INTELLIGENT DECISION SUPPORT SYSTEM FOR MARKET PARTICIPATION BY LOAD AGGREGATORS

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INTELLIGENT DECISION SUPPORT SYSTEM FOR MARKET PARTICIPATION BY LOAD AGGREGATORS

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Abstract

Many investigations and research works have been conducted on the National Electricity Market of Singapore (NEMS) by market regulators and researchers. It is well-known for its distinguished maturity and structure. A demand response (DR) program was implemented in April 2016 to further improve its efficiency and competitiveness. With the liberalization and deregulation of the electricity market, it becomes more flexible for demand side to actively participate in the wholesale electricity market. This has changed the way how electricity is traded.

In this research, a software-based intelligent decision support system is developed for participation of loads in the demand response program of the wholesale electricity market of NEMS.

This work focuses on the study of the Singapore demand response program featuring demand side bidding and incentive payments. The impact of demand side participation on the market is analyzed. The issues associated with the structure and operation of the electricity market are addressed in this research. In addition, DR can be employed in energy management to mitigate the difficulty brought by uncertain demand. Firstly, the current market clearing model (MCM) of Singapore, which is an assessment tool for the simultaneous allocation of energy and ancillary services, is introduced. The principle of transmission loss and nodal prices integrated in the MCM is presented as well. Secondly, the methodology of integration of demand side bidding with the newly introduced requirements is discussed. Finally, a stochastic optimization is proposed for the coordinated operation of generating units and demand response to adequately deal with the
uncertain electricity demand. The coordination is accomplished through a two-stage stochastic programming model.

In the future, the proposed modeling can be utilized for further evaluation on real time demand response (DR) and interruptible load (IL) strategies. The challenges to be addressed are also outlined while the solution is still under testing.
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Nomenclature

Constants

$\alpha_{Seg_{k,y}}$  Corresponding slope of linear segment $y$ of transmission line $k$

$\lambda_1$  Risk adjustment factor for reserve

$\lambda_2$  Reserve proportion factor

$c_{Seg_{k,y}}$  Intersection with the vertical axis of linear segment $y$ of transmission line $k$

$D^\text{Eng}$  Total forecast MW amount of energy demand

$D^\text{Reg}$  Total forecast MW amount of regulation

$D^\text{Eng}_k$  Demand forecast of $k_{th}$ scenario

$D^\text{Eng}_n$  Forecast MW amount of demand on energy at Bus n

$Flow^\text{Const}_{k,m}$  Vertical coordinate of point $m$ of line $k$

$Flow^\text{Max}_k$  Transmission capacity of line $k$

$INC_n$  Individual non-curtailable load of $n_{th}$ offer (MW)

$Loss^\text{Const}_{k,m}$  Horizontal coordinate of point $m$ of line $k$

$LP_{n,j}$  Price ($/\text{MWh}$) of $j_{th}$ tranche in $n_{th}$ load offer

$LQ_{n,j}$  Quantity (MW) of $j_{th}$ tranche in $n_{th}$ load offer

$M_{k,m}$  Constant points of flow and loss model of line $k$

$N$  Numer of load offers

$O_i$  Capacity (MW) of $i_{th}$ generating unit
\( P_{Eng}^{i,j} \) Energy price ($/MWh) of \( j^{th} \) tranche in \( i^{th} \) offer

\( P_{Reg}^{i,j} \) Regulation price ($/MWh) of \( j^{th} \) tranche in \( i^{th} \) offer

\( P_{Res}^{i,j} \) Reserve price ($/MWh) of \( j^{th} \) tranche in \( i^{th} \) offer

\( P_L \) Bidding price for energy from demand side (10×VoLL)

\( Q_{Eng}^{i,j} \) Energy quantity (MW) of \( j^{th} \) tranche in \( i^{th} \) offer

\( Q_{Reg}^{i,j} \) Regulation quantity (MW) of \( j^{th} \) tranche in \( i^{th} \) offer

\( Q_{Res}^{i,j} \) Reserve quantity (MW) of \( j^{th} \) tranche in \( i^{th} \) offer

\( Reg_{Max}^{i} \) Maximum output (MW) for \( i^{th} \) unit to dispatch regulation

\( Reg_{Min}^{i} \) Minimum output (MW) for \( i^{th} \) unit to dispatch regulation

\( S \) Number of supply offers

\( Segk,y \) Linear segments of flow and loss model of line \( k \)

\( T_{Eng}^{i} \) Number of energy tranches in \( i^{th} \) offer

\( T_{Reg}^{i} \) Number of regulation tranches in \( i^{th} \) offer

\( T_{Res}^{i} \) Number of reserve tranches in \( i^{th} \) offer

\( T_{DR}^{n} \) Number of price-quantity tranches in \( n^{th} \) load offer

\( TL_n \) Total load of \( n^{th} \) offer (MW)

\( TNC \) Total non-curtailable load (MW)

\( VoLL \) Value of lost load ($5000/MWh)

**Variables**

\( \Delta l_{n,j,k} \) Second-stage \( k_{th} \) scenario demand schedule (MW) of \( j^{th} \) tranche of \( n^{th} \) demand side bidding

\( \Delta q_{Eng}^{i,j,k} \) Second-stage \( k_{th} \) scenario energy schedule (MW) of \( j^{th} \) tranche of \( i^{th} \) offer

\( \Delta q_{Reg}^{i,j,k} \) Second-sage \( k_{th} \) scenario regualtion schedule (MW) of \( j^{th} \) tranche of \( i^{th} \) offer
\( \Delta_{i,j,k}^{Res} \) Second-stage \( k_{th} \) scenario reserve schedule (MW) of \( j^{th} \) tranche of \( i^{th} \) offer

\( \omega_{k,m} \) Weight coefficients of transmission line \( k \)

\( flow_{k} \) Power flow amount (MW) of line \( k \)

\( g \) Generation cost ($)

\( IP_{n} \) Incentive payment (MWh) for \( n^{th} \) load aggregator

\( l_{0} \) Accepted quantity of total non-curtailable load

\( l_{k} \) Accepted quantity of total non-curtailable load in \( k_{th} \) scenario of second stage

\( l_{n,j} \) Scheduled amount (MW) of load of \( j^{th} \) tranche of \( n^{th} \) load offer

\( LC_{n} \) Scheduled curtailment (MW) of \( n^{th} \) load offer

\( LCP \) Load curtailment price ($/MWh)

\( LCQ_{n} \) Load curtailment quantity (MWh) associated with \( n^{th} \) load offer

\( loss_{k} \) Power loss amount (MW) of line \( k \)

\( LSche_{n} \) Scheduled load consumption level (MW) of \( n^{th} \) load offer

\( q_{i,j}^{Eng} \) Energy schedule (MW) of \( j^{th} \) tranche of \( i^{th} \) offer

\( q_{i,j}^{Reg} \) Regulation schedule (MW) of \( j^{th} \) tranche of \( i^{th} \) offer

\( q_{i,j}^{Res} \) Reserve schedule (MW) of \( j^{th} \) tranche of \( i^{th} \) offer

\( q_{i}^{Eng} \) Scheduled energy (MW) of \( i^{th} \) supply offer

\( q_{i}^{Reg} \) Scheduled regulation (MW) of \( i^{th} \) supply offer

\( q_{i}^{Res} \) Scheduled reserve (MW) of \( i^{th} \) supply offer

\( u_{i,k}^{Reg} \) Binary variable of \( k_{th} \) scenario after second stage uncertainty relation:
1 if \( i^{th} \) unit schedules for regulation, 0 else

\( u_{i}^{Reg} \) Binary variable: 1 if \( i^{th} \) unit schedules for regulation; 0 otherwise
Chapter 1

Introduction

Modern power systems are transiting from traditional to renewable generation, which is an all-round solution to handle the issues of global energy crisis, environments, climates, economics and sustainable development.

High penetration of intelligent terminal devices such as smart meters and smart household appliances provides abundant dynamic response information for the security and stability control of the power system and makes it possible for consumers automatically control their energy consumption.

1.1 Background

National Electricity Market of Singapore, which was the first liberalized electricity market in Asia, aims to provide reliable and secure energy supply and promote effective competition. Recently, an important change in the market is the introduction of Singapore’s Demand Response (DR) program implemented in April 2016.

Demand Response (DR) is an important way to realize the bi-directional interaction between supply and demand. Through Advance Metering Infrastructure (AMI), bi-directional communication, and remote control techniques, consumers can adjust their electricity consumption by changing their power demand through the price signal or the incentive mechanism, thus the optimal allocation of electric-
ity generation and consumption resources can be achieved. Through DR, resources at the load side can fit into the normal electric power system dispatching.

The incorporation of DR in the electricity market provides new bidding opportunities for load aggregators to achieve incentive payments for load curtailment during high electricity price periods [2] and when selected by the market operator.

However due to the limited knowledge on the potential savings and a lack of information about the market operation, the end-users may have difficulty in participating in the DR programs. It is necessary to investigate if it is economical to provide flexible loads in DR programs. Load providers should only be engaged if they could get compensation or benefit.

1.2 Objectives and Scope

An intelligent decision support system is proposed to help schedule the load curtailment bids of DR programs in this work. It is essentially a mathematical description of the market clearing process which is employed to settle the energy and ancillary services.

This project focuses on the market clearing and demand response program to obtain an in-depth understanding of their implementation and operation. First, a mixed-integer linear programming (MILP) framework-based market clearing model integrating transmission system is provided. Numerical analysis is carried out to study the mechanism of market clearing model and to investigate the effects of transmission system on market clearing process and prices.

Second, the demand side bidding of the DR program is introduced and the intelligent decision support system with DR incorporated is developed. In addition, the incentive payment mechanism which concerns the reward of DR participants for their load curtailments is elaborated. The proposed support system for the demand response market is able to provide a platform for decision-making by load aggregators and end-users based on the long-term economic cashback.

Last, a two-stage stochastic market clearing strategy using Monte Carlo techniques is developed against demand uncertainty. It is demonstrated that DR can
effectively coordinate to accommodate demand uncertainty. The total social welfare is maximized with a solution which is viable for all scenarios.

The contribution of this work is twofold: 1) It takes into consideration the stochastic load demands. A two-layer stochastic simulation is employed to deal with the intrinsically random character of load demand. An optimal solution that is feasible for all scenarios is obtained. 2) This work is based on the Singapore wholesale electricity market and investigates the newly introduced DR program which is the first initiative in Singapore, with an emphasis on the salient features which are designed to smoothly introduce DR into the current regulated market. The key constraints imposed by PSO are highlighted.

The merits of the proposed model are verified through the MATLAB programming software.

1.3 Organizations

This report is organized as follows:

Chapter 2 provides the comprehensive literature review, including the overview of NEMS and various types of DR.

Chapter 3 discusses the detail mathematical formulation of the basic market clearing model which can also been regarded as the assessment tool. The incorporation of transmission losses is also presented.

Chapter 4 investigates the demand side participation. The market model incorporated with demand response is illustrated as well as the incentive payment calculation.

Chapter 5 presents the details of stochastic optimization. It focuses on two-stage stochastic market clearing model with scenarios construction via Latin Hypercube Sampling method.

Chapter 6 summarized the case studies and numerical analysis of different models.

Finally, the conclusion and plan for future research are presented in Chapter 7.
Chapter 2

Literature Review

2.1 Market Overview

National Electricity Market of Singapore (NEMS) is designed to promote efficient electricity with competitive price. Allowing privatization of certain government-owned properties and encouraging private investment in power system of Singapore open up the market to full competition [1].

2.1.1 Market Structure of NEMS

NEMS consists of a wholesale market and a retail market. The wholesale market mainly deals with the trading between generators and the wholesale buyers. Retail market intermediaries, such as market support services licensees, purchase electricity from the wholesale market and sell it to consumers. Main parties in the NEMS are:

**Regulator:** Energy Market Authority (EMA) is the regulator that governs the conduct of market participants in NEMS.

**Market operator:** The operation and administration of wholesale electricity market is under the care of Energy Market Company (EMC).

**Power system operator:** The PSO, a division of EMA, is responsible for ensuring the reliability and security of electricity supply. It is also in control of the dispatch of generating facilities.
**Chapter 2. Literature Review**

**Figure 2.1: Schematic diagram of NEMS (adapted from [1]).**

**Transmission licensee:** SP PowerAssets owns the transmission system and it is also in charge of the operation and maintenance.

**Generation licensees:** All dispatchable units with capacity of 10 MW or above should be licensed by the EMA and registered with the EMC. The purpose of mandatory participation is to ensure that all the significant-sized generating units comply with the market rules.

**Market support services licensees:** The MSSLs supply electricity to all non-contestable consumers and provide support services.

**Retail electricity licensees:** Retailers are allowed to sell electricity to contestable consumers. They may purchase from the wholesale market directly or obtain electricity through MSSLs.

**Consumers:** Consumers are categorized into two groups based on their annual electricity consumption. The contestable consumers may choose to purchase electricity directly from the wholesale market, a retailer or the MSSLs. The non-contestable consumers can only purchase electricity by MSSLs.

The institutions in the market are shown in Fig. 2.1.
2.1.2 Market Clearing

By definition, market clearing is a process indicating that the supply traded is equated to the demand in economics. It is essentially a pricing mechanism to settle energy and ancillary services in the electricity market.

For each half-hour dispatch period, a computer model is run to determine (i) market clearing prices (MCP), which are the settlement prices for the trade of the supply and demand, and (ii) optimal dispatch schedules that minimize the generation cost.

Fig. 2.2 illustrates a simplified market clearing process which only considers the energy bidding offer from suppliers. The staircase curve of generation bidding offer is arranged in ascending order. In the wholesale electricity market without demand response, the MCP is determined by the crossing of supply offer and system demand forecast as shown in the figure. The power system operator seeks to minimize the system cost.

Extensive research and review have been conducted on market clearing model (MCM). Detailed reviews on formulations and algorithms of economic dispatch and pricing schemes can be found in [3]. In particular, with increasing penetration
of renewable energy, novel MCMs are developed to incorporate offers from renewable energy resources [3], [4]. Reference [5] proposed a game-theoretical model to investigate the pricing strategies and market dynamics which lead to the supply uncertainty. An optimal wholesale investment and retail pricing strategy with closed-form is obtained for the market operator. In [6], the two-stage stochastic programming is employed to estimate the reserve capacity. This method provides a means to manage the failure of units and the uncertainty associated with renewable resources. Reference [4] presents a stochastic multi-layer agent-based model to analyze the interaction between market participants including renewable power plants. A case study containing wind power is carried out to validate the effectiveness of the proposed model.

Some market clearing models are formulated as quadratic programming (QP) methodology [7], [8], while many can be solved by linear programming (LP) [9]-[10]. The performance comparison of price-based economic dispatch between QP and LP method is presented in [8]. Different from many of the electricity markets in the literature, the market clearing model of Singapore schedules energy and ancillary services simultaneously.

2.2 Demand Response in the Energy and Ancillary Market

Recently, a significant change in the market is the introduction of a demand response (DR) program which was launched in April 2016.

The definition of Demand Response (DR) is the changes or shifts of electricity demand by consumers from their normal usage pattern, in response to the electricity prices, incentive payments or system constraints [11]. Its purpose is to reduce the high wholesale electricity price which exceeds a certain floor price through voluntary load curtailments by consumers. In 2013, the Energy Market Authority (EMA) issued the final determination paper on DR. The market rules were modified in December 2015 by Energy Market Authority (EMA) for the implementation
of DR program [2]. This DR program is the first initiative in Singapore to allow consumers actively manage their demand in response to market prices.

Various DR programs can be categorized into two groups: price based and incentive based. The incentive-based DR programs can be further categorized in to direct load control, interruptible load, load curtailment, and emergency demand response etc. While time-of-use, real time pricing, and critical peak pricing are considered as price-based programs [12].

Much effort has been dedicated to demand response as it can bring the electricity market system-wide benefits including lowering energy prices, curbing market power, enhancing system robustness and improving efficient investments [12], [13]. Ref. [14] presents an overview of demand response, some challenges and potential solutions for implementing DR under smart grid paradigm are summarized. The potential failures and difficulties faced by DR participants are summarized in [15]. A variety of market clearing models with DR incorporated have been developed for market clearing. They play an important role in power system economics. To maximize the load aggregators’ payoff and to determine the optimal DR dispatch schedule, mixed-integer linear programming is employed in [16] to formulate a price-based self-scheduling model for the day-ahead market. A decomposition algorithm is developed in [17] to address the optimization problem for market clearing with DR from end users. The appliance scheduling flexibility and user satisfaction are incorporated. Price elasticity matrices indicating the willingness of consumers to shift their energy consumption have been utilized in [18] in order for smart grids with demand response to increase market flexibility and efficiency. A heuristic optimization strategy is proposed in [19] for end residential users and load aggregators. Based on a pricing structure, it allows customers the choice of electricity supplier in a fully deregulated market circumstance. In [20], demand response operations via controlling ship speed are adopted to coordinate with power generation in an all-electric ship environment.

Conceptually, the DR program initiated in Singapore has certain similar characteristics as those DR works mentioned before. However, the Singapore DR program
has two distinct features that distinguish it from those discussed in the literature. The first is the form of demand bidding offers and the pre-processing of these offers. The load providers are required to bid on the energy curtailments rather than specific energy consumption. The incentive payment mechanism intended for promoting DR participation is the second feature. With these two features, this demand response plays an essential role in the liberalization electricity market in Singapore.

2.3 Stochastic Programming

Deterministic method alone is not always adequate due to inevitable uncertainties from vary demand and volatile renewable energy injection whose outputs may heavily deviate from the forecast. Optimization problems involving uncertainty can be accomplished via stochastic programming.

Ref. [21] summarized several situation where stochastic programming is needed. When the uncertain parameters are given some possible values, it makes sense to seek a solution that is feasible for all scenarios. Such methodology is usually dedicated to problems such as designing a least-weight bridge while tensile strength of steel are within some tolerance. Also these stochastic programming models are applied when decisions are made repeatedly in essentially the same conditions. An example would be designing truck routes for daily milk delivery to customers with random demand. The goal is to come up with a solution that will perform well on average. In situation where an on-off decision has to be made, stochastic programming is also frequently utilized, such as to construct an investment portfolio to maximize return. Another concern in this context is the choice of the objective function. It is less justifiable to maximize the expected return since the decision is only to be made once. Therefore, the attitude of decision-makers to risk becomes important.

Usually two-stage stochastic programming is recognized as the most widely applied stochastic programming models. Here in the first stage a set of decision
variables can be optimized irrespective of the uncertainty and are considered as "here and now" decisions. Then recourse operations, known as "wait and see" decisions, are made to compensate the first stage for any imperfect information. The two stage stochastic programming can be simply viewed as an optimization which defines a single first-stage decision and a series of recourse actions in response to each random outcome.

Recently, the discipline and applications of stochastic programming have grown and broadened, from fisheries management to financial investment [21]. In particular, great efforts have been devoted to the stochastic scheduling and energy management with renewable and demand uncertainty. In power dispatch, to account for the inherent random character of induced uncertainty is challenging. In Ref. [22], a stochastic unit commitment with uncertain availability DR is developed and a chance constraint is imposed to guarantee loss-of-load probability lower than a specified level. A distributed stochastic market clearing mechanism is proposed in [23]. To ensure robust allocation operations, a risk measure of the system re-dispatching cost, CVaR (Conditional Value at Risk) cost function is introduced. Ref. [24] presents a multi-layer agent-based model to analyze the interaction between market participants. The model is developed using a stochastic framework due to the uncertainties of customer behaviors and generation resources. Consumers who participate in DR programs are denoted as independent agents in the second layer. In [25], a multi-timescale stochastic control method is proposed due to various operation time interval of conventional voltage/var control devices and renewable energies.
Chapter 3

Market Clearing Model incorporating Transmission Losses

3.1 Basic Market Clearing Process

The trading of energy and other ancillary services among market participants in the wholesale electricity market is governed simultaneously by the market clearing model. In the specific circumstance of NEMS, energy, reserve and regulation are the three main products traded between the buyers and sellers. Reserve and regulation are the two major kinds among the various ancillary services. Reserve is defined as generation capacity required to cover the energy loss in case of an unexpected generator outage while regulation is required to cover the real-time deviation of load away from the forecast value. To briefly summarize the difference between these two ancillary services, reserve is reserved for emergency while regulation is scheduled for normal dispatch operation.

During every half-hourly dispatch period, gencos decide which generation units are to be in operation. Each dispatchable unit will simultaneously bid on energy, reserve and regulation with respective offers. Each offer comprises several price-quantity tranches indicating the product quantity that the unit is willing to produce at the corresponding prices. These bidding prices specified in the offers have to be within a given range such that those offers are able to be processed by the MCM. A simplified energy offer is shown in Fig. 3.1. Reserve and regulation offers are in a similar format.
Chapter 3. Market Clearing Model incorporating Transmission Losses

Figure 3.1: A simplified energy offer

Figure 3.2: Overall market clearing diagram

Once the bidding offers are submitted, market clearing model will take into consideration both forecast values of energy, regulation demand and generation offers to formulate a co-optimization problem seeking the optimal dispatch and the associated MCPs for three products. An overall market clearing diagram is summarized in Fig. 3.2.

3.2 Transmission Loss and Nodal Price

Transmission loss in the power line is caused by current and resistance. It is presented by the following formulation:

\[ L = I^2 \times R \]  \hspace{1cm} (3.1)

where \( I \), \( R \) and \( L \) respectively denote the current, resistance and the loss of the transmission line.

Since the current can be expressed by MW power flow and voltage in a relationship of \( I = F/V \), Eq (3.1) can be rewritten to:

\[ L = \frac{F^2}{V^2} \times R \]  \hspace{1cm} (3.2)

In power system equations, “per unit” system is frequently employed due to various reasons. By substituting per unit values for resistance and voltage in
Chapter 3. Market Clearing Model incorporating Transmission Losses

Figure 3.3: Flow and loss relation of a specific transmission line

equation (3.2), the transmission loss function is finally presented by the following mathematical formulation:

$$L = \frac{F^2 \times R_{pu}}{V_{pu}^2 \times F_b}$$

(3.3)

where, $V_{pu}$ and $R_{pu}$ are the transmission line “per unit” values of voltage and resistance, $F_b$ is the apparent power base value.

100 is used as the MVA base by National Electricity Market of Singapore and $V_{pu} = 1$. In addition, resistance values of all transmission lines in the power system have been submitted as the standing data registration (in per unit). Therefore, through adopting the above formula, the relation of transmission flow and loss can be readily obtained. However, using linear programming, this quadratic loss function cannot be solved. A “piece-wise linear” approximation is adopted.

Transmission line $k$ is shown in Fig. 3.3 as an example. It has eight linear segments $Seg_{k,y}$ ($y = 1,\ldots,8$) and nine constant points $M_{k,m}$ ($m = 1,\ldots,9$) from left to right in the quadratic curve. Coordinate values of point $M_{k,m}$ ($Flow_{k,m}^{Const}$, $Loss_{k,m}^{Const}$) associated with the transmission capability, $Flow_{k}^{Max}$, and “per unit” resistance value, $R_{pu,k}$, can be easily derived. Red lines represent the linear segments.

From the above approximation, power loss and flow relationship of transmission
line $k$ can be described with the linear equation as follows:

$$loss_k = \alpha_{Seg_{k,y}} \times flow_k + c_{Seg_{k,y}}$$ (3.4)

where, $\alpha_{Seg_{k,y}}$ is the associated segment slope with $flow_k$ in its horizontal coordinate interval, and $c_{Seg_{k,y}}$ is the intersection at the vertical axis.

A simple example is presented in Fig. 3.4 to illustrate the nodal price difference. The energy price at each node is calculated as the cost of providing an increment of load at that location. In the specific circumstance of determining the price difference between nodes A and B across a transmission line, the relationship is presented as (3.5):

$$Price_B = \left(1 + \frac{\partial loss_{A-B}}{\partial flow_B}\right) \times Price_A$$ (3.5)

where $flow_B$ denotes the MW flow into Node B. It can be derived in a relationship of $flow_B = flow_{A-B} - loss_{A-B}/2$ with power flow $flow_{A-B}$ and power loss $loss_{A-B}$.

![Figure 3.4: Price difference example](image)

It is well-accepted that power flows are measured at the middle point of the transmission line while losses are equally shared at the receiving and ending ends.

By introducing the linear segment slope illustrated in the previous section, the nodal price relationship between two nodes is expressed as formula (3.6). The detailed derivation process from (3.5) to (3.6) can be found in [26]. It is also presented in the Appendix section.

$$Price_B = \frac{2 + \alpha_{Seg_{A-B,y}}}{2 - \alpha_{Seg_{A-B,y}}} \times Price_A$$ (3.6)
3.3 Problem Formulation

In essence, the market clearing model is an optimization model that utilizes the linear programming method. It finds a solution of energy, regulation and reserve dispatch schedules that minimize the total cost of generation while satisfying the requirements, constraints and limits as a set of simultaneous linear equations. The formulation of the market clearing model is presented in the following section.

3.3.1 Generation Dispatch Schedule

The market clearing model will determine which units are to be operated and the scheduled capacities. Let \( q \) denote the vector consisting of the accepted quantities of price-quantity tranches of all offers. It can be defined as

\[
q = [q_{Eng}^1, ..., q_{Eng}^S, q_{Res}^1, ..., q_{Res}^S, q_{Reg}^1, ..., q_{Reg}^S]^T
\]  

(3.7)

where \( q_{Eng}^i = [q_{Eng,1}^i, ..., q_{Eng,T_i}^i] \), \( q_{Res}^i = [q_{Res,1}^i, ..., q_{Res,T_i}^i] \), \( q_{Reg}^i = [q_{Reg,1}^i, ..., q_{Reg,T_i}^i] \).

3.3.2 Constraints

Generator Capacity Constraint:

The total quantities of energy, reserve and regulation scheduled by each generating facility should not exceed its generation capability as Eq. (3.8) represents.

\[
\sum_{j=1}^{T_{Eng}} q_{Eng,j}^i + \sum_{j=1}^{T_{Res}} q_{Res,j}^i + \sum_{j=1}^{T_{Reg}} q_{Reg,j}^i \leq O_i, \forall i
\]

(3.8)

Constraint for Energy:

With the integration of transmission network, an energy balance constraint associated with the line flow and loss related to that bus must be respected. Eq. (3.9) specifies that the total energy scheduled should match the demand forecast value considering transmission losses, where \( LINES_n \) and \( OFFERS_n \) denote the sets of transmission lines and supply offers that are related to bus \( n \), respectively. Meanwhile, \( END_n \) and \( START_n \) are two sets of transmission lines of which flow ends
and starts at bus \( n \) respectively. They are the subsections that consist \( LINES_n \). Scheduled energy of each tranche is limited by constraint (3.10).

\[
\begin{align*}
& \sum_{i \in OFFERS_n} \sum_{j=1}^{T_{Eng}^i} q_{i,j}^{Eng} - D_{n}^{Eng} - \frac{1}{2} \times \sum_{k \in LINES_n} loss_k \\
& + \sum_{k \in END_n} |flow_k| - \sum_{k \in START_n} |flow_k| = 0, \forall n \\
& 0 \leq q_{i,j}^{Eng} \leq Q_{i,j}^{Eng}, \forall i, j
\end{align*}
\] (3.9)

**Constraint for Regulation:**

Generation schedules set every half hour. It can not match load second by second. Automatic Generation Control (AGC) systems change the output of generators providing “regulation” by small amounts to keep supply and demand in balance in real-time. The quantity of regulation required during each half hour is given. A generator usually can respond by 10 MW relative to its scheduled generation level.

There are mainly two constraints for regulation dispatch. The first regulation constraint is expressed in (3.11) like energy balance. Constraints (3.12) and (3.13) impose on the limit for a unit that is eligible for regulation: (i) the sum of scheduled energy and regulation by the unit should be less than a pre-defined maximum amount (denoted as \( Reg_i^{MAX} \)); and (ii) the amount of its scheduled energy should be greater than its scheduled regulation by a pre-defined minimum amount (denoted as \( Reg_i^{MIN} \)). Both \( Reg_i^{MAX} \) and \( Reg_i^{MIN} \) are specific to the unit.

It is noticed that the two constraints are valid only when the unit is selected for regulation. Therefore, binary variables \( u_{Reg} \) are introduced to indicate whether the units are selected for regulation. Similarly, constraint (3.14) represents the lower and upper bounds of the scheduled regulation for each tranche.

\[
\sum_{i=1}^{S} \sum_{j=1}^{T_{Reg}^i} q_{i,j}^{Reg} = D_{Reg}^{Reg}
\] (3.11)
Chapter 3. Market Clearing Model incorporating Transmission Losses

\[ \sum_{j=1}^{T_{Reg}} q_{i,j}^{Reg} \leq \left( -Reg_i^{Min} + \sum_{j=1}^{T_{Eng}} q_{i,j}^{Eng} \right) \times u_i^{Reg}, \forall i \quad (3.12) \]

\[ \sum_{j=1}^{T_{Reg}} q_{i,j}^{Reg} \leq \left( Reg_i^{Max} - \sum_{j=1}^{T_{Eng}} q_{i,j}^{Eng} \right) \times u_i^{Reg}, \forall i \quad (3.13) \]

\[ 0 \leq q_{i,j}^{Reg} \leq Q_{i,j}^{Reg}, \forall i, j \quad (3.14) \]

Constraint for Reserve:

If a generator or transmission line fails, reserve sources must be available to take up the slack. There are three kinds of reserve in the market. Primary Reserve arrests the initial fall in system frequency resulting from generation loss. In order to restore system frequency back to acceptable levels, Secondary Reserve is released. Contingency Reserve is activated manually by the Power System Operator to recover Primary and Secondary Reserves in preparation for the next generation loss. Since contingency reserve is of the imperial role, for simplicity this work only considers this type of reserve.

The total amount of reserve demand is given in a dynamic way which is much different from reserve and regulation. It depends on the largest generating facility capability and power system response. Thus contingency principle of N-1 is adopted, which implies that the reserve capacity should be sufficient to cover the loss resulted from failure of any one generator, including both its scheduled energy and reserve. The following figure illustrates how the reserve requirement is calculated.

To ensure a more secure and reliable supply, this value of scheduled energy and reserve is further scaled by the risk adjustment factor \( \lambda_1 \) set by the PSO. This is enforced by (3.15). Power system responses, up to turbine output damping, load damping and intertie contribution, are not considered in this work for simplicity. In addition, to prevent the situation that each unit provides too much reserve, the reserve scheduled from each unit is required not to exceed a certain ratio of the scheduled energy as imposed in individual reserve proportion constraint (3.16).
Also, the scheduled reserve of each tranche from each unit should fulfill constraint (3.17).

\[
\sum_{i=1}^{S} \sum_{j=1}^{T_{Res}^{i}} q_{i,j}^{Res} \geq \lambda_1 \times \max_{i=1,...,S} \left\{ \sum_{j=1}^{T_{Eng}^{i}} q_{i,j}^{Eng} + \sum_{j=1}^{T_{Res}^{i}} q_{i,j}^{Res} \right\} 
\]

(3.15)

\[
\sum_{j=1}^{T_{Res}^{i}} q_{i,j}^{Res} \leq \lambda_2 \times \sum_{j=1}^{T_{Eng}^{i}} q_{i,j}^{Eng}, \forall i 
\]

(3.16)

\[
0 \leq q_{i,j}^{Res} \leq Q_{i,j}^{Res}, \forall i, j
\]

(3.17)

Flow and Loss Constraint:

Constraints (3.18) and (3.19) describe the relationship between MW flow and loss. POINTSUB_B refers to the set of coordinate points in transmission line k. Corresponding calculations of coordinates Flow_{k,m}^{Const} and Loss_{k,m}^{Const} for transmission line k have been explained previously. The sum of weight coefficients, \omega_{k,m}, is equal to 1 as in (3.20). Constraint (3.21) describes the transmission capability.

\[
flow_k = \sum_{m \in \text{POINTSUB}_K} \text{Flow}_{k,m}^{Const} \times \omega_{k,m}, \forall k 
\]

(3.18)

\[
loss_k = \sum_{m \in \text{POINTSUB}_K} \text{Loss}_{k,m}^{Const} \times \omega_{k,m}, \forall k
\]

(3.19)
\[ \sum_{m \in \text{POINTSUB}_k} \omega_{k,m} = 1, \forall k \quad (3.20) \]

\[ -\text{Flow}_k^{\text{Max}} \leq \text{flow}_k \leq \text{Flow}_k^{\text{Max}}, \forall k \quad (3.21) \]

### 3.3.3 Objective Function

The objective of MCM of the Singapore electricity market is to maximize the net benefit of the market defined as the sum of producer surplus and consumer surplus. The bidding price from demand side is required when calculating the consumer surplus. Currently, bidding from the demand side does not occur in the Singapore electricity market. The bidding price is set at a very high value \( P_L \). The objective function is formulated as

\[
f = P_L \times \sum_{i=1}^{S} \sum_{j=1}^{T_{\text{Eng}}} q_{i,j}^{\text{Eng}} - g\]

where

\[
g = \sum_{i=1}^{S} \sum_{j=1}^{T_{\text{Eng}}} P_{i,j}^{\text{Eng}} \times q_{i,j}^{\text{Eng}} + \sum_{i=1}^{S} \sum_{j=1}^{T_{\text{Res}}} P_{i,j}^{\text{Res}} \times q_{i,j}^{\text{Res}} + \sum_{i=1}^{S} \sum_{j=1}^{T_{\text{Reg}}} P_{i,j}^{\text{Reg}} \times q_{i,j}^{\text{Reg}} \quad (3.23)\]

Note that \( P_L \) is a constant. Therefore, the objective function is equivalent to minimizing the overall generation cost (denoted as \( g \)) for the current MCM, although these two functions are totally different.

The current mathematical market clearing model is formulated as follows,

**Maximize**: \( f \) in (3.22)

**Subject to**: (3.8) – (3.21) and (3.23)

The market clearing prices for energy, reserve or regulation are the marginal costs of serving an extra unit of energy, reserve or regulation, respectively. These prices are derived when solving the model developed since they are the Lagrange multipliers or dual variables of corresponding balancing constraints.
Chapter 4

Formulation of Demand Response Program

The modified market clearing model with integration of demand response is proposed in this chapter. To study the effect of DR program specifically, transmission network incorporated in the last chapter is not considered.Demand-side bidding offers as well as the new constraints and objective function are formulated accordingly. Demand side participation proves to be efficient for reducing energy price and minimizing the generation cost compared to those market solutions without considering demand response.

4.1 Demand Side Bidding

4.1.1 Bidding Offers

Demand has been considered inelastic traditionally. However, with the implementation of the demand response program, consumers have the opportunity to reduce their consumption as a response to the wholesale electricity price. In this report, the consumers are required to submit the bidding offers as shown in Table 4.1. The bidding offers consist of the following information.

Price-quantity tranches

These tranches specify the load quantity that consumers are willing to curtail at the corresponding prices. These tranches are arranged in descending order as
shown in Fig. 4.1. With the energy clearing price increases, the consumers are more willing to get the load curtailed as they can also gain compensation.

To prevent the load aggregators from abusing the DR program such as submitting curtailment offers at very low prices that would have been carried out without DR, a price floor which is set at 1.5 times of the balance vesting price (BVP) is proposed (The BVP is a price designed to “approximate the long run marginal cost of a new entrant that uses the most economic generating technology in operation in Singapore and contributes to more than 25% of total demand” [1]). The bidding prices are required to be greater than the price floor, which is rather high. Therefore, the load aggregators representing consumers will only have a limited flexibility in changing their load pattern. Since the demand side bidding offers submitted to the PSO are screened before market clearing, the price floor is not taken as the constraint.

**Total load**

The total load is the conservative estimation of the energy demand without any curtailment. The load aggregator will be penalized if consumers use less energy. This prevents them from abusing the demand response program.

**Load ramp rate**

This specification indicates how fast the load can change. At the beginning of each dispatch period, the load aggregator is required to increase or decrease the load at the respective ramp rates to the instructed level and maintain at such a level until the end. Therefore, if the scheduled quantity for the previous dispatch period is provided, the load level range for the next period can be calculated.
4.1.2 Pre-process of Bidding Offer

The tranches in load offers specify the amount of load curtailment rather than the explicit amount consumed. In order to facilitate the clearing process, the demand offer mentioned before should be transformed into a form that indicates the specific energy consumption at the corresponding prices. To expound the pre-processing, the concept of individual non-curtailable load (INC) is introduced first. This refers to the amount of energy the consumer is going to use despite the electricity price. The individual non-curtailable load can be calculated by subtracting all the bidden curtailments from the total load as shown below:

\[
INC_n = TL_n - \sum_{j=1}^{T_{DR}} LQ_{n,j}, \forall n \tag{4.1}
\]

In particular, the bidding price for the individual non-curtailable load is set as \( P_L \), the same as that of MCM in chapter 3. The transformed offer is shown in Fig. 4.2. It can be observed that the tranche of individual non-curtailable load is the only difference between the original and transformed bidding offer.

The total non-curtailable load (TNC) is just the sum of all \( INC_n \). Another concern is that the estimation of total load consumption in the load offer is deemed
unreliable by PSO as the total load is specified by individual load aggregator to
give a baseline for calculating curtailment. Therefore, the energy demand forecast
from PSO is used.

The total non-curtailable load can be calculated by subtracting all the curtail-
ment offers of all the consumers (total possible curtailment of all consumers) from
energy demand forecast, with formula shown as below:

\[ TNC = D_{Eng} - \sum_{n=1}^{N} \sum_{j=1}^{T_{DR}} LQ_{n,j} \]  

(4.2)

With the introduction of non-curtailable load, each demand side bidding offer
can be separated into two groups. One group consists of the price-quantity tranches
that indicate all possible curtailments and another is the aggregated individual non-
curtailable load, \( TNC \), which is an artificial offer as it is processed to participate
in the market clearing rather than submitted directly by consumers.

4.2 Formulation with Demand Response

4.2.1 Dispatch schedule for consumers

To facilitate the mathematical description, the demand schedules which refer to
the set of accepted quantities of price-quantity tranches are introduced.

Let \( l_0 \) denote the accepted quantity of the total non-curtailable tranche and \( l_{n,j} \)
denote the accepted quantity of corresponding tranche. The total scheduled load
quantity \( l \) can be calculated.

\[ l = l_0 + \sum_{n=1}^{N} \sum_{j=1}^{T_{DR}} l_{n,j} \]  

(4.3)

4.2.2 Constraints associated with DR

Modified Constraint for Energy:

With the demand response incorporated, the demand is no longer a given fore-
cast value but a variable discussed in the previous section. To address this problem,
energy balance constraint stated in (3.9) is revised in (4.4). Constraints (4.5) and (4.6) limit the dispatchable load of each tranche.

\[
\sum_{i=1}^{S} \sum_{j=1}^{T_{\text{Eng}}} q_{i,j}^{\text{Eng}} = l_0 + \sum_{j=1}^{T_{\text{DR}}} l_{n,j}, \forall n \tag{4.4}
\]

\[
0 \leq l_{n,j} \leq LQ_{n,j}, \forall n, j \tag{4.5}
\]

\[
0 \leq l_0 \leq TNC \tag{4.6}
\]

**Load Ramping Constraint:**

Constraints (4.7) and (4.8) restrict the range of scheduled load quantity considering the load ramp rates \(RU_n\) and \(RD_n\) for each dispatch interval. After introducing the upper and lower bounds of the scheduled load, the load ramping constraint is formulated in (4.9).

\[
LQ_{\text{MAX}}_n = INC_{n}^{\text{PRE}} + l_{n}^{\text{PRE}} - INC_n + RU_n \times 30, \forall n \tag{4.7}
\]

\[
LQ_{\text{MIN}}_n = \max \{ INC_{n}^{\text{PRE}} + l_{n}^{\text{PRE}} - INC_n - RD_n \times 30, 0 \}, \forall n \tag{4.8}
\]

\[
LQ_{\text{MIN}}_n \leq \sum_{j=1}^{T_{\text{DR}}} l_{n,j} \leq LQ_{\text{MAX}}_n, \forall n \tag{4.9}
\]

### 4.2.3 Modified Objective Function

After integration of the consumer offer into the market clearing process, the objective function is revised as follows,

\[
f^* = P_L \times l_0 + b - g \tag{4.10}
\]

where \(g\) is defined in (3.23). The variable, \(b\), denoted as the aggregation of multiplications of the accepted load and bidding price, is given by

\[
b = \sum_{n=1}^{N} \sum_{j=1}^{T_{\text{DR}}} LP_{n,j} \times l_{n,j} \tag{4.11}
\]
Figure 4.3: Illustration on objective function

From the modified objective function (4.10), it can be seen that the total surplus to be maximized depends on not only generation dispatch schedules but also the demand schedules as well. The new model is summarized as

\[
\text{Maximize : } f^* \text{ in (4.10)}
\]

\[
\text{Subject to : } (3.8), (3.10) - (3.21), (3.23), (4.4) - (4.9) \text{ and (4.11)}
\]

The objective function of maximizing the social welfare from economic perspective is to minimize the total generation cost and to obtain the optimal dispatch schedules. Since the consumers’ demand is sensitive to price, the consumer may choose not to purchase when the price rises over a specific point.

A simple example of this double auction market clearing process is described in Fig. 4.3. There are two stair curves. One is the genco offer in ascending order and another is the transformed demand bidding offer in descending order.

As illustrated in Chapter 2, in the electricity market where demand bidding offers are absent, market clearing price is determined by the highest supply bidding with system load forecast. The shaded area denotes the social welfare consisting of consumer surplus and producer surplus, which corresponds to function (4.10).
By solving the above linear programming problem, the optimal dispatch schedules for supply and demand, which collectively lead to the maximum net benefit, can be obtained. Due to the energy balance constraint changes, currently the MCP for energy is the Lagrange multiplier corresponding to (4.4) rather than (3.9).

4.3 Incentive Payment

Incentive payment is paid to DR participants for the load curtailment and decrease of electricity price with the intention of promoting DR participation. One-third of the increased market surplus due to DR will be used as incentive payment.

For each demand bidding offer, after considering the upper bond of the achievable load level, the scheduled MW curtailment of each contestable consumer can be calculated,

\[ LC_n = \min \left\{ LQMAX_n, \sum_{j=1}^{T^{DR}} LQ_{n,j} \right\} - \sum_{j=1}^{T^{DR}} l_{n,j}, \forall n \]  

(4.12)

It is clear that \( LC_n \) is non-negative. And if \( \sum_{n=1}^{N} LC_n = 0 \), it means that there is no load curtailment selected and thus there is no need for incentive payment. As such, it is used as the trigger to the calculation of load curtailment quantity. The \( LC_n \) is the scheduled MW curtailment rather than the actual MWh curtailed. However, the incentive payment is based on the actual MWh curtailment. The rest of this section explains the calculation of the MWh curtailment and the incentive payment price at $/MWh.

4.3.1 Calculation of Load Curtailment Quantity

Each load aggregator should have reached its scheduled load consumption level (denoted as \( LSche_n \)) during each dispatch period, which can be calculated as
shown in (4.13), considering the upper bond of the scheduled level.

\[
LSche_n = \min \left\{ INC_n + \sum_{j=1}^{T_{DR}^n} LQ_{n,j}, \ INC_n + LQMAX_n \right\} - LC_n
\]

(4.13)

\[
= INC_n + \sum_{j=1}^{T_{DR}^n} l_{n,j}, \ \forall n
\]

The scheduled load consumption level of the immediately preceding period (i.e., \(LSche_{n}^{pre}\)) is considered as the starting load consumption level of the current period. Therefore, the load curtailment quantity in MWh, denoted as \(LCQ_n\), can be calculated as the MWh quantity difference between electricity consumption with and without curtailment (denoted as \(Q_{with_n}\) and \(Q_{without_n}\), respectively). The formulation is as follows,

\[
LCQ_n = Q_{with_n} - Q_{without_n}, \ \forall n
\]

(4.14)

It is assumed that the load consumption level can be instructed to ramp up or down to \(LSche_n + LC_n\) and \(LSche_n\) within the dispatch period. An example of the process of reaching the scheduled load consumption level with and without demand response is shown in Fig. 4.4.

Given the value of \(LSche_{n}^{pre}\), the calculation of the MWh scheduled load can be formulated.

\[
Q_{without_n} = \begin{cases} 
\frac{1}{2} \times (LSche_n + LC_n) + \frac{1}{2} \times \frac{(LSche_n + LC_n - LSche_{n}^{pre})^2}{RU_n \times 60} & \text{if } LSche_{n}^{pre} \geq (LSche_n + LC_n), \\
\frac{1}{2} \times (LSche_n + LC_n) - \frac{1}{2} \times \frac{(LSche_n + LC_n - LSche_{n}^{pre})^2}{RU_n \times 60} & \text{if } LSche_{n}^{pre} < (LSche_n + LC_n),
\end{cases}
\]

(4.15)

\[
Q_{with_n} = \begin{cases} 
\frac{1}{2} \times LSche_n + \frac{1}{2} \times \frac{(LSche_n - LSche_{n}^{pre})^2}{RU_n \times 60} & \text{if } LSche_{n}^{pre} \geq LSche_n, \\
\frac{1}{2} \times LSche_n - \frac{1}{2} \times \frac{(LSche_n - LSche_{n}^{pre})^2}{RU_n \times 60} & \text{if } LSche_{n}^{pre} < LSche_n.
\end{cases}
\]
4.3.2 Calculation of Load Curtailment Price

As mentioned before, one-third of the additional consumer surplus benefited from the price reduction is paid to DR participants.

To calculate the price reduction for energy, the market energy prices without and with DR are required by running the MCM twice. Therefore, the additional consumer surplus can be calculated as

\[
ACS = \max \left\{ \left( MCP^{ref} - MCP^{Eng} \right) \times \frac{1}{2} \times (D^{Eng} - \rho) \right\} \quad (4.16)
\]

where \( MCP^{ref} \) and \( MCP^{Eng} \) denote the nodal prices without and with DR respectively and \( \rho \), a known constant, is the sum quantity of those loads covered by regulatory contracts (e.g., vesting contracts). The uniform load curtailment price is formulated as

\[
LCP = \frac{1}{3} \times \frac{ACS}{\sum_{n=1}^{N} LCQ_n} \quad (4.17)
\]

Accordingly, the incentive payments to load aggregators can be calculated.

\[
IP_n = LCP \times LCQ_n, \forall n \quad (4.18)
\]
Chapter 5

Two-stage Stochastic Programming Model

In the presence of demand uncertainty, it becomes challenging but imperative for the power system operator and market participants to make efficient decisions, concerning market clearing problems, reserve requirements as well as economic dispatch [27].

Generation, demand scheduling and electricity pricing with uncertainties can be accomplished via stochastic programming. In this section, a stochastic market clearing model using Monte Carlo simulation will be developed.

5.1 Stochastic Model

The basic principle of two-stage stochastic programming is that decisions ought to be made on the basis of data available when decisions are made but not to depend on future observations [21].

A standard formulation of the two-stage stochastic linear program is:

\[
\min_{x \in X} \left\{ g(x) := c^T x + \mathbb{E}[Q(x, \xi)] \right\} \tag{5.1}
\]

where \( Q(x, \xi) \) is the optimal value of the second-stage problem,

\[
\min_y q^T y \tag{5.2}
\]
subject to $Tx + W y \leq h$ \hfill (5.3)

Equations (5.1) and (5.2) include both the two-stage variable sets, $x, y$. Firstly, $x \in \mathbb{R}^n$ represents the set of decision variables which can be optimized in the first stage before the realization of uncertain data $\xi$. While $y \in \mathbb{R}^m$ is a set of variables that can be adjusted in the second stage. $\xi = (q, T, W, h)$ expresses the uncertain variables.

The second-stage problem can be simply viewed as an optimization which describes the corresponding recourse actions in response to random outcomes of $\xi$. Term $q^T y$ is the cost of re-dispatching the system for compensation of the possible inconsistency of $Tx \leq h$. In this situation, the dispatchable generation outputs and demand response are adjusted in consequence of inaccurate estimation of actual load demand.

\subsection{Monte Carlo Simulation with LHS}
Monte Carlo simulation is a frequently used mathematical method to solve the optimization problems under uncertainties. Samples from input random variables are generated and then a deterministic problem is solved for each sample by Monte Carlo based techniques [28].

Second-stage uncertainty $\xi$, normally modeled as a random vector with a known probability distribution, is assumed to have a finite number of possible realizations, $\xi_1, ..., \xi_K$, called scenarios with respective probabilities $p_1, ..., p_K$. The objective of this problem is to seek a solution that is feasible for all possible parameters and to optimize the given function. Then the expectation function can be written as the summation:

$$\mathbb{E} [Q(x, \xi)] = \sum_{k=1}^{K} p_k Q(x, \xi_k) \hfill (5.4)$$

The original two-stage stochastic programming model can be reformulated:

$$\min_{x, y} c^T x + \sum_{k=1}^{K} q_k^T y_k \hfill (5.5)$$
Chapter 5. Two-stage Stochastic Programming Model

\[ s.t. \ x \in X, T_k x + W_k y_k \leq h_k, k = 1, ..., K \] (5.6)

By solving this problem, the first-stage optimal solution \( \bar{x} \) and the second-stage solution \( \bar{y}_k \) for each scenario can be obtained. \( \bar{y}_k \) provides the optimal recourse decisions regarding the uncertainty \( \xi_k \) of each corresponding scenario.

In the context of market clearing formulation proposed before, \( x \) is the variables vector for generation and DR dispatch, i.e. \( q, l \) and binary variables. The DR program is optimized and scheduled with a determined predicted load demand irrespective of the uncertainty. The here-and-now decisions are unmodifiable and then fixed in the second stage. In this way, the modified dispatch schedules coordinates with the pre-scheduled DR efficiently when maximizing profits in the presence of uncertain demand behavior.

The objective of this formulation is to maintain energy balance after the realization of demand uncertainty while maximizing the total social welfare or minimizing the generation cost. This accounts for the base generation cost as well as the cost incurred by re-dispatching the generation to compensate the demand forecast errors [23].

Simple random sampling is one of the most frequently used Monte Carlo techniques where samples are randomly generated from variables distributions [29]. Large sample size guarantees a specified accuracy, however, it suffers from long computation time and redundant repeated calculations when used in power system analysis [30].

Instead, Latin Hypercube Sampling (LHS) technique can be employed to obtain a large number of scenarios to approximate the uncertainties. It was first proposed in [31] as a stratified sampling technique, and was proved to be more efficient compared with random sampling method in the literature [30], [32]. In such a situation, Monte Carlo simulation with LHS is adopted to generate \( K \) samples of the demand forecast, \( D_1, ..., D_K \). Clearly, the sample average method will ensure approximates well for sufficiently large number of samples which can be justified by the Law of Large Numbers.

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5.1.2 Constraints Associated with Second-stage Variables

\[ \sum_{i=1}^{S} \sum_{j=1}^{T_i^\text{Eng}} (q_{i,j}^\text{Eng} + \Delta q_{i,j,k}^\text{Eng}) = l_k + \sum_{n=1}^{N} \sum_{j=1}^{T_i^\text{DR}} (l_{n,j} + \Delta l_{n,j,k}) , \forall k \]  
(5.7)

\[ \sum_{j=1}^{T_i^\text{Eng}} q_{i,j}^\text{Eng} + \sum_{j=1}^{T_i^\text{Res}} q_{i,j}^\text{Res} + \sum_{j=1}^{T_i^\text{Reg}} q_{i,j}^\text{Reg} \]  
(5.8)

\[ + \sum_{j=1}^{T_i^\text{Eng}} \Delta q_{i,j,k}^\text{Eng} + \sum_{j=1}^{T_i^\text{Res}} \Delta q_{i,j,k}^\text{Res} + \sum_{j=1}^{T_i^\text{Reg}} \Delta q_{i,j,k}^\text{Reg} \leq O_i , \forall i, k \]

\[ \sum_{i=1}^{S} \sum_{j=1}^{T_i^\text{Res}} (q_{i,j}^\text{Res} + \Delta q_{i,j,k}^\text{Res}) \geq \lambda_1 \times \max_{i=1,...,S} \]  
(5.9)

\[ \left\{ \sum_{j=1}^{T_i^\text{Eng}} (q_{i,j}^\text{Eng} + \Delta q_{i,j,k}^\text{Eng}) + \sum_{j=1}^{T_i^\text{Res}} (q_{i,j}^\text{Res} + \Delta q_{i,j,k}^\text{Res}) \right\} , \forall k \]

\[ \sum_{j=1}^{T_i^\text{Res}} (q_{i,j}^\text{Res} + \Delta q_{i,j,k}^\text{Res}) \leq \lambda_2 \times \sum_{j=1}^{T_i^\text{Eng}} (q_{i,j}^\text{Eng} + \Delta q_{i,j,k}^\text{Eng}) , \forall i, k \]  
(5.10)

\[ \sum_{i=1}^{S} \sum_{j=1}^{T_i^\text{Reg}} (q_{i,j}^\text{Reg} + \Delta q_{i,j,k}^\text{Reg}) = D^\text{Reg} , \forall k \]  
(5.11)

\[ \sum_{j=1}^{T_i^\text{Reg}} (q_{i,j}^\text{Reg} + \Delta q_{i,j,k}^\text{Reg}) \leq \left( -R_{i}^\text{Min} + \sum_{j=1}^{T_i^\text{Eng}} (q_{i,j}^\text{Eng} + \Delta q_{i,j,k}^\text{Eng}) \right) \times u_{i,k}^\text{Reg} , \forall i, k \]  
(5.12)

\[ \sum_{j=1}^{T_i^\text{Reg}} (q_{i,j}^\text{Reg} + \Delta q_{i,j,k}^\text{Reg}) \leq \left( R_{i}^\text{Max} - \sum_{j=1}^{T_i^\text{Eng}} (q_{i,j}^\text{Eng} + \Delta q_{i,j,k}^\text{Eng}) \right) \times u_{i,k}^\text{Reg} , \forall i, k \]  
(5.13)

\[ 0 \leq q_{i,j}^\text{Eng} + \Delta q_{i,j,k}^\text{Eng} \leq Q_{i,j}^\text{Eng} , \forall i, j, k \]  
(5.14)

\[ 0 \leq q_{i,j}^\text{Res} + \Delta q_{i,j,k}^\text{Res} \leq Q_{i,j}^\text{Res} , \forall i, j, k \]  
(5.15)

\[ 0 \leq q_{i,j}^\text{Reg} + \Delta q_{i,j,k}^\text{Reg} \leq Q_{i,j}^\text{Reg} , \forall i, j, k \]  
(5.16)

\[ 0 \leq l_{n,j} + \Delta l_{n,j,k} \leq LQ_{n,j} , \forall n, j, k \]  
(5.17)
The variables without subscript $k$ are related to the first-stage while those with $\Delta$ and subscript $k$ denote the second-stage $k^{th}$ scenario variables. Constraint (5.7) is the power balance after the uncertainty realization, where the scheduled generation and demand are adjusted. The generation limits of units when adjusting first-stage schedules are enforced by (5.8). The overall reserve requirement and proportion constraint are adjusted by constraints (5.9) and (5.10), respectively. Constraint (5.11) represents the regulation balance after the second-stage adjustment. Whether generating units are scheduled for regulation depends on the binary variable $u_{i,k}^{Reg}$ as stated in (5.12) and (5.13). The bounds of the second-stage variables are enforced by constraints (5.14)-(5.17).

### 5.1.3 Modified Objective Function

After introducing the second-stage variables, the modified objective function is formulated as follow:

$$f^* = c^T x + \mathbb{E}[Q(x, \xi)]$$  

(5.18)

where $c^T x$ is the corresponding optimal value of the first-stage problem, where expression is the same as Eq.(3.22).

Then the readily implementable approximation of the expectation objective function of the second-stage problem $\mathbb{E}[Q(x, \xi)]$ is its empirical estimate using Monte Carlo simulation with $K$ Latin Hypercube samples $\{D_k\}_{k=1}^K$.

$$\mathbb{E}[Q(x, \xi)] = \sum_{k=1}^K p_k Q(x, \xi_k)$$  

(5.19)

$$= \sum_{k=1}^K p_k \times (P_L \times l_k + b_k - g_k)$$

$b_k$ is defined as the $k^{th}$ scenario aggregation of multiplications of accepted load adjustment and demand bidding price in the second stage. $g_k$ is the $k^{th}$ scenario additional generation cost that is associated to the second stage variables. The expressions are as follows:
Chapter 5. Two-stage Stochastic Programming Model

\[ b_k = \sum_{n=1}^{N} \sum_{j=1}^{T_n^D} LP_{n,j} \times \Delta l_{n,j,k}, \forall k \]

\[ g_k = \sum_{i=1}^{S} \sum_{j=1}^{T_{i}^{Eng}} P_{i,j}^{Eng} \times \Delta q_{i,j,k}^{Eng} \]

\[ + \sum_{i=1}^{S} \sum_{j=1}^{T_{i}^{Res}} P_{i,j}^{Res} \times \Delta q_{i,j,k}^{Res} + \sum_{i=1}^{S} \sum_{j=1}^{T_{i}^{Reg}} P_{i,j}^{Reg} \times \Delta q_{i,j,k}^{Reg}, \forall k \]  

(5.20)
Chapter 6

Simulation and Results

6.1 MCM with Transmission Network

In this part, various numerical studies and case studies are conducted to demonstrate how the market clearing model integrated transmission system works. The numerical analysis focuses on the effects of power loss on the market clearing prices and generation dispatch schedules.

A 6-bus system adapted from [33] is employed to validate the effectiveness of the proposed market clearing model. As shown in Fig. 6.1, this network has 7

![Figure 6.1: A 6-bus network for test](image)
transmission lines and 8 generating units. The locational energy forecast, genco bidding offers and transmission line parameters are listed in Tables 6.1 and 6.2. Each bidding offer consists of two quantity-price tranches of energy, reserve and regulation. The power flows directions are also specified in Table 6.2. Regulation demand forecast is assumed to be 90 MW. Other parameters are set as: $\lambda_1 = 1.5$ and $\lambda_2 = 0.6$.

Table 6.1: Generation Offers and Locational Energy Demand Forecast

<table>
<thead>
<tr>
<th>Busn</th>
<th>$D_{Eng}^n$ (MW)</th>
<th>$O_i$ (MW)</th>
<th>$Reg_{Min}^i$ (MW)</th>
<th>$Reg_{Max}^i$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 1</td>
<td>1</td>
<td>210</td>
<td>600</td>
<td>100</td>
</tr>
<tr>
<td>i = 2</td>
<td>2</td>
<td>120</td>
<td>420</td>
<td>95</td>
</tr>
<tr>
<td>i = 3</td>
<td>3</td>
<td>240</td>
<td>380</td>
<td>80</td>
</tr>
<tr>
<td>i = 4</td>
<td>4</td>
<td>200</td>
<td>220</td>
<td>45</td>
</tr>
<tr>
<td>i = 5</td>
<td>5</td>
<td>210</td>
<td>210</td>
<td>40</td>
</tr>
<tr>
<td>i = 6</td>
<td>6</td>
<td>130</td>
<td>190</td>
<td>35</td>
</tr>
<tr>
<td>i = 7</td>
<td>7</td>
<td>170</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>i = 8</td>
<td>8</td>
<td>120</td>
<td>180</td>
<td>100</td>
</tr>
</tbody>
</table>

$Q_{Eng_{i,1}}$ (MW) | $Q_{Eng_{i,2}}$ (MW) | $Q_{Res_{i,1}}$ (MW) | $Q_{Res_{i,2}}$ (MW) | $Q_{Reg_{i,1}}$ (MW) | $Q_{Reg_{i,2}}$ (MW) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>170</td>
<td>350</td>
<td>100</td>
<td>50</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>120</td>
<td>180</td>
<td>100</td>
<td>60</td>
<td>18</td>
<td>18</td>
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<tr>
<td>110</td>
<td>170</td>
<td>70</td>
<td>65</td>
<td>23</td>
<td>14</td>
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<tr>
<td>90</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>5</td>
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<tr>
<td>80</td>
<td>80</td>
<td>50</td>
<td>15</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>40</td>
<td>15</td>
<td>10</td>
<td>15</td>
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<tr>
<td>110</td>
<td>70</td>
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<td>10</td>
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<td>10</td>
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<tr>
<td>120</td>
<td>180</td>
<td>80</td>
<td>80</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

$P_{Eng_{i,1}}$ ($/MWh$) | $P_{Eng_{i,2}}$ ($/MWh$) | $P_{Res_{i,1}}$ ($/MWh$) | $P_{Res_{i,2}}$ ($/MWh$) | $P_{Reg_{i,1}}$ ($/MWh$) | $P_{Reg_{i,2}}$ ($/MWh$) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>105</td>
<td>13</td>
<td>15</td>
<td>19</td>
<td>26</td>
</tr>
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<td>70</td>
<td>160</td>
<td>14</td>
<td>17</td>
<td>17</td>
<td>31</td>
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<td>80</td>
<td>130</td>
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<td>13</td>
<td>25</td>
</tr>
<tr>
<td>100</td>
<td>115</td>
<td>17</td>
<td>20</td>
<td>13</td>
<td>47</td>
</tr>
<tr>
<td>110</td>
<td>120</td>
<td>17</td>
<td>30</td>
<td>25</td>
<td>70</td>
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<tr>
<td>85</td>
<td>125</td>
<td>28.5</td>
<td>33</td>
<td>19</td>
<td>40</td>
</tr>
<tr>
<td>80</td>
<td>145</td>
<td>19.7</td>
<td>32.9</td>
<td>11</td>
<td>80</td>
</tr>
<tr>
<td>65</td>
<td>140</td>
<td>10</td>
<td>28</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Tables 6.3 and 6.4 summarize the simulation results of three different case studies. More details fo Tables 6.3 and 6.4 are well discussed in the following section.
Table 6.2: Transmission lines parameters

<table>
<thead>
<tr>
<th>Line $k$</th>
<th>From Bus $i$</th>
<th>To Bus $j$</th>
<th>$p_{uk}$</th>
<th>$\text{Flow}^\text{Max}_{ik}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>1</td>
<td>4</td>
<td>0.02</td>
<td>25</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>1</td>
<td>5</td>
<td>0.02</td>
<td>15</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>2</td>
<td>3</td>
<td>0.02</td>
<td>65</td>
</tr>
<tr>
<td>$k = 4$</td>
<td>2</td>
<td>4</td>
<td>0.02</td>
<td>55</td>
</tr>
<tr>
<td>$k = 5$</td>
<td>2</td>
<td>6</td>
<td>0.02</td>
<td>50</td>
</tr>
<tr>
<td>$k = 6$</td>
<td>3</td>
<td>5</td>
<td>0.02</td>
<td>35</td>
</tr>
<tr>
<td>$k = 7$</td>
<td>4</td>
<td>6</td>
<td>0.02</td>
<td>60</td>
</tr>
</tbody>
</table>

6.1.1 MCM without Transmission Network

To study the changes in market clearing process after incorporation of power loss, the base case without the transmission network is firstly conducted which results in uniform market clearing prices for energy, reserve and regulation. In this base case, there are no transmission network, thus the flow and loss constraints (3.18) - (3.21) mentioned before are not considered and the constraint for energy (3.9) should be rewritten as

$$\sum_{i=1}^{S} \sum_{j=1}^{T} q_{i,j}^{\text{Eng}} = \sum_{n=1}^{N} D_{n}^{\text{Eng}}$$  \hspace{1cm} (6.1)

It is observed from Table 6.3 that the market clearing prices for energy, reserve and regulation may be different from the bidding prices. The developed model is based on mixed-integer linear programming and the binaries render this optimization problem non-convex. Therefore, by solving the problem through intlinprog, no dual variables can be obtained directly. However, once the MILP is solved, the optimal values for the binaries will be obtained. The MILP problem is transformed into a LP problem through fixing the binaries at their optimal values and subsequently the MCPs can be obtained as the dual variables of the LP problem [34].

Another observation from Table 6.4 is that for each dispatchable unit, those bidding offers with lower prices have the priority to be dispatched. Nevertheless, among units the dispatch orders are not always in the price order. Capacities of
some units with higher prices may be dispatched first by the model before fully dispatching those units with lower prices. This kind of energy dispatch may seem to be out of merit order since the market clearing is a co-optimization process. It is this dispatch schedule that looks unreasonable but makes the solution optimal.

6.1.2 MCM with Transmission Network

Different cases studies are carried out. Case A, B and C are 3 illustrations on the modified market clearing model with the transmission system. Results in the base case, serving as reference, are compared with those cases with transmission loss integrated to analyze the effect of transmission loss.

Case A

All parameters and offers stay the same as those of base case except transmission system. Before solving for dispatch schedules, power losses and flows are unknown. The modified market clearing model takes transmission constraints into consideration and simultaneously find the optimal dispatch and power flows. The results clearly demonstrate that dispatch schedules and market clearing prices for three products are all changed with incorporation of transmission loss.

Case B

In this case, power flow capability of line 3, connects bus 2 and 3, decreases to 35 MW. Since the unstrained flow of line 3 is 50.1774 MW in Case A, it become congested. The transmission flow into bus 3 decreases, resulting in an flow increase of line 6 to supply energy demand. As a consequence, the outputs of unit 6 at bus 5 rises. The reduction of power flow into bus 3 results in an increase of flow in line 6 to supply the demand. Consequently, the outputs of unit 6 at bus 5 increases.

Case C

Energy demand at bus 3 rises to 280 MW in this case. Accordingly, the generation cost increases to $107430 which is the results of transmission loss cost and additional generation cost. Compared to that of the base case, case A and case B, it is raised by $6020, $4940 and $4900, respectively.
Table 6.3: Market Clearing Prices & Transmission Loss and Flow

<table>
<thead>
<tr>
<th>$Flow_k$ (MW)</th>
<th>Base</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>-16.0307</td>
<td>-25.0000</td>
<td>-6.2500</td>
<td></td>
</tr>
<tr>
<td>$k = 2$</td>
<td>-1.9967</td>
<td>6.0325</td>
<td>-11.7679</td>
<td></td>
</tr>
<tr>
<td>$k = 3$</td>
<td>50.1774</td>
<td>35.0000</td>
<td>62.9712</td>
<td></td>
</tr>
<tr>
<td>$k = 4$</td>
<td>41.2500</td>
<td>53.7729</td>
<td>31.4640</td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td>25.0855</td>
<td>25.0855</td>
<td>25.0000</td>
<td></td>
</tr>
<tr>
<td>$k = 6$</td>
<td>-8.0836</td>
<td>-23.1814</td>
<td>-27.5048</td>
<td></td>
</tr>
<tr>
<td>$k = 7$</td>
<td>-15.0000</td>
<td>-15.0000</td>
<td>-14.9151</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Loss_k$ (MW)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>0.0533</td>
<td>0.1250</td>
<td>0.0078</td>
<td></td>
</tr>
<tr>
<td>$k = 2$</td>
<td>0.0015</td>
<td>0.0100</td>
<td>0.0280</td>
<td></td>
</tr>
<tr>
<td>$k = 3$</td>
<td>0.5078</td>
<td>0.2519</td>
<td>0.7988</td>
<td></td>
</tr>
<tr>
<td>$k = 4$</td>
<td>0.3403</td>
<td>0.5814</td>
<td>0.2058</td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td>0.1261</td>
<td>0.1261</td>
<td>0.1250</td>
<td></td>
</tr>
<tr>
<td>$k = 6$</td>
<td>0.0141</td>
<td>0.1110</td>
<td>0.1532</td>
<td></td>
</tr>
<tr>
<td>$k = 7$</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0447</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Energy Price at Bus n ($/MWh)</th>
<th>Base</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>125.0938</td>
<td>124.7191</td>
<td>125.6580</td>
<td></td>
</tr>
<tr>
<td>$n = 2$</td>
<td>122.4023</td>
<td>120.9931</td>
<td>123.6943</td>
<td></td>
</tr>
<tr>
<td>$n = 3$</td>
<td>125.2189</td>
<td>126.0986</td>
<td>126.5407</td>
<td></td>
</tr>
<tr>
<td>$n = 4$</td>
<td>124.3144</td>
<td>123.3448</td>
<td>125.4068</td>
<td></td>
</tr>
<tr>
<td>$n = 5$</td>
<td>125.0000</td>
<td>125.0000</td>
<td>125.0000</td>
<td></td>
</tr>
<tr>
<td>$n = 6$</td>
<td>123.9419</td>
<td>122.3150</td>
<td>125.0312</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reserve Price ($/MWh)</th>
<th>Base</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>27.5528</td>
<td>28.5000</td>
<td>28.3448</td>
<td>28.5000</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>26.0000</td>
<td>26.0000</td>
<td>26.0000</td>
<td>26.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regulation Price ($/MWh)</th>
<th>Base</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>101,410</td>
<td>102,490</td>
<td>102,530</td>
<td>107,430</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Generation Cost g ($)</th>
<th>Base</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>101,410</td>
<td>102,490</td>
<td>102,530</td>
<td>107,430</td>
</tr>
</tbody>
</table>
It is also noticed that due to the large increase of energy demand at bus 3, the energy market clearing prices increase at all nodes except at bus 5. It stays the same as uniform market clearing price in Case A, B, C. This is because unit 6 at bus 5 is the marginal generation facility.

Table 6.4: Generation Dispatch Schedules

<table>
<thead>
<tr>
<th>Case</th>
<th>( q_{Eng}^{i,1} ) (MW)</th>
<th>( q_{Eng}^{i,2} ) (MW)</th>
<th>( q_{Res}^{i,1} ) (MW)</th>
<th>( q_{Res}^{i,2} ) (MW)</th>
<th>( q_{Reg}^{i,1} ) (MW)</th>
<th>( q_{Reg}^{i,2} ) (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>170 22 0 0 19 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>170 22 0 0 22 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>170 22 0 0 22 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>170 22 0 0 22 5</td>
<td></td>
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<td></td>
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<tr>
<td>6</td>
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<td></td>
</tr>
<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Nodal Price Difference

To illustrate the nodal price difference relationship as shown in Eq. (3.6), transmission line 5 that connects nodes 2 and 6 is employed. The capacity of this branch is 50 MW and the resistance is 0.02 per unit. Therefore, the flow and
loss tranches for this circuit can be constructed as shown in Table 6.5. In Case B, power flow of line 5 is 25.0855 MW with a loss of 0.1261 MW. The flow is accordingly on the tranche [7-8]. The segment slope of this tranche is calculated as $\alpha_{Seg,7} = (0.28125 - 0.125)/(37.5 - 25) = 0.0125$. Therefore the calculation of nodal price at bus 6 can is as shown in below

$$Price \ at \ Bus_6 = Price \ at \ Bus_2 \times \frac{2 + \alpha_{Seg,7}}{2 - \alpha_{Seg,7}}$$

$$= 120.9931 \times \frac{2 + 0.0125}{2 - 0.0125}$$

$$= 122.5150 \$/MWh$$

This price exactly matches the result in Table 6.3, validating the effectiveness of proposed market clearing model.

<table>
<thead>
<tr>
<th>POINT</th>
<th>$Flow_{5,m}^{Const}$</th>
<th>$Loss_{5,m}^{Const}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 1$</td>
<td>-50</td>
<td>0.5</td>
</tr>
<tr>
<td>$m = 2$</td>
<td>-37.5</td>
<td>0.28125</td>
</tr>
<tr>
<td>$m = 3$</td>
<td>-25</td>
<td>0.125</td>
</tr>
<tr>
<td>$m = 4$</td>
<td>-12.5</td>
<td>0.03125</td>
</tr>
<tr>
<td>$m = 5$</td>
<td>0</td>
<td>0.0125</td>
</tr>
<tr>
<td>$m = 6$</td>
<td>12.5</td>
<td>0.125</td>
</tr>
<tr>
<td>$m = 7$</td>
<td>25</td>
<td>0.28125</td>
</tr>
<tr>
<td>$m = 8$</td>
<td>37.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$m = 9$</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

### 6.1.3 Overall Comparison

Through comparing the dispatch schedules and energy market clearing prices of 3 cases, some conclusions can be obtained.

i) An significant feature of the nodal pricing can be made through comparing the power directions and nodal prices. The nodal price at sending end is always lower than that at receiving end. In other words, nodal price increases along the transmission flow.

ii) In Case B and C, merely the energy demand changes or transmission line restricts the energy flow, however, the market clearing prices dispatch schedules and for reserve and regulation change as well. This is due to the reason that energy dispatch is always cleared simultaneously with reserve and regulation in a co-optimized way.

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6.2 Numerical Analysis of DR Program

Demand side participation proves to be efficient for reducing energy price and lowering the generation cost compared to those market solutions without considering DR. The effect of demand side participation on the market clearing prices, the total cost and the benefit allocation between consumers and producers are illustrated in this section.

Firstly the simulated demand bidding offers are listed in Tables 6.6. All the prices in Table 6.6 are assumed to be higher than the floor price to prevent the DR participants from abusing DR [1]. Energy and regulation demand forecast in this DR program analysis is assumed to be 1,350 MW and 130 MW in this DR program analysis. Genco supply offers and other parameters for the upcoming dispatch period stay the same as those discussed in Case A in section 6.1.

The modified model focuses on illustrating how the new DR program is going to be incorporated into the market clearing model, whereas a few constraints, such as transmission network, are not considered for simplicity. After running the DR program, the dispatch schedules for both generators and consumers, the MCPs for energy, reserve and regulation, as well as the incentive payment are generated as outputs.

<table>
<thead>
<tr>
<th>Load Provider</th>
<th>$TL_n$ (MW)</th>
<th>$RU_n$ (MW/min)</th>
<th>$RD_n$ (MW/min)</th>
<th>$LP_{n,1}$ ($/MWh$)</th>
<th>$LP_{n,2}$ ($/MWh$)</th>
<th>$LQ_{n,1}$ (MW)</th>
<th>$LQ_{n,2}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 1$</td>
<td>510</td>
<td>10</td>
<td>10</td>
<td>185</td>
<td>175</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>370</td>
<td>10</td>
<td>10</td>
<td>175</td>
<td>170</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>470</td>
<td>10</td>
<td>10</td>
<td>170</td>
<td>160</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 1$</td>
<td>510</td>
<td>10</td>
<td>10</td>
<td>150</td>
<td>140</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>370</td>
<td>10</td>
<td>10</td>
<td>140</td>
<td>135</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>470</td>
<td>10</td>
<td>10</td>
<td>135</td>
<td>125</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>Case 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 1$</td>
<td>510</td>
<td>10</td>
<td>10</td>
<td>125</td>
<td>115</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>370</td>
<td>10</td>
<td>10</td>
<td>115</td>
<td>110</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>470</td>
<td>10</td>
<td>10</td>
<td>110</td>
<td>100</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>
Case 1, 2 and 3 are examples on the modified MCM with DR incorporated and they are in the increasing order of DR participation. Their details of bidding offers are shown in Table 6.6 and other associated parameter in Table 6.7.

With the supply offers and the demand side offers, the co-optimization problem described in (4.10) can be formulated and solved by *intlinprog*. The results are summarized in Tables 6.8 and 6.9. Incentives for Case 2 are presented in Table 6.10. Load curtailments and product prices for each case are also shown in Fig.6.2.

The observations on the simulation of MCM with DR are summarized as below:

1) It can be seen from Table 6.8 and 6.9 and Figure 6.2 that no curtailment is dispatched in Case 1. In Cases 2 and 3, as the DR participation level increases, there are different amounts of load curtailments and the energy prices drop accordingly. As the MCM with DR is a co-optimization process in order to minimize the overall cost of energy as well as the ancillary services, the incorporation of demand side bidding for energy will affect the MCP for reserve and regulation as well. This can be observed from Table 6.9 by comparing Case 2 or 3 with Case 1.

2) The additional consumer surplus (ACS) can be seen as the benefit of DR implementation. One third of the ACS is accrued to those load providers as an incentive payment for providing load curtailment. The rest of additional consumer surplus is allocated to other load providers. Such a compensation mechanism is designed to ensure a fair return to all the DR participants. The incentive payments can be recovered from load providers through an uplift charge (load providers pay for the electricity at a price of MCP with an uplift charge).

3) The bidding prices are required to be greater than the price floor. Therefore, the load aggregators only have a limited flexibility in changing their load pattern.

<table>
<thead>
<tr>
<th>Load Provider</th>
<th>$INC_{pre}$ (MW)</th>
<th>$lp_{pre}$ (MW)</th>
<th>$LSche_{pre}$ (MW)</th>
<th>$\rho$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>410</td>
<td>110</td>
<td>520</td>
<td></td>
</tr>
<tr>
<td>Case 1,2,3</td>
<td>$n = 2$</td>
<td>320</td>
<td>100</td>
<td>420</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>370</td>
<td>120</td>
<td>490</td>
<td></td>
</tr>
</tbody>
</table>
Furthermore, load curtailments will be scheduled only when the price spikes occur because of the high value of floor price.

The energy prices and BVPs (balance vesting prices) for the first half of 2016 in Singapore are presented in Table 6.11. It can be noted that (1) the average energy prices for this half year is much lower than the floor price, and (2) In January and May, the maximum energy price is very high. Price spikes may occur during these periods. The DR is introduced to lower the energy price.
Table 6.9: Market clearing prices with DR ($/MWh)

<table>
<thead>
<tr>
<th></th>
<th>Energy Price ($/MW)</th>
<th>Reserve Price ($/MW)</th>
<th>Regulation Price ($/MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>143.0714</td>
<td>42.2143</td>
<td>30.0000</td>
</tr>
<tr>
<td>Case 2</td>
<td>138.9286</td>
<td>29.7857</td>
<td>29.7857</td>
</tr>
<tr>
<td>Case 3</td>
<td>125.0002</td>
<td>27.9304</td>
<td>26.0001</td>
</tr>
</tbody>
</table>

Table 6.10: Incentive payments in Case 2

<table>
<thead>
<tr>
<th>$LC_n$ (MW)</th>
<th>$LCQ_n$ (MWh)</th>
<th>ACS ($)</th>
<th>$LCP$ ($/MWh$)</th>
<th>$IP_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>15</td>
<td>6.6025</td>
<td>1346.42</td>
<td>13.65</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>65</td>
<td>26.8125</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.11: Real energy price and BVP in Singapore

<table>
<thead>
<tr>
<th></th>
<th>2016</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Price ($/MWh)</td>
<td>Average</td>
<td>74.89</td>
<td>49.06</td>
<td>44.81</td>
<td>43.60</td>
<td>55.54</td>
<td>49.10</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>28.08</td>
<td>0.00</td>
<td>22.10</td>
<td>24.47</td>
<td>35.42</td>
<td>36.20</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>903.66</td>
<td>201.10</td>
<td>118.10</td>
<td>136.53</td>
<td>1010.75</td>
<td>76.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>BVP ($/MWh)</th>
<th>119.48</th>
<th>100.01</th>
</tr>
</thead>
</table>

However, due to lack of real market data, it is not possible for this work to simulate the impacts of the new DR program on these price spikes. Hence, the real economic gain of the new DR program is not analyzed in this work. Nonetheless, from the results, the DR program will help to bring down the energy price once the energy price is higher than the bidden prices (which are higher than the price floor) from demand side.

### 6.3 Simulation with Demand Uncertainty

Similarly, the stochastic programming is applied to the three cases in the previous section to study the incorporation of demand uncertainty.

It is assumed that the demand forecast variation is normally distributed where the expectation is forecasted demand value 1350 MWh and the standard deviation
is set to 10%. In ref. [30], Monte Carlo simulation with LHS of are carried out. Results of different sample sizes, 50, 100,..., 400, are compared. Therefore, a middle sample size of 200 is adopted. Latin Hypercube Sampling generates 200 samples, $D_1, ..., D_{250}$.

The dispatch schedules of energy generation that combines the two-stage optimization, the load curtailment and energy forecast are summarized in the following three pictures.

In each figure, the upper curve denotes the second-stage energy forecast of each scenario and the lower curve represents the actual energy demand amount with DR schedules. The areas between two curves are identified as the load curtailments.

![Figure 6.3: Two-stage stochastic optimization results of Case 1](image)

In Fig. 6.3, two curves almost coincide completely except for one scenario with relative high demand. This means the bidding prices from demand offers are higher than energy market clearing prices and no demand loads are curtailed consequently. It is also noticed that with the increase of DR participation which means more curtailment, the curves that represent generation dispatch are generally lower.
Different scenarios prices are also in Fig. 6.6. DR helps bring down the prices.

To verify the effectiveness of the two-stage stochastic market clearing model, the results of deterministic approach in section 6.2 and the two-stage stochastic model are compared. Solutions with deterministic approach are obtained through
substituting the outcomes in section 6.2 for the first-stage variables. The first-stage generation dispatch schedules of deterministic and stochastic approaches are summarized in Fig. 6.7. In addition, Table 6.12 compares the two different methods in terms of generation cost and social welfare. An intuitive comparison is presented in Fig. 6.8.

Figure 6.7: First-stage dispatch schedules of two different ways

To demonstrate the effects brought by DR participation directly, the profit
from non-curtailable load is deducted from the objective function. That is why the social welfare values presented in Table 6.12 and Figure 6.8 are negative.

Table 6.12: Comparison between different methods

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation Cost/$</td>
<td>132352.51</td>
<td>122665.78</td>
<td>111075.48</td>
<td>-101850.42</td>
<td>-107109.32</td>
<td>-108362.81</td>
</tr>
<tr>
<td>Social Welfare/$</td>
<td>-101530.49</td>
<td>-107002.28</td>
<td>-108344.11</td>
<td>132032.58</td>
<td>121433.84</td>
<td>111062.94</td>
</tr>
<tr>
<td>Change rate</td>
<td>-0.242%</td>
<td>-1.004%</td>
<td>-0.011%</td>
<td>0.314%</td>
<td>0.100%</td>
<td>0.017%</td>
</tr>
</tbody>
</table>

Figure 6.8: Comparison between stochastic and deterministic method

Fig. 6.8 compares the two methods in terms of social welfare and generation cost. For each generating facility, those capacities with lower prices will be dispatched first in the deterministic way. However, it is not the same case for the two-stage stochastic model as shown in Fig. 6.7. Although the first-stage dispatch results are much different, it can be seen that the proposed two-stage stochastic market clearing model outperforms the deterministic way in both terms. Generation cost decreases and social welfare increases accordingly compared to deterministic way. It is observed from Table 6.12 and Fig. 6.8 that the results between stochastic approach and deterministic approach look very close and the change rates are tiny, however, it is merely a half-hourly dispatch period.
Table 6.13: Accumulated cost savings and social welfare increase over a year

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation Cost Saving/$</td>
<td>$5.61 \times 10^6$</td>
<td>$2.16 \times 10^7$</td>
</tr>
<tr>
<td>Social Welfare Increase/$</td>
<td>$5.61 \times 10^6$</td>
<td>$1.88 \times 10^6$</td>
</tr>
</tbody>
</table>

As one day consists of 48 dispatch periods of the electricity market, the accumulated differences should be considerable. Table 6.13 shown above gives a simple calculation of accumulated cost saving and social welfare increase over the course of a year if the same percent change rates are applied. Although the calculation is not accurate and it is simply an estimation, it can demonstrate the advantage of the stochastic programming model to a certain extent.
Chapter 7

Conclusion and Future Work

7.1 Summary

This report provides a comprehensive and an in-depth study of the market clearing model of National Electricity Market of Singapore. The essential constraints on reserve and regulation are addressed. In addition, new constraints associated with the transmission network and demand response programs are explained in detail and interpreted into mathematical formulas. A two-stage stochastic market clearing strategy with demand uncertainty is developed at last.

The assessment tool for market participation, which is essentially the MILP framework based market clearing model, is developed to study the market prices and usage pattern changes. Several case studies with different DR participation levels are carried out to assess the effect of DR program on the market clearing prices and dispatch schedules. Results demonstrate that DR can effectively coordinate with conventional generation to accommodate the uncertain load demand. The total social welfare is optimized with a solution that is feasible for all scenarios. With increasing DR participation, the generation cost is considerably reduced.

In this new DR program, the consumers only have limited flexibility in managing their electricity usage in response to the price signals. The benefits of consumers may not be maximized in this situation. However, such a DR design enables a smooth and secure transition from the current regulated market to a deregulated one and therefore it is a reasonable compromise.
The proposed model in this report with incorporation of transmission loss and DR programs, based on the market design and piece-wise linearization of loss model, is specific to NEMS. Nevertheless, it serves as a valuable reference for other electricity markets employing the nodal pricing regime and initiating DR programs to achieve market deregulation.

7.2 Future Work

To extend the proposed framework, there are a number of research directions. It should be noted that the stochastic method is for a single period (half-hour). If a multi-period dispatch is considered, the temporal dependency of generation output and load demand should be well modeled. This can be done in the future. In addition, it is interesting to study the model extension with renewables injection, such as pv.

Another work scope worth further investigation could focus on the optimal bidding strategy for consumers. The next step is to co-optimize the demand response and interruptible load (IL) bidding. The assessment tool can be further enhanced for consumers’ decision-making to maximize their benefits.
Appendix

Nodal Price Difference Derivation

As mentioned in Chapter 3, the nodal price relationship between node A and B can be expressed as follows:

\[ Price_B = \left( 1 + \frac{\partial \text{loss}_{A-B}}{\partial \text{flow}_{B}} \right) \times Price_A \]  \hspace{1cm} (1)

After introducing the mid-point flow \( \text{flow}_{A-B} \), the derivative \( \frac{\partial \text{loss}_{A-B}}{\partial \text{flow}_{B}} \) can be represented as the product of two more convenient derivatives as follows:

\[ \frac{\partial \text{loss}_{A-B}}{\partial \text{flow}_{B}} = \frac{\partial \text{loss}_{A-B}}{\partial \text{flow}_{A-B}} \times \frac{\partial \text{flow}_{A-B}}{\partial \text{flow}_{B}} \]  \hspace{1cm} (2)

The piece-wise linear is employed in the loss function. The following equation can be obtained by taking the first derivative of Eq. 3.4:

\[ \frac{\partial \text{loss}_{A-B}}{\partial \text{flow}_{A-B}} = \alpha_{\text{Seg}_{A-B, \gamma}} \]  \hspace{1cm} (3)

As explained in Chapter 3, the MW flow into Node B \( \text{flow}_B \) can be derived via transmission flow \( \text{flow}_{A-B} \) and transmission loss \( \text{loss}_{A-B} \) in a relationship of

\[ \text{flow}_B = \text{flow}_{A-B} - \text{loss}_{A-B}/2. \]

Taking the first derivative of this equation, we can get:

\[ \frac{\partial \text{flow}_B}{\partial \text{flow}_{A-B}} = 1 - \frac{1}{2} \alpha_{\text{Seg}_{A-B, \gamma}} \]  \hspace{1cm} (4)
Thus, 

\[
\frac{\partial \text{loss}_{A-B}}{\partial \text{flow}_B} = \alpha_{Seg_{A-B,y}} \times \frac{1}{1 - \frac{1}{2} \alpha_{Seg_{A-B,y}}} \\
= \frac{2\alpha_{Seg_{A-B,y}}}{2 - \alpha_{Seg_{A-B,y}}}
\]  

(5)

Substituting (5) into (1) and rearranging the formula, the nodal price relationship is finally derived as follows:

\[
\text{Price}_B = \frac{2 + \alpha_{Seg_{A-B,y}}}{2 - \alpha_{Seg_{A-B,y}}} \times \text{Price}_A
\]  

(6)
Publication


References


REFERENCES


