DYNAMIC RESPONSES OF HUMAN BRAIN AND BRAIN MIMICKING GELS DURING IMPACT

ZHOU YOUJIN

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

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ZHOU YOUJIN

School of Mechanical and Aerospace Engineering
Nanyang Technological University, Singapore

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Abstract

Recently, more attention has been paid to the head injury studies. Among these head injuries, traumatic brain injury (TBI) has been specifically explored by many researchers due to its irreversible effects and high mortality or disability. On the prevention side, the study of mechanical properties of brain tissues is essential for a better understanding of injury mechanisms. The current research aims to study the mechanical behaviour of the brain tissue under dynamic loadings by using two typical mimicking gels, and to explore the injury mechanisms and the prevention of TBI. This research program has four parts.

Firstly, we conducted the compression tests and oscillatory shear tests (OSTs) to study the nonlinear and viscous behaviour of two brain mimicking gels, hydrogel and silicone gel. The compression results of the gels were compared with the experimental results of real brain tissues from literature to explore their feasibility in simulating the real brain. Time-Temperature Superposition (TTS) principle is used to obtain the master curves of storage and loss moduli in a wide range of frequency from the OSTs. An analytical method is developed to derive the compressive behaviour of the mimicking gels under different strain rates based on master curves. It is found that the two gels show nonlinear visco-elastic and significant strain rate-dependent properties and are suitable to mimic brain tissue in a range of the strain and loading rate.

Secondly, impact tests were conducted to study the dynamic responses of the mimicking gels at different velocities. The non-uniform deformation of the gels was captured by a high speed camera and analysed quantitatively by two methods, marker displacement measurement and Matlab analysis methods. An interesting phenomenon was observed that the gels compressed alternately at the two ends during the impact and the raised
lateral ring propagated like a wave at the lower velocity impact (2 m/s), but it disappeared at higher velocity impacts (6 and 20 m/s). Further investigation of the compression force demonstrates that compression velocity had a significant effect on the force-strain behaviour of the gels.

Thirdly, an analytical study is conducted on spherical wave propagation within soft materials. Based on the analytical model, a numerical study is conducted for the influence of the loading shape of particle velocity and mechanical parameters on the responses of the brain tissue. The analysis reveals that the loading shape of the particle velocity and bulk modulus have significant influences on the peak value and the attenuation of stress and strain, while viscosity shows little effect on them.

Finally, a detailed finite element (FE) head model is developed to simulate the scenario of front-rear head impact in football games. The brain injury risk is assessed based on the mechanical responses of the two heads and the risk curves. The effectiveness of headgear made of energy absorption foams is investigated in protecting players’ head. The simulation reveals that it is easy for bare head to get injured in two-head impact when the impact velocity $v \geq 2.5$ m/s and the proposed headgear can provide effective protection for the head at the impact velocity within 4 m/s.

Keywords: Traumatic brain injury, mechanical behaviour, brain tissue, soft gels, spherical wave propagation, head protection
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Table of Contents

Abstract ............................................................................................................................... I
Acknowledgement ........................................................................................................... III
Table of Contents ............................................................................................................ IV
List of Figures ............................................................................................................... VIII
List of Tables ................................................................................................................ XIV
List of Symbols ............................................................................................................... XV

Chapter 1  Introduction ................................................................................................. 1
  1.1  Background .......................................................................................................... 1
  1.2  Objectives ............................................................................................................. 3
  1.3  Scope .................................................................................................................... 4
  1.4  Outline .................................................................................................................. 6

Chapter 2  Literature Review ....................................................................................... 8
  2.1  Brain injury biomechanics ................................................................................... 8
  2.1.1  Experimental studies of head impact ............................................................ 8
  2.1.2  Finite element modelling of head impact .................................................... 14
  2.1.3  Analytical studies of head impact ............................................................... 17
  2.1.4  Criteria for brain injury ............................................................................... 20
  2.1.5  Head protection ........................................................................................... 24
  2.2  Mechanical properties of brain tissue ................................................................. 26
  2.2.1  Compression and tension studies ............................................................... 26
  2.2.2  Shear studies ............................................................................................... 30
  2.2.3  Wave propagation studies ......................................................................... 32
  2.2.4  Constitutive models of brain tissue ............................................................. 34
  2.3  Brain mimicking gels ......................................................................................... 37
  2.3.1  Applications of brain mimicking gels .......................................................... 38
  2.3.2  Mechanical properties of brain mimicking gels ........................................... 40
  2.4  Concluding remarks ........................................................................................... 43

Chapter 3  Dynamic Shear Behaviour and Constitutive Modelling of Brain Mimicking Gels ................................................................................................................. 46
3.1 Introduction ........................................................................................................ 46
3.2 Analytical methods............................................................................................. 46
  3.2.1 Maxwell-Wiechert model ........................................................................... 47
  3.2.2 Fourier transformation for visco-elastic model ........................................... 50
  3.2.3 Oscillatory shear test theory................................................................. 51
  3.2.4 Time–temperature superposition principle ............................................. 52
  3.2.5 Nonlinear visco-elastic constitutive model ............................................. 54
3.3 Experimental study ............................................................................................. 55
  3.3.1 Materials and sample preparation ........................................................... 56
  3.3.2 Compression test ................................................................................... 58
  3.3.3 Oscillatory shear test ............................................................................. 60
3.4 Results and discussion ........................................................................................ 61
  3.4.1 Compression test ................................................................................... 61
  3.4.2 Oscillatory shear test ............................................................................. 64
  3.4.3 Time–temperature superposition ............................................................ 65
  3.4.4 Complex shear modulus fitting ............................................................. 70
  3.4.5 Nonlinear visco-elastic constitutive modelling ....................................... 71
3.5 Conclusions ...................................................................................................... 73

Chapter 4 Dynamic Response of Brain Mimicking Gels during Impact .......... 75
4.1 Introduction ...................................................................................................... 75
4.2 Experimental study .......................................................................................... 75
  4.2.1 Materials and sample preparation ........................................................... 76
  4.2.2 Compression tests ............................................................................... 77
4.3 Results and discussion ..................................................................................... 79
  4.3.1 Deformation of the gels during the compression .................................... 79
  4.3.2 Quantitative analysis for the deformation of gels ................................... 86
  4.3.3 Compression force by quasi-static and impact tests ............................... 98
4.4 Conclusions .................................................................................................... 102

Chapter 5 Spherical Wave Propagation in Soft Materials ............................. 104
5.1 Introduction .................................................................................................... 104
5.2 Spherical wave propagation in soft materials .............................................. 105
5.2.1 Governing equations .................................................................................. 106
5.2.2 Solution by characteristic method ............................................................. 112
5.3 Numerical results and discussion ..................................................................... 113
  5.3.1 Validation of the analytical method .......................................................... 113
  5.3.2 Effect of the loading profile ...................................................................... 116
  5.3.3 Effect of the viscous properties ................................................................. 125
5.4 Analytical solution for the linear elastic case ................................................... 126
5.5 Discussion and conclusions .............................................................................. 132
  5.5.1 Discussion ................................................................................................. 132
  5.5.2 Conclusion ................................................................................................ 133

Chapter 6 Finite Element Study on Head Impact and Brain Protection .......... 134
  6.1 Introduction ...................................................................................................... 134
  6.2 Simulation work ............................................................................................... 135
    6.2.1 Description of FE head model ................................................................. 135
    6.2.2 Validation of the head model ................................................................. 139
    6.2.3 EPP foam headgear and the optimum thickness ..................................... 142
  6.3 Comparison of headgear made of EPP, EPS and CNT foams ......................... 155
    6.3.1 Mechanical properties of EPS and CNT foams .................................... 155
    6.3.2 Validation of the head model with headgear .......................................... 156
    6.3.3 Performance of headgear made of EPP, EPS and CNT foam ............... 157
  6.4 Discussion ........................................................................................................ 159
  6.5 Conclusions ...................................................................................................... 160

Chapter 7 Conclusions .......................................................................................... 164
  7.1 Conclusions ...................................................................................................... 164
    7.1.1 Dynamic shear behaviour and constitutive modelling of brain mimicking gels ........................................................................................................... 164
    7.1.2 Dynamic response of brain mimicking gels under impact .................... 165
    7.1.3 Spherical wave propagation in soft materials ....................................... 167
    7.1.4 Finite element study on head impact and brain protection ................. 167
  7.2 Summary of contributions .............................................................................. 168
  7.3 Future Work ................................................................................................... 169
List of Figures

Fig. 1.1 Classification of traumatic brain injury mechanisms: (a) penetrating injury; (b) contact injury; (c) acceleration or deceleration injury [1]. ........................................ 1

Fig. 2.1 (a) Fluid filled shell head model; (b) lumped-parameter head model [51, 55]. .. 19

Fig. 2.2 Schematic of wave propagation in the brain fibres [58].................................. 20

Fig. 2.3 Wayne State Tolerance Curve [62]. ................................................................. 21

Fig. 2.4 The compressive stress-stretch curves of brain tissues at low and moderate strain rates [2]......................................................................................................... 28

Fig. 2.5 Shear stiffness of healthy human brain measured by MRT at the frequency of 100 Hz [112]. ........................................................................................................ 33

Fig. 3.1 (a) Maxwell model; (b) Maxwell-Wiechert model.......................... 50

Fig. 3.2 Shifting of the modulus using time-temperature superposition [148]......... 53

Fig. 3.3 Cylinder samples of the gels: (a) 10% hydrogels (water content: 90%); (b) 8% hydrogels (water content: 92%); (c) 6% hydrogels (water content: 94%); (d) silicone gel. ........................................................................................................ 57

Fig. 3.4 Typical effect of the inertial force on the stress-strain curve due to the upper plate acceleration. ........................................................................................................ 59

Fig. 3.5 Oscillatory shear test on gels with the diameter of 40 mm and the thickness of 1 mm using rheometer. .............................................................. 61

Fig. 3.6 Nominal stress-strain curves of hydrogels and silicone gel under compression test and the results of real brain tissue from literature: (a) maximum nominal strain of 0.4, (b) zooming of the Fig (a) [77, 94].................................................. 63
Fig. 3.7 The storage modulus and loss modulus of the hydrogels and silicone gel by the oscillatory shear test at different temperatures. ................................................................. 65

Fig. 3.8 Master curves of two gels (reference temperature, 23 °C) obtained from the corresponding results of oscillatory shear test in Fig. 3.7. ........................... 68

Fig. 3.9 Comparison of storage and loss moduli between gels and real brains under different shear frequencies [6, 135, 159]. ......................................................... 69

Fig. 3.10 Nominal stress-strain curves of the two gels by modelling based on the corresponding master curves in Fig. 3.8. ......................................................... 73

Fig. 4.1 Cylinder gel sample with markers on the lateral surface............................................. 77

Fig. 4.2 Schematic of experimental setup for the gel impact test ........................................... 79

Fig. 4.3 Deformation images of gels: (a) 10% hydrogel, \( v=8 \times 10^{-4} \) m/s; (b) silicone gel, \( v=8 \times 10^{-4} \) m/s; (c) 10% hydrogel, \( v=8 \times 10^{-3} \) m/s; (d) silicone gel, \( v=8 \times 10^{-3} \) m/s. The numbers of 1-6 represent the nominal strains of 0, 0.2, 0.4, 0.6, 0.7 and 0.8, respectively. ........................................................................................................ 82

Fig. 4.4 Deformation images of gels at the impact velocity of 2 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel. The numbers of 1-5 represent the nominal strains of 0, 0.2, 0.4, 0.6, and maximum values, respectively. ....... 83

Fig. 4.5 Deformation images of gels at the impact velocity of 6 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel. The numbers of 1-5 represent the nominal strains of 0, 0.2, 0.4, 0.6, and maximum values, respectively. ....... 84

Fig. 4.6 Deformation images of gels at the impact velocity of 20 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel. The numbers of 1-5 represent the nominal strains of 0, 0.2, 0.4, 0.6, and maximum values, respectively. ....... 85

Fig. 4.7 Impact tests at the velocity of 20 m/s: (a) hydrogel broke into many small pieces, (b) silicone gel broke into four relatively symmetrical pieces from in axial
direction, quasi-static compression tests: (c) hydrogel broke into many small pieces, (d) silicone gel showed a crack in the middle of samples. ...................... 86

Fig. 4.8 Deformation histories of the gels: (a) v=2m/s; (b) v=6m/s; (c) v=20 m/s. .......... 88

Fig. 4.9 Schematic of the separation of the gel sample with three parts......................... 89

Fig. 4.10 Typical deformation of the three parts of the hydrogel: (a) v=2 m/s; (b) v=20m/s. ................................................................................................................................. 89

Fig. 4.11 Five typical shapes of hydrogel obtained by the ImageJ at the impact velocity of 2m/s................................................................. 91

Fig. 4.12 Volumes of hydrogel and silicone gel at the corresponding three impact velocities.................................................................................................................... 92

Fig. 4.13 Schematic of the separation of gel sample with six parts of the same volume using Matlab. ............................................................................................................ 93

Fig. 4.14 Deformation of the six parts of the gels at the impact velocity of 2 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel......................... 94

Fig. 4.15 Deformation of the six parts of gels at the impact velocity of 6 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel......................... 96

Fig. 4.16 Deformation of the six parts of gels at the impact velocity of 20 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel......................... 97

Fig. 4.17 Deformation comparison of the three parts between marker measurement and Matlab analysis. .................................................................................................................. 97

Fig. 4.18 Typical force histories at the two ends of the samples: (a) v=2 m/s (b) v=20 m/s. ................................................................................................................................. 99

Fig. 4.19 Compression force-strain curves of the gels at the back end: (a) v=8×10^{-4} m/s; (b) v=8×10^{-3} m/s; (c) v=2 m/s; (d) v=6 m/s; (e) v=20 m/s. ...................... 101
Fig. 4.20 The effects of the compression velocity on the force-strain behaviour of the gels. ................................................................. 102

Fig. 5.1 Schematic of wave propagation due to a uniform loading applied on the spherical inner surface .................................................. 106

Fig. 5.2 An infinitesimal element in spherical coordinates. .............................................. 107

Fig. 5.3 Validation of the analytical model by comparing the particle velocities between analytical results and experiments [166] in PMMA .................................................. 115

Fig. 5.4 Validation of the analytical model by comparing the particle velocities between analytical results and experiments [170, 171] in sierra white granite. .......... 116

Fig. 5.5 Input particle velocity versus time at \( r_0 = 5 \) mm. ................................................................. 119

Fig. 5.6 Sectional view of the spherical FE model with a spherical void .................... 119

Fig. 5.7 Particle velocity curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively. .................. 120

Fig. 5.8 Radial stress curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively. .................. 121

Fig. 5.9 Circumferential stress curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively. .................. 123

Fig. 5.10 Radial strain curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively. .................. 124

Fig. 5.11 Circumferential strain curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively. .................. 125

Fig. 5.12 Radial stress curves at \( r = 5, 10, \) and 25 mm, corresponding to the input in Fig. 5.5(a). ................................................................. 126
Fig. 5.13 Comparison of particle velocity curves with different bulk moduli at \( r_0 \), \( 2r_0 \), and \( 5r_0 \), corresponding to the input of constant particle velocity. ............................. 129

Fig. 5.14 Comparison of radial stress curves with different bulk moduli at \( r_0 \), \( 2r_0 \), and \( 5r_0 \), corresponding to the input of constant particle velocity. .............................. 131

Fig. 5.15 Comparison of circumferential stress curves with different bulk moduli at \( r_0 \), \( 2r_0 \), and \( 5r_0 \), corresponding to the input of constant particle velocity. ..................... 131

Fig. 6.1 FE model of an adult head .................................................................................. 138

Fig. 6.2(a) Loading time history curve used for the simulation; (b) acceleration comparison between experiments by Nahum’s and simulations by author and Wiliger [12, 184]. ................................. 141

Fig. 6.3 Comparison of pressure between experiments by Nahum’s and simulations by author and Wiliger at four locations: (a) frontal; (b)parietal; (c) occipital; and (d)posterior fossa [12, 184]. ............................................................................. 141

Fig. 6.4 (a) FE headgear model; (b) the modelling of front-rear head impact.............. 143

Fig. 6.5(a) Stress-strain curves for EPP foam of different densities; (b) zooming of the Fig (a) in vertical scale [188, 189]. ............................................................................. 146

Fig. 6.6 Acceleration curves of two heads with different head thicknesses (T) under the impact velocity of 2.5 m/s. ......................................................................................... 148

Fig. 6.7 Foam absorbed energy versus the displacement of the EPP headgear under the impact velocity of 2.5 m/s (u - displacement, T - initial thickness). ......................... 150

Fig. 6.8 The Mises stress contours of the headgear (T=10 mm) in the impact area under the impact velocity of 2.5 m/s............................................................................. 150

Fig. 6.9 (a) Maximum head acceleration; (b) HIC values of two heads. ..................... 151

Fig. 6.10 Injury risk curves for the brain [192]............................................................... 153
Fig. 6.11 Head injury risk under the impact velocities of 2.5, 3.0, 4.0, and 5.0 m/s: (a) moderate injury; (b) severe injury. ................................................................. 154

Fig. 6.12 Stress-strain curves for EPP, EPS and CNT foams [188, 189, 195, 196]. ...... 156

Fig. 6.13 Comparison of head acceleration between experiments and simulation for head impact with EPS foams [198]................................................................. 157

Fig. 6.14 Mechanical responses of the heads with the protection of foam headgear: (a) Maximum acceleration; (b) the corresponding HIC values......................... 162

Fig. 6.15 Head injury risk with the protection of EPP, EPS, and CNT foam headgear: (a) moderate injury; (b) severe injury. ................................................................. 163
List of Tables

Table 2.1 Proposed Head Injury Criterion for various dummy sizes [59] ......................... 22

Table 3.1 Detailed information of the gels for the compression test .......................... 58

Table 3.2 The detailed stress-strain values of Fig. 3.6 [77, 94] .................................. 63

Table 3.3 Parameters of the visco-elastic model obtained by fitting the master curves of complex modulus ............................................................. 72

Table 5.1 Parameters of the PMMA and granite used in this study [166, 169-171] ...... 115

Table 5.2 Mooney-Rivlin, visco-elastic constants, and parameters of brain tissue used in this study [115, 116] ................................................................. 117

Table 6.1 The weight and element information of brain tissue in FE head model ........ 137

Table 6.2 Material properties used in the FE head model [178, 180-182] ................. 139

Table 6.3 Material constants for the Prony series of six terms [180] .......................... 139

Table 6.4 Foam densities for the headgear according to the corresponding thicknesses ................................................................. 143

Table 6.5 Parameters for the EPP foam constitutive laws [188] ................................. 145

Table 6.6 Material parameters of foams used in the simulations [188, 189, 195, 196] .. 156
List of Symbols

Chapter 2

\(a(t), \alpha(t)\) Translational acceleration, angular acceleration

\(b, C_1, C_2\) Fitting constants of strain energy function

\(G_0, G_\infty\) Instantaneous shear modulus, long-time shear modulus

\(I_1, I_2\) Strain invariants in strain energy function

\(I_{jj}\) Moment of inertia

\(m\) Mass

\(t_1, t_2\) Initial, final integral time

\(t\) Time

\(T\) Total duration over calculated the acceleration or deceleration

\(W\) Strain energy

\(\lambda_i\) Principal stretch ratios

\(\tau_i\) Relaxation times

Chapter 3

\(A\) Cauchy-Green strain tensor

\(C\) Constant for deriving Maxwell model

\(C_{10}(t), C_{01}(t), C_1(t)\) Fitting constants of strain energy function

\(g_i, k_i\) Prony series parameters for shear and bulk moduli

\(G_0, G_\infty\) Instantaneous shear modulus, long-time shear modulus

\(G^*, G_S, G_l\) Complex modulus, storage modulus, the loss modulus

\(G_{exp}\) Experimental complex shear modulus

\(i_{lm}\) Imaginary number, \(i_{lm}^2 = -1\)

\(I_1, I_2\) Strain invariants in strain energy function

\(n_K\) Prony series number for bulk modulus

\(T, T_0\) Temperature, reference temperature
\(W\) Strain energy
\(w\) Frequency
\(a_T\) Horizontal temperature-dependent shift factor
\(\varepsilon, \varepsilon_s, \varepsilon_d\) Total strain, elastic strain, viscous strain
\(\lambda\) Stretch under uniaxial stress
\(\sigma(t)\) Total stress with the change of time
\(\sigma_0, \varepsilon_0\) Initial stress, initial strain
\(\sigma, \sigma_s, \sigma_d\) Total stress, elastic stress, viscous stress
\(\sigma_{spring}, \sigma_i\) Stress on the isolated spring and the Maxwell elements
\(\tau_i(T)\) Relaxation time at the temperature \(T\)
\(\tau_i, g_i\) Series of relaxation times and Prony series parameters
\(\tau_i^\theta, \tau_i^K\) Series of relaxation times for shear and bulk moduli
\(\varphi(w)\) Phase angle with the change of frequency

Chapter 4
\(\ddot{\varepsilon}\) Acceleration rate of the strain
\(\sigma_z, \sigma_{\theta}, \sigma_r\) Inertial axial stress, hoop stress, radial stress
\(r_0\) Radius of the samples

Chapter 5
\(c_L\) Wave velocity in linear elastic material
\(c_k(\varepsilon)\) Wave velocity
\(C_{10\infty}, C_{01\infty}, C_{10}\) Fitting constants
\(C_{01}, C_1\)
\(E_0\) Instantaneous elastic modulus
\(E_{eff}, G_{eff}\) Effective Young’s modulus, effective shear modulus
\(G_0, G_{\infty}\) Instantaneous shear modulus, long-time shear modulus
\(l_1, l_2\) Strain invariants

XVI
\( K \)
Bulk modulus

\( r, r_0 \)
Radius, initial loading radius

\( u(r, t) \)
Displacement in the radial direction

\( v(r, t) \)
Particle velocity in the radial direction

\( W \)
Strain energy

\( a, \gamma \)
Nonlinear elastic constants

\( \varepsilon_r, \varepsilon_\theta \)
Radial strain, circumferential strain

\( \lambda \)
Stretch under uniaxial stress

\( \lambda_1, \mu \)
Usual lame constants

\( \rho_0, \nu \)
Density, Poisson’s ratio

\( \sigma_{\text{eff}}(\varepsilon) \)
Effective stress

\( \sigma_r, \sigma_\theta, \sigma_\phi \)
Radial stress, two circumferential stress

\( \tau \)
Relaxation time

**Chapter 6**

\( a(t) \)
Translational acceleration

\( A, B \)
Density dependent parameters

\( C_{1,E}, C_{2,E} \), \( C_{1,A}, C_{2,A} \)
Material dependent parameters

\( C_{1,B}, C_{2,B} \)

\( E \)
Young’s modulus

\( g_i, \tau_i \)
Material constant, relaxation times

\( G(t), G_0 \)
Shear modulus at time \( t \), instantaneous shear modulus

\( m, n \)
Fitting parameters

\( t \)
Time

\( t_1, t_2 \)
Initial integral time, final integral time

\( \sigma, \varepsilon \)
Engineering stress, engineering strain

\( \rho \)
Density
Chapter 1 Introduction

1.1 Background

Traumatic brain injury (TBI) continues attracting a great deal of attention due to its irreversible effects and high mortality or disability. After TBI, people may suffer consciousness losing, mental depression, dizziness, nausea, blurred vision, and headache. Some symptoms happen immediately, whereas some may appear days, weeks or even years later. As shown in Fig. 1.1, the mechanisms of TBI can be classified into penetrating injury and closed injuries (contact injury and acceleration or deceleration injury), which may occur in traffic accidents, sports accidents, violence, terrorist attacks, wars, etc.

On the prevention side, the study of mechanical properties of brain tissues is essential for a better understanding of injury mechanisms. The quasi-static behaviour of the brain tissues has been studied systematically over the past several decades including compression, tension, shear, relaxation and indentation tests. However, human brains are

Fig. 1.1 Classification of traumatic brain injury mechanisms: (a) penetrating injury; (b) contact injury; (c) acceleration or deceleration injury [1].
often injured physically in the impacts or acceleration events, where brains are subjected to the high-rate loading and large deformation. Since brain tissue is a kind of strong strain rate-dependent material, the investigation on dynamic behaviour is essential. Some studies have been reported on the dynamic behaviour of brain tissue. However, many problems occurred in the dynamic studies associated with the tissue sample preparation, experiment operation, and large variability of the reported results. Given these problems, soft gels such as hydrogel and silicone gel were widely used to mimicking brain tissue in studying the TBI, due to the similar mechanical properties, wide availability, and stable properties [2-7].

Oscillatory shear test (OST) by the rheometer is a common method to characterize frequency effect on the elastic and viscous properties of the soft materials. However, the frequency is usually limited in the shear test by the conditions of commercial rheometers, which cannot cover the range of the deformation rate in some study conditions (e.g. impact, explosion, and wave propagation). In addition, split-Hopkinson pressure bar (SHPB) has been commonly used to obtain the dynamic behaviour of engineering materials, such as metals and concrete, since it can provide a nearly constant high strain rate compressive and tension ($> 1000 \text{ s}^{-1}$). However, when it is used for soft materials with low mechanical impedance, the stress-strain relationship obtained from the traditional SHPB will not be accurate. Non-equilibrium dynamic stress and non-uniform deformation are caused within the sample due to the short rise pulse and the low wave speed of the soft materials [8]. The radial inertia in soft materials has also been proven to have a significant effect on the uniaxial compressive stress. Thus, a more reliable and convenient method is needed to obtain the mechanical properties of soft materials under
high strain rate. In addition, to understand these problems fundamentally, it is necessary to study the dynamic responses of these soft materials during the impact. For example, how they deform non-uniformly; how the inertial influences the deformation and stress; and what the effects of the impact velocity and material stiffness on the deformation are.

When the materials are undertaken an impact or blast loading, stress wave is always a large factor that affects their response. Some studies have been reviewed in the literature on the shear and longitudinal wave propagation in brain tissue and soft gels. However, there are a handful of studies reporting the spherical wave propagation in these materials, whereas it is useful in injury detection by ultrasound and the understanding of deeper cerebral tissues [9-11].

With the increasing popularity of football, the number of people suffering the brain injuries is also increasing. To protect player’s head, it is necessary to study mechanical responses of the head during the impact, to evaluate the head injury risks, and to develop protective structures for the head. The mechanical responses of brain have been studied for the ball-to-head impact using head models, but few studies have been reported on the head-to-head impact, which has a very high possibility to cause injury.

1.2 Objectives

The objectives of the project are to present a basic understanding of the mechanical responses of the brain tissue under dynamic loadings by using two typical mimicking gels, and to explore the injury mechanisms and the prevention of TBI. For these purposes,
efforts are made to develop a reliable method to obtain the mechanical properties of the soft materials under high strain rate based on the OST on gels. Impact tests were conducted to investigate the influences of inertial effect, impact velocity, gel stiffness and viscosity on dynamic responses. Following the experimental work, an analytical model is developed to study the propagation characteristics of spherical waves in the soft materials under transient external loading. To better understand the mechanical response of head during impact, a detailed FE head model is developed to simulate head impact and investigate the protective ability of the football headgear with different foams. The objectives of this project can be briefly outlined, as follows,

(1) To investigate the dynamic behaviour of brain tissue under shear and compression tests by using mimicking gels.

(2) To develop a reliable method to derive the dynamic properties of the mimicking gels at high strain rates based on the OST.

(3) To better understand spherical wave propagation in nonlinear and viscous soft materials.

(4) To study mechanical responses of the head during the impact and to develop protective structures.

### 1.3 Scope

The research is carried out from three aspects including experimental studies, theoretical analysis, and Finite Element (FE) simulations.
Firstly, the efforts were made to investigate the mechanical behaviour of two brain mimicking gels. Quasi-static compression tests were carried to explore their feasibility in simulating the real brain. OSTs were conducted on the mimicking gels at different frequencies and temperatures. Given the difficulties in obtaining the dynamic properties of these soft materials under high strain rate, an analytical method is developed to derive the compressive stress-strain relationship of soft gels based on the OSTs. As brain tissues have been proven to have the similar temperature and rate-dependent properties [6], this method can be applied to explore the dynamic compressive behaviour of them.

Secondly, the impact tests were conducted to study the dynamic responses (e.g. deformation and impact force) of the gels at different velocities. The non-uniform deformation of the gels was captured by a high speed camera and the impact forces were measured at two ends of the cylinder samples. Deformation of the gels is analysed quantitatively by two methods. One is to measure the displacement of markers placed on the surface of the samples during the deformation. However, this method is not suitable for these soft materials due to the effect of the arc-shape surface by the friction. Another method is developed based on the incompressible property of the soft material. The deformation is analysed by integrating the volume using the Matlab, which provides more accurate and detailed deformation process.

Thirdly, an analytical study is conducted on spherical wave propagation within soft materials. The model is validated against the experiments previously performed as well as FE analysis. The analytical method is applied to investigate the effect of viscous property on the wave propagation behaviour (e.g., particle velocity, stress, and strain) of the brain tissue. Based on the analytical model, a numerical study is conducted to
investigate the influence of the loading shape of particle velocity on the responses of the brain tissue. In addition, as a simplified case, an analytical solution for the linear elastic wave propagation with the input of particle velocity is given.

Furthermore, a validated anatomical FE head model is developed to simulate a scenario of front-rear head impact in football games. The mechanical responses of the two heads are obtained to assess head injury risks based on the brain injury risk curves. The effectiveness of headgear made of energy absorption foams is investigated in protecting players’ heads during head impact. The thickness of the headgear is optimized based on the expanded polypropylene (EPP) foam. In addition, the carbon nanotube (CNT) containing foam is proposed to be used for headgear. The effectiveness of CNT foam as headgear is evaluated and compared with that of traditional foams, i.e., EPP and expanded polystyrene (EPS).

1.4 Outline

This thesis consists of seven chapters. The structure of this thesis is organized as follows.

Chapter 1 gives a brief introduction to the background and the objectives of the research.

Chapter 2 overviews the literature related to TBI studies by the experimental, FEM, and analytical methods. In addition, the studies on mechanical properties of the brain tissues have also been reviewed including, the compression, tension, shear, and wave based tests. Furthermore, a brief review of the application and mechanical property studies of brain mimicking gels are included.
In Chapter 3, quasi-static compression tests were carried out on brain mimicking gels to explore their feasibility in simulating the real brain tissue. Furthermore, OSTs were conducted on the mimicking gels to study the effects of shear frequency and temperature on the dynamic behaviour. An analytical method is developed to derive the compressive stress-strain relationship of soft gels under varied strain rates based on the OSTs.

In Chapter 4, the dynamic behaviour of the gels was further studied by impact tests based on the shear tests in Chapter 3. The non-uniform deformation of the gels was captured by a high speed camera and then analysed quantitatively by two methods. The impact forces were measured at two ends of the cylinder samples and compared at different impact velocities.

In Chapter 5, an analytical study is conducted on spherical wave propagation within soft materials. It is applied to investigate the effect of viscous property on the wave propagation behaviour of the brain tissue. Based on the analytical model, a numerical study is conducted on the influence of the loading shape of particle velocity and material parameters on the responses of the brain tissue.

Chapter 6 presents a simulation study on the scenario of front-rear head impact in football games. The brain injury is assessed based on the mechanical responses of the two heads and the injury risk curves. The effectiveness of headgear made of energy absorption foams is investigated in protecting players’ head.

Finally, Chapter 7 presents the general conclusions of this research and the brief suggestions in future work.
Chapter 2 Literature Review

The literature review provides an overview of the studies on TBI by the experimental, FEM, and analytical methods, and on mechanical properties of the brain tissues. In recent decades some brain substitute gels were widely used to study the TBI. A brief review of the application and mechanical property studies of brain mimicking gels were also included.

2.1 Brain injury biomechanics

2.1.1 Experimental studies of head impact

In the past decades, some experimental studies have been reported in investigating the impact and blast TBI by using human cadaver heads, animal heads and physical head models.

The experimental studies on the human cadaver heads were usually conducted by measuring the acceleration of heads, deformation of brain and skull, and the intracranial pressure of the brain under the impact and simulated blast loading. Pioneering studies on the human cadaver heads can be dated back to 1977 by Nahum and Smith [12]. In this early work, a series of human cadaver heads were impacted to the 45° anatomical plane of the frontal bone (to avoid the angular acceleration) by a cylindrical load cell with the velocities and mass varied from 4.36 to 12.95 m/s and 5.23 to 23.09 kg, respectively. Positive pressure was found at the impact site and the magnitudes decreased along the direction to the head back. Eventually, it became the negative pressure at the head back.
These impact test results were widely used to validate the FE head models. In 1978, a more comprehensive impact study was performed by Got et. al. [13] with 42 tests on perfused cadavers by drop impact. The head was impacted at the frontal, frontal-facial and temporal-parietal locations barely and with the protection of helmets.

Extensive studies have been carried out by Trosseille et al. [14] who conducted six tests involving three cadavers to validate the FE head model. The accelerations of the impactor and the cadaver head, and the intracranial pressure were measured during the impact. Besides the impact on the forehead, the impacts on the thorax and face were also carried out. Different from the impact tests by Nahum and Smith [12, 15], large angular acceleration was observed since the heads were hung by a harness. The intracranial pressure showed similar trends to that of the tests by Nahum and Smith [12, 15]. Rizzetti et al. [16] reported a study on the brain, cervical spine and head injuries by 14 head impacts at different locations. Skin lacerations and skull fractures were observed in the rigid impacts. The paddings were found to reduce the possibility of skull fractures, but have little effect on brain injury.

In addition to the head acceleration and intracranial pressure, more dynamic behaviour of the brain was obtained by using high speed x-ray. Stalnaker et al. [17] carried out the head impacts on 15 unembalmed cadavers to study the head injury mechanics by using accelerometers, high speed cineradiography based on x-ray and lead markers to analyse the 3D motion. The heads were repressurized for the tests, and the pressure was found to reduce the relative motion between the brain and skull, which indicated the difference between the tests on cadaver heads and in vivo. Similar impact tests were conducted by Nusholtz et al., [18], but on repressurized cadavers, live anesthetized and postmortem
Rhesus monkeys. The skull deformation and head angular acceleration showed potential importance on the injury of live Rhesus head. In 2007, Hardy et al. [19] used high-speed biplane x-ray and neutral density targets to investigate the dynamic deformation of the human brain during impact. The relative head motion and maximum principal and shear strain were analysed based on the target motions. A significant inertial effect was found on the brain deformation during the head acceleration. When the skull started to rotate after the impact, the brain kept static in the beginning and then accelerated slower than the skull, and when skull rotation slowed down, the brain moved faster than the skull. The intracranial pressure was affected significantly by the linear acceleration, but slightly by angular acceleration. In addition, the heads were impacted with the protection of American football helmet. The strain of brain was observed to increase with the helmet, whereas the mean pressure, linear and angular acceleration of the head were reduced. This indicated the helmet had little effect on the deformation of the brain during impact, which was similar to the report by Rizzetti et al. [16], where the paddings cannot reduce the possibility of the brain injury.

Up to now, a handful of studies have been reported on the TBI under the blast loading [20]. There are a few blast tests by a group from Wayne state University [21, 22]. The cadaver heads were subjected to the simulated blast by shock tube with different blast intensities. The deformation of the skull was measured by strain gauges at five locations of skull surface and intracranial pressures were measured by fibre optic pressure sensors at four locations. It was found the orientation to blast wave and the geometry of head showed significant influences on the intracranial pressure. The deformation of the skull
and brain was affected by the interaction of the skull/brain system and shock wave, which supported the multimodal skull flexure theory.

Over the past several decades, numerous animals have also been tested to understand the human TBI [23]. In addition to measuring the dynamic responses of the head, using the live animal heads, researchers can follow the state of consciousness of the animals after the injury. The impact tests were usually conducted by using a guided block to impact the skull or brain with free falling, where the impact energy can be adjusted by changing the drop weight and height.

In 1981, Feeney et al. [24] conducted drop tests on the dura of rat to study the evolution of cortical contusions after injury. The injury was found to progress from haemorrhages to necrotic cavity in the first 24 hours and then the cavitation started to expand over the next two weeks. Some deficits were detected in a long time beyond 90 days. Shohami et al. [25] investigated closed head injury by weight-drop tests on the rat heads. Some typical parameters (histopathology, cerebral edema, motor, blood–brain barrier and cognitive functions) were used to define the injury severity. The assessment using neurological severity score was performed to evaluate the alertness, seeking behaviour and neurological impairment of motor function. The severity score was found to have a high correlation with the brain injury severity. Drop tests on the rat heads were also be used to study the TBI caused by the motor vehicle accidents or falls [26]. Two group tests on 161 rats were conducted to establish skull fracture threshold and determine the primary cause of death. The simulated model for the rodent was proved to be able to produce a graded brain injury without causing brain stem damage.
In addition, there were some TBI studies by impacting monkey heads. As mentioned above, Nusholtz et al., [18] used the live anesthetized and postmortem Rhesus to investigate the effect of the skull deformation and angular acceleration on the brain injury. Both pure linear and angular acceleration impacts on the monkey heads were performed by Masuzawa et al. and Sekino et al. [27, 28] to evaluate the acceleration effects by producing brain contusion and concussion. The severity of the systemic blood pressure showed a correlation with the brain injury after the impact. The pure linear acceleration was proved to have little effect on the brain injury, whereas the pure angular acceleration produced visible brain damages. This study indicated the angular acceleration was the dominated factor to cause brain injury.

Various physical head models were also widely used to study TBI. Compared with cadaver and animal heads, physical head models have the advantages of the modest cost, availability, stable mechanical properties. The early work dates back to 1943 by Holbourn [29], who designed a simplified 2D head model with the wax skull and the gelatine brain to study the diffuse brain injury. He found that the major factors causing the brain injury were the angular acceleration and skull deformation, and the produced shear strain showed a significant damage to the brain. This was consistent with the finding by Masuzawa et al. and Sekino et al. [27, 28], that the angular acceleration dominated the brain injury. Kenner and Goldsmith [30] developed a simple head model, where the skull was mimicked by spherical aluminium and plastic shells and the brain was represented by water. The head models were impacted by steel spheres in the horizontal direction. The strain on the shell surface and the pressure of the water were measured by strain gauges and the tourmaline disks attached on the inner surface,
respectively. Instead of using the artificial skull, Margulies et al. [31] used one human and two baboon skulls to create the head model by injecting silicone gel into them. These models were used to study the TBI caused by angular acceleration. A high speed camera was used to capture the motion of the skull and the deformation of the simulant brain. Brain injury was determined by comparing overall deformation of the gel brain to the pathological portrait of real brain from animal studies. In recent decades, some more accurate 2D model and 3D head models were created by using the Magnetic Resonance Imaging (MRI) scan to obtain the anatomical structures of head [32, 33].

In this section, the experimental studies on the human cadaver, animal and physical heads have been reviewed in terms of head acceleration, brain and skull deformation, and the intracranial pressure under the impact and simulated blast loading. These experimental studies provided the fundamental understanding of mechanical responses of heads under dynamic loading. The intracranial pressure was affected significantly by the linear acceleration, but slightly by angular acceleration, whereas the angular acceleration was the dominant factor to cause brain injury.

Compared with the deformation and the intracranial pressure, the head acceleration is the most commonly used parameter in studying the head response due to its easy availability. Up to now, it is still used as the main index to assess the head injury risk in brain injury criteria. However, it cannot provide the detailed dynamic responses of the brain. To explore the underlying mechanism of brain injury and build the relationship between macroeconomic head injury and microeconomic brain responses, more technologies have been adopted to obtain the intracranial pressure of the brain and the deformation of the head and brain. Various physical head models have been widely used to study TBI.
Compared with cadaver heads and animals, they have the advantages of the modest cost, high availability, and stable mechanical properties.

2.1.2 Finite element modelling of head impact

With the development of computational technologies and three-dimensional (3D) images, FE head modelling has become a very efficient tool in studying the TBI with considering the complex geometry and multiple material compositions of the brain. The first 3D FE head model with actual skull geometry was proposed in 1973 by Hardy and Marcal [34], who simulated skull response under static frontal and lateral loading without considering the effect of brain. In 1974, Chan [35] developed an axisymmetric model with modelling brain and skull as a visco-elastic core bonded to a thin visco-elastic shell, to investigate the effect of large shear strain on the cerebral blood vessels and brain matter. Other early models were reported by Khalil et al. in 1974 and 1977 [36, 37], which used simplified 3D shells (spherical or ellipsoidal shell) to represent skull, and inviscid fluid, visco-elastic, or elastic solids to represent brain. In 1975, Shugar [38] developed a more realistic 3D FE head model where the skull was separated into several layers to represent the inner and outer tables and both the skull and brain were modelled as linear materials. All these models considered the head as only two major structures: skull or brain.

In order to account the effect of anatomical structures, Ward [39, 40] developed a 3D FE head model containing six main structures of the head: rigid skull, cerebrum, cerebellum, brainstem, ventricles, and dura membrane, for studying the frontal impact. This FE head model was validated by Nahum et al. [12], who compared simulated intracranial pressure
with that from experiments. Kumaresan and Radhakrishnan [41] used a 3D FE head model to investigate the influence of different anatomical brain structures on brain injury by analysing the intracranial pressure and shear stress under occipital impact loading.

Besides the effect of anatomy structures, some researchers investigate the influence of material parameters on the mechanical response of brain by using FE head model. Ruan et al. [42] found parameters, i.e., Young's modulus, bulk modulus, had a significant influence on the stress distribution of the brain under external loading by using linear elastic and visco-elastic constitutive models to represent the brain tissue, respectively. To represent the nonlinear and time-dependent properties of the brain under large deformation, Kleiven [43] used an isotropic hyper-visco-elastic constitutive model to simulate brain and found that the choice of the stiffness of brain had a significant effect on the brain injury prediction. In contrast to isotropic constitutive model, Colgan et al. [44] developed a FE head model with an anisotropic nonlinear visco-elastic brain to predict the mechanical response in case of high angular TBI. The results illustrated that the anisotropic model shows a variation in the predicted apoptosis at sites distal to the rotation centre. In addition, the influence of the skull property on the dynamic behaviour of head was studied by Kang et al. [45], who modelled the skull as an elastic brittle material to account the skull fracture in studying the head injury in motorcycle accidents.

Besides the simulative work listed, much more work has been reported in optimizing the FE head model by comparing the simulation and experimental results. Up to now, most of these models have been validated against the intracranial pressure data by Nahum and Trosseille et al. [12, 14], the brain displacements data by Hardy et al. [19], and the nasal response by Nyquist et al. [46].
In summary, with the help of MRI, more and more detailed head models have been developed with accurate skull and brain structures, and synthetic materials. It should be pointed out that the simulation accuracy of FE head model depends on the incorporation of the anatomical details, the layers of skull, the correct boundary conditions, the accurate constitutive representation of material properties, and the skull-brain interface conditions. Besides these factors, the complexity of brain structures and the number of finite elements also play important roles in the simulation. It is feasible and desirable to develop complex FE head models by incorporating more anatomical details of the human head and using nonlinear and time dependent constitutive models to represent brain tissues. An important feature regarding the model complexity and the number of finite elements is to capture the key anatomic information of the brain. The element number in these models varies from dozens [34] to millions [47]. Taylor and Ford [47] developed a head model with the fine grids of 1 mm$^3$ and element number of 6,850,560, which costed 31 hours with 64 CPU processors to simulate TBI under blast loading.

Although, the simulation study using a more detailed head model with more elements can obtain more accurate results, it is not suggested to develop FE head models as complex as possible. Because TBI research is different from medical applications which emphasize examine response and function of the complex brain tissue. Brain injury modelling focuses more on the brain-head interaction and brain constitutive models, which play the key roles in stress distribution. Thus, the requirement of anatomical details in FE head models is not as demanding as those in medical models. We need to take both the computational accuracy and the efficiency into consideration.
2.1.3 Analytical studies of head impact

Analytical studies on TBI can be divided into three groups, impact of fluid-filled shell head models, impact of lumped-parameter analytical models, wave propagation in head.

For the fluid-filled shell head model studies, some of the earliest work was done by Anzelius [48], who assumed the head as a rigid spherical vessel full-filled with the inviscid fluid. Due to the assumption of the rigid container, this study did not consider the influence of skull deformation. In 1969, Engin [49] proposed an analytical model to study the axisymmetric transient response of a fluid-filled shell subjected to delta-function impulsive loading. The shell representing the skull was considered to have the membrane and bending properties and the fluid representing the brain was assumed to be inviscid compressible. Based on Engin’s model, Talhouni and DiMaggio [50] developed a model considering the skull as a thin elastic prolate spheroidal shell to simulate the head injury by an explosive shock wave. It was found that the eccentricity of head had a significant effect on the distribution of stress in the shell and pressure in the fluid brain. During the last decade, Young [51] proposed a model combining the Hertzian contact stiffness and the effective bending stiffness of membrane for investigating the TBI as a result of the blunt head impact (Fig. 2.1(a)). Some global impact characteristics were derived as implicit function expressions including the duration of impact, the peak force, the peak acceleration of shell and the pressure in the head. Based on Young’s model, Heydari and Jani [52] developed a more realistic fluid-filled ellipsoidal shell model with a inconstant thickness to model the head.
In addition to considering the whole brain as fluid, some lumped-parameter models were proposed by representing the brain as concentrated mass. The early work was performed by Stalnaker and Fogle [53], who developed a two-degree-of-freedom model, where two masses were connected by a spring and damper in parallel to represent the human head. This model considered linear accelerations of the brain. In contrast, Low [54] proposed a rotational model with three masses connected by eight springs and dampers to account the angular accelerations. Based on these models, Zou et al. [55] developed a lumped-parameter model to evaluate the sensitivity of injury prediction to small changes in a few parameters. As shown in Fig. 2.1(b), each Kelvin element was kept a 45° angle with the horizontal in this model, which coupled the linear and angular movements of the brain with respect to the skull. Recent studies showed that intracranial wave propagation can generate significant intracranial pressure and strain in the brain, especially the wave produced by the blast loading and high local impact.

Recently, Rossikhin and Shitikova [56] used two elastic and visco-elastic spherical shell models to investigate the dynamic response of a human head impacted by another head or by some spherical objects. This model considered quasi-longitudinal and quasi-transverse shock wave propagation along the spherical shells after impact and took into account the age-related changes in the mechanical properties of the bone. Valdez and Balachandran [57] studied wave propagation through brain to understand the influence of nonlinear material properties of the tissues on the propagation characteristics of stress waves, i.e., propagation speed and attenuation. As shown in Fig. 2.2, the wave was assumed to propagate through the skull into the brain, and then propagate in one-dimensional brain
fibres. Rather than considered as fluid or a mass, the brain was modelled as nonlinear visco-elastic solid.

Analytical studies on TBI by developing fluid-filled shell head models, lumped-parameter analytical models, and wave propagation methods, have been reviewed. Model-based methods provide more mechanical analysis to the head acceleration and brain pressure, whereas wave-based methods focus on the stress and deformation behaviour of the brain.
2.1.4 Criteria for brain injury

Injury criteria play a very important role in predicting the probability of TBI under external loading. However, it is difficult to develop criteria which are suitable for everyone because people with different physiques have different injury tolerance levels. Many indirect methods have been tried to obtain injury tolerance information including human volunteers testing with low level, cadaver testing, animal testing, crash reconstructions, computer simulation, and dummy crashing testing [59]. Based on the accelerations of the head, many head level injury criteria have been developed including: Wayne State Tolerance Curve (WSTC), Gadd Severity Index (GSI), Head Injury Criterion (HIC), Rotational Injury Criterion (RIC), Generalized Acceleration Model for Brain Injury Tolerance (GAMBIT), and Head Impact Power (HIP).

The WSTC derived from cadaver and animal tests by Lissner et al. [60] has become the basis for most head injury criteria. Fig. 2.3 describes a relationship between the pulse
duration and effective acceleration (g). With a given duration, if the value of effective acceleration is above the line, the head injury will beyond the tolerance level of human. The time duration range was from 1 to 6 ms in the original data and then extended to above 6 ms by comparing animal and cadaver impact and with human volunteer restraint system tests. The WSTC also has the drawback that it only accounts the linear acceleration.

Based on the WSTC data, Gadd [61] plotted the duration and translational acceleration on log paper and got an approximate straight line function. It was developed to a weighted impulse criterion, Gadd Severity Index (GSI) as

\[
GSI = \int_0^T \left[ a(t) \right]^{2.5} dt \tag{2.1}
\]

where \( a(t) \) is the translational acceleration at the head centre of gravity, \( t \) is the time in microseconds, and \( T \) is the total pulse duration over which the acceleration or deceleration is calculated.

Fig. 2.3 Wayne State Tolerance Curve [62].
Based on Versace’s [63] analysis of the relationship between the WSTC and the GSI, in 1971, National Highway Traffic Safety Administration (NHTSA) developed a new criterion, Head Injury Criterion (HIC),

\[
HIC = \left[ (t_2 - t_1) \left( \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} a(t) \, dt \right)^{2.5} \right]_{\text{max}}
\]  

(2.2)

where \( t_1 \) and \( t_2 \) are the initial and final integral time.

HIC is calculated over \( t_1 \) and \( t_2 \), which are selected to maximize HIC. Duration of integration recommended by National Highway Traffic Safety Administration (NHTSA) in 1986, is 36 ms (\( t_1 - t_2 = 36 \text{ ms} \)) [64]. Corresponding limit of HIC value recommended by Federal Motor Vehicle Safety Standard (FMVSS 208) is 1000 [64], which is based on injury observations instead of tests. A recent report by NHTSA has improved this injury criterion by adopting the duration time to calculate the maximal HIC from 36 ms (HIC\(_{36}\)) to 15 ms (HIC\(_{15}\)) which specified the HIC value for the 50\(^{th}\) percentile male [59]. The corresponding limits of HIC value are shown in Table 2.1. Compared with HIC\(_{36}\), more limits of HIC value are added for very young children in HIC\(_{15}\), which shows smaller values than these of adults.

Table 2.1 Proposed Head Injury Criterion for various dummy sizes [59].

<table>
<thead>
<tr>
<th>Dummy Type</th>
<th>Large Male</th>
<th>Mid-Sized Male</th>
<th>Small Female</th>
<th>6 Year Old Child</th>
<th>3 Year Old Child</th>
<th>1 Year Old Infant</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIC(_{36}) limit</td>
<td>NA</td>
<td>1000</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>HIC(_{15}) limit</td>
<td>700</td>
<td>700</td>
<td>700</td>
<td>700</td>
<td>570</td>
<td>390</td>
</tr>
</tbody>
</table>
HIC is useful for assessing the probability of TBI under external loading or rapid acceleration or deceleration. However, HIC also has the drawback that cannot account angular acceleration. Ommaya et al. [65] developed an angular acceleration tolerance curve by analysing the test results of concussed and non-concussed monkeys. Based on the tolerance curve, Kimpara and Iwamoto [66] proposed Rotational Injury Criterion (RIC) as

\[
RIC = \left( t_2 - t_1 \right) \left[ \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \alpha(t) \, dt \right]^{2.5},
\]

where \( \alpha(t) \) is the resultant angular acceleration. Similar to HIC, it is derived by substituting resultant angular acceleration of \( \alpha(t) \) for translational acceleration of \( a(t) \).

In 2000, Newman [67] proposed the Head Impact Power (HIP) criterion based on the total kinetic energy of the head during impact, which accounted both angular and linear accelerations along all degrees of freedom. It is defined as,

\[
HIP = \sum_{i=1}^{3} ma_i \int a_i \, dt + \sum_{j=1}^{3} I_{ij} \int \alpha_j \, dt
\]

where \( m \) is the mass of the head, \( \alpha_i \) is the translational acceleration, \( \alpha_j \) is the angular acceleration, and \( I_{ij} \) is the moment of inertia. A HIP value of 12.79 kW represents a 50% probability of TBI [67].

In summary, the head level injury criteria comprised of translational and angular acceleration provide more accurate assessment than HIC. Nevertheless, HIC is still the
most widely used head level injury criteria for brain injury during sports and traffic accidents, attributing to its easy availability.

2.1.5 Head protection

In the past few years, a large number of the helmets and headgear have been developed and studied in protecting human head. Here, we review some studies on the protection of head in sports relating to our study (Chapter 6). For the helmets used for other aspects (i.e. combat and traffic), readers can refer to the reviews by Fernandes et al., [68], Kulkarni et al. [69] and National Research Council [70].

A typical helmet consists of outer shell, inner polymeric liner, and pad, where the shell and liner are used to absorb the impact energy. Vetter et al. [71] investigated the effect of thickness and materials of the shell and liner on the performance of American football helmets by experimental and FE methods. It was showed that the thickness and materials of the shell were not important factors on the helmet performance since the shell absorb a small portion of the impact energy. Extensive studies were conducted on the cricket helmets by McIntosh and Janda [72] to evaluate the performance with regard to helmet constructions and test conditions, where the helmets were impacted by ball with different velocities. At the low velocity impact (lower than 20 m/s), all of the helmets provided effective protection to the dummy headform, but the protective ability was not satisfactory when the impact velocity was higher than 30 m/s.

The soft headgear has also been developed to protect the human head in sports, such as football (soccer). In the United States, some national standards have been developed in an
effort to ensure the quality of football headgear [73]. “Full90 Headguard”, one of the major commercial products, has been used for football players and its performance was tested on dummy heads [73-75].

The heading scenario simulation by Posey [74] showed contact forces between the head and the ball were reduced with an average of 19.0%. Broglio [75] studied the effectiveness of headgear by projecting balls to a force platform mounted vertically with headgear. The results showed headgear provided protection by reducing the peak impact force by 12.5% at the impact velocity of 15.7 m/s. In Withnall et al.’s study [76], human volunteers’ heads and headforms were impacted by balls with maximum velocities of 8.4 m/s and 30 m/s, respectively. Headgear provided attenuation of impact force only when the impact velocity was lower than 20 m/s. This is probably because, for high speed impact, the deformation of the ball is so large that the compression of headgear makes little contribution in mitigating the impact [74]. For the head-to-head impact tests, two dummy headforms were impacted with the velocities from 2 to 5 m/s. The results demonstrated the head responses with headgear can achieve an overall 33% reduction in linear acceleration compared with that of bare heads [76]. In a population study, Delaney et al. [73] showed that 47.8% of 440 targeted players had experienced the symptoms of head injuries during the 2006 football season. 52.8% of those without headgear were injured, while it was reduced to 26.9% for those who wore, indicating an effective protection of headgear to players. The effect of headgear on the ball rebound trajectory or velocity has been investigated by projecting the football to Hybrid III headform at velocities from 2.7 to 19.2 m/s [74]. The study showed that the headgears had little effect on ball rebound properties.
In the studies of protecting player’s head, researchers have attempted to evaluate the head injury risk structures during the impact and to develop the protective equipment. The effects of the thickness and materials of the shell and liner on the performance of helmets have been studied by experiments and FE methods. The soft headgear shows some protective ability to players’ head and little effect on ball rebound behaviour. However, the head to head impact has seldom been reported.

### 2.2 Mechanical properties of brain tissue

The mechanical properties of the brain tissues have been studied over the past several decades by the compression, tension, shear, wave methods etc. [77-81].

#### 2.2.1 Compression and tension studies

The compression/tension studies can be divided into quasi-static, impact, and creep/relaxation studies.

Miller and Chinzei [77] studied the compressive behaviour of porcine brain tissues at the low strain rates from $6.4 \times 10^{-5}$ to $6.4 \times 10^{-1}$ s$^{-1}$. The brain tissues showed nonlinear stress-strain behaviour and strain rate-dependent property. Extensive studies on tensile behaviour of brain tissue were also performed by them with two loading rates of $6.4 \times 10^{-2}$ and $6.4 \times 10^{-1}$ s$^{-1}$ [82]. Similar to the compressive property, strain rate showed a significant effect on the stress-strain relationship. The compression/tension behaviour under the moderate strain rate was studied by Rashid et al. [83, 84], who conducted the
tests on porcine brains at the strain rate of 30, 60 and 90 \(s^{-1}\). It was found that the compressive/tensile stress increased significantly with the increase of strain rate, where at the nominal strain of 30\%, the compressive stresses were \(8.83 \pm 1.94\), \(12.8 \pm 3.10\) and \(16.0 \pm 1.41\) kPa under the strain rates of 30, 60 and 90 \(s^{-1}\), respectively. As shown in Fig. 2.4, more compressive work has been done to study the strain rate effect and nonlinear behaviour on brain tissue of human, porcine, calf at the low and moderate strain rates [2, 85-89]. All brain tissue showed nonlinear and strain rate-dependent properties except the tests by Estes and Mcelhaney [85], where the stress curves of human brain were too high to plot in order to show the detail of the others curves. The stress-strain relationship showed large variations even at the same or similar strain rates.

Up to now, a handful of studies have been conducted on compression behaviour of brain tissues at high strain rates. Different from the low and moderate strain rate studies, the tests on brain tissue at high strain rate have always been a big challenge due to their complex mechanical behaviour including low stiffness, large inertial effect, volumetric incompressibility, and low wave speed [90, 91]. SHPB is typically used to get the uniaxial stress-strain relationship under high strain rate \((> 1000 \text{ s}^{-1})\). This technique is well-established for stiff materials such as metals and concrete. However, problems will occur when it is used for soft materials with low mechanical impedance. Firstly, it is difficult to obtain an equilibrium dynamic stress and a uniform deformation within the sample due to the short rise time of incident pulse and the low wave speed of the soft materials [8]. Secondly, due to the low impedance property of soft materials, the stress transmitted through the sample to the transmitted bar is too weak for the normal strain
gauge to capture [92]. Thirdly, the radial inertia in soft materials has been proved to have a significant effect on the uniaxial compressive stress [93].

Fig. 2.4 The compressive stress-stretch curves of brain tissues at low and moderate strain rates [2].

Several modifications have been made to the conventional SHPB to solve these problems. Pervin and Chen [94] studied the dynamic behaviour of bovine brains by testing grey and white matters individually using modified SHPB. In this test, a thick layer of paper was employed as the pulse shaper between striker bar and the incident bar to extend the pulse rise time. The cross section of the sample was redesigned as annulus disc shape to reduce the radial inertia effect and the thickness was optimized to 1.7 mm to reduce the non-uniform deformation. More sensitive strain gauges (i.e. semiconductor strain gauge) and hollow aluminium bar were used to obtain the strain signal in the transmitted bar. The results showed both grey and white matters were highly strain rate sensitive and the white matter showed much higher stiffness than that of grey matter.
Creep studies on brain tissues can date back to 1962 by Dodgson [95], who conducted compressive creep tests on the rat brain. It was found that the compressive deformation showed a linear relationship with the creep time in the logarithmic scale. Extensive creep studies were carried out on the brain tissue of different animals including calf, porcine and rabbit by Koeneman [96]. Similar trends were observed to Dodgson’s [95] studies at different temperatures. In 1970, Galford [97] studied the visco-elastic properties of brain tissue by the compressive creep and relaxation tests on dura and brain of human and monkeys. The brain and dura showed linear visco-elastic property with the strain varied from 0 to 30± 10% based on the relaxation tests. A significant difference was found in the creep tests on the human brain and monkey brain, whereas similar trends were observed in the relaxation test. In recent years, more compressive tests were done to validate the constitutive model by ramp-and-hold stress relaxation [98], to investigate the loading rate effect by load–unload cyclic relaxation [89], and to develop the relationship between relaxation time and strain by high rate loading relaxation [99].

In this section, the compression/tension studies have been reviewed including quasi-static, impact, and creep/relaxation aspects. It is found that all brain tissues showed nonlinear and strain rate-dependent properties. However, for compression tests under high strain rate, the test accuracy is highly affected by the non-equilibrium stress, non-uniform deformation, and radial inertia. Researchers have made modifications to the conventional SHPB to partially reduce these effects. Nevertheless, there are few studies systematically investigating these effects.
2.2.2 Shear studies

The shear behaviour of brain tissues was usually studied by simple shear, shear relaxation and OSTs.

Donnelly and Medige [100] conducted the simple shear studies on human brain from twelve cadavers by two parallel plates with the strain rate varied from ~0 (quasi-static test) to 90 s\(^{-1}\). The results showed that the shear behaviour of brain tissue was highly dependent on strain rate, but was not on the location in the brain and sample conditions. This was different from the compressive impact studies by Pervin and Chen [94], where the grey matter and white matter showed significant different compressive behaviour.

Similar to the compressive relaxation test, shear relaxation test is to apply a constant shear strain on the sample, and measure the shear force or torque on the sample within a relatively long time. In 1995, Arbogast et al. [101] performed shear relaxation tests on the porcine brain with three relaxation strains from 2.5% to 7.5%. The shear behaviour of brain stem was found to be dependent on the direction of fibre orientation, which means it was an anisotropic material. Extensive tests were carried out by Bilston et al. [79], who tested the shear relaxation behaviour of bovine brain tissue with larger strains (up to 100%). With the increase of the applied strain, the relaxation showed a slower process.

OST by the rheometer is a common method to characterize viscous properties and the frequency effect on the brain tissue. The storage and loss moduli can be obtained over a range of frequency and temperature from the test. The storage modulus representing elastic property is important in studying the elastic deformation, whereas loss modulus standing for viscous property plays an important role in dynamic energy dissipation. The
first OST on brain tissue was reported by Fallenstein and Hulce in 1969 [102], who studied the dynamic shear behaviour of the human brain with the strain from 7% to 24.5% by an electro-mechanical test device. The strain level showed little influence on the dynamic modulus of the brain. Extensive tests were performed three year later by Shuck and Advani [103] to study the viscous properties of human brain tissue with a wider range of frequency from 5 to 350 Hz. It was found that the shear frequency had a significant effect on the yielding strain of the brain tissue. The brain tissue can sustain 3.5% shear strain at the frequency lower than 10 Hz, whereas it yielded at the strain of 1.3% when the frequency was higher than 60 Hz. Thibault and Margulies [104] found that both storage and loss moduli of brain tissue increased significantly with the increase of growth time by testing the brain tissue of neonatal and adult porcine. To obtain the dynamic behaviour of brain tissue under higher shear frequency (up to 1000 Hz), Brands et al. [105] adopted the Time-Temperature Superposition (TTS) principle to analyse OST (test frequency of 0.1 to 16 Hz) results of the porcine brain at the temperature from 4 to 38°C. In addition, the brain tissue was found to show nonlinear behaviour for the shear strain larger than 10%. More OSTs have been reported to study the dynamic behaviour of brain tissues of human, porcine, and bovine [79, 106, 107].

Based on the previous shear studies, including simple shear, shear relaxation, and OSTs, brain tissues were found to show rate-dependent and anisotropic properties, which is similar to the compressive behaviour. In addition, when the shear strain is larger than 10%, nonlinear behaviour can be observed.
2.2.3 Wave propagation studies

Recently, wave-based methods have also been developed to study the dynamic behaviour of brain tissue such as ultrasonic wave and magnetic resonance elastography (MRE). Different from the common compression/tension and shear tests, wave-based methods can measure the mechanical properties without contacting the brain tissue, so they can be used to conduct experiments in vivo. In 1981, Kremkau et al. [108] studied the effects of types, measured time from death, and age at the death of brain tissue on wave propagation speed and attenuation in 22 brain tissue samples from human. Etoh et al. [109] applied the longitudinal ultrasonic wave with the frequency range of 0.5-5 MHz to study internal friction effect of bovine brain tissue. Ultrasonic absorption and the apparent viscosity of the brain tissue increased with the decrease of the wave frequency. In addition to the wave speed and attenuation studies, more dynamic properties of brain tissue were studied by Lippert et al. [110] using "wave-in-a-tube" ultrasonic method with a wider frequency range from 100 kHz to 10 MHz. The results showed that with the increase of the wave frequency the complex Young’s and shear moduli increased and approached to an upper limit gradually, whereas the complex bulk modulus changed little. The MRE technique has also been developed to estimate the visco-elastic property of the brain tissue in recent years. The shear behaviour by the MRE test was usual in the linear range since the deformation caused by the wave was in microns level. McCracken et al. [111] used the MRE to quantify the shear stiffness of brain tissue by measuring cyclic displacements caused by shear waves. The results based on harmonic wave and transient impulse methods showed similar results. The shear stiffness of white matter was near 12 kPa and much higher than that of grey matter of 8 kPa. This was consistent with the
results from MRE tests on healthy human by Kruse et al. [112] and the compressive behaviour of brain tissue under high strain rate by Pervin and Chen [94]. In the study by Kruse et al. [112], MRE tests were conducted on the brain of 25 healthy adult volunteers to assess the effect of age on the shear modulus (Fig. 2.5). No significant difference was observed among the volunteers with different ages. In addition to measuring the mechanical properties of the human brain, wave-based technology can be used to detect the injured or abnormal parts, whose characteristics are altered due to the change of mechanical properties [113].

![Shear stiffness of healthy human brain measured by MRT at the frequency of 100 Hz](image)

**Fig. 2.5** Shear stiffness of healthy human brain measured by MRT at the frequency of 100 Hz [112].

Among these wave propagation studies, wave theory plays an important role in understanding wave tissue interaction. Based on spherical wave propagation method, the stereotactic theory was used to explain the deeper cortical injury by assuming the skull has a spherical shape and generates second spherical pressure wave due to the impact vibration [9, 10]. Galich and Rudykh [114] studied the effect of stiffening on elastic wave
propagation in soft materials subjected to finite deformation. They found in incompressible materials, wave direction and initial strain state exhibited a strong influence on the velocities of transverse wave, but little influence on that of longitudinal wave. This is different from highly compressible materials, where these factors showed significant effect on both transverse and longitudinal waves. Valdez and Balachandran [57] theoretically investigated nonlinear visco-elastic longitudinal wave propagation through axons of brain tissue to understand the effect of geometric features of axons on the longitudinal stress by changing the cross-sectional area.

In summary, with the decrease of the wave frequency, ultrasonic absorption, Young’s and shear moduli, and the viscosity increased, whereas the complex bulk modulus changed little. Based on the stereotactic theory, spherical wave propagation method can be used to explain the deeper cortical injury. For incompressible materials, wave direction and initial strain state showed strong influences on the velocities of transverse wave, but little influence on that of longitudinal wave.

2.2.4 Constitutive models of brain tissue

Due to the complex mechanical properties of soft materials, many mechanical constitutive models have been proposed to represent these properties, which can be generally classified as linear elastic, visco-elastic, nonlinear elastic, and nonlinear visco-elastic. Brain tissue was considered as linear elastic materials in early studies. However, the linear elastic models were found unable to represent the time-dependent and the nonlinear properties of brain tissue when the strain was larger than 1%.
Some hyperelastic constitutive models were used to represent the nonlinear property of the brain tissue [77, 115, 116]. One of most widely used hyperelastic models is Mooney-Rivlin constitutive model, which was the first general strain energy function proposed by Mooney [117] based on continuum theories, and then developed by Rivlin [118]. The constitutive model is composed of two strain invariants as,

\[ W = C_1 (I_1 - 3) + C_2 (I_2 - 3), \quad (2.5) \]

where \( W \) is the potential strain energy, \( C_1 \) and \( C_2 \) are fitting constants, \( I_1 \) and \( I_2 \) are strain invariants as,

\[ I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad \text{and} \quad I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2, \quad (2.6) \]

where \( \lambda_i \) are principal stretch.

Another simple hyperelastic constitutive model developed by Rivlin is Neo-Hookean model [119],

\[ W = C_1 (I_1 - 3). \quad (2.7) \]

It appears to be a special case of Mooney-Rivlin model, but was developed from different principles [120]. The two simple hyperelastic models, Mooney-Rivlin and Neo-Hookean, have been widely used to describe the nonlinear behaviour not only for brain tissues but also for many other soft materials. However, a common limitation exists for the two models that they cannot be used to predict very large strain behaviour.

Fung [121, 122] proposed another hyperelastic model to describe the tensile/compressive behaviour of the brain tissue based on the first strain invariant as
where $G_0$ is the instantaneous shear modulus, and $b$ is a fitting constant. This model was used by Rashid [83, 84] to fit the stress-strain curves of brain tissue for both tensile and compressive tests, and was found to provide excellent fitting.

In 1972, Ogden [123] developed a model to represent the nonlinear behaviour of rubberlike materials under uniaxial tension, simple shear and equibiaxial tension tests. This model was a linear combination of strain invariants and widely used for brain tissue with the form as [82-84],

$$W = \frac{G_0}{2b} \left[ e^{b(l_i-3)} - 1 \right], \quad (2.8)$$

Based on this model, Miller and Chinzer [82] developed a nonlinear visco-elastic model to describe the tensile and compressive behaviour of brain tissue under different strain rates.

Many more hyperelastic models have been developed based on the strain energy form, such as Reduced polynomial model [117], Gent model [124], Van der Waals (Kilian) model [125]. It is hard to determine which model can provide the best representation for brain tissue behaviour. To find an appropriate model, we should consider the test conditions, sample state of brain tissue, and the applications.

To represent the time-dependent/relaxation behaviour of brain tissue, the shear modulus is formulated in the exponential form as [126]
\[ G(t) = G_{\infty} + G_0 \sum_{i=1}^{n} g_i e^{-t/\tau_i} = G_0 - G_0 \sum_{i=1}^{n} g_i \left(1 - e^{-t/\tau_i}\right), \] (2.10)

where \( G_{\infty} \), \( G_0 \), and \( \tau_i \) represent the long-time shear modulus, instantaneous shear modulus, and relaxation times, respectively. Combining the hyperelastic constitutive law with visco-elastic constitutive equations, the new model can account both the nonlinear and time-dependent mechanical behaviour of brain tissue. For example, Miller and Chinzer, developed a nonlinear visco-elastic model as,

\[ W = \int_0^t \left\{ \sum_{i,j=1}^{N} G_{ij}(t-\tau) \frac{d}{d\tau} \left[ (I_1-3)(I_2-3)^i \right] \right\} d\tau, \] (2.11)

where \( G_{ij} = C_{ij\infty} + C_{ij} e^{-t/\tau_1} \). This model has been widely adopted for the simulation and analytical studies.

Many mechanical constitutive models representing the properties of the head have been reviewed. We do not recommend to choose the model as complex as possible just to obtain the excessively accurate head properties. The appropriate choose and development of the model should depend on the real application.

### 2.3 Brain mimicking gels

Given the problems associated with preparing the tissue samples, conducting experiments, and large variability of the reported results in using real brain tissues [2], some brain substitute materials were developed in recent decades to study the brain injury. Soft gels such as silicone gel and hydrogel were widely used in studying the TBI, due to the
similar mechanical properties to the brain tissue, wide availability, and stable properties [3-7].

### 2.3.1 Applications of brain mimicking gels

The silicone gel and hydrogel were widely used to mimic the brain in an artificial head to provide an insight into the response mechanisms of the brain for the head subjected to impact, ballistic and blast loading [3-7]. In addition, they can be used to provide validation in terms of the stress, pressure, and deformation of the brain for the simulation study.

Zhang et al. [127] used the silicone gel (DowCorning Sylgard 527) to simulate the ballistic brain injury caused by the ballistic penetration in a 3D model. Pressure transducers and high speed camera were used to capture the pressure history and the pulsation of the temporal cavity, respectively. It was found that the temporal cavity caused by 9-mm projectile was 1.5 times larger in size, 1.5 times longer in duration and three times higher in pressure than these created by the 25-caliber projectile. The same silicone gel was used to simulate the surgical process in a 3D physical brain without skull by Wittek et al. [128]. This model was further used to simulate the dynamic response of brain caused by hydrocephalus and ventriculostomy by inflating internal rubber membrane with pressure [129].

Bradshaw and Ivarsson [32] used silicone gel to simulate the brain in a 2D physical head and studied the acute subdural hematoma and diffuse axonal injury during head impact. Patrick markers were put into the gel to track the deformation. It was found that the
deformation of the simulant brain during the head deceleration showed an equivalent effect to that of the opposite side impact. Viano et al. [130] developed a 2D physical head to investigate the brain deformation during translational and angular acceleration by using silicone gel. A similar model was used by Ivarsson et al. [131] for the investigation of the natural protection by the cerebral ventricles to the brain during the head rotation. The cerebral ventricles were found to play an important role in protecting brain with reducing the strain by 40% and 25% during head acceleration and deceleration, respectively. Extensive experiments were conducted by them to study the effect of the lateral ventricles and irregular skull on the brain protection [132]. In this study, the head model was modified based on the previous model by designing a more detailed structure for the inferior part of a human skull. More studies have been reported by using the silicone gel to mimic human brain in a 2D physical head to investigate the fluid–structure interaction by transient loading [133], nonlinear material behaviour effect on brain response during eccentric rotation [134], and the influence of blood vessel on brain response during impact [135].

As a kind of hydrogel, ballistic gelatine was also widely used to mimic brain in TBI studies. The use of gelatine can date back to 1943 by Holbourn [29], who employed the gelatine to study the diffuse brain injury by angular accelerations. Matthew et al. [136] used polymethyl methacrylate (PMMA) shell as the skull and gelatine as the brain to simulate the brain deformation during the blast loading based on high speed imaging technology. The results showed that the maximum strain was produced at the interfaces and the primary blast effects dominated the TBI. Liu et al. [137] used gelatine mimicking brain to investigate the pressure and cavitation change during the impact. Air bubbles
were created in the brain model at different locations for calculating the pressure based on the volume change. Laksari et al. [138] studied the impact TBI based on the strain distribution and posterior gap between gelatine brain and skull. Shear wave propagation on brain tissue was investigated by Bayly et al. [4], who developed a magnetic resonance measurement technique based on MRI to capture the dynamic response of gel (gelatine) mimicking brain under transient angular acceleration. Thali et al. [139] studied ballistic injury by gunshot based on a skin–skull–brain model, where the scalp, skull and brain were mimicked by silicon cap, sandwich polyurethane sphere, gelatine with 10% concentration, respectively.

In summary, both silicone gel and hydrogel can be good carriers for promoting the fundamental mechanistic study to the brain injury. The stress, pressure, and deformation of the brain can be measured for the head under impact, ballistic and blast loadings. In addition, these gels can be applied to many other mechanical studies on the head injury, such as compressive and shear wave propagation under dynamic loadings.

2.3.2 Mechanical properties of brain mimicking gels

Before applying these gels to the brain tissue simulation, the understanding of mechanical behaviour of the gel and exploration of their feasibility to simulate the real brain are essential. Some studies have been reported to investigate the mechanical properties of these soft gels.

Pervin [94, 140] studied the dynamic behaviour of four candidate brain mimicking gels, agarose gel, Perma™ gel, ballistic gel, and collagen gel by compression tests at the strain...
rate varied from 0.01 to 3000 s\(^{-1}\) [8]. It was found that all gels showed significant nonlinear and strain rate-dependent behaviour. The stiffness of the gels increased with the increase of the gel concentration as well as strain rate. Among the four gels, agarose gel with the concentration of 0.4-0.6% showed the most similar property to the real brain tissue of bovine. In addition, to find suitable gels for brain mimicking, Brands et al. [6, 105] performed OSTs to study the viscous behaviour of a gelatine (4% and 20%) and a silicone gel (DowCorning Sylgard 527) at different strain and frequency. The two gels showed high-frequency sensitive shear behaviour, which was similar to the compressive behaviour of the gels in Pervin’s study [94]. Comparing them with the real brain, the two gels showed linear behaviour with the shear strain increased from 0.1% to 10%, whereas the brain tissue exhibited nonlinear shear behaviour at the strain larger than 1%. The gelatine selected was not suitable to mimic the brain tissue since it was much stiffer than the brain tissue and did not show viscous behaviour. Silicone gel showed similar modulus to that of brain tissue only at the shear frequency within 10 Hz. In addition, the friction force between thin tube and calf brain tissue and gelatine was measured under both static and dynamic conditions to simulate the process of pulling catheters magnetically through vivo brain tissue [141]. It was found that the frictional force in gelatine was near two times of that in brain tissue.

In addition, more studies have been reported on brain mimicking gels, but without comparison with real brain tissue. Kwon and Subhash [142] conducted the compression studies on gelatine with the strain rates from 0.0013 to 3200 s\(^{-1}\). The compressive strength of the gel increased dramatically with the increase of the strain rate due to the shear-thickening behaviour. The shear-thickening behaviour was further studied by the
shear tests at high strain rate using a double lab-shear test fixture adapted from a polymer SHPB [143]. It was found that the dynamic viscosity of the gel increased by more than five times with the strain rate varied from 2000 to 7000 s$^{-1}$. Namani and Bayly [144] studied effect viscous and anisotropic properties of white matter on shear wave propagation by using magnetically-aligned fibrin gel. The visco-elastic property of the gel played a very important role in preventing the wave propagating into the deep locations by dissipation effect. In addition, fibres direction showed a significant influence on the shear wave, where the wavelength along the fibre was longer than that perpendicular to fibre. The mechanical behaviour of silicone gel (Sylgard 527) was studied by Fontenier et al. [145] by indentation and relaxation tests. No significant difference was found in the indentation tests at the velocities of 1, 100 and 1000 mm/min. The force was qualitatively the same during the relaxation tests at the initial loading velocity of 100 mm/min and the viscous property can be neglected at the strain rate of that level.

In this section, both compressive and shear behaviour of candidate brain mimicking gels have been reviewed. It was found all of these gels showed significant nonlinear and rate-dependent properties under dynamic loadings. Gels can be used to mimic brain tissue in some ranges of strain and loading rate. Magnetically-aligned fibrin gel can be applied to study the direction influence on the shear wave propagation.
2.4 Concluding remarks

The literature review provides an overview of brain injury studies by the experimental, FEM, and analytical methods. Experimental studies on the human cadaver and animal heads have given a better understanding of dynamic responses of the head (acceleration) and internal brain (deformation and pressure) under impact or blast loadings. In addition, they can provide validation to the FE head model and analytical studies. Various physical head models were also widely used to study TBI. Compared with cadaver heads and animals, they have the advantages of the modest cost, availability, stable mechanical properties. One of the important applications is to study the protective ability of helmets in sports such as America football and soccer. The TBI have been studied for a ball-to-head impact using head models, but a handful of studies have been reported on the head-to-head impact, while it has a very high possibility to cause injury. FEM modelling is an efficient method to study the TBI due to its convenience and low cost. With the help of MRI, detailed head models have been developed with accurate skull and brain structures. These models have been used to simulate TBI under various sceneries including these that cannot be done in experiments. Current study employ validated anatomical FE head model to simulate a scenario of front-rear head impact in football games and study the protective ability of headgear.

The studies on mechanical properties of the brain tissues have also been reviewed including, the compression, tension, shear, wave based tests. The brain tissue showed nonlinear, viscous, and high strain rate sensitive properties. However, much variation was found among these results due to the individual difference of the head samples, storage conditions, and test conditions. Given the problems associated with preparing the tissue
samples, conducting experiments, and large variability in the reported results, soft gels such as silicone gel and hydrogel were widely used in studying the TBI. Some attempts have been made on SHPB to obtain an equilibrium stress and uniform deformation in impact tests of brain tissue and gels. However, the experimental process became more complicated and difficult, and the accurate of the results need to be validated further.

Up to now, most of those studies have focused on obtaining constitutive properties by reducing the non-equilibrium and non-uniform effects. However, only few studies have systematically examined these effects. To better understand the dynamic behaviour of the brain tissue and soft gels, the current study presents both shear and impact tests on brain mimicking gels. Shear tests were conducted to study the basic properties of the gels based on temperature and shear rate effects, and impact tests were carried out to investigate the non-equilibrium and non-uniform effects during impact.

As reviewed in Section 2.2.3, stress wave in soft materials is always a significant factor that affects their response when the materials are undertaken an impact or blast loading. Some literature has presented the studies about the shear and longitudinal wave propagation in brain tissues and soft gels. However, there are only a handful of studies reporting the spherical wave propagation in these materials, although it is useful for injury detection by ultrasound and for the understanding of deeper cerebral tissues [9-11]. The spherical wave propagation in soft materials is presented based on analytical and numerical methods in this thesis.

In the studies of protecting player’s head, researchers have attempted to explore the mechanical responses of the head, to evaluate the head injury risks and to develop
protective structures during the impact. Previous studies focused on the mechanical behaviour of the brain for the ball-to-head impact. However, in reality the head-to-head impact has a higher possibility to cause injury. Current study attempts to present a fundamental mechanistic understanding of the head-to-head impact by simulating the scenario of front-rear head impact in football games, and to evaluate the protective ability of the headgear made of different foams.
Chapter 3 Dynamic Shear Behaviour and Constitutive Modelling of Brain Mimicking Gels

3.1 Introduction

Brain tissue and mimicking gels are high strain rate sensitive materials. It is necessary to study the strain rate effect on the compressive behaviour and develop the corresponding strain rate-dependent constitutive models. In this chapter, an analytical method is developed to derive the compressive stress-strain relationship of soft gels under different strain rates based on the OSTs. Compression tests and OSTs were conducted to study the nonlinear and viscous behaviour of two brain mimicking gels, hydrogel and silicone gel. The compression results were compared with the experimental results of real brain tissues from literature. TTS principle is used to obtain the master curves of storage and loss moduli in a wide range of frequency from the OSTs. Using the derived analytical method, the nonlinear compressive stress-strain curves of the two gels as a function of strain rate are obtained based on the master curves. The analytical method is validated by comparing modelling results with the compression tests.

3.2 Analytical methods

An analytical method is developed to get the compressive behaviour of soft gels under different strain rates from the OST. This method contains four main parts: (1) conducting the OST to obtain the elastic and viscous properties at different temperatures using the rheometer; (2) using the TTS principle to get the master curves of storage and loss
moduli over a wide range by shifting the modulus curves obtained in part one; (3) fitting modulus curves to get the viscous parameters in Prony series from the Fourier transformations; (4) substituting the parameters obtained in part three to the nonlinear visco-elastic constitutive models and calculating the stress-strain curves at different strain rates.

### 3.2.1 Maxwell-Wiechert model

Fig. 3.1(a) shows a simple visco-elastic model, i.e., Maxwell model, where $E$ is the elastic modulus and $\eta$ is the viscosity. The stress on the spring and damper in series shows the same value as,

$$\sigma = \sigma_s = \sigma_d , \quad (3.1)$$

where $\sigma$ is the total stress in the Maxwell model, and $\sigma_s, \sigma_d$ are the elastic and viscous stress, respectively. The total deformation of a Maxwell model includes the displacement of the spring (linear elastic part) and damper (viscous part),

$$\varepsilon = \varepsilon_s + \varepsilon_d , \quad (3.2)$$

where $\varepsilon$ presents the total strain, and $\varepsilon_s, \varepsilon_d$ are the elastic and viscous strains, respectively.

The Eq. (3.2) is differentiated with respect to time,

$$\dot{\varepsilon} = \dot{\varepsilon}_s + \dot{\varepsilon}_d = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} . \quad (3.3)$$
Integration of Eq. (3.3) will give us,

\[ \sigma = C e^{\frac{E_t}{\eta}} \],

(3.4)

where the \( C \) is a constant. Based on the initial relaxation condition at time \( t = 0 \), an elongation is applied \( \varepsilon = \varepsilon_0 \), the relationship between stress and strain is obtained as,

\[ \sigma_0 = \varepsilon_0 E \],

where \( \sigma_0, \varepsilon_0 \) are the initial stress and strain, respectively. The Eq. (3.4) has a solution as

\[ \sigma = E \varepsilon_0 e^{\frac{E_t}{\eta}} \].

(3.5)

However, for most conditions, viscous materials cannot be represented by a single relaxation time. The relaxation property is dominated by the varying molecular segments, i.e., the simple and short segments have a fast relaxation process [146]. Therefore, Maxwell-Wiechert model was developed to approximate the relaxation time distribution, which contains a single spring element and multiple Maxwell elements in parallel (Fig. 3.1(b)). The total stress consists of the stress in the isolated spring and that in every Maxwell arm as,

\[ \sigma(t) = \sigma_{spring} + \sigma_1(t) + \sigma_2(t) + \cdots + \sigma_i(t) + \cdots + \sigma_n(t) \],

(3.6)

where \( \sigma(t) \) is the total stress, and \( \sigma_{spring}, \sigma_i \) are the stress by the isolated spring and the Maxwell elements, respectively. The relationship between the stress and strain in every Maxwell element can be expressed as,

\[ \dot{\varepsilon} = \frac{\dot{\sigma}_1}{E_1 \eta_1} + \frac{\sigma_1}{E_2 \eta_2} + \cdots + \frac{\dot{\sigma}_i}{E_i \eta_i} + \cdots + \frac{\dot{\sigma}_n}{E_n \eta_n} = \frac{\sigma_n}{E_n \eta_n} \].

(3.7)
Referring to the Eq. (3.5), the integration of Eq. (3.7) gives the solution as,

\[ \sigma_i(t) = \varepsilon_0 E_i e^{-\varepsilon_i / \eta_i} = \varepsilon_0 E_i e^{-t / \tau_i} , \]  

(3.8)

where \( \tau_i \) ( = \( \eta_i / E_i \)) are the series of relaxation times. Therefore, the total stress can be expressed as a time domain Prony series,

\[ \sigma(t) = \varepsilon_0 \left( E_\infty + \sum_{i=1}^{n} E_i e^{-t / \tau_i} \right) . \]  

(3.9)

A relaxation elastic modulus can be expressed as,

\[ E(t) = E_\infty + \sum_{i=1}^{n} E_i e^{-t / \tau_i} . \]  

(3.10)

Based on Eq. (3.10), the relaxation shear modulus \( G(t) \) and bulk \( K(t) \) are represented as,

\[ G(t) = G_\infty + G_0 \sum_{i=1}^{n} g_i e^{-t / \tau_i^G} = G_0 - G_0 \sum_{i=1}^{n} g_i \left( 1 - e^{-t / \tau_i^G} \right) , \]  

(3.11)

where \( G(t = 0) = G_0 = G_\infty + G_0 \sum_{i=1}^{n_0} g_i . \)

\[ K(t) = K_0 - K_0 \sum_{i=1}^{n} k_i \left( 1 - e^{-t / \tau_i^K} \right) = K_\infty + K_0 \sum_{i=1}^{n} k_i e^{-t / \tau_i^K} , \]  

(3.12)

where \( G_0 , G_\infty \) are the instantaneous and long-time shear modulus, respectively, \( \tau_i^G , \tau_i^K \) are series of relaxation times, \( g_i , k_i \) are Prony series parameters. Usually, for nearly incompressible material, Prony series number of bulk modulus, \( n_\kappa \) can be assumed to be 0, which means bulk modulus is assumed as a constant [147].
3.2.2 Fourier transformation for visco-elastic model

Shear modulus for viscous materials can be expressed as complex dynamic modulus containing the storage modulus $G_s$ and the loss modulus $G_l$,

$$G^* = G_s + i_{im} G_l ,$$  \hspace{1cm} (3.13)

where $i_{im}$ is an imaginary number, $i_{im}^2 = -1$.

In order to obtain the parameters in the shear modulus equation, OST is often used, but the expression needs to be transformed as a function of frequency using Fourier transformations. For the Maxwell element, after the Fourier transformation, the viscous parts of Eq. (3.11) can be expressed as [146],

![Fig. 3.1 (a) Maxwell model; (b) Maxwell-Wiechert model](image)
\[ G^*(w)_{\text{Maxwell}} = G_0 \sum_{i=1}^{n} \frac{i_{im}w}{1 + \tau_iw} = G_0 \sum_{i=1}^{n} \frac{i_{im}w\tau_i}{1 + i_{im}w\tau_i}, \quad (3.14) \]

where \( w \) is the angular frequency, \( \tau_i = \tau_i^G \) are a series of relaxation times, and \( n = n_G \) is the number of terms in the Prony series.

The Eq. (3.14) can be derived to a real part and an imaginary part as Eq. (3.13) by multiplying and dividing by the complex conjugate of the denominator,

\[ G^*(w)_{\text{Maxwell}} = G_0 \left( \sum_{i=1}^{n} \frac{g_i \tau_i^2 w^2}{1 + \tau_i^2 w^2} + i_{im} \sum_{i=1}^{n} \frac{g_i w \tau_i}{1 + \tau_i^2 w^2} \right). \quad (3.15) \]

For the elastic part, it can be expressed as,

\[ G^*(w)_{\text{spring}} = G_e = G_0 \left( 1 - \sum_{i=1}^{n} g_i \right). \quad (3.16) \]

The total complex dynamic modulus is described as,

\[ G^*(w) = G^*(w)_{\text{spring}} + G^*(w)_{\text{Maxwell}}. \quad (3.17) \]

### 3.2.3 Oscillatory shear test theory

The OST can be applied to characterize the dynamic properties of gels. In the tests, the imposed harmonically angle and the torque measured by sensor were used to estimate the peak oscillatory shear strain (Eq. (3.18)) and peak shear stress (Eq. (3.19)), respectively [4],

\[ \varepsilon = \text{Re} \left[ \varepsilon(w)e^{imw} \right], \quad (3.18) \]
\[
\sigma = \text{Re} \left[ \sigma(w) e^{i \omega t + \varphi(w)} \right],
\] (3.19)

where \( \varphi(w) \) is the phase angle. The complex shear modulus \( G_{\text{exp}}^* \) can be estimated at each frequency as follows,

\[
G_{\text{exp}}^*(w) = \frac{\sigma(w)}{2\varepsilon(w)} \left( \cos \varphi(w) + i \varepsilon w \sin \varphi(w) \right).
\] (3.20)

Using Matlab to fit the modelling modulus \( G^* \) with the experimental complex shear modulus \( G_{\text{exp}}^* \), we can obtain the parameters of \( \tau_i \), \( g_i \) and \( G_0 \).

### 3.2.4 Time–temperature superposition principle

For the model fitting of the complex shear modulus, the range of the relaxation time, \( \tau_i \), in Prony series is determined by the oscillatory frequency range. However, the frequency is usually limited in the shear test by the conditions of commercial rheometers, which cannot cover the range of the deformation rate under some conditions (e.g. impact, explosion, and wave propagation). To solve this problem, an extrapolation technique, TTS principle, is proposed. TTS is widely applied to the temperature-dependent materials, such as polymers and gels, which show similar trends of viscous properties with the change of temperature [148-151]. This method can be described that, lowering the temperature has the same effect of increasing the relaxation time, which slows down the process equivalently. For thermos-rheological simple materials, the relation between the time and temperature can be expressed in the same way as [148],

52
\[
\tau_i(T) = a_T \tau_i(T_0). 
\]  

(3.21)

where \(T_0\) is the reference temperature, \(\tau_i(T)\) is the relaxation time at the temperature \(T\), \(a_T\) is the horizontal temperature-dependent shift factor. By substituting Eq. (3.21) into Eq. (3.17), the complex shear modulus at temperature \(T\) can be obtained at the reference temperature \(T_0\),

\[
G^*(w) = G_0 (1 - \sum_{i=1}^{n} g_i) + G_0 \left[ \sum_{i=1}^{n} \frac{g_i w^2 a_T^2 \tau_i^2 (T_0)}{1 + w^2 a_T^2 \tau_i^2 (T_0)} + i \omega \sum_{i=1}^{n} \frac{g_i w a_T \tau_i (T_0)}{1 + w^2 a_T^2 \tau_i^2 (T_0)} \right]. 
\]  

(3.22)

Consequently, a single master curve can be obtained by shifting the oscillatory frequency data at different temperatures along the logarithmic frequency axis using the corresponding shift factors. This process has been illustrated in Fig. 3.2, where a reference curve is chosen at the temperature of \(T_0\), and then the curves measured at other different temperatures are shifted horizontally to obtain a smooth curve.

Fig. 3.2 Shifting of the modulus using time-temperature superposition [148].
3.2.5 Nonlinear visco-elastic constitutive model

To represent the nonlinear property, a hyper-elastic constitutive law is employed [77, 152, 153]. Coupling the viscous properties, the constitutive law can be given as a strain energy potential form,

\[ W = \sum_{i+j=1} C_{ij}(t) (I_1 - 3) (I_2 - 3) \]

where \( W \) is the potential strain energy, \( I_1 \) and \( I_2 \) are strain invariants,

\[
\begin{align*}
I_1 &= \text{tr}(A) \\
I_2 &= \frac{I_1 - \text{tr}(A^2)}{2I_3} \\
I_3 &= \sqrt{\det A} = 1
\end{align*}
\]

(3.24)

where \( A \) is a Cauchy-Green strain tensor. For the experiment conducted in one direction (tension or compression), considering the nearly incompressible property of the gels, this strain tensor can be written as,

\[
A = \begin{bmatrix}
\lambda^2 & 0 & 0 \\
0 & \lambda^{-1} & 0 \\
0 & 0 & \lambda^{-1}
\end{bmatrix}
\]

(3.25)

where \( \lambda \) is the stretch under uniaxial stress. Thus, the strain invariants are obtained as \( I_1 = \lambda^2 + 2\lambda^{-1} \), and \( I_2 = 2\lambda + \lambda^{-2} \). To simplify the problem, the constitutive equation only account the time effect in \( C_{ij}(t) \). Referring to Miller’s [77] study, we use the Mooney-Rivlin model to represent the nonlinear behaviour, so \( n \) is taken as 1. \( C_{10}(t) \) can
be assumed equal to \( C_{01}(t) \). Using \( C_1(t) \) to represent \( C_{10}(t) \) and \( C_{01}(t) \), Eq. (3.23) can be rewritten as,

\[
W = C_1(t) \left( \lambda^2 + 2\lambda + 2\lambda^{-1} + \lambda^{-2} - 6 \right)
= \left[ C_1 - C_1 \sum_{i=1}^{n} g_i \left( 1 - e^{-\lambda_1/t} \right) \right] \left( \lambda^2 + 2\lambda + 2\lambda^{-1} + \lambda^{-2} - 6 \right), \tag{3.26}
\]

where \( C_1 \) is an initial constant and equal to the \( 1/4 \) value of initial shear modulus \( G_0 \) based on the relation of \( G_0 = 2(C_{01} + C_{10})_{t=0} \) [77]. According to the equation \( \sigma = \frac{\partial W}{\partial \lambda} \) [154], the Lagrange stress can be derived as,

\[
\sigma = \frac{1}{4} \left[ G_0 - G_0 \sum_{i=1}^{n} g_i \left( 1 - e^{-\lambda_1/t} \right) \right] \left( 2\lambda + 2 - 2\lambda^{-2} - 2\lambda^{-3} \right), \tag{3.27}
\]

where \( \lambda = 1 + \epsilon \), and \( \epsilon \) is the nominal strain (engineering strain).

### 3.3 Experimental study

In this section, compression tests were carried out on two brain mimicking gels. The compression results of the hydrogel and silicone were compared with experimental data of real brain tissues from literature [77, 94]. In addition, the elastic and viscous properties of these two gels were studied by OST using the rheometer.
3.3.1 Materials and sample preparation

3.3.1.1 Materials

The basic selection criterion is that the material should have the density and mechanical properties similar to the brain tissue. Therefore, we chose two biological gels, i.e., hydrogel (gelatine, Knox, Camden, NJ) and silicone gel (Sylgard 527, Dow Corning, Midland, MI, USA), as candidates with the density of \( \sim 1000 \text{ kg/m}^3 \) and the stiffness of 1-5 kPa [4].

The hydrogel as a protein-derived polymer has commonly been used to mimic brain tissue by peer researchers [4, 6, 155]. The mechanical properties can be modified by adjusting the water content. Its structures (i.e., cross-link chains of polymer) provide similar mechanical behaviour (nonlinear visco-elastic) to the brain tissue. The silicone gel consisted of two components, including the gel base as part A and the catalyst as part B, is another candidate brain mimicking material in some studies [134, 156].

3.3.1.2 Sample preparation-compression test

The hydrogel was prepared by dissolving gelatine powder in deionized water at 80 °C. In order to obtain an appropriate brain mimicking hydrogel, we tried several different concentrations and tested these gel properties (compression tests) during the preliminary experiments. Finally, hydrogels with three concentrations of 10%, 8% and 6%, were proven to have the similar properties as the real brain, and therefore, were used to
compare with the results of real brain tissue from literature. The concentration 10% represents that the hydrogel is made up of 10% gelatine powder and 90% water by weight.

The mixed solution was poured uniformly into the moulds with a depth of 13 mm and a diameter of 30 mm, same as the size of brain tissue samples reported in literature [77]. After cooling down to the room temperature (~ 23 °C in the lab), the samples were cured in the moulds in a refrigerator with a constant degree of 7 °C for 30 minutes. Then the solid samples can be taken out easily from the moulds with a good shape (Fig. 3.3(a-c)).

The silicone gel was prepared by mixing A and B with a ratio of 1:1 by weight. As the hydrogel, the mixed liquid was poured into the moulds with the same size. Before pouring the liquid mixture, a thin layer of wax release agent was painted on the inner surface of the moulds. Because silicone gel was easy to stick on the moulds, and using the release agent made the samples easier to be taken out after cured. The gel samples were cured in the moulds in an oven with a constant degree of 65 °C for 4 hours [6]. Before any compression tests, all of the samples were cooled down to the room temperature (Fig. 3.3(d)). The details of the gels are listed in the Table 3.1.

![Fig. 3.3 Cylinder samples of the gels: (a) 10% hydrogels (water content: 90%); (b) 8% hydrogels (water content: 92%); (c) 6% hydrogels (water content: 94%); (d) silicone gel.](image-url)
3.3.1.3 Sample preparation - OST

Same as the compression test, the hydrogel was prepared by dissolving gelatine powder in deionized water at 80 °C. The gelatine mixture was poured directly into the space between the plates of the rheometer with a diameter of 40 mm [4]. The thickness of the sample was set to 1 mm by adjusting the distance between two shear plates. The hydrogel was cured at a temperature of 7 °C for 30 minutes. Before the tests, the sample was increased to the room temperature. The distance between two plates was re-adjusted slightly to account for the possible shrinkage during the sample curing. For the silicone gel (with a mass ratio of 1:1) preparation, the only difference from the hydrogel was the cure conditions (65 °C for 4 hours) [6].

<table>
<thead>
<tr>
<th>Gel samples</th>
<th>Hydrogel 10%</th>
<th>Hydrogel 8%</th>
<th>Hydrogel 6%</th>
<th>Silicone gel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Components</td>
<td>10%G+90%W</td>
<td>8%G+92%W</td>
<td>6%G+94%W</td>
<td>50%B+50%C</td>
</tr>
<tr>
<td>Cure condition</td>
<td>7°C -30 mins</td>
<td></td>
<td></td>
<td>65°C-4 hours</td>
</tr>
<tr>
<td>Properties</td>
<td>Sensitive to temperature, adjustable stiffness, viscous, accounting the water effect as brain tissue</td>
<td>Temperature stability, viscous, stick, hydrophobic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dimension</td>
<td>Diameter 30±1 mm, Thickness 13±0.5 mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: G, W, B, C stand for gelatin, water, gel base, and catalyst, respectively.

3.3.2 Compression test

Cylindrical gel samples were tested with a compression speed of 500 mm/min same as the brain tissue tested by Miller and Chinzer [77]. To reduce the friction between samples and plates, lubricating oil was painted onto the surfaces of the two compression plates.
However, inertial force of the upper plate was found to have a significant influence on the testing results (solid curve in Fig. 3.4). This is because the loading speed of 500 mm/min was very high compared with quasi-static compression. Before reaching that speed, there was a short acceleration process for the upper plate (connecting with the load cell) which caused a big inertial force and vibration. To eliminate the inertial effect on the final stress curves, we adjusted the starting point of the upper plate with a higher position than the upper surface of the samples and the strain was accounted as 0 when the upper plate touched the sample surface (dot curve in Fig. 3.4). Three repeated tests were conducted for each gel under the same compression condition.

![Figure 3.4](image.png)

**Fig. 3.4** Typical effect of the inertial force on the stress-strain curve due to the upper plate acceleration.
3.3.3 Oscillatory shear test

OSTs were conducted to investigate the effect of the shear frequency and temperature on the elastic and viscous properties of gels using a Discovery Hybrid-2 rheometer (TA Instruments, Delaware, USA). As shown Fig. 3.5, the rheometer consists of an upper plate which is free to rotate and move vertically, and a lower Peltier plate (for the temperature control with an accuracy of ±0.1 °C). Storage and loss moduli were measured over the frequency range of 0.16 - 16 Hz (1-100 rad/s) at a constant strain of 1%. The measurements were conducted on a logarithmic basis. To ensure the sample stabilization, we waited 10 mins after the initial temperature was set and 3 mins upon each change of condition (i.e., temperature) before each test. For the hydrogel, the repeated OSTs were carried out at the temperatures varied from 24 °C to 10 °C with an interval of 1 °C. These temperatures meet the requirements of TTS analysis for the investigation of the dynamic behaviour of gels. In our preliminary study, the temperature effect (24~10 °C) on the silicone gel was found not as large as that on the hydrogel, so a wider temperature range is necessary. Therefore a wider range of temperature (25 ~ -10 °C) with an interval of 5 °C was used in the OSTs. Three repeated tests were conducted for each gel under the same shear condition.
3.4 Results and discussion

3.4.1 Compression test

Fig. 3.6(a) shows the results of compression test on the hydrogel with the concentrates of 10%, 8%, 6%, and silicone gel, at the strain rate of 0.64 s\(^{-1}\). The curves were obtained by calculating the average values of three repeated tests. The compression testing data on the real brain tissue from literature [77, 94] are also presented for comparison. It is observed that the nominal stress of both gels and brain tissue showed a nonlinear increasing trend with the increase of nominal strain. The 6% hydrogel represented a similar behaviour to
the silicone gel and the brain tissue by Miller and Chinzer (1997) for the strain within 15%. For the strain larger than 15%, severe deviations can be seen between gels and brain tissue. Pervin and Chen [94] found the significant difference between the grey matter and the white matter of brain tissue by studying them individually. Although the compression strain rate applied by Pervin and Chen was different from that of the gels tested, we can still get a qualitative insight that white matter showed a more rapid increase than the gels. To have a better view, compressive curves with the normal strain between 0 and 0.1 are zoomed in Fig. 3.6(b). Although the curves were not as smooth as those in Fig. 3.6 (a) due to the vibration in measuring the small force, the trends were similar. In addition, Table 3.2 has been given to show the detailed information of Fig. 3.6.
Fig. 3.6 Nominal stress-strain curves of hydrogels and silicone gel under compression test and the results of real brain tissue from literature: (a) maximum nominal strain of 0.4, (b) zooming of the Fig (a) [77, 94].

Table 3.2 The detailed stress-strain values of Fig. 3.6 [77, 94].

<table>
<thead>
<tr>
<th>Items</th>
<th>Stress (MPa)</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Hydrogel-10%-0.64</td>
<td>0.0</td>
<td>225.2</td>
</tr>
<tr>
<td>Hydrogel-8%-0.64</td>
<td>0.0</td>
<td>185.1</td>
</tr>
<tr>
<td>Hydrogel-6%-0.64</td>
<td>0.0</td>
<td>115.4</td>
</tr>
<tr>
<td>Silicone gel-0.64</td>
<td>0.0</td>
<td>101.3</td>
</tr>
<tr>
<td>Brain tissue-0.64 (Miller and Chinzei, 1997)</td>
<td>0.0</td>
<td>75.2</td>
</tr>
<tr>
<td>Grey matter-0.1 (Pervin and Chen 2009)</td>
<td>0.0</td>
<td>20.2</td>
</tr>
<tr>
<td>White matter-0.1 (Pervin and Chen 2009)</td>
<td>0.0</td>
<td>455.3</td>
</tr>
</tbody>
</table>
3.4.2 Oscillatory shear test

Fig. 3.7 shows one of the three typical results of the OSTs on two gels with the frequency from 0.16 to 16 Hz at the corresponding temperature ranges of 24 ~ 10 °C for hydrogel and 25 ~ -10 °C for silicone gel. Note that storage and loss moduli of the gels increased with the decrease of temperature. This may be explained that the stress introduced by deformation is caused by the change of elastically chains inside the gel. The decreased temperature increased the chain number and slowed down the chain movement, resulting in more resistance for macromolecular rearrangement [157, 158]. Another common feature was that the modulus increased with the sweep frequency, which illustrated that the deformation rate had an effect on both elastic and viscous properties of gels. Similar phenomenon of temperature and frequency effects on gellan, polyvinyl alcohol gelatine was observed [150, 157].

Comparing the plots in Fig. 3.7(a-c), it is found both the storage and loss moduli increased with the increase of gelatine concentration. This made it possible to create a hydrogel with closer elastic and viscous properties to brain tissue by adjusting the gelatine concentration. In Fig. 3.7(d), both moduli of silicone gel versus the frequency showed larger slopes than these of hydrogel, but slower increase with the decreasing temperature. These results demonstrate that silicone gel is more sensitive to the deformation rate and less sensitive to the temperature compared with hydrogel. This is possibly because the crosslink chains of the gels played an important role on the stiffness and viscosity, while the main content of the hydrogel was water (≥90%), which was not significantly affected by the frequency. However, as the temperature changed, the number of crosslink chains in the hydrogel changed more significantly than that in the
silicone gel. Different from the hydrogel, the loss modulus of silicone gel showed much lower value than storage modulus at low frequency, but increased faster with the increasing frequency.

Fig. 3.7 The storage modulus and loss modulus of the hydrogels and silicone gel by the oscillatory shear test at different temperatures.

3.4.3 Time–temperature superposition

The TTS analysis is carried out for the gels. It is assumed that the shift factors representing the horizontal shifting distances along the frequency axis, should be the same for the storage and loss moduli at the same temperature. As shown in Fig. 3.8, the master curves of two gels are obtained at the reference temperature of 23 °C according to
the corresponding experimental data in Fig. 3.7. The scales of the vertical axis are set the same for the convenience of comparison. However, for the horizontal axis, the master curves of hydrogel show frequency range from $6.3 \times 10^{-2}$ to $2.5 \times 10^9$ Hz, which is much wider than that of silicone gel of ~0.1 to 21.1 Hz since the temperature effect on the hydrogel is much more significant than on the silicone gel. As can be seen, the hydrogel with different concentrations shows similar increasing trends, where the storage modulus curves are almost parallel to these of loss modulus. Different from the hydrogel, the two master curves of silicone gel show different increase slopes and intersected at about 20 Hz. This indicates that for the hydrogel, storage modulus always plays a main role in the complex modulus, whereas for silicone gel, the storage modulus dominates the complex modulus only at low frequency. Modulus values of silicone gel show much faster increase trends than that of the hydrogel, especially for the loss modulus. For the hydrogel, both the storage and loss moduli at $2.5 \times 10^9$ Hz are nearly ten times higher than that at $6.3 \times 10^{-2}$ Hz with all three concentrations. For the silicone gel, the storage and loss moduli increase from 686 Pa to 1600 Pa and from 66 Pa to 1597 Pa, which increased by 2.3 and 24 times, respectively, with the frequency varies from ~0.1 to 21.1 Hz.

Based on the master curves, the dynamic shear properties of the two gels were compared with those of real brains and silicone gel from literature [6, 135, 159]. For the storage modulus (Fig. 3.9(a)), with the increase of shear frequency, both hydrogels and brain tissues showed linear increase in the logarithmic coordinates, whereas silicone gels presented nonlinear trends. The silicone gel in this study showed higher storage modulus than these from literature, and a similar slope to the results by Parnaik et al. [135]. The
difference existed among the three tested results with the same kind of material and cure conditions [6]. This may be due to the difference of materials from different batches. Difference can also be observed for the repeated tests of real brain tissues. Considering the mimicking feasibility of gels, both 6% hydrogel and silicone gel showed similar storage modulus and trends to brain tissues with the frequency lower than 3 Hz. For the higher frequency, only 6% hydrogel presented the close elastic property.

For the loss modulus (Fig. 3.9(b)), silicone gel showed a larger increasing slope than brain tissues and hydrogels. The 10% hydrogel presented closer values to the real brain with the frequency lower than 20 Hz. For the higher frequency, significant deviations were observed between the gels and brain tissues. In general, both gels can be used to mimic the brain in mechanical studies in some appropriate conditions. Hydrogel is more suitable to mimic the brain tissue as its stiffness is adjustable via changing water content and its properties are closer to the brain tissue.
Fig. 3.8 Master curves of two gels (reference temperature, 23 °C) obtained from the corresponding results of oscillatory shear test in Fig. 3.7.
Fig. 3.9 Comparison of storage and loss moduli between gels and real brains under different shear frequencies [6, 135, 159].
3.4.4 Complex shear modulus fitting

As shown in Table 3.3, the parameters of $\tau_i$, $g_i$ and $G_0$ in viscous constitutive Eq. (3.15) are obtained by fitting the modulus $G^*$ with the experimental complex shear modulus $G^*_{\text{exp}}$. A nonlinear least-squares fit is performed to determine the Prony series parameter by minimizing the error function, $\chi$,

$$\chi^2 = \sum_{i=1}^{m_k} \frac{1}{G^*_0} \left[ G^* - G^*_{\text{exp}} \right]^2,$$

(3.28)

where $m_k$ is the total number of the fitting points. To improve the fitting efficiency and accuracy, the range of each relaxation time, $\tau_i$, in Eq. (3.15) is fixed in one decade. The number of the Prony series terms, $n$, is determined by the data points and frequency range, which is typically less than the half number of shear test points and close to the total number of frequency logarithmic decades. Total logarithmic decades are defined as $\log_{10}(w_{\text{max}}/w_{\text{min}})$, where $w_{\text{max}}$, $w_{\text{min}}$ are the maximum and minimum frequency obtained, respectively. According to the frequency range in Fig. 3.8, the numbers of terms for hydrogel and silicone gel are calculated as 11 and 3, respectively. The fitting results show that the initial shear modulus $G_0$ are 11.4, 8.92, 5.77 kPa for the 10%, 8%, and 6% hydrogels, respectively, and 6.01 kPa for silicone gel. It is affected by the complex modulus value and the highest frequency. The sum of $g_i$ is usually close to 1, but less than 1 since the long-time shear modulus is determined by $G_\infty = G_0(1 - \sum_{i=1}^{n} g_i)$, which should be positive. The $\tau_i$ representing relaxation time is inversely proportional to the corresponding frequency.
3.4.5 Nonlinear visco-elastic constitutive modelling

The nonlinear visco-elastic models for the gels are obtained by substituting the fitting parameters (Table 3.3) from the complex shear modulus to the constitutive Eq. (3.27). Nominal stress-strain relationship of the hydrogel at the strain rate of 0.64, 10, 100 and 1000 s\(^{-1}\) are calculated according to the constitutive model and plotted in Fig. 3.10. For the silicone gel, we cannot obtain a reasonable curve when the strain rate reaches 100 s\(^{-1}\) as shown in Fig. 3.10(d). This is because the strain rate range is limited by relaxation time in Prony series, which is determined by the frequency range in Fig. 3.8. The minimum relaxation time is 4.74×10\(^{-2}\) s for silicone gel, which is not short enough for high deformation rate. Strain rate has a significant effect on the compressive behaviour of both gels, and the effect on silicone gel is higher than that on the hydrogel due to the higher frequency effect on silicone gel in Fig. 3.8. The modelling results show good agreement with the compression test data at the strain rate of 0.64 s\(^{-1}\).
Table 3.3 Parameters of the visco-elastic model obtained by fitting the master curves of complex modulus

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hydrogel</th>
<th>Silicone gel</th>
<th>Parameter</th>
<th>Hydrogel</th>
<th>Silicone gel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0$ (kPa)</td>
<td>11.4</td>
<td>8.92</td>
<td>5.77</td>
<td>6.01</td>
<td>$\tau_1$</td>
</tr>
<tr>
<td>$g_1$</td>
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<td>0.009</td>
<td>0.020</td>
<td>0.010</td>
<td>$\tau_2$</td>
</tr>
<tr>
<td>$g_2$</td>
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<td>0.028</td>
<td>0.017</td>
<td>0.130</td>
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</tr>
<tr>
<td>$g_3$</td>
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<td>0.030</td>
<td>0.033</td>
<td>0.781</td>
<td>$\tau_4$</td>
</tr>
<tr>
<td>$g_4$</td>
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<td>0.055</td>
<td>0.048</td>
<td></td>
<td>$\tau_5$</td>
</tr>
<tr>
<td>$g_5$</td>
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<td>0.065</td>
<td>0.059</td>
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<td>$\tau_6$</td>
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<tr>
<td>$g_6$</td>
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<td>0.075</td>
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<td>0.109</td>
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<td>$g_8$</td>
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<td>$\tau_9$</td>
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<td>$g_9$</td>
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<td>$\tau_{10}$</td>
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<td>$g_{10}$</td>
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<td>$\tau_{11}$</td>
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Fig. 3.10 Nominal stress-strain curves of the two gels by modelling based on the corresponding master curves in Fig. 3.8.

### 3.5 Conclusions

An analytical method is developed to derive stress-strain relationship of soft gels under varied compressive strain rates from the rheometer test. In this method, the typical derivations adopted from the literature are combined, and the relationship is built between OSTs and the nonlinear viscoelastic constitutive model to find an effective way to obtain the model parameters. The range of available final strain rate is determined by the frequency range of the two master curves, which are affected by the testing temperature range and the temperature sensitivity of the materials.
Compression tests and OSTs were conducted to study the nonlinear and viscous behaviour of two brain mimicking materials. From the compression results, it was found that both gels were suitable to simulate the nonlinear behaviour of the brain tissue within the strain of 15%. For the strain larger than 15%, severe deviations can be seen between gels and brain tissue.

TTS principle is applied to extend the frequency range of storage and loss moduli of two gels based on the OST results. Based on the master curves, the dynamic shear properties of the two gels were compared with those of real brains and silicone gels from literature. Both gels can be used to mimic the brain in some appropriate conditions in mechanical studies. Hydrogel is more suitable to mimic brain tissues considering its closer properties and adjustable stiffness by changing water content.

Using the analytical method, the stress-strain curves of the two gels are calculated at different strain rates based on the master curves of storage and loss moduli. The results show that strain rate has a significant effect on the compressive behaviour of the gels. The analytical method is validated by comparing modelling results with the compression test.
Chapter 4 Dynamic Response of Brain Mimicking Gels during Impact

4.1 Introduction

Chapter 3 has presented the static compression and dynamic shear tests on the brain mimicking gels. It was found that these visco-elastic gels showed high shear rate-dependent properties. In this chapter, impact tests were conducted to study the dynamic responses of the two gels under high strain rates. The non-uniform deformation of the gels was captured by a high speed camera and analysed quantitatively by two methods. An interesting phenomenon was observed that the gels compressed alternately during the impact and the raised lateral ring propagated like a wave at the lower impact velocity (2 m/s), but disappeared at higher velocity impacts (6 and 20 m/s). Further investigation into the compression force demonstrates that compression velocity had a significant effect on the force-strain behaviour of the gels. It should be noted that this chapter focuses on the study of the dynamic responses of soft materials under the one-dimensional compression. It helps to understand the influence of the loading speed on the responses of non-uniform deformation, non-equilibrium stress and inertial effects, which is different from the impact study on a real head.

4.2 Experimental study

In this section, impact tests were carried out on two brain mimicking gels, i.e., hydrogel and silicone gel to study the dynamic behaviour of soft materials. To investigate the
effect of loading velocity on compression force, quasi-static tests were also conducted at two compression velocities.

4.2.1 Materials and sample preparation

4.2.1.1 Materials

The mimicking gels used in the impact and quasi-static compression studies were the same as these used in Chapter 3, two biological gels, i.e., hydrogel (Knox, Camden, NJ) and silicone gel (Sylgard 527, Dow Corning, Midland, MI, USA).

4.2.1.2 Sample preparation

The mixed solutions of the hydrogel and silicone gel were prepared in the same way as these in Chapter 3, but were poured uniformly into the moulds with a depth of 20 mm and a diameter of 30 mm. The cure conditions were also the same as these in the compression tests in Chapter 3.

As shown in Fig. 4.1, some markers were placed on the lateral surface of the samples to have a better view of the gel deformation during the impact. Due to the shrinkage or swell, the sample sizes were re-measured before tests, which were 20 ± 1 mm for thickness and 30 ± 1 mm for the diameter.
4.2.2 Compression tests

4.2.2.1 Quasi-static compression tests on gels

Quasi-static compression tests on both gels were conducted on the Instron machine with a load cell of a measuring range from -500 to 500 N. Two compression velocities, 0.8 and 8 mm/s, were adopted, which gave the nominal strain rate of 0.04 and 0.4 s\(^{-1}\), respectively. To have the same contacting condition as the impact tests, the surfaces of the plates were covered with the same tapes as the covers of PVDF piezoelectric films in impact tests, which provided the same friction between samples and plates.

4.2.2.2 Impact tests on gels

Fig. 4.2 shows the setup of the impact tests. The samples were placed between two aluminium plates with a diameter of 50 mm and a thickness of 11 mm. The right impacting plate was fixed on an aluminium bar with a diameter of 12.7 mm, which
comprised a “T” shape impactor with a mass of 0.15 kg and a length of 250 mm. Compared with the stiffness of the gels, the aluminium plates can be regarded as rigid targets. Two PVDF piezoelectric films with a thickness of 0.1 mm were placed between the samples and the plates to measure the impact force. The signal from PVDF piezoelectric films was recorded by a Yokogawa oscilloscope with a maximum recording frequency of 200 MHz. A high speed camera was used to capture the impact process on the gels. It was placed on the opposite side of the light source to get a better view of the gels since the gels were translucent material. The camera was set to record images at a rate of 50,000 frames/s, which was sufficient to capture the gel deformation process. The impact velocities were measured from the images by the camera, and then adjusted by changing the gas pressure of the gun in the preliminary tests.

For the impact process, an aluminium striker with a length of 400 mm and a diameter of 12.7 mm was pushed by the gas gun and then impacted on the T shape impactor. Then “T” shape impactor compressed the samples with an initial velocity. The striker rebounded slightly after the impact since it had a mass slightly smaller than the impactor. The impact tests were carried out at initial velocities of $2 \pm 0.2$, $6 \pm 0.5$ and $20 \pm 1$ m/s for the impactor, which provided the nominal strain rates of $\sim 100$, $\sim 300$ and $\sim 1000$ s$^{-1}$. Three repeated tests were conducted for each gel under both quasi-static and impact conditions.
4.3 Results and discussion

4.3.1 Deformation of the gels during the compression

Fig. 4.3 shows the deformations of hydrogel and silicone gel at the compression velocities of $8 \times 10^{-4}$ and $8 \times 10^{-3}$ m/s, respectively. For the hydrogel, we only presented the deformation images with the concentration of 10% since these of 8% and 6% showed similar deformation model. All gels showed a similar feature that the lateral sides expended in the middle of the cylinder samples due to the friction between the surfaces of samples and plates. In Fig. 4.3 (a) a crack was observed in the compressive direction and
developed from the lateral surface to the centre which was not observed in other three testing conditions (Fig. 4.3 (b-d)).

Fig. 4.4 shows the deformation processes of 10%, 8% and 6% hydrogels and silicone gel at the initial impact velocity of 2 m/s. In each figure, five images representing deformation of gels at the nominal strains of 0, 0.2, 0.4, 0.6 and maximum values in compression direction were presented. It was observed that all gels experienced non-uniform deformation during the impact. The diameter at impact end was much larger than that at the back end. It is attributed to the inertial effect of the soft materials where the gel of the impact end had a higher acceleration than the other end and deformed along the radial direction due to the nearly incompressible property. This phenomenon had also be observed in the SPHB studies on the bovine muscle tissues [160] and silicone rubbers [161] when the samples were not thin enough. The hydrogel with lower concentrations showed larger expansion rate at the impact end due to the lower stiffness which resulted in larger deformation in the radial direction. The silicone gel showed smoother lateral surface than that of hydrogel although the silicone gel showed a similar stiffness to the hydrogel in Chapter 3. This was possible because the stiffness and viscosity were dominated by crosslink chains in the gels. The main content of the hydrogel was water (≥90%), which provided large hydraulic pressure among crosslink chains in lateral direction during the impact and deformed to the radial direction more easily.

It was also found that the raised lateral ring propagated along the axial direction, and the position with the maximum diameter always changed during the compression. It propagated faster with the increase of hydrogel concentrations. It may be caused by the combination of compressive wave and shear wave, which was activated by the
longitudinal impact and radial expansion. Gels with higher modulus showed higher wave propagation velocity.

Fig. 4.5 shows the deformation processes of gels at the initial impact velocity of 6 m/s. All gels showed larger expansion rate at the impact ends than these in Fig. 4.4. The slopes of the lateral surface of hydrogel increased with the decrease of concentrations, but differences were less than these in Fig. 4.4. The lateral rings were not observed to propagate along the axial direction. This may imply that propagation velocity of the raised ring was lower than the impact speed. The gels showed larger maximum compression than these in Fig. 4.4. For higher velocity impact (20 m/s), as shown in Fig. 4.6, the difference of the expansion rate at impact end between the hydrogel with different concentrations and silicone gel were not obvious. All these gels experienced fully compressed at the impact end, while the back end still remained stationary. Similar to the deformation at 6 m/s, no lateral ring wave was observed. After the impact, the hydrogel broke into many small pieces, whereas the silicone gel broke into four relatively symmetrical pieces from the axial direction (Fig. 4.7(a-b)).

As shown in Fig. 4.7(c-d), in the quasi-static compression tests, the hydrogel also broke into many small pieces, whereas the silicone gel only showed a crack in the middle of samples. This was because the cross-links inside the hydrogel were undertaken a constant hydrostatic pressure during compression. Most of the cross-linked chains broke when the pressure reached the threshold. However, for the silicone gel, the break was caused by the maximum tensile strain on the lateral surface.
Fig. 4.3 Deformation images of gels: (a) 10% hydrogel, \( v = 8 \times 10^{-4} \) m/s; (b) silicone gel, \( v = 8 \times 10^{-4} \) m/s; (c) 10% hydrogel, \( v = 8 \times 10^{-3} \) m/s; (d) silicone gel, \( v = 8 \times 10^{-3} \) m/s. The numbers of 1-6 represent the nominal strains of 0, 0.2, 0.4, 0.6, 0.7 and 0.8, respectively.
Fig. 4.4 Deformation images of gels at the impact velocity of 2 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel. The numbers of 1-5 represent the nominal strains of 0, 0.2, 0.4, 0.6, and maximum values, respectively.
Fig. 4.5 Deformation images of gels at the impact velocity of 6 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel. The numbers of 1-5 represent the nominal strains of 0, 0.2, 0.4, 0.6, and maximum values, respectively.
Fig. 4.6 Deformation images of gels at the impact velocity of 20 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel. The numbers of 1-5 represent the nominal strains of 0, 0.2, 0.4, 0.6, and maximum values, respectively.
4.3.2 Quantitative analysis for the deformation of gels

In this section, the deformations of the gels during the impact were analysed quantitatively based on the images by the high speed camera. An image analysis software, ImageJ (National Institutes of Health, USA), was used to obtain the shape features of the gels.
4.3.2.1 Deformation of the gels by measurement

The deformation of gels was measured by the ImageJ and the corresponding time was obtained from the image information. Fig. 4.8 shows the deformation histories of gels at the impact velocities of 2, 6 and 20 m/s, respectively. The compression velocity on the gels can be considered as two stages, linear stage and deceleration stage. The linear stage showed much longer period than that of deceleration stage, which means that most of the compression were under constant strain rates. The final deformation increased with the increase of the impact velocity and the gels were fully compressed at 20 m/s.

To better understand the deformation, the samples were separated into three parts with similar thickness (Fig. 4.9) and each part was analysed individually. However, the markers were not at the position separating the samples with the same thickness, so some points of the markers were selected approximately as the boundary of each part. The distances between these markers were measured with the interval times of 0.04, 0.1 and 0.4 ms for the impact velocities of 2, 6 and 20 m/s, respectively. Fig. 4.10(a-b) show the typical deformation of the three parts at the impact velocities of 2 and 20 m/s, respectively. The total nominal strain calculated by dividing the deformed thickness samples by the original thickness of the whole sample and the nominal strain represented the individual strain in each part.
Fig. 4.8 Deformation histories of the gels: (a) $v=2\text{m/s}$; (b) $v=6\text{m/s}$; (c) $v=20\text{ m/s}$. 
Fig. 4.9 Schematic of the separation of the gel sample with three parts.

Fig. 4.10 Typical deformation of the three parts of the hydrogel: (a) \( v=2 \) m/s; (b) \( v=20 \) m/s.

At the impact velocity of 2 m/s, the strain in the back part (part 1) showed slower increase process than that of the impacted part (part 3) in the initial period, and increased faster after the total strain of 0.2. This trend was consistent with the position of raised lateral ring in Fig. 4.4, where the relative position of the largest diameter was always changing forward. The intersection at the total strain of 0.32 means the deformations and
the average diameters of part 1 and 3 were the same due to the separation of the same volume. However, part 2 showed a decreasing strain after 0.2, which contradicted our understanding to the normal compression test. It is attributed to the error of measuring the distance among the markers in obtaining the thickness of this part. As shown in Fig. 4.4, after the strain of 0.2, the lateral surface of part 2 was stretched to an arc-shape and the distance between the two markers cannot represent the true thickness of this part accurately. In addition, some markers were rolled into the interface between the samples and plates due to the friction, therefore cannot be used to measure the deformation (part 2 in Fig. 4.4). Thus, a more reliable method is needed to analysis the true deformation of the gels.

4.3.2.2 Deformation analysis of the gels by Matlab

The shape features of gels in the images were obtained by the ImageJ with the same interval times as in Fig. 4.10. Fig. 4.11 showed five typical shapes of the hydrogel during the compression at the impact velocity of 2m/s. The coordinate information of these figures was input into the Matlab. The lateral surface curves were fitted by a polynomial for each figure.

The volumes of the gels were integrated during the impact. Fig. 4.12 shows the dimensionless volumes of gels at the corresponding impact velocities, where $V$ is volumes during the impact and $V_0$ is the initial volume. The volume change for both hydrogel and silicone gel were all within 5%, which means the gels were nearly
incompressible during the impact. This validated the assumption of the incompressible property of hydrogels and silicone in the literature [128, 162].

Fig. 4.11 Five typical shapes of hydrogel obtained by the ImageJ at the impact velocity of 2 m/s.
Fig. 4.12 Volumes of hydrogel and silicone gel at the corresponding three impact velocities.

To obtain a better view of the gel deformation during the impact, the samples were separated into 6 parts with the same volume by integration. Due to the nearly incompressible property, the thickness of each part at the different time can be calculated by integrating the cross-section by Matlab to obtain the same volume as initial status. As shown in Fig. 4.13, the part with a larger radius showed a thinner thickness. This method provided more accurate and detailed deformation than that by measuring the distance among markers. In theory, a sample can be separated into as many parts as we need to get more detailed deformation information, which overcomes the limitations in measuring the displacement by markers. In addition, it avoided the problems occurred in Fig. 4.10, where deformation of the gel cannot be represented by the displacement of two markers due to the arc-shape effect.
Fig. 4.14 shows the deformation of gels at the initial impact velocity of 2 m/s. The total deformation can be divided into three stages. In the first stage, the parts near the impact end showed a much larger deformation than parts near the back end. However, this trend ended when the strain curves intersected. The intersected points represented that the average deformation of the two parts were the same. In the second stage, compression of the parts near the back increased to larger values than that of the parts near the impact, which is consistent with the phenomenon in Fig. 4.4 (the 4th figure in each group) where the radius at the left end (back end) was larger than that at the right end (impact end). In the third stage, the compression of the six parts showed similar values, which meant the deformation was uniform within the samples. These three stage deformation indicated the compression occurred alternately at the two ends.

Fig. 4.13 Schematic of the separation of gel sample with six parts of the same volume using Matlab.
Fig. 4.14 Deformation of the six parts of the gels at the impact velocity of 2 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel.

Comparing the figures in Fig. 4.14(a-d), it was observed the hydrogel with lower concentrations showed a larger difference of the deformation of the six parts due to the lower stiffness. The deformation of silicone gel was much more uniform than that of the hydrogel due to the smooth lateral surface (Fig. 4.4 (d)). The first intersection points for the part one and part six occurred at the total strain of 0.4, 0.48, 0.57 and 0.52 for the 10%, 8%, 6% hydrogels and silicone, respectively. This is consistent with the trend of ring propagation (Fig. 4.4), where gels with higher stiffness showed higher wave velocities.

Fig. 4.15 shows the deformation of the gels at the initial impact velocity of 6 m/s. Similar to 2 m/s impact, the compression near the impact end showed much larger values than
that near the back end. However, the difference among the six parts was much larger than that of 2 m/s, which means larger expansion rate and stronger non-uniform deformation occurred. Different from the three stage deformation of 2 m/s, after the first intersection both hydrogel and silicone gel deformed uniformly. Comparing the Fig. 4.15(a-d) with Fig. 4.14(a-d), the difference among the hydrogel with three concentrations was not as obvious as the impact of 2 m/s, except the intersection points. The intersections occurred at the total nominal strain of 0.67, 0.7, 0.73, and 0.6, respectively, which were delayed compared with these of 2 m/s due to the stronger non-uniform deformation in the first stage. Silicone gel showed much less delay than that of the hydrogel, which indicated the impact velocity had smaller effect on the strain of uniform deformation occurred of silicone gel.

Fig. 4.16 shows the deformation of the gels at the initial impact velocity of 20 m/s. The overall trends were similar to these of 6 m/s, but the intersections occurred at a larger total strain. In addition, the difference among the hydrogels and silicone gel was not obvious.

Fig. 4.17 shows the comparison of the gel deformation by two methods at the impact velocities of 2 and 20 m/s. The curves with filled symbols were obtained by measuring the displacement of markers, which are from Fig. 4.10. The curves with empty symbols were analysed by integration, which are average stains of part 1 and part 2, part 3 and part 4, part 5 and part 6 in Fig. 4.13, respectively. The measured strains were similar to these analysed by Matlab at the beginning at the impact velocity of 2 m/s (Fig. 4.17(a)). However, deviations appeared when the total strain was larger than 0.2 due to the error of
measuring the distance among the markers on the arc-shape lateral surface. When the impact velocity was increased to 20 m/s, the deviations became obvious (Fig. 4.17(b)).

Fig. 4.15 Deformation of the six parts of gels at the impact velocity of 6 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel.
Fig. 4.16 Deformation of the six parts of gels at the impact velocity of 20 m/s: (a) 10% hydrogel; (b) 8% hydrogel; (c) 6% hydrogel; (d) silicone gel.

Fig. 4.17 Deformation comparison of the three parts between marker measurement and Matlab analysis.
4.3.3 Compression force by quasi-static and impact tests

The impact forces at the two ends of the samples were measured by the PVDF piezoelectric films. Fig. 4.18(a-b) show two typical force curves at the impact velocities of 2 and 20 m/s, respectively. The forces at the impact end showed a spike at the beginning due to the inertial effect, which were produced to accelerate the soft gels at the impact end in both axial and radial directions. This was also observed in some SHPB compression tests on the soft materials, such as tissues and soft gels [93, 163]. The spike force showed some influence on the force of the back end at the velocity of 20 m/s, but little influence on that at the velocity of 2 m/s. For the incompressible material, the inertial force at the radius $r$ at the impact end was given by Forrestal et al. [164] as,

$$\sigma_z = \sigma_\theta = \sigma_r = \frac{\rho \left( r_0^2 - r^2 \right) \ddot{\varepsilon}}{4},$$

where $\sigma_z$, $\sigma_\theta$, and $\sigma_r$ are the inertial stress in the axial, hoop and radial directions, respectively, $\rho$ is the material density, $r_0$ is the radius of the samples, and $\ddot{\varepsilon}$ is the acceleration rate of the strain. From the expression, it is easy to find the inertial forces in the incompressible material have the same value in three directions, which are dependent on the density, position, radius and acceleration during the impact. The inertial force always exists until the whole sample deforms at a constant strain rate (uniform deformation). A higher acceleration rate leading to a higher inertial stress explains the trend that the increase of the impact velocity resulted in large expansion rate of gel deformation. In addition, since the densities of the hydrogel and silicone gel were very close, the inertial force should be similar under the same impact velocity, so the gel with
lower stiffness deformed more in the radial direction in the tests. The stress is also
dependent on position, where the inertial force is maximum at the centre of the cross-
section of the samples and reduces along the radius direction to 0 at the edge. This
provides a way to reduce the inertial effect by designing the gel as an annulus disc shape
[92, 93].

Fig. 4.18 Typical force histories at the two ends of the samples: (a) \( v = 2 \) m/s (b) \( v = 20 \)
m/s.

In this study, the relationship between the impact force and the deformation of the gels
was built up by the time. Due to the large inertial effect at the impact end, we just
selected the force at the back end for analysis and comparison, where the start point of
the force was determined by the rapid rise point of the force at the impact end.

Fig. 4.19 shows the compression force versus the nominal strain of the gels at the
compression velocities of \( 8 \times 10^{-4} \), \( 8 \times 10^{-3} \), 2, 6, and 20 m/s, respectively. All forces
showed slow increase trends at beginning due to the low stiffness of the gels. The
hydrogel with higher concentrations showed higher force. For the silicone gel, it showed
a similar increase trend to the hydrogel of 6% at the quasi-static tests, but increased to the
values close to the hydrogel of 8% in the impact tests. This indicated the compression velocity had a more significant effect on the silicone gel than on the hydrogel, which was consistent to the trend of the OST in Chapter 3, where the storage and loss moduli of silicone gel exhibited more rapid increase with the frequency than those of hydrogel. It was because the main component, i.e., water, in the hydrogel was less dependent on the deformation rate than the cross links. An obvious force drop occurred at the strain of 0.7 at the compression velocity of $8 \times 10^{-4}$ m/s due to the large crack along the compressive direction. When the compression velocity was increased to $8 \times 10^{-3}$ m/s, only a slight force drop at the strain of 0.83, and hydrogel started to break into pieces. The break strain of the hydrogel exhibited an increase trend with increase of compression velocity. For the silicone gel, no obvious break was found during the compression at both compression velocities, but a crack perpendicular to the compression direction was found after the tests (Fig. 4.7(d)). This crack showed little effect on the compression force of the silicone gel.

To have a better view of the compression velocity effects on the force, the force-strain curves of 10% hydrogel and silicone gel at different compression velocities were shown in Fig. 4.20 separately. The compression velocity showed a significant effect on the compression force. The inertial effect on the hydrogel was larger than that on the silicone gel.

For the impact tests, the spike force caused by the inertial force had little influence on the forces at the back end of the samples at 2 m/s, but the effect became larger when the impact velocities increased to 6 m/s. No break was observed for both hydrogel and silicone gel at the impact velocity of 2 m/s. When the impact velocity was increased to 6
m/s, 6% hydrogel broke at the strain of 0.8. At the impact velocity of 20 m/s, hydrogel with all the concentrations broke at the strain of ~0.9.

Fig. 4.19 Compression force-strain curves of the gels at the back end: (a) \( v = 8 \times 10^{-4} \) m/s; (b) \( v = 8 \times 10^{-3} \) m/s; (c) \( v = 2 \) m/s; (d) \( v = 6 \) m/s; (e) \( v = 20 \) m/s.
Fig. 4.20 The effects of the compression velocity on the force-strain behaviour of the gels.

4.4 Conclusions

In this chapter, the impact tests were conducted to study the dynamic responses (e.g. deformation and impact force) of two brain mimicking gels. The non-uniform deformation of the gels captured by a high speed camera was analysed quantitatively. In addition, quasi-static tests were carried out to compare with the impact tests in terms of compression force. Based on this study, following conclusions may be drawn.

In the impact tests, the inertial effect of soft gels had a significant influence on the deformation, which resulted in non-uniform deformation (raised lateral ring) at the impact end. With the increase of the impact velocity or the decrease of the stiffness, the expansion rate at the impact end became large. The inertial effect had more significant effect on the hydrogel than on the silicone gel, since main content of hydrogel was water (≥90%).

The deformation of the soft gels cannot be calculated accurately by measuring the displacement of the markers due to the influence of the arc-shape surface. Using the
Matlab to analyse the deformation by integrating the volume provides more accurate and detailed results. The gels were found to be compressed alternately at the two ends under the low velocity impact (2 m/s).

The raised lateral ring was observed to propagate at the impact velocity of 2 m/s. The wave propagation velocity is determined by the density and modulus of the incompressible soft materials.

The compression force increased significantly with the increase of the impact velocity. The inertial effect showed a significant effect on the impact force of soft gels at the impact end, which resulted in spike force in the beginning. Compared with the hydrogel, silicone gel was more sensitive to the compression velocity in force but less sensitive in non-uniform deformation. These are consistent to the shear test results in Chapter 3, where the shear frequency (shear rate) showed significant effect on the moduli and the silicone gel was more rate sensitive than that of the hydrogel.
Chapter 5 Spherical Wave Propagation in Soft Materials

5.1 Introduction

Previous work (Chapter 3 and Chapter 4) has presented both dynamic shear and compression studies on the brain mimicking gels by experiments. In the current chapter, an analytical study is conducted on spherical wave propagation within soft materials. It is applied to the brain tissue to investigate the effect of viscous property on the wave propagation behaviour. Based on the analytical model, a numerical study is conducted on the influence of the loading shape of particle velocity and mechanical parameters on the responses of the brain tissue. The analysis reveals that the loading shape of the particle velocity and bulk modulus has significant influence on the peak value and the attenuation of stress and strain, while viscosity shows little effect.

This chapter focuses on the study of the spherical wave propagation behaviour in brain-tissue-like soft materials rather than in a whole brain. It helps to understand how spherical wave propagates in these soft materials with nonlinear viscoelastic property. The study also promotes the development of the injury detection on human tissues by the stress-wave-based methods such as ultrasound. For example, the knowledge of the spherical ultrasonic wave propagation through human breast tissues was used to improve the design of imaging and therapy methods [165]. In addition, spherical pressure wave generated by vibration of the skull due to the impact has been considered to cause the deeper cortical injury. During the impact, the wave pressure propagates gradually to the centre of the brain and focuses its energy on the deeper cerebral tissues, which is actually spherical wave [9-11]. All
these studies were based on the understanding of the spherical wave propagation theory. Thus, it is necessary to understand the spherical wave behaviour in the soft materials.

5.2 Spherical wave propagation in soft materials

In this section, a spherical wave based analytical study is conducted to investigate the dynamic response of soft materials, such as tissues and gels. Spherical wave propagation within an infinite medium with an initial spherical void is considered. An initial particle velocity is applied on the inner surface (Fig. 5.1). The responses of particle velocity, stress, and strain tissue are analytically obtained, as a function of position and time. The analytical study contains two main parts. The first part is a description of the governing equations of a spherical wave, which include equation of mass conservation, equation of motion (in Section 5.2.1.1), and the constitutive equation. The second part is to use the characteristic method to solve these governing equations (in Section 5.2.2).
5.2.1 Governing equations

5.2.1.1 Motion equation and continuity equations

It is assumed that the spherical waves always keep the spherical shape during the propagation. We analyse the infinitesimal element in a spherical coordinate system (Fig. 5.2). The particle velocity $v$ and strain $\varepsilon_r$ in the radial direction are expressed as [166],

$$v(r, t) = \frac{\partial u(r, t)}{\partial t}, \quad \varepsilon_r(r, t) = \frac{\partial u(r, t)}{\partial r},$$  \hspace{1cm} (5.1)
where \( u(r,t) \) is the displacement in the radial direction. Since the motion is spherical, the principal strains have the same values in the two circumferential directions and can be determined by

\[
\varepsilon_{\phi}(r,t) = \varepsilon_{\theta}(r,t) = \frac{u(r,t)}{r}.
\] (5.2)

Also, \( \sigma_{\phi}(r,t) = \sigma_{\theta}(r,t) \). For convenience, we use \( v, \sigma_r, \sigma_\theta, \varepsilon_r, \) and \( \varepsilon_\theta \) to represent \( v(r,t), \sigma_r(r,t), \sigma_\theta(r,t), \varepsilon_r(r,t), \) and \( \varepsilon_\theta(r,t), \) respectively. \( \sigma_r \) and \( \sigma_\theta \) stand for the radial stress and circumferential stress, respectively.

Fig. 5.2 An infinitesimal element in spherical coordinates.
The governing equations of the spherical wave contain three parts, i.e., the motion equation, the continuity equations and the constitutive equation, which represent the momentum conservation, the mass conservation and the material properties, respectively.

The radial motion equation is given as,

\[ \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\theta)}{r} = \rho_0 \frac{\partial v}{\partial t}, \]

where \( \rho_0 \) is the material density. The continuity equations are represented as,

\[ \frac{\partial \varepsilon_r}{\partial t} - \frac{\partial v}{\partial r} = 0 \text{ and } \frac{\partial \varepsilon_\theta}{\partial t} - \frac{v}{r} = 0. \]

The material constitutive equation will be given in the next section.

5.2.1.2 Constitutive equation

The soft materials, tissues and gels, are assumed homogeneous, isotropic and nearly incompressible. For the nearly incompressible material, the stress can be expressed as two parts, a deviatoric part determined by shape change and a volumetric part depending on volumetric change only \([167, 168]\). To account for the time-dependent property of viscous materials, the shear modulus, \( G(t) \) for the deviatoric part, is expressed as an exponential form \([126]\),

\[ G(t) = G_\infty + (G_0 - G_\infty)e^{-t/\tau}, \]

\[ \tau, \]
where $G_0$, $G_\infty$, are the instantaneous and long-time shear modulus, respectively, and $\tau$ is the relaxation time. For the volumetric part, the bulk modulus, $K$, is usually assumed constant.

To represent the nonlinear property, a hyper-elastic constitutive law Mooney-Rivlin model is employed [77, 152, 153]. Coupling the viscous properties, the constitutive law can be given as,

$$W = \int_t^T \left\{ \sum_{i,j=1}^N G_{ij}(t-t_i) \frac{d}{dt_i} \left[ (I_1 - 3)(I_2 - 3)' \right] \right\} dt_i,$$

(5.6)

where $W$ is the strain energy, $I_1$ and $I_2$ are strain invariants, $G_{ij} = C_{ij\infty} + C_{ij}e^{-t/\tau}$. For simplicity, $N$ is taken as 1. Referring to Miller’s [77] study, the constants are assumed $C_{10\infty} = C_{01\infty}$ and $C_{10} = C_{01}$. Using $C_{1\infty}$ and $C_1$ to represent $C_{01\infty}$ and $C_{10}$, respectively, the Eq. (5.6) can be rewritten as,

$$W = \int_t^T \left\{ \left[ C_{1\infty} + C_i e^{-\left(t-t_i\right)/\tau} \right] \frac{\partial}{\partial t_i} \left[ I_1 - 3 + I_2 - 3 \right] \right\} dt_i$$

$$= \int_t^T \left\{ \left[ C_{1\infty} + C_i e^{-\left(t-t_i\right)/\tau} \right] \frac{\partial}{\partial t_i} \left[ \lambda^2 + 2\lambda^{-1} - 3 + \lambda^{-2} + 2\lambda^{-3} \right] \right\} dt_i,$$

(5.7)

where $\lambda$ is the stretch under uniaxial stress. According to the equation $\sigma = \frac{\partial W}{\partial \lambda}$ [154], the Lagrange stress can be derived as,

$$\sigma = \int_t^T \left[ C_{1\infty} + C_i e^{-\left(t-t_i\right)/\tau} \right] dN,$$

(5.8)

where $N = 2\lambda + 2 - 2\lambda^{-2} - 2\lambda^{-3}$. 

109
Using the integration by parts, Eq. (5.8) can be derived as,

$$\sigma = N\left[ C_{\infty} + C_1 e^{-(r-t)/\tau} \right] \left[ \int_0^t - \frac{1}{\tau} \int_0^t \left[ C_1 e^{-(r-t)/\tau} \right] N dt \right]$$

or

$$\sigma = CN - N_0 \left( C_{\infty} + C_1 e^{-t/\tau} \right) - \frac{1}{\tau} \int_0^t \left[ C_1 e^{-(r-t)/\tau} \right] N dt + N$$

where $C = C_{\infty} + C_1, N_0 = (2\lambda + 2 - 2\lambda^{-2} - 2\lambda^{-3})t = 0$, or in a differential form,

$$\frac{\partial \sigma}{\partial t} = C \frac{\partial N}{\partial t} + \frac{C_1}{\tau} N_0 e^{-t/\tau} - C_1 \left\{ \left[ -\frac{1}{\tau} \right] \int_0^t \left[ e^{-(r-t)/\tau} \right] N dt + N \right\}$$

Referring to Eq. (5.10), the integral term in Eq. (5.11) can be represented as,

$$-\frac{1}{\tau} \int_0^t \left[ C_1 e^{-(r-t)/\tau} \right] N dt = \sigma - CN + N_0 \left( C_{\infty} + C_1 e^{-t/\tau} \right).$$

Substituting Eq. (5.12) into Eq. (5.11), we obtain,

$$\frac{\partial \sigma}{\partial t} + \frac{\sigma}{\tau} = C \frac{\partial N}{\partial t} + \frac{C_1}{\tau} \left( N - N_0 \right).$$

The relationship between strain and stretch is given as $\varepsilon = \frac{1}{2} (\lambda^2 - 1)$ and $\frac{\partial \lambda}{\partial t} = \frac{1}{\sqrt{2\varepsilon + 1}} \frac{\partial \varepsilon}{\partial t}$, where $\varepsilon$ is the Green-Lagrangian strain. Now constitutive relationship can be expressed in a differential form,

$$\frac{\partial \sigma}{\partial t} + \frac{\sigma}{\tau} = M \frac{\partial \varepsilon}{\partial t} + \frac{C_{\infty}}{\tau} \left( N - N_0 \right),$$

110
where \( M = \mathcal{C} \left[ 2(2\varepsilon + 1)^{-\frac{1}{2}} + 4(2\varepsilon + 1)^{-2} + 6(2\varepsilon + 1)^{-\frac{5}{2}} \right] \).

Eq. (5.14) contains two main parts. The \( \frac{\partial \sigma}{\partial t} \) and \( M \frac{\partial \varepsilon}{\partial t} \) can be considered as the differential form of nonlinear elastic constitutive relationship, where \( M \) is the effective Young’s modulus. The \( \frac{\sigma}{\tau} \) and \( \frac{C_{1\infty}}{\tau} (N - N_0) \) can be considered as viscous constitutive relationship. As we know, the general form of Hooke's law for isotropic materials can be written as,

\[
\sigma_{ij} = \lambda_1 \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij},
\]

where \( \lambda_1 = \frac{\nu E}{(1+\nu)(1-2\nu)} \), \( \mu = G = \frac{E}{2(1+\nu)} \), and \( \delta_{ij} \) is Kronecker delta. The Hook's law can be divided into two parts volumetric law and distortional law as, \( \sigma_{kk} = \lambda_1 \varepsilon_{kk} \) and \( \sigma_{ij} = 2\mu \varepsilon_{ij} \ (i \neq j) \), respectively. Neglecting the volumetric viscosity [166], the volumetric law of nonlinear elastic constitutive can be expressed as differential form,

\[
\frac{\partial \sigma_r}{\partial t} + 2\frac{\partial \sigma_\theta}{\partial t} - 3K \left( \frac{\partial \varepsilon_r}{\partial t} + 2\frac{\partial \varepsilon_\theta}{\partial t} \right) = 0,
\]

For the distortional law, the viscosity cannot be neglected and the viscous constitutive relationship \( \left( \frac{\sigma}{\tau} \right. \) and \( \frac{C_{1\infty}}{\tau} (N - N_0) \) \) should also be considered. Hence we obtain,

\[
\left( \frac{\partial \sigma_r}{\partial t} - \frac{\partial \sigma_\theta}{\partial t} \right) - 2G_{\text{eff}} \left( \frac{\partial \varepsilon_r}{\partial t} - \frac{\partial \varepsilon_\theta}{\partial t} \right) + \left( \frac{\sigma_r - \sigma_\theta}{\tau} - \frac{\sigma_{\text{eff}} (\varepsilon_r) - \sigma_{\text{eff}} (\varepsilon_\theta)}{\tau} \right) = 0,
\]

where \( E_{\text{eff}} = M \), \( K \) is the bulk modulus which is assumed as a constant, \( G_{\text{eff}} = \frac{E_{\text{eff}}}{2(1+\nu)} \) is the effective shear modulus, and \( \sigma_{\text{eff}} (\varepsilon) = C_{1\infty} (N - N_0) \) is the effective stress.
5.2.2 Solution by characteristic method

A characteristic method is proposed by Wang et al. [166] to analyse the spherical wave propagation in engineering plastics. This characteristic method is employed in this study to solve the spherical wave propagation problem of soft materials. In this method a set of partial differential equations (motion equation, continuity equations and constitutive equations) are transformed to five ordinary differential equations. Then we obtain two characteristic lines as,

\[
\frac{dr}{dt} = \pm \sqrt{\frac{K + \frac{4}{3} G_{eff}}{\rho_0}} = \pm c_k(\varepsilon),
\]

(5.18)

where \(c_k(\varepsilon)\) is the wave velocity affected by the relaxation time and the strain, and \(K + \frac{4}{3} G_{eff}\) is defined as wave modulus. Along these two characteristic lines, the corresponding characteristic compatibility relationship is represented as

\[
d \sigma_r = \pm \rho_0 c_k^v \frac{2}{3} \left\{ \left( \sigma_r - \sigma_\theta \right) - \left[ \sigma_{eff} (\varepsilon_r) - \sigma_{eff} (\varepsilon_\theta) \right] \right\} c_k \tau, \quad (5.19)
\]

Furthermore, the third characteristic line is obtained as,

\[
dr = 0, \quad (5.20)
\]

and along this characteristic line, the characteristic compatibility relationships are expressed as,
\[
\begin{align*}
  d\sigma_r + 2d\sigma_\theta - 3K \left( d\varepsilon_r + 2d\varepsilon_\theta \right) &= 0 \quad (5.21) \\
  \left( d\sigma_r - d\sigma_\theta \right) - 2G_{\text{eff}} \left( d\varepsilon_r - d\varepsilon_\theta \right) + \frac{\left\{ \left( \sigma_r - \sigma_\theta \right) - \left[ \sigma_{\text{eff}} (\varepsilon_r) - \sigma_{\text{eff}} (\varepsilon_\theta) \right] \right\}}{\tau} dt &= 0 \quad (5.22)
\end{align*}
\]

\[
d\varepsilon_\theta = \frac{v}{r} dt , \quad (5.23)
\]

where Eq. (5.20) stands for the particle movement loci and Eqs. (5.21) and (5.22) reflect the material constitutive equations along the particle motion loci.

### 5.3 Numerical results and discussion

#### 5.3.1 Validation of the analytical method

Numerical results from this analytical method are obtained for validation of this method against the available experimental results from literature. Up to now, few experiments has been done to study the spherical wave propagation in soft material. Therefore, we found an experimental example of the spherical wave propagation in poly methyl methacrylate (PMMA), which is also a kind of nonlinear visco-elastic material, to validate our analytical method [166, 169]. In this experiment, spherical waves were produced by explosion of trinitrotoluene (TNT) at the centre of PMMA and the particle velocity was measured at different radii by circular-loop particle velocity gauges. Table 5.1 shows the parameters of PMMA used in this numerical study. Since the parameters given by the literature are in a polynomial form, some changes are made in our analytical expression, where effective Young’s modulus $E_{\text{eff}}$ and effective stress $\sigma_{\text{eff}}(\varepsilon)$ are represented as

113
\( E_{\text{eff}} = E_0 + E + 2 \alpha \epsilon + 3 \gamma \epsilon^2 \) and \( \sigma_{\text{eff}}(\epsilon) = E_0 \epsilon + \alpha \epsilon^2 + \gamma \epsilon^3 \), respectively. \( E_0 \) is the instantaneous elastic modulus, \( E \) is the constant elastic modulus, \( \alpha \) and \( \gamma \) are two nonlinear elastic constants.

Numerical study is conducted by solving Eqs. (5.18)-(5.23) using finite difference method (iteration using Matlab) to obtain the spherical wave behaviour in PMMA. \( v(r, t) = \sigma_r(r, t) = \sigma_\theta(r, t) = \epsilon_r(r, t) = \epsilon_\theta(r, t) = 0 \), and \( r_0 \leq r \leq \infty \) are taken as the initial conditions. For the boundary condition, the particle velocity versus time at the initial expansion radius, \( r_0 = 5mm \), is applied on the internal surface of a spherical cavity (Fig. 5.3). Fig. 5.3 shows particle velocity at different radii, obtained in the experimental and analytical studies using PMMA, which shows a good agreement.

This analytical method has also been validated by investigating the wave propagation in a linear elastic material, sierra white granite. The same initial condition and similar boundary condition are applied. The numerical studies are compared with the experimental results (in test 308) by Nagy et al. [170, 171]. In the theoretical study, for linear elastic material Young’s modulus \( E_{\text{eff}} \) and effective stress \( \sigma_{\text{eff}}(\epsilon) \) are constant.

The material parameters of sierra white granite are listed in Table 5.1. As shown in Fig. 5.4, the analytical results show a good agreement with the experimental ones in general. Some small differences can be found at the radii of 40 and 50 mm, where the wave occurrence from the theoretical study is delayed. This is because the bulk modulus in our numerical calculation is assumed as a constant, while in practice its value may change as a result of the radial and circumferential stress. This would lead to a variation of the wave speed, which affects the wave occurrence time.
Table 5.1 Parameters of the PMMA and granite used in this study [166, 169-171].

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_0$ (GPa)</th>
<th>$E$ (GPa)</th>
<th>$\alpha$ (GPa)</th>
<th>$\gamma$ (GPa)</th>
<th>$\tau$ ($\mu$s)</th>
<th>Density $\rho_0$ (kg/m$^3$)</th>
<th>Poisson’s ratio $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA [166, 169]</td>
<td>2.04</td>
<td>4.77</td>
<td>0.59</td>
<td>12.5</td>
<td>2.05</td>
<td>1040</td>
<td>0.35</td>
</tr>
<tr>
<td>Granite [170, 171]</td>
<td>62.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2640</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 5.3 Validation of the analytical model by comparing the particle velocities between analytical results and experiments [166] in PMMA.
Fig. 5.4 Validation of the analytical model by comparing the particle velocities between analytical results and experiments [170, 171] in sierra white granite.

5.3.2 Effect of the loading profile

In this section, brain tissues, as a kind of soft materials, are selected as the wave propagation study case. The study focuses on the understanding of the spherical wave propagation in brain-like materials instead of simulating the real head impact.

Numerical solutions are obtained by solving Eqs. (5.18)-(5.23) using finite difference method to study the spherical wave behaviour in nonlinear visco-elastic soft material, brain tissue. Based on the previous work by Mendis [115] and Kleiven [116], material parameters of brain tissue are given in Table 5.2.
The constant $C$ in Mooney-Rivlin model is the average value of $C_{01}$ and $C_{10}$ used by Kleiven [116]. To derive the value of $C_1$ and $C_{1\infty}$, the ratio of $C$ (instantaneous -term) to $C_{1\infty}$ (long-term) is taken as 5.26 [172]. 

$$v(r, t) = \sigma_r(r, t) = \sigma_\theta(r, t) = \varepsilon_r(r, t) = \varepsilon_\theta(r, t) = 0,$$

and $r_0 \leq r \leq \infty$ are taken as the initial condition. For the boundary condition, we apply particle velocity as a function of time on the internal surface of a spherical cavity of radius, $r_0$, corresponding to a transient internal expansion. Four different loading curves with the same peak value are applied at the same radius to evaluate the influence of the loading profile. The area under these particle velocity curves stands for the particle displacement applied at $r_0$. The particle velocity will be applied at sphere $r_0 = 5 \text{mm}$. The loading velocity in Fig. 5.5(a-c) is linearly varying, whereas it represents a nonlinear sine process in Fig. 5(d).

<table>
<thead>
<tr>
<th>Material</th>
<th>$C$ (Pa)</th>
<th>$C_1$ (Pa)</th>
<th>$C_{1\infty}$ (Pa)</th>
<th>Bulk modulus $K$ (GPa)</th>
<th>$\tau$ (s)</th>
<th>Density $\rho_0$ (kg/m$^3$)</th>
<th>Poisson’s ratio $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain</td>
<td>65</td>
<td>52.5</td>
<td>12.5</td>
<td>2.19</td>
<td>1/125</td>
<td>1040</td>
<td>0.499999</td>
</tr>
</tbody>
</table>

In the numerical calculation, the mechanical responses at different radii between 5 mm and 200 mm have been calculated. We find when the radii are larger than 25 mm, the responses there are much smaller compared with those at the initial radius, which can be neglected. Thus 5, 6, 10, 15, and 25 mm are used as the targeted radii. The mechanical responses at different radii are obtained.
In addition, finite element (FE) simulation is used to validate the analytical results (Fig. 5.6). As shown in Fig. 5.6, the initial loading of particle velocity in Fig. 5.5 is applied on the inner surface of the spherical void with a radius of 5 mm. To avoid the influence of the stress wave reflection, the outer boundary of the sphere in simulation is 100 mm, which is much larger than the radii we calculate. The model is meshed with the linear hexahedral elements with the total number of 465,728. The element size increases from 0.25×0.25×0.25 mm³ near the spherical void to 4.90×4.90×0.25 mm³ at the edge. The combination of “hyper-elastic” and “visco-elastic” constitutive models is adopted and the parameters used are the same as the numerical study in Table 5.2. The simulation is carried out by using ABAQUS/EXPLICIT, and the mechanical responses are output at the radii of 5, 6, 10, 15, and 25 mm.

As shown in Fig. 5.7, the particle velocity at these radii is calculated from the analytical studies and shows good agreement with the results in FEM. The amplitude of particle velocity attenuates during the wave propagation in radial direction, while the general shape is retained. The wave attenuation is because of the spherical dispersion and the viscous effects induced by the so-called internal dissipative force. In Fig. 5.7(c), the negative particle velocity in the end is caused by the rapid decrease of input particle velocity which needs force to resist the spherical inertial dispersion.
Fig. 5.5 Input particle velocity versus time at $r_0=5$ mm.

Fig. 5.6 Sectional view of the spherical FE model with a spherical void.
As shown in Fig. 5.8, radial stress at the different radii is calculated according to the respective input velocity in Fig. 5.5. Similar to the case for particle velocity, analytical studies give a similar trend and peak values to those from FEM, except for the peak value at \( r = 5 \) mm, where it is slightly higher. This may be because the stress from the FEM is the average value of a hexagonal finite element where 4 nodes in the element are at the location of \( r \geq 5 \) mm. Note that radial stress in all the figures experiences attenuation during the wave propagation. In terms of the effect of loading curve shape, the largest peak compressive radial stress is produced by loading profile (a), where the input particle
velocity has the most rapid increase process. For all four loading conditions, radial stress reduces to zero after the peak value and then changes to tension from compression, due to the inertia effect which leads to a tensile force to counteract it. The largest peak tensile stress is caused by loading curve (c) because of its largest deceleration. This can also be used to explain the occurrence of negative particle velocity in Fig. 5.7(c). The difference of loading profile shapes also causes significant difference of peak values of both compressive and tensile stress in Fig. 5.8, where the peak values of input velocity are the same. This demonstrates that the velocity change rate, which is the acceleration or deceleration value, has a significant influence on the peak values of radial stress.

Fig. 5.8 Radial stress curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively.
Circumferential stress in Fig. 5.9 shows similar plots to the radial stress in Fig. 5.8. This is due to the nearly incompressible property of the tissue whose shear modulus and Young’s modulus are $10^6$ times lower than bulk modulus and the Poisson’s ratio is close to 0.5. Another feature is noted that the compressive circumferential stress takes place under the transient expansion loading, whereas it should be tensile stress when subjected to a static pressure. We may find the reason from the last term of acceleration in motion equation Eq. (5.3). For a wave problem, when the acceleration term is sufficiently large, the significant inertia effect may cause the possibility of compressive circumferential stress, for a compressive radial stress. This indicates the inertial effect has a significant influence on the mechanical behaviour of soft materials under high rate deformation, which was validated by the impact tests in Chapter 4.
Fig. 5.9 Circumferential stress curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively.

The corresponding radial strain and circumferential strain are shown in Fig. 5.10 and Fig. 5.11, respectively. Both the radial strain and circumferential strain at different radii in analytical studies show consistent values with those in FEM. Some common features are noted in these figures. Firstly, both the radial strain and the circumferential strain reach the maximum value at the end of the loading. Secondly, the magnitude of radial strain is nearly twice of the circumferential strain under the four input situations. The reason is that the circumferential strain is determined by the integration of particle velocity at a radius (see Eq. (5.23)). The mechanical property of the tissue is approximated as
incompressible material. For compressible materials, the relationship between radial strain and circumferential strain can be expressed as 

\[(1 + \varepsilon_r)(1 + \varepsilon_\theta)^2 = 1\]

based on the constant volume of an infinitesimal element. We can obtain the relationship of \[\varepsilon_r = -2\varepsilon_\theta,\]

by neglecting the quadratic term.

Fig. 5.10 Radial strain curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively.
Fig. 5.11 Circumferential strain curves in analytical and FEM studies at different radii, corresponding to the four inputs in Fig. 5.5(a-d), respectively.

5.3.3 Effect of the viscous properties

To study the viscous effect on soft materials, we let the relaxation time, \( \tau \), approach infinite and then the property of brain tissue is reduced to be nonlinear elastic. As shown in Fig. 5.12, we obtain the radial stress at \( r = 5, 10, \) and \( 25 \text{ mm} \), corresponding to the input in Fig. 5.5(a). By comparing the radial stress in the brain tissue, we find the radial stress with nonlinear visco-elastic property shows similar results to those with nonlinear elastic properties, which means the viscous property has little influence on the radial stress. It is also similar to the one dimensional case studied by Valdez [57]. This is
because the stress relaxation may have little influence on the transient loadings due to the short time scales compared with experimental relaxation times [173]. However, it does not mean we can neglect the viscous property of the brain tissue. The effect of loss modulus should be accounted into the instantaneous complex shear modulus, where the shorter duration calculated, the higher instantaneous shear modulus should be used based on the shear tests in Chapter 3.

Fig. 5.12 Radial stress curves at $r = 5, 10, \text{ and } 25 \text{ mm},$ corresponding to the input in Fig. 5.5(a).

5.4 Analytical solution for the linear elastic case

An analytical solution for the linear elastic condition is obtained by a classical method. The same initial condition as in Section 5.3.2 is taken, i.e. particle velocity is applied at
the internal surface of a spherical cavity of radius \( r_0 \). The analytical solutions for spherical wave propagation in linear elastic materials were given by Achenbach [174], as follows,

\[
\begin{align*}
  u(r,t) &= -\frac{f'}{c_Lr} - \frac{f}{r^2} \\
  \sigma_r &= \frac{\rho c_L^2}{1-\nu}
  \left[
  \frac{(1-\nu)f^*}{c_L^2 r} + 2(1-2\nu)
  \left(
  \frac{f'}{c_L r^2} + \frac{f}{r^3}
  \right)
  \right] \\
  \sigma_\theta &= \frac{\rho c_L^2}{1-\nu}
  \left[
  \frac{\nu f^*}{c_L^2 r} - (1-2\nu)
  \left(
  \frac{f'}{c_L r^2} + \frac{f}{r^3}
  \right)
  \right],
\end{align*}
\]

where \( u(t) \) is the particle displacement, \( c_L \) is wave velocity in linear elastic material, which is a constant, and \( f \) is a term of potential function as,

\[
\varphi(r,t) = \frac{1}{r} f(s), \quad \text{where } s = t - (r - r_0)/c_L.
\]

Since the displacement should be continuous at the wave front, we can obtain the following boundary condition from Eq. (5.24)

\[
f(0) = f''(0) \equiv 0.
\]
By using Laplace transform method, we obtain the solution of Eq. (5.28) satisfying initial condition of Eq. (5.29) as,

\[ f = r_0^2 \int_0^s v_t \left( e^{-\alpha(s-\tau)} - 1 \right) d\tau = r_0^2 \left[ \int_0^s v_t e^{-\alpha(s-\tau)} d\tau - \int_0^s v_t d\tau \right], \]  

(5.30)

where \( \alpha = c_L/r_0 \). Here, we consider a special case,

\[ v_t = v_0. \]  

(5.31)

The results are,

\[ v = \left[ \frac{e^{-\alpha s}}{r} + \frac{r_0 \left( 1 - e^{-\alpha s} \right)}{r^2} \right] r_0 v_0 \]  

(5.32)

\[ \sigma_r = \frac{\rho c_L r_0 v_0}{(1 - \nu) r} \left[ (1 - \nu) e^{-\alpha s} + 2(1 - 2\nu) \left\{ \frac{r_0 \left( e^{-\alpha s} - 1 \right)}{r} - \frac{r_0^2 \left( e^{-\alpha s} - 1 \right) + c_L s r_0}{r^2} \right\} \right] \]  

(5.33)

\[ \sigma_\theta = \frac{\rho c_L r_0 v_0}{(1 - \nu) r} \left[ \nu e^{-\alpha s} - (1 - 2\nu) \left\{ \frac{r_0 \left( e^{-\alpha s} - 1 \right)}{r} - \frac{r_0^2 \left( e^{-\alpha s} - 1 \right) + c_L s r_0}{r^2} \right\} \right]. \]  

(5.34)

Up to now, most of researchers assume the bulk modulus of brain tissue is a constant of 2.1 GPa, which is approximately the value of water. However, few experiments have been performed to obtain an accurate value of this. Given this uncertainty, it is necessary to study the effect of the bulk modulus on spherical wave propagation. Considering the brain tissue as a linear elastic material, we can obtain the relationship among the particle velocity, radial stress, circumferential stress, and bulk modulus by using Eqs. (5.18), and (5.32) - (5.34). Calculation is conducted for bulk modulus equal to \( K \) and \( 4K \),
respectively, corresponding to an input of constant particle velocity $v_0$. To eliminate the effect of shear modulus, we assume the shear modulus as a constant in these two cases and the value is calculated from bulk modulus of $K$ according to the Poisson’s ratio in Table 5.2. For convenience in comparison, the time, particle velocity and stress are made to be dimensionless as $tc_L/r_0$, $v/v_0$, and $\sigma_r/v_0\rho c_L$, respectively. As shown in Fig. 5.13, particle velocity at $r_0$, $2r_0$, and $5r_0$ with corresponding to the bulk modulus of $K$ and $4K$ are compared. It is observed that in all the cases the particle velocity experiences attenuation with the propagation of wave. The particle velocity at $2r_0$ and $5r_0$ attenuates with time whereas at $r_0$ it is constant. At $2r_0$, the particle velocity with the bulk modulus of $4K$ has a smaller peak value than that with bulk modulus of $K$. In general, the value of bulk modulus has little influence on the peak value of particle velocity.

![Fig. 5.13 Comparison of particle velocity curves with different bulk moduli at $r_0$, $2r_0$, and $5r_0$, corresponding to the input of constant particle velocity.](image-url)
In Fig. 5.14, radial stress is compared at $r_0$, $2r_0$, and $5r_0$ with two different bulk modulus, corresponding to input of constant particle velocity. It is observed that the radial stress reaches the peak at the moment of loading and then reduces to zero. The peak radial stress with bulk modulus of $4K$ is nearly twice that with $K$, which demonstrates that the peak radial stress is proportional to the square of bulk modulus. The radial stress attenuation with bulk modulus of $4K$ is faster than that with $K$. It is also noted that peak stress is inversely proportional to the radius. In Fig. 5.15 circumferential stress shows similar plots to the radial stress in Fig. 5.14, which is the same as nonlinear visco-elastic cases in Section 5.3.2.

In the case presented here, we can find that the bulk modulus of brain tissue has little influence on the particle velocity, but significantly affects both the peak value and the attenuation of stress.
Fig. 5.14 Comparison of radial stress curves with different bulk moduli at $r_0$, $2r_0$, and $5r_0$, corresponding to the input of constant particle velocity.

Fig. 5.15 Comparison of circumferential stress curves with different bulk moduli at $r_0$, $2r_0$, and $5r_0$, corresponding to the input of constant particle velocity.
5.5 Discussion and conclusions

5.5.1 Discussion

An analytical study is presented for the wave propagation in a soft material under transient internal expansion. Based on the final ordinary differential equations, we conduct the numerical solution to study the propagation behaviour of spherical wave in a nonlinear visco-elastic soft material, brain tissue, by using characteristic method. This analytical study can be used to help us to understand the effect and sensitivity of the responses to the mechanical parameters of the soft material under dynamic loadings. In our analytical model, if any one of the five mechanical responses (particle velocity, radial stress, circumferential stress, radial strain, and circumferential strain) is known, the other four can be obtained, based on the characteristic equations and the corresponding compatibility equations. In addition, the input condition of a sudden internal expansion can be linked to the spherical wave source, which diffuses the wave energy from a point to the whole sample. An insight into this influence may be useful in understanding and mitigating potential injury to actual brain such as deeper cortical injury. For example, the particle velocity changing rate, which is the acceleration or deceleration, is found to have a significant effect on the peak radial stress. As shown in Fig. 5.8, the input velocities in conditions (a) and (c) produce the largest peak compressive and tensile stresses, respectively. Although, a comprehensive study of wave propagation in soft materials must take into account the outer boundary conditions, it is important to develop a fundamental understanding of their dynamic responses by isolating the relevant aspects firstly [173].
5.5.2 Conclusion

An analytical study is conducted for spherical wave propagation within soft materials. The model is validated against the experiments previously performed as well as FE analysis. Based on this study, following conclusions may be drawn.

Firstly, particle velocity, stress and strain experience attenuation with the wave propagation, due to the effect of spherical dispersion. The peak radial stress is found to be significantly affected by the loading shape of particle velocity, which is the acceleration or deceleration.

Secondly, compressive circumferential stress takes place in soft material under the transient internal expansion. This is different from the static result where it should be tensile stress when subjected to static pressure. Both the radial strain and circumferential strain reach the maximum at the end of the loading. The magnitude of radial strain is nearly twice compared with the circumferential strain under all the input conditions.

Finally, the viscous properties appear to have little influence on the mechanical response of soft material during rapid transient loading. Thus, the brain-like materials may be approximately regarded as nonlinear elastic under high rate deformation. However, the bulk modulus significantly affects both the peak value and the attenuation of stress and it is essential to obtain a reliable value in characterizing the brain-like soft material.
Chapter 6 Finite Element Study on Head Impact and Brain Protection

6.1 Introduction

Previous chapters presented the studies on the mechanical behaviour of brain mimicking gels. It is necessary to investigate the dynamic responses of a whole brain during the head impacts in consideration of the detailed structures and mechanical properties of skull and brain.

As one of the most popular sports, the football (soccer) game attracted about 270 million active players worldwide in 2006 and the number is still increasing these years [175]. Meanwhile, the number of players suffering the injuries also increases. Among these injuries, head injuries account for 22%, which have been attracting a great deal of attention due to negative influences to the players [176].

The main purposes of this chapter are from three aspects. Firstly, it is to investigate the mechanical responses and assess injury risks of heads during two-head impact in football games by using a validated anatomical FE head model. Secondly, it is to assess the energy absorption capacity of EPP foam as the headgear material in protecting human heads and to find out the optimum thickness based on HIC values and injury risks. Thirdly, it is to evaluate the feasibility of using CNT foam as the headgear material by assessing its performance via comparing with the traditional foams, e.g., EPP and EPS.
6.2 Simulation work

Simulations are carried out in ABAQUS/EXPLICIT to assess the energy absorption capacity of headgear made of EPP, EPS and CNT foams in terms of the impact duration, peak acceleration and head injury severity. In assessing the head injury severity, one of the most widely used head level injury criteria in sports and traffic accidents is Head Injury Criterion (HIC). It was proposed by Versace [177] and then developed by National Highway Traffic Safety Administration (NHTSA) in 1971 based on Wayne State Tolerance Curve and is defined as,

\[ HIC = \left( t_2 - t_1 \right) \left( t_2 - t_1 \right)^{2.5} \int_{t_1}^{t_2} a(t) dt \bigg|_{\text{max}} \]  

(6.1)

where \( a(t) \) is translational acceleration at the head’s centre of gravity and \( t_1 \) and \( t_2 \) are the initial and final integral times, which are selected to maximize HIC. \( \int_{t_1}^{t_2} a(t) dt / (t_2 - t_1) \) shows the average value of acceleration between \( t_1 \) and \( t_2 \). The index 2.5 in HIC represents a straight line approximation to Wayne State Tolerance Curve (WSTC) under logarithmic axes in the range of \( t_1 \) and \( t_2 \). The WSTC was obtained by fitting the experiments data of drop test of embalmed human cadaver heads, and hammer blowing or air blast tests of animal head based on skull fracture or concussion [60].

6.2.1 Description of FE head model

In this section, we develop (meshing element, defining material property, validating model etc.) a FE head model based on the geometric model (as shown in Fig. 6.1) by
Horgan and Gilchrist [178], where the head was scanned by CT with 0.3 mm increment in the coronal plane. This FE head model comprises scalp, 3-layered skull (cortical bone, trabecular bone), facial bones, dura, CSF (cerebral spinal fluid), pia, falx, corpus callosum, brain stem, cerebellum and brain (white matter and grey matter).

The element size we mesh is based on the study by Horgan and Gilchrist [178], where the effect of the element size has been investigated with the total element number varied from 9000 to 50,000. The simulation results were validated against the experimental data by Nahum and Simth [12]. The results showed that a coarsely mesh model (9000 elements) is adequate for studying the pressure response of the head. Considering that our study focused on the dynamic response of the whole head and the pressure responses in the brain, we used ~20,000 as the total element number considering the balance between the computational efficiency and accuracy.

In this model, the head is meshed with hexahedral solid and quadrilateral shell elements (as shown in Table 6.1). Since the shape of the head is irregular, it is difficult to mesh the different parts with the elements of regular shape and similar size. The membranes such as scalp, dura and pia are represented by four nodded shell elements with the sizes ranging from $2 \times 1$ to $9 \times 7$ mm and the thickness ranging from 1 to 3 mm. Facial bones are also modelled by shell elements with the sizes ranging from $6 \times 6$ to $9 \times 9$ mm and the thickness of 10 mm.

Other parts of the brain are modelled by solid elements. Skull is the most complex structure of the skeleton which supports the structure of the face and forms a cavity for the brain. To distinguish their difference of the mechanical properties, skull is modelled
with three layers, including two surface layers of cortical bones and one layer of
trebecular bones. Therefore, the thicknesses of skull elements are thinner than those of
brain and brain stem, and the sizes vary from $2 \times 1 \times 2$ to $9 \times 7 \times 2 \text{ mm}^3$. The thin layer
between the arachnoid membrane and pia mater, is filled with cerebrospinal fluid (CSF).
It acts as a cushion or buffer between skull and brain, which provides a basic mechanical
protection to the brain when the head is impacted. The CSF elements share the notes with
the inner layer of the skull in the interface, so the sizes of the elements are similar to that
layer of the skull. Similarly, the elements of the inner parts such as brain, brain stem, also
share the notes with their connected parts in the interfaces. Referring to the head models
in literature [178-180], the elements in this model are divergent from the centre of the
head to the skull. The sizes of the elements vary from $3 \times 3 \times 1$ to $11 \times 9 \times 9 \text{ mm}^3$.

Table 6.1 The weight and element information of brain tissue in FE head model.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Weight in FE head model (kg)</th>
<th>Type of elements</th>
<th>Number of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalp</td>
<td>0.184</td>
<td>Shell</td>
<td>1206</td>
</tr>
<tr>
<td>Facial bones</td>
<td>0.624</td>
<td>Shell</td>
<td>238</td>
</tr>
<tr>
<td>Skull (Cortical bones)</td>
<td>0.995</td>
<td>Solid</td>
<td>4000</td>
</tr>
<tr>
<td>Skull (Trebecular bones)</td>
<td>0.316</td>
<td>Solid</td>
<td>2000</td>
</tr>
<tr>
<td>Dura</td>
<td>0.091</td>
<td>Shell</td>
<td>2086</td>
</tr>
<tr>
<td>Pia</td>
<td>0.095</td>
<td>Shell</td>
<td>2272</td>
</tr>
<tr>
<td>Falx</td>
<td>0.027</td>
<td>Solid</td>
<td>318</td>
</tr>
<tr>
<td>Brain stem</td>
<td>0.094</td>
<td>Solid</td>
<td>558</td>
</tr>
<tr>
<td>Cerebellum</td>
<td>0.109</td>
<td>Solid</td>
<td>594</td>
</tr>
<tr>
<td>Brain (Grey matter and white matter)</td>
<td>1.353</td>
<td>Solid</td>
<td>5600</td>
</tr>
<tr>
<td>CSF</td>
<td>0.135</td>
<td>Solid</td>
<td>2260</td>
</tr>
<tr>
<td>Total</td>
<td>4.023</td>
<td></td>
<td>21514</td>
</tr>
</tbody>
</table>
According to the previous simulations [178, 180-182], a visco-elastic material model is chosen for brain stem, cerebellum, grey and white matter by employing Prony series of six terms as,

$$G(t) = G_0 \left[1 - \sum_{i=1}^{N} g_i (1 - e^{-\tau_i})\right],$$

where $G(t)$ is the shear modulus at time $t$, $G_0$ is the instantaneous shear modulus, which is calculated from the Young’s modules in Table 6.2, $g_i$ and $\tau_i$ are the material constants and relaxation times, respectively, which are listed in Table 6.3 [180]. The other parts of the head are modelled to be elastic and the corresponding material properties are listed in Table 6.2 [178, 180-182].
Table 6.2 Material properties used in the FE head model [178, 180-182].

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Material property</th>
<th>Elastic modulus (MPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalp</td>
<td>Elastic</td>
<td>16.7</td>
<td>0.420</td>
<td>1000</td>
</tr>
<tr>
<td>Facial bones</td>
<td>Elastic</td>
<td>5000</td>
<td>0.230</td>
<td>2100</td>
</tr>
<tr>
<td>Skull (Cortical bones)</td>
<td>Elastic</td>
<td>15000</td>
<td>0.220</td>
<td>2000</td>
</tr>
<tr>
<td>Skull (Trebecular bones)</td>
<td>Elastic</td>
<td>1000</td>
<td>0.240</td>
<td>1300</td>
</tr>
<tr>
<td>Dura</td>
<td>Elastic</td>
<td>31.5</td>
<td>0.450</td>
<td>1130</td>
</tr>
<tr>
<td>Pia</td>
<td>Elastic</td>
<td>11.5</td>
<td>0.450</td>
<td>1130</td>
</tr>
<tr>
<td>Falx</td>
<td>Elastic</td>
<td>31.5</td>
<td>0.450</td>
<td>1140</td>
</tr>
<tr>
<td>Brain stem</td>
<td>Visco-elastic</td>
<td></td>
<td>0.495</td>
<td>1060</td>
</tr>
<tr>
<td>Cerebellum</td>
<td>Visco-elastic</td>
<td></td>
<td>0.495</td>
<td>1060</td>
</tr>
<tr>
<td>Grey matter and white matter</td>
<td>Visco-elastic</td>
<td></td>
<td>0.495</td>
<td>1060</td>
</tr>
<tr>
<td>CSF</td>
<td>Elastic</td>
<td>0.1485</td>
<td>0.499</td>
<td>1040</td>
</tr>
</tbody>
</table>

Table 6.3 Material constants for the Prony series of six terms [180].

<table>
<thead>
<tr>
<th>i</th>
<th>( g_i )</th>
<th>( \tau_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.69×10⁻¹</td>
<td>1.00×10⁻⁶</td>
</tr>
<tr>
<td>2</td>
<td>1.86×10⁻¹</td>
<td>1.00×10⁻⁵</td>
</tr>
<tr>
<td>3</td>
<td>1.48×10⁻²</td>
<td>1.00×10⁻⁴</td>
</tr>
<tr>
<td>4</td>
<td>1.90×10⁻²</td>
<td>1.00×10⁻³</td>
</tr>
<tr>
<td>5</td>
<td>2.56×10⁻³</td>
<td>1.00×10⁻²</td>
</tr>
<tr>
<td>6</td>
<td>7.04×10⁻³</td>
<td>1.00×10⁻¹</td>
</tr>
</tbody>
</table>

6.2.2 Validation of the head model

Simulations are carried out to validate the FE head model compared with the head impact experiments by Nahum et al. [12, 15]. FEM simulation of the head impact is undertaken using ABAQUS/EXPLICIT. In the impact experiments by Nahum and Simth [12], unembalmed cadavers were used as experimental models due to their realistic mass distribution and geometry. From Nahum’s experimental recordings, No. 37 experiment is selected to validate the FE head model since it provided the detailed data. In this experiment, the cadaver is impacted by a cylinder with an initial velocity of 9.94 m/s and a mass of 5.59 kg. The thickness and the type of padding materials used to cover the
impactor surface were adjusted in different experiments. However, the geometry parameters and material properties of the impactor used in the experiments were not given out. In order to reproduce the impact conditions of the experiment, the loading are carried out by inputting force-time curve from the experiment which is shown in Fig. 6.2(a). The FE head model is oriented by rotating forward at 45° to the Frankfort plane, which is the same as Nahum’s experiments. A free boundary condition is defined by assuming that the neck had little effect on the head response due to the short impact duration (about 6ms) [183]. Simulation results are compared with the measured experimental data in terms of the head acceleration and intracranial pressures at four locations (frontal, parietal, occipital and posterior fossa), as well as with simulation recordings by Willinger et al. [184]. As shown in Fig. 6.2(b) and Fig. 6.3, the simulation results show a good agreement with the experimental data in general. Some errors in Fig. 6.3(c-d) may be because the pressures we export were not exact at the locations they measured since they did not provide the detail information about this. The differences of material parameters and model shapes between simulations and real tests might be the cause as well.
Fig. 6.2 (a) Loading time history curve used for the simulation; (b) acceleration comparison between experiments by Nahum’s and simulations by author and Wiliger [12, 184].

Fig. 6.3 Comparison of pressure between experiments by Nahum’s and simulations by author and Wiliger at four locations: (a) frontal; (b) parietal; (c) occipital; and (d) posterior fossa [12, 184].
6.2.3 EPP foam headgear and the optimum thickness

Simulations are conducted to assess the energy absorption capacity of EPP foam used in headgear (Fig. 6.4(a)) and with a fixed mass the most effective thickness is identified. Based on Withnall et al.’s video analysis [185], head-to-head impact comprised two general scenarios, front corner of one head to the rear of the other head impact (front-rear head impact) and front boss to the side impact (front-side head impact). The maximum impact velocity was observed to be 2.5 m/s in FIFA sanctioned matches. As shown in Fig. 6.4(b), scenario of front-rear head impact with velocity of 2.5 m/s is simulated to investigate the mechanical responses of two heads with the protection of headgear. In real game impact, both heads have initial velocities. In simulation, the initial velocities are applied at the back head only and the front head is kept stationary before impact. This is because the impacts happen mostly while two players are heading a ball, where two heads have opposite velocities. This indicates if the difference of two heads initial velocities is fixed, the maximum total kinetic energy of two head is the situation that one head is stationary. Further studies are also conducted to investigate the limitations of the headgear by increasing the impact velocity up to 5 m/s. Thicknesses of the headgear range from 2 to 14 mm. For convenience, the mass is kept the same and the density changes according to the thickness as in Table 6.4.
Table 6.4 Foam densities for the headgear according to the corresponding thicknesses.

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foam density (kg/m$^3$)</td>
<td>140.0</td>
<td>68.8</td>
<td>45.2</td>
<td>33.9</td>
<td>26.6</td>
<td>21.6</td>
<td>18.1</td>
</tr>
</tbody>
</table>

In real game impact, the neck should have some influence on the head response during head impact. However, according to the FE modelling study [186] and mathematical study [9], for short duration impacts, the influence of the neck is not significant which can be ignored. This has also been accepted by other authors in simulation studies [178, 179, 184]. In addition, Queen et al. [187] studied the effect of neck muscle activation conditions (infinite or zero stiffness) on head response during impact and found that there was no difference in the peak impact force and contact time under the two muscle conditions. Finally, the reaction force of the neck highly depends on the muscle activation levels. Up to now, no research has provided the stiffness under different muscles activation levels, since the neck muscle conditions change a lot during head moving. Therefore, we ignore the influence of the neck.
### 6.2.3.1 Mechanical properties of EPP foam

Generally, the stress-strain behaviour of cellular material can be divided into three stages, a short elastic stage, a non-linear or long plateau elastic stage, and finally a densification stage. The mechanical properties of EPP foams have recently been well expressed (R=0.969-0.999) by an empirical formula as [188, 189],

\[
\sigma = A \left[ 1 - e^{-(E/A)e^{(1-\epsilon)^n}} \right] + B \left( \frac{\epsilon}{1-\epsilon} \right)^n,
\]

where the first term is used to fit the first two stages, short elastic stage and long plateau elastic stage, while the second term is for the densification stage. \(\sigma\) and \(\epsilon\) are engineering stress and strain, respectively. \(m\) and \(n\) are two parameters which can be obtained from fitting one experimental curve. \(E, A\) and \(B\) are density dependent parameters, which are used to represent the slope of the elastic stage, the plateau stress (or yield stress), and densification coefficient of the curve, respectively. They are described as functions of the foam density,

\[
E = C_{1,E} \rho + C_{2,E} \rho^2
\]

\[
A = C_{1,A} \rho + C_{2,A} \rho^{3/2}
\]

\[
B = C_{1,B} \rho^{C_{2,n}}
\]

where, \(\rho\) is the foam density, \(C_{1,E}, C_{2,E}, C_{1,A}, C_{2,A}, C_{1,B},\) and \(C_{2,B}\), are constants dependent on matrix materials. For EPP foam, these constants are shown in Table 6.5 [188].
Fig. 6.5(a) shows the engineering compressive stress-strain curves of EPP foam corresponding to the densities in Table 6.4. To have a clear view of the low stress plate stresses, in Fig. 6.5(b), the vertical scale is zoomed to 0–0.5 MPa. The EPP foam is modelled using “crushable foams” constitutive model developed in ABAQUS [189-191].

Table 6.5 Parameters for the EPP foam constitutive laws [188].

<table>
<thead>
<tr>
<th>Foam</th>
<th>$C_{1,E}$</th>
<th>$C_{2,E}$</th>
<th>$C_{1,A}$</th>
<th>$C_{2,A}$</th>
<th>$C_{1,B}$</th>
<th>$C_{2,B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPP</td>
<td>$7.88 \times 10^{-2}$</td>
<td>$5.76 \times 10^{-4}$</td>
<td>$7.06 \times 10^{-4}$</td>
<td>$4.25 \times 10^{-4}$</td>
<td>$6.45 \times 10^{-6}$</td>
<td>$2.43$</td>
</tr>
</tbody>
</table>

Fig. 6.5(a) Stress-strain curves for EPP foam of different densities; (b) zooming of the Fig (a) in vertical scale [188, 189].
6.2.3.2 Head response with EPP foam headgear

The effectiveness of the proposed EPP foam headgear is assessed by comparing the dynamic responses of FE head model during two-head impact. Fig. 6.6 shows the acceleration curves of two heads under the impact velocity of 2.5 m/s, which are calculated according to the contact force divided by the mass of head. With the increasing of thicknesses from 2 to 14 mm, the densities decrease from 140 to 18.1 kg/ m³. The case of thickness equal to 0 represents two bare head impact without headgear. It is observed that the thickness of headgear has significant influence on the impact duration, where the thicker the headgear is, the longer the duration is. Head acceleration increased slowly at the beginning (several millisecond) and then grew rapidly in the following short period of ~1 millisecond. This phenomenon was more representative for thicker headgear. This is because the EPP foam has a long plateau elastic stage especially for low density foams, where resistance force increases little in this stage. When the foam reaches the stage of densification, the head-to-head contact will change from soft to hard. The maximum head acceleration with headgear shows much lower values than bare head. Especially at the thickness of 10 mm, the peak value is reduced by 57.7% compared with that of bare head. This indicates the headgear can provide effective protection to head in terms of the peak acceleration under the impact velocity of 2.5 m/s.
Fig. 6.6 Acceleration curves of two heads with different head thicknesses (T) under the impact velocity of 2.5 m/s.

Fig. 6.7 shows the foam absorbed energy during the head impact at velocity of 2.5 m/s. Since the impact area is roughly spherical, the foam displacements are not uniform in different areas. The foam displacements here are maximum values in whole impact area and made to be dimensionless as u/T, where u is the total displacement of two headgear. It shows that with the increasing of foam displacement, the energy absorbed by foam increases more and more rapidly. It is also observed that the trend of peak foam absorbed energy is contrast with peak head acceleration (Fig. 6.6). The maximum peak absorbed energy is at the thickness of 10 mm which explains the minimum peak head acceleration under this thickness. The maximum foam displacement increases with the decrease of foam density. This is because the maximum foam displacement is dominated by
compacted strain. Fig. 6.8 presents the Mises stress contours of the headgear with the thickness of 10 mm. The two headgear show similar displacements at every time point. In the end, both two headgear are nearly compacted in some area.

As shown in Fig. 6.9(a), impact velocities of 3, 4, and 5 m/s are also simulated. The lowest peak acceleration is no long at the thickness of 10 mm, but always between 8 and 12 mm with the increasing of velocities. The difference of accelerations between bare head and head with headgear reduces under high velocity impacts. At the impact velocity of 2.5 m/s, the headgear mitigates impact response by reducing peak accelerations from 284.3 to 120.2 g, which is 57.7% reduction. However, when the impact velocity is increased to 5 m/s, the maximum difference of peak accelerations drops to 136.7 g, which means only 24.2% reduction. Fig. 6.9(b) shows HIC values and acceleration curves under four impact velocities, which have similar plots to the peak acceleration. This result implies that the peak acceleration of the head dominates the head injury severity for two-head impact based on HIC (Eq. (6.1)) under this kind of short impact.
Fig. 6.7 Foam absorbed energy versus the displacement of the EPP headgear under the impact velocity of 2.5 m/s (u - displacement, T - initial thickness).

Fig. 6.8 The Mises stress contours of the headgear (T=10 mm) in the impact area under the impact velocity of 2.5 m/s.
Fig. 6.9 (a) Maximum head acceleration; (b) HIC values of two heads.
In order to assess the injury risk, brain injury risk prediction curves (as shown in Fig. 6.10) including moderate and severe head injury proposed by Marjoux et al. are used [192]. The injury risk curves were obtained by analysing the relationships between 61 real-world accidents of head injuries and injury criterion. They defined moderate neurological injuries when unconsciousness lasts more than 30 minutes, but less than 24 h and severe neurological injuries when unconsciousness lasting more than 24 hours [192]. Based on the injury risk curves and HIC values, the risks of moderate and severe head injury are shown in Fig. 6.11(a-b), respectively. The 5% injury risk is commonly considered as unlikely injury or no risk, while 95% injury risk may be interpreted as the almost certain injury [193].

In terms of moderate head injury, injury risks are very high for the bare head under these four impact velocities. At the impact velocity of 2.5 m/s, most of injury risks are lower than 5% with the protection of headgear except at the thickness of 2 mm which is 5.9%, slightly higher than 5%. However, when the impact velocity is raised to 3 m/s, the moderate injury risks increase a lot at the thickness of 2, 4, and 6 mm. For the \( v \geq 4 \) m/s, none of these thicknesses can provide enough protection that it is certain to appear moderate injury.

In terms of severe head injury, the risk is as low as 2% for the bare head at impact velocity of 2.5 m/s. This is consistent to Andersen et al’s [194] study that 1 of 28 players (3.5%) who experienced front-rear head impact got the concussion. For the impact velocities of 3 and 4 m/s, the headgear plays a very important role in reducing the probability of severe injury, where risks are controlled under 5% with the thickness of 10
or 12 mm. However, when the impact velocity is increased to 5 m/s, the risks are as high as 100% for all these thicknesses.

In summary, it is easy for bare head to get moderate injury in head-to-head impact at all these situations ($v \geq 2.5$ m/s). EPP foam headgear provides effective protection in avoiding moderate injury risk only when impact velocity $v \leq 3$ m/s. For the severe injury, EPP foam can reduce the risk at a great degree except at the velocity of 5 m/s. The optimum thickness of EPP foam is between 8 and 12 mm. When the impact velocity reaches 5 m/s, EPP foam headgear is no long suitable.

![Brain Injury risk curves for the brain](image)

Fig. 6.10 Injury risk curves for the brain [192].
Fig. 6.11 Head injury risk under the impact velocities of 2.5, 3.0, 4.0, and 5.0 m/s: (a) moderate injury; (b) severe injury.
6.3 Comparison of headgear made of EPP, EPS and CNT foams

In the recent years, some researchers have proposed the use of CNT foam as energy absorption material against impact due to its high plateau stress and being recoverable. Here, CNT foam is proposed as a candidate material for the headgear. The CNT foam used in this simulation is a kind of CNT arrays treated by post-growth chemical vapour deposition (CVD) for 125 min [195]. For comparison, the most commonly used foams in protective structures, EPP and EPS, are also simulated under the same impact velocities.

6.3.1 Mechanical properties of EPS and CNT foams

Based on the constitutive model proposed by Schraad and Harlow [196, 197], the stress-strain curve of EPS foam with density of 63 kg/m$^3$ is obtained. For comparison, EPP, EPS and CNT foams used in this simulation have the same density. In addition, experiments of headform impacted on EPS foam with density of 100 kg/m$^3$ by Gimbel and Hoshizaki is used to validate the FE model [198]. The values for parameters of these foams are listed in Table 6.6 [188, 189, 195, 196]. The densification strains of EPP and EPS foams are calculated according to the empirical formula of $\varepsilon_D = 1 - C_D \rho$ [188], where $C_D$ is the material constant. $C_D$ are given as $7.1 \times 10^{-4}$ and $1.2 \times 10^{-3}$ for EPP and EPS foams, respectively in modified Gibson’s model [188]. Fig. 6.12 shows the stress-strain curves under compression for EPP, EPS, and CNT foams. The stress-strain relationship of CNT foam is a smoothed constitutive curve based on Bradford et al.’s [195] experimental results. Similar to EPP foams, EPS and CNT foams are also modelled as “crushable foam” in ABAQUS [199, 200].
Table 6.6 Material parameters of foams used in the simulations [188, 189, 195, 196].

<table>
<thead>
<tr>
<th>Materials</th>
<th>Density (kg/m$^3$)</th>
<th>Young's modulus (MPa)</th>
<th>Plateau stress (MPa)</th>
<th>Densification strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPP foam [188, 189]</td>
<td>63</td>
<td>7.25</td>
<td>0.26</td>
<td>0.96</td>
</tr>
<tr>
<td>EPS foam [196]</td>
<td>63</td>
<td>12.75</td>
<td>0.45</td>
<td>0.92</td>
</tr>
<tr>
<td>EPS foam [196]</td>
<td>100</td>
<td>22.90</td>
<td>0.72</td>
<td>0.88</td>
</tr>
<tr>
<td>CNT foam [195]</td>
<td>63</td>
<td>10.00</td>
<td>0.56</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: the densification strain of CNT foam has not been obtained yet due to the lack of the experiments.

Fig. 6.12 Stress-strain curves for EPP, EPS and CNT foams [188, 189, 195, 196].

6.3.2 Validation of the head model with headgear

Simulation is compared with the experiment of headforms impacted on EPS foams by Gimbel and Hoshizaki [198]. In that experiment, headforms with different initial velocities (1 to 5 m/s) were impacted on EPS foams with the diameter of 50 mm and...
thickness of 25.4 mm. We choose two groups of experiment with foam densities of 63 kg/m$^3$ and 100 kg/m$^3$ and the headform mass of 4.09 kg to validate our model. As shown in Fig. 6.13, simulation results show good agreement with experiment results generally. It is also observed some differences exist. Those are because the mass of head model is 4.03 kg in simulation, but is 4.09 kg in experiment and the strain rate is not considered in simulation.

![Fig. 6.13 Comparison of head acceleration between experiments and simulation for head impact with EPS foams [198]](image)

6.3.3 Performance of headgear made of EPP, EPS and CNT foam

Simulations are conducted to study the effectiveness of the headgear of EPP, EPS and CNT foams. The simulated scenario is the front-rear head impact with impact velocities
from 1 to 5 m/s. The thickness of the headgear is kept the same at 8 mm for the three foams.

Fig. 6.14(a) shows the maximum acceleration of the two heads plotted against the initial impact velocity. All the curves have smooth upwards slopes for impact velocities between 1 and 3 m/s, while they show steeper increase at 4 and 5 m/s. This may be explained that the foams start to bottom out when impact velocity reaches 4 m/s. Among the three foams, EPP shows the highest values of maximum acceleration. This indicates that EPP is not as effective as EPS and CNT foams under high impact velocity ($v \geq 4 \text{ m/s}$ for two-head impact). The reason is that EPP has a lower plateau stress than that of EPS and CNT foams, leading to less impact energy absorption. Similar to the plots of maximum acceleration, HIC values for EPP foam in Fig. 6.14(b) are much higher than those for with the other two foams when the impact velocity reaches 4 m/s.

Fig. 6.15(a-b) show the moderate and severe head injury risk against the initial impact velocity, respectively. It is observed that the risks are kept at a low level for both moderate and severe head injuries when impact velocity $v \leq 4 \text{ m/s}$. However, when the impact velocity reaches 5 m/s, the moderate injury risks rise to much higher values, 100%, 98.7% and 78.1% for EPP, EPS and CNT foams, respectively. Different from moderate injury risk, the severe injury risks show significant difference between EPP and the other two foams at impact velocity of 5 m/s. It is also noted that the injury risk for CNT foam shows the lowest values, especially under high impact velocity.

In summary, with the density of 63 kg/m$^3$ and the thickness of 8mm, all these foams can provide effective protection for head-to-head impact when the impact velocity $v \leq 4 \text{ m/s}$. 

158
When the impact velocity increases to 5 m/s, only EPS and CNT foams can keep the severe risk at low levels, but they are not enough to prevent moderate injury. In general, CNT foam shows the most effective protection, especially compared with EPP foam.

6.4 Discussion

In this chapter, a validated anatomical FE head model is developed to simulate a scenario of front-rear head impact in football games [201]. The maximum impact velocity for head-to-head is found to be 2.5 m/s in FIFA sanctioned matches by Withnall et al.’s [185] video analysis. Based on this impact velocity, we found that the risk of suffering moderate injury is high and server injury is low; and EPP headgear can reduce the injury risk effectively. However, the highest impact velocity of 2.5 m/s is observed in video does not mean it is the maximum impact velocity in all football games. Thus, the maximum impact velocity used in this study is increased to 5 m/s. It is observed that this impact velocity has exceeded the protective capability of EPP foam headgear with all selected thicknesses under the certain mass. In addition, we propose the use of CNT foam which has higher plateau stress than EPP foam. The effectiveness of EPP, EPS, and CNT foams are compared under the same impact scenarios. CNT foam shows the highest protective ability, especially under high impact velocities.

Some additional points in this study should also be highlighted. Firstly, up to now, CNT foam has not been wildly used and the mechanical properties have not been comprehensively understood, especially under dynamic loading. If we just consider the energy absorption, CNT foam has much better performance than EPP foam, and slightly better performance than EPS foam. However, considering the recoverable property of
CNT foam, it will show more advantages than EPS foam, especially in football games where multiple impacts often happen. The cost of CNT foam is higher than other two traditional foams, which is expected to be overcome by improving the manufacturing technology. Secondly, in our simulation, the dynamic characteristics of EPP and EPS foams have not been considered. Although, 5 m/s is not the real high impact velocity, it may still affect the mechanical responses of foams. Thirdly, there may still be a long way before headgear gains widespread acceptance by players, because of some influences on them.

The use of headgears in football games is a new concept. The risk of head injury in football games may not be severe enough to urge players to wear headgears. However, due to the worldwide popularity of football, the number of head-injured players is increasing rapidly. The awareness of direct benefits obtained by players (especially for children and those who have the history of concussions) wearing headgears will certainly increase in the near future. Protection and convenience will be the dual goals when the headgears are used by the players.

### 6.5 Conclusions

In this Chapter, front-rear head impact in football games is simulated. The mechanical responses of heads are obtained and used to assess the injury severity. The effectiveness of EPP, EPS, and CNT foams used in headgear is evaluated based on biomechanical criteria (HIC) and injury risk. The main conclusions of this study can be summarized, as follows.
Firstly, it is easy for bare head to get injured in two-head impact when the impact velocity \( v \geq 2.5 \text{ m/s}. \)

Secondly, the optimum thickness is between 8 and 12 mm for EPP foam headgear. At the density of 63 kg/m\(^3\) and the thickness of 8mm, all these foams can provide effective protection for head-to-head impact when the impact velocity \( v \leq 4 \text{ m/s}. \) When the impact velocity is increased to 5 m/s, only EPS and CNT foams can keep the severe risk at low levels, but are not enough to prevent moderate injury.

Thirdly, CNT foam shows most effective protection in two-head impact. The energy absorption capacity of EPS foam is close to CNT foam. However, considering that the EPS foam is unrecoverable, CNT foam is more suitable for the multi-impact cases in football game.

In summary, as a new kind of energy absorption foams, CNT foam offers more advantages than traditional foams (e.g. effective energy absorption, rapid recovery after compression and stable properties in high temperature). However, further studies should be conducted to develop the manufacturing technology and understand the dynamic properties comprehensively.
Fig. 6.14 Mechanical responses of the heads with the protection of foam headgear: (a) Maximum acceleration; (b) the corresponding HIC values.
Fig. 6.15 Head injury risk with the protection of EPP, EPS, and CNT foam headgear: (a) moderate injury; (b) severe injury.
Chapter 7 Conclusions

This thesis presents a systematic study on dynamic behaviour of brain mimicking gels and brain during impact. A summary of current work is included and future work is suggested in this chapter.

7.1 Conclusions

7.1.1 Dynamic shear behaviour and constitutive modelling of brain mimicking gels

At the beginning, the efforts were made to investigate mechanical behaviour of the brain mimicking gels and their feasibility in simulating the real brain. From the compression and share test results, we found that both gels are suitable to simulate the nonlinear behaviour of the brain tissue in some appropriate conditions. Hydrogel is more suitable to mimic the brain tissue as its stiffness is adjustable via changing water content. In addition, based on the finding that the water in the hydrogel plays an important role in the mechanical behaviour such as, shear frequency behaviour, break model, strain rate effect, impact deformation etc., hydrogel can provide better simulation for the brain tissues, which also contain certain amount of water. OSTs were conducted on the mimicking gels within the shear strain of 1%. Similar to the brain tissue, significant viscous and strain rate-dependent behaviour were observed.

An analytical method is developed to derive the stress-strain relationship of soft gels under varied compressive strain rates based on OSTs. As brain tissues have been proven
to have the similar temperature and rate-dependent properties [6], this method can be
applied to explore the dynamic compressive behaviour of them under high strain rate.
The range of available final strain rate is determined by the frequency range of the two
master curves, which are affected by the testing temperature range and the temperature
sensitivity of the materials. Using the analytical method, the stress-strain curves of the
two gels are calculated at different strain rates based on the master curves of storage and
loss moduli. The results show that strain rate has a significant effect on the compressive
behaviour of the gels. The analytical method is validated by comparing modelling results
with the compression tests.

7.1.2 Dynamic response of brain mimicking gels under impact

Given the problems such as, non-uniform deformation, non-equilibrium stress and
significant inertial effect in testing the brain tissues by using SHPB, the impact tests were
conducted to understand the mechanisms of these problems using two gels. The non-
uniform deformation of the gels was analysed quantitatively. In addition, quasi-static
tests were carried out to compare with the impact tests in terms of compression force.

In the impact tests, the inertial effect of soft gels showed a significant influence on the
deformation, which resulted in non-uniform deformation (raised lateral ring) at the
impact end. With the increase of the impact velocity or the decrease of the stiffness, the
non-uniform deformation became more obvious. The inertial effect had a more
significant effect on the hydrogel than on the silicone gel, since main content of hydrogel
was water (≥90%).
In the deformation analysis, the traditional way that measuring the displacement of markers is not suitable for these soft materials due to the effect of the arc-shape surface by the friction. A new method is developed based on the incompressible property of the soft materials. The deformation is analysed by integrating the volume using the Matlab, which provides more accurate and detailed deformation process. An interesting phenomenon was observed that the gels were compressed alternately at two ends at the lower impact velocity (2 m/s), where the raised lateral ring was observed to propagate like a wave at the impact velocity of 2 m/s, but disappeared at higher impact velocity (6 and 20 m/s). The wave propagation velocity is determined by the density and modulus of the incompressible soft materials.

The compression velocity had a significant effect on the force-strain behaviour of the gels. The impact force was affected by inertial effect at the impact end, which resulted in spike force in the beginning. Compared with the hydrogel, silicone gel was more sensitive to the compression velocity in force but less sensitive in non-uniform deformation.

The tests in Chapter 3 and Chapter 4 demonstrate that the water in the hydrogel plays an important role in the mechanical behaviour such as, shear frequency behaviour, break model, strain rate effect, impact deformation, etc. It indicates that using the hydrogel as a mimicking material carries the potential for the mechanical understanding of the brain tissues, which also contain some water. In addition, the stiffness of the hydrogel is adjustable via changing water content and its properties are close to the brain tissue in dynamic shear tests.
7.1.3 Spherical wave propagation in soft materials

An analytical study is conducted for spherical wave propagation within soft materials. The derived method is applied to the brain tissue to investigate the effect of viscous property on the wave propagation behaviour.

The analytical study reveals that spherical dispersion dominates attenuation of the particle velocity, stress and strain during the wave propagation. Further study is performed for the effects of viscosity and bulk modulus. Viscosity appears to have little influence on the mechanical response of brain tissue during rapid transient loading, whereas the bulk modulus significantly affects both the peak value and the attenuation of stress. Consequently, it is essential to obtain a reliable bulk modulus in characterizing the brain-like soft materials. In addition, the shape of loading particle velocity significantly affects the peak radial stress, where compressive circumferential stress occurs under the transient internal expansion. This is different from the static result where it should be tensile stress when subjected to static pressure.

7.1.4 Finite element study on head impact and brain protection

Front-rear head impact in football games is simulated using the validated anatomical FE head model. The mechanical responses of heads are obtained and used to assess the injury severity based on HIC and brain injury risk curves by Marjoux et al. [192]. The results reveal it is easy for bare head to get injured in two-head impact when the impact velocity $v \geq 2.5$ m/s.
Subsequently, the effectiveness of EPP, EPS, and CNT foams used in headgear is evaluated. The optimum thickness of the headgear is between 8 and 12 mm based on EPP foam. A comparison is presented among the three foams. These headgear can provide effective protection for head-to-head impact when the impact velocity $v \leq 4 \text{ m/s}$. When the impact velocity is increased to 5 m/s, only EPS and CNT foams can keep the severe risk at low levels, but are not enough to prevent moderate injury. CNT foam shows most effective protection in two-head impact. The energy absorption capacity of EPS foam is close to CNT foam. However, CNT foam as a recoverable material is more suitable for the multi-impact cases in football game than EPS foam.

### 7.2 Summary of contributions

This study mainly focused on the investigations of the responses of the brain tissue and brain mimicking gels under dynamic loadings. The major contributions of this research are summarized as follows.

In the study of the dynamic compressive behaviour of brain tissues and brain mimicking gels, previous studies mainly focused on modifying the SHPB to reduce the effects of non-equilibrium stress, non-uniform deformation, and radial inertia. The current study has developed a method to obtain the dynamic compressive properties of these soft and temperature-dependent materials by building the relationship between OSTs and the compressive model.
Few previous studies were found to investigate the phenomenon of the non-equilibrium stress, non-uniform deformation, and radial inertia effects in the compression tests on soft materials. We studied the deformation of gels by experiments and analysed the non-uniform deformation quantitatively. The effects of impact velocity and inertia on compression force have also been explored on both hydrogels and silicone gel.

Spherical wave propagation within soft materials was investigated for the first time. The influences of viscosity, bulk modulus and loading profile on dynamic behaviour of the brain-tissue-like materials have been investigated.

Dynamic responses of two head impact have been simulated based on the FE head model, and the corresponding head injury risks have been assessed. The effects of the headgear made of different foams are obtained under the impact velocities of 1~5 m/s.

7.3 Future Work

Future studies will focus on the following issues:

The comparison has been done between the brain tissue and mimicking gels to explore the feasibility of the gels in simulating the real brain. However, it only contains the compression results under low strain rates. Furthermore, an analytical method is developed to derive the dynamic compressive behaviour of the brain tissue and mimicking gels based on OST. To obtain a comprehensive understanding of the feasibility of mimicking materials under dynamic loadings and validate the analytical method, more comparison is needed between gels and brain tissue under high strain rate.
In the gel impact study, an interesting phenomenon was observed that the gels were found to be compressed alternately at two ends and the lateral raised ring was observed at the impact velocity of 2 m/s. To have a better understanding, more FEM and analytical work need to be done for analysing the effect of impact velocity and material parameters on the behaviour of wave propagation.

In the study of spherical wave propagation in soft materials, the analytical method is validated by two case studies on stiffness solids. More experimental studies are needed to validate the method and better understand the wave behaviour in soft materials under transient loading.

A detailed 3D FE head model has been developed to simulate the two head impact. However, this work only focuses on the response of the whole head. More efforts can be made to study the responses of the different brain tissues during head impact and figure out the connections between brain tissue behaviour and whole head acceleration. Furthermore, based on the 3D head model and the understanding of gels’ properties, a physical head model with the gel brain can be developed for the experimental studies of head impact and protective ability of headgears.
References


44. Colgan, N.C., Gilchrist, M.D., and Curran, K.M., Applying DTI white matter orientations to finite element head models to examine diffuse TBI under high


54. Low, T.C., Rotational head injury model: a lumped parameter approach. 1986: Ohio State University.


90. Urban, M.W., Nenadic, I.Z., Qiang, B., Bernal, M., Chen, S., and Greenleaf, J.F., Characterization of material properties of soft solid thin layers with acoustic


173. Valdez, M.F. and Balachandran, B., Wave transmission through soft-tissue matter, in *Simulation-based Innovation and Discovery: Energetics Applications*, D. Anand,


List of Publications


