SYSTEMATIC STUDY ON REINFORCED CONCRETE STRUCTURES UNDER PROGRESSIVE COLLAPSE

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SYSTEMATIC STUDY ON REINFORCED CONCRETE STRUCTURES UNDER PROGRESSIVE COLLAPSE

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ABSTRACT

Due to the rapid increase of terrorist threats, the ability of a building to mitigate progressive collapse is of key interest to government agencies. Alternate Load Path approach is one of the direct methods to assess and quantify the resistance of building against collapse by evaluating the bridging resistance of the structure under notional removal of major load-bearing elements. This research conducted a systematic study on the structural behaviour and the development of secondary load-carrying mechanism in reinforced concrete (RC) structures starting from simple 2-D RC frames to more complete 3-D frame-slabs.

Three series of experimental tests, i.e. 2-D RC frame tests, 3-D RC frame tests, and 3-D RC frame-slab tests under single column removal scenario were conducted to investigate structural behaviour and development of load-resisting mechanisms in each configuration. The specimens were loaded on a single point above the removed column until distinct failure was observed. In 2-D RC frame tests, the effects of horizontal restraint and reinforcement detailing on the behaviour and load-carrying capacity of the double-spanning beam were investigated. In 3-D RC frame tests, the presence of direct and compatibility torsions, as well as the interactions among the 3-D connected beams were studied. Lastly, the contributions of slabs to the 3-D frame substructures were identified in 3-D RC frame-slab tests. From the tests, inadequate restraint (penultimate column removal scenario) and the presence of torsion hindered capacities of beams, especially catenary action. In frame and frame-slabs specimen with adequate restraint and negligible torsion, the significances on the development of catenary action in beams and tensile membrane action in slabs in increasing the progressive collapse resistance of RC structures were identified.

Numerical studies by employing fibre and plate elements in the modelling of beams and slabs, respectively, demonstrate the efficiency and reliability of this approach in simulating behaviour and resistance of RC substructures under progressive collapse (involving material and geometric non-linearity). The validated numerical models are employed to carry out further analyses such as multi-storey 2-D RC frames to identify the participation and interaction among
each storeys and unequal beams in 3-D RC frames (common in building) to identify the effect of span and reinforcement ratio.

Finally, a simplified analytical model is developed to predict the overall load capacity (response) of individual frames and slabs, as well as their combined capacities in frame-slab systems. This systematic study consisting of experimental, numerical, and analytical works of 2-D frames to 3-D frame-slabs is essential in supporting and improving the design approaches in current progressive collapse guidelines.
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<tr>
<td>$A$</td>
<td>Cross-sectional Area</td>
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<tr>
<td>$b$</td>
<td>Width</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth</td>
</tr>
<tr>
<td>$E$</td>
<td>Young modulus</td>
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<tr>
<td>$I$</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Concrete compressive strength</td>
</tr>
<tr>
<td>$K_{cat}$</td>
<td>Rate of Catenary action development</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
</tr>
<tr>
<td>$M_p, M_n$</td>
<td>Hogging and sagging moment resistance, respectively</td>
</tr>
<tr>
<td>$N_b, N_c$</td>
<td>Axial force in beam and column, respectively</td>
</tr>
<tr>
<td>$P_{CAA}, P_{CAT}$</td>
<td>Maximum capacity of Compressive arch and Catenary action in beam, respectively</td>
</tr>
<tr>
<td>$P_{c,fl}, P_{b,fl}, P_{s,fl}$</td>
<td>Flexural capacity of column, beam, and slab, respectively</td>
</tr>
<tr>
<td>$P_{TMA,F}$</td>
<td>Maximum tensile membrane action capacity in slab</td>
</tr>
<tr>
<td>$V_{Rd,c}$</td>
<td>Punching shear capacity in slab</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Axial strain</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Shear strain</td>
</tr>
<tr>
<td>$\Delta x, \Delta y, \theta$</td>
<td>Out-of-plane movement, in-plane movement, and twisting of the middle joint, respectively</td>
</tr>
<tr>
<td>$\varphi_y, \varphi_p$</td>
<td>Curvature at yielding and plastic stage, respectively</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>In-plane rotation of column joint</td>
</tr>
<tr>
<td>$\Delta_{cat,b}, \Delta_{cat,F}$</td>
<td>Displacement corresponding to commencement and final stage of Catenary action in beam, respectively</td>
</tr>
<tr>
<td>$\Delta_{b,fl}, \Delta_{b,fl,F}$</td>
<td>Displacement corresponding to attainment and final stage of beam flexural capacity</td>
</tr>
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</table>
$\Delta_{s,fl}$  Displacement corresponding to attainment of slab flexural capacity
LIST OF ACRONYMS

GENERAL
RC : Reinforced concrete
ALP : Alternate Load Path

LOAD MECHANISMS
CAA : Compressive arch action in beams
CAT : Catenary action in beams
CMA : Compressive membrane action in slabs
TMA : Tensile membrane action in slabs

LOCATION ALONG BEAMS
M : Middle joint-beam interface
CM : Curtailment point next to M
E : Column-beam interface
CE : Curtailment point next to E
CHAPTER 1: INTRODUCTION

1.1. Research background

With the recent occurrences of a number of building collapse incidents, either due to man-made or natural disasters, robustness of buildings to withstand or mitigate progressive collapse emerges as a key interest to building code authorities. Progressive collapse can be defined as a design situation where a local failure of a primary element (small area) leads to failure of adjacent elements resulting in a partial or total collapse of the whole building, or a disproportionately large part of it. Generally, it is conceived as a “domino effect”. From the analytical point of view, progressive collapse occurs when a structure is loaded beyond its capacity due to a sudden change in load patterns or boundary conditions [K3].

![Example of progressive collapse](image)

**Figure 1.1** Example of progressive collapse

Dusenberry and Juneja [D2] summarised that there had been totally three waves of interest in progressive collapse. The first wave of interest started in 1968 after the collapse of Ronan point apartment (22-storey precast concrete panel building) due to a gas explosion in a kitchen on the 18th floor as seen in Fig. 1.1a. A second wave of interest followed the terrorist attack (car bomb) on Alfred P.
Murrah Federal Building, USA, in 1995 (Fig. 1.1b). In the first two incidents (Ronan point and Murrah building collapses), although some countries had drafted design guidelines to prevent progressive collapse, they were mostly based on experience without any significant experimental work. The actual collapse mechanisms had not really been studied due to limited time. It was the third notable incidents, after the devastating impact from terrorist plane attack at the World Trade Centre, New York in 2001, which brought structural engineering communities worldwide to the highest level of interest in progressive collapse.

Progressive collapse may be a low probability event, but the detrimental effects and consequences cannot be neglected, especially due to increasing threat of terrorist attacks. To mitigate progressive collapse, two general threat independent approaches were proposed by Ellingwood and Leyendecker [E1]; namely, indirect and direct design method. Indirect design method implicitly considers structural resistance to progressive collapse through the provision of a minimum level of strength, continuity, and ductility, similar to the minimum requirements for “general structural integrity”. In direct design method, the resistance of a structure and its members against progressive collapse is explicitly considered by either providing certain primary elements with sufficient strength to resist failure from specific threats (local resistance method), or by assessing the capability of the structure to redistribute loads under notional removal of a single column (alternative load path method). The alternative load path (ALP) method relies on the development of secondary mechanisms in beams and slabs to increase the structural capacity and ductility beyond flexural mechanism. Although the requirement for general structural integrity in indirect method is a good strategy in providing passive resistance to progressive collapse, it does not provide a clear picture of redistribution of load paths following a local failure. Moreover, the development of the minimum tie-force requirements in the indirect method is actually based on the ALP method. Tie-force members are provided to increase the probability of the development of secondary ALP mechanisms in structural members such as through beams and slabs. Hence, in the study of progressive collapse, greater attention is paid to the ALP method, which allows direct quantification of progressive collapse resistance of a structure.
On the other hand, for progressive collapse scenarios induced by blast loading, the idea of considering an immaculate removal of a single column as a damage scenario while leaving the rest of a building undamaged is not realistic [K4]. After an explosion near a building, there will be an extensive localised damage, affecting more than a single column. Hence, the progressive collapse potential should be evaluated with the damaged structure as the initial state and at the end of nonlinear blast-structure interaction to obtain realistic prediction of the behaviour. However, it is challenging to determine the level of threat (i.e. explosive weight and standoff) in design guidelines. Furthermore, the assessment of progressive collapse potential of a building with a blast analysis is very complicated and time-consuming, which may not be suitable for practicing engineers. Therefore, to avoid blast analysis and facilitate design against progressive collapse, ALP method based on single column removal scenario is a good starting point to increase structural integrity and robustness. The structural performances of different components (i.e. beams, slabs, columns, connections, and etc) under a single column removal scenario can provide greater insight to simplified numerical modelling. Towards this end, it is imperative to experimentally investigate behaviour of structural components under a column removal scenario.

Developments of secondary load-resisting mechanisms, such as Compressive Arch Action (CAA) [F2,S1,S6,V1,V2,Y3-Y6] and Catenary Action (CAT) [K1,S1,S6,Y3-Y6] in beams, and Tensile Membrane Action in slabs (TMA) [B1,B2,P1] have been identified. The combination of secondary mechanism developments in beams and slabs has also been investigated in RC beam-slab systems [P5-P8,Q2,Q3]. These secondary load-resisting mechanisms are beneficial towards progressive collapse resistances of RC structures, as they may provide higher load capacities than flexural capacity, as well as increase the deformation capacity of the structure.
1.2. Problem statements

Although there are a number of progressive collapse design guidelines and research works providing recommendations or approaches to mitigate progressive collapse, some general issues and challenges concerning progressive collapses exist and are identified as follows:

a. Reliability of design codes

In the past decades, many design guidelines have been developed to seek the most effective way to design buildings against progressive collapse. However, the design parameters provided in the progressive collapse design guidelines are generally derived empirically, or are based on previous incidents. Moreover, the recommended design parameters varied among different design guidelines. Hence, it is important to quantify the actual progressive collapse resistance to ensure the safety of the designed RC structures.

b. Dynamic versus static approaches

Although progressive collapse is a dynamic event, performing a non-linear dynamic analysis under single column removal scenarios (ALP method) may not be practicable for engineers. Moreover, it can be challenging to obtain insight into the development of different structural mechanisms through dynamic tests. Therefore, in the numerical analyses and experimental studies of progressive collapse, non-linear static approaches are commonly taken. In a non-linear static approach, the dynamic effects such as amplification factors, inertia, and damping forces could not be considered [M5]. However, Izzuddin et al. [I2] presented an explicit way to incorporate the dynamic effects by converting the derived nonlinear static resistance to pseudo-static dynamic resistance through the balance of energy against work done. It should be noted that this method is only valid for the freefall scenario, excluding the effects of blast on the structure.

c. Field tests versus lab-based tests

Researches on the behaviour of reinforced concrete (RC) structures under column removal scenarios have been conducted on actual buildings [S1,S3], as well as lab-based experimental tests. Tests on actual buildings provide information on its
realistic behaviour. However, as it is difficult to gradually increase loading in an actual building and due to limited instrumentation, the development of load-resisting mechanisms and the structural behaviour cannot be studied in detail. On the other hand, with the proper scaling of specimens, a more controlled test to study the development of load-resisting mechanisms in different components (beam, slab, connection, and etc) can be conducted.

d. Improvement of deformation capacity of RC frames and pulling-in of outermost columns under penultimate column removal scenarios

Tests on 2-D RC double-spanning beams were mainly conducted under interior column removal scenarios with adequate horizontal restraint at each side of the beam. There were very few tests on RC double-spanning beams under penultimate column removal scenarios. Under this event, catenary action (CAT) developed in the beam may pull in the outermost column, leading to collapse. Researches on RC double-spanning beams with adequate horizontal restraints have identified the benefit of catenary action towards progressive collapse resistance of structures. Catenary action highly depends on deformation capacity, i.e. it will continue to develop and increase the load-carrying capacity until the final fracture of top reinforcing bars in the beam section near the supports. However, some tests [F2,Y5] showed that an early fracture of top reinforcing bars resulted in a low CAT capacity (below flexural capacity). Hence, it is worthwhile to investigate some ways to improve the deformation capacity of the beam especially near the support regions.

e. Beam and slab interactions in beam-slab systems

Research studies on the development of load-resisting mechanisms in beam-slab systems subject to single column removal scenarios have been conducted by a number of researchers. However, the interactions between beams and slabs have yet to be clearly reported due to the difficulty in isolating their co-existent mechanisms.

f. Development of analytical and numerical models

So far, there have not been any simple but reliable analytical models to predict the overall load resistance of RC beam-slab substructures subject to single column
removal scenarios. The development of these models may allow engineers to analyse the progressive collapse potential of a building, as well as to optimise structural design to mitigate progressive collapse.

1.3. Objective of research

The ultimate goal of the present work is to improve the understanding of the performance of RC substructures from the simplest 2-D RC frame level, through 3-D RC frame level, and then fully extended to RC frame-slab level. By systematically conducting substructure tests in this manner, one can obtain and establish relationships among the different levels of modelling. To achieve this, a series of experimental programmes is conducted to investigate the structural response of RC frame and frame-slab systems. Numerical models using fibre elements for beams and shell or plate elements for slabs are conducted and validated against the test results to demonstrate the reliability and to conduct some parametric studies. Based on the test and numerical results, simple analytical models are developed to predict the response of individual frames or overall frame-slab systems. The details of scope of work will be further elaborated in Chapter 2, after the identification of research gaps based on the review of previous works by other researchers.

1.4. Layout of Thesis

The thesis comprises seven chapters and three appendices. This first chapter gives an introduction on the research background of progressive collapse and the motivation of current research works. From a brief review of current trends and research needs on the 3-D response of a structure under progressive collapse, the objectives are established. Chapter 2 summarises a critical literature review on progressive collapse studies related to this research work. Design approaches and building design guidelines to mitigate progressive collapse are first introduced. This is followed by a review of previous researches on alternative load path (structural response) of RC structures or substructures under column removal scenarios. The scope of works in this PhD study will then be presented to overcome the identified research gaps.
Chapter 3 presents the experimental programme and test results of 2-D RC frames. The developments of ALP in 2-D RC frames, as well as the effects of detailing and horizontal restraints are discussed thoroughly at both the structural and member cross-sectional level. Chapter 4 presents the experimental programme and test results of 3-D RC frames, including the effect of torsion and transverse beams in the development of ALP. Chapter 5 presents the experimental programme and test results of 3-D RC frame-slab systems. The effects of slabs on the overall response can be elucidated through this comparison.

Chapter 6 presents the numerical work (validation and parametric studies) and the development of simplified analytical models in the predictions of non-linear responses of individual frames and frame-slab substructures. Finally, Chapter 7 concludes the research works and gives the recommendations for future works.
CHAPTER 2: LITERATURE REVIEW

2.1. Overview

In the mitigation of progressive collapse incidents, efforts are directed at code provisions and research works. This chapter will first look at the design approaches adopted in current guidelines to mitigate progressive collapse events, followed by research works on structural behaviour of RC members (beams and slabs) under progressive collapse. The findings and limitations from previous researches provide the rationale for the proposed research programmes in Chapters 3 to 5.

2.2. Guidelines on progressive collapse

The design strategies or methodologies to mitigate progressive collapse in most building codes [A3, C3, N1] can be divided into three main categories, i.e. 1) event control, 2) direct design and 3) indirect design as summarised in Fig. 2.1.

![Figure 2.1 Design methodologies in mitigating progressive collapse](image)

2.2.1. Event control

The basis of this approach is to reduce external hazards by controlling and preventing the occurrences of abnormal loadings through eliminating or minimising the consequence of an event, and protecting the structure against such an event. Some examples include installing a barrier around a column to prevent...
vehicle impact, the use of explosion venting to reduce pressure build-up within an enclosed space, and prohibiting vehicular access to a building area. Event control only minimises the effect but does not increase structural resistance of a building, since the design process does not directly address the strength, ductility, or continuity requirements. This approach is considered to be outside the scope of work of structural engineers [E1]. The remaining two approaches, i.e. indirect and direct design methods, on the contrary, are within the jurisdiction of structural engineers. These two methods have been incorporated into most current design codes against progressive collapse to enable engineers to analyse and design a building to achieve the desired level of structural performance.

2.2.2. Indirect design

This approach places a minimum requirement on the strength, continuity, and ductility in providing resistance to progressive collapse. The goal of increasing structural robustness is accomplished by providing a minimum level of connectivity among structural components, or by adopting general structural integrity measures throughout the design process of the building, including the selection of a suitable structural system, layout of columns, member sizing and joint detailing. Indirect design method has been traditionally favoured by many design guidelines as it can easily be applied to any structure where the risk of progressive collapse is relatively low. The indirect design method is expressed in terms of tie-force requirements, including horizontal and vertical tie forces [B4, C3, D1]. The underlying philosophy is that, if all members are mechanically connected by joints, which are of a specified capacity in tension, compression, or shear, under a column removal scenario, the load from the failed member may be redistributed to adjacent members providing adequate resistance to progressive collapse. This is often known as “tying a building together” by using an integrated system of ties in three directions along the principle lines of structural framing, as shown in Fig 2.2.

The limitation of the indirect approach is that it is only applicable to (a) regular-shape buildings which do not contain transfer girders, and (b) structures in low importance category. Moreover, it has been shown from some research works
[L2, I2] that neither structural integrity nor a uniform degree of safety can always be guaranteed with the indirect design approach since load redistribution of alternative paths cannot be identified by this method.

![Tie arrangements within frame structure](image)

Figure 2.2 Tie arrangements within frame structure [D1]

2.2.3. Direct design

In the direct design method, collapse resistance of a structure or its members is explicitly quantified. This approach relies heavily on structural analysis and may benefit from sophisticated analysis techniques, such as non-linear static and/or dynamic analyses, which are not commonly used in routine structural design. The direct design method is more rational and dependable compared to the indirect design method since the ability of a structure to resist consequences of a local failure is explicitly considered, and is directly related to performance-based criteria. The application of direct design method can be further divided into two categories, i.e. specific local resistance and alternative load path methods.

Local resistance

As the name implies, this method aims to increase local resistance of major structural bearing elements (columns and walls) that may be prone to terrorist attack, by either providing external protection or enhancing internal resistance. Local resistance method is considered as the most practical approach for
retrofitting an existing building since the compliance cost for other methods may be excessive, e.g. by complying with the tie-force requirements, a large part of the structure must be reconstructed in order to provide continuity among the members. It should be noted that only the resistances of several key load-bearing elements need to be increased to resist certain threat scenarios that could otherwise endanger more members. Due to its ease of application, local load resistance method has been adopted by many major design codes.

This method does not guarantee that the building may perform adequately under loading conditions that are different from the initial design situations or scenarios. In practice, it is challenging to determine the actual load intensity and to consider all the possible failure scenarios. However, building authorities will normally specify the threat scenario with a certain charge at a certain stand-off distance from the key elements. Besides this method, current design and research trends on progressive collapse are moving towards Alternative Load Path (ALP) approach.

Alternative Load Path (ALP)

In ALP method, the potential of progressive collapse is evaluated by (a) notionally removing a single or several major structural bearing elements, (b) ascertaining if any alternative load path mechanisms may develop to safely redistribute the loads and (c) preventing a major collapse as illustrated in Fig. 2.3a. In the building codes and design guidelines [B4, C3, D1, G1], only one column is required to be removed each time for one analysis. For a frame building, the available column removal scenarios are shown in Fig. 2.3b, including corner column, penultimate column, internal column, near penultimate column, edge column and near edge column scenarios [S5].

ALP is also known as a “threat-independent” approach, which does not require characterisation of the location and magnitude of damage and hence may be valid for any type of hazards or threats. Understandably, the results obtained from ALP method do not provide an accurate representation of the actual performance in the event of a local failure, since it assumes the immaculate removal (loss) of only a single load-bearing member, while in reality, more than one member may be
removed and adjacent structures may be badly damaged. Despite this, depending on the charge weight and stand-off distance, ALP can ensure the minimum level of integrity or robustness has been provided to allow redistribution of loads. In practice, both the indirect (tie provision) and the ALP methods are often used together during the design stage. Indirect method in fact relies on the development of ALP and hence, the tie provisions/requirements are specified to mobilise ALP development. After satisfying the tie-force provisions in the design, performance of a structure can be analysed and determined using the ALP method.

![Diagram of ALP under single column removal scenario and scenarios of column removals](image)

**Figure 2.3** Alternate load path (ALP) method in progressive collapse mitigation

Based on ALP method, four types of numerical analysis [M4] can be employed in evaluating progressive collapse resistance, i.e. Linear Static, Non-Linear Static (NLS) Linear Dynamic and Non-Linear Dynamic. As progressive collapse is a complex dynamic process wherein a system seeks alternate load paths and involves large deformations at the last stage, a Non-Linear Static analysis is the most accurate and realistic representation of progressive collapse event. However, this comes at a high computational cost and requires highly competent analysts to perform the analysis. Linear Static and Non-Linear analyses are generally preferred by engineers as it can be performed in most conventional design software. To account for dynamic and non-linear effects, a Linear Static analysis requires the use of a Load Increase Factor (LIF) to account for both dynamic and
non-linear effects, while a Non-Linear Static analysis only requires a Dynamic Increase Factor (DIF) to account for inertial effects [M3].

Based on extensive numerical analyses on RC frame structures, a DIF model was developed and incorporated into the UFC 4-023-03 as shown in Eq. (2-1):

\[
DIF = 1.04 + \frac{0.45}{\frac{\theta_{pra}}{\theta_y} + 0.48}
\]

(2 - 1)

where \( \theta_{pra} \) is the plastic rotation angle, and \( \theta_y \) is the yield rotation of RC members. The ratio of \( \theta_{pra}/\theta_y \) is an indicative ductility measure; the higher the ratio, the lower is the DIF. DIF for RC structural elements ranges from 1.0 to 1.4, rather than 2, which was used previously [D3]. This DIF model was developed by assuming bilinear elastic-plastic structural performance under static loading and, hence, ALP mechanisms in beams or frames such as Compressive Arch Action (CAA) and Catenary AcTion (CAT) were not considered.

Besides the codified approach, Izzuddin et al. [I2] proposed a simplified assessment framework to appraise progressive collapse resistance of multi-storey buildings due to a sudden column loss. After column removal, structural response at a higher level (for example, the whole building) can be obtained by assembling individual structural responses at a lower level (say, the floor system). Likewise, the structural response of a floor system can be obtained from the integrated responses of corresponding beams and connections, as well as slabs. At each assessment level, the pseudo-static response, which is transformed from non-linear static response based on energy approach, can be used to consider dynamic effects and assess the robustness of a structure. The DIF of an RC member or assembly can be evaluated by the ratio of pseudo-static resistance to quasi-static resistance for a given deflection [I1] as shown in Fig. 2.4a.
a) Characteristic responses and DIF for elastic-plastic system

Figure 2.4 DIF calculation based on ratio of non-linear-static to pseudo-static resistance [11]

The relationships between DIF and ductility measure (normalised deformation) obtained from the Izzuddin method based on bilinear material models, are compared with that from DIF model (UFC 4-032-03) in Fig. 2.4b. There is good agreement between the two. The advantage of the Izzuddin framework is that, it can be used to obtain DIF corresponding to actual structural performance (including ALP mechanism), as long as non-linear static response of the lowest level, such as beams and connections, can be ascertained either by simplified analytical models or by detailed numerical models. In addition, it is noteworthy that Figs. 2.5a and 2.5b indicate that in other types of non-linear static responses such as CAT, DIF does not monotonically decrease with increasing ductility. The increasing DIF with increasing ductility after a normalised deformation of 6 in Fig. 2.5a, and 9 in Fig. 2.5b, suggests unsafe predictions of DIF using Eq. (2-1).
2.3. Progressive collapse in global structure

2.3.1. Real incidents

Murrah Building (1995)

The collapse of Murrah building is a good example of progressive collapse. The 9-storey building, conventionally reinforced with one-way slab system was designed in the early 1970s. One-half of the total area collapsed after three columns supporting a transfer girder were destroyed by the blast wave from a car bomb (Fig. 2.6). It was recognised by structural engineering communities that the Murrah building collapsed progressively due to failure of the transfer girder (not allowing a hinge mechanism and subject to pulling-in action or torsion from the slab) rather than the direct effects of the explosion. Most agreed that by eliminating transfer girders [O2, S7], the extent of damage would have been reduced significantly.
This well-known 9/11 terrorist attack incident brought the structural engineering communities worldwide to the highest level of interest to mitigate progressive collapse incidents. Two Boeing 767 aircrafts crashed into the upper parts of the World Trade Centre towers at high speed, damaging the structures near the points of impact and causing intense fires within the buildings. The structures lost their ability to support the weight of members above the impact zone, which eventually led to collapse of the upper storeys. This in turn resulted in a progression of failures to each of the storeys below the impact zone (Fig. 2.7).
As designs checks and analyses showed that the towers satisfied the integrity requirement, this event in fact demonstrated that collapse was unavoidable under extreme scenarios, such as in this case. The twin towers were subjected to combined damages from blast, impact, additional plane weight and fire, even though the building was designed in accordance to the prevailing code at that time. Nevertheless, the ability of the twin towers to withstand the initial impact and subsequent fires demonstrated that the building design was probably adequate, allowing some time for evacuation of occupants in the floors below the impact zones.

2.3.2. Field tests

Sasani’s test on University of Aransas medical centre dormitory [S1]

The progressive collapse resistance of a 10-storey RC structure following the explosion of an exterior column (B5 in Fig. 2.8) was evaluated using a controlled test. As the building floor system consisted of one-way slabs supported by secondary beams, Vierendeel action of the RC frame was the major mechanism for redistribution of loads, as illustrated in Fig. 2.9. The large axial stiffness of columns above led to an almost identical vertical movement of different floors; 6.6 mm in the second floor and 6.0 mm in the fifth floor. However, the test could not capture the failure stage of the structure as additional dead and live loads were removed before conducting the test.

![Figure 2.8 Plan view of Aransas medical centre dormitory [S1]](image)
Sasani’s test on Hotel San Diego [S3]

In this study, the progressive collapse resistance of a 6-storey RC infilled-frame structure after sudden removal of two adjacent exterior columns was evaluated. The hotel had a non-ductile RC frame structure with hollow clay tile exterior infill walls with a floor system consisting of one-way joists running in longitudinal direction, as shown in Fig. 2.10. After the column removal, bi-directional Vierendeel (frame) action of transverse and longitudinal frames with participation of infill walls was identified as the major mechanism for redistribution of loads. Similarly, ultimate failure stage of the structure was not reached as superimposed dead and live loads were removed before the test commenced. This was evident by the small maximum vertical displacement of about 6.4 mm at the location directly above the removed column.
2.4. Research on ALP development in RC frames

The ALP method has been acknowledged by many building codes as a quantifiable model for designing robust buildings. Based on the modelling framework proposed by Izzuddin [I2], the potential of progressive collapse in a multi-storey building can be established from a non-linear static response analysis of the lowest level, such as beams and connections. As a result, the non-linear static response of the lowest level is the foundation of the whole simulation process. Hence, it is essential to ascertain the actual non-linear static response of members at the lowest level, i.e. beams (frames) and slabs from both experimental tests and detailed numerical analyses. Many researches in RC members (beams and slabs) subjected to column removal scenarios showed the development of ALP mechanisms, which significantly increased the ductility and load capacity beyond that of their flexural resistances.

2.4.1. ALP in 2-D RC beams or frames

Farhangvesali et al.’s test on development of arching action in longitudinally-restrained reinforced concrete beams [F2]

Six 2/5th scaled RC beam sub-assemblages were tested mainly to investigate the influence of longitudinal reinforcement ratio and configuration of stirrups to the development of arching action, which was obvious in each test as shown in the load versus vertical displacement curve in Fig. 2.11. The first few sharp drops in the loading capacity were due to rupture of tensile steel bars at the section at the adjacent of the centre stubs. Subsequently, with further downward vertical displacements, tensile steel bars at the section adjacent to the end supporting blocks failed. It was identified that the longitudinal reinforcing ratio has only minor effect on the beam capacity, whereas the influence of concrete strength is significant as seen in Fig 2.11a (the concrete strength of specimens 1 and 3 was 30 MPa, while that of specimen 2 was 27 MPa).
a) Applied load-displacement relationship of RC sub-assemblage tests

b) Horizontal reaction force-displacement relationship of RC sub-assemblage tests

**Figure 2.11** Experimental results of RC beam sub-assemblage subjected to middle column removal scenario [F2]

In addition, the effect of support stiffness and strain penetration on arching behaviour was studied using 1-D FE numerical model developed by Valipour et al. [V1] and calibrated against the experimental results. The numerical studies showed that neglecting the effect of strain penetration and fixed-end rotation in the supports may lead to overestimation of initial stiffness and ultimate loading capacity.

Yu et al.’s test on RC beam sub-assemblages [Y4]

Six one-half scaled RC beam-column sub-assemblages with enlarged column stubs at both-ends connected to a reaction wall via two horizontal steel restraints, were tested to investigate the development of ALPs in beams with varying reinforcement and beam span-to-depth ratios. The tests showed the mobilisation of CAA and CAT with adequate axial restraints, significantly increased structural resistance beyond the beam flexural capacity (Fig. 2.12). It was found that CAA was more beneficial to sub-assemblages with short span-to-depth ratios and lower reinforcement ratios, while CAT was more favourable to sub-assemblages with large span-to-depth ratios and high reinforcement ratios, particularly the top reinforcement content.
a) Applied load-displacement relationship of RC sub-assemblage tests

b) Horizontal reaction force-displacement relationship of RC sub-assemblage tests

Figure 2.12 Experimental results of RC beam sub-assemblage subjected to middle column removal scenario [Y4]

Yu et al.’s test on RC frames [Y5]

As previous tests had shown the successful development of CAA and CAT on RC beams with both-ends either fixed or adequately restrained horizontally, their developments were verified in Yu et al [Y5] for RC frames. The maximum CAT capacity in an RC frame with conventional non-seismic detailing was lower than that of CAA due to subsequent reinforcement bars rupture causing a drop in the load, demonstrating the ineffectiveness of CAT. This led to the test of three other RC frames designed with special detailing techniques at little additional construction cost, endeavouring to improve catenary action capacity at large deformations without reducing structural resistance at small deformations. The improvement of CAT by provision of special detailing is shown in Fig. 2.13.
a) Applied load-displacement relationship of RC frame

b) Axial force-displacement relationship of RC frame

**Figure 2.13** Experimental results of RC frame subjected to middle column removal scenario [Y5]

Yi et al.’s test on multi-storey RC frame [Y1]

Yi et al. tested a one-third scale, 4-bay and 3-storey RC frame under a middle column removal scenario, as shown in Fig. 2.14. Failure of the RC frame was controlled by rupture of steel bars in the beams at a vertical displacement of 456 mm (CAT stage) corresponding to beam chord rotation of 10.3°. Adequate axial restraints to the two-way beams were provided by adjacent bays to mobilise CAT as indicated in the load-displacement history in Fig. 2.14.

**Figure 2.14** Middle column load vs unloading displacement [Y1]
Pham et al.’s numerical and analytical works on RC frames [P4]

Based on extensive parametric studies on 2-D RC beams, Pham et al. proposed a simplified semi-analytical model for predicting the non-linear response (piecewise multi-linear curve in Fig. 2.15) of beams under middle column removal. The model is able to predict structural responses at different ALP stages, including rebar fracture, by accounting for boundary conditions (horizontal restraint stiffness) and beam properties (geometry, detailing and material properties).

![Diagram](image)

(a) Actual response  
(b) Simplified piecewise response

Figure 2.15 Simplified response of beam-column structures [P4]

2.4.2. ALP in 3-D RC beams or frames

Qian et al.’s test on RC beam sub-assemblages under corner column removal scenario [Q1]

Three one-third scaled beam sub-assemblages fixed at the beam ends were tested under corner column loss to investigate structural behaviour. CAA and, to a smaller degree, the development of CAT were observed in the tests (Fig. 2.16a). This is evidenced by horizontal reaction-displacement history in Fig. 2.16b. Failure was due to fracture of top reinforcement bars near to the fixed ends, causing vertical load resistance to drop to zero.
a) Test setup for RC beam sub-assemblage under corner column removal scenario

b) Load and horizontal reaction-displacement relationship

**Figure 2.16** Test setup and results of RC beam under corner column loss [Q1]

Qian et al.’s test on RC beam sub-assemblages under interior column removal scenario [Q2]

a) Test setup for RC beam sub-assemblage under interior column removal scenario

b) Load and horizontal reaction-displacement relationship

**Figure 2.17** Test setup and results of RC beam under interior column loss [Q2]

The tests on two one-third scaled beam sub-assemblies (fixed at beam ends) under interior column removal scenario (Fig. 2.17a) showed successfully the development of ALPs (CAA and CAT) along the double-spanning beams in both directions, as illustrated in Fig. 2.17b. One of the tests was stopped due to severe shear cracks near the middle joint of transverse beam (Fig. 2.17a) and the other was stopped at a vertical deflection of about 20% of the beam span accompanied
by severe concrete crushing in the plastic hinges. However, no fracture of reinforcing bars leading to failure of CAT was reported.

2.5. Research on ALP development in RC slabs

2.5.1. ALP in RC slabs

Park’s test on fully fixed slab subjected to uniform loads [P1]

Park reported a series of tests conducted on interior slab panels (fully restrained at all edges by very stiff steel frames) subjected to uniformly distributed loads (loaded by bags filled with water). Typical load-central displacement curves and crack pattern are shown in Figure 2.18. Compressive Membrane Action (CMA), which occurred at a deflection of about one-half of the slab depth, was followed by Tensile Membrane Action (TMA) until the slabs failed by fracture of reinforcement parallel to the short span.

![Figure 2.18 Load-deflection curves and crack patterns for fully fixed slab under UDL [P1]](image)

In lightly-reinforced concrete slabs, Park concluded that pure tensile membrane action did not occur. Loads were carried by a combination of bending moment and tensile membrane action, indicated by fewer cracks at the final stage rather than the flexural cracks developed at the yield-line stage. In heavily-reinforced concrete slabs, however, many additional cracks spreading over the whole area were formed indicating the development of tensile membrane action. Park
suggested that for a conservative estimation, the ultimate capacity for tensile membrane action could be taken as the load corresponding to a central deflection of 1/10 of the short span.

**Bailey’s test on simply supported slab subject to uniform loads [B1]**

In a simply supported slab, tensile membrane action can develop in the central region as the slab is able to form an outer compressive ring along its perimeter edges to equilibrate tensile membrane forces developed at the centre, as illustrated in Figure 2.19. As an attempt to apply this mechanism to structural fire design of composite structures, Bailey et al. conducted a number of scaled composite slabs at both ambient and elevated temperature conditions. The slabs were loaded to failure using airbags to represent uniformly distributed loads. The slabs were only supported vertically, with no lateral restraint to the perimeter edges.

![Airbag Test](image)

a) Fracture of reinforcement across shorter span  
b) Concrete crushing at the outer ring

**Figure 2.19** Two failure modes identified in Bailey’s tests [B1]

Two modes of failures were identified from the tests. For lightly reinforced slabs (i.e. $\rho = 0.198\%$), fracture of the reinforcement perpendicular to the shorter span was observed as shown in Figure 2.19a. This fracture was due to in-plane bending moments, which only occurred in slabs with laterally unrestrained boundary conditions. For moderately-reinforced slabs (i.e. $\rho = 0.274\%$ and $0.313\%$), compression failure of concrete occurred at the four corners of the slabs, as can be seen in Figure 2.19b. The maximum test loads ranged from 1.33 to 2.44 times the predicted yield-line load, showing the presence of TMA. The load-
displacement curves corresponding to the two typical failure modes showed a similar linear relationship towards the end of the tests, although the initial gradients were different (Fig. 2.20). The development of compressive arch action was observed to be insignificant in laterally-unrestrained RC slabs with high span-to-depth ratios (from 40 to 80).

a) Failure mode 1: fracture of bars

b) Failure mode 2: concrete crushing

Figure 2.20 Load-displacement curves according to the typical failure modes [B1]

From the observed failure modes in the tests, Bailey developed an analytical model for TMA based on equilibrium and estimation of in-plane forces, and assuming compressive membrane action in the perimeter of the slab and tensile membrane action in the central region as shown in Fig. 2.21a. The magnitudes of the in-plane forces, i.e. tension \(T_1\) and \(T_2\) and compression \(C\) in Fig. 2.21a were defined by constants \(k\) and \(b\). The constant \(k\) can be defined by equilibrium as given by Eq. (2-2)

\[
k = \frac{4n a^2 (1-2n)}{4n^2 a^2 + 1}
\]

where \(a\) is the aspect ratio and \(n\) is the horizontal projection length of diagonal yield line (Fig. 2.21a).

The value \(b\) was determined based on two failure modes witnessed from the tests, i.e. fracture of reinforcement across the shorter span of a rectangular slab, or compression (compressive ring) failure of concrete at the four corners of the slab. By considering the in-plane (membrane) resistance along line EF (Fig. 2.21b)
corresponding to the occurrence of a full crack and assuming all the reinforcement bars along EF is at its ultimate stress of \( f_u \) and the centroid of the compressive stress block was at location E at the edge of the slab, value of \( b \) was obtained as follows:

\[
b = \frac{1.1l^2}{8K(A + B + C + D)}
\]  
\[ (2 - 3) \]

The corresponding strain is given by

\[
\varepsilon = \frac{b}{l^2} K T_0 \text{ for } K T_0 \text{ in tension}
\]

where

\[
L = \sqrt{nL^2 + l^2/4}
\]

**a)** Assumed in-plane stress distribution for membrane action

**b)** Calculation of in-plane moment

**Figure 2.21** Assumed in-plane force in slab for TMA analytical prediction

\[
A = \frac{1}{2} \left( \frac{1}{1 + k} \right) \left[ \frac{l^2}{8n} - \frac{(l - nL)^2}{nL} \right] - \frac{1}{3} \left( \frac{1}{1 + k} \right) \left( nL^2 + \frac{l^2}{4} \right) \]  
\[ (2 - 4) \]

\[
B = \frac{1}{2} \left( \frac{k^2}{1 + k} \right) \left[ nL^2 \right] - \frac{k}{3(1 + k)} \left( nL^2 + \frac{l^2}{4} \right) \]  
\[ (2 - 5) \]

\[
C = \frac{l^2}{16n} (k - 1) \]  
\[ (2 - 6) \]

and

\[
D = \left( \frac{l}{2} - nL \right) \left( \frac{L}{4} - \frac{nL}{2} \right) \]  
\[ (2 - 7) \]

where \( L \) and \( l \) are the lengths of slabs in the long and the short span directions, respectively (Fig. 2.21a)

In the case of compression failure, by considering Fig. 2.21a, the maximum in-plane compressive stress (at the tip of C) was given by \( kbK T_0 \). In addition, the compressive force due to bending had to be considered. Assuming that the
maximum depth of the compressive stress-block was limited to 0.45\(d\), where \(d\) is
the average effective depth to the reinforcement in both orthogonal directions, the
following equilibrium equation was obtained:

\[
k_b k T_0 = 0.67 f_{cu} 0.45 \left( \frac{d_1 + d_2}{2} \right) - \left( K T_0 + T_0 \right)
\]

Solving for \(b\) led to

\[
b = \frac{1}{k k T_0} \left( 0.67 f_{cu} 0.45 \left( \frac{d_1 + d_2}{2} \right) - T_0 \left( K + 1 \right) \right)
\]

where \(d_1\) and \(d_2\) are the effective depths in the transverse and the longitudinal
directions, respectively and \(K T_0\) is the yield force in steel reinforcement per unit
width.

2.5.2. ALP in RC (frame) beam-slab substructures

Qian et al.’s test on beam-slab substructure under corner [Q1] and interior column
removal scenarios [Q2]

The tests on beam-slab structures were conducted by Qian et al. to study the
effect of slabs in providing additional capacity to the beams by comparing their
behaviour against that of earlier tests of beams subjected to corner and interior
column removal scenarios. The significant contribution from the slab could be
clearly observed from the load-displacement curves in Figs. 2.16b and 2.17b for
corner and interior column removal scenarios, respectively.
a) Beam-slab under corner column removal scenario  
b) Beam-slab under interior column removal scenario

Figure 2.22 Test setup on beam-slab sub-assemblages under different column loss

TMA developed in the corner scenario was beneficial as it increased the load by about twice and increased the ductility of the specimen. However, final failure of beam-slab substructure under corner column removal scenario was not reported. Similarly, slabs made significant contribution to the load-carrying capacity of beam-slab systems under interior column removal scenario. Failure was due to reinforcement bars fracture in the beams and punching shear failure at the corner joint region of the slab.

Pham et al.’s test on beam-slab substructure under penultimate interior [P7] and penultimate exterior column removal scenarios [P6]

Pham et al. conducted a series of tests under Penultimate Interior (PI) column removal (one-fourth scale) and Penultimate Exterior (PE) column removal scenarios (one-third scale). The rationale behind the tests was to investigate the development of membrane action in slabs that had inadequate horizontal restraints, i.e. penultimate column removal cases (Fig. 2.23). Test results showed that as long as the specimen was vertically supported and rotationally restrained (no torsional failure on supporting beams), a self-equilibrating mechanism in which a compressive ring supporting TMA in the central region was mobilised without the need of external horizontal restraints.
Pham reported that failure of the compressive ring to support CAT in the double-spanning beam after severe concrete crushing at the top surface of the slab corner resulted in continual inward movements of the outermost column. Despite this, sudden pulling-in was not observed indicating a ductile failure mode. In Figs. 2.2b (PI) and 2.25b (PE), it can be seen that although load was gradually decreasing due to failure of the compressive ring, the capacity was still relatively high at the end of the test, i.e. at about 75% of maximum load for both PI and PE specimens. Hence, final failure mode and the corresponding vertical displacement at the end stage were not captured.
a) Test setup for beam-slab specimens  

b) Load-displacement curve of specimens  

**Figure 2.24** Test setup and results for beam-slab sub-assemblages under penultimate interior column removal scenario [P7]

a) Test setup for beam-slab specimens  

b) Load-displacement curve of specimens  

**Figure 2.25** Test setup and results for beam-slab sub-assemblages under penultimate exterior column removal scenario [P6]

### 2.6. Review of previous research works

A critical review of a design guide [D1] and some selected research works on progressive collapse have been conducted to address the issues and challenges identified in the problem statements of Chapter 1. From review of previous research works, technical gaps are summarised as follows:

a) Reliability of design codes
The three main approaches to mitigate progressive collapse incidents, i.e. tie-force, local resistance, and alternate load path methods recommended by DoD [D1] are also commonly found in other design guidelines [A3,C3,G1]. However, the details of the three approaches, such as tie force capacity requirements and arrangements of reinforcement in tie-force method, dynamic increase factors for non-linear static analyses in ALP method, and design parameters in key element method, varied among the guidelines. This was because the design parameters are derived empirically or are based on previous collapse incidents, for example, the value of 34 kPa static design load for key element method adopted in EC1 [C3] was based on an estimate of gas explosion pressure causing failure of loadbearing flank walls at Ronan Point. The dynamic increase factor for the non-linear static analysis was derived empirically based on numerical predictions of frames, which may not be valid for beam-slab systems.

b) Dynamic versus static approaches

Under a dynamic scenario, whether actual collapse incidents or field tests, it is difficult to trace the development of different structural mechanisms. Moreover, the energy method proposed by Izzudin [I2] showed the reliability of converting non-linear static resistance to equivalent pseudo-static dynamic resistance.

c) Field tests versus lab-based tests

Besides the difficulty in tracing and capturing the structural behaviour due to limitation of instrumentation, field tests on actual buildings conducted by Sasani et al. [S1,S3] were still far from the progressive failure state. The residual deformations were small and hence, the progressive collapse resistance of the buildings had not been identified.

d) Improvement of deformation capacity of RC frames and pulling-in of outermost columns under penultimate column removal scenarios

Catenary action (CAT) is only mobilised at large deformation state, and hence, the provision of sufficient deformation or rotational capacities in beams is important to ensure successful development of CAT. It could be seen that CAT may be 2 to 3 times higher than flexural capacity [Y1,Y4,Y5], as long as the final fracture of reinforcement bars did not occur at an early stage [F2]. Tests on RC double-
spanning beams have shown that with adequate horizontal restraints provided at each side of the beam, axial tension (CAT) may develop in beams. The axial tension in the beam may induce pulling-in of the outermost column in the case of penultimate column removal scenarios, as the column may not have sufficient horizontal restraint to resist the axial tension in the beam.

e) Beam and slab interactions in beam-slab systems

Systematic studies have been conducted by Qian et al. under corner [Q1] and interior [Q2] column removal scenarios. 3-D beam sub-assemblages were compared with beam-slab substructures with similar configurations under the same column removal scenarios to investigate the slab contribution towards the behaviour of beam-slab substructures. However, there were some shortcomings in the tests. Firstly, the steel assembly meant for providing partial rotational restraints at the removed column joints in some way generated frictional forces to the specimen, i.e. horizontal (axial) restraints to support development of CAA and CAT, as well as restraints against out-of-plane actions. These restraints with the fixed column stubs at the beam-ends would increase actual ALP capacity. Secondly, the final failure mode in the interior column removal test was not clearly reported. Besides, Pham [P6, P7] did not clearly quantify the contribution of the slab. Moreover the final failure mode was not captured as the test was stopped at an early stage, i.e. at a vertical deflection of about 6% of the double-spanning beam length, and the load was still relatively high at the end of the test.

f) Development of analytical and numerical models

Analytical models to predict the load-displacement curve of secondary (ALP) mechanisms, i.e. CAA [Y6] and CAT [P4] in beams, and compressive membrane action (CMA) [P2] and TMA [B1] in slabs have been proposed. However, so far, there was no analytical model to predict a combination of ALP mechanisms in beam-slab systems. Besides, a simple but reliable numerical approach to predict the non-linear behaviour of RC beam-slab systems had yet to be proposed. Many numerical studies or analyses on beam-slab systems were conducted with 3-D continuum solid finite elements which required significant time and computing resources to build and run, and hence, impractical for engineers for most uses.
2.7. Scope of research

To fill the technical gaps identified from the critical review, the scope of works in this PhD study is introduced. It can be divided into four parts (Fig. 2.26) as follows:

1) 2-D RC frames

The objective of the first series of tests is to provide a clearer picture on the 2-D behaviour of RC frames before moving on to more complicated 3-D RC frames. This series of tests consists of one control specimen with conventional detailing, adequate horizontal restraints at each beam end and loaded at the middle joint of the double-spanning beam, to demonstrate the development of CAA and CAT in an idealised condition. Based on this control specimen, four additional specimens are designed to investigate the effects of detailing and boundary conditions (inadequate horizontal restraint) on the development of ALP.

2) 3-D frames

The objective of the second series is to study the 3-D joint response and the development of ALP under a more realistic scenario, i.e. in the presence of torsion and transverse beams. Two RC frame specimens consisting of a double-spanning beam each, are subjected to a point load acting at the middle joint applied at a known eccentricity from the beam longitudinal axis. This is to investigate the effect of direct torsion on ALP development. Two 3-D skeletal frame specimens under corner and exterior column removal scenarios are loaded at the middle joint to investigate 3-D actions including compatibility torsion and interactions between connected beams. The two scenarios are selected as they are among the most critical scenarios for analysis of structural resistance towards progressive collapse.

3) 3-D frame-slabs

Two frame-slab specimens under corner and exterior column removal scenarios loaded at the middle joint are tested to investigate the structural response, and at the same time, identify the slab contribution in improving or enhancing the overall load capacity. The test findings will aid engineers in
incorporating the slab contribution towards overall structural resistance to progressive collapse.

**Figure 2.26** Research framework

4) **Numerical and analytical models**

Numerical analyses using fibre elements for beams and columns and shell (plate) elements for slabs are performed and validated using the test results so that they can be applied with confidence to larger and more complex structures. The validated numerical models are further employed to investigate the adequacy of horizontal restraints provided in actual frames and the development of ALP in 3-D skeletal frames with unequal beam span on each side of the joint. Subsequently,
based on the test and the numerical results, analytical models are systematically
developed for 2-D RC frames, 3-D RC frames and 3-D RC frame-slab systems.

The size effect of specimens will not be studied although the specimens are
two-fifth scaled. This is because one-quarter scale is normally regarded as the
minimum scale for joint specimens fabricated with conventional deformed bars
and aggregate concrete mix [A1]. Besides, shear behaviour which is size-sensitive
is not dominant under a column removal scenario. Therefore, the size effect is
ignored in this research.

The problem statements addressed by the research works are summarised as
follows:

a. **Reliability of design codes**: the reliability of design parameters recommended
   in the design guidelines can be justified by conducting numerical and
   analytical studies based on the developed analytical and numerical methods
   (research work 4). Further numerical and analytical studies can also be
   conducted to develop a design guideline for progressive collapse.

b. **Dynamic versus static approaches**: all the tests (research works 1, 2, and 3)
   are conducted under non-linear quasi-static conditions. The dynamic
   resistances of the tested specimens are converted from their non-linear static
   resistances based on Izzuddin energy approach [I2].

c. **Field tests versus lab-based tests**: all the tests (research works 1, 2, and 3)
   are conducted systematically at component levels (beam, slab, and beam-slab
   systems) and under controlled conditions to trace and capture the behaviour
   and development of load-carrying mechanisms in RC structures under single
   column removal scenarios.

d. **Improvement of deformation capacity of RC frames and pulling-in of
   outermost columns under penultimate column removal scenarios**: addressed
   by 2-D RC frame tests (research work 1).

e. **Beam and slab interactions in beam-slab systems**: addressed by analysing and
   comparing 3-D frame (research work 2) and frame-slab (research work 3)
   tests under corner and exterior column removal scenarios.
f. Development of analytical and numerical models: The analytical and numerical models will be developed and calibrated based on the experimental results of the three series of tests (research works 1, 2, and 3).

2.8. Summary

The review on current design guidelines and research works has indicated some shortcomings and areas which require more comprehensive studies. Chiefly, many design guidelines have yet to account for slab contributions in load-carrying capacity of beam-slab systems and there are very few researches on an integrated study of structural behaviour which focuses on the interactions between beams and slabs in RC beam-slab systems. This leads to the systematic study in this research programme (2-D to 3-D RC frames and to 3-D RC frame-slabs, as summarised in Fig. 1.5 of Section 1.2) which facilitates the study of individual contributions and interactions among members (beams and slabs). An improved test setup and instrumentation system (based on some shortcomings from previous researches) is designed for the experimental programme to capture detailed structural behaviour of individual members (beams and slabs), as well as the whole frame-slab specimens. Based on the experimental results, simple numerical and analytical models to predict ALP resistances of RC structures are developed and demonstrated in Chapter 6. To the best of the author’s knowledge, currently, there are no analytical models that can consider the interaction of individual resistance of frames and slabs towards the overall non-linear static responses of frame-slab systems.

Hence, to improve understanding towards the development of ALP in RC structures under column loss scenario based on a detailed literature review, an experimental programme will be conducted at the Protective Engineering laboratory in Nanyang Technological University. The experimental results and analyses are described in Chapters 3 to 5. Finally, numerical and analytical models are developed in Chapter 6 to aid engineers in simulating structural behaviour at member, frame or substructure levels for different column removal scenarios.
CHAPTER 3: EXPERIMENTAL STUDIES OF 2-D RC FRAMES

3.1. Introduction

Alternative load path (ALP) method is used to evaluate progressive collapse resistance by removing one or several major structural bearing elements and analysing the remaining structure to determine if this initiating damage propagates from elements to elements. After the removal of a column, both the loading pattern and the load-resisting mechanism of the structure may change, which may not be considered in a conventional design. Prior to any possible collapse, structure will endeavour to balance the amplified gravity loads via different ALP mechanisms. If ALP developed in the substructure can resist the unbalanced load, progressive collapse is prevented. Otherwise, failure of this element may spread to adjacent substructures, resulting in progressive collapse. Hence, it is imperative to demonstrate and study the development of ALPs in RC structures under different column scenarios.

To achieve a better understanding on ALP developments in 3-D RC substructures (frame-slabs), a systematic test programme starting from the simplest 2-D RC frames, to 3-D RC frames, and finally extended to 3-D RC frame-slab systems was conducted. This chapter focuses on the development of ALPs in the most basic 2-D RC frames, as well as investigating the effects of reinforcement detailing and horizontal restraints on ALP developments. Five specimens were tested and presented in this chapter as follows (summarised in Table. 3.1):

(1) Full Restraint (FR) specimen – this specimen served as a control test to demonstrate the successful development of ALPs, i.e. CAA and CAT in most general (conventional detailing) and idealised conditions with adequate horizontal restraints at both ends of the beams and no out-of-plane movements, representing removal of an interior column as shown in Fig. 3.1a.
(2) Seismic Detailing (FR-S) specimen – this specimen aimed to improve the ALP capacity by confining the damage at the plastic hinges through provision of seismic detailing, i.e. closer stirrup spacing at the support region.

(3) Round bar detailing (FR-R) specimen – the test served to improve the ALP capacity, especially CAT, by increasing ductility through usage of round bars as longitudinal reinforcement, allowing more slip and larger fracture strain.

(4) Imperfect Restraint at 1 side (IR-1) specimen – the objective of this test was to investigate the development of ALP under imperfect restraint representing removal of a penultimate column as illustrated in Fig. 3.1b.

(5) Imperfect Restraint at 2 sides (IR-2) specimen – similar to IR-1, except that both ends of the beams were supported by outermost columns as illustrated in Fig. 3.1c. This scenario was not as common as FR and IR-1 specimens but it could be used to represent the extreme case of a slender structure with weak horizontal restraints, or in the event of damaged adjacent beams.

**Figure 3.1** Different scenarios represented by FR and IR specimens
Table 3.1 Specimens in 2-D RC frame tests

<table>
<thead>
<tr>
<th>Objective</th>
<th>No.</th>
<th>Specimen</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Specimen</td>
<td>1</td>
<td>FR</td>
<td>Full Restraint - normal detailing</td>
</tr>
<tr>
<td>Effect of Detailing</td>
<td>2</td>
<td>FR-S</td>
<td>Full Restraint - Seismic detailing</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>FR-R</td>
<td>Full Restraint - Round bar detailing</td>
</tr>
<tr>
<td>Effect of Restraint</td>
<td>4</td>
<td>IR-1</td>
<td>Imperfect Restraint at 1 side - normal detailing</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>IR-2</td>
<td>Imperfect Restraint at 2 sides - normal detailing</td>
</tr>
</tbody>
</table>

3.2. Test preparation

The 2/5 scaled specimens were extracted from a 5-storey, 4x6 bay prototype building designed based on Eurocode 2 [C1]. The similar scale was also adopted in the tests by Farhangvesali et al. [F2] and Vessali et al. [V2]. Detailed design of the prototype building and the scaling down procedure to the test specimens can be found in Appendix A.

3.2.1. Specimen design

The design and detailing of the control specimen FR were scaled down from the prototype building designed with conventional detailing under gravity loading as shown in Fig. 3.2.

Table 3.2 Geometric properties and reinforcement detailing

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>Concrete Cover</th>
<th>For all specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm)</td>
<td>Cross-section (depth x width)</td>
<td>length</td>
<td>Cross-section (depth x width)</td>
</tr>
<tr>
<td>2400 mm</td>
<td>180x100 mm</td>
<td>1530 mm</td>
<td>180x180 mm</td>
</tr>
<tr>
<td>Specimen</td>
<td>Reinforcement in Beam(^a)</td>
<td>Reinforcement in column(^a)</td>
<td></td>
</tr>
<tr>
<td>All except FR-R</td>
<td>1.52% (3T10)</td>
<td>1.01% (2T10)</td>
<td>1.78% (4T10+4T6)</td>
</tr>
<tr>
<td>FR-R</td>
<td>2.02% (4R10)</td>
<td>1.33% (2R10+1R8)</td>
<td>1.78% (4T10+4T6)</td>
</tr>
</tbody>
</table>

\(^a\) reinforcement ratio is calculated by \(\rho = A_s/bd\), in which \(b\) and \(d\) are the width and the effective depth of beam sections, respectively.
Previous tests on ALP in 2-D RC frames had shown that under large deformation stage, concrete surrounding reinforcement was highly damaged due to cracking, spalling, or splitting, which might reduce the member flexural capacity. Based on previous works on progressive collapse, deformation capacity played a vital role in the development of catenary action. An intuitive way to increase the deformation limit of frames was by allowing more slip or elongation of reinforcement bars before fracture occurred. Based on these two observations, modifications were made to conventional detailing so as to improve frame ductility, i.e. provision of stirrups in accordance to Seismic detailing rule to minimise concrete damage (FR-S specimen) and usage of longitudinal round bars instead of deformed bars to facilitate greater slip (FR-R specimen). The geometric properties and reinforcement detailing of the specimens are summarised in Table 3.2. It should be noted that the loss of bond in the use of round bars (FR-R specimen) may be a significant issue under flexural mechanism. Hence, the objective of FR-R was to investigate the improvement in catenary and deformation capacities, as well as to assess the influence of loss of bond under flexure. Note that the development length and the lap splice properties of round bars were not studied in the test, as continuous reinforcement was provided in the beam.

**Figure 3.2** Detailing of FR, IR-1, and IR-2 specimens
Note that the main objective of this programme was to focus on improvement of ALPs from detailing rather than on enhancement of strength. Hence, the flexural and axial capacities of the beam were maintained. FR-S specimen had the same longitudinal reinforcement arrangement as FR specimen, but with stirrup design based on seismic detailing in accordance to Eurocode 8 (EC8) [C2] for middle class ductility (DCM). The stirrup spacing was basically smaller at the support region as shown in Fig. 3.3. FR-R specimen had the same stirrup arrangement as FR, but slightly larger longitudinal reinforcement ratio (Fig. 3.4). This was to compensate for the lower strength of mild steel round bars (obtained from material test in Table 3.4) in order to maintain a comparable flexural and axial capacities of the beam. Note that the top bars were placed in two layers instead of one to avoid congestion at the beam-column joint region. Comparable comparisons of flexural capacity (bending moment resistance) and ultimate tension force between FR and FR-R specimens are given in Table 3.3.

Figure 3.3 Detailing of FR-S specimen
In the study of the effects of horizontal restraints, the design of IR-1 and IR-2 specimens were kept the same as that of FR specimen.

**Table 3.3** Comparison of flexural capacity and ultimate force among FR, FR-S, and FR-R specimens

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>FR and FR-S</th>
<th>FR-R</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexural resistance</td>
<td>(+) Moment at Mid-Joint</td>
<td>11.0 kNm</td>
<td>11.1 kNm</td>
</tr>
<tr>
<td></td>
<td>(-) Moment at Beam-End</td>
<td>16.5 kNm</td>
<td>14.2 kNm</td>
</tr>
<tr>
<td>Catenary resistance</td>
<td>Ultimate force at Bottom bar</td>
<td>94.2 kN</td>
<td>97.4 kN</td>
</tr>
<tr>
<td></td>
<td>Ultimate force at Top bar</td>
<td>141.3 kN</td>
<td>157.0 kN</td>
</tr>
</tbody>
</table>

**3.2.2. Test setup**

The setup of FR, FR-S, and FR-R specimens representing interior column removal scenario is shown in Fig. 3.5.
Figure 3.5 Typical test setup of 2-D RC frame specimens

It consisted of a double-spanning beam with column and beam extension at each side, as well as a short column in the middle representing the joint above the removed column. Note that in this thesis, the short column and its interfaces with the double-spanning beam above the removed column is referred to as “middle joint” and the side columns are simply referred to as “columns”. The supports were simplified as pin connections at contra-flexure points. The frame specimen sat on a simply supported base and was held by two steel horizontal restraints (top and middle) at each side to represent external restraints from adjacent members. Out-of-plane and rotational restraints (middle joint) were provided to prevent out-of-plane movements of the beam and in-plane rotations of the middle joint, respectively. To consider axial loads in the columns, an axial stress of $0.3f_c$ (concrete compressive strength) was applied onto the columns through a self-equilibrating steel configuration.

To represent the penultimate column removal scenarios, the horizontal restraint at the beam end was removed accordingly for IR-1 and IR-2 specimens as shown in Fig. 3.5. The rest of the setup was kept the same as that for FR specimen. All the specimens were loaded at the middle joint through an actuator in a
displacement-control mode at a relatively slow rate of 0.2 mm/s. The test was generally paused at every 0.1 beam depth (18 mm) for the purpose of recording.

3.2.3. Material properties

Concrete grade and reinforcing bar specification were the same for all series of specimens. During the stage of fabricating steel cages, several reinforcing bars were selected from the same batch for the frame specimens. During casting of specimens, concrete cylinders with 300 mm height and 150 mm diameter were cast on-site and later cured under the same condition as that for the frames. Note that since the specimens were scaled down, concrete chippings with the maximum aggregate size less than 10 mm were used. Reinforcing bar samples were used for tensile tests to determine the yield strength, elastic modulus, tensile strength and fracture strain of reinforcement. Compressive cylinder and split-cylinder tests were conducted to determine the compressive and tensile strength of concrete, respectively. All the material tests were conducted in accordance with corresponding ASTM specifications. The material properties are summarised in Table 3.4. Note that fracture strain in Table 3.4 refers to ultimate strain at failure and not to the strain corresponding to the maximum tensile strength of reinforcement. The yield strength for each reinforcing bar (T10, R10, R8 and R6) was obtained by taking the average value of yield strengths obtained from the tensile tests of three bars. The full stress-strain curves for each reinforcing bar are provided in Appendix D.

<table>
<thead>
<tr>
<th>Material</th>
<th>Size</th>
<th>Bar Type</th>
<th>Elastic Modulus (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Strength (MPa)</th>
<th>Fracture Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Reinforcement</td>
<td>T10</td>
<td>Deformed</td>
<td>200</td>
<td>507</td>
<td>609</td>
<td>11</td>
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<td></td>
<td>R10</td>
<td>Smooth</td>
<td>190</td>
<td>400</td>
<td>500</td>
<td>27</td>
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<tr>
<td></td>
<td>R8</td>
<td>Smooth</td>
<td>200</td>
<td>300</td>
<td>395</td>
<td>25</td>
</tr>
<tr>
<td>Stirrup</td>
<td>R6</td>
<td>Smooth</td>
<td>200</td>
<td>400</td>
<td>583</td>
<td>25</td>
</tr>
<tr>
<td>Concrete [150mm(dia.) x 300 mm (height)]</td>
<td>Compressive Strength (MPa)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Splitting Tensile Strength (MPa)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Elastic Modulus (GPa)</td>
<td>26.6</td>
<td></td>
</tr>
</tbody>
</table>
3.2.4. Instrumentation

Reaction measurements

Statically determinate tests were achieved by means of reaction measurements in all the supports. The reactions at bottom pin supports were recorded by load pins, i.e. vertical reaction ($V_r$) and horizontal reaction ($H_r$), the horizontal restraint axial forces ($N$) were measured by two-way load cells as shown in Fig. 3.6.

![Figure 3.6 Reaction measurements in RC frames](image)

Displacement measurements

Displacement measurements included Linear Variable Displacement Transducer (LVDT) and Line Transducer (LT) as shown in Fig. 3.7. B1 to B4 were used to measure the deflection profiles along beams and columns, respectively. J1 and J2 were used to obtain shear distortions of the joint; S1 to S4 to measure horizontal movements of columns and to determine the stiffness of horizontal axial restraints. Lastly, C1 to C4 and M1 to M4 were used to measure flexural and fixed end in-plane rotation at the column and the middle joint, respectively.
CHAPTER 3 EXPERIMENTAL STUDIES OF 2-D RC FRAMES

Figure 3.7 Displacement measurements in RC Frames

3.3. Experimental result of FR specimen

As FR specimen served as a control test to demonstrate the development of ALP in an idealised condition and as a basis of comparison with the other specimens, test result of FR was presented in detail at structural and cross-sectional levels to illustrate the typical development of ALP (CAA and CAT) in 2-D RC frames.

3.3.1. Structural level response

Structural response can be described by load-displacement history (vertical load versus displacement, horizontal reaction versus displacement), failure mode and physical observations through crack patterns. In the presentation of the test results in this chapter, the term “displacement” refers to the vertical displacement at the loaded middle joint measured by the actuator in-built displacement transducer, unless stated otherwise.

Load-displacement

The vertical load-displacement and the horizontal reaction-displacement relationships representing the overall structural behaviour are shown in Figs. 3.8 and 3.9, respectively. For every specimen, the vertical load at each support obtained from the load pin was summed up and checked against the vertical load obtained directly from the actuator built-in load cell. The horizontal reactions at each column obtained from two-way load cells (horizontal restraints) and load...
pins (base supports) were first verified against equilibrium before eventually summed up to obtain overall horizontal reactions. The horizontal reaction forces reflected the internal axial forces in the double-spanning beam, in which negative and positive values denote compression and tension, respectively. The vertical load and horizontal reaction in Figs. 3.8 and 3.9 from both columns indicated a good (symmetrical) test setup.

**Figure 3.8** Vertical load-displacement relationship of FR specimen

**Figure 3.9** Horizontal reaction-displacement relationship of FR specimen
In general, FR specimen went through two stages, i.e. flexural-CAA stage when the horizontal reaction was in compression, and CAT stage when it turned into tension. At small deformation, as the beam deflected downwards, the beam ends tried to push the two column outwards. This was prevented by the supports or horizontal restraints at the column sides, which in return generated the axial forces in the double-spanning beam itself. Due to the axial compression force in the beam in CAA stage, load resistance could be enhanced above the flexural capacity and it is commonly referred to as the “CAA capacity”. Note that the pure flexural capacity of the beam without considering axial force for FR and other specimens with similar reinforcement ratio was calculated to be 24.8 kN based on plastic hinge theory (details are presented in Section 6.3.2). After reaching the CAA capacity and as the beam deflected further, more concrete crushing occurred at the plastic hinge locations allowing the beam to rotate and pull in the two columns on the beam ends.

The columns which were initially pushed outward were being pulled in instead, gradually switching the axial force in the beam from compression to tension. As a result, CAT started to kick in when the axial force in the beam was in pure tension. A first drop in load was observed at displacement of 230 mm due to bottom reinforcing bar fracture at the beam-middle joint interface. CAT continued to resist the applied load with the axial tension force developed in the beam. CAT kept increasing until reaching the maximum capacity of 71 kN before eventually failing at displacement of 547 mm due to fracture of top reinforcing bars at the beam-middle joint interface causing discontinuity to the double-spanning beam as seen in Fig. 3.10. It was found that the top reinforcing bars fractured when the horizontal reaction forces reached the ultimate strength of reinforcing bar. The maximum horizontal force at each side was about 146 kN, resisted by three top T10 reinforcing bars (the bottom reinforcing bars were fractured earlier) with 49 kN each. Divided by 80 mm$^2$ cross sectional area, the stress in each reinforcing bar was equal to 613 MPa, which was close to the ultimate strength of 609 MPa.
Figure 3.10 Failure mode of FR specimen

Crack pattern

The development of cracks and crushing of concrete was divided into three stages as shown in Fig. 3.11:

(1) Stage 1: Prior to CAA capacity (black lines in Fig. 3.11a): flexural cracks which ran almost perpendicular to beam span and diagonal shear cracks were observed to form along the beam and the column, respectively.

(2) Stage 2: Between CAA capacity and the start of CAT (blue lines in Fig. 3.11b): Cracks from Stage 1 continued to widen and flexural cracks due to hogging moment started penetrating from the beam end to the middle joint region. Excessive concrete crushing was observed at both the middle joint and the beam end regions allowing the beam to rotate. Plastic hinges developed at the joint interfaces next to the beam end and the middle joint regions.

(3) Stage 3: CAT to final failure (red lines in Fig. 3.11c): since the beam was under axial tension, cracks continued to develop at roughly constant spacing along the beam especially at the curtailment points to allow more rotation for CAT. Due to axial tension of the beam at CAT stage, cracks started forming at the interior face of the column.
Figure 3.11 Crack patterns throughout the test of FR specimen

Displacement

Fig. 3.12 shows the vertical deflection profile of the double-spanning beam of FR specimen. The deflection profiles at both sides of the beam were identical at small displacement stage. The curvature profile of the beam was clearly observed indicating flexural mechanism. After the fracture of first reinforcing bar, i.e. from displacement of 252 mm onwards, there were some slight differences in the deflection profiles at both sides of the beam but generally the overall deflection profiles at both sides were quite symmetrical throughout the whole test. It could be seen that at CAT stage, the slopes of the deflection curves at the curtailment point from the beam end were larger than those of the beam end (at both sides of the beams), since flexural cracks were more extensively developed at the free end of the curtailed bars, as highlighted earlier in Fig. 3.11c.
Figure 3.12 Deflection profiles along the beams of FR specimen

Fig. 3.13 shows the horizontal deflection profile of the column at both sides throughout the test. It could be seen that from the beginning of load application up to the maximum axial compression, the columns at both sides deflected or were being pushed outwards by the beams. After the development of plastic hinges at the beam, the column was slowly moving back to its original position. It should be noted that the horizontal movements of columns at the two sides were slightly different from each other, e.g. at displacement of 252 mm, the left column was still moving outwards but the right column had started moving inwards. This could be due to differences of stiffness values of horizontal restraint systems as it was impossible to maintain perfectly symmetric condition throughout the test. However, when CAT started kicking in at displacement of 252 mm, both columns began to move inwards until the occurrence of final failure. The maximum outward and inward movements were about -3 mm and 10 mm, respectively.

For simplicity, the outward and inward movements of columns could be represented by horizontal movements of the column joints, obtained from the average value of two LVDTs (S1 and S2 in Fig. 3.7) located at the top and bottom of column joints. The horizontal movement-displacement relationships of the column joints at both beam ends are plotted in Fig. 3.14. Despite slight differences in the maximum horizontal movement and the transition point, the trends at both
beam ends were similar to each other denoted by outward-inward movements and a negligible increase in horizontal movement after displacement of 400 mm.

Figure 3.13 Horizontal movement profiles of columns in FR specimen

Figure 3.14 Horizontal movements of column joints in FR specimen
Two LVDTs (J1 and J2 in Fig. 3.7) were diagonally installed at the column joints to record shear distortions of the column joint panels. Based on geometric relationship, shear distortion could be obtained from the readings of the two potential meters as shown in Fig. 3.15. The terms $\Delta_1$ and $\Delta_2$ refer to the deformations of two LVDTs, positive sign for lengthening and negative sign for shortening, and $a = 210$ mm as indicated in Fig. 3.15. Shear distortion was affected by the formation and severity of cracks in the column joint panel. From Fig. 3.16, at flexural-CAA stage, shear distortion was very small (below $10\times10^{-4}$ rad) as only few (negligible) cracks were formed in the column joint (Fig. 3.11b).
More cracks were formed at the interior face of column joint at CAT stage which explained the increase in shear distortion after displacement of 300 mm, as can be seen in Fig. 3.16. As observed in the test, these cracks were not severe and relatively few, indicating low shear stress induced in the joint panels. This suggested that in frames with beam extensions and with adequate horizontal restraints, shear distortions of the column joint panels could be ignored.

3.3.2. Sectional level response

The cross-sectional level responses including bending moment, axial force, and shear force during the tests for all three specimens were calculated according to static equilibrium as illustrated in Fig. 3.17. It should be noted that since the position of neutral axis varied at different sections, all cross-sectional forces were calculated with reference to the geometrical centre of each section, i.e. based on deformed configurations.
Bending moment

The bending moments along the left and the right beams of FR specimen are presented in Figs. 3.18a and 3.18b, respectively. The profiles at both sides were quite similar which indicated a good set up of the specimen. In general, it can be noticed that at the initial stage, bending moments at both the middle joint and the beam end continued to increase due to axial compression until the flexural capacity was reached. Subsequently, due to concrete crushing and transition of axial compression to tension force, bending moment slowly reduced. After the commencement of catenary action (displacement of 250 mm), the non-zero bending moments suggested that at the CAT stage, the reinforcing bars at critical sections were not in uniform tension, i.e. some bars were under larger tension while others were under smaller tension, or even under compression. With further development of CAT, the bending moments gradually decreased to zero, indicating that the distribution of tension over bars at one beam section tended to be uniform. Eventually, when the neutral axis shifted to the geometric centre of the section (i.e. at the mid-depth), net tension in the top reinforcing bars induced the negative bending moment. It was observed that the moments at any location along the beam were converging towards the same value, which meant that flexural resistance was gradually diminishing.

Figure 3.18 Bending moments along the beams of FR specimen
Shear and axial force

At the middle joint, the vertical resistance could be decomposed into contribution of axial and shear forces via force equilibrium. The decomposition of axial and shear forces clearly illustrated the transition between flexural and the axial (CAT) mechanisms in resisting the vertical load. Bending and axial mechanisms were represented by shear and axial forces, respectively. The shear-axial force decomposition of FR specimen is plotted in Fig. 3.19. The applied load was initially resisted by shear force (flexural mechanism). When the flexural capacity started reducing, contribution of axial mechanism gradually became dominant and started replacing flexural mechanism as the main load-carrying mechanism. The two curves intersected at a displacement of 350 mm and from there onwards, the shear force capacity continued to decrease till zero. The axial force capacity acted as the sole load-carrying mechanism and kept increasing until final failure occurred by fracturing of top reinforcing bars.

![Figure 3.19 Shear and axial forces versus vertical load of FR specimen](image)

The development of ALPs, i.e. flexural-CAA and CAT, was also verified by the variations of strain profile along the beam reinforcement as shown in Figs. 3.20 and 3.21 for the top and bottom reinforcement bars in the left beam, respectively. For the top reinforcement, initially, at displacement 42 mm (flexural capacity), the strain was positive (tension) near the beam end and negative
(compression) near the middle joint, showing flexural mechanism. At displacement of 80 mm (CAA peak), the top reinforcing bar near the beam end yielded. The top reinforcing bar at the beam end continued to yield with increasing bending at the beam end. Flexural mechanism could be inferred by the same contra-flexure point from displacement of 0 to 228 mm. At displacement of 252 mm, when CAT started developing, the strain profile along the beam started moving up towards the positive strain region. At this stage, flexural mechanism was slowly replaced by CAT. At displacement of 342 mm, it can be seen all the top reinforcement along the beam was in pure tension.

**Figure 3.20** Strain profile along top reinforcement of FR specimen (left beam)
Figure 3.21 Strain profile along bottom reinforcement of FR specimen (left beam)

The same behaviour was also observed in the bottom reinforcement. Initially, the load was resisted by flexural mechanism, denoted by a contra-flexure point up to displacement of 252 mm. The contra-flexure point location was quite consistent at both top and bottom reinforcement, i.e. at about 750 mm from the middle joint. At displacement of 252 mm, the strain profile along the beam started moving upwards until the reinforcing bar was under pure tension. The strain at the bottom reinforcing bar near the middle joint reduced with increasing displacement (after displacement of 150 mm) due to damage of reinforcing bar. Basically, at CAT stage, all the top and bottom reinforcement bars were under tension.

3.4. Experimental results of FR-S and FR-R specimens

Test results of FR-S and FR-R specimens were first presented at structural and sectional levels (similar to FR) and subsequently based on these results, comparisons will be made between FR and these two specimens to illustrate the improvement in ALP due to reinforcement detailing.

3.4.1. Structural level response

Load-displacement

The vertical load-displacement and horizontal reaction-displacement relationships of FR-S (curve 2) and FR-R (curve 3) specimens are plotted together with FR specimen (curve 1) as seen in Figs. 3.22 and 3.23. The development of ALP was similar to that of FR, i.e. flexural-CAA at small deflection stage and CAT at large deflection stage. The initial stiffness of the three specimens (Fig. 3.22) was similar, indicating that same boundary conditions with external restraints and beam stiffness were achieved. FR-S specimen has a similar load-displacement profile as FR except for the sequence of bar fracture, i.e. in FR-S specimen, reinforcing bars fractured at a slightly later stage and progressively (one by one). Fracture of the first reinforcing bar occurred at the bottom reinforcement at the beam-joint interface. Final failure of FR-S was due to top reinforcing bar fracture at the beam-column interface (Fig. 3.24). This did not totally end CAT as tension force could still be transferred through bond stress.
The test was stopped due to a safety concern in solely relying on concrete to transfer load. Moreover, higher CAT capacity was not possible due to the fact that almost half of the reinforcing bars were fractured by then.

![Vertical load-displacement relationships of FR, FR-S, and FR-R specimens](image1)

**Figure 3.22** Vertical load-displacement relationships of FR, FR-S, and FR-R specimens

![Horizontal reaction-displacement relationships of FR, FR-S, and FR-R specimens](image2)

**Figure 3.23** Horizontal reaction-displacement relationships of FR, FR-S, and FR-R specimens

The maximum CAA capacity of FR-R was slightly lower as it had a lower flexural capacity (23 kN) to begin with. After concrete crushing occurring at the
beam end interfaces, the neutral axis depth in FR-R reduced further. This explained the relatively low negative horizontal force of FR-R specimen as compared to FR and FR-S specimens. The early commencement of CAT in FR-R specimen was due to the second layer of reinforcing bars which facilitated rotation at the support interface. The CAT capacity of FR-R specimen was significantly improved and enhanced due to an earlier commencement of CAT and fracture of first bottom reinforcing bar at the beam-joint interface was delayed to the very last stage (displacement of 550 mm). The specimen was yet to fail (due to large fracture strain in mild steel bars) and its capacity continued to increase at the moment the test was stopped, due to limitation of actuator stroke length.

![Failure Mode of FR-S specimen](image)

**Figure 3.24** Failure Mode of FR-S specimen

**Crack pattern**

The development of crack patterns in FR-R specimen is illustrated in Fig. 3.25. Each colour of the line represents different stages, i.e. black lines denotes from the start of test to CAA peak, blue line indicates from CAA peak to the commencement of CAT, and red line shows from CAT to the end of test. Note that the crack pattern of FR-S specimen was not drawn as it has similar development as that of FR specimen, except for slightly more distributed cracks along the beam. Due to closer stirrup spacing in FR-S specimen, localised damage was prevented and distributed to the remaining parts of the beam, thereby increasing the ductility. In FR-R specimen, moment transfer was not so effective due to lower bond stress of round bars. This resulted in only a few flexural cracks formed before CAA peak and none after. Only after the commencement of CAT, full depth tension cracks started to develop. The full depth cracks occurred along the beam with the largest crack width at the curtailment point as shown in Fig. 3.25b. The larger number of full-depth cracks developed in FR-R specimen.
allowed load to be distributed and more slip to occur, which in turn increased the deformation limit (ductility).

![Crack pattern in FR-R specimen](image1)

a) Crack patterns in FR-R specimen

![Crack patterns and opening in FR-R specimen](image2)

b) Crack patterns and opening in FR-R specimen

**Figure 3.25** Crack patterns of FR-R specimen

**Displacement**

The beam deflection profile for FR-R specimen is shown in Fig. 3.26. The beam deflection profile of FR-S specimen was not plotted as it has almost identical deflection to that of FR. In Fig. 3.26, it can be seen that there was almost no curvature in FR-R specimen and the double-spanning beam behaved exactly like multi-piecewise straight segments. The absence of curvature was due to ineffectiveness of moment transfer and the segment-type behaviour was due to full-depth cracks development along the double-spanning beam.
Figure 3.26 Deflection profiles along the beams of FR-R specimen

Figs. 3.27a and 3.27b show the column joints horizontal movements of FR-S and FR-R specimens, respectively. Generally, outward and inward movements of the column joint in both specimens were observed, except for a more rapid inward movement of FR-R column joints at CAT stage, since it was easier to mobilise CAT in FR-R specimen (straight line without curvature).

Figure 3.27 Horizontal movements of column joints in FR-S and FR-R specimens

The column joints shear distortions of FR-S and FR-R specimens are plotted together with that of FR specimen in Fig. 3.28 (Fig. 3.28a for the left column joint and Fig. 3.28b for the right column joint). The provision of closer stirrup spacing
in the column joint under seismic detailing was effective in minimising the shear damage in the joint as indicated by a significant reduction in shear distortion from about $30 \times 10^{-4}$ to $5 \times 10^{-4}$ rad. This was significant in the presence of large axial tension force in the beam, which in turn generated large shear forces in the column joints. Without proper confinement, shear cracks would develop and in turn weaken the joint capacity to support the axial tension force. No major difference was found in FR-R specimen as it had the same stirrup detailing as FR specimen and underwent the same mechanism, i.e. flexural-CAA followed by CAT.

![Shear distortions comparisons among FR, FR-S, and FR-R specimens](image)

Figure 3.28 Shear distortions comparisons among FR, FR-S, and FR-R specimens

3.4.2. Sectional level response

**Bending moment**

![Bending moments along the beams of FR-R specimen](image)

Figure 3.29 Bending moments along the beams of FR-R specimen
Shear and axial force

The axial and shear force interactions are plotted in Figs. 3.30a and 3.30b for FR-S and FR-R specimens, respectively. FR and FR-S had rather similar profiles, such as the same displacement corresponding to the shear and axial intersection, and similar axial force profile. Since the fracture of the middle bottom bar in FR specimen occurred earlier than FR-S specimen, it was observed that shear force tended to drop earlier as well, reducing its contribution to total vertical resistance. Between FR-R and FR specimens, it was observed that there was a similar axial force profile; the transition from flexural to CAT mechanisms, as well as the gradual reduction to zero after fracture of middle bottom bars. The similar axial profile was due to same setup with approximately the same axial restraint, beam cross section and reinforcement detailing in all three specimens. The higher capacity of FR-R was due to an earlier commencement of CAT and hence the transition/intersection point occurred at a smaller displacement. It was interesting to note that as the middle bottom reinforcing bar fracture was delayed till a very late stage, there was a reserve shear force capacity of about 5 kN from displacement of 400 to 550 mm, which provided a positive contribution to the total vertical resistance.

![Shear and Axial Force](image)

**Figure 3.30** Shear and axial forces versus vertical load of FR-S and FR-R specimens

The strain development profiles of FR-R specimen in Figs. 3.31 and 3.32 also show a transition from flexural to axial (CAT) mechanisms. Some observed differences from FR-S and FR specimens are as follows: (1) due to few flexural
cracks taking place, the strain profile did not show any significant changes from flexure to CAA and the contra-flexure point was not well defined; (2) at CAT stage, the increase in strain of FR-R was mainly in the region between the two curtailment points, whereas in FR and FR-S specimens, the strain increase was mainly in the plastic hinge region next to the beam-column interface.

Figure 3.31 Strain profile along top reinforcement of FR-R specimen (left beam)

Figure 3.32 Strain profile along bottom reinforcement of FR-R specimen (left beam)
3.4.3. Effect of detailing

Based on the test results at structural and cross-sectional levels, some enhancements from seismic detailing and longitudinal round bars could be observed. Generally, FR-S specimen contributed mainly to ductility by delaying the reinforcing bar fracture, the drop in shear capacity, as well as increasing the final deformation. No significant enhancement of ALP capacity was observed. This was because at the later stage, after bottom bar fracture and moderate damage to concrete had occurred, flexure was replaced by CAT mechanism which did not rely much on the presence of stirrups. Nevertheless, the provision of more stirrups at the support region showed the effectiveness in minimising shear damage in the column joint. This was significant in the presence of large axial tension force, which in turn generated a large shear to the column joint.

The enhancement in ALP capacity was more obvious in FR-R specimen, mainly due to an earlier commencement of CAT, a delayed in bottom bar fracture, larger vertical deflection, and large fracture strain of reinforcing bars. Note that the earlier commencement of CAT was due to an adjustment made in reinforcement ratio to compensate for the lower strength of round bars. The delay in bottom bar fracture allowed residual capacity of shear to contribute to overall strength. The fewer cracks at flexural-CAA stage were undesirable as it generated fewer slips, but this was compensated by full-depth cracks at CAT stage. In addition, the large strain capacity and lower bond stress of round bars allowed more slips to occur, contributing to vertical deflection.

3.5. Experimental results of IR-1 and IR-2 specimens

3.5.1. Structural level response

Load-displacement

The vertical load-displacement and horizontal reaction-displacement relationships of IR-1 (curve 2) and IR-2 (curve 3) specimens together with FR (curve 1) are plotted in Figs. 3.33 and 3.34, respectively.
Clearly, the vertical load and horizontal reaction profiles among the three specimens were identical at flexural-CAA stage. At CAT stage, it was observed that if axial tension force in the beam could not increase, then this would result in a flat plateau with the maximum vertical capacity of about 35 kN. This was due to inability of the outermost column to support the increase in beam axial forces after attaining its flexural capacity. Due to a lack of horizontal restraint, the
column was pulled inward excessively, which generated P-δ effect to the column. In addition, there was a danger that the hydraulic jack in the column might slip off from the rotated top end. Hence, the test had to be stopped for safety reason.

Crack pattern

Since the double-spanning beam and the interior columns of IR-1 and IR-2 had similar crack patterns to those of FR specimen, only the formation of cracks at the outermost column (with no middle horizontal restraint) is illustrated in Fig. 3.35. In IR-1 and IR-2 specimens, when the beams were pushed outwards during the flexural-CAA stage, flexural cracks were formed at the exterior face of the column, while in the CAT stage, cracks were formed at the interior face. Excessive cracking at the column joint was clearly observed in IR-1 and IR-2 specimens, which was due to missing horizontal restraints which in turn limited the development of CAT in the double-spanning beam.

![Crack patterns in exterior column of IR-1 specimen](image)

**Figure 3.35** Crack patterns in exterior column of IR-1 specimen

Displacement

The horizontal movement profiles of IR-1 columns are shown in Fig. 3.36. The outward movements at flexural-CAA stage (displacement from 0 to 65 mm) and inward movements at CAT stage (displacement of 257 mm) were also observed. The effects of imperfect restraints are clearly illustrated in Fig. 3.36. Obviously, due to a weaker restraint on the right column, it deformed more in both the inward and outward directions as compared to the column with sufficient restraint. As the displacement was progressively increased from 329 to 382 mm, there was only a slight pulling-in on the left column. However, there was an excessive pulling-in
on the right column and this raised the concern on the stability of the outermost column. The maximum outward and inward movements on the left column were -2 mm and 8 mm, respectively, whereas for the right column, the maximum outward and inward movements were -3 and 15 mm, respectively.

Figure 3.36 Horizontal movement profiles of columns in IR-1 specimen

The column joint horizontal movements of IR-1 and IR-2 are shown in Fig. 3.37. Due to missing restraint at the beam end, the right column joints in both IR-1 and IR-2 specimens were subjected to larger outward and inward movements. After the column flexural capacity was reached at displacement of about 350 mm and 400 mm for IR-1 and IR-2, respectively, the column joint was pulled in excessively with an increase in displacement.

For column joint shear distortion, obviously the right column joint (no middle horizontal restraint) was subjected to larger distortion due to more damages induced in the unrestrained column joint. At CAT stage, after column flexural failure had occurred, extensive cracks in the column joint resulted in a rapid increase in shear distortion. The maximum shear distortion of 110x10^-4 rad in IR-2 was about 3 times to that of FR (30x10^-4 rad).
3.5.2. Sectional level response

Bending Moment

The bending moment along the double-spanning beam of IR-2 specimen is presented in Fig. 3.39, which is representative of imperfect restraint condition since IR-1 had a similar bending moment profile with IR-2. The bending moment profile of IR-2 left beam (with adequate restraint) was similar to that of FR specimen, i.e. at the initial stage, the bending moments at the middle joint and the beam end reached the ultimate capacity (in the presence of axial compression). Then due to concrete crushing and transition of axial compression to tension force, the bending moment slowly reduced. Subsequently, the moment at the
middle joint interface reversed in sign, from a positive bending (tension in bottom reinforcement) to negative bending (tension in top reinforcement) indicating all reinforcing bars were in tension. And finally, the moments at any location along the beam were converging towards the same value indicating the end of flexural capacity.

On the right span, after the first drop of moment due to bar fracture, instead of converging towards one value, the moment along the beam was turning towards the positive moment. This was because the horizontal load on the right column of IR-2 was decreasing (Fig. 3.34) while the vertical load was somewhat increasing (Fig. 3.33). Hence, it can be concluded that due to inability of horizontal restraint to fully support the development of CAT, some flexural capacities remained to resist the vertical load.

![Bending moments along the beams of IR-2 specimen](image)

**Figure 3.39** Bending moments along the beams of IR-2 specimen

Shear and axial force

The shear and axial force interactions of IR-1 and the IR-2 specimens are shown in Figs. 3.40a and 3.40b, respectively. For IR-1 and the IR-2, a similar phenomenon to that of FR was observed from the beginning until the intersection of axial and shear forces. From this point onwards, although an increment in axial force was observed, it was much milder than the FR specimen and just barely compensated the drop in bending capacity. This was because the column without middle horizontal restraint had reached its maximum flexural capacity and was
unable to resist any further increase in horizontal reaction force. Hence, the total net vertical resistance was unable to be increased significantly.

Figure 3.40 Shear and axial forces versus vertical load of IR-1 and IR-2 specimens

Failure of Column

Column analyses were performed to shed light on the failure modes of IR-1 and IR-2 specimens. The columns were preloaded to 0.3$f_cA_c$ to simulate axial loads from upper storeys ($f_c$ is the compressive strength of concrete and $A_c$ is the column cross-sectional area).

Figure 3.41 Strain profile of column reinforcement at column joint region of IR-1 specimen

As the double-spanning beam entered into CAT phase, its tension force ($N_b$) was resisted by the columns on each side. Under the combined actions of $N_b$ and 0.3$f_cA_c$, the columns were pulled inwards and experienced P-δ effect. The
attainment of column flexural capacity (column flexural failure) had been verified from the readings of strain gauges located at the column joint, as shown in Fig. 3.41.

**Figure 3.42** Flexural failures in columns of IR-1 and IR-2 specimens

Based on plastic hinge theory and considering P-δ effect as illustrated in Fig. 3.42, at ultimate deformation $\Delta$, the calculated maximum value of $P_{c,fl}$ to attain column plastic moment of resistance ($M_p$) can be determined as follows:

$$P_{c,fl} = \frac{[L_1 + L_2][M_p - N_c * \Delta]}{L_1 L_2}$$

(3 – 1)

where $P_{c,fl}$ is the calculated axial force acting on the column, $\Delta$ is the column joint horizontal displacement, $M_p$ is the column plastic moment of resistance, and $N_c$ is the column axial load. The terms $L_1$ and $L_2$ are the lengths of the upper (720 mm) and lower columns (830 mm), respectively. The calculated values of $P_{c,fl}$ from Eq (3-1) and the corresponding measured beam axial forces $N_b$ at the same $\Delta$ are plotted in Fig. 3.43. Note that (1) the column flexural capacity $M_p$ would limit the development of beam axial force. Hence, the calculations for $P_{c,fl}$ were based on initially straight column configuration from the beginning of the test; (2)
variations of $P_{c,\beta}$ for IR-1 and IR-2 specimens were not the same due to slight differences in column axial force $N_c$, and (3) the beam axial force $N_b$ was assumed to act perpendicular to the column joint as the relative rotation between the beam and the column at the end of the test was about $11^\circ$ ($\cos 11^\circ = 0.98$). In Fig. 3.43, the values of $P_{c,\beta}$ for IR-1 and IR-2 specimens coincided with the beam axial force at $\Delta$ of about 9 mm and 17 mm, respectively, which corresponded to vertical displacement of 350 mm and 430 mm. These displacements corresponded to the instant when the axial force in the beam stopped increasing and approached column failure (Fig. 3.33).

Following the attainment of column flexural capacity (displacement of 350 mm of IR-1 and displacement of 430 mm of IR-2), the column joint continued to be pulled in (Fig. 3.44a) by $N_b$ and with both column ends rotating. It was obvious from Fig. 3.43 that there would not be any further increase in beam axial force $N_b$ for CAT as the column flexural capacity ($M_p$) had been attained by then. The misalignment between the column and the hydraulic jack also resulted in a partial release of column axial force $N_c$ as shown in Fig. 3.44b, which slightly reduced the P-δ effect in the column. However, if the tests were to be continued and the column axial force could be maintained, failure of the system would be governed by instability of columns.

Figure 3.43 Comparisons of $P_{c,\beta}$ and measured axial force in beam of IR-1 and IR-2 specimens
CHAPTER 3  
EXPERIMENTAL STUDIES OF 2-D RC FRAMES

3.5.3. Effect of horizontal restraint

At structural level, although all specimens went through two similar stages followed by similar cracking and crushing patterns, both IR-1 and IR-2 specimens were unable to fully utilise the capacity of CAT (until fracture of final reinforcing bars). Under an imperfect restraint condition, the beam axial tension force in CAT stage was limited by the column flexural capacity. Moreover, as the penultimate column moved in further, instability of column might occur. Therefore, failure of FR could be referred to as “beam failure”, whereas “column failure” occurred in IR-1 and IR-2 specimens. The CAA capacity of the three specimens was less affected by horizontal restraints due to large span-to-depth ratio of the double-spanning beam.

When comparing IR-1 with IR-2, although one may expect that the former should possess a slightly higher CAA and CAT capacities, the actual test results showed that the difference in resistance between these two specimens was negligible; IR-1 specimen basically behaved like IR-2. This could be supported by several reasons: (1) the differences between their CAA and CAT capacities were insignificant and (2) the slight differences could be due to the middle horizontal restraint force in IR-1 specimen as shown in Fig. 3.45 (decomposition of horizontal reactions along the restrained left column), which suggested that the

---

**Figure 3.44** Joint horizontal displacements and reduction in axial force of exterior columns of IR-1 and IR-2

- a) Column joint deformation ($\Delta$)-displacement relationship
- b) Column axial force ($N_c$)-Joint deformation ($\Delta$) relationship
presence of the middle horizontal restraint was insignificant. Fig. 3.45 also explained the small outward (-2 mm) and inward movements (8 mm) of the left column joint (adequate restraint) (in Fig. 3.37) as compared to those of FR specimen (in Fig. 3.14). The horizontal reaction at the middle restraint mainly contributed to the column joint horizontal movement since both (the column joint and middle restraint) were located at the same height. In the absence of horizontal reaction at the middle restraint, the column joint horizontal movement was also limited.

![Graph showing horizontal reaction forces and displacement](image)

**Figure 3.45** Decomposition of horizontal reaction forces at the restrained side (left column) of IR-1 specimen

From the shear-axial force interactions to vertical load, generally, the flexural mechanism (shear force) developed first, and axial tensile mechanism (CAT) would be mobilised later to replace the flexural mechanism. In FR specimen, the presence of horizontal restraints managed to support the successful development of CAT, which resulted in an increase of resistance until the fracture of top reinforcing bars. In IR-1 and IR-2 specimens, the slight contribution of axial tension force could not compensate the declining shear force or flexural mechanism. Hence, there was no significant increase in resistance.
3.6. Pseudo-Static response

Although only quasi-static tests were conducted, the dynamic behaviour of the specimens could be evaluated by converting the quasi-static load response into pseudo-static response based on energy approach suggested by Izzuddin et al. [12]. The corresponding gravity load (pseudo-static load) leading to dynamic displacement could be obtained by equating the external work done \( W_n \) and internal energy \( U_n \) as illustrated in Fig. 3.46. From the non-linear static load response (internal work), the level of dynamic gravity load \( \lambda_n P_o \) can be obtained as follows:

\[
W_n = \alpha \lambda_n P_o u_{d,n}; \quad U_n = \int_{0}^{u_{d,n}} \alpha P du_s
\]  

(3 – 2)

And since \( W_n = U_n \), the equation can be further simplified to:

\[
P_n = \lambda_n P_o = \frac{1}{u_{d,n}} \int_{0}^{u_{d,n}} P dU_s
\]  

(3 – 3)

This method is essential in practice as it is difficult to conduct experimental studies or numerical analyses under dynamic scenarios while at the same time considering progressive collapse scenarios. However, it should be noted that this method is only valid for the freefall scenario, but does not apply to blast effects on structures. The comparison between the converted pseudo-static response and the quasi-static response from the test is plotted in Fig. 3.47.

Figure 3.46 Static to Pseudo-static conversion procedure [12]
The pseudo-static response curve basically showed the corresponding displacement when the applied load had been stabilised. For example in FR specimen, if the load applied was 20 kN (below the first peak), when the column was suddenly removed, the specimen would deflect downwards and stop at about 100 mm (no collapse). If the load exceeded the first peak say 30 kN, the beam...
would continue to deflect until a corresponding displacement of 500 mm was reached. And finally, if it exceeded the maximum value, the beam would continue deflecting until collapse occurred. Fig. 3.47 also shows that the enhancement of CAT under dynamic (pseudo-static) condition may not be as high as in the static condition. The pseudo-static responses of all the 2-D RC frame specimens were converted from the test results and presented in Fig. 3.48. The actual loading on the frame can be checked against the pseudo-static capacity to determine the maximum deflection or if progressive collapse would occur.

3.7. Summary

In this chapter, the test results of 2-D RC frames subjected to a middle column removal scenario were discussed comprehensively. Based on the test findings, following conclusions are obtained:

1. **Load capacity and ALP development**: all five of the 2-D RC frame specimens exhibited similar ALP developments, i.e. flexural-CAA stage followed by CAT stage at large displacement. The curtailment point next to the beam end was helpful in improving the ductility of the beam as secondary hinges would take place there. These hinges allowed rotations and prevented localised damages at the primary plastic hinges (beam-column interface). To clearly show the effect of detailing and horizontal restraint on ALP development, the ALP capacities for all specimens are summarised in Table 3.5.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_{b,fl}$ (Flexural Capacity)</th>
<th>$P_{CAA}$ (Max. CAA Capacity)</th>
<th>$P_{CAT}$ (Max. CAT Capacity)</th>
<th>$P_{CAA}/P_{b,fl}$</th>
<th>$P_{CAT}/P_{b,fl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR</td>
<td>24.8 kN</td>
<td>27.5 kN</td>
<td>70.9 kN</td>
<td>1.12</td>
<td>2.89</td>
</tr>
<tr>
<td>FR-S</td>
<td>24.8 kN</td>
<td>27.5 kN</td>
<td>66.0 kN</td>
<td>1.12</td>
<td>2.66</td>
</tr>
<tr>
<td>FR-R</td>
<td>22.8 kN</td>
<td>24.5 kN</td>
<td>75.9 kN</td>
<td>1.07</td>
<td>3.33</td>
</tr>
<tr>
<td>IR-1</td>
<td>24.8 kN</td>
<td>27.2 kN</td>
<td>34.1 kN</td>
<td>1.11</td>
<td>1.39</td>
</tr>
<tr>
<td>IR-2</td>
<td>24.8 kN</td>
<td>25.8 kN</td>
<td>29.4 kN</td>
<td>1.04</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Seismic detailing seemed to have negligible effect on enhancing ALP capacity as the load-carrying capacity relied on axial force (CAT) in the beam which was independent of stirrup spacing. The usage of round bars seemed to be
effective in improving the capacity of CAT by increasing ductility (more slip and greater fracture strain). Larger displacement led to larger rotation, which resulted in greater tension force being converted to vertical resistance. The CAT capacity of 75.9 kN in FR-R specimen was the greatest among the 2-D RC frame specimens and may potentially be higher as its capacity was still increasing at the moment when the test was stopped. With regard to horizontal restraint, CAT capacity was significantly reduced and limited by the outermost column (no middle horizontal restraint) flexural capacity.

2. **Failure mode:** In the fully restrained frames (FR, FR-S, and FR-R specimens), CAT capacity was fully utilised until the fracture of final reinforcing bar. In IR-1 and IR-2 specimens, CAT capacity was limited to the outermost column flexural capacity. As displacement increased further, the column would be pulled in more and more by tension force in the beam accompanied by excessive damage of the column joint. Depending on column reinforcement details, shear failure might occur. Generally, failure in frames under penultimate column removal case would generally be governed by column failure as large tension force in the beam (during CAT stage) was not considered.

3. **Beam vertical displacement:** All specimens exhibited great rotational ductility. Neglecting IR-1 and IR-2 specimens (as it was difficult to determine displacement corresponding to a drop in load), FR and FR-S were able to reach a maximum of 550 mm which was 11.5% of double-spanning beam length and 13° of rotation.

4. **Column horizontal movement:** The columns which were fully restrained had only negligible horizontal displacements. In IR-1 and IR-2 specimens, it was observed that after columns had failed, they were pulled in rapidly with an increase in displacement due to severe weakening of column joints coupled by P-δ effect. This sudden increase in displacement coupled with failure of columns signalled the possibility of progressive collapse.

5. **Shear distortion in column joint panel:** The column joint distortion in fully restrained specimens was negligible. This was not the case in IR-1 and IR-2
specimens in which shear distortions increased to almost three times of those of FR or FR-R column joints. The effectiveness of increasing the stirrup ratio at the column joint based on seismic detailing (demonstrated by FR-S specimen) should be adopted for specimens with imperfect restraints to minimise shear distortions in the column joints.

From this study, several recommendations on the design of buildings against progressive collapse to improve ALP development and capacity could be established. The results of FR-S and FR-R specimens provided good insight on the way to improve the ductility and enhancement of ALPs. The provision of stirrups following seismic detailing rule can be easily applied. The usage of round bars as longitudinal reinforcement is uncommon in practice as the lower bond stress is ineffective in concrete-reinforcement stress transfer and will require much longer anchorage/ development length. However, in the development of ALPs, lower bond stress can be a desirable feature allowing larger slip and greater vertical deflection. Based on the test observations, the location of major cracks leading to slip usually occurs near the support interface and the curtailment of reinforcement. Hence, one of the ways to improve ALP in practice is by reducing bond stress of deformed longitudinal bar at these critical locations, e.g. by removing reinforcing bar ribs or enclosing the ribbed surface with a plastic sheath [Y5].

In the design of ALP under penultimate column removal scenarios, to avoid multiple analyses of different column detailing or usage of complex elements to capture different types of column failures (stability, shear, or flexural), one way is to obtain the ultimate CAT capacity (axial forces) under fully restrained condition. Subsequently, the column could then be designed to withstand a higher level of performance commensurate with the desired extent of CAT development in the double-spanning beam.
CHAPTER 4: EXPERIMENTAL STUDIES OF 3-D RC FRAMES

4.1. Introduction

Structural behaviour and ALP development in 2-D RC frames had been thoroughly studied in Chapter 3. Since progressive collapse involves global (large part of building) failures, it is imperative to extend the study of ALP development in 2-D RC frames to a 3-Dimensional (3-D) setting. In this series of tests, the behaviour of frames subjected to direct torsion and compatibility torsion (3-D skeletal frame) was investigated.

The objective of the direct torsion test was to investigate the effect of torsion in ALP development of the double-spanning beam. Previous test on 2-D RC frame with adequate horizontal restraints (FR specimen) had demonstrated the development of CAA and CAT to enhance structural capacity and increase ductility of RC frames. CAT in particular is of high interest among researchers as it may be activated at the last stage and sometimes may have a much greater capacity than pure flexural capacity. Since CAT controls the final failure load, it is important to study if CAT may be compromised due to other effects such as torsion. In 3-D structures, beams connected to the removed column are subjected to induced torsion from adjacent slabs, but such effects on ALP behaviour are hardly investigated. Moreover, actual collapse event (Murrah building collapse) [O2] and beam-slab substructure tests (penultimate column removal scenarios) by Pham et al. [P5, P6, P7] had reported collapses due to torsional failure. Tensile Membrane Action (TMA) from the floor generated twisting in the edge beam and eventually resulted in torsional failure in the affected beam above the removed column. The failed beam could no longer develop ALP (CAA or CAT) and support TMA of slabs, resulted in failure of the structure. The claims of edge beams’ torsional failures in the aforementioned cases leading to weakening of tensile membrane action in slabs, were solely based on observations without any quantification of torsion effects on structural resistance [O2, P5, P6, P7]. This was due to practical difficulty in isolating and quantifying behaviour of the edge beam
from the beam-slab system. Recent tests on 2-D RC frames subjected to contact
detonation \[Y2\] also revealed that horizontal forces from the explosive charges
induced out-of-plane actions, including torsion and lateral bending moment to the
double-spanning beam. It can be deduced that firstly, in a progressive collapse
scenario where the beam may be subjected to significant torsion or twisting,
development of ALP mechanisms solely based on in-plane loading of beam or
frame tests may not be realistic and unconservative. Secondly, it is essential to
conduct a thorough and detailed study on torsion-induced failures of beams. The
continuous torsional moment in the edge beam was idealised as a concentrated
point load acting at the middle joint (column removal location) at a certain load
eccentricity to simplify the test setup. Such simplified arrangement was adopted
due to physical constraints of the laboratory and the huge practical difficulty in
applying multiple point loads on the beam under a displacement-control mode,
especially when 3-D actions including twisting deformations were involved (viz.
axial tension, shear, bending and torsion). Two different eccentricities were
selected to quantify the torsion effects on the beam behaviour in the ALP phase,
i.e. T(0.5h) and T(1h) specimens with load eccentricities equal to half of total
beam depth (90 mm) and full beam depth (180 mm).

The objective of the 3-D skeletal frame tests is to demonstrate ALP
development in frames under a more realistic column removal scenario. Two
different column removal scenarios i.e. CORner (COR specimen) and EXTerior
(EXT specimen) column removal scenarios were studied, which are among the
most critical scenarios as both the corner and exterior columns (outermost edge of
the building) are prone to attacks, and the absence or lack of restraint from the
outermost edge may result in a lower structural resistance calculated based on 2-D
tests with adequate adjacent restraints. Table 4.1 summarises the four specimens
tested in this series.

<table>
<thead>
<tr>
<th>Objective</th>
<th>No.</th>
<th>Specimen</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Torsion</td>
<td>1</td>
<td>T(0.5h)</td>
<td>Direct Torsion with load eccentricity = 90mm (0.5 beam depth)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>T(1h)</td>
<td>Direct Torsion with load eccentricity = 180mm (1 beam depth)</td>
</tr>
<tr>
<td>Skeletal Frame</td>
<td>3</td>
<td>COR</td>
<td>Frame under CORner column removal scenario</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>EXT</td>
<td>Frame under EXTerior column removal scenario</td>
</tr>
</tbody>
</table>

Table 4.1 Specimens in 3-D RC Frame tests
4.2. Test preparation

4.2.1. Specimen design and material properties

As the objective was to study the 3-D effects on RC frames by comparing the current specimens with the 2-D RC frame control test (FR), the specimen details and material properties were kept the same as those of FR specimen (Section 3.2) as shown in Fig. 3.2 and Table 3.4, respectively.

4.2.2. Test setup

Direct torsion

The setup for these two specimens was similar to that for FR specimen, except that the point load was applied with a certain eccentricity at the enlarged middle joint. To prevent columns from toppling in the out-of-plane direction, a lateral steel restraint was provided at the mid-height of each column. A 3-D view of the test setup for T(0.5h) and T(1h) specimens is shown in Fig. 4.1. The out-of-plane and rotational restraints were removed to allow for out-of-plane and twisting of the double-spanning beam.

![Test setup for T(0.5h) and T(1h) specimens](image)

**Figure 4.1 Test setup for T(0.5h) and T(1h) specimens**

As mentioned in the introduction, application of multiple-point loads on the beam via a hydraulic actuator under a displacement-control mode would be far too complex in practice. Hence, the tests conducted in this study focused on the
behaviour of beams subjected to a combination of in-plane loading and torsion generated by an eccentric point load. To determine a reasonable range of load eccentricities which may represent a realistic combination of shear and torsion under typical column removal scenarios, preliminary numerical analyses were conducted. Two types of torsion may be induced by adjacent slabs to the beam in 3-D beam-slab systems under column removal situations, namely, direct torsion and compatibility torsion. In practice, direct torsion can take place in an exterior frame with a short cantilever slab (Scenario 1, Fig. 4.2a), while compatibility torsion will occur when an exterior frame is subjected to the pulling-in actions from internal slabs (Scenario 2, Fig. 4.2b). While it is difficult to measure the effect of compatibility torsion, it is relatively straightforward to conduct direct torsion tests, and thus, such tests on the edge beams were undertaken in this study. These two scenarios were used for the determination of torsion eccentricities, which was taken as the ratio of torsion-to-shear-forces on the affected beams.

![Diagram of torsion scenarios](image)

**Figure 4.2** Frames subjected to torsion scenarios

The cross-sectional forces on the affected beams were obtained based on linear elastic analyses in ETABS (Version 8) [E4] as the objective was to obtain some ideas on the magnitude of torsion and shear transferred to the affected double-spanning beam. Fig. 4.3a shows the ETABS model and the associated deformed shape for Scenario 1 corresponding to an exterior frame with a 1.2 m long
cantilever slab (half of the internal beam span), while Fig. 4.3b shows an exterior frame with pulling-in actions of an internal slab. The dimensions used in the modelling of beams and slabs were in accordance to the specimens. Note that for simplicity, all the beam ends were assumed to be fixed. Gravity and imposed loads of 7 kN/m² were applied to the slabs.

![Diagram](image1)

(a) Model and deformed shape for Scenario 1

![Diagram](image2)

(b) Model and deformed shape for Scenario 2

**Figure 4.3** Models used in preliminary analysis

The torsion and shear force diagrams along the affected double-spanning beam are plotted in Figs. 4.4 and 4.5 for Scenarios 1 and 2, respectively. Note that the diagrams are only plotted on a single span due to symmetry. Locations 0 and 2.4 refer to the middle joint (MJ) and the beam end (BE), respectively. As the beam basically sustains uniform distributed load (UDL) from the slab, the shear and torsion forces have the smallest value at the MJ (column removed location) but increase progressively towards the BE. The maximum shear force in Scenario 1 at the BE is greater than that of Scenario 2 due to triangular loads on scenario 2. On
the other hand, torsion is smaller in Scenario 1 as torsion effect at the larger span (2.4 m) is somewhat partially balanced by that of the shorter span (1.2 m).

Figure 4.4 Shear and torsion diagrams in affected beam for Scenario 1 (direct torsion)

Figure 4.5 Shear and torsion diagrams in affected beam for Scenario 2 (compatibility torsion)

Based on the torsion and shear force diagrams, the torsion-to-shear-force ratios at a few locations along the double-spanning beam are plotted in Fig. 4.6. Different torsion-to-shear-force ratio profiles are observed in the two scenarios, i.e. the maximum torsion-to-shear-force ratio for Scenario 1 takes place at the MJ. For scenario 2, it occurs at the BE. The range of torsion-to-shear-force ratios for Scenarios 1 and 2 vary from 0.17 m to 0.05 m, and from 0.1 m to 0.02 m, respectively. As a result, the two upper bound values, i.e. 0.17 m and 0.1 m were selected to investigate the torsional behaviour of the double-spanning beam, so that torsional failure could be observed in the beam and torsional effect on the development of ALPs could also be quantified. For ease of reference, the values
were slightly adjusted from 0.17 m to 0.18 m for T(1h) specimen and from 0.1 m to 0.09 m for T(0.5h) specimen.

**Figure 4.6** Torsion to shear force ratio in affected beam for Scenario 1 (direct torsion) and Scenario 2 (compatibility torsion)

**Figure 4.7** Ball and socket joint loading system in T(0.5h) and T(1h) specimens and associated degree of freedoms
An actuator in a displacement-control mode was employed to apply a point load at the middle joint of the specimens. The test was paused at 18 mm increment (1/10 beam depth) of displacement, so that there was time to trace cracks developing on the beam surfaces, take measurements, and check the condition of the specimen. It should be noted that “displacement” is defined as the magnitude of the actuator’s linear movement at the loading location and its value can be obtained from the in-built displacement sensor. For FR specimen, the actuator head was directly in contact with the middle joint. For T(0.5h) and T(1h) specimens, in order to maintain contact between the actuator head and the top of the middle joint, and to keep load eccentricity constant throughout the test while the middle joint experienced large twisting, a ball-and-socket loading system was employed on top of the MJ as shown in Fig. 4.7 to maintain verticality of applied load.

Skeletal Frame

The RC skeletal frame test configurations were designed based on FR specimen. The boundary or support conditions of RC skeletal frames were similar to those of FR, i.e. all the member ends were represented by pin supports at the contra-flexure points, and the axial restraints from adjacent structures were represented by two steel horizontal restraints, i.e. one placed at the column top and another at the beam end. In COR specimen (Fig. 4.8a), one side of the double-spanning beam in FR specimen was re-oriented to the transverse direction and in EXT specimen (Fig. 4.8c), a single-spanning beam in the transverse direction was added to the double-spanning beam of FR specimen. The labelling of the respective beams, i.e. Left (L), Right (R), and Transverse (T) and the coordinate axes, i.e. x, y, and z are included in Fig. 4.8a and 4.8c. A point load was applied on top of the middle joint by an actuator in a displacement-control mode. As twisting and out-of-plane movements were expected at the middle joint, a ball-and-socket system was also employed to maintain contact surface between the actuator and the middle joint (Fig. 4.8b). Note that column axial force was not applied in the RC skeletal frames (COR and EXT specimens). The test result of FR showed that under fully restrained conditions, load capacity and failure were
governed by the beam and hence the effect of axial force in the column was negligible.

Figure 4.8 Test setup and loading system in COR and EXT specimens
4.2.3. Instrumentation

Reaction measurements

In direct torsion specimens (T(0.5h) and T(1h)), in addition to reaction measurements on frames (similar to FR specimen), the axial forces of lateral (out-of-plane) restraints were measured by two-way load cells attached on them. As the RC skeletal frames had the same boundary conditions with those of FR specimen, the reaction measurement systems were also identical, i.e. load pin and two-way load cell were used to measure the reactions at bottom pin supports and the axial forces of horizontal restraints, respectively.

Displacement measurements

In 3-D RC frame tests (direct torsion and skeletal frame), in addition to in-plane displacement ($\Delta y$), the affected beam would undergo twisting ($\theta$) in the x-y plane and out-of-plane movement ($\Delta x$), resulting in three Degrees Of Freedom (DOFs).

![Displacement measurements in T(0.5h) and T(1h) specimens](image)

**Figure 4.9** Displacement measurements in T(0.5h) and T(1h) specimens

The measurements of in-plane movements of beams and columns in FR specimen (Fig. 3.7) were employed in these 3-D RC frame tests. Twisting along
the beam was measured by inclinometers placed at the four critical sections along one-span beam, i.e. near the Middle joint (M), Curtailment point next to the Middle joint (CM), Curtailment point next to the beam End (CE), and near the beam End (E) as indicated earlier in Fig. 3.7. The typical displacement measurements in T(0.5h) and T(1h) specimens are shown in Fig. 4.9.

In the specimens subjected to point load at the centre of the middle joint, i.e. FR, COR, and EXT specimens, it was obvious that the displacement of the actuator was equal to Δy at the middle joint. However, for T(0.5h) and T(1h), due to twisting of the beam, the value of Δy at the middle joint was not equal to displacement of the actuator as the applied point load did not act at the beam centroid. Hence, for COR and EXT specimens, vertical displacement at the middle joint (Δy) was directly obtained from the actuator reading, and twisting at the middle joint was measured by an inclinometer. However, for T(0.5h) and T(1h) specimens, the three DOFs (θ, Δx, and Δy) at the middle joint with respect to the beam longitudinal axis were obtained through a four-point measurement method (details are given in Appendix B). The values of the three DOFs could be obtained by solving three simultaneous compatibility relationships formed by the four-point measurement method.

Out-of-plane movement was only measured at the middle joint as its value was expected to be relatively small and hence the beam could be assumed to deform linearly in the out-of-plane direction. The out-of-plane movements at the middle joint for direct torsion (T(0.5h) and T(1h)) specimens and skeletal frames (COR and EXT) were obtained from four-point measurement method and direct measurement, respectively.

4.3. Experimental results of T(0.5h) and T(1h) specimens

4.3.1. T(0.5h) specimen (Torsion-Bending failure at M)

Structural level

The vertical load-displacement and horizontal reaction-displacement relationships of T(0.5h) (curve 2) specimen are plotted together with T(1h) (curve 3) and FR specimens (curve 1) in Figs. 4.10 and 4.11, respectively. It should be
noted that the term “displacement” refers to the vertical displacement at the loaded location (measured by the actuator built-in displacement transducer).

![Figure 4.10 Vertical load-displacement relationships of FR, T(0.5h), and T(1h) specimens](image1)

**Figure 4.10** Vertical load-displacement relationships of FR, T(0.5h), and T(1h) specimens

![Figure 4.11 Horizontal reaction-displacement relationships of FR, T(0.5h), and (1h) specimens](image2)

**Figure 4.11** Horizontal reaction-displacement relationships of FR, T(0.5h), and (1h) specimens

In terms of applied loads, two stages were observed in this specimen, i.e. flexural-CAA stage and CAT stage. At the first stage, vertical load and horizontal reaction versus displacement of T(0.5h) were similar to those of FR specimen. However, when the horizontal force switched to tension, a sharp reduction in load...
was observed in T(0.5h). The test was stopped at about 280 mm displacement due to several reasons. In particular, the vertical load did not show any sign of increase. Severe torsion damages were observed at M at both the right and the left spans of the double-spanning beam as shown in Figs. 4.12a and 4.12b, respectively. In addition, the loading point was no longer aligned with the actuator as shown in Fig. 4.12c.

Figure 4.12 Failure mode of T(0.5h) specimen
Fig. 4.13 shows that the twisting angle was quite linear up to displacement of 144 mm which corresponded to uniform development of torsion-shear cracks along the double-spanning beam. Rapid changes in the gradient between the middle joint (MJ) and M (left span) at displacement after 198 mm (Fig. 4.13) indicated the occurrence of localised twisting failure at this location (Fig. 4.12b) due to a rapid loss in torsional stiffness after severe cracking.

![Twisting profile](image)

**Figure 4.13** Twisting profiles along the double-spanning beam of T(0.5h) specimen

The variations of respective displacements in the three DOFs at the middle joint are presented in Figs. 4.14a, 4.14b, and 4.14c. In Fig. 4.14c, a straight line with a gradient of one (m=1) was plotted. The gap between this line and the vertical deflection profile ($\Delta y$) indicated the relative differences between actuator displacement and deflection of the middle joint (MJ) at the beam longitudinal axis ($\Delta y$). In general, all the three curves show that from the initial stage up to about 50 mm displacement, the effect of torsion was negligible, as evident by the almost zero out-of-plane movement ($\Delta x$) and twisting angle ($\theta$), as well as the almost linear deflection ($\Delta y$) at the middle joint. Thereafter, effects of torsion started to kick in, indicated by a small increase in $\Delta x$ and $\theta$, and a small deviation of the slope of $\Delta y$ from the straight line. At displacement of 200 mm (after occurrences
of severe torsional cracks), steep increases in $\Delta x$ and $\theta$, and a reduction in the slope of $\Delta y$ towards zero were observed. At the final stage, the MJ moved outwards and twisted significantly with every displacement increment. Moreover, $\Delta y$ gradually stopped increasing, indicating that with increasing displacement, MJ gradually stopped deflecting downwards but rather moved out-of-plane and twisted severely.

![Graph of out-of-plane movement ($\Delta x$)](image)

a) Out-of-plane movement ($\Delta x$)

![Graph of twisting ($\theta$)](image)

b) Twisting ($\theta$)
Sectional level

A capacity analysis based on a space-truss mechanism (strut and tie model) in accordance to EC 2 [C1] was performed to identify the cause of failure in T(0.5h) specimen (details are shown in Appendix C). The work was based on the analytical space-truss model developed by Mosley [M4]. In the space-truss model, external forces were resisted by truss actions of concrete struts acting at an inclined angle, together with vertical ties contributed by stirrups and horizontal tension chords from longitudinal reinforcement. The space-truss analyses were performed at four critical sections (E, CE, CM and M) along the affected beam (Fig. 4.9) to obtain the damage levels for the struts and ties. Failure was denoted when stresses exceeded the strength of each component. All internal forces were calculated by taking equilibrium of the measured reactions and deformed geometries. The analyses were performed at 200 mm displacement, right after the occurrence of severe torsional damage and are presented in Tables 4.2 to 4.4. Table 4.2 summarises the reinforcement provided and the predicted resistance based on EC2 space-truss model. The model predictions are the same for T(0.5h) and T(1h) specimens as they share the same reinforcement details. Table 4.3 summarises the actions including shear force, torsion, and bending moment.
applied at 200 mm displacement and required individual component resistance from longitudinal reinforcement, stirrups, and concrete struts at the four sections. Table 4.4 shows the component capacity comparisons between the actions (applied) and the resistance provided. Crack angle of 45° was employed assuming development of maximum torsion. Table 4.4 clearly indicates that the horizontal tension chord at the middle joint region (M) had reached its capacity, which strongly suggested that failure would eventually occur at this section. This explained why after this stage, no further increase in load was observed and localised failure due to extensive twisting indeed appeared at this location. The exceeded value of horizontal tension chord (1.14) was due to strain hardening effect of reinforcing bar was not taken into account and assumption of 45° diagonal crack which resulted in higher resistance or reinforcement required to resist the action.

Table 4.2 Resistance and reinforcement provided for T(0.5h) and T(1h) specimens

<table>
<thead>
<tr>
<th>Resistance</th>
<th>M</th>
<th>CM</th>
<th>CE</th>
<th>E</th>
<th>Reinf. Provided</th>
<th>M</th>
<th>CM</th>
<th>CE</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{Rd,max}$ [kNm]</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>$A_{sw}/s$</td>
<td>0.62</td>
<td>0.51</td>
<td>0.51</td>
<td>0.62</td>
</tr>
<tr>
<td>$V_{Rd,max}$ [kN]</td>
<td>83</td>
<td>83</td>
<td>83</td>
<td>83</td>
<td>$A_{sl(t)}$ [mm$^2$]</td>
<td>235.5</td>
<td>157</td>
<td>157</td>
<td>235.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A_{sl(b)}$ [mm$^2$]</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

$T_{Rd,max}$ and $V_{Rd,max}$ are concrete compressive strut capacity to Torsion and Shear, respectively $A_{sw}$ is stirrup Area provided; $s$ is stirrup Spacing $A_{ld}$ is longitudinal Area provided

(t) and (b) refer to top and bottom longitudinal reinforcement, respectively

Table 4.3 Action and resistance required for T(0.5h) specimen

<table>
<thead>
<tr>
<th>Action</th>
<th>M</th>
<th>CM</th>
<th>CE</th>
<th>E</th>
<th>Resistance Required</th>
<th>M</th>
<th>CM</th>
<th>CE</th>
<th>E</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>V [kN]</td>
<td>13.5</td>
<td>13.5</td>
<td>13.5</td>
<td>13.5</td>
<td>$A_{sw,V}$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>Stirrup</td>
</tr>
<tr>
<td>$T$ [kNm]</td>
<td>1.15</td>
<td>1.15</td>
<td>1.15</td>
<td>1.15</td>
<td>$A_{sw,T}$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>Long. Reinf.</td>
</tr>
<tr>
<td>M [kNm]</td>
<td>13</td>
<td>8</td>
<td>-5.5</td>
<td>-13</td>
<td>$A_{sl,M(t)}$ [mm$^2$]</td>
<td>0</td>
<td>0</td>
<td>66</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A_{sl,M(b)}$ [mm$^2$]</td>
<td>160</td>
<td>95</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

$V, T, M$ are Shear, Torsion, and Moment acting on the section, respectively $A_{sw,V}, A_{sw,T}$ are stirrup area required for Shear and Torsion, respectively $s$ is stirrup spacing $A_{sl,M}, A_{sl,T}$ are longitudinal reinforcement Area required for Moment and Torsion, respectively (t) and (b) is top and bottom longitudinal reinforcement, respectively
Table 4.4 Section analysis of T(0.5h) specimen

<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
<th>M</th>
<th>CM</th>
<th>CE</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete struts</td>
<td>( T/T_{Rd,max} + V/V_{Rd,max} \leq 1 )</td>
<td>0.44 ≤ 1</td>
<td>0.44 ≤ 1</td>
<td>0.44 ≤ 1</td>
<td>0.44 ≤ 1</td>
</tr>
<tr>
<td>Vertical ties</td>
<td>( [A_{sv,V} + 2A_{sv,T}]/A_{sv} \leq 1 )</td>
<td>0.60 ≤ 1</td>
<td>0.72 ≤ 1</td>
<td>0.72 ≤ 1</td>
<td>0.60 ≤ 1</td>
</tr>
<tr>
<td>Horizontal tension chords</td>
<td>( [A_{sl,M(t)} + 3A_{sl,T}/4]/A_{sl} \leq 1 )</td>
<td>0.20 ≤ 1</td>
<td>0.10 ≤ 1</td>
<td>0.71 ≤ 157</td>
<td>0.48 ≤ 1</td>
</tr>
<tr>
<td></td>
<td>( [A_{sl,M(b)} + A_{sl,T}/2]/A_{sl} \leq 1 )</td>
<td>(1.14 ≥ 1)</td>
<td>0.76 ≤ 1</td>
<td>0.16 ≤ 1</td>
<td>0.16 ≤ 1</td>
</tr>
</tbody>
</table>

The damage levels at the 4 selected sections (E, CE, CM and M) were also monitored by the reading of strain gauges installed on both the shear and longitudinal reinforcement bars as shown in Fig. 4.15. The longitudinal reinforcement strain gauges are labelled as B-x, where B refers to “Bending” and x indicates the critical section along the beam, i.e. E, CE, CM or M. Similarly, the label S-x-y is used for shear reinforcement, where S refers to “Shear”, x and y indicates the critical locations and the side of stirrup, respectively. Note that only the strain gauges on longitudinal reinforcement under tension (i.e. top bar at E and CE and bottom bar at M and CM) are presented as they are more critical.

Figure 4.15 Strain gauge locations on both shear and longitudinal reinforcement bars of T(0.5h) and T(1h) specimens

The strain gauge readings at stirrups and tension longitudinal bars at the selected sections are presented in Figs. 4.16 and 4.17, respectively. From Fig. 4.16, it can be seen that all the stirrups were far below yielding at the
commencement of severe torsional damages (displacement of 200 mm) and even until the end of the test. Note that the yield strains for R6 stirrups and T10 longitudinal reinforcement were 0.2% and 0.25%, respectively. Note that the strain gauge at S-CM-b was broken, and hence, the value remained zero in Fig. 4.16.

**Figure 4.16** Strain developments in shear reinforcement bars of T(0.5h)

**Figure 4.17** Strain developments in longitudinal reinforcement bars of T(0.5h)
The failure of horizontal tension chord (longitudinal bottom bar) at the middle joint is clearly illustrated in Fig. 4.17, denoted by the strain value of B-M which was way beyond yielding, while the strain values of tension longitudinal bars at the other sections were still below the yield strain.

All these observations tallied with the predictions from the section capacity analyses in Table 4.4. For example, the strains at all stirrups were about half of the yield strains (in Fig 4.16). This was also indicated by the normalised action to resistance ratio of vertical ties (Table 4.4) which were 0.6 (M and E sections) and 0.72 (CM and CE sections). Similarly, the section capacity analysis also showed that the provided longitudinal reinforcement at M had been fully utilised and hence yielded, eventually failing the tension longitudinal reinforcement. Following this, torsional stiffness was lost and the beam was simply twisted at the failure location (M) and vertical deflection curve ($\Delta y$) of the beam eventually stopped at around displacement of 200 mm (Fig. 4.14). CAT generally relies on the tension force provided by longitudinal reinforcement in the beam. As vertical deflection stopped increasing, CAT could not be further mobilised. Besides, capacity of the double-spanning beam was significantly decreased due to additional twisting damages along the beam.

4.3.2. T(1h) specimen (Torsion-Shear failure at CM)

Structural level

Contradictory to FR and T(0.5h) specimens, T(1h) specimen did not reach the maximum flexural capacity and the test was stopped at a very early flexural stage with similar reasons as in T(0.5h) specimen. Load was rapidly decreasing (curve 3 in Fig. 4.10) and an excessive localised rotation occurred at CM location on the left beam (Fig. 4.18), indicated by a rapid increase in rotation between the curtailment point CM and the beam-middle joint interface as shown in Fig. 4.19. From the twisting profile in Fig. 4.19, it could also be observed that even at small displacement (elastic stage), twisting along the beam was not linear due to different torsional stiffness and cracking along the beam.
In Figs. 4.14a to 4.14c (displacements at the middle joint), as soon as the test started, the effect of torsion on T(1h) specimen was visible as indicated by the non-zero values in all the three DOFs (\( \theta \), \( \Delta x \), and \( \Delta y \)). The middle joint was yanked outwards and eventually reached about 20 mm out-of-plane displacement at the end of the test. Severe damages due to torsion started at about 40 mm displacement right after the maximum capacity was reached. Following this, the load decreased immediately (Fig. 4.10) accompanied by excessive twisting of T(1h) specimen (Fig. 4.14b).
Sectional level

As torsional failure took place before flexural capacity was fully mobilised, to obtain a better understanding of torsion failure, space-truss analyses based on EC2 were again performed to determine the location of failure. Although experimental observation showed that failure occurred at the longitudinal reinforcement curtailment point or stirrup transition region (CM), analyses were performed at all four critical sections along the beam as the resistances and actions at these locations were different. The capacity analyses of T(1h) specimen at displacement of 40 mm (maximum capacity) are presented in Tables 4.5 and 4.6. The crack angle was taken as 45° for the four sections to account for maximum torsion.

Table 4.5 Action and resistance required for T(1h) specimen

<table>
<thead>
<tr>
<th>Action [kN]</th>
<th>M</th>
<th>CM</th>
<th>CE</th>
<th>E</th>
<th>Resistance Required [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>A_{sw,V}</td>
</tr>
<tr>
<td>T [kNm]</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>A_{sw,T}</td>
</tr>
<tr>
<td>M [kNm]</td>
<td>9</td>
<td>4.5</td>
<td>-4.5</td>
<td>9</td>
<td>A_{sl,M(t)} [mm²]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A_{sl,M(b)} [mm²]</td>
</tr>
</tbody>
</table>

The results shown in Table 4.6 confirm the failure of vertical ties at the curtailment points, indicated by the value of normalised action to resistance ratio which is close to one (underlined in Table 4b). Based on space-truss analysis, failures of vertical ties were expected at both CE and CM sections. However, it
should be noted that CE and CM sections were not subjected to identical actions, i.e. CE and CM sections were under sagging and hogging moments, respectively and hence simultaneous failures at both CE and CM did not occur. In this case, failure was concentrated at CM which was the weakest section. It was also noted that the horizontal tension chord at section M was about to reach the yield capacity. The test, however, did not show the full formation of diagonal crack at this location. This meant that the analysis based on the space-truss mechanism might not be fully applicable as the beam sections had by then undergone large deformations. Instead of the development of diagonal cracks at M, the first diagonal crack at the section with the lowest torsional rigidity (with a smaller reinforcement ratio and a larger stirrup spacing) occurred at CM. Since failure occurred first at this section, torsion could not be fully transferred to the other sections.

The strain readings at the four sections of T(1h) specimen are presented in Fig. 4.20 for stirrups and Fig. 4.21 for longitudinal bars. It can be seen from Fig. 4.21 that the stirrup strains at both CE and CM sections were considerably high and even attained yield strain. Due to localised failure at CM, the strain at this section was slightly higher than CE.

![Strain developments in shear reinforcement bars of T(1h)](image)

*Figure 4.20 Strain developments in shear reinforcement bars of T(1h)*
For the horizontal ties (longitudinal bars), the strains were highest at section M as indicated in the analysis but had not reached yielding yet. Both the section capacity analyses and strain readings confirmed failure of vertical ties at CM (Fig. 4.20), which led to excessive localised twisting there and prevented the development of ALPs.

Figure 4.21 Strain developments in longitudinal reinforcement bars of T(1h)

4.3.3. Effect of direct torsion

Comparing the experimental results between FR specimen with T(0.5h) and T(1h) specimens, torsion clearly hindered the development of ALPs. The maximum capacity of ALP was limited by the lowest component capacity of the equivalent truss model, namely, the horizontal tension chord (torsion-bending) component for T(0.5h) specimen and the vertical tie (torsion-shear) component for T(1h) specimen. Once the lowest truss component capacity was reached, failure took place at that section and at the same time the maximum ALP capacity was attained and could not be further increased. With further displacement increments, the load either remained constant or decreased, accompanied by localised twisting at the failed section. Note that the terms ‘torsion-bending’ and ‘torsion-shear’ failures did not literally mean that the failure was caused by a combination of only two actions. The ‘torsion-bending’ term was chosen as they
were the two main actions resisted by the failed horizontal tie component and similarly for the ‘torsion-shear’ term.

Before the cross-sectional failure occurred, besides the presence of torsion and twisting, the behaviour of all specimens was quite comparable. These included the development of beam axial compression (flexural-CAA) and the transition to axial tension (CAT for FR and T(0.5h) specimens only) in Fig. 4.11. In the tests, as adequate torsional stiffness was provided at the beam-column joint, there was no relative twisting at the beam-column interface. As the twisting or torsional moment was resisted by the top and bottom supports of the column, there were no transverse (out-of-plane) forces and movements at the beam-level of the column (Fig. 4.1), as indicated by negligible values of the out-of-plane two-way load cells and LVDTs at this location, respectively.

For T(1h) specimen, torsional cracks directly failed at CM section due to insufficient stirrups, ending its ability to transfer torsion to the other sections. This is clearly seen in Fig. 4.18, in which torsional cracks only developed at the curtailment point (CM) and not along the entire beam span. In T(0.5h) specimen, the cross-sectional capacities at the curtailment points (CM or CE) were adequate and torsional cracks developed uniformly throughout the beam span. Finally, the beam failed due to insufficient longitudinal reinforcement (horizontal tension chord failure) at the beam-middle joint interface. This in fact showed the difficulty of full CAT mobilisation even under a smaller torsion eccentricity of 0.5h. Vertical tie and compressive strut components could be strengthened by increasing the number of stirrups or the dimensions of cross sections. However, horizontal tension chord component might not be easily increased since the provided reinforcement would be fully utilised or yielded when flexural capacity was reached, leaving only some reserve capacity for torsion and almost none for CAT.

Based on the test results and analyses, two conclusions can be drawn on why CAT cannot be mobilised or only partially mobilised in the presence of torsion. Firstly, if significant amount of torsion is applied, such as in T(1h) specimen, local failure may occur through yielding of vertical ties. Secondly, if a smaller
amount of torsion is applied, such as in T(0.5h) specimen, even if vertical ties and concrete struts do not reach the maximum strength, the presence of torsion in addition to bending can still cause longitudinal reinforcement to yield and there will not be any reserve capacity to support axial tension for CAT. In both cases, after the beam loses its torsional rigidity, CAT will not be significantly mobilised as the failed section will be twisted locally.

4.4. Experimental results of COR and EXT specimens

4.4.1. COR specimen

Structural level

The vertical load and horizontal reaction of COR specimen are plotted against vertical displacement at the middle joint in Figs. 4.22 and 4.23, respectively. In Fig. 4.22, after reaching flexural capacity, the vertical load sustained by COR specimen remained flat with a slight decrease until the end of the test. The development of ALPs was monitored from horizontal reaction (reflecting the beam axial force), i.e. CAA-Flexural mechanism when the axial force is in compression (negative), and CAT mechanism when it turns tension (positive). The negligible horizontal reaction/ axial force of COR specimen in Fig. 4.23 showed that the load resistance was solely contributed by flexural mechanism of both single-spanning beams (T and L), which explained the flat plateau in vertical load in Fig. 4.22. The test was stopped at displacement of 370 mm, when the load reduced significantly due to excessive concrete crushing and spalling at the plastic hinge locations as shown in Fig 4.24b.

Figs. 4.24a and 4.24b show the front and back views of COR specimen at the final stage, respectively. Fig. 4.24c shows the development of cracks in COR specimen at two different stages, i.e. from the beginning of test to attainment of flexural capacity at displacement of 100 mm (indicated by black lines) and from this point onwards towards the end of test (indicated by blue lines). At the first stage, flexural cracks developed along the left and the transverse beams, except near the middle joint region. As the middle joint did not have sufficient rotational restraint, it rotated together with joining beams as a rigid body. At the second
stage, plastic hinge was formed at the curtailment point (CE) at both spans. At the end of test, excessive concrete crushing and spalling due to a combination of bending and twisting were observed at the plastic hinge location (CE). Minor or only a single crack was formed at the column joint, as it was not subjected to any significant horizontal force or axial force of the beam.

![Vertical load-displacement relationship of COR specimen](image)

**Figure 4.22** Vertical load-displacement relationship of COR specimen

![Horizontal reaction-displacement relationship of COR specimen](image)

**Figure 4.23** Horizontal reaction-displacement relationship of COR specimen
The developments of twisting at the middle joint and at the four critical sections along the left and the transverse beams are plotted in Figs. 4.25a and 4.25b. Twisting was present since the beginning of the test and it increased almost linearly along the beam with each displacement increment. After the formation of plastic hinge at CE of both spans (displacement of 100 mm), twisting at the beam
end (E) stopped and the twisting profile at the remaining locations (from CE up to the middle joint) became almost identical for both beams since twisting and bending were mainly concentrated at CE. Similarly, beam deflection was mainly concentrated at CE after the formation of plastic hinge as indicated by the beam vertical deflection profile in Fig. 4.25c. The maximum out-of-plane movement at the middle joint was about 70 mm in Fig. 4.25d. Note that $\Delta x$ and $\Delta z$ denote the horizontal movements of the middle joint along x (along left beam) and z axes (along transverse beam), respectively (details of axes are indicated in Fig. 4.8a). The column joint rotation and in-plane horizontal movement are plotted in Figs. 4.25e and 4.25f, respectively. The slight inward and outward movements of column joint throughout the test in Fig. 4.25f explained the absence of axial force development in the beam. Rotation with an average value of 8 mrad was observed at the column joint due to flexure.

![Graphs showing twisting and deflection profiles](image)

(a) Twisting along Transverse beam  
(b) Twisting along Left beam

c) Deflection profile  
d) Middle joint out-of-plan movement
e) Rotation of column joint  

f) Horizontal movement of column joint

Figure 4.25 Displacements of beams and columns of COR specimen

Sectional level

The internal forces, i.e. axial force, shear and bending moment were calculated from equilibrium based on the measured reactions and deformed geometries throughout the tests. Fig. 4.26 shows the shear and axial force curves of COR specimen, which can be used to identify the contributions of different ALP mechanisms (flexural-CAA and CAT) to overall capacity. Total vertical load was resisted by combination of shear and axial forces at the middle joint, in which shear and axial forces represent flexure and CAT, respectively. Clearly, the total applied load in COR specimen was mainly resisted by flexural mechanism (shear force). The contribution from axial force was insignificant compared to that of shear force. Axial force in the beam was not properly developed due to a lack of horizontal restraints. The bending moments at the middle joint and the beam end are plotted in Fig. 4.27a and 4.27b, respectively. Generally, both the bending moment profiles at the middle joint and the beam ends were similar to the shear force profiles in Fig. 4.26, as the load in COR was mainly resisted by flexure. The very small moment (0.6 kNm) at the middle joint in Fig. 4.27a accounted for the absence of flexural crack and plastic hinge at this location. The maximum bending moment at the beam end (Fig. 4.27b) was about 14 kNm, quite close to the design moment resistance of the beam section (16.5 kNm). Hence, it was concluded that the left and the transverse beams in COR specimen behaved like cantilevers.
The absence of bending moment at the middle joint region and development of CAT along the beam could be clearly indicated from the longitudinal strain profiles at top and bottom reinforcement bars along the beam, plotted in Fig. 4.28a and 4.28b, respectively. The insignificant or small moments at the middle joint regions were indicated by the almost zero strain at the top reinforcement and small strains (< 500 microstrain) at the bottom reinforcement. The commencement of CAT could be identified when the whole sections (both top and bottom reinforcement) along the beam were under tension (positive strain). This however, was not observed in the strain profiles of COR specimen even at the large deflection stage (displacement of 370 mm). In fact there was no much changes in the magnitude of strain and there was no sign of the strain profile
being shifted up to positive strain, especially after flexural capacity was reached at displacement of 100 mm.

**Figure 4.28** Strain profile along longitudinal reinforcement of COR specimen
As mentioned earlier, the out-of-plane action (twisting) did not affect ALP development and capacity. This could be confirmed from the small vertical strain values (below yielding) of stirrups at the four critical sections along the beam as shown in Fig. 4.29. The vertical strains were largest at the curtailment points (CEs) where plastic hinges were formed, as twisting and bending would be more localised or concentrated.

![Figure 4.29 Strain developments on stirrups of COR specimen](image)

4.4.2. EXT specimen

Structural level

The vertical load-displacement and horizontal reaction-displacement relationships of EXT are shown in Figs. 4.30 and 4.31, respectively. The negligible development of CAA (displacement of 120 mm), as shown in the vertical load-displacement relationship in Fig. 4.30 could be explained from the development of axial forces in the beam (Fig. 4.31), i.e. the relatively small compression force in the beam, as well as early transition to tension force. Along with the development of CAT, bottom reinforcement bars of the right span beam fractured at the middle joint interface (displacement of 390 mm), causing a drop to the load capacity. CAT continued to develop and increase the load capacity until the commencement of final failure (displacement of 590 mm) due to fracture of top reinforcement at the same cross-section (middle joint interface of right span...
beam). The discontinuity between the double-spanning beam (right and left beams) ended CAT stage.

![Figure 4.30 Vertical load-displacement relationship of EXT specimen](image)

**Figure 4.30** Vertical load-displacement relationship of EXT specimen

![Figure 4.31 Horizontal reaction-displacement relationship of EXT specimen](image)

**Figure 4.31** Horizontal reaction-displacement relationship of EXT specimen
CHAPTER 4  
EXPERIMENTAL STUDIES OF 3-D RC FRAMES

Figure 4.32 Crack patterns and failure mode of EXT specimen

The final stage and development of crack patterns of EXT specimen are shown in Figs. 4.32a and 4.32b, respectively. The development of cracks at different stages was indicated as follows: from the beginning of the test to attainment of CAA peak (displacement of 0 to 120 mm) indicated by black lines; CAA peak to the commencement of CAT indicated by blue lines; and from CAT to the end of test (displacement of 200 to 590 mm) indicated by red lines. At the first stage, flexural cracks developed along the left and the right beams, except near the middle joint region of the transverse beam. At the second stage, plastic hinges developed near the middle joint (M) and the beam-end (E) of the double-spanning beam, but only at E of the transverse beam. At the last stage, tension cracks at roughly constant spacing developed along the double-spanning beam indicating CAT. Concrete crushing at the plastic hinge locations became more severe and at
the end of the test, spalling was observed on the plastic hinges of both the left and the transverse beams due to excessive twisting.

The developments of twisting angle along the beams are presented in Figs. 4.33a, 4.33b and 4.33c for the left, right and transverse beams, respectively. Twisting along the double-spanning beam developed linearly at the beginning of the test. After the bottom reinforcement bars had fractured at M of the right span, rotational restraint about the transverse beam (z-axis) was lost, causing twisting of the transverse beam. Simultaneously, the twisting of the right span beam was now concentrated at M (weakest point). At the instant of top (final) reinforcement bars fractured at M of right span, an increase in the out-of-plane movement and twisting of the right and the transverse beams were observed but were not measured as they happened in a snap of time. Nevertheless, failure was marked by a sudden excessive spalling and reinforcement buckling at E of the left and the transverse beams at the end of the test. The maximum out-of-plane movement (Δz) at the middle joint due to pulling-in of transverse beam was about 50 mm as indicated in Fig. 4.33d. The rotations and in-plane horizontal movements of column joints are presented in Figs. 4.33e and 4.33f, respectively. The column joints did not move outwards but were instead pulled in at displacement of 70 mm, which explained the early transition of axial compression to tension forces in the double-spanning beam. The joint rotation increased with flexural capacity and remained constant after flexural capacity was reached.

a) Twisting along Left beam

b) Twisting along Right beam
CHAPTER 4
EXPERIMENTAL STUDIES OF 3-D RC FRAMES

Figure 4.33 Displacements of beams and columns of EXT specimen

Sectional level

In Fig. 4.34, shear and axial forces of the transverse beam are plotted separately to highlight the differences of ALP development between the double-spanning beam (left and right beams) and the transverse beam. In the transverse beam, due to the lack of horizontal restraint, CAA and CAT were hardly developed and the load capacity depended solely on flexural resistance. A subsequent drop in shear force of the transverse beam as compared to the double-spanning beam was due to twisting (after fracture of first reinforcing bar). In the double-spanning beam, CAT became the main load-carrying mechanism after a sudden reduction in flexural capacity due to fracture of reinforcing bar. The enhancement of CAA to flexural capacity was not obvious as the axial
compression force was very small indeed and it turned into tension at a relatively early stage.

![Graph showing shear and axial forces versus vertical load of EXT specimen](image)

**Figure 4.34** Shear and axial forces versus vertical load of EXT specimen

![Graph showing bending moments along the beams of EXT specimen](image)

**Figure 4.35** Bending moments along the beams of EXT specimen

The cantilever behaviour of the transverse beam can be clearly seen from the small bending moments at the middle joint of the transverse beam throughout the test as indicated in Fig. 4.35a. Ultimate moments were reached (about 17 kNm) at the plastic hinge locations, i.e. the middle joint (M) and the beam-end (E) of the double-spanning beam, and at E of the transverse beam as indicated in Fig. 4.35b.

The longitudinal strain profiles of the top and bottom reinforcement bars of the double-spanning beam in Figs. 4.36a and 4.36b clearly show the transition from flexure to CAT. For example, in the top reinforcement (Fig. 4.36a), initially the
middle joint (M) and the beam end (E) would be under compression and tension, respectively. The strains in the two regions (M and E) eventually yielded (2500 microstrain) implying the formation of plastic hinges. Subsequently, after the commencement of CAT, the overall strain profiles shifted upwards and were finally under net tension, as seen from displacement of 260 mm to 580 mm. The descriptions of the longitudinal strain profiles in the transverse beam (Fig. 4.36c and 4.36d) were similar to that in COR specimen, which clearly indicated the absence of ultimate moment at the middle joint and CAT.

![Graph showing strain profile](image-url)

a) Strain profile of top reinforcement (left and right beam)
b) Strain profile of bottom reinforcement (left and right beam)

c) Strain profile of top rebar (transverse)

d) Strain profile of bot rebar (transverse)

**Figure 4.36** Strain profile along longitudinal reinforcement of EXT specimen

In terms of vertical strains on stirrups in Fig. 4.37, similarly, it can be seen that the vertical strains were still far below yielding indicating the trivial effect of out-of-plane actions. Vertical strains on stirrups were generally higher at plastic hinge locations (E).
4.4.3. Comparison among 3-D skeletal frames (COR, EXT) and 2-D control specimen (FR)

The vertical load-displacement and horizontal reaction-displacement relationships of the three specimens are plotted together in Figs. 4.38 and 4.39. In general, COR specimen (curve 2) had the lowest load-carrying capacity. Load capacity of EXT specimen (curve 3) was initially the highest but became comparable with FR specimen (curve 1) after its first load drop at displacement of 390 mm. In COR specimen, the joint above the removed corner column could not provide sufficient restraints against rotation and horizontal movements. The lack of rotational restraint at the middle joint led the beams to behave as cantilevers which resulted in the lowest flexural capacity; the lack of horizontal restraint hindered the development of CAA at initial stage and CAT at large deformation stage. The higher load capacity of EXT as compared to FR specimen at the early stage was simply due to additional contribution from the flexural (cantilever) resistance of the transverse beam. The comparable load capacity between EXT and FR specimens at large deformation stage (displacement of 390 mm to end of test) can be explained from the shear-axial force interactions of EXT specimen in Fig 4.34. After the first bar fractured at displacement of 390 mm, shear forces at
the double-spanning beam as well as the transverse beam gradually diminished in values marking the end of flexural mechanism. As CAT was not developed in the transverse beam, EXT specimen relied solely on CAT of the double-spanning beam, which was exactly the same mechanism in FR. Current test results and analyses have yet to explain if the instant of first bar fractured (Fig. 4.38) and the differences in the horizontal reaction profiles at flexural-CAA stage (Fig. 4.39) between EXT and FR specimens were due to interaction of the transverse beam. The fracture of reinforcing bar was quite arbitrary, e.g. the first bar fracture occurred in FR specimen when horizontal reaction (axial force) was close to zero, but at a relatively high axial force of about 100 kN in EXT specimen. The slight negative horizontal reaction in EXT (Fig. 4.39) might be due to the presence of gaps between the specimen and the boundary conditions, which might occur in a 3-D test setup.

![Figure 4.38](image)

**Figure 4.38** Vertical load-displacement relationships of FR, COR, and EXT specimens
The 3-D actions showed the presence of twisting and out-of-plane movements along the beam. From the analysis of result, compatibility torsion generated from the single point load on the middle joint did not seem to affect much on the load-carrying capacity. In COR and EXT specimens, all the beams were able to reach their full flexural capacities, but the combination of bending and twisting induced excessive concrete spalling and crushing, which might accelerate the termination of flexural mechanism. In Fig. 4.39, EXT specimen had a comparable horizontal reaction profile to that of FR specimen, even up to the maximum horizontal reaction, showing that CAT was not affected by 3-D actions. The small magnitude of compatibility torsion (transferred from the almost zero moment at the middle joint of the single-spanning beam) was the main reason of this negligible effect.

4.5. Summary

The direct torsion tests focused on the torsion effect on ALPs in the double-spanning beam under middle column removal scenario, which was largely ignored in the past. T(0.5h) and T(1h) specimens subjected to reasonable torsional moments generated by point load eccentricities were tested and compared with a 2-D frame specimen (FR specimen) subjected to only in-plane loading to investigate the effect of torsion on ALP. The results are summarised as follows:
1) With sufficient amount of torsion, torsional failure may occur hindering the development of ALPs. Two types of failure modes corresponding to different point load eccentricities (torsion to shear ratios) were identified in the tests, namely, vertical tie (shear-torsion) failure due to insufficient stirrups, and horizontal tension chord (bending-torsion) failure due to insufficient longitudinal reinforcement which have been verified from experimental observations, section capacity analyses, and strain gauge readings of reinforcement.

2) Vertical tie failure in beams with high torsion-to-shear-force ratio (represented by T(1h) specimen) may be of great concern as it causes premature failure even before attaining flexural capacity. Capacity analysis should be performed to ensure that sufficient stirrup is provided against vertical tie failure.

3) The results from T(0.5h) specimen indicated the difficulty of CAT development in beams with lower torsion-to-shear-force ratio even though premature failure is prevented by sufficient provision of stirrups. After attainment of flexural capacity, the longitudinal reinforcement has exhausted, leaving hardly any reserve capacity to resist torsion and this may prevent full development of CAT. In the test, T(0.5h) specimen with 90 mm load eccentricity did not have sufficient reserve capacity to resist torsion, leading to horizontal tension chord failure. Subsequently, as the beam twisted locally without any further increase in vertical deflection, it was not possible to mobilise CAT.

4) Due to direct torsion, ALP (especially CAT) may not be fully mobilised. Thus, future studies should be focusing on 3D solid modelling of torsional failures in beam-slab substructures so as to quantify the maximum torsion eccentricity from compatibility torsion, beyond which CAT will not be mobilised.

The RC skeletal frame tests focused on the investigation of ALP development in 3-D frames under corner and exterior column removal scenarios. The main findings are as follows:

1) The lack of restraint at the middle joint hindered the development of plastic hinge at the middle joint and CAT, resulted in only cantilever resistance of the
single-spanning beam. However, the lack of restraint at the same time generated a small compatibility torsion which prevented significant damage or weakening to the load-carrying capacity.

2) In frames subjected to a point load above the removed column (without any external restraint), the vertical load was generally resisted by cantilever mechanism of single-spanning beams (such as COR specimen) and/or flexure and CAT of double-spanning beams (such as EXT specimen).
CHAPTER 5: EXPERIMENTAL STUDIES OF 3-D RC FRAME-SLABS

5.1. Introduction

As mentioned in Chapters 1 and 2, many of the current design guidelines on progressive collapse treat slabs as secondary members and do not take into account their contributions in collapse resistance of RC beam-slab systems. This may underestimate the actual capacity of the structure. Moreover, tests on RC beam-slab substructures had shown the significant contribution of slabs, not only from flexural capacity but also the development of Tensile Membrane Action (TMA) in increasing the capacity and ductility.

In this last series of tests, two statically determinate RC frame-slab tests i.e. S-COR and S-EXT specimens (Table 5.1) based on the tests of two skeletal RC frames under corner (COR) and exterior (EXT) column removal scenarios in Chapter 4 were conducted and compared to elucidate the slab contribution towards the overall behaviour and capacity of the frame-slab specimens. The “S” in the beginning of the label was to indicate the presence of Slab in the 3-D substructure. The quantification of slab contribution based on the test results and analyses is essential to developing an analytical model to account for reserve capacity from slabs on top of the capacity of the grillage of beams.

Table 5.1 Specimens in 3-D RC Frame-Slab tests

<table>
<thead>
<tr>
<th>Objective</th>
<th>No.</th>
<th>Specimen</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame-Slab</td>
<td>1</td>
<td>S-COR</td>
<td>Frame-Slab under CORner column removal scenario</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>S-EXT</td>
<td>Frame-Slab under EXTerior column removal scenario</td>
</tr>
</tbody>
</table>
5.2. Test preparation

5.2.1. Specimen design and material properties

ALP studies on two 2/5 scaled RC frame specimens under CORner (COR) and EXTerior (EXT) column loss scenarios had been conducted and reported in Chapter 4. Two RC frame-slab tests were designed based on the two aforementioned frame tests, i.e. S-COR from COR, and S-EXT from EXT. The frame details in S-COR and S-EXT specimens were identical to those in COR and EXT (also FR) specimens. Detailing of FR specimen in Chapter 3 is replotted as Fig. 5.1 for ease of reference.

The slabs were 80 mm thick with reinforcement details corresponding to the same loading condition in the design of frames as shown in Figs. 5.2a and 5.2b for S-EXT and S-COR specimens, respectively. Rotational restraints from adjacent slabs were represented by thicker slab portions (120 mm) extending 240 mm beyond the perimeter beams (shaded in Figs. 5.2a and 5.2b). The material properties followed those of FR specimen as shown in Table 3.4. The same R6 smooth bars for beam stirrups were used for the slab longitudinal reinforcement.

Figure 5.1 Frame detailing of S-COR and S-EXT specimens (replotted from Fig. 3.2)
a) Slab detailing of S-EXT  

b) Slab detailing of S-COR

Figure 5.2 Slab detailing of S-COR and S-EXT specimens

The beams and the columns are labelled in Figs. 5.2a and 5.2b for ease of reference. Primary beams “L”, “R”, and “T” were supported by primary columns “C-L”, “C-R”, and “C-T”, respectively. Secondary (perimeter) beams “LT-1” and “LT-2” were supported by a secondary column “C-LT”; and secondary beam “RT-1” and “RT-2” were supported by another secondary column “C-RT”.

5.2.2. Test setup

The test setups of S-COR and S-EXT specimens are respectively shown in Figs. 5.3a and 5.3b, in which the beam ends and the column tops were each pin supported to horizontal steel restraints and each column was seated on a pin base. Slabs were cast with the primary frames (connected directly to the middle joint) together with perimeter beams and columns. The perimeter (secondary) columns (C-LT and C-RT) were seated on circular hollow steel sections pinned to the
ground (inclined at 45° direction) without any horizontal restraints to simplify the test setup since they carried smaller loads compared to the primary frames and were not subjected to any significant horizontal movements. (Note that the slab extension is not included in Fig. 5.3 to show a clearer test setup)

![Test setups of S-COR and S-EXT specimens](image)

**Figure 5.3** Test setups of S-COR and S-EXT specimens

Similar to COR and EXT specimens, vertical point load was applied on the removed column location by an actuator in a displacement-control mode.

### 5.2.3. Instrumentation

**Reaction measurements**

The reaction measurements in the primary frames were similar to those of FR specimen. In the primary columns (C-L, C-R, and C-T), the axial force in each of the steel horizontal restraint was measured by two-way load cells, and both the vertical and horizontal reactions at the pin bases were measured by load pins as illustrated in Fig. 5.4a. For the secondary columns, the vertical and horizontal reactions at the pin supports were obtained via the readings of strain gauges installed on the affixed steel circular hollow sections as shown in Fig. 5.4b.
CHAPTER 5 EXPERIMENTAL STUDIES OF 3-D RC FRAME-SLABS

Figure 5.4 Reaction measurements in S-COR and S-EXT specimens

Four strain gauges were located around the circular steel column at the same height, two along and two perpendicular to the axis of rotation. This set of four strain gauges was installed at two different heights for verification purpose. The vertical reaction \( (V_r) \) of the column could be obtained by simply taking the average strain times \( (\varepsilon_{\text{ave}}) \) the young’s modulus \( (E) \) and cross section \( (A) \) as follows:

\[
V_r = \varepsilon_{\text{ave}} \times E \times A = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4}{4} \times E \times A
\]  

(5 - 1)

Based on moment equilibrium \( (M) \), horizontal reactions \( (H_r) \) at the bottom pins can be obtained by dividing the column bending moment (from strain gauges) by the height \( (H_c) \) corresponding to the strain gauge locations as follows:

\[
M = \frac{\varepsilon_1 - \varepsilon_{\text{ave}}}{r} \times E \times I
\]  

(5 - 2)
\[ H_r = \frac{M}{H_c} \quad (5 - 3) \]

If the specimen was set up properly, both the calculated vertical reaction \( (V_r) \) and horizontal reaction \( (H_r) \) at the two different heights within the same column section should be similar or close to each other.

**Displacement measurements**

Displacement measurements for the frame-slab specimens included twisting, out-of-plane and in-plane movements. The in-plane movements measured by LVDTs and LTs were similar on each of the primary members as shown in Fig. 5.5. Twisting was measured by an inclinometer at the middle joint and four critical sections along the primary beam (L, R, and T), i.e. near the Middle joint (M), near the beam End (E), Curtailment point next to the Middle joint (CM) and Curtailment point next to the beam End (CE). Out-of-plane movements were only measured at the middle joint. In addition to these, LVDTs and LTs were placed at certain regions below the slabs to measure relative deformations along a potential yield line as shown in Fig. 5.6. An LVDT was located at the secondary column to measure its horizontal movements.

**Figure 5.5** In-plane displacement measurements in primary frame of S-COR and S-EXT specimens (re plotted from Fig. 3.7)
Figure 5.6 Displacement measurements in slabs and secondary columns of S-COR and S-EXT specimens

Figure 5.7 Strain gauges layout in beams and slabs of S-COR specimen
Strain gauges were used extensively along the primary beams and on the slabs to capture the development of ALPs as illustrated in Figs. 5.7 and 5.8 for S-COR and S-EXT specimens, respectively. The labelling of strain gauges on the beam reinforcement consisted of three letters. The first letter refers to the beam span, i.e. Left (L), Right (R), and Transverse (T); the second letter refers to the reinforcement location, i.e. T (Top longitudinal reinforcement), B (Bottom longitudinal reinforcement), and V (shear reinforcement); the third letter refers to the strain gauge number. For example, TT1 refers to strain gauge at the first location on the Top reinforcement of the Transverse beam. The strain gauge labelling scheme on the slab reinforcement was similar to that of the beam. After the addition of letter “S” in the beginning followed by “-” indicating strain gauges on the Slab reinforcement, the next letter refers to the slab location, i.e. L (left slab) or R (right slab). Note that this term was not applicable to S-COR as there...
was only one slab. The last two letters refer to reinforcement location (T or B), and strain gauge's number, respectively. Hence, S-LB1 refers to strain gauge at first location on the Left Slab Bottom reinforcement.

5.3. Experimental result of S-COR specimen

Global behaviour (load-displacement relationship and crack development) of RC Frame-slab specimens are presented first, followed by the discussion on slab contribution via a comparison of S-COR and S-EXT test results vis-a-vis those of RC skeletal frames, i.e. COR and EXT, respectively.

5.3.1. Global response

Load-displacement

The developments of vertical load and horizontal reaction with increasing displacement are plotted in Figs. 5.9 and 5.10, respectively. In Fig. 5.9, the vertical load at each support was obtained from the load pin for the primary column and from the strain gauge readings for the secondary column, which were summed up and verified against the recorded total vertical load from the actuator built-in load cell. In Fig. 5.10, the horizontal reaction at each column, obtained from the instrumentation system (load pin, two-way load cell, strain gauge readings) was first verified against equilibrium and was later summed up to obtain overall horizontal forces. From Fig. 5.10, initially, horizontal reactions at all the supports were relatively small indicating the absence of CAT and TMA. Due to the absence of CAT in the beam and TMA in the slab, after the maximum vertical load (flexural capacity) of 21 kN was reached at displacement of 150 mm, the vertical load gradually decreased with crushing and spalling of concrete at the plastic hinges at the double-spanning beam and along the yield lines of the slabs. The test was stopped at displacement of 550 mm when the load dropped to 13 kN (60% of the peak load). At this point, the displacement was considerably large and there was no additional resistance beyond flexural capacity of the beam-slab specimen.
As point load was applied on the specimen, punching shear capacity of S-COR specimen was calculated based on EC2 [C1] to confirm that the decreasing load or
failure was not due to punching shear. The critical section for punching shear for S-COR specimen is illustrated in Fig. 5.11 and the punching shear resistance of slab without shear reinforcement ($V_{Rd,c}$) is calculated as follows:

$$V_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} u d$$

(5 - 4)

where $C_{Rd,c}$ is taken as 0.18, $k$ is a coefficient for slab thickness (the maximum value of 2 is used), $\rho_l$ is the combined reinforcement ratio (maximum value of 0.02 is used), $f_{ck}$ is the concrete compressive strength (30 MPa), $u$ is the critical perimeter of punching shear (484 mm), $d$ is the effective depth of slab (65 mm). By substituting these values, $V_{Rd,c}$ was calculated to be 39 kN, which was about twice the peak capacity of the specimen (21 kN).

**Figure 5.11** Critical sections for punching shear

Crack pattern

Fig. 5.12a illustrates the crack patterns in the frame and slab, and Figs. 5.12b and 5.12c show the top and bottom views of S-COR specimen at the final stage, respectively. The development of crack patterns could be divided into two stages, i.e. before the attainment of flexural capacity at displacement of 150 mm indicated by the black lines, and from this point onwards to the end of the test indicated by the blue lines. At the first stage, multiple yield lines running at 45° and parallel to one another were formed in the slab (fan mechanism), traversing perpendicular to the diagonal line “D1” joining C-LT and the middle joint as shown in Fig. 5.12a. These yield lines penetrated through the primary beams, generating flexural cracks along the beams. No yield line or flexural crack was developed near the column stub at the middle joint as it rotated together with the beams and the slabs due to a lack of rotational restraint. Flexural cracks were formed at the exterior
face of the primary columns (C-L and C-T), indicating the pushing out of primary beams (L and T) at the initial stage.

a) Crack patterns in S-COR

b) Final stage of S-COR (Top view)
At the second stage, these cracks further widened and localised bending was observed at two main yield lines along D1 (Fig. 5.12b). The crack patterns demonstrated flexural mechanism as the main load-carrying mechanism for corner frame-slab system (S-COR specimen).

### 5.3.2. Contribution of slab

**Physical behaviour**

The physical behaviour discussed herein included the development of crack patterns and 3-D actions, i.e. twisting, out-of-plane and in-plane movements of the frames and the slabs. A comparison of crack patterns between S-COR (Fig. 5.12a) and COR (Fig. 5.12d) specimens showed that the fan-shaped mechanism in the slab increased the number of flexural cracks and penetrated further towards the
middle joint region. The development of uniform flexural cracks in S-COR specimen facilitated the rotation of several segments along the beam and delayed localised rotations at the plastic hinges, which to a certain extent helped to increase the ductility of the specimen.

Figure 5.13 Displacements of frames and slabs in S-COR specimen

The almost similar twisting profile at the middle joint between COR and S-COR specimens in Fig. 5.13a indicated that the slab did not increase the twisting stiffness of the perimeter beams. As the yield lines on the slab traversed continuously to form flexural cracks on the beam, both the beam and the slab were actually deflecting together. Note that \( \theta_x \) and \( \theta_z \) in Fig 5.13a denote the rotation about x axis (along left beam) and about z axis (along transverse beam), respectively. These axes for COR and S-COR specimens are indicated in Figs. 4.8a and 5.3a, respectively. Similarly, \( \Delta x \) and \( \Delta z \) in Fig. 5.13b denote the
horizontal movements along x and z axes, respectively. A small reduction of the middle joint out-of-plane movement in Fig. 5.13b (S-COR) was due to the restraining effect of the slab in preventing horizontal movements of the primary beams (L and T).

Figs. 5.13c and 5.13d show the horizontal movements and rotations of S-COR column joints, respectively. The development of small axial forces was indicated by the small horizontal movements in the columns. The rotation profiles of primary column joints followed that of the flexural mechanism and had an almost similar magnitude to that of COR specimen in Fig. 4.25e. The rotation of C-LT was slightly smaller compared to C-T and C-L, as the yield line in the slab did not propagate to the perimeter beams and C-LT column.

Load capacity and ALP development

The contribution of slab towards the load-carrying capacity (curve 2) could be obtained by subtracting the vertical load-displacement profile of COR specimen (curve 1) from that of S-COR (curve 3) as shown in Fig. 5.14a.

![Graph showing load-displacement relationship](image-url)

**a) Load-displacement relationship of the overall specimen**
b) Load-displacement relationship at each column

**Figure 5.14** Vertical load- displacement comparisons between S-COR and COR specimens

The slab increased the peak capacity of the frame by about 55% from 13.5 kN (displacement of 130 mm) to 21 kN (displacement of 180 mm), as well as delayed the drop in the frame resistance, thereby increasing the ductility of S-COR specimen. The curves indicating the transfer of loads to each of the primary columns of S-COR (with slab) and COR (without slab) are plotted in Fig. 5.14b. In the presence of the slab, the primary columns (C-L and C-T) of S-COR specimen (curves 3 and 4) received almost twice the load (13 kN) compared to those in COR (curves 1 and 2) without the slab (6.5 kN).

Flexural mechanisms in both the primary beams and the slabs throughout the test were clearly evident from strain gauge readings along the left beam’s top reinforcement (LT1,2,3,4) and along the slab’s continuous bottom reinforcement next to the left beam (S-B1,2,3,13). They are shown in Figs. 5.15a and 5.15b, respectively. The almost zero value of strain at the middle joint indicated that the beam and slab behaved like cantilever and the absence of CAT was indicated by no sign of shifting up of tensile strains along the beam and slab reinforcement. Under cantilever mechanism, both the top and the bottom slab reinforcement next
to the beam worked together with the beam top reinforcement to resist hogging moment, which accounted for positive strain values along the slab bottom reinforcement.

Figure 5.15 Strain profile along beam and slab reinforcement in S-COR specimen

To impart a sense of the magnitude of vertical load contribution from the slab, its pure flexural capacity (without considering membrane forces) was calculated based on yield line theory. The comparable value between the calculated slab flexural capacity \( P_{s,fl} \) (details are presented in Section 6.3.3) of 7 kN (plotted in Fig. 5.14a) and the yield line capacity from the slab contribution of 8 kN (curve 2 in Fig. 5.14a) confirmed the sole contribution of flexural mechanism. Under corner column loss with lack of restraints, compressions between the end segments (formed by positive yield lines) to support the development of tensile membrane action in the central region could not be materialised. Instead, the slab was being pulled-in as it rotated along the positive yield line as shown in Fig. 5.12c. A gradual decrease in resistance of the slab near the end of the stage was due to excessive concrete crushing and cracking along the yield lines and plastic hinges.

5.4. Experimental result of S-EXT specimen

5.4.1. Global response

Load-displacement
The vertical load-displacement and horizontal reaction-displacement relationships of S-EXT specimen are plotted in Figs. 5.16 and 5.17, respectively.

**Figure 5.16** Vertical load-displacement relationship of S-EXT specimen

**Figure 5.17** Horizontal reaction-displacement relationship of S-EXT specimen

The load (Fig. 5.16) was mainly resisted by the primary frames, chiefly by the double-spanning beam, indicated by the vertical load resisted by the right (C-R)
and the left (C-L) columns (curves 2 and 3) followed by the transverse beam (C-T) (curve 4). The net negative (compressive) horizontal reactions in Fig. 5.17 indicated the formation of a compressive ring around the slab perimeter. From the start of load application, total vertical load increased until 93 kN at displacement of 320 mm, followed by a sudden drop due to fracture of beam bottom reinforcing bars at the right middle joint interface. The peak capacity of 93 kN might also be limited by the punching shear capacity of the slab, as indicated by detachment between the transverse beam and the slab in Figs. 5.18b and 5.18c. Punching shear without considering longitudinal and shear reinforcement from the beam was calculated to be 70 kN based on Eq. (5-4) and Fig. 5.11 by substituting the same input values as S-COR specimen except for $u$ (868 mm). Thereafter, the load continued to increase before finally failed at displacement of 540 mm, due to fracture of beam top reinforcing bars at the middle joint-right beam interface. This value was calculated without considering the shear reinforcement in the beam and hence the actual punching shear capacity of S-EXT specimen should be greater.

Crack pattern

The crack patterns, top and bottom views of S-EXT at the final state are shown in Figs. 5.18a, 5.18b, and 5.18c, respectively. The development of crack patterns was divided into three stages, viz. Stage 1 from the start of the test to displacement of 90 mm (one slab depth) indicated by the black lines; Stage 2 from displacement of 90 mm to the commencement of CAT (displacement of 250 mm) indicated by the blue lines; and Stage 3, from this point onwards till the end of the test at displacement of 540 mm indicated by the red lines. In stage 1, flexural cracks developed along the double-spanning beam. The fan-shaped yield lines on the slab propagated from the middle joint to the slab perimeter. In stage 2, more cracks were formed in both the beams and the slabs. The yield lines started penetrating towards the thickened slab extensions (120 mm thick). In stage 3, a few new cracks were formed in the beams but more cracks occurred in the slabs, especially near to the middle joint region. Approaching the end of the test, widening of cracks due to detachment between the slab and the beam resembling punching shear was observed along the fan-shaped yield lines at a distance of about 600 mm (curtailment length) from the middle joint (Fig. 5.18b).
a) Crack patterns of S-EXT

b) Final stage of S-EXT (Top view)
c) Final stage of S-EXT (bottom view)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.18}
\caption{Physical observations in S-EXT specimen}
\end{figure}

The detachment was most severe along the transverse beam accompanied by localised rotation and excessive concrete crushing at the first curtailment point (CM) in the transverse beam as seen in Fig. 5.18c. This created a disturbance to TMA which accounted for a mild increase in vertical load at displacement of about 350 mm (after fracture of beam bottom reinforcing bars). At the slab bottom, a number of positive yield lines were formed from the middle joint towards the perimeter of the slab as shown in Fig 5.18c. Flexural cracks were formed at the column exterior face due to pushing out of the compressive ring.
5.4.2. Contribution of slab

Physical behaviour

The major difference in the crack development between S-EXT and EXT specimens was the formation of flexural cracks in the column supporting the double-spanning beam. In the development of CAT in EXT specimen, the columns heavily supported the pulling-in and tension force of the double-spanning beam as indicated by flexural cracks along the column interior face (Fig. 5.18d). This might lead to a collapse under penultimate exterior column removal scenarios, as the outermost column might not be sufficiently stiff to resist the pulling-in action of the double-spanning beam under the development of CAT (as demonstrated by IR specimens in Section 3.5.2). In S-EXT specimen, TMA was developed in the slabs and the compressive ring was able to support CAT (tensile mechanism) in both beams and slabs without much reliance on the columns. This was indicated by the absence of new cracks in column at CAT stage (Fig. 5.18a). Besides, small positive horizontal reactions in Fig. 5.17 showed that columns C-R and C-T did not experience much horizontal forces as compared to vertical load in C-R and C-T in Fig. 5.16 at CAT stage.

Comparisons of twisting and out-of-plane movements at the middle joint between S-EXT and EXT specimens are plotted in Figs. 5.19a and 5.19b, respectively. The twisting trends in the double-spanning beam of EXT and S-EXT specimens were quite similar up to displacement of about 300 mm. Thereafter, due to a localised rotation at CM location on the transverse beam and its corresponding yield line, the twisting of S-EXT specimen was significantly increased. The out-of-plane movements were generally greater in S-EXT specimen, as more uniform cracks serving as axes of rotation were developed along the transverse beam, whereas rotation was mainly concentrated at the beam end in the transverse beam of EXT specimen. Similarly, after the formation of localised rotation at the transverse beam in S-EXT specimen, the middle joint moved horizontally inwards as it turned around the newly formed axes of rotation. Based on these findings, the presence of slab might not necessarily reduce twisting and out-of-plane movements of the frame-slab specimen due to
differences in the crack developments. However, it should be noted that for the same displacement, S-EXT specimen resisted about twice the load of that in EXT specimen. This might result in larger twisting and out-of-plane movement along the double-spanning beam of S-EXT specimen.

![Graphs showing displacement vs. rotation for S-EXT specimen](image)

a) Twisting of middle joint  
b) Out-of-plane movement of middle joint

c) Horizontal movement of column joint  
d) Rotation of column joint

**Figure 5.19** Displacements of frames and slabs S-EXT specimen

Figs. 5.19c and 5.19d show the horizontal movements and rotations of S-EXT column joints, respectively. In both columns of the double-spanning beam, pushing out and pulling-in of column joints were observed probably due to the development CAT. The outward movements of column joints (primary and secondary column) indicated formation of the compressive ring and as mentioned earlier, the compressive ring might overcome pulling-in of columns due to CAT. The joint rotation of the transverse column (C-T) was the largest, probably due to
the largest moment carried together by the transverse beam and the two adjacent slabs at the support end. The column joint rotations of the primary columns supporting the double-spanning beam were also comparable to those of EXT specimen in Fig. 4.33e (Section 4.4.2).

Load capacity and ALP development

Fig. 5.20a shows the vertical load-displacement profiles of S-EXT (curve 3), EXT (curve 1), and the slab contribution (S-EXT minus EXT, curve 2). The presence of slab increased the peak capacity of the frame by about 40%, from 67 kN (displacement of 590 mm) to 93 kN (displacement of 320 mm). Note that the load contribution of the slab from displacement of about 320 mm to 390 mm should be ignored as there was a series of load reductions due to bar fracture for S-EXT and EXT specimens at different displacements. A gradual decrease in the slab resistance after displacement of 390 mm was observed due to separation of the transverse beam from the slab, as well as separation between the concrete slab and its reinforcement, which greatly reduced TMA capacity.

(a) Load-displacement relationship of the overall specimen
The load resisted by each primary column of S-EXT and EXT specimens is plotted in Fig. 5.20b. During the early stage, when load was resisted by flexural mechanism, loads in the primary columns of S-EXT (C-L, C-R, and C-T) were about twice of those in the primary columns of EXT. This trend was consistent in the transverse column for both S-EXT and EXT specimens, in which the beam was mainly under flexure throughout the test. In the double-spanning beam, after attaining flexural capacity, the left and the right columns of S-EXT continued to resist the increase in loads due to TMA, and carried almost 2.5 times (40 kN in S-EXT to 16 kN in EXT) more vertical load compared to that in EXT at displacement of 320 mm. After the fracture of beam bottom reinforcing bars resulting in load reduction, and at the same time the detachment between the transverse beam and the slab at the middle joint region weakened the TMA, the main load-carrying capacity switched to CAT in the double-spanning beam. Thus, the final failure mode (fracture of top and bottom reinforcing bars on the right middle joint interface) of S-EXT and EXT were similar. The calculated slab flexural capacity of 21.5 kN (details are presented in Section 6.3.3) agreed well with what was observed in the load-displacement profile of the slab contribution. The flexural capacity of the slab was reached at displacement of 90 mm (one slab depth) and subsequently TMA developed to further increase the load-carrying
capacity until reaching a maximum peak of 56.5 kN (about 2.5 times of flexural capacity) at displacement of 320 mm.

The development of TMA was identified by the formation of a compressive ring at the perimeter region and a tensile zone at the central region. Physically, the positive yield lines at the bottom of the slab enabled the formation of a peripheral compressive ring from principal stress contours as illustrated in Fig. 5.18c. The strain profiles along the right span beam bottom reinforcement and along the slab bottom reinforcement located next to it in Figs. 5.21a and 5.21b confirmed the development of tensile action in both the beam and the slab reinforcement along the double-spanning beam direction, indicated by the shifting up of the strain profiles. In EXT specimen, CAT was not developed in the transverse beam due to a lack of horizontal restraint at the middle joint. In S-EXT specimen, the shifting up of the overall strain profiles along the transverse beam top reinforcement and the slab top reinforcement located next to it in Figs. 5.21c and 5.21d suggested axial tension development in the beam and the slab along the single-spanning beam (transverse) direction, in the presence of TMA. In addition, the strains in the slab bottom reinforcement along D1 (S-LB) and (S-RB) are plotted in Fig. 5.21e to demonstrate the variations of strains from the centre towards the outer region. The strains near the central region, i.e. S-LB4, S-LB5, S-RB3, and S-RB4, were generally of the highest values due to development of TMA and conversely, the strains were the smallest at the outer region (nearer to secondary columns), i.e. S-LB6 and S-RB7 due to presence of a compressive ring.

(a) Beam bot. rebar (Right span) (b) Slab bot. rebar (Right Span)
5.5. Summary

Through a thorough analysis and direct comparison between frame-slab and skeletal frame specimens under two different column removal scenarios, i.e. S-COR with COR specimens under corner column loss and S-EXT with EXT specimens under exterior column loss, slab contribution on ALP or behaviour of frame-slab systems subjected to a concentrated load above column loss was investigated. To facilitate the comparisons between skeletal frame and frame-slab specimens, their capacities and corresponding displacements at different stages (flexural and CAT/TMA mechanisms) are summarised in Tables 5.2 and 5.3 for
scenarios of corner column removal (COR and S-COR) and exterior column removal (EXT and S-EXT), respectively.

**Table 5.2 Key results of COR and S-COR**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Flexural Mechanism</th>
<th>Flexural capacity</th>
<th>Displacement</th>
<th>Final peak</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>COR</td>
<td></td>
<td>13 kN</td>
<td>100 mm</td>
<td>8 kN</td>
<td>370 mm</td>
</tr>
<tr>
<td>S-COR</td>
<td></td>
<td>21 kN</td>
<td>150 mm</td>
<td>13 kN</td>
<td>550 mm</td>
</tr>
</tbody>
</table>

-- Excessive spalling & load reduced to 60% max. capacity

**Table 5.3 Key results of EXT and S-EXT**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Flexural Mechanism</th>
<th>CAT and/or TMA</th>
<th>Flexural capacity</th>
<th>Displacement</th>
<th>1st Peak</th>
<th>Displacement</th>
<th>Final peak</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXT</td>
<td></td>
<td></td>
<td>32.5 kN</td>
<td>85 mm</td>
<td>44 kN</td>
<td>380 mm</td>
<td>590 mm</td>
<td>68.5 kN</td>
</tr>
<tr>
<td>S-EXT</td>
<td></td>
<td></td>
<td>60 kN</td>
<td>95 mm</td>
<td>93 kN</td>
<td>310 mm</td>
<td>540 mm</td>
<td>82 kN</td>
</tr>
</tbody>
</table>

-- Bottom rebar fracture at joint-beam interface

Important findings on the contribution of slab are summarised as follows:

**Corner column removal scenario:**

1. The presence of slab increased the ductility of the frame-slab specimen by forming multiple yield line cracks, which propagated to the beams, causing uniform distribution of cracks and delaying formation of localised cracks (plastic hinges).

2. The flexural capacity from the slab contributed to about 55% of additional capacity on top of the maximum flexural capacity of the frame. In the presence of the slab, the primary columns resisted double the amount of loads compared to the columns without the slab.

3. TMA could not be mobilised under corner column loss scenario as indicated by (1) the pulling-in of the slab segments (no formation of compressive ring) due to zero horizontal restraint, (2) strain profile along the slab reinforcement, and (3) a constant value trend of the slab resistance after attaining its flexural capacity.
4. Punching shear failure was unlikely to occur under column removal scenario as the load-carrying capacity depended solely on cantilever resistance, which was generally much lower than the punching shear capacity for thin slabs.

**Exterior column removal scenario:**

1. The presence of slab did not improve ductility of the frame-slab specimen as the failure mode was still governed by the fracture of reinforcing bars. Twisting and out-of-plane movements of the middle joint were similar in frame (EXT) and frame-slab specimens (S-EXT) at the early stage (up to 300 mm displacement), but after the detachment between the beam and the slab near the curtailment point next to the middle joint, localised rotation developed at this location leading to excessive twisting and out-of-plane movement of the middle joint.

2. TMA with about 2.5 times slab flexural capacity developed and greatly enhanced the maximum frame capacity by 40%. TMA development was confirmed from the strain profile along the slab, and the formation of positive yield lines in the slab. The formation of a compressive ring (self-equilibrating mechanism) around the perimeter region helped to anchor the development of CAT in both beams and slabs, and at the same time prevented the pulling-in of columns, which was generally observed in the frame tests.

3. During the early stage, the presence of TMA and flexural capacity of slab greatly increased the load resisted by the primary columns supporting the double-spanning beam. After the diminishing of TMA due to detachment between the transverse beam and the slab, the vertical load resisted by the primary columns was no longer benefiting the slab as CAT in the double-spanning beam resumed as the load-carrying mechanism.

4. Punching shear failure was possible under exterior column removal scenario if its capacity was exceeded by the enhancement from TMA and flexural capacity of slab.
CHAPTER 6: NUMERICAL AND ANALYTICAL MODELS FOR RC STRUCTURES

6.1. Introduction

In the analysis or design of buildings against progressive collapse via Alternative Load Path (ALP) method, evaluation of structural behaviour and resistance of buildings under different column removal scenarios [B4, C2, D1, G1] through experimental studies may not be cost or time effective. Engineers need to rely on numerical and analytical approaches. These two approaches are often used concurrently in the analysis or design. Numerical analyses are generally employed to obtain the global response or collapse mode of the entire, or majority of the structure. On the other hand, analytical models may give quick predictions of the local substructure (connected to the removed column) resistance to ascertain whether ALP can be mobilised and spreading of failure to adjacent members can be prevented. Hence, the objective of this chapter is to demonstrate a simple, quick and reliable numerical method which can be adopted in global analysis of building collapse and to develop a simplified analytical method to facilitate quick predictions of ALP capacity at the substructure level.

The numerical and the analytical models for the three series (levels) of tests (2-D RC frames, 3-D RC frames, and 3-D RC frame-slabs) are presented in Chapter 6, and separately from their respective experimental results (Chapter 3 to 5) as relationships among the different series or levels have been identified and established from the tests. In terms of numerical analysis, the models are validated from the lowest level (2-D frames), to 3-D frames, and finally to the 3-D RC frame-slabs. In the development of analytical models, ALP interactions among the different series (fully restrained and centrally loaded specimens) are identified as follows:

1) 2-D RC frames to 3-D RC frames: In the 3-D RC skeletal frames (COR and EXT specimens), load-carrying capacity (ALP) was contributed by flexural resistance of 2-D single-spanning beams and/ or flexure and CAT of 2-D
double-spanning beams (FR specimen). These were illustrated in Figs. 4.26 and 4.34 for COR and EXT specimens, respectively.

2) 3-D RC frames to 3-D RC frame-slabs: In Chapter 5, the overall load-carrying capacity of RC frame-slabs could be idealised as a summation of the 3-D skeletal frame capacity (beam flexure and/or CAT mechanisms) and the slab capacity (slab flexure and/or TMA mechanisms). These were illustrated in Figs. 5.14a and 5.20a for S-COR and S-EXT specimens, respectively.

In both numerical and the analytical studies, numerical analyses and analytical predictions, as well as their validations were conducted progressively from 2-D to 3-D RC frames and then extended to 3-D RC frame-slabs to demarcate and highlight the individual contribution from beams and slabs.

6.2 Numerical model

6.2.1. Introduction on numerical studies

While numerical modelling using continuum solid finite elements may give more accurate representations of structural behaviour which involves both geometric and material non-linearity, it is computationally expensive and inefficient. Besides, all the required parameters must be provided accurately. Towards this end, the concept of macromodel-based non-linear finite element analysis in which the beams and columns are modelled with fibre elements is a viable and more user-friendly alternative. Fibre elements can simulate both flexural and axial resistances which are the two main mechanisms observed from the tests. Moreover, this concept has been employed by many researchers [B3, K1, Y4] in simulating the behaviour of frames under column removal scenarios.

In the frame-slab specimens, the slabs are modelled with plate or shell elements in combination with fibre elements for frames. Engineer’s Studio software (Ver.1.06.03 English version) [F1] is employed as the platform for the numerical studies. Note that the direct torsion tests (T(0.5h) and T(1h) specimens) are excluded from the numerical analysis as the shear and the uni-axial behaviour of fibre elements are independent of each other in Engineer’s Studio. Hence, the weakening of flexural and CAT capacities due to torsion cannot be captured using this software.
The numerical studies started from the validation of 2-D to 3-D RC frames and finally extended to 3-D RC frame-slabs. In 2-D RC frames, the validated numerical model was employed for further parametric studies, i.e. to investigate the effects of boundary conditions and multi-storey interactions. In 3-D RC frames, the validated numerical model was extended for parametric studies on unsymmetrical beams with variations in span and reinforcement ratio. These two effects were later incorporated into the analytical model. The numerical studies of 3-D RC frame-slabs were conducted to demonstrate the coupling of fibre and plate (shell) elements in the ALP predictions of RC frame-slab structures.

The constitutive models of concrete and reinforcement developed by Maekawa’s research group [M2], namely COM3, are employed for uniaxial material properties of concrete and steel fibres, respectively. Concrete model of COM3 is an elasto-plastic fracture model of concrete in compression, and its tensile branch has incorporated the contribution of bond-slip into tension-stiffening for concrete in tension, as shown in Fig. 6.1a. The constitutive model of reinforcement is shown in Fig. 6.1b.

![Constitutive model of concrete (COM3)](image)

**a)** Constitutive model of concrete (COM3)
6.2.2. 2-D RC frames

Numerical modelling and validation

The numerical models were created accordingly to the three 2-D RC frame specimens (FR, IR-1, and IR-2). The same material and geometrical properties, and reinforcement details were applied in the 2-D RC frame models. Uni-axial material properties of concrete and steel fibres were simulated using constitutive models of concrete and reinforcement. The key parameters input for the concrete and steel models are summarised in Table 6.1. All the supports were pinned following the test setup. The steel horizontal restraints for the columns were modelled as springs as shown in Fig. 6.1. Note that only the numerical model of FR specimen is shown. The models for IR-1 and IR-2 specimens were created by simply removing the springs for horizontal restraints according to the respective test setup (Fig. 3.5 in Section 3.2.2). The spring stiffness values were obtained from the tests by normalising the axial forces from the two-way load cells with respect to the measured horizontal movements of horizontal restraints. For simplicity, the average spring stiffness value was used due to symmetry and comparable values at both sides of the horizontal restraints. The compression and
tension stiffness values of the springs connected to the top end of the columns were 1500 kN/m and 50000 kN/m, respectively. The compression and tension stiffness values of the springs connected to the beam ends were 2000 kN/m and 40000 kN/m, respectively.

Table 6.1 Input parameters for concrete and steel reinforcement

<table>
<thead>
<tr>
<th>Concrete model</th>
<th>COM3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength ($\sigma'_{ck}$)</td>
<td>32 MPa</td>
</tr>
<tr>
<td>Tensile strength ($\sigma_{bt}$)</td>
<td>3.5 MPa</td>
</tr>
<tr>
<td>Young Modulus ($E_c$)</td>
<td>26,000 MPa</td>
</tr>
<tr>
<td>Stiffening factor ($C$)</td>
<td>0.4</td>
</tr>
<tr>
<td>Poisson’s Ratio ($v_c$)</td>
<td>0.167</td>
</tr>
<tr>
<td>Initial Shear Modulus ($G_c$)</td>
<td>11,100 MPa</td>
</tr>
<tr>
<td>Expansion Coefficient ($\alpha_c$)</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Unit Weight ($\gamma_c$)</td>
<td>24.5 kN/m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Steel Model</th>
<th>COM3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Strength ($\sigma_{ys}$)</td>
<td>507 MPa</td>
</tr>
<tr>
<td>Broken Strength ($\sigma_{ub}$)</td>
<td>610 MPa</td>
</tr>
<tr>
<td>Young Modulus ($E_s$)</td>
<td>200 MPa</td>
</tr>
<tr>
<td>Poisson’s Ratio ($v_s$)</td>
<td>0.3</td>
</tr>
<tr>
<td>Initial Shear Modulus ($G_s$)</td>
<td>76,900 MPa</td>
</tr>
<tr>
<td>Expansion Coefficient ($\alpha_s$)</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Unit Weight ($\gamma_s$)</td>
<td>77 kN/m³</td>
</tr>
<tr>
<td>Yield elongation ($\varepsilon_{sh}$)</td>
<td>0.025</td>
</tr>
<tr>
<td>Hardening Modulus ($E_2$)</td>
<td>1500 MPa</td>
</tr>
</tbody>
</table>

Mesh analysis was conducted to select the most effective mesh for the numerical model. The beam was built using 20, 28, and 36 elements as illustrated in Fig. 6.2, and the numerical results are plotted in Fig. 6.3. When 20 elements (blue line) were used, the CAA peak of the numerical model slightly overestimated that of the test. By using a finer mesh (28 elements - red dotted line), the flexure profile, the first fracture of rebar, and the CAT profile slightly improved and were closer to the experimental results. When 36 elements (purple dotted line) were used, no much improvement was observed. However, the computational time was longer and the analysis was more difficult to converge at the CAT stage (indicated by some jumps at displacement of 470 mm and 500 mm). Hence, the numerical model with 28 elements was selected for further and
subsequent numerical analyses. The numerical model with 28 elements had a finer mesh at the critical regions (near the middle joint and the beam ends) compared to the model with 20 elements.

The double-spanning beam was divided into 28 beam elements with either one- or two-Gauss integration points along each element. The critical regions near the middle joint and the beam ends, in which plastic hinges were developed, were modelled with finer two-Gauss integration point elements (Fig. 6.2b), while the remainder of beams and columns were modelled with one-Gauss integration point elements to reduce computational time.

![Numerical model of FR specimen with (a) 20, (b) 28, and (c) 36 elements](image)

**Figure 6.2** Numerical model of FR specimen with (a) 20, (b) 28, and (c) 36 elements
Figure 6.3 Comparisons between numerical predictions with different number of elements and test result for FR specimen

The numerical results of vertical load-displacement and horizontal reaction-displacement for FR, IR-1, and IR-2 specimens are presented in Figs. 6.4 to 6.6. From these figures, clearly, both vertical load-displacement and horizontal reaction-displacement relationships between the tests and numerical results for all three specimens agreed well with each other, except for a higher initial stiffness in the numerical vertical load and an earlier transition of numerical horizontal reaction from compression to tension. This might be due to the imperfection in the modelling of horizontal restraints. In the test, the spring stiffness of the horizontal restraint was gradually increasing with an increase in horizontal displacement. However, the spring model in the numerical software (Engineer’s studio) could only model bilinear springs with decreasing stiffness. In the numerical model, an average stiffness value was used. Hence, the initial stiffness was overestimated in the numerical model. Nevertheless, the most important predictions which were the maximum vertical load and the final displacement were captured rather well in the simulations.
c) Vertical load-displacement  

Figure 6.4 Comparisons between numerical prediction and test result for FR specimen

(a) Vertical load-displacement  
(b) Horizontal reaction-displacement

Figure 6.5 Comparisons between numerical prediction and test result for IR-1 specimen

(a) Vertical load-displacement  
(b) Horizontal reaction-displacement

Figure 6.6 Comparisons between numerical prediction and test result for IR-2 specimen

In FR specimen (Fig. 6.4), both the predicted final CAT capacity and the displacement matched the test results quite well. In IR-1 (Fig. 6.5) and IR-2 (Fig.
6.6) specimens, although the fracture of reinforcing bar and CAT developed earlier in the numerical models, both the maximum CAA capacity at the early stage and the maximum CAT capacity at the later stage were predicted reasonably well. Note that, similar to the test, a total loss in load (drop to zero) was not observed in the numerical models of IR-1 and IR-2 specimens. The vertical loads and horizontal reactions became constant or increased mildly with increasing displacement. This supported the explanation on the limitation of outermost column capacity to CAT (Section 3.5). Based on these comparisons, it could be concluded that the models were accurate and reasonable while the software Engineer’s studio was capable of predicting the behaviour of 2-D RC frames under single column removal scenario.

**Boundary condition and multi-storey frames**

The validated numerical model was further extended from an idealised frame specimen to a two-storey frame structure as shown in Fig. 6.7 (based on FR model). The column was extended to the full height with the base fixed and the adjacent horizontal restraint was replaced by one adjacent bay. Two configurations, namely Configuration A and Configuration B, were created. Configuration A, in which there was no column above the lower missing column (Fig. 6.7a), was created to identify whether the test setup of the frame specimens (FR, IR-1, and IR-2) could simulate reasonable boundary conditions (close to actual) by comparing the numerical result of the frame specimens (FR, IR-1, and IR-2) with that of Configuration A. Configuration B, with a column above the lower missing column (Fig. 6.7b), was created to investigate the participation of the above storey in transferring and resisting the column load. The two configurations A and B were also created based on material properties and dimensions of IR-1 and IR-2 specimens.
A comparison between the test models and Configuration A at flexural-CAA stage shows that the vertical load profile matched well with each other for all three specimens (FR, IR-1 and IR-2) as shown in Figs. 6.8 to 6.10, except for a slightly more pronounced CAA variation in Configuration A due to stiffer compressive horizontal restraints provided by the adjacent bays. At CAT stage, the FR model and Configuration A of FR case (Fig. 6.8) matched closely, indicating that the horizontal restraint provided in the test was adequate to support the development of CAT. For IR-1 (Fig. 6.9) and IR-2 (Fig. 6.10) cases, instead of maintaining constant at the remaining CAT stage, the resistance of Configuration A of the extended model continued to increase, possibly due to a higher column flexural capacity of the full length column, contributed by column axial force from the above storey and the fixed base. Overall, the idealised boundary conditions in the test setup were able to represent restraints from adjacent bays throughout loading, except at the later part of CAT stage in IR-1 and IR-2.
Figure 6.8 Comparisons among different frame configurations for the extended FR models

Figure 6.9 Comparisons among different frame configurations for the extended IR-1 models
Figure 6.10 Comparisons among different frame configurations for the extended IR-2 models

A comparison between Configurations A and Configurations B at flexural-CAA stage shows that the vertical load profile of Configuration B was almost twice of that of Configuration A for all three cases (FR, IR-1 and IR-2), demonstrating that both the first and second storey beams utilised their full flexural and CAA capacities simultaneously in carrying the load. At CAT stage, the load ratios for all three specimens between Configurations A and B varied below 2. This was because while flexural and CAA capacities depended more on structural properties of the beam itself, CAT depended on horizontal restraints to develop axial tension forces in the double-spanning beam. In FR case, additional contribution from the second storey at CAT stage was about half of that of the first storey. As there was no other storey above, the horizontal restraint stiffness at the second storey was smaller than the first storey, leading to a smaller CAT capacity. In IR-1 and IR-2 cases, the maximum CAT capacities of both Configurations A and B were the same. Capacity of Configuration B dropped significantly after flexural stage while capacity of Configuration A slightly increased until reaching a similar value with Configuration B. This meant that structural capacity was limited by the capacity of the outermost column, which
was the same in both configurations. CAT in upper storeys could not be mobilised since there was inadequate horizontal restraints in IR-1 and IR-2, unlike FR. Hence, unlike CAT capacity in FR case which increased proportionally with multiple storeys, CAT capacities in IR-1 and IR-2 cases were limited by their outermost column capacity, regardless of the number of storeys or adjacent bays on the restrained side. This also suggests that under a penultimate column removal scenario, the greater the number of storeys, the greater the vulnerability to collapse, since CAT capacity is limited by the column which has to sustain a higher load from the greater number of storeys above it.

6.2.3. 3-D RC frames

Numerical models and validation

As the frames in COR and EXT specimens have the same details and boundary supports as FR specimen, the numerical models of COR and EXT specimens were built following FR specimen as shown in Figs. 6.11a and 6.11b, respectively. All the details of COR and EXT models, i.e. material properties, spring properties and element types were identical to those of FR specimen.

![Figure 6.11 Numerical models for 3-D RC Frames](image)

a) Numerical model of COR specimen    b) Numerical model of EXT specimen

Fig. 6.12a shows the vertical load-displacement comparisons between tests and numerical models of COR and EXT specimens. In COR specimen, the numerical prediction generally agreed quite well with the test results except that flexural capacity and the final displacement were slightly overestimated, possibly due to the
model’s limitation in predicting the formation of plastic hinges at the curtailment points and in considering additional damages from twisting. In EXT specimen, except for a higher initial stiffness, the overall development of ALP was captured well by the numerical model, e.g. the load profile, flexural and CAT capacities, as well as their corresponding displacements.

\[
\begin{align*}
\text{a) Comparisons between numerical models (original) and test results of COR and EXT} \\
\text{COR-Numerical} & \quad \text{COR-Test} \\
\text{EXT-Numerical} & \quad \text{EXT-Test}
\end{align*}
\]

\[
\begin{align*}
\text{b) Refinement of COR model}
\end{align*}
\]
c) Refinement of EXT model

**Figure 6.12** Comparisons between numerical models and test results of COR and EXT specimens

Refinement of numerical models

The numerical models of COR and EXT were refined to obtain a closer prediction of initial stiffness. Two approaches were undertaken, i.e. an increase in the number of beam elements (mesh studies) and a reduction in the Young’s modulus of concrete ($E_c$) to 80% (considering the weakening of concrete due to cracks). The improvements in numerical predictions based on the two approaches, i.e. an increase in the number of beam elements (curve 3) and a reduction in Young’s modulus of concrete (curve 2) are plotted in Figs. 6.12b and 6.12c for COR and EXT specimens, respectively. Note that (E) and (M) in Figs. 6.1b and 6.1c denote numerical analysis with a reduction in Young’s modulus of concrete and an increase in number of beam elements, respectively.

For both COR and EXT, the increase in the number of elements (two times the original number, i.e. 28 to 56 elements) did not significantly improve the initial stiffness, signifying that the number of elements in the original model had reached the optimum mesh size. The reduction in Young’s modulus only slightly reduced
or improved the initial stiffness of COR and EXT. However, the initial stiffness from numerical prediction was still relatively higher as compared to that of the test. It should be noted that the numerical analysis with a combination of an increase in number of elements and a reduction in $E_c$ was not plotted as the result was similar to that of numerical analysis with a reduction in $E_c$.

The higher initial stiffness in the numerical prediction of both COR and EXT as compared to the tests could be due to the test setup. In 3-D tests which involved a more complicated test setup, more imperfections would be introduced, such as connection gaps and misalignment of restraints. This might weaken the initial stiffness of the 3-D frames.

**Parametric studies on unequal beam of 3-D RC frames**

Based on the validated models of COR and EXT specimens, parametric studies were conducted to investigate the effects of unsymmetrical span and reinforcement ratio in beams, which are common in buildings. For each case of analysis, a single span (one side of the double-spanning beam) was subjected to either a reduction in beam span or an increase in reinforcement ratio, while the other beam span and longitudinal steel content remained unchanged. For each parameter, two cases were considered, i.e. the beam span was varied to half (0.5L) or three-quarter (0.75L) of the original span (2400 mm), and the reinforcement ratio (top and bottom) was increased by 25% (+0.25) or 50% (+0.5) as summarised in Table 6.2. In COR models, the left and the transverse beams were inter-changeable. In EXT models, EXT-T denotes changes in the Transverse beam and EXT-R denotes the changes in one side of the double-spanning beam (Right beam). A total of 12 cases (6 for each parameter) are summarised in Table 6.2.
### Table 6.2 Parametric studies on unsymmetrical beams in COR and EXT scenarios

<table>
<thead>
<tr>
<th>Case</th>
<th>Name</th>
<th>Adjusted beam</th>
<th>Span length</th>
<th>Curtailment length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>COR(0.75L)</td>
<td>Interchangeable</td>
<td>1800</td>
<td>540</td>
</tr>
<tr>
<td>2</td>
<td>COR(0.5L)</td>
<td>Interchangeable</td>
<td>1200</td>
<td>360</td>
</tr>
<tr>
<td>3</td>
<td>EXT-T(0.75L)</td>
<td>Transverse</td>
<td>1800</td>
<td>540</td>
</tr>
<tr>
<td>4</td>
<td>EXT-T(0.5L)</td>
<td>Transverse</td>
<td>1200</td>
<td>360</td>
</tr>
<tr>
<td>5</td>
<td>EXT-R(0.75L)</td>
<td>Right</td>
<td>1800</td>
<td>540</td>
</tr>
<tr>
<td>6</td>
<td>EXT-R(0.5L)</td>
<td>Right</td>
<td>1200</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>Original (COR, EXT)</td>
<td>-</td>
<td>2400</td>
<td>720</td>
</tr>
</tbody>
</table>

**Effect of span length**

**Effect of reinforcement ratio**

<table>
<thead>
<tr>
<th>Case</th>
<th>Name</th>
<th>Adjusted beam</th>
<th>Top reinforcement&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Bottom reinforcement&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>COR(+0.25)</td>
<td>Interchangeable</td>
<td>2T10+T13 (1.81%)</td>
<td>2T10+T8 (1.30%)</td>
</tr>
<tr>
<td>8</td>
<td>COR(+0.5)</td>
<td>Interchangeable</td>
<td>2T13+T10 (2.15%)</td>
<td>3T10 (1.52%)</td>
</tr>
<tr>
<td>9</td>
<td>EXT-T(+0.25)</td>
<td>Transverse</td>
<td>2T10+T13 (1.81%)</td>
<td>2T10+T8 (1.30%)</td>
</tr>
<tr>
<td>10</td>
<td>EXT-T(+0.5)</td>
<td>Transverse</td>
<td>2T13+T10 (2.15%)</td>
<td>3T10 (1.52%)</td>
</tr>
<tr>
<td>11</td>
<td>EXT-R(+0.25)</td>
<td>Right</td>
<td>2T10+T13 (1.81%)</td>
<td>2T10+T8 (1.30%)</td>
</tr>
<tr>
<td>12</td>
<td>EXT-R(+0.5)</td>
<td>Right</td>
<td>2T13+T10 (2.15%)</td>
<td>3T10 (1.52%)</td>
</tr>
<tr>
<td></td>
<td>Original (COR, EXT)</td>
<td>-</td>
<td>3T10 (1.52%)</td>
<td>2T10 (1.01%)</td>
</tr>
</tbody>
</table>

<sup>a</sup> reinforcement ratio is calculated by $\rho = \frac{A_s}{bd}$, in which $b$ and $d$ are the width and the effective depth of beam sections, respectively.

The effects of a reduction in the beam span of COR, a reduction in the transverse beam span of EXT, and a reduction in the right beam span of EXT are shown in Figs. 6.13a, 6.14a, and 6.15a, respectively. In both COR and EXT, generally, when the span was reduced, the initial load stiffness and the load capacity at flexural stage increased and a reduction in load due to bar fracture or excessive concrete crushing at flexural stage occurred at a smaller displacement. Note that “flexural stage” denotes the stage before the waning of flexural capacity. The increase in initial load stiffness was due to attainment of flexural capacity in the shorter beam span at a smaller displacement. For the same sectional moment...
resistance, shorter span beams (0.75L and 0.5L) could resist greater loads, accounting for the increase in capacity. Under the same displacement, a shorter span beam would sustain greater damages due to a larger rotation and hence the flexural capacity of the 3-D frames (COR and EXT) would reduce at a smaller displacement, indicated by an earlier reduction in load.

In EXT model at CAT stage (Figs. 6.14a), a reduction in the transverse beam span did not affect CAT response as the transverse beam only contributed to flexural capacity. After flexural capacity was exhausted, the frame solely depended on CAT of the double-spanning beam, which remained constant for all three spans of transverse beam. In Fig. 6.15a, it was observed that the rate of CAT development (indicated by the gradient of the curve) increased with a reduction in the right beam span. At CAT stage, load was resisted by the double-spanning beam acting as CAT as shown in Fig. 6.15c. At a certain displacement, the shorter span (right beam) with a larger rotation ($\alpha_2 > \alpha_1$) would require a greater tension force to balance that of the original span (left beam) in the horizontal direction. In return, the greater tension force in the shorter span with a larger rotation was able to sustain a greater vertical load, which explained the increase in the rate of CAT development. Consequently, the larger rotation of the shorter span beam compared to the full span beam resulted in more damages and an earlier fracture of reinforcing bar, ending the CAT stage.

Figure 6.13 Effects of unequal beams in COR specimen

![Figure 6.13 Effects of unequal beams in COR specimen](image-url)
a) Reduction in beam span  
b) Increase in reinforcement ratio

**Figure 6.14** Effects of unequal beams (Transverse beam) in EXT specimen

The effects of an increase in reinforcement ratio of COR and EXT models are plotted in Figs. 6.13b, 6.14b and 6.15b. An increase in the beam reinforcement
ratio contributed to a larger flexural capacity, which led to an increase in the overall load capacity, but did not affect the initial stiffness. A similar percentage increase in both the top and bottom reinforcement contents (Table 6.2) resulted in a comparable ultimate curvature among the double-spanning beams. As the beams of the same length but with different reinforcing steel contents had almost the same ultimate curvature, flexural capacity was reached at a similar displacement. Note that curvatures and span lengths are the two main parameters determining the ultimate deflection of beams.

In EXT models with increasing reinforcement ratios of the transverse beam, CAT response would not be affected since after the diminishing of flexural mechanism, the frame solely depended on CAT in the double-spanning beam. A similar rate of CAT development was observed with an increase in reinforcement ratio of the right beam. Based on equilibrium, the maximum tension force that could be developed in the double-spanning beam was governed by the single-spanning beam with the lower reinforcement content, and in this case, it was the left beam. In other words, the tension capacity of the right beam with a greater steel content would not be fully utilised. Beams with higher reinforcement ratios were less ductile leading to an earlier failure of CAT. Thus, the strength of double-spanning beams with unequal reinforcement ratios will be governed by the lower reinforcement content, while theirs ductility is negatively affected by the higher steel content.

6.2.4. 3-D RC frame-slabs

Numerical models and validation

The slabs were modelled with linear quadrilateral plate elements (based on Mindlin-Reissner theory) and incorporated into the existing models of the frame specimens (COR and EXT). The whole frame-slab specimens (including beams and slab extensions) are shown in Figs. 6.16a and 6.16b for S-COR and S-EXT specimens, respectively. The plate element in Engineer’s studio is a laminate structure consisting of multiple layers in the thickness direction [F1]. Non-linear concrete constitutive law (dispersed crack model) developed in the concrete lab at Tokyo University was incorporated into the plate element. Furthermore, it was
claimed that the plate elements could simulate not only non-linear behaviour of in-plane deformations but also out-of-plane deformations.

**Figure 6.16** Numerical models for 3-D RC Frame-slab specimens
Comparisons between the numerical and test results of S-COR and S-EXT specimens are plotted in Fig. 6.17. Generally, the numerical model was able to provide reasonable predictions of the initial stiffness of the load profile, except for S-COR specimen which was overestimated by the numerical model (similar to the case of COR specimen). Numerical iterations failed to converge after reaching the peak capacity due to unbalanced nodal forces and moments in the plate elements. Hence, the models could not capture the descending part or the drop in load and the final failure. This could be due to the limitations of the plate elements to account for large in-plane (axial) forces. Once a considerable amount of in-plane forces was mobilised, the plate element was unable to converge, unlike shell element which could resist the load by in-plane forces. Nevertheless, the numerical models for S-COR and S-EXT were still able to provide important parameters, such as the maximum capacity of the frame-slab specimens and the corresponding displacement.

**Figure 6.17** Comparisons between numerical models and test results of S-COR and S-EXT

The final strain values for S-COR and S-EXT models respectively plotted in Fig 6.16c and 6.16d, were indicative of crack formations on the slabs. The predicted crack formations agreed well with test observations, i.e. Figs. 5.12b and 5.18b for S-COR and S-EXT specimens, respectively. In S-COR specimen, both the
numerical and test results showed that the cracks mainly developed at the half region of the slab (from the middle joint to the diagonal line between C-T and C-L columns in Fig. 5.12 in Section 5.3.1), with the main cracks (yield lines) forming along the diagonal line between C-T and C-L columns, indicated by higher strains. The highest strains were located at the plastic hinge locations of the primary beam ends, which were observed in the test. In S-EXT specimen, formation of yield lines along the slab perimeter (compressive ring) was also captured numerically. Moreover, a relatively high strain in the transverse beam region (compared to other regions) was indicative of the beam-slab detachment (punching shear) observed in the test (Fig. 5.18 in Section 5.4.1), which significantly reduced the load-carrying capacity of the slabs.

6.3. Analytical model

6.3.1. Introduction on analytical studies

Simplified analytical models were developed based on existing analytical models and validated against the test results. The proposed analytical model was able to predict not only the ultimate capacity but also the overall non-linear static response of RC substructures. The analytical models covered flexural capacity, CAA and CAT in beams, and yield line capacity and TMA in slabs. Note that the analytical method was only applied to specimens with full restraint and without torsional failure. After obtaining the load response in the idealised condition (full restraint and no torsional failure), limiting capacities due to column failures (shear or flexural) or torsional failures could be calculated using the method presented earlier in Sections 3.5 and 4.3, respectively.

Similar to numerical models, the analytical model for ALP in RC frames will be presented first and validated against the 2-D and 3-D RC frames, followed by ALP in RC slabs. Subsequently, the respective analytical models for RC frames and slabs were applied to the prediction of ALP capacity for RC frame-slab specimens.
6.3.2. RC frames

From the test results, it was identified that single-spanning beams could only develop flexural resistance, whereas double-spanning beams could resist the load by flexure-CAA at the early stage and CAT at large deformation stage. Based on these findings, the author developed a simplified analytical model to predict structural resistance (non-linear static response) in RC frames (2-D and 3-D), which is illustrated in Fig. 6.16a. Basically, the overall resistances of frames were obtained by superposition among different ALP resistances, i.e. flexural resistance (blue line), CAA resistance (green line) and/or CAT resistance (red line) developed in each of the members.

The resistance of each mechanism was calculated by either adopting or modifying the existing models, i.e. plastic hinge theory for flexural capacity prediction, Yu’s model [Y6] for CAA capacity prediction and Pham’s model [P4] for CAT capacity prediction. Plastic hinge mechanism was applied based on conventional theory. Yu method’s [Y6] was based on analytical derivation, whereas Pham’s method [P4] was an empirical model obtained through comprehensive parametric studies. In double-spanning beams subject to a point load at the middle joint, bottom reinforcing bars at the middle joint-beam interface would fracture due to limited rotation capacity of the beam. This fracture facilitated rotation of the beam for the development of CAT. Load continued to increase until the fracture of top reinforcing bars (final failure). Based on previous tests, the first fracture of bottom reinforcing bars was quite arbitrary, and therefore, difficult to be predicted. For simplicity, in this analytical model, the first fracture at the middle joint-beam interface was assumed to occur concurrently with the loss of flexural capacity (due to crushing and spalling), at a displacement of \( A_{b,\theta,F} \). The final displacement corresponding to the fracture of top reinforcing bars was represented by \( A_{cat,F} \). The plastic curvature at fracture (\( \phi_{p,frac} \)) was defined as the fracture strain (\( \varepsilon_{s,frac} \)) over the distance from the top rebar to the end of concrete fibre as shown in Fig. 6.18d. The proposed analytical model was validated against the test results of RC frames (FR, COR, and EXT specimens) and numerical results of 3-D RC frame parametric studies (COR and EXT cases).
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(a) Analytical model for Flexure and CAT load profile in beams

(b) Free body diagrams of RC beams under CAA

(c) Rotations and deflections of beams
Compressive arch action

The CAA model by Yu [Y6] was presented first as this model could predict the vertical (flexural) resistance of the double-spanning beam in the presence of axial compression with increasing displacement. In other words, if the CAA model was applied, there was no need for flexural capacity prediction based on plastic hinge theory. The CAA analytical model was derived based on the concept of Compressive Membrane Action (CMA) in one way RC slabs proposed by Park [P2]. In Yu’s model, additional input parameters such as presence of axial connection gaps and rotational restraints were included. Fig. 6.18b shows the free body diagram of one-span of the double-spanning RC beam with plastic hinges formed at the middle joint and the beam end due to a concentrate load and in the presence of axial compression. Based on moment equilibrium, the vertical resistance $P$ at the middle joint can be expressed as:

$$P = 2V = \frac{2(M_{u1} + M_u - N\delta)}{l_n}$$  \hspace{1cm} (6-1)

where $V$ is the shear force acting at the joint interfaces; $M_{u1}$ and $M_u$ are ultimate bending moments acting on the beam ends and the middle joint interfaces, respectively; $N$ is the beam axial compression; $\delta$ is the beam deflection at the middle joint; and $l_n$ is the net span length of one-span of the double-spanning beam. Based on the compatibility condition, the ultimate moment ($M_u$) and axial force ($N$) at any displacement ($\delta$) are expressed in terms of neutral axis depth at middle joint ($c_u$) and beam end interfaces ($c_{u1}$). The two neutral axis values ($c_u$ and $c_{u1}$) can be solved by equating the axial compression force in the beam
\( N = N_{ul} = N_u \). Subsequently, the two neutral axis values can be used to obtain the internal force of the one-span beam \( (M_u \text{ and } N) \), which may then lead to the vertical resistance \( P \) of the double-spanning RC beam. Note that detailed derivation was not presented in this thesis as the author fully adopted the CAA model by Yu [Y6].

In addition to the material and geometric properties of the beam, the input parameters to account for the boundary conditions, i.e. axial spring stiffness, rotational spring stiffness and axial connection gaps measured from the tests (at CAA stage) were 50000 kNm, 5000 kNm/rad and 1 mm, respectively. The model was applied at deflection range of one-tenth (sufficient deflection to allow development of plastic hinges) to one beam depth (before development of CAT). Yu reported that the proposed model tended to predict smaller displacements corresponding to CAA capacity and maximum beam axial force, respectively. The CAA response was accurate after the attainment of maximum CAA. Hence, in this model, the load was assumed to increase linearly from zero to the maximum CAA capacity as shown in Fig. 6.18a (green line). CAA prediction was terminated when the calculated CAA capacity was equal to flexural capacity calculated based on plastic hinge theory.

Flexural action (plastic hinge)

The analytical model for CAA presented earlier might give a good prediction of flexural-CAA resistance in beams. However, the procedure on calculating \( P \) at each vertical displacement step involved several iterations, e.g. while solving for \( c_u \) and \( c_{ul} \), and were tedious, e.g. formulation of internal forces based on neutral axis depth and back calculations of internal forces after \( c_u \) and \( c_{ul} \) were obtained. As an alternative, CAA can be ignored and flexural capacity can be simply calculated based on plastic hinge theory as presented in this section. Besides, flexural capacity without considering enhancement from axial compression (CAA) yields more conservative predictions and CAA is negligible in beams with large span-to-depth ratios (as in these tests).
The analytical model for load-displacement profile of flexural mechanism is illustrated in Fig. 6.18a (blue line). The vertical load increases from zero to flexural capacity \( P_{b,fl} \) at displacement \( \Delta_{b,fl} \). The beam flexural capacity \( P_{b,fl} \) (ignoring axial force) for each side of the beam is calculated based on plastic hinge mechanism as follows:

\[
P_{b,fl} = \frac{M_p + M_n}{L}
\]  \hspace{1cm} (6 - 2)

The terms \( M_p \) and \( M_n \) are the ultimate positive and negative moments of resistance at plastic hinge locations, respectively, and \( L \) is the length between two plastic hinges, or between a plastic hinge and the applied load at the middle joint. The displacement at the attainment of flexural capacity \( (\Delta_{b,fl}) \) is obtained as follows:

\[
\Delta_{b,fl} = \theta_c L_T + \frac{\varphi_y}{n} L_T^2 + \varphi_p L_p L
\]  \hspace{1cm} (6 - 3)

The first, second and third terms represent displacement due to column rotation, yield displacement and plastic displacement at plastic hinge location, respectively, as illustrated in Fig. 6.18c. The term \( \theta_c \) is the column rotation obtained from Fig. 4.25e (Section 4.4.1) and Fig. 4.33e (Section for 4.4.2) for COR and EXT specimens, respectively. The term \( n \) is a measure of statical actions at the supports, i.e. 3 for cantilever and 6 for fully fixed support condition; \( L_p \) is the plastic hinge length taken as one effective beam depth (155 mm); \( \varphi_y \) is the yield curvature and \( L_T \) is the total length of one-span beam; \( \varphi_p \) is the plastic curvature obtained by subtracting \( \varphi_u \) (ultimate curvature corresponding to ultimate compressive strain of 0.004) from \( \varphi_y \) (Fig. 6.16c).

**Table 6.3** Flexural resistance calculations for COR (including parametric studies)

<table>
<thead>
<tr>
<th>Name</th>
<th>( M_p^* ) (kNm)</th>
<th>( M_n ) (kNm)</th>
<th>( L ) (mm)</th>
<th>( L_T ) (mm)</th>
<th>( \varphi_y ) (rad/mm)</th>
<th>( \varphi_p ) (rad/mm)</th>
<th>( \varphi_{p,spall} ) (rad/mm)</th>
<th>( P_{b,fl} ) (kN)</th>
<th>( \Delta_{b,fl} ) (mm)</th>
<th>( \Delta_{b,fl,F} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COR</td>
<td>0</td>
<td>11</td>
<td>1570</td>
<td>2220</td>
<td>3x10^{-5}</td>
<td>1.8x10^{-4}</td>
<td>1.4x10^{-4}</td>
<td>8x10^{-1}</td>
<td>116.7</td>
<td>372.9</td>
</tr>
<tr>
<td>COR(0.75L)</td>
<td>0</td>
<td>11</td>
<td>1170</td>
<td>1620</td>
<td>3x10^{-5}</td>
<td>1.8x10^{-4}</td>
<td>1.4x10^{-4}</td>
<td>8x10^{-1}</td>
<td>9.4</td>
<td>76.1</td>
</tr>
<tr>
<td>COR(0.5L)</td>
<td>0</td>
<td>11</td>
<td>750</td>
<td>1020</td>
<td>3x10^{-5}</td>
<td>1.8x10^{-4}</td>
<td>1.4x10^{-4}</td>
<td>8x10^{-1}</td>
<td>14.7</td>
<td>42.2</td>
</tr>
<tr>
<td>COR(+0.25)</td>
<td>0</td>
<td>12</td>
<td>1570</td>
<td>2220</td>
<td>3x10^{-5}</td>
<td>1.6x10^{-4}</td>
<td>1.4x10^{-4}</td>
<td>6x10^{-1}</td>
<td>7.6</td>
<td>111.7</td>
</tr>
<tr>
<td>COR(+0.5)</td>
<td>0</td>
<td>18.6</td>
<td>1570</td>
<td>2220</td>
<td>3.2x10^{-5}</td>
<td>1.4x10^{-4}</td>
<td>6x10^{-1}</td>
<td>11.8</td>
<td>109.9</td>
<td>300.8</td>
</tr>
</tbody>
</table>

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* $M_p$ of COR is zero as plastic hinge does not develop at the beam-middle joint interface (beams behaved like cantilevers).

**Table 6.4** Flexural resistance calculations for EXT (including parametric studies)

<table>
<thead>
<tr>
<th>Name</th>
<th>$M_p$ (kNm)</th>
<th>$M_n$ (kNm)</th>
<th>$L$ (mm)</th>
<th>$L_T$ (mm)</th>
<th>$\varphi_y$ (rad/mm)</th>
<th>$\varphi_p$ (rad/mm)</th>
<th>$\varphi_{p,spall}$ (rad/mm)</th>
<th>$P_{b,fl}$ (kN)</th>
<th>$\Delta_{b,fl}$ (mm)</th>
<th>$\Delta_{b,fl,F}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXT-T</td>
<td>0</td>
<td>16.5</td>
<td>2220</td>
<td>2220</td>
<td>3x10^{-5}</td>
<td>1.5x10^{-4}</td>
<td>6x10^{-4}</td>
<td>7.4</td>
<td>124.8</td>
<td>391.2</td>
</tr>
<tr>
<td>EXT-T(0.75L)</td>
<td>0</td>
<td>16.5</td>
<td>1620</td>
<td>1620</td>
<td>3x10^{-5}</td>
<td>1.5x10^{-4}</td>
<td>6x10^{-4}</td>
<td>10.2</td>
<td>81.3</td>
<td>275.7</td>
</tr>
<tr>
<td>EXT-T(0.5L)</td>
<td>0</td>
<td>16.5</td>
<td>1020</td>
<td>1020</td>
<td>3x10^{-5}</td>
<td>1.5x10^{-4}</td>
<td>6x10^{-4}</td>
<td>16.2</td>
<td>45.1</td>
<td>167.5</td>
</tr>
<tr>
<td>EXT-T(+0.25)</td>
<td>0</td>
<td>20.3</td>
<td>2220</td>
<td>2220</td>
<td>3.2x10^{-5}</td>
<td>1.6x10^{-4}</td>
<td>6x10^{-4}</td>
<td>9.1</td>
<td>131.6</td>
<td>394.4</td>
</tr>
<tr>
<td>EXT-T(+0.5)</td>
<td>0</td>
<td>24.1</td>
<td>2220</td>
<td>2220</td>
<td>3.4x10^{-5}</td>
<td>1.2x10^{-4}</td>
<td>5x10^{-4}</td>
<td>10.9</td>
<td>120.7</td>
<td>344.5</td>
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<tr>
<td>EXT-DS</td>
<td>11</td>
<td>16.5</td>
<td>2220</td>
<td>2220</td>
<td>3x10^{-5}</td>
<td>1.5x10^{-4}</td>
<td>6x10^{-4}</td>
<td>12.4</td>
<td>100.1</td>
<td>366.5</td>
</tr>
<tr>
<td>EXT-R(0.75L)</td>
<td>11</td>
<td>16.5</td>
<td>1620</td>
<td>1620</td>
<td>3x10^{-5}</td>
<td>1.5x10^{-4}</td>
<td>6x10^{-4}</td>
<td>17.0</td>
<td>68.2</td>
<td>262.6</td>
</tr>
<tr>
<td>EXT-R(0.5L)</td>
<td>11</td>
<td>16.5</td>
<td>1020</td>
<td>1020</td>
<td>3x10^{-5}</td>
<td>1.5x10^{-4}</td>
<td>6x10^{-4}</td>
<td>27.0</td>
<td>39.9</td>
<td>162.3</td>
</tr>
<tr>
<td>EXT-R(+0.25)</td>
<td>14.6</td>
<td>20.3</td>
<td>2220</td>
<td>2220</td>
<td>3.2x10^{-5}</td>
<td>1.6x10^{-4}</td>
<td>6x10^{-4}</td>
<td>15.7</td>
<td>105.3</td>
<td>368.2</td>
</tr>
<tr>
<td>EXT-R(+0.5)</td>
<td>16.5</td>
<td>24.1</td>
<td>2220</td>
<td>2220</td>
<td>3.4x10^{-5}</td>
<td>1.2x10^{-4}</td>
<td>5x10^{-4}</td>
<td>18.3</td>
<td>92.8</td>
<td>316.5</td>
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</tbody>
</table>

In Fig. 6.18a, after the attainment of beam flexural capacity ($P_{b,fl}$), the flexural resistance remains constant before eventually diminishes due to excessive concrete crushing or fracture of reinforcing bar. The final displacement $\Delta_{b,fl,F}$ corresponding to the end of flexural capacity is obtained as follows:

$$\Delta_{b,fl,F} = \theta_{c,spall} L_T + \frac{\varphi_p}{n} L_T^2 + \varphi_{p,spall} L_{p,F}$$  \hspace{1cm} (6 - 4)

where $L_{p,F}$ is the plastic hinge length at the final stage which was about 240 mm (1.5 times effective beam depth) as observed from the test (Fig. 3.11 in Section 3.3.1, Fig. 4.24 in Section 4.4.1 and Fig. 4.32 in Section 4.4.2 for FR, COR, and EXT, respectively). The terms $\varphi_{p,spall}$ and $\theta_{c,spall}$ refer to the plastic curvature in the beam and the column rotation at the failure stage due to occurrence of spalling, respectively. The term $\varphi_{p,spall}$ is obtained by assuming spalling up to the concrete cover (Fig. 6.18d). The input parameters for the calculations of $P_{b,fl}$, $\Delta_{b,fl}$, and $\Delta_{b,fl,F}$ for COR and EXT specimens (including parametric studies) are summarised in Tables 6.3 and 6.4, respectively. For unequal beams, overall flexural capacity was summed up by individual beam flexural resistance on each side of the middle joint.
Catenary action

In the analytical model of Pham [P4], the non-linear responses of 2-D RC frames were idealised as a piecewise multi-linear curve consisting of about 18 critical parameters, which might be too complicated and time-consuming. Moreover, the calculation for each parameter involved multiple steps. Hence, in this simplified analytical model proposed by the author, CAT profile was simply expressed as a single line. The important parameters for CAT included the displacement corresponding to the commencement of CAT \( \Delta_{\text{cat},I} \), gradient of increasing CAT capacity \( K_{\text{cat}} \), and final displacement of CAT \( \Delta_{\text{cat},F} \) as illustrated in Fig. 6.18a (red line). These important parameters are obtained based on Pham and Tan’s method [P4], as follows:

\[
\Delta_{\text{cat},I} = \left( 0.4 + 0.065K_{\text{span}} - 12K_{\text{flex}} \right) \left( \frac{1}{(1.07 + K_{\text{hor}})^{15}} + 1 \right) d_{\text{beam}} \tag{6-5}
\]

\[
K_{\text{cat}} = \left( 1.2 + 0.24K_{\text{span}} - 9K_{\text{topbar}} \right) \frac{F_{y,\text{topbar}}}{L_T} \tag{6-6}
\]

\[
\Delta_{\text{cat},F} = \frac{\varphi_p}{3} L_T^2 + \varphi_{p,\text{frac}} L_{p,\text{frac}} L_T + \int_{e_1}^{e_2} \varphi_{y,c} \left( 1 - \frac{x}{L_T} \right) x dx + \theta_c L_T \tag{6-7}
\]

The plastic curvature at fracture \( (\varphi_{p,\text{frac}}) \) was defined as the fracture strain \( (\varepsilon_{s,\text{frac}}) \) over the distance from top reinforcing bar to the end of concrete fibre as shown in Fig. 6.18d. The terms \( K_{\text{span}}, K_{\text{flex}}, K_{\text{hor}}, \) and \( K_{\text{topbar}} \) refer to span-to-depth ratio, normalised bending stiffness, normalised horizontal stiffness, and reinforcing steel-to-concrete stiffness ratio, respectively, which can be obtained as follows:

\[
K_{\text{span}} = \frac{L_T}{d_{\text{beam}}} \tag{6-8}; \quad K_{\text{flex}} = \frac{M_p + M_n}{E_{eq} I_g} \tag{6-10}
\]

\[
K_{\text{hor}} = \frac{K_s}{E_{eq} (A_c + A_s)/L_T} \tag{6-9}; \quad K_{\text{topbar}} = \frac{E_s A_{s,t}}{E_c A_c} \tag{6-11}
\]

All the input values and details are summarised in Tables 6.5 and 6.6 for EXT and parametric studies of EXT, respectively.
### Table 6.5 CAT calculations for EXT specimen

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
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<tr>
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<td>Span length</td>
<td>2220</td>
<td>2220</td>
<td>2220</td>
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<td>$d_{beam}$</td>
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<td>180</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>$I_g$</td>
<td>Moment of Inertia</td>
<td>$5 \times 10^7$</td>
<td>$17607.5$</td>
<td>$392.5$</td>
<td>$235.5$</td>
<td>$8 \times 10^4$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Concrete area</td>
<td>$17607.5$</td>
<td>$392.5$</td>
<td>$235.5$</td>
<td>$8 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>$A_{st}$</td>
<td>Steel area</td>
<td>$17607.5$</td>
<td>$392.5$</td>
<td>$235.5$</td>
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<td></td>
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<tr>
<td>$\psi_{p,\text{frac}}$</td>
<td>Plastic curvature at fracture</td>
<td>$17607.5$</td>
<td>$392.5$</td>
<td>$235.5$</td>
<td>$8 \times 10^4$</td>
<td></td>
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</table>

### Table 6.6 CAT calculations for EXT parametric studies

<table>
<thead>
<tr>
<th>Symbol</th>
<th>EXT-R (0.75L)</th>
<th>EXT-R (0.5L)</th>
<th>EXT-R (+0.25)</th>
<th>EXT-R (+0.5)</th>
<th>EXT-R (0.75L)</th>
<th>EXT-R (0.5L)</th>
<th>EXT-R (+0.25)</th>
<th>EXT-R (+0.5)</th>
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<td>1020</td>
<td>2220</td>
<td>2220</td>
<td>9.0</td>
<td>5.7</td>
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<td>12.3</td>
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<tr>
<td>$A_s$</td>
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<td>392.5</td>
<td>497</td>
<td>579.3</td>
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<td>0.01</td>
<td>0.03</td>
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<td>235.5</td>
<td>289.7</td>
<td>343.8</td>
<td>0.03</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
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<td>$M_p$</td>
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<td>11</td>
<td>14.6</td>
<td>16.5</td>
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<td>0.10</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>$M_n$</td>
<td>16.5</td>
<td>16.5</td>
<td>20.3</td>
<td>24.1</td>
<td>0.180</td>
<td>0.192</td>
<td>0.201</td>
<td>0.222</td>
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<tr>
<td>$\Delta_{cat,I}$</td>
<td>179</td>
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<td>192</td>
<td>181</td>
<td>367</td>
<td>311</td>
<td>475</td>
<td>455</td>
</tr>
</tbody>
</table>

In CAT predictions of unequal double-spanning beams, the analytical model by Pham and Tan [P4] was improved to account for unsymmetrical spans and reinforcement ratios in the double-spanning beam. Note that the critical parameters, i.e. $K_{span}$, $K_{hor}$, $K_{flex}$, and $K_{topbar}$ of the shorter span and/or with a higher steel content have been calculated to fall within the validity range of the analytical model (Table 6.5). In the case of a reduction in beam span, an increase in the gradient of CAT development due to different tensions in each beam was observed in numerical studies. Summing up of CAT capacity from individual beams might not be reasonable as the double-spanning beam acted as a single catenary system in which force equilibrium must be satisfied. However, it was
difficult to judge whether the development of tension force was governed by the shorter or longer span. Hence, average values of the three parameters for the two single-spanning beams, i.e. $\Delta_{\text{cat},I}$, $K_{\text{cat}}$, $\Delta_{\text{cat},F}$ were used in the prediction. In the case of unequal reinforcement ratio, the gradient of CAT development ($K_{\text{cat}}$) was governed by the beam with the lower tension steel content. However, $\Delta_{\text{cat},I}$ and $\Delta_{\text{cat},F}$ did not only depend on tension capacity but rather on beam geometry, i.e. beam span, depth and curvature. Similar to the case of unequal span, average values of $\Delta_{\text{cat},I}$ and $\Delta_{\text{cat},F}$ were taken since it was difficult to identify which single-spanning beam would govern the behaviour of EXT models.

**Comparison for 2-D Frame (FR) and 3-D Frame (COR and EXT) specimens**

The calculated resistance profiles of individual beams, i.e. flexural, CAA and CAT resistance (Tables 6.3, 6.4 and 6.5) were summed up to obtain the overall load-carrying capacity of RC frames. Comparisons between the analytical predictions and the test results of FR, COR, and EXT specimens are shown in Fig. 6.19. In EXT specimen, vertical load was contributed by flexural resistance in the transverse beam and the double-spanning beam at the initial stage, and CAT in the double-spanning beam at large deformation stage. Hence, the overall vertical load capacity was summed up from these three resistances as illustrated in Fig. 6.20. In the test of FR and EXT specimens (Fig. 6.20), flexural capacity in the double-spanning beam experienced a first drop due to bottom bar fracture at the middle joint, exhausting moment resistance ($M_p$) of the middle joint. Thereafter, the reserve flexural capacity slowly dropped to zero after excessive crushing and spalling at the beam ends. For simplicity, it was assumed that failures of both plastic hinges at the middle joint due to bar fracture and at the beam ends due to concrete spalling and crushing occurred simultaneously.
a) Comparisons between analytical predictions (including CAA) and test results of FR and EXT specimens

b) Comparisons between analytical predictions (ignoring CAA) and test results of FR, COR and EXT specimens

Figure 6.19 Comparisons between analytical predictions and test results of frame specimens
Fig. 6.19a shows the analytical predictions of FR and EXT including CAA. Fig. 6.19b shows the analytical predictions of FR, COR, and EXT ignoring CAA to represent a simplified approach. In FR and EXT specimens, obviously, the analytical model including CAA (curves 1 and 3) provided very close prediction to the test results at the initial stage since the increase in stiffness and vertical load due to axial compression was taken into account. In Fig. 6.19b, it can be seen that the analytical model for flexural resistance (ignoring CAA) might still give reasonable predictions of the test results for all 3 specimens. Moreover, this approach (ignoring CAA) was preferred as it was straight-forward and most importantly provided conservative estimations of the resistance of frames. Note that the analytical prediction ignoring CAA in COR matched well with the test results, since CAA was not mobilised in COR. It should be noted that the reasonable predictions between the analytical models excluding CAA and experimental results in FR and EXT were due to the low enhancement of CAA in the tested frames (with beam span-depth ratio of 12.3 and concrete strength of 30 MPa). In beams with lower aspect ratios and higher concrete strengths where the enhancement of CAA may be significant, the method proposed by Yu [Y6] could be adopted in the prediction of CAA.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.20.png}
\caption{Summation of flexural and CAT resistances in respective beams of EXT}
\end{figure}
In CAT stage of FR and EXT specimens, the relatively high load capacity between the displacements of 250 to 380 mm in the analytical model was due to the assumption of constant gradient of CAT in the prediction, whereas the actual gradient slowly increased as shown in Fig. 6.20. In general, the proposed analytical method (flexural and CAT) gave good predictions of the overall responses of the RC frame specimens.

Comparisons for unequal beam in 3-D RC Frames

It should be noted that the numerical results had higher capacity and initial stiffness than the analytical predictions as presented earlier in Section 6.2.3 (also illustrated in Figs. 6.12 to 6.15). Hence, the main challenge was in identifying whether the analytical method was able to capture the changes in load-carrying capacity due to the effects of beam span or reinforcing content, rather than the discrepancy between the numerical and analytical results. Note that CAA was not taken into account in the analytical prediction of 3-D frames with unequal beams for simplification, as the main intention was to capture the changes in load-displacement trends.

The analytical predictions for cases with a reduction in beam span are plotted in Figs. 6.21a, 6.22a and 6.23a. It is noteworthy that the predictions of CAT in EXT-R specimens calculated based on the proposed method, i.e. by taking average values of $\Delta_{cat,I}$ (displacement corresponding to commencement of CAT), $K_{cat}$ (gradient of CAT), $\Delta_{cat,F}$ (displacement corresponding to termination of CAT) for unequal span double-spanning beams and by taking average values of $\Delta_{cat,I}$, $\Delta_{cat,F}$, but $K_{cat}$ of beams with lower steel contents for unequal reinforcement ratio double-spanning beams have been compared with those calculated separately for “shorter and longer” spans and “higher and lower steel content” spans (details in Appendix E).” The proposed analytical method provided close prediction to the numerical result as compare to those calculated separately for each span. At flexural stage, it could be seen that the changes in trends with a reduction in span length were captured well, i.e. an increase in initial stiffness, flexural capacity and an earlier drop of load at a shorter span. The changes in CAT trends due to a reduction of right beam span were also captured well and they provided a good fit
to the numerical results shown in Fig. 6.15a. The analytical model predicted steeper CAT development and earlier failure in CAT stage due to a shorter span beam, thereby validating analytical model for unequal spans.

From Figs. 6.21b, 6.22b, and 6.23b (analytical predictions for unequal reinforcement ratios), it can be observed that the effects due to an increase of reinforcement content were demonstrated, i.e. enhanced flexural capacity with little changes in the initial stiffness and earlier failure of CAT.

**Figure 6.21** Analytical predictions of unequal beams in COR specimen

**Figure 6.22** Analytical predictions of unequal beams (Transverse beam) in EXT specimen
a) Reduction in beam span  
b) Increase in reinforcement ratio

Figure 6.23 Analytical predictions of unequal beams (Right beam) in EXT specimen

6.3.3. RC slabs

The analytical model for RC slabs included predictions of yield line capacity and Tensile Membrane Action (TMA), which were the two main load-carrying mechanisms observed in the frame-slab tests (Fig. 5.13a in Section 5.32 and 5.19a in Section 5.4.2 for S-COR and S-EXT, respectively). The slab flexural capacity was calculated based on yield line theory, whereas TMA prediction was developed based on Bailey’s model [B1].

Flexural action (yield line mechanism)

The yield line capacity of a slab was calculated based on the formation of yield line patterns which depended on various factors such as boundary condition, type of loading, reinforcement detailing, etc.
As yield line theory is an upper bound approach, the yield line corresponding to the lowest capacity is taken as the “correct kinematic or unique mechanism” for flexural capacity calculations. In S-COR, a diagonal yield line connecting C-L and C-T columns (Fig. 6.24a) was used in the calculation of yield line capacity, as observed from the test. In S-EXT, the lowest value of slab flexural capacity was obtained when the circular yield line passed through the curtailment point next to

**Figure 6.24** Yield line patterns for slab flexural capacity calculations
the beam end (CE) of the beam due to the lowest slab top reinforcement ratio as illustrated in Fig. 6.24b.

Using virtual work method (based on Fig. 6.24a), equating the external and internal work done, the yield line capacity \( P_{s,fl} \) can be expressed as follow:

\[
P_{s,fl}(S - COR) \times \delta = \frac{2}{3} \left[ m_{s,neg@150} \right] \times \sqrt{2}L \times \sqrt{\frac{2\delta}{L}} \quad (6 - 12)
\]

\( m_{s,neg@150} \) refers to the slab negative (top) moment capacity per unit width corresponding to a mesh spacing of 150 mm (along the yield lines). The ratio “2/3” considers the portion of yield line resisted by the slab top reinforcement; “\( \sqrt{2L} \)” is the length of negative yield line (joining C-T and C-L); and “\( \sqrt{2\delta/L} \)” is the rotation of the slab segment.

Similarly, based on the yield line in Fig. 6.24b, the slab flexural capacity \( P_{s,fl} \) for S-EXT can be calculated as follows:

\[
P_{s,fl}(S - EXT) = \pi \left[ m_{s,pos@150} + \frac{1}{3} \left( \frac{1}{2} m_{s,neg@150} + \frac{1}{2} m_{s,neg@100} \right) \right] \quad (6 - 13)
\]

The ratio “1/3” considers the portion of yield line resisted by the slab top reinforcement at two different regions i.e. 150 mm spacing at the region next to the perimeter beam (double-spanning beam) and 100 mm spacing at the internal beam (transverse beam) region as illustrated in Fig. 6.24b.

By considering a slab strip as a beam (Fig. 6.25), the displacement corresponding to the attainment of slab flexural capacity \( \Delta_{s,fl} \) can be obtained as follows:

\[
\Delta_{s,fl} = \frac{\phi_y}{n} L_T^2 \quad (6 - 14)
\]

where \( \phi_y \) is the yield curvature of the slab strip (top slab reinforcement at a spacing of 100 mm and bottom slab reinforcement at a spacing of 150 mm). The term \( n \) is taken as 3 (cantilever) to account for the single-spanning beam which is the last component to attain its flexural capacity. The term \( L_T \) is the slab span (2400mm). Detailed calculations for flexural resistances are summarised in Table 6.7. Note that the respective deflection due to plastic hinge rotation or column end
rotation was not considered as there was no distinct plastic hinge in the slabs or the columns at small displacements.

![Deflection of slabs](image)

**Figure 6.25** Deflection of slabs

<table>
<thead>
<tr>
<th></th>
<th>m_s,p@150</th>
<th>m_s,neg@150</th>
<th>m_s,neg@100</th>
<th>P_x,fl (S-COR)</th>
<th>P_x,fl (S-EXT)</th>
<th>( e_y )</th>
<th>( \Delta_s,fl )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>4.78 kN/mm</td>
<td>4.78 kN/mm</td>
<td>7.35 kN/mm</td>
<td>9 kN</td>
<td>21.5 kN</td>
<td>4.3 x 10^{-5}</td>
<td>82 mm</td>
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</table>

**Table 6.7** Flexural resistance calculations for slabs of S-COR and S-EXT specimens

**Tensile membrane action (TMA)**

An analytical method proposed by Bailey [B1] based on equilibrium was adopted to predict TMA capacity of S-EXT. As Bailey’s model was developed for interior slabs under UDL, the author made some modifications:

- In Bailey’s model which was developed based on a slab under UDL, the slab segments were clearly separated by distinct positive yield lines. Under the action of a concentrated load, multiple positive yield lines would be observed instead (Fig. 5.18c in Section 5.4.1). For simplicity, the diagonal line joining the perimeter columns to the middle joint was selected as the main positive yield line separating the segments.
- In the presence of beams, fracture of slab reinforcement near the middle joint would not occur and failure of the compressive ring is assumed instead. Under exterior column loss, the double-spanning beam supporting the slab was not restrained horizontally or rotationally which might possibly weaken the
formation of the compressive ring. Hence, to consider this as well as severe concrete crushing and cracking, the compressive ring capacity was reduced by half (since one half of the external constraint was lost).

- In the double-spanning beam, flexural resistance would gradually diminish when CAT was mobilised at large displacement. Hence, for a conservative estimation, enhancement of flexural capacity due to TMA was not considered.

Figure 6.26 Assumed (a) in-plane stress distribution and (b) failure mode for TMA based on Bailey’s method

The in-plane stress distribution and corresponding forces for Bailey’s model are shown in Fig. 6.26. Assuming an aspect ratio of 1, the enhancement of membrane action \( (e_m) \) towards load-carrying capacity can be calculated as follows:

\[
b = \frac{1}{T_0} \left[ \frac{1}{2} (0.67 f_c \times 0.45 d) - T_0 \right] \tag{6 - 14}
\]

\[
e_m = \frac{bw}{(3 + g_o)d} \tag{6 - 15}
\]
The term $b$ is a parameter for in-plane stress distribution, $T_o$ is the force in the steel reinforcement per unit width, $f_c$ is the concrete cylinder strength, $d$ is the slab effective depth, and $g_o$ is a factor of compressive stress block (0.85). Taking $T_o$ and $f_c$ as 90 kN/m and 30 MPa, respectively, $b$ is calculated to be 2.50. The TMA capacity was obtained by simply multiplying $e_m$ (at a certain displacement ($w$)) with the yield line capacity of the slab.

In this proposed analytical model, TMA was assumed to commence when $e_m$ was equal to 1 (TMA capacity = $P_{s/f}$). In the test, after reaching the peak capacity which was close to punching shear of the slab, detachment between the transverse beam and the slab started to occur, gradually but significantly reduced the TMA capacity. Based on this observation, the combined capacities of frames and slabs were limited to punching shear capacity (for a conservative estimation) in the proposed analytical model. Hence, after reaching the maximum capacity ($P_{TMA,F}$), TMA was assumed to diminish to account for punching shear in the slab. Note that calculations of punching shear in slabs based on EC2 [C1] have been demonstrated in Sections 5.3.1 and 5.4.1 for S-COR and S-EXT specimens, respectively.

Based on the presented analytical methods to predict flexural and TMA resistances in slabs, the author established a simplified analytical model for the overall load-displacement profile of slabs including flexural mechanism (blue line) and TMA (red line), as summarised in Fig 6.27. Although the presence of slab might increase ductility, the displacement corresponding to final failure was assumed to be governed by the frame (which failed at a smaller displacement) for a conservative estimation.
Figure 6.27 Analytical model for Flexural and TMA load profiles in slabs

Comparison for RC Frames-slab specimens (S-COR and S-EXT)

Comparisons between the analytical predictions and test results of S-COR and S-EXT specimens are plotted in Figs. 6.28 and 6.29, respectively. The predicted load-displacement profile of slab was added to that of the frame (Fig. 6.19) to obtain the overall load capacity profile of the frame-slab specimen. Basically, the analytical model for frames in Fig. 6.18a was combined with the analytical model for slabs in Fig. 6.27 to obtain overall resistance of frame-slab specimens.

Figure 6.28 Comparisons between analytical prediction and test result of S-COR specimen
In S-COR specimen, the slab flexural capacity \( P_{s,fl} \) agreed well with the test result. It was rather difficult to accurately predict the actual displacement corresponding to the attainment of full flexural capacity \( \Delta_{s,fl} \). This was due to the complex mechanism in a beam-slab system, e.g. the slab might only reach its flexural capacity after multiple cracking, or yield lines have been formed and could affect the beams. Nevertheless, the overall load-displacement profile of the frame-slab was predicted fairly well by the proposed analytical method. The slightly higher flexural capacity stemmed from an overestimation of the frame capacity.

In S-EXT specimen, on top of flexural resistance, the TMA profile was also captured well by the proposed analytical method justifying the assumption made in the modifications of Bailey’s model. Similarly, it was difficult to obtain the actual \( \Delta_{s,fl} \) due to the complex mechanism. The combinations of all the proposed analytical models (flexure, CAT, and TMA) seemed reliable as indicated by a good agreement of overall frame-slab load- displacement curves between the test result of S-EXT specimen and the analytical prediction in Fig. 6.29.

6.4. Comparisons among test results, numerical and analytical predictions
To demonstrate the accuracy and reliability of the proposed numerical and analytical methods, comparisons were made among the test results, numerical and analytical predictions. EXT and S-EXT specimens were selected as the representative of frame and frame-slab specimens, respectively. The comparisons among test results, numerical and analytical predictions of EXT and S-EXT are plotted in Figs. 6.28 and 6.29, respectively.

In Fig. 6.28 (EXT), generally, the numerical model (curve 2) tended to overestimate the initial stiffness, whereas the simplified analytical model (curve 4) (ignoring CAA) tended to underestimate the initial stiffness. The initial stiffness could be predicted accurately if CAA (curve 3) was considered in the analytical model. The critical parameters, e.g. the maximum flexural or CAA capacity, CAT capacity, and the maximum displacement were captured well by all the different methods, signifying the reliability of the proposed numerical and analytical models. Overall, the load-displacement responses of all the curves were in reasonable agreement among one another.

**Figure 6.30** Comparisons among test results, numerical and analytical predictions of EXT specimen (Frame)
In Fig. 6.29 (S-EXT), the numerical model slightly overestimated the actual capacity of the test. In the development of analytical models for frames and slabs, assumptions were made to provide conservative estimations. Hence, the analytical model slightly underestimated the actual capacity of the test. The shortcoming of the plate element in the numerical software was the limitation in predicting the softening of the frame-slab specimens. Nevertheless, the numerical model of S-EXT gave a good prediction of vertical load up to the maximum capacity and the analytical model gave a reasonable and conservative estimation of the load-displacement response of S-EXT, signifying the reliability of the proposed numerical and analytical models in predicting the ALP resistance of frame-slab systems.

6.5. Validations of analytical models with Lu et al tests [L3] on beam and beam-slab assemblage under exterior column removal scenarios.

The proposed analytical model was validated against the tests by Lu et al. [L3] on beam-slab substructures under edge (exterior) column removal scenarios. The validated specimens were 1 beam sub-assemblage (BE1), and 3 beam-slab sub-
assemblages (SE1, SE2, and SE4). All the design parameters of BE1 and SE1 were similar, except for the presence of slab in SE1 [L3]. The four specimens (aspect ratio of 1) had similar beam span of 2000 mm and concrete cube strength of 43 MPa. The differences of SE2 and SE4 from SE1 were the thickness of the slab and the reinforcement in the beam, respectively, as summarised in Table 6.8. Note that x is the direction of the double-spanning beam and y is the direction of the single-spanning (transverse) beam. The details of flexural and CAT prediction of the beams are provided in Tables 6.9 and 6.10, respectively.

Table 6.8 Details of specimens (SE1, SE2, and SE4) for analytical model validation [L3]

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<th>Specimen</th>
<th>Beam height (mm)</th>
<th>Beam width (mm)</th>
<th>Slab thickness (mm)</th>
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<th>X-direction (Top) Middle</th>
<th>Y-direction (Bottom) End</th>
<th>Y-direction (Bottom) Middle</th>
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Table 6.9 Flexural resistance calculations of beams in specimens SE1, SE2, and SE4

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<thead>
<tr>
<th>Name</th>
<th>(M_p) (kNm)</th>
<th>(M_n) (kNm)</th>
<th>(L) (mm)</th>
<th>(L_T) (mm)</th>
<th>(\varphi_y) (rad/mm)</th>
<th>(\varphi_p) (rad/mm)</th>
<th>(\varphi_{p,spall}) (rad/mm)</th>
<th>(P_{b,fl}) (kN)</th>
<th>(A_{b,fl}) (mm)</th>
<th>(A_{b,fl,F}) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE/BE-1(X)</td>
<td>2.9</td>
<td>5.5</td>
<td>2000</td>
<td>2000</td>
<td>1.7x10^{-5}</td>
<td>2.3x10^{-4}</td>
<td>8x10^{-4}</td>
<td>4.2</td>
<td>78</td>
<td>372</td>
</tr>
<tr>
<td>SE/BE-1(Y)</td>
<td>0</td>
<td>6.4</td>
<td>2000</td>
<td>2000</td>
<td>1.6x10^{-5}</td>
<td>2.3x10^{-4}</td>
<td>8x10^{-4}</td>
<td>3.2</td>
<td>90</td>
<td>385</td>
</tr>
<tr>
<td>SE-2(X)</td>
<td>2.9</td>
<td>5.5</td>
<td>2000</td>
<td>2000</td>
<td>1.7x10^{-5}</td>
<td>2.3x10^{-4}</td>
<td>8x10^{-4}</td>
<td>4.2</td>
<td>78</td>
<td>351.3</td>
</tr>
<tr>
<td>SE-2(Y)</td>
<td>0</td>
<td>6.4</td>
<td>2000</td>
<td>2000</td>
<td>1.6x10^{-5}</td>
<td>2.3x10^{-4}</td>
<td>8x10^{-4}</td>
<td>3.2</td>
<td>90</td>
<td>362.7</td>
</tr>
<tr>
<td>SE-4(X)</td>
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<td>15.1</td>
<td>2000</td>
<td>2000</td>
<td>1.8x10^{-5}</td>
<td>1.3x10^{-4}</td>
<td>6x10^{-4}</td>
<td>11.2</td>
<td>52</td>
<td>264</td>
</tr>
<tr>
<td>SE-4(Y)</td>
<td>0</td>
<td>15.1</td>
<td>2000</td>
<td>2000</td>
<td>1.7x10^{-5}</td>
<td>1.3x10^{-4}</td>
<td>6x10^{-4}</td>
<td>7.6</td>
<td>64</td>
<td>276</td>
</tr>
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</table>
### Table 6.10 CAT calculations of beams in specimens SE1, SE2, and SE4

#### Double-spanning beam of SE1 and SE2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$L_T$ (mm)</th>
<th>$d_{beam}$ (mm)</th>
<th>$I_g$ ($mm^4$)</th>
<th>$A_c$ ($mm^2$)</th>
<th>$A_s$ ($mm^2$)</th>
<th>$A_{st}$ ($mm^2$)</th>
<th>$\phi_{p,frac}$ (rad/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>Span length</td>
<td>Beam depth</td>
<td>Moment of Inertia</td>
<td>Concrete area</td>
<td>Steel area</td>
<td>(top) Steel area</td>
<td>Plastic curvature at fracture</td>
</tr>
<tr>
<td>Value</td>
<td>2000</td>
<td>170</td>
<td>$3.5 \times 10^7$</td>
<td>14286.7</td>
<td>163.3</td>
<td>106.8</td>
<td>12x$10^{-4}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>$E_c$ (GPa)</td>
<td>$E_s$ (GPa)</td>
<td>$E_{eq}$ (GPa)</td>
<td>$K_A$ (kN/mm)</td>
<td>$M_p$ (kNmm)</td>
<td>$M_n$ (kNmm)</td>
<td>$L_{p,F}$ (mm)</td>
</tr>
<tr>
<td>Definition</td>
<td>Concrete Modulus</td>
<td>Steel Modulus</td>
<td>Combined Modulus</td>
<td>Horizontal Stiffness</td>
<td>Positive moment</td>
<td>Negative moment</td>
<td>Plastic hinge at final stage</td>
</tr>
<tr>
<td>Value</td>
<td>27</td>
<td>225</td>
<td>28</td>
<td>100</td>
<td>2900</td>
<td>5500</td>
<td>220</td>
</tr>
<tr>
<td>Symbol</td>
<td>$K_{span}$</td>
<td>$K_{flex}$</td>
<td>$K_{hor}$</td>
<td>$K_{topbar}$</td>
<td>$K_{cat}$ (kN/mm)</td>
<td>$\Delta_{cat,I}$ (mm)</td>
<td>$\Delta_{cat,F}$ (mm)</td>
</tr>
<tr>
<td>Definition</td>
<td>Span-depth ratio</td>
<td>Normalised bending stiffness</td>
<td>Normalised horizontal stiffness</td>
<td>Steel-Conc stiffness ratio</td>
<td>Gradient of CAT</td>
<td>Initial Disp. of CAT</td>
<td>Final Disp. of CAT</td>
</tr>
<tr>
<td>Value</td>
<td>11.76</td>
<td>0.017</td>
<td>0.45</td>
<td>0.062</td>
<td>0.07</td>
<td>163</td>
<td>560</td>
</tr>
</tbody>
</table>

#### Double-spanning beam of SE4

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$L_T$ (mm)</th>
<th>$d_{beam}$ (mm)</th>
<th>$I_g$ ($mm^4$)</th>
<th>$A_c$ ($mm^2$)</th>
<th>$A_s$ ($mm^2$)</th>
<th>$A_{st}$ ($mm^2$)</th>
<th>$\phi_{p,frac}$ (rad/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>Span length</td>
<td>Beam depth</td>
<td>Moment of Inertia</td>
<td>Concrete area</td>
<td>Steel area</td>
<td>(top) Steel area</td>
<td>Plastic curvature at fracture</td>
</tr>
<tr>
<td>Value</td>
<td>2000</td>
<td>170</td>
<td>$3.5 \times 10^7$</td>
<td>14057.5</td>
<td>392.5</td>
<td>235.5</td>
<td>12x$10^{-4}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>$E_c$ (GPa)</td>
<td>$E_s$ (GPa)</td>
<td>$E_{eq}$ (GPa)</td>
<td>$K_A$ (kN/mm)</td>
<td>$M_p$ (kNmm)</td>
<td>$M_n$ (kNmm)</td>
<td>$L_{p,F}$ (mm)</td>
</tr>
<tr>
<td>Definition</td>
<td>Concrete Modulus</td>
<td>Steel Modulus</td>
<td>Combined Modulus</td>
<td>Horizontal Stiffness</td>
<td>Positive moment</td>
<td>Negative moment</td>
<td>Plastic hinge at final stage</td>
</tr>
<tr>
<td>Value</td>
<td>27</td>
<td>220</td>
<td>28</td>
<td>100</td>
<td>7200</td>
<td>10800</td>
<td>220</td>
</tr>
<tr>
<td>Symbol</td>
<td>$K_{span}$</td>
<td>$K_{flex}$</td>
<td>$K_{hor}$</td>
<td>$K_{topbar}$</td>
<td>$K_{cat}$ (kN/mm)</td>
<td>$\Delta_{cat,I}$ (mm)</td>
<td>$\Delta_{cat,F}$ (mm)</td>
</tr>
<tr>
<td>Definition</td>
<td>Span-depth ratio</td>
<td>Normalised bending stiffness</td>
<td>Normalised horizontal stiffness</td>
<td>Steel-Conc stiffness ratio</td>
<td>Gradient of CAT</td>
<td>Initial Disp. of CAT</td>
<td>Final Disp. of CAT</td>
</tr>
<tr>
<td>Value</td>
<td>11.76</td>
<td>0.034</td>
<td>0.45</td>
<td>0.135</td>
<td>0.109</td>
<td>130</td>
<td>560</td>
</tr>
</tbody>
</table>

For TMA prediction, the in-plane stress distribution parameters (b) of “SE1 and SE4” and “SE2” slabs were 2.6 and 4.4, respectively. TMA was terminated when the top reinforcing bars of the beam fractured, as reported from the test. The details for prediction of flexural capacity of the slabs are provided in Table 6.11.
Table 6.11 Flexural resistance calculations of slabs in specimens SE1, SE2, and SE4

<table>
<thead>
<tr>
<th>Slab Thickness</th>
<th>m_s,p@200 (kN/mm)</th>
<th>m_s,neg@200 (kN/mm)</th>
<th>m_s,neg@140 (kN/mm)</th>
<th>P_s,fl (kN)</th>
<th>( \varphi_y ) (( \times 10^5 ))</th>
<th>( \Delta_s,fl ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE1 and SE4 (Slab thickness = 50 mm)</td>
<td>2.1</td>
<td>2.1</td>
<td>2.9</td>
<td>9.1 kN</td>
<td>6.5 x 10^5</td>
<td>87</td>
</tr>
<tr>
<td>SE2 (Slab thickness = 75 mm)</td>
<td>3.15</td>
<td>3.15</td>
<td>4.4</td>
<td>13.5 kN</td>
<td>4.7 x 10^5</td>
<td>63</td>
</tr>
</tbody>
</table>

The comparisons between the test results and the proposed analytical model predictions are shown in Fig. 6.32 for BE1 and SE1, and Fig. 6.33 for SE2 and SE4. It should be noted that the proposed analytical models did not consider CAA in the beam and compressive membrane action (CMA) in the slab, and mainly focused on CAT and TMA (Red box in Figs. 6.32 and 6.33).

Figure 6.32 Comparisons between test results and analytical predictions of BE1 and SE1 specimens

For specimen BE1 (beam sub-assemblage) in Fig. 6.32, it can be seen that the proposed analytical model was able to provide a reasonable prediction of CAT profile (the commencement and the gradient of CAT) compared to the test result.
The higher peak capacity of CAT in BE1 test as compared to the analytical prediction was due to the first fracture of reinforcing bar at a larger displacement (580 mm) in the test. The fracture of reinforcing bars at a larger displacement was due to the use of mild steel round bars with higher fracture strain. In SE1 specimen, the analytical model provided a good prediction to the test result. The analytical model had a steeper gradient of TMA and CAT, but the maximum capacity and maximum displacement were well captured.

![Figure 6.33 Comparisons between test results and analytical predictions of SE2 and SE4 specimens](image)

The proposed analytical models provided reasonable agreement to the test result of SE4 (gradient of CAT and TMA, maximum capacity and displacement), but not SE2. The discrepancy between the analytical prediction and the test result in SE2 was due to the higher predicted gradients of TMA and CAT, as well as subsequent drops in load in the test. However, the actual cause of the drop in load was not clearly reported in the paper. The TMA prediction, as derived from Bailey’s method, was calculated based on the enhancement of slab flexural capacity. However, it was reported that despite the increases in slab thickness and
reinforcement content in SE2, TMA was essentially the same as that of SE1. Further investigation will be conducted on this matter.

6.6. Recommendations to assessment of progressive collapse resistance of RC structures

Based on a literature review and research works, as well as available analytical and numerical models, recommendations on simple but reliable ways to assess the progressive collapse resistance of RC substructures are presented, as summarised in Fig. 6.34. The proposed procedure involves iterations of design until adequate structural resistance can be achieved. As progressive collapse is a complex mechanism, there may not be a single definite way to calculate progressive collapse resistance. Engineers need to exercise some judgement when carrying out each step of the design.

The initial design of a structure starts with a conventional design based on tie-force requirements to ensure integrity and continuity of the structure. This is relatively simple without the need of numerical analysis or complicated calculations. Subsequently, progressive collapse resistance of the structure has to be analysed. Engineers may either do a quick check on local resistance of substructures through available analytical models, or directly conduct numerical analyses to obtain the local or global responses of the structure. In the analytical prediction of substructures, engineers may choose the appropriate level of sophistication commensurate with the required accuracy. For example, the simplest method is to treat frames as primary members and limit the maximum resistance to flexural capacity of beams. The most accurate approach is to include CAT of beams and TMA of slabs. Although the simplest way may provide conservative predictions, it may greatly underestimate the actual structural reserve and lead to an uneconomical design. Note that since the objective of analytical method is to facilitate quick and reliable predictions, the analytical method should not be too complicated and should provide conservative results.
As progressive collapse is a dynamic event, the pseudo-static (dynamic equivalent) resistance can be converted from the non-linear static resistance using energy approach [12]. If the predicted resistance is lower than the action effect, it is recommended to improve the structural design as it is highly possible for the local failure to spread to adjacent elements. If the predicted resistance based on analytical method is greater than the action effect with sufficient margin of safety, engineers can either claim that their design is adequate for the scenario considered, or conduct an independent numerical analysis for assessment.

The numerical analysis can be conducted at two different levels, i.e. on the substructure level to validate and support the analytical predictions, or on the structural level to obtain global response and resistance. Similarly, the global non-linear static resistance can be converted to pseudo-static resistance and checked against the global action effects. Lastly, if engineers want to understand the dynamic global response and resistance of the structure, a sophisticated non-linear dynamic numerical analysis can be conducted. The validated numerical models (from experiment test results or analytical predictions) can in fact be extended to conduct other single-column or multiple-columns removal scenarios. As it is
imperative to analyse the progressive collapse resistance of structure under different column removal scenarios, engineers need to ensure reliability and efficiency of their numerical platforms. From previous literature reviews as well as numerical studies conducted in this study, fibre and shell (plate) elements (for frames and slabs respectively) may provide reasonable numerical predictions without incurring exorbitant computational resources and time.

6.6 Summary

This chapter presents the numerical and analytical works essential for progressive collapse assessment of a structure. Following are the conclusions on the numerical work:

1. The numerical analyses using the model software Engineer’s Studio showed that non-linear fibre and plate finite elements were capable of simulating structural response of the test specimens (from 2- to 3-D RC frames and 3-D RC frame-slabs). In the frame specimens, the load responses of flexural resistance, CAA, and CAT were well predicted. The numerical models of the frame-slab specimens provided good predictions of the load response at the ascending stage, but the weakening and final displacement of the frame-slab specimens could not be captured due to convergence problem after failure in the plate element was detected due to unbalanced nodal forces and moments. Nevertheless, the maximum capacities and the crack patterns were well captured by the models.

2. The numerical responses of RC frames with adjacent bays (representing more realistic boundary conditions) confirmed the validity of the test setup. The global response indicated that at flexural-CAA stage, beams at each storey worked simultaneously in resisting load. In frames with adjacent bays or adequate restraints at both sides (FR specimen), under CAT stage, beams at each storey contributed to CAT capacity.

3. In frames with penultimate column removals (IR-1 and IR-2 specimens), CAT capacity might not increase with participation of remaining storeys as structural resistance was governed by the weakest outermost column. In other
words, with higher number of storeys, outer most columns would be subjected to potentially greater P-δ effects.

4. In 3-D RC skeletal frames, parametric studies were conducted to investigate the effects of reducing span or increasing reinforcement ratio of beams. Parametric studies showed that the two parameters certainly increased the capacity at the flexural stage of both COR and EXT specimens. At large deformation stage of EXT, the effect of a reduction in beam span or an increase in reinforcement ratio of the transverse beam was negligible as the load capacity relied solely on CAT of the double-spanning beam. A reduction in the span of one side of the double-spanning beam resulted in a corresponding increase of CAT and an earlier stage of failure. With an increase of reinforcement ratio, no change in CAT development was observed as structural resistance was governed by the lower reinforcement content. This increase in reinforcement content brought an earlier stage of failure due to a decrease in ductility.

5. As fibre and shell (plate) elements were reliable and computationally efficient (see Table 6.7 for computational time), they could be used to model a large scale structure subject to different column removal scenarios. Note that the computational time of the frame slab specimens (S-COR and S-EXT) were comparable to that of the frame specimens (COR and EXT) since the numerical analyses of the former, were not fully completed.

<table>
<thead>
<tr>
<th>Case</th>
<th>Time (hour)</th>
<th>Case</th>
<th>Time (hour)</th>
<th>Case</th>
<th>Time (hour)</th>
<th>Case</th>
<th>Time (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D RC Frame (FR, IR-1, IR-2)</td>
<td>1-1.5</td>
<td>COR</td>
<td>0.5-1</td>
<td>EXT</td>
<td>1-2</td>
<td>S-COR</td>
<td>0.5-1</td>
</tr>
<tr>
<td>2-D RC Frame (Multi-storey)</td>
<td>1.5-2</td>
<td>COR (unequal beam)</td>
<td>1-1.5</td>
<td>EXT-T</td>
<td>1-2</td>
<td>S-EXT</td>
<td>1-2</td>
</tr>
</tbody>
</table>

The conclusions for the simplified analytical method are as follows:

1. The proposed analytical model for frames consisting of beam flexural, CAA and CAT resistance provided reasonable predictions of the test results. The
model for CAT considered the effects of beam span and reinforcement ratio for both corner and exterior column removal scenarios.

2. The flexural and TMA capacities of slabs in the frame-slab specimens were predicted by yield line theory and modified Bailey’s model, respectively.

3. The superposition between the predicted ALP capacity from beams and slabs gave good agreement with overall capacity in frame-slab substructures under corner and exterior column removal scenarios (S-COR and S-EXT specimens).

4. The developed simplified analytical model was capable of considering the individual capacity of beams and slabs, as well as the total capacity of the substructure. This allowed engineers to account for the slab contribution in the capacity prediction of frame-slab systems. If the structure was previously designed by considering frames as primary members, the analytical predictions of slabs would give an indication of the available reserve capacity in the substructure for the scenario considered.
CHAPTER 7: CONCLUSIONS AND FUTURE WORKS

7.1 Conclusions

In this research programme, a systematic experimental investigation on structural behaviour and Alternative Load Path (ALP) development of Reinforced Concrete (RC) substructures under a single column removal scenario had been conducted, starting from the simplest 2-D RC frame level to the 3-D RC frame level and finally to the integrated 3-D frame-slab level. Structural behaviour and ALP developments from the start to the end of tests in all specimens were thoroughly and exhaustively analysed and studied from the structural level (physical observation, force and displacement measurement), cross-sectional level (internal force calculation), to the fibre level (strain gauge measurement). Based on the experimental findings, numerical and analytical models to predict the ALP capacities of RC structures were developed. The objectives and the main findings of the research program could be summarised as follows:

1. 2-D RC Frames (study on ALP development, improvement in detailing, and effect of horizontal restraint)
   
   - Test on 2-D RC frame (double-spanning beam) with adequate restraints at both sides, i.e. FR specimen under middle column removal scenario demonstrated the development of flexural-CAA at small deformation stage and CAT at large deformation stage. CAT development was indeed beneficial as it might reach about 3 times the flexural capacity and increase ductility to about 12% of the double-spanning beam length (4800 mm).

   - Usage of round longitudinal bars highlighted the potential advantages in increasing resistance and ductility of RC structures under progressive collapse. Reduction in bond-slips (especially at the plastic hinge regions) and usage of bars with large fracture strain might allow more slip leading to large vertical deflection before fracture. In addition to improvement in ductility, CAT capacity may also be further increased due to a larger component of tension force being resolved to vertical resistance at a larger beam rotation. Seismic detailing by provision of closer stirrup spacing at the support regions showed
no significant improvement as beam capacity was governed by CAT (axial mechanism) rather than shear or flexural mechanism at the final stage. In the test, seismic detailing slightly helped by producing a more ductile failure, i.e. milder concrete crushing and subsequent fracture of reinforcement bars at the same layers.

- CAT capacity was clearly limited by the capacity of the outermost column (with inadequate horizontal restraint), i.e. column flexural capacity in the test. The pulling-in of outermost column by the tension force (CAT) in the beam combined with the P-δ effect might generate instability problem to the column leading to total collapse of frames. Engineers may either ensure adequate capacity is provided to the column to resist the desired amount of CAT or ignore the contribution of CAT under penultimate column removal scenarios.

2. 3-D RC Frames (study on the effect of torsion and ALP development in more realistic scenario (3-D skeletal frame configuration))

- The presence of torsion clearly hindered the development of ALPs, i.e. CAA and CAT in the double-spanning beam. Two types of combined failure were observed, i.e. torsion-bending due to failure of longitudinal reinforcement in T(0.5h) specimen (90 mm eccentricity) and torsion-shear due to failure of stirrup in T(1h) specimen (180 mm eccentricity). The torsion-bending failure indicated the difficulty of CAT development since upon the attainment of flexural capacity, the longitudinal reinforcement may have been exhausted, leaving not much reserve capacity to resist torsion and to develop CAT.

- In 3-D skeletal frames subjected to a point load above the removed column, the single-spanning beam would resist load by cantilever mechanism and the double-spanning beam would resist the load by flexure and CAT (similar to 2-D RC double-spanning beams under middle column removal scenario). The lack of restraint at the middle joint caused the joint to twist and move together with the beams which at the same time resulted in small compatibility torsion which was negligible to the load-carrying capacity of frames.
3. **3-D RC Frame-Slabs (study on slab contribution and its interaction with beams)**

- The presence of slabs significantly improved the capacity of the frame-slab specimens, i.e. 60% of additional capacity (on top of frame maximum capacity) in S-COR specimen and 55% in S-EXT specimen. At a particular MJD, the maximum enhancement of slab could go up to 1.5 times of the frame capacity.

- S-COR specimen relied on the flexural resistance of both the beams and slabs. Whereas, S-EXT was greatly enhanced by flexure and CAT developed in beams and by flexure and TMA developed in slabs. The TMA capacity went up to about 2.5 times the slab flexural capacity before gradually dropped due to beam-slab detachment (punching shear).

- The formation of self-equilibrating mechanism (compressive ring) was detected in S-EXT specimen, which may be beneficial in supporting the development of TMA and CAT as well as preventing excessive pulling-in of outermost column under penultimate column removal scenarios.

4. **Numerical studies (reliability of numerical method and further numerical studies)**

- Non-linear FEM analyses by modelling beams and slabs with fibre and shell (plate) elements, respectively were conducted and validated against the test results, demonstrating the reliability and efficiency of this concept in predicting and analysing progressive collapse resistance in RC structures.

- The extended 2-D RC frames with adjacent bays confirmed the adequacy of horizontal restraints provided in the tests and in an actual structure to support development of CAT. The extended 2-D RC multi-storey frames indicated the identical participation of each storey in resisting loads during flexural stage. In a fully restrained condition, uppermost beam might contribute lesser CAT due to a lower horizontal restraint stiffness. In an imperfect restraint condition (penultimate column removal), CAT capacity might not increase with
participation of remaining storeys in carrying load as it was governed by the weakest outermost column.

- The parametric studies on 3-D skeletal frames with unsymmetrical beams, i.e. a reduction in beam span and an increase in beam reinforcement ratio showed that these two parameters increased the flexural capacity of frames under corner and exterior column removal scenarios. The reduction in the span of one side of the double-spanning beam increased the rate of CAT development but earlier stage of failure. In an increase of reinforcement ratio, the rate of CAT development was not affected (governed by beam with lower reinforcement) but similarly, it would result in an earlier stage of failure due to a decrease in ductility.

5. Analytical models (prediction of ALP capacity)

- Based on the relationship or interaction among the members (beams and slabs) established from the tests, a simple analytical method to predict ALP capacity in RC structures subjected to point load on the middle joint had been successfully developed.

- The analytical model was developed systematically starting from ALP capacity prediction in 2-D RC frames which consisted of flexure, CAA and CAT, which was further extended to 3-D RC frames. Finally, an analytical model for ALP capacity prediction in slabs consisting of flexure and TMA was developed and superposed with that of RC frames to predict the overall load-carrying capacity of frame-slab substructures.

- Validation with Lu tests [L3] showed reasonable prediction of the analytical models.

The major innovations of this systematic study are the developments of simplified analytical models to predict the overall load resistance of RC substructures, which are based on the interactions among beams in 3-D skeletal frames, and between beams and slabs in 3-D frame-slab systems, as observed in the tests and parametric numerical studies.
7.2 Future works

Application of low bond stress and large fracture strain reinforcement bars in practices

The test finding of FR-R specimen (round longitudinal bars) has demonstrated the advantages of bond stress reduction and large fracture strain bar in increasing ALP resistance and ductility (CAT in particular) in RC frames. Further study may focus on the application of this concept in an actual practice. For example, bond stress reduction of deformed longitudinal bars at the critical (plastic hinges) locations by removing reinforcing bar ribs or covering the perimeter of reinforcing bars with smooth or even surface material, or usage of round bars (high fracture strain properties) as longitudinal reinforcement supported by an effective way of anchorage or lap splicing (to minimise the development length).

Holistic behaviour of RC buildings under column removal scenario

The ALP development and structural behaviour of single storey 3-D frames and 3-D frame-slab substructures have been investigated in this research work. As progressive collapse involves global failures (large part of the building), further experiments or researches should be dedicated to study the holistic behaviour (global response) of RC buildings subjected to column loss scenarios. That is, specimens should include multiple bays, three dimensional actions and typical floor systems. Most importantly, experiments should be conducted to large deformations (final failure stage) to achieve both material and geometric nonlinearity of RC structures.

The tests of multiple-storey and/ or multiple-bay RC structures under single column removal scenarios may be combined and compared with the current research works (representing single storey) to establish a simplified assessment framework in the appraisal of progressive collapse resistance in multi-storey buildings. Furthermore, the corresponding test results could be used to validate or improve the magnitudes of horizontal and vertical tie forces specified in progressive collapse guidelines, which were previously determined without seemingly any apparent support from experimental data.
Torsional failure in RC frame-slab substructures

The successful indications on ALP interferences (especially CAT) in the presence of torsion (in the direct torsion) may generate concern to most building structures as torsion is commonly present in the beam while resisting loads in slabs. Future studies should be focusing on investigating torsional failure in beam-slab substructures so as to quantify the maximum torsion eccentricity beyond which CAT will not be mobilised. 3D solid modelling may be an alternative since it may be practically difficult to isolate and quantify the behaviour of beams from beam-slab systems in experimental tests.

Development of ALPs in RC frame-slab substructures under uniform distributed load (UDL)

A comprehensive study on RC substructures subjected to a point load (lower bound) above the removed column, i.e. investigation on ALP development, identification of slab contribution, demonstration of numerical analysis, and development of analytical model have been conducted in this research work. Similar approaches (systematic studies) should be conducted on RC substructures subjected to uniform distributed loads (upper bound) which are closer to actual loading conditions.

Holistic analytical models on load capacity of RC frame-slab substructures

Based on the future works mentioned above, holistic analytical models can be developed to cover a wider case of RC structural responses, such as:

- Prediction on ALP capacity in beam, i.e. flexural mechanism, CAA, and CAT under UDL.
- Prediction on ALP capacity in slab, i.e. flexural mechanism, TMA under UDL. Under UDL, the yield line pattern formed in the slab may vary affecting both flexural and TMA capacity. Moreover, different failure modes limiting TMA capacity may be observed and should be considered in the analytical model of TMA.
Assemblage of lower level structural response, e.g. frame system to a higher level, e.g. frame and slab system and finally the highest level, e.g. whole building in the quantification of progressive collapse potential in a building.

This work can be completed either experimentally or numerically (based on validated numerical model). The validated numerical models of RC frames and frame-slabs in this research work could be employed for further analysis or parametric studies for analytical model developments. For future studies, the slab will be modelled with shell elements to account for in-plane forces and hence a more complete load response (until failure) can be obtained.
REFERENCES


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APPENDIX A: DESIGN OF TEST SPECIMENS

A.1. Design information

A six-storey commercial building with 6x4 bay is selected as a prototype of the structure. The column spacing in both directions is 6000 mm. The height of a typical storey is 3000 mm, while the ground floor is 4500 mm. The 3-D and elevation views are shown in Fig. A1. The sizes of all the beams and columns are 250x450 and 450x450, respectively. Concrete cover thickness is taken as 30mm, thus the effective depth of beam and column is 410 mm. In the design of the prototype structure, C30/37 concrete is used for beams and columns, along with high yield bars with a nominal yield strength of 500 MPa as longitudinal reinforcement and mild yield bars with a nominal yield strength of 250 MPa as transverse reinforcement. The building is designed according to EC2 [C1].
For an office building, the possible dead and live loads are illustrated as follows:

- Live load for office = 5 kPa
- Imposed Dead loads:
  - Floor cover and tile = 2 kPa
  - Frame partitions = 1.5 kPa
  - Ceiling = 0.5 kPa
- Self-weight Dead loads:
  - Beam = 0.5x0.3x24 = 2.8 kN/m
  - Column = 0.5x0.5x24 = 4.9 kN/m
  - Slab = 0.2x24 = 3.6 kPa

The basic load combination based on EC2: U=1.35 Dead Load +1.5 Live Load

A.2. Gravity load design

A.2.1. Beam design

The structural elastic analysis is carried out by ETABS. After the determination of member forces under gravity, the flexural and shear resistances are designed.
based on EC2. Interior Beam (C3-D3 indicated in the elevation view) is selected for design due to the highest bending moment in the middle region.

**Design for Moment**

The moments along the selected beams are shown in Table A.1 (Note that Table A.1 shows the moment value in beams CD and DE, the purpose is to show that they are symmetric). Table A.2 shows the calculation for the required and provided longitudinal reinforcement.

**Table A.1** Moment along selected beams

<table>
<thead>
<tr>
<th></th>
<th>BR</th>
<th>CL</th>
<th>Mid-span</th>
<th>DR</th>
<th>DL</th>
<th>Mid-span</th>
<th>ER</th>
<th>FL</th>
</tr>
</thead>
<tbody>
<tr>
<td>MID</td>
<td>-47.43</td>
<td>-47.43</td>
<td>24</td>
<td>-47.43</td>
<td>-47.43</td>
<td>24</td>
<td>-47.34</td>
<td>-47.26</td>
</tr>
<tr>
<td>MU</td>
<td>-139.51</td>
<td>-170</td>
<td>88.3</td>
<td>-170</td>
<td>-140.05</td>
<td>88.3</td>
<td>-139.78</td>
<td>-139.51</td>
</tr>
<tr>
<td></td>
<td>-170</td>
<td>88.3</td>
<td>88.3</td>
<td>-170</td>
<td>88.3</td>
<td>-139.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure A.2** Minimum and maximum reinforcement requirement

**Table A.2** Longitudinal reinforcement design

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>C (-)</th>
<th>Mid-span (+)</th>
<th>D(-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mu (kNm)</td>
<td>-</td>
<td>170</td>
<td>88.3</td>
<td>170</td>
</tr>
<tr>
<td>k</td>
<td>( M/(bd^2f_{ck}) )</td>
<td>0.149</td>
<td>0.073</td>
<td>0.149</td>
</tr>
<tr>
<td>Z (mm)</td>
<td>( d\left[0.5 + \sqrt{(0.25 - K/1.134)}\right] )</td>
<td>329</td>
<td>360</td>
<td>329</td>
</tr>
<tr>
<td>A_{s,req,d} (mm²)</td>
<td>( M/(0.87f_{yk}Z) )</td>
<td>1190</td>
<td>570</td>
<td>1190</td>
</tr>
<tr>
<td>Bar</td>
<td>( n\times\pi/4\times d\times d )</td>
<td>2T25&amp;1T20</td>
<td>2T20&amp;1T16</td>
<td>2T25&amp;1T20</td>
</tr>
<tr>
<td>A_{s,pro,d} (mm²)</td>
<td>-</td>
<td>1296</td>
<td>829</td>
<td>1296</td>
</tr>
<tr>
<td>Design moment</td>
<td>( 0.87f_{yk}A_s(d - s/2) )</td>
<td>182</td>
<td>126</td>
<td>182</td>
</tr>
</tbody>
</table>

The reinforcement provided is in between the minimum and maximum reinforcement requirement (Fig. A.2). Hence, the design satisfies the requirement in design code.

**Design for shear**

The shear forces along beam C3-D3 are shown in Table A.3 and Fig A.3.
Table A.3 Shear forces along beam C3-D3

<table>
<thead>
<tr>
<th></th>
<th>BR</th>
<th>CL</th>
<th>Mid-span</th>
<th>DR</th>
<th>DL</th>
<th>Mid-span</th>
<th>ER</th>
<th>FL</th>
</tr>
</thead>
<tbody>
<tr>
<td>VID</td>
<td>42.23</td>
<td>42.23</td>
<td>3.73</td>
<td>42.23</td>
<td>42.23</td>
<td>42.23</td>
<td>42.23</td>
<td>42.23</td>
</tr>
<tr>
<td>VSD</td>
<td>27.52</td>
<td>27.52</td>
<td>3.82</td>
<td>27.52</td>
<td>27.52</td>
<td>27.52</td>
<td>27.52</td>
<td>27.52</td>
</tr>
<tr>
<td>VL</td>
<td>34.41</td>
<td>34.41</td>
<td>4.78</td>
<td>34.41</td>
<td>34.41</td>
<td>34.41</td>
<td>34.41</td>
<td>34.41</td>
</tr>
<tr>
<td>VU</td>
<td>146</td>
<td>146</td>
<td>17.3</td>
<td>146</td>
<td>146</td>
<td>146</td>
<td>146</td>
<td>146</td>
</tr>
</tbody>
</table>

\[
V_s = 146 - \frac{146 - 17.36}{3000} \times 225 = 136.35 kN
\]

Crushing strength \( V_{Rd,\text{max}} \) of diagonal strut, assuming angle \( \theta = 22^\circ \), \( \cot \theta = 2.5 \) is:

\[
V_{Rd,\text{max}(22)} = 0.124b_wd \left( 1 - \frac{f_{ck}}{250} \right) f_{ck}
\]

\[
= 0.124 \times 250 \times 390 \times \left( 1 - \frac{30}{250} \right) \times 30 = 319.2 > 136.35 kN
\]

(b) Shear links

At distance \( d \) from the face of support the design shear is:

\[
V_{Ed} = 136.35 - 42.88 \times 0.39 = 119.63 > 67.5 kN
\]

This value exceeded the value of \( V_{Rd,c} \) (shear strength provided by concrete), hence shear reinforcement is required at the end joint.

\[
\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78d_f y_k \cot \theta} = \frac{119.63 \times 10^3}{0.78 \times 390 \times 250 \times 2.5} = 0.63 \text{ mm}^2/\text{mm}
\]

Provide R10, two-leg stirrups, \( A_{sw} = 157 \text{ mm}^2 \), \( s = 225 \) (smaller than \( s_{l,\text{max}}/ \) maximum stirrup spacing):

\[
\frac{A_{sw}}{s} = \frac{157}{225} = 0.697 \text{ mm}^2/\text{mm}
\]
The ratio of shear reinforcement is supposed to be larger than:

\[ \rho_{w, \text{min}} = 0.08 \sqrt{f_{ck}/f_{yk}} = 0.08 \times \sqrt{30}/250 = 0.00175 \]

For this case,

\[ \rho_{w} = A_{sw}/(sb_{w}) = 157/(250 \times 250) = 0.0025 > 0.00175, \text{OK} \]

(c) Minimum links

\[ \frac{A_{sw, \text{min}}}{s} = \frac{0.08 f_{ck}^{0.5} b_{w}}{f_{yk}} = \frac{0.08 \times 30^{0.5} \times 250}{250} = 0.44 \]

Use R10 @ 300 spacing along the beam. \( A_{sw}/s = 0.523 \)

(d) Additional longitudinal tensile force

\[ \Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \times 111.87 \times 2.5 = 139.84 kN \]

Span-effective depth ratio:

\[ \rho = 100 A_{s,req}/bd = 100 \times 580/(250 \times 390) = 0.6 \]

From Fig. A.4, basic span-effective depth ratio = 18 (Take K=1 for conservative).

![Figure A4](image)

**Figure A4** Allowable span-effective depth ratio

Modified ratio = 18 x 829/580 = 25.7

Span – effective depth ratio provided = 6000/390 = 15.4 < 25.7, hence deflection requirement is satisfied.
A.2.2. Column design

Table A.4 Axial force and moment in selected columns

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P)</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>(M_x)</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>(M_y)</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

![Figure A.5 Design of reinforcement in column](image)

(a) Calculate the uniaxial moment. Since \(M_y/h'\) is equal to \(M_z/b'\), can use either \(M'_y\) or \(M'_z\).

\[
\frac{N_{Ed}}{bh_{f_{ck}}} = \frac{4000 \times 10^3}{450 \times 450 \times 30} = 0.66 \rightarrow \beta = 0.44
\]

\[
M'_y = M_y + \beta(h'/b')M_z = 80 + 0.44 \times (450/450) \times 80 = 115.2 \text{ kNm}
\]

(b) Design the reinforcement (\(N_{Ed}, M'_y\))

\[
\frac{N_{Ed}}{bh_{f_{ck}}} = \frac{4000 \times 10^3}{450 \times 450 \times 30} = 0.66 \quad \frac{M'_y}{bh_{f_{ck}}^2} = \frac{15.234 \times 10^6}{450 \times 450^2 \times 30} = 0.056 \quad \frac{d}{h} = \frac{390}{450} = 0.9
\]

From design chart, \(A_{fy}/bh_{f_{ck}} = 0.21\),

\[
A_s = 0.21 \times 450 \times 450 \times 30/500 = 2550
\]

Provide 8T20 longitudinal bars, \(A_s\) total = 2514 mm². This satisfies both the minimum and maximum area of longitudinal reinforcement.
(c) Design for shear

Under gravity loading condition, minimum shear reinforcement is to be provided in the column.

\[
\rho_{w,\text{min}} = 0.08 \sqrt{f_{ck} / f_{yk}} = 0.08 \times \sqrt{30/250} = 0.00175
\]

Try R10, two-leg stirrups, \( A_{sw} = 157 \text{ mm}^2 \), \( s = 175 \text{ mm} \).

\[
\rho_w = \frac{A_{sw}}{s b_w} = \frac{157}{175 \times 450} = 0.0020 > 0.00175, \text{OK}
\]

The spacing of transverse reinforcement should not exceed either 400 mm or the least dimension of the column cross section 500 mm. Thus, the stirrup spacing meets the detailing requirement.

**A.2.3 Slab design (Two-way slab, restrained)**

The slab is designed at the corner location with two discontinuous adjacent edges. The slab thickness is taken to be 180 mm.

(a) Slab division and moments

*Design load, \( n = 1.35 \times 9 + 1.5 \times 5 = 19.65 \text{ kPa} \)*

\[
l_y/l_x = 1 \rightarrow \beta_{sx} = 0.036; \quad \beta'_{sx} = 0.047
\]

\( \phi 10 \text{ bar}, d_x = 180 - 20 - 5 = 155, d_y = 155 - 10 = 145 \)

The slab is divided into the middle and edge strips:

(b) Bending in middle strip (position 1,2,4)

Mid-span (2), \( M_{sx} = \beta_{sx} n l_x^2 = 0.036 \times 19.65 \times 6^2 = 25.47 \text{ kNm/m} \)

Continuous edge (1,4), \( M_{sx}' = 0.047 n l_x^2 = 33.25 \text{ kNm/m} \)
Table A.5 Calculation for required and provided slab reinforcements

<table>
<thead>
<tr>
<th>Position</th>
<th>Moment</th>
<th>K</th>
<th>Z, mm</th>
<th>A_s,req</th>
<th>A_s,min</th>
<th>A_s,pro'd</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) Mid-span</td>
<td>25.47</td>
<td>0.035</td>
<td>147.25</td>
<td>398</td>
<td>233.7</td>
<td>411.3</td>
</tr>
<tr>
<td>(1)(4) Support</td>
<td>33.25</td>
<td>0.053</td>
<td>137.75</td>
<td>555</td>
<td>233.7</td>
<td>565</td>
</tr>
</tbody>
</table>

Position 3,5 (discontinuous edges)

\[
Top\ steel = 0.25 \times 411 = 102.8\ mm^2\ but\ minimum\ reinforcement
\]
\[
= 233.7\ mm^2
\]

Provide R12-350 (323 mm²) across the 4.5 m middle strip.

Edge strips:

Provide minimum reinforcement = 233.7 mm². R12-350 (323 mm²)

\[\begin{array}{c}
\text{Total area of torsion steel} = 300 \times 1.2 = 360\ mm^2 \rightarrow 4R12\ (452\ mm^2)
\end{array}\]

\[\begin{array}{c}
\text{Total area of torsion steel} = 150 \times 1.2 = 180\ mm^2 \rightarrow 2R12\ (226\ mm^2)
\end{array}\]

(c) Torsion reinforcement

Of length equal to \(l/5\) (=1.2 m) are needed at corners X and Y

- Corner X : \(A_s = 3/4A_{sx} = 3/4 \times 398 = 300\ mm^2\) per m width
  
  Total area of torsion steel = 300 \times 1.2 = 360 mm² \rightarrow 4R12 (452 mm²)

- Corner Y : \(A_s = 3/8A_{sx} = 3/8 \times 398 = 150\ mm^2\) over 1 m width
  
  Total area of torsion steel = 150 \times 1.2 = 180 mm² \rightarrow 2R12 (226 mm²)
Figure A.7 Design of torsion reinforcement in slab

(d) Check deflection – take $d = 145$ mm (conservative)

$$A_{s,req}/bd = 398 \times 100/1000/145 = 0.27\% \ < \ 0.35\%$$

$L/d$ basic = 39. Allowable $L/d$ ratio = $39 \times 411.3/398 = 40.3$

Actual $L/d$ ratio = $6000/145 = 41.4 \rightarrow OK$

(e) Reinforcement details

For C30 concrete, tension anchorage: $l_{bd} = 36 \times \Phi = 36 \times 12 = 432$ mm.

A.3. Seismic design

Note that for this research, one of the research interests is to investigate whether seismic detailing (rather than seismic design) will be able to increase the structural resistance against progressive collapse. The seismic detailing is provided in accordance to EC8 [C2] for middle ductility (DCM).

The geometrical constraints and materials have been checked to meet the requirement. The requirements are listed in Table A6 below.
Table A.6 Requirements for seismic design

<table>
<thead>
<tr>
<th>Material requirements</th>
<th>Geometrical constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Concrete &lt; C16/20 shall not be used in primary seismic elements</td>
<td>Beam: $b_w &lt; \min{b_c + h_w; 2d_c}, d_c=450, h_w=450$ 250&lt;900 (ok)</td>
</tr>
<tr>
<td>2. Only ribbed bars shall be used as reinforcing bar in critical regions, in exception to close stirrups</td>
<td>Column: Unless &lt; 0.1, cross-section &gt; one tenth of the larger distance between the point of contra-flexure and end of column 450<em>1/10</em>3375 =338 (ok)</td>
</tr>
</tbody>
</table>

A.3.1. Beam design

Detailing for local ductility:

1. The regions of a primary seismic beam up to distance $l_c=h_w$ shall be considered as critical region. $l_c=450\text{mm}$

2. The reinforcement at compression zone should not be less than half of the reinforcement provided at tension zone. The compression reinforcement ratio 1.012%) is larger than half of the tension reinforcement ratio (0.5*1.548%=0.774%).

3. The reinforcement ratio at the tension zone does not exceed value of:

$$\rho_{max} = \rho' + \frac{0.0018 f_{cd}}{\mu_{p} \epsilon_{dy} d_{yd}} = 1.012\% + \frac{0.0018 \times 20}{6.8 \times 0.002435} = 1.621\% > 1.548\% (\text{ok})$$

where: $\mu_{p} = 2q_0 - 1 = 2(3.9) - 1 = 6.8 $ and $q_0 = 3 \times 1.3(\text{for frame}) = 3.9$

4. Along the entire length of primary seismic beam, the reinforcement ratio of tension zone, shall not be less than the value of:

$$\rho_{min} = 0.5 \frac{f_{ctm}}{f_{yk}} = 0.5 \frac{2.9}{500} = 0.29\% < 1.06\% (\text{ok})$$

5. Within the critical regions of primary seismic beams, hoops satisfying the following conditions shall be provided:

a) The diameter $d_{bw}$ of the hoops shall not be less than 6. R10 provided

b) The spacing, $s$, of hoops shall not exceed:

$$s = \min\{h_w/4; 24d_{bw}; 225; 8d_{bl}\} = \min\{450/4; 24 \times 10; 225; 8\times20\} = 112.5 \text{ mm}$$
c) The first hoop shall be placed not more than 50 mm from the beam end section

A.3.2. Column design

Detailing for local ductility:

1. The length of critical region $l_{cr}$ may be computed from the following expression:

$$l_{cr} = \max\{h_c; l_{ct}/6; 0.45\} = \max\{450; 4500/6; 450\} = 750$$

2. If $l_c/h_c<3$, entire height shall be considered as critical region. $4500/450=10>3$

3. The designed column need to satisfy:

$$\alpha \omega_{wd} = 0.512 \times 0.32 = 0.164 \geq 30 \mu_p v_d \epsilon_{sy,d} \frac{b_c}{b_o} - 0.035$$

$$= 30 \times 6.8 \times 0.365 \times 0.002 \frac{500}{450} - 0.035 = 0.13$$

$$\omega_{wd} = \frac{\text{volume of confining hoops}}{\text{volume of concrete core}} \frac{f_{yd}}{f_{cd}} = \frac{3800\text{mm} \times 79\text{mm}^2 \times 45435}{4500 \times 450 \times 450} = 0.32$$

$$\alpha = \alpha_n \alpha_s = \left(1 - \sum b_l^2/6b_o h_0\right) \left(1 - \frac{s}{2b_o}\right) \left(1 - \frac{s}{2h_o}\right)$$

$$= \left(1 - \frac{8 \times 200^2}{6 \times 450^2}\right) \left(1 - \frac{150}{2 \times 450}\right)^2 = 0.512$$

4. Within critical regions, hoops of at least 6 mm diameter need to be provided. The hoop pattern shall be such that the cross section benefit from the tri-axial stress conditions produced by the hoops (confinement of concrete core)

5. The minimum conditions of the above clause are deemed to be satisfied if the following conditions are met.

a) The spacing, $s$, of hoops does not exceed:

$$s = \min\left\{\frac{b_o}{2}; 175; 8d_{pl}\right\} = \min\left\{\frac{450}{2}; 175; 8 \times 20\right\} = 160, \text{provide 150 mm}$$

b) The distance between consecutive longitudinal bars engaged by hoops does not exceed 200mm.
A.4. Mapping of prototype to two-fifth scaled model

A.4.1. Mapping of prototype beam and column to two-fifth scaled model

To facilitate the experimental test, the prototype frame is scaled down to two-fifth scale, in which the size of beam is 100 mm wide by 180 mm deep, and the column is 180 x 180 mm. Steel bars with a diameter of 10 mm and 6 mm are used as beam and column longitudinal reinforcement, and 6 mm rebars are adopted as stirrups in beams and columns. The same value of reinforcement ratio is used in both the prototype and model frames.

In the selection of rebar for the scaling down, there may be a slight difference on the reinforcement ratio between the prototype and model frames. The comparisons are displayed in the following tables.

Table A.7 Scaling down of prototype beam to scaled down beam

<table>
<thead>
<tr>
<th></th>
<th>Top reinforcement</th>
<th>Bottom reinforcement</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prototype beam</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(non-seismic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2T25&amp;1T20 (1265 mm²)</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{1265}{250 \times 390} = 1.297% )</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{829}{250 \times 390} = 0.85% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{157}{250 \times 250} = 0.251% )</td>
</tr>
<tr>
<td>Model beam (non-seismic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3T10 (240 mm²)</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{240}{100 \times 155} = 1.548% )</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{157}{100 \times 155} = 1.012% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{28}{100 \times 90} = 0.311% )</td>
</tr>
<tr>
<td><strong>Prototype beam</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(seismic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2T25&amp;1T20 (1265 mm²)</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{1265}{250 \times 390} = 1.297% )</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{829}{250 \times 390} = 0.85% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{157}{250 \times 125} = 0.502% )</td>
</tr>
<tr>
<td>Model beam (seismic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3T10 (240 mm²)</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{240}{100 \times 155} = 1.548% )</td>
<td>( \rho = \frac{A_s}{b_d w} = \frac{157}{100 \times 155} = 1.012% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{28}{100 \times 50} = 0.560% )</td>
</tr>
</tbody>
</table>
Table A.8 Scaling down of prototype column to scaled down column

<table>
<thead>
<tr>
<th>Prototype column (non-seismic)</th>
<th>Longitudinal reinforcement</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>8T20 (2514 mm²)</td>
<td>( \rho = \frac{A_s}{b_w d} = \frac{2514}{450 \times 390} = 1.432% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{157}{450 \times 175} = 0.199% )</td>
</tr>
<tr>
<td>4T10&amp;4T6 (427 mm²)</td>
<td>( \rho = \frac{A_s}{b_w d} = \frac{427}{180 \times 155} = 1.530% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{28}{180 \times 80} = 0.194% )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prototype column (seismic)</th>
<th>Longitudinal reinforcement</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>8T20 (2514 mm²)</td>
<td>( \rho = \frac{A_s}{b_w d} = \frac{2514}{450 \times 390} = 1.432% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{157}{450 \times 100} = 0.349% )</td>
</tr>
<tr>
<td>4T10&amp;4T6 (427 mm²)</td>
<td>( \rho = \frac{A_s}{b_w d} = \frac{427}{180 \times 155} = 1.530% )</td>
<td>( \rho = \frac{A_s}{b_w s} = \frac{28}{180 \times 40} = 0.388% )</td>
</tr>
</tbody>
</table>

Table A.9 Capacity of scaled down beam

<table>
<thead>
<tr>
<th>( A_{s,prov}' (\text{mm}^2) )</th>
<th>( s (\text{mm}) )</th>
<th>Equation</th>
<th>C (+)</th>
<th>D (+)</th>
<th>E (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>157</td>
<td>240</td>
<td>157</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>( s = 0.87 f_{yk} A_s/f_{ck} b )</td>
<td>34.8</td>
<td></td>
<td>22.8</td>
<td>34.8</td>
<td></td>
</tr>
<tr>
<td>Design moment (kNm)</td>
<td>( 0.87f_{yk} A_s(d - s/2) )</td>
<td>14.4</td>
<td>9.81</td>
<td>14.4</td>
<td></td>
</tr>
</tbody>
</table>

Table A.10 Capacity of scaled down column

<table>
<thead>
<tr>
<th>( A_{s,prov}' (\text{mm}^2) )</th>
<th>( A_{s,prov}' f_{yk} b h_f c )</th>
<th>Pure compression</th>
<th>Max. Bending</th>
<th>Pure Bending</th>
</tr>
</thead>
<tbody>
<tr>
<td>427</td>
<td>0.26</td>
<td>729</td>
<td>0</td>
<td>243</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M (kN)</td>
<td>N (kN)</td>
<td>M (kN)</td>
</tr>
</tbody>
</table>

A.4.2. Mapping of prototype slab to two-fifth scaled model

The thickness of the slab is scaled down by two-fifth to 80 mm with a concrete cover of 8 mm. The comparison between the prototype slab and model slab is summarised as follows:

Table A.11 Scaling down of prototype slab to scaled down slab

<table>
<thead>
<tr>
<th>Prototype slab</th>
<th>Middle strip (Bot.)</th>
<th>Interior strip (Top.)</th>
<th>Exterior strip (Top.)</th>
<th>Torsional reinforcement (X)</th>
<th>Torsional reinforcement (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R12@275</td>
<td>R12@200</td>
<td>R12@350</td>
<td>4R12</td>
<td>2R12</td>
<td></td>
</tr>
<tr>
<td>411/(155x10)</td>
<td>0.27%</td>
<td>565/(155x10)</td>
<td>323/(155x10)</td>
<td>452/(155x12) = 0.24%</td>
<td>226/(155x12) = 0.12%</td>
</tr>
<tr>
<td>R6@150</td>
<td>R6@100</td>
<td>R6@150</td>
<td>4R6</td>
<td>2R6</td>
<td></td>
</tr>
<tr>
<td>188/(65x10)</td>
<td>0.29%</td>
<td>282/(65x10)</td>
<td>188/(65x10)</td>
<td>113/(65x4) = 0.44%</td>
<td>56.5/(65x4) = 0.22%</td>
</tr>
</tbody>
</table>

Table A.12 Summary of test specimen design

<table>
<thead>
<tr>
<th>Test Specimens</th>
<th>Design Moment (kNm)</th>
<th>Capacity of scaled down beam (kN/m)</th>
<th>Pure Compression (kN/m)</th>
<th>Max. Bending (kN/m)</th>
<th>Pure Bending (kN/m)</th>
<th>Design Moment (kNm)</th>
<th>Capacity of scaled down beam (kN/m)</th>
<th>Pure Compression (kN/m)</th>
<th>Max. Bending (kN/m)</th>
<th>Pure Bending (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Specimens</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.13 Summary of test specimen design

<table>
<thead>
<tr>
<th>Test Specimens</th>
<th>Design Moment (kNm)</th>
<th>Capacity of scaled down beam (kN/m)</th>
<th>Pure Compression (kN/m)</th>
<th>Max. Bending (kN/m)</th>
<th>Pure Bending (kN/m)</th>
<th>Design Moment (kNm)</th>
<th>Capacity of scaled down beam (kN/m)</th>
<th>Pure Compression (kN/m)</th>
<th>Max. Bending (kN/m)</th>
<th>Pure Bending (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Specimens</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B: FOUR-POINT MEASUREMENT METHOD

The objective of the four-point measurement method is to calculate the three DOFs ($\Delta x$, $\Delta y$, $\theta$) of the beam at the middle joint by solving compatibility equations formed based on measurements made at four selected points at the mid joint. Note that theoretically, only three points are already sufficient to solve the three unknown DOFs. However, in order to enhance the accuracy of the calculation, an additional fourth point is included to provide one extra compatibility equation as redundancy to detect measurement errors and for validating the computed results. During the test, the movements of the four measurement points are monitored by Linear Transducers (LTs). The ends of the LTs’ wires are secured to the hook installed at each measurement point. LTs are employed as they are able to trace the 3-D movements of the measurement points.

The middle joint configuration and the locations of the four selected measurement points before and after the middle joint movement, as well as the corresponding three DOFs are shown in Fig. A1. In Fig. B1, the blue dots refer to the measuring points P1, P2, P3 and P4. Note that points P1 and P2 are located at each end of an extended steel plate (the pink line) which is attached to the joint. The purpose of this plate is to magnify the movements of P1 and P2 so that a larger difference between them could be measured. Obviously, this arrangement is important during the initial stage of the test when the twisting $\theta$ is small. In Fig. A1, the green solid lines and the green dotted lines represent the length of the LT before and after the movement of the joint, respectively. The original lengths of the LTs’ wires at the measuring point are denoted as $L_i$ for i=1 to 4 while $L_i'$ are their final lengths after the joint movement.

Compatibility equations can be formed by using Pythagoras theorem to relate the final length $L_i'$ with $X_i$ and $Y_i$ for i=1 to 4. $X_i$ and $Y_i$ are the projections of $L_i'$ on the horizontal axis and the vertical axis, respectively. In general, $X_i$ is a function of $\Delta x$ and $\theta$, whereas $Y_i$ is a function of $\Delta y$ and $\theta$. The expressions for $X_i$, $Y_i$ and the compatibility equations are:
\[ X_i = \Delta x + a \sin(b - \theta) + c \quad (B - 1) \]
\[ Y_i = \Delta y + d \sin(e - \theta) + f \quad (B - 2) \]
\[ L_i'^2 = X_i^2 + Y_i^2 = [\Delta x + a \sin(b - \theta) + c]^2 + [\Delta y + d \sin(e - \theta) + f]^2 \quad (B - 3) \]

In the compatibility Eq. (B-3), \( a, b, c, d, e \) and \( f \) are constants which depend on the dimensions of the joint and the initial LT wires lengths. For the set up employed in the test, the corresponding compatibility equation for \( P1 \) is given by:
\[ L1'^2 = [\Delta x - 322 \sin(17 - \theta) + L1 + 95]^2 + [\Delta y - 322 \sin(73 + \theta) + 307.5]^2 \]
\[ (B - 4) \]

Applying this to the other three points, one can obtain three more compatibility equations in the forms:
\[ L2'^2 = [\Delta x - 322 \sin(17 + \theta) + L2]^2 + [322 \sin(73 - \theta) - 136]^2 \quad (B - 5) \]
\[ L3'^2 = [\Delta x - 215 \sin(9 + \theta) + 35]^2 + [-\Delta y - 215 \sin(81 - \theta) + L3]^2 \quad (B - 6) \]
\[ L4'^2 = [\Delta x + 380 \sin(56 - \theta) - 315]^2 + [-\Delta y - 380 \sin(34 + \theta) + L4]^2 \quad (B - 7) \]

Since there are four compatibility equations and only three unknowns, at any loading step or AD, the three unknowns (DOFs), i.e. \( \Delta x, \Delta y, \) and \( \theta \) are first solved by three compatibility equations which correspond to those three points that have the three largest absolute values of \( L_i - L_i' \). The built-in solver tool in Microsoft Excel is employed to solve the three selected nonlinear equations. The results obtained are then substituted into the last equation to calculate its corresponding \( L_i' \) value. Subsequently, the calculated value is compared with the measured value of \( L_i' \) to check the accuracy of the solutions.
Figure B.1 The four-point measurement method
APPENDIX C: SPACE TRUSS ANALYSIS

This section describes the application of space-truss (strut and tie) model in the failure analysis. The space-truss or strut and tie model has been well established and incorporated in many design guidelines for the analysis of torsion. The explanation and details of the strut and tie model are mainly extracted from the book by Mosley et al. [20] which is compatible with Eurocode 2. In the presence of torsion, diagonal crack will form a spiral around the beam as shown in Fig. C1. The increasing torsional moment after cracking is carried by truss action with reinforcement acting as tension members, i.e. stirrup resisting vertical ties and longitudinal reinforcement resisting horizontal tension chords, and concrete as compressive struts between links as shown in Fig. C2. Capacity at each component should not be exceeded to prevent failure due to crushing of concrete along the diagonal crack (concrete struts), yielding of stirrup (vertical ties) or longitudinal reinforcement (horizontal tension chords).

![Figure C.1 Torsion force along the beam](image1)

![Figure C.2 Space truss component](image2)

The analysis of each component in the beam in the presence of torsion ($T$), shear ($V$), bending ($M$), and axial force ($N$) is as follow:

1. Concrete struts: The compressive force in the diagonal crack is mainly generated from shear and torsion forces. In the combination of shear and torsion, the combined action to resistance ratio should not exceeded 1 (to prevent failure) as follows:
\[ \frac{T}{T_{Rd,max}} + \frac{V}{V_{Rd,max}} \leq 1 \quad (C - 1) \]

where \( T \) and \( V \) are torsion and shear acting on the beam, respectively. \( T_{Rd,max} \) and \( V_{Rd,max} \) are the torsion and shear capacities of concrete, respectively.

Details for the calculation of \( T_{Rd,max} \) and \( V_{Rd,max} \) can be found in [20].

2. Vertical ties: Vertical ties failure is prevented if the stirrup area resisting torsion and shear do not exceed the provided stirrup area as follow:

\[ \frac{2A_{sw,T}}{s} + \frac{A_{sw,V}}{s} \leq \frac{A_{sw}}{s} \quad (C - 2) \]

where \( s \) is the stirrup spacing and \( A_{sw} \) is the two leg area of the provided stirrup.

\( A_{sw,T} \) is one leg area of stirrup resisting torsion (T), which can be obtained as:

\[ A_{sw,T}f_{ys} = \frac{T_s}{2A_k \cot \alpha} \quad (C - 3) \]

Similarly, \( A_{sw,V} \) is the two leg area of stirrup resisting shear (V), which can be obtained as:

\[ A_{sw,V}f_{ys} = \frac{V_s}{0.9d \cot \alpha} \quad (C - 4) \]

In Eq. (C-4), \( f_{ys} \) is the yield strength of the stirrup; \( A_k \) is the area enclosed within the centre line of hollow box section; \( d \) is the beam effective depth, and \( \alpha \) is the diagonal crack angle. For the purpose of analysis, Eq. (C-2) is normalised to:

\[ \frac{2A_{sw,T}}{A_{sw}} + \frac{A_{sw,V}}{A_{sw}} \leq 1 \quad (C - 5) \]

3. Horizontal tension chords: the longitudinal reinforcement mainly resists the horizontal tension from bending and torsion. The additional tension forces due to shear have been accounted for by the increasing of curtailment length during the design of specimen. Axial force is ignored in the calculation as from the test, it was observed that axial force was either under compression (beneficial) or negligible tension. In the calculation of longitudinal reinforcement to resist bending \( (A_{sl,M}) \), the beam is assumed to be singly reinforced for simplicity, expressed as follow:

\[ A_{sl,M} = \frac{M}{f_{yl}Z} \quad (C - 6) \]
where $f_{yl}$ is the yield strength of longitudinal bar and $z$ is the lever arm between the compressive and tension forces in the beam cross-section.

The additional longitudinal reinforcement to resist torsion ($A_{sl,T}$) can be expressed as follow:

$$A_{sl,T} = \frac{T u_k \cot \alpha}{2A_k f_y} \quad (C - 7)$$

Similar to the vertical tie, the longitudinal reinforcement area provided ($A_{sl}$) should not be lesser than the area required resisting bending ($M$) and torsion ($T$). Torsion is assumed to be resisted equally at the top and bottom sections (half of the value at each section). Hence, for the M and CM sections under sagging moment, the horizontal tension chords check can be expressed as:

$$(\text{Section M and CM}): A_{sl,M(b)} + \frac{A_{sl,T}}{2} \leq A_{sl(b)} \quad (C - 8)$$

The horizontal tension chords check at M and CM can be further normalised to:

$$(\text{Section M and CM}): \frac{A_{slM(b)}}{A_{sl(b)}} + \frac{A_{sl,T}}{2A_{sl(b)}} \leq 1 \quad (C - 9)$$

At sections CE and E under hogging moment, only two (continuous) out of the three top reinforcements are effective in resisting torsion. To account for this, the required area to resist torsion is multiplied by 3/2 as follow:

$$(\text{Section E and CE}): A_{sl,M(t)} + \frac{3A_{sl,T}}{2} \leq A_{sl(t)} \quad (C - 10)$$

Similarly, the horizontal tension chords check at E and CE can be further normalised to:

$$(\text{Section E and CE}): \frac{A_{slM(t)}}{A_{sl(t)}} + \frac{3A_{sl,T}}{4A_{sl(t)}} \leq 1 \quad (C - 11)$$

Capacity analysis could hence be performed on each component by substituting the actions, i.e. torsion ($T$), shear ($V$), bending ($M$) into the corresponding equations and comparing them against the resistances (capacity provided) to check against failure. The torsion force ($T$), shear force ($V$), bending ($M$), and axial force ($N$) at any sections could be obtained by taking equilibrium of forces in the deformed configuration (at any MJD) as illustrated in Fig. C3. $H_t$, $H_m$, and $H_b$ are the measured horizontal reaction at top two-way load cell, bottom two-way load cell, and bottom load pin, respectively. $V_b$ is the measured vertical reaction at
the bottom load pin. $T_E$ is torsion resistance at the beam end, which is assumed to
be equal to half of the applied torsion at each end. Based on force equilibrium, the
internal forces (actions) at any section along the beam with different vertical
deformation ($\delta_i$) are expressed as follow:

*(Bending moment)*: $M_i$

\[ M_i = VL_i - H_t(\delta_i + 720) - H_m(\delta_i) - H_b(\delta_i - 830) \quad (C - 12) \]

*(Axial force)*: $N_i = (V \tan \beta_i + H_t + H_m + H_b) \cos \beta_i \quad (C - 13)$

*(Shear force)*: $V_i = \frac{(V - N_i \sin \beta_i)}{\cos \beta_i} \quad (C - 14)$

*(Torsion force)*: $T_i = T_E \cos \beta_i \quad (C - 15)$

*Figure C.3* Calculations of internal forces (actions)
APPENDIX D: TENSILE STRESS-STRAIN CURVE

The yield strength for each bar (T10, R10, R8 and R6) was obtained based on the average value of yield strengths obtained from the tensile tests of three bars. The full stress-strain curves for each bar size are shown in Figs. D1, D2, D3, and D4 for T10, R10, R8, and R6, respectively. The material properties are summarised in Table D1.

Figure D.1 Stress-strain curve of T10

Figure D.2 Stress-strain curve of R10
APPENDIX D

TENSILE STRESS-STRAIN CURVE

Figure D.3 Stress-strain curve of R8

Figure D.4 Stress-strain curve of R6

Table D.1 Material properties of steel bars

<table>
<thead>
<tr>
<th>Material</th>
<th>Size</th>
<th>Bar Type</th>
<th>Elastic Modulus (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Strength (MPa)</th>
<th>Fracture Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T10</td>
<td></td>
<td>Deformed</td>
<td>200</td>
<td>507</td>
<td>609</td>
<td>11</td>
</tr>
<tr>
<td>R10</td>
<td></td>
<td>Smooth</td>
<td>190</td>
<td>400</td>
<td>500</td>
<td>27</td>
</tr>
<tr>
<td>R8</td>
<td></td>
<td>Smooth</td>
<td>200</td>
<td>300</td>
<td>395</td>
<td>25</td>
</tr>
<tr>
<td>Stirrup</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R6</td>
<td></td>
<td>Smooth</td>
<td>200</td>
<td>400</td>
<td>583</td>
<td>25</td>
</tr>
</tbody>
</table>
The objective of CAT prediction by separately calculating the “shorter and longer” spans and “higher and lower steel content” spans was to identify the governing span. The comparisons among the proposed analytical prediction by taking average values of CAT properties (as proposed in Chapter 6), the analytical prediction by calculating the CAT of shorter and longer spans separately, and the numerical result are shown in Figs. E1(a) and E1(b) for EXT-R(0.75L) and EXT-R(0.5L), respectively. Note that EXT-R(0.75L) and EXT-R(0.5L) refer to parametric studies of EXT frames (exterior column removal scenarios) with right beam span adjusted to half (0.5L) and three-quarter (0.75L) of the original span (2400 mm), respectively.

(a)
Figure E.1 Analytical comparisons of EXT unequal double-spanning beam length (a) EXT-R(0.75L) and (b) EXT-R(0.5L)

It was observed that the development of tension was neither governed by longer nor the shorter spans. Generally, the proposed method by taking average value of $\Delta_{cat,L}$, $K_{cat}$, $\Delta_{cat,F}$ provided a closer estimation to the CAT profile of the numerical result, i.e. the gradient and final displacement of CAT.
Figure E.2 Analytical comparisons of EXT with unequal double-spanning beam reinforcement content (a) EXT-R(+0.25) and (b) EXT-R(+0.5)
The comparisons among the proposed analytical prediction by taking average values of CAT properties (as proposed in Chapter 6), the analytical prediction by calculating the CAT of spans with different steel contents separately, and the numerical result are shown in Figs. E2(a) and E2(b) for EXT-R(+0.25) and EXT-R(+0.5), respectively. Note that EXT-R(+0.25) and EXT-R(+0.5) refer to parametric studies of EXT frames (exterior column removal scenarios) with the right beam’s reinforcement ratio increased by 25% (+0.25) and 50% (+0.5), respectively. In both cases, (EXT-R(+0.25) and EXT-R(+0.5)), the proposed analytical method provided the most consistent prediction in terms of the gradient and the final displacement of CAT. Although the peak CAT capacities calculated based on the beams with higher steel contents were closest to those of numerical results, the CAT gradients of the beams with higher steel contents overestimated those of the numerical results, which were not conservative.