DEVELOPMENT OF A SPHERICAL MOTOR WITH A 3-DOF SENSING SYSTEM

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ABSTRACT

The thesis proposes a new design of a spherical wheel motor (SWM) with three layers of permanent magnets (PMs) located both inside and outside of double layers of electrical magnets (EMs), so as to fully utilize the magnetic field generated by the EMs and enhance the inclination torque density. The inclination torque is generally weak due to the limited number and space for the PMs and EMs involved in inclination compared to spinning. Finite element modeling (FEM) has been utilized for simulation, and important design parameters are optimized to maximize the inclination torque. The FEM modeling has been verified by experiments with discrepancy of less than 10% achieved. The optimized design can generate strong inclination torque which can support heavy loadings. Iron stator cores are applied in the new design to improve magnetic torque, and the magnetic field distribution (MFD) and magnetic torque become nonlinear due to magnetic saturation characteristics of the iron cores. Based on the analysis of the MFD of the whole SWM applying different current inputs, the thesis has constructed a dynamic model for SWMs with current inputs within the working range. A multi-DOF non-contact sensing system based on magnetic sensors and neural networks (NNs) is proposed. NNs are applied to approximate the function between orientations and MFD. The proposed sensing system is simulated and verified by experimental investigations. The sensing error to working range ratio is about 1.4%, which verifies its feasibility. With the proposed SWM design with enhanced torque capability, dynamic model and sensing system, the present findings provide strong basis to realize an integrated system of an SWM and take it a step closer to industrial applications.
ACKNOWLEDGEMENT

First of all, I would like to express my heartfelt thanks and gratitude to my former supervisor, Asst/P Dr. Hungsun Son, for his continuous and patient guidance, generous help and valuable advice throughout the project. Prof. Hungsun Son is a kind, hardworking and humorous person who is knowledgeable and always coming up with new ideas, and I am deeply thankful for having the opportunity to work with him. Prof. Hungsun has been back to Korea, but as the Chinese saying goes, I always respect him as my mentor.

I am very grateful for my existing supervisors Assoc/P Ng Teng Yong and Assoc/Prof Seet Gim Lee, Gerald. Without their guidance, I may have already given up as the way to be a Doctor is always depressing and stressful. I will always remember Prof. Ng’s words to persuade me not to give up in any case, and Prof. Seet’s professional guidance on editing thesis.

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<tr>
<td>APDL</td>
<td>ANSYS parametric design language</td>
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<tr>
<td>BP</td>
<td>Back propagation</td>
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<tr>
<td>DC</td>
<td>Direct current</td>
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<tr>
<td>DMP</td>
<td>Distributed multi-pole</td>
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<tr>
<td>DOF</td>
<td>Degree of freedoms</td>
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<td>EM</td>
<td>Electromagnetic magnet</td>
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<tr>
<td>EMI</td>
<td>Electromagnetic interface</td>
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<tr>
<td>ESL</td>
<td>Equivalent single layer</td>
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<tr>
<td>FEM</td>
<td>Finite element modeling</td>
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<tr>
<td>FM</td>
<td>Full model</td>
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<tr>
<td>FPGA</td>
<td>Field-programmed gate array</td>
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<tr>
<td>GUI</td>
<td>Graphical user interface</td>
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<tr>
<td>HPC</td>
<td>High performance cluster</td>
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<tr>
<td>MFD</td>
<td>Magnetic field distribution</td>
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<tr>
<td>MFD_{relative noise}</td>
<td>Magnetic field distribution with relative noise</td>
</tr>
<tr>
<td>MFD_{absolute noise}</td>
<td>Magnetic field distribution with absolute noise</td>
</tr>
<tr>
<td>MFD_{noise}</td>
<td>Magnetic field distribution with both relative and absolute noise</td>
</tr>
<tr>
<td>MGXX, MGYY, MGZZ</td>
<td>X, Y, Z component of coercive force</td>
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<td>MSE</td>
<td>Mean square error</td>
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<tr>
<td>MN</td>
<td>Maximum negative</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>-------------</td>
<td>--------------------------------------------------</td>
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<tr>
<td>MP</td>
<td>Maximum positive</td>
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<tr>
<td>MRM</td>
<td>Multi-DOF Reconfigurable machine</td>
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<tr>
<td>NI</td>
<td>National Instruments</td>
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<tr>
<td>NNs</td>
<td>Neural networks</td>
</tr>
<tr>
<td>PD</td>
<td>Proportion Differentiation</td>
</tr>
<tr>
<td>PID</td>
<td>Proportion Integration Differentiation</td>
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<tr>
<td>PM</td>
<td>Permanent magnet</td>
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<tr>
<td>PMSM</td>
<td>Permanent magnet spherical motor</td>
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<tr>
<td>RMSE</td>
<td>Root mean square error</td>
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<tr>
<td>RNN</td>
<td>Robust neural networks</td>
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<td>SPM</td>
<td>Simplified model</td>
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<td>SWM</td>
<td>Spherical wheel motor</td>
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<tr>
<td>SUSM</td>
<td>Spherical ultrasonic motor</td>
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<tr>
<td>THD</td>
<td>Total harmonic distortion</td>
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<td>VRSM</td>
<td>Variable reluctance spherical motor</td>
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LIST OF SYMBOLS

**B** Magnetic flux density vector

**D** Displacement current density vector;

**E** Electric field intensity vector

**r**<sub>i</sub> (mm) Inner Radius

**r**<sub.o</sub> (mm) Outer Radius

**D**<sub>in</sub> (mm) Diameter of inner PM

**D**<sub>out</sub> (mm) Diameter of outer PM

**D**<sub>core</sub> (mm) Diameter of EM core

**L**<sub>in</sub> (mm) Length of inner PM

**L**<sub>out</sub> (mm) Length of outer PM

**δ**<sub>r</sub> (Deg) Angle between PMs in plain view

**δ**<sub>s</sub> (Deg) Angle between EMs in plain view

**A** (deg) Inclination angle of rotor from a Z-axis

**B**<sub>r</sub> (T) Residual Magnetism

**H** (Gauss) Magnetic field intensity

**e**<sub>n</sub> Normal unit vector;

**B**<sub>1</sub>, **B**<sub>2</sub> Magnetic flux density vector of two materials on boundary;

**H**<sub>1</sub>, **H**<sub>2</sub> Magnetic intensity vector of two materials on boundary;

**J**<sub>surf</sub> current vector on boundary surface.

**J** Total current density vector
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<td>J_s</td>
<td>Source current density vector</td>
</tr>
<tr>
<td>J_e</td>
<td>Induced eddy current density vector</td>
</tr>
<tr>
<td>L_{EM} (mm)</td>
<td>Length of EM</td>
</tr>
<tr>
<td>T_{in}</td>
<td>Turns of Inner EM</td>
</tr>
<tr>
<td>T_{out}</td>
<td>Turns of Outer EM</td>
</tr>
<tr>
<td>I (Amp)</td>
<td>Current on EM</td>
</tr>
<tr>
<td>( \beta_s ) (Deg)</td>
<td>Separation angle of EMs</td>
</tr>
<tr>
<td>( \beta_r ) (Deg)</td>
<td>Separation angle of PMs</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Space permeability</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>Relative permeability</td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>Permanent magnet permeability</td>
</tr>
<tr>
<td>M</td>
<td>Relative magnetism</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Scalar potential</td>
</tr>
<tr>
<td>T_e</td>
<td>Electromagnetic torque</td>
</tr>
<tr>
<td>u</td>
<td>Currents vector</td>
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<tr>
<td>( \rho )</td>
<td>Electric charge density</td>
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<tr>
<td>( \varphi_{jk} )</td>
<td>Separation angle between ( k^{th} ) PM and ( j^{th} ) EM</td>
</tr>
<tr>
<td>R_{ji+}, R_{ji-}</td>
<td>Distance from point P to the source and sink respectively</td>
</tr>
<tr>
<td>m_{ji}</td>
<td>Strength of the ( i^{th} ) source or sink of ( j^{th} ) loop</td>
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CHAPTER 1
INTRODUCTION

1.1 Motivation

A spherical wheel motor (SWM), unlike conventional motors of which motions are confined along a single axis, the SWM as a form of electromagnetic motor, offers a unique ability to control orientation of the rotor while the rotor can be continuously spinning. There are great potential applications for SWMs in fields such as laser cutting, robotic wrists, helicopters, omnidirectional wheels and computer vision systems.

Existing designs to realize multi-DOF motions are typically composed of plural single-axis motors; generally equal or more motors than motion DOFs are required and orientation control of the rotating shaft must be manipulated by an additional transmission mechanism. These multi-axis actuators are generally bulky, slow in dynamic response, and lack of dexterity in negotiating the orientation of the rotating shaft. As an alternative to conventional designs, an SWM has many advantages some of which are listed as followings:

- Compact size and simple design; a spherical rotor can achieve 3-DOF motions as a direct-drive motor without any transmission mechanism;

- Isotropic motion property without any kinematic singularity in the working range;

- Quick and smooth motion control in 3-DOF.
Recently, two types of spherical motors differentiated by operating principles are getting more attention, i.e. spherical ultrasonic motors (SUSMs) and SWMs based on electromagnetic principle. The idea of SUSMs comes from the extension of the typical single-axis ultrasonic motor which uses two natural vibrations of the stator to drive the rotor with frictional force. An SUSM is generally compact in size, simple in structure and able to generate high torque density. However, most SUSMs have problems such as weak preloading force, complicated in the view of manufacturing, requiring a power system of high voltage and frequency, which inhibit applications of such motors in industry. Another inherent drawback is that the maximum size of an SUSM is confined to guarantee its natural frequency ultrasonic, which causes the SUSMs not applicable in many conditions. Compared to SUSMs, there are many benefits for SWMs inheriting from conventional electromagnetic motors and corresponding control strategies. It can be operated at a relatively high speed with smooth motion control. Many researchers have been devoting to develop a compact SWM with strong output torque and high controllability. However, these studies are still infeasible for industrial applications due to challenges in two aspects, i.e. configuration design to improve torque capability especially for inclination torque and design of a fast and accurate non-contact sensing system.

An SWM consists of a number of PMs in a rotor and EMs in a stator with various designs for a range of motion, speed and torque. The number and size of PMs and combinations of magnetic polarities mainly affect performance with the same applied
current input. Since there are more PM-EM pairs involved to produce spinning torque than inclination torque, the design is mainly concerning with how to maximize the inclination torque with a given motor volume and a given current density limited by the heat dissipation ability of the SWM. In particular, the inclination torque as a key parameter should be maximized to support an external load.

The thesis has proposed a new design of SWM with PMs located both inside and outsider of EMs, so as to fully unitize magnetic field generated by EMs and reduce magnetic leakage. Important physical parameters affecting inclination torque have been optimized to achieve maximum torque within a given size. Iron stator cores are utilized to enhance magnetic torque, and effects of iron boundary of the SWM are also investigated. There is 135% increase of torque/volume ratio of the proposed SWM validated by experiments.

A multi-DOF sensing system is a prerequisite for development of an integrated SWM system. Several sensing strategies based on photoelectric encoders, laser range finders, magnetic sensors with analytical method have already been investigated by researchers. However, above existing methods have restrictions in applications. Sensing systems using photoelectric encoders belong to the contact type, and friction and backlash brought by the sensing system will damage motion controllability. A non-contact sensing system applying laser range finders is fast and accurate. However, the sensing strategy is only applicable to SWMs with a flat bottom. Researchers use magnetic sensors and closed-form expressions to solve the orientation from measured MFD.
However, due to complexity of the MFD, high-order items have to be neglected, resulting in low accuracy.

To improve sensing performance of orientation, the thesis has designed a non-contact, absolute, multi-DOF sensing system based on magnetic sensors, and NNs have been applied to compute orientation from MFD. Compared to existing closed-form solutions, NNs based computation strategy has the following advantages:

- NNs computation is more accurate, since closed-form solutions have to neglect high-order items of MFD;

- Accuracy of NNs based sensing system can be easily improved by adding more sensors or neurons;

- Closed-form solutions have modeling errors, while modeling is unnecessary for the proposed NNs based sensing system;

- NNs based sensing system is more adaptive to magnetic noise and detecting error of Hall Effect sensors.

The thesis contributes to making SWMs closer to industrial applications in mainly two aspects, i.e. configuration design to improve torque capability and design of a sensing system with characteristics of non-contact, fast speed (the sensing system should be fast enough to be applied in real-time feedback control, such as 3.6 kHz for the rotor with 600r/min spinning speed, so as to sense per degree), accurate and adaptive.
1.2 Background

The growing demands of devices with multi-DOF motions in industry have motivated many researchers to investigate various types of multi-DOF motors and corresponding designs, sensing and control methods.

Research of electromagnetic multi-DOF actuators can be dated back to 1950s in [1]. F.C. Williams et al. has designed a 2-DOF induction motor. Objective of the second DOF, which is realized by manually rotating the stator block along the stator axis, is to adjust the spinning speed.

As the cost of high coercive rare-earth permanent magnets goes low, many researchers have focused on PM based SWMs. Several designs have been proposed by Lee et al. in [2, 3], and by Yan et al. in [4]. These designs can spin the rotor on a spherical bearing as conventional electromagnetic motors, and there are multi-layer PMs and EMs to generate the inclination torque. The PM based SWM designs share the same working principle with difference in number and distribution of PMs and EMs.

![Stator axle](image)

Figure 1.1: Prototype of a 2-DOF Induction Motor by F.C. Williams et al. in [1]
A sensing system based on optical encoders as shown in Figure 1.3 has been designed for above PM based SWM designs. The sensing system is computation free and can be very accurate if high precision encoders are used. However, friction, backlash and increase of moment of inertia due to guide frames and encoders, have badly damaged the dynamic performance and motion controllability.
As shown in Figure 1.4, a magnetic sensor based 2-DOF sensing system is proposed by Son et al. in [6]. A cylinder PM is attached on the output shaft, and four single-axis sensors are located around the attached PM. The sensing system is able to measure 2-DOF inclinations but not the spinning. The PM and sensors are placed far away from the rotor PMs so MFD generated by rotor PMs can be neglected, which simplifies the computation from MFD to orientation. However, the sensing system has the following drawbacks:

- The sensing system is 2-DOF only and spinning cannot be measured;

- The sensing system is limited to be applied in SWMs with a long output shaft;

- High-order items of MFD have to be neglected to obtain a closed-form solution, which results in low accuracy.

Figure 1.4: A Magnetic Sensor Based 2-DOF Sensing System in [6]
1.3 Literature Review

The section offers as followings: several recent designs of SUSMs are presented first; as to spherical electromagnetic motors, more detailed reviews of previous reported researches are presented since our research belongs to this category. Various designs of spherical electromagnetic motors are presented first. Secondly, different methods for solving MFD and magnetic torque are illustrated. Thirdly, several sensing methods for measuring orientation of the output shaft or the sphere rotor are presented and their advantages and disadvantages are discussed. Other methods to realize multi-DOF motions in previous reports are presented finally in the literature review.

1.3.1 Spherical Ultrasonic Motor (SUSM)

A single-DOF ultrasonic motor combines two natural vibration modes of the stator to generate a single-DOF motion of the rotor by frictional force between the stator and the rotor. By appropriately exciting and combining three or more natural vibration modes of the stator of an SUSM, multi-DOF motions of the sphere rotor can be achieved. Most spherical ultrasonic motors have prominent characteristics such as compact structure, low weight, relatively high torque-to-volume ratio, absence of electromagnetic radiation and high holding torque for braking without electricity. Different types of SUSMs are proposed and studied in [7-26] and some prototypes of classic designs are shown as in Figure 1.5.

As presented in Figure 1.5 (a), a multi-DOF ultrasonic motor with a compact plate stator is proposed in [12] by K. Takemura et al. The motor has achieved an extremely compact
stator compared to previous designs. However, the difference of natural frequency of each vibration mode caused by manufacturing tolerance and material isotropy, the small contact area between the stator and the rotor, the weak preloading force provided by gravity, and the nonlinear and changing dynamics during operation weaken the torque capability and controllability of the spherical ultrasonic motor.

An SUSM with compact structure constructed of three annular stators and a spherical rotor has been designed as presented in Figure 1.5 (b) in [16]. The maximum torque and rotational speed of the prototype are 3.45N*mm and 65.4 rpm respectively with a size of 26mm. However, with two pairs of guides and rotary encoders to measure the orientation of the output shaft, the total size of the multi-DOF system is much larger and the dynamic property is also damaged.

As shown in Figure 1.5 (c), an SUSM in [20] which has a large spherical contact surface achieves high torque and compact structure, since contact surface is an important factor that influences the friction force between the stator and the rotor.

As shown in Figure 1.5 (d), a multi-DOF ultrasonic motor has been used in an auditory tele-existence robot in [13]. Several strained springs are used as a preloading mechanism and compensation for resistance torque caused by gravity of the head. Benefit by the sufficient preloading force provided by tensioned springs, the motor has a potential to generate as strong as 1 N*m torque. However, the relatively large size of the whole motor has limited its applications and the dynamic response is relatively slow.
Motivated by requirements of compact multi-DOF motors in minimally invasive surgery, a long and thin bar-shaped SUSM suitable for the small work space in the patient body has been proposed as shown in Figure 1.5 (e) as in [17]. The prototype can generate enough torque for free space motion; however, its torque capability needs to
be improved for contact tasks such as suturing and knotting.

The SUSMs as presented above, have not been applied to actual applications due to the following disadvantages:

- It is difficult to design a preloading mechanism which can produce sufficient normal force without any dragging force due to the spherical shape of the rotor, the rotational torque generated by frictional force. Therefore, the friction force needs to be improved.

- Generally, the SUSM works at the natural frequency of the stator. The maximum size of the stator is limited, since the natural frequency of the stator decreases as the size of the stator increases. And the natural frequency must be larger than the lower limit of ultrasonic sound, otherwise the motor will be very noisy. Since the torque capability depends on the stator size, such problem makes the potential maximum torque of ultrasonic motors too small for many applications.

- The dynamics of SUSMs is nonlinear and changing during operation since it is governed by friction force, which results in difficulty in control.

- A high voltage (sometimes 200V peak-to-peak voltage) and frequency (higher than 20 kHz) power system is necessary for most SUSMs.

- Although an SUSM itself can be very compact, the orientation sensing system is generally large in size.
Other drawbacks such as very low efficiency and wear of friction have also inhibited the SUSMs from labs to commercial usages.

1.3.2 Designs of Spherical Electromagnetic Motors

Most existing multi-DOF motors are in the type of electromagnetic motor in [1-6, 27-96]. Although SWMs are difficult to miniaturize compared to SUSMs, they inherit many advantages of single-axis electromagnetic motors which have been developed maturely with a long history.

There are many different types of SWMs such as spherical induction motor, variable reluctance spherical motor (VRSM), Direct Current (DC) servo motor, permanent magnet spherical motor (PMSM) and so forth.

Spherical induction motor

An induction motor, also named by asynchronous motor, produces electromagnetic torque by the interaction of rotating air-gap magnetic field and induction current on rotor. F. C. Williams et al. in [1] have proposed the first spherical induction motor, which could realize 2-DOF motions as shown in Figure 1.1. However, the second DOF is achieved by manually rotating the stator block along the stator axis, and the objective of the additional DOF is to adjust the spinning speed of the rotor.
A. Foggia et al. in [30] have presented a 3-DOF motor as in Figure 1.6, operation principle of which is identical to the one of induction motors. The motor consists of a moving armature of which inner surface is made of magnetic steel, with a thin layer of copper and three fixed inductors as marked by M1, M2, and M3. By activating the three inductors, three independent motions can be realized.

**Permanent magnet spherical motor (PMSM)**

Since exciting currents are not necessary for permanent magnet motors, such motors can work with high efficiency and compact structure. The permanent magnet spherical motor (PMSM) has attracted most researchers and many different designs emerge in recent decades.

Lee et al. present the design concept of a spherical motor based on the variable reluctance stepper in [31]. The spherical motor is composed of a hemispherical stator with many electromagnets and a spherical rotor with two permanent magnets. The
tangential force between PMs and neighbor energized stator coils drive the rotor in 3-DOF to minimize the magnetic reluctance. However, due to the small quantity of permanent magnets, the torque capability of proposed design is very weak. No prototype has been fabricated according to the design and an improved design of variable reluctance spherical motor has been proposed and fabricated by Lee et al. in [3] as shown in Figure 1.2 (a). Compared to previous designs, the improved design had a more compact structure which contains more PMs and larger EMs. The torque capability has been improved a lot due to the increase of involving PM-EM pairs; however, the inclination torque capability is the bottleneck since there are fewer PM-EM pairs producing inclination torque compared to the spinning torque.

A similar and further improved spherical permanent magnet motor has been proposed by Yan et al. in [4] as shown in Figure 1.2 (b). The motor has a compact structure with dihedral permanent magnets in rotor and conical-shaped (conical-shaped in design and cylinder in prototype) coils in stator. Two designs differentiated by layers of magnets (one or two layers) have been analyzed and compared. The proposed design has improved the isotropy of the electromagnetic torque in 3-DOF, i.e. the difference between the inclination torque and spinning torque is decreased for the proposed design with two layers of permanent magnets in rotor.

A PMSM whose schematic is shown in Figure 1.7 has been studied by W. Wang et al. in [45]. The motor is composed of a four-pole spherical permanent magnet rotor which contains two pairs of parallel magnetized quarter-spheres, and four pairs of stator
windings. By energizing several stator winding pairs with appropriate currents, the rotor can rotate in 3-DOF to minimize the system potential energy.

A spherical stepper motor with stator and rotor in a semi-regular packing as shown in Figure 1.8 has been proposed by Gregory S. Chirikjian et al. The stator overlaps less than half of the area of the rotor, leading to an advantage that such a design has large motion range compared to previous spherical stepper motors.

Figure 1.7: Schematic of a Spherical Permanent Magnet Motor in [45]

(a) Stator  (b) Rotor

Figure 1.8: Stator and Rotor Packing in [41]
1.3.3 Magnetic Field and Torque Analysis of Spherical Electromagnetic Motors

Recent methods about analyzing magnetic field of SWMs include mainly the followings: theoretical analysis, distributed multi-pole (DMP) model, FE analysis and experiments.

**Theoretical analysis**

In [54], the analytical expression of the magnetic field generated by rotor PMs of an air core SWM is obtained by solving the governing equations which in this case are Laplace’s equations. Lorentz force law is then employed to calculate the magnetic torque. Theoretical analysis for magnetic field distribution and torque calculation require surface and volume integrations which results in a heavy computation load and is not suitable for online computation.

**Distributed multi-pole (DMP) modeling**

The DMP model proposed in [52] offers an effective closed-form solution for precise calculation of magnetic field around a PM or an EM, and force and torque between them. For PMSMs fabricated with non-ferromagnetic material, such method can be easily expanded to analyze the whole PMSM under the superposition principle. The DMP modeling inherits many advantages of previous single dipole model such as closed-form, timesaving and visualized, is a great improvement of the previous one, since it can account for the shape and magnetization of PMs. Moreover, compared to single dipole models, there are no singularities in the space around PMs with DMP.
The magnetic field generated by a PM can be characterized by the superposition of effects of several pairs of sources and sinks since the magnetic field satisfies the superposition principle. The DMP modeling can obtain a pretty precise magnetic field distribution of a PM in closed-form with only very limited pairs of sources and sinks, which makes it suitable for online computation and real time control.

Three methods are provided to compute the magnetic torque between a PM and an EM, i.e. by Lorentz force law with a multi-layer coil, by Lorentz force law with an equivalent single layer (ESL) coil and by virtual displacement method with modeling the EM as a PM which has the same geometry as the ESL.

**FE analysis with ANSYS**

With the rapid improvement of the computation devices and commercial FE software, more and more researchers use the commercial software ANSYS to analyze the magnetic field and torque. ANSYS is used in [55] to solve the spinning torque of a permanent magnet spherical motor and presents a way to convert 3D to 2D modeling.

**Other methods**

The paper [64] presents a measurement-calculation approach to model the magnetic field distribution of a PM-based SWM. With Laplace’s equations as the governing equations, and measured data as the boundary conditions, the magnetic field distribution is reconstructed by using software COSMOL. The main advantage of such approach is that there is no requirements for the magnetic structure.
1.3.4 Sensing System Designs for Spherical Electromagnetic Motors

Unlike conventional single-axis motors whose motion is constrained along one axis, the spherical motors have multi-DOF motions and infinite postures, so it is crucial to find an appropriate sensing method to realize the closed-loop control. Recent sensing systems can be characterized to contact and non-contact categories. The former one has advantages of light computation burden, high accuracy but the sensing system will increase mass of inertia of the rotor, bring about friction and damage the dynamic performance of the spherical motor. More attentions have been paid to the non-contact one employing vision or magnetic sensors.

Contact measure systems

As shown in Figure 1.3, the contact-type orientation measurement system in [5] is realized by two guides and three optical encoders which can measure the pitch, roll and yaw of the output shaft. Such sensing systems need almost no computation and have very high accuracy. However, the volume and inertia of mass of the motor have been increased a lot. And the changing dynamics due to the friction has damaged the dynamic performance and controllability of the motor.

Non-contact sensing systems

As illustrated in Figure 1.9 in [4], the Hall Effect sensors are used to measure the magnetic field, and by comparison with the known magnetic field distribution in 3-DOF space which has already been obtained by DMP or theoretical analysis, the posture
of the rotor can be solved. In [45], six single-axis Hall Effect sensors are utilized to detect the rotor position in 3-DOF with the help of optical sensors since the magnetic field distribution of the rotor has symmetrical property. However, the accuracy of the sensing system may be insufficient due to the neglecting of high order harmonics in the rotor magnetic field and magnetic field due to the winding currents.

As illustrated in Figure 1.10 in [44], a two-color (black and white) pattern is fixed on the sphere’s surface and a number of light sensors are placed in known locations. The orientation of the sphere can be solved from the combination of the states which are determined by the detected color. However, such a sensing system is difficult to be precise enough, since it depends on the pattern density and number of sensors.

In [70], laser detectors are used to predict all the three Euler angles with a slotted plate at the bottom of the rotor. The laser-based noncontact measurement methodology measures the distance from several points on the bottom plate to the laser detector, and deduces the Euler angles from obtained value of displacements. Such a methodology has the intrinsic advantages of noncontact sensors as well as a light computation burden, but is only applicable to spherical motors with a flat surface.
1.3.5 Other Multi-DOF Actuators

Except from ultrasonic and electromagnetic designs as illustrated above, there are some other methods to generate multi-DOF motions as in [97-103]. Some of the following
designs can also be characterized to electromagnetic motors; however, since their working principles are different from driving a spherical rotor by magnetic interaction between the rotor and stator, they are introduced separately here.

As shown in Figure 1.11, a miniature spherical motor which consists of four rods of Galfenol (an alloy of iron and gallium, has amplified magnetostrictive effect compared to iron), a wound coil, a rotor on the edges of the rods, and a permanent magnet under the rotor has been proposed in [101]. The rod can be expanded or contracted when currents are applied on wound coils. By expanding a rod and contracting the rod in the opposite, the rotor can rotate by a tiny angle. Applying a saw tooth current which increases gradually and falls rapidly, the rotor can be rotated. When the current is slowly increased, the rotor rotates in accordance with the slow deformation of the rod because of the friction; when the current falls rapidly. However, the rotor cannot follow the rapid deformation of the rod, instead, a slippage between the rod and the rotor happens. The rotor can be rotated continuously by repeating these operations.

Another method proposed by Hirokazu Nagasawa et al. in [97] has been illustrated in Figure 1.12. The spherical motor is consisted of a spherical cell, a spherical concave shell and four wires. The spherical motor manipulates the rotor in three directions including pitch, roll and yaw by four wires. By altering lengths of the four wires with a simple algorithm, the posture of the rotor can be controlled.
1.4 Objective

The thesis research has two main objectives.

The first objective is to propose a design strategy to improve isotropic property of the torque of the SWM by maximizing the inclination torque. There are more PMs and EMs
involved in creating spinning torque compared to inclination torque. However, inclination torque is a very important performance parameter especially when the SWM has to support an external load. The thesis has proposed a new design with PMs of the rotor locating both inside and outside of EMs of the stator so as to fully utilize magnetic field generated by EMs. Ferromagnetic materials are applied in the coil cores to improve magnetic torque. Design optimization is carried out to optimize important physical parameters and maximize the inclination torque of the SWM within a given compact size.

The second objective is to develop an absolute 3-DOF sensing system of SWMs with advantages of non-contact, fast response speed and high accuracy. Contact type sensors such as one composed of three optical encoders can be fast and accurate. However, will damage dynamic performance of the SWM seriously. The proposed sensing system utilizes several Hall Effect sensors to measure MFD, and neural networks (NNs) to compute orientations from MFD. It is unnecessary for the proposed sensing system to estimate magnetic related physical parameters such as remnant magnetism or magnetic permeability of PMs, so there is no modeling errors.

1.5 Thesis Outline

The emphasis of the research work is placed on design of configuration to enhance torque capability, and design of a 3-DOF absolute sensing system based on magnetic sensors and neural network algorithm. Remaining contents of the thesis are organized as followings:
Chapter 2 proposes a new design of SWM with PMs located both inside and outside of EMs, and presents a single-objective optimization process to maximize the inclination torque with Finite Element (FE) analysis under certain constraints such as the outer radius and current density. Since the whole SWM contains dozens of PMs and EMs, a simplified model has been proposed to save computation load without losing accuracy. Optimization results have verified the great improvement of proposed design in inclination torque density.

The magnetic torque and dynamic model for SWMs with ferromagnetic stator cores are presented in Chapter 3. Existing dynamic models about SWMs are mostly concerning with SWMs with non-ferromagnetic materials only. For SWMs with ferromagnetic materials, MFD and magnetic torque becomes nonlinear to current inputs, so superposition principle is no longer applicable. The thesis has analyzed MFD of SWMs with different input currents, and concluded that as input currents are in the working range, magnetic torque on the rotor can be divided to two parts, i.e. a fixed part due to ferromagnetic stator cores, and a linear part proportional to input currents. Once magnetic torque is calculated by addition of the two parts, forward dynamic modeling- to calculate torque by given inputs, and inverse dynamic modeling- to obtain optimal inputs with desired torque is straightforward to build.

Chapter 4 proposes a 3-DOF sensing system based on magnetic sensors and NNs to measure the orientation in real-time. MFD of the whole SWM is very complicated with many PMs and EMs involved, so NNs have been used to approximate functions
between MFD and orientations. Optimal distribution of sensors, and important factors affecting sensing performance are investigated in Chapter 4.

Chapter 5 presents a prototype of the proposed SWM and experimental investigations to verify torque modeling and performance of proposed sensing system. Magnetic torque measured in experiments is in great agreement with torque computed by FEM modeling, which verifies our magnetic torque modeling and simplified model. For the sensing system, ratio of sensing error to measuring range in experiments is about 1.4%. Considering the 3% maximum error for magnetic sensors, and accuracy of reference system, the proposed sensing system has been proven to be good in performance.

Finally in Chapter 6, a conclusion of the thesis is presented and future work to realize an integrated system of SWM is proposed.
CHAPTER 2
DESIGN OPTIMIZATION FOR MAXIMIZING INCLINATION TORQUE

2.1 Overview

For multi-DOF motor designs composed of plural single-axis actuators, it is straightforward to enhance its torque capability by improving that of each actuator. However, for proposed PM based spherical motors in recent decades, it is still a challenge to obtain a high torque density. Chirikjian and David [41] have proposed a spherical motor with a stator covering only less than half of the rotor, and achieved a very wide range of motion. However, only less than half of PMs can interact with coils since most PMs are too far away from the coils, which greatly reduces the torque density. Wang et al. has done design optimization to maximize the output torque. However, there is only one parameter being optimized, i.e., the ratio of the rotor to the stator. There are two intrinsic drawbacks to obstruct improvement of the torque density, i.e., the non-negligible friction torque due to direct contact of the rotor magnet with the stator housing, and the limited number of PMs and EMs. Lee et al. [48] have proposed a spherical motor with improved torque density due to a high-density distribution of PMs and EMs. However, the inclination torque is weak due to a limited number of PMs for the inclination motion compared to the spin motion. And the cylinder shape of PMs and EMs simplifies the fabrication, but doesn’t take full advantage of the space. And there is large magnetic flux leakage from EMs to outer space. Yan et al. [78] have
proposed an improved design with conical-shaped coil and magnets to enhance utilization of space, and iron stator to reduce magnetic flux leakage, which enhance the torque density greatly.

A spherical motor consists of a number of PMs in a rotor and EMs in a stator with various designs for a range of motion, speed and torques. The different number of PMs and combinations of their magnetic polarities mainly affect the performances with the same applied current input. In particular, the inclination torque is a key performance parameter especially when it supports an external load. To increase the inclination torque with a certain current density input, the number and size of PMs should be increased to maximize magnetic field interaction between PMs and EMs. However, it is rather difficult within a limited space. Thus, recent designs are concerned with how to maximize the inclination torque within a compact volume.

Existing designs generally use one or two layers only of PMs inside of the stator, such designs have advantages of small moment of inertia of the rotor, but it is difficult to obtain a high torque-to-volume ratio especially for inclination torque. The thesis has proposed a new design of SWM with PMs located both inside and outside of the EMs to enhance magnetic interaction between stator and rotor, and improve inclination torque within a given spatial size. Compared to one or two layers of PMs used in the rotor for previous designs, to improve inclination torque ability, the thesis has proposed SWMs with three layers of PMs both inside and outside of a stator with two layers of EMs as in Figure 2.1. Only some of the PMs and EMs are shown for a clear presentation.
A single-objective design optimization to maximize the inclination torque within a given spatial size is carried out for both designs with two or three layers of PMs in the rotor, and optimization results show a great improvement of inclination torque, and a possibility to apply such multi-DOF motors in practice engineering areas.

### 2.2 Proposed Configurations of SWMs

Figure 2.2(a) presents a top view of a proposed design showing magnetic polarities of the SWM projected in the XY plane. Unlike most existing SWMs, PMs of which are only inside of the stator (such as in Figure 2.2(b)), the proposed designs consist of PMs at both inside and outside of the EMs with alternatively opposite polarities in radial circularly for spinning. Similar with the spin motion, the inclination motion also requires additional layers of pole pairs along the Z axis. The SWM can have a number of pole layers along the Z axis as shown in Figure 2.2(b)-(d).
Various designs with a different number of PMs in the rotor, but the same number of EMs in the stator have been proposed as shown in Figure 2.2. However, the operation principle is similar to each other although the numbers of PMs may be different; the PM pairs are mechanically connected to the rotor and rotated together on a universal bearing so that their magnetizations are always toward or against the center of rotation. The spinning motion along the z-axis is analogous to a conventional stepping motor but not for the inclination motion. The polarities of the PM pairs are alternatively opposite to each other so that the inclination torque is controlled by the push-pull principle; if one of PMs generates a force to push the EM, the other generates a force to pull the EM accordingly.

- Existing configuration: two layers of PMs inside of EMs
- Proposed Configuration A: two layers of PMs both in and outside of EMs
- Proposed Configuration B: three layers of PMs both in and outside of EMs
2.3 Electromagnetic Field Fundamentals

Maxwell’s equations as followings are applied as the basis for FEM analysis of electromagnetic field.

\[ \nabla \times \mathbf{H} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_s + \mathbf{J}_e + \mathbf{J}_v + \frac{\partial \mathbf{D}}{\partial t} \tag{2.1} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.2} \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (2.3) \]
\[ \nabla \cdot \mathbf{D} = \rho \quad (2.4) \]

Where:

\( \mathbf{H} \): Magnetic field intensity vector;

\( \mathbf{J} \): Total current density vector;

\( \mathbf{J}_s \): Source current density vector;

\( \mathbf{J}_e \): Induced eddy current density vector;

\( \mathbf{D} \): Displacement current density vector;

\( \mathbf{E} \): Electric field intensity vector;

\( \mathbf{B} \): Magnetic flux density vector;

\( \rho \): Electric charge density.

The above Maxwell’s equations are supplemented by a constitutive equation that describes the relationship between magnetic flux density vector and magnetic field intensity vector

\[ \mathbf{B} = \mu_0 \mu_r \mathbf{H} \quad (2.5) \]
Where: \( \mu_0 \) is magnetic permeability of free space with a value of \( 4\pi \times 10^{-7} \), and \( \mu_r \) is relative permeability of the material. For air and aluminum, \( \mu_r = 1 \); for ferromagnetic material such as iron, \( \mu_r \) depends on \( H \) instead of being a constant, generally a \( B-H \) curve is used to describe the relationship. For permanent magnets used in the rotor, the constitutive relation becomes

\[
B = \mu_0 \mu_r H + \mu_0 M_0
\]  
(2.6)

Where: \( M_0 \) is remnant intrinsic magnetization vector.

In our study, the time varying effects such as eddy currents are ignored, and Maxwell’s equations for static electromagnetic field are reduced to:

\[
\nabla \times H = J_s
\]  
(2.7)

\[
\nabla \cdot B = 0
\]  
(2.8)

For static electromagnetic field, the following boundary conditions apply on the boundary between different materials:

\[
e_n \cdot (B_1 - B_2) = 0
\]  
(2.9)

\[
e_n \times (H_1 - H_2) = J_{surf}
\]  
(2.10)

Where:

\( e_n \): Normal unit vector;
**B**₁, **B**₂: Magnetic flux density vector of two materials on boundary;

**H**₁, **H**₂: Magnetic intensity vector of two materials on boundary;

**J**ₘₐₓ: current vector on boundary surface.

The primary unknowns calculated by FEM are magnetic and electric potentials, and other electromagnetic field quantities are derived from the two potentials, such as magnetic flux density, energy, and magnetic force.

The thesis uses ANSYS- commercial software package of FEM to do the electromagnetic analysis and calculate the magnetic torque. Detailed procedures will be presented in the following sections.

### 2.4 Numerical Analysis by ANSYS

Finite element (FE) analysis is utilized to compare the magnetic field and the torque of different configurations. ANSYS parametric design language (APDL) is used to simulate the electromagnetic field of the SWM, since APDL is more suitable for parameters optimization compared to graphical user interface (GUI) and Workbench of ANSYS. As presented in last section, the primary unknowns that ANSYS calculates are magnetic and electric potentials, and magnetic flux density, forces, torques are derived from the potentials, and Maxwell’s equations are used as basis for electromagnetic field analysis. The ANSYS provides 2-D and 3-D modeling options for electromagnetic field analysis, and 2-D model can simplify original planar or axisymmetric 3-D models to 2-
D models. It is much easier to generate 2-D model and more timesaving to solve. However, in the thesis, 3-D model is selected to obtain high computation accuracy in design optimization.

To relieve computation load, mechanical structures of stator and rotor besides PMs and EMs are considered as made of non-ferromagnetic materials such as aluminum, and can be modeled as part of air as they share same magnetic related material properties. Main steps in the 3-D static electromagnetic field analysis to calculate magnetic torque on PMs generated by EMs (direct currents applied) contains the followings:

1) Set physical environment, including element types and related KEYOPT (Key Options) settings, material properties, real constants and so on. The settings are according to our model type - static electromagnetic model with direct current (DC) and ferromagnetic materials (for SWM with iron core).

2) Build the physical model including PMs, ferromagnetic core and air. Interested regions include electromagnetic parts of the SWM and surrounding air. Volumes representing PMs and iron cores are created first, and the air region volume is generated by Boolean operations. EMs are modeled by primitive sources, so actually no need to model in this step. Volumes are then meshed to generate elements and nodes.

3) Apply boundary conditions and loads. The outer surfaces of the air volume are meshed to INFIN47 elements, which are used to simulate the far field decay of
the magnetic field. Current excitations are supplied via SOURC36 elements. SOURC36 can be treated as a “dummy” element, and current source data such as amount of current, position and size can be specified by element real constants. Without meshing requirements, SOURC36 simplifies the modeling and reduces the computation time.

4) Solve the analysis.

5) Review the results and calculate the magnetic torque. There is no existing macro in ANSYS to compute the magnetic torque, so torque of each element of the rotor is calculated and added together to obtain the total torque.

Detailed procedures and setups of simulation with APDL in ANSYS are presented as follows.

1) **Selection of element types**

Three elements are employed to analyze the electromagnetic field.

For 3-D static electromagnetic field analysis, we select element SOLID96 with eight nodes and one degree of freedom on each node as magnetic potential. The PMs, ferromagnetic core and air region are characterized by SOLID96.

SOURC36 which contains three nodes is a primitive (consisting of predefined geometries) and it is applied to simulate current inputs; real constants are used to define
the circular shape of the coil, and amount of currents (amplitude multiply turns).

In ANSYS, generally the larger the boundary is, the better modeling accuracy will be. However, increasing the model volume will result in more elements if element sizes remain the same, and thus increase computation time. Boundary size is confined due to limited computation power. INFIN47, a four-node boundary element with a magnetic potential or temperature degree of freedom at each node, is used to model far field decay of the magnetic field. Verified by several trials, compared to setting far field magnetic potential to be zero, modeling with far field decay elements such as INFIN47 can produce better results.

2) Assigning material properties

Once element types are selected, material properties need to be assigned for PMs, air and ferromagnetic core.

It is assumed that the BH curve of the PMs is linear. The relative permeability of the PMs is $\mu_r = 1.16$, and the coercive force $H_c$ used to define PMs in ANSYS can be calculated by the following equation:

$$H_c = B_c / \mu_0 \mu_r$$  \hspace{1cm} (2.11)

There components in different directions of coercive force, named as MGXX, MGYY, MGZZ are defined to determine the magnetization axis of PMs.
For the air region, the relative permeability is 1.0.

For saturable magnetic material such as ferromagnetic core used in EMs, a BH curve as shown in Figure 2.3 is applied to describe the relationship between magnetic flux density and magnetic intensity. Lee et al. treat relative magnetic permeability of ferromagnetic materials as unlimited in [31] or a large constant like 1000 in [46], and a conclusion of linear characteristics of MFD and magnetic torque is drawn based on such assumptions. While in practice it should be along a BH curve as shown in the figure, the magnetic saturation happens when magnetic flux density arrives at a high level such as 1.3T. However, the demagnetizing curve cannot be described in ANSYS, it is regarded as coinciding the magnetizing BH curve.

3) Creating the physical model

First, PMs and ferromagnetic stator cores are geometrically modeled in the 3D space.
Since eddy currents are neglected in the static electromagnetic analysis, conductivity is not considered in material properties, and materials are classified by magnetic permeability only. Other structure components such as the rotor, stator are non-magnetic conducting materials and their magnetic permeability is the same as air, so can be modeled as air.

Then, a cylinder containing the whole electromagnetic components is created with Boolean operations, so that there is no overlapping between the air and other regions.

Suppose a cylinder with diameter D and height H is just enough to covers all the electromagnetic components; a cylinder with diameter of 2×D and height of 2×H is created to simulate the air surrounding the electromagnetic structure, where magnetic flux density is weaker but cannot be neglected. Larger air volume generally leads to higher accuracy of the analysis. However, the computation load will also increase as more elements and nodes are generated with the same meshing standard. The FEM solution must be a good tradeoff between modeling accuracy and computation time based on computation ability and design requirements. Magnetic field of the SWM decays and approaches to zero in the air region, and element INFIN47 is used to simulate the far field decay of the electromagnetic field. With application of INFIN47 elements, it is unnecessary to build a large air volume.
4) **Meshing with adaptive strategy**

The program meshes volumes to elements and nodes, which are basic units for FEM computation. The EM is not necessary to be meshed since it is already an element defined by three nodes and real constants. Analysis accuracy depends mainly on two factors- the model size and the element size. The quality of meshing significantly affects the accuracy of the results. Generally, a model meshed with more elements leads to a more precise calculation, while computation time is also increasing almost proportionally until running out of the RAM (random access memory) of the PC. Thus, the analysis requires us to employ different element sizes to mesh different parts in order to increase accuracy within affordable computation time. The magnetic field is strongest around the electromagnetic components, and decay radially in the air. Obviously, PMs and ferromagnetic cores, where magnetic flux density is the strongest, should be meshed with fine elements such as element size of 1mm. For air region, the further the air is, the weaker the magnetic flux density is, and thus the coarser the
meshing should be. Moreover, we should also pay attention to the sequence of meshing of different volumes so as to take advantage of self-adaptive meshing techniques in ANSYS. First we mesh the outer air volume with smart meshing with the ANSYS macro “SMRTSIZE”, which meshes the internal portion with fine elements and external portion with coarse elements; then the PMs and the stator core are meshed with much smaller element size; the inner air volume is meshed without assigned element size, and they will be meshed adaptively- the closer the air is to PMs and cores, the smaller the element size will be. Finally, the outmost surfaces are meshed with INFIN47 elements to simulate the far field decay of electromagnetic field as the boundary conditions. The INFIN47 is used to model magnetic flux decay in open boundary of the model, which lowers the size requirement of the outer air.

5) Applying flags before solving the analysis

The FEM model is to calculate magnetic torque on PMs which represent the rotor. First we group all the elements assigned with permanent magnet material properties into a component, and then an ANSYS command macro, FMAGBC, is used to apply virtual displacements and the Maxwell surface flag for calculating forces on the component.

6) Solve the model and compute magnetic torque

Once the model is solved, we can obtain the total force exerted on PMs (representing the rotor). However, there is no existing macro to calculate the torque in 3D static magnetic analysis in ANSYS. APDL are edited to calculate the cross product of the
force and the position vector for each node of the rotor, and cross products are then added together to obtain the total torque on PMs.

2.5 Simplified Model for Magnetic Torque

We simulate the torque model for SWM configurations with both the air core and the iron core. For configurations with the air core, the total torque can be obtained from the superposition of torque on each EM-PM pair, so only one EM-PM pair in the model needs to be simulated. The torque on one pair of PMs generated by one EM is shown in Figure 2.5. Refer to Figure 2.2, the modeling parameters are as follows: $r_i=25\text{mm}$ (limited by the spherical bearing), $r_o=65\text{mm}$ (the given total size); diameter and length of the inner PM are both 10mm, diameter and length of outer PM are 25mm and 5mm; inner and outer radius of the EM are 6mm and 16mm, length of the EM is 21mm, current density is 8A/mm$^2$ (based on heat dissipation capability); separation angle between PMs and the EM is from 0 to 40 degrees on XOY plane. Magnetic torque components on X and Y axis should be zero due to symmetry. However, due to the fact that size of element cannot be infinitesimal and meshing distortion exists, there will be nonzero torque on X and Y axis. Magnitude of torque components on X and Y axis, and torque component on Z axis when separation angle is zero, can be used as a reference of the computation accuracy. As presented in Figure 2.5, the average magnitude of theoretical zero components is less than 2% of maximum magnetic torque on Z axis. The model size and element size can be treat as a tradeoff between the computation time and the modeling accuracy. With computed torque-separation angle curve, total
magnetic torque on the rotor generated by all EMs can be calculated under the superposition principle.

However, for configurations with stator cores made of ferromagnetic materials, magnetic field and magnetic torque are nonlinear, and superposition principle is not applicable. Magnetic torque generated by an EM with iron core on a pair of PMs with different current density (same physical parameters as in last paragraph and separation angle is ten degrees) is presented in Figure 2.6. B-H curve is used to define magnetic permeability of the iron core, and magnetic saturation occurs with large current density applied. The magnetic torque is nonlinear to applied currents, and there is attraction forces between PMs and EMs with iron core even without current applied. Due to limited heat dissipation capability, applicable maximum current density is about 8A/mm². As shown in Figure 2.6, the torque-current density is almost along a straight line between -8 to 8A/mm², which indicates that for torque calculation, there is no need
to consider the saturation as long as the maximum current density is less than \( \pm 8 \text{A/mm}^2 \).

Magnetic torque on one pair of PMs generated by one EM with iron core is presented in Figure 2.7. As shown in Figure 2.7, there is an attraction force between PMs and EMs with iron core even no currents applied, and total magnetic torque cannot be obtained under superposition principle. However, the whole SWM of one proposed design contains 16 EMs and 30 pairs of PMs. Therefore it is very time consuming to simulate the whole model for design optimization.
In consideration of saving computation time without impairing too much modeling accuracy, a simplified model (SPM) is proposed to alleviate the computation cost, and the efficiency and accuracy are validated by the comparison between SPM and full model. Prior to torque computation with SPM, it is necessary to validate torque from the SPM against the full model (FM) for configurations with iron core. For calculation of the inclination torque of proposed Configuration B with ferromagnetic materials in EM cores, it is necessary to verify the following two SPMs first:

1) Torque generated by a single EM with six neighboring PMs (SPM1) and with all PMs (FM1) are compared.

2) The other comparison is between the torque produced by two EMs with neighboring PMs (FM2) and the sum of torques produced by each EM with neighboring PMs (SPM2).

The simulation results in Figure 2.8 indicate that the inclination torque generated by the SPM and FM is very close with difference less than 5%, proving that the superposition between PMs can be applied. In addition, the SPM1 reduces computational loads; the result from SPM1 with 463600 elements is compared to FM1 with 2078600 elements with the same element size, and the computation times for each separation angle are about 6.7 and 42.9 minutes respectively.

The simulation results in Figure 2.9 indicate that the inclination torque generated by an upper and a lower EM can be calculated by addition of torque generated by each
separately. In the FEM mode, the initial inclination angles for upper and lower EMs are 25 and -25 degrees respectively. The results verify that total magnetic torque can be calculated by addition of torque generated by each EM.

The comparison results indicate that torque calculated with the SPM composed of one EM and its neighboring PMs can be expanded to analyze the whole SWM.

Figure 2.8: Inclination Torque Calculated by SP1 and FM1
2.6 Design Optimization for Enhancing Inclination Torque

In FE Analysis, it is important to find an optimized mesh shape for satisfying accuracy and computational time simultaneously. In general, smaller element size, more elements and nodes produce more accurate results. However, the computation time is also increasing with number of elements and nodes and a good tradeoff between computation time and modeling accuracy must be found. Various shapes of mesh with different element sizes are compared by the nonzero torques at zero separation angle and computation time. Any non-zero torque at zero separation angle (PM and EM are aligned) is considered as an error. After several trials, we set the mesh to be composed of about 350000 tetrahedron elements for Configuration B, which takes about 10.3 minute for each computation, and confines the average errors to less than 1% of the maximum inclination torque.
Design procedures for maximizing the inclination torque are shown in Figure 2.10. It begins with defining constraints along with key design parameters and their desired ranges in motion for orientation control. The initial constraints with design specifications are set as following:

1) Inner radius $r_i$ and outer radius $r_o$ are initially set for all designs.

2) Air gap between a PM and EM is given since a smaller air gap is generally better and minimizes magnetic reluctance.

3) Two EMs with an equal length but different diameters are serially connected and become an EM to enlarge the magneto-motive force.

4) The separation angle of EMs, $\beta_s$ is set to avoid mechanical contacts between neighboring EMs.

Once general constraints are pre-determined, independent and dependent design parameters can be further classified based on their mechanical and magnetic coupling relations. The dependent parameters (such as diameters of PMs and EMs) are designed as large as possible as long as no mechanical contacts exist. Then, the following four physical parameters are chosen to be independent parameters:

1) Separation angle of PMs, $\beta_r$ affecting the sizes of PMs;

2) Diameter of the ferromagnetic EM core, $D_{\text{core}}$;
3) Lengths of inner PMs $L_{in}$, outer PMs $L_{out}$, and EMs $L_{EM}$ is then determined since the size of motor is given as a constraint.

<table>
<thead>
<tr>
<th><strong>Table 2.1 Design Parameters for Optimization</strong></th>
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<tbody>
<tr>
<td><strong>Parameters</strong></td>
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<tr>
<td>Inner PM length, $L_{in}$ (mm)</td>
</tr>
<tr>
<td>Outer PM length, $L_{out}$ (mm)</td>
</tr>
<tr>
<td>$\beta$, (deg)</td>
</tr>
<tr>
<td>Iron core diameter, $D_{core}$(mm)</td>
</tr>
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</table>

Figure 2.10: Optimization Flowchart
Four independent design parameters; and corresponding range are summarized in Table 2.1. Each parameter is uniformly digitized into four elements due to heavy computation and a number of their combinations. Four parameters are first paired as two \((L_{in}, L_{out})\) = ([6(initial):2(interval):12(final)]) (the vector format in Matlab is used for better illustration), [4:2:10]) and \((\beta_r, D_{core}) = ([26:2:32], [2.5:2.5:10])\). Then, all possible combinations are indexed with positive integer into two axes \((x, y)\); \((L_{in}, L_{out})=(6,4)\): \(x=1, (6,6): x=2, \ldots, (12,10): x=16 \). Similarly, \((\beta_r, D_{core}) = (26, 2.5): y=1, (26, 5): y=2\ldots (32, 10): y=16\). Thus, the total number of the index for each pair is finally 16 since each parameter has four elements, and the total number of design variations is \(16\times16 (=256)\) accordingly.

Figure 2.11 shows the resultant torque from proposed Configuration A and B. Each
configuration has its own constraints. $D_{\text{in}}$ and $D_{\text{out}}$ of Design B and C are limited mainly by $\delta_r$ and $\beta_r$ respectively once lengths of PMs are set (they are set to be as big as possible but avoid mechanical contact). Thus, Configuration A has 20% larger diameter PMs than Configuration B while Configuration B has 50% more PMs, indicating Configuration B is a more compact and efficient structure. By comparing the maximum sum of torque, the optimized designs can be found and the parameters are given in Table 2.2. The results show that after optimization, Configuration B can generate 124% higher inclination torque than Configuration A.

2.7 Summary

Chapter 2 presents conceptual designs of SWMs with introduction of ferromagnetic materials in coil cores and PMs locating both inside and outside of EMs. Compared to the number of PMs fully circulated along a spinning axis for spin motion, the number of PMs for inclination motion is limited and thus, inclination torque is relatively weak. To increase the inclination torque with a given current input and spatial size, the number of PMs should be radially increased to maximize magnetic field interaction between PMs and EMs. Two configurations with different layers of PMs located both inside and outside the stators with ferromagnetic stator cores are proposed. A single objective optimization for maximizing the inclination torque of both configurations is realized with FE analysis on HPC. To relieve computation burden, simplified models are proposed and verified. The simplified models have saved a lot of computation time while the simulation accuracies are maintained. Optimization results have shown that
key geometrical parameters have significant impact on torque capabilities of the SWM.

Design optimization results have also shown that the proposed design can generate much stronger inclination torque than existing designs and has improved isotropic property of magnetic torques. Based on enhanced inclination torque, the proposed SWM is able to be utilized in practice engineering applications in respect of torque ability.
CHAPTER 3
MAGNETIC TORQUE AND DYNAMIC MODELING

3.1 Overview

To realize real time control, the forward model describing relationship between inputs-currents applied on each EM and outputs- angular motions of the rotor has to be established first. Establishment of such relationship contains two procedures, i.e. magnetic torque modeling and dynamic modeling.

It has been a challenge to build the magnetic torque model for SWMs with ferromagnetic materials due to nonlinearity of the complicated magnetic field. SWM configurations without ferromagnetic materials can be regarded as a particular case of the ones with ferromagnetic materials, and torque model of SWMs with ferromagnetic materials can be easily extended to SWMs without ferromagnetic materials. For SWMs without ferromagnetic materials, it is already verified by Yan et al. in [4] that the MFD and magnetic torque is linear thus superposition principle can be applied to establish the torque model. As to SWMs with ferromagnetic materials. However, previous researches either used a constant as magnetic permeability of ferromagnetic materials, or set their relative magnetic permeability as infinite, did not adopted the actual magnetic induction curve, which is nonlinear and finite.

3.2 Magnetic Torque Model for SWMs with Iron Cores

The SPM with a constant current for SWMs with ferromagnetic stator cores has already
been verified in Chapter 2. For EMs with varying currents, the superposition principle is not satisfied due to the nonlinear magnetic permeability of ferromagnetic stator cores. However, a torque-current curve with the middle part (the applicable current range for SWMs) approximately as a straight line has been validated with FE analysis. The torque model of the whole SWM can be obtained from the torque-current curve and the SPM (one EM with surrounding PMs) which has been verified in last chapter. The SPM with optimized physical parameters obtained in Chapter 2 is used here to illustrate the relationship between magnetic field, torque and applied currents.

Considering the problem of heat dissipation, the maximum continuous current density applied on the coil should be less than 8A/mm². As shown in Figure 3.1, the core is divided into two regions: Region 1 as edges of the stator core which is closer to magnets and its magnetic flux density is mainly influenced by neighboring magnets, and Region 2 as interior of the stator core and its magnetic flux density is influenced by PMs as well as the coil. Comparing Figure 3.1 with Figure 3.2 (a) (b) (c) (d), we can conclude that the maximum magnetic flux density in Region 2 is smaller than 1.5T if the current density applied on the coil is in the range of -8 to 8A/mm².
Figure 3.1: Magnetic Flux Density of Ferromagnetic Stator Core with no Current Applied

(a) Current Density = -3 A/mm²
(b) Current Density = 3 A/mm²
(c) Current Density = -8 A/mm²
(d) Current Density = 8 A/mm²
The BH curve of the ferromagnetic core as shown in Figure 3.3 can be approximately treated as composed of two lines. So the magnetic flux density of Region 1 is varying along Line 2, while the magnetic flux density of Region 2 is varying along Line 1. When the current density is increased to 30 A/mm$^2$, the magnetic flux density of Region 2 is varying along from Line 1 to Line 2. With Lorenz force law, we can assume that the according magnetic torque-current curve is approximately along a straight line as
the current is within the range of -2.5 to 2.5 ampere, but not anymore as the current increases to 30 A/mm².

The torque-current curve, as expected from the magnetic field analysis and shown in Figure 3.4, is almost along a straight line while the current is -5 to 5 ampere (0.3mm diameter wire is used). Two methods for computing the magnetic force in ANSYS, i.e. virtual work and Maxwell’s equations are used, and the convergence value is as small as 1e-8.

PMs/ EMs which have collinear axis are treated as a group, and all the 60 PMs and 16 EMs are divided to fifteen groups and eight groups respectively. Once we compute the torque between a group of PMs and a group of EMs with ANSYS, we can obtain the total magnetic torque of SWMs with iron stator cores by the following equation:

\[ T = T_{\text{core}} + T_{\text{current}} \]  (3.1)
Where $T_{\text{core}}$ indicates torque due to iron material and $T_{\text{current}}$ torque due to input currents, expressed as Equations 3.2 – 3.5 respectively:

$$T_{\text{core}} = \begin{cases} 
- \sum_{j=1}^{m_j} \sum_{k=1}^{m_k} (f_i(\varphi)|_{\varphi=\varphi_{jk}} \frac{s_j \times r_k}{|s_j \times r_k|}) & \text{if } s_j \times r_k \neq 0 \\
0 & \text{if } s_j \times r_k = 0 
\end{cases}$$  \hspace{1cm} (3.2)

$$T_{\text{current}} = [K][I]^T = [K_1, K_2, \ldots, K_{m_k}][I_1, I_2, \ldots, I_{m_j}]^T$$ \hspace{1cm} (3.3)

$$K_j = \begin{cases} 
- \sum_{k=1}^{m_k} ((f_i(\varphi) - f_0(\varphi))|_{\varphi=\varphi_{jk}} \frac{s_j \times r_k}{|s_j \times r_k|}) & \text{if } s_j \times r_k \neq 0 \\
0 & \text{if } s_j \times r_k = 0 
\end{cases}$$ \hspace{1cm} (3.4)

$$f_i(\varphi) = \sum_{k=0}^{8} c_{ik}\varphi^k \quad (i=0, 1)$$ \hspace{1cm} (3.5)

$r_k$ and $s_j$ are the position vectors of the $k^{\text{th}}$ PM and the $j^{\text{th}}$ EM respectively; $f_0$ and $f_1$ are torque functions between a PM pole-pair and an EM pole-pair in terms of separation angle when input current is 0 or unit respectively as shown in Figure 3.5. $f_0$ is fixed and uncontrollable, and $f_1 - f_0$ can be treated as controllable unit. Noted that when the separation angle is larger than 40 degrees, the torque between PM and EM is close to zero. Referring to the SWM design of Son et al. in [52], the torque between a PM- and an EM pole pair is about 0.23Nm with 4A current applied in AWG29 (American wire gauge #29, 0.0642mm$^2$ cross area), and there are 8 PM- and 10 EM pole pairs with a total diameter of 76.2mm. Supposing the same current density is applied in an EM of the
proposed SWM (0.2042 mm² cross area, 12.7A current applied), the maximum controllable torque generated by a PM and an EM is about 2.5Nm, which is about 10.9 times of one in [52]. Considering the smaller total diameter and more PMs and EMs (30 PM pairs and 16 EMs for proposed SWM), the torque density of the proposed SWM is verified to be greatly improved.

### 3.3 Dynamics Modeling

X-Y'-Z'' Euler angles are applied to describe connection between stator coordinate
XYZ and rotor coordinate xyz. The rotor coordinate coincides with stator coordinate at first as presented in Figure 2.1, and then rotate along X axis for a degree of $\alpha$, and along new $Y'$ axis for a degree of $\beta$, finally along latest $Z''$ axis for a degree of $\gamma$. The rotor angular velocity in Euler angles is given by:

$$\omega = \dot{\alpha} e_x + \dot{\beta} e_y + \dot{\gamma} e_z$$  \hspace{1cm} (3.6)$$

While unit vectors of Euler angles can be expressed in rotor coordinate system in the form of $e_x$, $e_y$ and $e_z$ as

$$e_x = \begin{bmatrix}
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \beta & 0 & -\sin \beta \\
0 & 1 & 0 \\
\sin \beta & 0 & \cos \beta
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}$$  \hspace{1cm} (3.7)$$

$$e_y = \begin{bmatrix}
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \beta & 0 & -\sin \beta \\
0 & 1 & 0 \\
\sin \beta & 0 & \cos \beta
\end{bmatrix}
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}$$  \hspace{1cm} (3.8)$$

$$e_z = \begin{bmatrix}
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}$$  \hspace{1cm} (3.9)$$

Then we have the angular velocity of the rotor in rotor coordinate as

$$\omega = K\dot{\mathbf{q}}$$  \hspace{1cm} (3.10)$$

Where $\dot{\mathbf{q}} = \begin{bmatrix}
\dot{\alpha} \\
\dot{\beta} \\
\dot{\gamma}
\end{bmatrix}$ and
According to momentum theorem of the rotor as a rigid body, assuming the gravity center coincides with the rotor center and neglecting external torque, the dynamic equation of the rotor in rotor coordinate is given by

\[ M \ddot{\omega} + C(\omega) \dot{\omega} = T_e \]  

(3.12)

Where \( T_e \) is the electromagnetic torque in the rotor coordinate system, and

\[ K = \begin{bmatrix} \cos \beta \cos \gamma & \sin \gamma & 0 \\ -\cos \beta \sin \gamma & \cos \gamma & 0 \\ \sin \beta & 0 & 1 \end{bmatrix} \]  

(3.11)

Due to rotor symmetry, we can assume that \( I_{xx} = I_{yy} \), and

\[ C(\omega) = \begin{bmatrix} 0 & -I_{xx} \omega_z & I_{zz} \omega_y \\ I_{xx} \omega_z & 0 & -I_{zz} \omega_x \\ -I_{xx} \omega_z & I_{xx} \omega_x & 0 \end{bmatrix} \]  

(3.14)

To simplify the dynamic equations with Euler angles only, angular acceleration is given by

\[ \ddot{\omega} = \ddot{K} \dot{\omega} + K \dot{q} \]  

(3.15)

\[ \ddot{K} = \begin{bmatrix} -\sin \beta \cos \gamma \dot{\beta} - \cos \beta \sin \gamma \dot{\gamma} & \cos \gamma \dot{\gamma} & 0 \\ \sin \beta \sin \gamma \dot{\beta} - \cos \beta \cos \gamma \dot{\gamma} & -\sin \gamma \dot{\gamma} & 0 \\ \cos \beta \dot{\beta} & 0 & 0 \end{bmatrix} \]  

(3.16)
Substituting Equation 3.10 and 3.15 into 3.12, we have

\[(K^T gMgK)gq+(K^T MgK+K^T gCgK)gq=K^TT_e\] (3.17)

Or in a simple form as

\[[M]gq+C(q,\dot{q})=K^TT_e\] (3.18)

Since in the last section the electromagnetic torque is calculated in stator coordinate system, \(T_e\) is given by

\[
T_e = \begin{bmatrix}
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \beta & 0 & -\sin \beta \\
0 & 1 & 0 \\
\sin \beta & 0 & \cos \beta
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix}
\] (3.21)

Where \([T_x \ T_y \ T_z]^T\) is the electromagnetic torque calculated by the torque model in stator coordinate, and we have

\[
K^TT_e = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
\sin \beta & -\sin \alpha \cos \beta & \cos \alpha \cos \beta
\end{bmatrix}
\begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix}
\] (3.22)
3.4 Summary

Chapter 3 presents magnetic torque and dynamic modeling for SWMs with ferromagnetic stator cores. Ferromagnetic cores have improved the torque capability significantly. However, the nonlinear magnetic permeability of ferromagnetic materials has made the superposition principle not applicable for computation of magnetic torque. In the thesis, we analyze the MFD on ferromagnetic cores with different current inputs, and conclude that the torque-current curve is approximately along a straight line in our interested range, and magnetic torque on an EM can be divided to two parts due to ferromagnetic material and current inputs respectively. Combining with the SPM verified in Chapter 2, the total magnetic torque model of the SWM is obtained. In the remaining part of Chapter 3, dynamic modeling is presented. With magnetic torque model and dynamic model, the motion accelerations of the rotor under certain current inputs and optimum current inputs for given motion accelerations can be calculated, which lays the foundation of building an integrated closed-loop control system for an SWM in dynamics.
CHAPTER 4

NEURAL NETWORK BASED ORIENTATION MEASURE SYSTEM

4.1 Overview

Many researchers have been devoting to develop a compact, precise, high torque, and high speed integrated system of an SWM. However, the design of sensing system is one of the key challenges. Unlike single DOF motors, motion of which can be predicted easily, the three rotational angles of the rotor. However, due to coupling kinematics, are difficult to obtain in real-time. An accurate and fast speed sensing system is a prerequisite to realize closed-loop motion control for an SWM.

Previous sensing systems of SWMs can be divided into two main groups, i.e. contact and contact-less ones. Generally, a contact-type sensing system in [32] is realized by two mechanical guides and three optical encoders which can measure the pitch, roll and yaw of the output shaft. Such sensing systems have fast response without need of computation, and can be very accurate if high performance encoders are applied. However, the volume and inertia of mass of the motor increase a lot, and friction due to the guides has damaged the dynamic performance and controllability of the motor. Considering enhancing dynamic performance, a contact-less sensing system such as one composed of magnetic sensors is preferred.

The thesis focuses on research of a sensing system for SWMs without ferromagnetic
materials. Verified by Wang et al. in [45], in each spinning circle, there is one-to-one mapping between PM-based rotor orientation and the MFD, which indicates that a unique solution of orientation always exist with measured MFD. Orientation of the SWM can be described by three Euler angles such as ones in X-Y'-Z" rotation order. The chapter proposes a sensing system to obtain the three orientation angles of an SWM based on Hall Effect sensors and neural networks. Unlike the forward modeling- to solve MFD with given orientation, which can be solved under superposition principle once MFD of one or one pair of PMs is computed by analysis or FEM, the inverse modeling- to obtain three Euler angles from measured MFD is much more challenging, and it is very difficult to acquire a closed-loop equation unless high-order terms of the magnetic field are ignored. Neglecting high-order terms results in low accuracy, so neural networks [105-112] which can fit any practical function have been adopted here to approximate the relationship between inputs- measured MFD and outputs- orientation angles.

3-axis Hall Effect sensors as presented in Figure 4.1 are applied in the sensing system. A 3-axis Hall Effect sensor can output voltages in response to the magnetic field in 3 mutually perpendicular directions. The output voltages are proportional to magnetic flux density components in each direction over the working range of the sensors.

As presented in Figure 4.2, the working process of the proposed sensing system is as follows.
1) The SWM (rotor only) is equipped with encoder-based mechanical sensing system, and the Hall Effect sensors measure the MFD of the SWM at orientations evenly distributed in the working range.

2) The measured pairs of MFD-Euler angles are then extended by interpolation.

3) The extended MFD-Euler angles are used to train the neural network.

4) The neural network is applied to compute the Euler angles in real time.

The remainder of the chapter is organized as followings:

1) MFD generated by one PM is calculated by DMP. Compared to ANSYS or analysis,
DMP is able to provide a fast and closed-form solution. MFD of the SWM rotor is then calculated by superposition of MFD of each PM in simulation with MATLAB.

2) Position sensing of one PM in 3D has been simulated with neural networks.

3) Neural networks are used to solve orientation angles from total MFD of an SWM rotor. In the preliminary investigations, important factors of NNs affecting performance of the sensing system are studied.

4) To model practical MFD of the SWM, magnetic noise is considered and included in the simulation, and methods to alleviate influence of the noise are presented.

5) A large amount of MFD-orientation data is necessary to train the neural network. However, it is too time consuming and impractical to measure this amount of data in experiments. Numerical interpolation is used to obtain the training data from a small number of measured data. Influence of size of initial measured data on sensing accuracy is studied to balance measuring workload and sensing accuracy.

6) Computation time of the trained NNs is presented to verify the feasibility of the sensing strategy to be applied in a real-time control SWM system.

4.2 Magnetic Field Distribution of a PM Based on DMP

The SWM rotor is composed of three layers of 30 (pairs) of PMs in total, to solve the inverse modeling problem- to calculate orientation angles from measured magnetic field, the forward modeling- to obtain MFD under given orientations, needs to build
first. Here we choose DMP instead of ANSYS to build the forward model due to the following reasons:

1) DMP is closed-form with fast calculation speed.

2) DMP generates better results qualitatively. Take a cylinder PM which is magnetized along Z axis as shown in Figure 4.3 for example, x and y component of magnetic flux density at any point along z axis should be zero due to symmetry. However, FEM will output nonzero results because of meshing distortions and asymmetry, while there is no such problem with DMP.

3) MATLAB is used to carry out DMP algorithm, so the results from DMP can be used directly in MATLAB simulation for design of the sensing system.

Figure 4.3: DMP Modeling for a Cylinder PM in [52]
Three assumptions are set as followings:

1) Magnetic material of PM is isotropic and the PM is axisymmetric;

2) Magnetization is along Z axis and residual magnetism is a constant;

3) Relative permeability of PM is a constant such as 1.12;

The DMP uses multiple magnetic doublets to calculate MFD of a cylinder PM as shown in Figure 4.1, and magnetic potential due to each source or sink is given by the following equation:

\begin{equation}
\Phi_p = \frac{(-1)^{j+1}}{4\pi R} m
\end{equation}

Where \( j \) equals 1 or 0 depending on source or sink, \( m \) is strength of the source or sink, and \( R \) is distance between measured point and the source or sink. Under the superposition principle, the potential at point \( P (x, y, z) \) can be calculated by sum of potential due to all sources and sinks as following:

\begin{equation}
\Phi_p = \sum_{j=0}^{k} \sum_{i=0}^{n} m_{ji} \Psi_{ji}
\end{equation}

Where \( m_{ji} \) is the strength of the \( i^{th} \) source or sink of \( j^{th} \) loop, and \( \Psi_{ji} \) is given by

\begin{equation}
\Psi_{ji} = \frac{1}{R_{ji}} - \frac{1}{R_{ji}} \right) / (4\pi)
\end{equation}

\( R_{ji}^{+} \) and \( R_{ji}^{-} \) are distance from point \( P \) to the source and sink respectively as shown in
Given a potential field, the magnetic field intensity \( H \) and magnetic flux density \( B \) can be calculated by

\[
H = -\nabla \Phi \\
B = \mu_0 \mu_r H
\]

(4.4)

(4.5)

While \( \mu_0 \) is the permeability of free space, and \( \mu_r \) is relative permeability of PM. Generally more pairs of sources and sinks result in higher accuracy but also increase computation time. Values of unknown parameters such as number of loops, number of sources and sinks in each loop, positions and strength of each source and sink, are determined by minimizing the following error function under a certain requirement:

\[
E = \int [\Phi(z) - \Phi_\lambda(z)]^2 dz
\]

(4.6)

\( \Phi(z) \) and \( \Phi_\lambda(z) \) are magnetic potentials along the magnetization axis calculated by DMP and analysis respectively.

As illustrated in Figure 4.4, a point sensor and magnetization axis of a PM of the SWM rotor can define an XOY plane with rotating center of the SWM as center of the plane and magnetization axis as X axis. Position of point P can be given by distance \( R \) and separation angle \( \beta \). When \( R \) is set to be 55mm and separation angle is from 0 to 180 degrees, the magnetic flux density components generated by a PM with parameters in
Table 4.1 on a sensor 55mm away from rotating point are computed by DMP as shown in Figure 4.5. Separation angle $\theta = [0\text{(initial)}:1\text{(interval)}:179\text{(final)}]$, so magnetic flux density components at 180 separation angles have been calculated. The magnetic flux density $\mathbf{B}$ is expressed by polynomial as in Equation 4.7. Curve fitting by POLYFIT functions with a least square error in MATLAB has been carried out and the polynomial is with a degree of 12. Polynomial coefficients of $B_x$ and $B_y$ are listed in Table 4.2.

$$\mathbf{B} = p_1x^p + p_2x^{p-1} + \cdots + p_nx + p_{n+1} \quad (4.7)$$

Figure 4.4: XOY Plane Determined by Magnetization Axis of a PM and a Sensor

Figure 4.5: Magnetic Flux Density Generated by a Cylinder PM Calculated by DMP (R=50mm)
Table 4.1 PM Related Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM Diameter</td>
<td>13mm</td>
</tr>
<tr>
<td>PM Length</td>
<td>8mm</td>
</tr>
<tr>
<td>Permeability</td>
<td>1.12</td>
</tr>
<tr>
<td>Residual Magnetism</td>
<td>1.36T</td>
</tr>
</tbody>
</table>

Table 4.2 Polynomial Coefficients of MFD-Separation Angle

<table>
<thead>
<tr>
<th>Polynomial Coefficients of $B_x$</th>
<th>Polynomial Coefficients of $B_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$  -1.5278e-023</td>
<td>$P_1$  6.8551e-024</td>
</tr>
<tr>
<td>$P_2$  1.7819e-020</td>
<td>$P_2$  -7.0061e-021</td>
</tr>
<tr>
<td>$P_3$  -9.1847e-018</td>
<td>$P_3$  3.0160e-018</td>
</tr>
<tr>
<td>$P_4$  2.7491e-015</td>
<td>$P_4$  -6.9119e-016</td>
</tr>
<tr>
<td>$P_5$  -5.2732e-013</td>
<td>$P_5$  8.3291e-014</td>
</tr>
<tr>
<td>$P_6$  6.7309e-011</td>
<td>$P_6$  -2.7260e-012</td>
</tr>
<tr>
<td>$P_7$  -5.7175e-009</td>
<td>$P_7$  -6.6082e-010</td>
</tr>
<tr>
<td>$P_8$  3.0948e-007</td>
<td>$P_8$  1.0714e-007</td>
</tr>
<tr>
<td>$P_9$  -9.2702e-006</td>
<td>$P_9$  -7.2652e-006</td>
</tr>
<tr>
<td>$P_{10}$  7.2917e-005</td>
<td>$P_{10}$  2.2798e-004</td>
</tr>
<tr>
<td>$P_{11}$  2.7025e-003</td>
<td>$P_{11}$  -1.2608e-003</td>
</tr>
<tr>
<td>$P_{12}$  6.5624e-004</td>
<td>$P_{12}$  -6.8286e-002</td>
</tr>
<tr>
<td>$P_{13}$  -1.6453e+000</td>
<td>$P_{13}$  -3.2411e-003</td>
</tr>
</tbody>
</table>
With MFD generated by one PM, the MFD of the whole inner rotor is to be obtained under the superposition principle, i.e. MFD of the inner rotor is sum of MFD generated by each PM.

As shown in Figure 4.6 and Figure 2.2, initial location (without considering N or S polarity) of each PM in stator coordinate system (coincide with rotor coordinate initially) can be described as in Equation 4.8:

\[
P_j = [\cos((j-1) \times 36), \sin((j-1) \times 36), 0] \quad (j = 1, 2 ...10) \\
P_j = [\cos((j-1) \times 36) \times \cos(32), \sin((j-1) \times 36) \times \cos(32), \sin(32)] \quad (j = 11, 12 ...20) \\
P_j = [\cos((j-1) \times 36) \times \cos(-32), \sin((j-1) \times 36) \times \cos(-32), \sin(-32)] \quad (j = 21, 22 ...30) 
\]  

(4.8)

XYZ Euler angles as shown in Figure 3.6 are adopted to describe orientation of the rotor in stator coordinate system. The rotor rotates along X axis of the stator coordinate system for a degree of \(\alpha\) first, and then rotates along the new Y axis for a degree of \(\beta\), and finally rotates along the new Z axis for a degree of \(\gamma\). The three Euler angles \(\alpha\), \(\beta\) and \(\gamma\) can define all orientations of the rotor in the working range without singularity.
Rotations along X, Y and Z axis are describe by $\alpha$, $\beta$ and $\gamma$ respectively. Then relative locations of all PMs in stator coordinate system after orientation can be illustrated by Equation 4.9:

$$P_{j\text{ (new)}} = L_1 \times L_2 \times L_3 \times P_j$$  \hspace{1cm} (4.9)$$

$$L_1 = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where $$L_2 = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

Separation angle $\psi$ between $i^{th}$ sensor and $j^{th}$ PM can be calculated by Equation 4.10:

$$\varphi_{ij} = \cos^{-1}(\text{loc}_i \cdot P_{j\text{ (new)}})$$  \hspace{1cm} (4.10)$$

Where $\text{loc}_i$ is direction of the $i^{th}$ sensor.

Magnetic flux density $B_{ij}$ generated by $i^{th}$ PM on $j^{th}$ sensor is expressed as in Equation 4.11.

$$B_{ij} = (-1)^{i+j} \times \text{Poly}_1(\varphi_{ij}) \cdot P_{j\text{ (new)}} + (-1)^{i+1} \times \text{Poly}_2(\varphi_{ij}) \cdot \frac{(\text{loc}_i \times P_{j\text{ (new)}}) \times P_{j\text{ (new)}}}{|\text{loc}_i \times P_{j\text{ (new)}}| \times P_{j\text{ (new)}}}$$

if $\varphi_{ij} \neq 0$ \hspace{1cm} (4.11)$$

$$B_{ij} = (-1)^{i+j} \times \text{Poly}_1(\varphi_{ij}) \cdot P_{j\text{ (new)}}$$

if $\varphi_{ij} = 0$
The working range of the SWM is set to be $-15^\circ$ to $15^\circ$ for $\alpha$, $-15^\circ$ to $15^\circ$ for $\beta$, and $0$ to $36^\circ$ for $\gamma$, since MFD is periodic every $72^\circ$, and odd-symmetric every $36^\circ$ about $\gamma$ for the proposed SWM. MFD of the whole SWM at sensor [55mm, 0, 0] is presented in Figure 4.8.

4.3 Position Sensing of a Cylinder PM in 3D Based on Neural Networks

4.3.1 Introduction of Neural Networks

Neural networks composed of simple elements (called neurons) are inspired by biological nervous systems, and connections (weights and bias) between elements determine function of neural networks. These connections (weights and bias) can be trained to perform a particular function such as fitting a function, recognizing patterns, and clustering data. As illustrated in the following figure, a neural network with initial weight and bias values is performed to generate outputs with given inputs, then outputs
are then compared with the target, and weights and bias are adjusted according to the difference between outputs and targets. Through repeated adjustments of weights and bias values, the neural network with enough neurons is able to generate desired outputs within the scope of given tolerance.

Neural networks used in the thesis is to fit the function between inputs- measured magnetic flux densities and outputs- three Euler angles to describe the orientation. To solve MFD from an orientation is forward modeling and is relatively straightforward by FEM, DMP or analysis, while to solve an orientation from MFD is inverse modeling, which is more complicated and difficult to find a closed-form solution. Fortunately, neural networks are good at fitting functions and even a neural network with a simple structure is able to fit any practical function as long as it contains enough neurons.

A two-layer NN in MATLAB for simulation is shown in Figure 4.10 and illustrated in Equation 4.12.
4.3.2 Simulation of Position Sensing of 1 PM by DMP and NNs

Procedures of Simulation for position sensing of a PM by DMP and NNs are illustrated in Figure 4.11. A PM is located in the center of a Cartesian coordinate system, and three sensors are located on the X and Y axis on XOY plane. The PM is moving along X, Y and Z axis, and NNs are used to calculate the position of the PM with measured MFD of sensors.

To calculate the training sample, moving range of the PM is set as -10 to 10mm in x, y, z directions, and position on each axis is stepped by 2mm such as x=[-10(initial):2(interval):10(final)]; so there are 11*11*11=1331 positions in total in the training sample. Since the MFD of one PM is relatively simple for neural networks, here a two layer neural network with 50 neurons is adopted to calculate the position.

“Trainlm” training function which updates weights and bias values according to Levenberg-Marquardt optimization has been chosen as the network training function,

\[
a^2 = f^2(w^2 f^1(w^1 p + b^1) + b^2) \quad (4.12)
\]

Where a is the output, f is transition function, w is weight matrix and b is bias matrix; the superscripts represent the order of the layer.
since it is often the fastest back-propagation algorithm in the MABLAB toolbox, and this is also verified by several trials. During the training, Samples including 1331 positions are separated to three sets for training (80%), validation (15%) and test (5%).

The trained neural network is tested by position-MFD combinations which are not in the training sample in case of over-defined problem, i.e. neural networks work well for training sample but not for test sample, this happens when too many weights and bias compared to training data size are adopted. The training sample contains positions $x=[-10(\text{initial}):2(\text{interval}):10(\text{final})]$, $y=[-10(\text{initial}):2(\text{interval}):10(\text{final})]$. 

![Simulation Flowchart of Position Sensing for One PM](image-url)
Figure 4.12: Simulation Errors for Position Sensing of One PM

\[ z=\begin{cases} -10 & \text{(initial)} \\ 2 & \text{(interval)} \\ 10 & \text{(final)} \end{cases}, \]
while the test sample contains positions
\[ x=\begin{cases} -9 & \text{(initial)} \\ 3 & \text{(interval)} \\ 9 & \text{(final)} \end{cases}, \]
\[ y=\begin{cases} -9 & \text{(initial)} \\ 3 & \text{(interval)} \\ 9 & \text{(final)} \end{cases}, \]
\[ z=\begin{cases} -9 & \text{(initial)} \\ 3 & \text{(interval)} \\ 9 & \text{(final)} \end{cases}, \]
which makes sure that most positions in the test sample have not been used to train the neural network.

Since the position range on each axis is -10 to 10mm, while max error is less than 0.008mm, error/range ratio is about 0.04\%, which verifies good performance of the neural network and its strong ability to fit the function between MFD and positions. However, the training sample contains 1331 positions, which are too many for practical measurements. Two following requirements can be set to make it realistic in designing a position sensing system of a PM:

1) Number of positions to be measured cannot be too many, and the fewer the better to save measuring time;
2) Training sample must be large enough to make sure no over-defined problem, i.e. positions of training sample must be much more than unknown weights and biases.

To satisfy both requirements: fewer measurements and sample large enough for training, numerical interpolation is used to generate the training sample from the limited measured data. Interpolation is from measured model \((x=[-10\text{ (initial)}:5\text{ (interval)}:10\text{ (final)}])\) (the vector format in Matlab is used here for better illustration), \(y=[-10\text{ (initial)}:5\text{ (interval)}:10\text{ (final)}], z=[-10\text{ (initial)}:5\text{ (interval)}:10\text{ (final)}]\) to training sample \((x=[-10\text{ (initial)}:2\text{ (interval)}:10\text{ (final)}], y=[-10\text{ (initial)}:2\text{ (interval)}:10\text{ (final)}], z=[-10\text{ (initial)}:2\text{ (interval)}:10\text{ (final)}]\). Apparently, more measured positions to be interpolated result in smaller interpolation errors, and there is always a tradeoff between measuring times and interpolation accuracy.

(a) Interpolation Errors on the Whole Model
Numerical interpolation errors are shown as in Figure 4.13, the NN trained by
interpolation sample are then verified by the same test sample (without interpolation) and position errors of the NN are shown as in Figure 4.14. For simplicity, the Figures show one component only, while errors percentages are similar for other components due to the same interpolation strategy. The maximum position error is about 0.1mm, and error/range ratio is about 0.5%, which is larger than errors with original training sample, but can still be considered as accurate position sensing.

A polynomial of degree 3 are used to curve fit MFD-position, and the polynomial degree is chosen as 3 for the following reasons.

1) For polynomials with degrees less than 3, it is unable to fit well the complicated MFD-position curve;

2) For polynomials with degrees larger than 3 such as 4 or more, since each axis starts with five point only, so the polynomials may fit exactly with zero error for the 125 positions, but large errors are expected for other positions since the generalization is not as good as polynomials with degrees of 3.

To verify the assumption, polynomials of degrees 2 or 4 are also applied to curve fit MFD-position function and simulation results are shown in Figure 4.10. The max position errors are 1 and 0.03mm respectively, which verifies that polynomials of degree 3 has best performance based on given conditions.
Position sensing of a single cylinder PM is straightforward and can also be realized by closed-loop equation. However, it provides a basis for investigation of designing the sensing system for the SWM, which is presented in the following section.
4.4 Simulation of Sensing System for an SWM Rotor

4.4.1 Preliminary Investigations of Neural Network

Position sensing for a cylinder PM, actually can be realized by analytical closed-loop expression since the MFD is relatively simple. However, Due to coupling between the three Euler angles and highly complex MFD generated by dozens of PMs, it is very challenging to find a mathematical expression to obtain Euler angles from MFD for an SWM. Neural networks, which have been verified to be good at fitting functions, are applied to build the complicated relationship between orientation angles and measured MFD.

Four suppositional 3-DOF magnetic sensors are applied in the simulation, and directional vectors of the four sensors are set as \([1, 0, 0]\), \([\cos (45^\circ), \sin (45^\circ), 0]\), \([\cos (90^\circ), \sin (90^\circ), 0]\), \([\cos (135^\circ), \sin (135^\circ), 0]\), while all four sensors are 55mm away from the rotating center. There are 12 measured magnetic components in total at each orientation, and outputs are 3 Euler angles.

A Two-layer neural network are applied here to approximate the function between orientations and MFD. The two transfer functions are shown as in Equation 4.13:

\[
\begin{align*}
    f^1(n) &= \frac{2}{1 + e^{(-2n)}} - 1 \\
    f^2(n) &= n
\end{align*}
\]

Network training function TRAINLM in MATLAB is applied, so weight and bias values are updated according to Levenberg-Marquardt optimization. One hundred
neurons are applied in the hidden layer of the neural network; among all the orientation-MFD pairs, 85% of data is used for training, 10% is used for validation, and 5% is used for test. The grouping of the whole training sample is to avoid over-defined problem.

The error at each orientation is defined as in Equation 4.1, where $\alpha$, $\beta$, $\gamma$ are calculated results, and $\alpha_d$, $\beta_d$ and $\gamma_d$ are desired Euler angels.

$$\text{Error} = \sqrt{((\alpha - \alpha_d)^2 + (\beta - \beta_d)^2 + (\gamma - \gamma_d)^2)/3}$$  \hspace{1cm} (4.14)

The training errors of NN are shown in Figure 4.16. RMSE (Root Mean Square Error) is $2.76 \times 10^{-2}$ degrees.

The trained neural network are then applied to predict orientations from test sample. Three orientation angles of the test sample are chosen as $\alpha = [-15(\text{initial}):3.75(\text{interval}):15(\text{final})]$, $\beta = [-15(\text{initial}):3.75(\text{interval}):15(\text{final})]$, $\gamma = [0(\text{initial}):7.2(\text{interval}):36(\text{final})]$, so most orientation-MFD pairs are not used in
training. If we get similar sensing performance for the test sample, the trained neural network is workable without over-defined problem. Sensing error for testing orientation-MFD pairs are as shown in Figure 4.17, and RMSE is $4.52 \times 10^{-2}$ degrees. The testing error is bigger than the training error as expected, but is still in the same order of magnitudes, which verifies that the trained NNs can be extended to the whole working range of the SWM.

### 4.4.2 Sensors’ Location Optimization

In last section, directional vectors of the four sensors are set as $[1, 0, 0], [\cos(45^\circ), \sin(45^\circ), 0], [\cos(90^\circ), \sin(90^\circ), 0], [\cos(135^\circ), \sin(135^\circ), 0]$. Without tilting, the MFD is like a cosine wave every 36 degrees, and spinning angels 0, 45, 90 and 135 degrees are evenly distributed on the cosine wave. Son et al. in [6] use four sensors with directional vectors as $[1, 0, 0], [0, 1, 0], [-1, 0, 0], [0, -1, 0]$. Actually, if magnetic sensors
are located symmetrically in pairs, measured MFD is also odd-symmetrical, which means the information is repetitive and the effect of two sensors is similar or same as that of a single sensor. So in the thesis, preliminary selection of sensor locations has already considered taking full advantage of any sensor.

The distance between a sensor and the rotating point of the SWM has small influence on the variation trend of MFD. To avoid saturation and improve signal to noise ratio, the distance is chosen to be 55mm to make sure that there is no magnetic saturation in working range and the working range of selected Hall Effect sensor as ±7.3mT is fully utilized. With a step of 15°, the possible location for sensor in spherical coordinate are set to be [55mm, 0:15:30°, 0:15:165°]. So there are 36 candidate positions for sensors as shown in Figure 4.18, and $A_{in}^4 = 58905$ possible combinations. It is too time-consuming to try all combinations and compare, hence we have applied the following strategy to decide the locations for all sensors:

1) The first sensor is located at [55mm, 0, 0]. The distance of 55mm is chosen since at about this distance we can take full advantage of the working range of the Hall Effect sensors we are going to use.

2) All remaining 35 positions are used to combine with the first sensor. NNs are trained based on the two sensors, and testing errors as illustrated in Equation 4.15 are compared. The position generates the smallest testing error is selected to be the second location.
Testing Error = \sqrt{\sum_{i=1}^{n} (\alpha - \alpha_i)^2 + (\beta - \beta_i)^2 + (\gamma - \gamma_i)^2}\] (4.15)

3) The third and fourth location are selected based on the same principle.

Four locations [55mm, 0, 0], [55mm, 0, 45°], [55mm, 0, 60°], [55mm, 0, 75°] are selected subsequently by the above method.

Figure 4.18: Possible Positions for Hall Effect Sensors
Figure 4.19: Simulation Errors on Training Sample for Optimized Sensor Locations

Figure 4.20: Simulation Errors on Testing Sample for Optimized Sensor Locations
Sensing errors of simulation for training and testing sample are shown as in Figure 4.19 and Figure 4.20 respectively. RMSEs for training and testing are $2.01 \times 10^{-2}$ and $3.21 \times 10^{-2}$ degrees. Compared to preliminary selection of sensor locations, with the same NN and iterations times for training, the training and testing errors are decreased by 27.2% and 29% respectively. Since the preliminary sensor locations are settled with consideration of obtaining more information with each sensor already, the effectiveness of the sensor locations optimization is verified by the remarkable improvement of performance.

4.4.3 Enlarging Training Sample to Decrease Maximum Error

As shown in Figure 4.16-4.17, 4.19-4.20 in last section, global and local maximum errors always happen at the edge of the working range, such as $\alpha$ or $\beta$ equals $\pm 15^\circ$, or $\gamma$ equals 0 or $36^\circ$, at the edge of the measuring range. One solution to decrease the maximum errors on interested working range is to enlarge the range of training sample, so the maximum errors move to the edge of the training sample and maximum errors in interested range become smaller.

The training sample is chosen as $\alpha = [-17(initial):.1(interval):17(final)]$, $\beta = [-17(initial):.1(interval):17(final)]$, $\gamma = [-2(initial):1(interval):38(final)]$, so there are 50225 orientation-MFD pairs in the training sample. The training time will increase as larger amount of data is used. However, the training time is less important since training it offline. The training errors and testing errors are shown as in Figure 4.21 and Figure 4.22, and testing sample is same as the one in last section.
RMSEs for training and testing are $2.87 \times 10^{-2}$ and $2.89 \times 10^{-2}$ degrees respectively. The
training error with enlarged training sample is bigger than last section probably because of different initial values. However, the testing error is smaller even with a larger training error. And as presented in Figure 4.22, the peak errors are fewer in number and smaller in magnitude, which verifies our strategy.

### 4.4.4 Effect of Structure of NNs on Sensing Performance

To investigate effect of number of layers and neurons on sensing performance, 2-layer NNs with 50 and 150 neurons in hidden layer are applied, and sensing errors can be compared with 4.4.2. Training error and testing errors for a NN with 50 hidden layer neurons are presented as in Figure 4.23, and RMSEs for training and testing are 0.123 and $8.2 \times 10^{-2}$ degrees respectively. Training error and testing errors for a NN with 150 hidden layer neurons are presented as in Figure 4.24, and RMSEs for training and testing are $1.24 \times 10^{-2}$ and $1.95 \times 10^{-2}$ degrees respectively. As expected, NNs with more neurons generate more accurate results. However, the computation time also increases.

To investigate the effect of number of layers on the sensing performance, a 3-layer NN with 30 and 36 neurons in first and second hidden layer is applied in simulation. Number of neurons of hidden layers for the 3-layer NN is to make sure that the number of total unknowns is the same as the one in Section 4.4.2. Sensing errors are shown as in Figure 4.25, and RMSEs for training and testing samples are 0.0275 and 0.0195 degrees respectively, which indicate that the sensing performance of 3-layer NN is similar to the one of 2-layer NN.
Figure 4.23: Simulation Error for a 2-layer NN with 50 Neurons on the Hidden Layer

Figure 4.24: Simulation Error for a 2-layer NN with 150 Neurons on the Hidden Layer
4.4.5 Investigations of Influence of Noise

Relative noise

Relative magnetic field noise is defined by Equation 4.16 as following:

$$MFD_{\text{relative noise}} = MFD \times (1 + n \%)$$

(4.16)

Where $n$ is randomly distributed in the error range. For example, if up to ±3% relative noise is added on original MFD, then $n$ is a random value between -3 and 3. The same 2-layer NN with 100 hidden layer neurons as the one in Section 4.4.2 is applied. After same iteration times, the results are listed in Table 4.3 and compared. As shown in the table, the training errors increase a lot, which indicates that the inherent rules of the MFD are damaged, and makes it more difficult for NNs to find the correct mapping. Although sensing errors increase with relative noise, the neural network is still be able
to estimate reasonable orientations, which verifies the strong adaptability of the neural network.

Table 4.3: RMSE of Training & Testing for MFD with Relative Noise

<table>
<thead>
<tr>
<th>Relative Noise</th>
<th>RMSE of Training (Degree)</th>
<th>RMSE of Testing (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>2.01\times10^{-2}</td>
<td>3.21\times10^{-2}</td>
</tr>
<tr>
<td>1%</td>
<td>7.19\times10^{-2}</td>
<td>9.69\times10^{-2}</td>
</tr>
<tr>
<td>3%</td>
<td>1.39\times10^{-3}</td>
<td>1.75\times10^{-4}</td>
</tr>
<tr>
<td>5%</td>
<td>1.89\times10^{-4}</td>
<td>2.3\times10^{-4}</td>
</tr>
</tbody>
</table>

(a) Sensing Error for Sample with 1% Relative Noise
(b) Sensing Errors for Sample with 3% Relative Noise

(c) Sensing Errors for Sample with 5% Relative Noise

Figure 4.26: Sensing Errors with Relative Noise
Absolute noise

Absolute magnetic field noise is defined by Equation 4.17 as following:

\[ MFD_{\text{absolute~noise}} = MFD + N \] (4.17)

Where \( N \) is randomly distributed in the error range. For example, if up to ±0.03mT absolute noise is added on original MFD, then \( N \) is a random value between -0.03 and 0.03mT. Sensing RMSE of training and testing for MFD with absolute noise based on the same NN is listed in Table 4.4, and sensing error is presented in Figure 4.27.

<table>
<thead>
<tr>
<th>Absolute Noise (mT)</th>
<th>RMSE of Training (Degree)</th>
<th>RMSE of Testing (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 2.01 \times 10^{-2} )</td>
<td>( 3.21 \times 10^{-2} )</td>
</tr>
<tr>
<td>±0.02</td>
<td>( 1.12 \times 10^{-1} )</td>
<td>( 1.37 \times 10^{-1} )</td>
</tr>
<tr>
<td>±0.05</td>
<td>( 1.9 \times 10^{-1} )</td>
<td>( 2.16 \times 10^{-1} )</td>
</tr>
<tr>
<td>±0.1</td>
<td>( 3.26 \times 10^{-1} )</td>
<td>( 3.74 \times 10^{-1} )</td>
</tr>
</tbody>
</table>
(a) Sensing Errors for a Sample with 0.02mT Absolute Noise

(b) Sensing Errors for a Sample with 0.05mT Absolute Noise
(c) Sensing Errors for a Sample with 0.1 mT Absolute Noise

Figure 4.27: Sensing Errors for a Sample with Absolute Noise

Superposition of relative and absolute noise

Figure 4.28: Sensing Errors with both Relative and Absolute Noise
There are both relative and absolute noises in practice; MFD with both relative and absolute noises is given by

\[
MFD_{\text{noise}} = MFD \ast (1 + n \%) + N
\]  

(4.18)

For simplicity, here only one situation, i.e. 1% relative error and 0.02 mT absolute error is imposed on original MFD. The RMSE for training sample is \(1.24 \times 10^{-1}\) degrees, and \(1.53 \times 10^{-1}\) for testing sample. Sensing errors of NN are as shown in Figure 4.28.

**Methods to alleviate influence of noise**

Since an absolute sensing system is desired in the thesis, i.e. the orientation is computed by measured MFD components at that time only, without any historical information, there is no way we can separate information and noise. So we have to improve resistivity of the sensing system to noise to alleviate influence of the noise on sensing performance. To improve noise resistivity, one method is to apply additional sensors and the other one is to measure multiple times at each orientation. The noise is defined as 1% relative error and 0.02 mT absolute error, and improvement of introducing additional sensors and measuring multiple times at each orientation is studied respectively.

Sensing errors for sensing systems with additional sensors are shown as in Figure 4.29. The directional vector of the fifth and sixth sensors are selected as \([\cos (90^\circ), \sin (90^\circ), 0], [\cos (135^\circ), \sin (135^\circ), 0]\). RMSEs for sensing system with five 3-DOF sensors, are
$9.1 \times 10^{-2}$ and 0.120 degrees for training and testing sample. The number of hidden layer neurons is decreased to 84 to make sure that the same number of unknowns is applied as in last section. RMSEs for sensing system with six 3-DOF sensors, are $7.89 \times 10^{-2}$ and $9.75 \times 10^{-2}$ degrees for training and testing sample, and number of hidden layer neurons is decrease to 72. We can expect that the sensing errors will decrease if more sensor are included, and computation time of the sensing system can keep the same by adjusting number of hidden layer neurons, but hardware cost will increase.

(a) Sensing Errors for Sensing System with Five 3-D Sensors
Another way to improve the resistivity of sensing system to noise is to do multiple measurements at each orientation. The MFD measured is given by Equation 4.19:

\[
MFD_{\text{noise}} = \frac{1}{m} \sum_{i=1}^{m} MFD \times (1 + n_i \%) + N_i
\]  

(4.19)

As shown in Figure 4.30 and Table 4.6, MSEs of training and testing for different measuring times at each orientation are compared. The sensing errors decrease as
measuring times increase, and there is remarkable improvement from one to two times, but the improvement is very limited when measuring times increase from five to ten. Compared to increasing number of sensors, multiple measuring has better performance in enhancing resistivity of sensing system to noise.

(a) Sensing Errors for Two Measurement Times

(b) Sensing Errors for Five Measurement Times
(c) Sensing Errors for Ten Measurement Times

(d) Sensing Errors for 100 Measurement Times

Figure 4.30: Sensing Errors for Different Measurement Times
Table 4.6: RMSEs of Training & Testing for Multiple Measurements

<table>
<thead>
<tr>
<th>Measuring Times</th>
<th>RMSE of Training (Degree)</th>
<th>RMSE of Testing (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.24×10⁻¹</td>
<td>1.53×10⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>9.61×10⁻²</td>
<td>1.14×10⁻¹</td>
</tr>
<tr>
<td>5</td>
<td>7.98×10⁻²</td>
<td>1.05×10⁻¹</td>
</tr>
<tr>
<td>10</td>
<td>6.45×10⁻²</td>
<td>8.99×10⁻²</td>
</tr>
<tr>
<td>100</td>
<td>4.45×10⁻²</td>
<td>6.67×10⁻²</td>
</tr>
</tbody>
</table>

4.4.6 Numerical Interpolation to Reduce Measurement Workload

Above has presented the good performance of sensing system based on neural network and measured MFD, and training sample contains more than 3×10⁴ orientation-MFD pairs. However, in practice, it is too time consuming to measure MFD at so many orientations. Similar to position sensing of one PM, numerical interpolation is used to create the training sample from fewer measurements.

Interpolation from three different initial measuring samples is executed, and interpolation errors and sensing errors are compared. Three measuring samples are as followings:

Initial Sample 1: \( \alpha = [-15\text{(initial)}:7.5\text{(interval)}:15\text{(final)}], \beta = [-15:7.5:15], \gamma = [0:9:36] \), 125 orientation-MFD pairs in total;

Initial Sample 2: \( \alpha = [-15\text{(initial)}:6\text{(interval)}:15\text{(final)}], \beta = [-15:6:15], \gamma = [0:7.2:36] \), 216 orientation-MFD pairs in total;
Initial Sample 3: $\alpha = [-15(\text{initial}):5(\text{interval}):15(\text{final})], \beta = [-15:5:15], \gamma = [[0:6:36]],$ 343 orientation-MFD pairs in total.

The initial samples are then interpolated to interpolated sample: $\alpha = [-15(\text{initial}):1(\text{interval}):15(\text{final})], \beta = [-15:2:15], \gamma = [[0:1:36]],$ 3557 orientation-MFD pairs in total. Noted that polynomials of degree 3, 4, and 5 are applied respectively for sample 1, 2 and 3.

Two types of errors are compared between interpolations from different initial sample, i.e. interpolation errors as shown in Figure 4.31 and sensing errors based on the same neural network as shown in Figure 4.32. The sensing errors are for test sample, instead of interpolated sample.

![Figure 4.31: Interpolation Errors](image)
(a) Sensing Errors for Sample 1

(b) Sensing Errors for Sample 2
As expected, the more orientation-MFD pairs in the initial sample, the smaller errors of interpolation and training. However, bigger initial sample also means more measurements. So the suitable size of initial sample should be a tradeoff of accuracy.
and measuring time requirements of sensing system.

### 4.4.7 Investigation of Computation Time

There are two types of computation time about neural network, i.e. training time and running time. Training generally can be done offline, so running time to compute orientation is more important in consideration of real-time closed-loop control.

![Diagram of neural network computation procedures](image)

**Figure 4.33: Computation Procedures of a Neural Network**

The three types of transfer functions are defined as:

\[
\begin{align*}
tansig(n) &= \frac{2}{1 + \exp(-2n)} - 1 \\
logsig(n) &= \frac{1}{1 + \exp(-n)} \\
purelin(n) &= n
\end{align*}
\]  

(4.20)

The following factors have influence on the computation speed:

1. Size of inputs, which depends on number of sensors;
2. Structure of NN, i.e. number of layers and number of neurons on each layer;
3. Transfer functions of NN.

Computation is executed in MATLAB on a Quad-Core Laptop with 2.3 GHZ CPU.
frequency, and computation speed based on other real-time hardware can be estimated based on investigations of this section. Since MATLAB has strong ability in matrix operation for its special optimization for matrix computation, while other programs may not have, we have separated matrix operation to calculations such as additions and multiplications.

Suppose four 3-DOF sensors are used, so there are 12 MFD inputs. First step is normalization to transfer input values to the range of $\pm 1$. Based on MATLAB, computation time for 12 inputs is about $2.5 \times 10^{-2}$ second.

Refer to Equation 4.12, for a two-layer neural network with 80 neurons and 12 inputs, there are $80 \times 12 = 960$ multiplications and $960 + 80 = 1040$ additions before first input to transfer functions. Computation time for 960 multiplications and 1040 additions is about $8.7 \times 10^{-6}$ second.

First transfer function is Tansig or Logsig, take Logsig for example, there are 80 times of execution of Logsig function, and need about $1.9 \times 10^{-2}$ second. However, if we use look up table instead of calculation, the computation time of this step can be as short as about $6.7 \times 10^{-6}$ second.

Then before input to transfer function 2, there are 240 multiplications and 320 additions, which take about $2.5 \times 10^{-6}$ second. And transfer function 2 is Purelin of three inputs, the consuming time can be neglected.
So total computation time for NNs to calculate one orientation is about $1.815 \times 10^{-5}$ second, which means a measuring rate about 55KHZ sample depends on computation speed can be expected, which in most cases satisfies a practical SWM closed-loop control system. However, if SWM is controlled by other hardware such as NI controllers instead of a PC, the CPU speed may be one-tenth only of a PC, and measuring rate decreases to about 5.5 KHZ. Then the measuring rate need to be considered in control design in case of serious sensing delay.

4.5 Summary

The chapter has proposed a sensing system design of an SWM based on Hall Effect sensors and neural networks. For investigation of such a sensing system, MFD of SWM is built by DMP method to do simulation and provide useful observations.

3D position sensing of a cylinder PM is investigated first, and NNs have shown good performance. With interpolation method to get a training sample based on fewer measurements, the sensing system can be operated with fewer measuring times.

Preliminary investigations on NNs based sensing system of the SWM is done first, and then the sensing system is studied to be absolute, accurate, feasible, and with fast response. First, the spatial distribution of sensors are optimized first, and locations of the four sensors are settled subsequently. Second, important factors about the NNs such as layers, number of hidden layer neurons are investigated. Third, magnetic noises, which generally exist and cannot be neglected, are considered and methods to alleviate
the influence are presented. Numerical interpolation is then proposed to make the sensing system be feasible and practical by decreasing the measurement times. Finally, the computation time and according response rate is presented. With shown performance and estimated computation speed, the sensing system proposed in this chapter is able to be used in real time control of an SWM.
CHAPTER 5
EXPERIMENTAL INVESTIGATIONS

5.1 Overview

The proposed magnetic torque modeling and sensing system need to be validated by experimental investigations. A prototype is fabricated based on optimization results in Chapter 2, and magnetic torque measured in experiments are compared with the one calculated by FEM modeling. Experimental investigations about a 3-DOF positioning system of a cylinder PM is presented first. As MFD of a single cylinder PM is much simpler than MFD of the whole SWM, a better positioning accuracy is expected. Experiments on proposed sensing system for the SWM are then carried out, and strategies proposed in Chapter 4 to improve resistivity to magnetic noise, have been verified.

5.2 Prototype of Proposed SWM

Based on the optimization results in Chapter 2, SWM prototypes are fabricated and experimental investigations are carried out. The lengths of the PMs, separation angles between PMs, and diameter of the stator cores are determined from design optimization. Some parameters are predefined in design optimization such as \( r_i \) (distance from inner PM to rotating center), \( r_o \) (total radius of the SWM), etc. Other parameters are determined by maximizing the volume storing magnetic energy and minimizing the magnetic resistance, so as to improve the torque capability. The air gap is set to be 0.5mm based on fabrication and assembly skills as a small air gap helps to improve
magnetic torque. With total length and lengths of inner and outer PMs, the total length of EMs can be calculated. Each EM is separated to 2 coil with same length for fabrication and assembly simplification. The separation angle between the 2 layers of EMs is set as 48° to obtain the maximum diameters of the coils without mechanical contact. The cylinder permanent magnet made of NdFeB (N52, with the available highest residual magnetic flux density) is chosen for its high residual magnetic flux density and high coercive force.

Some parts and experiment setups are shown in Figure 5.1 and key geometric parameters are illustrated in Table 5.1.

For simplicity and economic considerations, #20 carbon steel is used to fabricate the ferromagnetic stator cores, although laminated silicon steel sheets are better choice for reducing eddy current losses and possess much higher relative magnetic permeability. In future study, laminated silicon steel should be used to fabricate the stator cores, to improve the magnetic permeability and reduce eddy current, which brings iron loss and makes it difficult to compute magnetic torque.
Figure 5.1: SWM Prototypes
Table 5.1 Geometric Parameters of Spherical Wheel Motor

<table>
<thead>
<tr>
<th>Rotor and PMs</th>
<th>Stator and EMs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_i ) : Distance from rotating center to inner surface of inside PMs;</td>
<td>( \beta_s ) : 24°</td>
</tr>
<tr>
<td>( r_o ) : Distance from rotating center to outer surface of outside PMs;</td>
<td>( \beta_r ) : 32°</td>
</tr>
<tr>
<td>( D_{in} ): Diameter of the inside PMs;</td>
<td>( L_{in} ): 10.3mm</td>
</tr>
<tr>
<td>( L_{in} ): Length of the inside PMs;</td>
<td>( D_{in-EM} ): 24mm</td>
</tr>
<tr>
<td>( D_{out} ): Diameter of the outside PMs;</td>
<td>( D_{out-EM} ): 32mm</td>
</tr>
<tr>
<td>( L_{out} ): Length of the outside PMs;</td>
<td>( I ): ± 2.5A</td>
</tr>
<tr>
<td>( B_r ): Magnetic field strength;</td>
<td>( D_{core} ): 10mm</td>
</tr>
<tr>
<td>( \beta_p ): Angle of the winding pattern;</td>
<td></td>
</tr>
</tbody>
</table>

Where:

- \( r_i \): Distance from rotating center to inner surface of inside PMs;
- \( r_o \): Distance from rotating center to outer surface of outside PMs;
- \( D_{in}, L_{in} \): Diameter and length of the inside PMs;
- \( D_{out}, L_{out} \): Diameter and length of the outside PMs;
- \( D_{in-EM} \): External diameter of the inside EM;
- \( D_{out-EM} \): External diameter of the outside EM;
$\beta_r$: Residual magnetism of both inside and outside PMs;

$\beta_p$: Separation angle between neighboring PMs on the middle layer;

$\beta_s$: Separation angle between upper or lower layer EMs and middle layer PMs;

$\beta$: Separation angle between upper or lower layer PMs and middle layer PMs;

$L_{EM}$: Length of the two parts of EMs;

$I$: currents applied on EMs in experiments;

$D_{core}$: Diameter of stator core.

Enameled copper wires with round section which can endure temperature as high as 180 degrees are used to wind coils. There are two considerations for determining the diameter of copper wires, i.e. the resistance $R$ of two pairs of EMs (an inner and outer EM compose a pair; the two pairs are placed diagonally) which are connected in series and activated by the same source, and the winding complexity which increases as the diameter increases. A common commercial high-power (about 50 to 100W continuous power) such as LM3886 from Texas Instruments can output highest power with 4 to 8 Ohm load. If the copper wires are winded ideally as presented in Figure 5.2, the space occupancy rate SOC can be calculated by Equation 5.1. However, the SOC is calculated in ideal condition. Based on the experience of the manufacturer, the real SOC is only 80-90% of the ideal one. The larger the diameter of copper wire, the lower the real SOC is. Here we use 85% of the ideal SOC, which is 77.1%, for estimation of coil resistance.

$$SOC = \frac{1}{4} \pi D^2 \left( \frac{\sqrt{3}}{4} D^2 - \frac{1}{8} \pi D^2 \right) = 90.7\%, \quad SOC_{real} = SOC \times 85\% = 77.1\% \quad (5.1)$$
Where $D$ is the diameter of the copper wires.

The resistance of a coil $R_{\text{coil}}$ with core diameter $D_{\text{core}}$, outer diameter $D_{\text{coil}}$, and length $L_{\text{coil}}$, can be calculated by Equation 5.2.

$$
R_{\text{coil}} = \rho_{\text{Cu}} \cdot \frac{n \cdot \pi \cdot \frac{1}{2} \left( D_{\text{core}} + D_{\text{coil}} \right)}{\frac{1}{4} \pi D_{\text{Cu}}^2}
$$

(5.2)

Where

$$
n = SOC_{\text{real}} \cdot \frac{L_{\text{coil}} \cdot (D_{\text{coil}} - D_{\text{core}})}{\frac{1}{4} \pi D^2}
$$

is the number of turns.

$D_{\text{Cu}}$ is the diameter of the copper of the wire, which is about 0.02-0.05mm smaller than $D$.

$\rho_{\text{Cu}}$ is the resistivity of copper, which is $1.71 \times 10^{-8}$ Ohm*m at 20°C. However, the resistivity is not a constant and is dependent on the temperature as in Equation 5.3.

$$
\rho_{\text{Cu}} = \rho_{\text{Cu,0}} \cdot (1 + \alpha t)
$$

(5.3)

Where

$\rho_{\text{Cu,0}}$ is the resistivity at 0°C, and $\alpha$ is temperature coefficient, which is about 0.0039.

Suppose the highest working temperature for copper wire is 80°C, then the resistivity at 80°C is calculated to be $2.08 \times 10^{-8}$ Ohm*m.

With external diameters of the inner coil and outer coil as presented in Table 5.1, and Equations 5.1, 5.2 and 5.3, the resistivity and diameter of copper wire is listed in Table 5.2. For maximizing output power capability of economical amplifiers (load should be
less than 8 Ohm), and reducing the complexity of windings coils (smaller diameter wire is better), the $D_{Cu}$ and D is chosen to be 0.5mm and 0.55mm. The final coils have 370 and 225 turns, which are fewer than expected especially for the smaller coil.

<table>
<thead>
<tr>
<th>$D_{Cu}$</th>
<th>$D$</th>
<th>$n_1$ (Turns of Inner EM)</th>
<th>$n_2$ (Turns of Outer EM)</th>
<th>$R$ (Temperature=20℃)</th>
<th>$R$ (Temperature=80℃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.225</td>
<td>2197</td>
<td>139</td>
<td>240.5</td>
<td>292.7</td>
</tr>
<tr>
<td>0.3</td>
<td>0.34</td>
<td>962</td>
<td>612</td>
<td>46.80</td>
<td>56.96</td>
</tr>
<tr>
<td>0.4</td>
<td>0.44</td>
<td>575</td>
<td>366</td>
<td>15.72</td>
<td>19.13</td>
</tr>
<tr>
<td>0.5</td>
<td>0.55</td>
<td>368</td>
<td>234</td>
<td>6.44</td>
<td>7.84</td>
</tr>
<tr>
<td>0.6</td>
<td>0.65</td>
<td>263</td>
<td>168</td>
<td>3.20</td>
<td>3.90</td>
</tr>
</tbody>
</table>

As illustrated in Figure 5.1 (g), the stator cores are composed of a screw and a nut, and two parts of EMs of different diameter are connected together as shown in Figure 5.1 (h). For consideration of the strength of the structure, the plate is 1mm thick between two parts of EMs. Since the magnetic field of the air gap between PMs and EMs is slightly influenced by the middle turns of EMs, the losses due to the 1mm thick plate can be neglected. And such a design solves the housing problem for stators, which becomes more difficult since different from existing designs, there are rotors both inside and outside of stators. Inner and outer rotors are connected by a bar as shown in Figure
5.1 (i) so all the PMs can rotates towards the center of the spherical rolling joint together. The diameter of the bar is determined so that it can support the outer rotor and does not reduce the inclination range.

A six-axis force/torque sensor Gamma SI-130-10 as shown in Figure 5.1 (k) is mounted on the output shaft of the motor and slides on a guide way to measure inclination torque at different separation angles. The measure range is 130N for force and 10N*m for torque. A data acquisition device NI PCI-6251 is used to transmit voltage information of the torque sensor to a computer, and a calibration matrix is used to transfer voltages to forces and torques. Take the torque on X axis as an example, since the asymmetry from manufacturing and assembling, the actual torque can be calculated by the following equation

\[ T_x = T'_x + F_x \times L \]  \hspace{1cm} (5.4)

Where

\( T_x \): Actual torque;

\( T'_x \): Measured Torque;

\( L \): Distance from the center of the spherical ball joint to the work surface of the torque sensor.
5.3 Experimental Verifications for Torque Simulation

Based on the SWM prototype, inclination torques are measured and compared with torques calculated by FEM model. The torque produced by experimental configurations with inner PMs only or with all PMs, with aluminum or iron material for the EM core and outer rotor as tabulated in Table 5.3 is measured and compared against simulations. The simulation utilizes the SPM, while the experiments use all the inner PMs or inner and outer PMs. Similar to the comparison between the SPM and FM in Chapter 2, the resultant torque from the simulations and experiments are close to each other with maximum discrepancy less than 10%, proving the SPM for optimization. Reasons for the error may include that we cannot obtain exact parameter values for PMs (magnetic permeability, coercive force, residual magnetic flux density), PMs are not fabricated totally homogeneous, and the fabrication and assembly accuracy problem of the measurement setup.

First, configurations D, E, F are compared with configurations A, B, C respectively. The main difference is that configurations D, E, F have both inner rotor and outer rotor, while configurations A, B, C have inner rotor only. The major improvement of torque-to-volume ratio that the inclination torque produced by configurations D, E, F is more than four times as the one produced by configurations A, B, C respectively while the volume of configurations D, E, F has increased by less than 70%. The 135% increase of torque density- ratio of torque to volume, argues the advantages of proposed configuration compared to existing designs.

Similarly, configurations with ferromagnetic stator core are compared with
configurations with aluminum stator core. Since the magnetic torque generated by configurations with ferromagnetic stator core is not proportional to currents, so both positive and negative currents are imposed and compared. Considering the sum or average of torque, ferromagnetic stator core configurations with both positive and negative currents can produce much stronger inclination torque than aluminum core configurations.

Finally, configurations D, E, F with aluminum outer rotor and configurations with ferromagnetic outer rotor G, H, I are compared. With ferromagnetic boundary, the magnetic circuit of the SWM is closed and magnetic field is mainly confined in the SWM, and results in much less magnetic flux leakage. As shown in the Figure, configurations G, H, I can produce torque 30% more than the one produce by configurations D, E, F respectively. Another advantage of SWMs with ferromagnetic boundary is that there is less undesired electromagnetic interference (EMI).

![Figure 5.3: Torque Comparisons between Experiments and Simulations](image)
### Table 5.3: Experimental Configurations

<table>
<thead>
<tr>
<th>Designs</th>
<th>PMs (inner (I) and/or outer (O))</th>
<th>EM Core (Aluminum A; Iron I)</th>
<th>Current (Amp)</th>
<th>Outer rotor (None N; A; I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I</td>
<td>A</td>
<td>2.5</td>
<td>N</td>
</tr>
<tr>
<td>B</td>
<td>I</td>
<td>I</td>
<td>2.5</td>
<td>N</td>
</tr>
<tr>
<td>C</td>
<td>I</td>
<td>I</td>
<td>-2.5</td>
<td>N</td>
</tr>
<tr>
<td>D</td>
<td>I and O</td>
<td>A</td>
<td>2.5</td>
<td>A</td>
</tr>
<tr>
<td>E</td>
<td>I and O</td>
<td>I</td>
<td>2.5</td>
<td>A</td>
</tr>
<tr>
<td>F</td>
<td>I and O</td>
<td>I</td>
<td>-2.5</td>
<td>A</td>
</tr>
<tr>
<td>G</td>
<td>I and O</td>
<td>A</td>
<td>2.5</td>
<td>I</td>
</tr>
<tr>
<td>H</td>
<td>I and O</td>
<td>I</td>
<td>2.5</td>
<td>I</td>
</tr>
<tr>
<td>I</td>
<td>I and O</td>
<td>I</td>
<td>-2.5</td>
<td>I</td>
</tr>
</tbody>
</table>

In conclusion, SWMs with both inner rotor and outer rotor, ferromagnetic stator core and outer rotor can produce maximum sum/average inclination torque and less EMI.

Figure 5.4 shows magnetic flux density distribution of Configuration H. The magnetic flux lines are along with the axis of the coil and confined within the iron boundary (yellow in Figure 5.4). It indicates the design has less leakage of magnetic field and magnetic flux density is strongest in iron core, which verifies the effect of iron on enhancing magnetic interaction between the stators and the rotors. Validated by experiments and FEM modeling, SWMs with iron core and iron boundary is to be chosen if maximum inclination torque is the object.
5.4 Experimental Investigations on Sensing System of SWM

5.4.1 Experimental Setup for Position Sensing of One PM

Position sensing of a PM which can move in three directions as shown in Figure 5.5 is studied here for its simplicity and generality. For SWMs without ferromagnetic materials, the sensing strategy of a PM can be extended to the whole SWM since superposition principle can be applied. The coordinate system is defined by the mobile platform, note that the PM is not necessarily to be in the center of the coordinate system. Here we are going to build the forward model through interpolation from measured magnetic flux densities, instead of analysis or DMP due to the following reasons:

1) It is difficult to measure the residual magnetism and material properties such as actual permeability;

2) The size of Hall Effect sensors cannot be neglected, which makes it difficult to treat actual sensors as point sensors;
3) Environmental conditions such as magnetic noise is unable to be calculated, and temperature, humidity … may affect performance of the sensor, let alone the sensor itself has up to 3% error.

The mobile platform can move in 3-DOF with ranges of 20mm in two planar directions and 10mm in vertical direction. Positioning resolution is 0.02mm in each direction.

As shown in Figure 5.5, three 3-DOF MFS-3A Hall Effect sensors are placed at [90mm, 0, 0], [0, 90mm, 0] and [-90mm, 0, 0] to measure x, y, z components of magnetic flux densities, so nine components in total are measured.

Specifications of MFS-3A are listed in the following table. As shown in Table 5.4, max magnetic flux density is ±7.3mT, so Hall Effect sensors cannot be put too close to the PM in case of saturation. And also, if sensors are put too far away from the PM, the
measuring errors due to magnetic noise will be relatively large. The best way is to make magnetic flux density on sensors just below the maximum measurable value. Outputs of MFS-3A are voltages in the range of 0.5 to 4.5V while measured magnetic flux density is in the range of ±7.3mT, and sensitivity is 280mV/mT. Magnetic flux density is to be obtained with linear transition, and we can also use measured voltages directly instead of transformed magnetic flux density in NNs.

Table 5.4 Specifications of MFS-3A

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Range</td>
<td>±7.3mT</td>
</tr>
<tr>
<td>Resolution</td>
<td>±10μT</td>
</tr>
<tr>
<td>Three Analog Outputs</td>
<td>0.5-4.5V</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>280mV/mT</td>
</tr>
<tr>
<td>Accuracy</td>
<td>±3%</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>100kHz</td>
</tr>
<tr>
<td>Size</td>
<td>10×13.5×12mm</td>
</tr>
<tr>
<td>Weight</td>
<td>2.5g</td>
</tr>
<tr>
<td>Power</td>
<td>36mA max at 5V</td>
</tr>
</tbody>
</table>

Measuring positions in experiments are chosen as $x=[-0(initial):5(interval):20mm(final)]$, $y=[0(initial):2(interval):8mm(final)]$, $z=[0(initial):5(interval):20mm(final)]$ according to the maximum working range of the mobile platform. And interpolation has extended the measured sample to $x=[-0(initial):1(interval):20mm(final)]$, $y=[0(initial):1(interval):8mm(final)]$, $z=[0(initial):1(interval):20mm(final)]$. 
\[ z = [0 \text{ (initial)} : 0.4 \text{ (interval)} : 20 \text{ mm (final)}], \] so \( 21 \times 21 \times 21 = 9261 \) positions in the interpolated training sample. Since the voltage outputs are fluctuating due to magnetic noise or sensor performance, we use average value of 40 measurements in each position to eliminate the influence of fluctuation. The influence of fluctuation on position sensing performance is presented later in this section.

The trained NN is tested by nine positions not used for interpolation in the working range, and errors are as shown in Figure 5.6. Maximum error is about 0.2mm which is larger than simulation in Chapter 4. The main reasons for the error include: limitations of sensors’ performance, environmental magnetic noise, modeling error due to interpolation, and approximation errors of the NN.

To test how much the fluctuations influence performance of the sensing system, a fixed position has been measured 10 times and positions estimated by the same trained NN are compared as presented in Figure 5.7. The difference between different measuring times can be as large as 0.2mm, superposition with errors due to other factors, the total error can be much larger than 0.2mm. And that is why we need to use average measured value to eliminate influence of fluctuations.
5.4.2 Experimental Investigation for 3-DOF Sensing System of an SWM

The sensing system of an SWM to measure the orientation is shown in Figure 5.8. The optical encoder based sensing system is used as reference. Several sensor standers are located around the rotor and can move in the radial to change distance between rotor and sensor.
Due to hardware limitations, five MFD-3A sensors are applied in the experiment, and fourteen magnetic flux density components are measured. The optical encoders generate 2000 pulses every circle, which means a resolution of $360/2000=0.18^\circ$. 216 orientations as $\alpha = [-14.4\text{ (initial)}:5.76\text{ (interval)}:14.4\text{ (final)}]$, $\beta = [-14.4:5.76:14.4]$, $\gamma = [0:7.2:36]$ are measured and then interpolated to 9261 orientations as $\alpha = [-14.4:1.44:14.4]$, $\beta = [-14.4:1.44:14.4]$, $\gamma = [0:1.8:36]$. The 9261 orientation-MFD pairs are used to train the neural network and finally, the trained neural network is tested by several randomly distributed orientations in the working range.

Each sensor outputs three voltages in a range of 0.5 to 4.5 V linear to the three MFD components. One voltage output of a sensor is as shown in Figure 5.9, and other voltage outputs have a similar distribution.
The measured sample \([14 \times 216]\) is then interpolated to training sample \([14 \times 9261]\), one component (one voltage output of one sensor at 9261 orientations) of the training sample is shown in Figure 5.10.
As shown in Figure 5.11, the smallest error occurs when 80 neurons are adopted.
Generally speaking, more neurons means higher approximation accuracy. However, as shown in Figure 5.11, NNs with 90 or 100 neurons outputs larger errors compared to NN with 80 neurons. This is because NNs with 90 or 100 neurons can be more accurate on training sample, but generalization ability is not as good as NN with less neurons. Test orientations are randomly selected in the working range and are not belonging to training sample, and maximum error as shown in the Figure for NN with 80 neurons is about 0.4°, the error/range ratio is about 1.4%, which verifies that feasibility of the sensing system. The error may due to the following reasons:

- Performance of Hall Effect sensors, the manual states that there might be up to 3% error;

- Magnetic noise due to environment, we have tested that even the rotor remain stationary, the sensor outputs still keep changing;

- The sensing system is limited by accuracy of the reference system, i.e. the optical encoder based reference system, since the resolution of optical encoders is 0.18°, the maximum error due to reference system is 0.18°, after accumulation, the total error contributed by reference system is even bigger;

- Approximation error due to neural networks.

Based on above analysis, to minimize error of the sensing system, we can do the followings:
• To adopt magnetic sensors with better performance such as higher stability;

• To use sensor with bigger magnetic flux density range, so noise/MFD ratio is smaller, also can use more sensors to offset the influence of the noise;

• To use optical encoders with higher resolution in the reference system, such as 6000 pulses per circle;

• To try NNs with more layers or neurons, and train NNs with sample with more orientations-MFD pairs.

5.4.3 Influence of Number of Measured Components on Sensing Performance

In last sector, fourteen components are used in the sensing system, here we are to investigate the influence of the number of measured components on sensing performance. 7, 8 … 14 measured components are applied and the sensing errors are compared. For time saving consideration, neural networks for all conditions are 2-layer with 80 neurons, and maximum training times are set to be 100. As shown in Figure 5.12, more measured MFD components generally result in higher sensing accuracy, which is reasonable since more measured components means more information, but the improvement is not so obvious when there are more than nine measured MFD components. So to save hardware cost, we can deduct number of measured MFD components to 9 or 10 without reducing much accuracy.
Figure 5.12: Sensing Errors Vs Number of Measured MFD Components
5.4.4 Influence of Measuring Times on Performance

Due to magnetic noise and Hall Effect sensors’ performance, the voltage outputs are always fluctuating. As shown in Figure 5.7, the fluctuations have influenced the positioning accuracy seriously. To enhance performance of the sensing system, we have used average outputs of 100 times to reduce influence of fluctuations. To investigate how much the fluctuations affect the sensing system, we adopt average outputs of 1, 10, 20, 40, 100 times respectively and comparison is as shown in Figure 5.12. From comparisons of sensing errors of different measuring times on each orientation, we can conclude that more measuring times can lower the influence of fluctuations, but the improvement becomes less obvious when measuring times at each orientation is more than 10. The sensing error is as large as $2.4^\circ$ if only one measurement is carried out at each orientation, which verifies the necessity of multiple measurements.

In design of a practical sensing system, we need to consider requirements on sample rate, sensing accuracy … so as to choose an appropriate times of measuring repetitions. The comparison as shown in Figure 5.13, can be a valuable reference in future.
5.5 Summary

A prototype of an SWM is fabricated based on optimization results. The SWM is designed to be easy in fabrication and inexpensive in cost, such as the wire diameter is chosen to be 0.5mm, so resistance of the coils is suitable for cost saving and common
amplifiers.

Magnetic torques of different configurations have been calculated and compared with FEM modeling results. The great agreement between modeling and experiments has validated the magnetic torque modeling with simplified model, and large improvement in torque density has proven the proposed design for SWMs in enhancing inclination torque.

Although limited by sensor specifications and accuracy of the reference measuring system, the proposed sensing system has achieved a 1.4% ratio of error to working range. Strategies proposed in Chapter 4 to alleviate influence of magnetic noise, have been validated by experiments. Based on experimental verification, proposed sensing system is feasible to be applied in an integrated SWM system with good performance.
CHAPTER 6
CONCLUSION AND FUTURE WORK

6.1 Accomplishments and Contributions

The thesis has presented three respects of research about an SWM, i.e. structure design optimization to maximize the inclination torque and improve torque isotropy, magnetic torque and dynamic modeling for SWMs with ferromagnetic stator cores, and design of non-contact sensing system for real-time measurement of orientation in 3-DOF. The novelties of the thesis lie mainly in 2 aspects as follows.

1. Proposal of a design optimization strategy, which including selecting key dependent physical parameters and their ranges, and optimizing their values by the numerical analysis method. The optimization strategy has been verified on the proposed new design of an SWM to maximize the inclination torque capability.

2. The design of 3-DOF Hall Effect sensor based sensing system, which applies neural networks to compute orientations from measured MFD in real-time. The sensing system is verified to be fast and accurate by experiments, which is important for feedback control.

6.1.1 Design Optimization to Enhance Inclination Torque

The thesis has presented a new design of an SWM with PMs both inside and outside of the EMs, so as to improve electromagnetic torque since Electromotive Force (EMF) at
both sides of the coils are applied and result in less MFD leakage. Inclination torque is generally weaker compared to spinning torque since less PM-EM pairs are involved in inclination torque. Design optimization is then applied to obtain optimal geometrical parameters to maximize inclination torque within a given volume. Finally, a prototype is fabricated based on optimization results and experimental investigations are executed to verify the simulation model.

Detailed work in structure design is listed as follows.

- Two configurations with two or three layers of PMs both inside and outside of EMs, different from previous designs with PMs only inside or outside of EMs are proposed.

- APDL in ANSYS is applied to calculate the electromagnetic torque, key geometrical parameters are selected, interpolated and compared to decide the best combinations which can generate maximum inclination torque. Since it is unfair to compare the two configurations with random geometrical parameters, both configurations are optimized and then compared.

- A prototype based on design optimization results is fabricated, and experiments are carried out to verify the simulation model. The experimental results shows that there is less than 10% discrepancy between FEM modeling and experimental measurements.

### 6.1.2 Magnetic Torque and Dynamic Modeling

MFD and magnetic torque of SWMs with ferromagnetic stator cores is nonlinear and
makes it challenging for modeling. MFD on ferromagnetic stator cores with different applied currents is generated and analyzed by FEM, and magnetic torque model is built based on MFD analysis and inspirations from the SPM. Although the magnetic torque is nonlinear to current inputs, it can be divided to two parts, i.e. one fixed part due to ferromagnetic stator cores and one part proportional to current inputs as long as the maximum currents density is less than 8A/mm$^2$, which is maximum current density limited by heat dissipation. Based on the magnetic torque and dynamic model, motions of the rotor can be anticipated with given inputs and optimal inputs can be computed with desired motions.

6.1.3 Design of a 3-DOF Non-contact Sensing System Based on MFD and NNs

A 3-DOF, real-time and absolute sensing system based on Hall Effect sensors and NNs is proposed and realized. Due to complicated MFD of the whole SWM, neural networks instead of analytical expressions are applied to approximate the relationship between MFD and orientations. Based on the fabricated prototype, the sensing system is verified by experiments.

Detailed work in sensing system design is listed as follows.

- MFD of a PM is computed by DMP, and based on superposition principle, MFD of the whole rotor is calculated.

- A solution to solve the inverse problem- to compute orientation from measured
MFD, based on neural networks is proposed.

- Parameters affecting the sensing system are studied and methods to improve performance of the sensing system are proposed. Practical conditions such as magnetic noise are also considered.

- Experiments are carried out to verify performance of the sensing system. Optical encoders based measuring system is used as reference, and accuracy of proposed sensing system is at the same level of the one of the reference system, which verifies the performance of the proposed sensing system.

6.2 Future Works

Recommendations for future work in the area of SWMs are as followings:

**Improvements on sensing system**

The sensing system can still be improved in the following aspects in future work:

- The thesis designs the sensing system based on SWMs with air stator cores, so MFD generated by current inputs can be deducted under superposition principle. However, for SWMs with ferromagnetic stator cores, nonlinearity of the MFD must be considered, which makes design of sensing system based on magnetic field more difficult. One important work future work is to extend the proposed sensing system to SWMs with ferromagnetic materials.
The magnetic field is periodic every 72 degrees, so sensing system based on MFD only is unable to distinguish if SWM spins 72 or more degrees. The proposed sensing system may need an auxiliary sensing method based on design requirements.

The neural networks of the sensing system in the thesis treat the MFD- inputs of NNs as an ordinary set of values. However, MFD of the SWM, which can be solved analytically, possesses some intrinsic characteristics. If we can fully make use of those characteristics, we are able to decrease the scale of the NN and improve computation speed while accuracy is maintained.

**Closed-loop controller design and realization**

The magnetic torque and dynamic modeling, and proposed design of the sensing system, have laid foundations for design of a closed-loop motion control system for SWMs. Future work about control of SWMs are desired so as to complete an integrated SWM system. Future work may focus on control simulations first, but the theories about the SWM in the thesis are better to be verified by successful motion control experiments.
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Appendix A

Design

Drawing of SWMs

The appendix presents design drawings of SWM prototype based on optimal design, design drawings of a miniaturized SWM, and design drawings of experimental setup for sensing system. All design drawings are final files for fabrication. For simplification, some simple structure parts are not presented.

The 1st drawing (Page 136) is a section drawing about the whole SWM; the 2nd drawing (Page 137) is about the inner rotor; the 3rd drawing (Page 138) is for the base to fix SWM on the working table; the 4th drawing (Page 139) is about the spherical bearing, which is not fabricated but bought from market; the 10th drawing (Page 145) is a miniaturized SWM, the 11th drawing (Page 146) is about the outer rotor of the miniaturized SWM; the 12th drawing (Page 147) is about the inner rotor of the miniaturized SWM; the 17th drawing (Page 152) is about the reference sensing system.