EXPLOITING STATISTICAL SIDE INFORMATION TO OPTIMIZE SECONDARY SPECTRUM ACCESS.

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Abstract

Unlicensed access of bands in the wireless spectra, that have been left under-utilized by the primary (or licensed) users, is the subject that addressed throughout this dissertation. Unlicensed users (or commonly known as secondary users) attempt to efficiently utilize these bands by exploiting the underlying spatial, spectral and temporal opportunities. We demonstrate that partial or complete knowledge of primary activity, transceiver locations and channel gains can be incorporated into the design of secondary access strategies to improve their throughput performance.

Surveys have shown that most of the bands auctioned to primary networks are under-utilized. To resolve the issue of spectrum scarcity, the idea of secondary access to the under-utilized bands has gained popularity over the years. Secondary networks are opportunistic in their attempts of acquiring resources, i.e. they search and acquire resources that are not in use by the primary network, and are expected to release them as primary transmissions resume. The quality-of-service of the primary system, and its throughput requirements supersede those of a secondary system. A logical consequence of a secondary system design is the efficiency of grasping and using under-utilized spectral opportunities.

Optimizing channel acquisition procedures of SUs for opportunistic access, by incorporating side information, is the essence of the contributions presented throughout this dissertation. Conventional sensing systems prove to be inadequate when it comes to streamlined secondary procedures. Firstly, we optimize a single SU’s decision making process that directly impacts the achievable capacity of secondary and primary systems. This decision is based on a threshold, and by incorporating spatial
(geographical) side information, the capacity maximization objective is achieved.

This dissertation further shows that partial or complete side information allows systems to break away from the conventional methods of channel selection. Typically, channels are defined by the primary activity on them, and are pursued by SUs in the same discrete manner. We show that if side information regarding interference is made available, this discrete method of channel selections becomes suboptimal. By relaxing the discrete constraints of the channel selections, and allowing users to select any band of frequencies, the contention and thus the interference among multiple SUs can be manipulated in a continuous manner.

Moving on to a multi-SU system, we shift our focus on the contentions among SUs that play a pivotal role in adversely affecting the transmission efficiency. These contentions result when multiple SUs, in a non-cooperative manner try to access premium frequency resources. Cooperative spectrum access methods minimize contention among SUs by performing orthogonal channel selections, for SUs. The extent of cooperation among SUs may be bounded by system constraints, but the operational conflict-minimization of these SUs can be improved. It has been shown that the thresholds, if carefully selected before sensing begins, can strike an optimal trade-off between premium-channel acquisition and contention minimization. We achieve this goal by modelling the throughput maximization problem to incorporate spatial and channel side information, if made available. Furthermore, a distributed sensing order selection approach is also proposed.

Finally, this dissertation also considers the secondary systems that allow SUs to sense multiple channels within a frame, before they can acquire any one of them for transmissions. The order or sequence in which the SUs perform sensing affects the contention among them. We show that if location side information is utilized in the design of sensing orders, the contentions among the SUs is minimized. Without side information, the SU system design has to rely on assumptions regarding the detection outcomes that result in a waste of valuable spatial opportunity. Numerical results and analytical proofs presented throughout this dissertation advocate our thesis of incorporating partial or complete side information for maximal exploitation of the
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$S$  Sensing order
$\mu$  Sensing duration in secs
$T$  Number of time slots in a frame
$\mathcal{T}$  Frame length in secs
$k$  SU’s bandwidth
$|S|$  The cardinality of the set $S$
$\mathbb{R}^{S}(1)$  Reward of sensing order $S$
$p^{S}_{u}(t)$  probability of SU $u$ acquiring a channel in time slot $t$ for sensing order $S$
$P_{r}(H_{i}|H_{j})$  Probability of hypothesis $H_{i}$ being detected when $H_{j}$ is the true hypothesis
$y(n)$  The $n^{th}$ sample of the received signal
$x(n)$  The signal component of the $n^{th}$ sample of the received signal
$e(n)$  The noise component of the $n^{th}$ sample of the received signal
$\beta$  pathloss coefficient
$d$  distance over which the pathloss occurs
$pl$  pathloss
$\gamma$  signal to noise ratio
$h$  channel gain
$R$  Bayesian risk function
$C_{i,j}$  Cost of detecting hypothesis $i$, when hypothesis $j$ is true
ε \quad \text{detection threshold}

\sigma^2_e \quad \text{noise power}

\sigma^2_x \quad \text{signal power}

\hat{f}_{\text{sam}} \quad \text{sampling rate}

\theta \quad \text{prior primary-free probability}

\mathcal{R} \quad \text{Transmission rate}
List of Abbreviations

- **SU**: secondary User
- **CSA**: channel sensing and acquisition
- **pdf**: probability density function
- **SNR**: signal to noise ratio
- **SO**: sensing order
- **SOS**: sensing order selection
- **pSOS**: pruning based sensing order selection
- **PD-SOS**: perfect detection based sensing order selection
- **SI-SOS**: side information based sensing order selection
Chapter 1

Introduction

1.1 Motivation and Objectives

The contributions of this dissertation are motivated by the loss of transmission opportunity by conventional secondary systems, resulting from their inability to utilize side information in the design of operational parameters. To understand this idea a discussion on the under-utilization of channels by the primary system, is presented. This under-utilization motivates the implementation of secondary systems to make use of the left-over transmission opportunity. This understanding will be followed by the performance analysis of the conventional secondary systems. The purpose of this performance analysis is to highlight the scenarios where secondary systems under-perform.

Primary users are licensed to utilize certain bands of the wireless spectra. Most primary systems have users spread over vast geographical regions. The user density in some regions may be very low, and their transmissions sporadic. This creates regions of under-utilized spectrum. The availability patterns of various systems have been studied over the past few years. The findings suggest that even the most favorable portions of the wireless spectra may experience sparse transmissions by its licensed users in certain regions (geographic). Numerous surveys show that the activity on these channels is not only scarce during certain periods of the day but also scant in
certain geographical regions [1–13]. The idea of secondary access originates from the need to benefit from these under-utilized resources.

Secondary network technologies make the under-utilized opportunity, in the spectral and spatial domains, available for access and transmissions. The constraint faced by such technologies is the condition that the primary network’s performance should remain unaffected by this access. The SU are expected to abort transmissions as soon as the primary users begin transmissions [1–12]. This implies that every now and then the SUs will have to stop transmissions and re-sense the channels for primary activity. If the operational time of SUs is to be divided into frames, then these frames are further partitioned into sensing and transmission slots. Dividing the frame time of a SU’s processes into sensing and transmission stages reduces the available time for transmissions [14], [15]. The rate of re-sensing of the occupied channels can be optimized using the prior availabilities probabilities of these channels. Learning primary activity patterns can also help optimize the sensing and transmission intervals.

As discussed, the bounds on the degradation of the primary user’s performance should not be violated by the secondary access [16]. Despite this constraint, our research has revealed that secondary performance can be further enhanced by careful selection of detection and access parameters. Conventional methods of sensing render the secondary access objective, suboptimal. Maximizing secondary throughput is the most sought after secondary access objective that can be achieved through smart exploitation of under-utilized resources. This dissertation shows that by incorporating side information, this gap can be tightened, leading to highly efficient and cognitive secondary systems.

Secondary systems can be distinctly classified as one that focusses on improving the secondary network’s performance by optimizing the access strategies. The benchmark for judging an SU’s performance is essentially the achievable throughput, measurable in bits per second. The network sum capacity can also be termed as the reward of the adopted policy. The cost of the proposed algorithms is measured in terms of the computational power required to achieve optimal results.

Sensor technologies have been around for a while, and using their sensing methods
for SU systems seems to be a straightforward approach. The detection and sensing procedures of these wireless systems have been perfected for accurate target tracking, localization purposes or just channel occupancy tests. The reason why it is not wise to directly use them for SU systems is that they have not been designed for secondary access. This is because these systems do not weigh the costs and the rewards of decisions when channel access is involved.

In the context of an SU system, the decision process can be categorized in to two very distinct types. The first type of decision is regarding the presence or absence of primary transmissions on a channel. SU operation is bound to check if the channel is primary-free before acquiring the channel. This limitation is to ensure interference-free primary operation. The second type of decision is regarding channel access, whether to acquire a channel or not. Some authors consider both the decisions to be the same. Furthermore, decisions regarding the presence or absence of contention and the channel gain also affect throughput of the SU system.

Channel sensing can be performed in a number of ways, the most popular of these ways are feature detection [17] and energy detection [18]. Feature detection implies that the SU receives a few samples of the signal and then tries to reconstruct the signal using estimation methods. This may require some knowledge of the signal characteristics beforehand. The estimated signal is then compared with already available statistics to determine if the received samples were just noise or did contain meaningful information.

On the other hand, in an energy detection procedure, the SU receives samples of the signal, for a short period of time. Then, the SU computes the average power of the received signal. The SU then compares this test statistic with a threshold, called the detection threshold. If the average received power is higher than the threshold, the SU decides that the channel is busy, otherwise, it decides that the channel is free. This procedure does not require any prior knowledge of the signal characteristics.

A number of contemporary authors have published on SU systems using these conventional methods. The purpose of this dissertation is to prove that such direct application of wireless sensor technologies does not result in an optimally performing
CHAPTER 1. INTRODUCTION

secondary network. The main contributions of this dissertation can be enumerated as follows.

1. To demonstrate that conventional design of the decision making process for an SU results in the loss of transmission opportunity by not considering the (partial or completely available) location side information into the risk and cost model.

2. To demonstrate that the decision making process of SUs can be further improved to maximize their throughput and minimize contention by incorporating the channel side information.

3. To show that in cooperative SU systems, optimization of the channel sensing parameters can considerably impact the throughput.

4. To demonstrate that secondary access can result in a higher throughput, and minimum contention even with minimal cooperation among SUs, if the problem formulation is altered to include this information.

5. To show that the decision of selecting the same channel for multiple SUs may not be a sub-optimal decision if spatial analysis and costs of detection errors are carefully computed.

6. Develop algorithms and procedures that demand a low computational power and contribute towards enhancing the secondary network’s sum capacity.

1.1.1 Adaptive Detection Threshold Design

In principle, the work presented in this dissertation was inspired by the lack of adaptability displayed by conventional sensing systems when it comes to including side information in the design. For example, a simple sensor just performs the task of sensing/detecting primary transmissions, and due to the lack of cognitive abilities, the sensor misjudges primary presence to be a sign of channel unavailability for secondary transmissions. The current study proves this notion to be wrong, and shows that if a SU can incorporate side information like the geographic distances in the
1.1. MOTIVATION AND OBJECTIVES

Design, the detection results can essentially be converted into a verdict regarding the availability of the channel. Primary presence or absence is not enough basis to rule out the possible acquisition of the channel by SUs. Using the side information, an SU may be able to determine the performance degradation that it would cause to the primary network, if it acquired the channel. This analysis may permit the SU’s acquisition of the channel, despite primary presence. For example, consider the scenario where the secondary transmit power is to be so low that the interference caused at the primary’s end is negligible (they may also be geographically distant from one another). This analysis motivates us to study the detection threshold design and convert the transmission detection threshold into a channel acquisition threshold.

1.1.2 Adaptive Thresholds with Continuous band selections

The channel structure used by the secondary network is one defined by the primary activity. The channel bandwidth and its location in the spectra is dictated by the pattern of primary transmissions on it. In other words, a channel is just a set of primary operating frequencies. Conventionally, SUs follow this structure (the division of the spectrum into channels) as it easy to deal with bands whose idle or busy characteristics are predictable. The current study presents the argument that although the SUs may perform detection following the same structure, they do not necessarily need to acquire channels in the same manner. This is called the continuous selections of bands. It is further shown that the contention and interference among the SUs can be better managed if the SUs acquire bands in a continuous manner. Furthermore, if the adaptive thresholds are also incorporated to the design the sum capacity of the network can further be increased. But this incorporation will require the threshold to be designed in a way that losses to multiple primary users be considered.

This study proposed the implementation of continuous selections when dealing with multi-channel systems. It is further shown that there may exist practical limitations of selecting a complete primary channel for an SU. These practical limitations are of the form of interference and throughput degradation at the primary end. There
exist constant rate primary systems where the users have to maintain a certain minimum rate. If the primary users fail to maintain this rate, the users stop transmissions all together. Selecting a complete primary channel for an SU may result in a very high interference at the primary’s end, whenever the SU fails to detect primary transmissions. As a result, the expected throughput may fall below the acceptable threshold. The purpose of this investigation is to find a channel for the SU that minimizes the loss in the primary’s throughput.

1.1.3 Sensing Order Design

Further, this study extends the scope to include systems where the SUs have the privilege of sensing multiple channels within a frame time. Each SU has scheduled multiple time slots within the sensing duration of a frame (the frame being divided into sensing and transmission phases). It is assumed that the SUs will not acquire bands using the continuous selection scheme, and follow discrete selections. During a frame time, no communication is allowed among SUs. SUs can communicate with themselves only at the beginning of a frame (or set of frames). The order in which a SU senses channels during its frame time is called the sensing order.

No existing sensing order design scheme incorporates channel side information or geographic information into the design. The existing schemes just assume that the probability of detection is 1 in the presence of primary or secondary transmissions. For practical scenarios where the distances among the SUs are large, this probability may be close to 0. Such an approach results in the loss of spatial opportunity. This motivates us to come up with a problem formulation that allows the incorporation of side information. Secondly, to the best of our knowledge, no algorithm or approach exists that can select optimal sensing orders. Any procedure that can guarantee an optimal solution has computational requirements that become intractable as the number of SUs or the sensing slots increase.
1.2 Contributions of this Dissertation

The analysis and proposals presented throughout this report are designed to be implemented in practical secondary systems. The key results of this study regarding the maximal exploitation of the spatial and spectral opportunities are presented in this section. The analysis begins with the study of a single SU’s performance. The main results from this work are

- **Incorporating geographic information in (Neyman-Pearson based) Threshold design:** A detection threshold has been designed that takes into account the interference caused at the primary and secondary ends. The consequence of this threshold design will allow the SUs to make use of the spatial opportunity present in the system. The derivation of the threshold incorporates partially or completely available location information of the primary and secondary geographic distances in the form of interference and transmission opportunity losses.

- **Incorporating system design preferences in Threshold design:** The presented design is such that the system designer can shift the emphasis between the primary and secondary performances by adjusting the weights associated with the losses.

- **Minimizing the Sensing duration:** A reduction in the sensing time of a SU has a direct impact on the fraction of frame time available for transmissions. A lot of effort by the research community is centered around this objective. It is shown that by using this side information, this objective can safely be achieved.

- **The use of primary receiver’s location:** Conventional primary systems have the geographic regions divided into service areas. These service areas serve primary receivers lying at certain distances from their transmitters. Even if the primary receivers are passive devices, the service area information provided by a third party can help determine an optimal detection threshold by estimating the possible interference loss caused at these receivers.
• A design tailored to the transmit power requirements of the secondary transceivers: Assisted by the knowledge of service areas and the expected interference loss, the transmission powers of the secondary transmitter can be selected to determine the optimal threshold and minimize losses.

After the performance enhancement observed at the single-SU, single channel level, the current study moves on to include multi-channel environments into the analysis.

The main results of this work are

• Continuous band selections: The novel idea of continuous band selections for secondary systems. These selections prove to be very effective in mitigating the impact of interference and contention caused at the primary’s receivers from the secondary transmitter. These selections also offer the advantage of catering to the needs of secondary systems where SUs require variable bandwidths.

• Threshold design with multiple primary channels: This work focuses on the design of a threshold, which unlike the previous work, considers the losses caused at multiple primary users by the SU. Since, continuous selections allow the overlap of an SU’s band with more than one primary channel, the SU’s detection errors may cause transmission losses to more than one primary user. The prior primary-free probabilities of more than one primary channel is to be included into the design. It is proven that the bayesian risk, as a function of the detection threshold, has a single minimum value that can be achieved by simple gradient descent approaches.

• Cooperative Spectrum Access with Joint optimization of Detection Thresholds: Cooperative systems can considerably manage the inter-SU interference if their thresholds are jointly optimized. Another aspect of such a design is that the overall interference at the primary user can also be regulated.

• Optimal Channel selections with location based threshold: Finally, to exploit the spatial opportunity in a given geographic layout, the SU can implement adaptive thresholds for energy detection of the primary transmissions in conjunction with continuous channel selections.
Proceeding with the analysis, a multi-SU system where SUs can sense multiple channels in the sensing duration of a frame time, is considered. The important contributions of this research are as follows

- **Sensing Orders with side information** The problem of sensing order selection is formulated to cater for the available side information. The interference power among SUs plays a decisive role in determining the sensing order as per our proposals. Previous works did not accommodate these interference parameters and ended up squandering spatial opportunity.

- **Sub-optimal sensing order design** We also present a low complexity sub-optimal approach. This approach differs from the conventional greedy algorithms presented in previous papers. The sub-optimality and its required complexity are also discussed.

- **Discrete band selection based Optimal sensing order design** Building on the sub-optimal approach, the optimal sensing order design algorithm, is presented. This algorithm proceeds in the forward direction, relinquishing band selection options based on the propositions presented. Owing to the analysis presented, by the last stage, the set of acceptable sensing orders is very small. Evaluating this small set of orders has a computational requirement at par with the sub-optimal approach.

- **Continuous band selections** The optimal channel selections for a secondary network that implements continuous band selections is also presented. The optimality of the approach has also been proven. These proposals result in a highly efficient mechanism of exploiting the opportunity that is revealed when side information is incorporated in the design.

- **Complexity Analysis** Finally, the complexity requirements of all these algorithms, is analyzed and compared. The algorithms and procedures presented have computational requirements that can be proven to be very close to the sub-optimal approaches.
1.3 Organization of the Report

The rest of this report is organized as follows: Chapter 2 highlights some important work relevant to the presented research. This literature survey helps not only to understand the contribution of the proposed algorithms, but also to establish benchmarks. Chapter 3 discusses the design of a detection threshold for a single SU. This threshold design is performed for two different channel selection strategies. Continuous channel selections require the adaptive threshold to be redesigned in a way that the losses due to detection errors are minimized. Chapter 4 is centered around the operation of a cooperative multi-SU system. The joint threshold design result in SU sum throughput maximization by managing the inter-SU interference. Chapter 5 considers sensing order design for an SU system. Chapter 6 gives a brief insight into the possible future extensions of this work and conclusions.
Chapter 2

Literature Survey

This chapter presents the contributions of various authors that fall in the subject area discussed throughout this dissertation. A good understanding of their results will help underline the significance of the contributions and the proposals presented in the later chapters. An emphasis on the differences between the existing literature and the contributions (presented in the later chapters), helps realize the significance of the approach and algorithms that are used to solve the spectrum utilization problem. Some of the methods mentioned in these papers are used as benchmark results to observe performance improvements that can be attributed to the presented contributions.

Secondary spectrum access have been a hot topic of research for the past decade. The pioneering works in this field were inspired by [10] and the papers [19–32]. The main objective was to develop systems that could retrieve spectral opportunities that were previously considered bounded to primary (licensed) users. Such an access was termed unlicensed, and therefore required bounds that would ensure the eminence of the primary performance metrics over those of the secondary system. The secondary systems explored in this dissertation are the ones that the primary systems remain unaware of. This implies that the primary system neither communicates with the secondary users nor is ready to accommodate them in any way. Despite the primary system being unaware to secondary presence, its performance can still be considerably
CHAPTER 2. Literature Survey

degraded, if the secondary system does not implement measures to prevent it.

In order for the secondary systems to sufficiently protect the primary systems, they have to rely on their own sensing and detection abilities to determine primary transmissions. These users are permitted to access the spectrum only when no primary transmissions have been detected or the performance degradation at the primary user’s end is within acceptable bounds. The secondary users are also required to abort transmissions and relinquish control of the channels once primary transmissions are detected. This requires the scheduling of recurring sensing phases after certain time intervals. At every sensing duration, decisions have to be taken regarding the presence or the absence of primary transmissions. This decision making procedure is an important aspect of the improvements presented in this dissertation. This decision making process, at the single SU level, is discussed in the next section.

2.1 Threshold Design

Sensing the primary transmissions is basically equivalent to a detection problem as discussed in [33–35]. After every sample collection, an SU has to make a decision regarding primary presence or absence. This decision is made after comparing the test statistic with a threshold. Previously, various authors have published on sensing and detection procedures of wireless sensor networks [36,37]. The sensor performance, challenges and objectives are similar to that of the secondary users, apart from the fact that secondary users attempt to acquire the channels. The standards presented in [16,25] for WRAN 802.22, focus on the cognitive radio systems, and highlight the importance of prioritizing primary activity. Opportunistic access to TV white spaces in certain geographic regions has also been explored in these articles.

As discussed earlier the SUs have to iterate between decision making (sensing) and transmissions phases [14] to make sure that the primary users are not active on that channel. A time frame can be divided into sensing and transmission phases. Sensing is performed at the beginning of each time frame. Based on the sensing results, channel acquisition may or may not take place. As a frame duration comes
to an end, the SU will have to abort transmissions and re-sense the channel. Shorter
the transmission phase, the earlier the SU will return to sensing to check whether the
primary users have resumed their transmissions.

The first step in a secondary system design is determining the decision making
(or the detection) procedure. Some systems implement energy detection while other
determine primary signal presence using feature detection [2, 38, 39]. If exact inform-
ation regarding the features of received primary signals is known beforehand, then
feature detection should be employed for detecting primary presence [39]. Energy de-
tection [2] becomes an interesting choice when much less is known about the primary
signal features.

Although the research in this dissertation is not bound to the detection method,
energy detection is adopted as the secondary user’s sensing procedure. This liberates
the analysis of any assumption regarding the primary signal’s features. Energy detec-
tion requires a number of samples collected over a period of time (within the sensing
slot). These energy samples are then compared with a threshold that has been drawn
using the Neyman-Pearson criterion [34]. The received signal level is to be compared
with noise samples. The details of the derivation of this threshold has been discussed
in [34]. The authors of [14] have also presented the derivation of this threshold and
studied the impact of sensing duration on the achievable throughput.

Detection is a hypothesis testing problem. The hypothesis that the primary user
may be active on the channel can be denoted by $H_1$. Similarly, the null hypothesis,
that the channel is primary-free, can be denoted by $H_0$. The probability that the
secondary user will detect primary presence is called the probability of detection,
and the probability that it will fail to detect presence is called the probability of
missed detection. The probability of the event where the SU detects primary presence
when it is not is called the probability of false alarm. These hypothesis can be
given as $H_0 : y(n) = e(n)$ and $H_1 : y(n) = x(n) + e(n)$, where $x(n)$ represents the
primary portion of the received signal and $e(n)$ represents the noise part of the signal.
Secondary systems limit the interference caused to the primary network by adjusting
the detection threshold. If this threshold is very low, the SU will erroneously detect
primary presence more often (even when the primary transmissions are absent). Such an event is called a false alarm. On the other hand, if this threshold is very high, the secondary user would frequently fail to detect primary presence, an error more commonly known as missed detection.

2.1.1 Incorporation of side Information

This section highlights the difference between the methods proposed (in the following chapters) and the related work on secondary systems. The induction of side information in the design of control strategies has been studied by many authors including the ones in [40–48]. The consequence of such a design is either to control the transmission power or the net transmission duration of the SUs. Such measures are intended to maximally utilize the secondary opportunity while maintaining a level of acceptable performance degradation at the primary network.

The authors in [40] discuss the impact of interference on the capacity of the primary and secondary networks. The work [40] suggests a strategy that keeps a check on the primary and secondary throughput at the same time by integrating them into a single objective function. A trade-off of one network’s capacity for another’s capacity is considered acceptable as long as the overall capacity is increased. The side information in [40] is the impact of geographic distances between users, interpreted in the form of interference loss or transmission-opportunity loss. The test statistic considered by these authors is different from the one proposed by us. The formulation of the problem studied later, brings sensing time into the analysis, an important secondary system parameter left out by this work.

Similarly, the work [48] considers the ratio of the actual distances to impact the threshold design. The problem with this approach is that it is unable to capture the impact of actual interference losses. The ratio of distances does not necessarily translate into a ratio of the actual losses. The proposed scheme considers the ratio of interference losses to that of the transmission opportunity lost due to false alarms. The same authors have modified their approaches in [41] and [42] to consider the actual losses. But these papers have left out a very important aspect from the design,
2.1. THRESHOLD DESIGN

![System model](image)

Figure 2.1: System model.

the sensing duration. The sensing duration, as shown by many authors, directly impacts the decision making process by considerably reducing the probabilities of error. Finally, the ideas proposed in this dissertation differ from [46] and [47] in the sense that these works propose power control to manage losses while the methods presented in the later chapters do not consider power control. Though the primary network’s capacity is not calculated in this dissertation, it does account for the increase in the transmission loss due to interference from secondary transmissions. The network layout under consideration is depicted in Figure 2.1. Furthermore, in this dissertation, sensing duration is considered to be an important aspect, defining the throughput of the primary and the secondary systems. The problem formulation, presented in the later chapters, considers this duration into the design.

The main motivation of studying the threshold design is considering a test statistic that can cater for the number of samples. Most articles on wireless sensor networks rely on the same criterion for keeping the interference at the primary receiver’s end under check by adjusting the threshold to have a fixed probability of false alarm. A higher probability of false alarm implies a higher protection of the primary network from secondary transmissions. But, it is shown later that this probability can be low as long as the interference loss at the primary’s end is under check.
CHAPTER 2. Literature Survey

2.2 Discrete and Continuous Channel Selections

Having introduced the threshold design for a single SU, in a single channel system, the case of threshold design in a multi-channel system is presented. The focus of this study is the spectral divisions that termed as channels. Conventional (primary) systems that follow narrow-band structures divide the spectrum into smaller chunks and allocate them to users. These channels may be allocated to different users, resulting in varying traffic and activity on them. The primary activity on a channel ( [49]) is an important feature that defines the secondary access to that channel. Previous works on cognitive users assume that the secondary network either knows about these channels or learns through sensing. A channel is described by its center frequency and bandwidth, which are defined by the primary activity. These channels can also be called the primary channels.

If a channel selection method is such that it selects bands for secondary users that have the center frequency and bandwidth of any one of the primary channels, then such a selection is called discrete. On the other hand, if the system selects bands that do not have the same center frequency or bandwidth of a primary channel, then this selection is called as continuous. The idea of overlapping bands is not new and has been explored before by [50–53]. The Figure 2.2 displays the discrete and continuous allocations.
selections of five SUs. The advantage of continuous selections is that the percentage of the interference and contention loss can be adjusted in a smooth fashion. This feature comes in handy when multiple SUs attempt to acquire the same band. The application of continuous selections in previous works has been confined to primary networks, especially wireless mesh networks.

The authors in [50] studied the impact of assigning partially overlapping bands to users in different systems. The effect of interference on the overall throughput of a network employing such allocations was studied. The network that employed these assignments is a primary network that follows the protocol laid down in either 802.11 or 802.16. The difference with 802.11 is that the users can access multiple channels. Signal attenuation as a result of spatial and spectral distances between the nodes was analyzed. The results of this work show an interesting improvement in wireless LAN networks as well as wireless mesh networks. The authors have also coined in the term continuous channels. Secondary systems are subjected to the same ideas (in the later chapter). These SU systems are assumed to be more resource deficient than the primary systems addressed in [50].

The authors of [51] have termed efficient routing and channel assignment schemes essential for capacity enhancement. The authors agree that although orthogonal assignments help mitigate the interference problem, a better solution may be possible with an understanding of interference. They also present partially overlapping assignments. The authors join different problems into a cross layer, mixed integer, network utility maximization problem. The performance of wireless mesh networks is of concern which is a primary network where sensing of channels is not required. The problem formulation presented in [51] does incorporate interference among SUs. The challenges faced by primary systems are very different from the ones faced by the secondary systems. The secondary systems have to check for the availability of channels before they are acquired. The spatial opportunity present in the secondary systems can be further utilized by employing continuous channel selections.

The paper [52] also presents the idea of partially overlapping channels for allocation in wireless mesh networks. Further, the authors present an interference model
that forms the basis of channel selection in a partially overlapping manner. The authors argue that orthogonal (non-overlapping) assignments can be used in conjunction of the partially overlapping structure to exploit the transmissions opportunities resulting from large geographic distances. Like the previous works, the users are not required to perform sensing. On the other hand, in a secondary system, prior availability probabilities of channels, detection imperfection of secondary users and the interference profile, determines the selection of channels.

Unlike the secondary systems studied in the later chapters, the authors of [53] focus on primary mesh networks. They propose an algorithm named Channel Assignment Exploiting Partially Overlapping Channels (CAEPO) that can select overlapping and non-overlapping bands for users. The metrics of concern are the packet loss ratio as well as the traffic based interference. The authors did not consider the impact of geographical distances or exploiting spatial opportunity. But the authors of [54] did bring into account the impact of distances. The paper [54] explores the greedy solution as well as the heuristic based genetic algorithm for the continuous channel assignment. Although no closed form solution or complexity analysis was presented for the proposed channel assignments, the numerical results advocated the continuous channel assignments to be a throughput enhancing approach.

2.3 Channel Selection in a Multi-User system

After proposing adaptive thresholds that can incorporate the impact of geographic distances between the transmitters and receivers, a study of their impact on multi-user systems is presented. Multi-SU system face the added challenge of managing channel access. If there exists no channel access or scheduling procedure for SUs, the SU transmissions face performance degradation as a result of contention and interference from one another. A conflict-free operation with minimal interference among the SUs is as crucial as identifying and utilizing new transmission opportunities.

In multi user systems, it is naturally preferable that the users acquire orthogonal channels. Such a selection intends to minimize the interference/contention loss at the
users. The prior availability probabilities of channels are assumed to be known beforehand. The analysis in the following chapters, shows that sometimes it is favorable to actually select the same channel for multiple users. The main objective of this work is also throughput maximization of the secondary network. The authors of [55] have discussed adjusting the sensing duration in multi-SU networks. They also assume a central controller that coordinates the acquisition of the channels. The channel selection procedure proposed in the later chapters may employ a central controller only before the sensing phase begins and not during the frame time.

The publications [56, 57] discuss multi user systems where the time frame of the secondary users are divided into sensing and transmission phases. The users sense only one channel in the sensing phase, the results of this sensing contribute towards the learning of the availability probabilities. The channel availability probabilities are assumed unknown. The users are to explore and exploit the spectrum at the same time [58]. The system proposed in this dissertation does not involve the learning phase, but is compatible with any learning procedure presented in the mentioned papers. As the confidence in the learnt statistics increases, the emphasis of the secondary network shifts towards exploitation of the spectrum. The results show that all these approaches are suboptimal in their attempt to maximally exploit the opportunity in the spectrum.

It would be ironic that secondary systems whose main purpose is to exploit wasted opportunities are themselves inefficient in their use of the resources. The sub-optimality of the above approaches stems from their inability to include side information into the problem formulation. The problem formulation of the channel selections presented in the later chapters is based on energy detection results. This implies that actual/estimated interference powers play a decisive role in the decision making process that lead to channel selection. The ratio of the average interference powers of secondary users is a depiction of the geographical distances between them. Moreover, the inclusion of adaptive thresholds helps increase the secondary throughput by improving the decision making process, and basing it on actual losses.

When multiple users try to acquire the same channel, contention takes place.
This contention needs to be resolved before any user attempts to acquire the channel. If the contention goes unresolved, the secondary transmissions would suffer from interference from one another throughout the frame time. The contention resolution procedure is intended to ensure only one user acquires the channel. The contention resolution events implemented in the later chapters have also been proposed in [59,60], similar to the IEEE 802.11 carrier sense multiple access (with collision detection). All existing works lack in terms of being able to cater for the side information. This procedure is illustrated in Figure 2.3. The flowchart shows that the SU performs sensing to find a primary-free channel. If the channel is busy, the SU stops the acquisition, otherwise, it performs contention detection to learn if this channel has been acquired by some other SU. If the channel is also contention-free, the SU acquires the channel. On the other hand, if the channel is found to be acquired by some other
SU, this SU backs-off for a random period of time. When the back-off period expires, the SU performs detection for the second time to ascertain if the channel is free. If the SU finds the channel to be contention-free, it acquires the channel, otherwise, it quits channel acquisition.

The problem formulation presented by all these works assume that the probability of detection of secondary presence equals 1 if two channels sense exactly the same channels. This is a MAC layer design approach that assumes perfect detection, also adopted in [59] and [61], suitable for wired systems where the signal strength is not an issue. But as the distances between users increase, so does the path-loss, thus creating an opportunity for users to exploit. For a system to be able to detect this opportunity and reflect its impact on channel selections, the problem formulation should be base on actual/estimated average received interference powers from other secondary users.

2.4 Sensing Order design

Proceeding in the same manner, an extension to systems that involve multiple time slots, is also discussed. The sensing phase of the frame time is divided into sensing time slots. Each user can sense a channel in each time slot. Figure 2.4 illustrates the operational frame structure of secondary users in such a system. The order in which each user senses channels during the sensing phase is called the sensing order [59,61–66]. Each user in the system follows the sensing order, and whenever it finds a channel available, it acquires the channel for transmissions. This transition from the sensing phase to the transmission phase is completely independent of any central controller’s intervention. As a matter of fact, the central controller plays no role during the frame time. The only way for these SUs to avoid contention is by a mutually agreed upon sensing order drawn-up before the frame begins.

The central controller, if any, plays a role in determining the sensing orders of each user. The selection of channels for the sensing order is dependent on the prior availability probabilities of channels. These probabilities, as assumed earlier, are
either learned over time or are provided by a third party. This prior information is the probability of the channel being free of primary transmissions. The most favorable or preferred channels are the ones that have the highest probability of being primary-free. If multiple users attempt to acquire the same channels, contention/interference losses may take place, resulting in a low throughput. On the other hand, if orthogonal channels selections are performed for the sensing orders, spatial opportunity may be lost.

A pioneering work in the direction of sensing order design was presented in [64]. According to the authors of this work the reward or the normalized expected throughput is the criterion to measure the efficiency of a sensing order. A multi-channel environment similar to wireless mesh networks was presented. The frame structure was similar to Figure 2.4, and the prior probabilities of channel availability were assumed known. This paper deals with the sensing order of a single user. This implies that contention/interference power among users is not a concern. The probability that the transceiver link (on the channel found primary-free) may be found to have a gain good enough to maintain a constant data rate $R$ is dependent on the fading model. It is not only necessary for the channel to be primary-free but it should also have a channel gain good enough to maintain data rate $R$. A dynamic programming solution
was proposed by the authors. This work does not study sensing orders in a multi-SU environment, hence, the challenges are completely different from the ones proposed in this dissertation.

The authors of [59] present a scenario where secondary users can sense multiple channels during a frame time. These secondary users can only acquire channels once they find them free of primary and secondary transmissions. The challenge of contention and interference among users was also addressed. Contention resolution strategies were used to resolve conflicts among the two users. The authors presented two different suboptimal approaches. The paper also brings to light the complexity requirements of these suboptimal approaches. This problem formulation differs from what is presented in this dissertation. The proposed design is based on actual or estimated average interference powers so that the impact of signal to interference noise ratio can also be incorporated into the analysis. This leads to the much desired spatial exploitation of the spectrum.

The authors of [63] also attempted to deal with the same problem but in a multiple user environment. The authors suggested the use of dynamic programming. Although, this approach can yield an optimal result for a single user scenario, it fails to reduce the complexity in a multi-user case. As a matter of fact, argue that the complexity of using dynamic programming for a multi-user problem is equivalent to evaluating all the possible sensing orders. The writers in [65] take on a different approach for sensing order selection. Instead of making hard selections, the authors suggest that preferences be set for each channel-user relationship. This preference is interpreted in terms of persistent acquisition approach. This method is useful when learning probabilities, like [56, 58] is one of the objective, and there exists a trade-off between exploration and exploitation. The collision probability of such an approach is lower bounded by the approach that uses hard selections.

The authors of [66] also address the sensing order design problem. They have also presented a suboptimal approach like [59], and illustrated the proximity of their search algorithm to the optimal approach in terms of the expected throughput achievable. The optimal results are acquired through brute-force search. The problem formulation
for this work also fails to incorporate any side information. The authors do not consider the fact that two or more SUs can have the same sensing orders if the interference losses are minimal among themselves.

The work [61] presents the idea of orthogonal selection of channels to minimize the contention among users. This work fails to reflect the impact of spatial information in the design. Although the algorithms presented have a low complexity, the focus is just reducing the likelihood of contentions among users. The authors have presented an algorithm that comes up with orthogonal channel selections that result in the least probability of conflict. The authors also study the impact of sensing imperfections and errors in channel information but they did not consider the blessing-in-disguise offered by the poor probability of detection, i.e. the unexploited transmission opportunity. The same authors have presented a persistent strategy [62] for achieving the same objective. But these results do not necessarily imply that throughput maximization coincides with conflict resolution, as demonstrated in the later chapters.

These contributions motivate us to explore the idea of spatial exploitation of the spectrum in non-cooperative secondary networks. The only cooperation that the SUs may have with one another is during the formation of the sensing order. This sensing order can also be formed in a distributed manner, by communication among the one-hop neighbors, as the throughput performance of a SU is only affected by the interfering SUs. Although, this problem can be solved by dynamic programming, but that approach is computationally expensive. Furthermore, the chapter discusses the greedy algorithm, and presents scenarios under which it may result in the optimal sensing order. The analysis also accounts for the impact of the number of SUs and the number of time slots on the reward of the sensing order.
Chapter 3

Adaptive Threshold Design

Secondary systems enable the identification and utilization of spectral opportunities. This aspect of the secondary networks paves the path for dealing with spectrum scarcity issues that choke the implementation of new wireless technologies. The idea is to access wireless resources that were considered off-limits, previously. This was because the spectrum regulatory authorities did not allow un-licensed access to these channels. These channels were considered to be solely at the disposal of licensed users, or primary users as some may refer. The prime reason for such a limitation was the fear of performance degradation in various quality of service parameters [1–12, 14, 15, 19–23]. Maximal transmission opportunity exploitation has been the corner stone of evolving secondary user (SU) technologies. Spectrum sensing is a part and parcel of SU systems [67]. The only way for these users to determine primary presence by actually implementing detection procedures.

The cognitive ability of SUs leads to a decision making stage where the SU has to chose between beginning transmissions or staying silent [68,69]. The acquisition of a channel by an SU is permitted only if the the quality of service of the primary system, is rendered minimal interference losses. Without any prior information regarding the geographic location of the primary transmitters and receivers, the SU has no way of judging the loss that the primary system would incur if the SU acquires the channel [70–72]. So, the most secure action on the SU’s part would be to relinquish
control of the channel as soon as primary transmissions are detected.

This chapter considers the SU systems that use energy detection as the means of determining whether a primary user is transmitting on a certain channel or not, are considered. Opportunistic access by SUs is based on spectrum sensing and detection of primary transmissions. An SU does this by comparing the received power against a threshold. This threshold is called the threshold of detection. The purpose this threshold is to detect primary activity with maximum possible accuracy to utilize the spatial, temporal and spectral transmission opportunities. This requires spectrum sensing to be a permanent feature of secondary systems. The availability of information regarding the primary transmitters and receivers impacts the process of opportunity utilization. Partial or complete information regarding the location of nodes can be incorporated in the design of sensing procedures to maximize the capacity of an SU.

A sensing strategy that can effectively utilize the geographic information to maximize the secondary throughput while minimizing the interference losses at the primary’s end, is studied in this chapter. Spectrum regulatory authorities require that the primary systems be sufficiently insulated from the interference caused due to secondary access. Moreover, our proposals help to effectively minimize the dependency of the secondary sensing results on sensing time. Sensing time takes up a big chunk of the operational frame of SUs, the remaining part of the frame time is used for transmissions.

Side information, in the SU system context, is of two types; 1) the primary activity probability on a channel, and 2) the average interference power received by an SU from any other SU. Authors either assume that this information is known [59,61,64], or learnt over time. The acquisition of this information is is not the scope of this work. How this information can be used, to maximize secondary and primary throughput, is the question that’s addressed throughout this chapter. It is reasonable to assume that the locations or transmit powers or average interference power can be estimated or provided by the network manager (central controller). SUs belonging to different secondary networks may have completely different operational structures, sensing
times, frame lengths etc. One SU network may treat the other SU networks activity as primary activity.

The first part of the chapter introduces the design of a detection threshold that makes use of side information. This design is followed by an extension in the scope of the problem to accommodate multiple channels. It is apparent that even if multiple channels are brought into the analysis, threshold design should be performed in the same way as presented in the first section. Further, we observe that for a system where continuous channel selection is allowed, the proposed threshold is no more the optimal option. As the idea of continuous channel selection is proposed in this chapter, it is shown that threshold design needs to be re-evaluated to account for the additional losses that such sort of selection may cause.

Continuous channel selection means that an SU can select any band of frequencies even if they overlap with more than one primary channel. The losses that occur at the primary and secondary terminals, due to sensing errors, with continuous channel selection, are taken into account while designing this new threshold. In the discrete channel selection scenario, the SU poses interference threat to a single primary user (assuming that a primary channel is occupied by a single SU). On the other hand, in a continuous channel selection scenario more than one SU may be at risk of interference from the SU.

Section 3.1 presents the idea of incorporating side information into the design of a threshold. This section also presents numerical results that compare popular existing methods of threshold design. Section 3.2 extends the work presented in the first section to accommodate continuous channel selection. Threshold design is reworked, and numerical results are provided with comparisons that show the efficacy of the methods presented. Further, it is shown that for certain systems, continuous channel selections may be a preferable option. Discrete channel selection is compared with continuous channel selection in terms of the achievable throughput of an SU, and the loss in the primary SU’s throughput.


3.1 Threshold Design with Side Information

3.1.1 Problem Motivation

An SU framework that divides its operational time into frames is studied throughout this chapter. Since, the primary transmitter’s activity may switch any time, it is necessary for the SU to schedule regular sensing time slots. For this purpose, each frame is further divided into sensing and transmission stages [1, 2]. The sensing time is an important aspect of the design. Larger the sensing duration, the lower would be the probability of error in detecting the primary presence. On the other hand, if the sensing time is reduced, the probability that detection may end up in an erroneous result increases, resulting in interference loss to the primary users.

The objective of this chapter is to maximize the secondary throughput by minimizing the sensing duration while limiting (or preferably reducing) the losses caused to the primary system to an acceptable extent. As discussed earlier, the detection process ends with a decision by the SU regarding the acquisition of the channel it sensed. This decision is based on a threshold of detection. Development of a strategy for the incorporation of (any partial or complete) geographic information into the design of the threshold for secondary throughput maximization is the main proposal studied in this work.

The references [40–42, 48] are closely related to the work presented in this section, except one major difference that completely changes the purpose of threshold design. The difference between their derivations and our work is that they considered a different test statistic for analysis. This difference results in a derivation of a detection threshold that enables us to study the impact of sensing duration on the achievable capacity of an SU. This aspect of the design not only allows throughput maximization by allowing a reduced sensing duration but also helps limit the interference at the primary receivers.

The main contribution of the research presented in this section is to create a relationship between the sensing duration and the geographic information for maximal secondary throughput. The main objective is to improve the decision making process
of an SU by incorporating side information. The underlying logic is that if the losses at the primary receiver, due to interference from the SU, are within acceptable limits, the SU should acquire the channel for its transmissions. The problem boils down to allowing the detection threshold of the SU’s energy detector to vary based on the primary user’s location information.

Section 3.1.2 presents the system model followed by the geographic information based detection threshold design in Section 3.1.3. The threshold design along with its optimality are the topic of discussion in this section. In Section 3.1.4, illustrates the significance of the proposals through numerical results. These numerical results evaluate the threshold design and study its impact on the sensing duration of SUs. Finally, the chapter concludes with a summary in Section 3.3.

3.1.2 Operational Framework

The framework for the SU threshold design is partitioned into two categories, the signal model and the system model. The signal model discusses the signal aspects like the energy in the signal, the impact of fading and noise. The system model on the other hand discusses the geographic data and its incorporation as side information in the design.

System Model

The system model depicted in Figure 3.1 illustrates the existence of primary and secondary systems operating side by side. The nodes in the systems operate in transceiver pairs. The location information is considered to be available (to the SU) a-priori, ideally provided by a third party vendor. Conventionally, primary systems create operational service areas around their base stations. If the exact location of the primary transmitters or receivers in not available, it is assumed that at least some knowledge of the service area is available to the SU.

Let \( r_p \) denote the radius of the service area around a primary transmitter. The primary receivers can be assumed to be uniformly distributed in this service area. The primary transmitter ensures at least a minimum power level at the edge of the
service area to meet the quality-of-service requirements of the system. Similarly, let the radius of the service area around a secondary transmitter be denoted by $r_s$. The secondary system also ensures a certain power level at the edge of its service area.

The loss due to interference that occurs at the primary receiver due to secondary transmission is of significance, and should be kept under check. The interference loss experienced by the secondary receivers is of no concern, as they are not the licensed users and are not entitled to the resource in the first place. The primary and secondary receivers are assumed to be uniformly distributed in the areas of radius $r_p$ and $r_s$ around their respective transmitters. Energy detection [73] is the method of detection employed by the SU to detect primary transmissions.

For the sake of consistency with practical wireless systems, a very small area of radius $\varrho$ (not depicted in Figure 3.1) is considered to be such that no secondary receiver falls in this region. For example, the density function of the uniform distribution of the secondary receiver’s location is given by

$$F_d = \frac{2d}{(r_s^2 - \varrho^2)}, \quad \varrho \leq l \leq r_s$$

$$F_{an} = \frac{1}{2\pi}, \quad 0 \leq an \leq 2\pi.$$  

where $d$ indicates the distance, and $an$ indicates the angle.

Figure 3.1: System model.
3.1. THRESHOLD DESIGN WITH SIDE INFORMATION

Signal Model

The secondary transmitter as shown in Figure 3.1 causes interference at the primary receiver, when it fails to detect the primary transmissions. The SU keeps on sensing the channel for a duration of $\mu$ secs. When this SU finds the channel free of primary transmission, it considers it primary-free and acquires it. This implies that sensing is a hypothesis testing problem. The null hypothesis can be denoted by $H_0$, and indicates the absence of primary transmissions. While the presence of these transmissions is the alternate hypothesis, denoted by $H_1$. The alternate hypothesis will be such that the SU detects primary signal plus the noise power. These hypotheses are given by

\begin{align*}
H_0 & : y(n) = e(n), \quad 0 \leq n < N \\
H_1 & : y(n) = x(n) + e(n), \quad 0 \leq n < N
\end{align*}

where $y(n)$ is the $n^{th}$ received sample, $N$ is the total number of samples, and the received samples are independent and identically distributed random variables. Also, $e(n)$ is assumed to be the $n^{th}$ sample of complex gaussian noise $e(n) \sim \text{CN}(0, \sigma_e^2)$. The primary signal, $x(n)$ is the $n^{th}$ sample of the received signal. The impact of path-loss attenuates the signal strength by the time it reaches the SU. The received signal component is distributed as $x(n) \sim \text{CN}(0, \tilde{\sigma}^2)$ where $E[|x(n)|^2] = \tilde{\sigma}^2$ and $E[|e(n)|^2] = \sigma_e^2$. So keeping this in view, the received signal to noise ratio (SNR) of the primary transmission at the secondary transmitter becomes $\gamma_{ps} = \tilde{\sigma}^2 / \sigma_e^2$.

In the rest of the chapter, the following channels and the SNRs are of interest: 1) The channel between the SU’s transmitter and the primary’s receiver denoted by $h_{sp}$; 2) The channel between the SU’s own receiver and transmitter denoted by $h_{ss}$; 3) The channel between the primary’s transmitter and SU’s receiver denoted by $h_{ps}$; and 4) The channel between the primary’s own transmitter and receiver given by $h_{pp}$.

Incomplete knowledge of the instantaneous channel gains implies that the analysis relies on their average SNR. Any channel can be represented by $h = pl \cdot \nu$, where $pl$ stands for the path-loss and $\nu$ stands for the small scale fading. The path-loss
component \( pl \) of the channel \( h \) is given by \( pl = K \cdot (1/d)^{\beta/2} \) where \( \beta \) is the path-loss exponent, \( d \) is the distance between a transmitter and a receiver, while \( K \) is a constant that depends upon the frequencies. Path-loss \( pl \) relies on the link distance, and that is what the analysis is based on. The proposed threshold design makes use of distances to enhance the SU throughput.

### 3.1.3 Adaptive Threshold Design

Errors that occur in the energy detection process result in losses. These errors are due to the imperfect nature of the energy detection procedure and the impact of noise. If an SU fails to detect primary presence, when the primary is actually transmitting on the channel, then this error is called a missed detection. This error is denoted by \( P_r(H_0|H_1) \); and this failure results in interference and loss of primary throughput. Similarly, if the SU decides that it has detected primary presence when the primary transmitter was actually idle, the error is called a false alarm. The probability of such an error is denoted by \( P_r(H_1|H_0) \). This error also results in a loss, a loss of transmission opportunity.

The SU aborts acquisition of the channel whenever it detects primary presence. This results in a loss of transmission opportunity, when the detection is due to a false alarm. The popular Bayes’ risk function [34] is given by

\[
R = \sum_u \sum_v C_{uv} P_r(H_u|H_v) P_r(H_v),
\]

(3.3)

where \( R \) is the total risk, \( C_{uv} \) is the cost of accepting hypothesis \( u \) when \( v \) was true, with probability \( P_r(H_u|H_v) \), and \( P_r(H_v) \) is the prior probability of hypothesis \( v \) occurring. In this case, we have

\[
R = w_{esp} \cdot \bar{C} \cdot P_r(H_0|H_1) \cdot P_r(H_1) \\
+ w_{ess} \cdot \bar{C} \cdot P_r(H_1|H_0) \cdot P_r(H_0),
\]

(3.4)

where \( w_{ess} \) and \( w_{esp} \) are the weights that a service provider may want to adjust for
setting the appropriate cost of $P_r(H_1|H_0)$ and $P_r(H_0|H_1)$, respectively. The cost of a false alarm, i.e. the loss of transmission opportunity, can be denoted by $\bar{C}$, while the cost of a missed detection, i.e. the loss of capacity of the primary system, can be denoted by $\tilde{C}$.

The cost of $P_r(H_u|H_u)$ will naturally be zero. It is important to note here that this risk function is directly usable in this form only when the exact location of the primary and secondary receivers are known. If these locations are not known then an expectation of the costs (of missed detection and false alarm) over the circular areas of radius $r_p$ and $r_s$, will need to be performed. The cost of missed detection can be given by the loss in the achievable primary throughput as

$$\tilde{C} = \log_2(1 + \gamma_{pp}) - \log_2(1 + \frac{\gamma_{pp}}{1 + \gamma_{sp}}),$$

(3.5)

where $\gamma_{pp}$ is the instantaneous received SNR of the primary transmission at the primary receiver, $\gamma_{sp}$ represents the interfering received SNR of the secondary transmission at the primary receiver. $\gamma_{pp}$ can be further written as $\gamma_{pp} = P_p \cdot |h_{pp}|^2/\sigma_e^2$, where $P_p$ is the transmit power of the primary transmitter. Similarly, the interference signal-to-noise ratio can be given by SNR $\gamma_{sp} = P_s \cdot |h_{sp}|^2/\sigma_e^2$ where $P_s$ is the transmit power of the secondary transmitter. In the same manner, the cost of a false alarm can be given by the loss of the achievable secondary throughput as

$$\bar{C} = \log_2(1 + \gamma_{ss}),$$

(3.6)

where $\gamma_{ss}$ is the received SNR of the secondary transmission at the secondary receiver.

Formulating the Bayesian probability ratio test [34] gives the condition of accepting hypothesis $H_1$ expressed as

$$\frac{p(x|H_1)}{p(x|H_0)} \overset{H_1}{\underset{H_0}{\gtrless}} \frac{(C_{10} - C_{00}) \cdot P_r(H_0)}{(C_{01} - C_{11}) \cdot P_r(H_1)},$$

(3.7)

where $p(x|H_1)$ is the probability density function (pdf) of having a sample $x$ under the hypothesis $H_1$ with $p(x|H_0)$ defined in a similar fashion, $C_{10}$ is the cost of accepting $H_1$
when $H_0$ was true, $C_{01}$, $C_{00}$, and $C_{11}$ can be defined in the same manner. Obviously the last two costs are 0 since there is no loss in making the right decision. This test in our case becomes

$$
\frac{\bar{C} \cdot p(y(n)|H_1) \cdot P_r(H_1)}{\bar{C} \cdot p(y(n)|H_0) \cdot P_r(H_0)} \overset{H_1}{\gtrless} 1.
$$

(3.8)

The variance of the received signal can be given by $E[|y(n)|^2] = \sigma_y^2 = \bar{\sigma}^2 + \sigma_e^2$ when the primary user is transmitting, otherwise $\sigma_e^2$ is the received power. The vector of received signal samples become $y = [y(0) \ y(1) \ldots \ y(N-1)]$. The received signal samples are assumed to be independent and identically distributed random variables and their distribution can be written denoted by $p(y|H_1)$, presented as

$$
p(y|H_1) = \frac{1}{\pi N(\bar{\sigma}^2 + \sigma_e^2)^N} \exp \left[ - \frac{1}{\bar{\sigma}^2 + \sigma_e^2} \sum_{n=0}^{N-1} |y(n)|^2 \right].
$$

(3.9)

In the same manner, we have

$$
p(y|H_0) = \frac{1}{\pi N(\sigma_e^2)^N} \exp \left[ - \frac{1}{\sigma_e^2} \sum_{n=0}^{N-1} |y(n)|^2 \right].
$$

(3.10)

Using expressions (3.9) and (3.10) to form the probability ratio test for a vector of received samples, we get

$$
\frac{\frac{1}{\pi N(\bar{\sigma}^2 + \sigma_e^2)^N} \exp \left[ - \frac{1}{\bar{\sigma}^2 + \sigma_e^2} \sum_{n=0}^{N-1} |y(n)|^2 \right] \lambda \bar{C} \overset{H_1}{\gtrless} 1,}{\frac{1}{\pi N(\sigma_e^2)^N} \exp \left[ - \frac{1}{\sigma_e^2} \sum_{n=0}^{N-1} |y(n)|^2 \right] (1 - \lambda) \bar{C} \overset{H_0}{\gtrless} 1},
$$

(3.11)

where $(1 - \theta)$ is the prior probability of primary’s presence given by $P_r(H_1)$ and $\theta$ is the prior probability of primary’s signal being absent given by $P_r(H_0)$. Rearranging the above expression and taking logarithm we get

$$
T(y) = \frac{1}{N} \sum_{n=0}^{N-1} |y(n)|^2 \overset{H_1}{\gtrless} \epsilon,
$$

(3.12)
where $T(y)$ forms the detection statistic and $\epsilon$ is the detection threshold, which can be derived to be

$$
\epsilon = \frac{\sigma_e^2}{\sigma^2} \left( \frac{\sigma_e^2 + \tilde{\sigma}^2}{\sigma^2} \right) \cdot \left[ \frac{1}{N} \ln \frac{\theta}{1-\theta} + \ln \frac{\sigma^2 + \sigma_e^2}{\sigma_e^2} + \frac{1}{N} \ln \frac{\bar{C}}{C} \right].
$$

(3.13)

Using $N = \mu \cdot \hat{f}_{sam}$, where $\hat{f}_{sam}$ and $\mu$ are the sampling frequency and sensing time, respectively. The detection threshold $\epsilon$ can be given by

$$
\epsilon = \sigma_e^2 \cdot \left( 1 + \frac{1}{\gamma_{ps}} \right) \cdot \left[ \frac{1}{\mu \cdot \hat{f}_{sam}} \cdot \ln \frac{\theta}{1-\theta} + \ln(1 + \gamma_{ps}) + \frac{1}{\mu \cdot \hat{f}_{sam}} \cdot \ln \frac{\bar{C}}{C} \right],
$$

(3.14)

where $\gamma_{ps}$ is the SNR of the primary transmission received at the secondary transmitter. The central limit theorem implies that if the number of samples $N$ become large, the detection statistic becomes gaussian distributed. Since, this work considers circularly symmetric complex gaussian noise, under the null hypothesis $H_0$, the mean and variance of the detection statistic becomes $\mu_0 = \sigma_e^2$ and $\sigma_0^2 = \frac{1}{N} \sigma_e^4$, respectively, giving a probability of false alarm, shown to be [14]

$$
P_r(H_1|H_0) = Q \left( \frac{\epsilon}{\sigma_e^2} - 1 \right) \sqrt{\frac{\mu \cdot \hat{f}_{sam}}{\gamma_{ps} + 1}}.
$$

(3.15)

Under the alternate hypothesis $H_1$, the mean and the variance of the test statistic become $\mu_1 = (1 + \gamma)\sigma_e^2$ and $\sigma_1^2 = \frac{1}{N}(1 + \gamma)^2\sigma_e^4$, respectively, making the probability of detection

$$
P_r(H_1|H_1) = Q \left( \frac{\epsilon}{\sigma_e^2} - 1 - \gamma_{ps} \right) \sqrt{\frac{\mu \cdot \hat{f}_{sam}}{2\gamma_{ps} + 1}}.
$$

(3.16)

Since, the threshold $\epsilon$ is a function of the distances between the transmission and receiving nodes, the error probabilities $P_r(H_1|H_1)$ and $P_r(H_1|H_0)$ are also a function of the distances. Such an analysis allows us to reach the conclusion that if primary receiver is closer to the secondary transmitter than the secondary receiver, the cost
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of a missed detection will be higher. To reduce this cost, the probability of missed
detection has to be reduced, which is only possible if the threshold of detection is
lowered.

On the contrary, if the primary receiver is far away as compared to the secondary
receiver, the interference loss experienced by the secondary system would be minimal,
implying that the cost of missed detection would be less than the cost of a false alarm.
As discussed earlier, that the cost of false alarm is the loss of transmission opportunity.
The solution to such a problem would be to raise the detection threshold. The above
derivation does not prove the existence of a single minimum. It may be possible
that the risk $R$ as a function of the threshold $\epsilon$ may have more than one minimum,
rendering the threshold (3.14) suboptimal. The following proposition addresses this
issue.

**Proposition 3.1.1.** The Risk $R$, as a function of the threshold $\epsilon$, has at most a single
minimum.

**Proof.** Refer to Appendix 3.4.1 for the proof of this proposition.

\[ \Box \]

3.1.4 Numerical Results

The proposed adaptive threshold is subjected to numerical evaluations in this section.
The exact location of the primary and secondary receivers is assumed unknown. The
receivers are considered to be uniformly distributed in their respective regions of
radius $r_p$ and $r_s$. For the first result, 1,500 realizations of the set $\{\theta, r_s, r_p, \mu\}$ are
generated such that $\theta \sim U[0,1]$, $r_s \sim \mathcal{N}(400m, 100m)$, $r_p \sim \mathcal{N}(1000m, 200m)$. The
received signal to noise ratio at the secondary receiver is a function of the distance
between the two nodes, and it decreases as the distance between them decreases.
The path-loss exponent is assumed to be $\beta = 2.1$, while $K = 1$. The transmit
powers of the secondary and the primary nodes are assumed to be such that they
are able to maintain at least 1 dB SNR at their respective boundaries $r_s$ and $r_p$.
Three popular methods of threshold computation are used as benchmark results:
1) the constant probability false-alarm [74], 2) [40], and 3) [48]. The distribution
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parameters mentioned above are chosen arbitrarily. Furthermore, the frame length is arbitrarily chosen to be 60ms.

Apart from the above mentioned methodologies, we have constant probability of detection method [74–76]. For this method, the detection threshold is adjusted to a value that corresponds to a certain probability of detection $P_d$ (a standard of $P_d = 0.9$ is used for the results presented). For the constant false alarm probability, the threshold is selected such that it results in a certain probability of false alarm $P_{fa}$ (for the results presented, we have $P_{fa} = 0.1$). At various inter-transmitter distance, $\epsilon$, $P_r(H_1|H_1)$, $P_r(H_1|H_0)$ and the secondary throughput are calculated. The throughput of the SU is computed as

$$C_s = (1 - \mu) \cdot \log_2 \left( 1 + \frac{\gamma_{ss}}{1 + \gamma_{ps}} \right) \cdot P_r(H_0|H_1) \cdot (1 - \theta) + (1 - \mu) \cdot \log_2 \left( 1 + \gamma_{ss} \right) \cdot (1 - P_r(H_1|H_0)) \cdot \theta,$$

(3.17)

where $\gamma_{ps}$ represents the received interfering signal to noise ratio at the secondary receiver due to the primary transmission.

Figure 3.2 is a comparison of the throughput of different methods of threshold calculation. For these results, the exact location of the primary and secondary receiver is assumed unknown. Figure 3.2 illustrates the achievable throughput of an SU using (3.17). It can be observed that as the distance among the primary and secondary networks increases, the spatial opportunity that the SU can exploit also increases. This opportunity is maximally exploited if the SU implements the detection threshold that has been proposed earlier.

Furthermore, another important result is the impact of different sensing durations. It can be observed that as the sensing duration is increased from 10ms to 20ms, the probabilities of detection increase (for the same threshold). If the threshold is kept constant, the secondary achievable throughput will decrease. Although, the expected throughput losses at the primary end would also decrease, the overall risk will become high if the threshold is not changed. The proposed scheme adjusts the threshold to allow a higher secondary throughput, while keeping the primary losses in
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Figure 3.2: Throughput of the SU.

Check (presented later). The other schemes [40] and [48] do not adjust the threshold resulting in a considerably high loss in secondary opportunity.

In Figure 3.3, the loss in overall capacity of the primary as well as the secondary system is shown. For this result, the exact location of the primary receiver along with the exact location of secondary receiver are assumed to be unknown. It is obvious that the risk $R$ for the proposed design is lowest compared to other schemes. The data points for this plot represent an expectation of losses over the 1,500 realizations of $\{\theta, r_s, r_p, \mu\}$. For the method that uses a fixed probability of detection to design the threshold, the majority of the losses are in the form of SU opportunity. The reason for this being that the primary user is over-protected by the scheme even when the distances have become very large, and the impact of interference very low. But as this scheme does not make use of side information, it is unable to detect the spatial opportunity. The same theory applies to the scheme that uses a fixed probability of false alarm. Although, the thresholds proposed by [40] and [48] can make use of side information and minimize the losses, these schemes are still not as good as the proposed method in curbing the losses.
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Figure 3.3: Loss in overall capacity of the primary and SU, as a function of the distances. The sensing duration is randomly selected such that $\mu \sim \mathcal{U}[5,20]$ms for each of the 1500 realizations of $\{\theta, r_s, r_p, \mu\}$.

Next, a comparison of the conventional threshold design (fixed probability of detection) against the side information based design, is presented. Figure 3.4 displays the average achievable throughput against the sensing time for different inter-transmitter distances. The $d$ in the legend denotes the distance between the SU receiver and the primary transmitter. It can be seen from Figure 3.4 as the sensing time increases, the corresponding transmission time decreases, resulting in a decreasing trend in the expected throughput of the SU. The throughput performance gap between the proposed scheme and the fixed probability of detection scheme is studied at different sensing times. Each data point represents an expectation over 1,500 realizations of the set $\{r_s, r_p, \theta\}$, where each parameter comes from the distributions described earlier. It is also interesting to observe that the largest difference in the performance of the proposed scheme and the fixed probability of detection scheme occurs at the maximum of the three simulated distances in Figure 3.4. As the distance decreases, the difference in their performance becomes smaller.
The impact of Incomplete Side Information on the Secondary and Primary Throughput

It should be noted that there are three different possibilities regarding the availability of side information:

1. Complete Side Information: The exact location of the primary and secondary receivers is known. This allows the calculation of the exact losses and costs.

2. Partial Side Information: The exact location of one of the receivers is unknown. In this case the cost is an expectation over the entire service area of the corresponding transmitter.

3. Unknown Side Information: Neither the primary receiver nor the secondary receiver’s location is known. The thresholds in this case will be based on costs that are basically expectations over the service areas of the corresponding transmitters.
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The previous results have demonstrated that the throughput performance of the proposed threshold design method is better than existing methods in making the best use of partial or complete side information. The next thing that we show is the degradation in performance that results from incomplete information. Since, the primary receiver is the node that experiences interference from the secondary network, the impact of its location being known or unknown plays a significant role in the computation of the primary losses.

Furthermore, as the secondary receiver (or the primary receiver) may be mobile, the impact of adaptive threshold on its throughput based on complete or partial information regarding its location, is also analyzed. Mobility is tested as an incomplete information scenario. The results presented in Figure 3.5 and Figure 3.6 are only to demonstrate the performance of the proposed threshold selection method in complete/partial information scenarios.

Figure 3.5 is a depiction of the impact of complete and partial side information on throughput. The thresholds for Figure 3.5 are designed based on the extent of information; 1) partially known (the location of one receiver is known), or 2) completely unknown (the location of no receiver is known). When the location of a node is unknown, all costs are averaged over the service area of the transmitters. The best-case performance in Figure 3.5 means that the location of the nodes is at a maximum distance from the other’s transmitter. This will result in minimum interference at the primary receiver. It can be observed that difference between the best and the worst case throughput performance is less for Figure 5.5a compared to Figure 5.5b. This is because of the information available while setting a detection threshold for the results in Figure 5.5a is more than the information available for the results in Figure 5.5b.

Figure 3.6 is an illustration of the losses that occur at the primary receiver due to the interference from the secondary transmissions. These losses are naturally higher for the case when the primary receiver is located closest to the primary transmitter (the worst case Figure 3.6b). On the other hand, these losses are minimum when the primary receiver is located farthest from the secondary transmitter (the best case Figure 3.6a). It can be observed that the losses are lower when the extent of
(a) The location of one of the receivers is known.

(b) Unknown primary and secondary receiver location.

Figure 3.5: The achievable throughput of the SU for different distances, as a function of sensing time. These plots also show the best case and the worst case throughput.

(a) Partial Information: The location of one receiver is known.

(b) Unknown Information: Unknown primary and secondary receiver location.

Figure 3.6: The losses at the primary end due to interference caused by the secondary transmissions, as a function of sensing time.
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Figure 3.7: Throughput comparison of the Adaptive threshold SU with other methods, at different sensing times. The highlight of this result is the throughput performance of the proposed threshold for a different ratio of $w_{sp}/w_{ss}$.

side information available is higher. Furthermore, the gap between the best and the worst case throughput performance is less for the partial information scenario when compared to the completely no information scenario.

Minimizing Losses to the Primary Users

The ability of the proposed scheme to control the interference at the primary’s end is demonstrated (numerically) in this section. As presented in (3.4), the weights $w_{sp}$ and $w_{ss}$ can shift the emphasis from one type of loss to another. Figure 3.7 is an illustration of this aspect of the design. The sensing time is 10ms, as in Figure 3.2. The only new line is the one that corresponds to a different ratio of the weights. It can be observed that the achievable secondary throughput drops. But at the same time, it can be observed in Figure 3.8 that there is a considerable decrease in the losses caused to the primary throughput by the secondary transmissions. This feature allows the system designer to control the interference according to the system constraints.
CHAPTER 3. Adaptive Threshold Design

3.2 Threshold Design with Side Information for Continuous Selections

The design of a detection threshold for throughput maximization was discussed in Section 3.1. The proposed problem formulation considered a single SU in the presence of a single primary transmitter. The threshold design, took into account the available geographic information in the form of path-loss, and proposed a detection threshold design that would maximize the SU’s throughput while causing minimum degradation at the primary system’s end. This effort was essentially a way to make the threshold a function of the losses (that occur due to sensing errors) incurred by the SUs and the primary user. The proposed design relied on the estimation of the throughput loss at the primary receiver, due to secondary interference. The numerical results showed that the adaptive threshold approach outperformed the existing methods of threshold design in terms of the achievable throughput.

This section considers an extension of the scope of the problem presented in Section
3.2. THRESHOLD DESIGN WITH SIDE INFORMATION FOR CONTINUOUS SELECTIONS

3.1. This extension considers a scenario where an SU is to acquire a channel from a set of available options. Multiple primary channels are available for sensing and subsequent acquisition, for the purpose of secondary transmissions. At this point, the problem may not seem very different from the previous one. The apparent solution to this problem is to use side information, specific to each channel, and then design detection thresholds for each one of them. The SU can then select a channel and use the threshold associated with that channel for sensing.

But, the problem becomes more complicated when the band sensed by an SU is selected in a continuous manner. Continuous channel selection means that the SU can select any set of frequencies even if this set overlaps with more than one primary channel. Further, this selection also implies that the SU can interfere with more than one primary user. The benefits of studying this problem are twofold: 1) To find the optimal threshold that can exploit side information in a continuous channel selection setting; 2) Find the band where the SU can achieve the highest throughput.

3.2.1 Problem Motivation

Implementing the straightforward approach of applying the same threshold design procedure to every channel in a multi-channel system would result in a total of $|M|$ detection thresholds, where $M$ is the set of primary channels. Despite the fact that utilizing prior information regarding the primary transmissions and geographic locations for the design of $|M|$ thresholds will contribute towards achieving a higher secondary throughput, it is only applicable if discrete selection of channels is implemented.

The purpose of this work is to manage the interference at the primary’s end by not only developing an adaptive threshold but also by controlling the percentage overlap between the band selected for the SU and the primary channel. It is to be shown that with the help of this feature, an SU can achieve an even higher throughput. Since, the overlap can vary from 0% to 100%, the interference losses and the throughput losses would also vary accordingly. This implies that there can be infinite possible channel selections for an SU. To understand such a channel selection, the construct
‘primary channels’ is analyzed.

Primary channels are defined by the primary activity on them. The bandwidth, as well as the center frequency of a primary channel is a primary system construct that allows allocation of discrete resources to primary users. Discrete allocation of channels to primary users gives rise to different primary channel availability probabilities (or prior primary-free probabilities). Apart from the availability probabilities, the channels also differ in terms of the primary signal to noise ratio experienced by an SU. Conventional secondary systems resort to the allocation of these channels in a discrete manner. Such channel allocation methods for the SUs imply that the bandwidth requirements of the SUs are to be the same as the primary users.

In this section, the argument that SUs do not need to sense and acquire primary channels in the discrete manner is discussed. In other words, the band of frequencies sensed by an SU maybe overlapping with more than one primary channel. This method of resource allocation is called continuous channel selection. The bandwidth sensed by an SU maybe the same as a primary channel, but because of being centered at a different frequency, may result in an overlap with two primary channels. Continuous channel selection for an SU further implies that the throughput of this SU will be a function of the prior availability probabilities of more than one primary channel. Also, the SU may cause interference at the receivers of two different primary users. Similarly, the SU’s throughput may be a function of more than one primary interferer’s activity.

The idea of continuous channel selections has been explored in works centered around wireless mesh networks and wireless lan networks ([50–54, 77–79]). To the best of our knowledge, the idea of partially overlapping channels has not been implemented in SU networks. In all the references mentioned above, the availability of the channels is assumed known while in SU networks, the channels have to be sensed before they are acquired. Furthermore, this chapter also designs an adaptive threshold for a SU that overlaps with multiple channels.

Threshold design for an SU that is to sense a channel that overlaps with more than one primary channel, involves many challenges. For example, with discrete selections,
the SU is to select a channel from a finite set of solutions. Computing the detection threshold of each channel in this set requires $|M| - 1$ more evaluations than the single channel threshold design. While for continuous channel selections, the band selected for an SU might be centered around any frequency. This implies that the possible set of channel selections for an SU, is infinite. Because of the infinite set, determining the detection threshold for each selection, beforehand is computationally impossible.

The most significant contribution, in the following sections, is the formulation a threshold that incorporates available side information to utilize the transmission opportunity, in a multi-channel continuous selection system. The next contribution is the demonstration of the performance improvement in a multi-SU scenario that results due to continuous channel selections and adaptive threshold design.

### 3.2.2 Operational Framework

As before, the framework for the SU threshold design is divided into two parts, the signal model and the system model. The signal model discusses the signal aspects like the energy in the signal, the impact of fading and noise. The system model on the other hand discusses the geographic data and its incorporation as side information in the design.

**System Model**

Consider a set of $M$ primary channels, each occupied by different primary transmitters. Primary activity is independent across channels and time frames. The SU is a transceiver pair that attempts to transmit over a channel only if it detects the channel to be primary-free with a sufficiently low probability of error. The distances between the SU and the primary transmitters determine the interference power received by one another (Figure 3.9). Formulating a problem that considers the impact of distances allows exploitation of the spatial opportunity in the system.

The system model depicted in Figure 3.9 illustrates the existence of primary and secondary systems operating side by side. The nodes in the systems operate in transceiver pairs. Further, consider the location information to be available (to the
SU) a-priori or provided by a third party vendor. Conventionally, primary systems create operational service areas around their base stations. If the exact location of the primary transmitters or receivers in not available, it is assumed that at least some knowledge of the service area is available to the SU.

As in previous sections, the radius of the service area around a primary transmitter can be given by $r_p$, where $p$ denotes the channel index. The primary receivers can assumed to be uniformly distributed in this service area. The primary transmitter ensures at least a minimum power level at the edge of the service area to meet the quality-of-service requirements of the system. Similarly, the radius of the service area around a secondary transmitter can be given by $r_s$. The secondary system also ensures a certain power level at the edge of its service area.

The loss due to interference that occurs at the primary receiver due to secondary transmission is of significance, and should be kept under check. The interference losses experienced by the secondary receivers are of no concern, as they are not the licensed users and are not entitled to the resource. The primary and secondary receivers are assumed to be uniformly distributed in the areas of radius $r_p$ and $r_s$. 

Figure 3.9: System model.
3.2. THRESHOLD DESIGN WITH SIDE INFORMATION FOR CONTINUOUS SELECTIONS

around their respective transmitters. As before, energy detection is the method of detection employed by the SU to detect primary transmissions.

Signal Model

Let the duration for which an SU performs sensing be denoted by \( \mu \) secs while the frame length by \( T \) secs. As a result of continuous channel selection, the channel sensed by an SU may overlap with more than one primary channel. The bandwidth requirement of an SU is considered to be the same as the primary channels. This implies that the channel selected for an SU can overlap with at most two primary channels.

As discussed in Section 3.1.2, sensing is a hypothesis testing problem. But unlike the previous sections, the possible hypothesis for continuous selections are \( H_0, H_1, H_2 \) and \( H_3 \). Let \( x_{p1}(n) \) represent the primary signal component of the \( n^{th} \) sample, of the signal received from the primary transmitter \( p1 \). The hypothesis are

\[
\begin{align*}
H_0 : y(n) &= e(n) \\
H_1 : y(n) &= x_{p1}(n) + e(n) \\
H_2 : y(n) &= x_{p2}(n) + e(n) \\
H_3 : y(n) &= x_{p1}(n) + x_{p2} + e(n)
\end{align*}
\]

where \( y(n) \) is the \( n^{th} \) received sample, \( N \) is the total number of samples, and the received samples are independent and identically distributed random variables. Also, \( e(n) \) is assumed to be the \( n^{th} \) sample of complex gaussian noise \( e(n) \sim CN(0, \sigma_e^2) \). The primary signal, \( x(n) \) is the \( n^{th} \) sample of the received signal. Path-loss attenuates the signal strength by the time it reaches the SU. The received signal component is distributed as \( x_i(n) \sim CN(0, \tilde{\sigma}_i^2) \) where \( E[|x_i(n)|^2] = \tilde{\sigma}_i^2 \) and \( E[|e(n)|^2] = \sigma_e^2 \). So keeping this in view the received signal to noise ratio (SNR) of the primary transmission at the secondary transmitter becomes \( \gamma_{ps} = \tilde{\sigma}_i^2/\sigma_e^2 \).

Let the primary channel \( p \) and the channel selected by an SU be denoted by their center frequency \( \bar{f}_p \) and \( f_s \), respectively. Let \( o(f_s, \bar{f}_p) \) represent the percentage overlap
between the channel selected for the SU and the primary channel \( p \), defined as

\[
o(\bar{f}_p, f_s) = \min\{(k - |\bar{f}_p - f_s|)/k, 1\}
\] (3.22)

where \( k \) is the bandwidth requirement of the primary and secondary users.

### 3.2.3 Threshold design

It has been discussed in Section 3.1.3 that adjusting the detection threshold of SUs based on the interference cost and the opportunity cost is a throughput enhancing approach. Furthermore, the directions for threshold selection for a single SU has been laid out in Section 3.1.3. Threshold design for a SU in the presence of continuous selections becomes a more challenging task. The reason being that the SU may now interfere with more than one primary user. Moreover, the availability probabilities of the channels may also be different.

### Channels and Path-loss Model

As before, the following channels are considered for the analysis: 1) The interference channel between the SU’s transmitter and the primary receiver (of some channel \( i \)) denoted by \( h_{si} \); 2) The interference channel between the primary transmitter and the secondary receiver \( h_{is} \); 3) The channel between secondary transmitter and secondary receiver \( h_{ss} \); 4) The channel between the primary transmitter and primary receiver \( h_{ii} \). The SNRs on these channels can be given by \( \gamma_{si}, \gamma_{is}, \gamma_{ss} \) and \( \gamma_{ii} \). Complete knowledge of all these instantaneous channel states may not be available at the secondary transmitter. Any channel can be represented by \( h = pl \cdot \nu \), where \( pl \) stands for the path-loss and \( \nu \) stands for the small scale fading. The path-loss component \( pl \) of the channel \( h \) is given by \( pl = K \cdot (1/d)^{\beta/2} \) where \( \beta \) is the path-loss exponent, \( d \) is the distance between the respective terminals, \( K \) is a constant dependent upon the frequencies. \( pl \) is dependent upon the knowledge of the link distance and that is what the analysis is based on. Using knowledge of these distances, it is shown in this section that the achievable throughput of a secondary system can be enhanced.
3.2. THRESHOLD DESIGN WITH SIDE INFORMATION FOR CONTINUOUS SELECTIONS

Cost Functions

The costs of sensing errors determines the Bayes’ risk. The cost formulation presented in this section is in its more general form, while the costs presented in Section 3.1.3 are based on specific system requirements. Every decision has a cost, whether it be a loss in the primary or secondary throughput. The weights associated with these costs can be adjusted by the system designer. The probability of erroneous detection can be given by $P_r(H_u|H_v)$, where $(u \neq v)$ and $(u,v \in \{0,1,2,3\})$. Let $C_{u,v}$ be the cost of detecting hypothesis $H_u$ when hypothesis $H_v$ is true. Then the risk function [34] can be given by

$$R = \sum_{u=0}^{3} \sum_{v=0}^{3} C_{u,v} P_r(H_u|H_v) P_r(H_v),$$  \hspace{1cm} (3.23)

where $R$ is the total risk. The costs $C_{u,v}$ where $(u,v \in \{1,2,3\})$ and $u \neq v$, do not result in a loss as the SU will stop transmissions in either case and no interference loss will occur. In other words, detecting two primary transmitters instead of one, or the other way around will not result in a loss as it will be considered a successful detection in a wireless sensor scenario. So, such costs can be given by

$$C_{u,v} = 0, \hspace{0.5cm} u,v \in \{1,2,3\}, u \neq v.$$  \hspace{1cm} (3.24)

Similarly, the costs $C_{u,v}$ where $(u = v)$ is also not a loss, and is considered as successful detection. For such events, we have

$$C_{u,v} = 0, \hspace{0.5cm} u,v \in \{0,1,2,3\}, u = v.$$  \hspace{1cm} (3.25)

Next, the costs that can be described as $C_{u,v}$ are considered, where $u \in \{1,2,3\}$, while $v = 0$. All these costs are the same and can also be termed as the cost of a false alarm. They will result in the loss of transmission opportunity which can be given by

$$C_{u,v} = I_{ss}, \hspace{0.5cm} u \in \{1,2,3\}, v = 0,$$  \hspace{1cm} (3.26)
where \( I_{ss} = \log_2(1 + \gamma_{ss}) \). The costs \( C_{u,v} \), where \( u = 0 \), while \( v \in \{1, 2, 3\} \) are the interference losses that will result due to the missed detection. These losses cause a reduction in the primary capacity and are given by

\[
C_{0,1} = I_{s1} = \log_2(1 + \gamma_{11}) - \log_2(1 + \frac{\gamma_{11}}{1 + o(f, f_1)\gamma_{s1}}), \tag{3.27}
\]

\[
C_{0,2} = I_{s2} = \log_2(1 + \gamma_{22}) - \log_2(1 + \frac{\gamma_{22}}{1 + o(f, f_2)\gamma_{s2}}), \tag{3.28}
\]

\[
C_{0,3} = I_{s1} + I_{s2}. \tag{3.29}
\]

The costs \( C_{0,1} \) and \( C_{0,2} \) are a function of a fraction of the interference power from the SU. This implies that the loss due to missed detection of primary transmissions (by an SU) will have a lesser impact on each primary user, as compared to the discrete selection case.

**Risk Functions and Threshold Design**

Risk is defined by the mapping of the received signal into a decision (on the presence or absence of primary transmissions). Let us define regions of samples \( y(n) \) that map into certain decisions [34] i.e., \( re_i = \{y(n); decide H_i\} \) such that \( \cap_{i=0}^{3} re_i = \emptyset \). The prior probabilities of each hypothesis can be given by

\[
P_r(H_0) = \theta_1 \theta_2, \quad P_r(H_1) = \theta_1 (1 - \theta_2), \quad P_r(H_2) = (1 - \theta_1) \theta_2 \quad \text{and} \quad P_r(H_3) = (1 - \theta_1)(1 - \theta_2). \]

Using the cost values, the risk function becomes

\[
R = C_{0,1} P_r(H_1) \int_{re_0} p(y(n)|H_1) dy(n) + C_{0,2} P_r(H_2) \int_{re_0} p(y(n)|H_2) dy(n) +
\]

\[
C_{0,3} P_r(H_3) \int_{re_0} p(y(n)|H_3) dy(n) + C_{1,0} P_r(H_0) \int_{re_1} p(y(n)|H_0) dy(n) +
\]

\[
C_{2,0} P_r(H_0) \int_{re_2} p(y(n)|H_0) dy(n) + C_{3,0} P_r(H_0) \int_{re_3} p(y(n)|H_0) dy(n). \tag{3.30}
\]
As per the definition of $re_i$, we have

$$1 - \int_{re_0} p(y(n)|H_0)dy(n) = \int_{r\epsilon} p(y(n)|H_0)dy(n) = \int_{re_1} p(y(n)|H_0)dy(n) +$$
$$\int_{re_2} p(y(n)|H_0)dy(n) + \int_{re_3} p(y(n)|H_0)dy(n), \quad (3.31)$$

where $r\epsilon = re_1 \cup re_2 \cup re_3$. Using the above expression and the weights, we have

$$R = C_{0,1}P_r(H_1)\int_{re_0} p(y(n)|H_1)dy(n) + C_{0,2}P_r(H_2)\int_{re_0} p(y(n)|H_2)dy(n) +$$
$$C_{0,3}P_r(H_3)\int_{re_0} p(y(n)|H_3)dy(n) + C_{i,0}P_r(H_0)\int_{r\epsilon} p(y(n)|H_0)dy(n) \quad (3.32)$$

$$= C_{0,1}P_r(H_1) + C_{0,2}P_r(H_2) + C_{0,3}P_r(H_3) +$$
$$\int_{r\epsilon} \left[ C_{i,0}P_r(H_0)p(y(n)|H_0) - C_{0,1}P_r(H_1)p(y(n)|H_1)$$
$$- C_{0,2}P_r(H_2)p(y(n)|H_2) - C_{0,3}P_r(H_3)p(y(n)|H_3) \right]dy(n). \quad (3.33)$$

In (3.33), the SU decides that the primary user is transmitting if the integrand is negative. Otherwise, the SU decides the band to be primary-free. For threshold design, we have

$$C_{i,0}P_r(H_0)p(y(n)|H_0) - C_{0,1}P_r(H_1)p(y(n)|H_1)$$
$$- C_{0,2}P_r(H_2)p(y(n)|H_2) - C_{0,3}P_r(H_3)p(y(n)|H_3) H_0 > 0. \quad (3.34)$$

Minimizing $R$ is equivalent to minimizing the integrand. Unlike (3.14), because of the many distribution functions involved in (3.33), a closed form solution for the threshold is not possible. An optimal detection threshold is one that can minimize the risk (3.33).

**Proposition 3.2.1.** If the detection threshold $\epsilon = 0$ does not minimize the Risk $R$,
then there will exist only one value of $\epsilon > 0$ for which the following equality holds.

\[
\nabla_\epsilon \int_{\mathbb{R}^2} \left( C_{i,0} p_r(H_0) p(y(n)|H_0) - C_{0,1} p_r(H_1) p(y(n)|H_1) 
- C_{0,2} p_r(H_2) p(y(n)|H_2) - C_{0,3} p_r(H_3) p(y(n)|H_3) \right) dy(n) = 0 \quad (3.35)
\]

Proof. We refer the reader to Appendix 3.4.1 for the proof of this proposition. 

### 3.2.4 A Case for Continuous Channel Selection

The threshold $\epsilon$ formed by the procedure described in Section 3.1.3 strikes a balance between the interference losses at the primary and the secondary throughput losses. If the channel selected for an SU is such that the interference losses caused by transmission (in the event of missed detection), at the primary receiver are negligible, then the threshold (3.14) will be high. On the other hand, if the interference losses are high, the threshold designed will be lower. Naturally, higher the threshold, higher will be the throughput of the SU. This section studies the impact of continuous selections on the SU’s throughput. Furthermore, scenarios where continuous selections outperform discrete selections in a Multi-SU network are explored.

**Continuous Channel Selection in a Single-SU system.**

Consider a channel selected for an SU with center frequency $f_s$, where $f_s \neq \bar{f}_i$ ($\forall i \in M$), while $\bar{f}_1 < f_s < \bar{f}_{[U]}$. If the primary and secondary throughputs are maximized at this selection, then clearly discrete selections is a suboptimal option. Let us consider an example for the sake of demonstration. Let Table 3.1 be the detection parameters of the primary and secondary users. The throughput threshold is the minimum required throughput by the primary system, failing to achieve this, the transmissions would seize.

Figure 3.10 shows the sum throughput of the primary users and the SU. For each data point in the Figure, 1,000 instances of $\bar{\theta} = \{\theta_1, \theta_3, \theta_3\}$ are randomly generated such that for each $i \in \{1, 2, 3\}$, $i \theta_i \sim \mathcal{U}[0, 1]$. The threshold is computed for each
instance using the adaptive threshold method and the fixed probability of false alarm
method. An expectation of the sum throughput of the SU and the primary system is
then plotted in Figure 3.10. It is obvious that the maximum throughput is achievable
with continuous selections. An explanation of the results presented in Figure 3.10 are

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{fsam}$</td>
<td>10.</td>
</tr>
<tr>
<td>$\theta_1, \theta_2, \theta_3$</td>
<td>generated randomly</td>
</tr>
<tr>
<td>$k$</td>
<td>10MHz.</td>
</tr>
<tr>
<td>$f_{p1}$</td>
<td>26MHz.</td>
</tr>
<tr>
<td>$f_{p2}$</td>
<td>36MHz.</td>
</tr>
<tr>
<td>$f_{p3}$</td>
<td>46MHz.</td>
</tr>
<tr>
<td>Throughput Threshold for the Primary users</td>
<td>0.285bps.</td>
</tr>
<tr>
<td>Detection Threshold for the fixed threshold case.</td>
<td>$\epsilon = 1.4052$, s.t. $P_r(H_1</td>
</tr>
<tr>
<td>Detection Threshold for the adaptive threshold case.</td>
<td>$\epsilon$, s.t. (3.34) is minimized.</td>
</tr>
<tr>
<td>$\gamma_{11}, \gamma_{ss}, \gamma_{22}$</td>
<td>0db.</td>
</tr>
<tr>
<td>$\gamma_{is}, \gamma_{si}$, $i \in {1, 2}$</td>
<td>$0.1\gamma_{ss}$.</td>
</tr>
</tbody>
</table>

Table 3.1: Channel Detection Parameters.

discussed, next. If initially channel $p_1$ is selected for the SU, then the SU is supposed
to transmit only when the channel is primary-free. But channel-detection errors
result in the SU interfering with the primary transmissions. The SU’s interference
power, received at the primary, may push the primary’s throughput below the primary
system’s required throughput threshold. For some primary systems, such a loss in
throughput may cause a complete switching-off of the primary transmissions.

On the other hand, if the channel selected for this SU has an $f_s$ such that $0 <
(k - |\bar{f}_{p1} - f_s|)/k < 1)$, then the interference power received by the primary, from
the secondary transmitter, will be less than what it was when $\bar{f} = f_s$. Such a channel
selection for the SU will result in lowering the throughput losses (due to interference)
at the primary transmitter $p_1$, and may bring the primary’s throughput within the
acceptable range. At the same time, the new channel selected for the SU will result
in a higher overlap with channel $p_2$, and cause a loss in its throughput.
The reduction in losses due to interference can also be observed in the systems that use a fixed-threshold, designed to maintain a fixed probability of false alarm. But the location information based threshold design for continuous selections presented in this chapter takes advantage of the knowledge of the received interference powers, and will always find a threshold that results in a higher primary and secondary throughput.

Continuous Channel Selection in a Multi-SU system.

The discussion presented in this section extends the case of continuous channel selections to a multi-SU scenario. Consider two primary channels, centered at $\bar{f}_{p1} = 800$ MHz and $\bar{f}_{p2} = 801$ MHz. The prior availability probabilities of these channels are $\theta_{p1}$ and $\theta_{p2}$, respectively. Let’s assume that the proposed scheme selects channels of 1 MHz centered around $f_{s1} = 800$ MHz and $f_{s2} = 800.5$ MHz for two SUs. This means that there exists a 50% overlap among the bands selected for both SUs. Assuming a very high $\gamma_{is}$ ($i \in \{p1, p2\}$), when the primary transmitter is present (with probability $(1 - \theta_i)$), the probability of detection of primary transmissions, given by
3.2. THRESHOLD DESIGN WITH SIDE INFORMATION FOR CONTINUOUS SELECTIONS

\( \hat{P} \), approaches 0. Also, the primary power received by the second SU, when the 801 MHz channel is busy (while the 800 MHz channel is free), is half of what it would have been if the second SU were selected in a discrete manner such that \( f_{s2} = 801 \) MHz. This leads to the notion that the probability of detection of a transmission opportunity (by the second SU), when the 801 MHz channel is busy, is greater than 0.

Let \( \overline{\phi}_1 \) be the probability that both the SUs acquire their bands when both the channels are primary-free. Similarly, let \( \overline{\phi}_2 \) be the probability that any one of the SU acquires its band, while the other fails. When both the SUs acquire their bands, the rate at which an SU will transmit (\( R \)) will be scaled by a factor \( \bar{\iota}_1 \zeta \), where \( \zeta \) is the probability of maintaining rate \( R \). The factor \( \bar{\iota}_1 \), where \( 0 < \bar{\iota}_1 < 1 \), depicts the impact of interference from the other SU, reducing the probability of maintaining rate \( R \). In the absence of an interferer, the expected rate of an SU is \( \bar{\iota}_2 \zeta R \). Naturally, \( \bar{\iota}_2 > \bar{\iota}_1 \), as the probability of maintaining rate \( R \) is higher in the absence of interference.

Let \( \bar{\phi}_1 \) be the probability that both the SUs acquire their bands when channel \( p_1 \) is primary-free while channel \( p_2 \) is busy. Similarly, let \( \bar{\phi}_2 \) be the probability that any one of the SUs acquires its band while the other fails, given the same primary activity. For such a scenario, when both the SUs acquire their bands the maximum probability that either of them would be able to maintain \( R \) is \( \bar{\iota}_1 \zeta \). Let \( \bar{\iota}_2 \zeta \) be the probability that a SU is able to maintain rate \( R \) when it acquires its band while the other SU fails. Also, \( \bar{\iota}_2 > \bar{\iota}_1 \), owing to the interference power received from one another.

The terms \( \bar{\phi}_1, \bar{\phi}_2, \bar{\phi}_1, \bar{\phi}_2, \bar{\gamma}_1, \bar{\iota}_2, \bar{\iota}_1 \) and \( \bar{\iota}_2 \) are functions of the overlap among the bands selected for the two SUs. Higher the overlap among the SUs, higher will be the interference power received from one another, and higher will be the probability that they detect contention from one another. The expected throughput can be given by

\[
\theta_1 \zeta R \left( (2\bar{\phi}_1 \bar{\iota}_1 + 2\bar{\phi}_2 \bar{\iota}_2)\theta_2 + (2\bar{\phi}_1 \bar{\iota}_1 + 2\bar{\phi}_2 \bar{\iota}_2)(1 - \theta_2) \right) + (1 - \theta_1)\theta_2 \zeta R \hat{P}.
\]

It is obvious that the expected throughput in the continuous selection case would equal \( (2\phi_1 \bar{\iota} + 2\phi_2)\theta_1 \zeta R \) if the overlap among the bands reaches 1 and approaches \( (\theta_1 + \theta_2)\hat{P} \zeta R \) when the overlap among the bands reaches 0. Selecting the bands 800
MHz and 800.5 MHz is an optimal decision if the following condition holds

\[
\frac{\hat{P} - (1 - \theta_1)\hat{P} - \theta_1(2\bar{\phi}_2\bar{i}_1 + 2\bar{i}_2)}{(1 - \theta_2)(2\bar{i}_1\bar{\phi}_1 + 2\bar{i}_2)} < \frac{\theta_1}{\theta_2} < \frac{(1 - \theta_1)\hat{P} + \theta_1(2\bar{\phi}_1\bar{i}_1 + 2\bar{\phi}_2)}{2(\bar{\phi}_1\bar{i}_1 + 2\bar{\phi}_2) - (1 - \theta_2)(2\bar{\phi}_1\bar{i}_1 + 2\bar{\phi}_2)},
\]

(3.37)

where the derivation of the above is straightforward. This example shows that channel selections based on interference and contention loss analysis can further be improved by implementing continuous channel selections.

### 3.2.5 Numerical Results

The effectiveness of adaptive threshold design based on side information, compared to the conventional methods of threshold design, has been demonstrated in Section 3.1.4. The proposals in Section 3.2 consider continuous channel selections with an adaptive threshold based on side information. Numerical results that demonstrate the efficiency of the continuous channel selections in a single SU and multi-SU settings are presented in this section. The results presented in this section are compared with conventional methods of threshold design and channel selection. The exact location of the primary and secondary receivers is assumed to be unknown.

The first result, presented in Figure 3.11, compares adaptive threshold with a threshold designed by the conventional method (for a fixed probability of false alarm), in the continuous channel selection setting. The x-axis in Figure 3.11 represents the center frequency of the channel selected for the SU. For the numerical results, four primary channels with center frequencies \(\bar{f}_{p_1} = 16\text{MHz}, \bar{f}_{p_2} = 26\text{MHz}, \bar{f}_{p_3} = 36\text{MHz}\) and \(\bar{f}_{p_4} = 46\text{MHz}\), are considered. The bandwidth requirement of an SU and the bandwidth of each primary channel is assumed to be 1MHz. The average SNRs are assumed to be such that \(\gamma_{ss} = \gamma_{11} = \gamma_{22} = \gamma_{33} = \gamma_{44} = 0\text{db}\). Further, if \(d_{si} = 1500m\) is the distance between the SU and primary transmitter \(i\), then the interference power received by the SU from this primary transmitter is \(0.1\gamma_{ss}\).

Let the set of primary availability probabilities of these primary channels be given
by \( \bar{\theta} = \{\theta_{p1}, \theta_{p4}, \theta_{p3}, \theta_{p4}\} \). Furthermore, let \( \bar{d} = \{d_{s1}, d_{s2}, d_{s3}, d_{s4}\} \) be the set of distances between the SU and the primary transmitters. For the results in Figure 3.11, 1,000 instances of the set \( \{\bar{d}, \bar{\theta}\} \) are generated such that \( d_{si} \ (i \in \{1, 2, 3, 4\}) \) is randomly generated such that \( d_{s4} \sim \mathcal{N}(400, 100) \), \( d_{s1} \sim \mathcal{N}(800, 100) \), \( d_{s2} \sim \mathcal{N}(1000, 100) \), \( d_{s3} \sim \mathcal{N}(1500, 100) \) and \( \theta_{i} \sim \mathcal{U}(0, 1) \) for all \( i \in \{1, 2, 3, 4\} \). Given the topology and the

instance of primary activity, a detection threshold and the resulting SU throughput (as in (3.17)) is calculated, for each channel selection (corresponding to the center frequency along the x-axis). An expectation is performed over each \( \{\bar{\theta}, \bar{d}\} \), and plotted. Figure 3.11 shows that not only does the adaptive threshold outperform the fixed threshold, but also that the peak in SU’s throughput occurs when the channel selected for the SU has an overlap with more than one primary channel.

Figure 3.12 is a depiction of the primary throughput under the effect of interference from the SU, when different thresholds are used in a continuous channel selection setting. As before, the x-axis represents the center frequency of the channel selected for the SU. Since, primary users do not interfere with one another (due to separate channels), any variation in the average primary sum throughput results from secondary activity. In the absence of interference power from the SU, each primary throughput

Figure 3.11: The SU throughput with Continuous channel selections.
user’s throughput is assumed to be \( \theta_{pi} \log_2(1 + \gamma_{pp}) \), where \( \gamma_{pp} = 0 \) db (the signal to noise ratio between the primary transmitter and receiver). Figure 3.12 shows that

![Graph showing primary throughput with continuous channel selections.]

the primary throughput is maximized when the channel selection performed for SU is done in a continuous manner. Each data point in the figure represents an expectation of the primary sum throughput over all 1,000 instances of the set \( \{\bar{\theta}, d\} \). Figure 3.12 basically shows the impact of the interference caused by the SU, at the primary end. The figure also shows that the primary throughput is maximized when the channel selected for the SU is such that it overlaps with more than one primary channel. This result is different from the one shown in Figure 3.10 as no throughput threshold is considered for primary transmissions. The gain of continuous selections becomes more pronounced for systems that have a throughput threshold for primary transmissions.

Figure 3.13 presents the thresholds in discrete and continuous channel selections for the adaptive design. The stem plots represent the detection threshold if channels were selected for the SU in a discrete manner i.e., \( f_s \in \{16, 26, 36, 46\} \) MHz. A higher threshold implies that the SU will achieve a higher throughput at these channel selections. The data points in Figure 3.13 represent the average thresholds over the
3.2. THRESHOLD DESIGN WITH SIDE INFORMATION FOR CONTINUOUS SELECTIONS

1,000 different primary availability and topology scenarios, randomly generated.

Figure 3.13: Thresholds in Discrete and Continuous Selections.

Figure 3.14: The Sum throughput of the SUs if the channel selected for SU 1 is given by the x-axis, while the channel selected for SU 2 is 46MHz.

Figure 3.14 presents the impact of continuous channel selection on the SU throughput in a multi-SU setting. For this result, the channel selected for SU 1 is fixed to
be $f_{s1} = 46\text{Mhz}$. While, the x-axis represents the center frequency of the channel selected for SU 2. Let $\hat{d}$ be the distance between the two SUs. As assumed for the primary, if $\hat{d} = 1500\text{m}$, then the interference power received by an SU from another is $0.1\gamma_{ss}$. For this result, 1,000 instances of the set $\{\tilde{d}, \tilde{\theta}\}$ are generated (from the same distributions as described earlier), for three different inter-SU distance $\hat{d}$. The SU-primary distances in the set $\tilde{d}$ refer to the distances between SU 2 and the primary transmitters. For the sake of simplicity, for all instances of the set $\{\tilde{d}, \tilde{\theta}\}$, the distances between the SU 1 and the primary transmitters are assumed to be fixed at 900m.

When the number of SUs that try to acquire overlapping bands, is more than 1, they interfere with one another. This interference will result in a reduction in the SU’s ability to maintain a constant data rate $R$. The SU system considered for the numerical results is such that the SUs stop transmissions if the achievable rate falls below $R$. Furthermore, to test continuous selections, it has been assumed that the SUs do not communicate with one another. Figure 3.14 shows that continuous selections result in a higher secondary network throughput especially if the channels acquired by the SUs overlap.

Figure 3.14 further highlights the fact that if the distances between the SUs are large enough to result in negligible interference, then selecting the same channel for both the SUs may result in a higher reward (compared to selecting different channels for the SUs). This may happen because of the low interference power among SUs and the high availability of the channel.

Figure 3.15 shows the performance difference between discrete and continuous channel selections for different channel availability scenarios. The set of distances can be given by $\tilde{d} = \{d_{sp}\}_{s\in\{1,2\}, p\in\{1,2,3,4\}}$. For this result, 1,000 instances of the set $\{\tilde{\theta}, \tilde{d}\}$ were randomly generated, such that each $\theta_p \in \tilde{\theta}$ comes from a uniform distribution. The number of primary channels are 4, and the uniform distribution of each $\theta_p$ is shown in the figure. The distances between the primary transmitter and the SUs are generated from a gaussian distribution with mean 1000m. and variance 400.
3.2. THRESHOLD DESIGN WITH SIDE INFORMATION FOR CONTINUOUS SELECTIONS

Figure 3.15: The SU sum throughput for different interference levels.

Two very important conclusions that can be drawn from Figure 3.15 are related to the distribution of opportunity and the inter-SU interference. Figure 3.15 shows that if the transmission opportunity in the spectrum is concentrated in a few channels, continuous channel selection is the optimal way of exploiting it. This is because all possible discrete channel selections are just a subset of the possible continuous channel selections. Furthermore, as the inter-SU distance increases, the interference power decreases, and the sum throughput increases.

Figure 3.16 presents the impact of continuous and discrete channel selections on a multi-SUs system. For this result, 4 primary channels were considered, for each $p$, $\theta_p \sim U[0, 1]$ for all $p \in \{1, 2, 3, 4\}$. The distances $\hat{d} = \{d_{sp}\}_{s \in U, p \in \{1,2,3,4\}}$ were assumed to be such that $d_{sp} \sim \mathcal{N}(1000, 400)$. For this result, let $\check{d}_{u,v}$ denote the distance between SUs $u$ and $v$. Let $\bar{d}$ be the mean of the set $\{\check{d}_{u,v}\}_{u,v \in U, u \neq v}$. Each data point in Figure 3.16 represents an expectation over 1,000 randomly generated instances of the set $\{\check{\theta}, \check{d}\}$.

Brute force approach is used to find the channel selections for discrete and continuous methods. The channel selections for the discrete method are optimal because the brute force approach has to evaluate a finite amount of combinations. But the results for the continuous channel selection method are suboptimal.
CHAPTER 3. Adaptive Threshold Design

3.3 Summary

The main objective of the contributions in this chapter is SU throughput maximization and reduction of interference at the primary network (caused due to SU transmissions). These contributions are centered around the idea of incorporating any information regarding the geographic layout of the transceivers. The study begins with the design of a detection threshold, for an SU, that relies on energy detection to determine the presence of primary transmissions. The proposals presented in this chapter advocate the idea of making use of location side information in the design of the threshold. The impact of the geographic layout of primary and secondary systems plays a significant role when it comes to computing losses. The losses are divided into two types: 1) the interference losses that cost primary throughput; and 2) the transmission opportunity that the SU fails to avail. The detection threshold is formulated such that it becomes a function of these losses.

The chapter then explores the efficacy of the adaptive threshold design in a multi-channel scenario. The idea of continuous channel selections followed by the adaptive
threshold design with such selections is then presented. It is shown that the losses incurred by the primary and SU networks in the continuous channel setting are different from the losses in the discrete channel selection scenario. A redesign of the detection threshold for continuous selections is then performed. This derivation results in the threshold design problem becoming an optimization problem. A discussion on the solution of the optimization problem to find the optimal threshold is also presented.

Having discussed threshold design for continuous channel selections, the focus of the chapter turns towards a multi-SU setting. A multi-SU problem introduces further challenges like inter-SU contention and interference. The scope of this chapter was not to find algorithms for optimal channel selections but to show that continuous channel selection method with adaptive thresholds is beneficial even in the multi-SU context. The numerical results presented test the proposals under a range of network conditions. SU and primary throughput performance has been studied under different channel availability distributions. Further, the proposals were tested for randomly generated geographic layout of transmitters and inter-SU interference. The proposed methods of SU system design outperform the existing methods.

3.4 Appendix

3.4.1 Proof of Proposition 3.1.1 and 3.2.1

Discrete Selection Case

The costs $C_{i,j}$ along with the probabilities $P_r(H_j)$ are independent of the threshold $\varepsilon$. Further, the probability of false alarm decreases, while the probability of missed detection increases as $\varepsilon$ increases. If the power received from the primary transmitter is 0, then the probability of detection $P_r(H_1|H_1)$ (3.16) and the probability of false alarm $P_r(H_1|H_0)$ (3.15) are equal. Since, $1 - P_r(H_1|H_1) = P_r(H_0|H_1)$, these tail probabilities are such that the following relationship holds

$$\nabla_\varepsilon (P_r(H_0|H_1) + P_r(H_1|H_0)) = 0, \quad \gamma_{ps} = 0.$$

(3.38)
The probability of missed detection \( (P_r(H_0|H_1)) \) increases with an increase of the detection threshold. The sum of the two tail probabilities, such that one is monotonically increasing, while the other monotonically decreasing, has been studied by the authors in [80] and [81]. Let \( IN_1 \) be the inflection point of the decreasing tail probability, and \( IN_2 \) be the inflection point of the increasing tail probability. Then, the sum of these two functions will have two maxima if the point of intersection (denoted by \( INT \)) between the two curves is such that

\[
IN_1 < INT < IN_2. 
\]  

(3.39)

Figure 3.17 is an illustration of the sum that does not result in two maxima. The two curves in Figure 3.17 intersect at the inflection points. On the other hand, Figure 3.18 is an illustration of the sum that does result in two maxima. The sum \( ((1 - P_r(H_1|H_1)) + P_r(H_1|H_0)) \) will initially decrease, then increase, as \( \varepsilon \) increases, resulting in a single minimum point for the Risk function.
3.4. APPENDIX

Continuous Selection Case

For continuous channel selections, \( \int_{\mathcal{F}} p(y(n)|H_j)dy(n) \) (for \( j \in \{1, 2, 3\} \)) represents the probabilities of missed detection. Each term \( (1 - \int_{\mathcal{F}} p(y(n)|H_j)dy(n)) \) (for \( j \in \{1, 2, 3\} \)) will equal the probability of false alarm \( \int_{\mathcal{F}} p(y(n)|H_j)dy(n) \) (\( j = 0 \)), for \( \gamma_{ps} = 0 \). The cumulative probability of interference losses (due to missed detection) is higher compared to the probability of SU’s transmission losses (due to false alarm), for the continuous selection case. The Risk function may even have its minimum for a lower threshold value \( \varepsilon \). The lowest value of the Risk may occur at \( \varepsilon = 0 \). It has been shown in the previous section that the sum of probabilities \( P_r(H_0|H_1) \) and \( P_r(H_1|H_0) \) can have a single minimum. For the continuous case, the only difference would be the addition of more monotonically decreasing functions (of the threshold). It can be shown that such an addition will make the not affect the fact that the Risk has a single minimum as a function of \( \varepsilon \). For such a situation, as \( \varepsilon \) increases, the interference loss probabilities increase while the throughput loss probability decreases, ruling out the possibility of another R minimum. This completes the proof.
Chapter 4

Spectrum Access with Cooperative Threshold Design

The previous chapter presented adaptive threshold design for a single SU in a discrete channel selection setting, continuous channel setting and multi-SU scenarios. The purpose was to incorporate side information and maximally utilize spectral opportunities that lie in the spatial domain. The proposed methods did not address a very important issue that will be extensively discussed along with the solution, in this chapter. This issue is contention and interference management in a multi-SU system. The proposals of Section 3.2 touch on this issue by proposing channel selections that minimize contention and interference among SUs.

In the previous chapter, all SUs make use of side information to select a threshold. This threshold is selected to minimize the total risk. The total risk is the expected cost of sensing errors. These sensing errors disrupt the operation of the primary user (in the form of interference) and the SU’s operation (in the form of lost transmission opportunity). The first important aspect that the previously proposed design misses out on is the contention and interference that the SUs may face from one another. The second important aspect is the collective interference that the primary user may face from all the SUs.

Adaptive threshold design, discussed in the previous chapter, selects a threshold
that strikes a balance between the losses incurred by the primary user (as a result of this SU’s performance) and the losses incurred by the SU (lost opportunity). This design does not discuss the impact of multiple SUs trying to acquire the same channel. The threshold designed in Section 3.2 does consider multiple primary users in the analysis, but not multiple SUs.

The interference losses that degrade the primary user’s performance were associated with the energy detection process of a single SU, in the previous chapter. This chapter considers threshold design for multiple SUs. If all SUs were to design a detection threshold without any cooperation among one another, then the objection function would be similar to the ones presented in the previous chapter. But, as shown later in the numerical results, such a non-cooperative operation leads to very high losses.

Another aspect that is presented in this chapter is that of network topology. The inter-SUs distances would be different in practical wireless system, leading to different levels of interference from one another. Similarly, the distances between the SUs and primary users will also differ. This implies that the spatial opportunity in the channel would vary for different SUs. The numerical results presented later in this chapter evaluate the proposals over a wide variety of network topologies and interference scenarios.

Section 4.1 presents the motivation and relevant literature for the work. Section 4.2 discusses the operational framework which includes the system and the signal model. Section 4.3 formulates the threshold design problem and Section 4.4 presents readily solvable approximations. Section 4.5 presents the numerical evaluations that bring to light the significance of this work in comparison to famous benchmark results. Section 4.6 summarizes the contributions of this chapter.

4.1 Problem Motivation

As described earlier, cooperation among SUs has the following two advantages: 1) improved sensing performance; and 2) improved spectrum access. This chapter also
4.1. PROBLEM MOTIVATION

presents a discussion on how conventional cooperative sensing and exploiting spatial opportunity for maximizing SU throughput might be conflicting objectives. Some works on cooperative sensing are discussed in this section to understand the contribution of cooperation in threshold design.

Cooperative sensing was discussed in [82] to enhance the secondary and primary throughput. The authors in [82] have considered cooperation among the primary and secondary networks. The work presented in this dissertation does not consider such an extent of cooperation. Weighted decision fusion method was explored in [75] to allow SUs make accurate decisions regarding primary presence. But as discussed in this chapter, some SUs of the secondary network may be located out of the interference range of the primary users. If spectrum access decision is based on this sensing result, then it amounts to ignoring the spatial opportunity.

Cooperative sensing that considers utilizing spatial opportunity has also been explored by [83]. The authors of [83] optimize the power and sensing duration for optimal performance of decision strategies. The decision is made by the central controller based on decision fusion methods. The purpose of this work is accurate detection of the primary transmissions. This work differs from the work presented in this chapter in terms of the objectives sought from cooperation. The contributions of this chapter are focused on maximizing spatial opportunity. Secondary transmissions even in the presence of primary activity are considered acceptable if the expected interference losses are within acceptable range.

Another popular contribution on cooperative sensing was presented in [84]. The authors of [84] weigh different decision making methods like soft decision fusion and hard decision fusion. All SUs receive primary transmissions and they either make a decision on the primary presence, locally, or send the received energy level information to a central controller. Channel access is based on the central controllers allocation of channels to SUs. Similar efforts have been discussed by the authors of [85]. The thesis presented in this dissertation is that optimal detection of primary transmissions and optimal secondary channel access should be treated separately.

Popular channel assignment works like [86] and [87] discuss secondary access using
swarm methods. The main objective that is sought in these papers is contention-free and interference-free channel assignments. These works neither address the issue of designing the detection threshold nor do they incorporate location side information in the proposed channel assignment algorithms. Further, the authors of these papers do not comment on optimal channel selections.

The decision making problem that we discuss involves deciding between different hypotheses. These various hypotheses correspond to different number of SUs (or primary user) transmitting on the channel of interest. A similar problem, but for a completely different setting and detection method, was studied in [88]. The authors in [88] present the case for cyclostationary detection method. The contributions of this chapter differ from the contributions of [88] in terms of the problem formulation and the solution presented. The contributions of this chapter can be summarized as follows.

2. Minimizing losses that occur due to sensing errors by leveraging the network layout information.
3. Results on the joint threshold design (optimization) problem.
4. Formulation of an approximation of the joint threshold design problem that is easily solvable by gradient based algorithms.

### 4.2 Operational Framework

The operational frame of SUs can be assumed to be divided into a sensing phase and a transmission phase. The SUs start sensing at the beginning of the sensing phase. During this sensing procedure they receive signal samples that may have been transmitted by the primary user or other SUs. Each SU compares the average of the received samples against a detection threshold. If the average received power goes higher than the detection threshold, then the SU aborts the acquisition of the channel.
If the average received power does not go higher than the detection threshold throughout the sensing phase, then the SU assumes that the channel is free of any transmissions, and available for acquisition. But more than one SU may arrive at this conclusion and attempt to acquire this channel. Such a scenario would lead to contention and interference losses among the SUs. Communication control links between the SUs are assumed to be absent, implying that all power received by an SU from any other SU would be noise.

Further, SUs are assumed to posses no contention resolution mechanism throughout this section. SU systems with contention resolution abilities will be discussed in the Chapter 4. This implies that if two or more SUs do acquire the same channel, then they will interfere with one another’s transmission throughout the frame time. As a result the overall throughput of the SU system would be adversely affected. Let \( \bar{\epsilon}_p = \{\epsilon_{u,p}\}_{u \in \omega_p} \) denote the set of detection thresholds of all the SUs sensing channel \( p \). Given \( a = (a_i)_{i=1}^{[a]} \) and \( b = (b_i)_{i=1}^{[b]} \), let \( \text{dis}(a, b) = (\sum_i(a_i - b_i)^2)^{1/2} \) be a measure of the difference between these two vectors. If \( x \) is a set, then let the cardinality of this set be denoted by \(|x|\).

Furthermore, one consequence of inter-SU interference is a reduction in the SU throughput. SUs are assumed to lack the ability of processing signals from other SUs. This implies that the interference power adds on to the noise and results in a decrease in achievable throughput. Most SU systems have a lower bound on the SU throughput. If the SU throughput falls below this bound because of interference power, then the SU stops transmitting. This minimum throughput is denoted by \( \mathcal{R} \), and is affected by fading. Let the average power received by an SU \( u \)'s receiver from its transmitter be denoted by \( \bar{\Gamma}_u \). Then, the probability that this SU will be able to maintain rate \( \mathcal{R} \) can be given by

\[
\zeta(\bar{\Gamma}_u) = \int_{Ra(\Gamma) > \mathcal{R}} f(\bar{\Gamma}_u)d\Gamma,
\]

(4.1)

where \( Ra(\Gamma) \) is the achievable rate.
Consider a set of $M$ primary channels, each occupied by different primary transmitters. Primary activity is independent across channels and time frames. The SU is a transceiver pair that attempts to transmit over a channel only if it detects the channel to be primary-free with a sufficiently low probability of error. The distances between the SU and the primary transmitters determine the interference power received by one another (Figure 4.1). Formulating a problem that considers the impact of distances allows exploitation of the spatial opportunity in the system.

As in the previous chapter, the system model depicted in Figure 4.1 illustrates the existence of primary and secondary systems operating side by side. The nodes in the systems operate in transceiver pairs. Further, consider the location information to be available (to the SU) a-priori or provided by a third party vendor. Conventionally, primary systems create operational service areas around their base stations. If the exact location of the primary transmitters or receivers in not available, it is assumed that at least some knowledge of the service area is available to the SU.

Figure 4.1: System model. Multiple SUs trying to acquire the same channel.

### 4.2.1 System Model
4.2. OPERATIONAL FRAMEWORK

As in previous sections, the radius of the service area around a primary transmitter can be given by $r_p$, where $p$ denotes the channel index. The primary receivers can be assumed to be uniformly distributed in this service area. The primary transmitter ensures at least a minimum power level at the edge of the service area to meet the quality-of-service requirements of the system. Similarly, the radius of the service area around a secondary transmitter can be given by $r_s$. The secondary system also ensures a certain power level at the edge of its service area.

The loss due to interference that occurs at the primary receiver due to secondary transmission is of significance, and should be kept under check. The interference losses experienced by the secondary receivers are of no concern, as they are not the licensed users and are not entitled to the resource. The primary and secondary receivers are assumed to be uniformly distributed in the areas of radius $r_p$ and $r_s$ around their respective transmitters. As before, energy detection is the method of detection employed by the SU to detect primary transmissions.

4.2.2 Signal Model

As before, the sensing time is denoted by $\mu$ secs. When this SU finds the channel free of primary transmission, it considers it primary-free and acquires it. This implies that sensing is a hypothesis testing problem. The null hypothesis can be denoted by $H_0$, and indicates the absence of primary transmissions. While the presence of these transmissions is the alternate hypothesis, denoted by $H_1$. The alternate hypothesis will be such that the SU detects primary signal plus the noise power. These hypotheses are given by

$$H_0 : y(n) = e(n), \quad 0 \leq n < N \quad (4.2)$$

$$H_1 : y(n) = x(n) + e(n), \quad 0 \leq n < N \quad (4.3)$$

where $y(n)$ is the $n^{th}$ received sample, $N$ is the total number of samples, and the received samples are independent and identically distributed random variables. Also, $e(n)$ is assumed to be the $n^{th}$ sample of complex gaussian noise $e(n) \sim \text{CN}(0, \sigma_e^2)$. 
The primary signal, $x(n)$ is the $n^{th}$ sample of the received signal. The impact of path-loss attenuates the signal strength by the time it reaches the SU. The received signal component is distributed as $x(n) \sim CN(0, \tilde{\sigma}^2)$ where $E[|x(n)|^2] = \tilde{\sigma}^2$ and $E[|e(n)|^2] = \sigma^2_e$. So keeping this in view, the received signal to noise ratio (SNR) of the primary transmission at the secondary transmitter becomes $\gamma_{ps} = \tilde{\sigma}^2 / \sigma^2_e$.

4.3 Joint Threshold Design for Multi-SU Systems

This section presents the joint threshold design of a multi-SU system. This effort requires cooperation among the SUs as the threshold also incorporates side information to limit the interference at the primary receivers and the SUs. A central controller selects channels for SUs, and then designs detection thresholds for each SU based on the side information. The objective is contention-free SU transmissions during the frame time. Furthermore, the side information allows capitalizing on the spatial opportunity in the network.

The first formulation that is presented follows the line of reasoning discussed in the previous chapters. It is shown that the problem is nothing more than risk minimization where the costs are the losses that occur due to error. The optimization variable is the threshold of each SU in the secondary network. Further, it is shown that the problem is not readily solvable using standard optimization methods. To resolve this issue a new formulation is presented that can reach the optimal thresholds using simple gradient based algorithms.

Sections 3.1.3 and 3.2.3 described the derivation of detection thresholds under two different scenarios i.e., discrete and continuous channel selections. It has been further demonstrated that adjusting the detection threshold of SUs, based on the interference cost and the (SU transmission) opportunity cost, is a throughput enhancing approach. Furthermore, the directions for threshold selection for a single SU has been laid out in Section 3.1.3. Sections 3.1.3 and 3.2.3 further showed that the threshold design in the presence of a single SU and the threshold design in the presence of multiple SUs results in different detection thresholds.
4.3. JOINT THRESHOLD DESIGN FOR MULTI-SU SYSTEMS

Threshold design in the presence of multiple SUs, attempting to acquire the same channel, is the topic of analysis throughout this section. The following subsections explore the impact of network topology in terms of the channel gains and path-loss. Further, the representation of costs is also extensively discussed to provide the reader an insight into the losses that shape threshold design for a multi-SU system. Some important aspects of the problem are discussed next.

Before discussing the problem formulation, the reader should take note of one significant difference that separates the contributions of the previous chapter from the proposals of this chapter. This difference is that cooperative joint threshold design becomes a necessity only when multiple SUs attempt to acquire the same channel. If multiple SUs are not attempting to acquire the same channel, then threshold design should proceed as described in the previous chapter.

Channels and Path-loss Model

As before, the following channels are considered for the analysis: 1) The interference channel between the SU’s transmitter and the primary receiver (of some channel \( i \)) denoted by \( h_{si} \); 2) The interference channel between the primary transmitter and the secondary receiver \( h_{is} \); 3) The channel between secondary transmitter and secondary receiver \( h_{ss} \); 4) The channel between the primary transmitter and primary receiver \( h_{ii} \). Unlike previous sections, a new channel, the channel between SUs, denoted by \( \bar{h}_{u,v} \) (where \( u \neq v \) and \( u, v \in U \)), is also considered.

The SNRs on these channels can be given by \( \gamma_{si} \), \( \gamma_{is} \), \( \gamma_{ss} \), \( \gamma_{ii} \) and \( \gamma_{uv} \). Complete knowledge of all these instantaneous channel states may not be available at the secondary transmitter. Any channel can be represented by \( h = pl \cdot \nu \), where \( pl \) stands for the path-loss and \( \nu \) stands for the small scale fading. The path-loss component \( pl \) of the channel \( h \) is given by \( pl = K \cdot (1/d)^{\beta/2} \) where \( \beta \) is the path-loss exponent, \( d \) is the distance between the respective terminals, \( K \) is a constant dependent upon the frequencies. \( pl \) is dependent upon the knowledge of the link distance and that is what the analysis is based on. Using knowledge of these distances, it is shown in this section that the achievable throughput of a secondary system can be enhanced.
CHAPTER 4. Spectrum Access with Cooperative Threshold Design

Cost Functions and Risk

Bayes’ risk, as discussed earlier, is a function of the cost of decision making errors. Earlier, it was assumed that these losses originate when an SU fails to detect primary presence or absence. Unlike previous sections, the errors accounted for in this section include the interference losses that SUs may inflict upon one another in case of a failure to detect transmissions from one another.

Every decision is assumed to have a cost associated with it, whether it be a loss in the primary or secondary throughput. The weights associated with these costs can be adjusted by the system designer. When some SU $u$ attempts to acquire a channel, it may receive interference power from any number of SUs, that are also attempting to acquire the same channel. If $\omega_p$ is the set of SUs that attempt to sense and acquire channel $p$, then $|\omega_p| - 1$ is the total number of SUs that can interfere with SU $u$ (such that $u \in \omega_p$).

As discussed in earlier sections, let $P_r(H_i|H_j)$ be the probability that some SU detects hypothesis $H_i$ to be true while $H_j$ was the actual hypothesis. This is a detection error and carries a penalty. Let $P_r^u(H_i|H_j)$, with superscript $u$ added, denote the detection probability associated with SU $u$. Then, this probability is given by

$$P_r^u(H_i|H_j) = \int_{re_i} p^u(y(n)|H_j) dy(n),$$

(4.4)

where $re_i$ is the region of the probability density function $p^u(y(n)|H_j)$ that maps into the decision that hypothesis $H_i$ is the actual hypothesis. Naturally, $P_r^u(H_i|H_j)$ is a function of the detection threshold $\epsilon_{u,p}$ and the received power. Let $\delta_i$ be a set of channel acquisition decisions made by all the SUs, after sensing, where $\delta_i(u) \in \{0, 1\}$, and the subscript $i$ indicates any one decision set of the $|\omega_p|^2$ possible decision sets. Further, $\delta_i(u) = 1$ implies that SU $u$ found the hypothesis to be $H_1$. Let $\xi$ be the set of all possible decision sets. Let $\bar{H}_i$ denote the hypothesis that $\delta_i$ is the set of decisions made by the SUs.

Let $P(\bar{H}_i|H_j)$ be the probability of $\bar{H}_i$ being the hypothesis detected by the SUs,
when $H_j$ was the actual hypothesis. Consider an example where 3 SUs sense the same channel when it was primary-free (i.e., $H_1$ was the true hypothesis) and two of them find it primary-free while the last one found it busy. Then, $\tilde{H}_i = \{H_0, H_0, H_1\}$, $\delta_i = \{0, 0, 1\}$ and probability of such a scenario can be given by

$$P(\tilde{H}_i | H_0) = P_r(\tilde{H}_i | H_0) P_r(H_0 | H_0) P_r(H_1 | H_0)$$

$$= \int_{r\epsilon_0} p^1(y(n) | H_0) dy(n) \int_{r\epsilon_0} p^2(y(n) | H_0) dy(n) \int_{r\epsilon_1} p^3(y(n) | H_0) dy(n),$$

where $r\epsilon_1$ is the region of the pdf that maps into the decision that the primary user is active.

If the number of SUs sensing channel $p$ are such that $|\omega_p| = 1$, then $\tilde{H}_i = H_i$ would represent the decision of a single SU to acquire or not to acquire channel $p$. But, in a multi-SU system, the possible decisions form a very large set. Each hypothesis carries a cost. Given $H_1$ to be the actual hypothesis, any $\tilde{H}_j$, for which $\sum_u \delta_j(u) < |\omega_p|$, will result in interference at the primary. Owing to the spatial layout, given $H_1$, some $j \in \xi$ may cause more interference to the primary compared to others. Further, the probability $P_r(\tilde{H}_i | H_j)$ is a function of $\epsilon_p$.

Let $C_{j,i}$ be the cost associated with deciding hypothesis (by all SUs of set $\omega_p$) $\tilde{H}_j$, when $H_i$ is the actual hypothesis. Then risk [34], is a function of the decisions made by SUs. The overall risk can be given by

$$R = \sum_{j \in \xi} \sum_{i \in \{0, 1\}} C_{j,i} P(\tilde{H}_j | H_i) P_r(H_i),$$

where $R$ is the total risk. The cost $C_{j,i}$ can be given by

$$C_{j,1} = 0, \quad \text{if } \sum_{u \in \omega_p} \delta_j(u) = 0.$$ 

This is because no SU failed to detect primary transmissions. Given $H_1$, any $j$ for which $\sum_{u \in \omega_p} \delta_j(u) < |\omega_p|$ will result in an interference loss at the primary user. The
interference cost that results due to missed detection can be given by

$$C_{j,1} = \log_2(1 + \gamma_{pp}) - \log_2\left(1 + \frac{\gamma_{pp}}{\sigma_e^2 + (1 - \delta_j)\hat{\sigma}}\right),$$  \hspace{1cm} (4.8)

where $\hat{\sigma} = (\sigma^2_{p,u})_{u \in \omega_p}$ is a vector of powers received by the primary from the SUs. All interference powers are treated as noise. Similarly, the losses that occur when $H_0$ is the actual hypothesis but SUs reach a different conclusion. Conventionally, the cost in such an event is the loss of secondary opportunity given by

$$C_{j,0} = \sum_{u \in \omega_p} \delta_j(u) \log_2(1 + \gamma_{uu}).$$  \hspace{1cm} (4.9)

But this cost does not include the impact of inter-SU interference. Further, the achievable throughput is also a function of the fading model. With these issues in view, the cost of a false alarm error is reformulated to be

$$C_{j,0} = \sum_{u \in \omega_p} (1 - \delta_j(u))\zeta(\bar{\Gamma}_{u,j})\mathcal{R} - \sum_{u \in \omega_p} (1 - \delta_j(u))\zeta(\bar{\Gamma}_{u,j})\mathcal{R},$$  \hspace{1cm} (4.10)

where $\tilde{j} \in \xi$ is such that $\sum_{u \in \omega_p} \delta_j(u) = 0$ i.e., no SU faces a false alarm, and all transmit. Similarly, $\bar{\Gamma}_{u,j}$ is the average power received by receiver of SU $u$ when $j \in \xi$ represents the transmissions decisions of all the SUs $u \in \omega_p$. As discussed earlier, all interference powers are treated as noise. The risk function along with all the associated costs can be derived to be

$$R = P_r(H_1)\left(\sum_{j \in \xi} C_{j,1} \prod_{u \in \omega_p} \int_{re0} p^u(y(n)|H_1)dy(n)\right)$$

$$+ P_r(H_0)\left(\sum_{j \in \xi} C_{j,0} \prod_{u \in \omega_p} \int_{re1} p^u(y(n)|H_0)dy(n)\right).$$  \hspace{1cm} (4.11)

The problem of joint threshold selection can be expressed as
The next section discusses the different approaches that can be employed to find the optimal solution of Problem 1.

4.4 Optimal Threshold Selection

Joint threshold selection such that the risk is minimized has been presented as Problem 1. This section explores the issues related with such a formulation, popularized by existing literature on risk. Further, this section explores the structure of Problem 1 and proposes a solution for optimal threshold selection. An important result on the structure of the problem is presented in the next proposition.

Proposition 4.4.1. The risk in Problem 1, as a function of the threshold $\bar{\epsilon}_p$, is non-convex.

Proof. The Risk $R$ is a summation of the probabilities of detection (in the presence and absence of primary transmissions). These probabilities are essentially gaussian tail probabilities [14], given by

$$P = Q((\frac{\epsilon}{\sigma^2_e} - \gamma - 1)\sqrt{\frac{\mu\hat{f}_{\text{sam}}}{2\gamma + 1}}),$$

(4.14)

where $\mu$ is the sensing duration, $\hat{f}_{\text{sam}}$ is the sampling frequency, $\gamma$ is the signal to noise ratio and $\sigma^2_e$ is the noise power. These tail probabilities are monotonic non-convex functions of $\epsilon_{p,u}$, and the sum of their products scaled by logarithmic cost functions (as in (4.11)) is also non-convex. This completes the proof. □

Many heuristic and non-heuristic algorithms require the continuity of the objective function to find the optimal solution. The continuity of the objective function in
Problem 1, is discussed next.

**Proposition 4.4.2.** The objective function in Problem 1 is continuous, and the solution set, convex.

*Proof.* Proving that the gradient $dR/d\bar{\epsilon}_p$ is bounded is sufficient proof of the continuity of the risk $R$ function. The risk function $R$ is based on the probabilities of missed detection and false alarm which are functions of the threshold. These probabilities are defined for every threshold $\bar{\epsilon}_p > 0$. Let $R_i$ be the risk associated with threshold $\bar{\epsilon}_p^i$. Further, let $dis(\bar{\epsilon}_p^1, \bar{\epsilon}_p^2)$ be the difference between any two thresholds, $\bar{\epsilon}_p^1$ and $\bar{\epsilon}_p^2$, such that

$$R_1 < R_2.$$  \hfill (4.15)

If $R$ is a continuous function, then for any constant $co$ such that $0 < co < dis(\bar{\epsilon}_p^1, \bar{\epsilon}_p^2)$, there will always exist a corresponding threshold $\bar{\epsilon}_p^3$ (in the vicinity of $\bar{\epsilon}_p^1$) (where $dis(\bar{\epsilon}_p^1, \bar{\epsilon}_p^3) < dis(\bar{\epsilon}_p^1, \bar{\epsilon}_p^2)$) such that

$$R_1 < R_3 < R_2.$$  \hfill (4.16)

It can be easily shown that there will always exist a threshold $\bar{\epsilon}_p^3$ where inequality (4.16) will hold. This shows that the risk is a continuous function of the thresholds. Furthermore, the solution set is convex because all thresholds that lie between any two thresholds from the same solution set, are feasible. This completes the proof. \hfill \square

The result Proposition 4.4.1 implies that no convex method can be applied to Problem 1. Some problems, despite being non-convex, are uni-modal. This aspect of the function allows the application of gradient based methods. Gradient based methods, in such scenarios can result in the globally optimal solution. The next result discusses this feature of the problem.

**Proposition 4.4.3.** In the absence of inter-SU interference, the Gradient-Descent approach will result in an optimal solution of Problem 1.
4.4. OPTIMAL THRESHOLD SELECTION

Proof. If an optimization problem satisfies any one of the following conditions, then gradient based methods will result in a suboptimal solution. The conditions that lead to the suboptimality of gradient based approaches are as follows:

1. the solution set is non-convex;

2. the objective function is not continuous;

3. the objective function has multiple maximum and minimum points.

Problem 1 has been shown to be continuous, and the solution set, convex, in Proposition 4.4.2. In order to prove this proposition, it is necessary to demonstrate that the objective function in Problem 1, has one minimum, under the given conditions.

The risk $R$ defined in (4.11) consists of two distinct parts. The first part is related to the interference losses (that occur due to missed detection) while the second part is related in to the throughput losses (that occur due to false alarm). First, we consider the part related to the missed detection event given by

$$P_r(H_1) \left( \sum_{j \in \xi} C_{2,1} \prod_{u \in \omega} \int_{r=0} p^u(y(n)|H_1)dy(n) \right).$$

The losses given by the expression (4.17) are 0 when the thresholds are set to 0. This is because the probability of missed detection is maximum when the detection threshold of each SU is set to 0. As the thresholds increase, the probability of missed detection by each individual SU also increases.

Let the cost of missed detection by a single SU be denoted by $C_{0,1}^u$. The relationship (4.20) can also be derived from the fact that the interference loss caused by a single SU $u$ increases as the selected threshold increases i.e.,

$$\frac{d}{d\epsilon_u} P_r(H_1) C_{0,1}^u \int_{r=0} p^u(y(n)|H_1)dy(n) > 0.$$
The sum of probabilities of all $j \in \xi$ can be given by

$$\sum_{j \in \xi} \prod_{u \in \omega_p} \int_{re_0} p^u(y(n)|H_1)dy(n) = 1, \quad \text{for any} \ \bar{\epsilon}_p. \quad (4.19)$$

Further, the cost $C_{j,1}$ is maximum for $j \in \xi$ where more SUs miss detection, and the probability of missed detection increases with the threshold. Since, the loss of missed detection caused by each SU just depends on the detection threshold of that SU, we have

$$\frac{d}{d\bar{\epsilon}_p} P_r(H_1)(\sum_{j \in \xi} C_{j,1} \prod_{u \in \omega_p} \int_{re_0} p^u(y(n)|H_1)dy(n)) > 0. \quad (4.20)$$

Thus, the losses (4.17) are continuously increasing functions of the threshold with minimum at $\bar{\epsilon}_p = 0$.

Next, we move on to the second part of the risk function. The losses that result due to false alarm can be given by

$$P_r(H_0)(\sum_{j \in \xi} C_{j,0} \prod_{u \in \omega_p} \int_{re_1} p^u(y(n)|H_0)dy(n)). \quad (4.21)$$

For a single SU system (or a system with no inter-SU interference), the cost $C_{1,0}$ equals $\zeta(\Gamma_u)\mathcal{R}$. These losses would increase if the probability of false alarm increases, and these losses are maximum at $\bar{\epsilon}_p = 0$. In the absence of inter-SU interference, the false alarm loss of each SU is independent of one another, and we have the relationship

$$\frac{d}{d\epsilon_{u,p}} P_r(H_1)C_{1,0}^{u} \int_{re_1} p^u(y(n)|H_0)dy(n) < 0. \quad (4.22)$$

The inequality (4.22) further implies that in the absence of inter-SU interference, the losses (4.21) are continuously decreasing functions of the threshold $\epsilon_{u,p}$. Another conclusion, that can be derived from the above discussion is that for each individual
SU there exists only one $\epsilon_{u,p}$ that minimizes the problem

$$\arg\min_{\epsilon_{u,p}} P_r(H_1)C^u_{\epsilon_{u,p}} \int_{\mathbb{R}_0} p^u(y(n)|H_1)dy(n) + P_r(H_0)C^u_{\epsilon_{u,p}} \int_{\mathbb{R}_1} p^u(y(n)|H_0)dy(n),$$

(4.23)

s.t. $\epsilon_{u,p} \geq 0$. (4.24)

The above mentioned optimization problem clearly attempts to minimize the risk of a single SU, independent of any other SU. In the absence of inter-SU interference, the above optimization problem will provide the threshold of each individual SU that can minimize the overall risk $R$. This implies that in the absence of inter-SU interference, a gradient descent algorithm, initialized anywhere in the solution set, will converge to the same minimum. This completes the proof.

Proposition 4.4.4. In the presence of inter-SU interference, the Gradient-Descent approach of optimizing Problem 1, is suboptimal.

Proof. A single example would be sufficient to prove the suboptimality of any gradient based approach in solving Problem 1, in the presence of inter-SU interference. Consider an SU system with two SUs sensing the same channel $p$. Further, the system is assumed to be such that it causes negligible interference to the primary user. Such an assumption would imply that

$$P_r(H_1)\left(\sum_{j \in \xi} C_{j,1} \prod_{u \in \omega_p} \int_{\mathbb{R}_0} p^u(y(n)|H_1)dy(n)\right) = 0. \quad (4.25)$$

Given the equality (4.25), the problem reduces down to selecting the thresholds that maximize the SU throughput.

Let $\tilde{\epsilon}_p^1 = \{0, 0\}$ and $\tilde{\epsilon}_p^2 = \{th, th\}$ (assumed to be the maximum allowable threshold). Further, let threshold $th$ be such that the probability of a false alarm, in the absence of primary transmissions, is very close to 0. As has been shown earlier, the losses (4.21) will be minimized at $\tilde{\epsilon}_p^2$, in the absence of inter-SU interference.

In the presence of inter-SU interference, the interference among the SUs will be
maximum at $\bar{\epsilon}^2_p$. The probability of maintaining rate $\mathcal{R}$ is affected by this interference, and so is the overall SU sum throughput. This implies that there will exist some other threshold, denoted by $\bar{\epsilon}^3_p$ such that

$$\bar{\epsilon}^3 = \arg \max_{\bar{\epsilon}_p} (\zeta(\bar{\Gamma}_{1,0}) + \zeta(\bar{\Gamma}_{2,0}))\mathcal{R}$$

(4.26)

$$\text{s.t. } dis(\bar{\epsilon}^3, \bar{\epsilon}^1) < dis(\bar{\epsilon}^1, \bar{\epsilon}^2),$$

(4.27)

where $\zeta(\bar{\Gamma}_{u,i})$ is the probability of maintaining rate $\mathcal{R}$ by SU $u$, under hypothesis $i$. For the example under study, the individual thresholds in the set $\bar{\epsilon}^3_p$ do not matter as much as the separation between the individual thresholds. Consider another threshold $\bar{\epsilon}^4$ such that $\bar{\epsilon}^3_{u_1,p} = \bar{\epsilon}^4_{u_2,p}$ and $\bar{\epsilon}^3_{u_2,p} = \bar{\epsilon}^4_{u_1,p}$. Further, as $\gamma_{i,j}$ is the SNR on the transceiver link of SU $i$, it has been assumed in this section that $\gamma_{u,u} = \gamma_{v,v}$ for all $u,v \in \omega_p$. The SU sum throughput, given by

$$G(\bar{\epsilon}_p) = P_r(H_1) \left( \sum_{j \in \xi} (\zeta(\bar{\Gamma}_{u,j}) + \zeta(\bar{\Gamma}_{v,j})) \prod_{u \in \omega_p} \int_{\text{rea}} p^u(y(n)|H_1)dy(n) \right)$$

$$+ P_r(H_0) \left( \sum_{j \in \xi} (\zeta(\bar{\Gamma}_{u,j}) + \zeta(\bar{\Gamma}_{v,j})) \prod_{u \in \omega_p} \int_{\text{rea}} p^u(y(n)|H_0)dy(n) \right).$$

(4.28)

Further, it can be shown with little effort that

$$G(\bar{\epsilon}^3_p) = G(\bar{\epsilon}^4_p).$$

(4.29)

This means that the individual thresholds of the two SUs, for this example, can be swapped to get the same SU sum throughput. Furthermore, (4.28) will reduce as the interference among the SUs increases. This implies that the threshold selection problem is multi-modal, proving that inter-SU interference makes gradient descent a suboptimal approach.

Clearly, no gradient based approach is directly applicable to solve the problem. The simple and straightforward solution proposed in the remainder of this section is to approximate the original problem with a new formulation. Then divide the
solution set of the new formulation into regions such that each region has at most one optimum point. The next proposition presents a result that outlines the structure of the solution. Assuming that the interference power received by the primary user from the SUs is directly proportional to the distances, we have the following result.

**Proposition 4.4.5.** Given that $d_{u,p} < d_{u+1,p}$ for all $u \in \omega_p$, where $d_{u,p}$ is the distance between SU $u$ and the primary user on channel $p$, then the optimal threshold selection will be such that

$$
\epsilon_{u,p} < \epsilon_{u+1,p}.
$$

(4.30)

**Proof.** Consider an SU system with two SUs denoted by $u_1$ and $u_2$ such that $d_{u_1,p} < d_{u_2,p}$ and $u_1, u_2 \in \omega_p$. It has been discussed in Proposition 4.4.4 that more than one $\bar{\epsilon}_p$ may result in the same SU sum throughput. Let $\bar{\epsilon}_1^1$ and $\bar{\epsilon}_2^2$ be two thresholds that lead to the same SU sum throughput. These thresholds are such that

$$
\epsilon_{u_1,p}^1 > \epsilon_{u_1,p}^2,
$$

(4.31)

$$
\epsilon_{u_2,p}^1 < \epsilon_{u_2,p}^2.
$$

(4.32)

As per the proposition, $\bar{\epsilon}_1^1$ cannot minimize the risk as much as $\bar{\epsilon}_2^2$. The interference losses, that occur due to missed detection, at the primary that are given by

$$
L(\bar{\epsilon}_p) = P_r(H_1)(\sum_{j \in \xi}(C_{j,1}^{u_1} + C_{j,1}^{u_2}) \prod_{u \in \omega_p} \int_{r=0}^p p^u(y(n)|H_1)dy(n)),
$$

(4.33)

But, because of the fact that SU $u_1$ is closer to the primary user than SU $u_2$, and owing to the inequalities (4.31) and (4.32), it can be shown that

$$
L(\bar{\epsilon}_1^1) > L(\bar{\epsilon}_2^2).
$$

(4.34)

The inequality (4.34) implies that the optimal threshold selections will always be such that (4.30) holds. This completes the proof.

It has been shown in the previous results that Problem 1 is not only non-convex but
gradient based algorithms will also result in a suboptimal (locally optimal) solution. To deal with this issue, a new criterion, called the reward, is introduced, and given by

$$R = \sum_{j \in \xi} \sum_{u \in \omega_p} (1 - \delta_j(u)) \zeta(\bar{\Gamma}_{u,j}) R.$$  \hspace{1cm} (4.35)

The reward $R$ is essentially the sum expected throughput of all the SUs attempting to acquire channel $p$. Instead of focussing on minimizing the risk, Problem 1 can be approximated by the following problem definition.

**Problem 2.** If any two SUs $u$ and $v$ are such that $d_{u,p} < d_{v,p}$, then the optimal threshold selection problem is

$$\max R$$

s.t. $P_r(H_1) \sum_{j \in \xi} C_{j,1} \mathbb{P}(\bar{H}_j | H_1) \leq co$, \hspace{1cm} (4.37)

$$\bar{\epsilon}_p \geq 0,$$  \hspace{1cm} (4.38)

$$\epsilon_{u,p} < \epsilon_{v,p}.$$  \hspace{1cm} (4.39)

The constraint (4.37) in Problem 2 essentially sets the missed detection errors to stay less than a constant $co$. Further, constraint (4.39) implies that the optimal solution would be such that the SU that is the highest interferer to the primary user, should have the lowest threshold. The next result in an important step towards developing an optimal approach for threshold selection.

**Proposition 4.4.6.** The solution set for Problem 2 is non-convex.

**Proof.** Problem 2 has a solution set bounded by the constraints (4.37), (4.38) and (4.39). For the solution set to be convex under constraint (4.37), a linear increase in threshold should be followed by a linear increase in the missed detection losses. Consider an SU system of two SUs, sensing channel $p$. Further, consider a detection
threshold $\bar{\epsilon}_p$ ($\bar{\epsilon}_p = \{\epsilon_{u,j,p}^i\}_{i \in \{1,2\}, j \in \{1,2,3,4\}}$) to be such that

$$\epsilon_{u_1,p}^1 > \epsilon_{u_2,p}^1.$$  

(4.40)

Let $\text{co}$ denote the interference losses that result from $\bar{\epsilon}_p$ given by

$$P_{r}(H_1)(\sum_{j \in \xi} C_{j,1} \prod_{u \in \omega_p} \int_{r=0}^{\rho(p)(y(n)|H_1)dy(n)}) \leq \text{co.}$$  

(4.41)

Next, consider another threshold, denoted by $\bar{\epsilon}_p^2$ such that

$$\epsilon_{u_1,p}^1 < \epsilon_{u_1,p}^2.$$  

(4.42)

For $\bar{\epsilon}_p^2$ to result in the same or less missed detection losses of $\text{co}$, the thresholds $\bar{\epsilon}_p^1$ and $\bar{\epsilon}_p^2$ should be further related as

$$\epsilon_{u_2,p}^1 > \epsilon_{u_2,p}^2.$$  

(4.43)

Given (4.42), (4.43) has to hold for the missed detection losses of $\bar{\epsilon}_p^1$ and $\bar{\epsilon}_p^2$ to be the same. Any threshold on the line connecting the two thresholds, $\bar{\epsilon}_p^1$ and $\bar{\epsilon}_p^2$, should have the same (or less) missed detection losses, given by $\text{co}$, for the constraint to conserve convexity. The solution set is non-convex because an increase in one SU’s threshold is followed by a decrease in the other SU’s threshold, but the corresponding increase and decrease in respective missed detection errors may not be proportional. This completes the proof.

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Proposition 4.4.7. Gradient ascent (or commonly known as Hill-Climbing), initialized at some $\bar{\epsilon}_p$ for which $\epsilon_{u,p} = \epsilon_{v,p}$ for all $u, v \in \omega_p$, results in the optimal solution of Problem 2.

Apparently non-convexity of the solution set does not affect the optimality of the solution that we propose. Next, the optimality of a Hill-Climbing approach is presented.
Proof. Consider an SU system with two SUs, sensing the same channel $p$. Assuming that there is no interference among the SUs and the SU-primary link, then the maximum SU sum throughput will occur at the maximum individual thresholds. Let $\bar{\epsilon}_p = \{th, th\}$ be such a threshold, where $th$ is the maximum allowable threshold. Clearly, we would have $\text{dis}(th, th) = 0$.

But, if the inter-SU interference is not negligible, then the optimal threshold, $\bar{\epsilon}_p^1$, must be such that $\text{dis}(\epsilon_{u1,p}^1, \epsilon_{u2,p}^1) > 0$. This is because as the threshold of one SU decreases, the inter-SU interference decreases, maximizing the overall SU sum throughput. Clearly, the same $\text{dis}(\epsilon_{u1,p}^i, \epsilon_{u2,p}^i)$ can occur for two sets of thresholds. Let these two thresholds be denoted by $\bar{\epsilon}_p^1$ and $\bar{\epsilon}_p^2$. These two thresholds are such that

$$\text{If, } \epsilon_{u1,p}^1 < \epsilon_{u1,p}^2,$$

$$\text{then } \epsilon_{u2,p}^1 > \epsilon_{u2,p}^2.$$  \hspace{1cm} (4.44) \hspace{1cm} (4.45)

The relationships (4.44) and (4.45) further imply that there will be at least $|\omega_p|!$ maxima within the solution space.

The constraint (4.39) divides the solution space in a manner that each one of the $|\omega_p|!$ maxima is limited to one region. Proposition 4.4.5 proves that only one region among all these regions will contain the global maximum. Constraint (4.39) ensures that the gradient ascent is initialized in the region that contains the global maximum.

The challenge is that constraint (4.39) makes this region non-convex. To deal with this issue the gradient ascent algorithm is initialized at the point where all individual thresholds are equal and the missed detection losses are less than or equal to $co$. Naturally, the algorithm will progress along the path where the $\text{dis}(\epsilon_{u1,p}^1, \epsilon_{u2,p}^1)$ becomes larger, as it means that the SU sum throughput is increasing. This is because it has been shown earlier that (in the presence of inter-SU interference), the optimal threshold is where

$$\text{dis}(\epsilon_{u1,p}^1, \epsilon_{u2,p}^1) > 0,$$  \hspace{1cm} (4.46)
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The algorithm will terminate at the point where either

\[ P_r(H_1) \sum_{j \in \xi} C_j \mathbb{P}(\tilde{H}_j | H_1) = co, \]  

or

\[ \frac{d\mathcal{R}}{d\epsilon_p} = 0. \]  

In either case, the terminal point of the gradient ascent algorithm will be the global optimal solution. This proves that the gradient ascent algorithm will result in the globally optimal solution of Problem 2. This completes the proof.

\[ \square \]

### 4.5 Numerical Results

This section explores the effectiveness of the proposals presented in the cooperative threshold design context. The numerical evaluations are performed over a large set of primary activity scenarios and network topologies. The first result is a comparison of different methods of threshold selection. The proposed method is compared with the case when no threshold design is implemented. This result brings out the effectiveness of location based joint threshold design for optimal performance.

The primary availability probability was assumed to be such that \( \theta_p \sim \mathcal{U}[0.2, 1] \). Further, a Rayleigh fading model is considered. Let \( \tilde{d} = \{d_{u,v}\}_{u,v \in \omega_p, u \neq v} \) and \( \tilde{d} = \{d_{u,p}\}_{u \in \omega_p} \) be the sets representing the distances between various nodes. Each transceiver pair is assumed to be such that in the absence of interference power, the power received by a receiver, from its transmitter, if it is located at the edge of the service area, is 0db (arbitrarily chosen).

Furthermore, the path-loss parameters were also arbitrarily chosen to be \( K = 1 \) and \( \beta = 1.5 \). The service area of the primary, \( r_p = 1000m \), and the service area of the SUs, is arbitrarily chosen to be \( r_s = 400m \). For the numerical evaluations, 1,500 scenarios, of \( \{\mu, \theta_p, \tilde{d}, \tilde{d}\} \), were generated randomly. Throughout this section, the frame time is assumed to be 20ms. These random scenarios were such that
Figure 4.2: The effectiveness of joint threshold design against increasing number of SUs.

Figure 4.3: The effectiveness of joint threshold design against increasing number of SUs.
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Figure 4.4: The effectiveness of joint threshold design against increasing number of SUs.

Figure 4.5: The effectiveness of joint threshold design against increasing number of SUs.
### Figure 4.6: The effectiveness of joint threshold design against increasing number of SUs.

\[
\theta_p \sim U[0.2, 1], \quad \mu \sim U[2, 15] \text{ms}, \quad d_{u,p} \sim \mathcal{N}(1000, 400) \quad \text{and} \quad d_{u,v} \sim \mathcal{N}(600, 200) \text{ for 3 plots} \quad \text{and} \quad d_{u,v} \sim \mathcal{N}(1000, 200) \text{ for the remaining 3 plots.} \]  

The SU sum throughput was calculated for each scenario, followed by an expectation over all the scenarios. For the method that solves Problem 2, the interference losses were set to be the same as the interference losses that result from optimizing Problem 1. It is to be noted that the solution for Problem 1 is a suboptimal one while the solution for Problem 2 has been found using the gradient ascent approach.

Figure 4.9 is a comparison of the losses incurred by the primary user from the SUs. These losses are the ones that occur due to missed detection errors. For the results in Figure 4.9, 1,500 realizations of the set \(\{\theta_p, \mu, d, \bar{d}\}\) were generated randomly. These random scenarios were such that \(\mu \sim U[2, 15] \text{ms}, \quad \theta_p \sim U[0.2, 1], \quad d_{u,p} \sim \mathcal{N}(1000, 400)\) and \(d_{u,v} \sim \mathcal{N}(600, 200)\). The rest of the simulation parameters were the same as used for the previous figure. The losses incurred by the primary user, for each scenario were computed followed by an expectation over the entire set of realizations.

Figure 4.9 shows that as the number of interfering SUs increase, so does the losses
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**Figure 4.7:** The interference losses incurred by the primary user, as a result of SU’s missed detection errors.

**Figure 4.8:** The interference losses incurred by the primary user, as a result of SU’s missed detection errors.
Figure 4.9: The interference losses incurred by the primary user, as a result of SU’s missed detection errors.

incurred by the primary user. It can be seen that the losses are maximum for the non-cooperative scheme that does not selects thresholds for the SUs in a joint manner. All thresholds selected for the SUs (by the non-cooperative method) are based on the location information based threshold design of the previous section. Minimum losses at the PU’s end are caused by the cooperative threshold selection method that optimizes Problem 1.

To generate the results in Figure 4.9, the SU sum throughputs of the both the cooperative methods were constrained to be the same, for each individual realization. Further, it can also be observed that even though the method that solves Problem 2 can result in the optimal thresholds (for Problem 2), it results in a higher loss at the primary user’s end. This is primarily because Problem 2 is an approximation of Problem 1. Figure 4.10 is a comparison of different cooperative sensing scheme with the ones proposed in this chapter. For this figure, a network of 9 SUs is assumed to be in search of transmission opportunities. Let $\bar{\theta}$ be the set of primary availability probabilities of all channels. For each data point a total of 1,500 realizations of the
set \{\mu, \bar{\theta}, \tilde{d}, \bar{d}\} are generated randomly according the the distributions defined earlier.

For each data point, channel selection needs to be performed such that the SU sum throughput is maximized. For the first line in the plot tagged as Optimal Non-Cooperative Channel Selections, brute force method is used to find the optimal channel selections. The thresholds for each SU on a channel is based on side information, as described in the previous chapter. No cooperative threshold selection is performed.

For the cooperative joint threshold design method that optimizes Problem 1, the greedy approach of channel selections is employed. This means that an SU is arbitrarily picked, and a channel that results in the maximum increase in reward, is selected for it. If a channel is selected for more than one SU, then the thresholds are jointly optimized using side information according to Problem 1. After this optimization, if the channel selections results in a higher reward, then this channel is selected for both the SUs. The same procedure is repeated for all the SUs till a channel selections have been performed for all SUs. The same greedy approach is used for the method that solves Problem 2.

The next channel selection method is the one described in [61]. The method

Figure 4.10: The SU sum throughput for different cooperative schemes.
presented in [61] is basically a sensing order selection problem, but it becomes equivalent to our problem when the number of sensing slots are set to 1. The authors in [61] do not incorporate any side information into the design. The final comparison with the method presented in [59]. The work in [59] is also a sensing order selection problem, but for two SUs. If the number of sensing slots are set to 1, the problem becomes equivalent to the one considered in this section. Both [61] and [59] require a cooperation to come up with the optimal channel selections.

Figure 4.11 is a performance comparison of cooperation. The cooperation proposed in this chapter is for developing a detection threshold, not for cooperatively determining primary presence. Nevertheless, questions regarding the performance of such a cooperation arise. The SUs make separate decisions on primary presence (based on the proposed threshold). While for other methods namely or, and, majority [14] and weighted decision fusion [75], the SUs collectively make a decision. The losses at the primary user caused by the weighted cooperative decision making scheme proposed in [75] and the conventional decision (hard decision) making schemes mentioned in [14] are compared with the losses caused by the threshold design scheme.
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Figure 4.12: The SU throughput comparison of Decision Rules with the proposed Cooperative Threshold Design.

For the result in Figure 4.11, a primary user with transmit power such that it can maintain an expected throughput of 15 bits/sec/Hz (arbitrarily chosen), is considered. As before, $r_p = 1000m$ and $r_s = 400m$. A network of 5 SUs is assumed to be located in the vicinity. A total of 1,500 realizations of the set $\{\mu, \theta_p, \tilde{d}, \bar{d}\}$ are generated as per the distributions, described earlier. The loss in the throughput of the primary, as a result of interference from the SUs is computed followed by an expectation over all scenarios.

It can be observed that the losses (at the primary user) caused by the SU, by using the proposed cooperative method, are higher. But, at the same time it is to be noted that the proposed method does not perform cooperative decision making. Despite the fact that all SUs make their own decisions (based on the proposed threshold), the performance is at par with cooperative decision making schemes. Figure 4.12 presents the SU sum throughput performance of different cooperative scheme. Naturally, the proposed scheme has a higher SU throughput for the reason that all SUs make separate
decisions and that they make use of side information in adjusting their detection thresholds. For each data point in Figure 4.12, a total of 5,000 realizations of the set \( \{ \mu, \theta_p, \tilde{d}, \bar{d} \} \) were generated according to the previously defined distributions such that the mean of all the distances in the set \( \{ \tilde{d}, \bar{d} \} \) correspond to the x-axis.

It should be noted that although the proposed method has a very high SU sum throughput, the losses of missed detection for such a scheme are also very high compared to conservative conventional methods of threshold and channel selection. But, another thing to take notice is that the cooperative spectrum access by threshold design does not actually require communication among the individual SUs, on a regular basis. This implies that due very low overhead communication for the spectrum access method proposed in this chapter, the SU sum throughput should be even higher.

### 4.6 Summary

This chapter has shifted the focus of the threshold design work from single SU system to multiple-SU systems. The idea of cooperative spectrum access has been explored in a relatively different context, when compared to the existing literature on cooperative SU systems. The incorporation of side information regarding the SU locations has helped reduce not only the interference among the SUs but also the interference losses incurred by the primary user.

Further, the formulation of the joint threshold design was studied. It was shown that the problem is non-convex and not amenable to gradient based methods. To find the solution of such problems, heuristic based search algorithms are most commonly employed. The problem with these heuristic based search algorithms is that they have a high complexity. Furthermore, the convergence of such algorithms at a globally optimal solution cannot be guaranteed.

The problem was reformulated for the purpose of making it solvable by known methods that can guarantee convergence to a global optimal solution. The results were then compared with popular cooperative SU schemes that are used to make SU systems more efficient in terms of their sum throughput and primary detection. The
numerical results showed that the proposed joint threshold design method resulted in a high throughput performance.

Cooperative spectrum access through joint threshold design enables the SUs to free opportunity among themselves, and assign it to SUs that can result in a higher increase in SU sum throughput. Further, it has been shown that SUs that can cause very high interference at the primary user are assigned very low thresholds by the optimal solution. This is to minimize their probability of missed detection. But, this decrease in threshold also causes a high probability of false alarm. To balance the loss in SU throughput, the optimal solution raises the threshold of some other SU, the one that does not pose very high interference threat to the primary user.
Chapter 5

Optimal Channel Selections in a Multi-SU System

Proposals regarding threshold design and continuous channel selections were presented and put to test in the previous chapter. This chapter extends the scope of research presented in the previous chapter to consider optimal design and channel selections. Further, the argument that the cognitive abilities of SUs not only encompass their sensing abilities but also their capacity to ensure contention-free operation with better coordination among the nodes, is also studied in this chapter. In a multi-SU network, adaptive threshold design (discussed in the previous chapter) is not enough to maximally utilize the transmission opportunities available in the spatial domain. Adaptive threshold design contributes towards throughput maximization by increasing access of the SU to the spatial opportunity in the network. All this is done while maintaining an acceptable level of performance degradation at the primary’s end. Despite this fact, adaptive threshold design is unable to resolve the contention that may arise when multiple SUs attempt to acquire the same channel.

Transmission opportunities left unused in the physical layer can be exploited by optimizing spectrum sensing and access of secondary users (SUs). Unlike conventional models, the proposals in this chapter exploit a priori side information regarding mutual interference and contention resolution, for SU channel selections, is presented.
Mutual interference and contention between multiple SUs result when more than one SU tries to acquire the same primary channel. The results in this chapter show that by the modeling of sensing imperfections and incorporating channel side information, the spectral opportunity that would otherwise be neglected, can be utilized.

Channel selection for a single sensing slot is a special case of sensing order (SO) selection. An SO is a set of channels that an SU can sense in a sequence, during its frame time. The general problem of sensing order selection (SOS) is addressed in this chapter. SOS is an off-line procedure, performed before sensing begins. A central controller may select SOs and communicate them to all SUs. The next round of SOS is not necessary as long as the channel availability statistics remain unchanged. SOS relies on channel availability probabilities, assumed available a-priori.

A set of SUs (located within interfering range from one another) sensing the same channel may find it primary-free, and try to acquire it. This may lead to contention and throughput loss due to interference. To mitigate this loss, SOs are conventionally designed to minimize contention. But if side information regarding the average received (interference) powers among SUs is made available beforehand, then the interference losses can be computed. As long as the interference losses are within bounds, the SOS procedure may select SOs that result in a higher expected throughput despite the contention among SUs.

The optimization of the SO to maximize the expected sum throughput of the SU network. The assumption of a non-cooperative setting makes this optimization more complicated. The SUs depend on energy detection of the primary channels, in their channel acquisition process. The contributions include a novel, low complexity algorithm for the optimal SOS for SUs. An analysis of the suboptimal greedy approach is also analyzed. Different scenarios where the greedy approach can result in the optimal SO, are also presented. The greedy solution is, by definition, a low complexity solution. The complexity of this (greedy) approach is used as a benchmark to evaluate the complexity requirements of the proposed optimal algorithm.

An analysis of the problem formulation, presented later, shows that the formulated optimization problem is an intractable dynamic programming problem. Numerical
results suggest that the proposed optimal SOS strategy results in a higher network throughput by effectively making use of the spatial information made available in the form of side information (a-priori). Analytical results are presented to prove the optimality of the proposals. The analysis further evaluates the complexity requirements of the proposed optimal algorithm under different inter-SU interference scenarios. The objective of this chapter is to provide a comprehensive framework for optimal SOS that can exploit spatial opportunity by incorporating side information. This work shows that by optimizing channel SOs in the proposed framework, substantial gains in the SUs' sum throughput can be achieved in a multi-channel secondary network.

5.1 Motivation

SUs deployed in the form of networks incur performance degradation that manifests itself in the form of interference losses. As discussed briefly, in the previous chapter, the challenges faced by multi-SU networks are far more intricate and cumbersome when compared to a single SU system. These SUs, as discussed earlier, either have very limited licensed frequency resources or do not have a licensed resource at all, at their disposal. Such networks rely solely on spectrum sensing for opportunities. It may happen that more than one SU may find the same channel free of primary transmissions, and try to acquire it. This acquisition attempt may lead to contention among the SUs. These complications arise due to the greedy nature of the SUs, in a non-cooperative setting. This lack of cooperation among the SUs proves detrimental to the throughput maximization objective.

The most convenient way for the SUs to resolve the interference issue is by the help of a central controller [75,89–97]. All SUs in such applications [75,89–97] sense channels, and then send their decisions or test statistics to the central controller. The central controller, by exploiting the diversity, ascertains whether the primary user is present or absent. The purpose of considering a multi-SU scenario in this chapter is not to advocate cooperative sensing, but to highlight the significance of optimal (non-cooperative) channel access. It is important to note that cooperation
among SUs has two benefits: 1) channel sensing; and 2) managing contention-free channel access. If the channel sensing results are good enough, then the only reason cooperation among SUs would be required is to minimize contention and interference by managing channel access. But this type of channel access is plagued with overhead transmissions. This chapter shows that channel SO can be designed by a central controller and assigned to SUs, before the sensing and transmission frame. During the frame, the SUs can operate in a completely non-cooperative manner (without the need for overhead transmissions) and sense (and acquire) channels according to the SOs.

Assuming that the results of cooperative sensing are as good as the results of non-cooperative sensing, cooperative schemes are a computational burden on the secondary system’s resources. The SUs may sense different channels in the same sensing slot. But this does not imply that the central controller becomes redundant. Despite the discontinuation of the cooperative sensing task, the central controller may still be of much use. If multiple SUs sense the same channel and find it available, they will attempt to acquire it. This may lead to contention and interference among the SUs. At this stage the central controller can allocate the resources to all SUs in a manner such that the contention and interference is minimized while maximizing the sum throughput.

As discussed earlier, the drawback of a strategy that relies on a central controller is that it requires a communication link between each SU and the central controller. This link is required every time an SU performs sensing and attempts to acquire a channel. Most of the time, owing to the nature of secondary networks, no such control channel may be available for this communication. And even if such control channel is available, the communication link may not be operational at all times, due to channel imperfections like fading and shadowing.

The next logical approach would be to adopt a distributed structure [98–105] that permits SUs to reach a resolution without requiring a central controller. Most of the work on distributed design in the context of cognitive radios is focussed on the objective of cooperative sensing rather than the optimization of optimal channel
5.1. MOTIVATION

Figure 5.1: 5 primary users transmitting on different channels with 5 different SUs trying to opportunistically access the channels.

access. But references like [106–109] present a distributed framework for channel selections in a multi-SU environment using algorithms based on swarming methods [110] or heuristic approaches like [80, 111], solely for the purpose of interference and contention minimization.

Having discussed the centralized and distributed approaches, the final approach (found in SU literature) to resolve the contention and interference issue among SUs is without the use of any communication among SUs. The proposals presented in this chapter are based on this approach. This means that systems rely on channel selections, performed before the sensing-transmission frame (Figure 2.4) of the SU network begins. As demonstrated in this work, channels selected in a smart manner can add to the overall reward. Further, adaptive threshold design can considerably increase the capacity of the secondary network. To the best of our knowledge, no existing work has attempted channel selections by incorporating side information. Such a design is a desired feature as it greatly enhances the cognitive abilities of SU network.
5.2 The Significance of Sensing Order Design

In a secondary system, (SUs) are allowed to access the channels licensed to primary users as long as the degradation in the latter’s performance is limited to lie within acceptable bounds. Figure 5.1 is an illustration of a multi-SU system with multiple primary channels to select from. This is the layout of the problem discussed throughout this chapter. Clearly, the power received by an SU from each primary transmitter would not be the same. Similarly, the power received by SUs from one another will also vary. SUs sense radio frequencies to find under-utilized primary-free bands for transmissions [1–3, 19–23, 112].

Most secondary networks lack the ability to calculate the exact degradation in the primary network’s performance as a result of their transmissions. Thus, the SUs acquire channels only when they detect them to be free of primary transmissions. Moreover, SUs are expected to abort transmissions as soon as they detect resumption of primary activity. Each SU senses multiple bands, one at a time, till it finds a channel satisfying a stopping policy [11, 113, 114]. The order in which an SU senses channels, in a frame, is called its SO [59, 61–66, 115–117]. Different SUs (located within interfering range from one another) sensing the same channel may find it primary-free and try to acquire it. This may lead to contention and throughput loss due to interference. To mitigate this loss, sensing orders are designed to minimize contention. However, completely avoiding contention amongst SUs may not be optimal, as shown in the following example.

An Illustrative Example

Consider two channels of 1 MHz bandwidth, centered at 800 MHz and 801 MHz, with primary-free probabilities of $\theta_1$ and $\theta_2$, respectively. Most existing sensing order approaches [59, 61] will select the 800 MHz channel for one SU and the 801 MHz channel for the other, for sensing and acquisition. Clearly, these selections will result in no overlap and the SUs would not interfere with one another. Further, suppose that each SU transmits at a constant rate $R$ when it acquires a channel. Suppose that
conditioned on a channel being primary-free, each SU is able to detect an opportunity with probability $P$, and maintain rate $R$ with probability $\zeta$. Then, the expected sum throughput of both SUs is $(\theta_1 + \theta_2)P\zeta R$. On the other hand, suppose that the two SUs are located spatially apart so that the mutual interference between them is limited.

Consider now the scheme that selects the 800 MHz channel for both SUs. Assume that in acquiring the channel, the SUs implement some contention resolution protocols like [118–121] when they detect contention from each other. Suppose that this contention resolution process results in both SUs acquiring the same channel with probability $\phi_1$, and with probability $\phi_2$, only one of the SUs acquires the channel while the other fails in its acquisition and does not transmit. With probability $1 - \phi_1 - 2\phi_2$, both SUs fail and do not transmit. If both SUs successfully acquire the same channel, then because of mutual interference, each SU’s throughput is now reduced to $\iota R$, where $0 \leq \iota \leq 1$.

The expected sum throughput of both SUs is given by $(2\phi_1\iota + 2\phi_2)\theta_1\zeta R$, which can be shown to be greater than $(\theta_1 + \theta_2)P\zeta R$ if $2\phi_1\iota + 2\phi_2 > P(1 + \theta_2/\theta_1)$. As the interference power between the SUs decreases, $\phi_1 + 2\phi_2 \to 1$, and $\iota \to 1$, while $P(1 + \theta_2/\theta_1)$ remains constant. Therefore, if $\theta_1$ is much greater than $\theta_2$, and the SUs’ mutual interference is sufficiently low, it is better for both SUs to select the 800 MHz channel. The intuition in this example is clear: it is better for SUs to share a channel that has a high probability of being primary-free if the mutual interference between SUs is sufficiently low, thus maximally utilizing the spatial opportunity.

One contribution of this work is to provide a comprehensive framework for optimal sensing order design that incorporates various physical layer attributes like interference, fading and other side information. This has also been done in [122], but for a single SU system. The results show that by optimizing channel SOs, substantial gains in the SUs’ sum capacity can be achieved in a multi-channel secondary network, as compared to SOIs that avoid contentions between SUs. The problem of optimally selecting channels for multiple users in a network has been extensively addressed in publications on wireless mesh networks [123,124]. These papers and many
CHAPTER 5. Optimal Channel Selections in a Multi-SU System

more discuss selections based on routing, power and interference challenges. SUs in such networks do not face the challenge of sensing imperfections and channel unavailability due to primary activity. In secondary systems, the need of sensing before acquisition leads to a logical extension towards systems that require SUs to sense multiple channels in a frame.

Multi-channel sensing in an SU network has been explored by many researchers including [56, 59, 61–66, 115–117, 125] and the references within. The channel selection process involves two steps: learning the channel availability statistics, and then scheduling the resources for sensing and subsequent acquisition. The a-priori primary channel availability profiles are either assumed known [59, 64] or they are learned over time [56, 63, 125, 126]. Learning the profile is exploration while transmitting on it is its exploitation. Systems are faced with the challenge of dealing with this trade-off. As the confidence in the learned availability probabilities increases, SUs shift towards exploitation of these resources.

To the best of our knowledge, no existing scheme formulates the sensing order design problem in a manner that can exploit channel side information (other than [122] for a limited scenario). This is because these methods do not incorporate physical layer information like the channel gains, interference or contention, and geographic distances among the SUs. Another way to exploit spatial opportunity is by adjusting the detection threshold as discussed in [127], by building on the ideas presented in [34, 40]. The proposals presented in this chapter are compatible with the adaptable threshold design of [127], and both these works can be used in conjunction to capitalize on the geographic opportunity.

The formulation for the reward of an SO as a function of interference powers and contention among SUs was also discussed in [128]. As [128] studies a system with only one sensing slot scheduled for sensing, it can be considered a special case of the problem addressed in this chapter. The authors of [64] and [59] present channel selection for sensing and acquisition by one and two SUs respectively. These papers perform channel selection assuming perfect detection and channel gain knowledge. The contributions in this chapter are based on a more practical physical layer scenario...
where imperfect detection and unknown channel gains limit the maximal utilization of resources on the one hand, and offer exploitation of spatial opportunity on the other hand.

Selecting orthogonal channels for different SUs has been explored to avoid contention among SUs in [61,62]. However, the work presented in this chapter shows that the reuse and overlap among the channels sensed by SUs can actually enhance the throughput when the interference between SUs is low. Similarly, [56,125] address systems that sense once before the transmission phase of a frame. These papers optimize the sensing-throughput trade-off. The proposals and the formulation for the expected throughput that are presented in this chapter is compatible with their framework as well. However, by incorporating side information in sensing order optimization, the proposed algorithm is able to achieve a higher spectrum utilization.

The goal of this chapter is the development of a channel selection strategy for all SUs in a secondary network such that the achievable sum capacity is maximized. The main contributions are as follows:

1. The modelling of sensing imperfections, contention, interference and noise along with available channel gain and received-power side information in the problem formulation. The expected throughput of the secondary network, along with the above factors, is presented as a function of the SO. This modeling enables the exploitation of transmission opportunities present in the spectral and spatial domain, which would otherwise have been left unused.

2. Unlike previous works on SOS, an optimal algorithm that incorporates side information for SOS, has been proposed. The numerical results demonstrate that SO designed as per the presented proposals can result in as high as a two-fold increase in expected throughput even for a small network of 5 SUs. This improvement in the expected throughput becomes more pronounced as the network size increases.

3. Further analysis of the problem structure enables a reduction in the complexity of the optimal algorithm. Our procedure proceeds by sequentially selecting
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channels so as to yield a small subset of sensing orders, amongst which the optimal one can be found. Our proposed procedure considerably prunes the number of feasible options at every stage.

To summarize the literature, we classify the SOS methods into two groups; 1) the ones that perform SOS assuming perfect detection [59, 61, 62, 64] (we call these the PD-SOS strategies), and 2) the ones that assume imperfect detection and incorporate side information as discussed in the current paper and [122, 128] (we call these the SI-SOS strategies). The channel acquisition process that takes place during the frame time is not affected by the SOS strategies, other than the fact that the SOs used by the SUs may be different (for different strategies). Although, PD-SOS strategies seem convenient as they do not require any side information, they fail to utilize the transmission opportunities that arise due to the spatial attributes of the network. The rest of this chapter is organized as follows. The system model and the assumptions are presented in Section 5.3. The problem formulation is presented in Section 5.4. The formulation is followed by the Suboptimal approach of SOS in Section 5.5. Based on the results derived from the suboptimal approach, the optimal algorithm is presented in Section 5.6. The proposals are subjected to numerical evaluation in Section 5.8. The chapter concludes with the highlights of the contributions summarized in Section 5.9.

5.3 Operational Framework

The system model is studied in this section. This model plays a crucial role in any strategy that implements SOS. Clearly, the expected sum throughput depends on the number of SUs, sensing duration, transmission time and the number of channels that can be sensed by an SU. The next few sections discuss the different components of the system model under study. If \( x \) denotes a set, then throughput this text, the notation \(|x|\) indicates the cardinality of set \( x \).
5.3. OPERATIONAL FRAMEWORK

5.3.1 Transmission Model and Sensing Orders

Consider a set of SUs denoted by $U$, operating in a time-synchronized network [112] where a set $M$ of primary channels can be accessed by the SUs. Every SU is a transceiver pair. The frame time, $\mathcal{T}$ secs in length, is divided into a sensing and transmission phase [14]. The sensing stage of each frame is further divided into multiple sensing slots of duration $\mu$ secs for channel detection, illustrated in Fig. 5.3. Within a time frame, each SU is scheduled to sense $T$ channels, where $T < \mathcal{T}/\mu$. If an SU acquires a channel in the $t^{th}$ sensing slot where $t \in [1, T]$, the fraction of the frame time that it gets for transmission is $c_t = 1 - t\mu/\mathcal{T}$.

For the sake of simplicity, the primary channel’s bandwidth and the bandwidth requirement of an SU are assumed to be the same, as in [14, 59, 61, 64, 125]. We assume primary activity to be independent across channels and time frames. Further, the primary activity status on a channel is assumed to remain the same throughout a frame time [59, 64, 66]. For each channel $p \in M$, let the probability of it being...
primary-free be denoted by $\theta_p \in [0,1]$. Let $L_t$ be the set of channels sensed by all SUs of set $U$, in sensing slot $t$, with $L_t(u)$ denoting the channel sensed by SU $u$. If $S_u$ is the sensing order of SU $u$, then the sensing order of the entire network can be given by $S = (S_u)_{u=1}^{U} = (L_t)^T_{t=1}$.

The SU system that we consider allows a fixed transmission rate (as assumed by [66], [59]) denoted by $R$. If an SU is unable to maintain $R$ because of interference or poor channel gain, it does not acquire the channel. As before, let $\tilde{\sigma}_{u,i}^2$ be power received by SU $u$ from SU $i$. Similarly, $\tilde{\sigma}_{u,p}^2$ be the power received by SU $u$ from primary transmitter $p$. Further, the wireless SU network that we study relies on energy detection for SUs to determine the busy or free state of channels. An SU cannot differentiate between the power received from an SU and that received from a primary transmitter. An SU can detect a channel’s status, by comparing the received power against the detection threshold.

### 5.3.2 Non-Cooperative SOS

The SU model that we study comprises of two phases; 1) the SOS, and 2) the sensing and transmission frame. Before the SU network begins sensing, a central controller selects SOs for all SUs. This process takes place before sensing begins, and relies solely on the prior statistical side information. The need for a new SO will arise only
if there is a change in the prior information. The SOS phase is then followed by the sensing and transmission frame where each SU attempts to acquire a channel in a completely non-cooperative manner. This non-cooperative nature implies that the SUs do not share sensing results or perform any sort of communication.

5.3.3 Model Symmetry

For any SOS algorithm to find the optimal SO (among the set of $M^{U|T}$ possible options), it should be able to compute the reward of selecting an SO. SUs, later use the selected SO to sense channels, in the $T$ sensing slots. PD-SOS and SI-SOS strategies differ in terms of how they compute this reward. Since, channel sensing and acquisition (CSA) is a random process, each possible realization of this process has a probability associated with it. The probability of detection ($P_d$) of transmission is an integral part of the probability of each realization. PD-SOS strategies assume perfect detection i.e., $P_d = 1$ if some other user is also transmitting on the same channel, and $P_d = 0$ otherwise. SI-SOS strategies, on the other hand, assume imperfect detection, where $P_d$ is determined by system attributes like noise, fading, received powers and threshold of detection.

Since, PD-SOS strategies assume perfect detection, it implies that reward computation is independent of the system attributes. This further implies that for such strategies, all network topologies are the same (as inter-SU distances do not matter) i.e., symmetric in nature. While for the SI-SOS strategies, any change in network topology means a change in the inter-SU interference. If the prior side information regarding this change is provided, SI-SOS strategy of reward computation may result in a different optimal SO.

5.3.4 Channel Acquisition Process

During each sensing sensing slot $t \in [1, T]$, the following steps occur in the given order (see Figure 5.4). These steps influence the probability that a channel will be acquired by an SU.
Channel detection

An SU that fails to acquire a channel in sensing slots 1 through \((t - 1)\), waits for the beginning of sensing slot \(t\) to sense a channel according to their SO. Each SU performs energy detection, and each SU’s detection process is independent of one another. The SUs that fail to acquire a channel in all sensing slots, stay silent throughout the frame, and attempt sensing in the next frame. All SUs perform channel detection before they acquire a channel. The detection procedure implemented by the SUs is energy detection [14], and depends on the powers received from the primary and secondary transmitters. Let \(\hat{A}_t\) be the set of SUs that acquire the channels that they sensed in sensing slot \(t\). The Table 5.1 lists all the sets that are significant to the channel acquisition process.

Contention detection before and after back-off

The SUs that find their channels primary-free may face contention from one another if they attempt to acquire the same channel. This contention is detected by SUs in an independent manner, either by energy detection or by the RTS/CTS ([119], [121]) procedure of CSMA/CA. The SUs that detect contention, back-off for random periods.
of time. The back-off periods are different for each SU with probability 1. When the back-off period of an SU expires, it reattempts to acquire the same channel. During this acquisition reattempt, if the SU detects the primary channel to be busy (i.e., it either made a missed detection in the previous channel detection step or another SU has already acquired the channel before this SU’s back-off period expires), it aborts the acquisition process and waits till the beginning of the next sensing slot to sense some other channel.

**Channel gain**

The SUs that find their channels primary free and do not detect contention from other SUs, before or after back-off, have to make sure that the channel gain is good enough to maintain a transmission rate of $R$ [59]. The SUs that fail to find the channel gain good enough, do not acquire the channel. The probability of finding the channel gain good enough is given by

$$
\zeta(\bar{\Gamma}) = \int_{R_a(\Gamma)>R} f(\Gamma) d\Gamma,
$$

where $\bar{\Gamma}$ is the average channel gain, $R_a(\Gamma)$ is the achievable rate as a function of the channel gain and $f(\Gamma)$ is the probability density function of the fading model considered. The model $f(\Gamma)$ is considered to be a Rayleigh fading model.

### 5.3.5 Detection Probabilities

Consider an SU $u$ sensing a channel, according to the order $S$, in sensing slot $t$. Let $\omega_{L_t(u)}(S)$ denote the set of SUs scheduled to sense channel $L_t(u)$ (in some sensing slot). The set of SUs that are already transmitting on the channel $L_t(u)$ can be given by

$$
\hat{I}_{u,t} = \{ \hat{u} | \hat{u} \in \bigcup_{j=1}^{t-1} \hat{A}_j, \hat{u} \in \omega_{L_t(u)}(S) \},
$$

where $\hat{A}_t$ is the set of SUs that have acquired the channels that they sensed in sensing slot $t$. Table 5.1 lists all the sets that are significant to the CSA. If $\tilde{l}$ be a set where each element of the set given by $\tilde{l}(p) \in \{0,1\}$ (for all $p \in M$) denotes the primary busy or free status. The status $\tilde{l}(p) = 0$ means that the primary channel $p$ is free of primary transmissions. The probability of a
primary activity scenario \(\tilde{l}\) can be given by
\[
P(\tilde{l}) = \prod_{p \in M} \theta_p (1 - \tilde{l}(p)) + (1 - \theta_p) \tilde{l}(p).
\]

The probability that SU \(u\) will successfully detect primary or secondary presence on the channel can be denoted by \(P_{u,t}^{s}(\tilde{l}, \hat{I}_{u,t})\). Let \(\gamma_{u,t}(g_1, g_2)\) be the signal to noise ratio, to be detected by a SU \(u\) in sensing slot \(t\), where \(g_1\) indicates the primary presence or absence, and \(g_2\) is the set of SUs that have already acquired their channels, then this probability is given by
\[
P_{u,t}^{s}(\tilde{l}, \hat{I}_{u,t}) = Q((\frac{\epsilon}{\sigma_e^2} - \gamma_{u,t}(\tilde{l}, \hat{I}_{u,t}) - 1) \sqrt{\frac{\mu \hat{f}_{sam}}{2\gamma_{u,t}(\tilde{l}, \hat{I}_{u,t}) + 1}}),
\]
where \(\mu\) is the sensing duration, \(\hat{f}_{sam}\) is the sampling rate, \(\sigma_e^2\) is the noise power and \(\epsilon\) is the threshold of detection. The probability of false alarm is independent of the received power and hence independent of the sensing slot, and the interfering SUs. A higher probability of false alarm ensures better protection of the primary network from secondary SUs. The energy to be detected is
\[
\gamma_{u,t}(\tilde{l}, \hat{I}_{u,t}) = \frac{\tilde{l}(L_t(u))\tilde{\sigma}^2 + |\hat{I}_{u,t}|\tilde{\sigma}^2}{\sigma_e^2},
\]
where \(\tilde{\sigma}^2\) denotes the power received by an SU from the primary transmitter on channel \(L_t(u)\), \(\sigma^2\) is the power received from another SU, and \(\tilde{l}(L_t(u))\) is the activity status of channel \(L_t(u)\). Let \(\{U \setminus \bigcup_{j=1}^{t-1} A_j\}\) be the set of SUs that have failed to acquire a channel in sensing slots 1 through \((t - 1)\), and are to perform channel detection in sensing slot \(t\). We denote by \(\hat{A}_t\) the set of SUs that find their channels primary-free as a result of this detection, with probability
\[
\mathbb{P}(\hat{A}_t | \tilde{l}, \hat{I}_{u,t}) = \prod_{u \in \hat{A}_t} P_{u,t}^{s}(\tilde{l}, \hat{I}_{u,t}) \cdot \prod_{u \notin \hat{A}_t, u \in U \setminus \bigcup_{j=1}^{t-1} A_j} (1 - P_{u,t}^{s}(\tilde{l}, \hat{I}_{u,t})).
\]

The SUs that find their channels primary-free, attempt to acquire them. Let \(\tilde{I}_{u,t} = \{\tilde{\check{u}} | \tilde{\check{u}} \in \bigcup_{j=1}^{t-1} A_j \cup \hat{A}_t, \tilde{\check{u}} \in \omega_{L_t(u)}(S)\}\) be the set of SUs that will be transmitting on the channel when SU \(u\) attempts to acquire it. This acquisition process leads to contention. Let \(\check{A}_t\) be the set of SUs that detect contention and back-off for random
periods of time with probability

\[ P(\bar{A}_t|\tilde{l}, \bar{I}_{u,t}) = \prod_{u \in \bar{A}_t} P^S_{u,t}(\tilde{l}, \bar{I}_{u,t}) \cdot \prod_{u \notin \bar{A}_t, u \in \hat{A}_t} (1 - P^S_{u,t}(\tilde{l}, \bar{I}_{u,t})). \] (5.5)

The SUs that back-off for random periods of time may reattempt acquisition only when the back-off periods expire. As these back-off periods expire, these SUs reattempt in different sequences. For a particular channel \( p \), the probability of each sequence is the same. Let the realization of the sets of sequences of all channels be denoted by \( b \), and the probability of a \( b \) by \( \mathbb{P}(b|\bar{A}) \). The probability of some SUs having the same back-off time is negligible because we assume that the back-off times are picked from a very large set of options. As the back-off periods of the SUs expire, they perform contention detection for the second time to ascertain if the channel is free from other SUs’ transmissions. Of the SUs that backed-off, let \( B_{u,t}(b) \) be the set of SUs that acquire the channel before SU \( u \) because of the reattempt sequence \( b \). Then the set of SUs that interfere with SU \( u \) can be given by

\[ \bar{I}_{u,t} = \{ \hat{u}|\hat{u} \in \bigcup_{j=1}^{t-1} \bar{A}_t \cup \{ \hat{A}_t \setminus \bar{A}_t \} \cup B_{u,t}(b), \hat{u} \in \omega_{s_u(t)}(S) \}. \]

The probability that a set

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>The set of SUs that sense and find their channels primary-free.</td>
</tr>
<tr>
<td>( I_{u,t} )</td>
<td>Interfering SUs that have already acquired channel ( s_u(t) ) when SU ( u ) senses in sensing slot ( t ).</td>
</tr>
<tr>
<td>( A_t )</td>
<td>The set of SUs that acquire their channels at sensing slot ( t ).</td>
</tr>
<tr>
<td>( I_{u,t} )</td>
<td>Interfering SUs that will be transmitting on channel ( L_t(u) ) when SU ( u ) attempts to acquire it.</td>
</tr>
<tr>
<td>( \hat{A}_t )</td>
<td>The set of SUs that detect contention and back-off.</td>
</tr>
<tr>
<td>( B_{u,t}(b) )</td>
<td>The set of SUs that acquire channel ( L_t(u) ) before SU ( u ), after back-off.</td>
</tr>
<tr>
<td>( I_{u,t} )</td>
<td>Interfering SUs that will be transmitting on channel ( L_t(u) ) when SU ( u ) attempts to sense after back-off.</td>
</tr>
<tr>
<td>( \hat{A}_t )</td>
<td>The set of SUs that find their channels primary and contention-free after they perform sensing, after back-off.</td>
</tr>
</tbody>
</table>

Table 5.1: Sets of SUs in the Channel Acquisition Process.
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\( \hat{A}_t \) of SUs find their channels primary and contention free is given by

\[
P(\hat{A}_t \mid \tilde{l}, \hat{l}_{u,t}) = \prod_{u \in \hat{A}_t} \left( 1 - P_{u,t}^{S}(\tilde{l}, \hat{l}_{u,t}) \right) \cdot \prod_{u \notin \hat{A}_t, u \in \tilde{A}_t} (P_{u,t}^{S}(\tilde{l}, \hat{l}_{u,t})). \tag{5.6}
\]

All SUs in the sets \( \hat{A}_t \) and \( \tilde{A}_t \setminus \hat{A}_t \) are to check whether the channel gain is good enough to maintain rate \( \mathcal{R} \). The probability that a SU finds the channel gain good enough to maintain rate \( \mathcal{R} \) is also dependent in the fading model considered, and is given by

\[
\zeta(\Gamma_{u,t}(\tilde{l}, \hat{l}_{u,t})) = \int_{\mathcal{C}(\Gamma) \geq \mathcal{R}} \frac{1}{\Gamma_{u,t}(z)} \exp^{-\gamma/\Gamma_{u,t}(l, I_{u,t})} d\Gamma. \tag{5.7}
\]

where \( \mathcal{C}(\Gamma) \) is the normalized achievable throughput dependent on the received \( \Gamma \), and the average signal to noise ratio can be given by \( \Gamma_{u,t}(\tilde{l}, \hat{l}_{u,t}) \)

\[
\Gamma_{u,t}(\tilde{l}, \hat{l}_{u,t}) = \frac{\sigma^2}{l_{Lt(u)} \tilde{\sigma}^2 + |\hat{l}_{u,t}| \tilde{\sigma}^2 + \sigma_e^2}. \tag{5.8}
\]

The probability that a set \( \bar{A}_t \) of SUs finally acquire the channels that they sensed is given by

\[
P(\bar{A}_t \mid \tilde{l}, \hat{l}_{u,t}) = \prod_{u \in \bar{A}_t} \left( \zeta(\Gamma_{u,t}(\tilde{l}, \hat{l}_{u,t})) \right) \cdot \prod_{u \notin \bar{A}_t, u \in \hat{A}_t} (1 - \zeta(\Gamma_{u,t}(\tilde{l}, \hat{l}_{u,t}))). \tag{5.9}
\]

A realization of a sequence of these random sets, each corresponding to a step in the channel acquisition process, defines a realization of the channel acquisition process. Let \( z = \{ (\hat{A}_t)_{t=1}^T, (\tilde{A}_t)_{t=1}^T, (\hat{A}_t)_{t=1}^T, (\tilde{A}_t)_{t=1}^T \} \) be one such realization while \( \mathcal{Z} \) be the collection of all possible realizations of the CSA process. Given \( z \), the product of all event probabilities can be denoted by \( F^{S}(\tilde{l}, z) \). Given \( z \), the realization of various steps in the channel acquisition process can conveniently be denoted by \( \hat{A}_t(z), \tilde{A}_t(z), \hat{A}_t(z) \) and \( \bar{A}_t(z) \).
5.4 Problem Formulation

Decisions made by SUs (by the energy detection method), based on channel sensing, result in SUs acquiring channels or moving on to other sensing slots. The entire channel sensing and acquisition (CSA) process is essentially a sequence of these decision-making events (random in nature) where the decisions made in the early stages influence the ones in later stages. The CSA process is similar to a discrete time, discrete level stochastic process. As defined earlier, \( \mathcal{Z} \) is the set of all possible realizations of this process. Each CSA realization determines the subset of SUs acquiring channels in various sensing slots and the resulting sum throughput. Reward of an SO is an expectation of the sum throughput over the set \( \mathcal{Z} \).

Once, the SO has been selected and assigned to the secondary network, the SUs can operate in a completely non-cooperative manner. For computing the reward, the controller requires; 1) the prior availability probabilities of channels, 2) the average interference power received by SUs from one another, 3) the average channel gains and the fading model. Let \( F^S(\tilde{l}, z) \) represent the probability of a CSA process realization \( z \in \mathcal{Z} \). Given a \( \tilde{l} \) and \( S \), we have \( \sum_{z \in \mathcal{Z}} F^S(\tilde{l}, z) = 1 \).

Consider a network of two SUs, SU 1 and SU 2. The SOs selected for both the SUs are \( S_1 = (1, 2) \) and \( S_2 = (1, 3) \). We consider a primary activity scenario where \( \tilde{l}(1) = \tilde{l}(3) = 0 \) (channel 1 and 3 are primary-free), with probability \( \mathbb{P}(\tilde{l}) = \theta_1 \theta_3 \). Further, we consider a \( z \in \mathcal{Z} \), the details of which have been listed in Table 5.2.

As the probability of detection of transmissions is a function of the received power and the detection threshold, for the sake of convenience, we denote (5.2) by \( P^S_{u,j}(\epsilon, \sigma^2) \). The only difference with the previous notation is that instead of using the signal to noise ratio, we use the received power \( \sigma^2 \). The design of a detection threshold can be studied at [14]. As per this scenario, SU 1 and SU 2 both find channel 1 primary-free with probability \( (1 - P^S_{1,1}(\epsilon, \sigma^2))(1 - P^S_{2,1}(\epsilon, \sigma^2)) \), where \( \sigma^2_e \) is the noise power. Both SUs attempt to acquire it but detect contention from one another with probability \( P^S_{1,1}(\epsilon, (\sigma^2_e + \sigma^2_{1,2}))(1 - P^S_{2,1}(\epsilon, (\sigma^2_e + \sigma^2_{2,1}))) \), and back-off.

As the back-off periods of both the SUs are random (and selected from a very large pool), the probability that one SU has a back-off period shorter than the other
approaches 1/2. As per the scenario (of Table 5.2), SU 1 returns from back-off before SU 2, finds the channel available with probability \((1 - P_{1,1}^S(\epsilon, \sigma_n^2))\), and the channel gain good enough to maintain \(R\) with probability \(\zeta_{1,1}(\Gamma_1)\), where \(\Gamma_1\) is the average power on the transceiver link. SU 1 acquires the channel. SU 2 returns from back-off to find the channel acquired with probability \(P_{2,1}^S(\epsilon, (\sigma_n^2 + \bar{\sigma}_2^2))\), and moves on to the next sensing slot to sense channel 3. SU 2 finds channel 3 primary-free with probability \((1 - P_{2,2}^S(\epsilon, \sigma_n^2))\) and acquires it if it finds the channel gain acceptable (the probability of this being \(\zeta_{2,2}(\Gamma_3)\) defined as (5.1)). For the scenario presented in Table 5.2 we have,

\[
\mathbb{P}(\tilde{l}) F_S(\tilde{l}, z) = \mathbb{P}(\tilde{l})(1 - P_{1,1}^S(\epsilon, \sigma_n^2))(1 - P_{2,1}^S(\epsilon, \sigma_n^2))P_{1,1}^S(\epsilon, \sigma_n^2 + \bar{\sigma}_{1,2})P_{2,1}^S(\epsilon, \sigma_n^2 + \bar{\sigma}_{2,1}) \frac{1}{2} (1 - P_{1,1}^S(\epsilon, \sigma_n^2))\zeta_{1,1}(\Gamma_1)P_{2,1}^S(\epsilon, \sigma_n^2 + \bar{\sigma}_{2,1})(1 - P_{2,2}^S(\epsilon, \sigma_n^2))\zeta_{2,2}(\Gamma_3).
\]

(5.10)

For any \(z \in Z\), as \(\tilde{A}_t(z)\) is the set of SUs that acquire a channel in sensing slot \(t\), the total expected sum throughput can be given by

\[
\mathbb{R}^S(1) = \sum_{i} \mathbb{P}(\tilde{l}) \sum_{z \in Z} F_S(\tilde{l}, z) \sum_{t=1}^T |A_t(z)| c_t R.
\]

(5.11)

If \(p_u^S(t)\) denotes the probability that SU \(u\), in sensing slot \(t\), acquires a channel, then

\[
p_u^S(t) = \sum_{i} \mathbb{P}(\tilde{l}) \sum_{z \in Z \mid u \in A_t(z)} F_S(\tilde{l}, z),
\]

where \(Z \subset Z\) is such that \(\sum_{t} |A_t(z)| > 0\) (for any \(z \in Z\)). The reward becomes

\[
\mathbb{R}^S(1) = \sum_{u \in U} \sum_{t=1}^T p_u^S(t) c_t R.
\]

The problem of optimal SO selection is then equivalent to finding

\[
S^* = \max_S \mathbb{R}^S(1) \quad \text{s.t.} \quad L_t(u) \in M, \forall u \in U, \ t \in [1, T].
\]

(5.12)

The optimization problem (5.12) is a mixed integer non-linear programming problem.
( [129,130]). Next, we study algorithms to find a solution for (5.12).

<table>
<thead>
<tr>
<th>( t )</th>
<th>Event</th>
<th>Decision by SU1</th>
<th>Power received by SU1</th>
<th>Decision by SU2</th>
<th>Power received by SU2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Primary Detection</td>
<td>No primary</td>
<td>( \sigma_n^2 ) (no primary)</td>
<td>No primary</td>
<td>( \sigma_n^2 ) (no primary)</td>
</tr>
<tr>
<td></td>
<td>Contention Detection</td>
<td>Detected</td>
<td>( \sigma_n^2 + \bar{\sigma}_{1,2}^2 )</td>
<td>Detected</td>
<td>( \sigma_n^2 + \bar{\sigma}_{2,1}^2 )</td>
</tr>
<tr>
<td></td>
<td>Contention Detection after back-off</td>
<td>Not Detected</td>
<td>( \sigma_n^2 )</td>
<td>Detected</td>
<td>( \sigma_n^2 + \bar{\sigma}_{2,1}^2 )</td>
</tr>
<tr>
<td></td>
<td>Channel Gain</td>
<td>Acceptable</td>
<td>Avg. gain= ( \Gamma_1 )</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>Primary Detection</td>
<td>N/A</td>
<td>N/A</td>
<td>No primary</td>
<td>( \sigma_n^2 ) (no primary)</td>
</tr>
<tr>
<td></td>
<td>Contention Detection</td>
<td>N/A</td>
<td>N/A</td>
<td>Not Detected</td>
<td>( \sigma_n^2 )</td>
</tr>
<tr>
<td></td>
<td>Contention Detection after back-off</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Channel Gain</td>
<td>N/A</td>
<td>N/A</td>
<td>Acceptable</td>
<td>Avg. gain= ( \Gamma_3 )</td>
</tr>
</tbody>
</table>

Table 5.2: One possible scenario of the Channel Acquisition Process. An SU system where \(|U| = 2\), \(|M| = 3\), and \( T = 2 \).

## 5.5 Sensing Order Selection: The Suboptimal Approach

We present the popular greedy approach and then prove some results that pave the way towards forming the optimal algorithm. The optimal algorithm, presented in the next section, is compared with this approach in terms of the computational complexity and the achievable reward. Next, we present a condition to judge the optimality of
CHAPTER 5. Optimal Channel Selections in a Multi-SU System

an SOS approach.

**Condition 1.** A necessary condition for any SO’s optimality is that given all other channel selections, the channel selected for any slot \(\{u,t\}\), is such that

\[
\hat{L}_t(u) = \arg \max_{L_t(u)} R^S(1), \quad \forall u, t.
\]  

(5.13)

where \(\hat{L}_t(u)\) is the channel selected for slot \(\{u,t\}\) in the SO \(\hat{S}\).

Proof. Given all channel selections, we assume that the channel selected for some slot \(\{u,t\}\) is not the one that results in the maximum increase in reward. And, there exists some other channel that can result in a higher increase in reward. Then, by definition, SO \(\hat{S}\) is suboptimal. \(\square\)

The Greedy algorithm that we present was discussed in [59] (in the SOS context), has been illustrated in Algorithm 3. This approach tries to maintain fairness among the SUs.

**Definition 1.** Initially, channels are randomly selected for each \((u,t)\)-slot. Then, for each \((u,t)\)-slot, given all other channel selections, the channel that results in the highest increase in overall throughput, is selected. Multiple iterations, of this channel selection, for all \((u,t)\)-slots, is performed till Condition 1 is satisfied.

Algorithm 1 has a computational complexity of \(O(|U|T|M|)\), where \(O(1)\) is the complexity of computing the reward of a single SO.

**Definition 2.** Given an NSO, denoted by \(S_U\), where \(S_u(t) = p\), the availability of selecting a channel at slot \((u,t)\) is defined as

\[
a_p(u,t,S_U) = \frac{R(S_U) - R(\{S_U \setminus S_u(t)\})}{1 - \sum_{i=1}^{t-1} p^S_u(i)},
\]  

(5.14)

where \(\{S_U \setminus S_u(t)\}\) is a set of NSO with nothing selected for slot \((u,t)\).

A relationship between Condition 1 and Approach 1 is studied in the next Proposition. The challenges imposed by model asymmetry are highlighted by this result.
5.5. SENSING ORDER SELECTION: THE SUBOPTIMAL APPROACH

Algorithm 1 Greedy SSS

1: Randomly generate an $S_U$. Denote it by $S_U^1$, the superscript is the iteration index.
Further, let $S_U^0 = \emptyset$.
2: while $S_U^i \neq S_U^{i-1}$ do
3:     $i \leftarrow i + 1$
4:     for $j = 1$ to $\eta(U)T$ do
5:         $(u,t) \leftarrow V(j)$, the $j^{th}$ slot, where $V$ is the set of all slots.
6:         $S_u^i(t) = \arg \max_{S_u^i} R(S_u^i) \text{ s.t. } S_u^i(t) \in M$.
7:     end for
8: end while

Proposition 5.5.1. In an asymmetric SU system, Approach 1 may result in a SO that does not satisfy Condition 1.

Proof. Refer to Appendix 5.10.1

Clearly, Approach 1’s inability to find an SO that satisfies Condition 1 is related to the asymmetry of the system. The proof of this proposition also demonstrates why this problem will not be faced by symmetric systems, studied in the previous papers. The issue presented in Proposition 5.5.1 suggests that Algorithm 3 has to be modified to result in an SO that satisfies Condition 1. A straightforward solution for this problem is to iterate over every single $\{u,t\}$-slot in search of the one that is not (locally) optimal, given all other selections, and then find and replace the one that results in the highest increase in reward. After multiple iterations over all the $\{u,t\}$-slots, each channel selection will be such that it is optimal, given all other selections, satisfying Condition 1. The next result presents the conditions for which a greedy approach of SOS can result in an optimal solution, an SO with the maximum reward.

Proposition 5.5.2. The Greedy approach of SOS is optimal only for systems where the expected throughput is not a function of contention or interference among SUs.

Proof. Refer to Appendix 5.10.2 for the proof of this proposition.

Proposition 5.5.2 further implies that certain non-greedy selections may result in an increase of availabilities. But, not every non-greedy selection that results in an
increase of availability may lead to the optimal SO. Furthermore, it has been shown in Proposition 5.5.2, that a non-greedy selection, that does not increase the availability of a channel, will certainly not result in the optimal SO.

Another aspect of our work that differs from previous papers is that previous papers select the same set of channels for all SUs, in different orders ([59, 61]). We show that the transmission opportunity in channels is bounded. Any further selection of a channel, once its utilization has reached the upper bound, will not result in a significant increase in reward. This proposition will make use of the sum of acquisition probabilities of a channel, defined next.

**Definition 3.** Given $S$, the sum of probabilities of all events for which a channel $p$ will be acquired during a frame time, is called the utilization of that channel. The utilization is defined as $\alpha_p(S) = \sum_{\{u,t\} \in \nu_p(S)} p^S_{u}(t)$.

The following proposition demonstrates the upper bound on the transmission opportunity offered by a channel. Since, systems with perfect detection are symmetric, we drop the subscripts of the received power and denote them by $\tilde{\sigma}^2$ and $\bar{\sigma}^2$, for the sake of simplicity. Let $v_p(S)$ and $\omega_p(S)$ denote the set of $\{u,t\}$-slots and the set of SUs for which channel $p$ has been selected, respectively.

**Proposition 5.5.3.** Assuming perfect detection, we consider a sensing order $S$ where a channel $p$ is selected for a set $\omega_p(S)$ of SUs. If channel $p$ is selected for all $T$ slots of each SU ($\in \omega_p(S)$), then the utilization of this channel is given by

$$\alpha_p(S) = \begin{cases} 
|\omega_p(S)|, & \text{if } \tilde{\sigma}^2 = \bar{\sigma}^2 = 0, \\
\theta_p|\omega_p(S)|, & \text{if } \tilde{\sigma}^2 = 0, \\
1, & \text{if either } |\omega_p(S)| = 1, \tilde{\sigma}^2 = 0, \text{ or } |\omega_p(S)| > 1, \bar{\sigma}^2 > \sigma^2_e, \tilde{\sigma}^2 = 0, \\
\theta_p, & \text{if either } |\omega_p(S)| = 1, \text{ or } |\omega_p(S)| > 1, \bar{\sigma}^2 > \sigma^2_e. 
\end{cases}$$

(5.15)

**Proof.** Refer to Appendix 5.10.3. □

This shows that the opportunity offered by a channel is limited by the received primary and secondary powers. This fact remains unchanged even in the presence
of sensing imperfections. This proposition implies that the SOS methods that use orthogonal SOs (as in [61]), or the ones that select a channel for all SUs (as in [59]) are suboptimal. Based on the insights presented in this section, the next section proposes an optimal algorithm.

5.6 Sensing Order Selection: The Optimal Approach

Having discussed the suboptimal approach in the previous section, we present an algorithm called Pruning based Sensing Order Selection (PSOS) to find an optimal SO. We further show that the optimality of PSOS comes at the cost of a slightly higher complexity, when compared to the greedy approach. The alternative methods to PSOS are Brute-force or heuristic based search algorithms like Particle Swarm Optimization or the Genetic algorithm. These algorithms not only have a very high complexity, but often their convergence to the optimal solution cannot be guaranteed. Before we discuss PSOS, we show that this problem is solvable using the DP approach, as well.

**Proposition 5.6.1.** Consider a single SU scheduled to sense in $T$ sensing slots. Let $S^* = (L^*_1, \ldots, L^*_T)$ be an optimal SO that maximizes $R^S(1)$. Then, for any $t \in [1, T]$, $(\bar{L}_t \ldots L_T)$, and $\bar{S} = (L^*_1, \ldots, L^*_{t-1}, \bar{L}_t \ldots \bar{L}_T)$, we have

$$R^{S^*}(t) \geq R^{\bar{S}}(t).$$

(5.16)

**Proof.** The two SOs $S$ and $S^*$ differ from one another in terms of the channel selection, from sensing slots $t$ through $T$. We prove the proposition by contradiction. Suppose, that despite $S^*$ being optimal, the reward from sensing slot $t$ through $T$ is such that
we have $\mathbb{R}^{S^*}(t) < \mathbb{R}^{\bar{S}}(t)$. We have,

$$\mathbb{R}^{S^*}(t - 1) = \sum_{i=t-1}^{T} \sum_{u \in U} p_u^{S^*}(t)i \mathcal{R} = \sum_{u \in U} p_u^{S^*}(t-1)\mathcal{R} + \mathbb{R}^{S^*}(t) < \sum_{u \in U} p_u^{S^*}(t-1)\mathcal{R} + \mathbb{R}^{\bar{S}}(t) = \mathbb{R}^{\bar{S}}(t - 1),$$

(5.17)

where the last inequality follows from $\mathbb{R}^{S^*}(t) < \mathbb{R}^{\bar{S}}(t)$. The conclusion (5.17) also uses the fact that the sensing orders are the same from sensing slot 1 to $(t - 1)$, implying that $\sum_{u \in U} p_u^{S^*}(t-1)\mathcal{R} = \sum_{u \in U} p_u^{\bar{S}}(t-1)\mathcal{R}$. Proceeding in the same way, it can be shown that $\mathbb{R}^{S^*}(1) < \mathbb{R}^{\bar{S}}(1)$, which contradicts the assumption that $S^*$ is an optimal sensing order. This completes the proof.

Proposition 5.6.1 shows that the DP approach as discussed in [131], can be used to solve (5.12). But this approach becomes intractable as the number of SUs or the sensing slots increase. Further, at every stage, the reward contributed by each channel depends on all previous selections (highlighted in Proposition 5.5.3). Consequently, the minimum number of evaluations required for the optimal solution is $|M||U||T|$ i.e., the entire solution set.

We have shown earlier that non-greedy selections may result in the optimal solution. Unless, we can differentiate between the selections that can lead to an optimal SO, and the ones that cannot, we will have to evaluate all possible channel combinations. This is an exhaustive exercise and becomes computationally intractable as the number of SUs, the channels, or the sensing slots increase. A rule that effectively allows us to discard certain channel selections without the need to evaluate them, is studied next.

Let $\bar{S}$ be the greedy sensing order such that $\bar{S} = (Q_1, Q_2)$, where $Q_1 = (\bar{L}_t)_{i=1}^{t-1}$ and $Q_2 = (\bar{L}_t)_{i=t}^{T}$. Let $A = \{p \in M|p \neq p_1, u_p(Q_2) \neq \emptyset\}$ denote the set of channels other than $p_1$. Let the availability of the channel selected at the last stage of the
5.6. Sensing Order Selection: The Optimal Approach

Greedy algorithm be $\bar{a}(\bar{S})$. By definition of the greedy algorithm, $\bar{a}(\bar{S})$ is the lowest availability.

Algorithm 2 Pruning based Sensing Order Selection (PSOS)

Initialize $G_0 = \emptyset$.

for $t = 1$ to $T$

for $i = 1$ to $|G_{t-1}|$

Given $Q_1 = G_{t-1}(i)$, let $Q_2 = (\bar{L}_t, \ldots, \bar{L}_T)$ be the greedy result.

if $\bar{a}^2 = 0$ or Channel-reselection is not allowed then

Given $Q_1$, the greedy $L_t$ is optimal (Proposition 5.5.2), and $G_t \leftarrow \{G_t, (Q_1, L_t)\}$.

else

Let $SS$ be a set of sensing slot selections $L_t$, where $L_t$ is defined as in Proposition 5.6.2.

For each $L_t \notin SS$, we form $(Q_1, L_t)$. Then, $G_t \leftarrow \{G_t, (Q_1, L_t)\}$.

end if

end for

end for

Return set $G_T$.

Proposition 5.6.2. Given $Q_1$, any $L_t$ is strictly suboptimal if

$$a_{p_1}(u, t, (Q_1, L_t)) < \bar{a}(\bar{S}), \quad \forall \{u, t\} \in v_{p_1}(L_t) \text{ s.t. } \{u, t\} \notin v_{p_1}(\bar{L}_t),$$

(5.18)

where $\bar{L}_t$ is such that $v_{p_1}(\bar{L}_t) \subset v_{p_1}(L_t)$ and $\alpha_{p_1}(Q_1, \bar{L}_t) \geq \alpha_{p_1}(\bar{S})$.

Proof. Refer to Appendix 5.10.4

Algorithm 2 initializes at sensing slot $t = 1$, with no channel selected for any SU. At any sensing slot $t$, given previous selections $Q_1 = (L_{i=1}^{t-1})$, PSOS uses the greedy algorithm to select channels from sensing slot $t$ through $T$, denoted by $Q_2 = (L_{i=t}^{T})$. Using these selections and the result presented in Proposition 5.6.2, PSOS forms a set of selections at sensing slot $t$ that can lead to the optimal SO. At every sensing slot $t$, given $Q_1$, the more the number of $t$-slot selections that satisfy the criterion (of Proposition 5.6.2), the more will be the number of branches that sprout from each node. PSOS essentially implements pruning, based on Proposition 5.6.2, to reduce
the number of branches, and the overall complexity. At sensing slot $t$, given previous selections, the algorithm selects a small set of selections (that do not belong to set $SS$). Proposition 5.6.2 is also a proof of the optimality of this approach.

### 5.7 Distributed SOS for Complexity Reduction

This section presents the idea of distributed SOS. The purpose of introducing a distributed implementation is to find an SO for each SU at a minimum cost of reward but with a considerable decrease in complexity. The reduction in computational cost is independent of the optimal algorithm presented in the previous section. It is demonstrated that the implementation of a brute force search instead of PSOS can still result in considerable complexity reduction. This section also presents an analysis on the throughput and complexity performance of the network under a distributed implementation. The proposed approach that forms a distributed implementation is called D-SOS. The complexity-throughput tradeoff is also highlighted by this analysis.

Since, a reduction in computational complexity is one of the key features of this contribution, we investigate its motivation. Consider a secondary network divided into $v$ number of clusters of SUs. If $k_i$ is the set of SUs in cluster $i$, then this distributed implementation is such that $\bigcup_{i=1}^{v} k_i = U$, and $\bigcap_{i=1}^{v} k_i = \emptyset$. The total number of possible SO for a cluster is $|M|^{\sum_{i} |k_i|}T$, while the total number of SO for the entire network is $\sum_{i \in v} |M|^{\sum_{i} |k_i|}T$. It can be shown that for any $v > 0$, we have

$$\sum_{i \in v} |M|^{\sum_{i} |k_i|}T < |M|^{\sum_{i} |U|}T.$$  \hspace{1cm} (5.19)

The inequality (5.19) implies that the complexity of finding the optimal solution through a brute force search is higher for the centralized scheme compared to a distributed one. In a distributed implementation, the optimal CSO (cluster sensing order) is selected for each cluster, by the cluster controller, assuming no inter-cluster interference. Naturally, a communication link would be required between the cluster controller and the SUs within the cluster. This requirement is unlike [59, 117, 122]
where a link would be required between all SUs and the controller. The next proposition presents the impact of distributed SO selection on spectral utilization. It is shown that even if each cluster performs brute force search for the optimal SO, transmission opportunity will not be completely utilized.

Let $\bar{\eta}(S)$ be the unique number of channels in the SO $S$.

**Proposition 5.7.1.** If $S_u^*$ is the optimal SO selected independently by SU $u$, then we have

$$\min_{u \in U} \bar{\eta}(S_u^*) \leq T \leq \bar{\eta}(\bigcup_{i=1}^{v} S_{k_i}^*) \leq \bar{\eta}(S_U^*),$$

(5.20)

where $\bar{\eta}(S)$ represents the unique number of channels in $S$.

**Proof.** Consider an SU that has to select channels for its $T$ sensing slots without considering the contention or interference among all SUs. As a result of greedy selection, we have

$$\bar{\eta}(S_u^*) = T.$$  

(5.21)

Because of imperfect sensing, there exists a non-zero probability that the SU (ignoring contention and interference) may find re-sensing a channel more rewarding compared to sensing a new channel. For such a scenario, we have

$$\bar{\eta}(S_u^*) < T.$$  

(5.22)

If $\tilde{\sigma}_{u,1}^2$ is the average power received by SU $u$ from primary transmitter on channel $M(1)$, and $\tilde{\sigma}_{u,1}^2 = \tilde{\sigma}_{v,1}^2$ for all $u, v \in U$, then given that the prior availability probabilities are the same, we have

$$S_u^* = S_v^*,$$

for all $u, v \in U$  

(5.23)

$$\Rightarrow \bar{\eta}(\bigcup_{i \in U} S_i^*) = \bar{\eta}(S_u^*),$$

for any $u$  

(5.24)
where the equality (5.23) is possible because each SU selects its own SO independently, and with exactly the same information. If all SUs select their SO greedily by themselves, then the SU network is essentially a $|U|$-cluster network i.e., a network with the maximum possible clusters. The equality (5.24) holds for such a cluster. On the other hand, if the received powers are not the same, SUs may end up selecting different SO for themselves (still independent of one another). This will result in

$$S^*_i \neq S^*_j, \quad \text{for all } u, v \in U, \text{ where } u \neq v$$  \hspace{1cm} (5.25)

$$\Rightarrow \bar{\eta}(\cup_{i \in U} S^*_i) > \min_{u \in U} \bar{\eta}(S^*_u).$$  \hspace{1cm} (5.26)

The inequalities (5.24) and (5.26) prove the leftmost part of (5.20). Next, we investigate a network with less than $|U|$ number of clusters. Having a network with less than $|U|$ number of clusters implies that there will be at least one cluster with more than one SU. Let such a cluster be denoted by $k_1$. Furthermore, let $M(1)$ be the channel most frequently selected i.e., $M(1) = \arg \max_{i \in M} |v_i(\{S^*_u\}_{u \in k_1})|$. We further assume that channel $M(1)$ has been selected by some SU $u_1 (u_1 \in k_1)$ in sensing slot $t$. Let $Z_1$ be the set of CSA realizations where SU $u_1$ fails to acquire a channel before sensing slot $t$, and acquires channel $M(1)$ in $t$. We further divide set $Z_1$ into $\bar{n}$ number of subsets (each indexed by $i$ $(1 \leq i \leq \bar{n})$ and denoted by $Z_{1,i}$) such that the power received by SU $u_1$, in each of these subsets can be denoted by $\sigma^2_{z,i}$ (where $\sigma^2_{z,i} \neq \sigma^2_{z,j}$, for $i \neq j$).

Since, the probability of contention detection is a function of the received power, higher the received power, higher will be the contention probability and lower will be the channel acquisition probability. Once, sensing has reached slot $t$, the probability that SU $u_1$ will acquire channel $M(1)$ can be denoted by $\hat{p}^Q_{z,i}(u_1, t, M(1))$, where $Q = \{S^*_u\}_{u \in U}$. Owing to the channel resolution process considered, we have

$$\hat{p}^Q_{z,i}(u_1, t, M(1)) \propto (1 - P_1(z, i)^2).$$  \hspace{1cm} (5.27)

Furthermore, let the sets $Z_{1,i}$ be such that $P_1(z, i) > P_1(z, i + 1)$. If $\hat{p}^Q_{z}(u_1, t)$ (where $Q = \{S^*_u\}_{u \in k_1}$) is the probability that SU $u1$ fails to acquire in all slots before
5.7. DISTRIBUTED SOS FOR COMPLEXITY REDUCTION

Let \( t \), then the probability that SU \( u_1 \) will acquire a channel in slot \( t \), can be given by

\[
I_Q(M(1), u_1, t) = \theta_p \sum_{i=1}^{\hat{n}} \sum_{z \in Z_{1,i}} \hat{p}_z^Q(u_1, t) \hat{p}_{z,i}^Q(u_1, t, M(1)) + (1 - \theta_p) \sum_{i=\hat{n}+1}^{\tilde{n}} \sum_{z \in Z_{1,i}} \bar{p}_z^Q(u_1, t) \hat{p}_{z,i}^Q(u_1, t, M(1)),
\]

(5.28)

where \( \tilde{n} = 2\hat{n} \). For a system where the inter-SU interference is assumed to be zero, \( \hat{p}_{z,i}^Q(u_1, t, M(1)) = \hat{p}_{z,j}^Q(u_1, t, M(1)) \), where \( j \neq i \). For such a system, the increase in reward that results from the selection of channel \( M(1) \) can be denoted by

\[
\bar{I}_Q(M(1), u_1, t) = \theta_p \cdot |\hat{n}| \cdot |Z_1| \cdot \hat{p}_z^Q(u_1, t) \cdot \hat{p}_z^Q(u_1, t, M(1)) + (1 - \theta_p) \cdot |\hat{n}| \cdot |Z_1| \cdot \bar{p}_z^Q(u_1, t) \cdot \hat{p}_z^Q(u_1, t, M(1)),
\]

(5.29)

where the index \( i \) has been dropped for the sake of simplicity. Since, for any \( i \),

\[
\hat{p}_z^Q(u_1, t, M(1)) \geq \hat{p}_{z,i}^Q(u_1, t, M(1)),
\]

we have

\[
\bar{I}_Q(M(1), u_1, t) > I_Q(M(1), u_1, t)
\]

(5.30)

This shows that having considered the interference, the increase in reward that results from the reselection of a channel is lower than the reward of the no-interference scenario. Since, in a \(|U|\)-cluster network, each SU assumes the inter-SU interference to be zero, it miscalculates the increase in reward that results from the reselection of channel \( M(1) \). Let

\[
G_{1,1} = \sum_{i=1}^{\hat{n}} \sum_{z \in Z_{1,i}} \hat{p}_z^Q(u_1, t) \hat{p}_{z,i}^Q(u_1, t, M(1))
\]

(5.31)

\[
G_{1,2} = \sum_{i=\hat{n}+1}^{\tilde{n}} \sum_{z \in Z_{1,i}} \bar{p}_z^Q(u_1, t) \hat{p}_{z,i}^Q(u_1, t, M(1)).
\]

(5.32)

If there exists some channel \( M(2) \) such that it has not been selected before, and the
power received by SU $u_1$ is such that $\tilde{\sigma}_{u_1,1} = \tilde{\sigma}_{u_1,2}$, then we have

$$G_{1,1} + G_{1,2} < G_{2,1} + G_{2,2}. \tag{5.33}$$

The above inequality holds because $\hat{p}^Q_{z,i}(u_1, t, M(2)) \leq \hat{p}^Q_{z,i}(u_1, t, M(1))$, for all $i$. Furthermore, for some $\theta_2/\theta_1$, we have

$$\theta_1 G_{1,1} + (1 - \theta_1)G_{1,2} < \theta_2 G_{2,1} + (1 - \theta_2)G_{2,2}. \tag{5.34}$$

The above inequality implies that the selection of channel $M(2)$ will result in a higher increase in reward compared to the selection of channel $M(1)$. This further shows that the number of channels selected in a cluster is always greater than or equal to the number of channel selected by SUs for themselves (ignoring the inter-SU interference). As a result, the maximum number of channels will be selected by a cluster that takes into account the interference among all SUs. This completes the proof. \hfill \Box

If $K^{(j)} = \{k^{(j)}_i\}_{i=1}^{v_j}$ is a distributed implementation of a network with $v_j$ number of clusters, then the NSO of this network can be denoted by $Q_j = \{S^*_{k^{(j)}_i}\}_{i=1}^{v_j}$. The complexity of optimal SOS, by the brute search approach, for a distributed implementation $K^{(1)}$, equals to $O(\sum_{i \in v_1} |M|^{||k^{(1)}_i||})$. Higher the number of clusters, lower will be the computational complexity of finding the optimal solution. Furthermore, higher the number of clusters, higher will be the unaccounted-for inter-cluster contention and interference losses, and lower will be the reward.

**Proposition 5.7.2.** Let $K^{(1)}$ be such that for any two of its clusters, $k^{(1)}_i$ and $k^{(1)}_j$, either the inter-cluster interference is negligible or $S^*_{k^{(1)}_i} \cap S^*_{k^{(1)}_j} = \emptyset$. Then, we have

$$\mathbb{R}^{Q_1} = \mathbb{R}^{S^*_U}, \tag{5.35}$$

where $S^*_U$ is the NSO formed by the central controller, for all SUs in the network.
Proof. One way the equality (5.35) is only possible if

\[ Q_1 = S^*_U. \] (5.36)

The only way (5.36) fails to hold is if some cluster controller erroneously estimates the opportunity in one or more channels. As neither the prior availability probability nor the estimate of the probability of detection are affected by the clustering (defined in the proposition), (5.36) will hold, and so will (5.35). This completes the proof. \( \square \)

The next proposition is a result on the utilization of the channel with the highest prior availability probability. Let \( S \) be the greedy SO selected for all SUs of the network.

**Proposition 5.7.3.** If \( p_1 = \arg \max_p \{ \theta_p \} \), then for any distributed implementation with NSO given by \( Q_1 \), we have

\[ \alpha_{p_1}(Q_1) \geq \alpha_{p_1}(S), \] (5.37)

given that \( v_1 > w_{p_1}(S) \).

Proof. Channel \( p_1 \) would be selected by at least \( v_1 \) number of SUs, in the first sensing slot, within the distributed implementation. Let \( \hat{v}_{p_1}(Q_1) = \{(u,t) | t = 1, Q_1(u,t) = p_1\} \) be the set of such SUs. Furthermore, there would be other SUs for which channel \( p_1 \) maybe selected for other than the first slot. Let the set of these SUs be denoted by \( \hat{v}_{p_1}(Q_1) = \{(u,t) | t > 1, Q_1(u,t) = p_1\} \).

Let \( lg \subset \hat{v}_{p_1}(Q_1) \) be a set of SUs such that \( \eta(lg) = \eta(w_{p_1}(S)) \). If \( y_p(x, S) \) is the vector of probabilities that sensing reaches channel \( p \) (for each SU in \( x \)), then we have

\[ y_{p_1}(lg, Q_1) \geq y_{p_1}(v_{p_1}(S), \hat{S}). \] (5.38)

The inequality (5.38) holds because channel \( p_1 \) may not be selected for the first sensing slot, for all SUs \( u \in S \), by the greedy approach. The utilization of channel \( p_1 \), within
the distributed implementation, can be given by

\[
\alpha_{p_1}(Q_1) = \sum_{(u,t) \in \hat{\nu}_{p_1}(Q_1)} p_u^{Q_1}(t) + \sum_{(u,t) \in \nu_{p_1}(Q_1)} p_u^{Q_1}(t) > \sum_{(u,t) \in \nu_{p_1}(Q_1)} p_u^{Q_1}(t) > \sum_{(u,t) \in \nu_{p_1}(\hat{S})} p_u^{\hat{S}}(t) = \alpha_{p_1}(\hat{S}).
\]

(5.39) (5.40) (5.41) (5.42)

where inequality (5.41) comes from the fact that \( \eta(\hat{\nu}_{p_1}(Q_1)) > \eta(w_{p_1}(\hat{S})) \). Furthermore, (5.42) holds because of (5.38). The inequality (5.42) proves that the utilization of channel \( p_1 \) will definitely be higher for NSO \( Q_1 \) compared to its utilization in the NSO \( \hat{S} \), under the conditions described in this proposition. This completes the proof.

The following proposition presents a condition where the computational complexity of a greedy approach of NSO selection is lower than that of a distributed implementation, while the achievable reward is higher.

**Proposition 5.7.4.** If for some distributed implementation \( K^{(1)} \), we have

\[
\bar{\eta}(\hat{S}) > \bar{\eta}(Q_1),
\]

(5.43)

\[
v_1 > w_{p_1}(\hat{S}),
\]

(5.44)

where \( p_1 = \arg \max \{ \theta_p \}_{p \in M} \), then for any distributed implementation \( j \), such that \( v_j \geq v_1 \), we have

\[
\mathbb{R}^\hat{S} > \mathbb{R}^{Q_1}.
\]

(5.45)

**Proof.** To prove this proposition, we consider the contrary to be true i.e., given the
5.7. DISTRIBUTED SOS FOR COMPLEXITY REDUCTION

conditions (5.43) and (5.44), we have

\[ R^\mathcal{S} \leq R^{Q_1}. \]  (5.46)

For (5.46) to hold, \( Q_1 \) should at least satisfy the necessary condition of optimality. Because, if the condition does not satisfy, then by definition there exists a greedy NSO that has a higher reward than \( Q_1 \). And, for such a scenario, the distributed implementation is not only computationally expensive but will also have a poor throughput performance.

The inequality (5.43) suggests that there is at least one channel (denoted by \( p_2 \)), in the set \( \bar{\eta}(\hat{\mathcal{S}}) \), that has not been selected for any SU, in the distributed implementation. Let \( v_{p_2}(\hat{\mathcal{S}}) \) be the set of \((u, t)\)-slots it has been selected for, by the greedy approach. Then, for all \((u, t) \in v_{p_2}(\hat{\mathcal{S}})\), given all other \(|\hat{\mathcal{S}}| - |v_{p_2}(\hat{\mathcal{S}})|\) selections, we have

\[ a_{p_2}(u, t, \hat{\mathcal{S}}) > a_p(u, t, \hat{\mathcal{S}}), \quad (u, t) \in v_{p_2}(\hat{\mathcal{S}}), p \neq p_2. \]  (5.47)

Having discussed the availability of channel \( p_2 \), we discuss the NSO for distributed implementation under the conditions mentioned in the proposition. Since, \( p_2 \notin \bar{\eta}(Q_1) \), then the only way \( Q_1 \) can satisfy the necessary condition of optimality is if for all \( p \neq p_2 \), we have

\[ a_p(u, t, Q_1) > a_{p_2}(u, t, Q_1), \quad \forall(u, t). \]  (5.48)

But, Proposition 5.7.3 shows that a channel \( p_1 \) (defined in the Proposition) will have a higher utilization in the distributed implementation compared to \( a_{p_1}(\mathcal{S}) \). The utilization will be higher even when selected for \(|w_{p_1}(\hat{\mathcal{S}})|\) number of SUs, and can be expressed as

\[ \alpha_{p_1}(\hat{\mathcal{S}}) \leq \sum_{(u, t) \in \lg} p_u^{Q_1}(t), \]  (5.49)

where \( \lg \) is a set of \((u, t)\)-slots spanning \( \eta(w_{p_1}(\hat{\mathcal{S}})) \) number of SUs. It has been shown in
(5.47) that for the greedy NSO, the availability of channel $p_1$ (among other channels) was less than that of $p_2$, for a utilization given by $\alpha_{p_1}(\hat{S})$. An increase in channel $p_1$'s utilization, as a result of the distributed implementation will result in an even lower availability due to an increase in contention and interference. This will result in

$$a_{p_1}(u, t, Q_1) < a_{p_2}(u, t, Q_1), \quad \forall u \notin w_{p_1}(\hat{S}). \quad (5.50)$$

Despite the inequality (5.50), channel $p_1$ will be selected for at least $v_1$ number of clusters, due to the lack of cooperation among cluster controllers. This suggests that there will always exist a greedy NSO with a reward higher than the reward of the NSO i.e.,

$$R^{Q_1} < R^\hat{S}. \quad (5.51)$$

Furthermore, any distributed implementation with the number of clusters more than $v_1$ will have channel $p_1$ selected even more, resulting in an even higher utilization. This will always lead to an NSO that does not satisfy the necessary condition of optimality. This completes the proof.

Based on the analysis and the results presented in this section, we develop a distributed algorithm. Finally, we address the case not covered by the previous propositions. This is the case where the the inter-SU interference is very high and the number of channels along with the opportunity is very low. For such a scenario, it can be shown that the reward of a greedy algorithm approaches that of the brute-force search.

The performance of the approach proposed in this paper is numerically compared to the approaches proposed in the previous papers. Each SU is a transceiver pair, and throughout this section we assume that the power received by the receiver of each SU, from its transmitter is $\sigma_{ss}^2$. The signal to noise ratio on the link between the transmitter and its receiver is 2db. The first result is the comparison of SOS approaches,
5.7. DISTRIBUTED SOS FOR COMPLEXITY REDUCTION

Algorithm 3 Clustering for Distributed SOS (D-SOS)

1: Group SUs into clusters s.t. $\bar{\sigma}_{u,v}^2 = \bar{\sigma}_{v,u}^2 = 0$, for all $u \in k_j, v \in k_i$, and $i \neq j$ (Proposition 5.7.2 shows that this will not impact the reward).
2: Divide the clusters s.t. $k_i \cap k_j = \emptyset$ (Proposition 5.7.2 shows that this will not impact the reward).
3: Further divide each individual cluster till any further division results in the greedy approach having a higher reward (as shown in Proposition 5.7.4).
4: If the SUs cannot be divided into clusters because of high opportunity and very low opportunity, then use one of the greedy solutions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>200ms</td>
</tr>
<tr>
<td>$\mu$</td>
<td>10ms</td>
</tr>
<tr>
<td>sampling frequency</td>
<td>6MHz</td>
</tr>
<tr>
<td>$R$</td>
<td>5bits/sec/Hz</td>
</tr>
<tr>
<td>Threshold</td>
<td>for which $P_{fa} = 0.1$</td>
</tr>
</tbody>
</table>

Table 5.3: Simulation Parameters

distributed and centralized. Let $\kappa_M = \{\theta_p\}_{p \in M}$ be the set of prior availability probabilities. Similarly, let $\hat{\kappa} = \{\hat{\sigma}_{p,u}^2\}_{p \in M, u \in U}$ and $\bar{\kappa} = \{\bar{\sigma}_{u,v}^2\}_{u,v \in U, u \neq v}$. For this result, 10000 realizations of $\kappa_M$ are generated such that each $\theta_p \sim \mathcal{U}[0, 0.9]$. Furthermore, for each $\kappa_M$, 1000 realizations of $\{\hat{\kappa}, \bar{\kappa}\}$ were generated such that $\hat{\sigma}_{p,u}^2 \in \mathcal{N}(0.1\sigma_{ss}^2, \sigma_{ss}^2)$. Finally, the x-axis represents $mean(\bar{\kappa})$, and for each value of $mean(\bar{\kappa})$, 1000 realizations of $\bar{\kappa}$ are generated such that each $\bar{\sigma}_{u,v}^2 \sim \mathcal{N}(0.1\sigma_{ss}^2, \sigma_{ss}^2)$. For the results presented in Fig. 5.5a and Fig. 5.5b, the number of channels are $|M| = 5$. Fig. 5.5a is a comparison of the different SOS approaches and the proposed D-SOS approach for a 10 SU network. As the number of SUs are increased to 60 for Fig. 5.5b, the throughput performance of the proposed D-SOS approaches that of brute-force search and the greedy approach. This is because of the scarcity of opportunity.

Fig. 5.7 is a depiction of the impact of increase in network size on the achievable throughput of the SO selected for SUs. It can be observed that for very less number of SUs, the throughput performance of all approaches is close to one another. This is because of ample number of channels, the probability of contention decreases
CHAPTER 5. Optimal Channel Selections in a Multi-SU System

Figure 5.5: The achievable reward comparison of different Distributed and Centralized SOS approaches for different network sizes.
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Figure 5.6: Throughput comparison of different approaches as the network size increase.

among SUs. For intermediate number of SUs, the D-SOS performs much better than the greedy approach. This is because D-SOS is able to manage the contention among SUs much better than other approaches. Finally, for very high number of SUs, the opportunity is already very low, and the performance of D-SOS, brute-force and greedy approach comes close to one another. The other approaches randomly select SO, and are unable to manage contention among the SUs as well as the other approaches.

Next, we observe the impact of the distribution of opportunity in the spectrum. At times, the opportunity in the channels may be highly skewed i.e., the availability probability of a very few channels being very high while that of the remaining being very low. Similarly, there may be scenarios where the opportunity in the channels may be uniformly distributed. For the results presented in Fig. 5.8, we have $|M| = 10$. 
The networks are randomly generated as before.

It can be observed in Fig. 5.8 that as the throughput performance of D-SOS approaches that of the brute-force search if the opportunity in the spectrum is uniformly distributed. This is because D-SOS can form more clusters and each cluster can select non-overlapping sets of channels. Finally, we study the computational requirements of the proposed D-SOS approach. The computational complexity of the brute force search, the greedy approach and the approaches based on random SOS does not depend on the opportunity in the spectrum or the inter-SU interference. But, the computational complexity of the proposed approach is a function of the spectral opportunity distribution. Fig. 5.9 shows that if the opportunity is uniformly distributed, the complexity is lower compared to the complexity when the opportunity is skewed. Furthermore, it can be seen that for very low and very high inter-SU interference, the computational requirement is low.

5.8 Numerical Results

The numerical results presented in this section include comparisons with previous works under different network sizes, network topologies and primary availability probabilities. Benchmark methods (proposed in the previous papers) when compared with our proposals, bring forth the advantage offered by PSOS in particular, and SI-SOS strategies in general. The first part of the numerical results compare the SOS
strategies (SI-SOS and PD-SOS), while the second part compares the optimal and suboptimal algorithms in terms of reward and complexity.

### 5.8.1 Comparing SOS strategies- PD-SOS and SI-SOS

The procedures that perform SI-SOS require detection parameters to determine probabilities of channel detection (during the SOS phase for different CSA process realizations $z \in Z$). These detection parameters used for numerical evaluations are listed in Table 5.4. Algorithms based on PD-SOS, while computing the reward, do not require a detection threshold as they consider the probability of detection to be 1, when a channel is busy. The channel gain (on the transceiver link of each SU) for each channel is assumed to be the same throughout the frame time. Let $\sigma^2$ be the average

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame length $T$</td>
<td>10 sec</td>
</tr>
<tr>
<td>Number of sensing slots $T$</td>
<td>10</td>
</tr>
<tr>
<td>Sensing duration $\mu$</td>
<td>0.1 secs</td>
</tr>
<tr>
<td>Transmission Rate $R$</td>
<td>100 bps</td>
</tr>
<tr>
<td>Detection Threshold $\epsilon$</td>
<td>For which the probability of false alarm is 0.1.</td>
</tr>
<tr>
<td>$\mu f_s$</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 5.4: Channel Detection Parameters.
Figure 5.10: The impact of interference power on secondary throughput for the SI-based and the PD-based formulations. The results for PD-SOS were generated through brute-force search.

interference power received by an SU from any other SU. Throughout this section, given the parameter values in Table 5.4, we assume that the probability of contention detection by an SU is such that $1 \geq P_{S_i}^{S}(\epsilon, 0.9\bar{\sigma}^2) \geq 0.9$. Let $\bar{\kappa} = \{\bar{\sigma}_{u,v}\}_{u,v \in U}$ be a set of individual interference powers received by SUs from one another, known as the interference profile. Similarly, let $\bar{\sigma} = \{\bar{\sigma}_{u,i}\}_{u \in U, i \in M}$ be the set of powers received from the primary transmitters. A set of prior availability probabilities of all channels can be denoted by $\bar{\theta} = \{\theta_p\}_{p \in M}$, and is called an activity profile. The number of channels is $|M| = 6$. Throughout this section, we identify network topologies by the mean interference powers among SUs in the network. For example, two different SU topologies may have different interference profiles $\bar{\kappa}$, but these profiles may have the same mean interference power $\bar{\sigma}^2$.

The impact of Interference Power among SUs.

In Fig. 5.10, the x-axis represents the average power received by an SU from another SU, while the y-axis is the expected normalized throughput. Given the average interference power $\bar{\sigma}^2$, we randomly generate 10,000 scenarios, where each scenario can be identified by a unique set $\{\theta, \bar{\kappa}, \bar{\sigma}\}$. All scenarios are such that they correspond to
5.8. NUMERICAL RESULTS

<table>
<thead>
<tr>
<th>No. of Users,</th>
<th>U</th>
<th>Expected Secondary Throughput (bits/sec/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.1 (\sigma^2 ), (T=1), PD-SOS (PD = 1 assumption [59, 61, 62, 64])</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>0.1 (\sigma^2 ), (T=1), SI-SOS (proposed)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.4 (\sigma^2 ), (T=2), PD-SOS (PD = 1 assumption [59, 61, 62, 64])</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>0.4 (\sigma^2 ), (T=2), SI-SOS (proposed)</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.4 (\sigma^2 ), (T=1), PD-SOS (PD = 1 assumption [59, 61, 62, 64])</td>
</tr>
<tr>
<td>3.5</td>
<td>3.5</td>
<td>0.4 (\sigma^2 ), (T=2), SI-SOS (proposed)</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.4 (\sigma^2 ), (T=1), PD-SOS (PD = 1 assumption [59, 61, 62, 64])</td>
</tr>
<tr>
<td>4.5</td>
<td>4.5</td>
<td>0.4 (\sigma^2 ), (T=2), SI-SOS (proposed)</td>
</tr>
</tbody>
</table>

Figure 5.11: The impact on the expected throughput as the number of SUs is increased. The results for PD-SOS were generated through brute-force search.

The average interference power level along the x-axis. The optimal SO and its reward, for each scenario, is determined using two different SOS strategies; 1) PD-SOS, and 2) SI-SOS. A reward expectation is performed over all 10,000 scenarios for both the strategies. These rewards are then plotted against the average interference levels.

The Impact of the Number of slots.

This section highlights the throughput performance gap of the SOS methods under increasing network sizes. Fig. 5.11 presents the expected normalized throughput of SOs, as the number of SUs in the network increases. The results are presented for \(T = 1\) and \(T = 2\). A total of 10,000 scenarios are randomly generated such that the interference profile of all the scenarios have the same average interference power of \(\bar{\sigma}^2\). Optimal SOs are selected for each scenario by the PD-SOS and SI-SOS strategies, and reward expectation is performed over all the scenarios. The results show that the proposed scheme always outperforms the greedy approach even for a system of small number of SUs.
CHAPTER 5. Optimal Channel Selections in a Multi-SU System

5.8.2 The Optimal and Suboptimal Approaches for SI-SOS

The focus of this section is performance measurement of SI-SOS based algorithms (optimal and suboptimal). All strategies for SOS selection (in existing literature), whether they be based on SI-SOS or PD-SOS, are suboptimal greedy algorithms. In order to bring to light the effectiveness of PSOS, we test the throughput performance and complexity performance against the popular greedy algorithm.

Reward

Fig. 5.12 is a performance comparison, in terms of the achievable reward, among the PSOS algorithm and the greedy algorithm. The x-axis indicates the interference power among the SUs. The reward is evaluated for different interference power levels. As before, a total of 10,000 scenarios that differ in terms of the primary activity profile and the network topology, are generated. SOs are selected for SUs, using both PSOS and the greedy algorithm (based on a SI-SOS strategy), and the corresponding rewards are also computed. Fig. 5.12 shows that as the number of \{u, t\}-slots increases, the impact of the PSOS algorithm becomes more pronounced. At the same time, it can be observed that for very high and very low interference powers, the reward that results from the PSOS and the greedy approaches is almost the same.

Figure 5.12: Reward comparison of the optimal (PSOS) algorithm and the suboptimal (greedy) algorithm presented in [59].
Figure 5.13: The complexity of PSOS compared with the greedy approach (presented in [59]), at different interference power levels.

Complexity

One of the most important contributions of this work is the reduction in computational requirements of finding the optimal SO. Prior to our work, the only method for optimal SOS, was brute force search. Fig. 5.13 is a comparison of the computational power required by the greedy approach and the proposed PSOS approach. The y-axis in Fig. 5.13 shows the number of evaluations required by the proposed optimal approach as a multiple of $O(|M||U|T)$ (the complexity of the greedy algorithm). As before, each data point represents the number of evaluations required by PSOS, averaged over 10,000 randomly generated $\{\bar{\theta}, \bar{\kappa}, \bar{\kappa}\}$. Fig. 5.13 shows that the computational requirements of PSOS are very close to the greedy approach when the interference power among the SUs is either very high or low.

5.9 Summary

The problem of sensing order selection for an SU network where imperfect sensing, interference and contention among SUs hampers the throughput maximization objective, has been presented in this chapter. This problem presented in this paper is a straightforward extension of the ideas presented in the previous chapter. The previous
chapter discussed the general idea of a multi-SU system performing discrete (or continuous) channel selections with the adaptive threshold. The focus of this chapter has been discrete selections and finding the optimal channels for all SUs. Furthermore, the problem of discrete selections was generalized to sensing order design, where the number of sensing slots are more than 1. The solution for such a problem, discussed in the previous chapter was brute force search. The high complexity of such an approach makes it intractable as the number of SUs, channels or sensing slots increase. The chapter further touches the issue of asymmetry of the network layout and how existing literature fails to find a solution for it.

The transmission opportunities that arise due to the spatial attributes of multi-channel wireless networks are analyzed. It is to be noted that sensing order selection takes place before the sensing and transmission frame of SUs. SUs sense and then make decisions based on sensing results without communicating with one another during the frame time. This completely decentralized, communication-free operation requires that the SOs be selected in a way that maximizes throughput. The motivation behind the problem formulation presented in this chapter is to make probabilities of detection a function of the received powers, the pathloss, the distance and the fading model. Such an effort introduces sensing imperfections. On the one hand sensing imperfections lead to sensing errors and cause the corresponding losses, while on the other hand, these imperfections pave the way towards utilizing spatial opportunity.

The chapter further highlights the performance of the greedy algorithm and analyzes scenarios where this algorithm can result in the optimal SO. Unlike all previous works that can be classified as SI-SOS strategy, we introduce an optimal algorithm (PSOS) for the sensing order selection problem that incorporates side information and imperfect sensing. Although, the optimal algorithm PSOS, has a higher complexity than the greedy algorithm, the throughput performance of SOs selected by PSOS can be significantly higher for different interference scenarios and network topologies.
5.10 Appendix

5.10.1 Proof of Proposition 5.5.1

An example that shows that Algorithm 3 can result in an SO that does not satisfy Condition 1, would be sufficient to prove this proposition. Consider an SU system where $|U| = 2$ and $T = 2$. Further, let the system be such that the relationship between the primary received powers and the inter-SU interference powers be given by

\begin{align*}
\tilde{\sigma}_{1,1} & \gg \tilde{\sigma}_{2,1}, \\
\tilde{\sigma}_{1,2} & = \tilde{\sigma}_{2,2}, \\
\tilde{\sigma}_{2,1} & = \tilde{\sigma}_{2,2}, \\
\theta_2 & > \theta_1.
\end{align*}

(5.52) (5.53) (5.54) (5.55)

Given a set of selections, for sensing slot 1 ($L_1$), where channel 1 and 2 have not been selected, we discuss the channel selection for sensing slot 2 (denoted by $L_2$). Two possible selections at sensing slot 2 are $\hat{L}_2 = (2, 2)$ and $\tilde{L}_2 = (2, 1)$. Naturally, SUs would not experience contention and interference losses from one another if $\hat{L}_2$ is the channel selection performed.

As has been described earlier, $p_{L_1}^u(1)$ is the probability that SU $u$ acquires a channel in sensing slot 1, while sensing according to $L_1$. If we assume that $p_{L_1}^u(1) - p_{L_2}^u(1) > 0$, then Approach 1 would first pick SU 2, and then SU 1 for channel selection. Because of (5.54) and (5.55), it is straightforward to show that the relationship between the availabilities of channel 1 and 2 can be given by

\[ a_2(2, 2, L_1) > a_1(2, 2, L_1), \]

(5.56)

resulting in channel 2’s selection for SU 2. Further, when the Algorithm moves to
select a channel for SU 1, (5.52) and (5.55) imply that

\[ a_2(1, 2, (L_1, L_2(2))) > a_1(1, 2, (L_1, L_2(2))). \]  

(5.57)

This relationship arises because of the fact that the power received from primary user 1, by SU 1, is very high. This high received power implies that the probability of detection of these transmissions will be very high. A high probability of detection in turn implies that the probability that this SU will miss detection and start transmitting will be very low. Although, the transmissions due to missed detection are not permitted, they do contribute to the SU sum throughput. Naturally, this will result in channel 2’s selection for SU 1. This means \( \hat{L}_2 = (2, 2) \) is picked (Algorithm 3).

Let \( p_{2}^{(L_1, \hat{L}_2)}(2) - p_{2}^{(L_1, \hat{L}_2)}(2) = \epsilon_3 \), and as Algorithm 3 has selected \( \hat{L}_2 \), we have \( \epsilon_3 > 0 \). Further, let \( p_{1}^{(L_1, \hat{L}_2)}(2) - p_{1}^{(L_1, \hat{L}_2)}(2) = \epsilon_4 \). Since, \( \hat{L}_2 \) has been selected instead of \( \tilde{L}_2 \), we have

\[ \epsilon_4 > 0. \]  

(5.58)

This is because of the loss in SU 2’s throughput at sensing slot 2 due to interference. As \( \theta_2/\theta_1 \) approaches 1, \( \epsilon_3 \) decreases. For a certain \( \theta_2/\theta_1 \), we will have

\[ \epsilon_4 - \epsilon_3 \geq 0. \]  

(5.59)

At this point, SO \((L_1, \hat{L}_2)\) will have a higher reward than SO \((L_1, \tilde{L}_2)\). Nevertheless, Algorithm 3 will still select \((L_1, \hat{L}_2)\) because of (5.54) and (5.55). As the SOs \((L_1, \hat{L}_2)\) and \((L_1, \tilde{L}_2)\) differ by a single channel selection, SO \((L_1, \hat{L}_2)\) clearly does not satisfy Condition 1, which can be expressed as

\[ \hat{L}_2(2) \neq \arg \max_{L_2(2)} R^{(L_1(1), L_1(2), L_2(1))}(1). \]  

(5.60)

This completes the proof that Algorithm 3 can result in an SO that fails to satisfy Condition 1.

The main implication of this result is that Algorithm 3 will fail to completely
utilize the channels. This also implies that there may exist certain sensing slot selections that can maximally utilize channels. Such a problem only arises in asymmetric systems. On the other hand, in symmetric systems, the all channels are the same to each SU in terms of the primary received primary power. Further, the probability of missed detection on a channel, is the same for all SUs.

This would further imply that in order to find the sensing slot selection that maximally utilizes the opportunity in channels, different sequence of picking SUs, for channel selection need to be tested. To find the \( L_t \) that results in the highest increase in reward, for an asymmetric wireless system, the maximum number of such possible sequences will be \(|U|!\).

### 5.10.2 Proof of Proposition 5.5.2

In the absence of interference among SUs, the CSA process of one SU does not affect the CSA of the other. No contention and interference implies that the order in which an SU senses its channels does not impact the expected throughput of any other SU, even if it senses the same channels. Consider a system where no channel reselections are allowed i.e., \(|v_p(S)| \leq 1, \forall p \in M\). For any \( u \in U \), and \( t \) (\( 1 \leq t \leq T \)), if \( L_t(u) = p \), the probability that this channel will be acquired is

\[
y_t = \theta_p(1 - P^S_{u,t}(\epsilon, \gamma_1))\zeta_{u,t}(\Gamma_p) + (1 - \theta_p)(1 - P^S_{u,t}(\epsilon, \gamma_2))\zeta_{u,t}(\Gamma_p). \tag{5.61}
\]

where \( \gamma_1 = \sigma_n^2 \), \( \gamma_2 = \bar{\sigma}^2_{u,p} \), and \( \Gamma_p \) is the average power received on the transceiver link. Similarly, the probability of acquisition failure is

\[
1 - y_t = \theta_p(1 - (1 - P^S_{u,t}(\epsilon, \gamma_1))\zeta_{u,t}(\Gamma_p)) + (1 - \theta_p)(1 - (1 - P^S_{u,t}(\gamma_2))\zeta_{u,t}(\Gamma)). \tag{5.62}
\]

Using (5.61) and (5.62) the expected throughput of SU \( u \) can be written as

\[
(y_1c_1 + (1 - y_1)y_2c_2 + \ldots + \prod_{t=1}^{T-1}(1 - y_t)c_T)R, \tag{5.63}
\]
where the expression (5.63) is valid only in the absence of interference. If $c_t = c_{t+1}$, \( \forall t \), (5.63) is not affected by the sequence in which the channels are selected, for each SU. On the other hand, if $c_t > c_{t+1}$, \( \forall t \), and no channel is to be selected more than once, then the greedy approach of selecting the channel with the highest $\theta_p$ for the earlier sensing slots is clearly optimal. Consequently, the SOS problem for a $|U|$-SU network becomes equivalent to an SOS problem of $|U|$ single-SU networks. Since, $c_t > c_{t+1}$ (for all $t$), the greedy selection of a channel with the higher $\theta_p$ at an earlier sensing slot is optimal (also proven in [64]).

To complete the proof of the proposition, we show that in the presence of contention and interference powers, the greedy approach is suboptimal. An example would be sufficient to prove this suboptimality. Consider an SO, denoted by $S_1$, formed by the greedy approach, where $S_1 = (L_1, L_2) = ((1, 2), (4, 3))$. We assume that $\theta_1 \geq \theta_2 \geq \theta_3 \geq \theta_4$. Naturally, the greedy approach will select slots in the sequence $\{u_1, t_1\}, \{u_2, t_1\}, \{u_2, t_2\}$ and $\{u_1, t_2\}$, where $u_1, u_2 \in U$, $t_i < t_{i+1}$ (for $i \geq 0$). Next, we consider a non-greedy SO, denoted by $S_2$, where $S_2 = (S_1, S_2) = ((4, 3), (1, 2))$).

Since, no channel is reselected in $S_1$ and $S_2$, the reward for a single SU can be given by (5.63). Clearly, this reward is maximized if the channel with a higher availability is selected at an earlier slot, for each SU. Furthermore, for $S_1$ and $S_2$, given $c_t = c_{t+1}$ (1 $\leq t < T$), the SOs can be related as

$$R_{S_1}^{S_1}(1) = R_{S_1}^{S_2}(1).$$  \hspace{1cm} (5.64)

Since, channel $p = 1$ has a lower probability of being acquired in $S_2$, compared to $S_1$, we have

$$a_1(u_2, t_2, S_2) > a_1(u_2, t_2, S_1).$$  \hspace{1cm} (5.65)

Depending on $\theta_1, \theta_2, \theta_3, \theta_4$ and the ratio $c_t/c_{t+1}$, we may have

$$a_1(u_2, t_2, S_2) > a_2(u_2, t_2, S_2).$$  \hspace{1cm} (5.66)
The inequality (5.66) would imply that $S_2$ does not satisfy Condition 1, and the reward could be further maximized by selecting channel $p = 1$ for slot $\{u_1, t_1\}$ in $S_2$. After performing this selection, the new SO becomes $S_3 = (S_1, S_2) = ((4, 3), (1, 1))$, and assuming $c_t = c_{t+1}$, we have

$$R^{S_1}(1) = R^{S_2}(1) \leq R^{S_3}(1).$$

The inequality (5.67) will hold for a range of values of $c_t/c_{t+1}$ close to 1. This analysis implies that the greedy approach is suboptimal when the availability of channel selections are affected by interference powers. This completes the proof.

### 5.10.3 Proof of Proposition 5.5.3

Perfect detection implies that the probabilities of false alarm and missed detection equal 0. Further, for the sake of simplicity, we assume that $\zeta_{u_1, t}(\Gamma) = 1$ in the absence of any primary and secondary transmissions, while $\zeta_{u_1, t}(\Gamma) = 0$ in their presence. We consider an SO for SU $u_1$ such that $S = (L_t(u_1))_{t=1}^T$. The utilization of channel $p$, denoted by $\alpha_p(S)$, can be given by

$$\alpha_p(S) = \sum_{t=1}^{T} p_{u_1}^S(t) = \sum_l \mathbb{P}(\bar{l}) \sum_{z \in Z} F^S(\bar{l}, z)

= \theta_p \sum_{t=1}^{T} \prod_{i=1}^{t-1} \left( 1 - (1 - P_{u_1, t}^S(\epsilon, \gamma_1))\zeta_{u_1, t}(\Gamma_1) \right) \left( 1 - P_{u_1, t}^S(\epsilon, \gamma_1) \right) \zeta_{u_1, t}(\Gamma_1) +

(1 - \theta_p) \sum_{t=1}^{T} \prod_{i=1}^{t-1} \left( 1 - (1 - P_{u_1, t}^S(\epsilon, \gamma_2))\zeta_{u_1, t}(\Gamma_2) \right) \left( 1 - P_{u_1, t}^S(\epsilon, \gamma_2) \right) \zeta_{u_1, t}(\Gamma_2),$$

where $Z \subset \mathcal{Z}$ is the set of scenarios where channel $p$ gets acquired, $\gamma_1 = \sigma_{u_1, p}^2 + \sigma_n^2$ and $\gamma_2 = \sigma_n^2$. Similarly, $\Gamma_1$ is the average power received by the SU on the transceiver link, in the absence of interference from the primary transmitter, and $\Gamma_2$ is the average power in its presence. The acquisition probability of this channel is maximized as
\( \tilde{\sigma}_{u_1}^2 \rightarrow 0 \). In such a scenario, the probabilities \( P_{u_1,t}^S(\epsilon, \gamma_1) = P_{u_1,t}^S(\epsilon, \gamma_2) \) (and we can use \( \gamma \) for both \( \gamma_1 \) and \( \gamma_2 \)), and \( \zeta_{u_1,t}(\Gamma_1) = \zeta_{u_1,t}(\Gamma_2) \). The equality (5.68) becomes

\[
\alpha_p(S) = \sum_{t=1}^{T} \prod_{\hat{t}=1}^{t-1} (1 - (1 - P_{u_1,t}^S(\epsilon, \gamma))) \zeta_{u_1,t}(\Gamma)) (1 - P_{u_1,t}^S(\epsilon, \gamma)) \zeta_{u_1,t}(\Gamma). \tag{5.69}
\]

Since, the probability \( P_{u_1,t}^S(\epsilon, \gamma) = 0 \) (perfect detection) and \( \zeta_{u_1,t}(\Gamma) = 1 \) we have

\[
\alpha_p(S) = (1 - P_{u_1,t}^S(\epsilon, \gamma)) \zeta_{u_1,t}(\Gamma) = 1. \tag{5.70}
\]

On the other hand, if the power received on the channel is non-zero, the probability that the SU fails to detect primary transmissions equals 0. The maximum acquisition probability becomes

\[
\alpha_p(S) = \theta_p \sum_{t=1}^{T} \prod_{\hat{t}=1}^{t-1} (1 - (1 - P_{u_1,t}^S(\epsilon, \gamma_1))) \zeta((\hat{I}_1, \hat{I}_{u_1,t})))(1 - P_{u_1,t}^S(\epsilon, \gamma_1)) \zeta_{u_1,t}(\Gamma_1). \tag{5.71}
\]

Given the perfect detection assumption and \( \tilde{\sigma}_{u_1}^2 \rightarrow 0 \), the probability of false alarm also approaches 0. As a result, the probability that the SU acquires a channel in the first time slot approaches 1. Under such conditions, we have

\[
\alpha_p(S) = \theta_p \prod_{t=1}^{t-1} (1 - \zeta_{u_1,t}(\Gamma_1)) \zeta_{u_1,t}(\Gamma_1) = \theta_p. \tag{5.72}
\]

**Selection of a channel by multiple SUs**

Next, we consider a case where this channel is selected for all SUs in all sensing slots i.e., \( S = (L_t(u) = p)_{u \in U, t \in [1,T]} \). If the power received by an SU from any other SU or the primary transmitter is zero, it will acquire the channel sensed in the first sensing slot with probability 1. Then using (5.70), the utilization of this channel can be
derived to be

\[
\alpha_p(S) = \sum_{u \in U} \sum_{t=1}^{T} p_u^S(t) = \sum_{u \in U} \sum_{t=1}^{T} \prod_{i=1}^{t-1} (1 - \zeta_{u_1,i}(\Gamma)) \zeta_{u_1,t}(\Gamma) = |\omega_p(S)|. \tag{5.73}
\]

Similarly, if the primary power received by the SUs is greater than 0, then owing to perfect detection, we have

\[
\alpha_p(S) = \sum_{u \in U} \sum_{t=1}^{T} p_u^S(t) = \sum_{u \in U} \theta_p \sum_{t=1}^{T} \prod_{i=1}^{t-1} (1 - \zeta_{u_1,i}(\Gamma)) \zeta_{u_1,t}(\Gamma) = \theta_p |\omega_p(S)|. \tag{5.74}
\]

Finally, if both the received primary and secondary powers are non-zero, then the probability that an SU fails to detect any transmissions becomes zero. Channel \( p \)'s utilization becomes

\[
\alpha_p(S) = \sum_{u \in U} \sum_{t=1}^{T} p_u^S(t) = \theta_p \sum_{z \in Z} F^S(\tilde{l}, z), \tag{5.75}
\]

where the probability that any SU fails to detect primary presence when the primary channel is busy, is 0. The term \( \sum_{z \in Z} F^S(\tilde{l}, z) \) is the probability that SU \( u \) acquires the channel. For the set of all possible channel acquisition process scenarios, \( Z \), we have

\[
\sum_{z \in Z} F^S(\tilde{l}, z) = 1. \tag{5.76}
\]

We define a set \( Z_1 \subset Z \), where SU \( u_1 \) acquires a channel in any sensing slot. Once, two or more SUs find their channels primary-free, and attempt acquisition of the channel, the probability that any one of them fails to detect contention (from the other) is negligible, because of perfect detection. The probabilities of contention detection before and after back-off approach 1 (in the presence of received power). This implies that a scenario where SUs fail to detect contention from one another (when they attempt to acquire the same channel simultaneously) has zero probability. If \( Z_i \cap Z_j \) represents a set of channel acquisition events where both SU \( i \) and \( j \) acquire channel
CHAPTER 5. Optimal Channel Selections in a Multi-SU System

$p$, then $Z_i \cap Z_j = \emptyset, \forall \{i, j\} \in U$, where $i \neq j$. Further, we form a set $\hat{Z} = \bigcup_{i=1}^{U} Z_i$, such that $\hat{Z} \subseteq Z$. The utilization (5.75) can be shown to be

\[
\alpha_p(S) = \theta_p \sum_{u \in U} \sum_{z \in \hat{Z}} F^S(\tilde{l}, z) = \theta_p \sum_{z \in \hat{Z}} F^S(\tilde{l}, z) = \theta_p \sum_{z \in Z} F^S(\tilde{l}, z)
\]

(5.77)

where the inequality (5.77) arises from the fact that the probability of the event where no SU acquires channel $p$ is zero, because of perfect detection. Using the same line of reasoning, it can be shown that when the power received from the primary transmitters is negligible while that received from other SU is non-zero, we have

\[
\alpha_p(S) = 1.
\]

(5.79)

The results (5.70), (5.72), (5.73), (5.74), (5.78) and (5.79) prove (5.15). This completes the proof.

5.10.4 Proof of Proposition 5.6.2

Given $Q_1$, we consider a set of selections at time slot $t$, denoted by $\bar{L}_t$ where $|\nu_{p_1}(\bar{L}_t)| = |\nu_{p_1}(Q_2)|$. The selections are such that the utilization of channel $p_1$ is higher than (or equal) to what it was in $Q_2$ i.e., $\alpha_{p_1}(Q_1, Q_2) \leq \alpha_{p_1}(Q_1, \bar{L}_t)$. We are to show that any further selection of channel $p_1$ in this time slot is strictly suboptimal. For any slot $\{u_1, t\} \notin \nu_{p_1}(Q_1, Q_2), \nu_{p_1}(Q_1, \bar{L}_t)$, owing to the increased utilization of channel $p_1$ (in $\bar{L}_t$), we have

\[
a_{p_1}(u_1, t, (Q_1, \bar{L}_t)) \leq a_{p_1}(u_1, t, (Q_1, Q_2)) < \bar{a}.
\]

(5.80)

The above inequality holds because an increase in utilization of channel $p_1$ increases the probability of contention with other SU (for which this channel is selected). Furthermore, an increase in one channel’s utilization means that one or more channels will be selected for slots with less weight (i.e., the probability that sensing reaches
these slots is less). This implies that an increase in one channel’s utilization results in a decrease in the utilization of one or more channels. Since, channel $p_1$’s utilization has increased, the utilization of other channels would decrease.

In $(Q_1, Q_2)$, all channels (other than $p_1$) have an availability higher than $\bar{a}(S_1)$ when selected by the greedy algorithm. Now, after a decrease in their utilization, these channels will definitely have a higher availability (if selections are performed by the greedy algorithm). We have

$$a_p(u, t, (Q_1, Q_2)) \geq \bar{a}(S_1), \quad (5.81)$$

for all channels $p \in A = \{p \in M|p \in Q_2, p \neq p_1\}$, and $Q_2$ being the selections performed by the greedy algorithm, given $\bar{L}_t$. Given $Q_1$, and the selection of channel $p_1$ (for $|v_{p_1}(Q_2)|$ number of slots) at time slot $t$, let $B$ be the $\{u, t\}$ slots for which channels are to be selected from time slot $t$ through $T$. Consider an algorithm (denoted by $C_1$) that does not select channel $p_1$ for more than $|v_{p_1}(Q_2)|$ number of slots from time slots $t$ through $T$. Let $\beta_1$ be the set of all SOs, formed by Algorithm $C_1$, where $|v_{p_1}(\bar{S}_t, \ldots, S_T)| = |v_{p_1}(Q_2)|$. Furthermore, let $\bar{S}_1 = \max_S \beta_1$. Now, consider another algorithm (denoted by $C_2$) that does select channel $p_1$ for some slot in $B$ i.e., $|v_{p_1}(\bar{L}_t, \ldots, L_T)| = |v_{p_1}(Q_2)| + 1$, and let $\bar{S}_2 = \max_S \beta_2$. Then, we have

$$\sum_{\{u, t\} \in v_A(\bar{S}_1)} p^\bar{S}_1(t) \geq \sum_{\{u, t\} \in v_A(\bar{S}_2)} p^\bar{S}_2(t), \quad (5.82)$$

where $v_A(\bar{S}_i) = \bigcup_{p \in A} v_p(\bar{S}_i)$. The above inequality holds because $v_A(\bar{S}_2) \subset v_A(\bar{S}_1)$. The purpose of this proposition is to show that algorithm $C_2$ and the SO formed by this algorithm is suboptimal. But, if this algorithm were to be optimal, then we would have $R^\bar{S}_1(1) \leq R^\bar{S}_2(1)$, which implies that

$$\sum_{\{u, t\} \in v_A(\bar{S}_1)} p^\bar{S}_1(t) - \sum_{\{u, t\} \in v_A(\bar{S}_2)} p^\bar{S}_2(t) \leq p^\bar{S}_2(t_1) \quad (5.83)$$

where $\{u_1, t_1\} \in B$ and $s_{u_1}(t_1) = p_1$. For the inequality (5.83) to be true, the
availability of channel $p_1$ at each of its slots $\{u,t\} \in \nu_{p_1}(\bar{S}_2)$ has to be such that

$$a_{p_1}(u,t,\bar{S}_1) \geq \bar{a}(S_1). \quad (5.84)$$

As the inequality (5.84) cannot hold (shown earlier), (5.83) is not satisfied. This implies that

$$\mathbb{R}^{\bar{S}_1}(1) > \mathbb{R}^{\bar{S}_2}(1). \quad (5.85)$$

This shows that the selection of channel $p_1$ for $|\nu_{p_1}(Q_2)| + 1$ number of slots such that the utilization of the $|\nu_{p_1}(Q_2)|$ number of slots has already been increased, is suboptimal.
Chapter 6

Conclusion and Future Work

6.1 Conclusion

This dissertation demonstrates the significance of incorporating side information in SU system design. It has been shown that by carefully including the average received powers into the design, a significant increase in the secondary network’s throughput can be achieved. The usefulness of this side information manifests itself in the form of lowered contention and interference among SUs and lowered interference at the primary.

Having discussed the need for low complexity, and efficient procedures that can result in the optimal solutions, algorithms that converge to the optimal solution, are presented. Secondary systems are required to be efficient in their pursuit of transmission opportunities. Apart from the cognitive abilities, an effective cooperation plays a role that cannot be neglected. The important results and their impact on the secondary system design are discussed next.

- Chapter 3:

The design of a detection threshold was discussed in this section. The main contributions were to include side information into the risk function. The costs in the risk, were functions of the estimated losses at the secondary and the
primary end. The purpose of this work was to demonstrate that the transmission opportunity available to the secondary system is more than what conventional systems have been able to utilize. Scenarios arise where the existing methods of threshold design loose opportunity. The proposals in the chapter make use of information that may be easily available to a secondary system.

The chapter shifted the focus on a single SU’s threshold, but in a multi primary channel environment. The threshold design under such circumstances is challenging as the number of losses to account for increase. The chapter further discussed the risk as a function of the threshold. The risk with continuous selections was also analyzed. The results showed that there may exist practical scenarios where the selection of continuous bands results in an increase in reward that is higher than the discrete channel selections. Further, analytical cases were presented to justify the use of continuous channel selections in a multi-SU system.

• Chapter 4:

Cooperative channel access was included into the design, to extend the scope of the contributions. The SUs cooperatively selected their detection thresholds to limit the losses that occur at the primary user’s end. Further, it was shown that such a design also leads to SU sum throughput maximization. The optimality of the different throughput maximization problems was studied. Different approaches of reaching the optimal solution for the problems were presented. The effectiveness of heuristic based and gradient based algorithms was analyzed.

The issues that arise due to non-convex solution sets were also put to the analytical test. Furthermore, it was shown that joint optimal detection thresholds could easily be developed despite non-convex constraints and objective functions. Numerical results showed that the approximate objective functions could even result in a higher expected throughput because of the reason that optimal solutions were attainable for them.
• **Chapter 5:**

The objective of this work is also the same as the previous chapters i.e., to make use of the side information to maximize throughput. This side information may be partially available, but its use can lead to the SU system exploiting the opportunity in the physical layer. This side information helps in optimally assessing the interference losses among the SUs, if the same channels are selected. Moving on from a single SU system to a multi-SU design, the impact of optimal channel selections, is analyzed. The problem considered assumes that multiple sensing slots are available to SUs, before the transmission phase. The lack of cooperation among the SUs raises the challenge of avoiding interference among the SUs. To deal with this challenge we come up with a framework where channels are selected for all SUs, to sense in all their time slots, beforehand. This implies that the system has to design the sensing orders without any sensing results, and relying completely on the prior probabilities of channels. Side information regarding the average interference power among the SUs is also incorporated into the problem formulation. This allows the system to make use of the transmission opportunity that would otherwise have been lost, if perfect detection were assumed (like previous works).

## 6.2 Future Work

The work presented in this dissertation has the potential to be extended to practical problems for which optimal solutions are sought. The field of secondary system design is an ever-increasing and developing direction that will require attention from researchers to make the SU design more cost-effective. We have identified the following areas.
• **Joint Threshold design with Continuous channel selections**

The work presented in this dissertation extensively explores detection threshold design for single SU and multiple SU systems. Threshold design in a multi-SU system with continuous selections to minimize contention and maximize the utilization of opportunity has not been presented. The detection threshold of an SU controls the access of the SU to the channel being sensed. By carefully selecting this threshold and the overlap among bands, the contention among the SUs can be controlled. Such a design would further encourage continuous band selections and allow more SUs to sense the same or partially overlapping channels.

• **Distributed implementation of Joint Threshold Design**

This dissertation studies the impact of joint threshold design on the overall throughput of the SU system. Communication links between all the SUs may not be available, at times. For many secondary user systems, the only communication that an SU is capable of establishing is with its one-hop neighbors. Studying a distributed implementation may lead to solutions that are more readily implementable in wireless systems.
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