Tunable Mid-infrared and Far-infrared Surface
Plasmons in Doped Semiconductor and Semimetal
Structures

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Abstract
Surface plasmons (SPs) are the electromagnetic (EM) waves due to the light coupled with surface charge oscillations at the interface between metal and dielectric. SPs are widely investigated in the visible and near-infrared wavelengths range due to their capabilities to enhance, guide, and manipulate EM wave on the subwavelength scale. At these wavelengths they have been served as enabling mechanisms from fundamental science to many applications, such as nonlinear harmonic generation, cavity quantum electrodynamics, high-performance sensor, ultracompact interconnects, high resolution microscopy and photolithography. Successful adoption of SP-inspired solutions in the mid-infrared (IR) and far-IR regions will give benefits to fields of environment and health, security and defense, communication and detection such as chemical sensing, thermal imaging, beam shaping and steering, and high-performance detectors. However, the existing knowledge of SPs in visible/near-IR wavelengths cannot directly used in mid- and far-IR regimes as the optical responses to the metal in these two wavelength regions are quite different. In the mid-IR wavelength and beyond, the conventional noble metals resemble perfect electrical conductors, and SPs mode weakly penetrates into metal and the interaction between electrons in metal and the oscillating EM wave is instantaneous. As a result, most the EM field penetrates to the dielectric side and SP mode is weekly confined, which limits many important applications needing strong field confinements.

In this thesis, we proposed several approaches to guide and manipulate surface plasmons in mid- and far-IR region with strong field confinement. We mainly used doped semiconductors of indium antimonide (InSb) and graphene for these wavelengths regimes. In the chapter 2, we proposed tunable subwavelength terahertz plasmonic stub waveguide filters based on InSb whose permittivity is similar to that of
metals at optical frequencies, thus it provides good field confinement. In addition, the permittivity of InSb can be modified by varying temperature, dopant concentrations and magnetic fields which can be used to realize the tunability of plasmonic devices. We used the transmission line theory and the Finite Different Time Domain to investigate the optical responses of the single-stub and multiple-stub waveguide structures. The results show that the proposed structure can realize tunable narrow-and wide-stop band filtering functions.

In the following sections of this thesis, we mainly discussed graphene surface plasmons. The two dimensional semimetal — doped graphene, has also been found as a promising platform for plasmonic applications in the IR frequency regime owing to an unprecedented spatial confinement and tunability by electrostatic gating. In addition, graphene exhibits a relatively large conductivity which translates into long optical relaxation times, and thus could potentially provide a large plasmon wave propagation distance. Efficient excitation of plasmons on graphene still remains a challenge owing to the large wave-vector mismatch between the optical beam in air and graphene plasmon. Current approaches to excite graphene plasmons mainly depend on the grating coupling method, which requires a nanoscale patterning of either graphene itself or the underlying substrate, but the scattering of plasmons from these patterned edges will significantly reduce the plasmon lifetime. In chapter 3, we presented a novel scheme capable of exciting graphene plasmons on a flat suspended graphene by using only \textit{s}-polarized optical beams through four-wave mixing (FWM) process, where the graphene surface plasmons fields were derived analytically based on the Green's function method, under the basis of momentum conservation.

Much attention has been focused on localized graphene surface plasmons resonance with the incident light from free space, such as nano-ribbons, nano-disks, graphene-
metal plasmonic antennas and graphene metamaterials. It is also interesting and indispensable to study the propagation properties of graphene plasmon waves. In the chapter 4, we proposed and numerically analyzed a plasmonic Bragg reflector formed in graphene waveguide. The results show that the graphene plasmonic Bragg reflector can produce a broadband stopband, which can be tuned over a wide wavelength range by a small change in Fermi energy level of graphene. By introducing a defect into the Bragg reflector, we can achieve a Fabry-Perot-like microcavity with a quality factor of 50 for the defect resonance mode formed in the stopband.

Plasmon losses remain the main obstacle for implementation of such devices. In the chapter 5, we performed near-field microscopic experiments at the wavelength of 10 μm and showed that a substantial reduction of plasmon damping can be achieved by placing a nanometric polymer nano-dots spacer between the graphene layer and the supporting silicon oxide slab making graphene quasi-suspended. We argued that reduction of plasmon losses is attributed to weaker coupling with substrate phonons in the quasi-suspended graphene.
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1 Introduction

1.1 Overview and background

As a realization of physicist R. P. Feynman's famous statement “There's Plenty of Room at the Bottom”[1], Nanophotonics is an exciting field dealing with light-matter interaction in a nanoscale, providing platforms for fundamental research and opportunities for creating new technologies. Plasmonics is a study of the interaction between electromagnetic waves and conduction electrons at metallic interface or in subwavelength metallic structures, rapidly forming a major part of fascinating field of nanophotonics. The metallic nanostructures were employed to impart vibrant colors on glass artifacts by ancient artisans, such as the famous 4th-century Lycurgus cup. The scientific study of plasmonics starts from the beginning of the 20th century. In the year 1902, R. W. Woods first observed an anomalous intensity dips in optical reflection spectra of metallic gratings [2]. J. Zenneck analyzed a solution of Maxwell's equation that expresses a "surface wave" in 1907 [3]. In the following year, German physicist G. Mie developed the now widely used theory to describe the electromagnetic field scattering by spherical particles [4]. In 1968, E. Kretschmann and H. Raether generated surface plasmon waves using a prism coupling method [5], making surface plasmons experimentally accessible to researchers. In the recent two decades, plasmonics field has grown rapidly due to the development of new techniques, such as electron-beam lithography (EBL), focused ion beam (FIB), scanning near-field optical microscope (SNOM), that allow a possibility of the fabrication and characterization of objects, as well as the modeling tools that allow the simulation of their performances. Particularly, after the first report of the extraordinary optical transmission phenomenon by T. Ebbesen in 1998, there is a sudden increase of studies in plasmonics field. Surface plasmons (SPs) have been widely investigated in the visible and near-IR range.
from fundamental science to many applications, such as nonlinear harmonic generation [6-8], cavity quantum electrodynamics (QED) [9, 10], high-performance sensors [11-14], ultracompact interconnects [15-19], high resolution microscopy and photolithography [20, 21], due to the capabilities to enhance, guide, and manipulate EM waves on the subwavelength scale.

In the first section of this chapter, we will introduce Drude model to describe the optical properties for the conductive materials, dispersion relation for SPs on single interface of metallic and dielectric layers, and several excitation methods for SPs as well as SPs dispersion in metal-dielectric-metal structures.

1.1.1 Drude model for conductive materials

SPs occur at the interface between the dielectric and metallic (or conductive) materials. It is necessary to know the dispersive properties of metal which is the wavelength dependence on complex permittivity function. Hence, we start with optical properties of the metallic material. The interaction between the light and metallic structures is due to the collective oscillations of the photons with the free electrons inside the metallic materials, which results in the energy exchange between the photons and electrons. Such an interaction can be quantitatively described by the motion equation [22]:

\[
m^* \frac{d^2 \vec{r}}{dt^2} + m \Gamma \frac{d \vec{r}}{dt} = e \vec{E}
\]

(1.1)

where \( m^* \) is the effective mass of electrons, \( e \) is the charge unit. \( \vec{E}(t) = \vec{E}_0 e^{-i\omega t} \) is the time dependent electric driving field. If we assume the solution of the Eq. (1.1) has the form \( \vec{r}(t) = r_0 e^{-i\omega t} \), the amplitude of the oscillated electrons can be expressed as
with $\Gamma$ the collision rate of the free electrons. The polarization which is defined as the dipole moment per unit volume can be expressed as $P = eN\vec{r}(t) = \varepsilon_0\chi\vec{E}$. Here $N$ and $\varepsilon_0$ are the density of the free electrons and the permittivity in free space, respectively. And $\chi$ is the susceptibility. Then the relative permittivity function can be written as

$$\varepsilon_{\text{Drude}}(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}$$

where $\omega_p^2 = \sqrt{Ne^2/(\varepsilon_0m^*)}$ is the plasma frequency. $\varepsilon_\infty$ is the permittivity of the background. Here we can see that the plasma frequency depends on the density of the carriers in the metallic medium.

Figure 1-1 shows the complex permittivity for silver from visible to mid-IR ranges. The Drude model fits well with the experimental data. From the figure, one can see that the real part of the complex permittivity for Ag is negative which is essentially due to the collective oscillations of free electrons. In the conventional plasmonic materials such as gold, silver, the electrons density is fixed ($N \sim 10^{22}\text{cm}^3$). However, carrier density in the doped semiconductors, such as transparent conducting oxides, selicides, indium antimonide (InSb), and graphene, can be changed to realize tunable functional devices from near-IR to the far-IR regions. And these materials are considered as alternative plasmonic materials due to their low loss, tunable carrier density, easy fabrication, and complementary metal-oxide-semiconductor (CMOS) compatibility [25].
Fig. 1-1 The real part (blue) and imaginary part (red) of dielectric permittivity for silver measured by Johnson and Christy (squares and circles) [23]. The Drude model fits to the experimental data (Solid lines), (a) from visible and near-IR wavelength range, (b) and (c) from near-IR to mid-IR range (adapted from Ref [24]).

1.1.2 Surface plasmon polaritons on single interface of metallic and dielectric layers

We proceed to derive the dispersion relation of surface plasmon polaritons and investigate their properties [26-28]. Let's assume the interface of metallic material interface lies on the $x$-$y$ plane, the metallic material (permittivity $\varepsilon_z = \varepsilon'_z + i\varepsilon''_z$) is filled in semi-infinite space of $z < 0$, and the dielectric is filled in semi-infinite space of $z > 0$, as shown in Fig. 1-2.

![Schematic for metal/dielectric interface](image-url)

Fig. 1-2 Schematic for metal/dielectric interface.
According to the properties of the SPs that the propagation surface waves confined to the interface with evanescent decay in the z-direction, let's set the trial transverse magnetic (TM) solution as:

\[
H_1 = (0, H_y, 0)e^{i(k_{1x}x + k_{1z}z)}, \quad E_1 = (E_{1x}, 0, E_{1z})e^{i(k_{1x}x + k_{1z}z)}, \quad z > 0 \quad (1.4)
\]

with \( k_{1z} = \sqrt{\epsilon_1 k_0^2 - k_{1x}^2} \), and

\[
H_2 = (0, H_y, 0)e^{i(k_{2x}x + k_{2z}z)}, \quad E_1 = (E_{2x}, 0, E_{2z})e^{i(k_{2x}x + k_{2z}z)}, \quad z < 0 \quad (1.5)
\]

with \( k_{2z} = \sqrt{\epsilon_2 k_0^2 - k_{2x}^2} \). By Substituting the Eqs. (1.4) and (1.5) into the Maxwell equation with the tangential continuity \( H_1(x, z = 0) = H_2(x, z = 0) \) at the interface of \( z = 0 \), we can know that \( k_{1x} = k_{2x} = k_{pp} = \beta \). And we can get the electric field from Maxwell equation \( \nabla \times H = -ik_0 \epsilon E \) at \( z = 0 \):

\[
\frac{1}{\epsilon_1} \frac{\partial H_1}{\partial z} = \frac{1}{\epsilon_2} \frac{\partial H_2}{\partial z} \quad (1.6)
\]

Then we can have

\[
\frac{k_{1z}}{\epsilon_1} + \frac{k_{2z}}{\epsilon_2} = 0 \quad (1.7)
\]

with \( k_{jz}^2 = \epsilon_j k_0^2 - k_{pp}^2, (j = 1,2) \). By Combing Eqs. (1.6) and (1.7), we obtain the dispersion relation of SPs propagating on the planar metallic surface.

\[
k_{pp} = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}} \quad (1.8)
\]

and the wavevector component normal to the interface:
From Eqs. (1.7) and (1.8), we can know that the existing condition for SPs on the planar metallic structures are:

\[ \varepsilon_i \varepsilon_j < 0, \text{ and } \varepsilon_i + \varepsilon_j < 0 \]  

(1.10)

For metallic structure, its permittivity is negative \( \varepsilon_2 < 0 \), and absolute value is larger than that of dielectric \( |\varepsilon_2| > \varepsilon_1 \). Therefore, the SP wavevector \( k_{\text{sp}} \) is larger than free space wavevector, which is associated with the confinement of SPs to the surface. Here, we don't consider the case of the transverse electric (TE) polarization, because there is no solution from the Maxwell equation. Physically, the electrons cannot be collectively oscillated as the electric field component is parallel to the plane of the surface. SP mode only exists for TM mode as shown in Fig. 1-3(a). As the wavevectors normal to the interface are imaginary, the surface plasmon polaritons are confined very close to the interface. Figure 1-3(b) shows a typical dispersion curve for SPs which is calculated from Eq. 1.8. As we can see in the higher frequency (visible and near IR range), the wavevector of SPs is larger than that of the light in free space.
Fig. 1-3 (a) Surface plasmons field (TM) and surface charge distribution at dielectric/metal interface and with magnetic component in the y direction. (b) Dispersion curve for surface plasmons. (c) The electric field ($E_z$) distribution along z direction (adapted from [29]).

After obtaining the dispersion relation of SPs, we will discuss their length scales which are SP wavelength $\lambda_{spp}$, penetration depth $\delta_m$ and $\delta_d$ in the mediums, and SP propagation distance $\delta_{spp}$ [30].

From the Eq. (1.8) the real part of the SP wave vector can be expressed as $k_{spp}^r = k_0 \sqrt{\varepsilon_1 \varepsilon_2 / (\varepsilon_1 + \varepsilon_2)}$ as the permittivity of the metal is complex. Then we can get the SP wavelength,

$$\lambda_{spp} = 2\pi / k_{spp}^r = \lambda_0 \sqrt{\varepsilon_1 + \varepsilon_2 \over \varepsilon_1 \varepsilon_2} \quad (1.11)$$

We can see that $\lambda_{spp}$ is always smaller than the free space wavelength $\lambda_0$, which also reflects that the confinement nature of the SP mode on a planar metallic surface. The imaginary part $k_{spp}^i = k_0 \sqrt{\varepsilon_2^i / 2(\varepsilon_1^i)^2} \sqrt{\varepsilon_1 \varepsilon_2 \over \varepsilon_1 + \varepsilon_2}$ of SP wave vector is related to propagation loss. The propagation distance which is defined as the distance of mode intensity reduce to $1/e$ can be expressed as:
From the Eq. (1.9), we can easily obtain the penetration depth in the two media:

\[
\delta_{\text{app}} = \frac{1}{2k_{\text{app}}} = \lambda_0 \frac{(\varepsilon'_2)^2}{2\pi\varepsilon_2} \sqrt{\frac{\varepsilon'_1 + \varepsilon'_2}{\varepsilon'_1}} 
\]

(1.12)

In the visible and near-IR range, the SP field confines on the metal surface and decays exponentially with the distance from the interface as the featured in Fig. 1-3 (c). However, from the mid-IR to THz frequencies, the real part of metal permittivity is very huge compared to that of the dielectric. From the Eq. (1.13), we can know that the penetration depth \(\delta_m\) into the metal is too small that the photons hardly interact with the free electrons. Therefore the SP mode extends many wavelengths into the dielectric side. In addition, from Fig. 1-3(b), we can see that at the lower frequencies, the dispersion curve of SP wave can hardly be distinguished from that of the light line in free space. In these frequencies regimes, the less confinement features limits utility of SP for applications. Actually, they are also known as Sommerfield-Zenneck waves [3, 26, 31, 32].

1.1.3 Methods for excitation of surface plasmons

From above discussions, we know that the wavevector for SP is larger than that of the light in free space, which is so called momentum mismatch. In order to excite the SPs, the momentum mismatch has to be compensated. There are four main techniques used to provide the missing momentum. The first is to use the prism coupling as shown in
Fig. 1-4(a)-(b), which is also called attenuated total reflection (ATR). There are Kretschmann configuration and Otto configurations [5, 33]. In the geometry of the Kretschmann, the metal film is directly deposited on the prism surface. When the incident angle of light is larger than the critical angle, the ATR occurs at the interface between the prism and metal, which will generate evanescent waves. The wave vector $k_i = n_p k_0 \sin \theta$ depends on the incident angle would be possible to match the wavevector of the SPs. In the Otto configuration, there is an air gap between the prism and metal, the total reflected evanescent waves at the interface of prism and air could satisfy the momentum matched conditions to generate SPs at a metal/air interface. By scanning the incident angle spectrum for SPs excitation in these two configurations have been widely used for chemical and biological sensing applications [34-37]. The second makes use of grating coupling as shown in Fig. 1-4(c). By introducing a periodic corrugation on the metal surface, the free space wavevector can be increased by $k_g = 2N\pi / \Lambda$ to offset the momentum mismatch. In addition, the scattering from the defects on gold surface can also provide a way to generate SPs. The third approach makes of near field excitation. As is shown in Fig. 1-4(d), a subwavelength tip is used to approach to the metal surface in near-field. The tip radius determines the probing momentum that is in the order of $1/r$, where $r$ is the tip radius. Therefore, by appropriately designing the radius of the tip makes it possible to the excitation of SPs on any metallic structures. We will used this method to couple light into graphene plasmons in chapter 5.
1.1.4 Surface plasmons in the metal-insulator-metal structures.

In this section, we derive the SP wave dispersion in metal-insulator-metal (MIM) or metal-dielectric-metal (MDM) waveguide structure providing a good confinement [38, 39], which is used for terahertz plasmonic waveguide filters in chapter 2. This configuration consists of dielectric core with width of $w$ sandwiched in two metal claddings as shown in Fig. 1-5. The electromagnetic field has the forms,

$$
E(x, y, z) = (E_x \hat{x} + E_y \hat{y} + E_z \hat{z}) e^{j(\beta z - \omega t)}
$$

$$
H(x, y, z) = (H_x \hat{x} + H_y \hat{y} + H_z \hat{z}) e^{j(\beta z - \omega t)}
$$

(1.14)

with $E_x, E_y, H_y$ identically nonzero for TM polarization and $E_x, H_x, H_z$ identically nonzero for TE polarization. Here, we only consider the component of $H_y$. The wave equation of magnetic field component can be derived from the Maxwell equations:

$$
\frac{\partial^2 H_y}{\partial x^2} - k^2 H_y = 0
$$

(1.15)
where \( \kappa^2 = \beta^2 - \varepsilon \varepsilon_0^2 \). \( \varepsilon \) is the permittivity of the corresponding material in the regions. \( k_0 = 2\pi / \lambda_0 \) is the free space wavevector.

Fig. 1-5 Schematic of MIM waveguide structure.

We assume that \( n_1 > n_2 > n_3 \), and \( n_1^2 k_0^2 > \beta > n_2^2 k_0^2 > n_3^2 k_0^2 \), in the region (I), the EM field components can be written as

\[
H_y^{(I)} = Ae^{i\kappa_1 x} + Be^{-i\kappa_1 x}
\]
\[
E_z^{(I)} = \frac{-\kappa_1}{\omega \varepsilon_0 \varepsilon_1} (A e^{i\kappa_1 x} - B e^{-i\kappa_1 x})
\]
\[
E_x^{(I)} = \frac{\beta}{\omega \varepsilon_0 \varepsilon_1} (A e^{i\kappa_1 x} + B e^{-i\kappa_1 x})
\]

where \( \beta \) is the propagation constant for the SP wave in the MIM waveguide structures.

In region (II) the EM field components can be written as

\[
H_y^{(II)} = Ce^{-\kappa_2 (x-w/2)}
\]
\[
E_z^{(II)} = \frac{i \kappa_2}{\omega \varepsilon_0 \varepsilon_2} Ce^{-\kappa_2 (x-w/2)}
\]
\[
E_x^{(II)} = \frac{\beta}{\omega \varepsilon_0 \varepsilon_2} Ce^{-\kappa_2 (x-w/2)}
\]

In region (III), the field components can be written as
\[ H_y^{(III)} = D e^{\kappa_3 (x+w/2)} \]
\[ E_z^{(III)} = \frac{i \kappa_3}{\omega \varepsilon_0 n_3^2} D e^{\kappa_3 (x+w/2)} \]
\[ E_x^{(III)} = \frac{\beta}{\omega \varepsilon_0 n_3^2} D e^{\kappa_3 (x+w/2)} \]

where

\[ \kappa_2^2 = \beta^2 - \varepsilon_2 k_0^2, \kappa_1^2 = \varepsilon_1 k_0^2 - \beta^2, \kappa_3^2 = \beta^2 - \varepsilon_3 k_0^2 \]  

Applying the boundary conditions at the metal-dielectric interface:

\[ H_y^I = H_y^II \big|_{x=w/2}, H_y^I = H_y^III \big|_{x=-w/2} \]
\[ E_z^I = E_z^II \big|_{x=w/2}, E_z^I = E_z^III \big|_{x=-w/2} \]
\[ \varepsilon_1 E_x^I = \varepsilon_2 E_x^II \big|_{x=w/2}, \varepsilon_1 E_x^I = \varepsilon_3 E_x^{III} \big|_{x=-w/2} \]

Then we have

\[ Ae^{i \kappa w/2} + Be^{-i \kappa w/2} = C \]
\[ \frac{\kappa}{\varepsilon_1} (A e^{i \kappa w/2} - B e^{-i \kappa w/2}) = i \frac{\kappa_2}{\varepsilon_2} C \]
\[ Ae^{-i \kappa w/2} + Be^{i \kappa w/2} = D \]
\[ -\frac{\kappa}{\varepsilon_1} (A e^{-i \kappa w/2} - B e^{i \kappa w/2}) = i \frac{\kappa_3}{\varepsilon_3} D \]  

If the above equations have solutions, we can get the dispersion relation for the SPs wave in the MIM structure with \( \varepsilon_2 = \varepsilon_3 \):

\[ \sqrt{\varepsilon_1 k_0^2 - \beta^2} w - 2 \arctan \frac{\sqrt{\beta^2 - \varepsilon_2 k_0^2} \varepsilon_1}{\sqrt{\varepsilon_1 k_0^2 - \beta^2} \varepsilon_2} = m \pi \]  

where \( m \) is the order of the mode. From Eq. (1.22), we will find that each mode has its own cut-off frequency and cut-off width:

The cut-off frequency of the \( m_{th} \) mode can be expressed:
\[
\omega = \frac{m \pi c}{w \sqrt{\varepsilon_1}} - \frac{2c}{w \sqrt{\varepsilon_1}} \arctan \sqrt{-\frac{\varepsilon_1}{\varepsilon_2}}
\]  \hspace{1cm} (1.23)

And the cut-off width of the $m_{th}$ mode can be expressed:

\[
w = \frac{m \lambda}{2 \sqrt{\varepsilon_1}} - \frac{\lambda}{\pi \sqrt{\varepsilon_1}} \arctan \sqrt{-\frac{\varepsilon_1}{\varepsilon_2}}
\]  \hspace{1cm} (1.24)

For $m=0$, we have the TM$_0$ mode (fundamental mode) from Eq. (1.22)

\[
\sqrt{\varepsilon_1 k_0^2 - \beta^2} w = 2 \arctan \frac{\beta^2 - \varepsilon_1 k_0^2 \varepsilon_1}{\sqrt{\varepsilon_1 k_0^2 - \beta^2 \varepsilon_2}}
\]  \hspace{1cm} (1.25)

For MIM structure, $n_2 < 0$, $n_1 > 0$, so $\kappa_1$ has no solution in the region of $n_1 k_0 > \beta$. Therefore, the assumption of $n_1 k_0 > \beta$ is not correct. The waves in the lateral direction of the slit are not a superposition of two oscillating waves, but two attenuated waves. i.e. $n_1 k_0 < \beta$. Then Eq. (1.25) can be written as

\[
tanh \left( \frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2}}{2} - w \right) = -\frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2 \varepsilon_1}}{\sqrt{\beta^2 - \varepsilon_1 k_0^2 \varepsilon_2}}
\]  \hspace{1cm} (1.26)

which is the dispersion relation for the symmetric plasmon mode. Fig. 1-6 shows dispersion relation of the fundamental mode of surface plasmons in silver/air/silver waveguide structure.

For $m=1$, we can have the dispersion relation for TM$_1$,

\[
\frac{\sqrt{\varepsilon_1 k_0^2 - \beta^2}}{2} w = \frac{\pi}{2} + \arctan \frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2 \varepsilon_1}}{\sqrt{\varepsilon_1 k_0^2 - \beta^2 \varepsilon_2}}
\]  \hspace{1cm} (1.27)

Because $\varepsilon_2 < 0$, according to $\arctan(1/x) = -\pi/2 - \arctan(x)$ with $x < 0$, we have
\[
\tan \left( \frac{\sqrt{\varepsilon_1 k_0^2 - \beta^2}}{2} w \right) = -\frac{\sqrt{\varepsilon_1 k_0^2 - \beta^2} e_2}{\sqrt{\beta^2 - \varepsilon_2 k_0^2} e_1}
\] (1.28)

If \( \beta/k_0 > \varepsilon_1 \), we obtain the dispersion relation for asymmetric mode:

\[
\tanh \left( \frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2}}{2} w \right) = -\frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2} e_2}{\sqrt{\beta^2 - \varepsilon_2 k_0^2} e_1}
\] (1.29)

or

\[
\coth \left( \frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2}}{2} w \right) = -\frac{\sqrt{\beta^2 - \varepsilon_1 k_0^2} e_1}{\sqrt{\beta^2 - \varepsilon_2 k_0^2} e_2}
\] (1.30)

Therefore, we know that the existing of symmetric mode doesn't depend on the width of the slit. When it becomes wider (at \( w_{\varepsilon_1} = \frac{2}{k_0 \sqrt{\varepsilon_1}} \arctan \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \)), for \( \beta_{TM1}/k_0=0 \), TM1 mode becomes a guiding mode. And when \( \beta_{TM1}/k_0 > \varepsilon_1 \), we have \( w_{\varepsilon_1} = -\frac{2 \varepsilon_2}{k_0 \sqrt{\varepsilon_1 - \varepsilon_2}} \), the TM1 mode becomes SPs wave.

![Fig. 1-6](image)

Fig. 1-6. Dispersion relation of the fundamental mode of surface plasmons in silver/air/silver waveguide structure. The width of the slot is 100 nm(broken gray curve), 50 nm (broken black curve).
and 25 nm (continuous black curve). The gray curve is for the dispersion of surface plasmon polaritons at single silver/air interface. The gray line is the air light line. (adapted from [40])

1.2 Scanning near-field optical microscope

As the properties of plasmonic structures or devices are intimately related to the activity and the distribution of SP field. The spatial resolution of the conventional far-field techniques such as camera is limited to the diffraction limit of about half of the wavelength. The scanning near-field optical microscope (SNOM) was first proposed to image an object optically below the diffraction limit by E. Synge in 1928 [41]. Since the 1980s, various configurations of the SNOM system have been developed, such as aperture-type SNOM and apertureless SNOM. They are emerging as an effective technique for investigating optical responses in the nanoscale. The applications of the SNOM range from biochemical detection and imaging [42, 43], laser technology [44-47] to nanophotonics [48, 49] and material science [50-52]. Normally, the aperture type SNOM uses a metallic coated tapered fiber with an aperture opened on the tip to collect the transmitted or reflected light from the surface of the sample. The near-field images can be obtained by scanning the aperture over the sample surface. The spatial resolution depends on the aperture size of the tip. However, with reduction of the aperture size, the power of the light transmitted through the tip decreases significantly, which causes the decrease of the signal-to-noise ratio. Furthermore, the aperture type SNOM is only suitable for the visible and near IR range with the sufficiently transmission efficiency in the fibers, while is it not suitable for operation with mid-IR and long wavelength ranges due to the unavailable fibers in those regimes. The apertureless or scattering type SNOM has been identified as an only solution for the nanoscale spatial resolution in the mid-IR and long wavelength spectral ranges. As we
focus on the investigation of SPs in mid- and far-IR regions in this thesis, we use a commercial scattering type SNOM system from Neaspec GmbH [53, 54].

![Fig. 1-7 Principle of the scattering type SNOM. Adapted from [55].](image)

The Neaspec SNOM uses a standard atomic force microscope (AFM) tip (T) (normally silicon tip or metal-coated tip) as shown in Fig. 1-6, which is illuminated by a laser beam focused by a parabolic mirror. The illuminated tip can generate a nano-focused field (N) at their apexes as a illumination source to probe the sample. The illuminated method is also used as an approach for excitation of SPs. The tip can provide a large momentum to match that of the SPs, as the tip radius determines the momentum \( k \) that is in the order of \( 1/r \), where \( r \) is the tip radius. In chapter 5, we will employ this method for excitation of graphene plasmons.

The size of the nano-focused field depends on the radius of the tip apex, which is also valid for mid-IR and terahertz range. Thus the scattering type SNOM gives rise to a spatial resolution as small as 10 nm throughout the whole optical spectrum range. The near-field interaction between the tip and the samples will elastically backscatter the plasmon field or the light containing the properties of materials into the far-field, which is collected by the sample parabolic mirror used for focusing to the detector. The AFM tip vibrates vertically at the mechanical resonance frequency \( \Omega \) of the cantilever. And the near-field signal is demodulated with pseudo-heterodyne
interferometer at a higher harmonic $n\Omega$, which will suppress the overwhelming background scattering from the tip and sample. In addition, the topography of the samples is recorded simultaneously with the near-field image.

1.3 Motivations

SPs are widely investigated in the visible and near IR wavelengths range due to their capabilities to enhance, guide, and manipulate EM wave on the subwavelength scale. At these wavelengths they have been serving as enabling mechanisms from fundamental science to many applications, such as nonlinear harmonic generation [6-8], cavity quantum electrodynamics (QED) [9, 10], high-performance sensors [11-14], ultracompact interconnects [15-19], high resolution microscopy and photolithography [20, 21]. Successful adoption of SPs-inspired solutions in the mid-IR and long-IR regions will give benefits in fields of environment and health, security and defense, communication and detection such as chemical sensing, thermal imaging, beam shaping and steering, and high-performance detectors [56]. However, the existing knowledge of SPs in visible/near-IR wavelengths cannot directly translate to in mid- and far-infrared regimes as the optical responses to the metal in these two wavelength regions are quite different. In the mid-IR wavelength and beyond, the conventional noble metals resemble perfect electrical conductors, and SP mode weakly penetrates to metal and the interaction between electrons in metal and the oscillating EM wave is instantaneous. As a result, most the EM field penetrates to the dielectric side and SP mode is weekly confined, which limits many important applications needing strong field confinements. So the new schemes on effectively and efficiently manipulation of SPs in mid-and far-IR with good confinements are interesting and necessary.
1.4 Objectives and methodologies

We aim to propose several approaches to guide and manipulate SPs in mid- and far-IR region with strong field confinement, using both conventional doped semiconductors and emerging graphene, which also constitute the main objectives of this thesis:

1. As we mentioned in the section of motivations that terahertz SP mode is weakly confined on the conventional metallic structures, whose charge carrier concentration is fixed. We propose the tunable subwavelength THz plasmonic stub waveguide filters based on InSb whose permittivity is similar to that of metals at optical frequencies, thus it provides good field confinement. The transmission line theory and the FDTD are used to investigate the optical responses of the single-stub and multiple-stub waveguide structures, as well as tunable properties.

2. Semimetal graphene, has also been found as a promising platform for plasmonic applications in the IR frequency regime owing to an unprecedented spatial confinement and tunability by electrostatic gating. In addition, graphene exhibits a relatively large conductivity which translates into long optical relaxation times, and thus could potentially provide a large plasmon wave propagation distance. Efficient excitation of plasmons on graphene still remains a challenge owing to the large wave-vector mismatch between the optical beam in air and graphene surface plasmons. We study a novel scheme capable of exciting graphene surface plasmons on a flat suspended graphene by using only $s$-polarized optical beams through four-wave mixing (FWM) process to overcome the large wave-vector mismatch between the optical beam in air and graphene plasmons. The graphene plasmons fields were derived analytically based on the Green's function method, under the basis of momentum conservation. By incorporating the merits of nonlinear optics, the
presented scheme avoids the scattering of plasmons from patterning of either graphene or substrate.

3. Much attention has been focused on localized graphene surface plasmons resonance with the incident light from free space, such as nano-ribbons, nano-disks, graphene-metal plasmonic antennas and graphene metamaterials. We investigate the propagating graphene plasmons' responses on plasmonic Bragg reflector formed in graphene waveguide as most of the attentions have been focused on localized plasmons. And also study Fabry-Perot-like microcavity effect by introducing a defect into the Bragg reflector structure.

4. Losses remain the main obstacle for implementation of graphene plasmonic devices. By direct mapping of GPs with scattering-type scanning near-field optical microscope (s-SNOM), we study an approach to reduce the graphene plasmon damping by placing a nanometric polymer nano-dots spacer between the graphene layer and the silicon oxide substrate. In addition, we compare our experimental results with numerical simulations based on a developed numerical model.

**1.5 Thesis overview**

The thesis is organized as following chapters: Chapter 2 introduces the characteristics of the subwavelength InSb plasmonic slot waveguide, including the mode refractive index, propagation loss, and field distributions. Transmission line theory and FDTD are employed to analyze the single-stub and multiple-stub structures. The results show the proposed structures can realize a tunable narrow-and wide-stop band filtering function. Chapter 3 introduces the basics of the graphene surface plasmons, including the permittivity of graphene, and dispersion of graphene plasmons as to better understand the operation principle. We then present a novel scheme capable of
exciting graphene plasmons on a flat suspended graphene by using only s-polarized optical beams through four-wave mixing (FWM) process, where the graphene plasmon fields were derived analytically based on the Green's function method, under the basis of momentum conservation. Chapter 4 studies the propagation properties of graphene plasmon waves. We numerically analyze a plasmonic Bragg reflector formed in graphene waveguide and a high quality factor Fabry-Perot-like microcavity by introducing a defect into the Bragg reflector. Chapter 5 experimentally studies a scheme to reduce the graphene surface plasmon damping by placing a nanometric polymer nano-dots spacer between the graphene layer and the silicon oxide substrate. Chapter 6 provides a summary for the thesis and points out to future research directions.
2. Tunable subwavelength terahertz plasmonic stub waveguide filters

2.1 Introduction

Terahertz (THz) technology is now drawing extensive attentions because of its potential applications in biochemical sensing, spectroscopy, and high-speed communication. With the rapid development of THz sources and detectors [57], there is a high demand for THz components, such as waveguides, polarizers, filters, and collimators [58]. Waveguide filter is probably the most basic component of all. Various waveguide filters have been demonstrated within the last few years. Most of these are based on conventional guiding structures, such as metal tubes [59, 60], metal wires [61, 62], and dielectric waveguides [63, 64]. Confining and manipulating electromagnetic waves at dimensions much smaller than the wavelength are of great importance for miniaturizing integrated photonic circuits. The minimum confinement of a guided mode in conventional waveguides is limited by the diffraction limit.

Plasmonic devices, based on SPs propagating at metal-dielectric interface, have shown great potential to guide and manipulate light by metallic nanostructures at sub-wavelength scales [18]. Most of the efforts in plasmonic devices have been concentrated at optical and near-IR frequencies [65-67], and the tuning mechanisms of active plasmonic devices are based on changing the permittivity of the dielectric materials [68] through thermo-optic [69], electro-optic [70] and nonlinear optical effects rather than changing permittivity of the metal because of the large charge carrier concentration (\(\sim 10^{22}\)cm\(^{-3}\) of metals), which makes it difficult to change its permittivity by varying the carrier density. In the THz region, both the real and imaginary values of the metal permittivity are huge, with typical absolute values to the order of \(10^5\). From the analysis in Section 1.1.2, we can know that these values of
permittivity give a rise to weak coupling of the electromagnetic field to the electrons in the metal. Actually, the metal resembles a perfect electric conductor (PEC), where the electromagnetic waves hardly penetrate into the metal, instead of hundreds of wavelengths in to the dielectric medium [71]. Therefore, SPs in the real sense do not exist on planar metal/dielectric interface in THz range. Spoof SPs based on corrugated metallic structures can provide THz wave localization and slow light localization by introducing localized electromagnetic modes strongly interacting with structured surfaces [72]. A dynamic terahertz SPs switch based on resonance and absorption has been proposed [73]. As we mentioned that alternative materials for exciting low frequency SPs on planar structures are doped semiconductors in the section 1.1.1. Some semiconductors have a permittivity at THz range close to that of metal at optical range. In addition, the permittivity of semiconductors can be modified [48] by varying temperature, dopant concentrations and magnetic fields which can be used to realize the tunability of plasmonic devices.

In this chapter, we propose planar tunable subwavelength plasmonic stub waveguide filters in THz range using indium antimonide (InSb) as the “metal” material. Characteristics of deep-subwavelength InSb plasmonic slot waveguide are first investigated. The single-stub InSb slot waveguide filters are proposed and analyzed by transmission line theory and Finite Different Time Domain (FDTD) simulation [74]. The results obtained from both methods agree well. As an extension to the tunable single-stub filter, a multiple-stub structure is also demonstrated with more functionalities. These structures may have great potential for ultra-compact THz integrated circuits.
2.2 Characteristics of subwavelength InSb plasmonic slot waveguide

The inset of Fig. 2-1(a) shows the InSb slot waveguide structure composed of two parallel InSb plates with a dielectric core. The dispersion relation of the fundamental SPs mode in the InSb slot waveguide is given by [75], and we also give its detail derivation in section 1.1.4.

\[
\tanh\left(\sqrt{n_{\text{eff}}^2 - \varepsilon_d k_0 w / 2}ight) = \frac{-\varepsilon_d \sqrt{n_{\text{eff}}^2 - \varepsilon_{\text{insb}}(\omega, T)}}{\varepsilon_{\text{insb}}(\omega, T) \sqrt{n_{\text{eff}}^2 - \varepsilon_d}}
\]

(2.1)

where \( n_{\text{eff}} \) is the effective refractive index of the SPs in the waveguide and \( k_0 = 2\pi/\lambda_0 \) is the free-space wave vector. \( \varepsilon_d \) denotes the permittivity of medium in the slot guide region and the permittivity \( \varepsilon_{\text{insb}}(\omega, T) \) of InSb can be described by the Drude model [76, 77] approximation,

\[
\varepsilon_{\text{insb}}(\omega, T) = \varepsilon_\infty - \frac{\omega_p^2(T)}{\omega[\omega + i\Gamma(T)]}
\]

(2.2)

where \( \omega \) is the angular frequency of the incident electromagnetic radiation, and \( \varepsilon_\infty = 15.75 \) is the high-frequency permittivity; \( \omega_p(T) \) and \( \Gamma(T) \) are the plasma frequency and collision rate of the charge carriers respectively. The plasma frequency \( \omega_p(T) = \sqrt{Ne^2 / \varepsilon_0 \varepsilon_0 m^*} \) depends on the density of carriers, where \( e \) is the effective charge unit, \( \varepsilon_0 \) is the permittivity in vacuum, and \( m^* \) is the effective mass of electrons. Different from metal, the plasma frequency of the InSb significantly depends on the temperature \( T \). The intrinsic concentration in the InSb is described by the formula \( N = 5.76 \times 10^{14} T^{1.5} \exp(-0.129/K_B T) \text{cm}^{-3} \) [78], where \( K_B \) and \( T \) are the Boltzmann constant and temperature, respectively. At 295 K, the permittivity of InSb at 1 THz is \(-44.19+15.58i\), which is similar to that of metals at optical frequencies. The InSb permittivity is changed to \(-26.69+10.18i\) at temperature of 280 K. The dielectric
constant $\varepsilon(\omega, T)$ of InSb material is affected by the variation of the temperature in the terahertz range because the plasma frequency and collision rate of the charge carriers are both dependent on the temperature. Therefore, tunable plasmonic filters based on thermal tuning can be expected.

Figure 2-1 (a) Real part of the effective index of InSb slot waveguide as a function of slot widths at different temperatures for incident frequency at 1 THz. The inset: schematic structure of an InSb slot waveguide. (b) Imaginary part of the effective index of InSb slot waveguide.

In the InSb slot waveguide, the fundamental transverse magnetic mode ($\text{TM}_0$) always exists regardless of the waveguide width, while other high-order modes have a cutoff waveguide width. To satisfy the single mode propagation condition [79], the width of the waveguide must be smaller than $\lambda_0 \arctan(\sqrt{-\text{Re}(\varepsilon_{\text{ref}})\varepsilon_d} / \pi \sqrt{\varepsilon_d})$. In this chapter, the dielectric in the guided region is assumed to be air with a permittivity $\varepsilon_d=1$. Figures 2-1 (a) and (b) show the dependence of the real part and imaginary part, respectively, of the effective refractive index of InSb slot waveguide on the slot widths at different temperatures for an incident light at 1 THz. From Figure 2-1(a), one can see that the real part of effective refractive index of the waveguide decreases with increasing $w$ at a certain temperature, and for a given width it decreases as the temperature rises. The waveguide introduces higher losses for smaller slot width, as indicated by the imaginary part of waveguide refractive indices in Fig. 2-1(b). So there is a trade-off
between strong mode confinement and high attenuation for the slot waveguide. And for the slot waveguide with a given width, the imaginary part of the effective refractive index increases as the temperature decreases, which means the propagation loss is higher at a lower temperature.

For the InSb slot waveguide, the $H_y$ field component distribution can be written as [80]

$$
H_y = \left\{ \begin{array}{ll}
\frac{\varepsilon_{\text{inb}}^{1/2}}{\omega} \left( e^{ik_1 y/2} + e^{-ik_1 y/2} \right) e^{-ik_2(x-w/2)} & x > w/2 \\
\frac{\varepsilon_{d} k_{0} c}{\omega} \left( e^{ik_1 x} + e^{ik_2 x} \right) & |x| \leq w/2 \\
\frac{\varepsilon_{\text{inb}}^{1/2}}{\omega} \left( e^{ik_1 y/2} + e^{-ik_1 y/2} \right) e^{-ik_2(x+w/2)} & x < -w/2
\end{array} \right.
$$

with

$$
k_1^2 = \beta^2 - \varepsilon_{\text{inb}} (T) k_0^2, k_2^2 = \varepsilon_d k_0^2 - \beta^2
$$

Here $[\text{Re}(k_1)]^1$ and $[\text{Re}(k_2)]^1$ determine the skin depths of SPs in the dielectric guided region and the InSb region, respectively; $\beta$ is the propagation constant of SP wave that can be represented by $\beta = k_0 \mu_{\text{eff}}$. Figure 2-2 shows the normalized $H_y$ profiles as a function of the distance from the waveguide median for the incident wave at 1 THz. One can see from the figure that the majority of the SPs energy is localized inside the dielectric guided region. The $H_y$ profile has nearly a parabolic shape in the slot, reaching a maximum value at the interface between the slot and the InSb material. Outside the guided region, $H_y$ field decays exponentially. It should be noted that, the penetration depth of the electromagnetic wave in InSb decreases when the temperature rises, which means bigger part of the THz wave propagates in the dielectric slot region than in the InSb material. Therefore, the propagation constant decreases (moves close to the propagation constant in air) with the increase of the temperature. As a
consequence to the smaller skin depth, SPs are less dissipated, which gives rise to a larger propagation distance.

![Normalized H_y profiles for the InSb-Air-InSb waveguide with slot width of 50 µm at different temperatures for incident frequency of 1 THz.](image)

**Fig. 2-2** Normalized $H_y$ (y component of the magnetic field) profiles for the InSb-Air-InSb waveguide with slot width of 50 µm at different temperatures for incident frequency of 1 THz.

### 2.3 Analysis of InSb plasmonic stub waveguide filters

#### 2.3.1 Single-stub structure filter

The InSb plasmonic waveguide filter with a single-stub structure is shown in Fig. 2-3(a). $P$ and $Q$ are respectively the power monitor positions of the incident and transmission field. $w_s$ and $L$ are the width and length of the stub, respectively. The plasmonic waveguide can be modeled as a transmission line with characteristic impedance. Figure 2-3(b) shows the transmission line equivalent circuit of the single-stub structure. $Z_0$ is the characteristic impedance of the waveguide. The characteristic susceptance of the single-stub structure is given $Y_s = -jY_0 \tan(\beta(w_s)L)$, where $j = \sqrt{-1}$, $Y_0 = 1/Z_0$, $\beta(w_s)$ and $\beta(w)$ are the propagation constant of the stub and input waveguide [81]. The transmittance of the single-stub structure can be expressed as
where $L_{SPP} = (2\text{Im}\beta)^{-1}$ is the characteristic propagation length of the SP mode [82]. The equation above exhibits a clear physical process: the first part accounts for interference between the incident wave and the wave reflected from the stub. The exponential factor describes the attenuation of the SPs. When the phase delay satisfies $2\pi n_{\text{eff}} L / \lambda = (m + 1/2) \pi (m = 0, 1, 2, ...) , \quad \text{the transmittance } T_s \text{ reaches the minimum.}$ Therefore, the wavelength $\lambda_m$ of the notch of the transmittance is determined as follows:

$$\lambda_m = \frac{2n_{\text{eff}} L}{m + 1/2}. \quad (2.6)$$

It can be seen that the wavelength $\lambda_m$ is linear to the stub length $L$ and the effective refractive index of the stub.

Fig. 2- 3(a) Schematic structure of an InSb plasmonic slot waveguide filter with the single-stub structure.
(b) Transmission line equivalent circuit of the single-stub structure.

It should be noted that the transmission model cannot be very accurate as the phase changes at the junction of the stub and incident waveguide are not included in the
model. In the following part, FDTD method is used to investigate the transmission response of the stub filters. The perfectly matched layer absorbing boundary conditions is set at all boundaries of the simulation domain. Since the width of the InSb slot waveguide is much smaller than the operating wavelength in the structure, only fundamental waveguide mode is supported. The incident wave for excitation of SP mode is a TM-polarized (the magnetic field is parallel to \( y \) axis) fundamental mode. The width \( w \) of the incident waveguide is set to be 50 \( \mu \)m and the distance \( L \) is fixed to 120 \( \mu \)m.

The transmission spectra of the InSb single-stub filter are shown in Fig. 2-4(a), which are obtained by the transmission line model (red dash curve) and FDTD (black solid curve and black dot curve correspond to the lossy and lossless case respectively). For the lossy case, one can see that there are two transmission notches at the frequencies of 0.213 THz (corresponding to \( m=0 \)) and 0.642 THz (corresponding to \( m=1 \)), with minimum transmittance of 2\% and 3\% respectively. The maximum transmittance is \(~80\%) due to the Ohmic loss of InSb, corresponding to an insertion loss of 10.5 dB and the full width half maximum (FWHM) are respectively 0.136 THz and 0.137 THz, for the notch frequencies of 0.213 THz and 0.642 THz. While for the lossless case, the maximum transmittance is 1, and FWHMs at two notch frequencies is 0.127 THz and 0.118 THz, which are smaller than those of lossy case. It is found that the FDTD results almost agree well with those calculated by the transmission line model. There is a small deviation at the positions of the transmission notch because the phase changes at the junction of the stub and incident waveguide are not considered in the transmission line model. Submitting \( \omega=0.213 \) THz into Eq. (2.6) gives a total phase change \( \Delta \phi=0.019\pi \) for the first notch with the stub length \( L=300 \) \( \mu \)m, and the effective index \( n_{\text{eff}}=1.1302 \). The second resonance frequency can be approximately calculated as
0.661 THz by the formula neglecting the frequency dependence on the phase change. The magnetic fields ($H_y$) at the frequencies of 0.213 THz and 0.43 THz corresponding to the transmission notch and peak in Fig. 2-4(a) are displayed in Fig. 2-4(b).

Fig. 2-4 (a) Transmission spectra of the InSb single-stub filter with the waveguide width $w=50 \, \mu m$, the stub width $w_s=50 \, \mu m$, and the length $L=300 \, \mu m$ at temperature of 295 K. (b) The corresponding simulated magnetic field ($H_y$) distributions for frequencies at 0.213 THz (one notch in Fig. 2-4(a)) and 0.43 THz (one peak in Fig. 2-4(a)), respectively.

Figure 2-5(a) shows the transmission spectra of the single-stub InSb slot waveguide structure functioning as a notch filter with different stub lengths $L$. It reveals that the central wavelength moves to a longer wavelength with the increase of the stub length $L$. Figure 2-5(b) shows that the central wavelength of the notch has a linear relationship with the length of the stub as expected from Eq. (2.6).
Fig. 2-5 (a) Transmission spectra of the single-stub InSb slot waveguide filter with different stub lengths $L$. (b) Relationship between the frequency (wavelength) of the transmission notch and the stub length. The width of the waveguide is $w=50 \, \mu m$, and the stub width is $w_s=50 \, \mu m$ at temperature of 295 K.

Figure 2-6 shows the central wavelength (frequency) of the first transmission notch versus the stub width of $w_s$ for stub length of 250 $\mu m$ at temperature of 295 K. It can be seen that the initial notch of the transmission moves quickly to the shorter wavelength (blue-shift) with the increase of $w_s$ for $w_s<30 \, \mu m$, and the shift becomes small after $w_s>30 \, \mu m$. As revealed in Eq. (2.6), the above relationship between the notch position and $w_s$ is mainly due to the contribution of inverse-proportion-like dependence of the effective index $n_{\text{eff}}$ on the stub width $w_t$ as shown in Fig. 2-1(a). Therefore, one can realize filtering at various required frequencies by properly choosing the width and/or the length of the stub structure.

![Graph showing the relationship between frequency and stub width](image)

Fig. 2-6 Relationship between the frequency (wavelength) of the first transmission notch and the stub width. The width of the waveguide is $w=50 \, \mu m$, and the stub length $L=250 \, \mu m$ at temperature of 295 K.
2.3.2 Actively controlled SPs in single-stub structure

As discussed in the section 2.2, the permittivity of the InSb and the effective index of the InSb slot waveguide are affected by the variation of the temperature at the THz range. One can expect that the filtering characteristics of the InSb single-stub waveguide filter can be controlled by tuning the temperature. Figure 2-7(a) shows the transmission spectra of the InSb single-stub waveguide filter at temperatures of 225 K and 325 K. The other parameters of the structure are set as: slot waveguide width $w=50 \mu\text{m}$, stub width $w_s=30 \mu\text{m}$, stub length $L=200 \mu\text{m}$. From Fig. 2-7(a), it can be seen that the central frequency of the transmission notch shifts from 0.231 THz to 0.305 THz as temperature increases from 225 K to 325 K. The transmission of the pass band at 225 K is lower than that at 325 K because the imaginary part of the effective index the InSb slot waveguide is higher at lower temperatures (shown in Fig. 2-2(b)) which causes higher transmission losses in the waveguide. Figure 2-7(b) shows the relationship between the central frequency of the transmission notch and the temperature. One can see that the central frequency of the transmission notch increases with the increase of the temperature. This is because the wavelength of the transmission notch has a linear relationship with the effective index of the stub which decreases with the increase of the temperature (shown in Fig. 2-1(a)). Therefore, active control of SP wave in InSb slot stub waveguide structure can be realized by tuning the temperature.
2.4 Periodic multiple-stubs structure filter

It is straightforward to expand the plasmonic single-stub waveguide structure to periodic multiple-stubs structure, as shown in Fig. 2-8(a). The InSb slot waveguide width \(w\) and the distance \(d\) are fixed to be 50 μm and 100 μm. \(A\) and \(P\) are the period length and the number of stubs, respectively. Figure 2-8(b) shows the transmission spectra of the multiple stubs InSb slot structure for different period numbers with \(w=50\) μm, \(w_s=50\) μm, length \(L=300\) μm and the distance \(d=100\) μm at temperature \(T=295\) K, which is obtained by the FDTD method. Two wide stopbands occur around 0.221 THz \((m=0)\) with a bandwidth of 0.08 THz (defined as the difference between the two frequencies at each of which the transmittance is equal to 1%) and 0.646 THz \((m=1)\) with bandwidth 0.09 THz for a period number \(P=3\). The second stop-band appears with a better filtering performance. It can be seen that the width of the stopband broadens with the increase of the period number. The bandwidths are 0.108 THz and 0.057 THz for \(P=4\) and \(P=2\), respectively. It should also be noted that large period number induces a high transmission losses. The maximum transmittance of pass band decreases from 69% to 52% as the period number increases from 2 to 4.
Therefore, there is a tradeoff between the bandwidth and the period number. For the multiple stub structure with the parameters of \( \text{Re}(n_{\text{eff,stub}}) = 1.1094 \) for the width of \( D = L + w = 350 \, \mu\text{m}, \text{Re}(n_{\text{eff,w}}) = 1.1222 \) for \( w = 50 \, \mu\text{m} \), and the stub period of \( A = 100 \, \mu\text{m} \) in the z axis direction (Fig. 8). One can see that the \( \text{Re}(n_{\text{eff,stub}}) + (A - w_s) \text{Re}(n_{\text{eff,w}}) \approx 111.58 \, \mu\text{m} < 464.4 \, \mu\text{m}/2 \) (Bragg wavelength \( \lambda_{\text{bragg}} \)). Thus, the structure does not follow the Bragg condition in the z-axis direction. The filtering feature of the stub structures can be attributed to the multiple-interference of the reflected and transmitted fields from each of the three stubs structures.

![Diagram](image1)

Fig. 2-8 (a) Schematic structure of a multiple stubs InSb slot waveguide. (b) Transmission spectra of the multiple stubs InSb slot waveguide with different period numbers with the slot width \( w = 50 \, \mu\text{m} \), the stub width \( w_s = 50 \, \mu\text{m} \), the length \( L = 300 \, \mu\text{m} \), the distance \( d = 100 \, \mu\text{m} \), period length \( A = 100 \, \mu\text{m} \) and the temperature \( T = 295 \, \text{K} \).
Figure 2-9 reveals that the center wavelength of the second stop-band (corresponding to \( m=1 \)) has a linear relationship with the length of the stub as expected from Eq. (2.6). The parameters of the structure are set as: input waveguide width \( w=50 \) μm, stub width \( w_s=50 \) μm, distance \( d=100 \) μm, period number \( N=3 \) and temperature \( T=295 \) K. The center wavelength of the stop-band moves to a longer wavelength with the increase of the stub length \( L \). In addition, similar to the InSb slot single stub waveguide structure, the central wavelength of the stop-band of multiple-stubs structures can be tuned by temperature. Figure 2-10 shows the relationship between the central wavelength of the second transmission stopband and the temperature for the incident waveguide width \( w=50 \) μm, stub width \( w_s=50 \) μm, distance \( d=100 \) μm, period number \( N=3 \). The central frequency of the transmission stopband increases with the increase of the temperature. The frequency tuning range is 0.13 THz for temperature from 225 K to 325 K. Therefore, one can realize wide stopband filtering function at the desired terahertz.
range in multiple-stubs InSb slot waveguide structures by properly choosing the structure parameters, such as the stub length and width.

![Graph showing the relationship between central frequency of the second transmission stop-band and temperature](image)

Fig. 2-10 Relationship between the central frequency of the second transmission stop-band and the temperature. The parameters include the waveguide width $w=50 \mu m$, the stub width $w_s=50 \mu m$, the distance $d=100 \mu m$, and the period number $N=3$.

Plasmon dissipation is a critical issue for designing the plasmonic waveguide filter. A possible solution to improve the performance of the filter is to combine surface plasmons with electrically and optically pumped gain media such as semiconductor quantum well embedded into the structure. The emergence technology such as graphene technology is also to provide loss compensation for terahertz spectral range [83, 84].

### 2.5 Conclusion

In summary, we propose planar tunable subwavelength plasmonic stub waveguide filters in terahertz range. The InSb plasmonic slot waveguide has a strong confinement and guides THz waves beyond the diffraction limit. The single-stub InSb plasmonic slot waveguide structure can operate as a notch filter with its central wavelength
designed by choosing the width and/or the length of the stub structure. As an extension to the single-stub structure, the multiple-stubs InSb slot waveguide structure can realize a wide stop-band filtering function. These filtering characteristics of the stub structures can be actively controlled through temperature tuning. The proposed structures open a possibility for future applications in ultra-compact THz integration circuits.
3 Basics of graphene plasmons and the excitation by s-polarized four-wave mixing

3.1 Basics of graphene plasmons

Graphene is a carbon atom plane arranged in two-dimensional (2D) hexagonal lattice, which was first discovered by Novoselov and Geim [85] in 2004. This atomic layer of carbon has attracted great interest due to its unique electronic and mechanical properties [86-89], such as the electron mobility is highest compared with any currently known materials, which is due to the electrons in graphene are of zero effective mass which transport without scattering. These electrons are also called Dirac fermions [90], which have spurred great interests on its electronic applications such as high-frequency transistors [91, 92], transparent electrodes [93-95]. Besides the excellent electronic properties, graphene has been utilized in developing ultrafast photonic devices, such as the optical modulator [96], and photodetectors [48]. Graphene has also been found as a promising platform for plasmonic applications in the IR frequency regime. Graphene exhibits a relatively large conductivity which translates into long optical relaxation times ($\tau \sim 10^{-13}$ s), and thus could potentially provide a large plasmon wave propagation distances [97, 98]. The carrier density or equivalent Fermi energy level $E_f$ relative to the Dirac point of graphene can be adjusted chemically or through bias voltage applied on a field effect transistor (FET) [99] in less than a nanosecond [96]. In addition, graphene surface plasmons have a strong optical field confinement, which has been verified by experiments [100-102]. Graphene surface plasmon resonances have strong oscillator strength at room temperature, in comparison, low temperatures (4.2K) were required for conventional two-dimensional electron gas [103, 104].
3.1.1 The permittivity of graphene

Graphene sheet can treated as an anisotropic material with thickness $t = 0.5$ nm, where the out of plane permittivity is 2.5 based on the graphite dielectric constant. The optical conductivity of the graphene sheet is calculated with the random-phase approximation (RPA) [105, 106] as

$$\sigma(\omega) = \frac{2ie^2k_B}{\pi\hbar^2(\omega+i\tau)^{-1}}\ln\left[2\cosh\left(\frac{E_f}{2k_B T}\right)\right]$$

$$+ \frac{e^2}{4\hbar}\left\{\frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{\hbar\omega - 2E_f}{2k_B T}\right) - \frac{i}{2\pi}\ln\left[\frac{(\hbar\omega + 2E_f)^2}{(\hbar\omega - 2E_f)^2 + (2k_B T)^2}\right]\right\}$$

(3.1)

where $k_B$ is the Boltzmann constant, $T$ is the temperature, $\omega$ is the light frequency, $\tau$ is the carrier relaxation time from the impurities in graphene, and $E_f = \hbar V_f(\pi n)^{1/2}$ is the Fermi energy level, where $n$ is the charge carrier concentration, $V_f = 10^6$ m/s is the Fermi velocity, $\mu$ is the carrier mobility in graphene. The first and second terms of Eq. (3.1) are respectively attributed to the intraband transition and interband transition.

The in-plane graphene permittivity is characterized by a dielectric function of $\varepsilon_{||} = 2.5 + i\sigma(\omega)/\varepsilon_0\omega$. The carrier relaxation time $\tau = \mu E_F eV_F^2$ is determined by the Fermi energy level and carrier mobility in graphene [107].

Figures 3-1 (a) and (b) show the Real part and imaginary parts of the graphene permittivity with different carrier concentrations at room temperature. As we can see that the real part is negative in the mid-IR spectral range and it decreases as a increase of the carrier concentration. Graphene can function as an electrical or carrier concentration tunable plasmonic material in IR ranges.
3.1.2 The dispersion of graphene plasmons

Next, we derive the dispersion relation of graphene surface plasmons (GSPs), which is very important for graphene plasmonics study.

H \_y \_1 = A e^{-k_z z} e^{i\beta_z}, \text{ for } z>0, \\
H \_y \_2 = B e^{k_z z} e^{i\beta_z}, \text{ for } z<0. \quad (3.2)

Through the Maxwell's equation
\[
\frac{\partial H_z}{\partial z} = i\omega \varepsilon_0 \varepsilon_{1,2} E_x \quad (3.3)
\]

we can obtain the expression of the electric field:

\[
E_{x1} = -\frac{A_k}{i\omega \varepsilon_1} e^{-k_1 z}, \text{for } z>0, \\
E_{x2} = \frac{B_k}{i\omega \varepsilon_0} e^{-k_2 z}, \text{for } z<0. \quad (3.4)
\]

After applying the continuity of the \(x\) components of the electric field at the interface of \(z=0\), we have

\[
\frac{-A_k}{\varepsilon_1} = \frac{B_k}{\varepsilon_2} \quad (3.5)
\]

The wavevectors perpendicular and parallel to the interface are conserved in the following relations

\[
k_{1,2}^2 + \varepsilon_{1,2} k_0^2 = \beta^2, \quad (3.6)
\]

and the graphene plasmons follow the following boundary conditions:

\[
H_{y1} = H_{y2} + \sigma E_{x1} \quad (3.7)
\]

which includes the optical conductivity of the 2D graphene \(\sigma\).

Combining the Eqs. (3.5-3.7), we obtain the dispersion relation of the TM modes:

\[
\frac{\varepsilon_1}{\sqrt{\beta^2 - \varepsilon_1 k_0^2}} + \frac{\varepsilon_2}{\sqrt{\beta^2 - \varepsilon_2 k_0^2}} = -\frac{i\sigma(\omega)}{\omega \varepsilon_0} \quad (3.8)
\]

Since the graphene plasmons satisfies \(\beta \gg k_0 = \omega/c\), the dispersion relation simplifies to
\[ \beta = \varepsilon_0 \varepsilon_1 + \varepsilon_2 \frac{2i\omega}{\sigma(\omega)} \]  

(3.9)

Fig. 3-3 shows the dispersion relation of graphene plasmons, which is the graphene plasmon wavevector \( k_{sp} \) (equals to graphene plasmon propagation constant) normalized by free space wave vector \( k_0 \) for different carrier concentrations. The substrate is assumed to be SiO\(_2\) with permittivity \( \varepsilon_2 = 4 \), and the dielectric above is assumed to be air with permittivity \( \varepsilon_1 = 1 \). Fig. 3-4 shows the graphene plasmons wavelength as a function of free space wavelength with different carrier concentrations. One can see that the graphene plasmons wavelength is much shorter than that of the free space. As a result, the graphene can have a highly confined electric field. Also, it is a challenge to excite graphene plasmons due to strong wavevector mismatch between graphene plasmons and light in free space as shown in Fig 3-3.

![Dispersion relation of graphene plasmons with different carrier concentrations. The carrier relaxation time is 0.4 ps.](image)
3.2 Scheme for exciting graphene surface plasmons using four-wave mixing

Despite these promising GSPs capabilities, methods to efficiently excite GSPs by free-space optical beams are on demand, where such a challenge is due to the large wave-vector mismatch between GSPs and optical beams. In recent experimental demonstrations so far, a sharp atomic force microscope (AFM) metallic tip that acts as a resonant optical antenna to mimic a vertically-polarized dipole, was used for launching GSPs [100, 101, 108]. Another approach is to use grating coupling method, for example the patterned silicon gratings placed beneath the graphene layer [109, 110] or graphene grating created by its elastic vibration due to the flexural wave or electrically generated surface acoustics [111, 112]. While, the vertical-dipole method needs the AFM tip contact with graphene surface which could change the properties of the graphene, and the grating coupler method could suffer from scattering of plasmons on the patterned edges. Terahertz surface plasmon excitation by the nonlinear difference frequency generation in graphene and topological insulators have been theoretically studied [113]. Moreover, all these approaches for exciting GSPs requires
the polarization state of the incident optical field to be p-polarized incidence as is also the case for other plasmonic materials, such as gold and silver [114]. In this case, a fundamental question of interest arises whether it is possible to excite plasmons using *only s-polarized* light.

In this section, we present a novel scheme that is capable of exciting GSPs on a flat suspended graphene by using *only s-polarized* optical beams through nonlinear four-wave mixing (FWM) process. We present the detailed theoretical derivations by using the Green's function analysis, which enables us to obtain the required conditions of third-order susceptibility tensors for plasmon excitation on graphene. The proposed scheme provides a new route for graphene plasmon excitation with a potential for pure optical switching and modulation of GSPs.

The proposed scheme for exciting GSPs using FWM process is shown in Fig. 3-5, where two *s-polarized* optical beams (i.e. the electric field polarized along \( x \)) are with the oscillation frequencies of \( \omega_1 \) and \( \omega_2 \), respectively. The corresponding incident angles are denoted as \( \theta_1 \) and \( \theta_2 \), where the incident angle \( \theta \) is measured with respect to the surface normal direction clockwise. The graphene layer is suspended in air and the dielectric environment surrounding graphene in this case is denoted as \( \varepsilon_1 = \varepsilon_2 = 1 \).

Because the nonlinear FWM process involves three incident photons, the oscillation frequencies of the resultant optical beams will be either \( 2\omega_1 - \omega_2 \) or \( 2\omega_2 - \omega_1 \). Here, to simplify the analysis, we focus on one of the oscillation frequencies without the loss of generality, where

\[
\omega_{\text{app}} = 2\omega_1 - \omega_2
\]  

(3.10)

Nonlinear FWM process has been used to excite surface plasmon polaritons on gold film using two *p-polarized* optical beams [115, 116], where the detailed analytical
expressions of the required third-order nonlinear susceptibility is difficult to obtain due to the 3D nature of bulk gold. In comparison, graphene has an atomic layer thickness, which simplifies the analysis significantly. This simplification enables us to investigate the novel excitation schematic, i.e. pure \( s \)-polarized incident condition.

![Diagram](image)

Fig. 3-5 Scheme for exciting GSPs by using FWM process. The incident optical beams from free-space are having the oscillation frequencies of \( \omega_1 \) and \( \omega_2 \), respectively, where the incidence angles are denoted as \( \theta_1 \) and \( \theta_2 \). The figure illustrates the resonant incident angles (blue arrows) and the direction of GSPs propagation (purple arrow).

Figure 3-6 presents the dispersion curve of GSPs with a Fermi energy level of \( E_F = 0.4 \) eV (solid red line), where the in-plane GSPs wave vector is given as

\[
k_{spp}(\omega_{spp}) = \frac{\epsilon_1 + \epsilon_2}{2} \frac{2i\omega_{spp}}{\sigma(\omega_{spp})}.
\]

This expression was obtained by solving the dispersion relation of GSPs [117]. We also give its detail derivation in section 3.1.2. Here, \( \sigma \) is the optical conductivity calculated using RPA, and the detailed expression of \( \sigma \) is given by Eq. (3.1). We can see that the GSP wave vector is much larger than the one of the free-space optical
beams with the same oscillation frequency. In order to excite GSPs with free-space optical beams, the momentum mismatch has to be satisfied. Based on the conservation of momentum as shown in Fig. 3-6, we have

\[
\pm \text{Re}\{k_{\text{app}}(\omega_{\text{app}})\} = 2k_i \sin \theta_1 - k_2 \sin \theta_2
\]

(3.12)

where the upper sign of \(k_{\text{app}}(\omega_{\text{app}})\) is corresponding to a solution of \(\theta_2 > \theta_1\), and the lower sign is corresponding to \(\theta_1 > \theta_2\).

Fig.3-6 Dispersion curve of GSPs with the Fermi energy level of \(E_F = 0.4 \text{ eV}\) (solid red line). Dashed line represents the light line in free-space, where \(k_i = (\pi n)^{1/2}\) is the Fermi wave vector. The carrier mobility used is \(\mu = 10000 \text{ cm}^2/(\text{V} \cdot \text{s})\). The GSP dispersion curve is lying beyond the light line, which prohibits the plasmon to be excited by a single incident beam. The vectorial sum of three incident photons, as shown solid line in pink and purple, make it possible to couple light into GSPs in the red line.

In order to satisfy the momentum conservation condition as specified by Eqs. (3.10) and (3.12), the relationship between \(\theta_1\) and \(\theta_2\) are solved numerically as shown in Fig. 3-7, where the incident optical beam has the free-space wavelengths of \(\lambda_i = 40 \mu\text{m}\) and
\( \lambda_2 = 25 \, \mu \text{m} \) respectively. From Fig. 3-7, one can see that within the solutions of the angular regions, the incident angles can be tuned by adjusting the Fermi energy levels, which principally enable to realize the functions of ultrafast electrically controlled switches.

![Fig.3-7 Relationship between the incident angles of \( \theta_1 \) and \( \theta_2 \) for exciting GSPs using FWM with different Fermi energy levels of graphene. The incident wavelengths are \( \lambda_1 = 40 \, \mu \text{m} \) and \( \lambda_2 = 25 \, \mu \text{m} \).](image)

### 3.3 Derivation of nonlinear optical field distribution for exciting GSPs

In this subsection, we shall present the detailed analytical investigation for exciting GSPs using FWM with only \( s \)-polarized optical beams. The electric field components for the two incident optical beams with the oscillation frequencies of \( \omega_1 \) and \( \omega_2 \) are written as:

\[
E_{1,x} = E_1 e^{ik_{1,x}x} e^{i\omega_1 t} e^{-i\lambda_1 z}, \\
E_{2,x} = E_2 e^{ik_{2,x}x} e^{i\omega_2 t} e^{-i\lambda_2 z},
\]

(3.13)
where $E_1$ and $E_2$ are the electric field amplitude of the incident waves. $k_1$ and $k_2$ vectors are determined by the angle of incidence $\theta$ according to $k_{1,2} = \omega_{1,2} \sin \theta / c$ and $k_{1,2} = \omega_{1,2} \cos \theta / c$.

The nonlinear polarization at the FWM frequency $\omega_{\text{app}}$, i.e. $P_i(r)$, due to the two incident optical beams on graphene itself can be expressed as

$$P_i(r) = \chi_{(3),xxx} E_{1,x} E_{1,x}^* E_{2,x}^*(\vec{r}),$$

(3.14)

where $\chi_{(3),xxx}$ denotes a component of third-order susceptibility tensor. At the FWM frequency of $2\omega_1 - \omega_2$, the beam at the oscillation frequency "$\omega_1$" contributes two photons, where the other beam at the oscillation frequency "$\omega_2$" contributes one photon. Similarly, at the FWM frequency of $2\omega_2 - \omega_1$, the condition will be vice versa.

The electric field distribution at the FWM frequency can be calculated from the current source based on the Green's function $\tilde{G}(\vec{r}, r')$ as [114]

$$\tilde{E}(r) = i \omega \mu_0 \int \tilde{G}(\vec{r}, r') \tilde{j}(r') dV',$$

(3.15)

where the current source is given by (see Eq. A (1) in Appendix for details)

$$\tilde{j}(r') = -i \omega \tilde{P}(r').$$

(3.16)

After inserting Eqs. (3.14) and (3.16) into Eq. (3.15), we obtain

$$\tilde{E}(r') = i \omega \mu_0 \int G_0(r', r') \tilde{j}(r') dV' dx' dy' dz'$$

$$= i \omega \mu_0 \int \int \int G_0(r', r') \tilde{j}(r') \delta(x', y', z') \delta(z'-z_0) dx' dy' dz'$$

$$= i \omega \mu_0 \int \int \int \chi_{(3),xxx} \delta(x', y', z') \delta(z'-z_0) dx' dy' dz',$$

(3.17)
where we use $\delta(z' - z_0)$ here to represent the 2D dimension characteristic of graphene.

$\hat{a}_x, \hat{a}_y$ and $\hat{a}_z$ are unit vectors along $x$, $y$ and $z$ directions, respectively. Substituting the Green's function as shown in Eq. (A (4)) into Eq. (3.17), we can obtain the analytical expression for the GSPs field:

$$
\overrightarrow{E}(r) = -\frac{\omega \mu_0}{2} \begin{bmatrix} M_{xx}(k_x, k_y = 0)\chi^{(3),xxx}_x + M_{yx}(k_x, k_y = 0)\chi^{(3),zzz}_x \\ M_{yx}(k_x, k_y = 0)\chi^{(3),zzz}_x + M_{yy}(k_x, k_y = 0)\chi^{(3),xxx}_y \\ M_{zz}(k_x, k_y = 0)\chi^{(3),xxx}_z + M_{zx}(k_x, k_y = 0)\chi^{(3),zzz}_z \end{bmatrix} e^{i(2k_x x + k_y y)} e^{ik_z z} \times E_1 e^{-i\omega t},
$$

(3.18)

where $M_{xx}, M_{yx}, M_{zx}, M_{zz}$ are the components of matrix $\overrightarrow{M}$ (see Eq. A (5) in Appendix for details). The optical fields generated at the FWM frequency have two sets of independent fields:

a) One is transverse electric (TE) field component consisted of $E_x, H_y$, and $H_z$, which is not a surface mode wave.

b) The other one is the transverse magnetic (TM) field component consisted of $E_y, E_z$, and $H_x$, which is corresponding to the GSPs mode.

As shown in literature [117], for graphene surface plasmon field for $z>0$, we have

$$
E_y = A e^{ik_y y} e^{-ik_z z}, E_x = 0, E_z = B e^{ik_y y} e^{-ik_z z},
$$

(3.19)

with $k_z = 0$, where $A$ and $B$ are the electric field amplitude.

By comparing Eqs. (3.18) and (3.19), we can obtain relation for the $M$ matrix components in order for the FWM component to satisfy the GSPs condition:

$$
\frac{M_{xx}(k_x, k_y = 0)\chi^{(3),xxx}_x + M_{yx}(k_x, k_y = 0)\chi^{(3),zzz}_x}{M_{xx}(k_x, k_y = 0)\chi^{(3),xxx}_x + M_{yx}(k_x, k_y = 0)\chi^{(3),zzz}_x} = \frac{A}{B} = D.
$$

(3.20)
After submitting the $M$ matrix components in to Eq. (3.20), we obtain the condition for GSPs excitation by $s$-polarized beams as:

$$[k_{1,spp}^2 + D k_{y,1,spp}] \chi_{(3),xxx} = [D k_{x}^2 + k_{y,1,spp}] \chi_{(3),zzz},$$

(3.21)

where $k_{y,1} = k_{spp}(\omega_{spp})$. We can get the numerical relationship between $\chi_{(3),xxx}$ and $\chi_{(3),zzz}$, as plotted in Figs. 3-8 with the monolayer graphene of having different Fermi energy levels. By introducing an elastic or plastic deformation to graphene, its symmetry class can be changed and thus the values of the third-order susceptibility might be able to be tuned for experimental realization of the required susceptibility values [118].

![Graph showing the relationship between the two components of the third-order susceptibilities with different monolayer graphene Fermi energy levels.](image)

Fig. 3-8 The relationship between the two components of the $\chi_{(3),xxx}$ and $\chi_{(3),zzz}$ with different monolayer graphene Fermi energy levels of $E_F = 0.2$ eV, 0.4 eV, 0.6 eV and 0.8 eV.

Figures 3-9(a) and 5(b) show the electric field wavefront distributions of $E_x$ with the pump lasers with the incident wavelengths of 40 µm and 25 µm, and under the incidence angles of 50° and 25.2° respectively, where the ratio of $\chi_{(3),xxx}$ over $\chi_{(3),zzz}$ is 0.014 as calculated in Fig. 3-8. The location of the monolayer graphene is labeled by
the dash lines. Figure 3-9(c) shows the calculated electric field distribution of $E_z$ for the GSPs, as excited by the FWM process. One can see that the GSPs has a wavelength of 47.8 µm, which is less than half of the wavelength of light in free space. In addition, the electric field is tightly confined on the graphene surface.

![Electric field wavefront distribution](image1)

Fig.3- 9(a)-(b) Electric field wavefront distribution of $E_x$ for the two pump lasers with incident wavelength of 40 µm and 25 µm, and incident angles of 50° and 25.2°, respectively. (c) Electric field distribution of $E_z$ for the excited GSPs on a monolayer graphene sheet with a Fermi energy level of $E_f = 0.4$ eV. The ratio of $\chi_{(3),xxx} / \chi_{(3),zzz}$ is 0.014.

We have also investigated the third-order susceptibility tensors relation for multilayer graphene with $N>1$. Here, the optical conductivity for $N$-layer graphene is $N\sigma$ [119, 120]. In Fig. 3-10, the relationship between the two components of the $\chi_{(3),xxx}$ and $\chi_{(3),zzz}$ for monolayer ($N=1$), bilayer ($N=2$), and four-layer ($N=4$) graphene with a Fermi energy level of $E_f = 0.4$ eV is shown. We can see that the proposed scheme is also working for the multi-layer graphene as well.
Lastly, we would like to mention that GSPs could also be excited by FWM at $p$-polarized incidence condition, where the required conditions could be derived similarly by using the theoretical framework as formulated in Eqs. (3.13)-(3.18). The detailed investigation on the $p$-polarized incidence condition might be beyond the scope of the current work and will be reported elsewhere. In addition, we also would like to mention a bit on the experimental feasibility, where the required laser power for exciting the four-wave mixing process of graphene optoelectronics is in the level of several hundred $\mu$W [121].

3.4 Conclusion

In conclusion, we have proposed a scheme capable of exciting plasmons on a flat suspended graphene through the FWM process. Based on the analytical derivations, we have shown it is possible to excite surface plasmons on graphene sheet by only $s$-polarized optical beams with certain incident angles over a broadband frequency and
the graphene surface plasmon are tunable by varying the electrical gating, or graphene doping. The proposed concept contributes a new possibility for the study of graphene surface plasmons, and this scheme can also be used for pure optical modulation and switching applications in the infrared regime.
4 Graphene-based tunable plasmonic Bragg reflector with a broad bandwidth

4.1 Introduction

Much attention has been focused on localized graphene surface plasmons resonance with the incident light from free space, such as nano-ribbons [102, 107, 122, 123], nano-disks [124], graphene-metal plasmonic antennas [125, 126], and graphene metamaterials [127]. A diffractive silicon grating has been demonstrated to excite plasmonic waves in graphene [128]. It is also interesting and indispensable to study the propagation properties of graphene plasmon waves. Guided plasmon waveguiding and hybridization in individual and paired nano-ribbon have been analyzed [98]. Recently, graphene nano-ribbon bended waveguides and splitters have been investigated [129].

In this chapter, we propose a plasmonic Bragg reflector structure formed in graphene waveguides and investigate its performance. We show that periodic stacks of plasmonic graphene-silicon and graphene-air waveguides can be utilized to design effective filtering effects around Bragg wavelength. The tunability of the filtering stopband by electrostatic and defect cavity mode are also studied. Such graphene Bragg reflector can be used to build high speed integrated modulators with broadband bandwidth. In addition, we introduce a defect into the Bragg reflector to achieve a defect cavity mode formed in the stopband with a high-Q of 50. Such a defect microcavity may be used to build graphene-based resonators for various applications.
4.2 The optical properties of GSPs on a graphene sheet deposited on silicon and air substrates

We first investigate the optical properties of SP waves on a graphene sheet deposited on silicon and air substrates, as shown in the inset of Fig. 4-1(b). By solving the dispersion relation (Eq. 3.9) of TM mode in graphene sheet, we can obtain the effective refractive index $n_{\text{eff}}$ of the surface plasmon waveguide mode on graphene sheet. In the calculations and simulations, graphene sheet is treated as an anisotropic material with thickness $t = 0.5$ nm, where the out of plane permittivity is 2.5 based on the graphite dielectric constant. The optical conductivity of the graphene sheet is calculated with the random-phase approximation (RPA) [105, 106] as introduced in section 3.1.1. In our following calculations, the optical conductivity contribution of interband transition is neglected because the Fermi level is always above half of the photon energy in the simulated mid-infrared spectral range and under this circumstance the intraband transition dominates the conductivity.

![Graphene-plasmon modes](image)

Fig. 4-1 The effective refractive index of a graphene plasmon mode at graphene-silicon waveguide and graphene-air waveguide as a function of optical wavelength for different Fermi energy levels. (a) Real ($n_{\text{eff}}$), and (b) Imag ($n_{\text{eff}}$). The depth of the air trench is 20 nm and the carrier mobility used is $\mu = 10000$ cm$^2$/V·s.
Figure 4-1 (a) shows the real part of effective refractive index of the surface plasmon mode supported by graphene sheet on silicon and air substrate for different Fermi energy levels. The effective refractive index at the Fermi energy level $E_f = 0.4$ eV is larger than 100, indicating that the mid-infrared plasmon wavelength is 2 orders of magnitude smaller than its wavelength in free space and the surface plasmon wave is highly localized. As the Fermi energy level increases, the real part of surface plasmon wave effective index decreases, which can be utilized in the tunable plasmonic device in the following sections. In addition, the effective refractive index of the surface plasmon mode on the silicon substrate is larger than that on the air substrate for a given Fermi level energy which can provide an effective index contrast. The imaginary part of effective refractive index of surface plasmon mode at different Fermi energy level is also plotted in Fig. 4-1(b). One can see that Imag $(n_{\text{eff}})$ decreases with increasing Fermi energy level for a given wavelength, indicating that the propagation loss is smaller at higher Fermi energy level as the propagation distance is given by $L = 1/[2k_0\text{Im}(n_{\text{eff}})]$, where $k_0 = 2\pi/\lambda_0$ (wavelength in the vacuum $\lambda_0$). As shown in Fig. 4-1(a), there is a high effective index contrast for the surface plasmon mode between the graphene on silicon and air substrate. Thus, by periodically modulating the effective index along the graphene sheet which can be realized by alternatively stacking graphene-silicon and graphene-air waveguide, a Bragg reflector will be formed. In addition, by electrostatic tuning properties of graphene, a fast tunable device can be expected. Figure 4-2 shows the schematic of graphene plasmonic Bragg reflector formed in silicon gratings, for which the Bragg condition is formulated as:

$$L_1\text{Real}(n_{\text{eff}1}) + L_2\text{Real}(n_{\text{eff}2}) = m\lambda_b / 2$$  \hspace{1cm} (4.1)$$

where $\lambda_b$ is the Bragg wavelength and $m$ is an integer. $L_1$ and $L_2$ are respectively the lengths of the graphene-silicon and graphene-air plasmonic waveguide as indicated in
Fig. 4-2. Real($n_{\text{eff1}}$) and Real($n_{\text{eff2}}$) are the real parts of the refractive indices of the graphene-silicon and graphene-air plasmonic waveguide, respectively. If Eq. (4.1) is satisfied, surface plasmon wave propagation through the structure at the Bragg wavelength will be prohibited. The FDTD simulations are used to calculate the optical response of the graphene Bragg reflector. Perfectly matched layer absorbing boundary condition is set at all boundaries of the simulated domain.

![Schematic of graphene plasmonic Bragg reflector formed in silicon grating substrate. The depth of air trench is 20 nm.](image)

Figure 4-3(a) shows the simulated transmission spectra of graphene plasmonic reflector with the Fermi energy level $E_f = 0.6$ eV, 0.7 eV, and 0.8 eV, and period number of Bragg cell $N = 10$. One can see that there are two wide stopbands with near-zero transmissions around the Bragg wavelength of 9.1 µm (corresponding to $m = 2$) and wavelength of 7.3 µm (corresponding to $m = 3$) for $E_f = 0.6$ eV which shows good filtering characteristics. The bandwidths of the stopband (defined as the difference between the two wavelengths at each of the transmittance is equal to 1%) are respectively 368 nm and 310 nm for the Bragg wavelength of 9.1 µm and 7.3 µm. Broadband and faster electrostatic tunability are the most intriguing properties of graphene materials. Simulated spectra in Fig. 4-3(a) shows that the stopband shifts to a shorter wavelength with an increase of the Fermi energy level. Fig. 4-3(b) shows the
center wavelength of Bragg reflector as a function of Fermi energy levels, which confirms the broad tuning range with a small change of Fermi level. The FDTD simulations results agree very well with the theoretical curve obtained from Eq. (4.1).

Figure 4 shows the transmission spectra of the graphene Bragg reflector for different period numbers. In principle, Bragg scattering exists at any number of Bragg periods. In the case of the graphene Bragg reflector, it is found that the minimum $N$ required to realize transmission less than 0.4% in the stopband is $N = 10$. The spectrum displays some sidelobes out of the stopband which, we believe, are due to light scattering at the abruptly disappearing boundary at the end of the Bragg gratings. As shown in Fig. 4-4, the increase of period number gives rise to higher propagation losses. The reflectance at the Bragg wavelength of 7.9 µm is as high as 96.4%, while the transmittance is below 0.4%. In addition, we calculate the transmittances of the graphene Bragg reflector at different carrier relaxation times and find that the transmittances at the
wavelength of 8.4 µm are 0.4%, 25%, 49% for \( \tau = 1.6 \times 10^{-13} \text{s} \), \( 4 \times 10^{-13} \text{s} \), \( 8 \times 10^{-13} \text{s} \), which are corresponding to the carrier mobilities of 2000 cm\(^2\)/(V\cdot S), 5000 cm\(^2\)/(V\cdot S), 10000 cm\(^2\)/(V\cdot S) \) of graphene with periods \( N=10 \), Fermi energy level \( E_f = 0.8 \text{ eV} \). One can conclude that with a large relaxation time, graphene Bragg reflector has a lower propagation loss.

Fig. 4-4 Transmission spectra of graphene Bragg reflector consisting of 6, 8 and 10 periods with Fermi energy level \( E_f = 0.8 \text{ eV} \), and reflection spectrum for periods of 10. Other structure parameters are the same as in Fig. 4-3.

To further investigate the properties of the graphene plasmonic Bragg reflector, we introduce a defect into the graphene Bragg plasmonic structure by decreasing the length \( L_d \) of the central graphene-silicon waveguide. Figure 4-5(a) shows the transmission spectrum when the 4\(^{th}\) graphene-silicon waveguide length is reduced to 50 nm. The cavity length Q-factor is defined as \( Q = \frac{\lambda_0}{\Delta \lambda} \), where \( \lambda_0 \) and \( \Delta \lambda \) are respectively the central resonance wavelength and full width at half maximum of the defect mode. It describes the ratio of the energy stored in the microcavity at resonance to the energy escape from the cavity per cycle of oscillation. The Q-factor for the
cavity in Fig. 4-5 (a) is found to be about 50, which is higher than the nano-ribbon resonance cavity [102] in the literature. One can see that the stopband of defect cavity is little larger than the Bragg reflector, which is attributed to the destructive interference inside of the Fabry-Perot-like microcavity. As shown in Fig. 4-5(b), the peak wavelength of the defect mode can be adjusted by changing the length of the cavity. In addition, the peak wavelength can also be easily shifted by electrically tuning the graphene carrier density.

Fig. 4-5 (a) Transmission spectra of Bragg reflector with period number \( N = 8 \) and the microcavity formed by introducing a defect length \( L_d = 50 \) nm. (b) Peak wavelength as a function of defect length. The Fermi energy level \( E_f = 0.8 \) eV, and the lengths of the graphene-silicon, graphene-air waveguide are \( L_1 = 80 \) nm, \( L_2 = 40 \) nm, respectively.

Figure 4-6 shows the field profiles of the graphene surface plasmons propagation through the Bragg reflector and the defect microcavity. Figure 4-6(b) shows that the incident wave is reflected at the wavelength of 9.1 \( \mu \)m, while it transmits through the structure at the wavelength of 10.5 \( \mu \)m which is out of the stopband [Fig. 4-6(a)]. As displayed in Fig. 4-6 (c), the surface plasmons wave at the wavelength of 7.6 \( \mu \)m couples into the microcavity structure, which agrees well with the spectrum shown in Fig. 4-5(a).
Fig. 4-6 (a) and (b) Simulated electric field intensity profiles of the plasmonic Bragg reflectors at the incident wavelength at 10.5 µm and 9.1 µm, and the structure parameters are the same as Fig. 4.3(a) with Fermi energy level of 0.6 eV. (c) corresponds to the defect resonance mode in Fig. 4.5(a) at 7.6 µm.

4.3 Conclusion

In conclusion, we have proposed and numerically analyzed plasmonic Bragg reflector formed in graphene waveguides. The plasmonic Bragg reflector can achieve a broadband fast-tunable stopband. This structure can also be used as a modulator, for a Bragg reflector with structure parameters in Fig. 4-3, a modulation depth of 21.4 dB can be achieved at the wavelength of 8.34 µm when the Fermi level is increased from 0.7 eV to 0.8 eV, which is the appealing property of the device compared with metallic plasmonic Bragg structures [130, 131]. By introducing a defect in the Bragg reflector, we can obtain a defect resonance mode with a high-Q factor. Those proposed structures could find applications in building active integrated photonic circuits.
5 The reduction of surface plasmon losses in quasi-suspended graphene

5.1 Introduction

Graphene has a high charge carrier mobility that potentially promises GSPs with a low loss and a relative large propagating distance. The graphene mobility strongly depends on the properties of the substrate/environment surrounding the graphene, which directly affects the performance of graphene based devices [132, 133]. Recently, mid-IR spectroscopic studies uncovered the crucial role of substrate optical phonons on GSPs damping [134, 135]. So far, the direct experimental mapping of propagating mid-IR GSPs have demonstrated only a very short propagation distance (about several plasmon wavelengths) [100, 101]. Therefore, further study of GSPs damping mechanisms and approaches to increase the propagating distance are highly important for the development of future on-chip mid-IR plasmonic devices. GSPs damping originates from the coupling with the substrate optical phonons have been proposed to be reduced by using an excitation wavelengths out of the corresponding phonon resonant region [134].

In this chapter, we experimentally study the effect of quasi-suspending graphene on GSPs. The monolayer graphene is placed above a chemically engineered nanometric spacer (NS) on the substrate, that leads to a “quasi-free-standing”-like graphene. By direct mapping of GSPs with scattering-type scanning near-field optical microscope (s-SNOM), we find that the nanostructured approach can reduce the damping of mid-IR plasmons in graphene. In addition, we compare our experimental results with numerical simulations based on a developed numerical model.
5.2 Near-field measurement results and discussions

To experimentally image GSPs, we use scanning plasmon interferometry technique as we introduced the setup in the section 1.2, which was first used for observation of the mid-IR propagating plasmons in graphene [101]. The technique utilizes a sharp metalized tip to overcome the large plasmon-photon momentum mismatch for excitation of GSPs. The tip strongly confines incident light and launches propagating plasmons on graphene as shown in Fig. 5-1(a). In the experiment, the tip is illuminated by either a quantum cascade or carbon dioxide laser with wavelength of $\lambda_1 = 10 \ \mu m$ or $\lambda_2 = 11.2 \ \mu m$ correspondingly in $p$-polarization. The Pt-coated Si tip generates cylindrical GSPs travelling in all directions along the graphene flake during the scanning at the tapping frequency $\Omega$ of about 250 kHz and the tapping amplitude of about 60 nm. Launched GSPs reflected back from the graphene edges are collected by the tip and scattered out in the far-field for detection. Pseudo-heterodyne interferometric detection is used to record the tip-scattered light $E_s$, as well as topography. We use the 4\textsuperscript{th} harmonic signal due to its good signal to noises ratio.
Fig. 5-1(a). A schematic of the principle for s-SNOM measurements. (b,d) Optical near-field images of exfoliated graphene with $\lambda_2 = 11.2 \, \mu m$ and $\lambda_1 = 10 \, \mu m$, respectively. (c,e) Corresponding cross-sections along dotted yellow lines in images (b,d). $E_s$ is the strength of tip-scattered field. Green and pink dotted lines mark GSPs’ field magnitudes at the first maximum close to the boundary ($E_{s_{pl}}$), and inside the inner part of graphene crystals (background value, $E_{s_{bg}}$).

We first measure the samples without applied spacer using an excitation wavelength $\lambda_1 = 10 \, \mu m$ where the strong coupling between GSPs and SiO$_2$ optical phonons [135]. All graphene samples are fabricated by mechanical exfoliation of graphite with a Scotch tape. Figure 5-1(b, d) show two typical near-field images of the edge of a graphene flake fabricated on a clean 285-nm thermally oxidized Si-SiO$_2$ wafer at $\lambda_1 = 10 \, \mu m$ and $\lambda_2 = 11.2 \, \mu m$. Corresponding cross-sections of $E_s$ along yellow dotted lines are
plotted in Fig. 5-1(c, e). We can see that GSPs appear as half GSP-wavelength spaced fringes at $\lambda_2 = 11.2 \mu m$, while plasmon field is significantly damped at $\lambda_1 = 10 \mu m$ and nearly invisible due to the strong interaction with substrate phonons. We use the ratio $R$ between the first maximum of the electric field (close to graphene edge) and the field magnitude in the inner part of the flake, $E_{sp}/E_{bg}$, to represent the damping rate of GSPs. Moderately damped GSPs at $\lambda_2 = 11.2 \mu m$ (out of SiO$_2$ phonon line) is characterized by $R \approx 1.5$, while for $\lambda_1 = 10 \mu m$, the ratio $R \approx 1$. In the following study we will use the nanometric spacing approach to increase the parameter $R$ for the case of excitation at $\lambda_1$.

We use the following fabrication processes to produce graphene-SiO$_2$ spacer: (1) A thermally oxidized Si-SiO$_2$ wafer (285 nm oxide thickness) was covered with AZ 5214E photoresist with thickness of 1.6 $\mu m$ by spin-coating at 4000 rpm for 30 s. (2) The wafer was then softbaked for 100 s at 105°C. (3) The resulting sample was dry-etched for 4.5 min in the Plasma-Therm 790 series RIE (CF$_4$ etchant, pressure 80 mtorr, RF power 100 W). The dry-etching process leads to a higher degree of cross-linking, within the novolac-based polymer component of the photoresist due to a possible overheating/UV exposure from the plasma. The dry-etching does not remove the photoresist film completely with the chosen etching time. (4) Then the sample was sonicated in NI555 stripper for 5 min at room temperature, and finally, it was rinsed thoroughly with 2-propanol and dried with nitrogen.
Fig. 5-2 (a) Atomic force microscopy (AFM) image and a corresponding height profile along the dotted white line. (b) Side-view schematic of the graphene on top of NS-SiO$_2$. (c) Zoomed-in AFM topography of the area inside the dotted cyan square in image (a). (d) Height histogram of the data in the image (c); inset shows a height profile along the white dotted line in the image (c).

Then graphene is mechanically exfoliated on top of the fabricated spacer. Fig. 5-2(a) shows a typical AFM image and the height profile of a graphene flake above NS. The tapered-ribbon shape of the flake of graphene is clearly defined in the middle of the image. Graphene appears as a sheet with a thickness less than 1 nm, partially conformed to the engineered nano-dots. Additional scratching and AFM measurements of NS close to the flake location, as well as presented further optical s-SNOM data, show that nano-dots have been grown directly on SiO$_2$ without any continuous base-layer of polymer beneath. Zoomed-in AFM topography inside the cyan dotted square in Fig. 5-2(a) is presented in Fig. 5-2(c). The corresponding height histogram (Fig. 5-2(d)) and cross-sections analysis (inset in the Fig. 5-2(d)) show that fabricated NS on top of SiO$_2$ surface is formed by randomly placed nano-dots with an
average height of 1.2 nm, a typical size of 20 - 40 nm and a typical spacing of 10 - 100 nm.

Fig. 5-3 Near-field imaging of the graphene on NS-SiO$_2$, performed at $\lambda_1 = 10$ μm. (a) Near-field optical image. The yellow arrow marks an optical ring structure around a single “extra-high” NS dot beneath graphene. (b) Corresponding cross-section along the dotted yellow line in image (a). Green and pink dotted lines mark GSPs field magnitudes at the first maximum close to graphene edges, and inside the inner part of the graphene. Red arrows highlight a distance between first and second maxima of GSPs field distribution.

Fig. 5-3(a) shows a typical optical near-field image of graphene supported by engineered NS on SiO$_2$ (at $\lambda_1 = 10$ μm). The image corresponds to the bottom part of the tapered ribbon displayed in Fig. 5-2(a). As can be seen from data the electric field is concentrated close to the edges of NS-supported graphene flake in a much stronger fashion than in the case of bare SiO$_2$ substrate (Fig. 5-1(d, e)). The estimated ratio $R$ from the corresponding cross-section in Fig. 5-3(b)), reaches the value of about 1.3. The tip of the graphene ribbon taper localizes plasmon fields stronger than graphene edges that are distant from the tip part. These observations agree with the previously reported typical pictures of mid-IR GSPs for the case of moderate damping [100]. This
directly supports the benefits of NS as an efficient approach for the reduction damping. Nanostructured spacer increases corresponding visibility of GSPs in s-SNOM from a crucially-damped level (Fig. 5-1(d, e)) to a moderately-damped and visible case (Fig. 5-3).

A typical feature of the near-field images of NS-supported graphene is the lack of polymer dot optical fingerprints in the field distribution inside the flake areas. The substrate part (Fig. 5.3(a)) appears as a uniform background (SiO\textsubscript{2} signal) with randomly distributed black spots which represent the near-field signal of polymer dots. Therefore, we conclude that the roughness in topography observed within NS beneath the flake does not cause considerable reflections/scattering of GSPs field. This agrees with previously reported observations of GSPs reflections at nanometer-size steps in quasi-free-standing graphene on SiC [136], where authors derived a critical step-height of about 1.5 nm below which, no reflection appears. In our observations, as it is seen from Fig. 5-2, the majority of the polymer dots have a height within the 1.5 nm range. As an exception, marked with a white arrow in the Fig. 5-3(a) we observe the reflection of GSPs around an “extra-high” polymer dot with a height of about 3.5 nm. The field distribution around the dot features a center minimum and surrounding bright ring structure. A distance $\Delta$ between the center of the minimum and the ring is about 75 nm. We suggest that the estimated $\Delta$ is related to the half wavelength ($\lambda_{gp}/2$) of propagating GSPs reflected from the dot. Additionally, as it can be seen from Fig. 5-3(b) that the optical field profile features two faint secondary maxima at the distance of about 85 nm from first maxima at both sides of the flake, that we ascribe to the plasmon interferometry. All mentioned regularities are a typical attribute related to propagating mid-infrared GSPs, thus demonstrating the beneficial effect of employing NS for damping reduction at $\lambda_1 = 10 \, \mu$m.
Fig. 5-4 Demonstration of GSPs damping suppression in graphene, exfoliated directly at the boundary between SiO$_2$ and NS-SiO$_2$. (a) AFM topography; the pink and green colors highlight graphene on NS-SiO$_2$ and graphene on approximately clean SiO$_2$, respectively. (b) Near-field optical image (recorded at $\lambda_1 = 10$ μm) of the graphene, corresponding to the region marked with a cyan dotted frame in image (a); (c, d) Corresponding cross-sections along green and red dotted lines in image (b); green and pink dotted lines mark GSPs field magnitudes at the first maximum close to the graphene edge, and inside the inner part of the graphene.

To further analyze the effects of NS on GSPs, we study a single exfoliated graphene flake which locates directly on the boundary of NS and almost clean SiO$_2$ regions. Figure 5-4(a) shows the topography of the sample, where graphene on NS and
graphene on SiO$_2$ are marked with pink and green colors, respectively. As it can be seen from the data, the green area contains only several nano-dots, therefore can be considered, approximately, as bare SiO$_2$; while the pink region is covered with well-developed nanostructured layer. Grey part represents the SiO$_2$ substrate, which is not covered with graphene. An optical near-field image (Fig. 5-4(b)) of the boundary has been recorded for the region marked with a cyan dotted rectangle in Fig. 5-4(a). White lines highlight the boundary between graphene-NS and graphene-SiO$_2$ areas. On this boundary the flake is divided into two regions: with a higher and a lower near-field scattered signal strength. The electric field distribution close to the edge of the flake has a typical maximum with a magnitude switching while passing across the boundary. Field cross-sections, $E_s$, along dotted green and red lines are plotted in Fig. 5-4(c,d) correspondingly. It is important to mention that not only the field magnitude, but also the damping-related factor $R$ increases along with passing across the boundary transition. The value $R$ in graphene-SiO$_2$ area is about 1.1, while in graphene-NS region it reaches the value of more than 1.3, thus directly demonstrating GSPs damping decrease in the same graphene as a result of NS implementation.

Finally, the simulations of mid-IR plasmon damping and propagation in graphene on NS-SiO$_2$ at $\lambda_1 = 10$ µm are conducted by FDTD method. To describe spacing and partial suspension effects on GSPs, we implement a simplified two-dimensional model as schematically displayed in the inset of Fig. 5-5. The optical conductivity of a graphene sheet is calculated in the RPA model. The thickness of the polymer spacer is set as 1.2 nm, based on the average experimental value. The Fermi energy and mobility of graphene are taken as: $E_F = 0.4$ eV and $\mu = 10000$ cm$^2$/V∙s; the refraction index of the polymer $n = 1.45$. The GSPs are launched in the graphene from the left-hand side. The evolution with the distance of the absolute value of vertical component
of plasmonic electric field is plotted in the graph in Fig. 5-5. We characterize the damping rate of GSPs by a ratio between the fifth and the first maxima of the field, which is presented (by colored circles, triangles and squares in Fig. 5-5) for several spacer parameters in the same graph (right-hand side axis). This data shows an increase of this parameter by about 4-5 times for all calculated NS geometries, compared to graphene on bare SiO$_2$, that represents a significant suppression of damping and agrees with the experiment. Our model describes only the effect of spacing and partial suspending of graphene sheet from SiO$_2$, while the value of the mobility is fixed.

Fig.5-5 Numerical simulations of GSPs propagation and damping on NS-SiO$_2$ at $\lambda_1 = 10$ μm. Inset shows a sketch of simplified 2-D model of the spacer; where $d$ is the nano-dot size, $g$ is the gap between neighboring dots, and blue arrow represents the direction of the plasmon launching. Light green curve is calculated for the random geometry of the spacer (from the left to the right: $d_1 = 40$ nm, $g_1 = 45$ nm, $d_2 = 60$ nm, $g_2 = 40$ nm, $d_3 = 45$ nm, $g_3 = 60$ nm, $d_4 = 30$ nm, $g_4 = 30$ nm, $d_5 = 45$ nm, $g_5 = 60$ nm, $d_6 = 45$ nm). Left-hand side axis displays an absolute value of the vertical component of electric field, taken at 3 nm distance above graphene. Right-hand side axis represents the damping strength, which is defined here as a ratio between fifth and first maxima of the field.
5.3 Conclusion

In summary, we demonstrate that lifting exfoliated graphene from the silicon dioxide surface with an ultra-thin nanostructured polymer spacer helps with the control of mid-IR plasmon damping and propagation. Polymer nano-dots cause spacing and partial suspension of graphene that is beneficial for remote phonons screening. Owing to the ultra-small thickness of the spacer, the nanoscale roughness does not lead to strong chaotic reflections of GSPs at the polymer nano-dots, that is one of requirements for unperturbed performance of possible devices fabricated in graphene on top of NS. Numerical simulations of plasmons propagation in graphene placed above silicon dioxide covered with polymer nano-dots show an increase of the propagation length and a suppression of damping that is in agreement with the experiment observations.
6. Conclusions and Future work

6.1 Conclusions

In this thesis, we study on guiding and manipulation of SPs in mid-and far-IR region, overcoming the weak confinement and non-tunability of SPs on conventional noble metals. In the chapter 2, we propose planar tunable subwavelength plasmonic stub waveguide filters in THz range, which provides a strong confinement and guides THz waves beyond the diffraction limit. The calculated results show that the single-stub InSb plasmonic slot waveguide structure can operate as a notch filter with its central wavelength designed by choosing the width and/or the length of the stub structure. As an extension to the single-stub structure, the multiple-stubs InSb slot waveguide structure can realize a wide stop-band filtering function. In addition, those filtering characteristics of the stub structures can be actively controlled through temperature tuning. In the rest of the chapters, we focus on studying the SP properties on the graphene, which is a promising platform for plasmonic applications in the IR frequency regime due to its attractive properties, such as high field confinement, long lifetime, and fast electrical gating tunability. In chapter 3, we introduce the basic knowledge of graphene plasmon and propose a scheme capable of exciting plasmons on a flat suspended graphene through the FWM process, which provides a new method to overcome the momentum mismatch between the GSPs and free space light. Based on the analytical derivations, we have shown it is possible to excite surface plasmons on graphene sheet by only s-polarized optical beams with certain incident angles over a broadband frequency. The proposed concept contributes a new possibility for the study of graphene surface plasmons, and this scheme can also be used for pure optical modulation and switching applications in the infrared regime.
In addition to the existing free space graphene plasmonic devices or structures, in the following chapter 3, we have proposed and numerically analyzed plasmonic Bragg reflectors formed in graphene waveguides. This structure can also be used as a modulator, for a Bragg reflector with structure parameters in Fig. 4-2, a modulation depth of 21.4 dB can be achieved at the wavelength of 8.34 μm when the Fermi level is increased from 0.7 eV to 0.8 eV, which is the appealing property of the device compared with metallic plasmonic Bragg structures. By introducing a defect in the Bragg reflector, we can obtain a defect resonance mode with a high-Q factor. In the last chapter, we use the SNOM system to demonstrate that lifting exfoliated graphene from the silicon dioxide surface with an ultra-thin nanostructured polymer spacer, which helps with the control of mid-infrared plasmon damping and propagation. Polymer nano-dots result in spacing and partial suspension of graphene that is beneficial for remote phonons screening. Numerical simulations of plasmons propagation in graphene placed above silicon dioxide covered with polymer nano-dots show an increase of the propagation length and a suppression of damping that is in agreement with the experiment.

6.2. Future work

There are still some aspects that should be further studied. We list some points that could be done for the future works:

1. Developing a new type of metamaterials in the mid- and far-IR regions. As shown in near field measurement of Fig. 6-1, we can see the graphene plasmons exist in the gold cavity on graphene films. We could expect to use the gold-cavity on graphene as a basic metamaterials unit-cell, whose resonance wavelength is determined by the size of the cavity. In combination of the tunable properties of graphene, proposed metamaterials have the benefits of ultra-small size, monolithic, fast-tunability.
Fig. 6-1 Near-field measurement of gold cavity on graphene. (a) AFM topography. (b) graphene plasmon amplitude. (c) graphene plasmon phase.

2. Near-field investigation on the high coupling efficient structure for the excitation of graphene surface plasmons, such as coupled subwavelength grating assisted method.

3. Experiment observation of Bragg effect in plasmonic graphene waveguide structure by near-field measurement as we designed in Chapter 4, as well as defect cavity confined mode. We could use this structure as an integrated ultrafast switch in mid-IR and longer wavelength range.
Appendix

The relation between current source and the polarization components [114]:

\[
    \overrightarrow{j}(r') = -i\omega \varepsilon_0 \left[ \varepsilon(r') - \varepsilon_{ref}(r') \right] \overrightarrow{E}(r')
    = -i\omega \varepsilon_0 \Delta \varepsilon(r') \overrightarrow{E}(r')
    = -i\omega \overrightarrow{P}(r')
\]

(A1)

The detailed derivation for the electric field term in Eq. (3.14) is shown below:

\[
    E_{1,x}E_{2,x} = E_1^2 e^{i(2k_{1y}y)} e^{-i2\omega t} \times E_2^2 e^{i(2k_{1z}z)} e^{-i2\omega t}
    = E_1^2 E_2^2 e^{i(2k_{1y}y)e^{i(2k_{1z}z)}} e^{-i2\omega t}
    = E_1^2 E_2^2 e^{i(2k_{1y}y)e^{i(2k_{1z}z)}} e^{-i2\omega t}.
\]

(A2)

By substituting Eq. (A2) into Eq. (3.16), we obtain,

\[
    P_x(r') = \chi \overrightarrow{E_x} \overrightarrow{E_x} e^{i(2k_{1y}y)e^{i(2k_{1z}z)}} e^{-i2\omega t},
\]

(A3)

\[
    P_y(r') = \chi \overrightarrow{E_y} \overrightarrow{E_y} e^{i(2k_{1y}y)e^{i(2k_{1z}z)}} e^{-i2\omega t},
\]

(A3.a)

\[
    P_z(r') = \chi \overrightarrow{E_z} \overrightarrow{E_z} e^{i(2k_{1y}y)e^{i(2k_{1z}z)}} e^{-i2\omega t},
\]

(A3.b)

The Green's function is expressed as [114]

\[
    \overrightarrow{G}_0(r,r') = \frac{i}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overrightarrow{M}(k_x,k_y) e^{ik_x(x-x')} e^{ik_y(y-y')} e^{-i(\varepsilon-\varepsilon_0)\omega t} dk_x dk_y,
\]

(A4)

and the \(M\) matrix is given as
\[
\overline{M}(k_x, k_y) = 0 = \frac{1}{k_{_l,app}^2 - k_{z,app}^2} \left[ \begin{array}{ccc}
 k_{_l,app}^2 - k_y^2 & 0 & \mp k_z k_{z,app} \\
 0 & k_{_l,app}^2 & 0 \\
 \mp k_z k_{z,app} & 0 & k_y^2 
\end{array} \right]. \quad (A5)
\]

It can be seen that some terms in matrix $\overline{M}$ have two different signs. This sign difference originates from the absolute values $|z - z_0|$. The upper sign corresponds to $z > z_0$ and the lower sign to $z < z_0$. As graphene is an anisotropic material, the relation of the wavevector in both direction is given by:

\[
k^2_y \varepsilon_y + k^2_z \varepsilon_z = \left( \frac{\omega}{c} \right)^2 \varepsilon_y \varepsilon_z. \quad (A6)
\]

By substituting Eqs. (A5) - (A6) into Eq. (3.18), we have,

\[
\overline{E}(r) = i \omega \mu_0 \int \int \int \int \int \int \frac{i}{8\pi^2} \int \overline{M}(k_x, k_y) e^{ik_x(x-x')} e^{ik_y(y-y')} e^{-ik_zz'}
\times \left[ \hat{a}_{x X(3),xxx} + \hat{a}_{y X(3),xxx} + \hat{a}_{z X(3),xxx} \right] E^2 \overline{E}^* e^{i[2k_x-k_z]y'} e^{-i\omega \mu_0 t} dk_x dk_y dy',
\]

\[
= i \omega \mu_0 \int \int \int \int \int \int \frac{i}{8\pi} \overline{M}(k_x, k_y) e^{ik_x x'} e^{-ik_y y'} e^{ik_z z'}
\times \left[ \hat{a}_{x X(3),xxx} + \hat{a}_{y X(3),xxx} + \hat{a}_{z X(3),xxx} \right] E^2 \overline{E}^* e^{i[2k_x-k_z]y'} e^{-i\omega \mu_0 t} 2\pi \delta(k_z) dk_x dk_y dy',
\]

\[
= - \frac{i \omega \mu_0}{4\pi} \int \int \overline{M}(k_x, k_y) = 0 e^{ik_x x'} e^{-ik_y y'} e^{ik_z z'}
\times \left[ \hat{a}_{x X(3),xxx} + \hat{a}_{y X(3),xxx} + \hat{a}_{z X(3),xxx} \right] E^2 \overline{E}^* e^{i[2k_x-k_z]y'} e^{-i\omega \mu_0 t} dk_x dk_y dy',
\]

(A7)

where

\[
\int_{-\infty}^{+\infty} e^{i[2k_x-k_z]y'} e^{-ik_y y'} dy' = 2\pi \delta[2k_y - k_{y1} - k_{y2}]. \quad (A8)
\]

Then, we can obtain:
By submitting Eq. (A6) into Eq. (A10), we have the analytic expressions for the optical field at the FWM frequency:

\[
\overrightarrow{E}(r) = -\frac{\omega \mu_0}{4\pi} \int_{-\infty}^{\infty} M(k_x, k_y = 0) e^{ik_{1y}} e^{ik_{2y}} [\hat{a}_y \chi_{(3),xxx} + \hat{a}_y \chi_{(3),yyy} + \hat{a}_z \chi_{(3),zxx}] \\
\times E_1^* E_2^* 2\pi\delta[2k_{1y} - k_{2y} - \omega t] e^{-i\omega t} dk_y \\
= -\frac{\omega \mu_0}{2} M(k_y = 2k_{1y} - k_{2y}, k_x = 0) e^{i(2k_{1y} - k_{2y})y} e^{ik_{2y}} [\hat{a}_y \chi_{(3),xxx} + \hat{a}_y \chi_{(3),yyy} + \hat{a}_z \chi_{(3),zxx}] \times E_1^* E_2^* e^{-i\omega t}. \quad (A9)
\]

\[
\overrightarrow{E}(r) = -\frac{\omega \mu_0}{2} \begin{bmatrix}
M_{xx}(k_x, k_y = 0) & 0 & M_{x}(k_x, k_y = 0) \\
0 & M_{yy}(k_x, k_y = 0) & 0 \\
M_{zz}(k_x, k_y = 0) & 0 & M_{zz}(k_x, k_y = 0)
\end{bmatrix} \times e^{i(2k_{1y} - k_{2y})y} e^{ik_{2y}} \begin{bmatrix}
\chi_{(3),xxx} \\
\chi_{(3),yyy} \\
\chi_{(3),zxx}
\end{bmatrix} \times E_1^* E_2^* e^{-i\omega t}. \quad (A10)
\]
Publications

Journal


Conference


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48. (!!! INVALID CITATION !!)).


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