Wearable Human Motion Tracking and Analysis System Based on Wireless Sensor Network

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

.............................. ..............................
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Abstract

Measurement of human body movement has become a widely used clinical tool for evaluating and quantifying musculoskeletal functions based on the information from kinematic and kinetic data. It has a myriad of applications ranging from rehabilitation to gait analysis. Existing methods of motion tracking include visual, mechanical, magnetic and inertial tracking. Visual systems are more sensitive to changes in light, clutter and shadow. For mechanical and inertial tracking systems, they may be cumbersome which hinder the natural movements. In this thesis, we first developed a wearable motion tracking system using ultra wideband (UWB) radios, and then a low-cost motion analysis system using wireless ultrasonic sensor network was proposed to overcome the limitations of these existing methods.

The first part of the thesis describes a new method for measuring and monitoring human body joint angles, which uses wearable UWB transceivers mounted on body segments. This work is motivated by the high accuracy of UWB technology in ranging and positioning, which makes it a promising candidate for human motion monitoring. The model is based on providing a high ranging accuracy (inter-sensor distance) between a pair of transceivers placed on the adjacent segments of the joint centre of rotation. The measured distance is then used to compute the joint angles based on the law of cosines. The proposed joint angle measurement system used only one pair of transceivers to measure the flexion/extension angles in the sagittal plane. The performance of the method was compared with a flexible goniometer by simultaneously measuring joint flexion-extension angles at different angular velocities, ranging between 8°/s and 90°/s. The measurement errors were evaluated by
the average differences between two sets of data (ranging from 0.8° for slow movement to 2.8° for fast movement), by standard deviation (ranging from 1.2° to 4.2° for various movement speeds) and by the Pearson correlation coefficient (greater than 0.99) which demonstrates the very good performance of the UWB based approach. The experimental results have shown that the system has sufficient accuracy for clinical applications, such as rehabilitation. Measurement of three-dimensional (3D) motion requires more transceivers. However, it is difficult to sample the received signals in real time due to large bandwidth of UWB pulse. Furthermore, the clock between transmitters and receivers should be strictly synchronized, because even a small clock drift would produce significant measurement error due to its high transmission speed.

This gives the motivation to find new low cost techniques to track human movement. At the second stage of our research, a wearable wireless ultrasonic sensor network was developed to measure the 3-dimensional foot trajectory during walking, and to extract some spatial-temporal gait parameters, including stride length, stride duration, stride velocity, stride cadence, and stride symmetry, for gait analysis. The proposed ultrasonic motion analysis system uses the wireless sensor network concept with all the mobile nodes communicating wirelessly with the coordinator enabling patients to be monitored in an unrestrained environment. It consists of an ultrasonic transmitter (mobile) and four receivers (anchors) with fixed known positions. The foot motion was obtained from the movement of the ultrasonic sensor placed on the subject’s heel, by adapting spherical positioning technique which finds the intersection area of circles centred at each anchor with radius equal to the measured distance from the transmitter to each anchor. Based on the measured foot displacements, a methodology has been developed to extract spatial-temporal gait parameters including stride length, stride duration, stride velocity, stride cadence, and stride symmetry. The performance of this system is validated against a camera-based system in the laboratory. Numerical results show the feasibility of the proposed system with average error of 2.7% for all the estimated gait parameters. In addition, we developed a gait phase detection algorithm that reliably measured
the transition periods during different phases. The algorithm performance is also examined by comparing with a commercial optical motion tracking system with ten healthy subjects and two foot injured subjects. Accurate estimates of gait cycle (with an error of -0.02 ± 0.01 s), stance phase (with an error of 0.04 ± 0.03 s), and swing phase (with an error of -0.05 ± 0.03 s) compared to the reference system are obtained.

The proposed ultrasonic system was also used to track lower extremity joint angle during a squat exercise. Only one ultrasonic sensor is required which not only minimizes the discomfort for users, but also avoids complex calibration procedures and synchronization issues. The Kalman filter is applied to estimate the displacements in vertical and horizontal direction of the ultrasonic sensor, and then the recorded displacements together with known joint constraints are used to estimate joint angles of the trunk using damped least-squares-based technique for the singularity avoidance problem of redundant systems. The performance of the proposed ultrasonic measurement system was validated against a camera-based tracking system on 8 healthy subjects performing planar squat exercise. Joint angles estimated from the ultrasonic system showed a Root Mean Square Error (RMSE) of 2.85° ± 0.57° with the reference system. Statistical analysis indicated great agreements between these two systems with Pearson’s correlation coefficient (PCC) value larger than 0.99 for all joint angles estimation. These results show that the proposed ultrasonic measurement system is useful for applications such as rehabilitation and sports.

Experiments had been conducted to evaluate the performance of the proposed wearable wireless ultrasonic motion analysis system with the gold standard, camera based motion capture reference system (Motion Analysis Eagle System) with eight high speed cameras. Experimental results showed that the proposed method was able to track the foot trajectory and detect gait phases as accurate as the reference system. Similarly, the performance of the joint angle tracking during squat exercise is also comparable to the reference system.
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Chapter 1

Introduction

1.1 Motivation

Tracking of human movement has become a widely used clinical tool for evaluating and quantifying musculoskeletal functions based on the information from kinematic and kinetic data [5]. In its widespread applications, such as rehabilitation [6], gait analysis [7], conventional methods paid more attention to qualitative analysis of rehabilitation progress or gait deviations, and are largely subject to the observers’s experience [8]. Although simple visual observation is a useful clinical tool, quantitative analysis can further provide some detailed information in investigating complex gait disorders, monitoring of progress in rehabilitation, identifying and separating primary causes of gait abnormalities from secondary causes, and providing the required information for treatment plans [9].

Presently, human motion tracking system are based on various methods, including mechanical, visual, audio, radar, magnetic and inertial tracking [10]. The most accurate measurement systems employ one or more cameras to track the spatial movement (3D) of many markers positioned at particular anatomical sites on the body segments, while a software outputs some kinematics parameters [11]. However, camera-based systems require highly skilled operators and a dedicated motion
laboratory, where complexity and cost are often prohibitive for routine applications. Additionally, they are more sensitive to changes in lighting, clutter and shadow [12].

Over the last decade, many research groups developed some innovative human motion tracking systems by using non-traditional methods. An alternative to camera-based systems is magnetic tracking systems, which also enable 3D kinematic analysis of human movements [11]. These systems take advantage of the earth’s magnetic field, provide information on azimuth, and are now widely used in commercial systems [13]. However, it is extremely difficult to measure orientation in typical clinical environments due to heterogeneity of the earth’s magnetic field inside a room [13].

In order to obtain kinematics and spatial parameters for human motion analysis indirectly, these systems use laser technology or measure near-body air flow [11]. Also, electronic carpets or wearable force sensors are used for estimation of ground reaction forces, centre of pressure, and temporal parameters. Unfortunately, these systems are not sufficiently robust in various environments [14].

The development of microelectromechanical system technology has produced many low cost and small inertial sensors which can be used in human tracking system, such as accelerometer and gyroscope [2, 15, 16]. In [17], Brandes et al. use the accelerations measured at the lower trunk to extract spatial-temporal gait parameters with healthy children. Daniel et al. [18] proposed an ambulatory measurement system, using a triaxial accelerometer and a dual-axis gyroscope, to assess the vertical displacement of foot during walking. Although these inertial sensors overcome the shortcomings of camera-based tracking system, it requires integration to estimate position and orientation of human motion [19]. Even small measurement error or noise would accumulate, thus making the estimation error to increase exponentially for long time monitoring [20].

The growth uncertainty that arises from the integration of acceleration error is mitigated by periodical corrections of sensor drift based on the intrinsic property of pedestrian walking such as zero velocity update (ZUPT) at motion-less period (stance phase) in many literatures [4, 21, 22]. Although the removal of the accel-
eration bias error can significantly improve the accuracy of motion tracking, the assumption that there is a heel-strike at the initial contact or a toe-off at the terminal contact should be valid. In fact, however, it is a difficult task to detect initial contact in some type of abnormal gait such as Parkinson’s disease [23], and the presence of measurement errors could be still problematic for long-term monitoring [18].

To eliminate the accumulated error and improve tracking performance of IMU-based system over long measurement duration, some researchers fuse IMU with some local positioning techniques or sensors, such as camera or imaging sensors [24], Radio Frequency Identification (RFID) technology [25] and ultrasound [26]. Li et al.[27] presented a vision-aided inertial tracking system using an IMU and a stereo camera. Ruiz et al.[25] took advantage of known positions of the RFID beacons to reduce the bias and drift associated with IMU-based method. Girard et al.[26] and Fischer et al.[28] have focused on using inertial sensors in combination with ultrasound range sensor to decrease the error accumulation. Among all the mentioned technologies, the performance of the tracking system is improved with additional instrumentations resulting in high complexity and cost of the overall system.

UWB radio is an emerging technology that has attracted significant interest in recent years due to its high data rate transmission, robustness to fading, security, low loss penetration, low power spectral density, multiple access, and scalability feasibility [29]. Particularly, wearable UWB radios are good candidates for human motion tracking, since they can provide high ranging and positioning accuracies and offer low-power consumption and robust performance in multipath environment [30]. However, in UWB system, where sub-nanosecond pulse is used, the required synchronization is extremely challenging. Highly accurate synchronization clock that provides precise timing information is required for tracking which is difficult to be achieved in low cost systems [31]. Furthermore, it is difficult to sample the received signal in real time with current ADC technology due to the large bandwidth of IR-UWB pulse [32].
It is therefore of interest to design a low cost and accurate motion tracking system for routine applications. Ultrasonic sensors are among the most commonly used techniques in gait analysis due to its safety, low cost, and high accuracy and resolution for low range measurement. In this thesis, the use of ultrasonic sensors for determining the human body motion is investigated. Specifically, the spatial-temporal gait parameters are examined as they are helpful to diagnose impairments in balance control, and useful to gait analysis.

1.2 Related Works

1.2.1 Gait Analysis

Gait analysis is important for investigating complex gait disorders, monitoring of progress in rehabilitation, identifying and separating primary causes of gait abnormalities from secondary causes, compensatory mechanisms and providing the required information for treatment plans [9]. Gait abnormalities occur due to the delay between applied stimulus and the resulting force, variability in the stimulus-force relation, unwanted spasms or reflexes, and the ballistic movements of the leg during swing phase [33]. Typically, the identification of abnormal gait is commonly evaluated by the measurement of the temporal and spatial parameters of human
locomotion biomechanics [34]. Fig. 1.1 shows some samples of different abnormal gaits [35]. The key factor to detecting abnormal gait is recognizing the symmetry of movement, where patients should be observed while walking for some distance [33, 36].

The biomechanical approach to human movement analysis can be both qualitative and quantitative [1, 3]. A qualitative analysis is a nonnumeric evaluation of motion based on direct observations, which is directly related to the observer’s experience. Nevertheless, a quantitative analysis is a numeric evaluation of motion based on the measurement of different movement aspects. In most cases, quantitative information is important since it accurately described the human movements required for rehabilitation [3]. Human biomechanical analysis can be analyzed either by motion characteristics (kinematics) or determining the cause of the motion (kinetics). Kinematics analysis can directly describe the movement from a spatial and temporal perspective by the position, velocity and acceleration of interest, while kinetics that examines the forces causing movement is more abstract since forces cannot be seen[3]. However, the examination of both the kinematic and kinetic components is necessary to have a thorough comprehension of movement [1, 3].

Walking gait is generally defined when one foot touched the ground and ends when that same foot contacts the ground [37]. Walking gait of a healthy person is usually broken down into two main phases, stance and swing [38, 39]. Stance phase accounts for approximately 60 percent, and swing phase for approximately 40 percent, of a single gait cycle, shown in Fig. 1.2. The stance phase can be subdivided into five phases, namely, Initial Contact, Loading Response, Midstance, Terminal Stance and Preswing. Loading Response begins with Initial Contact, the instant the foot contacts the ground. (For those who have normal gait, their heels touched the ground first. In contrast, patients who demonstrate pathological gait patterns will contact the ground first with their entire foot or the toes at the beginning of Loading Response.) Loading response ends with contralateral toe off, when the opposite extremity leaves the ground. Thus, loading response corresponds to the gait cycle’s first period of double limb support [40, 41, 42]. Midstance begins with con-
tralateral toe off and ends when the center of gravity is directly over the reference foot. Terminal stance begins when the center of gravity is over the supporting foot and ends when the contralateral foot contacts the ground. During terminal stance, around 35 percent of the gait cycle, the heel rises from the ground. Preswing begins at contralateral initial contact and ends at toe off. Thus, preswing corresponds to the gait cycle’s second period of double limb support [40, 41, 42]. The stance phase is followed by swing phase which can be further broken down into Initial & Mid Swing and Terminal Swing [43]. It is reported that most falls in older adults result from inappropriate foot placement and clearance over ground during swing phase [44].

![Figure 1.2: A scheme of gait cycle of a normal gait [1].](image)

The universally used method of describing human movement is based on a system of planes [3] which is shown in Fig. 1.3. This position is also known as the anatomical position which is commonly used as a standard reference point.

### 1.2.2 Overview of UWB Technology

According to the US Federal Communications Commission (FCC) rules, the UWB signal is defined as having at least 500MHz for absolute bandwidth or the fractional bandwidth that is greater than 20% at -10 dB level [30, 29]. However, narrow band communication systems have fractional bandwidths less than 1%. The fractional
bandwidth is defined as the ratio of the energy bandwidth and the center frequency and is expressed as:

\[
\text{fractional bandwidth} = \frac{2\left(f_H - f_L\right)}{f_H + f_L} \tag{1.1}
\]

The most common and traditional way of generating an UWB signal is by radiating pulses with duration of the order of fractions of nanoseconds. This is called Impulse Radio UWB (IR-UWB). This transmission technique does not require additional modulation thus reducing power consumption required in the high speed Fast Fourier Transform (FFT) processes [45]. On the other hand, IR-UWB signals require fast switching times and highly precise synchronization.

### 1.2.3 Overview of Ranging Techniques

Ranging in a wireless system involves the collection of distance information from radio signals propagating between a transmitter and a receiver [45]. Rangin-
1.2. RELATED WORKS

mines the distance between a pair of nodes by first obtaining the signal propagation delay. Suppose \( s(t) \) is a transmitted signal, the corresponding received signal takes the form:

\[
    r(t) = h(t) * s(t) + n(t)
\]

where \( h(t) \) is the channel impulse response and \( n(t) \) is thermal noise. Suppose the signal is propagating over a perfect channel described by the impulse response

\[
    h(t) = A_d \delta(t - \tau_d)
\]

Then, the received signal is given by:

\[
    r(t) = A_d s(t - \tau_d) + n(t)
\]

Equation (1.4) shows that ranging can be estimated using received signal strength indication (RSSI) \( A_d \) or delay \( \tau_d \).

1.2.3.1 Received Signal Strength Based Ranging

RSSI technique is based on measuring the power of received signal to indirectly obtain the separation distance. This technique is usually used in low cost systems with coarse ranging requirements. Furthermore, RSSI-based systems are less complex and more power efficient since it does not require template generation. In RSSI-based system, the strength of the received signal is estimated by theoretical or empirical path-loss models and then translated to a distance estimate [46].

A commonly used model to characterize the RSSI is given by [47]

\[
    P_r(d) = P_0 - 10\gamma \log_{10} \left( \frac{d}{d_0} \right) + S
\]

where \( P_r(d) \) (dBm) is the received power, \( P_0 \) is the received power (dBm) at the reference distance \( d_0 \), \( d \) (meters) is the separation distance between transmitter and receiver, and \( S \) (dB) denotes the large-scale fading variations, which is assumed to
be Gaussian random variable with zero mean. The parameter $\gamma$ is the path loss exponent, which typically have the values between 2 and 6 [47, 48]. Although RSSI-based ranging reduce the system complexity and power consumption, the ranging accuracy is far from being acceptable for some localization systems which require precise ranging [30].

1.2.3.2 Time-Based Ranging

The Time-Based technique computes the separation distance between a pair of nodes, which can be measured by obtaining the signal propagation delay. The distance $d$ is then calculated by the following equation:

$$d = \tau \times c$$  \hspace{1cm} (1.6)

where $c = 299792458$ m/s is the speed of electromagnetic waves. There are three main methods for Time-Based approaches: one-way Time of Arrival (TOA), two-way TOA, and Time Difference of Arrival (TDOA).

![Figure 1.4: One-Way TOA Ranging.](image)

**One-Way TOA Ranging**  At time $t_0$, transmitter (Tr) sends out a packet that contains the time information to receiver (Re). The receiver receives the packet at time $t_1$. The whole process of the TOA ranging is shown in Fig. 1.4. If transmitter and receiver are perfectly synchronized to a common clock, it is apparent that the propagation delay $\tau$ can be estimated by equation (1.7).

$$\tau = t_1 - t_0$$  \hspace{1cm} (1.7)

Therefore, synchronization is a major issue in ranging error using this method.
1.2. RELATED WORKS

**Two-Way TOA Ranging**  In two-way ranging, the transmitter first transmits a packet with time information at time $t_0$ to receiver. The receiver replies with an acknowledgement packet after a response delay $\Delta$ [49, 50]. Finally, the transmitter receives the reply packet at time $t_1 = t_0 + 2\tau + \Delta$, and the propagation delays is given as follows:

$$\tau = \frac{t_1 - t_0 - \Delta}{2} \quad (1.8)$$

Although two-way ranging eliminates the error due to imperfect synchronization between transmitter and receiver, relative clock drift still affects ranging accuracy. Additionally, the response delay is another issue that will introduce time estimation error. For some short distance measurement, the propagation is of the order of nanoseconds, thus even a small clock drift will cause a relatively large ranging error.

**TDOA Ranging**  TDOA-based ranging is a modification of two-way TOA technique, to remove effects of response delay. In TDOA-based ranging system, two packets are exchanged between transmitter and receiver. The first packet is transmitted and received in the same way as two-way TOA. After a random time delay, the transmitter tries to send out the second packet at time $t_2$, which is also acknowledged by the receiver. However, this response delay maybe two times of $\Delta$. Hence, the transmitter received the second reply packet at time $t_3 = t_2 + 2\tau + 2\Delta$. Then,
the propagation delay is estimated using the following equation:

\[ \tau = t_1 - t_0 - \frac{t_3 - t_2}{2} \]  

(1.9)

Comparing with equation (1.8), the response delay term is eliminated. Hence, the effect of response delay is removed in TDOA-based ranging system. However, the relative clock drift still affects the accuracy of ranging in this system. The comparison of different approaches for TOA estimation is summarized in table 1.1.

1.2.4 Overview of Localization Techniques

Range-based localization commonly involves two steps. The first is range measurement, which is conducted between the target and a number of reference nodes. Ranging data can be estimated from four commonly used techniques, such as received-
1.2. RELATED WORKS

signal-strength-indicator (RSSI), one-way TOA, two-way TOA, and TDOA [51, 48]. RSSI is relatively simple to implement with lower power consumption compared to other techniques. However, RSSI is unable to provide the required ranging accuracy [45]. Due to the limitation of the RSSI ranging accuracy, we will focus on time-based ranging estimation.

The second step is the position calculation of the target node based on the TOA-based ranging data. There are two major localization techniques, spherical and hyperbolic positioning. When a common time reference is available to both target and reference nodes, the position of target can be determined by the intersection of circles centered at each reference node, with radius equal to the measured distance. This is known as the spherical positioning, also called TOA positioning. Unfortunately, in most practical cases, the synchronization of target and references is impossible due to misalignments and clock drifts [45]. The hyperbolic positioning technique, also know as Time Difference of Arrival (TDOA), resorts to relative range measurement to determine the location of the target as it only requires receivers synchronized and eliminates the clock bias.

1.2.4.1 Spherical Positioning

The spherical positioning technique is efficient under the ideal case of free-of-error distance estimations. Suppose there are N anchors (reference nodes) and L mobile nodes (target nodes) in a 2-dimensional space. The distance \(d_{ji}\) between anchor \(j\) and mobile node \(i\) determines a circle of radius centered in anchor \(j\). Therefore, the position \((x_i, y_i)\) of the mobile node \(i\) is determined by the intersection of the \(k\) circles of radius \((d_{1i}, d_{2i}, ..., d_{ki})\). In order to find the common point of intersection for these circle trajectories, at least 3 anchors with known positions are required to retrieve a 2D-position and at least 4 anchors with known positions are required to retrieve a 3D-position. Additional number of reference nodes are required for positioning estimation affected by ranging errors. The intersection between the k
circle trajectories can be computed by solving the following system of equations:

\[
\begin{bmatrix}
\sqrt{(x_1 - x_i)^2 + (y_1 - y_i)^2} \\
\sqrt{(x_2 - x_i)^2 + (y_2 - y_i)^2} \\
\vdots \\
\sqrt{(x_k - x_i)^2 + (y_k - y_i)^2}
\end{bmatrix}
= \begin{bmatrix} d_{1i} \\
               d_{2i} \\
               \vdots \\
               d_{ki} \end{bmatrix}
\]

(1.10)

Figure 1.7: Spherical positioning of a mobile node \((x_m, y_m)\) in a 2-dimensional space with three anchors \(A_1 (x_1, y_1)\), \(A_2 (x_2, y_2)\), \(A_3 (x_3, y_3)\).

Spherical positioning can be used only when both mobile node and anchors share the same clock, and then the perfect ranging estimation can be obtained. It is unfortunately not the case in many practical situations due to misalignments and clock drifts. But it is possible to provide a common time reference among those anchors (reference nodes). The hyperbolic positioning technique determines the
position of mobile nodes based on the difference between times of arrival from the
$k$ reference nodes and the mobile one.

### 1.2.4.2 Hyperbolic Positioning

Suppose that we have $l$ targets and $n$ reference points, and that the clock at each
target is delayed by a time $\sigma$ with respect to a common time reference. Then the
time $\sigma$ is removed through the subtraction between TOA from different reference
receivers, shown as follows:

\[
\begin{align*}
\epsilon_{ki} &= \tau_{ki} + \sigma \\
\epsilon_{(k-1)i} &= \tau_{(k-1)i} + \sigma \\
\tau_{k(k-1)i} &= \epsilon_{ki} - \epsilon_{(k-1)i} = \tau_{ki} - \tau_{(k-1)i}
\end{align*}
\]  

(1.11)

where $\tau_{k(k-1)i}$ is the differences in the TOA of $k$th and $(k-1)$th reference points
with respect to $i$th mobile, and $\tau_{ki}$ and $\tau_{(k-1)i}$ are the TOA of $k$th and $(k-1)$th
reference points with respect to $i$th mobile.

![Hyperbolic positioning of a mobile node (x_m, y_m) in a 2-dimensional space with three anchors A1 (x_1, y_1), A2 (x_2, y_2), A3 (x_3, y_3).](image)

Figure 1.8: Hyperbolic positioning of a mobile node $(x_m, y_m)$ in a 2-dimensional space with three anchors A1 $(x_1, y_1)$, A2 $(x_2, y_2)$, A3 $(x_3, y_3)$. 

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The position of mobile node \( i \) in 2-dimensional space is then determined by finding the common point of the intersection of hyperbolic trajectories, as described by the following system of equations:

\[
\begin{bmatrix}
\sqrt{(x_2 - x_i)^2 + (y_2 - y_i)^2} - \sqrt{(x_1 - x_i)^2 + (y_1 - y_i)^2} \\
\sqrt{(x_3 - x_i)^2 + (y_3 - y_i)^2} - \sqrt{(x_2 - x_i)^2 + (y_2 - y_i)^2} \\
\vdots \\
\sqrt{(x_k - x_i)^2 + (y_k - y_i)^2} - \sqrt{(x_{k-1} - x_i)^2 + (y_{k-1} - y_i)^2}
\end{bmatrix} = 
\begin{bmatrix}
d_{2i} - d_{1i} \\
d_{3i} - d_{2i} \\
\vdots \\
d_{k_i} - d_{(k-1)i}
\end{bmatrix}
\] (1.12)

where \( k \geq 3 \). Fig. (1.8) shows an example of hyperbolic localization in a 2-dimensional space.

Hyperbolic positioning requires an accurate common time reference between the reference nodes, but does not depend on precise synchronization between reference nodes and the target node [45]. Both the spherical and hyperbolic positioning techniques require error-free ranging estimation to provide solutions for the equation (1.10) and (1.12). However, external factors such as clock drift and thermal noise will introduce some errors in ranging, leading to imperfect estimations of distance between nodes. In such case, the analytical solution of the two equations may not exist. The effect of errors in ranging estimations on the accuracy of localization can be effectively reduced by using minimization procedures such as Least Square Error (LSE), or introducing more reference nodes, applying some filters like extended Kalman filter [52]. This will be discussed in the following chapter.

### 1.3 Thesis Organization

The rest of the thesis is organized as follows.

Chapter 2 introduces the proposed joint flexion/extension angle measurement system based on wearable UWB radios. It provides an overview of the proposed ranging
technique and analyzes the lower bound of the common TOA ranging estimator. The performance of the wearable system is evaluated experimentally against a goniometer.

Chapter 3 studies the localization technique using UWB radios. Specifically, it derives the theoretical lower bound for TOA and TDOA-based localization, and evaluates the effect of the geometry of anchors and the number of anchors on the positioning accuracy. Both simulation results and experimental results for evaluating the performance of a recursive Taylor series (Newton-Gauss) positioning algorithm are provided.

Chapter 4 presents the development of a wearable wireless ultrasonic system and proposes the foot trajectory measurement in 3-dimensional space. The system configuration is first described, followed by 3 steps tracking algorithm using the combination of recursive Newton-Gauss method and Kalman filter. Then, the performance of the proposed method is investigated by comparing with a camera-based motion capture system.

In Chapter 5, a methodology has been developed to extract spatial-temporal gait parameters including stride length, stride duration, stride velocity, stride cadence, and stride symmetry from 3-dimensional foot displacements. We first introduced the autocorrelation procedure to analyze the gait data. Then, experiments under a number of subjects are conducted against the camera-based motion capture system with metrics, such as mean difference, standard deviation and root mean square error (RMSE). Finally, statistical analysis such as analysis of variance (ANOVA) and two-sample t-tests, are used to assess the significance of change in these gait parameters with respect to speed.

Chapter 6 proposes a new highly accurate gait phase detection system using the 3-dimensional foot displacement during walking. First, we give a brief overview of walking gait, and describes the gait phase detection algorithm. Then, different experiments are performed with different subjects to validate the proposed algorithm against the camera-based motion capture system. Bland & Altma statistical analysis
has been applied to quantitatively measure the agreement between the proposed system and the reference system.

In Chapter 7, a wireless wearable sensor system based on ultrasound for monitoring squat exercise is proposed. A brief overview of biomechanical model of leg is first given. This is followed by the introduction of direct and inverse kinematic model for redundant system. The damped-least square method is introduced to overcome the problem of inverting differential kinematics in the neighbourhood of a singularity. Meanwhile, the performance of the proposed system is benchmarked by the reference camera-based motion capture system with different subjects.

Chapter 8 concludes the thesis by summarizing our contributions and suggesting possible directions for future works.
Chapter 2

UWB Based Human Joint
Flexion/Extension Measurement

This chapter proposes a new clinically acceptable ambulatory and accurate system for measuring joint flexion/extension angle based on a pair of transceivers placed on the adjacent segments of the joint center of rotation described in Section 2.2. This is followed by measurement approach and system design in Section 2.3. The theoretical measurement error is investigated in Section 2.4. Section 2.5 presents experimental results from simultaneous joint angle measurements using UWB radios and a flexible goniometer. Finally, conclusions are made in Section 2.6.

2.1 Introduction

In this Chapter, a new method for measuring and monitoring human body joint angles, which uses wearable UWB transceivers mounted on body segments, is proposed and investigated. The objective is to allow patients to wear equipments as light and small as possible and to be monitored under a natural environment. The patient only need to wear two antennas with small form factor and light weight. It is low cost compared to the optical system. The model is based on providing a high
2.2. BIOMECHANICS OF HUMAN MOVEMENT AND SYSTEM DESCRIPTION

ranging accuracy (intersensor distance) between a pair of transceivers placed on the adjacent segments of the joint center of rotation. The measured distance is then used to compute the joint angles based on the law of cosines. The performance of the method was compared with a flexible goniometer by simultaneously measuring joint flexion-extension angles at different angular velocities, ranging between 8°/s and 90°/s. The experimental results have shown that the system has sufficient accuracy for clinical applications, such as rehabilitation.

2.2 Biomechanics of Human Movement and System Description

Almost all of the freely movable joints on body allow flexion and extension movements, as shown in Fig. 2.1, including head, shoulder, elbow, wrist, trunk, finger, hip, knee and ankle [3]. Flexion is a bending movement which decreases the relative joint angle between the adjacent segments, while extension is a straightening movement in which the relative angle of joint between two adjacent segments increases as the joint returns to the reference anatomical position [3, 53].

The system is based on wearable UWB radios mounted on body segments to measure distance between two points during human movement. The joint angles are first computed using the law of cosines, and then converted to corresponding flexion/extension angles. Fig. 2.2 shows a simplified diagrammatic representation for the placement of transceivers. Accordingly, when a transmitter and a receiver were attached on human body as in Fig. 2.2, a triangle is formed by the pair of transceivers and a joint, i.e. elbow, shown in the left of Fig. 2.2. \( \alpha \) and \( \beta \) are the elbow or knee flexion/extension angle, which is defined as the relative angle between two adjacent segments of knee or elbow joint, respectively. Since the two sides of the triangle, i.e. \( d_1 \) and \( d_2 \), are constant during human movement with transceivers fixed on body, the joint angles can be estimated using the law of cosines.
As shown in Fig. 2.2, we are interested in finding $\alpha$ and $\beta$, i.e. the elbow/knee flexion/extension angle, which can be computed using the measured distance. To capture the segment’s rotational motion, two phases are considered. First, an initial or static measurement phase should be taken to get the values of $d_1$ and $d_2$, which can be evaluated by bending segments with special angles, $\alpha$ or $\beta = 0^\circ$ and $\alpha$ or $\beta = 90^\circ$, respectively. Then, ranging data are collected between different nodes during segment’s movement through the estimation of the propagation delay between transmitter and receiver. Time-of-arrival (TOA) of the first arrival path is the most commonly used distance estimation method. Finally, we convert the distance estimation to an angle estimate, as illustrated in Fig. 2.2. The details are described in the following sections.
2.3 Measurement Approach and System Design

In order to simplify the description of the method, we focus on the elbow flexion/extension angle $\alpha$. As shown in Fig. 2.2, the different green squares on human body are the attachment of the UWB transceivers, which help to measure the distances between transmitter and receiver.

![Figure 2.2: The configuration of elbow/knee flexion/extension angle $\alpha$ angle $\beta$ respectively. $d_1$ and $d_2$ denote the distance between elbow joint and transceivers respectively; $d$ is the ranging data required in the proposed system.](image)

2.3.1 Geometric model

Transmitter and receiver were used to measure the distance $d$ which will be converted to angle estimation by the law of cosines. As shown in Fig. 2.2, elbow joint acts as a virtual point that consists a triangle with transmitter and receiver. The following equation demonstrates how angle $\alpha$ is calculated from the triangle in Fig. 2.2.

During segment movement, modelling the variation of the arm yields

$$\cos \alpha = \frac{-d_1^2 + d_2^2 - d^2}{2d_1d_2}$$

(2.1)

where $d_1$ and $d_2$ have been measured in the initial or static phase, $d$ is the transmitter-receiver separation distance. Hence, equation (2.1) has only one variable, i.e. $d$, and then the angle $\alpha$ is calculated.
2.3.2 Ranging Based on Modified Phase-Only Correlator

Ranging in a wireless system involves the collection of distance information from radio signals travelling between a transmitter and a receiver [45]. Ranging among two nodes based on the estimation of the first arrival path can be difficult in multipath channels, especially under non-line-of-sight (NLOS), but the nodes in our system have been attached with LOS links [54]. Suppose \( s(t) \) is a transmitted UWB pulse waveform, and its spectrum is \( S(f) \). When passing through a multipath channel, the signal will be delayed and attenuated. Therefore, the received signal takes the form:

\[
r(t) = s(t|\tau_i) + n(t) \quad (2.2)
\]

where

\[
s(t|\tau_i) = \sum_{i=1}^{n} \alpha_i s(t - \tau_i) \quad (2.3)
\]

is the received pulse, \( \alpha_i \) is the attenuation, \( \tau_i \) is the propagation delay of \( i \)th arrival path respectively, and \( n(t) \) is thermal noise. The Fourier spectrum \( R(f) \) of the received signal can be written as:

\[
R(f) = \sum_{i=1}^{N} \alpha_i S(f)e^{-j2\pi f \tau_i} + N(f) \quad (2.4)
\]

where \( N \) is the total number of paths in the channel, and \( N(f) \) is the noise spectrum. The propagation delay \( \tau_i \) can be estimated in a number of ways such as detecting the instance of the maximum envelope of \( s(t) \), finding the mean arrival time, or correlating with a locally generated template [55, 56]. In order to achieve a high ranging accuracy, we choose template correlation for TOA detection. The correlation process in frequency domain is [57]:

\[
C(f) = R(f) \cdot S(f)^* \quad (2.5)
\]

where \([\cdot]^*\) denotes complex conjugate.
As in Modified Phase-Only Correlator (MPOC), both the local template and received signal are normalized by the amplitude of \( S(f) \) first to yield:

\[
MPOC(f) = \frac{R(f) \cdot S(f)^*}{|S(f)|^2} = \sum_{i=1}^{N} \alpha_i e^{-j2\pi f \tau_i} + \frac{N(f)}{S(f)} \tag{2.6}
\]

From equation (2.6), the first term gives the frequency response of the multipath channel. The first arrival path \( \tau_1 \) can be given by Inverse Fast Fourier Transform (IFFT) of the channel impulse response. The second term, \( N(f)/S(f) \), is additive noise and can be suppressed by setting the high peak values of \( N(f)/S(f) \) in the out-of-band frequencies to zero [57, 58].

In the estimation of TOA, the distance \( d \) between the transmitter and receivers can be measured by the following equation:

\[
d = (\tau_1 - \tau_0) \times c \tag{2.7}
\]

where \( \tau_0 \) is the time when transmission begins, i.e. system delays, and \( c = 299792458 \) m/s is the speed of light in free space.

### 2.3.3 System Performance and Ranging Accuracy

The primary motivation for using UWB technology is the ability of UWB pulses to provide high temporal resolution with large bandwidth [30, 59]. Based on Shannon’s sampling theorem, the performance of a communication system can be improved by increasing either the effective signal bandwidth of transmitted pulse or pulse-energy-to-noise-ratio at the receiver [60]. Cramer-Rao lower bound (CRLB) gives a theoretical lower bound in terms of the variance of ranging estimates [54, 61].

#### 2.3.3.1 The Cramer-Rao Lower Bound

Using estimation theory, non-linear estimation problems such as MSE \( \sigma_d^2 \) of any unbiased estimate \( \hat{d} \) are bounded by the Cramer-Rao inequality, in which a maximum
likelihood estimator approaching the CRLB closely for high SNRs is used [30, 54, 62]:

\[
\sigma^2 \hat{d} = E\{ (\hat{d} - d)^2 \} \geq CRLB
\]  

where \( E\{ \cdot \} \) defines the statistical expectation.

In the ideal case of no multipath effects [54, 63],

\[
\sigma^2 \hat{d} \geq CRLB = \frac{c^2}{2\kappa^2 SNR}
\]  

where \( SNR = E_r/N_0 \), \( E_r \) is the energy of the received pulse and \( \kappa \) is the second moment of the spectrum \( R(f) \) of the received pulse \( r(t) \) defined by

\[
\kappa^2 = \frac{\int_{-\infty}^{\infty} (2\pi f)^2 |R(f)|^2 df}{E_r}
\]  

Gaussian pulse is used in our measurements. A Gaussian pulse in terms of pulse width \( T_r = 2\tau_r \) has the form [45]:

\[
r_0(t) = e^{-\frac{2\pi t^2}{\tau_r^2}}
\]  

where \( \tau_r \) is the shape factor of the Gaussian pulse. Further Gaussian derivatives yield additional zero crossings, one additional zero crossing for each additional derivative [45]. In this case, we have [56]

\[
r(t) = \sqrt{E_r} r_0^{(n)}(t) \sqrt{\frac{(n-1)!}{(2n-1)!\pi^n\tau_r^{1-2n}}} \]  

where \( r_0^{(n)}(t) \) is the n-th order Gaussian pulse in the form of \( r_0(t) \) with respect to \( t \).

Note that the bound can be minimized by increasing either SNR or \( \kappa \). Furthermore, the value of \( \kappa \) depends on the shape of the pulse in (2.10). Using (2.10) and (2.12), it is easy to show that \( \kappa_{(n)} \) simplifies to [54]

\[
\kappa_{(n)}^2 = \frac{2\pi(2n + 1)}{\tau_r^2}
\]
2.3. MEASUREMENT APPROACH AND SYSTEM DESIGN

Figure 2.3: Graphical representation of the template used in knee movement estimation and the normalized received pulse.

Figure 2.4: Graphical representation of the template used in arm motion estimation and the normalized received pulse.

Generally, it is essential that the CRLB needs estimators to be unbiased. Furthermore, the CRLB is an accurate estimator only for large SNRs but too optimistic for moderate SNRs under severe multipath conditions [54, 64]. As compared to CRLB, Ziv-Zakai lower bound (ZZLB) provides accurate results for both very large and very low SNRs under multipath phenomena without need the estimator to be unbiased [30, 61, 64].
2.3.3 The Ziv-Zakai Lower Bound

The performance limit of correlation-based TOA estimators using ZZLB is affected by different templates used in coherent receivers, and the most accurate coherent detector is optimal template, which is highly matched to the received pulse [56]. The ZZLB transforms the performance evaluation of an estimation problem into a binary detection problem, as given in the form of trapezoid formula [54]

\[
ZZLB = \frac{c^2 T_o^2}{N^2} \sum_{k=1}^{N-1} \left( k \cdot \frac{k^2}{N} \right) P_{\text{min}} \left( \frac{k T_o}{N} \right)
\]

(2.14)
where $T_a$ is the observation interval $[0, T_a]$ and $P_{\text{min}}(z)$ is the minimum attainable probability of error, which can be expressed as [30]

$$P_{\text{min}}(z) = Q\left(\sqrt{\frac{E_r}{N_0}(1 - \rho_p(z))}\right)$$ \hspace{1cm} (2.15)

where $Q(.)$ is the Gaussian Q-function and $\rho_p(z)$ is normalized auto-correlation function.

In multipath scenario, the ZZLB is evaluated using experimentally measured channel impulse responses or Monte-carlo simulation [54].

$$P_{\text{min}}(z) \approx \frac{1}{N_{ch}} \sum_{k=1}^{N_{ch}} Q\left(\sqrt{\frac{SNR}{2}d_{k,i}(z)^2}\right)$$

$$\approx Q\left(\sqrt{\frac{SNR}{2}d_{\text{min}}^2(z)}\right)$$ \hspace{1cm} (2.16)

where $d_{\text{min}}(z) = \min_k (d_{k,i}(z))$ is the minimum normalized distance, $d_{k,i}(z)$ is the normalized distance between multipath components, $k$ is the argument of the minimization decided by the number of channel realizations $N_{ch}$.

First, note that the radiation patterns of the antennas used in our experiment will be different when placed on the different sites of human body [65]. Furthermore, the unexpected reflection, diffraction or other disturbances will increase the complexity of the propagation channel. Hence, based on experiments under indoor environments, we choose different templates for different joints’ flexion and extension angles, the first order derivative Gaussian pulse for knee flexion/extension angle estimation with width $T_p = 0.28 \text{ns}$, and sixth order for elbow flexion/extension angles estimation. Fig. 2.5 shows the root MSE (RMSE) of distance bound related to the CRLB and ZZLB using the 1st and 6th order Gaussian pulses for an Additive White Gaussian Noise (AWGN) channel and multipath channel respectively. It is evident that CRLB provides loose bound even for the AWGN single path model, except for high SNRs. ZZLB, however, is an accurate estimator for both very large and very low SNRs.

As concluded in [29], the ranging accuracy is independent of the transmitter-receiver
separation distance under LOS condition. Therefore, the ranging error in the investigated system will not be affected by human movement. The signal-to-noise-ratio is assumed to be 15 dB, since this value is achievable in our experiment setup, described in Section V. Based on the analysis and results shown in the Fig. 2.5, a ranging accuracy of 0.058 cm is achievable with an SNR=15 dB for elbow motion. In addition, an accuracy of 0.093 cm is possible for knee movement estimation.

2.4 Error Analysis

Based on equation (2.1), there are three parameters contributing to the error term, i.e. $d$, $d_1$ and $d_2$. Suppose $e_1$ and $e_2$ are caused by misalignment in attaching the transceivers on human body and $e$ is the ranging error. A more accurate realistic angle estimation is given by

$$\cos \alpha_r = -\frac{(d_1 + e_1)^2 + (d_2 + e_2)^2 - (d + e)^2}{2(d_1 + e_1)(d_2 + e_2)}$$ (2.17)

The general length of forearm or upper arm is about 20 cm to 25 cm, and the measurement errors for $e_1$, $e_2$ and $e$ are less than 1 cm in most cases. Therefore, $d_1^2 \gg e_1^2$, $d_2^2 \gg e_2^2$ and $d^2 \gg e^2$. By ignoring second order terms, we can simplify equation (2.17) as follows:

$$\cos \alpha_r = \frac{d_1 d_2 \cos \alpha - e_1 d_1 - e_2 d_2 + e d}{d_1 d_2 + e_1 d_2 + e_2 d_1}$$ (2.18)

In following subsections, we will discuss the angle measurement error caused by misalignment of antennas and ranging error respectively.

2.4.1 Misalignment Error

Suppose we have correct measured distance, i.e. ranging error $e=0$, then the two parameters, $e_1$ and $e_2$, are used to estimate the misalignment error.
Figure 2.6: Variation of angle error with misalignment $e_1$ and $e_2$. 
2.4. ERROR ANALYSIS

Since $e = 0$, we can simplify (2.18) as follows:

$$
\cos \alpha_r = \frac{d_1 d_2 \cos \alpha - e_1 d_1 - e_2 d_2}{d_1 d_2 + e_1 d_2 + e_2 d_1} = f(e_1, e_2) \tag{2.19}
$$

where $f(e_1, e_2)$ denotes the value of $\cos \alpha_r$ with respect to two variables $e_1$ and $e_2$, and must satisfy the restriction $|f(e_1, e_2)| \leq 1$. From (2.19), the misalignment error $\Delta \alpha$ related to the actual angle $\alpha_r$ and measured one $\alpha$ can be expressed by:

$$
\Delta \alpha = \arccos(f(e_1, e_2)) - \alpha \tag{2.20}
$$

Using (2.20), the angle error $\Delta \alpha$, caused by misalignment, versus the variation in the value of $e_1$ and $e_2$ is shown in Fig. 2.6. The ranges of value for $e_1$ and $e_2$ are from -3 cm to 3 cm for an arbitrary value of $\alpha = 60^\circ$, $d_1 = 25$ cm and $d_2 = 30$ cm.

The subplots on the bottom of Fig. 2.6 show the error with respect to $e_1$ with fixed values $e_2=0$ and 3 cm, respectively. For the fixed value of $e_1$, the error is minimized at $e_2=0$. From the subplots, the errors $e_1$ and $e_2$ have similar effect on the angle error. Furthermore, the error is approximately linear to the variation of $e_1$ for fixed $e_2$ as well as $e_2$ for fixed $e_1$. The combine effect of $e_1$ and $e_2$ on the angle error produces the curved surface as shown on the top of Fig. 2.6. The result of this curve shows that the joint angle measurement is very sensitive to the placement of transceivers on body.

2.4.2 Error Caused by Ranging

We first evaluate the effect of misalignment on the performance of the system under study, then we focus on investigating the effect of the other factor, the ranging error, on the angle error. Equation (2.18) describes the relationships between measured angle and actual angle with respect to ranging error and misalignment. Suppose misalignment error $e_1=0$ and $e_2=0$ in (2.18), we have

$$
\cos \alpha_r = \cos \alpha + \frac{ed}{d_1 d_2} = f(e) \tag{2.21}
$$

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where $ed/d_1d_2$ is the error term, and $f(e)$ represents the value of $\cos \alpha_r$ with respect to the variable $e$, and must satisfy the restriction $|f(e)| \leq 1$.

From (2.21), it is observed that the difference between $\cos \alpha_r$ and $\cos \alpha$ is affected by the magnitude of $d$, $e$, $d_1$ and $d_2$. This suggests that the effects of the ranging error can be minimized by placing the pair of transceivers far from the centre of rotation. With the limitation of the length of legs, the values of $d_1$ and $d_2$ cannot be enlarged infinitely. Additionally, the distance between transmitter and receiver also increases, which will undermine the performance of the system due to the attenuation of pulse transmitted over a long distance. Therefore, the most efficient way to improve the performance of our system is designing suitable parameters for UWB radios combined with maximum achievable length of $d_1$ and $d_2$.

Using (2.21), the estimated error $\Delta \alpha$ caused by ranging error can be expressed by:

$$\Delta \alpha = \arccos(f(e)) - \alpha$$  \hspace{1cm} (2.22)

The angle error $\Delta \alpha$ versus ranging error $e$ is plotted in Fig. 2.7. The range of value for $e$ is from 0 cm to 6.5 cm and $\alpha$ at some fixed value.
From the plots, it is observed that the estimated angle error increases exponentially as $e$ increases. However, the effect of ranging error on the angle estimation is decreasing as the joint flexion/extension angle becomes larger.

Based on the error analysis, the measurement error is likely to increase linearly with misalignment and ranging error. Using equation (2.18), we combine these two error sources together, and provide a theoretical error bound. Since $e_1d_2 + e_2d_1 \ll d_1d_2$, we ignore the term $e_1d_2 + e_2d_1$ in the denominator and take the error term in the form of

$$E = \frac{e_1d_1 + e_2d_2 - ed}{d_1d_2}$$

As discussed in section III, the ranging error of 0.058 cm for arm and 0.093 cm for leg is achievable in our system. The absolute error bound due to misalignment and ranging in the range of $0^\circ$ to $80^\circ$ is shown in Fig. 2.11. The performance is evaluated using real measurements and discussed in the following section.

### 2.5 Experimental Validation

#### 2.5.1 Experiment Setup and Measurement Site

In order to investigate the accuracy of the proposed approach, real on-body UWB measurements were obtained. A pair of UWB transceivers were attached to the thigh and shank, or forearm and upper arm, of the test subject in order to estimate the distance between the two points based on TOA of the received pulses.

The original UWB pulse generated from Picosecond Pulse Labs 3500D impulse generator has a null-to-null pulse width of around 200ps. However, the signal in the free space was broadened and distorted due to the dispersive behavior of antennas and cables across the entire frequency spectrum [66]. Therefore, we have different received pulses, shown in Fig. 2.3 and Fig. 2.4, when placed on different parts of the body. The centre frequency of the received pulse from the antennas attached on arms, as shown in Fig. 2.4, is around 4.5 GHz with a -10 dB bandwidth that
is approximated to 3 GHz. The average power of the received pulse is about -45.9 dBmW. In on-body wireless channel, there are also some other propagation disturbances between the antennas, including the reflection and diffraction around the human body surface. It has been shown that different UWB antennas for on-body use have different effects on the channel behavior [67, 68]. Skycross SMT-3TO10M UWB antennas have been used in [65] due to their small-size (25.0 mm × 18.5 mm × 1 mm) and lower-profile characteristics, which precisely match body sensor requirements. They cover a wide frequency range between 3.1 GHz to 10 GHz. Therefore, they are also used in our experiments. The transmitter sends out UWB pulses every 1000 ns. The received waveform is sampled by Real Time Oscilloscope DSO80804B with sampling rate of 40 GHz. 16 periods of the captured signal are averaged to reduce the effect of noise.
2.5. EXPERIMENTAL VALIDATION

Figure 2.9: Distance variation during the forearm movement and relative flexion/extension angle for goniometer and UWB system.

2.5.2 Comparison of proposed approach with goniometer measurements

In section IV, we have discussed the theoretical error of the UWB radios for angle determination. To benchmark the performance of the proposed system, we compare with a goniometer probe (PS-2137 from PASCO) attached on a human leg/arm. The goniometer consists of two metal arm links and a potentiometer. As the angle between the leg/arm changes, the resistance of the potentiometer changes [53]. The accuracy of the equipment is $\pm 1^\circ$ when calibrated, with resolution of $0.042^\circ$ at a sampling rate of 500 Hz [53].

We first show that simultaneous goniometer and UWB-based system measurements do not affect the results of the UWB-based system. During the experiments, we are only interested in the direct path of received signal. Under LOS condition, direct path is always the first arrival path and the strongest path. UWB-based measurements with and without goniometer probe attached on body are conducted. The goniometer probe does not block the direct path. Hence, measurements show that the signal of direct path does not change, but it has some small sidelobes when goniometer probe is added. However, these sidelobes do not affect our correlation...
The subject wearing UWB antennas and goniometer probe was instructed to perform flexion and extension of his forearm and shank for 1 min, then repeated 10 times. The attachment of the goniometer and UWB antennas to the arm and leg is as shown in Fig. 2.8(a), (b). Goniometer was mounted on the lateral side of the leg at the centre of knee joint. Simultaneously, the data were recorded with the oscilloscope for later analysis. The data logger for the goniometer is also shown on the left of Fig. 2.8(b). The UWB antennas are attached on the body covered by the straps and attached to the side of the goniometer so that the measurements can be made with respect to the same reference frame for comparison. The synchronization between goniometer system and UWB system was done by maximizing the correlation between the angle. The results of the subject for the arm motion and leg movement are shown in Fig. 2.9 and Fig. 2.10.

In the experiments, we evaluate the effect of misalignment on the angle error by slightly changing the position of UWB antennas on body. Fig. 2.11 shows the statistical representation of the experimental results versus the theoretical error bound. This result is similar to the theoretical analysis in section IV. The effect of misalignment can be minimized by calibration.
2.5. EXPERIMENTAL VALIDATION

In order to provide a more systematic validation, we concentrate on the analysis of knee joint flexion/extension angle. The subject was instructed to vary his speed of motion using a metronome as a reference. All data were recorded by the oscilloscope. The mean error and standard deviation over the computed mean angular speed for these experiments are shown in Fig. 2.12. The accuracy of the proposed system was also compared in terms of the Pearson’s product-moment correlation coefficients (PCCs) with the goniometer results. PCC values range between -1 and 1, where 1 represents the best possible similarity between the two sets of angles, as shown in the bottom of Fig. 2.13.

As shown in Fig. 2.12, the mean difference in angle between the proposed system and goniometer is proportional to mean angular velocity (ranging from nearly 0° at a mean angular velocity of 10°/s to 3.2° with mean angular speed of 160°/s). The standard deviation also increases linearly from 1.2° to 7.5° over the range of mean angular velocities. The performance of our system is more robust at relatively high moving speed than the system proposed in [53]. Apart from the misalignment and ranging error, several other factors contribute to the error. The results are based on
2.6 Conclusion

In this chapter, a wearable system based on on-body fixed sensors using UWB radios for tracking human motion has been proposed and investigated. This system
is capable of taking indoor and outdoor measurements and is suitable for assessing mobility diseases, pervasive healthcare and rehabilitation engineering. To evaluate the performance of the proposed system, we theoretically analysed the error sources and experimentally compared with the goniometer system. It is shown that the proposed system has a similar performance trend with a slight increase in error, which demonstrates a sufficient accuracy for rehabilitation engineering where the angular velocity of the leg/arm movement is less than $20^\circ/s$.

As compared with other existing approaches, the proposed system provides high ranging accuracy for clinical applications. Furthermore, the on-body antenna is much smaller compared to some current equipments used in human motion tracking system, which enables the patients to be monitored in a natural environment instead of a specialized laboratory in the hospital.
Chapter 3

Localization

Section 3.1 explains the derivation of the theoretical lower bound for TOA and TDOA-based localization. Moreover, a reliable localization using the joint TOA/TDOA technique via UWB, is described in Section 3.2. This is followed by measurement schemes and mathematical modeling in Section 3.3. Section 3.4 presents both simulation results and experimental results for evaluating the performance of the proposed algorithm. Finally, conclusions are made in Section 3.5.

3.1 Lower Bound For TDOA and TOA Positioning

Determination of the position accuracy is a fundamental problem in localization system. CRLB has been frequently used to benchmark the performance of the positioning algorithms [63]. CRLB, which can be obtained by the maximum likelihood (ML) estimator is a theoretical minimum variance of any unbiased estimator. It provides us insight into the system design [59, 70].
3.1. LOWER BOUND FOR TDOA AND TOA POSITIONING

3.1.1 Range Error Modeling

Without loss of generality, we consider the problem of localization in the 2D-plane. Suppose that there are $N$ locators in the localization system and the position vector of the $i^{th}$ receiver is $\mathbf{p}_i = [x_i, y_i]^T$. The target node $\mathbf{p} = [x, y]$ that needs to be found requires at least 3 receivers in bidimensional space, and a minimum of 4 receivers in tridimensional space respectively. The range model between the target node and the $i^{th}$ receiver is:

$$\hat{d}_i = d_i + d_{bias} + \omega_i = \|\mathbf{p}_i - \mathbf{p}\| + d_{bias} + \omega_i$$  \hspace{1cm} (3.1)

where $\hat{d}_i$ and $d_i$ are the estimated and true distance from the target to the receiver, respectively. $d_{bias}$ indicates the distance bias caused by the clock offset between target and receivers. Assuming the range measurements are conducted under Line-of-Sight (LOS) condition, we can define the random error term $\omega_i$ is zero expectation Gaussian. With the attenuation of the transmitted signal, the range error is dependent on the distance between transmitter and receivers. As mentioned in Chapter 2 in Section 2.3.3.1, the ranging accuracy is inversely proportional to the SNR, which is attenuated with the signal propagation distance by a path loss exponent $\alpha$ [63]. Therefore, the variance of the range error is:

$$\text{var}(\omega_i) = \sigma_i^2 = \sigma_0^2 \frac{d_i^\alpha}{d_0^\alpha}$$  \hspace{1cm} (3.2)

where $\sigma_0^2$ is the variance of the range error at the reference distance $d_0$.

As discussed in Section 1.2.4, spherical positioning, also known as TOA-based Localization system, has $d_{bias} = 0$ in equation (3.1) and the variance of the range error is indicated by equation (3.2). However, if the receiver and transmitter fail to share same common reference clock, that is $d_{bias} \neq 0$, only hyperbolic localization, TDOA positioning, can be applied, which uses the range difference to eliminate the clock drift and then locate the target node. In TDOA-based positioning system, the range
difference between the \( i^{th} \) and \( j^{th} \) locator is defined as:

\[
\hat{d}_{ij} = \hat{d}_i - \hat{d}_j = \|\mathbf{p} - \mathbf{p}_i\| - \|\mathbf{p} - \mathbf{p}_j\| + \omega_i - \omega_j \tag{3.3}
\]

Suppose the range measurements of these receivers are independent, then we have the variance for hyperbolic positioning is

\[
\text{var}(\omega_i - \omega_j) = \sigma_i^2 + \sigma_j^2 = \sigma_0^2 \left( \frac{\hat{d}_i + \hat{d}_j}{d_0} \right) \tag{3.4}
\]

### 3.1.2 Theoretical Lower Bound

CRLB gives a theoretical lower bound of the variance of the position estimates and it is defined as the inverse of the Fisher Information Matrix (FIM) [63]. Let’s assume \( \hat{C}_p \) is the covariance matrix of the position estimate \( \hat{\mathbf{p}} = [\hat{x}, \hat{y}]^T \), denoted by:

\[
\hat{C}_p = E[(\hat{\mathbf{p}} - \mathbf{p})(\hat{\mathbf{p}} - \mathbf{p})^T] \gtrsim \text{CRLB} = \text{tr}[\hat{I}^{-1}(\mathbf{p})] \tag{3.5}
\]

where \( \text{tr}[\cdot] \) denotes the operation of sum of the main diagonal elements in square matrix, and \( \hat{I}(\mathbf{p}) \) is the FIM defined as:

\[
[I(\mathbf{p})]_{ij} = -E\left[ \frac{\partial^2 \ln f(\hat{\mathbf{d}}, \mathbf{p})}{\partial \mathbf{p}_i \partial \mathbf{p}_j} \right] \tag{3.6}
\]

where \( \hat{\mathbf{d}} \) is the estimated distance, and \( f(\hat{\mathbf{d}}, \mathbf{p}) \) is the Probability Distribution Function (PDF) of the ranges based on the target node.

#### 3.1.2.1 TOA-based Localization

In TOA-based localization system, \( \hat{\mathbf{d}} \sim N(\mathbf{d}, \mathbf{C}) \) and \( \hat{\mathbf{d}} = [\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_N]^T \), and the PDF is

\[
f(\hat{\mathbf{d}}, \mathbf{d}) = \frac{1}{(2\pi)^{N/2}|\mathbf{C}|^{1/2}} \exp\left( -\frac{(\hat{\mathbf{d}} - \mathbf{d})^T \mathbf{C}^{-1} (\hat{\mathbf{d}} - \mathbf{d})}{2} \right) \tag{3.7}
\]
44  3.1. LOWER BOUND FOR TDOA AND TOA POSITIONING

Figure 3.1: Positioning system in polar coordinate system.

where $\mathbf{d}$ is the true distance vector and $\mathbf{C}$ is the error covariance matrix:

$$
\mathbf{C} = \text{diag}(\sigma_0^2 \frac{d_1^2}{d_0^2}, \sigma_0^2 \frac{d_2^2}{d_0^2}, \ldots, \sigma_0^2 \frac{d_N^2}{d_0^2})
$$

(3.8)

For range measurements with Gaussian error, the FIM in (3.6) can be calculated as [63]:

$$
[I(p)]_{ij} = \left[ \frac{\partial \mathbf{d}}{\partial p_i} \right]^T \mathbf{C}^{-1} \left[ \frac{\partial \mathbf{d}}{\partial p_j} \right] + \frac{1}{2} tr \left[ \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial p_i} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial p_j} \right]
$$

(3.9)

In order to show the calculation clearly, we derive the CRLB in polar coordinate system, as shown in Fig. (3.1). As mentioned in equation (3.5), CRLB is the trace of inverse FIM. Therefore, the CRLB for TOA-based positioning system can be expressed as:

$$
\text{CRLB} = \frac{I_{11} + I_{22}}{I_{11} I_{22} - I_{12} I_{21}}
$$

(3.10)

where

$$
[I(p)]_{11} = \sum_{i=1}^{N} \frac{\cos^2 \theta_i}{\sigma_0^2 d_i^2} + \sum_{i=1}^{N} \frac{\alpha^2 \cos^2 \theta_i}{2d_i^2}
$$

$$
[I(p)]_{22} = \sum_{i=1}^{N} \frac{\sin^2 \theta_i}{\sigma_0^2 d_i^2} + \sum_{i=1}^{N} \frac{\alpha^2 \sin^2 \theta_i}{2d_i^2}
$$

$$
[I(p)]_{12} = [I(p)]_{21} = \sum_{i=1}^{N} \frac{\cos \theta_i \sin \theta_i}{\sigma_0^2 d_i^2} + \sum_{i=1}^{N} \frac{\alpha^2 \cos \theta_i \sin \theta_i}{2d_i^2}
$$

(3.11)

If it is distance independent (DI) range error case (i.e. $\alpha = 0$), where the variance

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3.1. LOWER BOUND FOR TDOA AND TOA POSITIONING

![Graph showing CRLB for both TOA and TDOA positioning under decentralized locator geometry.]

Figure 3.2: CRLB for both TOA and TDOA positioning under decentralized locator geometry. $\sigma_0 = 0.01 \text{ m}, \alpha = 3$

of the TOA range is constant for any distances, the elements in the FIM are

\[
[I(p)]_{11} = \sum_{i=1}^{N} \frac{\cos^2 \theta_i}{\sigma_0^2}, \\
[I(p)]_{22} = \sum_{i=1}^{N} \frac{\sin^2 \theta_i}{\sigma_0^2}, \\
[I(p)]_{12} = [I(p)]_{21} = \sum_{i=1}^{N} \frac{\cos \theta_i \sin \theta_i}{\sigma_0^2}
\] (3.12)

3.1.2.2 TDOA-based Localization

In TDOA-based localization system, $\hat{r} \sim N(\mathbf{r}, \mathbf{C}_{RS})$ and $\hat{r} = [\hat{d}_2 - \hat{d}_1, \hat{d}_3 - \hat{d}_1, \cdots, \hat{d}_N - \hat{d}_1]^T$. \(\mathbf{r}\) is the range difference vector and \(\mathbf{C}_{RS}\) is the error covariance matrix:

\[
\mathbf{C}_{RS} = \text{diag}(\sigma_0^2 \frac{d_2^\alpha}{d_0^\alpha}, \sigma_0^2 \frac{d_3^\alpha}{d_0^\alpha} + \frac{d_1^\alpha}{d_0^\alpha}, \cdots, \sigma_0^2 \frac{d_N^\alpha + d_1^\alpha}{d_0^\alpha})
\] (3.13)
Since the range measurements in TDOA localization are also Gaussian model, the FIM thus can also be calculated using the following equation:

\[
[I(p)]_{ij} = \left[ \frac{\partial r}{\partial p_i} \right]^T C_{RS}^{-1} \left[ \frac{\partial r}{\partial p_j} \right] + \frac{1}{2} \text{tr} \left[ C_{RS}^{-1} \frac{\partial C_{RS}}{\partial p_i} C_{RS}^{-1} \frac{\partial C_{RS}}{\partial p_j} \right]
\] (3.14)

Then, the elements in the FIM can be easily derived in polar coordinate system:

\[
[I(p)]_{11} = \sum_{i=2}^{N} \frac{(\cos \theta_i - \cos \theta_1)^2}{\sigma_0^2(d_i^{-1}(d_i^{-1} + d_1^{-1}))} + \sum_{i=2}^{N} \frac{\alpha^2(d_i^{-1} \cos \theta_i + d_1^{-1} \cos \theta_1)^2}{2(d_i^2 + d_1^2)^2}
\]

\[
[I(p)]_{22} = \sum_{i=2}^{N} \frac{(\sin \theta_i - \sin \theta_1)^2}{\sigma_0^2(d_i^{-1}(d_i^{-1} + d_1^{-1}))} + \sum_{i=2}^{N} \frac{\alpha^2(d_i^{-1} \sin \theta_i + d_1^{-1} \sin \theta_1)^2}{2(d_i^2 + d_1^2)^2}
\]

\[
[I(p)]_{12} = [I(p)]_{21} = \sum_{i=2}^{N} \frac{(\cos \theta_i - \cos \theta_1)(\sin \theta_i - \sin \theta_1)}{\sigma_0^2(d_i^{-1} + d_1^{-1})} + \sum_{i=2}^{N} \frac{\alpha^2(d_i^{-1} \cos \theta_i + d_1^{-1} \cos \theta_1)(d_i^{-1} \sin \theta_i + d_1^{-1} \sin \theta_1)}{2(d_i^2 + d_1^2)^2}
\] (3.15)

Figure 3.3: CRLB for both TOA and TDOA positioning under centralized locator geometry. \( \sigma_0 = 0.01 \ m, \alpha = 3 \)
If it is distance independent (DI) range error case (i.e. $\alpha = 0$), the elements in the FIM can also be simplified to:

\[
[I(p)]_{11} = \sum_{i=2}^{N} \frac{(\cos \theta_i - \cos \theta_1)^2}{2\sigma_0^2}
\]

\[
[I(p)]_{22} = \sum_{i=2}^{N} \frac{(\sin \theta_i - \sin \theta_1)^2}{2\sigma_0^2}
\]

\[
[I(p)]_{12} = [I(p)]_{21} = \sum_{i=2}^{N} \frac{(\cos \theta_i - \cos \theta_1)(\sin \theta_i - \sin \theta_1)}{2\sigma_0^2}
\]  

(3.16)

Both the FIMs $I(p)$ derived here are applicable for the distance independent range error case as well as distance dependent (DD) case.

### 3.1.3 Discussion

The CRLB is not only benchmark the performance of the positioning system, but also provides insights for system design. The accuracy of localization system is dependent on locator geometry and the range measurement approaches.

First, we evaluate the effect of the distributed geometry of the locators on the positioning accuracy. There are five locators at $(0, 10); (-10, 0); (10, 0); (-5, -10); (5, -10)$. This case is called decentralized locator geometry, where the target node and locators are not close to each other [70]. All units are in meters, and the CRLB for the target positions are scaled within the area $-10 \leq x \leq 10; -10 \leq y \leq 10$, as shown in Fig. 3.2.

Next, we also examine the CRLB for a centralized locators. The positions of these five locators are $(0, 0); (-0.25, 0.1); (0.25, 0.1); (0.5, 0.2); (-0.5, 0.2)$ and the corresponding CRLBs are plotted in Fig. 3.3.

From Fig. 3.2 and Fig. 3.3, TOA-based localization can potentially achieve much better accuracy than positioning with TDOA measurements. Furthermore, the decentralized locator geometry also has a lower CRLB for the same area of interest than that of the centralized locator geometry.
3.1. LOWER BOUND FOR TDOA AND TOA POSITIONING

There are two main factors that affect the positioning accuracy of both TOA-based and TDOA-based localization system. As in the aforementioned part, we have discussed the effect of the distribution of reference geometry. In the following part, we will examine the influence of distance dependent (DD) or distance independent (DI) on localization system. We divided this issue into two parts: decentralized locator geometry and centralized locator geometry. Each part comprises of four cases: DI-TOA range error, DI-TDOA range error, DD-TOA range error and DD-TDOA range error. To illustrate the positioning error lower bound more clearly, line \( x = 0 \) and \( y = 0 \) is used to cut the CRLB which are scaled within the area \((0, 10); (-10, 0); (10, 0); (-5, -10); (5, -10)\). The part of decentralized locator is shown in Fig. 3.4 and Fig. 3.5 is the part of centralized locator. As shown

![Figure 3.4: Comparison of CRLB for TOA and TDOA distance-dependent (DD) and distance-independent (DI) cases with centralized locator distribution. The two figures of first column are DD cases and the rest two figures are DI cases.](image)

in equation (3.12) and (3.16) for case DI: , the elements in FIM are inversely proportional to the error variance. The simulation results, Fig. 3.4 and Fig. 3.5, also
Figure 3.5: Comparison of CRLB for TOA and TDOA distance-dependent (DD) and distance-independent (DI) cases with decentralized locator distribution. The two figures of first column are DD cases and the rest two figures are DI cases.

indicate that the TOA-based localization has a better CRLB than that in TDOA-based localization system. In case DD, the expression for the elements of FIM has more terms in equation (3.9) and (3.14), that means the effect of distance terms cannot be eliminated as equation (3.12) and (3.16). Fig. 3.4 and Fig. 3.5 shows that for TOA-based localization can again achieve better accuracy than localization with TDOA Measurements.

3.2 System Description

We propose a reliable localization system using the joint TOA/TDOA technique via UWB radios. A flow chart of the UWB localization system is shown in Fig. 3.6. It is divided into three parts: UWB communication, TOA estimation, and Hyperbolic (TDOA) Positioning using Taylor Series.
The proposed system acquires the distance between the transmitter and receivers by multiplying signal velocity to the TOA estimation of first arrival path. Both coherent and noncoherent receivers can be used in TOA estimation. Non-coherent receiver architectures are simpler as they do not need to generate correlation-based templates and thus reducing the receivers complexity [30], the ranging accuracy provided by them may not be acceptable to the system under study. Therefore, coherent receivers are preferred in our system.

Based on the preceding TOA ranging procedure, we can compute the range differences between the mobile and reference nodes. Even though traditional Linear Square Error (LSE) can be applied to reduce the effect of ranging errors on the accuracy of positioning due to its easy implementation and computationally efficient, it is limited to centimetre-resolution for localization precision. To improve
the accuracy, Taylor-series estimation, that is an iterative scheme for solution of
the simultaneous set of algebraic position equations, starting with a rough initial
guess and updating the guess at each iteration step by computing the local linear
least-sum-squared-error correction is applied to the system under study [71].

3.3 Measurement Schemes and Modeling

3.3.1 Mathematical model of the hyperbolic positioning

Suppose that we have \( l \) targets and \( n \) reference points, and that the clock at each
target is delayed by a time \( \sigma \) with respect to a common time reference. Then the
time \( \sigma \) is removed through the subtraction between TOA from different reference
receivers, shown in equation (1.11).

In order to simplify the mathematical model, we assume that there are only one
target node \((x, y, z)\) and \( n \) reference nodes arbitrarily distributed in a tridimen-
sional space with coordinates \((x_1, x_2, ..., x_n; y_1, y_2, ..., y_n; z_1, z_2, ..., z_n)\). The distance
\( d_i \) between a mobile (target node) and the \( i \)th receiver is determined indirectly by
multiplying the TOA \( \tau_i \) by the signal velocity \( c \), i.e. light speed, as shown in (3.17).

\[
d_i = c \times \tau_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \quad (3.17)
\]

We need to solve for these three unknown variables \( x, y \) and \( z \) (target position).
Therefore, other reference nodes’ locations are required. Even though only three
equations are needed for three unknowns, the solution requires perfect distance
estimation. Additionally, having more equations greatly simplifies the solutions for
\( x, y, z \) and improves performance of localization.

Suppose that one of the reference nodes is the origin \( O \) with \((0, 0, 0)\), then we have

\[
d_0 = c \times \tau_0 = \sqrt{x^2 + y^2 + z^2} \quad (3.18)
\]
where $d_0$ is the distance from origin to the mobile. Therefore, range differences from reference to the mobile are

$$
\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - \sqrt{x^2 + y^2 + z^2} = R_{i0} = d_i - d_0 = c \times (\tau_i - \tau_0)
$$

(3.19)

$$
\sqrt{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2} - \sqrt{x^2 + y^2 + z^2} = R_{j0} = d_j - d_0 = c \times (\tau_j - \tau_0)
$$

(3.20)

where $R_{i0}$ and $R_{j0}$ are range difference from the mobile to reference system, converted from the measured TOA differences. Transposing the second terms to the right side of (3.19) and (3.20), simplifying, and we have

$$
2x_i x + 2y_i y + 2z_i z - D_i^2 + R_{i0}^2 = -2R_{i0} \sqrt{x^2 + y^2 + z^2}
$$

(3.21)

$$
2x_j x + 2y_j y + 2z_j z - D_j^2 + R_{j0}^2 = -2R_{j0} \sqrt{x^2 + y^2 + z^2}
$$

(3.22)

where $D_i^2 = x_i^2 + y_i^2 + z_i^2$ and $D_j^2 = x_j^2 + y_j^2 + z_j^2$. These two equations represent two hyperboloids of revolution with foci at origin O, reference $i$ and origin O, reference $j$, respectively.

It is convenient to combine equations (3.21) and (3.22) into linear form,

$$
a_{ij} x + b_{ij} y + c_{ij} z = d_{ij}
$$

(3.23)

where

$$
a_{ij} = x_i R_{j0} - x_j R_{i0}
$$

$$
b_{ij} = y_i R_{j0} - y_j R_{i0}
$$

$$
c_{ij} = z_i R_{j0} - z_j R_{i0}
$$

$$
d_{ij} = \frac{D_i^2 R_{j0} - R_{j0} R_{i0}^2 - D_j^2 R_{i0} + R_{i0} R_{j0}^2}{2}
$$

(3.24)
The preceding discussion shows that when TDOA to three reference nodes are known, a plane intersected by two hyperboloids on which the mobile lies can be computed. However, additional information is required to determine the accurate position of the target nodes. One more reference node is added, a new plane where the mobile lies will be introduced, and the position is at the intersection of at least three such planes.

The position of mobile in a 3D space is determined by the following equations:

\[ Au = h \]  \hspace{1cm} (3.25)

with

\[
A = \begin{bmatrix}
a_{12} & b_{12} & c_{12} \\
\vdots & \ddots & \vdots \\
a_{ij} & b_{ij} & c_{ij} \\
\vdots & \ddots & \vdots \\
\end{bmatrix}^{(n-2)(n-1)}
\]

\[
u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}^T \quad h = \begin{bmatrix} d_{11} \\ d_{12} \\ \vdots \\ d_{ij} \cdots d_{(n-2)(n-1)} \end{bmatrix}^T
\]

where \( n \geq 5 \) for the non-singularity of matrix \( A \) and \( i = 1, 2, \cdots, n - 1; j = i + 1, i + 2, \cdots, n - 2 \).

As compared to the mathematical model describe in Fang’s paper [72, 73], the proposed method can select reference nodes randomly in 3D space. However, the reference nodes proposed by Fang [72] should be restricted in the XY plane, i.e. \( z = 0 \). This method produces a poor estimation in Z axis and the model for calculation of Z coordinate is nonlinear and complex.

### 3.3.2 Error minimization procedure by iterative Taylor series (Newton-Gauss method)

The primary motivation for using UWB technology is the ability of UWB pulses to provide very accurate ranging data using TOA estimation [30]. Note that the
introduction of additional reference nodes is necessary to reduce the effect of errors in ranging estimations on the accuracy of positioning, or the analytical solution may not exist for equation (3.25).

Traditional LSE can be used to minimize the effect of ranging errors due to its easy implementation and computationally efficient. However, it provides no significant improvement in accuracy. The method proposed in [71] is an effective and accurate error minimization technique to calculate the position of mobile. It begins with an initial guess, then applies least-sum-squared-error to solve the navigational equations, and finally computes the position of the mobile.

Suppose \((x, y, z)\) is the true position of mobile, \((x_k, y_k, z_k)\) is the position of the \(k\)th reference node with respect to \(k = 1, 2, \ldots, n - 1\), and \(n\) is the number of fixed reference points. Rewrite equation (3.23) in the form of algebraic relation

\[
f_{ij}(x, y, z; x_k, y_k, z_k) = d_{ij} = m_{ij} - e_{ij}
\]

(3.27)

where \(d_{ij}\) is the correct value of measured quantity, \(m_{ij}\) is the measured value, and \(e_{ij}\) is error in the \(m_{ij}\) measurement. We assume that the errors have zero expectation \(E(e_{ij}) = 0\), then the \(mn\)th element in the error covariance matrix is \(r_{mn} = E(e_m e_n)\). Therefore, the error covariance matrix is \(R = [r_{mn}]\).

Let’s assume that \((x_0, y_0, z_0)\) is the initial guess position of mobile, and takes the form of

\[
\begin{align*}
x &= x_0 + \Delta x \\
y &= y_0 + \Delta y \\
z &= z_0 + \Delta z
\end{align*}
\]

(3.28)

Ignoring the second and higher order terms in Taylor series, we have

\[
f_{ij0} + \alpha_{ij} \Delta x + \beta_{ij} \Delta y + \gamma_{ij} \Delta z \simeq m_{ij} - e_{ij}
\]

(3.29)
where

\[ f_{ij0} = f_{ij}(x_0, y_0, z_0; x_k, y_k, z_k) \]

\[ \alpha_{ij} = \frac{\partial f_{ij}}{\partial x}|_{x_0, y_0, z_0} \]

\[ \beta_{ij} = \frac{\partial f_{ij}}{\partial y}|_{x_0, y_0, z_0} \]

\[ \gamma_{ij} = \frac{\partial f_{ij}}{\partial z}|_{x_0, y_0, z_0} \]  

(3.30)

Combining (3.25) with (3.29), we define the approximate relations of (3.27)

\[ A\Delta = h - f - e \]  

(3.31)

where

\[ \Delta = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}^T \]

\[ f = \begin{bmatrix} f_{110} & f_{120} & \cdots & f_{ij0} & \cdots & f_{(n-2)(n-1)0} \end{bmatrix}^T \]

\[ e = \begin{bmatrix} e_{110} & e_{120} & \cdots & e_{ij0} & \cdots & e_{(n-2)(n-1)0} \end{bmatrix}^T \]  

(3.32)

with respect to \( i = 1, 2, \ldots, n - 1; j = i + 1, i + 2, \ldots, n - 2. \)

The solution for equation (3.31) that is given in terms of covariances of the measurement errors and weighted least-sum-squared error is

\[ \Delta = [A^T R^{-1} A]^{-1} A^T R^{-1} (h - f) \]  

(3.33)

Then the initial guess will be updated by equation (3.28) with computed errors. The iterations will have converged when they satisfy

\[ |\Delta x| + |\Delta y| + |\Delta z| \leq \xi \]  

(3.34)

Finally, the position of mobile can be obtained by the preceding algorithm. Although an initial guess is required in the beginning of the minimization procedure, the convergence to the desired solution is independent of the initial value and guaranteed [71].

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3.4 Simulation and experiment results

3.4.1 Simulation results

In order to evaluate the performance of the proposed method, we simulate as follows: a set of eight reference nodes randomly distributed in a cubic space with volume of 100 m×100 m×100 m. The variance of ranging error is $\sigma^2 = 1 \text{ m}^2$, and the position estimation by iterative Taylor series with ranging error is shown in Fig. 3.7.

Additionally, the iterative Taylor-series-based error minimization is compared with the method proposed in [72]. A relation between the ranging error and the positioning error can be obtained by iterating the steps above different values of $\sigma^2$. We repeat the trials for 10000 times with specific value of $\sigma^2$, then get the average positioning error by averaging over the 10000 trials. Furthermore, we also estimate the effect of the number of reference nodes on the positioning error. These results are shown in Fig. 3.8 and Fig. 3.9.

Fig. 3.8 shows that the positioning error can be effectively minimized by employing iterative Taylor series, and when the ranging error increases, the positioning error
3.4. SIMULATION AND EXPERIMENT RESULTS

Figure 3.8: The performance comparison of two methods.

increases exponentially, thus reducing the positioning accuracy. Fig. 3.9 shows that the positioning error can be reduced by increasing the number of reference nodes.

3.4.2 Experimental Validation

In order to investigate the performance of the proposed approach, real UWB measurements were conducted. The measurement system consists of one transmitter and eight receivers, and the pulse used in the UWB system is Gaussian pulse.

The original excited pulse generated from the Impulse Generator 3500D manufactured by Picosecond pulse labs is shown in Fig. 3.10, which has a null-to-null pulse width of around 200 ps. However, the signal in the free space was broadened and distorted due to the dispersive behavior of antennas and cables across the entire frequency spectrum [66]. Nine Skycross antennas SMT-3TO10M, one for transmitter and others for receivers, are used in the experiments. The signals are captured by Agilent Real Time Oscilloscope DSO80804B with sampling rate of 40 GHz.

In order to simplify the TOA detection process, we set up the eight reference nodes in the same plane. Before measurement, calibration must be done to remove the clock bias caused by the different length of cables. As computed, the offset value for these
8 receivers is [0, 0, 0.01, 61.39, 1.48, 1.49, 1.48, 2.98] cm. We select the first receiver as the origin of the reference system, thus the positions for these 8 receivers are [0, 0, 0; 10, 0, 0.01; 20, 0, 0.01; 0, 10, 0.01; 20, 10, 0.01; 15, 20, 0.01; 5, 25, 0.01; 0, 20, 0.01] cm. The value in z axis cannot be zero, or the matrix $A$ in equation (3.31) will be singular, thus determining the wrong position of the target. In the experiment, the actual position of mobile is $p = [5, 5, 0.01]$, and the detection of TDOA is shown in Fig. 3.11.

For the given reference system, two positioning algorithms are used to determine the transmitter position. If the traditional LSE estimation is used with the model proposed in [72, 73], the measured position is $p_t = [5.4414, 5.3645, 3.0288]$. As can be seen, the estimated values in $x$ axis and $y$ axis are accurate, but the measured value in $z$ has relatively large error. To improve the positioning accuracy, we refine the LSE position estimation recursively by Taylor series algorithm with model proposed in section (III). Then we can get the result, $p_t = [5.7089, 5.703, 0]$, which has a very high accuracy. The positioning error defined in Euler distance is about 1cm.

To further evaluate the performance of the proposed method, we change the position of the transmitter in the plane formed by the same reference nodes, and obtain the
result shown in Table (3.1). Due to its high-temporal resolution, multipath immu-
nity, and simultaneous ranging and transmission capability, UWB-based measure-
ment approach, using Taylor-series minimization algorithm, provides high accuracy
for localization with mean Euler error and standard deviation $(0.7198 \pm 0.271)\text{cm}$.

Table 3.1: Estimation of positions of different targets with respect to same reference
system

<table>
<thead>
<tr>
<th>Actual Position (cm)</th>
<th>Estimated Position (cm)</th>
<th>Euler Error (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(10,10,0.01)$</td>
<td>$(9.5633,9.8164,0)$</td>
<td>0.4738</td>
</tr>
<tr>
<td>$(10,0,0.01)$</td>
<td>$(10.847,0.4888,0)$</td>
<td>0.978</td>
</tr>
<tr>
<td>$(0,10,0.01)$</td>
<td>$(0.8884,10.229,0)$</td>
<td>0.9175</td>
</tr>
<tr>
<td>$(5,15,0.01)$</td>
<td>$(5.4213,14.91,0)$</td>
<td>0.4309</td>
</tr>
<tr>
<td>$(5,5,0.01)$</td>
<td>$(5.7089,5.703,0)$</td>
<td>0.9984</td>
</tr>
<tr>
<td>$(10,15,0.01)$</td>
<td>$(10.383,14.648,0)$</td>
<td>0.5203</td>
</tr>
<tr>
<td><strong>Mean Error</strong></td>
<td><strong>0.7198</strong></td>
<td><strong>Standard Deviation 0.271</strong></td>
</tr>
</tbody>
</table>
3.5 Conclusion

In this chapter, we presented a hyperbolic position measurement system based on absolute range (TOA) measurements. By selecting one receiver as the reference origin and computing the differential TOA between the direct signal from the transmitter and receivers, we are able to eliminate the requirement for precise synchronization between reference nodes and the target node.

However, some disadvantages of UWB-based positioning technique are: 1) only TDOA-based localization technique can effectively overcome the strict clock synchronization issue between transmitter and receivers with a penalty of decreasing positioning accuracy. As seen in section 3.1, TOA-based positioning technique has higher performance than TDOA-based localization technique. In human motion tracking system, we prefer higher tracking accuracy to accurately measure the required gait parameters. 2) in experimental part, we had to dedicate around 20 seconds to acquire 20 samples. Therefore, the sampling rate is far from high enough for human movement tracking. This is because of the large bandwidth of IR-UWB pulse, and the use of eight receivers in our experiment. Even though the Agilent Real time Oscilloscope used here is with the sampling rate of 40 GHz for one chan-
nel acquisition, the sampling rate is halved when more than one channel is working at the same time. 3) although the experimental results show an acceptable positioning accuracy, the performance is improved with additional instrumentations (8 receivers) resulting in high complexity and cost of the overall system.
Chapter 4

Development of Wearable Wireless Ultrasonic Motion Analysis System

In our previous chapter, the use of the wearable UWB radios for localization in 3-dimensional space has been described. In UWB system, where sub-nanosecond pulse is used, the required synchronization is extremely challenging. Highly accurate synchronization clock that provides precise timing information is required for tracking which is difficult to be achieved in low cost systems [31]. Furthermore, it is difficult to sample the received signal in real time with current ADC technology due to the large bandwidth of IR-UWB pulse [32].

As such, a wearable wireless ultrasonic sensor network based on ultrasound sensors for the measurement of 3-dimensional foot trajectory is proposed to overcome these limitations. The objective is to allow patients to be monitored under an unrestrained environment. The proposed approach makes use of the wireless sensor network concept with all the mobile sensor nodes communicating wirelessly with the coordinator. These sensor nodes are light and small for attaching to the human foot. Furthermore, it is low cost as compared to the camera based motion capture system. RF module used in our system is only to transmit collected data to a central
processing unit e.g. laptop, and to provide synchronization signal between the ultrasonic transmitter and the receiver. All the distance information is collected from the ultrasonic transmitter/receiver. The foot motion was obtained from the movement of the ultrasonic sensor placed on the subject’s heel, by adapting spherical positioning technique which finds the intersection area of circles centered at each anchor with radius equal to the measured distance from the transmitter to each anchor. Then, Kalman filter is applied to enhance the performance of the tracking technique.

The chapter is organized as follows: a brief overview of the configuration of the wireless wearable ultrasonic system is given in Section 4.1. Section 4.2 describes the proposed measurement system using state-space methods to continuously track foot displacement during walking. This is followed by 3 steps tracking algorithm using different approaches, such as Least-Square (LS) method, recursive Newton-Gauss method, extended Kalman Filter (EKF), and the combination of Newton-Gauss method and Kalman filter to reduce the effect of range measurement errors on the foot displacement measurement in Section 4.3. Section 4.4 investigates the performance of the ultrasonic tracking system by comparing with camera based motion capture system using different tracking algorithms during different walking speeds. Finally, conclusions are made in Section 4.5.

4.1 System Description and Configuration

4.1.1 System Description

The proposed measurement system uses ultrasonic sensor to track and localize the foot motion during walking. As shown in Fig. 4.1, vertical displacement, also called foot clearance, is defined as the height between a foot and the ground during walking. The distance during walking is widely used in many fields extending from gait analysis to rehabilitation. General requirements for gait analysis are that the
device attached to human body should be as small and as light as possible. Any bulky devices heavier than about 1-2 percent of the subject’s mass may potentially disturb normal gait [74].

![Image of foot displacement measurement during gait](image)

Figure 4.1: The prototype of foot displacement measurement during gait [4].

### 4.1.2 System Configuration

![General configuration of the system](image)

Figure 4.2: General configuration of the system

Fig. 4.2 shows the general configuration of the system. The system comprises of a number of sensor nodes, coordinator, data transmission module, anchors and a mobile. The mobile consists of an ultrasound generator board, as shown in Fig.
4.1. SYSTEM DESCRIPTION AND CONFIGURATION

4.3(a), and a controller board, shown in Fig. 4.4, comprising of a microcontroller unit (MCU), a RF module and a temperature sensor. An anchor is composed of an ultrasound receiver, as shown in Fig. 4.3(b), and a controller board. The anchor receives ultrasonic signals from the mobile device and computes distance estimates to the mobile using time-of-arrival of the ultrasonic signal.

A single ranging cycle is shown in Fig. 4.5. Distance measurement is activated by a trigger input generated by the control board on the mobile. Then, the ultrasound generator sends out an ultrasonic burst including 8 square pulses with a frequency of 40kHz. Meanwhile, the RF module on the mobile transmits its own address and synchronization signal to the RF module on the control board of anchors. The device uses the STC12LE5612AD microprocessor with high crystal frequency (Fosc=22.1184 MHz), which provides high time resolution, resulting in a theoretical...
resolution of 0.031 mm (340 m/s / (Fosc/2)). Additionally, the ultrasound generator is designed to make the maximum propagation distance of about 20m, and so 16-bit counters are used to ensure that there is no overflow occurring during counting. These 16-bit counters start counting the moment the RF module receives the synchronization signal from the mobile. The transmission time of the RF signal is negligible, since the speed of light is much faster than the speed of sound. The corresponding counter will be immediately stopped when the transmitted burst is received by the anchor. When all the four anchors stop counting, the counted steps will be converted to distance, and then it is transferred wirelessly through the RF module to wireless data transmission module upon the end of the measurement cycle. It is then forwarded to a personal computer through RS232 cable for postprocessing.

The distance between the mobile and the anchor can be calculated from:

\[ d = t \cdot v_s \]  

(4.1)

where \( d \) is the distance in meters, \( t \) is the propagation delay in seconds and \( v_s \) is the speed of ultrasound in air. The ultrasound velocity can be approximated to [75]:

\[ v_s = 331.5 + 0.6T_c \]  

(4.2)

Figure 4.5: Single ranging cycle. Distance measurement is initiated by sending a pulse to the trigger input. The receiver will respond with a pulse with a duration corresponding to the propagation delay.

The distance between the mobile and the anchor can be calculated from:
where \( T_c \) is the air temperature in degree Celsius. Together with the known positions of these anchors, the position of the mobile is located using the TOA-based tracking technique which finds the intersection area of circles centered at each anchor with radius equal to the measured distances. The tracking algorithm is discussed in the following section.

### 4.2 System Modeling

#### 4.2.1 Motion Model

We assume that the mobile with position \( \mathbf{p} = [x \ y \ z]^T \) sends out ultrasonic signals to anchors after each trigger signal. The positions of anchors are known with \( \mathbf{p}_i = [x_i \ y_i \ z_i]^T \), respectively. Then, we design an EKF using a state vector with six components, three Cartesian coordinates \((x, y, z)\) and their velocity components \((\dot{x}, \dot{y}, \dot{z})\). Therefore, the state of the mobile target for time step \( k \) can be expressed as

\[
\mathbf{x}(k) = [x(k) \ y(k) \ z(k) \ \dot{x}(k) \ \dot{y}(k) \ \dot{z}(k)]^T
\]

(4.3)

Then the state transition equation from time step \( k - 1 \) to \( k \) is given by:

\[
\mathbf{x}(k) = A\mathbf{x}(k - 1) + \mathbf{q}(k - 1)
\]

(4.4)

where the state transition matrix \( A \) from the respective kinematics equations is

\[
A = \begin{bmatrix}
I_{3\times3} & T \cdot I_{3\times3} \\
O_{3\times3} & I_{3\times3}
\end{bmatrix}
\]

(4.5)

where \( I_{3\times3} \) is the identity matrix and \( O_{3\times3} \) is the matrix with all elements zero. \( T \) is the sampling interval. The process noise is \( \mathbf{q}(k - 1) \sim N(0, Q(k - 1)) \). The covariance matrix \( Q(k - 1) \) accounts for the un-modeled factors of the system that
will be treated as random noise. It becomes:

\[
Q(k - 1) = \begin{bmatrix}
\frac{T^3}{3}Q_s & \frac{T^2}{2}Q_s \\
\frac{T^2}{2}Q_s & TQ_s
\end{bmatrix}
\]

(4.6)

where \( Q_s = \text{diag}(q_x^2, q_y^2, q_z^2) \). In most cases, \( q_x, q_y \) and \( q_z \) can be considered as standard deviations of the velocity noise in x, y and z directions, respectively.

### 4.2.2 Measurement Model

We let \( d_i \) denote the absolute distance measured at \( i \)th anchor using the following equation:

\[
d_i = ||p - p_i|| + \tilde{d}_i = r_i + \tilde{d}_i
\]

(4.7)

where \( \tilde{d}_i \) is range measurement noise and \( r_i \) is actual distance. Stacking all the distance information, we have the measurement model expressed as:

\[
D(k) = F(x(k))x(k) + \tilde{D}(k)
\]

(4.8)

where

\[
D(k) = \begin{bmatrix} d_1 & d_2 & \cdots & d_n \end{bmatrix}^T
\]

\[
F(x(k)) = \begin{bmatrix} F_1 & F_2 & \cdots & F_n \end{bmatrix}^T
\]

\[
F_i = \begin{bmatrix}
\frac{\partial d_i}{\partial x} & \frac{\partial d_i}{\partial y} & \frac{\partial d_i}{\partial z} & 0 & 0 & 0
\end{bmatrix}
\]

(4.9)

\[
\tilde{D}(k) = \begin{bmatrix} \tilde{d}_1 & \tilde{d}_2 & \cdots & \tilde{d}_n \end{bmatrix}^T
\]

\( n \) is the number of anchors, and \( \tilde{D}(k) \sim N(0, R(k)) \) is measurement errors. \( R(k) = \text{diag}(e_1^2, e_2^2, \cdots, e_n^2) \) is the covariance matrix of measurement errors. \( e_i \) is always considered as the standard deviations of the measurement error of anchor \( i \).
4.3 Tracking Algorithm

We first apply pre-filtering of the range measurements to reduce the errors in tracking and localization. Then, we will apply some minimization technique to improve the positioning accuracy [76].

4.3.1 Prefiltering

Since the range measurements may have some large errors or outliers, these outliers can result in significant errors for mobile target tracking. It is not necessary to have a very rigorous pre-filtering since Kalman filter is a robust estimator. Extremely large error in range measurements can be easily eliminated. The method used involves combining past distance measurements and the maximum moving speed at which the mobile is expected to move.

First note that the previous distance measurement for \( i \)th anchors is stored in \( d_i \). The mobile target can either be moving away from the \( i \)th anchor or approaching it. In approaching case, the distance between the mobile and \( i \)th anchor will decrease as given by equation (4.10) for \( d_{imax}^- \). Otherwise, the distance from the \( i \)th anchor will increase over time as indicated by equation (4.11) for \( d_{imax}^+ \).

\[
d_{imax}^- = d_i - T \cdot v_{max} \quad (4.10)
\]

\[
d_{imax}^+ = d_i + T \cdot v_{max} \quad (4.11)
\]

where \( v_{max} \) is the maximum possible moving velocity of the mobile. Subtracting (4.11) by (4.10) results in the following equation:

\[
\Delta d_i = d_{imax}^+ - d_{imax}^- = 2 \cdot T \cdot v_{max} \quad (4.12)
\]

Equation (4.12) provides a threshold to eliminate large range measurement errors.

The current range measurement is only used for tracking and localization when
4.3. TRACKING ALGORITHM

the absolute difference between the current range measurement and the previous
measurement is smaller then the predefined threshold $\Delta d_i$, as illustrated by equation
(4.13).

$$|d_{ik} - d_{i(k-1)}| \leq \Delta d_i = 2 \cdot T \cdot v_{max} \tag{4.13}$$

4.3.2 Minimization Procedure

In this section, we are trying to use different approaches to improve the performance
of TOA-based tracking algorithm.

4.3.2.1 Approach I: Least Square (LS) Method

As the measurement model used in equation (4.8), we have assumed that the mea-
sured distance are independent. Therefore, the position of the mobile at time step
$k$ can be determined as:

$$p = \arg \min_p (A p - \tilde{D})^T (A p - \tilde{D}) = (A^T A)^{-1} A^T \tilde{D} \tag{4.14}$$

The above mentioned LS estimation method can be used to minimize the ranging
errors due to its easy implementation and computationally efficiency. However, the
lower tracking accuracy is not suitable for some high precision system. Another
commonly used method is Newton-Gauss iterative method as discussed in section
3.3.2. Here summarized as follows.

4.3.2.2 Approach II: Newton-Gauss (NG) Method

Newton-Gauss iterative method is commonly used to solve the nonlinear optimiza-
tion problem (4.7) [59]. It begins with an initial guess and is followed by least-sum-
square-error minimization [77]. For the $jth$ iteration, we have

$$p^{j+1} = p^j - T^j \Theta^j \tag{4.15}$$
until the condition
\[ \|p^{j+1} - p^j\| < \varepsilon \] (4.16)
is satisfied.

\[ T^j = [F^T(p^j)F(p^j)]^{-1}F^T(p^j) \]
\[ \Theta^j = D^j - \|p - p_i\| \]
\[ D^j = [d_1^j d_2^j \cdots d_n^j]^T \]
\[ F(p^j) = [F(p^j)_1 F(p^j)_2 \cdots F(p^j)_n]^T \]
\[ [F(p^j)]_i = \left[ \frac{\partial \|p^j - p_i\|}{\partial x} \frac{\partial \|p^j - p_i\|}{\partial y} \frac{\partial \|p^j - p_i\|}{\partial z} \right] \]

where \( n \) is the number of anchors, and \( \varepsilon \) is a prescribed threshold. The initial value for \( k \)th step is
\[ p^0 = \begin{bmatrix} x^0 \\ y^0 \\ z^0 \end{bmatrix} = \begin{bmatrix} \hat{x}(k) \\ \hat{y}(k) \\ \hat{z}(k) \end{bmatrix} \] (4.18)
where \([\hat{x}(k) \hat{y}(k) \hat{z}(k)]^T\) are the position of the mobile estimated by LS estimator.

Initial guess is important to guarantee the convergence of such recursive method. Therefore, the position information of LS estimator has been used as initial guess [78]. The experiment shows that such method converges after a few iterations.

4.3.2.3 Approach III: Extended Kalman Filter (EKF)

The basic idea of the extended Kalman filter is that the filter uses prior knowledge of all distance information to predict and produce an estimate of position of where the mobile might be in the next time step. Once the next distance sample arrives, the filter first corrects the state based on the actual distance information [79, 80]. In each iteration, the current state \( x(k-1) \) is used to predict the velocity and position at the time step \( k - 1 \). The error covariance \( \hat{P}(k) \) is also predicted using the state space model in equation (4.4). This is followed by the computation of Kalman filter gain \( K(k) \). Once \( K(k) \) is computed, the range measurements \( D(k) \) are calibrated by the predicted state of the mobile and are used to update the mean and covariance.
4.3. TRACKING ALGORITHM

of the state $x(k)$ and $P(k)$, respectively. The standard expression is given below [76]

- **Prediction:**
  $$
  \hat{x}(k) = A x(k-1) \\
  \hat{P}(k) = A P(k-1) A^T + Q(k-1)
  $$ (4.19)

- **Update:**
  $$
  \Phi(k) = D(k) - F(\hat{x}(k))\hat{x}(k) \\
  S(k) = F(\hat{x}(k))\hat{P}(k)F^T(\hat{x}(k)) + R(k) \\
  K(k) = \hat{P}(k)F^T(\hat{x}(k))S(k)^{-1} \\
  x(k) = \hat{x}(k) + K(k)\Phi(k) \\
  P(k) = \hat{P}(k) - K(k)S(k)K(k)^T
  $$ (4.20)

where $\hat{x}(k)$ and $\hat{P}(k)$ are the predicted mean and covariance of the state, respectively, for time step $k$ before getting measurement result; $x(k)$ and $P(k)$ are the estimated mean and covariance of the state, respectively for time step $k$ after getting measurement result; $K(k)$ is the Kalman filter gain.

EKF is the most commonly used nonlinear state estimator using the first or second order terms of the Taylor series expansion, which is most appropriate when the noise statistics is Gaussian distribution, to linearize the state and observation models [76]. However, for some highly nonlinear dynamics, the linearization of EKF insufficiently characterizes the relationship. Therefore, We mentioned another approach to overcome these limitations, i.e. the combination of Newton-Gauss method and Kalman filter.

4.3.2.4 Approach IV: Combination of NG and Kalman Filter (KF-NG)

Once a new measurement of $Y_k = D(k)$ has been obtained, the iterative procedure starts with an initial guess position $p_k^0 = [x_k^0 \ y_k^0 \ z_k^0]^T$. Then, the position of moving target is first obtained by applying iterative Newton Gauss method. Once the
prelocalization is done, we need to update the measurement model in equation (4.8) with the result of prelocalization result \( \hat{p}_k = [\hat{x}_k \ \hat{y}_k \ \hat{z}_k]^T \). The following linear equation represents the state transition:

\[
\hat{p}_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} X_k + v_k \\
= BX_k + v_k
\] (4.21)

Finally, the state of moving target can be evaluated from the model as described below using Kalman filter. The tracking algorithm has been summarized as follows:

For each time step \( k \), before getting the measurement \( Y_{k+1} \),

1. Predict the state \( \hat{X}_{k+1|k} \) for the next time instant using the current state \( X_{k|k} \)

\[
\hat{X}_{k+1|k} = A_k X_{k|k}
\] (4.22)

2. Update the error covariance matrix \( P_{k+1|k} \) for the next time step

\[
P_{k+1|k} = A_k P_{k|k} A_k^T + G_k Q G_k^T
\] (4.23)

3. Compute the true distance after getting the measurement \( Y_{k+1} \)

\[
r_{i(k+1)}^0 = ||p_{k+1}^0 - p_i||
\] (4.24)

with respect to the initial guess position \( p_{k+1}^0 = [\hat{x}_{k+1|k} \ \hat{y}_{k+1|k} \ \hat{z}_{k+1|k}]^T \).

4. Update the error matrix \( \Theta \) using Newton Gauss method to linearize \( h(X_{k+1}) \)

\[
\Theta = [\Delta x \ \Delta y \ \Delta z]^T \\
= (\Gamma_{k+1}^T \Gamma_{k+1})^{-1} \Gamma_{k+1}^T (Y_{k+1} - h^0(X_{k+1}))
\] (4.25)
where
\[ \Gamma_{k+1}^i = \left[ \frac{\partial \| p_{k+1} - p_i \|}{\partial x} \frac{\partial \| p_{k+1} - p_i \|}{\partial y} \frac{\partial \| p_{k+1} - p_i \|}{\partial z} \right] \] (4.26)

with \( i = 1, 2, 3, 4 \).

5. Correct the initial guess position by
\[ x_{k+1}^1 = x_{k+1}^0 + \Delta x \]
\[ y_{k+1}^1 = y_{k+1}^0 + \Delta y \]
\[ z_{k+1}^1 = z_{k+1}^0 + \Delta z \] (4.27)

6. Repeat step 4, 5, and 6 until the condition
\[ \| p_{k+1}^{j+1} - p_{k+1}^j \| \leq \varepsilon \] (4.28)
is satisfied, where \( \varepsilon > 0 \) is a prescribed threshold.

7. Compute Kalman gain \( K_{k+1} \)
\[ S_{k+1} = BP_{k+1\mid k}B^T + V_{k+1} \] (4.29)
\[ K_{k+1} = P_{k+1\mid k}B^T S_{k+1}^{-1} \] (4.30)

8. Update the predicted state and error covariance
\[ X_{k+1\mid k+1} = \hat{X}_{k+1\mid k} + K_{k+1}(\hat{p}_{k+1} - B\hat{X}_{k+1\mid k}) \] (4.31)
\[ P_{k+1\mid k+1} = P_{k+1\mid k} - K_{k+1} S_{k+1} K_{k+1}^T \] (4.32)

9. End for

There are two issues to be addressed here. First, the initial guess should be selected
4.4 EXPERIMENTAL VALIDATION

carefully to guarantee the convergence of the iterations [78]. As discussed in our previous work [81], the first three elements of the predicted state $\hat{X}_{k+1|k}$ of the Kalman filter is selected as initial guess position. Simulation and experiment shows that such selection results in high convergence rate. Another issue is the typical value of $\varepsilon$. The value of $\varepsilon$ also determines the positioning accuracy and convergence rate of the iterative Newton Gauss method. Large value of $\varepsilon$ gives high convergence rate but results in low tracking accuracy. Small value of epsilon gives accurate positioning results but low convergence rate. Typical value of $\varepsilon$ is 10 mm in our method to provide rough estimates of the mobile position which will be further refined by the following Kalman filter.

4.4 Experimental Validation

In this section, we use the experimental results to compare the proposed ultrasonic motion analysis system with the camera based motion capture system. The foot 3-dimensional displacements are estimated using different approaches, such as LS approach, purely NG method, purely EKF, and the combination of NG and KF (KF-NG).

4.4.1 Experiment Setup

To provide a more systematic validation, we conducted the experiments in a motion analysis lab with eight high speed cameras (Motion Analysis Eagle System, Santa Rosa, CA, USA) in the School of Mechanical and Aerospace Engineering at Nanyang Technological University. The Motion Analysis Eagle System consists of Eagle Digital Cameras and Cortex software, which captures complex 3D motion with extreme accuracy. System calibrations of the reference system should be done at both static (with 4-point calibration L-frame) and dynamic process (with 3-point calibration wand) to ensure an acceptable accuracy of the reference system. In our
4.4. EXPERIMENTAL VALIDATION

Figure 4.6: Wireless unit with embedded ultrasonic sensor attached on foot, and three reflective markers fixed on toe, heel and shank for reference camera based motion capture system.

experiments, the accuracy of the reference system is $0.43 \pm 0.18 \text{ mm}$ (Average ± Standard deviation).

Ten healthy subjects (age $25.7 \pm 1.4$ years, height $171.4 \pm 6.5$ cm, and weight $62.8 \pm 5.6$ kg) were used to test the performance of the proposed system. All subjects were required to walk on treadmill without shoes or wear regular shoes without high heels, and repeat several times at slow, normal and fast speeds. The camera system tracked the position of three reflective markers placed on toe, heel and shank extremities of subjects’s foot or shoe according to Fig. 4.6. Actually, one reflective marker on heel was enough to provide a reference data for validation. The other two reflective markers were used to create a template for better tracking in camera based motion capture system.

There were four anchors used in our experiment with positions ($p_1 = [0 \ 0 \ 0]^T$, $p_2 = [0.324m \ 0 \ 0]^T$, $p_3 = [0.324m \ 0.230m \ 0]^T$, $p_4 = [0 \ 0.230m \ 0]^T$). The ultrasonic transmitter was attached to the heel of the foot pointing towards the four anchors, using elastic straps. In our method, only one ultrasonic sensor (transmitter) is needed which is to be attached to the foot, which minimizes the discomfort for users and avoids complex calibration procedures and synchronization issues. All the data transmission between anchors, coordinator and transmitter are done wirelessly.
through the RF module. Therefore, it does not restrict the movement of subjects. The ultrasonic sensor data were acquired at 50Hz. Data from the reference system were captured at 200Hz. The difference between the sampling rate of these two systems were compensated by linear interpolation. All data were low-pass filtered by second order low-pass Butterworth filter at 10Hz.

4.4.2 Parameters Identification

The process and measurement noise statistics should be estimated for the system models in Section 4.2. We first conduct experiments with the mobile target moving at a given trajectory. These experiments help to find suitable values of process noise $Q_s$ and measurement noise $R(k)$. We take a sensor and run $M$ tests with the same trajectory. The actual distance for test $i$, $r_i$, is known, and there are $N$ measurement samples $m_{ij}$ collected for each test, where $j = 1, \cdots, N$.

4.4.2.1 Process Noise Statistics in Kalman Filter

As the process noise in EKF is an independent variable, it is difficult to get an exact value [80]. Here, we consider it as a velocity noise in $x$, $y$ and $z$ directions with mm/sec unit. A metric, Net Root Mean Square Error (Net RMSE), as defined by equation (4.34), is used to select a reasonable value of $Q_s$.

$$RMSE = \sqrt{\frac{\sum (Actual - Estimated)^2}{Number of Estimates}}$$

$$Net RMSE = \sqrt{X_{RMSE}^2 + Y_{RMSE}^2 + Z_{RMSE}^2}$$

The process noise $Q_s$ was estimated by using numerical methods. By varying the values of $q_x$, $q_y$, and $q_z$, we will get the corresponding trajectory of the mobile to compute the Net RMSE value. Typical values of $q_x$, $q_y$, and $q_z$ will be selected when their corresponding Net RMSE is minimal. The typical values of $Q_s$ used in our experiments is $q_x = 36$, $q_y = 30$, $q_z = 9$.

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4.4.2.2 Measurement Noise Statistics in Kalman Filter

It is reasonable to assume that all the anchors have independent distributed noise. Then, the mean and covariance of the measurement noise can be evaluated. Using the data obtained from the experiments, mean and variance of the measurement errors given by

\[ u = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (m_i^j - r_i) \]

\[ e^2 = \frac{1}{M (N - 1)} \sum_{i=1}^{M} \sum_{j=1}^{N} (m_i^j - u)^2 \]  

are computed. Typical value of \( R(k) \) used in our experiments is \( R(k) = \text{diag}(9, 9, 9, 9) \) with the units as \( mm^2 \).

4.4.3 Performance Comparison

To validate our models, we first investigate the performance of LS-based tracker, purely NG-based tracker, purely EKF-based tracker, and then study the combination of NG and KF (KF-NG) for target tracking.

There are two metrics used for evaluate the performance of our system. One, RMSE, is described in section 7.4. This metric computes the tracking errors for x, y and z directions separately [80]. The other metric is the mean error and standard deviation of the discrepancy, considered as accuracy and precision, between the 3-dimensional displacement measured with the camera based system, for LS, NG, EKF, and KF-NG approaches.

Good correspondence between proposed system and the reference camera based system is shown in Fig. 6.2. Table 4.1 provides numerical comparison of the 3-dimensional displacements in horizontal, vertical and lateral directions by applying different approaches (LS, NG, EKF, and KF-NG). LS and NG approaches do not use the information of previous state and are sensitive to the geometry distribution of anchors, while EKF-based tracker uses prior knowledge of noise characteristics.
to filter out the noise. Furthermore, it is independent of the geometry distribution of anchors. Here, we combine the NG and KF approach to offer superior tracking performance. It is shown in the numerical results that KF-NG based tracker has a better performance to track the foot movement among all these approaches.

Horizontal displacement was obtained with an error of $-0.10 \pm 39.76$ mm (expressed as the mean ± STD of the set of difference with the reference camera based system) for KF-NG tracker and was smaller than other methods. The best absolute accuracy and precision observed was the vertical displacement with $0.62 \pm 7.48$ mm using KF-NG tracker. The net RMSE value in 2D model ($\sqrt{X_{RMSE}^2 + Y_{RMSE}^2}$) of 40.46 mm shows the KF-NG-based method gives a better estimate than the purely EKF- and NG -based methods of 50.96 mm and 63.39 mm, respectively, which are much better than LS-based estimation of 72.76 mm. LS-based approach fails to identify the displacement in lateral direction due to the setup of the 4 anchors in x-y plane. Therefore, there is no degree of freedom in the lateral direction. In other words, the result indicates that Kalman filters are robust and not sensitive to the geometry distribution of anchors. In addition, KF-NG-based tracker can achieve net RMSE value of 41.79 mm in 3-dimensional space.
Table 4.1: Errors of foot movement in 3-dimensional space compared with camera based motion capture system using different approaches

<table>
<thead>
<tr>
<th></th>
<th>Mean (mm)</th>
<th>STD (mm)</th>
<th>RMSE (mm)</th>
<th>Net RMSE (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>7.89</td>
<td>66.37</td>
<td>66.83</td>
<td></td>
</tr>
<tr>
<td>NG</td>
<td>1.06</td>
<td>60.99</td>
<td>61.00</td>
<td></td>
</tr>
<tr>
<td>EKF</td>
<td>0.61</td>
<td>49.27</td>
<td>49.26</td>
<td></td>
</tr>
<tr>
<td>KF-NG</td>
<td>-0.10</td>
<td>39.76</td>
<td>39.76</td>
<td></td>
</tr>
<tr>
<td><strong>Vertical</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>-9.58</td>
<td>27.11</td>
<td>28.76</td>
<td>72.76</td>
</tr>
<tr>
<td>NG</td>
<td>0.56</td>
<td>17.24</td>
<td>17.25</td>
<td>63.39</td>
</tr>
<tr>
<td>EKF</td>
<td>0.52</td>
<td>13.01</td>
<td>13.02</td>
<td>50.96</td>
</tr>
<tr>
<td>KF-NG</td>
<td>0.62</td>
<td>7.48</td>
<td>7.50</td>
<td>40.46</td>
</tr>
<tr>
<td><strong>Lateral</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>NG</td>
<td>-1.86</td>
<td>10.65</td>
<td>10.80</td>
<td>64.3</td>
</tr>
<tr>
<td>EKF</td>
<td>1.29</td>
<td>9.75</td>
<td>9.83</td>
<td>51.90</td>
</tr>
<tr>
<td>KF-NG</td>
<td>-1.82</td>
<td>10.30</td>
<td>10.46</td>
<td>41.79</td>
</tr>
</tbody>
</table>
4.4.4 Influence of Walking Speed on Foot Displacement

Table 4.2: Foot movement in 3-dimensional space at different walking speeds

<table>
<thead>
<tr>
<th>Speed (mile/h)</th>
<th>Slow (1.0)</th>
<th>Normal (2.0)</th>
<th>Fast (3.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NG</td>
<td>1.01</td>
<td>-0.71</td>
<td>2.99</td>
</tr>
<tr>
<td>EKF</td>
<td>0.86</td>
<td>2.39</td>
<td>0.32</td>
</tr>
<tr>
<td>KF-NG</td>
<td>0.13</td>
<td>0.93</td>
<td>0.23</td>
</tr>
<tr>
<td>Vertical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NG</td>
<td>-0.30</td>
<td>0.84</td>
<td>-0.22</td>
</tr>
<tr>
<td>EKF</td>
<td>0.31</td>
<td>0.78</td>
<td>0.71</td>
</tr>
<tr>
<td>KF-NG</td>
<td>0.27</td>
<td>0.35</td>
<td>1.03</td>
</tr>
<tr>
<td>Lateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NG</td>
<td>-0.99</td>
<td>-1.52</td>
<td>-5.49</td>
</tr>
<tr>
<td>EKF</td>
<td>-0.18</td>
<td>-1.53</td>
<td>2.79</td>
</tr>
<tr>
<td>KF-NG</td>
<td>-1.00</td>
<td>0.38</td>
<td>-0.82</td>
</tr>
<tr>
<td>Net RMSE (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NG</td>
<td>50.39</td>
<td>62.10</td>
<td>88.23</td>
</tr>
<tr>
<td>EKF</td>
<td>43.64</td>
<td>49.89</td>
<td>62.72</td>
</tr>
<tr>
<td>KF-NG</td>
<td>41.19</td>
<td>41.52</td>
<td>42.99</td>
</tr>
</tbody>
</table>

The subjects are instructed to vary the speed of walking on treadmill at slow, normal, and fast speeds. Table 4.2 shows the mean and STD value of the difference between foot trajectory estimated using ultrasonic system and reference camera based system. The influence of speed on the 3-dimensional measurement of foot trajectory was found to be insignificant for Kalman filter, whereas it was significant for NG-based approach. Overall, the STD and net RMSE were found to be smaller with slight increase in our proposed KF-NG method at all walking speeds. This can be interpreted as lower temporal resolution at higher speed. The results show the proposed KF-NG method is more robust than purely EKF- and NG-based approaches.

4.5 Discussion and Conclusion

We use state-space methods together with Newton-Gauss method to estimate the 3-dimensional displacements of foot during walking using one ultrasonic transmitter
and four receivers. To estimate the 3-dimensional displacements of foot, we use the combination of Newton-Gauss method and KF which is more computationally efficient compared to other filters.

Some of the estimation errors might be attributed to the vibrations of reflective markers or sensors mounted on body, especially at high walking speed. Marker occlusion is another factor that affects the results of camera based motion capture system. The recording of the marker will be discarded when a significant percentage of markers have not been detected by more than 3 cameras. When one of the three markers is nonvisible, the position of such marker will be estimated from the other two markers by interpolation.

In our experiments, subjects are instructed to walk over several minutes period with slow, normal and fast speeds. Other researchers [53, 82] have also investigated foot displacement using inertial sensors (accelerometers, gyroscopes or both). Their experiments were conducted under slow movements or a limited number of consecutive strides to eliminate the error accumulation over time, since the displacements are estimated by either integrating the velocities or double integrating the measured accelerations. However, our proposed system does not have such limitations and does not have error accumulation even for prolonged measurement durations.

In summary, this study proposes a novel measurement system using wearable ultrasonic sensor to measure the foot displacement continuously during walking in 3-dimensional space. To evaluate the performance of the proposed system, the 3-dimensional foot displacement was measured and validated against the reference camera based motion capture system. The experiments have been conducted at different walking velocities with several healthy subjects. These experiments demonstrate that the results from the proposed ultrasonic measurement system have high correspondence with the results from camera based motion capture system over long walking period. Additionally, the proposed system is easy to wear and to use. It does not restrict the movement of patients or subjects with bulky cables.
Chapter 5

Estimation of Spatial-Temporal Gait Parameters Using Estimated Foot Trajectory

In Chapter 4, the use of ultrasonic sensors for measuring the 3-dimensional foot trajectory is described. The use of these sensors offers a practical and low cost method to monitor human movement objectively. The 3-dimensional foot displacements are estimated using different approaches, such as LS approach, purely NG method, purely EKF, and the combination of NG and KF (KF-NG). This chapter describes the determination of some gait parameters including stride length, stride duration, stride velocity, stride cadence, and stride symmetry from the 3-dimensional foot displacements. The performance of the developed wearable wireless system implementing the gait parameters estimation is evaluated experimentally under a set of common benchmarks, namely mean difference, standard deviation, and RMSE.

The significance of spatial-temporal gait parameters measurement has been addressed in many research papers [82, 83, 84]. The quantitative analysis of such gait parameters can be helpful to diagnose impairments in balance control [23], to monitor the progress in rehabilitation [85], and to predict the risk of falling [44, 86].
Such parameters include stride length, walking velocity, stride cadence, stride duration and asymmetry of stride. In particular, stride asymmetry has been shown to be more indicative of the underlying impairments and walking stability [87, 88]. Therefore, having instruments, which are capable of measuring these gait parameters about patients’ walking ability, is useful in many clinical applications [89].

We first describes the idea of analyzing gait data by autocorrelation procedure in section 5.1. This is followed by the estimation of stride regularity and symmetry in section 5.2. Then, the estimation of gait parameters is provided in section 5.3. Section 5.4 investigates the performance of the ultrasonic motion capture system by comparing with camera based motion capture system during different walking speeds. Finally, discussion and conclusion are made in section 5.5.

## 5.1 Autocorrelation Procedure

The idea of analyzing gait data by autocorrelation procedure is first proposed by Barrey et al.[90] and Auvinet et al.[91]. Then, the difference between biased and unbiased autocorrelation procedure for gait data analysis has been discussed by Moe et al.[88]. Here we summarize the autocorrelation procedure as follows.

Autocorrelation coefficient shows the degree of similarity between the given observations $a_i$ ($i = 1, 2, ..., N$) as a function of the time lag over successive time intervals, as given by:

\[
A = \sum_{i=1}^{N-m} a_i a_{i+m}
\]  

(5.1)

where $m$ is the phase shift in the number of observations. The autocorrelation coefficients of a periodical signal will produce peak values for lag time equivalent to the cycle of the signal, which is the stride durations. Therefore, visual assessment of autocorrelation from the time series plot can be used to inspect the structure of a cyclic component.

As discussed in [88], either biased or unbiased autocorrelation coefficient can be computed for gait data analysis, but biased autocorrelation is not suitable for comparing
autocorrelation coefficient over different time lags. The biased autocorrelation is the result of the raw autocorrelation coefficient $A$ divided by the number of the observations in equation (5.1):

$$A_{biased} = \frac{1}{N} \sum_{i=1}^{N-m} a_i a_{i+m}$$  \hspace{1cm} (5.2)

From the equation (5.2), the denominator $N$ is the number of samples in observation $a_i$, and is independent of the time lag $m$. It means that the number of samples in the numerator will decrease with the increasing value of time lag $m$, and then the autocorrelation coefficient will attenuate. However, this is not the case in unbiased autocorrelation estimator, expressed as:

$$A_{unbiased} = \frac{1}{N - m} \sum_{i=1}^{N-m} a_i a_{i+m}$$  \hspace{1cm} (5.3)

Since the number of terms in the numerator $N - m$ is always equal to the value of denominator, there is no noticeable attenuation in unbiased estimator.

Figure 5.1 shows the two different estimators for horizontal displacement during normal gait.
treadmill walking. The biased estimator shows clear periodicity but with attenuated amplitudes, while the unbiased estimator introduces no obvious attenuation except a deteriorated curve at the tails.

5.2 Estimation of Stride Regularity and Symmetry

Figure 5.2 shows the normalized unbiased autocorrelation of horizontal and vertical foot displacement during treadmill walking. Since the first peak from the zero phase represents a phase shift of one stride duration, the autocorrelation coefficient at the periodic phase shift is defined as the regularity of the stride between neighboring strides, referred as $hR_i$ for horizontal displacement and $vR_i$ for vertical displacement. Therefore, either for horizontal or vertical displacement, closeness of $hR_{i+1}/hR_i$ or $vR_{i+1}/vR_i$ reflects the stride symmetry. Figure 5.3 demonstrates an example of asymmetric gait showing the unbiased autocorrelation sequence of the horizontal and vertical displacements.
5.3 Estimation of Gait parameters

From the estimated foot displacements by the proposed algorithm, the following spatial-temporal gait parameters can be obtained. With respect to the $j$th gait cycle, the estimators of the spatial-temporal gait parameters are as follows.

- Stride Length, $S$:
  \[
  S(j) = 2St(j) \\
  St(j) = \text{Max}(x_j) - \text{Min}(x_j)
  \]  
  where the function $\text{Max}(x)$ and $\text{Min}(x)$ returns the maximum and minimum value of the variable $x$, and $x_j$ is the horizontal displacement in $j$th gait cycle.

- Normalized Stride Length, $NS$:
  \[
  NS(j) = S(j)/n
  \]  
  where $NS$ is defined as the stride length normalized by the number of strides $n$. 

Figure 5.3: Horizontal and vertical unbiased autocorrelation plots of abnormal gait
5.4. EXPERIMENTAL VALIDATION

- Stride Duration, $T$:

\[ T(j) = Index(max(x_{j+1})) - Index(max(x_j)) \]  

(5.6)

where the function $Index(max(x_j))$ returns the location of the maximum value in $x_j$.

- Stride velocity, $V$:

\[ V(j) = S(j)/T(j) \]  

(5.7)

- Normalized Velocity, $NV$:

\[ NV(j) = V(j)/n \]  

(5.8)

where the normalized speed is the speed as percentage of the number of strides $n$.

- Cadence, $C$:

\[ C(j) = 1/T(j) \]  

(5.9)

where the cadence is the number of strides in a second.

5.4 Experimental Validation

5.4.1 Experiment Setup

The proposed method was tested on 10 healthy subjects (age 25.7 ± 1.4 years, height 171.4 ± 6.5 cm, and weight 62.8 ± 5.6 kg) walking 5 minutes on the treadmill at slow, normal, and fast walking speeds, whose results are presented in this Chapter. The subjects were recruited among students of Nanyang Technological University and none of them had a history of pathological gait disorders. To provide a more systematic validation, we conducted the experiments in a motion analysis lab with
eight high speed cameras (Motion Analysis Eagle System, Santa Rosa, CA, USA) in the School of Mechanical and Aerospace Engineering at Nanyang Technological University. The Motion Analysis Eagle System consists of Eagle Digital Cameras and Cortex software, which captures complex 3D motion with extreme accuracy. System calibrations of the reference system should be done at both static (with 4-point calibration L-frame) and dynamic process (with 3-point calibration wand) to ensure an acceptable accuracy of the reference system. In our experiments, the accuracy of the reference system is $0.43 \pm 0.18$ mm (Average ± Standard deviation). All the experiment setup has been described in Chapter 4.

5.4.2 Processing of Measured Data

In order to compare the estimated spatial-temporal gait parameters at each recorded gait cycle, the foot trajectory estimate with proposed ultrasonic sensors was temporally delayed to match the trajectory estimated by the camera reference system, by finding the maximum values of cross-correlation between these two trajectories. To quantify the performance of the proposed system against the camera reference system, the mean and standard deviation (std) were calculated on the data sets of difference, as well as the Root Mean Square Error (RMSE). This is followed by using the analysis of variance (ANOVA) to test differences in means of ten subjects for statistical significance. Finally, walking speed was estimated using the proposed ultrasonic sensor configuration to check significant changes over different speeds. Two-sample t-tests were performed on the walking velocity and the extracted gait parameters to assess the significance of change in these gait parameters with speed, and thus investigate the effect of walking velocity on the difference between proposed and reference system in gait parameters estimation.

5.4.2.1 Performance Comparison

The mean and standard deviation in stride length, stride duration, and stride velocity estimation between the proposed and reference system together with RMSE
value are reported in Table 5.1.5.2, and 5.3 for all subjects walking at normal speed. On average, across all subjects, the estimates of stride length from the proposed method were 0.001m less than the reference measurements. The overall RMSE value is about 0.027m which is 2.3% of the mean estimated stride length of the reference system. The mean and standard deviation of stride duration at normal walking speed is reported as 1.18 ± 0.02s by the reference system and 1.18 ± 0.04s by the proposed system, which shows no mean difference between the two systems. The average error across all subjects of RMSE of the estimated stride duration is 0.035s with 3% percent error. The mean and standard deviation in the estimation of the stride velocity is reported in table 5.3, which shows that the proposed estimates of stride velocity were slightly overestimated by 0.001 m/s with an RMSE value of 0.036 m/s, occupying 3.6% of the proposed estimates of stride velocity.

Table 5.1: Mean and standard deviation (in meters) of the reference (Ref) and proposed (Pro) system and RMSE in detecting stride length are reported for each subject. Averaged values across the ten subjects are also reported.

<table>
<thead>
<tr>
<th>subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref</td>
<td>1.147</td>
<td>1.071</td>
<td>1.158</td>
<td>1.421</td>
<td>1.117</td>
<td>1.137</td>
<td>1.101</td>
<td>1.276</td>
<td>1.041</td>
<td>1.224</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.147</td>
<td>1.070</td>
<td>1.157</td>
<td>1.420</td>
<td>1.116</td>
<td>1.136</td>
<td>1.100</td>
<td>1.274</td>
<td>1.040</td>
<td>1.223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>std</td>
<td>0.047</td>
<td>0.019</td>
<td>0.019</td>
<td>0.037</td>
<td>0.046</td>
<td>0.056</td>
<td>0.041</td>
<td>0.035</td>
<td>0.055</td>
<td>0.034</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pro</td>
<td>1.147</td>
<td>1.070</td>
<td>1.157</td>
<td>1.420</td>
<td>1.116</td>
<td>1.136</td>
<td>1.100</td>
<td>1.274</td>
<td>1.040</td>
<td>1.223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.147</td>
<td>1.070</td>
<td>1.157</td>
<td>1.420</td>
<td>1.116</td>
<td>1.136</td>
<td>1.100</td>
<td>1.274</td>
<td>1.040</td>
<td>1.223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>std</td>
<td>0.057</td>
<td>0.025</td>
<td>0.027</td>
<td>0.057</td>
<td>0.050</td>
<td>0.064</td>
<td>0.034</td>
<td>0.038</td>
<td>0.067</td>
<td>0.035</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.029</td>
<td>0.022</td>
<td>0.013</td>
<td>0.033</td>
<td>0.029</td>
<td>0.024</td>
<td>0.020</td>
<td>0.029</td>
<td>0.042</td>
<td>0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.027</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Mean and standard deviation (in seconds) of the reference (Ref) and proposed (Pro) system and RMSE in detecting stride duration are reported for each subject. Averaged values across the ten subjects are also reported.

<table>
<thead>
<tr>
<th>subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref</td>
<td>1.237</td>
<td>1.109</td>
<td>1.134</td>
<td>1.344</td>
<td>1.160</td>
<td>1.155</td>
<td>1.114</td>
<td>1.309</td>
<td>1.050</td>
<td>1.192</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.236</td>
<td>1.108</td>
<td>1.134</td>
<td>1.341</td>
<td>1.161</td>
<td>1.155</td>
<td>1.114</td>
<td>1.308</td>
<td>1.051</td>
<td>1.190</td>
<td></td>
<td></td>
</tr>
<tr>
<td>std</td>
<td>0.031</td>
<td>0.014</td>
<td>0.014</td>
<td>0.015</td>
<td>0.021</td>
<td>0.025</td>
<td>0.020</td>
<td>0.020</td>
<td>0.030</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pro</td>
<td>1.236</td>
<td>1.108</td>
<td>1.134</td>
<td>1.341</td>
<td>1.161</td>
<td>1.155</td>
<td>1.114</td>
<td>1.308</td>
<td>1.051</td>
<td>1.190</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.236</td>
<td>1.108</td>
<td>1.134</td>
<td>1.341</td>
<td>1.161</td>
<td>1.155</td>
<td>1.114</td>
<td>1.308</td>
<td>1.051</td>
<td>1.190</td>
<td></td>
<td></td>
</tr>
<tr>
<td>std</td>
<td>0.053</td>
<td>0.031</td>
<td>0.033</td>
<td>0.024</td>
<td>0.049</td>
<td>0.039</td>
<td>0.044</td>
<td>0.045</td>
<td>0.050</td>
<td>0.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.042</td>
<td>0.030</td>
<td>0.027</td>
<td>0.026</td>
<td>0.046</td>
<td>0.029</td>
<td>0.034</td>
<td>0.035</td>
<td>0.033</td>
<td>0.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.035</td>
<td>0.046</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We have elaborated how gait cycle periodicity of foot displacement data can be used to extract stride regularity and symmetry by unbiased autocorrelation procedure in Section 5.2. Table 5.4 and 5.5 shows the mean and standard deviation of the reference and the proposed system together with RMSE values in detecting horizontal
and vertical stride symmetry respectively for each subject. The mean and standard deviation of horizontal stride symmetry are 1.001 ± 0.021 by the reference system and 0.999 ± 0.027 by the proposed system, which shows that the ultrasonic-based estimates of horizontal stride symmetry were underestimated by a negligible error of 0.002. An RMSE of 0.013 with 1.3% percent error is also reported for the estimates of horizontal stride symmetry across all subjects. In the contrary, the ultrasonic-based estimates of vertical stride symmetry were overestimated by 0.007. Here, the RMSE value is 0.034 with percent error 3.5%. In summary, all the numerical results show a clinical acceptable accuracy of the proposed system with an average percent error of 2.7% for all the estimated gait parameters.

Table 5.3: Mean and standard deviation (in meters per second) of the reference (Ref) and proposed (Pro) system and RMSE in detecting stride velocity are reported for each subject. Averaged values across the ten subjects are also reported.

<table>
<thead>
<tr>
<th>subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref</td>
<td>mean</td>
<td>0.927</td>
<td>0.966</td>
<td>1.021</td>
<td>1.057</td>
<td>0.964</td>
<td>0.984</td>
<td>0.989</td>
<td>0.974</td>
<td>0.992</td>
<td>1.027</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.032</td>
<td>0.017</td>
<td>0.011</td>
<td>0.025</td>
<td>0.039</td>
<td>0.043</td>
<td>0.043</td>
<td>0.028</td>
<td>0.049</td>
<td>0.034</td>
<td>0.032</td>
</tr>
<tr>
<td>Pro</td>
<td>mean</td>
<td>0.928</td>
<td>0.966</td>
<td>1.021</td>
<td>1.060</td>
<td>0.962</td>
<td>0.984</td>
<td>0.989</td>
<td>0.975</td>
<td>0.991</td>
<td>1.029</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.042</td>
<td>0.032</td>
<td>0.027</td>
<td>0.042</td>
<td>0.049</td>
<td>0.053</td>
<td>0.046</td>
<td>0.038</td>
<td>0.064</td>
<td>0.053</td>
<td>0.045</td>
</tr>
<tr>
<td>RMSE</td>
<td></td>
<td>0.038</td>
<td>0.033</td>
<td>0.024</td>
<td>0.021</td>
<td>0.049</td>
<td>0.032</td>
<td>0.035</td>
<td>0.031</td>
<td>0.047</td>
<td>0.053</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Table 5.4: Mean and standard deviation of the reference (Ref) and proposed (Pro) system and RMSE in detecting horizontal stride symmetry (hS) are reported for each subject. Averaged values across the ten subjects are also reported.

<table>
<thead>
<tr>
<th>subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref</td>
<td>mean</td>
<td>1.002</td>
<td>1.003</td>
<td>1.000</td>
<td>1.012</td>
<td>1.002</td>
<td>0.996</td>
<td>1.000</td>
<td>0.995</td>
<td>0.999</td>
<td>1.000</td>
<td>1.001</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.021</td>
<td>0.010</td>
<td>0.007</td>
<td>0.021</td>
<td>0.016</td>
<td>0.033</td>
<td>0.006</td>
<td>0.014</td>
<td>0.067</td>
<td>0.009</td>
<td>0.021</td>
</tr>
<tr>
<td>Pro</td>
<td>mean</td>
<td>1.004</td>
<td>1.001</td>
<td>1.001</td>
<td>1.011</td>
<td>1.001</td>
<td>0.991</td>
<td>1.000</td>
<td>0.996</td>
<td>0.989</td>
<td>1.000</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.027</td>
<td>0.007</td>
<td>0.008</td>
<td>0.023</td>
<td>0.022</td>
<td>0.049</td>
<td>0.013</td>
<td>0.016</td>
<td>0.089</td>
<td>0.018</td>
<td>0.027</td>
</tr>
<tr>
<td>RMSE</td>
<td></td>
<td>0.010</td>
<td>0.004</td>
<td>0.004</td>
<td>0.013</td>
<td>0.011</td>
<td>0.021</td>
<td>0.009</td>
<td>0.008</td>
<td>0.038</td>
<td>0.010</td>
<td>0.013</td>
</tr>
</tbody>
</table>

5.4.2.2 Statistical Analysis

In this part, the ANOVA has been performed to test differences in means (for ten subjects) for statistical significance. We base this test on a comparison of the variance due to the between-groups variability (called Mean Square Effect, or $MS_{effect}$)
with the within-group variability (called Mean Square Error, or \( MS_{\text{error}} \)). Before applying ANOVA, whether the distribution of the data is normal or not should be checked. Results are reported in Table 5.6 and Figure 5.5. In Table 5.6, \( H = 0 \) indicates that the null hypothesis ("mean is zero") cannot be rejected at the 5% significance level. \( p \)-value is the probability of observing the given result by chance if the null hypothesis is true. Large value of \( p \) shows the validity of the null hypothesis. As in Table 5.6, not only all values of \( H \) are equal to zero, and values of \( p \) are equal to one, but also the means of estimates are located in the 95% confidence interval. Therefore, the estimated parameters are normally distributed.

Under the null hypothesis (that there are no mean differences among subjects), we compare the \( MS_{\text{effect}} \) and \( MS_{\text{error}} \) via the F-test, which tests whether the ratio of the two variance estimates is significantly greater than 1. Otherwise, we will accept the null hypothesis of no differences between means, i.e. the means (in the population) are not statistically different from each other. Figure 5.6 shows the boxplots of stride length, stride duration, stride velocity, horizontal stride symmetry and vertical stride symmetry for each subject. And the analysis of variance is

Table 5.5: Mean and standard deviation of the reference (Ref) and proposed (Pro) system and RMSE in detecting vertical stride symmetry (vS) are reported for each subject. Averaged values across the ten subjects are also reported.

<table>
<thead>
<tr>
<th>subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref</td>
<td>mean</td>
<td>1.012</td>
<td>1.000</td>
<td>0.995</td>
<td>0.999</td>
<td>1.002</td>
<td>1.002</td>
<td>1.004</td>
<td>1.000</td>
<td>1.007</td>
<td>0.995</td>
<td>1.012</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.045</td>
<td>0.009</td>
<td>0.019</td>
<td>0.181</td>
<td>0.016</td>
<td>0.051</td>
<td>0.010</td>
<td>0.018</td>
<td>0.065</td>
<td>0.043</td>
<td>0.181</td>
</tr>
<tr>
<td>Pro</td>
<td>mean</td>
<td>1.009</td>
<td>1.002</td>
<td>0.997</td>
<td>0.996</td>
<td>1.003</td>
<td>1.002</td>
<td>1.012</td>
<td>0.997</td>
<td>1.011</td>
<td>1.002</td>
<td>1.012</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.042</td>
<td>0.038</td>
<td>0.017</td>
<td>0.069</td>
<td>0.038</td>
<td>0.022</td>
<td>0.044</td>
<td>0.088</td>
<td>0.030</td>
<td>0.043</td>
<td>0.088</td>
</tr>
<tr>
<td>RMSE</td>
<td>mean</td>
<td>0.017</td>
<td>0.036</td>
<td>0.023</td>
<td>0.079</td>
<td>0.028</td>
<td>0.031</td>
<td>0.020</td>
<td>0.039</td>
<td>0.048</td>
<td>0.034</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td>0.011</td>
<td>0.013</td>
<td>0.012</td>
<td>0.023</td>
<td>0.014</td>
<td>0.016</td>
<td>0.015</td>
<td>0.017</td>
<td>0.019</td>
<td>0.015</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 5.6: Normality test of gait parameters

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std</th>
<th>( H )</th>
<th>( p )</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>S (m)</td>
<td>1.147</td>
<td>0.093</td>
<td>1.000</td>
<td>1.137</td>
<td>1.158</td>
</tr>
<tr>
<td>T (s)</td>
<td>1.164</td>
<td>0.080</td>
<td>0.000</td>
<td>1.155</td>
<td>1.173</td>
</tr>
<tr>
<td>V (m/s)</td>
<td>0.986</td>
<td>0.046</td>
<td>0.000</td>
<td>0.980</td>
<td>0.991</td>
</tr>
<tr>
<td>hS</td>
<td>1.000</td>
<td>0.028</td>
<td>0.000</td>
<td>0.997</td>
<td>1.003</td>
</tr>
<tr>
<td>vS</td>
<td>0.999</td>
<td>0.048</td>
<td>0.000</td>
<td>0.994</td>
<td>1.005</td>
</tr>
</tbody>
</table>
summarized in Table 5.7. As shown in Table 5.7, for all estimated gait parameters, the small value of between-groups Sum of Squares likely indicates no differences among the subjects. Additionally, the values of F are less than 1, which indicates that the means of all gait parameters are not statistically different.

Table 5.7: Summary of the analysis of variance of stride length, stride duration, stride velocity, horizontal stride symmetry, and vertical stride symmetry.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>0.01804</td>
<td>9</td>
<td>0.002</td>
<td>0.14</td>
<td>0.9984</td>
</tr>
<tr>
<td>Within groups</td>
<td>1.29563</td>
<td>90</td>
<td>0.0144</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.31367</td>
<td>99</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>0.00446</td>
<td>9</td>
<td>0.0005</td>
<td>0.05</td>
<td>1</td>
</tr>
<tr>
<td>Within groups</td>
<td>0.81343</td>
<td>90</td>
<td>0.00904</td>
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<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.81789</td>
<td>99</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
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<th>Degrees of Freedom</th>
<th>Mean Square</th>
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<th>Degrees of Freedom</th>
<th>Mean Square</th>
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</tbody>
</table>

5.4.2.3 The Effect of Walking Speed on the Measurement of Gait parameters

Table 6.1 provides the numerical results of estimated gait parameters by the proposed system compared with those obtained from the reference system using the pair t-test. Significant difference was assumed when the null hypothesis can be rejected.
at p-value smaller than 0.05. The walking speed, on average, across all subjects was significantly different ($p < 0.001$ for the two measurement system) among slow ($0.54 \pm 0.02$ m/s), normal ($0.99 \pm 0.04$ m/s), and fast ($1.40 \pm 0.04$ m/s) speed. The influence of walking speed on all spatial-temporal gait parameters was tested by the mean and standard deviation values for the proposed and reference system.

![Figure 5.4](image.png)

**Figure 5.4:** The effect of walking speeds on spatial-temporal gait parameters at slow, normal, and fast walking speed. * indicates the significant differences with normal speed ($p < 0.05$)

The measurement errors of estimated $S$, $NS$, $NV$, $C$, and $vS$ were not affected significantly by the changes in walking velocity ($p > 0.05$). Particularly, there is no difference in cadence estimation between the proposed and reference system. The influence of speed on the measurement errors of stride duration $T$ was found to be significantly higher ($p < 0.05$) at fast speed, but it was not significant for the $V$ and $hS$. This can be interpreted as the lower temporal resolution at higher walking speed. Figure 5.4 shows significant changes in $T$ and $C$, but there is no significant changes in other parameters. Although the means of both horizontal stride symmetry and vertical stride symmetry are not statistically significant, the largest variations at slow speeds were observed. Therefore, the stride symmetry can be used as a warning sign of walking disorders.
5.4. EXPERIMENTAL VALIDATION

Table 5.8: Foot parameters at different walking velocities, * indicates the significant differences with normal speed (p<0.05).

<table>
<thead>
<tr>
<th>Speed</th>
<th>Slow</th>
<th>Normal</th>
<th>Fast</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
<td>p</td>
</tr>
<tr>
<td>S (m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref</td>
<td>0.884</td>
<td>0.095</td>
<td>0.000*</td>
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<tr>
<td>Pro</td>
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<td>0.095</td>
<td>0.000*</td>
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<tr>
<td>RMSE</td>
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<td>0.012</td>
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<tr>
<td>Pro</td>
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<td>0.329</td>
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<td>RMSE</td>
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<td>T (s)</td>
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<td>Ref</td>
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<td>0.192</td>
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<td>0.000*</td>
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<td>RMSE</td>
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<td>0.592</td>
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<td>0.023</td>
<td>0.000*</td>
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<td>0.692</td>
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<td>0.040</td>
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<tr>
<td>RMSE</td>
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<td>0.001</td>
<td>0.693</td>
</tr>
<tr>
<td>C (stride/s)</td>
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<td>0.614</td>
<td>0.073</td>
<td>0.000*</td>
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<tr>
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<td>0.073</td>
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<td>NaN</td>
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<tr>
<td>hS</td>
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<tr>
<td>RMSE</td>
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<td>0.111</td>
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</table>
5.5 Discussion and Conclusion

In this chapter, a low-cost ultrasonic motion analysis system using an ultrasonic transmitter and four receivers to track the foot displacement in 3D space is developed. The proposed motion analysis system has been validated against camera based system with 10 healthy subjects, and shown to have accurate estimates of some spatial-temporal gait parameters including stride length with RMSE value of 0.027m (2.3%), stride duration with RMSE value of 0.035s (3%), stride velocity with RMSE value of 0.036m/s (3.6%), horizontal stride symmetry with RMSE value of 0.013 (1.3%) and vertical stride symmetry with RMSE value of 0.034m (3.5%). We have further evaluated the influence of walking speed on these gait parameters by paired t-test.

The proposed system includes some ultrasonic sensors and micro-controllers, estimated today at about a cost of $100, which is inexpensive compared to current commercial camera based system. With the rapid development of technology, the performance of these sensors will continue to improve with even lower price. Therefore, low-cost in-home monitoring system for clinical applications is possible.

As the work stated here is a first step to evaluate the feasibility of the proposed ultrasonic system, only ten healthy subjects participated in the experiments and were instructed to walk 5 minutes on treadmill at different speeds. The walking experiments were chosen on treadmill due to the limited measurement volume of the reference camera based system. In addition, we can get a cyclic signal on horizontal displacement to analyze the stride symmetry. Although the proposed ultrasonic motion analysis system also has such limitations, the maximum propagation distance of the ultrasonic signal used in our system is to 20m, which is large enough for indoor applications.

Although the positive results showed the feasibility of applying such a system for in-home monitoring, there is an issue to be addressed in further research. That is how to deal with the multipath propagation. All the experiments in this study are
conducted under line-of-sight condition, where the ultrasonic transmitter faces all the receivers without any obstacles between them. Therefore, for 3D displacements, according to spherical positioning technique, minimum 4 anchors with known positions are required. The method used in our experiment to mitigate the multipath propagation is by setting an inhibit time, i.e. the ultrasound detector will be disabled within the inhibit time to detect an ultrasound signal. Then, it will be enabled after this inhibit duration. Another possible solution is that we can use more receivers, which can not only account for multipath propagation, but also increase the measurement volume and accuracy of the proposed system [77].

Long-term monitoring is expected to be more challenging as demonstrated in some studies [18, 4]. In [18, 4], foot clearance measurement using inertial sensors is proposed and investigated. The displacement estimation requires double integration of measured accelerations from inertial sensors, which involves error accumulation over long time monitoring. Even though the growth uncertainty that arises from the integration of acceleration error can be mitigated by periodic corrections like ZUPT, the prerequisite is that the initial and/or terminal contact should be detected correctly, but it maybe difficult in some type of abnormal gait. Although not specifically studied under long term monitoring, the proposed system does not have significant error accumulation for a 5 minute walk.

In summary, we use a low-cost ultrasonic motion analysis system to extract spatial-temporal gait parameters, and test the feasibility of the system against a reference camera based system. The positive results demonstrate a great potential in using this low-cost system for clinical applications such as rehabilitation, gait analysis, and sports. For further work, experiments conducted with patients in collaboration with a hospital are being planned using our system.
Figure 5.5: Histogrmes of all estimates of stride length, stride duration, stride velocity, horizontal stride symmetry and vertical stride symmetry.
Figure 5.6: Boxplots of stride length, stride duration, stride velocity, horizontal stride symmetry and vertical stride symmetry for each subject.
Chapter 6

Assessment of Foot Parameters for Human Gait Phase Detection Using Estimated Foot Trajectory

In Chapter 4, several algorithms were proposed using spherical positioning technique to track the 3-dimensional foot displacements using wearable wireless ultrasonic sensor network. Based on the estimated 3-dimensional foot displacements, a methodology has been developed to extract spatial-temporal gait parameters including stride length, stride duration, stride velocity, stride cadence, and stride symmetry in Chapter 5. In this chapter, we developed a gait phase detection algorithm that reliably measure the transition periods during different phases.

The remainder of this chapter describes the gait phase detection algorithm in section 6.1. This is followed by Section 6.2 which performs the experimental validation by comparing with the gold standard camera based motion capture system. Finally, discussion and conclusion are given in Section 6.3.
6.1 Gait Phase Detection

The gait phase detection system divides the gait cycle into stance and swing phase, as shown in Fig. 6.1. Stance phase in our system consists of the instant where the heel rises from the ground, defined as Heel Off (HO). Therefore, \{HS, HO, TO\} are stacked as the temporal events of stance. Swing phase encapsulates the instant where the heel achieves its maximum height, referred as Mid Swing (MS). \{TO, MS, HS\} are defined as the temporal events of swing.

The following procedure to obtain information about different gait events is performed using the foot displacements by ultrasonic sensor. The symbols, \(x\) and \(y\), represent the displacements of heel in sagittal plane, since the displacement in lateral direction is negligible and discarded in our gait phase detection algorithm.

We discuss the gait phase detection algorithm with respect to the \(jth\) gait cycle. Assume the initial posture of subjects is HS, which is the beginning of stance phase or the ending of previous swing phase. In the HS state, the algorithm awaits the coming HO phase. The duration from the HS state to HO phase is defined by \(T1(j) = HS(j) - HO(j)\). It is assumed that the transition occurs when the condition satisfies

\[
y_j \geq TH_{HO}
\]

(6.1)

\(TH_{HO}\) is a given threshold foot clearance (vertical displacement) of heel. The time
6.1. GAIT PHASE DETECTION

Instant $T_{HO}(j)$ is defined by:

$$T_{HO}(j) = \text{Index}(y_j > TH_{HO}) \quad (6.2)$$

When the heel is raised, the body weight moves ahead of the forefoot, thus pushing the subject forward. The next coming state is $TO$, when the whole foot leaves the ground and starts to move forward, thus increasing the horizontal displacement. The transition is represented by $T2(j) = HO(j) - TO(j)$ at time instant $T_{TO}(j)$ when the foot displacement starts to increase.

$$T_{TO}(j) = \text{Index}(\min\{x_j\}) \quad (6.3)$$

It is assumed that the transition $T3(j) = TO(j) - MS(j)$, referred as early swing, occurs at the time instant $T_{MS}(j)$ when the heel clearance reaches the maximum value. During the transition, the forward momentum provides push-off action.

$$T_{MS}(j) = \text{Index}(\max\{y_j\}) \quad (6.4)$$

In the early swing phase, the algorithm awaits the transition to the next HS state. The transition $T4(j) = MS(j) - HS(j + 1)$, referred as late swing, occurs when the foot horizontal displacement achieves its maximum value and the heel touches the ground again. Therefore, the time instant of HS is defined as:

$$T_{HS}(j + 1) = \text{Index}(\max\{x_j\}) \quad (6.5)$$

With respect to the $j$-th gait cycle, the estimator of the temporal gait parameters are as follows.

- Cycle time, $CT$:

$$CT(j) = T_{HS}(j + 1) - T_{HS}(j) \quad (6.6)$$
• Duration of stance phase, $T_{\text{stance}}$:

$$T_{\text{stance}}(j) = T_{\text{TO}}(j) - T_{\text{HS}}(j) \quad (6.7)$$

• Duration of swing phase, $T_{\text{swing}}$:

$$T_{\text{swing}}(j) = CT(j) - T_{\text{stance}}(j) \quad (6.8)$$

• Relative swing, $RS$:

$$RS(j) = T_{\text{swing}}(j)/CT(j) \quad (6.9)$$

### 6.2 Results

An experimental study was conducted with 10 healthy adults (mean age 25.7±1.4 years, mean weight 62.8 ± 5.6 kg, mean height 171.4 ± 6.5 cm) and two subjects (mean age 25.5± 0.5 years, mean weight 68 ± 7.0 kg, mean height 177.0 ± 7.0 cm) with athletic ankle injuries are also included. To quantify the performance of the proposed method, different experiments were carried out under different scenarios using the the hardware platform described in Chapter 4 [81]. In the first experiment, the accuracy of the proposed gait phase detection system was benchmarked by the optical motion capture system. The second experiment was used to test the influence of walking velocities on accuracy and gait temporal parameters. Finally, the experiment was conducted to extract more information about gait characteristics under wide walking speed range.

### 6.2.1 Calibration of Motion Capture System

All experiments were conducted in a motion analysis lab (Motion Analysis Eagle System, Santa Rosa, CA, USA) with eight high speed cameras in the School of Mechanical and Aerospace Engineering at Nanyang Technological University. The Motion Analysis Eagle System consists of Eagle Digital Cameras and Cortex software,
Figure 6.2: Foot trajectory during walking in 3-dimensional space (a) Horizontal displacement, (b) Vertical displacement (foot clearance), and (c) Lateral displacement from ultrasonic sensor are compared with the camera based motion capture system. Camera refers to the reference system (Ref), and ultrasound refers to the proposed system (Pro).

which captures complex 3D motion with extreme accuracy. System calibrations of the reference system should be done at both static (with 4-point calibration L-frame) and dynamic process (with 3-point calibration wand) to ensure an acceptable accuracy of the reference system. In our experiments, the accuracy of the reference system is $0.43 \pm 0.18 \text{ mm}$ (Average $\pm$ Standard deviation).

6.2.2 Experiment I: Comparison of the proposed system with motion capture system

In the first experiment, all healthy subjects were asked to do a 2 minutes walking at self-selected speed on a treadmill with three reflective markers attached to their shank, heel, and toe. The cameras were used to record the trajectories of the three reflective markers. Meanwhile, the 3-dimension displacement of ultrasonic sensor were obtained based on the algorithm described in Chapter 4. Then, the gait phases can be identified by the gait phase detection algorithm in Section 6.1.

A representative result of the 3-dimensional displacement of foot recorded by both reference (Camera) and proposed (Ultrasound) systems during the walking of one
subject is shown in Fig. 6.2. Fig. 6.3 plots the horizontal and vertical placements of both Ref (Reference) and Pro (Proposed) system, which shows that all points lie along the line of equality. This indicates the high degree of agreement between the two systems [92]. The correlation coefficient between the two systems is calculated as 0.97 (p<0.001) for horizontal displacement and 0.97 (p<0.001) for vertical displacement, where the p-value is the probability that the measurements by the two methods are not linearly related. As the probability is very small, we can safely conclude that the correlations of displacement measurements by the two systems are significant.

Fig. 6.4 shows the Bland & Altman plot, which was used to quantitatively measure
Figure 6.4: Bland-Altman plot with mean and difference between two values estimated by ultrasonic sensor and reference system. Limits of agreement is specified as average difference ± 2 standard deviation of the difference.
Figure 6.5: Based on both proposed and reference measurements of heel trajectories in the sagittal plane, we extracted a reference gait phase signal, which is used to benchmark the performance of proposed gait phase detection algorithm.

The agreement between the two systems, for horizontal and vertical displacements and the limit of the 95% confidence interval (±2 standard deviation (std)) around perfect agreement [92]. The limits of agreement are small enough for us to be confident to carry out further gait analysis. The gait events detected by both Ref and Pro system are shown in Fig. 6.5. To analyze the accuracy of the gait detection, we have conducted more detailed experiments as explained in the following part.

6.2.3 Experiment II: Three walking speeds

The purpose of this experiment is to test the performance of the gait event detection system during different walking speeds, and to evaluate the influence of walking speeds on gait temporal parameters. All healthy subjects were instructed to walk at three speeds (slow (0.5 m/s), normal (1 m/s), and fast (1.5 m/s)) for a duration of 2 minutes on treadmill.

Table 6.1 provides numerical comparison for instructed walking speeds. The variations of the gait cycle, stance, swing, T1, T2, T3, and T4 with speed are investigated by the mean and std value. The errors between Ref and Pro systems are also computed. The p-value here defines the probability of the null hypothesis that the two samples have same mean value, what means that if p-value here is small enough
6.2. RESULTS

(common significance level is 0.02), the means of two samples are significantly different.

First, we focus on the influence of walking speed on the measurement errors for estimating gait events temporal parameters. Since all the p-values of the errors for the estimated gait events parameters are larger than 0.02, we conclude that the walking velocity does not affect the measurement errors significantly. Overall, the mean errors are found to be small for all temporal parameters at slow, normal and fast walking speed, indicating that the proposed ultrasonic system has a capable of being a useful tool for gait analysis.

We next turn our attention to the duration of the different gait phases. We can see from Table 6.1 that the influence of the speed on the durations of stance, swing and T4 is statistically significant since $p < 0.02$. Furthermore, the duration of these phases tend to be shorter as the speed increases, which is consistent with our expectation. By contrast, $p > 0.02$ for T2 and T3, suggesting that the influence of the speed on the duration of T2 and T3 is small. Additionally, we can observe that in comparison with the case of normal speed, the mean durations of T2 and T3 corresponding to the low speed yields a larger difference than those corresponding to the high speed. This can be explained by the fact that the duration of T2 and T3 in the normal speed case is already very short and can not be further decreased greatly with the increase of the speed. On the other hand, the results for T1 are mixed: $p < 0.02$ in the case of low speed, while $p > 0.02$ in the case of high speed.

Fig. 6.6 shows some significant changes in gait cycle, stance and swing and T4, which are decreasing with speed. Although the decreasing tendency is also observed in T1, it is not statistically significant. We can observe gait cycle ranging from 1.7s at slow speed to 1.2s at normal speed and 1.0s at fast speed and stance phase ranging from 1.1s, 0.7s, 0.6s, respectively. Therefore, the relative stance phases are 63% at slow speed, 58% at normal speed, and 57% at fast speed, respectively. There are not enough samples to show a clear relation between the speed and the relative stances, thus driving us to do more experiments with wide speed range as discussed.
6.2. RESULTS

Gait Stance Swing T1 T2 T3 T4
0
0.5
1
1.5
2
Duration [s]

Slow Normal Fast

Figure 6.6: The effect of walking speeds on gait temporal parameters at slow, normal, and fast walking speed. ∗ indicates the significant differences with normal speed (p<0.02)

in the following section.

6.2.4 Experiment III: Wide speed range

The aim of these experiments is to determine the effect of walking speeds on the relative gait events, and validate that whether the proposed system can work under some patients with athletic ankle injuries. Both healthy subjects and subjects who has athletic ankle injuries were instructed to walk on the treadmill. All the ten healthy subjects are required to gradually increase the walking speed from a slow speed (0.2 m/s) to a maximum speed of (1.6 m/s). Since the injured subjects can not walk fast, the maximum speed for them is 1.3 m/s. For each walking task, subjects were asked to walk for a period of 3 mins. The ultrasonic transmitter is attached to the injured foot of both two foot impaired subjects.

We list the results for healthy and injured subjects respectively in Fig. 6.7 and Fig. 6.8. It can be observed that for both healthy and injured subjects, the durations of gait cycle and stance phase decreases with the increase of the speed. However, three major differences between injured and healthy subjects can be gleaned from the results. First, comparing Fig. 6.7(a) with Fig. 6.8(a), we can find that the durations of gait cycle and stance phase for the injured subjects are shorter than
### Table 6.1: Gait phases at different walking velocities

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>Slow (0.5)</th>
<th>Normal (1.0)</th>
<th>Fast (1.5)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
<td>p</td>
</tr>
<tr>
<td>Cycle (s)</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.35</td>
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</tr>
<tr>
<td>Pro</td>
<td>0.13</td>
<td>0.02</td>
<td>0.76</td>
</tr>
<tr>
<td>Error</td>
<td>0.01</td>
<td>0.02</td>
<td>0.53</td>
</tr>
<tr>
<td>T4 (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref</td>
<td>0.39</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Pro</td>
<td>0.48</td>
<td>0.06</td>
<td>0.00</td>
</tr>
<tr>
<td>Error</td>
<td>-0.09</td>
<td>0.04</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Figure 6.7: (a) and (b) are relationships between estimated durations of different gait events and walking speeds for healthy subject. (c) shows the relative gait phases versus wide speed range. At each speed, the data are averaged across all subjects. The regression curve was derived from a second-order polynomial model.
Figure 6.8: (a) and (b) are relationships between estimated durations of different gait events and walking speeds for injured subject. (c) shows the relative gait phases versus wide speed range. At each speed, the data are averaged across all subjects. The regression curve was derived from a second-order polynomial model.
Figure 6.9: (a), (b) and (c) are relationships between relative gait phases and walking speeds with healthy (referred as 'H') and injured (referred as 'I') subjects, respectively. Rt1: Relative T1, Rt2: Relative T2, Rt3: Relative T3, Rt4: Relative T4.
those for the healthy ones. One possible explanation is that the patient is less likely
to use the injured foot while walking, resulting in the shorter duration. Second, for
healthy subjects, the durations of the sub-phases can be typically listed as T1, T4,
T2, and T3, in the order of duration length (see Fig. 6.7(b)), but such relation is not
held for injured subjects (see Fig. 6.8(b)). Finally, as shown in Fig. 6.7(c) and Fig.
6.8(c), with the increase of the stride speed, relative T1 decreases more rapidly for
the injured subjects than for the healthy ones. probably because it is more difficult
to support the body weight using the injured ankle as people walk faster.

Table 6.2: Relative gait events of healthy subjects at different walking speeds

<table>
<thead>
<tr>
<th></th>
<th>Reference Range</th>
<th>Proposed Range</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min (%) max (%)</td>
<td>min (%) max (%)</td>
<td></td>
</tr>
<tr>
<td>Rst</td>
<td>58.1 73.1</td>
<td>58.3 71.5</td>
<td>3.7</td>
</tr>
<tr>
<td>Rsw</td>
<td>26.9 41.9</td>
<td>28.5 41.7</td>
<td>-3.7</td>
</tr>
<tr>
<td>Rt1</td>
<td>27.2 56.4</td>
<td>25.0 55.0</td>
<td>-2.5</td>
</tr>
<tr>
<td>Rt2</td>
<td>16.2 34.1</td>
<td>13.5 33.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Rt3</td>
<td>6.4 10.9</td>
<td>5.8 12.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Rt4</td>
<td>21.1 28.6</td>
<td>25.7 34.6</td>
<td>-4.7</td>
</tr>
</tbody>
</table>

Table 6.3: Relative gait events of injured subjects at different walking speeds

<table>
<thead>
<tr>
<th></th>
<th>Reference Range</th>
<th>Proposed Range</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min (%) max (%)</td>
<td>min (%) max (%)</td>
<td></td>
</tr>
<tr>
<td>Rst</td>
<td>54.8 69.2</td>
<td>56.0 68.6</td>
<td>4.1</td>
</tr>
<tr>
<td>Rsw</td>
<td>30.8 45.2</td>
<td>31.4 44.0</td>
<td>-4.1</td>
</tr>
<tr>
<td>Rt1</td>
<td>21.6 60.8</td>
<td>23.9 61.0</td>
<td>2.1</td>
</tr>
<tr>
<td>Rt2</td>
<td>6.8 33.9</td>
<td>6.8 33.2</td>
<td>2.0</td>
</tr>
<tr>
<td>Rt3</td>
<td>6.2 11.8</td>
<td>6.8 12.2</td>
<td>-1.7</td>
</tr>
<tr>
<td>Rt4</td>
<td>23.5 34.6</td>
<td>23.2 33.0</td>
<td>-2.4</td>
</tr>
</tbody>
</table>

We further depict the relative gait phases as a function of walking speed in Fig. 6.9.
Here, we define the relative gait phase as the ratio between the duration of a certain
gait phase and the duration of the entire gait cycle. For example, relative swing can be computed as in eq. (6.9). By plotting such values with response to the walking speed, we can tell more differences between the injured and healthy subjects. We can see from Fig. 6.9(a) that the difference between relative stance and relative swing for injured subjects is smaller than that for healthy ones, regardless of the speed variation. Fig. 6.9(b) shows that the relative T1 and the relative T2 change more rapidly for the injured subjects as the speed increases. As we mentioned before, the patient are more likely to use the forefoot to support their body than the injured ankle, when the speed becomes higher. Fig. 6.9(c) further demonstrates that the relative T4 of the injured subjects is slightly larger. This is because the relative swing phase for injured subjects is larger than that for healthy subjects, meanwhile small difference is observed from the relative T3 for injured subjects and healthy subjects.

The mean difference and standard deviation of errors between the reference and the proposed system are reported in table 6.2 and table 6.3. On average, across all healthy subjects, the proposed system estimates of Relative stance (Rst) are $3.7 \pm 1\%$ larger than reference system. On the contrary, the proposed system estimates of Relative swing (Rsw) are $-3.7 \pm 1\%$ smaller than the reference measurements. For injured subjects, the error of the estimates of Rst and Rsw are $4.1 \pm 1.1\%$ and $-4.1 \pm 1.1\%$, respectively. The errors of other relative gait events for both healthy and injured subjects are also shown in table 6.2 and table 6.3.

### 6.3 Discussion and Conclusion

A method for detecting gait phases during treadmill walking using wearable wireless ultrasonic sensor network was presented and validated. The commercial optical motion tracking system with eight high speed cameras has been used to benchmark the performance of the proposed system. As for normal walking, both horizontal and vertical experiments have a great correspondence of correlation coefficient of 0.97
and great agreement between proposed and reference system. It is able to obtain accurate estimates of gait cycle (with error of -0.02 ± 0.01 s), stance phase (with an error of 0.04 ± 0.03 s), swing phase (with an error of -0.05 ± 0.03 s), and other gait events (shown in Table 6.1). We have further tested this proposed system with different healthy subjects and injured subjects, and evaluated the influence of walking speeds on the durations of gait events. The statistical analysis also indicates that there is no significant difference between both systems.

Foot displacements were obtained from the movement of an ultrasonic transmitter attached to the subject’s heel, by employing spherical positioning technique which finds the intersections of several circles centered at each anchor with radius equal to the measured distance from the transmitter to each anchor. Then, we use recursive Newton Gauss method together with Kalman filter to enhance the performance of displacement estimation.

Some other methods to estimate foot displacement, using inertial sensors (accelerometers, gyroscopes or both), have been proposed in the literature [82, 93, 18, 89]. A common feature of these methods is that the displacement estimates suffer from error drift due to the double integration of accelerations. They minimize the drift by either shortening the integration window [18] or periodical updates of the signal at motion-less period [4, 89], which means the method should correctly identify the gait events first. Such gait events, as assumed in [4], a heel-strike at the initial contact and toe-off at the terminal contact, may change or even disappear in many type of pathologic and abnormal gait. Fortunately, there is no integration process needed in the proposed method, thus the experiment can be conducted for a long observation time without any additional techniques.

Of course, some important factors and error sources should be carefully considered before the implementation of the proposed ultrasonic motion analysis system. First, the measurement space is limited using the proposed ultrasonic motion analysis system. However, the maximum propagation distance of ultrasonic signal is 20 m, which is large enough for indoor applications. Additionally, we can also increase
the number of anchors enabling a corresponding increase in the measurement space. The second factor is how to deal with the multipath propagation. All the experiments in this study are conducted under line-of-sight condition, where the ultrasonic transmitter faces all the receivers without any obstacles between them. The method used in our experiment to mitigate the multipath propagation is by setting an inhibit time, i.e. the ultrasound detector will be disabled within the inhibit time to detect an ultrasound signal. Then, it will be enabled after this inhibit duration. Third, some of the estimation errors come from the manual placement of the reflective markers on the foot, which shape will be affected by the soft tissue deformation and the rotation of joint, especially at high walking speed. Finally, another factor which has the influence on the estimates of optical motion analysis system is marker occlusion. Only those markers that have been detected by at least 2 cameras will be recorded, or the position of such marker will be estimated by interpolation from other markers.

Based on the estimated displacements, we further developed a new gait phase detection algorithm that reliably measured the transition periods, stance, swing, T1, T2, T3, and T4. The performance of the proposed gait phase detection algorithm was evaluated against a standard optical motion analysis system. Testing of the system revealed its capability of detecting gait events during different walking speed with high degree of accuracy, as shown in table 6.1, 6.2 and 6.3.

We validate the proposed gait phase detection method on the experimental data of both healthy and injured subjects. The temporal parameters of gait phases, such as the duration of T1, can offer a quantitative assessment of the pathological state of the injured subjects. Such assessment may help clinicians and doctors to identify pathological gait impairment, monitor the progress of rehabilitation, prescribe treatment, and assess the improvements in response to therapeutic intervention.

In summary, we use low-cost ultrasonic sensors to develop a human monitoring system for the estimation of spatial-temporal parameters in a human gait. The system firstly estimated the position of the foot-worn wireless ultrasonic sensor to extract
gait temporal information about the different gait events (Heel-strike, Heel-off, Toe-off, and Mid-swing). Then, the relative gait events about different walking speeds are evaluated and validated against a reference optical motion analysis system. Our experimental results demonstrate the high feasibility of the proposed system in detecting gait phases under long term monitoring in home. In addition, only one light and low-cost ultrasonic transmitter is needed to be placed on human body, which enables patients or subjects to move above in unrestrained environment. The possibility of the proposed system for clinical applications has been analysed for both healthy and injured subjects. The experimental results indicate that the estimated gait phases have the potential to become indicators for sports and rehabilitation engineering.
Chapter 7

Lower Extremity Joint Angle Tracking with the Proposed Wireless Ultrasonic Sensors During a Squat Exercise

This Chapter presents an unrestrained measurement system based on the proposed wearable wireless ultrasonic motion analysis system to track the lower extremity joint and trunk kinematics during a squat exercise with only one ultrasonic sensor attached to the trunk. As mentioned in Chapter 1, only one ultrasonic sensor (transmitter) is needed to be attached on human trunk, which minimizes the discomfort for users and avoids complex calibration procedures and synchronization issues. The proposed system measures the horizontal and vertical displacement, together with known joint constraints to estimate joint flexion/extension angles using inverse kinematic model based on damped least-squares technique.

Squat exercise is an effective training exercise for maintaining mobility and improving lower-limb muscle function [94]. Therefore, it has been included as part of athletic training and rehabilitation. Squat exercise can be investigated using a wide variety of measured and estimated parameters. In [95], the hip and knee torques
combined with the position of knee have been used for the assessment of barbell squat. Ankle, knee, and hip joint kinematics have been used in [96] to quantitatively evaluate the motion of squat. In many of these applications, it is essential to monitor the squat exercise under natural environment without hindrance. Therefore, this entails the need of an unrestrained low cost motion tracking system.

The Chapter is organized as follows: Biomechanical model of leg is given in section 7.1. This is followed by the description of inverse kinematic model in section 7.2. Section 7.3 introduces the damped least-square scheme which is used for overcoming oscillations and shakings of the inverse kinematic problem in the neighbourhood of a singularity. Section 7.4 investigates the performance of our ultrasonic tracking system by comparing with camera-based tracking reference system. Finally, discussion and conclusion are given in Section 7.5.

Figure 7.1: 3 degrees-of-freedom kinematic model of the leg.

7.1 Biomechanics of Human Body

The commonly used method for modeling human rigid body is based on a sequence of links connected by joints. This model could better represent the movement of any parts on human body [97]. In order to describe the position and orientation between
7.1. BIOMECHANICS OF HUMAN BODY

adjacent links, Denavit and Hartenberg (D-H) in 1955 proposed a systematic notation for assigning right-handed orthogonal coordinate frames to each link [98]. The transformations between two adjacent frames can be described by a \( 4 \times 4 \) homogeneous transformation matrix. Then, the transformation is described by four parameters associated with each link, known as D-H parameters \( (L_k = (\theta_k, d_k, a_k, \alpha_k)) \). The first one is the joint angle \( \theta_k \), which is defined as the angle between \( x^{k-1} \) and \( x^k \) axes about the \( z^{k-1} \) axis. The second parameter is \( d_k \), which is the distance between \( x^{k-1} \) and \( x^k \) axes along the \( z^{k-1} \) axis. The third one is \( a_k \), which is defined as the distance between \( z^{k-1} \) and \( z^k \) axes along the \( x^k \) axis. The last parameter is \( \alpha_k \), which is the twist angle between \( z^{k-1} \) and \( z^k \) axes about the \( x^k \) axis.

Fig. 7.1 and Table 7.1 show the biomechanical model of human body used in our study and D-H parameters for the leg kinematics, respectively, which correspond to a kinematic chain comprising of three rigid segments (shank \( (a_1) \), thigh \( (a_2) \), and torso \( (a_3) \)).

Given the D-H model and parameters presented before, we then proceed to investigate the orientation and position of the sensor point, where the ultrasonic sensor will be attached. The transformation matrix between two adjacent links \( k - 1 \) and \( k \) is defined by the following expression [98]:

\[
T_{k-1}^k = \begin{bmatrix}
C\theta_k - C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\
S\theta_k & C\alpha_k C\theta_k - S\alpha_k C\theta_k & a_k S\theta_k \\
0 & S\alpha_k & C\alpha_k & d_k \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (7.1)

where \( Sx = \sin x \) and \( Cx = \cos x \). In order to compute the leg matrix, we can multiply the transformation matrices in turns: [98]:

\[
T_{\text{torso}}(L) = T_{\text{ankle}}(L_1) \cdot T_{\text{knee}}(L_2) \cdot T_{\text{hip}}(L_3)
\]  \hspace{1cm} (7.2)
Once an expression for the leg matrix is available, the leg equation is given by:

\[ T_{\text{torso to ankle}} = \begin{bmatrix} R(L) p(L) \\ 0 0 0 1 \end{bmatrix} \]  

(7.3)

Table 7.1: Denavit-Hartenberg parameters of the leg model

<table>
<thead>
<tr>
<th>Joint</th>
<th>( \theta_k )</th>
<th>( d_k )</th>
<th>( a_k )</th>
<th>( \alpha_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ankle ((L_1))</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>( a_1 )</td>
<td>0</td>
</tr>
<tr>
<td>Knee ((L_2))</td>
<td>( \theta_2 )</td>
<td>0</td>
<td>( a_2 )</td>
<td>0</td>
</tr>
<tr>
<td>Hip ((L_3))</td>
<td>( \theta_3 )</td>
<td>0</td>
<td>( a_3 )</td>
<td>0</td>
</tr>
</tbody>
</table>

The \( 3 \times 3 \) submatrix \( R(L) \) specifies the orientation of the torso, while the \( 3 \times 1 \) submatrix \( p(L) \) specifies the position of the torso, that is the location of the ultrasound sensor. Then the orientation and position of the torso (or ultrasonic sensor) can be calculated with respect to the ankle frame as follows

\[
R = \begin{bmatrix}
C_{\theta_123} & -S_{\theta_123} & 0 \\
S_{\theta_123} & C_{\theta_123} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
p = \begin{bmatrix}
a_3C_{\theta_123} + a_2C_{\theta_12} + a_1C_{\theta_1} \\
a_3S_{\theta_123} + a_2S_{\theta_12} + a_1S_{\theta_1} \\
0
\end{bmatrix}
\]

(7.4)

where the orientation angle \( \phi = \theta_{123} = \theta_1 + \theta_2 + \theta_3 \) and \( \theta_{12} = \theta_1 + \theta_2 \). \( \theta_1 \), \( \theta_2 \), and \( \theta_3 \) are the ankle, knee, and hip joint angles, respectively.
7.2 Inverse Kinematic Model

The direct kinematic mapping of interest for the system is given by

\[ p = f(\theta) \]
\[ \dot{p} = J(\theta)\dot{\theta} \]  

where \( \theta = [\theta_1 \, \theta_2 \, \theta_3]^T \), \( \dot{\theta} \) and \( \dot{p} \) are joint velocities and Cartesian velocities, respectively, \( f \) is the nonlinear function described in equation (7.4) and \( J \) is a Jacobian matrix defined as \( J(\theta) = \left[ \frac{\partial f}{\partial \theta_1} \, \frac{\partial f}{\partial \theta_2} \, \frac{\partial f}{\partial \theta_3} \right]^T \). The system under study is kinematically redundant during a planar squat exercise, since there is a 3-Degree-of-Freedom (DoF) movement with only 2 displacements known.

A general solution in terms of a generalized inverse of the Jacobian matrix is [99, 100]

\[ \dot{\theta} = F(p - \hat{p}_e) + (I - FJ)\Phi \]  

where \( \hat{p}_e \) is the estimated position obtained from the forward kinematic model and \( p \) is the Cartesian coordinates (target position) acquired by the ultrasonic sensor. \( F \) is a generalized inverse matrix of Jacobian matrix, \( I - FJ \) is the matrix which projects the vector \( \Phi \) in the null space of \( J \), \( J(\theta) \) is simplified to \( J \), and

\[ \Phi = \xi_0 \left( \frac{\partial \omega(\theta)}{\partial \theta} \right)^T \]  

where \( \xi_0 > 0 \) and \( \omega(\theta) \) is a scalar (objective) function of joint variables. Since the solution moves along the direction of the gradient of \( \omega(\theta) \), it attempts to locally maximize subject to kinematic constraint. The objective function is based on limited joint range from lower \( \theta_{\min} \) to upper \( \theta_{\max} \) limits, shown in Table 7.2 [94]

\[ \omega(\theta) = -\frac{1}{6} \sum_{i=1}^{3} \left( \frac{\theta_i - \bar{\theta}_i}{\theta_{i\max} - \theta_{i\min}} \right)^2 \]

\[ \bar{\theta}_i = \frac{\theta_{i\max} + \theta_{i\min}}{2} \]
where $\tilde{\theta}_i$ is the middle value of the joint range; thus redundancy is exploited to keep the joint variables $\theta$ as close as possible to the centre of their ranges [101].

Table 7.2: Range of Motion and Constraints Imposed in the System

<table>
<thead>
<tr>
<th></th>
<th>Upper limit (deg)</th>
<th>Lower limit (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$ ankle joint</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>$\theta_2$ knee joint</td>
<td>5</td>
<td>-120</td>
</tr>
<tr>
<td>$\theta_3$ hip joint</td>
<td>110</td>
<td>-15</td>
</tr>
</tbody>
</table>

7.3 Damped Least-Squares Scheme

The system is close to singular point near/at the beginning and ending of the squat cycle, since the Jacobian matrix does not have full rank [102]. The unavoidable measurement errors in displacement estimations using spherical positioning algorithm will produce some unreachable targets, which leads to oscillations and shakings of the inverse kinematic problem. An alternative solution overcoming the problem of inverting differential kinematics in the neighbourhood of a singularity is damped least-squares inverse (DLS)

$$F = J^T(JJ^T + \kappa^2 I)^{-1}$$

where $\kappa$ is a damping factor that ensures continuity and good shaping of the solution near/at singularity and for out-of-reach targets. The damping factor should be critically determined for obtaining good performance over the entire workspace as following

$$\kappa^2 = \begin{cases} 0 & \text{when } \sigma_{\text{min}} \geq \varepsilon \\ \left(1 - \left(\frac{\sigma_{\text{min}}}{\varepsilon}\right)^2\right)^{\kappa_{\text{max}}} & \text{otherwise} \end{cases}$$
7.3. DAMPED LEAST-SQUARES SCHEME

where \( \kappa_{\text{max}} \) is used to adjust the solution in the neighborhood of a singularity, \( \sigma_{\text{min}} \) is the estimate of the smallest singular value, and \( \varepsilon \) is defined as the size of the singular region [103].

To gain more insight into the features of the DLS, the singular value decomposition [102, 103] of the Jacobian matrix is introduced, that is

\[
J = \sum_{i=1}^{3} \sigma_i u_i v_i^T
\]  

(7.11)

where \( u_i \) and \( v_i \) are output and input singular vector, and \( \sigma_i \) are the singular values. Thus, the solution of DLS can be expressed in the form of

\[
F = J^T (JJ^T + \kappa^2 I)^{-1} = \sum_{i=1}^{3} \frac{\sigma_i}{\sigma_i^2 + \kappa^2} v_i u_i^T
\]  

(7.12)

From the equation (7.12), the components for which \( \sigma_i \gg \kappa \) are little affected by the damping factor since for large \( \sigma_i \), \( \sigma_i/(\sigma_i^2 + \kappa^2) \approx 1/\sigma_i \). On the contrary, when \( \sigma_i \) is of the similar or even smaller magnitude as \( \kappa \), then the associate component of the solution is driven to zero by the factor \( \sigma_i/\kappa^2 \). Therefore, the DLS method tends to effectively smooth out the performance in the neighborhood of singularities.

Once the solution of (7.6) is found, the joint angles can be initially computed using previous joint angles and the estimated joint velocities

\[
\theta_{k+1} = \theta_k + \dot{\theta}
\]  

(7.13)

The above joint angles estimation can be further refined iteratively: For \( j = 0, 1, \cdots \), compute

\[
\begin{align*}
p_{k+1}^{j+1} &= f(\theta_{k+1}^j) \\
\dot{\theta}_{k+1}^{j+1} &= f_{k+1}^j(p - \dot{p}_{k+1}^{j+1}) + (I - f_{k+1}^j J) \Phi_{k+1}^j
\end{align*}
\]  

(7.14)

until

\[
\|p - p_{k+1}^{j+1}\| \leq \xi
\]  

(7.15)

and \( \xi \geq 0 \) is a predefined threshold.

Nanyang Technological University Singapore
7.4 Experimental Validation

7.4.1 Experimental Setup

To compare the performance of the proposed measurement system with a conventional measurement system, experiments were conducted in a motion analysis lab with eight high speed cameras in the School of Mechanical and Aerospace Engineering, Nanyang Technological University. Eight healthy subjects (mean age 24.5 ± 1.73 years, mean weight 61.3 ± 6.40 kg, mean height 171.8 ± 5.80 cm) volunteered in the experiments. All subjects were required to wear a ultrasound transmitter on torso and five reflective markers, as shown in Fig. 7.2, to perform an unrestrained squat exercise at a self-selected speed, keeping their feet flat on the ground. Each subject was asked to repeat the squat exercise five times. The segment length and joint angles with respect to the kinematic model in Fig. 7.1 were calculated using the instantaneous positions of the above-mentioned five reference markers.

There were four anchors used in our experiment with positions \( p_1 = [0 \ 0 \ 0]^T \), \( p_2 = [0.324m \ 0 \ 0]^T \), \( p_3 = [0.324m \ 0.230m \ 0]^T \), \( p_4 = [0 \ 0.230m \ 0]^T \). In our method, only one ultrasonic sensor (transmitter) is needed which is to be attached to the foot, which minimizes the discomfort for users and avoids complex calibration procedures and synchronization issues. All the data transmission between anchors, coordinator and transmitter are done wirelessly through the RF module. Therefore, it does not restrict the movement of subjects.

The synchronization between the proposed ultrasonic and reference camera measurement system was done by maximizing the correlation between the horizontal displacements of the torso point estimated by both systems. The proposed ultrasonic sensor data were acquired at 50Hz. Data from the reference system were acquired at 200Hz. All data were low-pass filtered by second order low-pass Butterworth filter at 5 Hz. The positions of ankle, knee, hip and torso markers and related joint angles estimated from motion capture system were considered as a reference data and used for the evaluation of the ultrasound-based system.
7.4. EXPERIMENTAL VALIDATION

7.4.2 Data Analysis

To verify the joint angles estimation accuracy, which is implemented in the hardware platform as described in Chapter 4, the mean and standard deviation value of the difference between the parameters extracted with optical tracking reference system, together with Root Mean Square Error (RMSE), were computed. Pearson’s correlation coefficient (PCC) is also used in gauging the degree of agreement between these two systems. Furthermore, a statistical method, Bland-Altman analysis [92], for assessing the level of agreement between the proposed and reference systems was applied. Limits of agreement is referred as the mean $\pm 1.96$ times the standard deviation of the difference between the two systems.

7.4.3 Results

In Fig. 7.3, a randomly selected trial for the estimated ankle, knee, and hip joint flexion and extension angles during a squat exercise is shown. Fig. 7.3 shows the effect of different damping factors on the joint angle estimation. We compared with pseudo-inverse method ($\kappa = 0$) and DLS method. It shows clearly that
7.4. EXPERIMENTAL VALIDATION

Figure 7.3: The joint angles obtained from both ultrasonic and optical tracking system during 15s consecutive planar squat exercise. Black line makes use of pseudo-inverse method, that is $\kappa = 0$. Blue line is Damping Least-squares method. Red line is reference result.

pseudo-inverse method performs poorly because of instability near singularities ($\sigma_i$ approaches zero). However, DLS method tends act similarly to the pseudo-inverse method away from singularities and effectively smooths out the performance of pseudo-inverse method in the neighborhood of singularities. All the joint angles estimated by the DLS method show a great correspondence with reference system.

For the clarity of Bland-Altman plots, we randomly selected 4 trials to draw these figures, as shown in Fig. 7.4. The data plotted in Fig. 7.4(I) shows that all points lie along the line of equality, which means high degree of agreement between measurements. The PCC values between the two system are also calculated for the data in Fig. 7.4(I). For all these joint angles, the correlations are all equal to or larger than 0.99 ($p < 0.001$), where the p-value is the probability that the measurements by the two methods are not linearly related. As the p-value is extremely small, we can safely conclude that the measurements of joint angles by these two methods are significant. Fig. 7.4(II) shows the Bland-Altman plot for the selected data by plotting the difference between the two systems against the mean values of ankle, knee, and hip joint angles. This plot is used to gauge the limits of agreement between two methods. The 95% confidence intervals were determined to be $(-6.48^\circ, 6.56^\circ)$.
7.5. DISCUSSION AND CONCLUSION

for ankle angles, $(-5.34^\circ, -0.32^\circ)$ for knee angles, and $(-1.42^\circ, 4.7^\circ)$ for hip angles.

Table 7.3: Comparison of errors in measurements for proposed and reference system. The mean difference and relevant standard deviation (std), together with PCC values are shown. Bland-Altman limits of agreement are also reported.

<table>
<thead>
<tr>
<th>HD: Horizontal Displacement, VD: Vertical Displacement</th>
<th>Difference</th>
<th>Peak</th>
<th>RMSE</th>
<th>PCC</th>
<th>Limits of agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean std</td>
<td>min</td>
<td>max</td>
<td>mean std</td>
<td>mean std</td>
</tr>
<tr>
<td>HD(mm)</td>
<td>1.08 6.07</td>
<td>4.99</td>
<td>7.15</td>
<td>6.50 2.08</td>
<td>0.94 0.04</td>
</tr>
<tr>
<td>VD(mm)</td>
<td>0.75 9.65</td>
<td>8.90</td>
<td>10.40</td>
<td>9.90 4.79</td>
<td>0.94 0.06</td>
</tr>
<tr>
<td>ankle(deg)</td>
<td>0.57 3.10</td>
<td>2.53</td>
<td>3.07</td>
<td>3.10 1.10</td>
<td>0.99 0.01</td>
</tr>
<tr>
<td>knee(deg)</td>
<td>-2.46 1.82</td>
<td>0.64</td>
<td>4.26</td>
<td>3.28 0.39</td>
<td>1.00 0.00</td>
</tr>
<tr>
<td>hip(deg)</td>
<td>1.49 1.56</td>
<td>0.07</td>
<td>3.05</td>
<td>2.18 0.21</td>
<td>1.00 0.00</td>
</tr>
</tbody>
</table>

All the collected data from 8 subjects are used to compute the mean and standard deviation of difference, RMSE, PCC values, and limits of agreement, which are reported in Table 7.3. Horizontal displacement was obtained with a difference of $1.08 \pm 6.07$ mm, an RMSE of $6.50 \pm 2.08$ mm, and a high PCC value of $0.94 \pm 0.04$. The mean difference and STD of the vertical displacement was $0.75 \pm 9.65$ mm, which had a high similarity of $0.94 \pm 0.06$ with the results of reference system. For all the collected data, the joint angles of interest were estimated with a mean difference from the reference data of $2.85^\circ$ (RMSE) and with high similarities greater than $0.99$, where all p-values are smaller than $(0.001)$. For all three estimated joint angles, hip joint has the smallest error, and consequently, tighter, lower and upper limits of agreement, more closely to zero, than ankle and hip joint. In summary, the great agreement of all joint angles estimated by the two systems is enough for us to be confident that the proposed measurement system can be used for further clinical applications.

7.5 Discussion and Conclusion

This study has demonstrated that one ultrasound transmitter (attached to human body) and four receivers can be used to obtain high accuracy ankle, knee, hip joint
Figure 7.4: I. Joint angles estimated by proposed ultrasonic and reference camera-based measurement system, plotting with line of equality. (a) Ankle joint angle, (c) Knee joint angle, and (e) Hip joint angle. II. Bland-Altman plot with mean and difference between two values estimated by proposed ultrasonic and reference camera-based measurement system, plotting with limits of agreement which are the average difference ± 1.96 standard deviation of the difference. (b) Ankle joint angle, (d) Knee joint angle, and (f) Hip joint angle.
flexion/extension angles estimates. The spherical positioning technique is applied to estimate the displacements in vertical and horizontal direction of the ultrasonic sensor, and then the recorded displacements together with known joint constraints are used to estimate joint angles of the trunk using damped least-squares-based technique for the singularity avoidance problem of redundant systems.

The proposed measurement system was compared to a camera-based system, and the results were averaged over eight subjects. These metrics (difference, RMSE, PCC values, and limits of agreement) are shown in Table 7.3 to illustrate the differences between these two systems. The experimental results showed that the horizontal and vertical displacement of ultrasonic sensor and the joint angles (hip, knee, and ankle) can be estimated with relatively small differences. Bland-Altman analysis indicated a great agreement between these two systems with PCC value higher than 0.99. The average RMSD of the estimate of the lower extremity joint angles is $2.85 \pm 0.57^\circ$, which is similar to what has been obtained in other studies using inertial sensors to estimate these joint angles [94, 19].

A key issue in the system is the determination of damping factor in equation (7.9), since it plays an important role in finding solution to the inverse kinematic model. Larger $\kappa$ makes the solution for joint velocities well behaved in the neighbourhood of a singularity, but results in low tracking accuracy and low convergence rate. Small values of damping factor give accurate solutions but not robust to the singular or near-singular region. Hence, it is essential to select suitable values for the damping factor.

In summary, this chapter presents an unrestrained measurement system based on wearable wireless ultrasonic sensor network to track the lower limb joint and trunk kinematics during a squat exercise with only one ultrasonic sensor attached to the trunk. The camera-based tracking reference system was used to benchmark the performance of the proposed ultrasonic motion analysis system. It demonstrates that the accuracy of this system is sufficiently good for clinical applications, such as rehabilitation, pervasive healthcare, and sports surveillance. Additionally, the
proposed system is easy to wear, to use and much cheaper than current camera system. It does not restrict the movement of patients or subjects with bulky cables.
Chapter 8

Conclusions and Recommendations

In this thesis, the development of human motion tracking system based on wearable UWB radios and wireless ultrasonic sensor network has been presented. This chapter closes this thesis by summarizing the main contributions and suggesting directions for future research.

8.1 Summary of Contributions

The contributions of the thesis are as follows:

- **A novel approach to determine the joint flexion/extension angles based on on-body fixed sensor using UWB radios**

A new method for measuring and monitoring human body joint angles, which uses wearable UWB transceivers mounted on body segments, is proposed and investigated. The model is based on providing a high ranging accuracy (inter-sensor distance) between a pair of transceivers placed on adjacent segments of the joint center of rotation. The measured distance is then used to compute
the joint angles based on the law of cosines. The performance of the method was compared with a flexible goniometer by simultaneously measuring joint flexion-extension angles at different angular velocities, ranging between $8^\circ/s$ and $90^\circ/s$. The measurement errors were evaluated by the average differences between two sets of data (ranging from $0.8^\circ$ for slow movement to $2.8^\circ$ for fast movement), by standard deviation (ranging from $1.2^\circ$ to $4.2^\circ$ for various movement speeds) and by the Pearson correlation coefficient (greater than 0.99) which demonstrates the very good performance of the UWB based approach. The experimental results have shown that the system has sufficient accuracy for clinical applications, such as rehabilitation.

- **Development of Wearable Wireless Ultrasonic Motion Analysis System**

A novel system based on a wearable wireless ultrasonic sensor network for 3-dimensional foot trajectory measurement has been proposed. The system consists of an ultrasonic transmitter (mobile) and several receivers (anchors) with fixed known positions. In order not to restrict the movement of subjects, RF module is used for wireless data transmission. The same RF module also provides the synchronization clock between mobile and anchors. The proposed system measures the Time-of-Arrival (TOA) of the ultrasonic signal from mobile to anchors. Together with the knowledge of the anchor’s position, the absolute distance the signal travels can be computed. Then, the range information defines a circle centered at this anchor with radius equal to the measured distance, and the mobile resides within the intersections of several such circles. Based on the TOA-based tracking technique, the 3-dimensional foot trajectories are validated against a camera based motion capture system for 10 healthy subjects walking on a treadmill at slow, normal and fast speeds. The experimental results have shown that the ultrasonic system has sufficient accuracy of net root mean square error (4.2 cm) for 3-dimensional displacement, especially for foot clearance with accuracy and standard devia-
tion (0.62 ± 7.48 mm) compared to the camera-based motion capture system. The small form factor and lightweight feature of the proposed system make it easy to use. Such a system is also much lower in cost compared to camera-based tracking system.

- Estimation of spatial-temporal gait parameters using the proposed ultrasonic motion analysis system

With the proposed system, the measurement is extended to measure the displacements of foot in 3-dimensional space using the combination of Kalman filter and Newton-Gauss method. Using the measured displacements, some gait parameters including stride length, stride duration, stride velocity, stride cadence, and stride symmetry are extracted. The performance of this system is validated against a camera based system in the laboratory with 10 healthy volunteers. Numerical results show the feasibility of the proposed system with an average error of 2.7% for all the estimated gait parameters. The influence of walking speed on the measurement accuracy of proposed system is also evaluated. Statistical analysis demonstrates its potential to be used as a gait assessment tool for some medical applications.

- Development of a new gait phase detection approach using the displacements of foot during walking

A methodology for detecting gait phases during walking is based on the estimated foot trajectories. It is capable of detecting the phases: heel-strike, heel-off, toe-off and mid-swing. The proposed gait phase detection algorithm can reliably measured the transition periods, such as stance and swing. Testing of the system revealed its capability of detecting gait events during different walking speeds with high degree of accuracy. The performance of the proposed algorithm is examined by comparing with a commercial optical motion tracking system with ten healthy subjects. Accurate estimates of gait cycle (with an error of -0.02 ± 0.01 s), stance phase (with an error of 0.04 ± 0.03 s), and swing phase (with an error of -0.05 ± 0.03 s) compared to the reference system.
are obtained. We have also investigated the influence of walking velocities on the performance of the proposed gait phase detection algorithm. Statistical analysis also shows that there is no significant difference between both systems during different walking speeds.

- **Development of a lower extremity joint angle tracking system using the combination of inverse kinematic model and joint constraints**

Lower limb joint and trunk kinematics during a planar squat exercise with only one ultrasonic sensor is proposed and investigated. The displacements of ultrasonic transmitter together with known joint constraints are used to estimate joint angles of the trunk. The damped least-squares (DLS) technique is used for the avoidance of singularity of redundant system. The main advantages of the system: 1) The use of only one ultrasonic sensor not only minimizes the discomfort for users, but also avoids complex calibration procedures and synchronization issues. 2) The DLS method can effectively smooth out the performance in the neighborhood of singularities. 3) Measurement of the lower extremity angle can be limited to a single plane without the need for additional optimization constraint. The performance of the proposed ultrasonic measurement system was validated against a camera-based tracking system on 8 healthy subjects performing planar squat exercise. Joint angles estimated from the ultrasonic system showed a Root Mean Square Error (RMSE) of $2.85^\circ \pm 0.57^\circ$ with the reference system. Statistical analysis indicated great agreements between these two systems with Pearson’s correlation coefficient (PCC) value larger than 0.99 for all joint angles estimation. These results show that the proposed ultrasonic measurement system is useful for applications such as rehabilitation and sports.

### 8.2 Recommendations for Future Works

This section outlines the potential extensions for future research.
8.2. RECOMMENDATIONS FOR FUTURE WORKS

8.2.1 3-dimensional Displacement of Center of Gravity

Since we have successfully measured the 3-dimensional foot trajectory using the proposed ultrasonic motion analysis system, we can continue the work by attaching the mobile node to different part of human body. As an example, monitoring Center of Gravity (COG) behavior is the simplest way to study human gait. The displacement pattern of the COG is mainly estimated by optical systems, which requires complex experiment setup and high cost. However, the proposed system based on wireless ultrasonic sensor is capable of providing a high positioning accuracy, which also exceeds the accuracy provided by current locomotion tracking system.

8.2.2 Extension of the Measurement of Joint Angle into Three Degrees of Freedom

The determination of flexion/extension angle information with only a single degree of freedom is not sufficient for clinical applications which require quantitative measurements of complex movements of human limbs. Therefore, it is important to extend our measurement technique for two or three degrees of freedom. In order to extend the proposed system in capturing more complex motion, more mobile nodes are needed to be placed on human body.

It is possible to have two or more mobile nodes in our system because each mobile node has its own ID address. The coordinator can identify the specific mobile with this address. Multiple mobiles require time division multiplexing as only one mobile node can transmit ultrasound signal at any time. They can only work in sequence, which will require much higher speed processor.

8.2.3 Wireless Sensor Network

Wireless sensor network is a promising field that integrated wireless communication system, embedded system and sensor technologies together to produce a portable,
8.2. RECOMMENDATIONS FOR FUTURE WORKS

<table>
<thead>
<tr>
<th></th>
<th>WLAN (802.11b)</th>
<th>Bluetooth (802.15.1)</th>
<th>ZIGBEE (802.15.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>30-100 m</td>
<td>2-10 m</td>
<td>10-100 m</td>
</tr>
<tr>
<td>Power Consumption</td>
<td>Medium</td>
<td>Low</td>
<td>Lower</td>
</tr>
<tr>
<td>Size</td>
<td>Larger</td>
<td>Smaller</td>
<td>Smallest</td>
</tr>
<tr>
<td>Data Rate</td>
<td>1-11 Mb/s</td>
<td>1Mb/s-3 Mb/s</td>
<td>10 kb/s-250 kb/s</td>
</tr>
<tr>
<td>Frequency Bands</td>
<td>2.4 GHz</td>
<td>2.4 GHz</td>
<td>868 M/915 M/2.4 GHz</td>
</tr>
</tbody>
</table>

Table 8.1: Comparison of different wireless communication standards.

low power consumption and low cost system for human movement tracking. There are several wireless communication standards for short-range wireless networking, including IEEE 802.11 Wireless Local Area Network (WLAN) and Bluetooth-based WLAN (802.15.1), ZigBee (802.15.4). A summary of comparison among these protocols is given in Table 8.1.

As can be seen from Table 8.1, both WLAN (802.11b) and Bluetooth (802.15.1) offer higher data rate (larger than 3 Mb/s) than ZigBee (802.15.4) (up to 250 kb/s) at the expense of larger form factor and higher power consumption. However, the ZigBee standard is specifically developed to address the need for very low cost implementation of low-data-rate wireless networks with ultra-low power consumption [104]. Therefore, in our future work, we can collect health related information by having patients wear ZigBee devices that interfaces with our proposed system that can measure complex human movements that are required in home health care applications. Then, the data can be wirelessly transmitted to a local server, such as a personal computer inside the patients home, where initial analysis is performed. Finally, the vital information is sent to the patients nurse or physician via the Internet for further analysis.
Author’s Publications

Journal Papers:


NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE
8.2. RECOMMENDATIONS FOR FUTURE WORKS

Conference Papers:


Bibliography


[96] Vincent Bonnet, Claudia Mazza, Philippe Fraisse, and Aurelio Cappozzo. A least-squares identification algorithm for estimating squat exercise mechanics


