MODELING, DESIGN AND CONTROL OF DISH-STIRLING SOLAR-THERMAL POWER GENERATING SYSTEM

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Summary

Dish-Stirling (DS) solar-thermal generating system is one type of renewable energy technology in which a parabolic dish-like reflector is used to concentrate sunlight to a small area located at the focal point of a number of mirrors. The high temperature achieved at the focal point is used as a heat source for a Stirling engine, which is a type of closed-cycle external heat engine with high thermal efficiency and no emissions. Unfortunately, the intermittent and uncontrollable nature of the solar irradiance makes the control of the harnessed energy most challenging. Quality and reliability of the supply could be degraded to unacceptable level if the harnessed energy is fed directly into an electrical power grid in significant amount. Various types of technical and economic issues for the DS system must be solved in order to compete with the other type of renewable technologies in the dynamic market. Hence, appropriate system design and operations of the DS power plant are called for.

The existing models of the most important component of the DS system – the Stirling Engine, are not suitable for the studies of the DS solar-thermal power generation, in terms of the compatibility of the computer simulation step size and control system design. Proper average-value models for the four-cylinder double-acting kinematic Stirling engine, including a general model for variable-speed operation and a specific model for constant-speed operation are developed based on thermodynamic analysis for the studies of the DS system. With the derived models, a suitable model for such a generating system is developed. Linearized model of the constant-speed DS system is derived for temperature control system design. New temperature control schemes with transient droop compensation and feedforward compensation are proposed to overcome the problems caused by the non-minimum phase and nonlinear characteristic of the system model.
The potential of the maximum energy harness of the DS system is analyzed and discussed, and the results show that variable-speed operation of the DS system is a viable method to extract considerable more energy. The overall configuration of the variable-speed DS system using doubly-fed induction generator (DFIG) is proposed and the model of the entire system will be derived especially for the design of temperature control while achieving maximum power harnessing. The computer simulation results are given to show the performance of the proposed temperature controllers to overcome the problems introduced by the continuous engine/generator speed variation which is not seen in the case of the conventional constant-speed DS system.

A DS-DFIG simulator has been developed in the author’s laboratory with the view to study the maximum energy harness ability of the DS-DFIG system. The developed system is based on the use of a separately-excited dc motor to generate the equivalent mechanical torque from the DS under specific insolation level and engine/motor speed. The dc motor therefore emulates the torque vs. speed behavior of the DS system, and the produced torque is to drive a coupled DFIG in the laboratory.

In order to meet the grid code requirements for the potential integration of the large-scale DS system, the ability to provide frequency support from the DS-DFIG system is investigated. An overall system configuration for the DS-DFIG plant is proposed with the objectives to enhance the dynamic performance and to reduce the entire cost of the plant. Based on the model and the control system derived previously, various methods to provide inertia and frequency support from the DS-DFIG system are proposed and discussed. Numerical examples are included to show the availability of the proposed schemes.
# Table of Contents

Summary ............................................................................................................................................... i
Table of Contents .............................................................................................................................. iii
List of Symbols ................................................................................................................................. ix
List of Abbreviations ......................................................................................................................... xiv
List of Figures ..................................................................................................................................... xvi
List of Tables ....................................................................................................................................... xxii

## Chapter 1 Introduction ................................................................. 1

1.1. Background ......................................................................................... 1

1.1.1. Potential of Concentrated Solar Power ........................................... 1
1.1.2. Concentrated Solar Power Technologies ....................................... 3
1.1.3. Dish-Stirling Concepts ................................................................. 5

1.2. Motivation .......................................................................................... 7

1.3. Objectives .......................................................................................... 11

1.4. Main Contributions of the Thesis .................................................... 12

1.5. Organization of the Thesis ............................................................... 14

## Chapter 2 Dish-Stirling Concentrated Solar Power Technology – A Review .... 17

2.1. Introduction ....................................................................................... 17

2.2. Literature Review ................................................................................ 17

2.2.1. Development of Stirling Engine .................................................... 17
2.2.2. Modeling of the Stirling Engine .................................................... 19
2.2.3. Control of the Stirling Engine ....................................................... 20
2.2.4. Modeling of the concentrator and receiver ................................... 20
2.2.5. Dish-Stirling system ................................................................. 21

2.3. Preliminary Considerations on the Modeling of Stirling Engine ........... 22

2.4. Thermodynamics of Stirling Engine ..................................................... 22

2.4.1. Ideal Stirling Engine Cycle .......................................................... 22
2.4.2. Ideal Adiabatic Model ................................................................. 25
2.4.3. Ideal Adiabatic Model Considering Temperature and Mass Variation

2.4.4. Double-Acting Kinematic Configuration of Stirling Engine

2.5. Solution of Ideal Adiabatic Model and Relevant Issues

2.6. Summary

Chapter 3 Development of Stirling Engine Model for Dish-Stirling Solar-Thermal Systems

3.1. Introduction

3.2. Normalized Average-Value Adiabatic Model

3.2.1. Total Per-Cycle Input Heat of Four-Cylinder Double-Acting Stirling Engine

3.2.2. Total Per-Cycle Output Work of Four-Cylinder Double-Acting Stirling Engine

3.2.3. Mean Pressure versus Total Mass Relationship

3.2.4. Normalization of the Stirling Engine Model

3.2.5. Transfer Functions for Constant-Speed Operation

3.3. Quasi-Static Representation of Losses

3.3.1. Effects of Losses

3.3.2. Multivariate Polynomial Coefficients

3.3.3. Thermal Efficiency of Stirling Engine

3.4. Model Comparison and Case Study

3.5. Summary of the Developed Model of the Stirling Engine

3.5.1. General Normalized Average-Value Model

3.5.2. Normalized Average-Value Model for Constant-Speed Operation

3.6. Summary

Chapter 4 Constant-Speed Dish-Stirling Solar-Thermal System

4.1. Introduction
4.2. Modeling of Constant-Speed Dish-Stirling Solar-Thermal System

4.2.1. Modeling of Concentrator and Receiver

4.2.2. Modeling of Mean Pressure Control

4.2.3. Modeling of Stirling Engine

4.2.4. Modeling of Induction Generator

4.2.5. Model Normalization

4.3. Linearized Model of Constant-Speed DS system

4.3.1. Linearized Model Considering Engine Speed as a Disturbance

4.3.2. Linearized Model Considering Electromagnetic Torque as a Disturbance

4.4. Temperature Control Design of Constant-Speed DS System

4.4.1. Transient Droop Compensation

4.4.2. Feedforward Compensation during Periods of Speed Variations

4.5. Maximum Energy Harness by Utilizing the DS Systems

4.5.1. Output Power versus Insolation under Constant Speed Operation

4.5.2. Feasible Operating Regime under Variable Speed Operation

4.5.3. Variable Speed Operation and Maximum Power Harness

4.6. Illustrative Example

4.6.1. Comparison of Detailed and Average-Value Adiabatic Models under Step Change of Engine Speed

4.6.2. Comparison of Detailed and Average-Value Adiabatic Models Under Grid Fault

4.6.3. Temperature Controller Design

4.7. Conclusions

Chapter 5 Variable-Speed Dish-Stirling Solar-Thermal System

5.1. Introduction

5.2. Modeling of Variable-Speed DS System

5.2.1. Configuration of Variable-Speed DS Systems
5.2.2. Modeling of the Prime Mover: Dish and Stirling Engine .......... 89
5.2.3. Modeling of DFIG ................................................................. 90
5.2.4. Modeling of PMSG ................................................................. 90
5.2.5. Modeling of Back-to-Back PWM Converters with LC filter ....... 91
5.3. Double-Loop Feedback Control of Variable-Speed DS Systems .......... 94
  5.3.1. Current Control of DFIG ...................................................... 94
  5.3.2. Speed Control of DFIG ...................................................... 96
  5.3.3. Control of PMSG ................................................................. 98
5.4. Optimal Speed for Maximum Output Power .................................. 98
5.5. Linearized Model of Variable-Speed DS Systems .......................... 103
  5.5.1. Linearized Model of the Prime Mover .................................. 103
  5.5.2. Linearized Model of Speed Control ..................................... 104
  5.5.3. Small-Signal State-Space Equation ..................................... 105
5.6. Control of Receiver Temperature in the Variable-Speed DS Systems .. 106
  5.6.1. Overall Strategy for Temperature Control .............................. 106
  5.6.2. Temperature Control with Droop Characteristics ..................... 107
  5.6.3. Local Full State-Feedback with Integral Control ..................... 110
  5.6.4. Fuzzy Supervisory Control Scheme ................................... 113
5.7. Overall Control Structure .......................................................... 114
5.8. Illustrative Example ................................................................. 115
  5.8.1. Simulation Model of the Test DS-DFIG System ....................... 115
  5.8.2. Case 1: Transient Response under Insolation Change .............. 115
  5.8.3. Case 2: Maximum Power Point Tracking .............................. 119
5.9. Conclusions .................................................................................. 120
Chapter 6 Laboratory Development of a DS-Simulator .......................... 121
  6.1. Introduction ................................................................................. 121
  6.2. Modeling and Control of DC Motor .......................................... 121
    6.2.1. Modeling of DC Motor ....................................................... 121
6.2.2. Torque/Current Control of the DC Motor ........................................ 123
6.2.3. Model Simplification and Analysis .................................................. 125
6.3. Dynamic DS-Simulator ........................................................................ 126
6.4. Steady-State Analysis of the DS-Simulator .......................................... 128
6.5. Conclusions .......................................................................................... 131

Chapter 7 Frequency Control of Variable-Speed Dish-Stirling Solar-Thermal System Using DFIG ................................................................. 133
7.1. Introduction ........................................................................................... 133
7.2. Dish-Stirling Solar-Thermal Power Plant ............................................ 134
7.3. Aggregated Model of DS-DFIG System .............................................. 135
7.4. Frequency Support of DS-DFIG System .............................................. 136
  7.4.1. Frequency Support from Synchronous Generators ...................... 136
  7.4.2. Inertia Response from Speed Control Loop ................................. 138
  7.4.3. Frequency Response from Temperature Control ........................ 140
  7.4.4. Primary Frequency Control DS-DFIG by De-Loading ............... 142
  7.4.5. Secondary Frequency Control of DS-DFIG ................................. 147
7.5. Illustrative Example .............................................................................. 148
  7.5.1. Test System Configuration for Frequency Support .................... 148
  7.5.2. Case 1: Frequency Response of DS-DFIG System Under Primary Frequency Control of Synchronous Generator .......................... 148
  7.5.3. Case 2: Frequency Response of DS-DFIG System Under Secondary Frequency Control of Synchronous Generator .......... 149
7.6. Conclusions .......................................................................................... 152

Chapter 8 Conclusions and Recommendations ....................................... 153
8.1. Conclusions .......................................................................................... 153
8.2. Recommendations ................................................................................. 154

Appendix A Parameters of the Dish and Stirling Engine ............................. 156
A.1. Dish & Receiver Parameters ............................................................... 156
A.2. Stirling Engine Parameters ................................................................. 156
A.3. Control Parameters ............................................................................ 157
Appendix B  Machine Parameters ............................................................... 158
  B.1. SCIG Parameters ............................................................................... 158
  B.2. DFIG Parameters ............................................................................... 158
  B.3. DC Motor Parameters ........................................................................ 158
Appendix C  Multivariate Polynomial Coefficients ..................................... 159
  C.1. Output Power Coefficient ................................................................. 159
  C.2. Input Power Coefficient ..................................................................... 162
Author’s Publications ................................................................................... 165
Bibliography ................................................................................................. 166
List of Symbols

Thermodynamic Analysis of the Stirling Engine

D Derivative operator with respective to crank angle \(=d/d\phi\).

\(M\) Total mass of working gas in the cylinder (kg).

\(P_m\) Output mechanical power (W).

\(Q_h\) Heat transferred via heater (J).

\(Q_k\) Heat transferred via cooler (J).

\(Q_r\) Heat transferred via regenerator (J).

\(R\) Gas constant \([\text{J/(kg·K)}]\).

\(T_c\) Compression space temperature (K).

\(T_{ck}\) Interface temperature from compression space to cooler (K).

\(T_e\) Expansion space temperature (K).

\(T_h\) Heater temperature (K).

\(T_{he}\) Interface temperature from heater to expansion space (K).

\(T_k\) Cooler temperature (K).

\(T_{kr}\) Interface temperature from cooler to regenerator (K).

\(T_r\) Regenerator temperature (K).

\(T_{th}\) Interface temperature from regenerator to heater (K).

\(V_{cl}\) Volume of clearance space (m\(^3\)).

\(V_h\) Volume of heater (m\(^3\)).

\(V_k\) Volume of cooler (m\(^3\)).

\(V_r\) Volume of regenerator (m\(^3\)).

\(V_{sw}\) Volume of swept space (m\(^3\)).

\(W\) Work (J).

\(c_p\) Specific heat capacities of the gas at constant pressure \([\text{J/(kg·K)}]\).

\(c_v\) Specific heat capacities of the gas at constant volume \([\text{J/(kg·K)}]\).
Mass flow rate (kg/s).

Interface mass flow rate from compression space to cooler (kg/s).

Interface mass flow rate from heater to expansion space (kg/s).

Interface mass flow rate from cooler to regenerator (kg/s).

Interface mass flow rate from regenerator to heater (kg/s).

Mass of the working gas in compression space (kg).

Mass of the working gas in expansion space (kg).

Mass of the working gas in heater (kg).

Mass of the working gas in cooler (kg).

Mass of the working gas in regenerator (kg).

Instantaneous Pressure of the working gas in the cylinder (Pa).

Mean pressure of the working gas (Pa).

Volume of compression space (m$^3$).

Volume of expansion space (m$^3$).

Initial crank angle of compression space (rad).

Initial crank angle of expansion space (rad).

Specific heat ratio (-).

Efficiency (-).

Mechanical torque from the Stirling engine shaft (N·m).

Crank angle (rad).

Rotational speed of Stirling engine shaft (rad/s).

**Dish-Stirling Systems: Thermodynamic Part**

Projection area of the concentrator (m$^2$).

Aperture area of the receiver (m$^2$).

Permanent droop of temperature control (p.u.).

Transient droop of temperature control (p.u.).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_b$</td>
<td>Equivalent radiative conductance [W/(m²·K)].</td>
</tr>
<tr>
<td>$I$</td>
<td>Solar insolation (W/m² or p.u.).</td>
</tr>
<tr>
<td>$K_{con}$</td>
<td>Concentrator gain (p.u.).</td>
</tr>
<tr>
<td>$K_h$</td>
<td>Input adiabatic gain of the Stirling engine (p.u.).</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Output adiabatic gain of the Stirling engine (p.u.).</td>
</tr>
<tr>
<td>$K_{rec}$</td>
<td>Receiver gain (p.u.).</td>
</tr>
<tr>
<td>$M$</td>
<td>Total mass of working gas in the cylinder (p.u.).</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Mechanical power (p.u.).</td>
</tr>
<tr>
<td>$U$</td>
<td>Convection-conduction heat loss coefficient [W/(m²·K)].</td>
</tr>
<tr>
<td>$T_R$</td>
<td>Resetting time constant (s).</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Ambient temperature (p.u.).</td>
</tr>
<tr>
<td>$T_h$</td>
<td>Heater temperature (p.u.).</td>
</tr>
<tr>
<td>$T_{rec}$</td>
<td>Receiver time constant (s).</td>
</tr>
<tr>
<td>$T_{se1}, T_{se2}, T_{se3}$</td>
<td>Stirling engine time constants (s).</td>
</tr>
<tr>
<td>$a_{ij}$</td>
<td>Input multivariate polynomial coefficients of Stirling engine (p.u.).</td>
</tr>
<tr>
<td>$b_{ij}$</td>
<td>Output multivariate polynomial coefficients of Stirling engine (p.u.).</td>
</tr>
<tr>
<td>$c$</td>
<td>Input valve reference command (p.u.).</td>
</tr>
<tr>
<td>$gA$</td>
<td>Mass flow rate (p.u.).</td>
</tr>
<tr>
<td>$p_{max}$</td>
<td>Maximum mean pressure of the working gas (p.u.).</td>
</tr>
<tr>
<td>$p_{mean}$</td>
<td>Mean pressure of the working gas (p.u.).</td>
</tr>
<tr>
<td>$p_{min}$</td>
<td>Minimum mean pressure of the working gas (p.u.).</td>
</tr>
<tr>
<td>$q_I$</td>
<td>Solar thermal power in the insolation (p.u.).</td>
</tr>
<tr>
<td>$q_L$</td>
<td>Heat transfer losses to the atmosphere (p.u.).</td>
</tr>
<tr>
<td>$q_{L,conv_cond}$</td>
<td>Convection and conduction losses to the atmosphere (p.u.).</td>
</tr>
<tr>
<td>$q_{L,radi}$</td>
<td>Radiation loss to the atmosphere (p.u.).</td>
</tr>
<tr>
<td>$q_h$</td>
<td>Heat transferred rate via heater (p.u.).</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Damping ratio (-).</td>
</tr>
</tbody>
</table>
\( \sigma \) Stefan-Boltzmann constant \([\text{W/(m}^2\cdot\text{K}^4)]\)

\( \eta_{\text{con}} \) Energy efficiency of the concentrator (-).

\( \eta_h \) Input energy efficiency of the Stirling engine (-).

\( \eta_m \) Output energy efficiency of the Stirling engine (-).

\( \eta_{\text{th}} \) Thermal efficiency of the Stirling engine (-).

\( \tau_m \) Mechanical torque from the Stirling engine shaft (p.u.).

\( \omega_m \) Rotational speed of Stirling engine shaft (p.u.).

**Dish-Stirling Systems: Electro-mechanical and Electromagnetic Part**

\( F \) Friction factor (p.u.).

\( H \) Inertia constant (s).

\( J \) Moment of inertia (kg·m²).

\( L_{\text{ds}}, L_{\text{qs}} \) \( d \)-axis and \( q \)-axis synchronous inductances (p.u.).

\( L_{\text{ds}}, L_{\text{qs}} \) Stator and rotor leakage inductances (p.u.).

\( L_m \) Magnetizing inductance (p.u.).

\( L_{\text{rs}}, L_{\text{fr}} \) Stator and rotor self-inductances (p.u.).

\( R_{\text{rs}}, R_{\text{fr}} \) Stator and rotor winding resistances (p.u.).

\( f \) System frequency (Hz).

\( i_{\text{ds}}, i_{\text{qs}}, i_{\text{dr}}, i_{\text{qr}} \) Stator and rotor current components in the \( d-q \) reference frame (p.u.).

\( p_n \) Number of pole pairs.

\( s \) Slip.

\( v_{\text{ds}}, v_{\text{qs}}, v_{\text{dr}}, v_{\text{qr}} \) Stator and rotor voltage components in the \( d-q \) reference frame (p.u.).

\( z \) Output of the integrator of the speed controller (p.u.).

\( \psi_{\text{ds}}, \psi_{\text{qs}}, \psi_{\text{dr}}, \psi_{\text{qr}} \) Stator and rotor flux-linkage components in the \( d-q \) reference frame (p.u.).

\( \tau_L \) Mechanical torque from the generator shaft (p.u.).
\( \tau_e \)  
Electromagnetic torque (p.u.).

\( \omega_s \)  
Synchronous speed (p.u.).

\( \omega_r \)  
Rotor electrical angular speed (p.u.).

\( \omega_{sl} \)  
Angular slip frequency (p.u.).

**DS-Simulator**

\( D \)  
Duty Cycle (-).

\( E_a \)  
Counter electromagnetic force (V).

\( F_{DCM}^{\text{DCM}} \)  
Friction coefficient of dc motor (N·m·s/rad).

\( F_{eq} \)  
Equivalent friction factor of dc motor and DFIG (p.u.).

\( H_{eq} \)  
Equivalent inertia constant of dc motor and DFIG (s).

\( I_a \)  
Armature current (A).

\( i_{DCM}^{D} \)  
Moment of inertia of dc motor (kg·m²).

\( K_e \)  
Torque/voltage constant (N·m/A).

\( k_i^{D} \)  
Proportional gain of torque/current controller.

\( k_p^{D} \)  
Integral gain of torque/current controller.

\( L_a \)  
Armature inductance (H).

\( R_a \)  
Armature resistance (Ω).

\( T_{DCM} \)  
Time constant of the dc motor (s).

\( V_a \)  
Armature voltage (V).

\( \tau_e^{D} \)  
Electromagnetic torque of dc motor (N·m).

\( \tau_m^{D} \)  
Mechanical torque of dc motor shaft (N·m).

\( \omega_m^{D} \)  
Rotor mechanical angular speed of dc motor shaft (rad/s).
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>CEMF</td>
<td>Counter Electromagnetic Force</td>
</tr>
<tr>
<td>CSDS</td>
<td>Constant-Speed Dish-Stirling</td>
</tr>
<tr>
<td>CSP</td>
<td>Concentrated Solar Power</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DCM</td>
<td>DC Motor</td>
</tr>
<tr>
<td>DFIG</td>
<td>Doubly-Fed Induction Generator</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generation</td>
</tr>
<tr>
<td>DNI</td>
<td>Direct Normal Irradiance</td>
</tr>
<tr>
<td>DS</td>
<td>Dish-Stirling</td>
</tr>
<tr>
<td>FSF</td>
<td>Full State-Feedback</td>
</tr>
<tr>
<td>FPSE</td>
<td>Free-Piston Stirling Engine</td>
</tr>
<tr>
<td>FVT</td>
<td>Final Value Theorem</td>
</tr>
<tr>
<td>GSC</td>
<td>Grid-Side Converter</td>
</tr>
<tr>
<td>HTF</td>
<td>Heat-Transfer Fluid</td>
</tr>
<tr>
<td>IG</td>
<td>Induction Generator</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated-Gate Bipolar Transistor</td>
</tr>
<tr>
<td>LHP</td>
<td>Left Half Plane</td>
</tr>
<tr>
<td>LPF</td>
<td>Low Pass Filter</td>
</tr>
<tr>
<td>LVRT</td>
<td>Low-Voltage Ride-Through</td>
</tr>
<tr>
<td>MP</td>
<td>Multivariate Polynomial</td>
</tr>
<tr>
<td>MPC</td>
<td>Mean Pressure Control</td>
</tr>
<tr>
<td>MPP</td>
<td>Maximum Power Point</td>
</tr>
<tr>
<td>MPPT</td>
<td>Maximum Power Point Tracking</td>
</tr>
<tr>
<td>MSC</td>
<td>Machine-Side Converter</td>
</tr>
<tr>
<td>OTC</td>
<td>Optimal Torque Control</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
</tr>
<tr>
<td>PCU</td>
<td>Power Conversion Unit</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional-Differential</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional-Integral</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Differential</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Lock Loop</td>
</tr>
<tr>
<td>PMSG</td>
<td>Permanent Magnetic Synchronous Generator</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>RHP</td>
<td>Right Half Plane</td>
</tr>
<tr>
<td>RPM</td>
<td>Revolutions Per Minute</td>
</tr>
<tr>
<td>RSC</td>
<td>Rotor-Side Converter</td>
</tr>
<tr>
<td>RMS</td>
<td>Root-Mean-Square</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root-Mean-Square Error</td>
</tr>
<tr>
<td>SCIG</td>
<td>Squirrel-Cage Induction Generator</td>
</tr>
<tr>
<td>SG</td>
<td>Synchronous Generator</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
</tr>
<tr>
<td>SMIB</td>
<td>Single Machine Infinite Bus</td>
</tr>
<tr>
<td>SPWM</td>
<td>Sinusoidal Pulse Width Modulation</td>
</tr>
<tr>
<td>SSC</td>
<td>Stator-Side Converter</td>
</tr>
<tr>
<td>SVOC</td>
<td>Stator Voltage Oriented Vector Control</td>
</tr>
<tr>
<td>SVPWM</td>
<td>Space-Vector Pulse Width Modulation</td>
</tr>
<tr>
<td>TES</td>
<td>Thermal Energy Storage</td>
</tr>
<tr>
<td>TITO</td>
<td>Two-Input Two-Output</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage-Source Converter</td>
</tr>
<tr>
<td>WECS</td>
<td>Wind Energy Conversion System</td>
</tr>
<tr>
<td>WTG</td>
<td>Wind Turbine Generator</td>
</tr>
</tbody>
</table>
# List of Figures

Figure 1.1. Worldwide installed and cumulative concentrated solar power capacity since 1984 ......................................................................................................................... 2
Figure 1.2 Four types of concentrated solar power generation technologies .......... 4
Figure 1.3. Schematic of the typical dish-Stirling system ...................................... 6
Figure 1.4. Representation of the general model of the DS systems for investigation of grid-related issues ......................................................................................... 10
Figure 2.1. Thermodynamic state diagrams of ideal Stirling cycle: (a) $p-V$ diagram (b) $T-S$ diagram .................................................................................................................. 23
Figure 2.2. Engine arrangement and piston operation ............................................. 23
Figure 2.3. Arrangement of the compartments for ideal adiabatic analysis ............ 26
Figure 2.4. Temperature distribution of ideal adiabatic model ............................... 26
Figure 2.5. A generalized cell of the working space .............................................. 26
Figure 2.6. Four-cylinder double-acting kinematic configuration of Stirling engine. 31
Figure 2.7. Waveforms of initialization of ideal adiabatic model .............................. 32
Figure 3.1. Waveforms of the original and approximate waveforms of the instantaneous pressure .................................................................................................................. 38
Figure 3.2. Power and efficiency curves of Stirling engine considering different types of thermal and mechanical losses ................................................................. 47
Figure 3.3. Typical power curves of Stirling engine ............................................... 47
Figure 3.4. Test model for the comparison of the Stirling engine models ................ 50
Figure 3.5. Step response under constant heater temperature for engine speed change. ............................................................................................................................... 51
Figure 3.6. Ramp response under constant heater temperature for total mass/mean pressure change ................................................................................................. 52
Figure 3.7. Block diagram of the general normalized average-value model of the Stirling engine ............................................................................................................. 53
Figure 3.8. Block diagram of the normalized constant-speed average-value model of the Stirling engine. ................................................................. 54
Figure 4.1. Components of constant-speed dish-Stirling systems. ............... 56
Figure 4.2. Block diagram of the concentrator and receiver. ....................... 59
Figure 4.3. Block diagram of dish-Stirling absorber temperature control scheme. .... 60
Figure 4.4. Block diagram of MPC within the temperature control scheme. .......... 60
Figure 4.5. Direction convention of stator and rotor currents. ......................... 62
Figure 4.6. Schematic of the SCIG and the interaction with grid and the prime mover. .................................................................................................. 65
Figure 4.7. Per-unit system conversion of dish-Stirling system. ....................... 66
Figure 4.8. Linearized plant model for the design of temperature control of the heat absorber in the Stirling engine. ................................................................. 68
Figure 4.9. Block diagram of the linearized model considering electromagnetic torque as a disturbance. ................................................................. 69
Figure 4.10. Pressure versus temperature relationship with droop characteristic. .... 70
Figure 4.11. Block diagram of the droop temperature controller. ..................... 70
Figure 4.12. A new temperature controller with transient droop and feedforward compensation. ................................................................. 71
Figure 4.13. $P_m – I$ relationship under constant speed and variable speed operations of the dish-Stirling system. ................................................................. 74
Figure 4.14. Steady-state feasible operating area of dish-Stirling system. ........... 75
Figure 4.15. Testing system: conventional temperature control system of DS system. .................................................................................................. 78
Figure 4.16. Simulation results for model comparison with small change on engine speed $\Delta \omega_m = 0.1$ p.u. ........................................................................... 79
Figure 4.17. Simulation results for model comparison with large change on engine speed ($\Delta \omega_m = 0.4$ p.u.). ................................................................. 80
Figure 4.18. Simulink model of dish-Stirling solar thermal plant and grid system. .... 81
Figure 4.19. Comparison between the simulation results using the ideal adiabatic and the average-value adiabatic models.................................................................82
Figure 4.20. Open-loop Bode diagram of the systems with and without transient droop compensation. ........................................................................................................83
Figure 4.21. Comparison between the simulation results using the traditional and improved temperature controllers under insolation variation. .........................84
Figure 4.22. Comparison between the simulation results using the traditional and improved temperature controllers during grid-fault condition. Solid line: with transient droop and feedforward compensation; Dashed line: with transient droop compensation only; Dotted line: without transient droop and feedforward compensation..............................................................................................85
Figure 5.1. Schematic diagram of one (a) DS-DFIG unit and one (b) DS-PMSG unit in a large-scale DS solar-thermal power plant. .................................................................88
Figure 5.2. Block diagram of the general normalized average-value model of the prime mover: dish and Stirling engine. .................................................................................90
Figure 5.3. Topology of back-to-back PWM voltage source converters. ...............91
Figure 5.4. Schematic of the GSC and LC filter. .........................................................92
Figure 5.5. Block diagram of the per-unit model of dc-link, GSC and LC filters. .....93
Figure 5.6. Block diagram of the $d$-axis and $q$-axis current control loop of DFIG (generator base). .......................................................................................................95
Figure 5.7. Simplified double-loop feedback control of DFIG (generator base). .......97
Figure 5.8. Simplified double-loop feedback control of PMSG (generator base). .....98
Figure 5.9. Operating region and MPPT curve of DS-DFIG system. .......................99
Figure 5.10. Steady-state relationship between optimal engine speed and insolation for DS-DFIG system. .................................................................101
Figure 5.11. Steady-state relationship between optimal engine speed and insolation for DS-PMSG system.........................................................................................102
Figure 5.12. Linearized variable-speed DS plant model for temperature control system design (engine base). ................................................................. 107
Figure 5.13. Block diagram of classic control method with droop characteristic. ... 107
Figure 5.14. Generalized closed-loop root locus as insolation changes. (a) $D_p = 0.04$ p.u.; (b) $D_p = 0.036$ p.u.; (c) with transient droop compensation. ................. 109
Figure 5.15. Design of the local temperature controllers using pole placement method. ............................................................................................................. 111
Figure 5.16. The pertinence function of the insolation level. ....................... 113
Figure 5.17. Schematic of the overall control structure for the DS-DFIG system (PCU side). ........................................................................................................ 114
Figure 5.18. Schematic of the overall control structure for the DS-PMSG system (PCU side). ........................................................................................................ 114
Figure 5.19. Simulink model of variable speed dish-Stirling solar thermal plant using DFIG. ........................................................................................................ 115
Figure 5.20. DS-DFIG responses to 0.05 p.u. step increase in insolation $I$ at $I_0 = 0.9$ p.u. Solid line: fuzzy supervisory control; dashed line: droop control; dotted line: full state feedback control tuned at $I = 0.6$ p.u. (medium level). ....................... 117
Figure 5.21. DS-DFIG responses to a ramp increase in insolation $I$ at 0.1 p.u./s. Solid line: fuzzy supervisory control; dashed line: droop control; dotted line: full state feedback control tuned at $I = 0.6$ p.u. (medium level). .............................. 118
Figure 5.22. Comparison of generated power between variable- and constant-speed DS systems. ........................................................................................................ 119
Figure 6.1. Block diagram of the dc motor model with one-mass shaft system.... 122
Figure 6.2. Schematic diagram of the dc motor.............................................. 122
Figure 6.3. Schematic of several commonly used dc power supplies. (a) Buck-boost dc-dc chopper (b) Diode bridge rectifier with three-phase variable ac source (c) Active rectifier with constant ac source.............................................. 124
Figure 6.4. Block diagram of the torque/current control system of the dc motor.... 125
Figure 6.5. Block diagram of a dynamic DS-Simulator ................................................. 127
Figure 6.6. Comparison between the simulation results from the dynamic DS-simulator and the original DS model. ................................................................. 128
Figure 6.7. Schematic of experimental setup for DS-DFIG system ......................... 130
Figure 6.8. Experimental setup of the DS-DFIG system with steady-state DS simulator ............................................................................................................. 130
Figure 6.9. Experimental results obtained at maximum power operation when I = 0.626 p.u. (a) dc armature voltage and current (b) AC stator voltage and current. ............................................................................................................. 131
Figure 7.1. Proposed scheme of a DS-DFIG system using a common GSC unit. .... 134
Figure 7.2. Aggregated effects of the DS-DFIG plant, the unit of the horizontal axis is second ................................................................. 136
Figure 7.3. Block diagram of the speed regulation system of synchronous generator. ............................................................................................................. 137
Figure 7.4. (a) Supplementary inertia control loop for optimal torque control scheme to provide frequency response (b) Proposed supplementary inertia control loop for feedback PI control to provide frequency response, where $K’_{sc}>0$ ........................................... 139
Figure 7.5. Supplementary control loop from temperature control to provide frequency response ............................................................................................................. 141
Figure 7.6. Simulation result of: dashed lines: no frequency response from temperature control system; solid lines: with frequency response from temperature control system. The unit of the x-axis is $second$ ............................................................................................................. 142
Figure 7.7. Operating region and tracking curve when de-loaded. ....................... 144
Figure 7.8. Proposed droop control loop to provide primary and secondary frequency support ............................................................................................................. 145
Figure 7.9. (a) Droop constant $K_{droop}$ vs. insolation level $I$; (b) Slope $k_{I}$ vs. insolation level $I$. ............................................................................................................. 146
Figure 7.10. $\Delta f$ surface as a function of $I$ and $\Delta P_{load}$ ............................................ 147
Figure 7.11. A 5-bus test system for the study of frequency support (Base power: 1000 MVA). ................................................................. 148

Figure 7.12. Simulation results with and without frequency support from DS-DFIG system when synchronous generator performs primary frequency control. The unit of the x-axis is second. ................................................................. 150

Figure 7.13. Simulation results with and without supplementary control loop from DS-DFIG system when synchronous generator performs secondary frequency control. The unit of the x-axis is second. ................................................................. 151

Figure C.1. Mechanical power $P_m$ vs. engine speed $\omega_m$ characteristics .......... 160

Figure C.2. Mechanical power $P_m$ vs. mean pressure of the working gas $p_{\text{mean}}$ characteristics ................................................................. 161

Figure C.3. Mechanical torque $\tau_m$ vs. engine speed $\omega_m$ characteristics .......... 161

Figure C.4. Mechanical power $P_m$ surface as a function of mean pressure of the working gas $p_{\text{mean}}$ and engine speed $\omega_m$ ................................................................. 162

Figure C.5. Absorbed heat transfer rate $q_h$ vs. engine speed $\omega_m$ characteristics .... 163

Figure C.6. Absorbed heat transfer rate $q_h$ surface as a function of mean pressure of the working gas $p_{\text{mean}}$ and engine speed $\omega_m$ ................................................................. 164
List of Tables

Table 1-I National concentrated solar power capacity in 2013..........................3
Table 1-II Cost, worldwide generating capacity, and performance data of CSP
    technologies........................................................................................................5
Table 6-I Results of experimental from DS-DFIG system.................................131
Table C-I Data of Stirling engine (Part 1)............................................................159
Table C-II Data of Stirling engine (Part 2)...........................................................163
Chapter 1  Introduction

1.1. Background

In recent years, the power systems have experienced drastic structural changes in the manner of the emergence of renewable energy sources, restricting of the utility markets and ever-increasing importance of the distributed generation (DG). The reasons for these changes are mainly the limited fossil fuel resources, rising fuel prices and concerns for the environment pertaining to pollution and carbon dioxide emission. According to the World Coal Institute, the current coal reserves will only support human activities for 130 years, natural gas for 60 years and oil for 42 years, at current rate of consumption [1]. Hence there are many opportunities for the development of clean, emission-free and grid-connected distributed generation sources such as solar, wind, fuel cells and other alternative power sources.

1.1.1. Potential of Concentrated Solar Power

Among the various types of renewable energy sources, solar power generation has shown itself to be a viable technology and has sufficient quantity to meet the entire energy demand of human beings. The amount of available solar power on the earth surface is about 5000 times the current world energy needs. There are two dominant types of solar power collection technologies: photovoltaic (PV) solar cells and solar thermal collectors. Photovoltaic solar cell technology has been developed and commercialized for many years and its market booms over the last few years. It converts the energy in sunlight radiation directly into electricity using semiconductor solar panels. While the second technology, commonly called solar thermal or concentrated solar power (CSP), is to use lenses or mirrors to focus sun radiation onto a smaller area, where very high temperature is obtained. The thermal energy with this high temperature can be used directly or indirectly by heat engines and electric generators to produce electricity, normally with higher energy conversion efficiency compared with the PV technology. It has been identified that the solar thermal technology has the potential to become one of the dominant solutions for the future global energy demand due to the ubiquity and large amount of the solar energy source [1].
The amount of the concentrated solar power generation can make up a significant portion of the global renewable generation capacity. Figure 1.1 shows the worldwide installed and cumulative CSP capacity since 1984. Global installed CSP capacity has increased ten times since 2004 and grew nearly 50 percent per year during the last five years [2-4]. Often the CSP have a larger generating capacity than PV installations since solar thermal is used exclusively in utility applications. PV, on the other hand, is used in residential, commercial, industrial, and utility applications, but typically has a generating capacity much smaller than solar thermal technologies. PV has a higher expected annual growth rate (8.6%) than solar thermal technologies (2.2%) through 2035, but the predicted levelized cost of electricity (LCOE) for solar thermal (256.6 $/MWh) is lower than PV (396.1 $/MWh) for plants entering service in 2016 [5]. The National Renewable Energy Laboratory (NREL) maintains a database of solar thermal projects in the U.S. [6], and gives a much higher capacity of solar thermal installations in the next few years than given in [5]. Nevertheless, growth in solar power generating capacity is expected due to an increasing need for clean and renewable energy.

Table 1-I shows the total and installed concentrated solar power capacity in 2013 by country. Worldwide installed capacity increased by 36% or nearly 0.9 GW to more
than 3.4 GW. Spain and the United States remained the global leaders, while the number of countries with installed CSP was growing. There is a notable trend towards developing countries and regions with high solar radiation [2].

Table 1-I National concentrated solar power capacity in 2013.

<table>
<thead>
<tr>
<th>Country</th>
<th>Total (MW)</th>
<th>Added (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>2300</td>
<td>+350</td>
</tr>
<tr>
<td>United States</td>
<td>882</td>
<td>+375</td>
</tr>
<tr>
<td>United Arab Emirates</td>
<td>100</td>
<td>+100</td>
</tr>
<tr>
<td>India</td>
<td>50</td>
<td>+50</td>
</tr>
<tr>
<td>Algeria</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>Egypt</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Morocco</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Australia</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>China</td>
<td>10</td>
<td>+10</td>
</tr>
<tr>
<td>Thailand</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

CSP requires direct normal irradiance (DNI) to generate electricity, which is different from the photovoltaic technologies that can use diffuse sunlight. DNI is the portion of the sunlight comes directly from the sun. Nevertheless the potential resource for CSP generation is very significant. Take the U.S. as an example, the southwest of the U.S. has a solar generation capacity of more than 11 million MW that is suitable for CSP [7], while the entire electricity generation capacity of the U.S. is slightly more than 1 million MW.

1.1.2. Concentrated Solar Power Technologies

Concentrated solar power harnesses thermal energy from sunlight to produce electricity. A large number of mirrors are arranged in such a way to concentrate sunlight onto a small area, where the high-temperature heat is converted into electrical power by driving a heat engine which is coupled to an electric generator. As illustrated in Figure 1.2, currently, four types of CSP technologies have been developed for electrical power generation, including [8]:

3
Figure 1.2 Four types of concentrated solar power generation technologies.

1) Parabolic trough
2) Fresnel reflector
3) Solar power tower
4) Dish-Stirling

Parabolic troughs and Fresnel reflectors use single-axis-tracking parabolic mirrors to focus the sunlight to heat up a heat-transfer fluid (HTF, usually oil) which in turn boils water to produce steam and drives a turbine. A solar power tower utilizes dual-axis tracking reflectors (heliostats) to concentrate the sunlight onto a central receiver atop a tower. On the other hand, dish-Stirling (DS) system uses a parabolic reflector to concentrate the sunlight to drive a closed-cycle, external combustion heat engine called Stirling engine, where thermal energy is converted into mechanical energy directly. Then electricity is generated from the electrical generator coupled to the heat engine. Among these four technologies, parabolic trough is the most developed CSP technology. A number of parabolic trough systems have been built and operated commercially for many years in the U.S. and Spain, among other
countries. Studies about the potential and the future of CSP from Greenpeace International have shown that CSP could account for up to 25% of the world’s energy needs by 2050 [9].

Table 1-II shows a comparison of the four types of CSP [5, 10]. It can be seen that dish-Stirling technology has demonstrated the highest instantaneous and annual efficiency amongst the CSP technologies [11]. Although dish-Stirling technology is not as mature as the trough and tower designs currently, and it is undergoing several technical and economic difficulties in commercializing in recent years, its advantage of higher energy conversion efficiency and the potential for low-cost mass-production level makes the technology a significant competitor in the future renewable energy markets.

Table 1-II Cost, worldwide generating capacity, and performance data of CSP technologies

<table>
<thead>
<tr>
<th></th>
<th>Capacity by 2013 (MW)</th>
<th>Capacity by 2018 (MW)</th>
<th>Largest Installation (MW)</th>
<th>LCOE ($/MWh)</th>
<th>Annual Efficiency</th>
<th>Installed Cost ($/kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parabolic Trough</td>
<td>3869</td>
<td>4560</td>
<td>354</td>
<td>50-110</td>
<td>13-17%</td>
<td>2805-4900</td>
</tr>
<tr>
<td>Fresnel Reflector</td>
<td>46.65</td>
<td>52.65</td>
<td>31.4</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Power Tower</td>
<td>469.4</td>
<td>961.8</td>
<td>392</td>
<td>40-256.6</td>
<td>14-18%</td>
<td>2500-6800</td>
</tr>
<tr>
<td>DS</td>
<td>1.5</td>
<td>2.5</td>
<td>1.5</td>
<td>60-400</td>
<td>20-26%</td>
<td>3000-8600</td>
</tr>
</tbody>
</table>

1.1.3. Dish-Stirling Concepts

Dish-Stirling system tracks the sun and focus solar energy into a receiver where it is absorbed and transferred to a Stirling engine and then an electric generator. Figure 1.3 shows a schematic of a typical DS system. The large concentrator consists of a great number of mirrors arranged in the shape of a paraboloid. The direct normal irradiance is reflected and concentrated onto a focal point located at the receiver. The receiver is a hollow chamber, whose walls consist of a large number of metallic tubes called the absorber. The absorber tubes provide the heater volume and function as the heat source for the Stirling engine. In the working volumes of the Stirling engine,
light-weighted working gas (hydrogen or helium) is compressed and expanded alternatively by pistons, supplying the work for a piston-crankshaft drive mechanism. The Stirling engine acts as the prime mover and drives an electric generator, where electricity is generated and transmitted to the electric power grid via step-up power transformers.

Currently the dish-Stirling system is still at the pre-commercial stage and a lot of demonstrating projects have been established located worldwide, most of them in the U.S. and Europe. Although other types of prime movers such as the Brayton engine and micro-turbines have been considered and tested on the dish concentrator to generate mechanical power, and some new-type Stirling engines have been developed in recently years, the kinematic Stirling engine has dominate the application in the existing dish-Stirling projects for its high thermal-to-mechanical efficiency. The pilot 25-kW Vanguard system built by ADVANCO in Southern California achieved a solar-to-electric conversion efficiency of 29.4% [12]. Other dish-Stirling projects were built and maintained by Schlaich-Bergermann und Partner (SBP), McDonnell Douglas Aerospace Corporation (MDAC), Stirling Energy Systems (SES), and Cummins Power Generation [13], etc.

![Figure 1.3. Schematic of the typical dish-Stirling system.](image)
1.2. Motivation

Compared to other types of CSP technologies, currently the DS faces some technical and economic issues which hamper the mass-production and rapid development of large-scale DS solar-thermal power plants. One major reason is that the construction and maintenance of the Stirling engine is costly. While on the other hand, the lack of research on the impact of the DS system on the power grid prevents the further efforts on performance enhancement and cost reduction via better design of the system configuration and the control systems. When large-scale DS systems are incorporated into existing electric power grid, a number of technical requirements must be met [14]:

- Dispatchability as an intermittent energy source (Real power balancing, spinning reserve);
- Power quality (Frequency control, voltage control, flicker mitigation, three-phase balancing, etc.);
- Fault ride-through ability (LVRT, protection);
- System stability.

Such requirements and related issues are often overlapped and interact with each other.

Although there is no commercial dish-Stirling project presently, one should attempt to solve the abovementioned issues before full preparation of the implementation of the large-scale dish-Stirling systems. At the first stage, the studies must start from computer simulation. In any model-based simulation studies, the modeling of physical system is the most important part. It is well-known that the model must comply with the nature of the issues under investigation. The model must be able to represent the physical nature accurately but it is unnecessary and impossible to represent every detail of it. The complexity of the model depends on the research objectives. For the problem in hand, the dish-Stirling system can be divided into two parts:

1. Electromechanical system, including electric generator, power electronic converter, power grid and the corresponding control systems.
2. Thermodynamic system, which governs the behavior of the prime mover for
the electric generator, including solar concentrator, receiver, the Stirling
engine and the corresponding control systems.

It is not necessary to represent all of them in investigating issues pertaining to
grid-connected DS solar-thermal power plant. The complexity of the
electro-mechanical and thermodynamics aspects of the power plant should be
simplified appropriately. Over-detailed description of the model will lead to heavy
computation burden on the simulation study and produces results which are not of
interest. Hence, in the modeling of the solar-thermal power plant, there are

- Thermodynamic model of the Stirling engine: Simplified steady-state and
dynamic models will be derived which are appropriate for the investigation of
grid-connected issues.
- Heat transfer model of the concentrator and receiver/absorber: The thermal
inertia on the absorber is quite large and this fact shall be considered in the
modeling process.
- Shaft system: No existing literature has been found on the topic of torsional
forces of Stirling engine. For the low-rating Stirling engine and electric
generator, the shaft is often considered as completely stiff. This aspect can be
investigated later, if proves necessary.
- Generator models: They shall be similar to those used in wind turbine study.
- Control system for Stirling engine considered includes mean pressure control,
stroke control and phase angle control.
- Protective relay, the characteristic of relay may have impacts on the low-voltage
ride-through (LVRT).
- Power electronic converters for variable speed operation
- Reactive compensation and energy storage

Most of the above components have been properly modeled in previous studies, as
the electromagnetic part of the DS systems have similar configuration as that of the
existing wind energy conversion systems (WECS). The major gap is in the lack of
proper model of the main component of the prime mover, i.e. the Stirling engine.
Although the thermodynamic behavior of the Stirling engine is studied to high degree
and many models with different levels of complexity have been derived, it is unclear which models are more appropriate for the investigation into each specific issue when the dish-Stirling system is connected to the power grid.

Currently there is scarce literature dealing with grid-connected problems of dish-Stirling system. Although several papers [15, 16] have initialed the studies, there are still several challenges concerning the model of Stirling engine:

- Some researchers use ideal adiabatic model of Stirling engine which is too complex for the studies in hand. This model consists of many linear and nonlinear differential and algebraic equations, which only can be solved via numerical method depending on specific configurations and operating conditions. It is not an initial-value problem; rather it is a boundary problem whose initial values cannot be found from the steady state analysis. In fact, there are no steady states for the internal variables as they are cyclic. Although normally the equations can be solved by assigning arbitrary initial conditions but it cannot be guaranteed that the result can converge;

- Representation of internal thermodynamic behavior is unnecessary. The ideal adiabatic model is derived from the thermodynamic analysis of the real engine, so its internal thermodynamic behavior is similar to the real engine. These states include instantaneous value of mass flow rate, pressure and temperature of each working space and heat exchanger. However, those internal details are not important in the studies of grid-connected issues;

- The accuracy of the simulation result obtained from ideal adiabatic model depends heavily on the step size. To obtain accurate result, it requires very small step size which is computationally time-consuming;

- This model is so idealized. The effects of most type of thermal and mechanical losses in the Stirling engine are ignored. However losses are inevitable and they greatly influence the input and output power characteristics and thermal efficiency. Both power and efficiency quantities obtained from this model will deviate from the experimental data significantly. There are other improved models which do consider the effects of losses by thermodynamic analysis, but they are even more complex and the solution of such models is even more difficult to converge. The main research objective of the previous work is to find
Chapter 1 Introduction

the factors that influence on the characteristics of Stirling engine and how to optimize the engine performance;

- Model-based classic control system analysis and design cannot be carried out used such models.

In summary, the computation load is heavy and the simulation results may vary from the real data if the ideal adiabatic model is used. Moreover, no systematic analysis on the design and performance of control system has been found as the existing models are so complex.

In view of the above, two major parts of the present works are:

- Developing proper models of dish-Stirling systems for the investigation of different grid-connected issues;
- Applying these models to study the issues and find the corresponding solutions.

The general representation of DS systems for the investigation of grid-connected issues is shown in Figure 1.4. In this figure, the blocks in solid lines represent the common components used in existing DS systems. The blocks in dashed lines represent the potential areas that can be investigated to improve the system performance and to mitigate grid-related problems. The major topics addressed in this
thesis consist of the blocks with gray background, which will be discussed in greater detail. Amongst them, modeling of the Stirling engine is one of the key components of the studies.

1.3. Objectives

According to the problems in hand, the main research objectives of this thesis includes

- To develop suitable mathematical models of the DS system for different research purposes in the field of power engineering, including prime mover control, real power control and frequency support, in terms of the proper model complexity and compatible computer simulation step size;
- To analyze the characteristics of the DS systems with different system configurations in order to reveal the potential of maximum power harness and to find the factors to limit the power generation capacity;
- To investigate the ability to provide ancillary services from DS system;
- To design multi-objective advanced control system for the DS system in order to enhance the system dynamic performance, increase energy conversion efficiency, and thus reduce the cost of the DS system.

For the conventional constant-speed DS system, the complete models of the dish-Stirling system should be derived. Temperature control of the receiver/absorber is crucial as it ensures the stable and secure operation of the entire system. While the existing models do not reveal the useful characteristic of the system, to derive the transfer functions for temperature control is essential for the control system design based on classical techniques. The control system requires the ability to reject unexpected disturbances, such as the inevitable variations of the solar irradiance and speed variation due to the grid disturbances. Thus, the second objective of the research in this thesis is to analyze and design the control system of the prime mover, to achieve superior temperature control for the constant-speed DS system, and thus enhance the performance of the DS system.

As the constant-speed DS system uses a squirrel-cage induction generator (SCIG), the system is unable to provide the ability for dynamic voltage and frequency support to the grid. Thus, variable-speed DS system is proposed using a doubly-fed induction
generator (DFIG) or a permanent magnet synchronous generator (PMSG). The introduction of variable-speed operation brings difficulties in the temperature control system design. Advanced multi-objective control system is required as the classical controller shall be shown to perform rather poorly for the high-order and highly nonlinear system.

To improve the energy conversion efficiency of the DS system is another objective in this research. The steady-state speed versus power characteristics reveals the potential of higher energy harness from the variable-speed operation of DS system than its constant-speed operation counter-part. Other than the optimization of the design of individual component such as the Stirling engine and the generator, only minimal expense shall be incurred by the proper design of the control system strategies in order to achieve maximum power point tracking.

If large-scale DS system is integrated into an existing power grid, or a DS system is to be used within a small isolated system, the dynamics of the DS system shall have great impact on the external system. High penetration of such renewable energy generating system requires that the DS system to provide enough frequency support to the grid as the conventional frequency control role of the synchronous generating units is displaced. Thus, the design and analysis of the frequency control of the DS system is another objective of the research in this thesis.

1.4. Main Contributions of the Thesis

The main contributions of this thesis are summarized in this section which mainly concerns the following three parts.

- Modeling of the DS solar-thermal power system. In order to provide appropriate models for computer simulation and for the control system design, the complete models for both the conventional constant-speed and variable-speed DS systems are derived. This includes simplified and normalized models of the prime mover.

- Analysis of the steady-state model of DS system. The characteristics for the maximum energy harness and de-loading scheme to provide spinning reserve are designed.
• Control of constant-speed and variable-speed DS systems. Based on the derived models and through analysis, classical and advanced modern control schemes are proposed.

In this thesis, two dynamic models of the double-acting kinematic Stirling engine commonly used in the DS system are derived for computer simulation and control system design. The models only represent the aggregated quantities such as the output power, input heat transfer rate and the mean pressure of the working gas, which are essential for grid-related studies. No internal dynamics are simulated as they are not of major concern. Various thermal and mechanical losses are included by using multivariate polynomial methods. Information on the losses can be obtained from experimental data or the specifications of the Stirling engine provided by the manufactures. The proposed general model of Stirling engine is suitable for the simulation studies of prime mover control system design and to investigate electromagnetic and electromechanical issues including the frequency support, voltage support, system stability analysis. The use of this model reduces the computational burden greatly as the existing model requires very small step size and the computation of the existing models cannot be easily initialized from any given steady state. Normalization of the prime mover is carried out to make it compatible to the existing models of the power generating units. The other proposed model is in the form of transfer function, which is derived by considering the constant speed of the induction generator. This model is suitable for classical control system design method such as PID feedback control, feedforward control and linear compensator. The complete model of the dish-Stirling also includes the concentrator, the receiver, electric generator, power converer and the grid interface. Configurations of variable-speed DS system are proposed using DFIG and PMSG respectively for better energy harness and better system control ability. A small-signal state-space model of the variable-speed DS is derived for advanced control system design by linearizing the plant at specific operating points.

Based on the derived models, the steady-state analysis of the DS system is carried out. The steady-state characteristic of the DS system mainly depends on the characteristics of the Stirling engine in the form of performance maps. From the derived results, it is shown that the constant-speed operation is not harnessing the maximum power. The maximum power point tracking curve requires the Stirling
Chapter 1 Introduction

An engine/generator to operate in a variable-speed manner. The long-term ability of optimal energy utilization is analyzed and compared to the conventional constant-speed operation. It shows an increase amount of energy can be extracted. The tracking curve is converted accordingly into the insolation versus speed profile for the implementation in the speed control system. However, if spinning reserve is required to support the system frequency, the tracking profile needs to be revised. The system must operate under a de-loading condition.

The control system of the DS system is designed based on the derived model and the steady-state analysis. The temperature controller of the constant-speed operation is designed using the transfer functions of the temperature control loop. The performance of the traditional temperature is thus able to be analyzed and designed, while in the past only empirical methods can be used to tune the controller parameters. A transient droop compensator is proposed to damp the system transient caused by the inevitable variations of the solar insolation during the normal operation, and a feedforward compensator is proposed to reject the temporal speed variations of the squirrel-cage induction generator due to disturbances from the external power grid. In the proposed variable-speed DS system, the continuous variations of engine/generator speed cannot be compensated using the above methods. To overcome this problem, a fuzzy supervisory temperature controller is proposed using the derived state-space model, considering the variations of the solar insolation are much slower than most of the dynamics of the DS system.

1.5. Organization of the Thesis

There are eight chapters in this thesis which is organized as follows:

Chapter 1 introduces the background and motivations of the thesis. The basic concept of DS system is introduced. The objectives and the major contributions of this thesis have also been presented.

In Chapter 2, some literature which address associated research work with this thesis are reviewed. The preliminary considerations and basic thermodynamics of the Stirling engine are presented for the derivation of the model of dish and Stirling engine suitable for grid-related studies.

In Chapter 3, based on thermodynamic analysis given in Chapter 2, two average-value models for four-cylinder double-acting kinematic Stirling engine,
including a general model for variable-speed operation and a specific model for constant-speed operation are developed for the studies of DS solar-thermal power generation. Model comparison based on computer simulation with several case studies will be given to validate the accuracy of the developed models.

The focus of Chapter 4 is to develop a suitable model for such a generating system and then design the appropriate control system to achieve temperature control of the receiver/absorber. Linearized model of the constant-speed DS system is derived for temperature control system design. A new temperature controller with transient droop compensation and feedforward compensation is proposed to solve the problem caused by the non-minimum phase and nonlinear characteristic of the system model. Computer simulation shows that the system is well damped at the occurrence of system disturbance such as the insolation variation and grid fault. From the steady-state analysis, the potential of maximum energy harness of DS system is discussed and it shows that variable speed operation of DS system is a viable method to extract considerable more power.

In Chapter 5, the overall configuration of a DS-DFIG system is proposed and the model of the system is derived especially for the design of temperature control while achieving potential maximum power harnessing revealed in Chapter 4. The computer simulation results are given to shown the performance of the proposed temperature controllers to overcome the problems introduced by the continuous engine speed variation which is not seen in the case of the constant-speed DS system.

In Chapter 6, a DS-DFIG simulator is developed in the author’s laboratory with the view to study the system without using real a Stirling engine, which is difficult to construct in the current research stage. The developed system is based on the use of a separately-excited dc motor to generate the equivalent mechanical torque from the DS under the specific insolation level and engine/motor speed. Using this DS-DFIG simulator, the maximum energy harness ability of the DS-DFIG system is examined.

In Chapter 7, the ability of frequency support of the DS-DFIG system is discussed. An overall system configuration for the DS-DFIG plant is proposed for large-scale integration. Based on the model derived in the previous chapters, various methods to provide inertia and frequency support from the DS-DFIG system are proposed and discussed. Numerical examples are included.
The conclusions of the thesis and some recommendations for future work are presented in Chapter 8.
2.1. Introduction

In this chapter, some existing literature addressing associated research studies with this thesis will be reviewed. The preliminary considerations and basic thermodynamics of the Stirling engine will be presented for the derivation of the model of the dish-Stirling system suitable for grid-related studies.

2.2. Literature Review

2.2.1. Development of Stirling Engine

A Stirling engine is a closed-loop external combustion engine. It operates by cyclic expansion and compression of a light-weighted working gas such as helium or hydrogen at different temperature. The Stirling engine technology has been around since it was first invented by Robert Stirling in 1816 and was patented in [17]. In 1853, John Ericsson built a large Stirling engine with brake power 220 kW. After that, however, for many years, due to technology and material limit the development of Stirling engine is slow, as many difficulties could not be resolved easily. These difficulties include, amongst others, lubrication of the pistons, seals and sealing, adequate and uniform heat transfer at high temperature to the working gas, and regenerator contamination, etc. In the meanwhile, internal combustion engines and electric machines became the mainstream due to the giant strides had been made for centuries [18].

During the past few decades, many of difficulties have been largely resolved as the technologies are improved on many fields such as materials and manufacturing technologies. People began to find the advantages of the Stirling engine:

- High thermal efficiency
- Low noise
- Low water consumption
- Availability to use various heat sources including fossil and renewable fuels, solar fuels and waste heat
- No CO₂ emission
Since 1937 when great process was made on the development of Stirling engine by Philips Research Laboratory in Holland, the performance of the Stirling engine has been enhanced greatly. The specific power of 102C engine of 1952 was 30 times that of the old Stirling engines [19]. Recent application of the dish-Stirling solar-thermal generating system is of great interest as its attributes of high efficiency and no harmful gas emission make it a suitable technology for future power supply [13].

The components of a Stirling engine include two volumes at different temperature connected to each other through a regenerative heat exchanger and auxiliary heat exchanger. The gas dynamic and heat transfer process needs to change periodically to fulfill thermodynamic. Thus, the drive mechanism of a Stirling engine must be able to reproduce the volumetric changes. A comprehensive listing of possible engine arrangement of Stirling engine is given in [20]. Various parameters needs to be considered when selecting proper Stirling engine configuration for different applications, including engine cylinder arrangement, engine mechanism, heater type, displacer and piston construction, type and size of regenerator and crankcase construction [21]. The cylinder arrangement can be broadly classify into single-acting and double-acting arrangements. The double-acting engines use both sides of the piston to move the working fluid from one space to another. Most of the developed Stirling engines use double-acting arrangement of cylinders. Piston coupling types include slider crank drive, rhombic drive [22], Ross rocker [23] and Ringbom type [24] of swash plate [25].

Above kinematic configurations have made use of drive mechanism in which the piston is connected to the connecting rod by some form of mechanical linkage, which in turn is attached to a power output shaft. However, Stirling engine can operate without the piston being coupled mechanically. The concept is firstly realized in practice by William Beale and called free-piston Stirling engine (FPSE). [26] have shown that these engines can be used for generating power by using waste. FPSE is suitable for pumping water reciprocating pumps and linear alternator [27]. [28] have recently tried a cam mechanism for conversion of reciprocating motion in to rotary motion for Stirling engines. The mechanism may prove its suitability for engine operation for variety of type of engine configurations. However, no existing application in dish-Stirling solar thermal power generating is found currently.
2.2.2. Modeling of the Stirling Engine

Research on the Stirling engines can be greatly helped by establishing suitable steady-state and dynamic mathematical models for different research objectives. For the objective of design and optimization of the engine performance, per-cycle (cyclic) analysis is often used. The existing thermodynamic models of the Stirling engine are commonly categorized into three groups:

1) Zero-Order Model

A zero-order model is usually obtained from empirical observation and described as the rudimentary “back of the envelope” type calculations. Its genesis lies in empirical observation and experience more than mathematical and scientific principles. The equations for these models featured with some empirical parameters to characterize the performance of Stirling engine, including the famous Beale Number and West Number [29].

2) First-Order Model

The mathematical expressions of the first-order models are similar to that of the zero-order models, while they are developed analytically based on the isothermal analysis developed by German mathematician Gustav Schmidt [30]. Example of such models are the Schmidt equations developed by Mayer [31], Senft [32], Cooke-Yarborough [33], Walker [20], Meijer [34], Finkelstein [35], etc.

3) Second-Order Model

A second-order simulation model refers to the model derived from cyclic analysis focusing on cyclic thermodynamic and gas dynamic behavior. It is often called the nodal or finite-cell analysis and strives to find the numerical solutions to the various thermodynamic variables, e.g. temperature and pressure, as the function of location and time throughout the engine [36]. A most referred second-order model was developed in [37] using a one-dimensional approach with complete differential equations of continuity. [38] continued to improve on the work [37], e.g. by including viscous dissipation in the working gas and identifying errors and unnecessary assumptions and proposed corrections thereof. Abovementioned methods simulate the engine dynamics based on the ideal adiabatic cycle with loss mechanisms included and parasitic losses are only considered afterwards. These methods are less accurate while they are particularly suitable for preliminary design of engine [39]. A model
Chapter 2 Dish-Stirling Concentrated Solar Power Technology – A Review

with simplified variation of this approach was developed in [30] that used as a basis for the course at Ohio University in Athens, Ohio, on Stirling cycle machine analysis. In [40], a Heinrici Stirling engine with this simplified variation approach was simulated and the results are compared with experimental data, although Heinrici Stirling engine is not a high-efficiency engine. In recent years various other dynamic models were developed based on the second-order models, such as [41-44], with the efforts to represent the thermal losses more accurately. However, most of these research works focused on the design and optimization the performance of the engine. Literature [45] attempted to simplify the dynamic model of Stirling engine by approximating all the fast-varying oscillating variables such as instantaneous temperature, mass and pressure with sinusoidal signals. This method reduces the computation load for computer simulation. Other improvements on the Stirling engine models can be found in [46-59].

2.2.3. Control of the Stirling Engine

Control systems are necessary to regulate the torque and power of the Stirling engine. Traditionally, the rotating speed of the engine shaft is kept constant with varying load condition in the case of stationary-constant speed, fixed frequency electric power generators using a squirrel-cage induction generator. Mean pressure control (MPC) is most commonly used [60] by supplying and dumping working fluid using valve system. This quite simple conceptual system becomes more complex in practice, since the timing and degree of valve opening has to be carefully controlled and monitored. Increase in the dead volume within the Stirling engine system results in a loss of power but not necessarily a reduction in efficiency. By providing extra spaces the dead volume can be increased or decreased. [61] analyzed the effect of dead volume variation on Stirling engine performance. The engine was operating in a closed regenerative thermodynamic cycle by using polytrophic processes for the power and displacement pistons. General Motors adopted a power control system based on phase angle variation for their V-Eight engine. However, this method is not applicable to double-acting engines.

2.2.4. Modeling of the concentrator and receiver

The research work on heat transfer and thermal model of the concentrator and receiver is reported in a number of literature. An analytical model for estimation of
convective heat losses from cavity receivers was presented in [62]. A thermal model of energy conversion of EuroDish dish-Stirling system was presented in [63], and the simulation gave good results compared to the experimental measurements. [64] investigated the performance of the system based on the linearized heat loss model of the concentrator.

### 2.2.5. Dish-Stirling system

Existing literature on grid-connected application is limited as no commercial DS project is built up at present and the model of Stirling engine for such applications is not well-studied. Most of such literature based on the steady-state model focus on the analysis of the performance of energy conversion efficiency such as [13, 65, 66]. Literature [14, 67] presented some related interconnection issues of traditional constant-speed DS system, including power factor correction, self-excitation over voltage, low voltage ride through, and reactive power and voltage control. In order to study such issues and find proper solutions, an integrated DS system model was first proposed in [15] and its control system was investigated in [16, 60] based on mean pressure control technique, where the main control objective of constant-speed DS is to maintain the receiver/absorber temperature as certain high value to ensure its efficient and secure operation. While in [16] it proposed an over-speed control loop to enhance the system’s grid fault ride-through capacity, the receiver temperature will rise which would damage the materials. The effects of the variable solar irradiance on the reactive power compensation for large DS solar farm is investigated in [68]. Literature [69] proposed a variable-speed dish-Stirling system using doubly-fed induction generator to achieve maximum power point tracking. However, the detail thermodynamic model of the dish and Stirling engine is simply reduced to a first-order transfer function and no corresponding prime mover control system is given. Thus, the impact of the various grid dynamics on the prime mover is unknown. A DS system based on permanent magnet synchronous generator is studied in [70] to find similar higher power harness results, while it use the detailed second-order model of Stirling engine same as that in [15]. Recently another DS system based on a novel high-temperature superconducting linear synchronous generator was proposed and designed [71]. Hybrid systems with other energy sources was proposed in literature [72, 73], of which the complete modeling and control strategy with a wind energy conversion system was given in [72] and a hybrid gas/solar DS system was designed
in [73]. Again, detailed model of the Stirling engine was used of the computer simulation.

2.3. Preliminary Considerations on the Modeling of Stirling Engine

To study dish-Stirling solar-thermal generating system, suitable models of the Stirling engine are required. Although there are many newer types of Stirling engine being studied in recent years [74-76], most of the reported works focus on developing the suitable models for the kinematic type of Stirling engine using ideal adiabatic analysis and with the inclusion of various forms of thermal losses [30, 41, 77]. The main purpose of these detailed models is for the design and optimization of engine performance. However, high-frequency cylindrical motions of the internal variables in Stirling engine are of no interest for most grid-related studies, in which the Stirling engine is considered as part of the prime mover which provides mechanical power for the electrical generator. Furthermore, the detailed thermodynamic models consist of many high-order nonlinear and discontinuous differential and algebraic equations and they require small step size (e.g. 2 µs) to yield accurate solutions [30]. The required step size is incompatible with many electromechanical power system studies and would incur heavy computational load. Besides, detailed models are boundary-value problems, the solution of which can only be obtained by assigning arbitrary initial conditions. By incorporating the simulation program of Stirling engine into the power system model, the solution may not even converge. Hence, using the detailed models in power system-related studies may be problematic. Also, another noticeable drawback of the detail models is that in the context of model-based design of control system, the detailed models are so complex that they cannot reveal any external dynamic characteristics of Stirling engine. Thus very often, the design of the control system is based on empirical methods.

2.4. Thermodynamics of Stirling Engine

2.4.1. Ideal Stirling Engine Cycle

The ideal thermodynamic cycle of Stirling engine consists of four processes and it is commonly explained [20, 36, 78] using the thermodynamic state diagrams shown in Figure 2.1. These processes consist of two isentropic processes (process 1–2 and process 3–4) and two isothermal processes (process 2–3 and process 4–1). The
physical diagrams of the full operation cycle of the cylinder and pistons can be described with the aid of Figure 2.1, considering a cylinder containing two opposing pistons with a regenerator between the pistons.

![Figure 2.1. Thermodynamic state diagrams of ideal Stirling cycle: (a) p-V diagram (b) T-S diagram.](image)

![Figure 2.2. Engine arrangement and piston operation.](image)
In Figure 2.2, the left space is heated by some form of heat source, thus it is commonly called the heater. In the dish-Stirling applications, the heat source is from the thermal receiver where solar thermal energy is concentrated. The right space is cooled by any methods and it is called the cooler. The mesh-like substance in the middle is called regenerator which functions like a thermal sponge alternatively absorbing and releasing heat. It is a matrix of finely divided metal in the form of wires or strips. The space between the left piston and the regenerator is called expansion space, and the one between the right piston and the regenerator is called compression space. The temperature gradient of $T_h - T_k$ between the ends of regenerator is maintained.

To start with a cycle, it is assumed that the compression space piston is at outer dead point and the expansion space piston is at inner dead point close to regenerator. All working fluid is in the cold compression space. The compression volume is at maximum and the pressure and temperature are at their minimum values represented by point 1 on the $P-V$ and $T-S$ diagrams in Figure 2.1. The four processes of the thermodynamic cycle are described below:

**Process 1–2 Isothermal compression process**: At the very beginning, the left piston rest at the inner point next to the left side of regenerator. The volume of expansion space is zero. During the compression process, the right piston move to the left but the left piston remains still. The working gas is compressed, pressure is increased and the temperature does not change due to the isothermal nature of this process.

**Process 2–3 constant volume regenerative transfer process**: When the working fluid in the compression space cannot be compressed any more, the left piston will move to the left together with the right piston. The working fluid will go through the regenerator to the expansion space. The total volume of working gas is kept constant, but as the temperature is increasing by heat transfer from the regenerator, the pressure also increases, which can also be seen in the P-V diagram of Figure 2.1.

**Process 3–4 Isothermal expansion process**: The right piston will stop moving when it reaches the rightmost point of the regenerator. However, the left piston will still move to the left as the working gas is expanding due to the heat transfer from
receiver. The volume of the total working space increases which means the working fluid is doing work.

**Process 4–1 Constant volume regenerative transfer Process:** Both pistons move simultaneously to the right and transfer the working fluid from expansion space to compression space. Heat is stored in the regenerator, thus the internal energy decreases which results in the decrease of temperature.

This Stirling cycle is a highly idealized thermodynamic cycle, which consists of two isothermal and two constant volume processes and the cycle is thermodynamically reversible. The first assumption of isothermal working and heat exchange implies that the heat exchangers are required to be perfectly effective and to do so, infinite rate of heat transfer is required between the cylinder wall and working fluid. The second assumption requires zero heat transfer between the walls and working fluid. Both assumptions remain invalid in actual engine operation.

### 2.4.2. Ideal Adiabatic Model

Historically different types of thermodynamic models of the Stirling engine have been developed. A widely accepted model based on ideal adiabatic analysis is most referred in the literature on development of the model of Stirling engine. This model, with non-isothermal working spaces, was first analyzed in [79] and further explored in [80]. The non-isothermal analysis assumes finite heat transfer in the working spaces which results in temperature variations. The engine is configured as a five-component serially-connected model. These five components are called

- Compression volume (c),
- Cooler volume (k),
- Regenerator volume (r),
- Heater volume (h) and
- Expansion volume (e).

The gas in the cooler and heater volumes is maintained under isothermal condition at $T_k$ and $T_h$ respectively. The regenerator matrix, which functions as thermal sponge, and the gas in the regenerator valid volume have the identical linear temperature distribution, the gas flowing through the regenerator/cooler interface being at the cooler temperature $T_k$, and the regenerator/heater interface at the heater temperature $T_h$. 


The working spaces are assumed to be in the ideal adiabatic condition, and thus the temperatures $T_c$ and $T_e$ will vary over the cycle. The arrangement of the compartments and temperature profile are shown in Figure 2.3 and Figure 2.4 respectively.

Figure 2.3. Arrangement of the compartments for ideal adiabatic analysis.

Figure 2.4. Temperature distribution of ideal adiabatic model.

Figure 2.5. A generalized cell of the working space.
To analyze the thermodynamic of the compartment, a generalized cell of the working space is used, as shown in Figure 2.5, where \( m, p, T, V \) are the mass, pressure, temperature and volume of the working fluid respectively. \( Q \) is the absorbed heat and \( W \) is the work done by the cell. Energy equations applied to this generalized cell can be described as

\[
DQ + (c_p T m_i - c_v T o m_o) = DW + c_v D(mT) \tag{2.1}
\]

\[
pV = mRT \tag{2.2}
\]

where \( c_p \) and \( c_v \) are the specific heat capacities of the gas at constant pressure and constant volume respectively and \( R = c_p - c_v \) (Mayer’s relation) is gas constant. The operator \( D \) denotes the derivative with respect to the crank angle \( \phi \), i.e. \( D = d/d\phi \).

The detailed thermodynamic analysis of each space and derivation of the complete ideal adiabatic model can be found in [29, 30] considering the different natures of the spaces. Equation set of the ideal adiabatic model of Stirling engine based on cyclic analysis is given below:

**Volume equations for sinusoidal volume variation:**

\[
v_c = V_{cl} + 0.5V_{sw}[1 + \cos(\phi - \alpha_c)] \tag{2.3}
\]

\[
v_e = V_{el} + 0.5V_{sw}[1 + \cos(\phi - \alpha_e)] \tag{2.4}
\]

\[
Dv_c = -0.5V_{sw}\sin(\phi - \alpha_c) \tag{2.5}
\]

\[
Dv_e = -0.5V_{sw}\sin(\phi - \alpha_e) \tag{2.6}
\]

**Pressure equation:**

\[
Dp = -\gamma p \left( \frac{Dv_c}{T_{ck}} + \frac{Dv_e}{T_{he}} \right) \tag{2.7}
\]

\[
\frac{v_c}{T_{ck}} + \gamma \left( \frac{V_{ch}}{T_k} + \frac{V_{ce}}{T_r} + \frac{V_{he}}{T_h} \right) + \frac{v_e}{T_{he}}
\]

**Mass equations:**

\[
Dm_c = \frac{pDv_c + v_c Dp / \gamma}{RT_{ck}} \tag{2.8}
\]
\[ m_k = \frac{pV_k}{RT_k} \quad (2.9) \]
\[ m_r = \frac{pV_r}{RT_r} \quad (2.10) \]
\[ m_h = \frac{pV_h}{RT_h} \quad (2.11) \]
\[ m_e = M - (m_c + m_k + m_h + m_r) \quad (2.12) \]

Temperature equations:
\[ T_c = \frac{pV_c}{(Rm_c)} \quad (2.13) \]
\[ T_r = \frac{pV_r}{(Rm_r)} \quad (2.14) \]
\[ T_r = \frac{T_h - T_k}{\ln(T_h/T_k)} \quad (2.15) \]

Mass accumulation and mass flow equations:
\[ \frac{Dm_k}{Dk} = \frac{m_k Dp}{p} \quad (2.16) \]
\[ \frac{Dm_r}{Dr} = \frac{m_r Dp}{p} \quad (2.17) \]
\[ \frac{Dm_h}{Dh} = \frac{m_h Dp}{p} \quad (2.18) \]
\[ gA_{ck} = -Dm_c \quad (2.19) \]
\[ gA_{cr} = gA_{ck} - Dm_k \quad (2.20) \]
\[ gA_{rh} = gA_{cr} - Dm_r \quad (2.21) \]
\[ gA_{he} = gA_{rh} - Dm_h \quad (2.22) \]
\[ \frac{Dm_e}{De} = gA_{he} \quad (2.23) \]

Conditional temperature equations:
\[ T_{ck} = \begin{cases} T_c, & gA_{ck} > 0 \\ T_k, & gA_{ck} \leq 0 \end{cases} \quad (2.24) \]
\[ T_{he} = \begin{cases} T_h, & gA_{he} > 0 \\ T_e, & gA_{he} \leq 0 \end{cases} \quad (2.25) \]
**Energy equations:**

\[ DW = p(Dv_e + Dv_r) \]  \hspace{1cm} (2.26)

\[ DQ_k = \frac{c_v}{R}Dp - c_p(T_{ck}gA_{ck} - T_{kr}gA_{kr}) \]  \hspace{1cm} (2.27)

\[ DQ_r = \frac{c_v}{R}Dp - c_p(T_{kr}gA_{kr} - T_{rh}gA_{rh}) \]  \hspace{1cm} (2.28)

\[ DQ_h = \frac{c_v}{R}Dp - c_p(T_{rh}gA_{rh} - T_{he}gA_{he}) \]  \hspace{1cm} (2.29)

where \( \gamma = c_p/c_v \) is the specific heat ratio. \( V_{sw} \) and \( V_{cl} \) are the swept and clearance volumes of the working spaces respectively.

**2.4.3. Ideal Adiabatic Model Considering Temperature and Mass Variation**

In deriving the original ideal adiabatic model (2.3)–(2.29), the heater temperature \( T_h \) and the total mass of the working gas \( M \) are assumed unchanged. However, as shall be explained in latter section on the dish-Stirling system, because the solar insolation \( I \) is time-varying, the imbalance of the thermal power would cause variations in the receiver/absorber and heater temperature \( T_h \). An effective power regulation method is through changing the mean pressure of the working gas by supplying and dumping the working gas using valve system. This will change the total mass \( M \) of the working gas. Considering varying \( T_h \) and \( M \), a revised model was given in [81], where the following six equations are used to replace (2.7), (2.8), (2.23), (2.17), (2.18) and (2.15) in the original ideal adiabatic model respectively. The relevant equations are:

**Pressure equation:**

\[ \frac{\gamma R}{T_{ck}} DM - \gamma p \left( \frac{Dv_e}{T_{ck}} + \frac{Dv_e}{T_{he}} - \frac{V_rDT_r}{T_r^2} - \frac{V_hDT_h}{T_h^2} \right) \]

\[ Dp = \frac{v_e}{T_{ck}} + \gamma \left( \frac{v_k}{T_k} + \frac{v_e}{T_r} + \frac{V_h}{T_h} \right) + \frac{v_e}{T_{he}} \]  \hspace{1cm} (2.30)

**Mass equations:**
Chapter 2 Dish-Stirling Concentrated Solar Power Technology – A Review

\[
Dm_c = \frac{pDv_c + v_c Dp / \gamma}{RT_{ck}} \left( T_k - T_{ck} / T_{ck} \right) DM
\]  

(2.31)

\[
Dm_c = \frac{pDv_c + v_c Dp / \gamma}{RT_{hc}}
\]  

(2.32)

**Mass flow equations:**

\[
Dm_r = m_r \left( \frac{Dp}{p} - \frac{DT_r}{T_r} \right)
\]  

(2.33)

\[
Dm_h = m_h \left( \frac{Dp}{p} - \frac{DT_h}{T_h} \right)
\]  

(2.34)

**Temperature equations:**

\[
DT_r = \frac{DT_h \ln \left( \frac{T_h}{T_k} \right) - DT_h + \frac{T_k}{T_h} DT_h}{\ln \left( \frac{T_h}{T_k} \right)^2}
\]  

(2.35)

---

### 2.4.4. Double-Acting Kinematic Configuration of Stirling Engine

In the application of Stirling engine, multi-cylinder configuration engine is often used to achieve higher and smoother output power. In the dish-Stirling solar-thermal systems, the double-acting kinematic configuration of the Stirling engine is most commonly studied [13, 82], where four cylinders are interconnected in series, as shown in Figure 2.6 [15].

In this configuration, the upper expansion space of one cylinder is connected to the lower compression space of the adjacent cylinder. Each piston moves in pure sinusoidal reciprocating motion with 90° phase difference between the adjacent pistons. Using the subscript \( i = 1–4 \) to denote the cylinder number, the aggregated quantities of the Stirling engine from the four cylinders are given as

**Input Heat Transfer Rate:**

\[
DQ_{h,\text{total}} = \sum_{i=1}^{4} DQ_{h,i}
\]  

(2.36)

**Output Mechanical Power:**
\[ DW_{\text{total}} = \sum_{i=1}^{4} DW_i \]  

(2.37)

The mean pressure of the working gas is defined as the average of the pressure in the four cylinders, i.e.

\[ p_{\text{mean}} = \frac{1}{4} \sum_{i=1}^{4} p_i \]  

(2.38)

![Diagram of Stirling engine](image)

Figure 2.6. Four-cylinder double-acting kinematic configuration of Stirling engine.

### 2.5. Solution of Ideal Adiabatic Model and Relevant Issues

It can be seen that the equation set of the ideal adiabatic model consists of nonlinear differential and algebraic equations, including two conditional equations (2.24) and (2.25). Only numerical solution can be obtained using specific computational methods. Only two differential equations (2.7) and (2.8) in the variables \( p \) and \( m_c \) are independent and need to be solved simultaneously with the algebraic equations.

As the ideal adiabatic model is not an initial-value problem, but instead a boundary-value problem, only cyclic steady state can be obtained by setting arbitrary initial conditions. For most configurations, starting from some guessed initial condition, the system can solved iteratively to get the converged solution through five to ten cycles [30].
Example time-domain simulation waveforms of the instantaneous pressure $p$, pressure change rate $dp/dt$, input heat transfer rate $q_h$ and the output mechanical torque $\tau_m$ are shown in Figure 2.7. The initial pressure $p_0$ of the four cylinders are all set to the maximum pressure (i.e. 1.0 p.u.), and the initial mass $m_{c0}$ in each compression space are set to zero.

![Waveforms of initialization of ideal adiabatic model.](image)

From Figure 2.7 it can be seen that from the selected arbitrary initial conditions, the system would reach the cyclic steady state in about four cycles. However, the initial absorbed heat transfer rate is very much larger than the nominal condition ($q_h = 1.0$ p.u.) and the mechanical torque goes to negative. In the computer simulation for power system related studies, such unrealistic initial setting of the power quantities of the prime mover would lead to unexpected simulation results such as the divergence of the solution process.
2.6. Summary

In practice, the internal thermodynamic process of the Stirling engine does not need to be explicitly represented in power system studies. This is because the Stirling engine is considered as part of the prime mover to convert the thermal into the mechanical energy to drive the electrical generator. It can be seen the aggregate heat and torque from the four cylinders are quite smooth. Thus, only the mean or average values of the Stirling engine variables would affect the external electrical-mechanical system. In power system studies, researchers are more concern about the variables which have major impacts on the efficient conversion and delivery of electrical power, rather than the internal thermodynamic behaviors of the Stirling engine.

Furthermore, for control system design, the ideal adiabatic model is unlikely to be suitable for use in theoretical analysis as it is too complex. Simplification on the complex model is needed to reveal its characteristic. In the next few sections, average-value models of Stirling engine will be derived for the grid related studies of dish-Stirling solar-thermal power generating system.
Chapter 3 Development of Stirling Engine Model for Dish-Stirling Solar-Thermal Systems

3.1. Introduction

As explained in Section 1.2, the existing models of the Stirling Engine are not suitable for the studies of dish-Stirling solar-thermal power generation because the complexity of the models brings heavy computational load and these models cannot be started from the steady state in most of the power system simulation software. Furthermore, the system cannot be analyzed for the design of control system based on classic methods. Thus, proper simplification on these complex models is needed to reveal their characteristics.

In this chapter, a general normalized average-value model of double-acting kinematic Stirling engine is derived based on the most-referred ideal adiabatic model for dish-Stirling solar-thermal system. Continuous speed variations caused by grid fault have been considered in the development of the model. Next, transfer functions for the conventional constant-speed dish-Stirling system, in which squirrel-cage induction generator (SCIG) is usually connected to, are derived for the purposes of software simulation and control system design. The developed models are compared with the existing detailed adiabatic model under different conditions. The simulation results show that both the general average-value model and the model for constant-speed operation can represent the average values of the input and output powers, where the high-frequency oscillating components are not shown. The general average-value model has more accurate steady-state and dynamic response to large change of engine speed and total mass flow rate than the average-value model for constant-speed operation. Part of the materials contained in this chapter has appeared in the author's publications [83, 84].

3.2. Normalized Average-Value Adiabatic Model

The motivation of developing the average-value models is to extract minimum necessary information to present the steady-state and dynamic characteristic of the Stirling engine without consideration of the internal dynamics such as the instantaneous pressure, temperature and mass flow rate in each compartment of a single cylinder. In power system related studies, average-value or mean value
variables such as the total input heat transfer rate \( q_h \) and the total output power \( P_m \) are of major concern when the dish and Stirling engine is considered as a prime mover to drive the coupling electric generator.

In this section, the time-domain, normalized and average-value expressions of the absorbed heat transfer rate \( q_h \) and mechanical power \( P_m \) will be derived from the per-cycle expressions \( DQ_h \) and \( DW \) of a single cylinder given in Section 2.4.2 respectively. The preliminary considerations and equations used to develop the models are presented in Section 2.3.

3.2.1. Total Per-Cycle Input Heat of Four-Cylinder Double-Acting Stirling Engine

The total per-cycle input heat of the Stirling engine is defined as the amount of heat absorbed by the metallic tubes (absorber) from the heat source in one period of analysis.

For a single cylinder, substituting (2.23) and (2.34) into (2.22), and using the ideal gas equation (2.2), the mass flow rate from the regenerator to the heater can be expressed as

\[
gA_{th} = gA_{he} + Dm_h
= Dm_e + m_h \left( \frac{Dp}{p} - \frac{DT_h}{T_h} \right)
= Dm_e + \frac{V_h}{RT_h} Dp - \frac{V_h}{RT_h^2} pDT_h
\]  

(3.1)

Substituting (2.32) and (3.1) into (2.29) and assuming \( T_{ih} = T_h \) [29], one obtains another expression of the absorbed heat, i.e.

\[
DQ_h = \frac{c_v V_h}{R} Dp - c_p (T_h gA_{th} - T_{he} gA_{he})
= \frac{c_v V_h}{R} Dp - c_p [T_h (Dm_e + \frac{V_h}{RT_h} Dp - \frac{V_h}{RT_h^2} pDT_h) - T_{he} \frac{pDV_e + v_e Dp / \gamma}{RT_{he}}]
= \frac{c_v V_h}{R} Dp - c_p T_h Dm_e - c_p \frac{V_h}{RT_h} Dp + c_p \frac{V_h}{RT_h} pDT_h + \frac{c_p}{R} (pDV_e + v_e Dp / \gamma)
= \left( c_v - c_p \right) \frac{V_h}{R} Dp - \frac{c_p}{R} T_h Dm_e + \frac{c_p}{R} (pDV_e + v_e Dp / \gamma) + \frac{c_p V_h}{R} DT_h p
\]  

(3.2)
Considering $c_p - c_v = R$, from (3.2) one obtains

$$DQ_h = -V_h Dp - c_p T_h Dm_e + \frac{c_p}{R} (p Dv_e + v_e Dp / \gamma) + \frac{c_p V_h}{R} \frac{DT_h}{T_p} p$$  \tag{3.3}

Now considering the four cylinders in the double-acting kinematic configuration described in Section 2.4.4. From (3.3), the total heat absorbed is the sum of the heat from each cylinder, which can be expressed as

$$DQ_{h,total} = \sum_{i=1}^{4} DQ_{h,i} = -V_h \sum_{i=1}^{4} Dp_i - c_p T_h \sum_{i=1}^{4} Dm_{e,i}$$

$$+ \frac{c_p}{R} \sum_{i=1}^{4} (p_i Dv_{e,i} + v_{e,i} \frac{Dp_i}{\gamma}) + \frac{c_p V_h}{R} \sum_{i=1}^{4} \left( \frac{DT_{h,i}}{T_{h,i}} p_i \right)$$  \tag{3.4}

where the subscript $i = 1\sim 4$ indicates the cylinder number.

To simplify (3.4), the following assumptions are considered:

1) The temperature distribution in the absorber/heater head is uniform, i.e. $T_{h,i} = T_h, DT_{h,i} = DT_h$
2) Temperature variation is much smaller than its nominal value, i.e. $T_h = T_{h,max}$;
3) The mean pressure value is equal to the averaging of the four instantaneous pressure values, i.e. $p_{mean} = \frac{1}{4} \sum_{i=1}^{4} p_i$ and thus $Dp_{mean} = \frac{1}{4} \sum_{i=1}^{4} Dp_i$;
4) The temperature of the expansion space is approximately equal to the heater temperature, i.e., $T_{e,i} = T_h, DT_{e,i} = DT_h$.

The expressions of $p_i, Dp_i, v_{e,i}, Dv_{e,i}$ and $Dm_{e,i}$ are given in Section 2.4.2 and Section 2.4.3. However, from (2.30) it can be seen that the expressions of the instantaneous pressure $p$ and thus its derivative $Dp$ are very complex. Thus these expressions cannot be readily incorporated into the model. Fortunately from the observation of the $p_i$ waveforms, e.g. as shown in Figure 3.1, it can be seen that the instantaneous pressures are quite sinusoidal [85]. By ignoring the second- and higher-order components, and denoting the magnitude and phase of the fundamental component of $p_i$ as $\Delta p_i$ and $\theta$, and the average value of $p_i$ as $p_i$, therefore, $p_i$ can be approximately expressed as
\[ p_i \approx p_i + \Delta p_i \cos(\phi - \alpha_{e,i} - \theta) \]  
(3.5)

As shown in [30], \( \Delta p_i \) is proportional to \( p_i \). Thus, \( p_i \) can be expressed as

\[ p_i \approx p_i [1 + b \cos(\phi - \alpha_{e,i} - \theta)] \]  
(3.6)

where \( b \) and \( \theta \) are assumed to be constant. The values of \( b \) and \( \theta \) can be estimated using Schmidt method and based on the dimension of the engine and the working gas temperatures [16], i.e.

\[ b = \frac{V_{sw} \sqrt{(T_h \cos \alpha_e + T_k \cos \alpha_e)^2 + (T_h \sin \alpha_e + T_k \sin \alpha_e)^2}}{2T_h T_k K} \]  
(3.7)

\[ \theta = \tan^{-1} \left( \frac{T_h \sin \alpha_e + T_k \sin \alpha_e}{T_h \cos \alpha_e + T_k \cos \alpha_e} \right) + \frac{\pi}{2} \]  
(3.8)

where

\[ K = \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_{cl} + 0.5V_{sw}}{T_k} + \frac{V_{cl} + 0.5V_{sw}}{T_h} \]  
(3.9)

Using the parametric values given in Appendix A, one can show that \( b = 0.2 \), \( \theta = 2.8385 \text{ rad} (162.64^\circ) \). However, the Schmidt method is based on isothermal analysis. While in the adiabatic analysis, the mean value of the expansion space temperature is not the same as the heater temperature. Thus, to obtain more accurate values of \( b \) and \( \theta \), \( T_h \) and \( T_k \) should be replaced with the mean values of \( T_e \) and \( T_c \) respectively. In doing so, \( b = 0.22 \), \( \theta = 2.8089 \text{ (160.94^\circ)} \).

The original and the corresponding approximate waveforms of the instantaneous pressure are shown in Figure 3.1.
Differentiating (3.6) with respect to the crank angle \( \phi \), thus
\[
\Delta p_i \approx -b\sin(\phi - \alpha_{c,i} - \theta) + [1 + b\cos(\phi - \alpha_{c,i} - \theta)]\Delta p_i
\]  
(3.10)

With (3.6), (3.10), (2.4) and (2.6), one obtains
\[
p_i \Delta v_{r,i} = (V_i + \bar{V}_{i,j})\Delta p_i
\]  
(3.11)
\[
v_{r,i} \Delta p_i = (V_2 + \bar{V}_{2,j})p_i + (V_3 + \bar{V}_{3,j})\Delta p_i
\]  
(3.12)
\[
p_i v_{r,d} = (V_4 + \bar{V}_{4,j})\Delta p_i
\]  
(3.13)

where \( V_1 = -V_2 = -0.25V_{sw}b\cos \theta, V_3 = V_4 = V_d + 0.25V_{sw}b\sin \theta \), and \( V_d = V_{cl} + 0.5V_{sw} \). \( \bar{V}_{1,j}, \bar{V}_{2,j}, \bar{V}_{3,j}, \) and \( \bar{V}_{4,j} \) can be expressed as linear combination of \( \sin \theta \), \( \cos \theta \), \( \sin 2\theta \) and \( \cos 2\theta \). As the adjacent piston has a \( \pi/2 \) phase difference, it can be readily shown that
\[
\sum_{j=1}^{4} \bar{V}_{1,j} = \sum_{j=1}^{4} \bar{V}_{2,j} = \sum_{j=1}^{4} \bar{V}_{3,j} = \sum_{j=1}^{4} \bar{V}_{4,j} = 0
\]  
(3.14)

Thus, using (3.11)–(3.14) and assuming \( p_i = p_{\text{mean}} \), one can obtain

Figure 3.1. Waveforms of the original and approximate waveforms of the instantaneous pressure.
\[
\sum_{i=1}^{4} D_{p_i} = 4D_{p_{\text{mean}}} \quad (3.15)
\]
\[
\sum_{i=1}^{4} p_i D_{v_{e,i}} = 4V_1 p_{\text{mean}} \quad (3.16)
\]
\[
\sum_{i=1}^{4} v_{e,i} D_{p_i} = -4V_1 p_{\text{mean}} + 4V_3 D_{p_{\text{mean}}} \quad (3.17)
\]
\[
\sum_{i=1}^{4} p_i v_{e,i} = 4V_3 p_{\text{mean}} \quad (3.18)
\]

Hence, using (3.16) and (3.17), the term \( \odot \) in (3.4) can be expressed as
\[
\sum_{i=1}^{4} (p_i D_{v_{e,i}} + v_{e,i} D_{p_i}/\gamma) = \frac{4R}{c_p} V_1 p_{\text{mean}} + \frac{4c_v}{c_p} V_3 D_{p_{\text{mean}}} \quad (3.19)
\]

The term \( \odot \) in (3.4) can be readily evaluated as
\[
\sum_{i=1}^{4} \frac{D_{T_{h,i}}}{T_{h,i}} p_i \approx \frac{D_{T_h}}{T_h} \sum_{i=1}^{4} p_i = \frac{D_{T_h}}{T_{h,\text{max}}} p_{\text{mean}} \quad (3.20)
\]

Finally, the term \( \odot \) in (3.4) can be evaluated in the following way. As
\[
D_{m_{e,i}} = D\left(\frac{p_i v_{e,i}}{RT_{e,i}}\right) \approx \frac{p_i D_{v_{e,i}}}{RT_h} + \frac{v_{e,i} D_{p_i}}{RT_h} - \frac{p_i v_{e,i} D_{T_h}}{RT_h^2} \quad (3.21)
\]

thus,
\[
\sum_{i=1}^{4} D_{m_{e,i}} = \frac{1}{RT_h} \sum_{i=1}^{4} p_i D_{v_{e,i}} + \frac{1}{RT_h} \sum_{i=1}^{4} v_{e,i} D_{p_i} - \frac{D_{T_h}}{RT_h^2} \sum_{i=1}^{4} p_i v_{e,i} \quad (3.22)
\]

Substituting (3.16)–(3.18) into (3.22), it yields
\[
\sum_{i=1}^{4} D_{m_{e,i}} = \frac{4V_3}{RT_h} D_{p_{\text{mean}}} - \frac{4V_3}{RT_h^2} p_{\text{mean}} D_{T_h} \quad (3.23)
\]

Then substituting (3.15), (3.19), (3.20) and (3.23) into (3.4), and assuming \( T_h = T_{h,\text{max}} \), one can finally obtain the expression of total cyclic input heat from the Stirling engine, i.e.
\[
DQ_{h,\text{total}} = 4V_{T}p_{\text{mean}} - 4(V_{h} + V_{j})Dp_{\text{mean}} - 4(V_{h} + V_{j})\frac{c_{p}P_{\text{mean}}}{RT_{h,\text{max}}} DT_{h} \tag{3.24}
\]

Equation (3.24) describes the dynamics of the input heat from four cylinders of the double-acting kinematic engine within a single cycle.

### 3.2.2. Total Per-Cycle Output Work of Four-Cylinder Double-Acting Stirling Engine

Similarly to the method used to derive the total per-cycle input heat shown in Section 3.2.1, the expression of total cyclic output work from the Stirling engine can be derived by considering four cylinders together, i.e.

\[
DW_{\text{total}} = \sum_{i=1}^{4} DW_{i} = \sum_{i=1}^{4} p_{i}Dv_{c,i} + \sum_{i=1}^{4} p_{i}Dv_{e} \tag{3.25}
\]

With (3.6), (3.10), (2.4) and (2.6), thus

\[
\sum_{i=1}^{4} p_{i}Dv_{c,i} = -V_{sw}b\cos(\theta - \frac{\pi}{2})p_{\text{mean}} \tag{3.26}
\]

Using (3.16) and (3.26), the total cyclic output work from the four cylinders can be derived as

\[
DW_{\text{total}} = -V_{sw}b\cos\theta p_{\text{mean}} - V_{sw}b\cos(\theta - \frac{\pi}{2})p_{\text{mean}} \\
= \sqrt{2}b\sin(\theta - \frac{3}{4}\pi)V_{sw}p_{\text{mean}} \tag{3.27}
\]

### 3.2.3. Mean Pressure versus Total Mass Relationship

As explained in Section 3.2.1, the mean pressure of the working gas \(p_{\text{mean}}\) is assumed to be the same as the average pressure of the four cylinders, and it is proportional to the total mass of the working gas [16], i.e.

\[
p_{\text{mean}} = K_{p}M \tag{3.28}
\]

where the coefficient \(K_{p}\) is expressed as

\[
K_{p} = \frac{R/(1-b)}{K} \frac{1-b}{\sqrt{1+b}} \tag{3.29}
\]
The expressions of \( b \) and \( K \) are given in (3.7) and (3.9) respectively. Normally, \( K_p \) can be seen to be constant when the temperature quantities are near their normal operating values.

**3.2.4. Normalization of the Stirling Engine Model**

In power system related studies, it is usually convenient to use a normalized or per-unit model. This is because well-chosen normalized system can minimize computational efforts, simplify evaluation, and facilitate the understanding of the system characteristics. In this thesis, the base quantities used to derive a normalized model of the Stirling engine are

- \( p_{\text{base}} = p_{\text{max}}, \text{Pa} \), the maximum mean pressure;
- \( m_{\text{base}} = M_{\text{max}}, \text{kg} \), the total mass of the working gas when pressure is maximum;
- \( T_{\text{base}} = T_{h, \text{max}}, \text{K} \), the maximum temperature of receiver;
- \( \omega_{\text{base}} = \omega_{m, N}, \text{rad/s} \), the nominal engine speed;
- \( P_{\text{base}} = P_{m, N}, \text{W} \), the generated nominal mechanical power.

In doing so, the normalized mean pressure would be the same as the normalized total mass of the working gas, i.e.

\[
\bar{p}_{\text{mean}} = \bar{M}
\]  

(3.30)

where the overbar “\( \bar{\} \)" indicates the normalized or per-unit quantities.

Considering (3.30) and using the above base quantities to normalize (3.24), one obtains

\[
DQ_{h, \text{total}} - \frac{DQ_{h, \text{total}}}{P_{m, N}} = \frac{4p_{\text{max}}}{P_{m, N}} \left[ V_1\bar{p}_{\text{mean}} - (V_h + V_3)\Delta\bar{p}_{\text{mean}} - (V_h + V_3)\frac{cp\bar{p}_{\text{mean}}}{R} \right] \Delta T_h
\]

(3.31)

For the time-domain simulation, the derived results derived in Section 3.2.1 and Section 3.2.2 based on the cyclic analysis need to be converted accordingly. By considering \( d\phi/dt = \omega_m \), the total absorbed heat transfer rate from the adiabatic analysis is
\[ \tilde{q}_{h,\text{adi}} = D \tilde{Q}_{h,\text{total}} \times \frac{d\phi}{dt} = D \tilde{Q}_{h,\text{total}} \times \omega_m \]
\[ = \frac{4p_{\text{max}}}{P_{m,N}} \left[ V_1 \bar{p}_{\text{mean}} \omega_m - (V_h + V_3) \frac{d\bar{p}_{\text{mean}}}{dt} - (V_h + V_3) \frac{c_p \bar{p}_{\text{mean}} d\bar{T}_h}{R} \right] \quad (3.32) \]
\[ = K_h \bar{p}_{\text{mean}} \bar{\omega}_m + A gA + C \bar{p}_{\text{mean}} \frac{d\bar{T}_h}{dt} \]

where

\[ K_h = \frac{4p_{\text{max}} \omega_{m,N}}{p_{m,N}} V_1 \]
\[ A = -\frac{4p_{\text{max}}}{P_{m,N}} (V_h + V_3) \]
\[ C = \frac{4p_{\text{max}} c_p}{P_{m,N} R} (V_h + V_3) = \frac{\gamma}{1 - \gamma} A \]
\[ \frac{gA}{d\bar{M}} = \frac{d\bar{p}_{\text{mean}}}{dt} \]

Similarly, the expression of total mechanical power can be derived using (3.27), i.e.
\[ \bar{P}_{m,\text{adi}} = \bar{r}_{m,\text{adi}} \bar{\omega}_m = K_m \bar{p}_{\text{mean}} \bar{\omega}_m \quad (3.33) \]

where

\[ K_m = \frac{p_{\text{max}} \omega_{m,N}}{p_{m,N}} \sqrt{2} \sin(\theta - 0.75\pi) V_{nv} \]

In the steady state, the thermal efficiency of the Stirling engine can be obtained as
\[ \eta_{th,\text{adi}} = \frac{\bar{P}_{m,\text{adi}}}{\tilde{q}_{h,\text{adi}}} = \frac{K_h \bar{p}_{\text{mean}} \bar{\omega}_m}{K_h \bar{p}_{\text{mean}} \bar{\omega}_m} = \frac{K_m}{K_h} = \frac{\sqrt{2} \sin(\theta - 0.75\pi)}{-\cos \theta} = 1 + \tan \theta \quad (3.34) \]

Equation (3.34) provides a convenient way to estimate the engine thermal efficiency based on the adiabatic analysis according to the heater and cooler temperatures. For example, for the Stirling engine in Section 3.2.1 where \( \theta = 2.8089 \) (160.94°), the calculated adiabatic thermal efficiency is 65.54%. While the Carnot efficiency which denote the theoretical maximum thermal efficiency of a heat engine is
\[
\eta_{\text{th,carnot}} = 1 - \frac{T_h}{293 K} = 1 - \frac{293 K}{1033 K} = 71.64\%
\] (3.35)

The difference between \(\eta_{\text{th,adi}}\) and \(\eta_{\text{th,carnot}}\) results from the consideration of losses from adiabatic analysis. The steady-state thermal efficiency of an actual Stirling engine is much lower than that forecasted from adiabatic analysis [30].

### 3.2.5. Transfer Functions for Constant-Speed Operation

For the constant-speed operation of Stirling engine, the engine speed \(\bar{\omega}_m\) is very close to its nominal value, i.e. \(\bar{\omega}_m \approx 1.0\) p.u. Thus, using (3.32) and (3.33), one can readily show that the transfer functions between the input heat transfer rate \(\bar{q}_{\text{h,adi}}\) with the mean pressure \(\bar{p}_{\text{mean}}\) of the working gas, and between the output power \(\bar{P}_{m,\text{adi}}\) with \(\bar{p}_{\text{mean}}\) are in the following form:

\[
\begin{cases}
G_h(s) = \frac{\bar{q}_{\text{h,adi}}(s)}{\bar{p}_{\text{mean}}(s)} = K_h \bar{\omega}_m \frac{1 - T_{se1}s}{1 + T_{se2}s} \\
G_m(s) = \frac{\bar{P}_{m,\text{adi}}(s)}{\bar{p}_{\text{mean}}(s)} = K_m \bar{\omega}_m \frac{1}{1 + T_{se3}s}
\end{cases}
\] (3.36)

where the main Stirling engine time constant \(T_{se1}\) is defined as

\[
T_{se1} = -\frac{A}{K_h \bar{\omega}_m} = \frac{V_s + V_i}{V_i \bar{\omega}_m}
\] (3.37)

The values of \(T_{se2}\) and \(T_{se3}\) are selected to embody the effect of the fast oscillation components on the power quantities and they should be compatible with the power system phenomenon studied. The expression of \(T_{se2}\) and \(T_{se3}\) is

\[
T_{se2} = T_{se3} = 0.5 \times \frac{1}{f_{HF}} = 0.5 \times \frac{1}{4 \times (\omega_n / 2\pi)} = \frac{p_n}{4 f_N}
\] (3.38)

where \(f_{HF}\) is the frequency of the fast oscillation components, \(p_n\) is the number of pole pairs of the coupling induction generator and \(f_N\) is the nominal frequency of the grid. With \(p_n = 2, f_N = 60\), thus \(T_{se2} = T_{se3} = 0.0021\) s.
The derived transfer functions (3.36) also can be used for control system design which will presented in the later chapters.

3.3. Quasi-Static Representation of Losses

Ideal adiabatic model does not consider most types of thermal losses in the Stirling engine except the losses caused by the adiabatic process. Hence the model will introduce steady-state errors. These losses will be briefly discussed in this section.

From (3.32) and (3.33), the steady-state values of the absorbed heat transfer rate and the mechanical power are

\[
\begin{align*}
\tilde{q}_{h,\text{adi}}(0) &= K_h \bar{p}_{\text{mean}} \bar{\omega}_m \\
\bar{P}_{m,\text{adi}}(0) &= K_m \bar{p}_{\text{mean}} \bar{\omega}_m
\end{align*}
\]  
(3.39)

3.3.1. Effects of Losses

It is clear from (3.39) that the steady-state values of the mechanical power and absorbed heat are directly proportional to the product of the engine speed \( \bar{\omega}_m \) and mean pressure \( \bar{p}_{\text{mean}} \). In practice, due to the thermal and frictional losses, \( \tilde{q}_{h}(0) \) and \( \bar{P}_{m}(0) \) will deviate from the calculated using (3.39). The steady-state losses can be categorized into three groups, based on the manner by which the losses vary with \( \bar{\omega}_m \) and \( \bar{p}_{\text{mean}} \) [20]:

1) **Static thermal losses**

Static thermal losses include

- Conduction loss: thermal conduction along the cylinder walls and other conduction paths
- Shuttle heat transfer loss
- Heat loss by convection and radiation

Static thermal losses are mainly caused by the temperature differences between the working gas and cylinder wall. Thus they will affect both the input and output powers. The static thermal losses are relatively independent of the engine speed and can be expressed as
\[
\begin{align*}
\Delta \bar{q}_0 &= k'_0 + k'_{10} \bar{P}_{\text{mean}} \\
\Delta \bar{P}_0 &= k_0 + k_{10} \bar{P}_{\text{mean}}
\end{align*}
\] (3.40)

2) Linear thermal losses

The main components of the linear thermal losses are

- Stack loss: arises from the heat carried off by the exhaust gas, which is proportional to the absorbed power;
- Heat exchanger temperature potential (Hysteresis loss);
- Imperfect regeneration.

They will affect both the output and input powers. All these losses will increase linearly as the speed or mean pressure. They can be expressed as

\[
\begin{align*}
\Delta \bar{q}_1 &= k'_0 \bar{\omega}_m + k'_{11} \bar{P}_{\text{mean}} \bar{\omega}_m \\
\Delta \bar{P}_1 &= k_0 \bar{\omega}_m + k_{11} \bar{P}_{\text{mean}} \bar{\omega}_m
\end{align*}
\] (3.41)

3) Losses caused by friction

Friction consists of

- Mechanical friction;
- Aerodynamic friction: internal windage losses caused by fluid dynamic drag (pressure drop).

Generally losses caused by friction only reduce the output power and they can be considered as proportional to the square of the fluid velocity, i.e.

\[
\Delta \bar{P}_2 = k_{02} \bar{\omega}_m^2 + k_{12} \bar{P}_{\text{mean}} \bar{\omega}_m^2
\] (3.42)

In the above equations, each type of losses is considered as linear to the mean pressure \( \bar{P}_{\text{mean}} \). This is reasonable and can be observed in many experimental data.

Generally losses are influenced by \( \bar{T}_h \) significantly over the full temperature range. Again, during the period of normal operation, an important objective of the control system is to keep the temperature near the maximum value. Thus temperature can be treated as constant in the following analysis.
A new set of more accurate expressions for \( \bar{q}_h(0) \) and \( \bar{P}_m(0) \) can be derived by subtracting the losses shown in (3.40)–(3.42) from the steady-state values \( \bar{q}_{h, \text{adi}}(0) \) and \( \bar{P}_{m, \text{adi}}(0) \) derived from adiabatic analysis. Henceforth, the subscript “adi” shall be omitted from \( \bar{q}_{h, \text{adi}}(0) \) and \( \bar{P}_{m, \text{adi}}(0) \) to signify that the analysis to follow has included the losses. Thus,

\[
\bar{q}_h(0) = \bar{q}_{h, \text{adi}}(0) - \Delta \bar{q}_0 - \Delta \bar{q}_l = (K_h - k_{11}') \bar{p}_{\text{mean}} \bar{\omega}_m - k_{00}' - k_{10}' \bar{p}_{\text{mean}} - k_{01}' \bar{\omega}_m \tag{3.43}
\]

\[
\bar{P}_m(0) = \bar{P}_{m, \text{adi}}(0) - \Delta \bar{P}_0 - \Delta \bar{P}_1 - \Delta \bar{P}_2 = (K_m - k_{11}) \bar{p}_{\text{mean}} \bar{\omega}_m - k_{00} - k_{10} \bar{p}_{\text{mean}} - k_{01} \bar{\omega}_m \tag{3.44}
\]

\[-k_{02} \bar{\omega}_m^2 - k_{12} \bar{p}_{\text{mean}} \bar{\omega}_m^2 \]

A family of \( \bar{P}_m - \bar{\omega}_m \) curves can be obtained by considering different types of losses and setting \( \bar{p}_{\text{mean}} \) at a specific value. Similarly, the \( \bar{q}_h - \bar{\omega}_m \) and \( \bar{P}_m - \bar{p}_{\text{mean}} \) curves can be obtained. The corresponding power and efficiency curves considering different types of thermal and mechanical losses are shown in Figure 3.2.

In Figure 3.2, the solid curves with subscript ‘adi’ denote those from adiabatic analysis without considering any type of thermal and mechanical losses that is not considered in the adiabatic analysis. The second curves with dash-dotted line denote those considering only static thermal losses. The dotted curves denote those considering both static and linear thermal losses. The last curves denote those considering all three types of thermal and mechanical losses discussed above. From Figure 3.2 (d) it can be seen in this case the nominal engine speed is selected at where the engine thermal efficiency is the maximum.

These power curves of a real Stirling engine can be obtained from Stirling engine manufacturers or from experimental results. An example of the \( \bar{P}_m(0) - \bar{\omega}_m \) curves is shown in Figure 3.3 [86].
Figure 3.2. Power and efficiency curves of Stirling engine considering different types of thermal and mechanical losses.

Figure 3.3. Typical power curves of Stirling engine.
3.3.2. Multivariate Polynomial Coefficients

Equations (3.43) and (3.44) show that \( \bar{q}_h(0) \) and \( \bar{P}_m(0) \) are multivariate polynomials (MPs) of \( \bar{p}_{\text{mean}} \) and \( \bar{\omega}_m \). The coefficients \( k'_{ij} \) and \( k_{ij} \) can be considered as constant during the normal operation of the Stirling engine. Instead of estimating their values through the thermodynamic and heat transfer analysis, a more convenient approach is to determine the polynomial functions (3.43) and (3.44), which best-fit the engine steady-state performance maps of \( \bar{q}_h(0) \) and \( \bar{P}_m(0) \) against \( \bar{p}_{\text{mean}} \) and \( \bar{\omega}_m \), obtained from laboratory experiments or provided by the Stirling engine manufacturers. The general representation of the input and output powers using the MP model are thus

\[
\begin{align*}
\bar{q}_h(0) &= \sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} \bar{p}_{\text{mean}}^i \bar{\omega}_m^j \\
\bar{P}_m(0) &= \sum_{i=0}^{2} \sum_{j=0}^{2} b_{ij} \bar{p}_{\text{mean}}^i \bar{\omega}_m^j 
\end{align*}
\]

where \( a_{ij} \) and \( b_{ij} \) are the respective dimensionless MP coefficients. Second-order approximation is considered sufficiently accurate to represent the effect of losses around the operating state. By comparing (3.45) with (3.43) and (3.44), two new parameters \( \hat{K}_h \) and \( \hat{K}_m \) are defined and they will be used to replace \( K_h \) and \( K_m \) in (3.32) and (3.33) of the normalized average-value adiabatic model, respectively, i.e.

\[
\begin{align*}
\hat{K}_h(\bar{p}_{\text{mean}}, \bar{\omega}_m) &= \sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} \bar{p}_{\text{mean}}^{i-1} \bar{\omega}_m^{j-1} \\
\hat{K}_m(\bar{p}_{\text{mean}}, \bar{\omega}_m) &= \sum_{i=0}^{2} \sum_{j=0}^{2} b_{ij} \bar{p}_{\text{mean}}^{i-1} \bar{\omega}_m^{j-1} 
\end{align*}
\]

In Appendix C, the typical values of the MP coefficients are calculated using the data of a Stirling engine reported in [20]. The root-mean-square error (RMSE) between the calculated \( \bar{q}_h(0), \bar{P}_m(0) \) and those obtained in [20] for \( \bar{p}_{\text{mean}} = 0.33 \sim 1.0 \)
p.u. is less than 2.5%. This indicates that the developed steady-state model is sufficiently accurate.

Regarding to the constant-speed condition, submitting $\bar{\omega}_m = 1.0$ into (3.46), thus

$$
\begin{align*}
\dot{K}_h &= (a_{10} + a_{11}) \bar{p}_{\text{mean}} + a_{00} + a_{01} \\
\dot{K}_m &= (b_{10} + b_{11} + b_{12}) \bar{p}_{\text{mean}} + b_{00} + b_{01} + b_{02}
\end{align*}
$$

(3.47)

### 3.3.3. Thermal Efficiency of Stirling Engine

From first equation of (3.45), at the nominal operating point, $\bar{p}_{\text{mean}} = 1.0$ p.u., $\bar{\omega}_m = 1.0$ p.u. and $\bar{p}_m(0) = 1.0$. Thus,

$$
\sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij} = 1
$$

(3.48)

Dividing $\bar{p}_m(0)$ by $\bar{q}_h(0)$ which is obtained from the second equation of (3.45) yields

$$
\frac{\bar{p}_m(0)}{\bar{q}_h(0)} = \eta_{\text{th},N} \frac{\sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij}}{\sum_{i=0}^{1} a_{ij}} = \frac{1}{\sum_{i=0}^{1} \sum_{j=0}^{2} a_{ij}}
$$

(3.49)

Thus, (3.49) allows the thermal efficiency $\eta_{\text{th},N}$ of the Stirling engine at the nominal operating state to be more accurately evaluated as $a_{ij}$ are known and the various losses have been accounted for.

Also, this losses effect can be represented using two efficiency coefficients, which are defined as

$$
\begin{align*}
\eta_h(\bar{p}_{\text{mean}}, \bar{\omega}_m) &= \frac{\bar{q}_h(0)}{\bar{q}_{\text{th},N}(0)} = \frac{\sum_{i=0}^{1} \sum_{j=0}^{2} a_{ij} \bar{p}_{\text{mean}}}{K_h \bar{p}_{\text{mean}} \bar{\omega}_m} = \frac{\dot{K}_h}{\dot{K}_h} \\
\eta_m(\bar{p}_{\text{mean}}, \bar{\omega}_m) &= \frac{\bar{p}_m(0)}{\bar{p}_{\text{th},N}(0)} = \frac{\sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij} \bar{p}_{\text{mean}}}{K_m \bar{p}_{\text{mean}} \bar{\omega}_m} = \frac{\dot{K}_m}{\dot{K}_m}
\end{align*}
$$

(3.50)
3.4. Model Comparison and Case Study

In order to validate the developed models of the Stirling engine, simulation under various input changes shall be carried out using the developed models and the results are compared with those obtained using the existing detailed Stirling engine models. The test system is shown in Figure 3.4, where the input variables include the engine speed $\bar{\omega}_m$, the change rate of the total mass $\bar{g} \bar{A}$ and the average heater temperature $\bar{T}_h$. The two output variables are the total absorbed heat transfer rate $\bar{q}_h$ and the mechanical power $\bar{P}_m$ or torque $\bar{\tau}_m$.

![Figure 3.4. Test model for the comparison of the Stirling engine models](image)

As shown in Figure 3.4, the detail adiabatic model and the developed average-value models can be considered as four-input and two-output system. In the real world, the total absorbed heat transfer rate $\bar{q}_h$ is not independent to the average heater temperature $\bar{T}_h$ due to the heat transfer process between the heater and the absorber of the Stirling engine. This process, which determines the variation of the heater temperature, is not fully represented in the models. In this initial attempt, the heater temperature is assumed to be constant. Under this condition, responses to step changes of the engine speed $\bar{\omega}_m$ and that of the total mass change rate $\bar{g} \bar{A}$ of the working gas are shown in Figure 3.5 and Figure 3.6.

Figure 3.5 shows the response of the Stirling engine when the engine speed decreases from 1.0 p.u. to 0.6 p.u. The test case shown in Figure 3.6 is that of the net mass change rate $\bar{g} \bar{A} = \frac{d\bar{M}}{dt}$ of the total mass increases from 0 p.u./s to 10 p.u./s and then returns back to zero after 0.01 s. This causes a fast ramp change of $\bar{M}$ and thus the mean pressure $\bar{p}_{\text{mean}}$ of the working gas. From the observation of the waveforms of the output power and input heat, it can be seen that both the general
average-value model and the model for constant-speed operation can represent the average values of the input and output power, where the high-frequency components are not represented. The general average-value model has closer steady-state and dynamic response to the step change of engine speed and mean pressure of working gas than the average-value model for constant-speed operation.

![Diagram showing step response under constant heater temperature for engine speed change.](image)

Figure 3.5. Step response under constant heater temperature for engine speed change.

However, the heater temperature \( \bar{T}_h \) is not independent to the heat transfer rate \( \bar{q}_h \) in the real world. The heat transfer between the working gas, the absorber tubes and the receiver chamber governs the dynamics of variation of heater temperature \( \bar{T}_h \). The validation and comparison the developed Stirling engine models by considering the temperature variation will be presented in Chapter 4 after the model of the receiver and concentrator is derived.
3.5. Summary of the Developed Model of the Stirling Engine

The Stirling engine models developed in this chapter may now be summarized into the following expressions.

3.5.1. General Normalized Average-Value Model

The derived general normalized average-value model of Stirling engine is summarized as follows:

\[
\bar{q}_h = \eta_b \left[ K_h \bar{p}_m \bar{\omega}_m + A(gA) + C_{p_{\text{mean}}} \frac{dT_h}{dt} \right]
\]  

(3.51)
\[ \bar{p}_m = \bar{\rho}_m \bar{\omega}_m = \eta_m K_m \bar{p}_{\text{mean}} \bar{\omega}_m \]  

(3.52)

**Mass flow equation:**

\[ \frac{d\bar{M}}{dt} = \frac{d\bar{p}_{\text{mean}}}{dt} \]  

(3.53)

**Thermal and mechanical efficiency equations:**

\[ \eta_h = \frac{\sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} \bar{p}_{\text{mean}}^i \bar{\omega}_m^j}{K_h \bar{p}_{\text{mean}} \bar{\omega}_m} \]  

(3.54)

\[ \eta_m = \frac{\sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij} \bar{p}_{\text{mean}}^i \bar{\omega}_m^j}{K_m \bar{p}_{\text{mean}} \bar{\omega}_m} \]  

(3.55)

The block diagram of the general normalized average-value model is shown in Figure 3.7. During steady-state, the average heater temperature \( T_h \), the total mass \( M \) and thus the mean pressure \( \bar{p}_{\text{mean}} \) of the working gas in the four cylinders are constant, and the total mass change rate \( gA \) and temperature change rate \( dT_h/dt \) of the working gas are zero. Both the absorbed heat transfer rate \( q_h \) and the mechanical torque \( \tau_m \) is only related to the mean pressure \( \bar{p}_{\text{mean}} \) and the engine speed \( \omega_m \). It can be observed from Figure 3.7 that, the nonlinearity of the model results from two multipliers and the thermal efficiencies \( \eta_h \) and \( \eta_m \).

![Figure 3.7. Block diagram of the general normalized average-value model of the Stirling engine.](image-url)
3.5.2. Normalized Average-Value Model for Constant-Speed Operation

The derived normalized average-value model of the constant-speed Stirling engine is given as follows:

\[
G_h(s) = \frac{\bar{q}_{h,ad}(s)}{\bar{p}_{mean}(s)} = \hat{K}_h \hat{\phi}_m \frac{1 - T_{se1}s}{1 + T_{se2}s} \tag{3.56}
\]

\[
G_m(s) = \frac{\bar{p}_{m,ad}(s)}{\bar{p}_{mean}(s)} = \frac{\hat{K}_m \hat{\phi}_m}{1 + T_{se3}s} \tag{3.57}
\]

\[
\ddot{g}A = \frac{d\bar{M}}{dt} = \frac{d\bar{p}_{mean}}{dt} \tag{3.58}
\]

\[
\hat{K}_h = (a_{10} + a_{11})\bar{p}_{mean} + a_{00} + a_{01} \tag{3.59}
\]

\[
\hat{K}_m = (b_{10} + b_{11} + b_{12})\bar{p}_{mean} + b_{00} + b_{01} + b_{02} \tag{3.60}
\]

The corresponding block diagram is shown in Figure 3.8. The model consists of an integrator and two transfer functions which represents the thermal and mechanical dynamics respectively. Equation (3.56) is a non-minimum phase system with a zero located on the left half plane (LHP) of the complex plane.

![Figure 3.8. Block diagram of the normalized constant-speed average-value model of the Stirling engine.](image)

3.6. Summary

In this chapter, a general normalized average-value model of Stirling engine is derived based on the most-commonly referred ideal adiabatic model for variable-speed dish-Stirling solar-thermal system. The dish and the Stirling engine acts as the prime
mover of an electric generator. The transfer functions for the conventional constant-speed operation of dish-Stirling system, where a squirrel-cage induction generator is connected to the Stirling engine, are derived for the purposes of software simulation and control system design. The developed models are compared with the existing detailed adiabatic model under both constant and variable temperature conditions, and the simulation results show that both the general average-value model and the model for constant-speed operation can represent the average values of the input and output power, where the high-frequency components are not considered. The general average-value model appears to be able to predict more accurately the steady-state and dynamic response than the average-value model for constant-speed operation. This is not surprising especially when the engine speed deviations are no longer insignificant.
Chapter 4  Constant-Speed Dish-Stirling Solar-Thermal System

4.1. Introduction

The focus of Chapter 3 has been on the Stirling engine and as has been explained in Section 1.1.3, essentially, three energy conversion processes are involved in a typical dish-Stirling solar-thermal power generating system [13]. First, the direct normal irradiance from the sunlight collected by the dish concentrator is reflected onto a small hollow chamber called receiver. In the receiver, the harnessed solar energy is in turn converted to thermal energy and absorbed by a large number of metallic tubes called the absorber. Second, the Stirling engine converts the absorbed thermal energy into mechanical energy by compressing and expanding a working gas, such as hydrogen and helium. Finally, the Stirling engine drives an electric generator, which converts the mechanical energy into electricity.

Although the concept sounds straightforward, dish-Stirling power generating system is rather complex and the analysis of it requires knowledge from areas of thermal, material, electrical, mechanical and control engineering. The relationship between the main components of the dish-Stirling system is shown in Figure 4.1.

![Figure 4.1. Components of constant-speed dish-Stirling systems.](image)

Conventionally, squirrel-cage induction generators (SCIGs) are commonly used in such systems. As the normal operating speed of the SCIG is closed to the synchronous speed with very small slip, such a system is herewith referred as constant-speed DS solar-thermal generating system.

The variation of the insolation and the grid dynamics may have negative impact on the performance and stability of the constant-speed DS system. The focus of this chapter is to develop a suitable model for such a generating system which is
compatible with the simulation model of the power system, and then to design high-performance control system to achieve effective and efficient temperature control of the receiver/absorber under system disturbances. Linearized model of the constant-speed dish-Stirling is derived for temperature control system design. A new temperature controller with transient droop compensation and feedforward compensation is proposed to solve the problem caused by the non-minimum phase and nonlinear characteristic of the system model. Computer simulation shows that the system is well damped at the occurrence of system disturbance such as the insolation variation and grid fault. From the steady-state analysis, the potential of maximum energy harness of dish-Stirling system is discussed and it shows that variable speed operation of dish-Stirling is a viable method to extract considerable more power.

Part of the materials contained in this chapter also appears in the author’s publication [83].

4.2. **Modeling of Constant-Speed Dish-Stirling Solar-Thermal System**

4.2.1. **Modeling of Concentrator and Receiver**

The parabolic concentrator of the dish-Stirling system collected the solar irradiance over a large area and concentrates and reflects it onto the receiver. The concentrated solar power intercepted by the receiver is converted into heat $q_I$, viz.

$$ q_I = \eta_{\text{con}} A_{\text{con}} I $$

(4.1)

where $A_{\text{con}}$ is the projection area of the concentrator and $I$ is the insolation level. $\eta_{\text{con}}$, is the overall efficiency of the concentrator and its value depends on the reflectivity of the concentrator surface, the intercept fraction of the concentrated sunlight by the receiver, and the tracking system performance, among other factors [82]. Normally, $A_{\text{con}}$ and $\eta_{\text{con}}$ can be considered as constants. Note the symbol $q$ is used here to represent thermal power quantity in unit of watt, to replace the symbol $Q$ used in some literature to avoid possible confusion.

The receiver is the interface between the concentrator and the Stirling engine. It converts the input concentrated solar power $q_I$ into thermal energy to supply the input heat for the Stirling engine. Only part of $q_I$ shall be absorbed by the heater of the Stirling engine. It is denoted as $q_h$ and its value is governed by the dynamic characteristics of the Stirling engine and is traditionally calculated using the complex
high-order nonlinear thermodynamic models, e.g., the ideal adiabatic model. The balance of the thermal power \( (q_L) \) will be lost to the surrounding atmosphere through conduction, convection and radiation. According to Fourier’s law of heat conduction and Newton’s law of cooling, the sum of the conduction and the convection losses can be expressed as [82]

\[
q_{L, \text{conv-cond}} = A_{\text{rec}} U (T_h - T_a)
\]

(4.2)

where \( A_{\text{rec}} \) is the area of the receiver aperture, \( U \) is the convection-conduction heat loss coefficient, \( T_h \) denotes the temperature of the absorber tubes. \( T_h \) is considered as uniform in the whole space of the absorber walls and equals to the average temperature of the working gas in the heater tubes of the Stirling engine [16]. \( T_a \) is the temperature of the atmosphere.

According to Stefan-Boltzmann’s Law, the thermal loss by radiation can be expressed as [82]

\[
q_{L, \text{radi}} = \sigma F_b (T_h^4 - T_a^4) = \sigma F_b (T_h^2 + T_a^2)(T_h + T_a)(T_h - T_a)
\]

(4.3)

where \( \sigma \) is the Stefan-Boltzmann radiant-energy-transfer constant and \( F_b \) equivalent radiative conductance.

Thus, the overall thermal losses can be expressed as the sum of (4.2) and (4.3), i.e.

\[
q_L = q_{L, \text{conv-cond}} + q_{L, \text{radi}} = A_{\text{rec}} U (T_h - T_a) + \sigma F_b (T_h^2 + T_a^2)(T_h + T_a)(T_h - T_a)
\]

(4.4)

where

\[
K_L = A_{\text{rec}} U + \sigma F_b (T_h^2 + T_a^2)(T_h + T_a)
\]

(4.5)

The coefficient \( K_L \) determines the main characteristics of the thermal losses. In (4.4), at very high temperature, the variation of \( K_L \) is much smaller than that of the term \( (T_h - T_a) \). In this condition, \( K_L \) can be seen as a constant.

The imbalance between \( q_I \) (input power) and \( q_h + q_L \) (output power) will cause a temperature change on the absorber [63]. The dynamics of the heat transfer process of the receiver can be described by
\[
K_r \frac{dT_h}{dt} = q_i - q_L - q_h \tag{4.6}
\]

where \( K_r \) is a constant whose value depends on the volume and thermal characteristics of the absorber tubes [87].

Substituting (4.1) and (4.4) into (4.6), and in the \( s \)-domain representation, the concentrator/receiver model is

\[
T_h(s) = \frac{\eta_{\text{con}} A_{\text{con}} I(s) + K_L T_a - q_h(s)}{K_L + K_r s} \tag{4.7}
\]

The block diagram describes the dynamic of the concentrator and receiver is shown in Figure 4.2.

![Block diagram of the concentrator and receiver.](image)

Unlike other applications of the Stirling engine, the “fuel” supplied to the dish-Stirling system, i.e., the solar insolation \( I \), is intermittent. It can be seen from (4.7) that a change of \( I \) will cause variation in the temperature \( T_h \). As controlling \( T_h \) to an acceptable value is one of the most important tasks in the operation of the dish-Stirling system, \( q_h \) must be regulated appropriately.

### 4.2.2. Modeling of Mean Pressure Control

Among the many types of Stirling engine and methods to control \( q_h \) described in e.g. [29], only the double-acting kinematic engine with variable mean pressure control (MPC) scheme is studied here as it is the most developed and effective configuration in current applications [13]. The basic concept of the temperature control scheme based on MPC can be explained using Figure 4.3.
In Figure 4.3, the concentrator and receiver model is that shown in Figure 4.2. For the Stirling engine model, the three variables, mean pressure $p_{\text{mean}}$ of the working gas, engine speed $\omega_m$ and heater temperature $T_h$, describe the behavior of the Stirling engine. $p_{\text{mean}}$ is shown to be approximately proportional to the total mass $M$ of the working gas in the cylinders of the Stirling engine [30], which is discussed in Section 3.2.3. Thus, by operating a set of valves, $p_{\text{mean}}$ can be controlled by either supplying or dumping the working gas. The MPC system could therefore be studied using the block diagram shown in Figure 4.4, where the approximately linear relationship between $p_{\text{mean}}$ and $M$ is represented by the constant gain $K_p$, as expressed using (3.28).

The single-lag block is used to represent the fast-responding solenoid valves [12]. Solenoid valves use pulse width modulation (PWM) techniques regulate the mass flow rate $gA = dM/dT$, where the valves are turned on and off successively, according to the pressure command $c$. The process is modeled as [88],

$$T_v \frac{d(gA)}{dt} = -(gA) + K_v c$$  \hspace{1cm} (4.8)
where $K_i$ and $T_i$ are the gain and time constants of the solenoid valves, and $c$ is the input reference command of the solenoid valves to control the working gas.

The proportional controller $G_p$ regulates $p_{\text{mean}}$. Tuning method for $G_p$ is given in [16] based on the desired damping ratio $\zeta$, i.e.

$$G_p = \frac{1}{4T_v \xi^2 K_i K_p} \quad (4.9)$$

The transfer function $G_{\text{MPC}}(s) = \frac{p_{\text{mean}}(s)}{p_{\text{mean}}(s)}$ is

$$G_{\text{MPC}}(s) = \frac{p_{\text{mean}}(s)}{p_{\text{mean}}(s)} = \frac{1}{T_p T_p s^2 + T_p s + 1} \quad (4.10)$$

where $T_p = 1/(G_pK_iK_p) = 4\xi^2 T_v$. For $\xi = 0.707$, $T_p = 2T_v$.

Noting that for the fast-responding solenoid valves, $T_v \ll 1$, thus $G_{\text{MPC}}(s)$ can be simplified to a first-order transfer function for control system design, i.e.

$$G_{\text{MPC}}(s) = \frac{p_{\text{mean}}(s)}{p_{\text{mean}}(s)} \approx \frac{1}{T_p s + 1} \quad (4.11)$$

MPC can be considered as the inner loop of the temperature control scheme shown in Figure 4.3.

### 4.2.3. Modeling of Stirling Engine

Thus far in the open literature, the design of the temperature controller is carried out in an empirical manner as no suitable model of the Stirling engine has been developed for this purpose. Hence in [16] and as shown in Figure 4.4, only the approximate linear mean pressure and total mass model of the Stirling engine is used. The normalized mathematical model explored in Section 3.2.5 and is recalled here

$$G_n(s) = \frac{q_h(s)}{p_{\text{mean}}(s)} = \hat{K}_h \hat{\omega}_n \frac{1-T_{s1}s}{1+T_{s2}s} \quad (4.12)$$

$$G_m(s) = \frac{\bar{p}_n(s)}{p_{\text{mean}}(s)} = \hat{K}_m \hat{\omega}_n \frac{1}{1+T_{s3}s} \quad (4.13)$$

where
\[
\hat{K}_h = \sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} \bar{P}_{\text{mean}}^{i-1} \bar{\omega}_m^{j-1} \tag{4.14}
\]

\[
\hat{K}_m = \sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij} \bar{P}_{\text{mean}}^{i-1} \bar{\omega}_m^{j-1} \tag{4.15}
\]

The relationship governing \( \bar{q}_h \) and \( \bar{P}_{\text{mean}} \) is utilized to advantage in the control of \( \bar{T}_b \), as shown next.

### 4.2.4. Modeling of Induction Generator

The complexity of the model depends on the objectives of studies. In the time-domain simulation of many grid-related studies, fifth-order induction machine model [69] [89] has acceptable accuracy and are commonly used. The per-unit fifth-order induction generator model in \( d-q \) reference frame is derived using the base power rating \( S_N \) of the machine, the rated terminal voltage \( V_N \) of the stator and the nominal system frequency \( f_N \). Two modifications are made:

1) The directions of the currents: the stator current \( I_s \) flows out of the stator windings and the rotor currents \( I_r \) flows into the rotor windings, as indicated in Figure 4.5;

2) The direction of the torques: as for a generator the mechanical torque \( \tau_L \) is the driving torque and the electromagnetic torque \( \tau_e \) is the braking torque, the sign of \( \tau_L \) should be positive in the motion equation of the generator. By doing so, the \( \tau_L \) and \( \tau_e \) are positive for the induction generator.

![Figure 4.5. Direction convention of stator and rotor currents.](image-url)
The complete equations of the per-unit model of induction generator in $d$-$q$ reference frame are given below:

**Voltage equations:**

\[
v_{ds} = -R_{ds}i_{ds} + \omega_{m}\psi_{qs} - \frac{1}{\omega_{m}} \frac{d\psi_{ds}}{dt}
\]  
(4.16)

\[
v_{qs} = -R_{qs}i_{qs} - \omega_{m}\psi_{ds} - \frac{1}{\omega_{m}} \frac{d\psi_{qs}}{dt}
\]  
(4.17)

\[
v_{dr} = R_{dr}i_{dr} - \omega_{m}\psi_{qr} + \frac{1}{\omega_{m}} \frac{d\psi_{dr}}{dt}
\]  
(4.18)

\[
v_{qr} = R_{qr}i_{qr} + \omega_{m}\psi_{dr} + \frac{1}{\omega_{m}} \frac{d\psi_{qr}}{dt}
\]  
(4.19)

**Flux-linkage equations:**

\[
\psi_{ds} = L_{d}i_{ds} - L_{m}i_{dr}
\]  
(4.20)

\[
\psi_{qs} = L_{q}i_{qs} - L_{m}i_{qr}
\]  
(4.21)

\[
\psi_{dr} = L_{d}i_{dr} - L_{m}i_{ds}
\]  
(4.22)

\[
\psi_{qr} = L_{q}i_{qr} - L_{m}i_{qs}
\]  
(4.23)

**Electromagnetic torque equation:**

\[
\tau_{e} = \frac{L_{m}}{L_{d}} (\psi_{qs}i_{dr} - \psi_{dr}i_{qr})
\]  
(4.24)

**Lumped-mass motion equation:**

\[
2H \frac{d\omega_{r}}{dt} = \tau_{L} - \tau_{e} - F \omega_{r}
\]  
(4.25)

**Power equations:**

\[
P_{e} = v_{ds}i_{ds} + v_{qs}i_{qs}
\]  
(4.26)

\[
Q_{e} = v_{qs}i_{ds} - v_{ds}i_{qs}
\]  
(4.27)
\[ P_r = v_{dq}i_{dq} + v_{qr}i_{qr} \]  \hspace{1cm} (4.28)

\[ Q_r = v_{qr}i_{dq} - v_{dq}i_{qr} \]  \hspace{1cm} (4.29)

where \( v_{ds}, v_{qs}, v_{dr} \) and \( v_{qr} \) are the stator and rotor voltage components in the \( d-q \) reference frame, \( i_{ds}, i_{qs}, i_{dr} \) and \( i_{qr} \) are the stator and rotor current components in the \( d-q \) reference frame. \( \psi_{ds}, \psi_{qs}, \psi_{dr} \) and \( \psi_{qr} \) are the stator and rotor flux-linkage components in the \( d-q \) reference frame. \( R_s \) and \( R_r \) are stator and rotor resistances respectively. \( L_s, L_r \) and \( L_m \) are the stator self-inductance, rotor self-inductance and magnetizing inductance respectively. \( \omega_s \) is the synchronous speed, \( \omega_{sl} \) is the angular speed of slip frequency and \( \omega_r \) is the rotor electrical speed of the induction generator. \( H \) and \( F \) are the equivalent inertia constant and friction factor of a single-lumped-mass shaft system respectively. \( P_s, Q_s, P_r \) and \( Q_r \) are the stator and rotor real and reactive powers.

The following base quantities of stator are selected for the per-unit system:

- Base power: rated power, \( S_B = S_N \), (VA);
- Base voltage: peak value of rated phase voltage, \( V_B = \sqrt{2/3}V_N \), (V);
- Base frequency: nominal system frequency, \( f_B = f_N \), (Hz).

Using the above three quantities, other base values can be set:

- Base current: \( I_B = S_B / V_B \), (A);
- Base impedance: \( Z_B = V_B / I_B \), (\( \Omega \));
- Base inductance: \( L_B = Z_B / \omega_B \), (H);
- Base flux linkage: \( \psi_B = V_B / \omega_B \), (Wb).
- Base electrical angular speed: \( \omega_B = 2\pi f_B \), (rad/s);
- Base mechanical angular speed: \( \omega_{mb} = \omega_B / p_n \), (rad/s);
- Base torque: \( \tau_B = S_B / \omega_{mb} \), (N\( \cdot \)m).

For the SCIG used in the constant-speed DS system, the rotor windings are short-circuited. Thus, in (4.18) and (4.19) the rotor voltage components \( v_{dr} \) and \( v_{qr} \) in \( d-q \) reference frame are zero. Moreover, from (4.28) and (4.29) it can be seen that the
rotor real and reactive powers are both zero. The total injected power from the generator is provided by the stator. The complete model can be represented using the schematic in Figure 4.6, where the relationship between the power grid and the prime mover is shown.

![Figure 4.6. Schematic of the SCIG and the interaction with grid and the prime mover.](image)

### 4.2.5. Model Normalization

In order to integrate the developed normalized Stirling engine model into the dish-Stirling system for computer simulation and control system design, the unit system of the other components need to be converted properly. In the following context, symbol ‘−’ indicates the quantities based on Stirling engine per-unit system.

Using the same base values for the normalized model of the Stirling engine derived in Section 3.5, the normalized equation of (4.7) is

$$
\bar{T}_h(s) = \frac{K_{\text{rec}} [K_{\text{con}} \bar{T}(s) - \bar{q}_h(s)] + \bar{T}_a}{1 + T_{\text{rec}} s}
$$

(4.30)

where \(\bar{T}_a\) is the normalized temperature of the atmosphere, which is assumed to be constant over the studied period. The three constants \(T_{\text{rec}}, K_{\text{rec}}\) and \(K_{\text{con}}\) are expressed as [83]

$$
K_{\text{con}} = \eta_{\text{con}} A_{\text{con}} \frac{I_{\text{max}}}{P_{m,N}}
$$

(4.31)

$$
K_{\text{rec}} = \frac{1}{K_{L} T_{h,\text{max}}}
$$

(4.32)
\[ T_{\text{rec}} = \frac{K_r}{K_L} \]  

(4.33)

The normalized transfer function of the pressure control loop is the same as (4.11), i.e.

\[ G_{\text{MPC}}(s) = \frac{\bar{p}_{\text{mean}}(s)}{p_{\text{mean}}(s)} = \frac{1}{T_p s + 1} \]  

(4.34)

For the induction generator, as shown in Section 4.2.4, the base quantities are typically selected different from that for the prime mover. Thus, the torque and speed need to be converted accordingly. The relationship is shown in Figure 4.7.

![Per-unit system conversion](image)

**Figure 4.7.** Per-unit system conversion of dish-Stirling system.

The torque and speed conversion ratio are defined as

\[ K_{\text{TC}} = \frac{\tau_L(\text{generator})}{\tau_m(\text{engine})} = \frac{(2\pi f_N / p_n)P_{m,N}}{\omega_{m,N} S_N} \]  

(4.35)

\[ K_{\text{SC}} = \frac{\omega_r(\text{engine})}{\bar{\omega}_m(\text{generator})} = \frac{(2\pi f_N / p_n)}{\omega_{m,N}} \]  

(4.36)

where \( P_{m,N} \) and \( \omega_{m,N} \) are the nominal output power and speed of Stirling engine, \( S_N \) is the rated power of induction generator and \( f_N \) is the nominal grid. After the conversion, the motion equation of the stiff shaft (4.25) becomes

\[ 2H' \frac{d\bar{\omega}_m}{dt} = \bar{\tau}_m - \bar{\tau}_e' - F' \bar{\omega}_m \]  

(4.37)

where \( H' \), \( F' \) and \( \bar{\tau}_e' \) are the quantities that have been converted from those based on nominal values of the induction general to the same bases as those used in the
normalized model of the DS developed in Section 3.2. The relationship between them is

\[ H' = \left( \frac{\omega_{m,N}}{\omega_{m_b}} \right)^2 \frac{S_N}{P_{m,N}} H = \frac{1}{K_{TC} K_{SC}} H \]  

(4.38)

\[ F' = \left( \frac{\omega_{m,N}}{\omega_{m_b}} \right)^2 \frac{S_N}{P_{m,N}} F = \frac{1}{K_{TC} K_{SC}} F \]  

(4.39)

\[ \bar{\tau}_c' = \frac{1}{K_{TC}} \tau_c \]  

(4.40)

### 4.3. Linearized Model of Constant-Speed DS system

#### 4.3.1. Linearized Model Considering Engine Speed as a Disturbance

Figure 4.3 shows the two major disturbances in the temperature control loop are the variations of \( I \) and \( \omega_m \), the latter would include the consequence of a disturbance in the external power grid system. Considering a small deviation from an arbitrary steady-state operating point \( \bar{p}_{\text{mean}} = \bar{p}_0 \in [\bar{p}_{\text{min}}, \bar{p}_{\text{max}}] \), \( \bar{T}_h = \bar{T}_{h0} \in [\bar{T}_{h,\text{min}}, \bar{T}_{h,\text{max}}] \), and \( \bar{\omega}_{m0} = 1.0 \) p.u.

From (4.30), therefore variations of \( \bar{T}_h \) can be expressed as

\[ \Delta \bar{T}_h(s) = \frac{K_{\text{rec}} K_{\text{con}} \Delta I(s) - \Delta \bar{q}_h(s)}{1 + T_{\text{rec}} s} \]  

(4.41)

The change of the absorbed heat transfer rate \( \Delta \bar{q}_h \) of the Stirling engine due to engine speed \( \Delta \bar{\omega}_m \) and the mean pressure of the working gas \( \Delta \bar{p}_{\text{mean}} \) changes can be obtained using (4.12) to (4.15) and expressed as

\[ \Delta \bar{q}_h = \frac{\partial \bar{q}_h}{\partial \bar{p}_{\text{mean}}} \bigg|_{\bar{p}_{\text{mean}}, \bar{\omega}_m} \Delta \bar{p}_{\text{mean}} + \frac{\partial \bar{q}_h}{\partial \bar{\omega}_m} \bigg|_{\bar{p}_{\text{mean}}, \bar{\omega}_m} \Delta \bar{\omega}_m \]  

(4.42)

By considering \( \bar{\omega}_{m0} = 1.0 \) p.u. The coefficients in (4.42) can be expressed approximately as

\[ \frac{\partial \bar{q}_h}{\partial \bar{p}_{\text{mean}}} \bigg|_{\bar{p}_{\text{mean}}, \bar{\omega}_m} = \left( a_{10} + a_{11} \right) \frac{1 - T_{\text{se1}} s}{1 + T_{\text{se2}} s} \]  

(4.43)
\[ \frac{\partial \bar{q}_h}{\partial \bar{\omega}_m} \bigg|_{p_e, \theta_{\text{m}}} = a_{01} + a_{11} \bar{p}_0 \]  

(4.44)

Combining (4.41)–(4.44), the normalized linearized model of the Stirling engine for the heat absorber temperature control is illustrated in Figure 4.8. The relevant expressions used in deriving the model are also shown in the figure.

\[ \sum_{\text{rec}} K_{\text{con}} \frac{1}{1 + T_{\text{sec}} s} \Delta \bar{I} \]

\[ \sum_{\text{rec}} \frac{1}{1 + T_{\text{sec}} s} \Delta \bar{I}_{h} \]

Figure 4.8. Linearized plant model for the design of temperature control of the heat absorber in the Stirling engine.

4.3.2. Linearized Model Considering Electromagnetic Torque as a Disturbance

Similarly, the change of the mechanical torque \( \Delta \bar{\tau}_m \) of the Stirling engine due to the variation of the engine speed \( \Delta \bar{\omega}_m \) and the mean pressure \( \Delta \bar{p}_{\text{mean}} \) of the working gas can be obtained using (4.13) and (4.15), i.e.

\[ \Delta \bar{\tau}_m = \frac{\partial \bar{\tau}_m}{\partial \bar{p}_{\text{mean}}} \bigg|_{p_e, \theta_{\text{m}}} \Delta \bar{p}_{\text{mean}} + \frac{\partial \bar{\tau}_m}{\partial \bar{\omega}_m} \bigg|_{p_e, \theta_{\text{m}}} \Delta \bar{\omega}_m \]  

(4.45)

By considering \( \Delta \bar{\omega}_{m0} = 1.0 \text{ p.u.} \), the coefficients in (4.45) can be expressed approximately as

\[ \frac{\partial \bar{\tau}_m}{\partial \bar{p}_{\text{mean}}} \bigg|_{p_e, \theta_{\text{m}}} = \left( b_{10} + b_{11} + b_{12} \right) \frac{1}{1 + T_{\text{sec}} s} \]  

(4.46)

\[ \frac{\partial \bar{\tau}_m}{\partial \bar{\omega}_m} \bigg|_{p_e, \theta_{\text{m}}} = b_{01} + 2 b_{02} + \left( b_{10} + b_{11} + 2 b_{12} \right) \bar{p}_0 \]  

(4.47)

According to (4.37)–(4.47), the block diagram of the linearized model considering electromagnetic torque \( \Delta \bar{\tau}_e \) as a disturbance is shown in Figure 4.9,
Figure 4.9. Block diagram of the linearized model considering electromagnetic torque as a disturbance.

In Figure 4.9, the engine speed is the internal variable. The electromagnetic torque $\Delta \varphi_e$ is a disturbance in this condition. $\Delta \varphi_e$ may cause by the grid dynamics such as abnormal operation of grid fault. This model can be used for further analysis for linear temperature control design.

4.4. Temperature Control Design of Constant-Speed DS System

Temperature control is critical because it ensures the efficient and secure operation of the dish-Stirling system. The temperature of the receiver/absorber should be maintained as high as possible to maximize the thermal efficiency of the Stirling engine, while it must not exceed the thermal limit of the materials of the metallic tubes and the receiver walls. Due to the non-minimum phase characteristic of the Stirling engine as shown in (4.12), careful design of temperature control system is required to properly damper the system and prevent large temperature variation during inevitable disturbance introduced from the system.

4.4.1. Transient Droop Compensation

The basic concept of the temperature control base on mean pressure control is shown in Figure 4.3. In the control of the absorber temperature in the Stirling engine, the average temperature on the absorber is conventionally measured using temperature sensors such as thermocouples. The pressure reference setting $\bar{p}_{\text{mean}}^*$ is often
calculated based on empirical static temperature-pressure droop characteristic, such as that shown in Figure 4.10 [16].

![Diagram of droop temperature controller]

Figure 4.10. Pressure versus temperature relationship with droop characteristic.

The permanent droop $D_P$ is defined as the slope

$$D_P = \frac{\Delta \bar{T}_h}{\Delta \bar{p}_{\text{mean}}} = \frac{\bar{T}_{h,\text{max}} - \bar{T}_{h,\text{min}}}{\bar{p}_{\text{max}} - \bar{p}_{\text{min}}} = \frac{1 - \bar{T}_{h,\text{min}}}{1 - \bar{p}_{\text{min}}}$$

(4.48)

With the permanent droop $D_P$ as defined in (4.48), $\bar{T}_h$ is thus not maintained constant over the whole range of the insolation level. This is equivalent to the introduction of a proportional controller (P controller) with gain $1/D_P$ into the temperature control system. The structure of this droop controller is shown in Figure 4.11.

![Diagram of droop temperature controller block diagram]

Figure 4.11. Block diagram of the droop temperature controller.

Combining Figure 4.8 and Figure 4.11, the open-loop transfer function of the temperature control loop can be obtained as

$$G_{\text{open-loop}}(s) = \frac{K_{\text{rec}}(a_{10} + a_{11})(1 - T_{\text{set}}s)}{D_p(1 + T_p s)(1 + T_{\text{set}} s)(1 + T_{\text{rec}} s)}$$

(4.49)
By considering \( T_{\text{rec}} >> T_p \) and \( T_{\text{rec}} >> T_{\text{se2}} \), (4.49) can be simplified to

\[
G_{\text{open-loop}}(s) \approx \frac{K_{\Sigma}(1-T_{\text{se}1}s)}{D_p(1+T_{\Sigma}s)}
\]

(4.50)

where \( K_{\Sigma} = K_{\text{rec}}(a_{10}+a_{11}), T_{\Sigma} = T_p + T_{\text{se2}} + T_{\text{rec}}. \)

Apply Hurwitz-Routh Stability Criteria on the resulting closed-loop transfer function, the range of \( D_p \) for which stable operation is guaranteed is

\[
D_p > \frac{K_{\Sigma}T_{\text{se}1}}{T_{\Sigma}}
\]

(4.51)

Due to the non-minimum phase nature of \( G_h(s) \), however, the change in \( \bar{P}_{\text{mean}} \) will cause an initial \( \bar{q}_h \) change in a direction opposite to that sought. Thus, \( \bar{T}_h \) will require a longer time to reach its new steady state following sudden \( T \) and \( \bar{\omega}_m \) variations. A solution to reduce the settling time is to increase \( D_p \), but it will lead to an increase in the steady-state error in \( \bar{T}_h \) and significant reduction in the engine efficiency.

Figure 4.12. A new temperature controller with transient droop and feedforward compensation.

To overcome this problem, a transient droop compensation scheme is now proposed and shown in Figure 4.12. The transfer function of the proposed temperature controller is

\[
G_c(s) = \frac{1}{D_p} \frac{1+T_{R}s}{1+(1+D_p/T_{R})T_{R}s}
\]

(4.52)
where \( D_T \) denotes the temporary droop setting and \( T_R \) is the reset time. As \( t \to 0 \), \( s \to \infty \), and \( G_w(s) \) becomes

\[
G_w(\infty) = \frac{1}{D_p} \frac{1}{1 + \frac{D_T}{D_p}} = \frac{1}{D_p + D_T}
\]  

(4.53)

As \( t \to \infty \), \( s \to 0 \), thus,

\[
G_w(0) = \frac{1}{D_p}
\]  

(4.54)

From (4.53), it can be readily shown that the equivalent droop is temporarily increased to \( D_p + D_T \) at the initial stage of the transients. The open-loop transfer function of temperature control loop is

\[
G_{\text{open-loop}}(s) \approx \frac{K_T(1 - T_{\text{set}}s)(1 + T_R s)}{D_p(1 + T_s s)[1 + (1 + \frac{D_T}{D_R})T_R s]}
\]  

(4.55)

The range of \( D_p \) for which closed-loop stability is guaranteed is

\[
D_p > \max \left\{ \frac{K_T T_{\text{set}}}{T_s} \frac{1}{1 + D_T/D_p}, \frac{K_T (T_{\text{set}} - T_R)}{T_s + (1 + D_T/D_p)T_R} \right\}
\]  

(4.56)

It can be seen that the maximum \( D_p \) determined from (4.56) is smaller than that obtained in (4.51). The proposed scheme actually provides lead-lag compensation and leads to improved system stability.

### 4.4.2. Feedforward Compensation during Periods of Speed Variations

As can be seen from the above analysis, variations in the engine speed \( \bar{\omega}_m \) would result in the absorber temperature change. During normal operation, \( \bar{\omega}_m \) tends to be small and the temperature control functions described in the previous sub-section functions well. However, if \( \bar{\omega}_m \) varies greatly during e.g. severe system disturbance condition, \( \Delta \bar{\omega}_m \) needs to be compensated for and this can be achieved using a feedforward controller. By measuring \( \bar{\omega}_m \) and \( \bar{p}_{\text{mean}} \), a compensation term \( \bar{p}_{\text{ff}} \) is used to cancel the effect of \( \Delta \bar{\omega}_m \), as reflected by the second term in (4.42). Whence
\[ \bar{p}_{\text{ff}} = \frac{a_{01} + a_{11} \bar{p}_0}{a_{10} + a_{11}} \Delta \bar{\omega}_m = \frac{a_{01} + a_{11} \bar{p}_{\text{mean}}}{a_{10} + a_{11}} \Delta \bar{\omega}_m \] (4.57)

The feedforward control loop is also featured in Figure 4.12.

4.5. **Maximum Energy Harness by Utilizing the DS Systems**

In this section, the potential to achieve maximum solar energy harness using the DS system shall be examined.

4.5.1. **Output Power versus Insolation under Constant Speed Operation**

Steady-state operation of the dish-Stirling system will be examined next. As \( T_h \) can be effectively maintained nearly constant by the improved control scheme described in Section 4.4, the effect of \( \Delta T_h \) on the engine performance will not be included in the analysis. Substitute (3.51) and (4.48) into (4.7), the steady-state relationship governing \( \bar{p}_{\text{mean}}, \bar{T} \) and \( \bar{\omega}_m \) is obtained:

\[
\bar{p}_{\text{mean}} = \frac{K_{\text{con}} \bar{T} - a_{01} \bar{\omega}_m - a'_{00}}{a_{10} \bar{\omega}_m + a'_{00}} 
\] (4.58)

where

\[
a'_{00} = a_{00} - \frac{\bar{T}_a - \bar{T}_{h,\text{max}} + D_p \bar{p}_{\text{max}}}{K_{\text{rec}}} = a_{00} - \frac{\bar{T}_a - 1 + D_p}{K_{\text{rec}}} 
\]

\[
a'_{00} = a_{10} + \frac{D_p}{K_{\text{rec}}} 
\]

In practice however, \( \bar{p}_{\text{mean}} \) must be higher than the minimum pressure \( \bar{p}_{\text{min}} \). For constant engine speed operation, thus (4.58) is only applicable above the minimum insolation level \( \bar{T}_{\text{min}} \) where \( \bar{T}_{\text{min}} \) is obtained by substituting \( \bar{p}_{\text{min}} \) into (4.58), i.e.,

\[
\bar{T}_{\text{min}} = \left( a_{11} \bar{p}_{\text{min}} + a_{01} \bar{\omega}_m + a'_{10} \bar{p}_{\text{min}} + a'_{00} \right) K_{\text{con}}^{-1} 
\] (4.59)

Also, by substituting (4.58) into (3.45), the relationship between the steady-state output power \( \bar{p}_m, \bar{\omega}_m \) and \( \bar{T} \) can be obtained:

\[
\bar{p}_m = m_1 \bar{T} + n_1 
\] (4.60)
where

$$m_1 = \frac{b_{10} + b_{11} \omega_m + b_{12} \omega_m^2}{a_{11} \omega_m + a_{10}^\prime} K_{\text{con}}$$  \hspace{1cm} (4.61)$$

$$n_1 = b_{00} + b_{01} \omega_m + b_{02} \omega_m^2 - (b_{10} + b_{11} \omega_m + b_{12} \omega_m^2) \frac{a_{01} \omega_m + a_{00}^\prime}{a_{11} \omega_m + a_{10}^\prime}$$  \hspace{1cm} (4.62)$$

For constant-speed operation with (say) \( \omega_m \) at the nominal speed of 1.0 p.u., from (4.60), \( P_m \) is seen to be linear with respect to \( T \). This is as shown in Figure 4.13 by the line AD. If \( T \) is less than \( T_{\text{min}} \), \( \bar{T} \) will rise as the power balance cannot be maintained by reducing \( \bar{p}_{\text{mean}} \) any further. \( \bar{T} \) is then not controllable and may even exceed its maximum set value: the dish-Stirling system is to shut down and the output power is zero.

![Figure 4.13. \( P_m - I \) relationship under constant speed and variable speed operations of the dish-Stirling system.](image)

If one ignores losses in the mechanical to electrical energy conversion process in the generator, the linear power-insolation characteristic is similar to the DS system output electrical power-solar insolation relationship derived from the linear interpolation of the experimental data shown in [90]. The present approach is, however, advantageous in that it provides a direct means in deriving the linear relationship (4.60) based on the proposed MP model and in conjunction with (4.61) and (4.62). Hence, the approach is particularly useful during the feasibility evaluation or planning stage of specific dish-Stirling solar-thermal scheme.
4.5.2. Feasible Operating Regime under Variable Speed Operation

Equation (4.58) shows explicitly how steady-state $\bar{p}_{\text{mean}}$, $\bar{\omega}_m$ and $\bar{T}$ are related. As shown in (3.52) and (3.55), $\bar{p}_{\text{mean}}$ and $\bar{\omega}_m$ are the two important variables that govern $P_m(0)$. The potential of controlling $P_m(0)$ through the manipulation of $\bar{\omega}_m$ shall now be examined. Using (3.45) and (4.58), a family of typical $P_m(0) - \bar{\omega}_m$ curves for different insolation levels is depicted in Figure 4.14. Note that at each of the constant-$I$ curves, there is a corresponding maximum $P_m(0)$. Also each point on the curve has the corresponding $\bar{p}_{\text{mean}}$, although $\bar{p}_{\text{mean}}$ has not been explicitly indicated on the figure.

Operation of the dish-Stirling system is limited by several practical constraints placed on $\bar{p}_{\text{mean}}$, $\bar{\omega}_m$ and $\bar{T}$. These limits define the feasible operating boundary ABCGFA shown on Figure 4.14. The various sectors of the boundary shall now be described.

Firstly, the boundaries FA and GC, corresponding to the engine operating under the maximum and minimum mean pressures respectively, can be derived by setting $\bar{p}_{\text{mean}} = \bar{p}_{\text{max}}$ and $\bar{p}_{\text{mean}} = \bar{p}_{\text{min}}$ in (3.45) for the speed range $\bar{\omega}_{\text{min}} \leq \bar{\omega}_m \leq \bar{\omega}_{\text{max}}$. Next, the remaining boundaries shall depend on the operating speed range, as follows.

![Figure 4.14. Steady-state feasible operating area of dish-Stirling system.](image)
4.5.3. Variable Speed Operation and Maximum Power Harness

Figure 4.13 shows that for the constant-speed operation, there will be a minimum insolation level \( I_{\text{min}} \) below which power generation is not possible. The reason for this has been explained in Section 4.5.2. Suppose the engine operates at the nominal speed \( \bar{\omega}_m \) of 1 p.u. The engine operating state at the minimum insolation level will then correspond to the point D in Figure 4.14 where \( I_{\text{min}} \) is obtained using (4.58) and \( \bar{p}_{\text{mean}} = \bar{p}_{\text{min}} \). As \( I \) increases and maintaining \( \bar{\omega}_m \) constant at 1.0 p.u., the operating state moves from D toward A where \( \bar{I} = \bar{I}_{\text{min}} \). The constant speed operation is therefore described by the line DA. On the other hand, if the dish-Stirling system is able to operate at variable speed and by reducing \( \bar{\omega}_m \) to below 1 p.u., it is still feasible to operate the dish-Stirling system in the shaded area EDG where the insolation level is lower than \( I_{\text{min}} \) the minimum insolation level determined under the constant speed operating mode. The corresponding minimum speed \( \bar{\omega}_{\text{min}} \) at point G can be calculated by substituting \( \bar{P}_{m}(0) = 0 \) and by setting \( \bar{p}_{\text{mean}} = \bar{p}_{\text{min}} \) into (3.45). Then using (4.58), the minimum insolation level \( I_{\text{min}} \) under variable-speed operation can be readily shown to be

\[
I_{\text{min}} = \frac{\bar{p}_{\text{min}}(a_{11}\bar{\omega}_{\text{min}} + a_{10}a_{10}) + a_{10}\bar{\omega}_{\text{min}} + a_{10}a_{10}}{K_{\text{con}}}
\]  

(4.63)

The nominal operating speed of the dish-Stirling system is often selected to be at the maximum thermal efficiency point at maximum insolation. This is because this operating state can be easily found from the specifications or experimental data of the Stirling engine. However, Figure 4.14 shows that the Stirling engine is then not harnessing the maximum power at the nominal speed. The theoretical maximum power point (MPP) locus could be obtained by setting \( \partial \bar{P}_{m}/\partial \bar{\omega}_m = 0 \) in (4.60), but the locus is a complex nonlinear equation. Instead, an approximate expression of the MPP locus can be derived by firstly, ignoring the terms containing the small-value coefficients \( b_{00}, b_{01}, b_{02} \) and \( a_{10}' \) in (4.60)–(4.62). The approximate expression of \( \bar{P}_{m} \) is
\[
\bar{P}_m = \frac{b_{10} + b_1 \bar{\omega}_m + b_2 \bar{\omega}_m^2}{a_{11} \bar{\omega}_m} (K_{\text{con}} \bar{T} - a_{10} \bar{\omega}_m - a'_{10}) \quad (4.64)
\]

At high insolation level, \( K_{\text{con}} \bar{T} >> a_{10} \bar{\omega}_m + a'_{10} \). The speed \( \bar{\omega}_{m,\text{opt}} \) by which maximum power harness occurs can be obtained by setting \( \frac{\partial \bar{P}_m}{\partial \bar{\omega}_m} \bigg|_{\bar{\omega}_m=\bar{\omega}_{m,\text{opt}}} = 0 \), whence

\[
\bar{\omega}_{m,\text{opt}} \approx \frac{b_{10}}{b_{12}} \quad (4.65)
\]

Operation at constant speed \( \bar{\omega}_{m,\text{opt}} \) is represented by the vertical line to the left of the nominal constant speed operating line AD in Figure 4.14. The theoretical MPP locus is also shown there. In practice, due to the maximum pressure limit, the theoretical MPP is not achievable above the point H. The achievable maximum power is governed by the curve HA when \( \bar{p}_{\text{mean}} = \bar{p}_{\text{max}} \). As \( b_{12} < b_{10} < 0 \) and as \( I \) decreases, the MPP locus deviates increasingly from the vertical line AD as the condition \( K_{\text{con}} \bar{T} >> a_{10} \bar{\omega}_m + a'_{10} \) becomes less and less valid. At insolation level below that corresponding to that at point H, the straight line HK can be used to approximate the MPP locus instead. HK is of the form:

\[
\bar{P}_m \approx c \bar{\omega}_m + d \quad (4.66)
\]

Equation (4.66) can be used to advantage in the design of a speed control strategy to achieve maximum power harness.

The overall \( \bar{P}_m \) versus \( \bar{T} \) MPP curve AHK under variable speed operation is also shown in Figure 4.14. Clearly the amount of energy extracted under MPP depends on the characteristics of the \( \bar{P}_m - \bar{\omega}_m \) curve of the Stirling engine and the selection of the nominal value of \( \bar{\omega}_m \). If the nominal speed for the constant speed operation is much above \( \bar{\omega}_{n,\text{opt}} \), the variable speed operation of the DS system can be expected to be able to extract much more energy than that under the constant speed operation.
4.6. Illustrative Example

4.6.1. Comparison of Detailed and Average-Value Adiabatic Models under Step Change of Engine Speed

In this section, the purpose is to show the comparison of the model under different step changes of the engine speed. Under the normal operation of the constant-speed operation of the DS system, the value of the engine/generator speed varies with a very small range. While under the fault condition, or the DS system is operating in variable speed mode which will be discuss in Chapter 5, the engine/generator speed would change in a greater and faster manner.

The configuration of the test system is shown in Figure 4.15, where the conventional temperature control system with MPC, without any compensation or damping system, is added to the control loop. Three models of the Stirling engine are used including the detailed model, the general average-value model and the constant-speed average model developed in Chapter 3.

![Figure 4.15. Testing system: conventional temperature control system of DS system.](image)

The simulation starts with the insolation level $\bar{T} = 1.0$ p.u. and the simulation results are shown in Figure 4.16 and Figure 4.17. In Figure 4.16, the engine speed $\bar{\omega}_m$ drops from 1.0 p.u. to 0.9 p.u. while that of Figure 4.17 pertains to the engine speed decreases to 0.6 p.u. It can be seen that the high-frequency oscillation from the detailed model is not found in the simulation results of the average-value models. The general average-value model presented in Section 3.5.1 is able to reflect the behavior of the Stirling engine more accurately than the model assuming constant-speed operation as described in Section 3.5.2, because during large speed variation, the assumption of constant speed operation is no longer valid.
Chapter 4 Constant-Speed Dish-Stirling Solar-Thermal System

Figure 4.16. Simulation results for model comparison with small change on engine speed $\Delta \omega_m = 0.1$ p.u.
Figure 4.17. Simulation results for model comparison with large change on engine speed ($\Delta \omega_m = 0.4$ p.u.).
4.6.2. Comparison of Detailed and Average-Value Adiabatic Models Under Grid Fault

In this section, the purpose is to show the comparison between the results obtained using the ideal adiabatic model with that obtained using the developed average-value model under grid fault condition.

Consider a 25-kW dish-Stirling system connected to a large grid via a 480V/22-kV step-up transformer and 20-km distribution line. The short-circuit level at the 22-kV connection point is assumed to be 1 MVA, yielding an equivalent system impedance of 0.1+j1 p.u. on 1.0 MVA base. The Stirling engine is connected to a 25-kW induction generator (of 2 pole-pairs). A 15.6 kVar static capacitor bank is connected at the terminal of the generator for reactive power compensation purpose.

The parameters of the dish-Stirling system are: \( \eta_{\text{con}} = 0.88 \), \( A_{\text{con}} = 88.7 \text{ m}^2 \), \( K_r = 200 \), \( K_L = 14.83 \), \( K_v = 1.0 \), \( T_v = 0.02 \text{ s} \), \( V_{sw} = 95 \text{ cm}^3 \), \( V_{cl} = 10 \text{ cm}^3 \), \( V_h = 33.08 \text{ cm}^3 \), \( \theta = 0.89\pi \text{ rad} \), \( b = 0.2 \). Nominal parameters: \( p_{\text{max}} = 20 \text{ MPa} \), \( T_{h,\text{max}} = 1033 \text{ K} \), \( I_{\text{max}} = 1000 \text{ W/m}^2 \), \( P_{m,N} = 27 \text{ kW} \), \( \omega_{m,N} = 190.63 \text{ rad/s} \). These parameters were obtained from [81].

The model of the power system, including that of the dish-Stirling system, was established in MATLAB/Simulink and is illustrated in Figure 4.18. Due to space constraint, it is simply stated herewith the ideal adiabatic model of the double-acting kinematic Stirling engine used in the study is described in [30] and [81].

The disturbance is an on-off 3-cycle 3-phase to ground fault applied at the 22 kV connecting point. As sun insolation level is not expected to change significantly over the fault duration, \( I \) can be considered constant over the study period.
The simulation results based on the ideal and average-value adiabatic models are shown in Figure 4.19. It can be seen that using the average-value adiabatic model, only the average values of the input and output powers of the Stirling engine are evaluated. The high-frequency components (at twice the power frequency) caused by the engine piston motion are not represented. The average values reflect accurately the responses in terms of the output power and temperature. The corresponding step size used was 50 µs and the simulation required 33.1 s to complete. Using the ideal adiabatic model, however, it required a 2-µs step size and the simulation time was
much higher at 333.2 s. Thus using the average-value adiabatic model can speed up the simulation without greatly compromising the accuracy required in the study of power system dynamics.

### 4.6.3. Temperature Controller Design

The relevant parameters for the absorber temperature controller design can be calculated using the corresponding equations in Section 4.4: $K_{con} = 2.856$, $K_{rec} = 1.756$, $T_{rec} = 13.44$ s, $T_p = 0.056$ s, $T_{se1} \approx 0.11$ s at $\bar{\omega}_m = 1.0$ p.u., $T_{se2} = 0.03$ s. The minimum permanent droop calculated from (4.51) for the conventional droop temperature control system is 0.027, and its corresponding minimum temperature limit $T_{h,\min}$ must be less than 1008 K. In practical application, $T_{h,\min}$ is selected as 993 K using the droop controller, to guarantee stability of the system. However, from the Bode diagram shown in Figure 4.20, the corresponding phase margin is only 8.1° and the cut-off frequency ($\omega_c$) is 8.4 rad/s. When a large disturbance occurs, it will require a relatively long time for the temperature and output power to return to their steady states.

![Bode Diagram](image)

Figure 4.20. Open-loop Bode diagram of the systems with and without transient droop compensation.
Using the transient droop compensation, in order to enhance the transient response, the corresponding break frequency of the compensator is selected to be higher than $1/T_{rec}$ and lower than $\omega_c$, i.e.

$$\frac{1}{\omega_c} < T_R < (1 + \frac{D_T}{D_P})T_R < T_{rec} \quad (4.67)$$

By selecting $T_R = 0.5$ s and $D_T = D_P$, the new phase margin is 37.1°, which results in a much improved design over the conventional controller.

This expected improvement could be verified by studying the performance of the proposed temperature control scheme under rapid insolation variation condition. Typically, the rate of change of 30 W/(m²·s) is considered high in photovoltaic system study [91]. Figure 4.21 shows the results of a study in which has been ramped down and then up at a rate of 100 W/(m²·s) or 0.1 p.u./s. Even at such a rapid change, the proposed temperature controller with transient droop and feedforward compensation is able to maintain the absorber temperature to within an acceptable range and with negligible over- and under-shoots.
Figure 4.22. Comparison between the simulation results using the traditional and improved temperature controllers during grid-fault condition. Solid line: with transient droop and feedforward compensation; Dashed line: with transient droop compensation only; Dotted line: without transient droop and feedforward compensation.
The proposed design is further tested under the grid-fault condition considered earlier. The responses of the dish-Stirling system based on the conventional droop and the proposed transient droop reduction controllers are compared in Figure 4.22. There are persistent power oscillations after the fault clearance in the case of using the conventional droop controller. Similar phenomenon is also observed in the simulation result shown in [16]. Such power oscillations are undesirable, if a large-scale DS system is incorporated into the power grid. By introducing the transient droop compensation, the transients in the output power and temperature excursions are very rapidly subdued. Further improvement is seen with the addition of the proposed feedforward compensation.

4.7. Conclusions

Based on the developed linearized model, an improved temperature controller with transient droop reduction and feedforward compensation has been proposed. This new temperature controller is effective in reducing the temperature excursions following a disturbance. It also leads to improved damping in the output power of the DS solar-thermal power plant.

Furthermore, based on the steady-state analysis of the DS system, feasible operating area of the power plant has also been obtained. It is shown that to maximize the harness of the solar energy, it requires the DS system to operate under a variable speed mode.
Chapter 5 Variable-Speed Dish-Stirling Solar-Thermal System

5.1. Introduction

The previous chapter has examined the modeling, design and control of constant-speed dish-Stirling solar-thermal power system. In developing the dish-Stirling model, the steady-state relationship between the engine speed and the generated power has been derived. It shows that the harnessed energy from the sun can be maximized if variable-speed operation of the dish-Stirling system can be realized. As shall be shown in latter sections, variations of the engine speed would impact negatively on the control of the temperature of the receiver: strict control of the temperature is crucial to ensure safe and effective operations of the dish-Stirling system. The performance of the temperature control system would have impact on the output power and thus the dynamics of the entire system. Furthermore, in adopting variable-speed operations for the dish-Stirling system, there are two preferred choices of the electric generator: doubly-fed induction generator (DFIG) and permanent magnetic synchronous generator (PMSG). DFIG is attractive due to its economic advantage over other types of variable-speed generators, while PMSG has the advantage of larger operating speed range and better speed regulation capability. While reported works on speed and power control of DFIG and PMSG for other power source such as wind energy conversion system is abound, the present investigation shall show that the design of control schemes for such a DS-DFIG and DS-PMSG systems is more complex because the power regulating ability of the generating system will be impacted by the receiver operating temperature.

In this chapter, the overall configuration of DS-DFIG and DS-PMSG systems will be proposed and the models of the systems will be derived especially for the design of temperature control while achieving maximum power harnessing. The computer simulation results will be given to shown the performance of the proposed temperature controllers to overcome the problems introduced by the continuous engine speed variation which is not seen in the case of the constant-speed dish-Stirling system. Part of the materials contained in this chapter have appeared in the author’s publications [92].
5.2. **Modeling of Variable-Speed DS System**

5.2.1. **Configuration of Variable-Speed DS Systems**

In the DS solar-thermal power generation scheme considered in [15, 16, 83], the dish and Stirling engine function as heat engine to convert solar energy into mechanical energy to drive a constant-speed squirrel-cage induction generator. In contrast, in the variable-speed DS system equipped with DFIG or PMSG considered in the present investigation, a back-to-back converter is added as an interface between the generator and the external power grid. For the DS-DFIG system, a low-rating back-to-back converter connects the rotor windings of the DFIG with the grid through a rotor-side converter (RSC), a grid-side converter (GSC), a smoothing dc-link, a LC filter and a step-up transformer, as shown in Figure 5.1 (a). While for the DS-PMSG system, a full-rating back-to-back converter connects the stator windings of the PMSG with the grid through a stator-side converter (SSC), a GSC, a smoothing dc-link, a LC filter and a step-up transformer, as shown in Figure 5.1 (b).

![Diagram of DS-DFIG system](image1)

![Diagram of DS-PMSG system](image2)

**Figure 5.1.** Schematic diagram of one (a) DS-DFIG unit and one (b) DS-PMSG unit in a large-scale DS solar-thermal power plant.
Although the configurations of the generators are similar to that commonly seen in wind energy conversion systems, the challenge is on developing a credible model for the prime mover of the generator, i.e. the dish and Stirling engine. This work has been done in Chapter 3 where a general model for the prime mover is derived from basic heat transfer and thermodynamics principles that can be used for both DS-DFIG and DS-PMSG systems. This is unlike [69] where the authors utilized a simple first-order transfer function to model the DS system, and without clear justification on how the model can be sufficiently accurate for the purpose of designing the control system for the DS system.

5.2.2. Modeling of the Prime Mover: Dish and Stirling Engine

Recall the derived general normalized average-value model of the Stirling engine in Section 3.2 and the normalized model of the concentrator, receiver/absorber in Section 4.2.5. The overbar symbol “–” to denote the normalized values is omitted in the remaining part of this thesis.

**Stirling Engine:**

\[
q_h = \eta_h \left[ K_h p_{\text{mean}}\alpha_m + A(gA) + C_p \frac{dT}{dt} \right] 
\]

\[
P_m = \eta_m K_m p_{\text{mean}}\alpha_m
\]

\[
gA = \frac{dM}{dt} = \frac{dp_{\text{mean}}}{dt}
\]

\[
\eta_m = \frac{\sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij} p_{\text{mean}}^i \alpha_m^j}{K_m p_{\text{mean}} \alpha_m}
\]

\[
\eta_h = \frac{\sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} p_{\text{mean}}^i \alpha_m^j}{K_h p_{\text{mean}} \alpha_m}
\]

**Concentrator and Receiver/Absorber:**

\[
T_{rec} \frac{dT}{dt} = K_{rec}(K_{con}^l - q_h) - (T_h - T_a)
\]

**Solenoid Valves:**

89
\[ T_v \frac{d(gA)}{dt} = -(gA) + K_v c \quad (5.7) \]

The block diagram of the general normalized average-value model of the dish and Stirling engine is shown in Figure 5.2 based on (5.1)–(5.7).

5.2.3. Modeling of DFIG

The per-unit DFIG model in the \( d-q \) reference frame is presented in Section 4.2.4, where a different base value selection scheme is applied from that used in Section 5.2.2. The rotor windings of DFIG are open-circuit. Thus, in (4.18) and (4.19) the rotor voltage components in \( d-q \) reference frame \( v_{dr} \) and \( v_{qr} \) are non-zero. The generated real power of the rotor is

\[ P_r = v_{dr}i_{dr} + v_{qr}i_{qr} \quad (5.8) \]

5.2.4. Modeling of PMSG

The per-unit PMSG model in the \( d-q \) reference frame is given as [93]

**Stator Voltage Equations:**

\[
\begin{align*}
    v_d &= -R_i i_d - \frac{L_d}{\omega_h} \frac{di_d}{dt} + \omega_r L_q i_q \\
    v_q &= -R_i i_q - \frac{L_q}{\omega_h} \frac{di_q}{dt} + \omega_r (\psi_f - L_q i_d)
\end{align*}
\quad (5.9)
\]
Chapter 5 Variable-Speed Dish-Stirling Solar-Thermal System

Electromagnetic Torque Equation:
\[ \tau_e = i_q [\psi_f - i_d (L_d - L_q)] \]  
(5.10)

Lumped-Mass Motion Equation:
\[ 2H \frac{d\omega_r}{dt} = \tau_e - F \omega_r \]  
(5.11)

Power Equations:
\[ P_s = v_d i_d + v_q i_q \]  
(5.12)

If \( L_d = L_q \), the electromagnetic torque equation (5.10) is reduced to
\[ \tau_e = \psi_f i_q \]  
(5.13)

where \( v_d \) and \( v_q \) are the \( d \)-axis and the \( q \)-axis voltage components in \( d-q \) reference frame, \( i_d \) and \( i_q \) are the \( d \)-axis and the \( q \)-axis currents components in \( d-q \) reference frame. \( \psi_f \) is the rotor field flux, \( R_s \) is the stator winding resistance, \( L_d \) and \( L_q \) are the \( d \)-axis and \( q \)-axis synchronous inductances. \( \omega_r \) is the rotor electrical angular speed of PMSG. The base quantities are selected similar to those used for the per-unit DFIG model.

5.2.5. Modeling of Back-to-Back PWM Converters with LC filter

The back-to-back PWM voltage source converters (VSCs) enable variable-speed operation of the DS system by decoupling the power system electrical frequency from the generator/engine mechanical frequency. It consists of two four-quadrant three-phase full-controlled ac-dc rectifier/inverter bridges and a dc-link capacitor, which are shown in Figure 5.3.

![Figure 5.3. Topology of back-to-back PWM voltage source converters.](image-url)
In Figure 5.3, the RSC of DS-DFIG system and the SSC of DS-PMSG system are denoted by the machine-side converter (MSC) for convenience. $P_{mc}$ and $P_{gc}$ are the real power of the MSC and GSC respectively, $V_{dc}$ is the dc-link voltage and $C_{dc}$ is the capacitance of the dc-link capacitor. $I_{mc}$ and $I_{gc}$ are the dc currents on the branches of the MSC and GSC sides respectively. The detail topology and comprehensive analysis of the switching dynamics of the converter bridges can be found in [94]. For the problems in hand, the converters are modeled without considering the high-frequency switching dynamics and losses [95]. Furthermore, the model has been converted into the per-unit system in $d$-$q$ reference frame using the same base quantities for the generator, i.e.

$$P_{mc} = V_{dc} I_{mc}$$  \hspace{1cm} (5.14)

$$P_{gc} = V_{gc} I_{gc} = v_{d,gc} i_{d,gc} + v_{q,gc} i_{q,gc}$$  \hspace{1cm} (5.15)

$$C_{dc} \frac{dV_{dc}}{dt} = I_{mc} - I_{gc} = \frac{P_{mc}}{V_{dc}} - \frac{P_{gc}}{V_{dc}}$$  \hspace{1cm} (5.16)

where $v_{d,gc}$, $v_{q,gc}$, $i_{d,gc}$ and $i_{q,gc}$ are the ac voltage and current components in $d$-$q$ reference frame at the ac side of the GSC. For the DFIG and PMSG, $P_{mc}$ is $P_r$ and $P_s$ respectively, which are expressed in (5.8) and (5.12).

As shown in Figure 5.1, a LC filter is connected between the GSC and Bus 1. Bus 1 is the low-voltage side of the step-up transformer. Schematic of the LC filter in the $d$-$q$ reference frame is shown in Figure 5.4.

$$\bar{V}_{gc} = v_{d,gc} + jv_{q,gc}$$

$$\bar{V}_1 = v_{d1} + jv_{q1} = \bar{V}_1 \angle \theta_1$$

The model of LC filter in the $d$-$q$ reference frame can be expressed as

92
\[
\begin{align*}
\begin{cases}
  v_{d,gc} = R_f i_{d,gc} + \frac{L_f}{\omega_B} \frac{di_{d,gc}}{dt} - \omega_f L_f i_{q,gc} + v_{d1} \\
  v_{q,gc} = R_f i_{q,gc} + \frac{L_f}{\omega_B} \frac{di_{q,gc}}{dt} + \omega_f L_f i_{d,gc} + v_{q1}
\end{cases}
\end{align*}
\] (5.17)

\[
\begin{align*}
\begin{cases}
  i_{d,c} = \frac{v_{d1}}{R_c} + \frac{C_f}{\omega_B} \frac{dv_{d1}}{dt} - \omega_f C_f v_{q1} \\
  i_{q,c} = \frac{v_{q1}}{R_c} + \frac{C_f}{\omega_B} \frac{dv_{q1}}{dt} + \omega_f C_f v_{d1}
\end{cases}
\end{align*}
\] (5.18)

where $L_f$ and $C_f$ are the inductance and capacitance of LC filter. $R_f$ and $R_c$ is the series and parallel resistances respectively.

When the system is directly connected to a single machine infinite bus (SMIB), the external grid is an ideal voltage source with constant voltage magnitude $|V_1|$ and phase angle $\theta_1$. In this case, the capacitor branch can be omitted.

The block diagram of the per-unit model of the dc-link, GSC and the LC filter is shown in Figure 5.5.

![Figure 5.5. Block diagram of the per-unit model of dc-link, GSC and LC filters.](image_url)

\[
\begin{align*}
\begin{cases}
  v_{ds} = R_{sys} i_{d1} + \frac{L_{sys}}{\omega_B} \frac{di_{d,sys}}{dt} - \omega_f L_{sys} i_{q,sys} + v_{d,sys} \\
  v_{qs} = R_{sys} i_{q1} + \frac{L_{sys}}{\omega_B} \frac{di_{q,sys}}{dt} + \omega_f L_{sys} i_{d,sys} + v_{q,sys}
\end{cases}
\end{align*}
\] (5.19)
5.3. **Double-Loop Feedback Control of Variable-Speed DS Systems**

Traditionally, current control forms the inner loop of the traditional double-loop feedback control of DFIG or PMSG whereas the outer loop is for speed/power control loop. With appropriate decoupling techniques, the design of the inner and outer loop PI controllers can be carried out separately and the real power and reactive power of the DFIG can be controlled independently.

For DFIG, stator-voltage oriented vector control (SVOC) technique is commonly applied [96, 97]. This is convenient because the stator of DFIG is connected directly to the grid. Different from other vector control techniques such as stator flux oriented control [98-100] and air gap flux oriented control [101], in SVOC, the stator voltage vector is aligning with the $d$-axis of the synchronous reference frame, which results in

\[
\begin{align*}
    v_{sq} &= \bar{V}_s \\
    v_{ds} &= 0
\end{align*}
\]  

(5.20)

\[
\begin{align*}
    \psi_{d} &\approx 0 \\
    \psi_{q} &\approx |\bar{\psi}_s|
\end{align*}
\]  

(5.21)

where $|\bar{V}_s|$ and $|\bar{\psi}_s|$ are the magnitudes of stator voltage and flux linkage respectively.

5.3.1. Current Control of DFIG

Using (4.18)–(4.23), (5.20), and (5.21), then ignoring the terms of derivative of the flux linkage, one obtains

\[
\begin{align*}
    v_{dr} &= R_d i_{dr} + L_R \frac{di_{dr}}{dt} - \omega_d (\sigma L_r i_{qr} - \frac{L_{dl}}{L_r} |\bar{\psi}_s|) \\
    v_{qr} &= R_r i_{qr} + L_R \frac{di_{qr}}{dt} + \omega_d \sigma L_d i_{dr}
\end{align*}
\]  

(5.22)

where $\sigma = 1 - \frac{L_{dl}^2}{L_r L_{dr}}$ and $L_R = \frac{\sigma L_d}{\omega_d}$.

From (5.22) it can be seen the $d$-axis and $q$-axis currents are coupled to each other. Thus, the current control system consists of feedback PI controllers and feedforward compensation for the purpose of decoupling. The expressions of the controllers are
where $K_i$ and $\tau_i$ are the gain and time constant of the PI current regulators respectively, $v_{dr}', \ v_{qr}'$ are the outputs of the current regulators, $i_{dr}^*, \ i_{qr}^*$ are the current references, $v_{dr}^*, \ v_{qr}^*$ are the voltage references. The block diagram of the current control loop is shown in Figure 5.6.

In Figure 5.6, the first-order lag transfer function with time constant $T_{PWM}$ denotes the dynamics of the power electronic converter, where $T_{PWM}$ is the reciprocal of the PWM switching. The transfer function between the $d$-axis current $i_{dr}$ and its reference is

$$i_{dr}(s) = \frac{K_i (\tau_s s + 1)}{\tau_s s (1 + T_{PWM}s) (R_s + L_{Rs}s) + K_i (\tau_s s + 1)}$$  \hspace{1cm} (5.25)

In order to eliminate the pole introduced by the rotor inductance, let

$$\tau_i = \frac{L_R}{R_s}$$  \hspace{1cm} (5.26)
Substituting (5.26) into (5.25), thus,

\[
\begin{align*}
\frac{i_{dr}(s)}{i_{dr}^2(s)} &= \frac{K_i}{L_i T_{PWM}} \frac{s^2 + \frac{1}{T_{PWM}} s + \frac{K_i}{L_i T_{PWM}}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\end{align*}
\]

(5.27)

where \(\zeta\) is the damping ratio of the second-order system, and

\[
\omega_n = \frac{1}{2\zeta T_{PWM}} = \sqrt\frac{K_i}{L_i T_{PWM}}
\]

(5.28)

\[
K_i = \frac{L_i}{4\zeta^2 T_{PWM}}
\]

(5.29)

By selecting \(\zeta = 0.707\), one obtains

\[
K_i = \frac{L_i}{2T_{PWM}}
\]

(5.30)

Substituting the expressions of the control parameters (5.26) and (5.30) into (5.25), thus

\[
\begin{align*}
\frac{i_{dr}(s)}{i_{dr}^2(s)} &= \frac{L_i}{2L_{PWM} T_{PWM}} \frac{R_{r} \tau_i s^2 + R_{r} \tau_i s + 1}{K_i s^2 + 2T_{PWM} s^2 + 2T_{PWM} s + 1} \approx \frac{1}{2T_{PWM} s^2 + 1}
\end{align*}
\]

(5.31)

From (5.31) it can be seen, for a PWM switching frequency of 5000 Hz, the equivalent time constant of the current control loop is \(2\times T_{PWM} = 2/5000 = 0.0004\) s.

### 5.3.2. Speed Control of DFIG

Substituting (5.21) into the electromagnetic torque equation (4.24), one can obtain

\[
\tau_e = \frac{L_m}{L_s} |\Psi| i_{dr}
\]

(5.32)

From (5.32), it can be seen that the \(d\)-axis rotor current \(i_{dr}\) is proportional to the electromagnetic torque \(\tau_e\). Thus, it is possible to use \(i_{dr}\) to control \(\tau_e\) and thus the generator speed \(\omega_r\). Usually the inertia time constant of DFIG is around 1.0 s, which is much larger than that of the current control loop as shown in Section 5.3.1. Hence, the dynamics of the inner (electromagnetic) loop are much faster than that of the outer (electromechanical and thermo-mechanical) loop. In the design of the speed and temperature controllers, the inner loop can be modeled as an ideal current loop, i.e.
\(i_{\alpha} = i^*_{\alpha}.\) Similarly, the inner loop also can be seen as an equivalent torque source if the \(d\)-axis rotor current reference is calculated from

\[
i^*_{\alpha} = \frac{1}{\psi_s} L_s \tau^*_e
\]

(5.33)

where torque reference \(\tau^*_e\) is the output of the speed regulator. The simplified speed control loop is shown in Figure 5.7.

In Figure 5.7, the PI speed regulator for the closed-loop feedback control is expressed as

\[
\tau^*_e = k_{p,w} (\omega_r - \omega^*_r) + z
\]

(5.34)

\[
\frac{dz}{dt} = k_{i,w} (\omega_r - \omega^*_r)
\]

(5.35)

where \(k_{p,w}\) and \(k_{i,w}\) of the PI controller can be tuned using classical control techniques. \(z\) is the output of the integrator of the PI controller. Torque reference \(\tau^*_e\) is generated as the output of a speed regulator, and the corresponding \(d\)-axis current reference can be calculated as

\[
i^*_{\alpha} = \frac{L_s}{L_m |\psi_s|} \tau^*_e
\]

(5.36)

On the other hand, the control of \(q\)-axis rotor current \(i_{q}\) depends on the strategy of reactive power/terminal voltage control. Design methods are available and interested readers may wish to refer to [93] and this aspect shall not be elaborated in this thesis.
5.3.3. Control of PMSG

For the speed control of PMSG, similar vector control scheme can be applied as the control scheme of DFIG. The current control loop can be simplified to the unit gain block as the dynamic of it is much faster than the speed control loop. From (5.13) it can be seen that the electromagnetic torque of PMSG is proportional to the $q$-axis stator current $i_q$ of PMSG. Thus, the $q$-axis stator current reference can be calculated as

$$i^*_q = \frac{1}{\psi_f} \tau^*_e$$  \(5.37\)

The simplified double-loop feedback control of PMSG is shown in Figure 5.8

![Figure 5.8. Simplified double-loop feedback control of PMSG (generator base).](image)

5.4. Optimal Speed for Maximum Output Power

The expression for the steady-state mean pressure of the working gas can be obtained by substituting (5.1) into (5.6) and let $dT_h/dt = 0$, thus

$$p_{\text{mean}} = \frac{K_{\text{con}} I - a_{01}\omega_m - a_{00} - (1 - T_a) / K_{\text{res}}}{a_{11}\omega_m + a_{10}}$$  \(5.38\)

In arriving (5.38), one assumes the temperature is maintained at the maximum value, i.e. $T_h = 1.0$ p.u. This is because the thermal efficiency of the engine increases with the temperature of the working gas. However, the temperature must not exceed a pre-set limit so as to prevent damage to the absorber tubes. Substituting (5.38) into (5.2), one can obtain a family of the steady-state mechanical power $P_m$ vs. engine speed $\omega_m$ curves of DS-DFIG system at various solar irradiance levels $I$, as shown in Figure 4.14.
In Figure 4.14, the feasible operating area without consideration of the type of connected electric generator is limited by the pressure, insolation and engine speed limits. Similarly, the steady-state operations of the DS-DFIG system are somewhat constrained due to practical limits placed on the mean pressure $p_{\text{mean}}$ of the working gas, insolation and speed of DFIG. This is because the speed range of a DFIG is usually smaller than a Stirling engine. The constraints have to be reflected on Figure 5.9. In Figure 5.9, the $P_m - \omega_m$ curve under maximum $p_{\text{mean}}$ operation is shown by the line A-J whereas that at the minimum $p_{\text{mean}}$ is the curve C-E. A-B is part of the $P_m - \omega_m$ curve corresponding to the maximum $I$. Furthermore, as the operating speed range of a Stirling engine is typically much wider than that of a DFIG, feasible speed range of a DS-DFIG system is thus constrained by the speed range of the DFIG. Typically, DFIG operates within the range of 0.65 to 1.3 p.u. of the synchronous speed. If no spinning reserve or frequency support is required from the DS-DFIG, the nominal Stirling engine speed can be designed to match the maximum DFIG speed, i.e. 1.3 p.u. of the synchronous speed. Thus, the corresponding minimum engine speed is reduced to $\omega_{m,\text{min}} = 0.65/1.3 = 0.5$ p.u. of the nominal engine speed. Thus, the feasible steady-state operating engine speed is between 0.5 to 1 p.u. of the nominal engine speed. By incorporating this engine speed range constraint into the $P_m - \omega_m$ plot, it is clear that the feasible operating state of the DS-DFIG system is within the area A-D-G-F-A shown in Figure 5.9.
Within the feasible operating area, maximum power harness can be realized by the control of the engine/generator speed. In Figure 5.9, the maximum power point tracking (MPPT) locus is indicated by the line A-H-K-G which is made up of three line sections: A-H governs the maximum power harness under relatively high $I$ but is restricted by the maximum $p_{\text{mean}}$ limit of the Stirling engine, H-K is the theoretical MPPT locus at intermediate $I$ level, and when $I$ drops below the level corresponding to the point K, the speed of the DFIG reaches its minimum limit.

For a given $I$, the corresponding speed on the MPPT curve is defined as the optimal speed $\omega_{m,\text{opt}}(I)$. Since A-H-K-G is a known function of $I$, $\omega_{m,\text{opt}}(I)$ can be calculated. So for real-time MPPT application, the engine speed can be regulated to track $\omega_{m,\text{opt}}(I)$ since $I$ can be measured on-line and is known.

Denote the MPPT locus by the function $P_{m,\text{opt}} = f_{\text{MPPT}}(\omega_{m,\text{opt}})$. Substituting (5.38) into (5.2), the steady-state relationship between $\omega_{m,\text{opt}}$ and $I$ of section H-K can be shown to be

$$P_{m,\text{opt}} = f_{\text{MPPT}}(\omega_{m,\text{opt}}) = (b_{00} + b_{01}\omega_{m,\text{opt}} + b_{02}\omega_{m,\text{opt}}^2) +$$

$$+ (b_{10} + b_{11}\omega_{m,\text{opt}} + b_{12}\omega_{m,\text{opt}}^2) \frac{K_{\text{com}}I - a_{01}\omega_{m,\text{opt}} - a_{00}'}{a_{11}\omega_{m,\text{opt}} + a_{10}'}$$  \hspace{1cm} (5.39)

The $\omega_{m,\text{opt}} - I$ curve corresponding to the MPPT curve A-H-K-G can be derived by solving (5.39) and it is shown in Figure 5.10. From (5.39) and Figure 5.10, it can be seen that $\omega_{m,\text{opt}}$ is a complex and nonlinear function of $I$. For the purpose of analysis and control system design, this curve can be approximated by three linear sections: A-H corresponds to the maximum pressure section; H-K is the theoretical MPPT section while K-G is that of the minimum speed. This revised MPPT curve can be expressed as

$$\omega_{m}^* = \omega_{m,\text{opt}} = k_{\text{MPPT}}I + b_{\text{MPPT}}$$  \hspace{1cm} (5.40)

where $k_{\text{MPPT}}$ and $b_{\text{MPPT}}$ are the slope and y-intercept of the $\omega_{m,\text{opt}} - I$ curve at the corresponding insolation level. Equation (5.40) indicates that one only needs to set the engine speed reference signal $\omega_{m}^*$ in the DS speed control system to $\omega_{m,\text{opt}}$ in order to achieve MPPT. In (5.40), $k_{\text{MPPT}}$ is the slope of the $\omega_{m,\text{opt}} - I$ curve at the corresponding insolation level. Thus,
Chapter 5 Variable-Speed Dish-Stirling Solar-Thermal System

\[
\begin{align*}
  k_{\text{MPPT}} &= \begin{cases} 
  k_{\text{AH}} = \frac{1 - \omega_{mH}}{1 - I_H}, & I_H < I \leq 1.0 \\
  k_{\text{HK}} = \frac{\omega_{mH} - \omega_{mK}}{I_H - I_K}, & I_K < I \leq I_H \\
  k_{\text{KG}} = 0, & I_G < I \leq I_K
  \end{cases} \\
  b_{\text{MPPT}} &= \begin{cases} 
  1 - k_{\text{AH}}, & I_H < I \leq 1.0 \\
  \omega_{mH} - k_{\text{HK}} I_H, & I_K < I \leq I_H \\
  \omega_{m,\text{min}}, & I_G < I \leq I_K
  \end{cases}
\end{align*}
\]  

(5.41)

(5.42)

where \(I_H\), \(I_K\) and \(I_G\) are the insolation level at point H, point K and point G respectively. \(\omega_{mH}\) and \(\omega_{mK}\) are the engine speed at point H and point K respectively.

Equation (5.40) indicates that one only needs to set the engine speed reference signal \(\omega_m^*\) in the DS speed control system to \(\omega_{m,\text{opt}}\) in order to achieve MPPT.

Figure 5.10. Steady-state relationship between optimal engine speed and insolation for DS-DFIG system.

To implement the optimal speed scheme proposed above to DS-PMSG system, some modification needs to be carried out. The major difference is the speed range of a PMSG is much wider than that of a DFIG. Usually the PMSG can operate at very low speed and in our analysis the minimum operating speed of the PMSG is assumed to be zero. Thus, the MPPT curve for DS-PMSG system expands to A-H-E in Figure 5.9. This tracking curve consists of three sections: a maximum pressure curve A-H
and two theoretical MPPT curves H-K and K-E. From Figure 5.9 it can be seen that the DS-PMSG system is able to generate power at even lower insolation level as PMSG can operate at lower speed than DFIG. More power is expected to be produced by DS-PMSG system.

The steady-state relationship between the engine speed and insolation for DS-PMSG system is shown in Figure 5.11.

![Steady-state relationship between optimal engine speed and insolation for DS-PMSG system.](image)

The linearized expression of this curve is similar to (5.40), while $k_{\text{MPPT}}$ and $b_{\text{MPPT}}$ are expressed as

$$
k_{\text{MPPT}} = \begin{cases} 
\frac{1 - \omega_{\text{nh}}}{1 - I_H}, & I_H < I \leq 1.0 \\
\frac{\omega_{\text{mh}} - \omega_{\text{nk}}}{I_H - I_K}, & I_K < I \leq I_H \\
\frac{\omega_{\text{mk}} - \omega_{\text{mh}}}{I_E - I_H}, & I_E < I \leq I_K
\end{cases}
$$

(5.43)

$$
b_{\text{MPPT}} = \begin{cases} 
1 - k_{\text{AH}}, & I_H < I \leq 1.0 \\
\omega_{\text{mh}} - k_{\text{HK}} I_H, & I_K < I \leq I_H \\
\omega_{\text{mk}} - k_{\text{KE}} I_K, & I_E < I \leq I_K
\end{cases}
$$

(5.44)
5.5. Linearized Model of Variable-Speed DS Systems

5.5.1. Linearized Model of the Prime Mover

The variable-speed DS system is expected to operate under constant changes in the solar irradiance $I$. Thus it would be meaningful to firstly construct a suitable model which describes the behavior of the solar-thermal plant under such small-disturbance condition. For a small displacement (denoted using symbol $\Delta$) about any arbitrary initial operating point $I = I_0$, $p_{\text{mean}} = p_0$, $\omega_m = \omega_{m0}$ and $T_h = T_{h0}$, one obtains

$$I = I_0 + \Delta I$$
$$p_{\text{mean}} = p_0 + \Delta p_{\text{mean}}$$
$$\omega_m = \omega_{m0} + \Delta \omega_m$$
$$T_h = T_{h0} + \Delta T_h$$

(5.45)

Using (5.1), (5.5), (5.6) and (5.45), one can obtain the small-signal equations governing the working gas pressure and heat transfer dynamics

$$T'_{\text{rec}} \frac{\Delta dT}{dt} + \Delta T_h = K_{\text{rec}} [K_{\text{con}} \Delta I - \eta_{h0} \Delta \Delta + \frac{dp_{\text{mean}}}{dt}]$$
$$-(a_{10} + a_{11}\omega_{m0}) \Delta p_{\text{mean}} - (a_{01} + a_{11} p_0) \Delta \omega_m$$

where $T'_{\text{rec}} = T_{\text{rec}} + \eta_{h0} K_{\text{rec}} C_p t$ and $\eta_{h0}$ is the corresponding thermal efficiency at the operating point calculated using (5.5). From (5.46), the following transfer functions can be obtained:

$$\frac{\Delta T_h(s)}{\Delta p_{\text{mean}}(s)} = -K_{\text{rec}} K_{\text{hp}} \frac{1 - T_{\text{sc1}} s}{1 + T_{\text{rec}}' s}$$

(5.47)

$$\frac{\Delta T_h(s)}{\Delta \omega_m(s)} = -K_{\text{rec}} K_{\text{hw}} \frac{1}{1 + T_{\text{rec}}' s}$$

(5.48)

$$\frac{\Delta T_h(s)}{\Delta I(s)} = K_{\text{rec}} K_{\text{con}} \frac{1}{1 + T_{\text{rec}}' s}$$

(5.49)

where $K_{\text{hp}} = a_{10} + a_{11} \omega_{m0}$, $K_{\text{hw}} = a_{01} + a_{11} p_0$.

Similarly, using (5.2), (5.3), (5.4) and (5.7), the following small-signal equations can also be obtained

$$\frac{d \Delta p_{\text{mean}}}{dt} = \Delta gA$$

(5.50)
\[
\Delta \tau_m = K_{mp} \Delta p_{\text{mean}} + K_{mw} \Delta \omega_m \quad (5.51)
\]

\[
T_v \frac{d \Delta (gA)}{dt} = -\Delta (gA) + K_c \Delta c \quad (5.52)
\]

where \(K_{mp} = b_{10}/\omega_0 + b_{11} + b_{12}\omega_0,\) \(K_{mw} = -b_{00}/\omega_0^2 - b_{10}p_0/\omega_0 + b_{02} + b_{12}p_0.\)

### 5.5.2. Linearized Model of Speed Control

As the purpose of the present investigation is to design temperature and speed control systems for the variable-speed DS system, the model used in [69] for the generators and converter can be simplified by ignoring the much faster electromagnetic dynamics. The electromagnetic torque \(\tau_e\) is assumed to be exactly the same as its reference value, i.e. \(\tau_e^* = \tau_e.\) As a result, the electromechanical dynamics of the DFIG contains only the equation of motion. As discussed previously, by converting (4.25) from the base of the DFIG to that of the Stirling engine, (4.37) is derived. The small-signal equation of (4.37) is thus

\[
2H' \frac{d \Delta \omega_m}{dt} = \Delta \tau_m - \Delta \tau_e' - F' \Delta \omega_m \quad (5.53)
\]

where inertia constant \(H',\) friction factor \(F'\) and electromagnetic torque \(\tau_e'\) are the quantities that have been converted from those based on DFIG rated values to the same bases as those used in the normalized model of the DS developed in Section 3.2.

Equations (5.46) and (5.50)–(5.53) constitute the linearized small-signal model of the DS-DFIG power plant which can be used for controller design. The fourth-order model has two inputs \(\Delta c\) and \(\Delta \tau_e',\) and two outputs \(\Delta T_h\) and \(\Delta \omega_m,\) while \(\Delta I\) is the disturbance. Classical speed control techniques of DFIG and PMSG are well-established and the technique can reject the torque disturbance \(\Delta \tau_m\) introduced by the prime mover. However, the traditional mean pressure control technique for regulating the DS temperature is designed without considering speed variation \(\Delta \omega_m.\)

As shall be shown later, \(\Delta \omega_m\) does contribute toward \(\Delta T_h.\) Thus, in this investigation, the speed controller can be designed first and then the temperature controller can be designed by considering the speed as a disturbance, as shown in the following sections.
Similarly, the speed controller described in (5.34) and (5.35) needs to be converted to the same bases for the prime mover. The small-signal representation of this speed regulator is

$$\Delta \tau''_e = k'_{p,w} (\Delta \omega_m - \Delta \omega^*_m) + \Delta z'$$

(5.54)

$$\frac{d \Delta z'}{dt} = k'_{i,w} (\Delta \omega_m - \Delta \omega^*_m)$$

(5.55)

where $k'_{p,w}$, $k'_{i,w}$, $\tau''_e$ and $z'$ are also converted to the same bases as those are used in the normalized model, i.e.

$$k'_{p,w} = \frac{1}{K_{TC}K_{SC}} k_{p,w}$$

(5.56)

$$k'_{i,w} = \frac{1}{K_{TC}K_{SC}} k_{i,w}$$

(5.57)

$$\tau''_e = \frac{1}{K_{TC}} \tau'_e$$

(5.58)

$$z' = \frac{1}{K_{TC}} z$$

(5.59)

From (5.40), the small-signal representation of the revised MPPT curve can be expressed as

$$\Delta \omega^*_m = \Delta \omega_{m, opt} = k_{MPPT} \Delta I$$

(5.60)

### 5.5.3. Small-Signal State-Space Equation

Using (5.46), (5.50)–(5.55) and (5.60), the small-signal state-space model of a SISO system at operating point $x = x_0 = [gA_0 \ p_0 \ T_{h0} \ \omega_{nd0} \ z'_0]$ is obtained:

$$\begin{align*}
\frac{d}{dt} \Delta x(t) &= A \Delta x(t) + B \Delta u(t) + \Delta d(t) \\
\Delta y(t) &= C \Delta x(t)
\end{align*}$$

(5.61)

where
\[ \Delta x(t) = \left[ \Delta gA(t) \quad \Delta \rho_{\text{mean}}(t) \quad \Delta T_g(t) \quad \Delta \omega_s(t) \quad \Delta \zeta'(t) \right]^T, \quad \Delta u(t) = \Delta c(t), \quad \Delta y(t) = \Delta T_h(t), \]

\[ \Delta d(t) = \begin{bmatrix} 0 & 0 & \frac{K_{\text{com}}}{T_{\text{rec}}} & \frac{k_{\text{MPPT}} k'_{p,w}}{2H'} & -k_{\text{MPPT}} k'_{l,w} \end{bmatrix}^T \Delta I(t) \]

\[ A = \begin{bmatrix} \frac{1}{T_v} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\frac{K_{\text{rec}} K_{\text{hp}}}{T_{\text{rec}}} & 0 & \frac{K_{\text{rec}}}{T_{\text{rec}}} & -\frac{K_{\text{rec}}}{T_{\text{rec}}} & 0 \\ 0 & \frac{K_{\text{hp}}}{2H'} & 0 & \frac{K_{\text{mv}} - F' - k'_{p,w}}{2H'} & -1 \\ 0 & 0 & 0 & \frac{1}{k'_{l,w}} & 0 \end{bmatrix} \]

\[ B = \begin{bmatrix} K_v / T_v & 0 & 0 & 0 \end{bmatrix}^T \]

\[ C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \]

### 5.6. Control of Receiver Temperature in the Variable-Speed DS Systems

#### 5.6.1. Overall Strategy for Temperature Control

As explained in Section 5.5.2, usually the speed control loop can reject the torque disturbance caused by temperature variation in the prime mover. Thus, the speed control system of DFIG can be designed first using well-established methods and the speed control loop becomes part of the whole plant for which the design of the temperature controller is to be dealt with next. Thus the original two-input-two-output plant model governed by (5.46) and (5.50)–(5.53) is reduced to a single-input single-output (SISO) system with a single disturbance (\( \Delta I \)). The block diagram of the reduced variable-speed DS plant model for temperature controller design is shown in Figure 5.12.
5.6.2. Temperature Control with Droop Characteristics

The temperature control based on droop characteristic for the conventional constant-speed DS system has been introduced in Section 4.4. To solve the problem caused by the non-minimum phase characteristics of the Stirling engine, transient droop compensation can be used to improve on the damping level of the system. The block diagram of the overall control system is shown in Figure 5.13.

This is a double-loop feedback control system. In the inner loop, mean pressure is regulated by a proportional controller with a gain $G_p$. The droop characteristic introduces steady-state error of the working temperature, which causes the temperature of the receiver at periods of low $I$ would be lower than that at high $I$. The transient droop compensation is equivalent to a series lead-lag compensator which can improve damping for the non-minimum phase system.
Using (5.51) (5.54) (5.55) and (5.53), the transfer function \( \frac{\Delta \omega_m(s)}{\Delta p_{\text{mean}}(s)} \) can be derived

\[
\frac{\Delta \omega_m(s)}{\Delta p_{\text{mean}}(s)} = \frac{K_{\text{mp}} s}{2H s^2 + (F' - K_{\text{mw}} + k_{p,w}') s + k_{i,w}'}
\] (5.62)

\[
\frac{\Delta \omega_m(s)}{\Delta I(s)} = \frac{k_{\text{MPPT}} (k_{p,w}' s + k_{i,w}')} {2H s^2 + (F' - K_{\text{mw}} + k_{p,w}') s + k_{i,w}'}
\] (5.63)

\[
\Delta T_h(s) = \left[ -K_{\text{hp}} (-T_{\text{se1}} s + 1) \Delta p_{\text{mean}}(s) - K_{\text{hw}} \Delta \omega_m(s) + K_{\text{con}} \Delta I(s) \right] \frac{K_{\text{rec}}}{T_{\text{rec}} s + 1}
\] (5.64)

\[
\Delta T_h(s) = -\frac{K_{\text{hp}} (-T_{\text{se1}} s + 1)(2H s^2 + F'' s + k_{i,w}')} {2H s^2 + F'' s + k_{i,w}'} \frac{K_{\text{rec}}}{T_{\text{rec}} s + 1} \Delta p_{\text{mean}}(s)
\]

\[
+ \frac{K_{\text{con}} (2H s^2 + F'' s + k_{i,w}') - k_{\text{MPPT}} K_{\text{hw}} (k_{p,w}' s + k_{i,w}')} {2H s^2 + F'' s + k_{i,w}'} \frac{K_{\text{rec}}}{T_{\text{rec}} s + 1} \Delta I(s)
\]

(5.65)

Using (5.62), (5.47) and (4.10), the open-loop transfer function of the temperature control loop is derived, i.e.

\[
\frac{\Delta T_i(s)}{\Delta p_{\text{mean}}(s)} = -K_{\text{rec}} \frac{K_{\text{hp}} (-T_{\text{se1}} s + 1)(2H s^2 + F'' s + k_{i,w}')} {T_{\text{rec}} s^2 + T s + 1}(T' s + 1)(2H s^2 + F'' s + k_{i,w}')
\] (5.66)

\[
\frac{\Delta T_i(s)}{\Delta I(s)} = \frac{K_{\text{con}} (2H s^2 + F'' s + k_{i,w}')} {2H s^2 + F'' s + k_{i,w}'} - k_{\text{MPPT}} K_{\text{hw}} (k_{p,w}' s + k_{i,w}') \frac{K_{\text{rec}}}{T_{\text{rec}} s + 1}
\] (5.67)

where \( F'' = F' - K_{\text{mw}} + k_{p,w}' \).

From (5.66) it can be seen the plant is a fifth-order system with five poles. As the coefficients \( K_{\text{hp}}, K_{\text{hw}}, K_{\text{mp}}, K_{\text{mw}} \) and \( T_{\text{rec}} \) are functions of the steady-state operating pressure \( p_0 \) and engine speed \( \omega_{\text{md0}} \), and thus the insolation level \( I_0 \), one can choose \( I \) as the independent variable and the generalized root locus with different control configuration can be obtained. An example of this is shown in Figure 5.14.
Figure 5.14. Generalized closed-loop root locus as insolation changes. (a) $D_p = 0.04$ p.u.; (b) $D_p = 0.036$ p.u.; (c) with transient droop compensation.
In Figure 5.14 (a), the droop $D_p$ is set to 0.04 p.u., which is normally used in constant-speed DS system. It can be seen that, without transient droop compensation, as the insolation increases, a pair of roots will move from the left-hand plane to the right-hand plane, which means the system becomes less stable as the insolation increases in this strategy.

By reducing the value of $D_p$, the steady-state error becomes smaller. However, it will cause the system to become less stable. Figure 5.14 (b) shows the root locus when the droop is reduced to 0.036, i.e. 90% of the previous setting of 0.04.

Figure 5.14 (c) shows the closed-loop root locus using the transient droop compensation. It can be seen that with this control system, the system is always stable.

In order to increase the thermal efficiency of the DS system, the droop of the temperature controller needs to reduce but at the expense of a deterioration in system damping. Steady-state error cannot be eliminated using the transient droop compensation technique. Besides, the speed feedforward compensator introduced in Section 4.4.2 is designed for constant-speed operation where the steady-state engine speed is always about 1.0 p.u., which is not suitable for the variable-speed operation. To solve these problems, a new temperature controller will be proposed in the following sections.

### 5.6.3. Local Full State-Feedback with Integral Control

In order to maximize the thermal efficiency of the variable-speed DS system, temperature control system with zero steady-state error performance is necessary. The fifth-order small-signal state-space model of a SISO system linearized at operating point $\mathbf{x} = \mathbf{x}_0 = [gA_0 \ p_0 \ T_{h0} \ \omega_{m0} \ z'_0]$ is shown in Section 5.5.3.

All the state variables in $\mathbf{x}$ are measurable: For example, thermocouple sensors, pressure gauges and speed sensors are used to measure $T_h$, $p_{\text{mean}}$ and $\omega_m$ respectively in the DS system described in [12], whereas $gA$ can be calculated as it is the derivative of $p_{\text{mean}}$, and $z'$ can be generated as it is the output signal of the integrator of the speed regulator. Thus, full state feedback (FSF) control strategy can be realized to achieve temperature control. The design of the controller could be based on the placement of closed-loop poles in desirable locations in the complex plane. In addition, an integral control is introduced to eliminate the steady-state error in $T_h$ due to step change in $I$.

The block diagram of this control scheme is shown in Figure 5.15.
Figure 5.15. Design of the local temperature controllers using pole placement method.

The structure of the controller is expressed as

$$\Delta u(t) = -K_1 \Delta x(t) - K_2 \int [\Delta y(t) - \Delta y_r(t)] dt$$  \hspace{1cm} (5.68)$$

which can also be expressed as

$$\Delta u(t) = -K_1 \Delta x(t) - K_2 \Delta v(t)$$  \hspace{1cm} (5.69)$$

where $y_r$ is the reference for $T_h$. $K_1 = [K_{11} \quad K_{12} \quad K_{13} \quad K_{14} \quad K_{15}]$ and $K_2$ are the gain parameters of the state feedback and integral control loops respectively, and their values can be calculated using well-established pole-placement technique. The output of the integrator introduces a new state $v$, i.e.

$$\frac{d \Delta v(t)}{dt} = \Delta y(t) - \Delta y_r(t)$$  \hspace{1cm} (5.70)$$

The new state-space equation is

$$\begin{bmatrix} \frac{d \Delta x(t)}{dt} \\ \frac{d \Delta v(t)}{dt} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} \Delta x(t) \\ \Delta v(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u(t) + \begin{bmatrix} \Delta d(t) \\ -\Delta y_r(t) \end{bmatrix}$$  \hspace{1cm} (5.71)$$

Indeed, by following the analysis method shown in [102], it can be shown that if the following requirements are met, placement of the poles at any desirable locations in the complex plane is achievable and the DS heater temperature $T_h$ will reach its set value following a step disturbance of $I$. The requirements are:

- The original system (5.61) is controllable;
- Matrix $R = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}$ is full rank.

Check for compliance of the two requirements can be achieved using existing computational tool such as MATLAB.
Whence from \((5.61)\) and \((5.68)\), a new state-space equation can be derived by substituting \((5.69)\) into \((5.71)\). It yields

\[
\begin{bmatrix}
\frac{d}{dt} \Delta x(t) \\
\Delta v(t)
\end{bmatrix} =
\begin{bmatrix}
A' & B' \\
\Delta y_r(t)
\end{bmatrix} +
\begin{bmatrix}
\Delta d(t) \\
0
\end{bmatrix} \quad (5.72)
\]

where

\[
A' = \begin{bmatrix}
A - BK_1 & -BK_2 \\
C & 0
\end{bmatrix} \quad B' = [0 \ 0 \ 0 \ 0 \ 0 \ -1]^T \quad C' = [0 \ 0 \ 1 \ 0 \ 0 \ 0]
\]

Considering step change of disturbance and the reference signal, and taking Laplace transform for \((5.72)\),

\[
\begin{bmatrix}
\Delta x(s) \\
\Delta v(s)
\end{bmatrix} = (sI - A')^{-1} \begin{bmatrix}
\Delta d(s) / s \\
-\Delta y_r(s) / s
\end{bmatrix} \quad (5.73)
\]

According to the final value theorem (FVT), in the time domain,

\[
\lim_{t \to \infty} \begin{bmatrix}
\Delta x(t) \\
\Delta v(t)
\end{bmatrix} = \lim_{s \to 0} \begin{bmatrix}
\Delta x(s) \\
\Delta v(s)
\end{bmatrix} = (-A')^{-1} \begin{bmatrix}
\Delta d(\infty) \\
-\Delta y_r(\infty)
\end{bmatrix} \quad (5.74)
\]

From \((5.74)\) it can be seen that \(\Delta v(t)\) will approach a constant and thus,

\[
d\Delta v(t) / dt \quad \text{will approach zero, i.e.}
\]

\[
\frac{d}{dt} \Delta v(\infty) = \Delta y(\infty) - \Delta y_r(\infty) = 0 \quad (5.75)
\]

Above procedures prove that the temperature \(\Delta T_h = \Delta y\) will reach its setting value \(\Delta y_r\) at the present of the signal disturbance \(\Delta d\).

The transfer function \(g(s)\) between the output \(y\) and the input reference \(y_r\) can be derived:

\[
g(s) = \frac{\Delta y(s)}{\Delta y_r(s)} = C' \left(sI - A'\right)^{-1} B' \quad (5.76)
\]

By selecting the closed-loop poles at \(s = \lambda_1 \sim \lambda_6\), \(K_1\) and \(K_2\) can be obtained by solving
\[
\left| C_i(sI - A_i)^{-1}B_i \right| = \prod_{n=1}^{6}(s - \lambda_n)
\]  
(5.77)

There are various methods to obtain the solutions of (5.77). Interested readers may refer to [102] for the details.

### 5.6.4. Fuzzy Supervisory Control Scheme

From (5.61), it can be seen that the system matrix \( A \) are affected by the parameters \( K_{hp}, K_{lw}, K_{mp}, k_{mppt}, T'_{rec} \) and these parameters are nonlinear functions of the state variables \( p_{mean} \) and \( \omega_m \), and in turn, the insolation level \( I \). In practice, \( I \) tends to vary slowly compared to the thermodynamics of the Stirling engine and electromagnetic dynamics of the power system. Thus, gain scheduling technique can be used in this instance [103]: A fuzzy supervisory control scheme is proposed by designing several local temperature controllers at selected steady-state operating points. The plant operating state can be characterized by \( I(t) \), measured using insolation sensors such as pyranometers. As an illustration, suppose the insolation level is partitioned into three operating sections. For each section, a corresponding controller is designed using the full state feedback and integral control method described earlier. A Takagi-Sugeno fuzzy supervisor is used to calculate the average output of the controllers according to the measured \( I \). For example, if triangular membership functions are used to characterize \( I \) as shown in Figure 5.16, the global output of the controllers can be expressed as

\[
u(t) = \mu^{(low)}(I)u^{(low)}(t) + \mu^{(medium)}(I)u^{(medium)}(t) + \mu^{(high)}(I)u^{(high)}(t)
\]  
(5.78)

where \( u^{(low)}, u^{(medium)}, u^{(high)} \) are the output of the controllers designed for \( I = 0.3, 0.6 \) and \( 0.9 \) p.u. respectively.

![Figure 5.16. The pertinence function of the insolation level.](image-url)

\[113\]
5.7. **Overall Control Structure**

The overall control system of the RSC and the temperature control of the DS-DFIG system and DS-PMSG system are shown in Figure 5.17 and Figure 5.18 respectively.

![Figure 5.17](image1.png)

**Figure 5.17.** Schematic of the overall control structure for the DS-DFIG system (PCU side).

![Figure 5.18](image2.png)

**Figure 5.18.** Schematic of the overall control structure for the DS-PMSG system (PCU side).

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**Chapter 5 Variable-Speed Dish-Stirling Solar-Thermal System**

114
5.8. Illustrative Example

5.8.1. Simulation Model of the Test DS-DFIG System

In this example, a 9-MW DS-DFIG system is connected to a large grid through an equivalent impedance of 0.005+j0.028 p.u., on 100 MVA base. The DFIG is represented by a fifth-order model shown in Section 5.2.3 [89]. The back-to-back converters are modeled without the consideration of harmonics and losses. The prime mover is represented by the general average-value model presented in Section 3.5.1. Parameters of the DS-DFIG, converters and the control systems are given in Appendix C. The overall schematic of the control structure of the power conversion unit is shown in Figure 5.17. Simulation was carried out in MATLAB/Simulink, and the configuration is shown in Figure 5.19.

Figure 5.19. Simulink model of variable speed dish-Stirling solar thermal plant using DFIG.

5.8.2. Case 1: Transient Response under Insolation Change

This example is to compare the dynamic performances of the temperature controllers designed using different approaches. Figure 5.20 shows the engine speed \( \omega_m \) and temperature \( T_h \) responses following a 0.05-p.u. step change of \( I \) at the initial operating point \( I_0 = 0.9 \) p.u. The control parameters of the state-feedback controller is tuned at \( I = 0.6 \) p.u. Hence, it is not surprising to note that the perturbations in \( T_h \) are poorly damped. On the other hand, the fuzzy supervisory controller is seen to provide much
improved dynamic performance, as it is able to adjust the control parameters according to the measured insolation level.

Figure 5.21 shows the dynamic response of the DS-DFIG system when $I$ is assumed to increase from 0.25 p.u. to 1.0 p.u. at a rate of 0.1 p.u./s, i.e. 100 W/(m$^2$·s). Normally, a rate of 30 W/(m$^2$·s) is considered to be fast [91] and hence, this is an extremely strenuous test. It can be seen as $I$ increases from 0.3 to 1 p.u., the engine accelerates from 0.5 p.u. to 1.0 p.u. of nominal engine speed, i.e. from −35% to +30% of synchronous speed. With droop controller, the temperature is not at the maximum level until $I$ is at the maximum. Hence, the droop controller cannot ensure the DS-DFIG system is operating at the maximum efficiency level. This short-coming is alleviated using the state feedback and fuzzy supervisory controllers as they allow the steady-state error in the temperature to be reduced. As the state feedback controller with a single integrator can only eliminate the steady-state errors following a step change disturbance, temperature rises during the ramp change of $I$. The maximum temperature rise depends on the maximum rate of change of $I$. From Figure 5.21, although all the controllers have produced similar dynamic performances at low and medium $I$ levels, the fuzzy supervisory controller can maintain the temperature at the set value more effectively than the state feedback controller at higher $I$ levels.
Figure 5.20. DS-DFIG responses to 0.05 p.u. step increase in insolation $I$ at $I_0 = 0.9$ p.u. Solid line: fuzzy supervisory control; dashed line: droop control; dotted line: full state feedback control tuned at $I = 0.6$ p.u. (medium level).
Figure 5.21. DS-DFIG responses to a ramp increase in insolation $I$ at 0.1 p.u./s. Solid line: fuzzy supervisory control; dashed line: droop control; dotted line: full state feedback control tuned at $I = 0.6$ p.u. (medium level).
5.8.3. Case 2: Maximum Power Point Tracking

This example is to demonstrate the maximum power tracking capability of the designed system. For such long-term studies, steady-state analysis was carried out. Generated powers from the proposed DS-DFIG MPPT scheme and traditional constant-speed DS system described in [83] are compared. In this example, the hourly measured insolation level of 2005 and that of every minute on Aug 5 and 6, 2005 were downloaded from [104]. The location where the measurements were taken is latitude $45.587^\circ$ and longitude $10.44^\circ$. Aug 5 2005 was a clear day whereas it was cloudy on the next. Figure 5.22 (a) and (b) show the generated power on the clear and cloudy days respectively. It can be seen that through the proposed MPPT scheme, the DS-DFIG system is able to extract more power and hence, more energy from the sun.
In this case, the energy harness increases by 3.63\% and 10.83\% in the clear and cloudy days respectively. From the number of hours vs insolation level distribution profile shown in Figure 5.22 (c), the total amount of energy harness in 2005 can be calculated under the proposed DS-DFIG MPPT and constant-speed operations. In this example, the proposed MPPT scheme can increase energy harness by about 5.6\%.

5.9. Conclusions

A general model of the variable speed dish-Stirling system has been derived for the study of variable-speed operation of the DS system either using DFIG or PMSG. The proposed variable-speed DS system is to realize maximum energy harness. A supervisory fuzzy control scheme has been identified as a suitable means because it can effectively mitigate the impacts of the engine speed variations on the DS receiver temperature. Simulation results have demonstrated the effectiveness of the proposed approach in maximizing the solar energy harness while ensuring satisfactory control of the receiver temperature of the variable-speed DS system.
Chapter 6 Laboratory Development of a DS-Simulator

6.1. Introduction

In this chapter, a DS-DFIG simulator has been proposed and developed in the author’s laboratory with the view to study the effectiveness of the control system design of the DS-DFIG system such as the temperature control and MPPT scheme developed in Chapter 5. The DS simulator would be a useful tool to verify the performance of the control system without costly actual Stirling engine construction. The simulator would generate the equivalent mechanical torque from the real DS under specific insolation level and specific engine/motor speed. The dc motor therefore emulates the torque speed behavior of the DS system, and the produced torque is to drive a DFIG in the laboratory. This approach of using a prime mover emulator is similar to that often used in the studies of variable-speed wind power conversion systems [95]. Part of the materials contained in this chapter have appeared in the author’s publications [92].

6.2. Modeling and Control of DC Motor

6.2.1. Modeling of DC Motor

Due to the simplicity of the control, a dc motor is a good solution to emulate the dish and Stirling engine. The mathematical model of the separate-excited dc motor in steady state is [105]:

Voltage Equation:

\[ V_a = L_a \frac{dl_a}{dt} + R_a l_a + E_a \]  \hspace{1cm} (6.1)

Counter-Electromagnetic Force Equation:

\[ E_a = K_e \omega_{mo}^{DCM} \]  \hspace{1cm} (6.2)

Electromagnetic Torque Equation:

\[ \tau_e^{DCM} = K_e l_a \]  \hspace{1cm} (6.3)

Motion Equation:
\[
J_{\text{DCM}} \frac{d\omega_m^{\text{DCM}}}{dt} = \tau_m^{\text{DCM}} - \tau_e^{\text{DCM}} - F_{\text{DCM}}^{\text{DCM}} \omega_m^{\text{DCM}}
\]  

(6.4)

where \( E_a \) is the counter-electromagnetic force (CEMF), \( K_e \) is the torque constant which is equal to the voltage constant, and \( V_a, I_a, R_a \) are the armature voltage, current and resistance respectively. \( J_{\text{DCM}} \) and \( F_{\text{DCM}} \) are the moment of inertia and the viscous friction of the dc motor respectively. The block diagram of the dc motor is shown in Figure 6.1 and the schematic diagram of the dc motor is shown in Figure 6.2. The superscript ‘DCM’ denotes the quantities of the dc motor, which may differ from the quantities of the interconnected DFIG.

![DC Motor (Electromagnetic Part)](image)

Figure 6.1. Block diagram of the dc motor model with one-mass shaft system.

![Schematic diagram of the dc motor](image)

Figure 6.2. Schematic diagram of the dc motor.

In order to model a stiff-shaft interconnection in a motor-generator configuration, (6.4) needs to be combined with the motion equation (4.25) of the DFIG. Suppose there is no gearbox between the shaft of the dc motor and the DFIG, the rotating speed \( \omega_m^{\text{DCM}} \) of the dc motor is equal to the rotating speed \( \omega_r \) of the DFIG. The electromagnetic torque \( \tau_e^{\text{DCM}} \) of the dc motor is assumed to be the same as the
mechanical torque $\tau_L$ of the DFIG. Thus, from (4.25), the new normalized motion equation is expressed as

$$2H_{eq} \frac{d\omega}{dt} = \tau_e^{DCM} - \tau_e - F_{eq}\omega$$

where the equivalent inertia constant $H_{eq}$ and friction constant $F_{eq}$ are

$$H_{eq} = H + \frac{0.5\omega_{mb}^2}{S_B} J^{DCM}$$

$$F_{eq} = F + \frac{0.5\omega_{mb}^2}{S_B} F^{DCM}$$

The overbar ‘$\bar{}$’ denotes the normalized quantities using the based values $\tau_B$, $\omega_{mb}$ of the DFIG introduced in Section 4.2.4.

According to above discussion, the block diagram of the dc motor model including the one-mass stiff shaft system is shown in Figure 6.1.

### 6.2.2. Torque/Current Control of the DC Motor

In order to simulate the steady-state and dynamic performance of the DS, the dc motor is to generate the mechanical torque equal to that of the DS under given insolation $I$ and engine speed $\omega_m$. Thus, the dc motor functions as a torque generator. As stated in the previous section, the mechanical torque of the DFIG $\tau_L$ is assumed to be same as the electromagnetic torque $\tau_e^{DCM}$ generated by the dc motor. From (6.3), it can be seen that $\tau_e^{DCM}$ is directly proportional to the dc armature current $I_a$. Thus, the torque can be regulated by current control scheme using a dc voltage source converter. Various dc voltage control scheme can be found in literature. The most commonly used schemes includes

1) A dc-dc buck-boost converter;
2) A uncontrolled ac/dc rectifier with a variable ac voltage source;
3) An active ac/dc rectifier with a fixed ac voltage source.

The schematic of these commonly used dc supply schemes are shown in Figure 6.3. Generally, for the purpose of control system design, these converters can be simply modelled as an ideal voltage source, i.e.

$$V^* = \bar{V}_a$$

(6.6)
A PI controller is used as the feedback torque regulator, which can be expressed as

$$V_a(s) = V_a^*(s) = (\tau_{ma}^{\text{DCM}} - \tau_{e}^{\text{DCM}})(k_p^{\text{DCM}} + \frac{k_i^{\text{DCM}}}{s})$$  \hspace{1cm} (6.7)
where \( k_p^{DCM} \) and \( k_e^{DCM} \) are the control parameters of the torque PI controller. The block diagram of the dc motor torque control system is shown in Figure 6.4.

![Block diagram of the torque/current control system of the dc motor.](image)

Figure 6.4. Block diagram of the torque/current control system of the dc motor.

In Figure 6.4, a feedforward control is added to enhance dynamic performance of the torque control considering the influence of the CEMF variation.

### 6.2.3. Model Simplification and Analysis

In order to investigate the dynamic and steady-state characteristics of the torque control system, the model is to be simplified. Using (6.1)–(6.3), one obtains

\[
\tau_e^{DCM} = \frac{K_e}{R_a + sL_a} (V_a - K_e \omega_m)
\]  

(6.8)

Equation (6.5) can be expressed as

\[
\omega_m = \frac{\tau_e^{DCM}}{\tau_B} - \frac{\omega_{ab}}{2H_{eq}s + F_{eq}}
\]  

(6.9)

Substituting (6.7) and (6.9) into (6.8), thus

\[
\tau_e^{DCM} = \frac{K_e}{R_a + L_a s} \left[ \left( \tau_m^{*DCM} - \tau_e^{DCM} \right) \left( k_p^{DCM} + \frac{k_e^{DCM}}{s} \right) - K_e \frac{\tau_e^{DCM}}{\tau_B} \frac{\omega_{ab}}{2H_{eq}s + F_{eq}} \right]
\]  

(6.10)

Re-arranging terms,
\[
\tau_{\text{DCM}}^{\text{eq}}(s) = \frac{K_e}{k_p^{\text{DCM}} + k_i^{\text{DCM}}} + \frac{K_i^{\text{DCM}}}{s} + \frac{K_o^{\text{DCM}}}{2H_{\text{eq}}s + F_{\text{eq}}} + \frac{K_{\omega_{\text{mb}}} / \tau_B}{2H_{\text{eq}}s + F_{\text{eq}}}
\]

where

\[
G_1(s) = \frac{K_e}{L_a s^2 + R_a s + K_e (k_p^{\text{DCM}} s + k_i^{\text{DCM}})} + \frac{sK_e^{\omega_{\text{mb}}} / \tau_B}{2H_{\text{eq}}s + F_{\text{eq}}}
\]

To simplify analysis, ignore friction i.e., assume \( F_{\text{eq}} = 0 \), thus,

\[
G_1(s) = \frac{K_e}{L_a s^2 + R_a s + K_e (k_p^{\text{DCM}} s + k_i^{\text{DCM}})} + \frac{sK_e^{\omega_{\text{mb}}} / \tau_B}{2H_{\text{eq}}s + F_{\text{eq}}}
\]

The time constant \( T_{\text{DCM}} \) of this second-order transfer function \( G_1(s) \) is

\[
T_{\text{DCM}} = \frac{1}{\sqrt{\frac{K_e^{\omega_{\text{mb}}} / \tau_B}{2H_{\text{eq}}L_a}}}
\]

Using the typical parametric values of a 1-kW dc motor in Appendix B, \( T_{\text{DCM}} \) is about 1 s. The time constant of the dish-Stirling model is much larger than this value. This means the dynamics of the dc motor is much faster than that of the dish-Stirling engine. The dynamics of the dc motor is thus negligible.

6.3. Dynamic DS-Simulator

As mentioned above, the dynamics of the dc motor is relatively faster than that of the DS. Thus, a dynamic DS-simulator can be developed using a dc motor and the DS model given in Section 4.2 and a dc motor to simulate the real Stirling engine and its control system. The block diagram of the dynamic DS-simulator is shown in Figure 6.5.
The required torque reference $\tau_m^{\text{DCM}}$ is calculated using the DS and its temperature control system models developed in the previous chapters, with the assumed insolation level and measured speed of the dc motor as two input variables. The electromagnetic torque of the dc motor is calculated using (6.3), and the dc/dc buck-boost chopper and the dc motor are realized using hardware. Using the PWM technique, the gate signal of the switch (IGBT) of the dc/dc chopper, as shown in Figure 6.3 (a), is obtained by comparing the duty ratio $D$ with the triangular carrier wave. The duty ratio is calculated by

$$D = \frac{V_a^*}{V_{\text{dc, const}}} \quad (6.15)$$

where $V_{\text{dc, const}}$ is the voltage provided by the constant dc voltage source as shown in Figure 6.3 (a).

The effectiveness of the dynamic DS-simulator is investigated via computer simulation. The simulation results by using the dc motor and the DS model are shown in Figure 6.7. The system configuration is the same with that described in Section 5.8. The simulated insolation rises from 0.9 p.u. to 0.95 p.u. at $t = 0.5$ s and fuzzy supervisory control proposed in Section 5.6.4 is used for heater temperature regulation. It can be seen that by incorporating the dc motor and the torque control system models into the original mathematical model developed in Chapter 5, the resulting waveforms are quite similar. With the high performance torque control of the dc motor, the DS-simulator shall be able to emulate the dynamics of the prime mover in terms of the external mechanical characteristics such as the mechanical
torque and the engine/dc motor rotating speed. On the other hand, the thermodynamic variables, such as the heater temperature and the mean pressure of the working gas, are not real physical quantities in the simulator and have the similar waveforms as well.

![Graphs showing temperature and torque over time](image)

Figure 6.6. Comparison between the simulation results from the dynamic DS-simulator and the original DS model.

### 6.4. Steady-State Analysis of the DS-Simulator

The steady-state normalized mechanical torque $\tau_m^*$ of the Stirling engine under given normalized insolation $I$ and normalized engine speed $\omega_m$ can be calculated using (5.2), (5.4) and (5.38), viz.

$$
\tau_m^* = \sum_{i=0}^{1} \sum_{j=0}^{2} b_{ij} \left( \frac{K_{\text{con}} I - a_{01} \omega_m - a_{00} (1 - T_q) / K_{\text{rec}}}{a_{11} \omega_m + a_{10}} \right) \omega_m^{j-1}
$$

(6.16)
Due to the hardware resource constraint in the laboratory, the dc armature voltage $V_a$ is manually varied as shown in Figure 6.3 (b), instead of using a dc drive controlled by digital controller, such as the dc voltage supply schemes shown in Figure 6.3 (a) and Figure 6.3 (c). It can be readily shown that the steady-state relationship between the normalized mechanical torque of Stirling engine $\tau_m^*$, normalized Stirling engine speed $\omega_m$ and dc armature voltage $V_a$ is

$$V_a = \frac{K_e \omega_m^{DCM} + \sqrt{\tau_m^{DCM} \omega_m^{DCM} R_a}}{2}$$

$$= \frac{K_e (\omega_m, \omega_m, N) + \sqrt{(\tau_m^* P_{m, N} / \omega_{m, N}) (\omega_m, \omega_m, N) R_a}}{2}$$

$$= \frac{K_e (\omega_m, \omega_m, N) + \sqrt{\tau_m^* \omega_m P_{m, N} R_a}}{2}$$  \hspace{1cm} (6.17)

The mechanical power is assumed to be the same as the input dc power of dc motor. Thus, using (6.16) and (6.17), the steady-state dc armature voltage $V_a$ can be calculated and its value is then manually set for the DS simulator so that the required output torque from the dc motor-drive can be obtained.

The schematic of the constructed DS-DFIG system is shown in Figure 6.7. In the author’s laboratory set-up, a 1.0-kW separately-excited dc motor is connected to a 1.0-kW DFIG with a RSC. The speed and current control systems of DFIG, (6.16) and (6.17) were implemented under dSpace (DS1104) platform, with MATLAB/Simulink in real-time interface. Figure 6.8 is a photograph of the experimental setup.

From the laboratory set-up, extensive steady-state measurements were carried out. Samples of the experimental results are shown in Table 6-I. The measured results are normalized using the based values of DFIG and compared with the theoretical results, as shown in Figure 5.9. It can be seen that the measured (identified by the symbol $\Delta$) results are in close agreement with those obtained from the theoretical analysis.
Figure 6.7. Schematic of experimental setup for DS-DFIG system.

Figure 6.8. Experimental setup of the DS-DFIG system with steady-state DS simulator.

Samples of the steady-state waveforms of the armature voltage and current are shown in Figure 6.9. It shows that at $I = 0.626$ p.u., the DFIG speed is controlled at the optimal speed $\omega_{m,\text{opt}} = 0.75$ p.u. By regulating the armature voltage at 206.2 V, as determined by (6.16) and (6.17), the measured armature current is about 2.6 A. The resulting power of 0.536 p.u. (on 1 kW base) compares favourably with the value calculated using (5.39), which is 0.5338 p.u.
Figure 6.9. Experimental results obtained at maximum power operation when $I = 0.626$ p.u. (a) dc armature voltage and current (b) AC stator voltage and current.

Table 6-I Results of experimental from DS-DFIG system

<table>
<thead>
<tr>
<th>$I$ (p.u.)</th>
<th>$\omega_m$ (p.u.)</th>
<th>$\tau_m$ (p.u.)</th>
<th>$V_a$ (V)</th>
<th>$P$ (W)</th>
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<tbody>
<tr>
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<td>0.9677</td>
<td>158.5</td>
<td>743.8</td>
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<tr>
<td>0.6</td>
<td>0.75</td>
<td>0.8663</td>
<td>176.6</td>
<td>749.7</td>
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<td>0.6</td>
<td>1.05</td>
<td>0.6112</td>
<td>233.6</td>
<td>740.7</td>
</tr>
<tr>
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</tr>
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<td>544.3</td>
</tr>
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<td>269.9</td>
<td>525.4</td>
</tr>
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<td>0.4881</td>
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</tr>
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<td>0.4224</td>
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<td>0.2283</td>
<td>266.9</td>
<td>329.3</td>
</tr>
</tbody>
</table>

6.5. Conclusions

In this chapter the overall configuration of a DS-simulator is proposed for the testing of the effectiveness of the control system design. The developed system with the DS-simulator is based on the use of a separately-excited dc motor to generate the equivalent mechanical torque from the DS under specific insolation level and
engine/motor speed. The dc motor therefore emulates the torque speed behavior of the DS system, and the produced torque is to drive a DFIG in the laboratory. This approach of using a prime mover emulator is similar to that often used in the studies of variable-speed wind energy conversion systems.
Chapter 7 Frequency Control of Variable-Speed Dish-Stirling Solar-Thermal System Using DFIG

7.1. Introduction

The scale of grid-connected conventional constant-speed DS system, using a squirrel-cage induction generator, is restricted due to the lack of control ability to meet the grid code requirements such as to provide frequency and voltage support. In contrast, a variable-speed DS system proposed in Chapter 5 using doubly-fed induction generator (DFIG) or permanent magnet synchronous generator (PMSG) and back-to-back converters, with vector control techniques, make the independent control of the real power and reactive power possible. These contribute to the system frequency and voltage support respectively. DFIG is attractive for economic reasons as it is relatively cheaper than PMSG. When large-scale DS-DFIG system is incorporated into the existing power system, or within a small, isolated system, the contribution to the system inertia and frequency support of the DS system would be extremely important because it determines the sensitivity of system frequency to supply demand imbalance. If no extra measures are taken, the DFIGs would not respond to the variation of system frequency due to the present of the speed/power control system of DFIG. The speed of the DFIG is fixed and it would not change much to release stored kinetic energy [106]. Works [107-109] on frequency control of wind turbine generator (WTG) using DFIG show that by amending the control strategy, DFIG can provide significant inertia response to the system frequency variation. However, because the inertia constant of the DS-DFIG system are usually much smaller than most commercial WTGs, the ability to provide inertia response by DS-DFIG system needs to be investigated independently.

In this chapter, an overall system configuration for the DS-DFIG plant will be proposed. Based on the model derived in the previous chapters, various methods to provide inertia and frequency support from the DS-DFIG system will be proposed and discussed. Numerical examples are included to show the effectiveness of the proposed approach for frequency support. Part of the materials contained in this chapter have appeared in the author's publication [110].
7.2. *Dish-Stirling Solar-Thermal Power Plant*

The energy conversion process of the conventional DS system consists of three parts: first, a concentrator collects, reflects and concentrates the solar irradiance into a small hollow chamber called the receiver. Secondly, a Stirling engine converts high temperature heat into mechanical energy. Finally, an electrical generator which is driven by the Stirling engine converts mechanical energy into electricity. Conventionally squirrel-cage induction generators are commonly used in such constant-speed operation systems [111]. While in the proposed variable-speed DS system using a wound-rotor DFIG, a back-to-back converter is needed to connect the rotor windings of the DFIG and the grid. By considering that the power rating of a single Stirling engine, e.g. 25 kW, is much smaller than most commercial DFIG-based WTG, Figure 7.1 proposed a possible design of a large-scale DS solar-thermal power plant, if one were to use the multi-string-converter configuration with common dc-link and common grid-side converter (GSC) similar to that of the PV plant as described in [112].

![Figure 7.1. Proposed scheme of a DS-DFIG system using a common GSC unit.](image)

In Figure 7.1, a local power conversion unit (PCU) consists of a prime mover (DS), a DFIG and a rotor-side converter (RSC) and corresponding control system. The dc terminals of RSCs from a cluster of $N$ PCUs are connected to a common dc bus. A common GSC unit consists of the GSC, dc-link capacitor, the LC filter, and corresponding control systems. The components of the common GSC unit have
much higher power ratings than those individual ones. The advantages of this proposed configuration scheme include the following aspects [112]:

- Local power control can be achieved for individual PCU;
- Larger power-rating GSCs are relatively cheaper, in terms of the specific cost (cost/kW) than lower-rating ones;
- Lower installation cost due to reduction of the number of GSC unit (dc-link, brake chopper, GSC, LC filter, measuring and monitoring devices, control system, protective device, etc.);
- Smoothing effect due to the geographical diversity of the insolation among the PCUs is expected to yield smaller power variations on the common dc bus. This would in turn facilitate the control of GSC.
- Enhanced functionality of GSC such as power quality and reactive power control are more economical for increasing nominal rated converter.

The role of each RSC is to achieve real power control for each PCU and to provide reactive power to the external grid. As long as the dc-link voltage is effectively maintained by the common GSC, the RSC can be controlled individually. The drawback of the common GSC is, when it fails, the dc-link voltage cannot be maintained and it may increase to some unacceptable level at high-insolation period because the incoming power cannot be delivered to the grid. In this situation energy storage system or braking resistor is need to dissipate the energy. Other solution includes using an ac crowbar to short-circuit the rotor windings and the DFIG operates likes a SCIG. However, this situation is not considered in this thesis.

7.3. Aggregated Model of DS-DFIG System

In order to study the interaction between the large-scale DS-DFIG plant and the grid, an equivalent per-unit single-machine model of DS-DFIG system would be used instead of representing each PCU individually [113]. An average or aggregated insolation level will used as a single insolation input, which is defined as

\[ I_{agg} = \frac{1}{N} \sum_{k=1}^{N} I_k \text{ (p.u.)} \]  

(7.1)

where \( k \) denotes the individual PCU in the DS plant. Figure 7.2 shows a comparison between the simulation results from a three-unit configuration model and an
aggregated model. The insolation levels vary randomly and they are different for each PCU. The results show that the waveforms from the aggregated model are very close to the arithmetic averaging of individual quantities. Thus, the aggregated model can be used to study the frequency control of DS-DFIG system, from which the aggregated quantities, such as the engine speed $\omega_{agg}$ in Figure 7.2 (b) and receiver temperature $T_{h,agg}$ in Figure 7.2 (c), embody the average value of the engine speed and temperature of the individual units, while the aggregated real power $P_{DS,agg}$ in Figure 7.2 (d) is the total power of the DS-DFIG plant.

![Figure 7.2](image_url)  
Figure 7.2. Aggregated effects of the DS-DFIG plant, the unit of the horizontal axis is second.

### 7.4. Frequency Support of DS-DFIG System

#### 7.4.1. Frequency Support from Synchronous Generators

When large-scale DS-DFIG systems are integrated into the existing power system, or as a type of distributed generation resource within a small insolated system, the total system inertia will be reduced greatly if conventional synchronous generators
(SGs) are replaced. For the SGs used in conventional thermal power plants, the frequency control concept can be simply explained by the block diagram of an equivalent generator represented in Figure 7.3 [89], where the meaning of the symbols are explained.

In Figure 7.3, the control system to regulate the system frequency consists of a speed governor and an integral control loop. The speed governor provides a speed droop using a proportional controller with a gain $1/R$, and the integrator with a gain $K_I$ adjusts the load reference $P_{ref}$. $F_{HP}$, $T_{RH}$ and $T_{CH}$ are the constants of the governor and reheat steam turbine. $H_{SG}$ and $D_{SG}$ are the inertia constant and damping constant of the equivalent SG respectively. For the primary frequency control, the system frequency $f$ will deviate from its nominal value under the non-frequency sensitive load change. The additional mechanical power generated by the synchronous generator due to the frequency change $\Delta f$ is

$$\Delta P_m = -\frac{1}{R} \Delta \omega = -\frac{1}{R} \Delta f \neq \Delta P_L$$ (7.2)

The secondary frequency control is carried out by the integrator, and this process is much slower than the primary frequency control. The secondary frequency control is able to bring the system frequency back to its nominal value $f_N = 1.0$ p.u. as long as there is enough reserve in the power system.

However, for the DFIG based system, the control system of DFIG decouples the mechanical and the electrical systems, and the DFIG will not respond to the frequency changes [106]. Both capabilities of the primary and secondary frequency controls would be reduced if large-scale DS-DFIG systems are integrated into the conventional system. Thus, ancillary service providing frequency support needs to be designed for
Chapter 7 Frequency Control of Variable-Speed Dish-Stirling Solar-Thermal System Using DFIG

the DS-DFIG system to avoid negative impact on the power grid. In the following sub-sections, several methods shall be examined for the DS-DFIG system to provide frequency support.

7.4.2. Inertia Response from Speed Control Loop

Kinetic energy stored in the rotating mass of the generator and Stirling engine can provide inertia response when system frequency varies. The magnitude of the inertia response depends on the extent by which the rotational speed changes in response to changing system frequency [107]. In most literature regarding to frequency control of DFIG based generation system, as shown in Figure 7.4 (a), a supplementary inertia control loop is added to provide the required decelerating torque $\tau_{sc}$ in order to release the stored kinetic energy, i.e.

$$
G_{sc}(s) = \frac{\tau_{sc}(s)}{f(s)} = \frac{K_{sc}s}{1 + T_{sc}s}
$$

(7.3)

where $\tau_{sc}$ is proportional to the change rate of the system frequency $f$, and control parameters $K_{sc}$ and $T_{sc}$ determine the magnitude of the response. Selection of these parameters must be careful to make sure that some constraints must not exceed such as the current limits. A proper $K_{sc}$ is found to be the twice of the inertia constant $H$ in [106, 107]. However, this scheme is especially designed for the optimal torque control as described in [106, 107], where the torque reference $\tau^e$ is obtained from the measured shaft speed $\omega_m$ directly, as shown in Figure 7.4 (a). If this inertia control loop is added directly to the feedback speed control loop using a PI controller described in Section 5.3.2, as shown in Figure 7.4 (b), the well-tuned PI controller would reject $\tau_{sc}$ because $\tau_{sc}$ is treated as a disturbance in such a feedback control loop. The shaft speed $\omega_m$ would hardly change much to release required kinetic energy stored in the rotating mass.

In order to provide sufficient inertia response in the proposed feedback speed control scheme, the original inertia control loop will be revised as shown in Figure 7.4 (b). In this strategy, when the system frequency drops, $\omega_{m,\text{inertia}} < 0$, thus the revised inertia control loop should attempt to increase the speed reference $\omega^*_m$, which helps to release the kinetic energy in the rotating mass. The design criteria for the new inertia control loop $G'_{sc}(s)$ is to generate the equivalent inertia response $\Delta\omega_m$ under specific
frequency change $\Delta f$. The transfer function of the original and the new inertia control loop are

\[
\frac{\Delta \omega_m(s)}{\Delta f(s)} = -\frac{1}{2Hs + D + k_{OTC} T_{sc}s + 1} \frac{K_{sc}s}{K_{sc}s + k_{p} s + k_{i} T_{sc}s + 1}
\]  

(7.4)

\[
\frac{\Delta \omega_m(s)}{\Delta f(s)} = -G_{\omega,cl}(s) K'_{sc}s T_{sc}'s + 1 = -\frac{k_{p} s + k_{i} T_{sc}s + 1}{2Hs^2 + (D + k_{p})s + k_{i} T_{sc}'s + 1}
\]  

(7.5)

where $G_{\omega,cl}(s) = \omega_m(s)/\omega'_m(s)$ is the close-loop transfer function of the speed control system, and $k_{OTC}$ is the slope of the tracking locus calculated from steady-state analysis of torque-insolation relationship. Comparing (7.5) with (7.4), it can be seen that as long as the stability of $G_{\omega,cl}(s)$ is ensured, the adding of the new inertia response loop will not affect the stability of the whole control system. And by selecting $K'_{sc} = K_{sc}$ will produce the same inertia response under the same frequency change.

Figure 7.4. (a) Supplementary inertia control loop for optimal torque control scheme to provide frequency response (b) Proposed supplementary inertia control loop for feedback PI control to provide frequency response, where $K'_{sc}>0$. 

139
It is noted that usually a large flywheel is commonly added to the shaft of the Stirling engine, for the purpose of providing stored energy to produce smooth output power [114]. Using the data in [115], the calculated inertia constant varies from 0.2 s to 5.7 s, depending on the design of the flywheel. Increasing the size of the flywheel, Stirling engine is able to provide more kinetic energy for inertia response to frequency change. In this article, the inertia constant is assumed to be 0.7 s, which is much smaller than the inertia constants of the conventional generation units.

7.4.3. Frequency Response from Temperature Control

Theoretically, any forms of stored energy within the system can be used as frequency response. Thus, frequency response can be provided by the temperature control system of the DS system by adding another supplementary control loop shown in Figure 7.5, where the temperature controller presented in Section 5.6 is also included. In Figure 7.5, if the system frequency drop, the temperature setting point $T_h^*$ will be reduced to some lower value. From (4.7) it can be seen that in order to reduce the temperature, i.e. $dT_h/dt<0$, the Stirling engine shall absorb more heat $q_h$ from the heat source. Equation (5.1) shows that when the engine speed $\omega_m$ is kept unchanged, the result of absorbing more heat is that the mean pressure $p_{\text{mean}}$ of the working gas would increase, which in turn increase the output mechanical torque $\tau_m$ of the Stirling engine. However, $p_{\text{mean}}$ would return to its original steady-state value once $T_h$ reaches its new set point $T_{h,\text{max}} - \Delta T_h^*$, and so does the mechanical torque $\tau_m$. Thus, extra mechanical torque is temporally provided at the initial stage of the frequency drop. The temperature would stay at the new set point unless the system frequency is brought back to its nominal value by the secondary frequency control of the power system. In this way, the Stirling engine can release it thermal energy stored in the high-temperature and high-pressure working gas by reducing its temperature, which is similar to the concept of inertia response provided by releasing the kinetic energy stored in the rotating mass discussed in Section 7.4.2.
Based on the above observation, the proposed primary frequency support scheme via temperature control is shown in Figure 7.5: when system frequency drops, the temperature set point will be reduced from $T_{h,max} = 1.0$ p.u. by $\Delta T_h^*$. $\Delta T_h^*$ is proportional to the change of system frequency $\Delta f$, i.e.

$$\Delta T_h^* = K_T \Delta f = \frac{T_{h,max} - T_{h,min}}{\Delta f_{max}} \Delta f \quad (7.6)$$

where $T_{h,min}$ is the selected normalized minimum temperature for the normal operation of the Stirling engine, and $\Delta f_{max}$ is the maximum frequency change that the DS-DFIG can respond to.

On the other hand, when system frequency rises, the temperature set point will increase. Normally the material of the Stirling engine is designed to tolerate short-time overheat beyond the maximum continuous working temperature $T_{h,max}$ up to 5%. Long-time overheat would definitely shorten the lifetime of the material of the receiver and the heat tubes which must be prevented. In this article, short-time overheat temperature is assumed within the tolerable range for safe operation of the Stirling engine, otherwise extra mechanism is required to protect the devices. Figure 7.6 shows the effect of the proposed control scheme to provide frequency response from temperature control system, where frequency drops about 0.5 Hz at $t = 2$ s. DS system would inject some amount of power to support the recovering of the system frequency by reducing the temperature as shown in Figure 7.6 (c) and (e).
7.4.4. **Primary Frequency Control DS-DFIG by De-Loading**

Primary frequency control is required by several grid codes [116] for non-conventional generation units when they are connected to the main power grid. Because there are differences between the working principles of SG and DFIG, the structures of their primary frequency controllers are of great difference. For the
DS-DFIG system, the primary frequency support can be provided by de-loading the original tracking curve for MPPT to provide sufficient spinning reserve. However, the de-loading is limited by the steady-state operating speed range of DFIG. This can be illustrated using Figure 7.7, where the original allowable operating region derived in Section 5.4 is the area A-F-G-D, and the MPPT locus is A-H-K-G. The calculated de-loaded tracking curve for x% spinning reserve shall shift from the original MPPT curve to its right-handed side, denoted by the gray dashed line A’-G’ in the figure. Thus, the frequency support requirement cannot be met above some insolation level at point Y due to the maximum speed limit. The worst case happens at the maximum insolation level \( I = 1.0 \text{ p.u.} \) (Point A), where no spinning reserve can be provided at all.

In order to meet the requirement of the grid code for all insolation levels, one solution is to select a DFIG with higher nominal speed in the design period of the DS-DFIG plant. Suppose the speed range of DFIG is \([\omega_{m,\text{min}}, \omega_{m,\text{max}}]\) in per-unit (normally from 0.7 to 1.3), the corresponding speed range based on the nominal Stirling engine speed should be \([\omega_{m,\text{min}}^G, \omega_{m,\text{max}}^G, 1.0]\) for the MPPT purpose. For the proposed new de-loading scheme, the maximum engine speed line moves from A-D (\(\omega_m = 1.0\)) to A’-D’ (\(\omega_m = \omega_{m,\text{max}}\)). The value of \(\omega_{m,\text{max}}\) depends on the requirement of specific grid code, i.e. at point A, where \( I = 1.0 \text{ p.u.} \), the system must be able to provide \(x\%\) spinning reserve. The minimum Stirling engine speed \(\omega_{m,\text{min}}\) can be calculated by

\[
\omega_{m,\text{min}} = \frac{\omega_{m,\text{min}}^G \omega_{m,\text{max}}^G}{\omega_{m,\text{max}}^G} \quad (7.7)
\]

From Figure 7.7 it also can be seen that due to the minimum speed limit of DFIG, part of the original MPPT locus \(F’-H-K-G\) may move outside of the new allowable operating area \(A-A’-D’-G’-F’\). Thus, in this situation, the steady-state maximum power limits for frequency support consists of the maximum pressure line \(A-F’\) and the minimum speed line \(F’-G’\), and the corresponding de-loaded locus needs correction according to this new maximum power locus, denoted by the black dashed line A’-G’ in Figure 7.7.
In order to implement the de-loaded operating locus $A'-G'$ in the control system, $\omega_m - I$ relationship needs to be pre-calculated via steady-state analysis, which is also shown in Figure 7.7 as line $A'-G'$. From Figure 7.7 it can be seen that the de-loaded speed $\omega_{m,DL}$ is higher than the original MPPT case and it is approximately linear to insolation $I$. The slope $k_{DL}$ shall replace the parameter $k_{MPPT}$ in (5.61) to form the system matrix for the fuzzy supervisory control scheme presented in Section 5.6 to regulate the receive temperature. This type of supervisory control based on state feedback control which can guarantee the system stability and dynamics performance when the frequency control is incorporated.

When system frequency drops, the control system should force the DS-DFIG to moves from the original operating points on line $A'-G'$ towards the point on the maximum power line $A-F'-G'$ under along the equal-insolation line. In order to do this, a droop control loop will be proposed as shown in Figure 7.8.
In Figure 7.8, the droop constant $K_{\text{droop}}$ determines the capability for primary frequency support. The larger $K_{\text{droop}}$ is, the lower speed and more power can be generated to support the system frequency, while $K_{\text{droop}}$ cannot be too large otherwise the minimum speed and maximum pressure limits would exceed. Suppose the maximum system frequency change that DS-DFIG is able to respond to is $\Delta f_{\text{max}}$, the speed difference between the point on the new MPPT locus and that of the de-loaded line is $\Delta \omega_{\text{m,DL,max}}$. $K_{\text{droop}}$ is defined as

$$K_{\text{droop}}(I) = \frac{\Delta \omega_{\text{m,DL,max}}(I)}{\Delta f_{\text{max}}} = \frac{\omega_{\text{m,DL}}(I) - \omega_{m,AFG'}(I)}{\Delta f_{\text{max}}}$$  \hspace{1cm} (7.8)

The steady-state $K_{\text{droop}} - I$ relationship is shown in Figure 7.9 (a). From Figure 7.9 (a), it can be seen the $K_{\text{droop}}$ calculated using (7.8) is not a constant over the whole insolation levels. Look-up table or gain scheduling can be used to determine $K_{\text{droop}}$ according to the measured insolation $I$. The amount of power change in response to the system frequency change is

$$\Delta P_m = -k_I \Delta \omega_m = -k_I K_{\text{droop}} \Delta f$$  \hspace{1cm} (7.9)

where $k_I$ is the slope of the equal-insolation line under insolation level $I$. The $k_I - I$ relationship is shown in Figure 7.9 (b), and $k_I$ is determined by

$$k_I(I) = \frac{P_{m,DL}(I) - P_{m,AFG'}(I)}{\omega_{\text{m,DL}}(I) - \omega_{m,AFG'}(I)} < 0$$  \hspace{1cm} (7.10)
Chapter 7 Frequency Control of Variable-Speed Dish-Stirling Solar-Thermal System Using DFIG

Figure 7.9. (a) Droop constant $K_{\text{droop}}$ vs. insolation level $I$; (b) Slope $k_I$ vs. insolation level $I$.

As an example, the system is assumed operating at point $P'$ as shown in Figure 7.7, where the insolation level is 0.8 p.u. and its corresponding operating point on the MPPT locus is $P$. Suppose the system frequency drops by $\Delta f$ due to a positive resistive load change $\Delta P_{\text{load}}$, the proposed primary frequency control system of DS-DFIG will force the operating point to move from point $P'$ to some point $P''$ by reducing the engine/DFIG speed. The system frequency change can be calculated by

$$
\Delta f = \frac{-\Delta P_{\text{load}}}{1 - \frac{k_I}{R} K_{\text{droop, max}} \frac{S_{N,DS}}{S_{N,eq}} + D_{eq} \frac{S_{N,DS}}{S_{N,eq}}} \quad \text{(p.u.)} \quad (7.11)
$$

where $S_{N,eq}$ is the rating of the equivalent SG and $S_{N,DS}$ is the rating of the DS-DFIG system. $\beta$ is the composite frequency response characteristic expressed in SG per-unit system. From (7.11) it can be seen $\Delta f$ depends on the load change $\Delta P_{\text{load}}$, the insolation level $I$, and the ratio of DS-DFIG and equivalent SG ratings $S_{N,DS}/S_{N,eq}$. The relationship can be illustrated using the $\Delta f$ surface in Figure 7.10.
7.4.5. Secondary Frequency Control of DS-DFIG

Normally grid codes do not request any secondary frequency control from the non-conventional power generation plants. However in a small isolated system, such frequency support would be helpful for the system stability. When there are other generating units performing secondary frequency control in the main grid, the primary frequency control of DS-DFIG discussed above does not work. This is because if the secondary frequency control brings the system frequency back to its nominal value, from (7.9) it can be seen that no additional power would be generated from DS-DFIG, i.e. in Figure 7.7, the DS-DFIG would always stay at line $A'-G'$ as long as there is enough reserve in the power system to provide secondary frequency control.

To solve this problem, an integrator with gain $K_{I,DS}$ is proposed to be added to the droop control loop discussed in Section 7.4.4, which is also shown in Figure 7.7. This control structure is similar to that of the synchronous generator shown in Figure 7.3 and the performance of the supplementary control loops will be shown in next section.
7.5. Illustrative Example

7.5.1. Test System Configuration for Frequency Support

In this configuration, as shown in Figure 7.11, a 900 MW (0.9 p.u. based on 1000 MVA) large-scale DS-DFIG system is connected to the high-voltage transmission grid directly via a step-up transformer, without the distribution system for those small-scale DS systems. The system frequency is mainly supported by an equivalent 5000 MVA (5.0 p.u.) synchronous generator. The description of the models of the synchronous generator and its speed governing and excitation systems can be found in [89] and will not be elaborated here. The inertia time constant of the synchronous generator is \( H_{SG} = 3.7 \) s and permanent droop of its speed governor is 0.05. A resistive load of 5000 MW (5.0 p.u.) is connected to the transmission grid. The initial insolation level is 0.8 p.u., i.e., at Point \( P' \) in Figure 7.7. The injected real powers from the synchronous generator and from the DS-DFIG system are 4434 MW (4.434 p.u.) and 619.6 MW (0.6196 p.u.) respectively. Corresponding parameters of the grid elements are shown in Figure 7.11. The parameters of the DS and DFIG are given in Appendix B.

![Diagram](image)

Figure 7.11. A 5-bus test system for the study of frequency support (Base power: 1000 MVA).

7.5.2. Case 1: Frequency Response of DS-DFIG System Under Primary Frequency Control of Synchronous Generator

At \( t = 2 \) s, the resistive load increases from 5000 MW to 5500 MW. In order to study the frequency response of the DS-DFIG system, no secondary frequency control is provided by the speed governor of the synchronous generator, because secondary frequency control will bring the frequency back to its nominal value. The maximum frequency change that the DS-DFIG can respond to is set to \( \Delta f_{\text{max}} = 0.5 \) Hz. The
simulation results with and without the supplementary control loop of frequency support are shown in Figure 7.12.

From Figure 7.12 (a), it can be seen due to the increase of load, system frequency will drop by about 0.3 Hz in the steady state. The DS-DFIG without any supplementary loop has least response to this frequency drop. When the inertia control loop or the droop control loop is added, the control system attempts to reduce the engine speed to produce more power to support the frequency, which is shown in Figure 7.12 (b) and Figure 7.12 (c). The difference is, with the droop control loop, the steady-state frequency is higher than that in the cases without it, while the inertia control loop only increase the power until all the energy stored in the rotating mass is release. From Figure 7.12 (d) it can be seen with the droop control loop, the generated power from the synchronous generator is less than in the other cases. Thus, with the droop control loop the DS-DFIG system is able to share the duty of the primary frequency control of the equivalent grid synchronous generator. From Figure 7.12 (e), it can be seen that the proposed temperature controller works well when any supplementary control loop is added. From Figure 7.12 (d) and Figure 7.12 (e) it can be seen there is the power spike injected into the grid at the beginning stage of the frequency drop at the expense of reducing the temperature.

Because the frequency drop is about 0.3 Hz, which is less than the maximum frequency that can respond to (Δf_{max} = 0.5 Hz), the new operating point will not hit the maximum power lines A-F'-G' but at some point P'' between point P and Point P' in Figure 7.7.

7.5.3. Case 2: Frequency Response of DS-DFIG System Under Secondary Frequency Control of Synchronous Generator

Similar to Case 1, the resistive load increases by 500 MW at t = 2 s. In this case the synchronous generator performs both primary and secondary frequency control, which will bring the system frequency back to its nominal value. The simulation results are shown in Figure 7.13.

It can be seen from Figure 7.13 (a) and Figure 7.13 (c) that the frequency support from DS-DFIG system can provide better frequency response with the supplementary control loops and it is able to share the duty of secondary control of the synchronous generator also by providing additional power.
Figure 7.12. Simulation results with and without frequency support from DS-DFIG system when synchronous generator performs primary frequency control. The unit of the x-axis is second.

- With three supplementary control loops ($K_{\text{sc}}' = 0.5, K_I = 0.0774, K_{\text{droop}} \neq 0, K_{\text{IDS}} = 5$)
- With the droop control loop only ($K_{\text{sc}}' = 0, K_I = 0, K_{\text{droop}} \neq 0, K_{\text{IDS}} = 5$)
- With the inertia control loop only ($K_{\text{sc}}' = 0.5, K_I = 0, K_{\text{droop}} = 0, K_{\text{IDS}} = 0$)
- With the inertia control from temperature control only ($K_{\text{sc}}' = 0, K_I = 0.0774, K_{\text{droop}} = 0, K_{\text{IDS}} = 0$)
- Without any supplementary control loops ($K_{\text{sc}}' = 0, K_I = 0, K_{\text{droop}} = 0, K_{\text{IDS}} = 5$)
Figure 7.13. Simulation results with and without supplementary control loop from DS-DFIG system when synchronous generator performs secondary frequency control. The unit of the x-axis is second.

- - - - Without any supplementary control loops ($K'_{sc} = 0$, $K_T = 0$, $K_{droop} = 0$, $K_{IDS} = 0$)
- - - - - With the inertia control from temperature control only ($K'_{sc} = 0$, $K_T = 0.0774$, $K_{droop} = 0$, $K_{IDS} = 0$)
- - - - - - With the inertia control loop only ($K'_{sc} = 0.5$, $K_T = 0$, $K_{droop} = 0$, $K_{IDS} = 0$)
- - - - - - - With the droop control loop only ($K'_{sc} = 0$, $K_T = 0$, $K_{droop} \neq 0$, $K_{IDS} = 5$)
- - - - - - - - With three supplementary control loops ($K'_{sc} = 0.5$, $K_T = 0.0774$, $K_{droop} \neq 0$, $K_{IDS} = 5$)
7.6. Conclusions

In this chapter, an overall system configuration for the DS-DFIG plant is proposed for large-scale integration. A single-machine aggregated model is shown to be sufficient for the studies of the frequency control. Based on the model derived in the previous chapters various methods to provide inertia and frequency support of DS-DFIG system are discussed. The proposed supplementary control loops are able to provide inertia response, primary and secondary frequency control under the variation of the system frequency. The utility-scale DS farms have the capability to provide frequency support for the main power grid.
Chapter 8  Conclusions and Recommendations

8.1. Conclusions

As introduced in Chapter 1, dish-Stirling (DS) solar-thermal generation system is a type of renewable energy technology. Stirling engine is capable of operating at high efficiency and releases no emissions. Unfortunately, the often random and uncontrollable nature of solar irradiance makes the control of the harnessed energy most challenging. Hence, appropriate system design and operations of the DS power plant are called for.

In view of the above, firstly in this thesis, models of the Stirling engine are developed which would be suitable for power system study. Two average-value models for four-cylinder double-acting kinematic Stirling engine, including a general model for variable-speed operation and a specific model for constant-speed operation of DS system, are derived in this thesis based on thermodynamic analysis. With the developed models, design of the appropriate control system to achieve temperature control of the receiver/absorber is examined. Accordingly, from the derived linearized model of the constant-speed dish-Stirling system, a new temperature controller with transient droop compensation and feedforward compensation is proposed to overcome the problem caused by the non-minimum phase and nonlinear characteristic of the model.

From the steady-state analysis of the derived model, the potential of maximum power operation of the DS system is examined. It shows that variable speed operation of dish-Stirling is a viable method to extract considerably more energy from the sun. The overall configuration of the variable-speed DS system using doubly-fed induction generator is proposed. The DS system model is then derived specifically for the design of temperature control while achieving maximum power harness. Simulation results show that the proposed temperature controllers can readily overcome the problems introduced by the continuous engine speed variations.

A DS-DFIG simulator has also been developed in the author’s laboratory, with the view to study the maximum energy harness ability of the DS-DFIG system. The developed system is based on the use of a separately-excited dc motor to generate the equivalent mechanical torque from the DS under specific solar insolation level and
engine/motor speed. The dc motor therefore emulates the torque speed behavior of the DS system, and the produced torque is to drive a DFIG in the laboratory.

Finally the ability of frequency support provided by the DS-DFIG system is examined. An overall configuration for large-scale operation the DS-DFIG plant is proposed. Based on the DS model derived earlier, various methods to provide inertia and frequency support from the DS-DFIG system is demonstrated.

8.2. Recommendations

Notwithstanding the progress made so far in this thesis, there are several potential areas for further works. These are:

**Model validation**

The Stirling engine models developed in this thesis have been compared with the adiabatic model on its dynamic behavior. However, the accuracy of the adiabatic model to represent the dynamic behavior of a real engine still needs further investigation. In Chapter 3, the gas temperature in the heater is assumed to be the same as the temperature of heater wall, assuming infinite thermal conductance of the material of the heat exchangers. In practice, there will be a difference between them. This needs to be investigated. Furthermore, the thermal and mechanical losses incorporated in the model are based on the steady-state analysis, and the temperature effects on the engine performance are ignored by considering that the working temperature is sufficiently high. Thus, when the engine is operating under abnormal condition when the working temperature is quite low, the output power will be greatly reduced. Further modification on the model of the Stirling engine needs to be carried out for the studies of fault condition when the working temperature is not within the assumed range.

In this thesis, the research is based on the most commonly used kinematic Stirling engine. More recent applications of Stirling engines for solar power have made used of free-piston Stirling engines, which are very different than the kinematic type discussed in the thesis. With continued research, the author is hopeful the potential of this promising technology can be exploited to the fullest.
Advanced control system design

The dish-Stirling is a highly nonlinear system. In this thesis, the control system is design mostly based on the small-signal analysis. At different insolation levels, the system matrix varies and fuzzy supervisory control is used considering the relatively slow variation of the insolation. The ability to deal with large disturbances caused by grid-fault (for example) by this type of control system may be less effective as the dynamic is rather fast. Advanced nonlinear controller can provide better system damping and could be one way to mitigate the difficulty. This could be another area for fruitful investigation.

Hybrid systems with thermal energy storage

One of the virtues of the Stirling engine is its inherent capability of being able to use any energy source that can provide heat at an appropriate temperature level. In fact, when the Stirling engine was invented, it was designed to absorb the heat from combustion of fossil fuel.

The multi-fuel capability of Stirling engine can be realized if the concept of thermal energy storage (TES) can be applied. Currently, thermal energy storage systems have been integrated into some commercial CSP projects in the U.S. However, these projects mainly use parabolic trough technology which uses thermal power to heat the steam and then drives a traditional steam turbine. Thus the thermal energy storage can be easily realized by storing the high temperature steam in huge tanks. For the dish-Stirling systems, the energy must be stored and be available at temperature in the range of around 600–800 ºC. For all practical purposes, the storage material must have a high gravimetric and volumetric energy storage capability. The energy stored in the TES is then extracted and used as the heat source for the Stirling engine. Once the temperature of the storage material has fallen below some minimum temperature value, it can be recharged from primary heat sources such as solar insolation, other fuels and even from electric heating during the off-peak load period (as part of the smart grid). The concept is often termed as a heat accumulator or thermal battery.
Appendix A Parameters of the Dish and Stirling Engine

A.1. Dish & Receiver Parameters

Concentrator parameters: $A_{\text{con}} = 87.8 \text{ m}^2$, $\eta_{\text{con}} = 0.88$;
Receiver parameters: $K_r = 200$, $K_L = 14.83$;
Atmosphere temperature: $T_a = 298 \text{ K (25 °C)}$;
Maximum insolation level: $I_{\text{max}} = 1000 \text{ W/m}^2$
Normalized dish/receiver constants:
$K_{\text{con}} = 2.865$, $K_{\text{rec}} = 1.756$, $T_{\text{rec}} = 13.44 \text{ s}$, $T_a = 0.288$

A.2. Stirling Engine Parameters

Swept volume: $V_{\text{sw}} = 95 \text{ cm}^3$
Clearance volume: $V_{\text{cl}} = 10 \text{ cm}^3$
Heater volume: $V_h = 33.08 \text{ cm}^3$
Cooler volume: $V_k = 86.47 \text{ cm}^3$
Regenerator void volume: $V_r = 134.73 \text{ cm}^3$
Nominal mean pressure: $p_{\text{mean,N}} = 20 \text{ MPa}$
Nominal engine frequency: $f = 30 \text{ Hz}$
Maximum heater temperature: $T_{h, \text{max}} = 1033 \text{ K (760 °C)}$
Minimum heater temperature: $T_{h, \text{min}} = 993 \text{ K (720 °C)}$
Cooler temperature $T_k = 323 \text{ K (50 °C)}$
Gas constant (Hydrogen): $R = 4120.0 \text{ J/(kg·K)}$
Specific heat capacities of the gas at constant pressure $c_p = 14420 \text{ J/(kg·K)}$
Specific heat capacities of the gas at constant volume $c_v = 10300 \text{ J/(kg·K)}$
Specific heat ratio: $\gamma = 1.4$

Multivariate polynomial coefficients and normalized Stirling engine constants:
$a_{00} = 0.045$, $a_{10} = 0.068$, $a_{01} = 0.20$, $a_{11} = 2.14$
$b_{00} = -0.038$, $b_{10} = -0.072$, $b_{01} = 0.055$, $b_{11} = 1.21$, $b_{00} = -0.026$, $b_{11} = -0.13$

Normalized Stirling engine constants:
$A = -0.2735$, $C = 0.8752$
A.3. **Control Parameters**

Solenoid valves gain: \( K_v = 1.0; \)
Solenoid valve time constant: \( T_v = 0.02 \text{ s}; \)
Mean pressure controller gain: \( G_p = 25; \)
Temperature control droop: \( D_p = 0.043; \)

Fuzzy supervisory control gains:
\[
K_1^{(\text{low})} = [1.9, 1046, -22485, -1384, -20.2], \quad K_2^{(\text{low})} = -46057,
\]
\[
K_1^{(\text{medium})} = [2, 751, -20508, -1144, -19.8], \quad K_2^{(\text{medium})} = -56509,
\]
\[
K_1^{(\text{high})} = [2.3, 708, -18888, -244, -5.8], \quad K_2^{(\text{high})} = -59677.
\]
Appendix B  Machine Parameters

B.1.  SCIG Parameters

Nominal apparent power: $S_N = 29.23$ kVA
Nominal line-to-line voltage: $V_N = 460$ V (rms)
Nominal stator frequency: $f_N = 60$ Hz
Stator winding resistance: $R_s = 0.0279$ p.u.
Stator self inductance: $L_s = 3.16$ p.u.
Rotor winding resistance: $R_r = 0.0123$ p.u.
Rotor self inductance: $L_r = 3.16$ p.u.
Magnetizing inductance: $L_m = 3.06$ p.u.
The inertia time constant $H = 0.6078$
The friction constant $F = 0.0243$
The number of pole pairs: $p_n = 2$

B.2.  DFIG Parameters

Nominal line-to-line voltage: $V_N = 575$ V (rms)
Nominal stator frequency: $f_N = 60$ Hz
Stator winding resistance: $R_s = 0.023$ p.u.
Stator self inductance: $L_s = 3.08$ p.u.
Rotor winding resistance: $R_r = 0.016$ p.u.
Rotor self inductance: $L_r = 3.06$ p.u.
Magnetizing inductance: $L_m = 2.9$ p.u.
The inertia time constant $H = 0.685$
The friction constant $F = 0.01$
The number of pole pairs: $p_n = 3$
Speed controller parameters: $k_{p,w} = 128, k_{i,w} = 4200$;

B.3.  DC Motor Parameters

Armature resistance $R_a = 13.5$ Ω
Armature inductance $L_a = 132.5$ mH
Torque/voltage constant $K_e = 1.4$
Appendix C Multivariate Polynomial Coefficients

C.1. Output Power Coefficient

Different types of Stirling engines for solar thermal generation have similar power curves. The most commonly used structure is the double-acting kinematic configuration. The power rating depends on the dimension of engine and working condition. Normalized or non-dimensional power curves are useful to represent the characteristic of Stirling engine. Table C-I is an example to calculate the power coefficient $b_{ij}$ and $a_{ij}$ of MP models. The data are obtained from [20].

Table C-I Data of Stirling engine (Part 1)

<table>
<thead>
<tr>
<th>Point</th>
<th>$p_{\text{mean}}$ (MPa)</th>
<th>$\omega_m$ (rpm)</th>
<th>$P_m$ (kW)</th>
</tr>
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<td>13</td>
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<tr>
<td>24</td>
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</table>

Nominal values:

$p_{\text{mean},N} = 15 \text{ MPa}$, $\omega_{m,N} = 1500 \text{ rpm}$, $P_{m,N} = 32 \text{ kW}$
First, convert the original data to per-unit representation by selecting the nominal values as the base quantities. Then use the normalized multivariate polynomial model introduced in Section 3.3.2, i.e.

\[ P_m(0) = b_{00} + b_{10} P_{\text{mean}} + b_{11} P_{\text{mean}} \omega_m + b_{02} \omega_m^2 + b_{12} P_{\text{mean}} \omega_m^2 \]  

(C.1)

The coefficient \( b_{ij} \) can be calculated by multiple linear regression using least square method. Using the MATLAB Surface Fitting Tool (sftool), one obtains

\[
\begin{align*}
  b_{00} &= -0.038 & b_{10} &= -0.072 & b_{01} &= 0.055 \\
  b_{11} &= 1.21 & b_{02} &= -0.026 & b_{12} &= -0.13
\end{align*}
\]

It can be verified that the sum of \( b_{ij} \) is approximately one.

The normalized data points and estimated output power vs. speed, output power vs. mean pressure and torque vs. speed curves are shown in Figure C.1, Figure C.2 and Figure C.3 respectively. It can be seen that the MP model can describe the Stirling engine performance most satisfactorily.

![Figure C.1. Mechanical power \( P_m \) vs. engine speed \( \omega_m \) characteristics](image)
Figure C.2. Mechanical power $P_m$ vs. mean pressure of the working gas $p_{\text{mean}}$ characteristics.

Figure C.3. Mechanical torque $\tau_m$ vs. engine speed $\omega_m$ characteristics.
Appendix C Multivariate Polynomial Coefficients

Figure C.4. Mechanical power $P_m$ surface as a function of mean pressure of the working gas $p_{\text{mean}}$ and engine speed $\omega_m$.

C.2. Input Power Coefficient

Again, first convert the original data of input power as shown in Table C-2 into per-unit representation, then use the normalized multivariate polynomial model of input power introduced in Section 3.3.2, i.e.

$$q_h(0) = a_{00} + a_{10}p_{\text{mean}} + a_{01}\omega_m + a_{11}p_{\text{mean}}\omega_m \quad \text{(C.2)}$$

The coefficient $b_{ij}$ can be calculated by multiple linear regression using least square method. Using the MATLAB Surface Fitting Tool (`sftool`), one obtains

$$a_{00} = 0.045 \quad a_{10} = 0.068$$
$$a_{01} = 0.20 \quad a_{11} = 2.14$$

The normalized data points and estimated input power vs. speed is shown in Figure C.5. Again the agreement between the two sets of results is excellent.
Appendix C Multivariate Polynomial Coefficients

Table C-II Data of Stirling engine (Part 2)

<table>
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<th>Point</th>
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<th>$\omega_m$ (RPM)</th>
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<th>$P_m$ (kW)</th>
<th>$DQ_h$ (kW)</th>
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</table>

Figure C.5. Absorbed heat transfer rate $q_h$ vs. engine speed $\omega_m$ characteristics.
Figure C.6. Absorbed heart transfer rate $q_h$ surface as a function of mean pressure of the working gas $p_{\text{mean}}$ and engine speed $\omega_{\text{me}}$. 
Author’s Publications


Bibliography


Bibliography


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