An Investigation into Systemic Risks and its Mitigation Based on Complex Network and Agent-Based Modelling

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Abstract

Significant events, such as crises created by breakdowns of financial systems, natural hazards, failures of the infrastructure systems, production shortfalls within supply chains create enormous amount of losses to affected communities, whose impact is not only confined to a single country or economic sector but more. Their statistics on social and economic damages are sobering and urge us to a better understanding of the systems we live in, identifying the sources and drivers of systemic risks, and finding measures to mitigate crises.

To make a contribution toward this goal we present two projects using agent-based models. In the first project, we simulate a dynamical supply chain network and investigate whether diversification within the supply chain can help mid-level firms survive prolonged financial crises. To do so, we analyze the lifetime distribution of undiversified and diversified firms and compare the average lifetimes and the rates at which the midsections and tails of the cumulative lifetime distribution decay. We have found that the main benefit of diversification does not lie in increasing profits or lower operating costs, but helps mid-level firms more effectively manage the systemic risks they are exposed to, and help them survive longer in a competitive and uncertain business world.

In the second project we are particularly interested to investigate the interbank market in Singapore. To do that we develop a numerical scheme that allows us to reconstruct the Singaporean interbank network structure based on day-to-day data of interbank borrowing and lending recorded by the Monetary Authorities of Singapore. We then simulate an interbank network, where banks are connected via interbank loans and invest into individually optimized portfolios comprising of one risky and one risk-free asset. This allows us to investigate the systemic risk introduced by leverage and interbank loans. To understand the effectiveness of regulations on bank liquidity and leverage introduced by Basel III, we compare the lifetime of banks for three different regulation scenarios: (1) unregulated leverage (2)
penalized leverage and (3) forbidden leverage. We observe that, although banks in our model have complete market information they do not perform well due to highly leveraged investments. In addition we investigate the consequences of incomplete or wrong market information. We identify that the over- and underestimation of probabilities for successful risky investments has similar severe consequences with a sharp drop in lifetimes. Our results show that a maximum in lifetimes is reached when regulations completely forbid leverage. Additionally we find that interbank lendings have a stabilizing effect on the network. This can be concluded since the reduction of linkages between banks reduces the lifetime of banks. On the other hand, if the network size and therefore the number of linkages per bank are increased we observe increased interbank lending accompanied by longer lifetimes.
Chapter 1

Introduction

Today, almost six years after the financial crisis hit in summer 2007 we still wonder how it could take us by surprise. Policy-makers and economic experts in governments, were also caught unprepared. They succeeded neither in anticipating the impact of the crisis nor guiding us out of the mess. Defending their ignorance they argued that ‘this time is different’ and the crisis to be an ‘once-in-a-century event’ as Alan Greenspan the former Federal Reserve chairman said in September 2008. Interestingly, earlier that year Carmen M. Reinhart and Kenneth S. Rogoff published two papers arguing the converse [1, 2]. The papers, which are based on historical records of financial crises of various regions including the U.S., show that ‘crises frequently emanate from the financial center’, hence ‘the recent US sub-prime financial crisis is hardly unique’ and ‘the global dimension of the current crisis is neither new nor unique to this episode’.

This made it abundantly clear that current economic models in use are not sophisticated enough to account for the development of crises. In fact, strong accusations has been made claiming economic research to be ‘spectacularly useless at best, and positively harmful at worst’ and policy-makers ‘flying the economy by the seat of their pants’ [3, 4]. In other words, there exists fundamental shortcomings both in previous social science models and graph theory. In social science, one of the major concerns is that empirical statistical models are fitted to past data, such as in econometrics. While this method can forecast the economy the next few quarters if everything stays the same, it fails if great changes occur. Another major concern is that social science models assume a perfect world with actors who are perfectly informed, has infinite computing capacity, and is maximizing a fixed exogenous utility function. Such models rule out all consequences of heterogeneity. Also models such as game theory or general equilibrium theory are preoccupied with static equilibria and ignore time dynamics. Moving on to pure graph theory, it is principally concerned with the combinatorial properties of artificial constructs instead of the structure of naturally-occurring complex
networks. In these networks interactions between agents (such as individuals or institutions) are non-linear. While each agent is influenced by the whole system, the system is itself influenced by the agents. Since causes and effects are not proportional to each other, behaviors become rather difficult to control or predict. Typically, unexpected and unwanted side effects emerge after new regulations or subsidies have been introduced to the system [5, 6, 7]. Furthermore, interactions have the potential to spread failure throughout the entire system, transforming local risks into a large-scale disaster. This phenomenon is also referred to as systemic risk, whose impact is not confined to a single country or economic sector but frequently escalates to a global scale [8]. It can be observed during natural hazards [5, 9] such as tsunamis, earthquakes or hurricanes. We experience it during failures of infrastructure systems such as the transportation, communication and power systems or gas and water supply [10, 11, 12, 13, 14]. In economic systems the cascading effects can be found during financial crises where illiquidity of a single bank can trigger failure of other banks hence posing a threat to the entire banking system [15, 16, 17, 18, 19]. Shortfalls cascade through supply chains, where firms between various production levels are connected to each other as suppliers and customers are also observed in real life [20, 21, 22]. Similar cascades can be seen in social networks when sick people spread diseases through interactions with their neighbors to create epidemics [14].

All these crises create enormous losses to affected communities and to our whole society. As in 2011, the Centre for Research on the Epidemiology of Disasters have recorded 332 natural disasters, where 30,773 people were killed and 244.7 million people became victims. For that year, the overall economic loss was about US$ 366 billion [23, 24]. According to the UN Office for Disaster Risk Reduction, in the past ten years from 2000 to 2012, natural and biological disasters caused US$ 1.7 trillion damage, killed 1.2 million people and affected another 2.9 billion people [25]. In addition to the damage caused by natural hazards, the financial crisis of 2007 has created output losses of over US$ 10 trillion in the U.S. itself within only two years [26]. Also according to further estimations by the Congressional Budget Office the losses in GDP could reach US$ 5.7 trillion by 2018 [27]. In fact, it has been shown that in general after severe financial crises, unemployment rate increases by an average of 7%, output falls by an average of 9%, and government debt rises by an average of 86% [28].

These statistics are sobering and urge us to better understand the systems we live in, identifying the sources and drivers of systemic risks, and finding measures to mitigate crises. We are not entirely powerless but capable of controlling our vulnerability to a certain extent and use self-organising behavior of certain systems to attain favourable performances and stability [5, 9]. And although a lot of effort has already been put into understanding systemic risks in financial systems, infrastructure systems and other socio-economic systems, problems are
ever-changing and we need to constantly explore new solutions. With this thesis I hope to make a contribution toward this goal.

In this thesis I will present two agent-based models. The first model simulates a dynamical supply chain network. In this project, my co-authors and I investigate whether diversification within a supply chain can help middle-men firms survive prolonged financial crises. From the simulations we extract the lifetime distributions of undiversified firms, and of firms adopting forward vertical integration, backward vertical integration and horizontal merger. We then compare the average lifetimes and the rates at which the midsections and tails of the cumulative lifetime distributions decay for these four types of firms.

In the second project we investigate the interbank market of Singapore. To do that we first develop a numerical technique that allows us to reconstruct the underlying interbank network based on the day-to-day interbank borrowings and lendings. Our algorithms applied on interbank lending data collected from artificial networks show remarkable agreement between the reconstructed and the original network. In our simulations we allow banks to invest into two types of assets: (1) risk-free and (2) risky. In addition we include leverage which means that both assets can be sold short as well. Counterparty risks are introduced through interbank loans. Based on this model we investigate the systemic risk introduced by leverage and interbank loans. In market situations when banks do highly leveraged investments we observe high rates of failing banks. To understand the effectiveness of regulations on bank liquidity and leverage introduced by Basel III, we compare the lifetime of banks for three different regulation scenarios: (1) unregulated leverage (2) penalized leverage and (3) forbidden leverage. We find that the most strict regulation where no leverage is allowed results in the most stable financial system with a maximum in bank lifetimes. In addition we investigate the consequences of incomplete or wrong market information. To do that we let banks speculate based on slightly wrong real investment probabilities. We identify that the over- and underestimation of probabilities for successful risky investments has similar severe consequences with a sharp drop in lifetimes. Furthermore we found that the number of defaults increases when the number of interbank linkages is reduced i.e. the access to interbank loans decreases. Support for the importance of interbank linkages is found from studies on larger networks. By increasing the number of banks in the network interbank lending increase resulting in increased bank lifetimes.

The thesis is organized as follows. In chapter 2, I will provide an overview of existing literature on supply chain models and interbank networks. Both literature reviews can be found in their respective subsections of chapter 2. The description and results of both projects can be found separately in chapters 3 and 4. The conclusion on both topics can be found in chapter 5.
Chapter 2

Literature

2.1 Supply chain

With growing globalization and companies expanding into international locations, effective management of supply chain is essential to deal with ever-changing challenges. In fact, Supply-Chain Management or SCM is viewed as the key business improvement area for minimizing inventory and logistic costs [29]. Designing supply chains means facing decision problems such as the optimal number of facilities, their locations as well as capacity of each facility in a given location. Making these decisions becomes even more difficult as a supply chain expands internationally. Furthermore, managers used to design supply chains that only include corporate, but are including more and more supplier facilities into their supply chain as outsourcing has grown as a usual practice. As a result, supply chain design decision problems are accompanied by a supplier selection decision problem nowadays. In this section we will review literature on supply chain research and classify it into three categories. The review on the first category is based on research that has been done to help find solutions to supply chain design problems in light of the emerging issues of facility selection, supplier selection and others [30, 31, 32, 33]. We will then continue reviewing methods used and SCM tools designed across supply chain studies. While a lot of research is solely based on finding an optimal strategy upon a given supply chain, we will also review studies that try to understand the complex network itself by investigating inherent risk and instabilities.

Beside the design of corporate supply chains, there are also studies investigating optimal designs of non-corporate supply chains comprising individual companies that are intermediates to each other. For example, Tzafestas and Kapsiotis [34] used a supply chain with one manufacturer and several suppliers and sub-suppliers to test optimization scenarios. In their paper they tried to find the most effective strategy to reduce the costs of holding and ordering in a
supply chain. Using a similar non-corporate design Towill, Naim and Wikner [35] studied methods to reduce demand amplification by comparing different business strategies. The authors concluded that in order to achieve an improvement in supply chain dynamics, firms should consider taking over intermediates. However, the more cost-efficient alternative to this is collaboration between all agents in the supply chain. This transparency decreases buffer inventories and reduces the distortion of demand in the supply chain. Swaminathan, Sadeh and Smith [36] investigated whether sharing supplier information can reduce costs in a supply chain. In fact, they found that costs caused by left-over stock and short-falls between manufacturer and cheaper supplier can be lowered by it. While this is true when information sharing is gratis, the manufacturer is better off with no sharing if costs are introduced into information sharing and if they are especially high. A well studied topic in supply chain design is the bull-whip effect. Given that distribution of products in a supply chain is forecast-driven, this effect arises if inventory of firms is growing with the length of the supply chain. As a consequence, production processes become inefficient and participants experience excessive stock. Hence, minimizing the bull-whip effect is, amongst others, an important goal for SCM. Ahn and Lee [21] proposed a dynamic supply chain model, where agents share information via collaborations. Information are used to estimate market situations and to provide efficient production planning, which in turn theoretically minimizes the bull-whip effect. To simulate restriction through distance between firms, information can only be shared between directly connected agents. Before information sharing takes place, agents establish the underlying supply chain network by choosing their efficient suppliers. By providing information sharing the authors showed that the model is able to minimize the bull-whip effect and increase the overall performance of the supply chain. Nagatani and Helbing [22] investigated different management strategies by using the linear stability condition to address the bull-whip effect in their work. If firms observe and forecast their present and future inventory closely the production system can be stabilized. A better strategy, however, for a firm is to not only have an eye on its own inventory but also anticipate the inventory level of its supplier and customer. Their results agree with [21] showing that if firms have enough information they can successfully hedge against the bull-whip effect. To address the causes of fluctuations in inventories, several models are also used that were original developed for transportation dynamics [45, 46]. The authors mapped transportation problems onto the material flow in supply chain networks. The result of these model suggests that business cycles has their roots in the instability of supply chains and that the bull-whip effect arises due to the resonance effect. In [46] the network exhibits growing oscillations, even though no loops are present.

Apart from investigating optimal supply chain designs in order to reduce cost
or maximize profit is, another important challenge in the literature is to develop decision support tools for managers. As an example, Swaminathan, Smith and Sadeh [37] presented a framework for the development of a decision support tool for supply chains, which can be customized and reused for various alternatives. This is achieved by incorporating three distinct supply chain network types. Based on them the authors were able to identity processes that were common to all three networks and developed an ‘universal’ model. Julka et al. [38, 39] also developed an agent-based model that can adapt to different firms as well as industries. The model not only can simulate an overall supply chain, but it is also able to adapt to and simulate different levels of the supply chain. Beside the main goal of maximizing the profit or equivalently minimizing the costs, research on supply chain has also been addressing the adaptation of supply capacity to future demand. Akanle and Zhang [40] introduced a model that is able to simulate both static and dynamic supply chains with the aim to provide an optimal strategy to deal with future demands. The authors tested a simplified version of this approach on test data and found they could find an optimal network structure that satisfies orders optimally. By combining all optimal network structures for all order, the model was able to find a compromised network structure to deal with all orders. Going even a step further, Anosike and Zhang [41] inferred the optimal usage of an existing supply chain structure in addition to searching for the optimal supply chain design. The developed model is able to suggest new network configurations when demand pattern changes which can no longer be properly satisfied by the old network structure. To address the challenges of supply chain management, a decent amount of research has been done on the analytical front. For example, Ambrosino and Scutella [47] addressed facility location, warehousing, transportation and inventory decisions using integer linear programming. Wang et al. [48] used integer linear programming with non-negative constraints to investigate the performance of a non-linear inventory system, i.e. no return of excess inventory. However, using either stochastic processes, discrete events, linear and non-linear programming, game theory or empirical studies [49, 50, 51, 52] are rather restricting when dealing with dynamics of supply chains. Hence, simulations of supply chain gained more attention instead as they provide mechanisms to understand these dynamics. In particular, the use of agent-based models or ABM have become popular in recent years due to its ability to give insight into the formation of collective behaviour from interactions in a complex system such as a supply chain network.

Apart from searching for the optimal supply chain design and finding methods to adapt to dynamic uncertainties, another application of supply chain modelling is to understand the instabilities that are inherent in this complex network. Bak et al. [42] tried to understand how individual shocks, mostly small in size and constrained to certain sectors only, can influence the overall economic situation.
The model is a multi-sector and multi-layer supply chain, where a large group of producers is selling goods to a small group of customers and buying goods from a small group of suppliers. Shocks into the system are incorporated as fluctuations in demand of the final product. Since products are assumed to be independent to each other, their fluctuations are as well. If the distribution of fluctuations are similar and the number of final products is large, then aggregate production of final product producers stops being variable. Furthermore, the fluctuations triggered stop being independent if fluctuations occur on the level above the final product. In this case the number of affected producers is small if the number of final products is large. It is shown that aggregate production is less variable when the number of final products increases. Also using a multi-layer network comprising of firms and banks interacting with each other, Gatti et al. [43] investigated business fluctuations. It is assumed that firms are financially fragile. The authors found that fluctuations are an outcome of the changing network structure of firms. Furthermore the model is able to produce the following universal laws: The distribution of firm sizes follows a power law and is skewed. Also the change rate of aggregate outputs and as well as the output of firms have a similar Laplace distribution. The authors also showed that under certain assumptions the distribution of the firm sizes is linked to the distribution of growth rate. In fact, the former is the source of the latter. Hence, when bankruptcy occur in the system it triggers an avalanche of bankruptcies through the entire system. Wagner et al. [44] were interested in positive default dependencies between suppliers, where the default of one may be harmful for the other one. By measuring the default dependencies and correlation of automotive suppliers using empirical data on automotive supply chain, the authors concluded that the suppliers are connected to each other in a way where the default of one has significant consequences for others.

For our project we address the emerging issue of exploring optimal supply chain designs by investigating different management strategies. Using a supply chain network, where firms on the same tier are in the same industry and the same stage of production and thus are competitors to each other, we let firms face dynamic changes in the structure of the supply chain network as well as include internal shocks in the form of production shortfalls. As goods are being sold to and bought from each other, production shortfalls interrupt the delivery of production. Through the cascading effect these shocks can influence the entire economic situation of the network. In the presence of ever-changing network structure and internal shocks we test the efficacy of management strategies. As existing analytical methods are too restricted when dealing with dynamic changes in a complex supply chain network, we design an agent-based model to deal with its complex interactions. In fact, the use of ABM has become increasingly popular in supply chain research. However, we observe shortcomings in the literature
regarding the interpretation of an optimal functioning supply chain design, which is in general restricted to maximizing profit or minimizing costs only. To close this gap and to offer a balanced view of supply chain design we measure performance by the ability in lifetime proliferation. To ground our study on realistic stimulations, we use the dynamic supply chain model proposed by Mizgier, Wagner and Holyst [20] as our basis model. An important feature in this model is price dispersion, which assigns different prices to the same goods. This serves as an incentive for manufacturers in the model to always search for cheaper suppliers. As manufacturers change suppliers the supply chain also changes, making the network structure dynamic. Another important feature in this model is production shortfall. In practice production quota cannot always be met due to various reasons. In their model, production shortfall appears randomly in any manufacturer and cascades through the supply chain creating losses to connected customers. Lastly, the authors also incorporated technological innovations and changes in the labour market. To simulate this agents in the supply chain experience a change in their production costs after a certain period of time. Incorporating all these features, at equilibrium the model is able to reproduce a rather realistic network structure with only a few but large raw material suppliers and retailers at both the bottom and top layer, however many small and medium sized manufacturers on the middle layers of the supply chain.

2.2 Interbank market

Understanding the features of financial crises and finding efficient government provisions have always been a hotly debated issue in our financial history. One prominent phenomenon that has been thoroughly studied is bank runs. During a bank run, depositors panic because they expect the bank will fail, and proceed to withdraw all their deposits. This high amount of withdrawal of funds within a short time can lead to collapse of the bank and the financial system. In fact, many observers and studies view the financial crisis of 2007 to be reminiscent of a bank run [54]. However, traditional bank runs in the 19th or 20th centuries (and still occurring in emerging markets [55]) are different from the current one. During historical bank panics depositors ran on their banks to exchange their checking accounts for cash, while during the current panic financial institutions were running each other by letting contracts expire or increasing contract margin [56].

The first model on bank runs was developed by Diamond & Dybvig [57]. They showed in their model that banks offering demand-deposit contracts has two Nash equilibria. In the first equilibrium demand-deposit achieves optimal risk sharing. The second equilibrium results in a bank run, which causes even
‘healthy’ banks to collapse. The authors argue that the change from the first ‘good’ equilibrium to the second ‘bad’ equilibrium occurs due to some external shock, such as the change in depositors’ expectation, representing the irrational behaviour of humans. Investigating traditional measures to prevent bank runs, where a bank temporarily suspends the withdrawal of deposits, they found that it not only prevents bank run but also provides optimal risk sharing. Furthermore, when government deposit insurance is available bank runs can be successfully eliminated. Since [57], a lot of studies has been done based on or related to it. In another model featuring the demand-deposit contract, Postlewaite & Vives [58] show that there exists an unique equilibrium of bank runs without exogenous events affecting the behaviour of depositors. Unlike in Diamond & Dybvig [57], a bank run is the outcome of a Prisoner’s Dilemma game, where depositors withdraw money strategically because of self-interest instead of random irrational behaviour. The result is that all equilibria have a positive probability of bank runs. Hence, there are no generally optimal standard deposit contracts in this model. In order to put in place contracts superior to the standard deposit contracts, the withdrawal behaviour of people must be more closely monitored, and the amount of cash withdrawn in the present and future must be predicted. This results in much higher costs than in a standard contract. Naturally, the feasibility and optimality of these more elaborate deposit contracts is limited by the fact that saving and consumptions strategies and personal preferences among depositors are private information. Unlike in [57], where banks provide deposit rate higher than the primary yield in both short and long terms, banks in [59] offer primary security rates that are equal to primary security yield. Waldo tries to explain the increase in short-term interest rates and the decrease in deposit-currency ratio during bank runs. It has been observed that the increase of short-term interest rates are rather sharp during runs, while the deposit-ratio falls before runs. There are also U.S. data showing that after establishing the Federal Reserve System, the increase of short-term interest rates was less sharp than before despite more severe runs. The model has multiple equilibria, and bank runs are possible in all but one of these. The author explains the rise in the short-term rate during a bank run as a result of banks selling off their long-term securities earlier to meet the increasing withdrawal. The decrease in deposit-currency ratio can be explained by depositors withdrawing their money to protect themselves against possible runs.

While some studies such as those described above are based on expectation-based panic runs, others assume information-based runs. Jacklin & Bhattachary [60] showed that with depositors who are less risk-averse in the model, pure panic runs do not occur as no one is worried about the eventual liquidity shortage of the bank. However, information-based runs do happen, but do not pose any threat to the bank or the banking system. Interestingly, the combination of depositors
who are more risk-averse and dire news of banks do not necessarily lead to bank runs as people tend to reverse hedge. Overall, the authors’ findings are that information-based runs will always exist regardless of the risk aversion of depositors. However, bad prospects for a few banks will not dry out funds in the whole system as depositors will simply deposit their withdrawn money into another bank. Chari & Jagannathan [61] have found that information-based bank runs are an equilibrium phenomena. While ‘bad’ information trigger bank runs, runs also occur when no information are present. The social costs of such information-based runs can be limited through suspending the conversion of illiquid deposits.

Important as they are, bank runs (or their institutional equivalents) are not the causes of the 2007 financial crisis. In fact, Mishkin [62] showed that during pre-war financial crises panics almost always happen after the start of a recession. Given that stock prices declined and financial institution failed prior to a panic, the author concludes that bank panics were not the trigger of most of the financial crises. Instead, most of them started with a stock market decline, increase in interest rate and interest rate spread. Also, there are evidence that panics are preceded by the failure of financial institutions. This failure causes a deterioration of trust and hence leads to withdrawal of deposits. This in turn causes further severe economic failure. The panic would abate after solvent financial institutions have been separated from the insolvent ones. At this stage the economy undergoes a recovery. Based on these observations, Gorton [63] argues that panics are predictable, and stock price as well as interest rate spread may be useful in predicting them.

Aside from studies that investigate how failure arises in the first place, a lot of research has also been done on analysing the propagation of failure. A very well studied topic is overnight interbank lending, which allows banks with liquidity shortage to borrow from banks with liquidity surplus. On the one hand, interbank lending is crucial for providing efficient capital redistribution within the interbank market, but on the other hand it also acts as a medium for failure spreading across the banking network. The work by Allen and Gale [65] investigates the channel of contagion, when banking systems from different regions have claims on each other. In their model agents have complete information and small shocks in one region spread into other regions infecting the whole system even-
tually. The authors observe that the degree of completeness and connectedness of connections between banks from different regions — the market structure — tells a lot about the degree of contagion. If the market structure is complete, then by increasing the number of regions contagion can be eventually eliminated. Hence, with increasing size of the economy and complete connections between banks the banking system becomes more resilient against shocks. However, if the market structure is incomplete, then initial shocks in one region becomes uncontrollably large through spreading, which is independent of the size of the economy. Dasgupta [66] supports these findings by investigating financial contagion through cross holding of deposits by banks. The results show that in general, higher interconnectedness leads to stable banking systems, where contagion is rare. However, when stable systems experience contagion, the outcomes are more severe than in unstable ones. On the other hand, Gai and Kapadia [67] show that interbank lending also act as a medium for failure spreading. In their model banks are connected through interbank markets and payment systems and it allows two channels of contagion: the spread of losses through the network after an initial unexpected default and also the knock-on effect of financial distress. The authors find that financial systems are robust-yet-fragile networks, which means the probability of contagion may be low, but when contagion does arise the impact is severe. Furthermore, their results show that contagion from idiosyncratic shocks never occur. However, aggregate shocks make the financial system susceptible to contagion risk. In particular, a system with structural weakness will be less resilient against shocks at this particular pressure point, albeit it may be generally robust to shocks. When introducing a decrease in asset price after a bank default, the results show that the decrease potentially makes more banks vulnerable to shocks or even causes them to fail. The authors conclude that liquidity risk amplifies the scope of contagion when it exists. However, the authors also point out that depressed asset prices not only occur after the default of a bank but more likely before defaults in practice. Hence, liquidity risk also has the potential to increase the probability of initial default in a financial system.

But financial institutions are not only exposed to counterparty risks but also experience common shocks. In fact, there are a lot of studies [68, 69, 70, 71] that investigate which one of both exposures has the largest potential to cause financial instability. Iori et al. [68] simulated a financial system, where banks do risk-free investments and experience deposit fluctuations. The authors considered two different interbank market scenarios, where in one banks are homogeneous and in the other banks are heterogeneous. In the homogeneous case banks all have the same size or equivalently the same starting capital. In the heterogeneous case the sizes of banks are different. The results show that in both cases, increasing interbank connections translate into more stability of the financial system and more banks survive. If bigger banks experience lower fluctuations than smaller
banks then increasing heterogeneity makes the network more stable. However, the network becomes unstable when all banks experience the same withdrawal fluctuations. In further simulations, the authors considered the effect of unequal investment opportunities. They observed that those with higher opportunities have more excess liquidity. In a network without interbank connections, banks with higher opportunities never fail. However, with interbank connections they become victims of counterparty risks. To be precise, with increasing connections the number of defaulting banks that would otherwise be liquid increased. Still, the total number of defaults with interbank lending is smaller than without interbank lending. In the random interbank network model by Nier et al. [69] each bank holds external assets and internal assets. External assets are borrowings that originate outside the interbank market, whereas internal assets are borrowings by other banks. To simulate a shock, the model cuts down a certain percentage of external assets of a single bank. The results show that as net worth increases the number of default banks decreases. But the decrease of contagion is not a linear function of the increase in percentage of net worth. The level of contagion remains stable until net worth surpasses a certain percentage, where the level of contagion starts to drop. If the percentage of interbank assets changes, the results show two opposing effects on the stability of the network. Very low percentage of interbank assets leads to no contagion. However, when the percentage surpasses a certain level, contagion starts to increase until it reaches a stable value. While interbank connections act as channels for the transmission of shocks, they also redistribute risks and allows the absorption of default through multiple banks. The higher the percentage of net worth and the higher the interbank connectivity the lower is the level of contagion. Changing the size of the interbank network, while keeping the number of total assets constant shows that a higher number of banks provide higher resilience of the network. Given fixed total assets, a lower number of banks increases the impact of each bank. By including a decrease in asset pricing as banks default, number of defaults increases. Even with high net worth, liquidity tend to lead to total default of the banking system. Co-Pierre [71] extends the model by [68] by incorporating a central bank into the interbank network. Furthermore, banks not only invest into risk-free but also risky assets. As a result, banks not only experience liquidity shocks through deposit withdrawals but also through investment risks. The author compares the impact of two different shocks to the system. The first shock is letting the largest bank fail. The second shock is a given percentage default on all banks’ assets. It is shown that the second shock makes more banks become vulnerable to deposit fluctuations and defaults of other banks, which makes the overall system vulnerable and increases the number of defaulting banks. However, the first shock has more impact on the interbank market liquidity than the second shock.
While some of above discussed literature investigated artificial interbank networks, others study networks reconstructed from real-world interbank lending data. The spread of counterparty risk in the former in general is investigated by simulating the interacting behaviours in an agent-based model, while in the latter the consequences usually are measured when a random bank is removed from an underlying static network. However, not much work has been done to compare the impact of counterparty risk and common shock [71]. Our plan was thus to introduce an agent-based interbank model recreated using real-world Singapore interbank lending data, where both counterparty risk and common shock are considered. While we let counterparty risk propagate through interbank lending, at the same time we expose banks to deposit shocks. Moreover, we investigate the effect of leverage in our financial model. It has been observed that prior to financial crises leverage increased significantly [81, 82, 83] and it also has been identified as one of the possible causes of the 2007 financial crisis. De La Motte et al. [6] and Wallison [7] argue that subsidies and regulations are implicated in every financial crisis. These distort domestic supply or demand in the long run, which then lead to mispricing of assets and misevaluation of risks. The underestimation of risks leads to higher ratings for risky assets, which then encourage investors to go in with a higher leverage [81]. To address the increasing leverage, Basel III requires higher bank capital, and more strict regulations on bank liquidity and leverage. The goal of our project is to understand the impact of leverage on an interbank network and how failure could arise in the first place. Additionally, we believe that more attention should be given to analyse the effectiveness of interventions and regulations. Therefore, we also examine the effects Basel III, or rather a simplified implementation of it, will have on the stability of a network of banks. The aim of this study is to design network-based regulations and interventions that can effectively prevent liquidity shocks and arrest cascading failures.
Chapter 3

Supply chain network

In the global financial crisis of 2007, many big firms with long histories went bankrupt. General Motors and Chevrolet, went the way of dinosaurs from the ensuing slowdown. While news of these giants going under occupied our consciousness, the demise of many more smaller firms went largely unreported.

According to the American Bankruptcy Institute, the number of business bankruptcy filings in the U.S. in 2009 reached 16,014 new bankruptcy filings in the second quarter, the highest since 1994 [72]. In total, bankruptcy filings increased from 43,546 in 2008 to 60,837 in 2009. Along with the increased rate of bankruptcies, the U.S. Bureau of Labor Statistics reported that the annual unemployment rate greatly increased from 5.9% in 2008 to 9.3% in 2009 [73].

This loss of employment due to firms folding adds stress to society. If the number of bankruptcies can be minimized, either through government support during crisis, or through better management practices, it might be possible to reduce the number of job losses. Surely, this would be a desirable economic and social outcome.

The work by Mizgier, Wagner and Holyst [20] adapted the dynamical supply chain model of Weisbuch and Battiston [74] to introduce more realistic features. At equilibrium, the model has few large bottom level suppliers and few large top level customers, but many small and medium sized firms in the middle of the supply chain. This proliferation of middlemen has been observed in many supply chains, in particular by Popp [75] in the apparels industry. Through case studies, Popp concluded that intermediation by middlemen lower information costs arising from distance and volatility. Mentzer et al. [76] found that strategic alliances involving middlemen is advantageous, because of the creation of customer value. From an ecological perspective, middlemen firms evolve naturally during good
times in a sophisticated economy, where they fill the niche of specialized firms producing a small number of products highly efficiently. However, because of their specialization, these middlemen firms are also most susceptible to sudden changes in the economic climate and conditions.

In this project, we extend the model studied by Mizgier, Wagner and Holyst [20], referred as MWH, by allowing middlemen firms to diversify, either by (i) buying over one of its suppliers, or (ii) by buying over one of its customers or (iii) by merger. The aim is to explore whether such management practices can enhance a middlemen firm's chances of surviving an economic downturn.

3.1 The supply chain model

This section gives a detailed description of the model that is used for the supply chain project. Since, it is an extension of the agent-based model of [20], a detailed explanation on how the two models differ is given here.

3.1.1 The supply chain network

There are five stages in the network from stage 0, the consumers, to stage 4, the raw material suppliers. Within this network 50 firms are assumed in each stage, so that there are a total of $N = 250$ firms in the whole supply chain.

Firms of adjacent stages are connected by edges, which represent input and output flows. Input flow is defined as the flow of orders, whereas output flows stands for the flow of production. If a firm in stage $s$ and a firm in stage $s + 1$ are connected through an edge, it means that the former places orders to, and receives goods from the latter. In this case, the firm in stage $s$ is the customer of the firm in stage $s + 1$. Consequently, the firm in stage $s + 1$ is the supplier to the firm in stage $s$. Firms within the same stage are not connected.

3.1.2 The simulation

To recreate realistic features similar to those observed in real world supply chains, this dynamic supply chain model has incorporated dispersion in pricing products, production costs that are passed along the supply chain, production shortfalls, bankruptcies of existing firms and the periodic re-evaluation of suppliers so as to improve cost effectiveness. These features will be discussed in detail in the next paragraphs.
Figure 3.1: Schematic diagram of the MWH supply chain. This supply chain consists of five stages, going from the top-level consumer (s = 0) to raw material suppliers (s = 4). Each stage s contains n = 50 firms, making up N = 250 firms in total for the supply chain. Firms on stage s are linked only to firms on stage s - 1 (customers) and firms on stage s + 1 (suppliers). In our simulations, orders flow from the consumer to the raw material suppliers, whereas products flow from the raw material suppliers to the consumers.

Price dispersion and dynamics: In a real market, we often find different prices assigned to same goods by different sellers. To model this price dispersion, the model assigns random sales prices drawn from a log-normal distribution, \( \ln N(\mu, \sigma^2) \), to firms at the beginning of each simulation, as proposed by [20]:

\[
 f(x, \mu, \sigma) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}},
\]

where \( x > 0 \), \( \mu \) is the expected value, and \( \sigma \) the standard deviation of \( \ln x \). We then further include mark-ups of 0.5, 0.4, 0.3, 0.2, 0.1 to the unit sales prices for \( s = 0, 1, 2, 3, 4 \). Here, the mark-up — a function of stages — reflects the fact that
raw materials have a lower retail price than produced goods. In the real world
the shape of the mark-up function can be convex, linear or concave depending
on the industry. Since we are not simulating the supply chain of any industry
specifically, we chose to use a linear mark-up function.

To allow changes in unit sales price at every time step, \( p_{s,i}(t) \) of firm \( i \) on stage
\( s \) at time \( t \) is given by

\[
p_{s,i}(t) = c_{s,i}(t) + \lambda + \frac{l}{2} P,
\]

(3.2)

where \( c_{s,i}(t) \) is the unit cost of production, \( \lambda \) is the capital interest rate, \( l \) is the
number of stages in the supply chain and \( P \) is the failure probability. This price
dynamics, proposed by [77], ensures that firms not only cover unit production
costs but also have a marginal reserve to cope with production shortfalls.

In the model this dynamics is simplified in a way, such that \( p_{s,i}(t) \) remains con­
stant for firms \( i \) on stage \( s \) unless their profit in 3.7 becomes negative. In this
case they will raise their unit sales prices \( p_{s,i}(t) \) according to equation 3.2.

**Network configuration:** Given the difference in prices caused by price dis­
ersion, firms are constantly searching for the cheapest suppliers. To assign con­
nections between firms in the supply chain model the following linking algorithm
is used at each time step:

1. Starting from stage 0, randomly choose one firm \( A \) on stage \( s \) and then
randomly choose three firms of the stage \( s + 1 \) above it.

2. Connect the cheapest firm \( B \) of the three in \( s + 1 \) with \( A \) in \( s \) only if the
sales price of \( B \) is lower than the sales price \( A \) adopts for its products, such
that there is potential to profit from the difference.

3. Repeat until linkages are established in the last stages.

The network starts out without any edges and uses above linking algorithm to
connect the vertices.

Beside searching for cheapest suppliers in order to maximize the profit, firms
also try to minimize risks caused by shortfalls by maintaining multiple suppliers.
While there is an upper limit on the number of suppliers each firm can have, it is
assumed that once firms find cheaper suppliers they can exchange their existing
suppliers for these new and cheaper ones, instead of just adding a new supplier
to its list of suppliers.
To keep this feature realistic, it is further assumed that the more suppliers a firm has the more likely it will switch to new ones. Whereas, firms with only a few suppliers are less likely to reconfigure their linkages since they face higher risks of production shortfalls during the reconfiguration in real life. Hence, the probability of reconfiguration is as follows

\[ P_{\text{reconf}} = \tanh(ks_i/a), \]  

(3.3)

where \( a = 5 \) can be interpreted as the typical number of links, and \( k_{s,i} \geq 0 \) is the number of suppliers the firm \( i \) on stage \( s \) has. Mizgier et al. [20] have shown that with \( a = 5 \) and \( k_{s,i} = 12 \) the probability value \( P_{\text{reconf}} \approx 0.95 \).

**Production dynamics:** The firm’s production function is a linear function of its working capital. Furthermore, orders of consumers are limited by the production capacity. It is assumed that a consumer \( i \) on stage 0 places its order \( Y_{0,i}(t) \), according to

\[ Y_{0,i}(t) = mA_{0,i}(t), \]  

(3.4)

where \( m \) is the technological proportionality coefficient and \( A_{0,i} \) is the working capital of consumer \( i \). Orders of \( i \) are then evenly distributed between all its suppliers on stage 1.

Hence, the sum of orders received by a firm \( i \) at time \( t \) in stages \( s \geq 1 \) is

\[ Y_{s,i}(t) = \sum_{i' \in v} Y_{s-1,i'}^{r}(t), \]  

(3.5)

where \( v \) stands for the set of customers of \( i \). After receiving \( Y_{s,i}^{r}(t) \), firm \( i \) then places its own order. It is \( \min\{Y_{s,i}(t), Y_{s,i}^{r}(t)\} \), i.e. the minimum between its productions capacity given by equation 3.4 and \( Y_{s,i}^{r}(t) \).

As soon as the orders reach the raw material suppliers at the last stage of the supply chain, the production process starts.

The equation for delivered production is

\[ Y_{s,i}(t) = \epsilon(t) \sum_{i' \in v'} Y_{s+1,i'}^{d}(t) \frac{Y_{s,i}(t)/k_{s,i}}{Y_{s+1,i'}^{r}(t)}, \]  

(3.6)

where \( v' \) stands for the set of suppliers of \( i \). It simulates shortages in a firm’s production by including a stochastic coefficient, \( \epsilon(t) \). \( \epsilon(t) \) is either 0 or 1, where we have no deliveries at all if \( \epsilon(t) = 0 \). In case of a production shortfall, it will cascade through the supply chain.
Hence, 3.6 states that the delivered production of a firm $i$ on stage $s$ depends on the products delivered by its suppliers on stage $s + 1$. When there is a production shortfall, and $i$ cannot complete the orders of all its customers, the amount delivered to each customer will be proportional to its order.

Beside the production process, the working capital of each firm has to be updated to reflect the profits and losses they make. The profit is

$$\Pi_{s,i} = p_{s,i}(t) Y_{s,i}^d - c_{s,i}(t) Y_{s,i}^d - \lambda A_{s,i},$$  \hspace{1cm} (3.7)

where $p_{s,i}(t)$ is the unit sales price at time $t$, $\lambda$ is the constant interest rate for the capital, and $c_{s,i}(t)$ is the unit cost of production at time $t$, which is given by

$$c_{s,i}(t) = \sum_{l' \in u'} \frac{p_{s+1, l'}(t) Y_{s+1, l'}^d(t) / k_{s,i}}{Y_{s+1, l'}^d(t)}. \hspace{1cm} (3.8)$$

**Costs of production:** The original model in [20] assumes that firms are changing their cost of production over time, which is a result of changes in the labor market and technological innovations. This time-dependent total cost of production $\bar{c}_{s,i}(t)$ of firm $i$ on stage $s$ at time $t$ is

$$\bar{c}_{s,i}(t) = c_{s,i}(t) + q(t), \hspace{1cm} (3.9)$$

where $q(t)$ is the stochastic term representing technological innovations and other changes, whose logarithmic change follows a normal distribution, $N(-0.1, 0.1)$.

In this project, we are interested in simulating the response of the supply chain to sudden (and prolonged) financial crisis. As such, we neglect slow technological innovations or changes in the labor market, and set $q(t) = 0$.

**Bankruptcies and recovery:** Firms tap into their working capital to pay taxes and wages. Hence, it is a reference for the state of a firm, i.e., if it is too low, the firm will not be able to pay taxes and wages, and will therefore go bankrupt.

When a firm goes bankrupt we sever all its links, and replace it with a completely new firm a few time steps later. This new firm $A'$ will be initialized with new links to its suppliers and customers. To choose the number of such linkages we first choose another random firm $A'$ on the same level of the supply chain. We then count the number of suppliers, $k_-(A')$, and the number of customers, $k_+(A')$, $A'$ has. The number of suppliers, $k_-(A)$, the new firm $A$ will
have is then \( \min[k_-(A'), 3] \), while the number of customers, \( k_+(A) \), it will have is \( \min[k_+(A'), 3] \). This ensures that a new player does not immediately become more successful than the established firms.

**Diversification and merger:** During financial crises, we allow firms on the middle level of our supply chain to diversify and merge.

There are two diversification strategies which are implemented as follows.

- **Backward vertical integration:** Randomly choose one firm \( A \) on stage \( s = 2 \) and let it buy over one of its linked suppliers \( B \) on stage \( s = 3 \). After integration \( A \) still places its orders in the usual way, whereas \( B \) now has an obligation to fulfill.

  In case \( B \) experiences a production shortfall, and cannot meet the demands of all its customers, priority is given to \( A \). That means when \( B \) is not able to produce enough to meet overall demand of its customers including \( A \), then \( B \) must first complete \( A \)'s orders. Any remaining products are then proportionally delivered to the rest of the customers, based on the size of their orders.

- **Forward vertical integration:** Randomly choose one firm \( A \) on stage \( s = 2 \) and let it buy over one of its linked customers \( C \) on stage \( s = 1 \).

  After integration, \( A \) delivers its products in the same way as before, without giving priority to \( C \), whereas \( C \) cancels its links to all other suppliers. That is to say, after integration the obligation of \( C \) is to place orders only with \( A \).

Besides comparing forward and backward vertical integration we also observe the outcome of the horizontal merger of firms. The algorithm for merging two firms is as follows.

- **Horizontal merger:** Randomly choose two firms on stage \( s = 2 \) to merge. Combine their working capital and their linkages to both stages \( s + 1 \) and \( s - 1 \).

**Financial crisis:** During ‘normal times’ customers on stage 0 initiate the order flow by placing orders according to equation 3.4. During financial crises, it is assumed that the order flow is cut by half. This is done by first determining \( Y_{0,i}(t) \) from 3.4 and then halve it.
3.1.3 Parameter setting

The model is calibrated with parameter settings described in Mizgier et al. [20]. As their model is again an adoption from the model by Weisbuch and Battiston [77], respective parameter values are taken from the latter. Unless otherwise stated in previous sections, the following paragraphs give the parameter settings for the simulation.

**Price dispersion and dynamics:** Random unit sales prices are drawn from a log-normal distribution \( \ln N(1, 0.0001) \). We then add 0.5, 0.4, 0.3, 0.2, 0.1 to the unit sales prices for the stages \( s = 0, 1, 2, 3, 4 \). This serves the purpose to assign cheaper raw materials but expensive produced goods.

As for the price dynamic, we set the capital interest rate to \( \lambda = 0.002 \), the number of stages in the supply chain to \( l = 5 \) and the failure probability to \( P = 0.05 \).

**Production dynamics:** During ‘normal times’ we set \( m = 1 \) for 3.4 while change this coefficient to \( m = 0.5 \) when the financial crisis starts. The initial working capital of each firm is sampled from a uniform distribution \( U(1.0, 1.1) \).

For 3.6 \( \varepsilon(t) \) is 0 with probability \( P = 0.05 \), while \( \varepsilon(t) = 1 \) with probability \( 1 - P \). For 3.7 we set the constant interest rate for the capital \( \lambda = 0.002 \).

3.2 Results

We are interested in investigating management strategies, such as forward integration, backward integration and horizontal merger and explore whether they can enhance a middlemen firm’s chances of surviving an economic downturn. To do this, we simulate a supply chain network, to obtain the distributions of lifetimes of such firms. Finally, we compare these lifetimes against those of undiversified firms, and also of firms that choose to manage system risk through mergers.

In particular, we simulated the supply chain network under normal economic conditions and during a financial crisis. When simulating the crisis we allowed firms in the middle level of the supply chain to merge, and to forward or backward integrate vertically. In each simulation we record the lifetimes of undiversified and diversified firms. Combining the records from a large number of simulations, we determine the cumulative distribution of lifetimes, where \( F_s(n) \) denote the number of firms on stage \( s = 0, 1, 2, 3, 4 \) with lifetimes at least \( n \) time steps long.
Figure 3.2 shows the typical behaviour of cumulative lifetime distribution of firms at various stages $s$ of the supply chain. It decays exponentially for a while before accelerating to a Gaussian-like decay. To facilitate quantitative comparisons, we fit the midsection of the cumulative lifetime distributions to exponential decays of the form

$$F_s(n) = \alpha \exp(-\lambda n),$$

and the tails of the cumulative lifetime distributions to Gaussians. We measure the cumulative lifetime distribution instead of the lifetime distribution $f_s(n)$ (denoting the number of firms on stage $s$ with lifetime equal to $n$ time steps) because the former is less noisy.

<table>
<thead>
<tr>
<th>Supply chain stage</th>
<th>Normal times</th>
<th>Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 0$</td>
<td>$\lambda_0 = (7.916 \pm 0.008) \times 10^{-4}$</td>
<td>$\lambda_0 = (8.736 \pm 0.004) \times 10^{-4}$</td>
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<tr>
<td></td>
<td>$n_0 = 590.7 \pm 2.6$</td>
<td>$n_0 = 701.9 \pm 2.9$</td>
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<tr>
<td>$s = 1$</td>
<td>$\lambda_1 = (9.783 \pm 0.008) \times 10^{-4}$</td>
<td>$\lambda_1 = (7.704 \pm 0.008) \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_1 = 2553 \pm 5$</td>
<td>$\sigma_1 = 2118 \pm 5$</td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_1 = 624.7 \pm 2.6$</td>
<td>$\bar{n}_1 = 714.8 \pm 3.2$</td>
</tr>
<tr>
<td>$s = 2$</td>
<td>$\lambda_2 = (1.02 \pm 0.09) \times 10^{-3}$</td>
<td>$\lambda_2 = (9.003 \pm 0.004) \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_2 = 1613 \pm 5$</td>
<td>$\sigma_2 = 1550 \pm 3$</td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_2 = 563.8 \pm 1.9$</td>
<td>$\bar{n}_2 = 595.5 \pm 2.1$</td>
</tr>
<tr>
<td>$s = 3$</td>
<td>$\lambda_3 = (9.892 \pm 0.011) \times 10^{-4}$</td>
<td>$\lambda_3 = (9.177 \pm 0.009) \times 10^{-4}$</td>
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<tr>
<td></td>
<td>$\sigma_3 = 1509 \pm 8$</td>
<td>$\sigma_3 = 1629 \pm 7$</td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_3 = 470.5 \pm 1.7$</td>
<td>$\bar{n}_3 = 458.1 \pm 3.3$</td>
</tr>
<tr>
<td>$s = 4$</td>
<td>$\lambda_4 = (7.642 \pm 0.009) \times 10^{-4}$</td>
<td>$\lambda_4 = (7.517 \pm 0.015) \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_4 = 1590 \pm 8$</td>
<td>$\sigma_4 = 1707 \pm 8$</td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_4 = 296.1 \pm 0.9$</td>
<td>$\bar{n}_4 = 298.7 \pm 1.0$</td>
</tr>
</tbody>
</table>

Table 3.1: Lifetime decay rates and average lifetimes of undiversified firms during normal and crisis times, where $\lambda$ is the midsection decay rate, $\sigma$ is the tail decay rate, and $\bar{n}$ is the average lifetime. The smaller $\lambda$ or larger $\sigma$ are, the slower the respective decays.

### 3.2.1 Undiversified firms

Figures 3.2, 3.3 and Table 3.1 show lifetime decay rates and average lifetimes of undiversified firms during normal and crisis times at various stages $s$ of the supply chain. Because the cumulative lifetime distribution of undiversified firms on $s = 0$ shows an exponential decay that extends nearly all the way, we do not
Figure 3.2: Cumulative distributions of the lifetimes of undiversified firms on stages (a) $s = 0$, (b) $s = 1$, (c) $s = 2$, (d) $s = 3$, and (e) $s = 4$ of the dynamical supply chain during normal times. These distributions are extracted from 10,000 simulations, running for 10,000 time steps each.

fit its tail to a Gaussian. Except for $s = 0$, the midsections of these cumulative lifetime distributions all decay slower during a financial crisis. For $s = 2, 3, 4$, these decays are only slightly slower, whereas for $s = 1$, this decay is significantly slower. The cumulative lifetime distribution of $s = 0$ decays significantly faster during a financial crisis. For $s = 1, 2$, we find their Gaussian-like tails decay faster during a financial crisis, whereas for $s = 3, 4$, we find their Gaussian-like tails decaying slower during a financial crisis.

We also measured the average lifetimes, which received contributions from both the short-lived and long-lived ends of the lifetime distributions. We found the average lifetimes of firms on stages $s = 0, 1, 2$ to be slightly shorter during normal times than during crisis. In contrast, the average lifetimes for firms on stage $s = 3$ is longer in normal times, while they remain more or less unchanged on stage $s = 4$.

The exponential decay on stage $s = 0$ and Gaussian decays on stages $s = 1$
and $s = 2$ were measured for lifetimes between 4,000 and 10,000 time steps. We believe that the reduced demand during crisis weeds off long-lived firms – firms living more than 4,000 time steps – more effectively than it weeds off those firms living less than 4,000 time steps. The firms weeded off pile up at the section of the cumulative lifetime distribution, where lifetime is between 1,000 and 4,000 time steps, thus flattening the lifetime decay between 1,000 and 4,000 time steps and sharpening the long-lived tail of the distribution at the same time. This new composition of short-lived and long-lived firms slightly increases the average lifetime of firms in the supply chain. On stage $s = 3$, as mentioned before, average lifetime is slightly shorter during crisis while exponential and Gaussian decay of the cumulative lifetime distribution, however, is slower. We believe this is due to the fact that the Gaussian fit did not properly account for the elongated tail of the cumulative lifetime distribution. As Figure 3.2 shows in (d) the tail of the cumulative distribution has only been fitted until roughly 8,000 time steps in normal times, ignoring several firms living longer than that. However, for crisis times (see Figure 3.3 (d)) the tail has been properly fit until 9,000 time steps. In this case, the comparison of Gaussian fits is not reliable. Instead, we refer to the average lifetimes here. Going up the supply chain to the raw material suppliers on stage $s = 4$, average lifetime does not change significantly during times of low demand.

The algorithm in our model is designed to help firms with the best profit/cost ratio grow. During normal times, there are a few large and many small retailers at the bottom of the supply chain. Reducing the demand seems to induce a shift in the composition. Low demand creates a hostile environment for large long-lived firms by lowering profits and equivalently slowing down the growth of capital. At the same time, low demand means fewer product orders that do not entirely use up working capital, leaving more buffer to cope with costs caused by production shortfalls. As a result, firms on average live longer than during normal times. As we move up the supply chain, low demand seems to lose its effects. Having customers that order less each time but live longer on average seems to create an as stable income for suppliers as in normal times.

### 3.2.2 Diversified firms

Comparing the midsections of the cumulative lifetime distributions of the undiversified, forward vertically integrated, backward vertically integrated, and horizontally merged middlemen firms, we find slower decays for the diversified firms.

Figure 3.4 and Table 3.2 show $F_2(n)$ for the three diversification strategies during crisis. Instead of decaying at a rate of $\lambda = (9.003 \pm 0.004) \times 10^{-4}$ for the undiversified firms during a financial crisis, the midsections decay with $\lambda_m =$
Figure 3.3: Cumulative distributions of the lifetimes of undiversified firms on stages (a) \( s = 0 \), (b) \( s = 1 \), (c) \( s = 2 \), (d) \( s = 3 \), and (e) \( s = 4 \) of the dynamical supply chain during crisis times. These distributions are extracted from 10,000 simulations, running for 10,000 time steps each.

(8.124 ± 0.006) \( \times 10^{-4} \) (horizontal merger), \( \lambda_f = (6.668 ± 0.011) \times 10^{-4} \) (forward vertical integration), and \( \lambda_b = (6.315 ± 0.013) \times 10^{-4} \) (backward vertical integration). From Table 3.2, we also see the tail of the cumulative lifetime distribution for horizontally merged middlemen firms decaying significantly faster (\( \sigma_m = 1021 ± 7 \)) than that for undiversified middlemen firms (\( \sigma = 1550 ± 3 \)). The tails of the cumulative lifetime distributions for the vertically integrated middlemen firms, on the other hand, decay significantly slower (\( \sigma_f = 2148 ± 12 \) for forward vertical integration, and \( \sigma_b = 1894 ± 16 \) for backward vertical integration).

During normal times, the midsections of the cumulative lifetime distributions of diversified firms decay slower (see Figure 3.5 and Table 3.3, \( \lambda_f = (7.393 ± 0.011) \times 10^{-4} \), \( \lambda_b = (7.263 ± 0.013) \times 10^{-4} \), \( \lambda_m = (9.249 ± 0.005) \times 10^{-4} \)) compared to that seen for undiversified firms (\( \lambda = (1.02 ± 0.09) \times 10^{-3} \)). Additionally, the tail of the cumulative lifetime distribution for horizontally merged firms again decays faster (\( \sigma_m = 1152 ± 14 \)), while the tails of the cumulative lifetime distributions for vertically integrated firms decay slower (\( \sigma_f = 1765 ± 13 \) and
Table 3.2: Lifetime decay rates and average lifetimes of diversified middleman firms during crisis times, where $\lambda$ is the midsection decay rate, $\sigma$ is the tail decay rate, and $\bar{n}$ is the average lifetime. The smaller $\lambda$ or larger $\sigma$ are, the slower the respective decays.

<table>
<thead>
<tr>
<th>Management strategy</th>
<th>Diversified firms (crisis)</th>
<th>Undiversified firms (crisis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward vertical integration</td>
<td>$\lambda_f = (6.668 \pm 0.011) \times 10^{-4}$</td>
<td>$\lambda_2 = (9.003 \pm 0.004) \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_f = 2148 \pm 12$</td>
<td>$\sigma_2 = 1550 \pm 3$</td>
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<tr>
<td></td>
<td>$\bar{n}_f = 1744 \pm 170$</td>
<td>$\bar{n}_2 = 595.5 \pm 2.1$</td>
</tr>
<tr>
<td>backward vertical integration</td>
<td>$\lambda_b = (6.315 \pm 0.013) \times 10^{-4}$</td>
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<tr>
<td></td>
<td>$\sigma_b = 1894 \pm 16$</td>
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<tr>
<td></td>
<td>$\bar{n}_b = 1474 \pm 69$</td>
<td></td>
</tr>
<tr>
<td>horizontal merger</td>
<td>$\lambda_m = (8.124 \pm 0.006) \times 10^{-4}$</td>
<td></td>
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<tr>
<td></td>
<td>$\sigma_m = 1021 \pm 7$</td>
<td></td>
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<tr>
<td></td>
<td>$\bar{n}_m = 932 \pm 52$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Lifetime decay rates and average lifetimes of diversified middleman firms during normal times, where $\lambda$ is the midsection decay rate, $\sigma$ is the tail decay rate, and $\bar{n}$ is the average lifetime. The smaller $\lambda$ or larger $\sigma$ are, the slower the respective decays.

<table>
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<td>$\lambda_2 = (1.02 \pm 0.09) \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_f = 1765 \pm 13$</td>
<td>$\sigma_2 = 1613 \pm 5$</td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_f = 1602 \pm 73$</td>
<td>$\bar{n}_2 = 563.8 \pm 1.9$</td>
</tr>
<tr>
<td>backward vertical integration</td>
<td>$\lambda_b = (7.263 \pm 0.013) \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_b = 4096 \pm 175$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_b = 1671 \pm 68$</td>
<td></td>
</tr>
<tr>
<td>horizontal merger</td>
<td>$\lambda_m = (9.249 \pm 0.005) \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_m = 1152 \pm 14$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\bar{n}_m = 1033 \pm 633$</td>
<td></td>
</tr>
</tbody>
</table>

$\sigma_b = 4096 \pm 175$) than the tail of the cumulative lifetime distribution of undiversified firms ($\sigma = 1613 \pm 5$).

Again, another way to gauge the impact diversification has on the lifetimes of the middlemen firms is to compare the average lifetimes. Unlike the exponential decay constant $\lambda$ of the intermediate lifetimes and the Gaussian standard deviation $\sigma$ of the long lifetimes, the average lifetime also receives contributions from
the short-lifetime end of the distribution.

Here, we find the average lifetime of undiversified firms is $\bar{n} = 595.5 \pm 2.1$ (based on 7,900,000 lifetimes recorded), whereas those for forward vertical integration, backward vertical integration, and horizontal merger are $\bar{n}_f = 1744 \pm 170$ (based on 10,000 lifetimes recorded), $\bar{n}_b = 1474 \pm 69$ (based on 11,000 lifetimes recorded), and $\bar{n}_m = 932 \pm 52$ (based on 11,000 lifetimes recorded) respectively. We see that all three strategies increased the average lifetime.

As discussed in Subsection ‘Undiversified firms’ when comparing the cumulative lifetime distribution of normal times and times of low demand, the latter reduces the number of long-lived firms in the mid-level of the supply chain while increasing the number of short-lived and medium-lived ones. This then results in an overall slight increase in average lifetime.

However, by allowing mid-level firms to forward or backward vertical integrate during crisis not only increases average lifetime significantly but also the number of long-lived firms. In fact, both are even higher than they are in normal times for undiversified firms. While, horizontal merger also increases average lifetime it is not at all as effective in saving long-lived firms than the other two strategies.

Figure 3.4: Cumulative lifetime distributions of the diversified firms during a financial crisis. We ran 10,000 simulations for 10,000 time steps.
Out of all three management strategies, in times of low demand forward vertical integration – having customers obliged to order only from their parent company – maintains the largest average lifetime and the highest number of long-lived firms. These firms that buy over customers during crisis need to compete less for the already lower order volume than firms using other strategies. This suggests, that securing orders when demand is low – in general – is safer than securing supplies or increasing working capital.

This picture however changes if demand increases. By testing all three strategies for normal times we have found that this time the favourable strategy is no longer forward vertical integration. In fact, it even performs slightly worse than during crisis times in terms of average lifetime and number of long-lived firms. Instead, the other two remaining strategies increased their performance with backward vertical integration being the most advantageous strategy. While some orders can be secured for the parent company by forward vertical integration, the bought-over subsidiary company loses the potential to order with the cheapest and also decreases its ability to hedge against possible production shortfalls. Then in times of higher demand it seems that the disadvantage of the subsidiary company out-weights the advantage of the parent company. This agrees
with the fact that backward vertical integration becomes the better strategy. The greater the order volume is in our model the higher a production shortfall can be, if it occurs. Hence, hedging against shortfalls by buying over suppliers that become obliged to only supply their parent company becomes – in general – the better strategy.

### 3.2.3 Tagged comparison

As mentioned in Subsection ‘Diversified firms’ securing orders when demand is low is, in general, safer than securing supplies. However, if demand increases then hedging against shortfalls is, in general, the preferable strategy.

We say ‘in general’ as until now we have not explicitly looked at the actual change of lifetime of those firms, that have decided to diversify. Because the supply chain is highly heterogeneous, the environments of some middlemen firms may be better than others on the same level. A direct comparison of the lifetime distributions will not tell us how much longer a middlemen firm is expected to live, had it decided to diversify. Also, in the untagged simulations diversified firms lived out their lives entirely within normal or crisis conditions. It would be interesting to see what effects diversification has on middlemen firms living
through the start of a financial crisis. We therefore use the following more precise method to compare the lifetimes of these firms.

For the tagged comparison we used ten different supply chain networks in equilibrium state. Starting from each of these ten equilibrium networks we ran 1,000 tagged simulations. In each tagged simulation, we randomly pick and tag a firm from the middle stage of our supply chain. We then ran two parallel simulations of 10,000 time steps, with the first 2,500 time steps under normal economic conditions, and the next 7,500 time steps under crisis conditions. In one simulation our tagged middlemen was left undiversified. In the second simulation we allow the tagged middlemen to diversify.

We then compare the lifetime distributions of the tagged middlemen in the two simulations. In Figure 3.6 we show the histograms for diversified lifetime minus the undiversified lifetime, for all three diversification strategies. Comparing the average lifetime differences, we observe that backward vertical integration is the most effective strategy, because it increases lifetime by an average of $104 \pm 79$ time steps, whereas forward vertical integration is the least effective in helping middlemen firms survive a sudden downturn, as the lifetime decreases by an average of $495 \pm 227$ time steps.

### 3.2.4 Summary

During crisis times firms in the middle of the supply chain are allowed to (1) forward vertically integrate by buying over one of its customers, (2) backward vertically integrate by buying over one of its suppliers, or (3) horizontally merge with a competitor to pool capital and resources. After extracting the lifetime distributions of undiversified firms, and of firms adopting the three diversification strategies, we then compare the average lifetimes and the rates at which the mid-sections and tails of the cumulative lifetime distributions decay for these four types of firms. If firms do not diversify in times of low demand, our results show a decrease in the number of long-lived firms in the lower to middle level of the supply chain in low demand times and, at the same time, a decrease of average lifetime of firms in the upper levels. Instead, the results suggest that securing orders when demand is low, in general, is safer than securing supplies or increasing working capital. Firms buying over customers during crisis need to compete less for the already lower order volume than firms using other strategies. Further, we have found diversification not only is advantageous through crisis times but also should be considered even in times of higher demand. In this case, hedging against shortfalls by buying over suppliers that become obliged to only supply their parent company becomes, in general, the better strategy. Furthermore, the additional results on tagged comparisons suggest that backward vertical integration is the
preferable strategy if firms live through the transition of high demand to low demand times. Average lifetime can be slightly increased this way.

Overall, the combination of our results suggest firms to always diversify. In times of higher demand or normal economic situation, firms should concentrate their efforts in hedging against production shortfalls by backward vertical integration. This should also be the main effort when the economic situation undergoes a turnaround. After the demand has shifted from high onto low, firms should apply forward vertical integration instead. In this case the advantage of securing orders in times of lower order volume becomes more crucial instead.
Chapter 4

Interbank network

4.1 Motivations

In the literature review in Chapter 2, we saw that much work have been dedicated to understanding the impact of contagion and systemic risks on a banking system, whose normal function is to efficiently redistribute capital. In most of these studies, regulations and interventions are tested by making assumptions on how the interbank market works, and how the interbank network looks like. It is only recently that empirical studies by Furfine [78], Uppers and Worm [79], Iori et al. [68], and Battiston et al. [19] mapped out the interbank networks in Germany, Italy, and U.S. Needless to say, access to interbank transaction data is critical in such studies. Unfortunately, in most other countries, this data is either not made available to researchers, or simply non-existent. However, for regulatory and reporting purposes, most central banks and regulatory bodies do have information on the state of the banks.

This is why, in this project, we would like to start from a system of banks reporting daily balance sheets to their regulators. Based on these mandatory reports, we infer the underlying interbank network, and design network-based regulations and interventions that can effectively prevent liquidity shocks and arrest cascading failures. If we can successfully achieve these objectives, the insights from our study can be implemented in countries and banking systems where loan contracts between banks may be considered data that are too sensitive for regulators to collect. Since something like this has never been attempted, we design the project to follow two parallel tracks as shown in Figure 4.1. In the virtual track, we plan to first demonstrate that interbank network estimation is in principle possible, and then use the estimated interbank network to analyze different scenarios of regulations and interventions. Using confidence and insights gained from the simulation study, we plan in the real-world track to acquire reporting data from the Monetary Authority of Singapore (MAS), which plays the role of
the central bank in Singapore, to estimate the interbank network in Singapore. We aim then to incorporate regulation and intervention simulations developed in the virtual track to develop a scenario-based decision support system.

Assuming that MAS will not go far beyond present-day reporting standards, we simulate an artificial interbank network on which banks manage the savings of depositors, use this money to invest, and thereafter pay interest to depositors and dividends to shareholders. At all times, banks comply with regulations and pay up any shortfall in required reserves. Sometimes, when investment returns fall short of expectations or banks experience spikes in withdrawals, they will borrow money from other banks to get through the day. This borrowed money must be returned the next day, according to an overnight rate agreed upon within the interbank network. These transactions are condensed into daily balance sheets, which are then reported to the regulatory body. Based on these reports, or weekly or monthly aggregates, we will design a statistical procedure to estimate the underlying interbank network. This statistical estimation will be described in detail in section 4.3, but the basic idea is very simple: if a bank lends money to other banks, its asset will grow, whereas if a bank borrows money from other banks, its liability will grow. We will therefore draw tentative links between banks with growing assets on the one hand, and banks with growing liabilities on the other hand. As expected, many tentative links will be wrong, but after repeated estimation, true links will get reinforced, whereas false links will decay and be eventually deleted. Once the interbank network is estimated, we will simulate different intervention and regulation scenarios, to evaluate which measures will be the most effective in maintaining liquidity.

Once the estimation algorithms are developed, we plan to apply them to reporting data from MAS to estimate the interbank network in Singapore. After the network structure has been determined, we will also need to fix the microscopic parameters. We start with an initial guess for these parameters and then run simulations to generate the observable time series. Then, we compare these to the actual time series we obtain from the MAS, and adjust the parameters until the simulated time series converge onto the observable time series. More importantly, in this real track, we will simulate and analyze different regulations and interventions, to determine which measures are the most effective according to standard banking market performance indicators. In particular, the recently introduced Basel III called for higher bank capital requirements, and more stringent regulations on bank liquidity and leverage. Such financing constraints are expected to slow down economic growth, but are touted to give in return improved financial stability in the long term. Some economists however are skeptical about the effectiveness of Basel III [80]. In fact, De La Motte et al. [6] and Wallison [7] argue that subsidies and regulations are implicated prior to every financial crisis. They distort domestic supply or demand in the long run, which then lead to mis-
Figure 4.1: The original plan for this project is divided into two parallel sets of two work packages. The virtual track (left) consists of the development of a robust procedure for estimating interbank networks, and scenario analysis of intervention and regulation measures. As this virtual exploration is in progress, we will also source for data in the real-world track (right) to estimate the real-world interbank network, and apply insights gained from the scenario analysis of the artificial interbank network to build a scenario-based decision support system.
pricing of assets and misevaluation of risks. As a result of increasing asset prices it has been observed that prior to financial crises leverage increased significantly [81, 82, 83]. At the same time, underestimation of risks leads to higher ratings for risky assets, which encourage investors to go in with a higher leverage [81]. One of our goals is to therefore examine what effects Basel III, or rather a simplified implementation of it, will have on the stability of our complex agent network of banks.

4.2 The interbank network model

In this section I will describe the ABM used to simulate the investment behavior of banks and their interactions with each other through interbank loans. In the model banks are represented by nodes and linkages between nodes represent transaction channels where interbank lending and borrowing take place. In this simplified version of interbank network, banks receive deposits which are put into investments. Beside doing investments, banks pay dividends and interest to depositors and are liable to keep a minimum required reserve at the central bank. If a bank suffers from illiquidity it will ask for interbank loan from banks it is connected to. These borrowings have to be returned the next day. If a bank fails to do so, it defaults.

Subsection 4.2.1 will provide a more detailed description of the used artificial network. It is then followed by more details on the simulation in Subsection 4.2.2. Thereafter I will list in Subsection 4.2.3 the parameters used in my simulations.

4.2.1 The interbank network

According to MAS, the Singapore interbank network comprises of 32 banks, of which 6 are local and 26 are foreign banks (see Table 4.1 and 4.2). We expect the Singaporean banking network to consist of two major clusters: (1) local banks and (2) foreign banks. The amount of linkages within the local bank cluster and the foreign bank cluster is high, while the two clusters are only weakly connected. Hence, we build an artificial network of 32 banks where we assume that the probability of interbank connection within local banks is 100%, 60% within foreign banks and 5% between local and foreign banks. The artificial network is shown in Figure 4.2.

In this chapter, we also investigate as a benchmark random interbank networks with the same $N = 32$ banks. In these networks, all banks are labeled and their sizes are sampled from a normal distribution $N(\mu, \sigma^2)$. Each link represents a possible lending and borrowing channel between two banks. To simulate a diverse ecology of financial institutions comprising risk-seeking investment banks and
Table 4.1: List of 6 local banks in Singapore.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Country</th>
<th>Market capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of Singapore Ltd</td>
<td>private</td>
<td>Singapore</td>
<td>N/A</td>
</tr>
<tr>
<td>DBS Bank Ltd</td>
<td>public</td>
<td>Singapore</td>
<td>USD 33.30 billion</td>
</tr>
<tr>
<td>Far Eastern Bank Ltd</td>
<td>public</td>
<td>Singapore</td>
<td>USD 26.80 billion</td>
</tr>
<tr>
<td>Oversea-Chinese Banking Cor-</td>
<td>public</td>
<td>Singapore</td>
<td>USD 27.17 billion</td>
</tr>
<tr>
<td>poration Ltd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Singapore Island Bank Ltd</td>
<td>public</td>
<td>Singapore</td>
<td>N/A</td>
</tr>
<tr>
<td>United Overseas Bank Ltd</td>
<td>public</td>
<td>Singapore</td>
<td>USD 25.26 billion</td>
</tr>
</tbody>
</table>

Figure 4.2: Artificial network with 100% linkage between local bank, 60% linkage between foreign banks and 5% linkages between local and foreign banks.
### Table 4.2: List of 26 foreign banks in Singapore.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Country</th>
<th>Market capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia and New Zealand Banking Group Ltd</td>
<td>public</td>
<td>Australia</td>
<td>USD 76.14 billion</td>
</tr>
<tr>
<td>Bangkok Bank Public Company Ltd</td>
<td>public</td>
<td>Thailand</td>
<td>USD 10.73 billion</td>
</tr>
<tr>
<td>Bank of America</td>
<td>public</td>
<td>U.S.</td>
<td>USD 152.77 billion</td>
</tr>
<tr>
<td>Bank of China Ltd</td>
<td>public</td>
<td>China</td>
<td>USD 270.1 billion</td>
</tr>
<tr>
<td>The Bank of East Asia Ltd</td>
<td>public</td>
<td>Hong Kong</td>
<td>USD 8.97 billion</td>
</tr>
<tr>
<td>Bank of India</td>
<td>public</td>
<td>India</td>
<td>USD 132.77 trillion</td>
</tr>
<tr>
<td>The Bank of Tokyo-Mitsubishi UFJ Ltd</td>
<td>public</td>
<td>Japan</td>
<td>USD 85 billion</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>private</td>
<td>France</td>
<td>USD 82.01 billion</td>
</tr>
<tr>
<td>CIMB Bank Berha</td>
<td>public</td>
<td>Malaysia</td>
<td>USD 17.67 billion</td>
</tr>
<tr>
<td>Citibank</td>
<td>public</td>
<td>U.S.</td>
<td>USD 148 billion</td>
</tr>
<tr>
<td>Citibank Singapore Ltd</td>
<td>public</td>
<td>U.S.</td>
<td>USD 148 billion</td>
</tr>
<tr>
<td>Credit Agricole Corporate and Investment Bank</td>
<td>private</td>
<td>France</td>
<td>USD 26.22 billion</td>
</tr>
<tr>
<td>HL Bank</td>
<td>public</td>
<td>Malaysia</td>
<td>USD 7.75 billion</td>
</tr>
<tr>
<td>The Hongkong and Shanghai Banking Corporation Ltd</td>
<td>public</td>
<td>Hong Kong</td>
<td>USD 212.09 billion</td>
</tr>
<tr>
<td>ICICI Bank Ltd</td>
<td>private</td>
<td>India</td>
<td>USD 1,562.96 trillion</td>
</tr>
<tr>
<td>Indian Bank</td>
<td>public</td>
<td>India</td>
<td>USD 46.27 trillion</td>
</tr>
<tr>
<td>Indian Overseas Bank</td>
<td>public</td>
<td>India</td>
<td>USD 62.61 trillion</td>
</tr>
<tr>
<td>JP Morgan Chase Bank</td>
<td>public</td>
<td>U.S.</td>
<td>USD 191 billion</td>
</tr>
<tr>
<td>Malayan banking Berhad</td>
<td>public</td>
<td>Malaysia</td>
<td>USD 24.06 billion</td>
</tr>
<tr>
<td>Mizuho Corporate Bank Ltd</td>
<td>public</td>
<td>Japan</td>
<td>USD 49 billion</td>
</tr>
<tr>
<td>PT Bank Negara Indonesia (Persero) Tbk</td>
<td>public</td>
<td>Indonesia</td>
<td>USD 5.9 billion</td>
</tr>
<tr>
<td>PHB Bank Berhad</td>
<td>public</td>
<td>Malaysia</td>
<td>USD 1.09 billion</td>
</tr>
<tr>
<td>Standard Chartered Bank</td>
<td>public</td>
<td>United Kingdom</td>
<td>USD 62.76 billion</td>
</tr>
<tr>
<td>State Bank of India</td>
<td>public</td>
<td>India</td>
<td>USD 1.02 trillion</td>
</tr>
<tr>
<td>Sumitomo Mitsui Banking Corporation</td>
<td>private</td>
<td>Japan</td>
<td>USD 61.02 billion</td>
</tr>
<tr>
<td>UCO Bank</td>
<td>public</td>
<td>India</td>
<td>USD 64.94 trillion</td>
</tr>
</tbody>
</table>
risk-avoiding business banks, each bank is assigned a random constant relative risk aversion, $\frac{1}{\beta}$, from a uniform distribution $U(a, b)$.

### 4.2.2 The simulation

The overall dynamics of the model is as follows: cash deposited at the bank by customers is channeled into investments. Leftover liquidity may then be loaned out to other banks with debts through the interbank market. These over-night loans have to be returned with interest the next day. Banks with debts or unable to repay previous loans at the end of a day fail and are removed from the network.

**Initialization at time step $= 0$:** At the beginning of each simulation, we set the starting deposit $A^k_0$ of bank $k$ equal to its size, $A^k_0 = s^k$ and the borrowings are set to zero, $B^k_{-1} = 0$. The starting liquidity $L^k_0$ is initialized to be equal to the first deposit for each bank. At $t = 0$, banks also pay the minimum required reserve, invest or lend and borrow, according to the rules for $t > 0$, which are described as follows.

**Deposit fluctuation and interest payment:** At each time step customers withdraw money from or deposit money into banks, causing unpredictable liquidity shocks. Let $A^k_t$ denote the cash deposit of bank $k$ at time step $t$. We assume bigger banks experience less deposit fluctuations than smaller banks. The new deposit is determined by

$$A^k_t = s^k + \sigma_s \sqrt{s^k} \epsilon_t,$$

(4.1)

as proposed by [68], where $\epsilon_t \sim N(0, 1)$.

Banks further have to pay interest to their customers, which is $r_a$ times their deposit from previous time step. Hence, the amount of interest to be paid is

$$D^k_t = r_a A^k_{t-1}.$$

(4.2)

**Minimum required reserve:** Given the deposit $A^k_t$, banks are required to deposit a 'minimum required reserve' at the central bank.

$$R^k_t = \beta A^k_t,$$

(4.3)

where $\beta < 1$. This required reserve is consider illiquid at time $t$. Furthermore, the central banks does not pay interest for it to banks [84].
Liquidity: The amount of liquidity, $\hat{L}_t^k$, a bank has at the beginning of each time step $t > 0$ is given by

$$\hat{L}_t^k = L_{t-1}^k,$$

(4.4)

where $L_{t-1}^k$ is the end-of-period liquidity of bank $k$ at time step $t - 1$.

Within a time step, a bank’s liquidity changes due to returns and payment obligations. This liquidity status is called the intra-period liquidity $\tilde{L}_t^k$. Starting from $\hat{L}_t^k$, intra-period liquidity is updated as follows:

1. banks get back ‘minimum required reserve’ from last time step;
2. banks experience deposit fluctuations;
3. banks pay interest to depositors;
4. banks receive returns of on-going and recently matured portfolios; and
5. banks get back the principal of matured portfolios.

The equation for intra-period liquidity $\tilde{L}_t^k$ is then

$$\tilde{L}_t^k = \hat{L}_t^k + R_t^k + (A_t^k - A_{t-1}^k) - D_t^k$$

$$+ \sum_{s=1}^{\tau} \left[ r_b \left( 1 - y_{t-s}^* \right) I_{t-s}^k + \rho^{+/-} y_{t-s}^* I_{t-s}^k \right] + I_{t-\tau}^k,$$

(4.5)

where $r_b$ is the return of risk-free asset, $y^*$ is the portfolio weight of risky asset, $I^k$ is the portfolio volume, and $\rho^{+/-}$ is the return/loss of risky asset. These parameters will be explained later under ‘Investments’.

Debt certificates and liquidity update: After determining the intra-period liquidity in the previous subparagraph, banks pay the ‘minimum required reserve’, $R_t^k$. If after payment the remaining liquidity is $\left( \tilde{L}_t^k - R_t^k \right) < 0$, banks need to issue debt certificates of the same amount. These certificates have to be exchanged for cash through borrowings by the end of a time step, otherwise they lose their value. If a bank is not able to redeem these certificates, it fails.

If $\left( \tilde{L}_t^k - R_t^k \right) > 0$, banks need to repay their creditors if there are any open obligations from last time step. The payment of the borrowings, $Br_t^k$, plus interest of $r_c$, has to be paid in full and in cash. If a bank does not have enough cash to repay, it has to issue debt certificates and pay back after it has redeemed the full cash amount. The updated intra-period liquidity is therefore

$$\tilde{L}_t^k = \tilde{L}_t^k - R_t^k + Br_t^k - r_c R_t^k.$$

(4.6)

Note that borrowing, $Br_t^k$, can have either a positive or negative value. It is positive if bank $k$ lends money to other banks. It is negative if bank $k$ borrows money from other banks.
**Lenders and borrowers:** To simulate monetary transactions within the inter-bank network, it is important to identify lending banks and borrowing banks. The status ‘Lender’ and ‘Borrower’ needs to be updated every time the intra-period liquidity of a bank is updated. After the update of the intra-period liquidity, a bank with surplus liquidity is called ‘Lender’. A ‘Borrower’ is a bank with liquidity deficit or existing debt certificates.

**Dividends:** After the intra-period liquidity update in Equation (4.6), if \( \bar{L}_t^k > 0 \) banks pay their dividends according to the scheme proposed by [68]. To determine which bank needs to pay dividends, the intra-period capital, \( \bar{V}_t^k \), of a bank needs to be calculated first. \( \bar{V}_t^k \) consists of \( \bar{L}_t^k \) plus the required reserve, principals of all on-going portfolios minus the deposit, i.e.

\[
\bar{V}_t^k = \bar{L}_t^k + R_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - A_t^k.
\]  

Next, we set a minimum capital/deposit ratio, \( \lambda \). The capital/deposit ratio is

\[
E_t^k = \frac{\bar{V}_t^k}{A_t^k}.
\]  

Those banks with \( E_t^k > \lambda \) pay dividends.

To determine the amount of dividends, \( C_t^k \), to be paid, we use

\[
C_t^k = \max \left[ 0, \min [a, b, c] \right],
\]

where

\[
a = \sum_{s=1}^{\tau} \left[ r_b \left( 1 - y_{t-s}^* \right) I_{t-s}^k + \rho^{+/-} y_{t-s}^* I_{t-s}^k \right] - D_t^k,
\]

\[
b = \bar{L}_t^k,
\]

\[
c = \bar{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1 + \lambda) A_t^k.
\]

Equation (4.9) is a guideline for dividend payment, that is bound to the three conditions \( a \), \( b \), and \( c \). The equation restricts the payment to the net investment income \( a \) if it does not violate either the liquidity \( b \) or the capital/deposit target \( c \). Once dividends are paid, if any, intra-period liquidity has to be updated once again as well as the liquidity status ‘Lender’ and ‘Borrower’.
Investments: If after dividend payment left-over liquidity is still positive, i.e. \((L^k_t - C^k_t) > 0\), banks plan their portfolios and do investments. Banks are offered risk-free investments alongside risky investments. Both originate from the outside the interbank market. To simplify the simulation, we assume that both investments have the same maturity, \(\tau\). Investments made at time \(t\) mature at time \(t + \tau\), and banks receive their returns at \(t + 1, \ldots, t + \tau\).

To simulate risky investments, we let risky assets yield a successful return \(p^+ > 0\) with probability \(p\), and a failed return \(p^- < 0\) with probability \((1 - p)\). Risk-free assets are assumed to always give a successful return of \(r_b > 0\). Banks calculate the optimal portfolio weight of risky asset, \(y^*\) as described in Appendix A.0.9.

Since \(y^*\) does not depend on the portfolio volume, see Equation (A.17), we consider a portfolio volume, \(I^k\), which depends on the bank’s size and is constrained by the available liquidity. To determine \(I^k\) we assign a maximum total dollar investment,

\[
\omega^k_t = \left| \bar{\omega} + \sigma \eta_t \right|,
\]

as proposed by [68], where \(\bar{\omega} = \delta s^k, 0 < \delta < 1\), and \(\eta_t \sim N(0, 1)\). Then the portfolio volume is

\[
I^k = \min \left[ L, \omega^k \right]. \quad (4.10)
\]

If \(y^*\) is negative, banks are selling short \(y^* I^k\) risky assets, whereas if \((1 - y^*)\) is negative, banks are selling short \((1 - y^*) I^k\) risk-free assets.

In the model, assets borrowed by banks with short positions originate from outside the interbank market, as well as the buyers of borrowed assets. It is further assumed that banks opting for short positions will always find assets to borrow and buyers to sell the borrowed assets to. Short positions made at time step \(t\) have to be closed out at \(t + \tau\). Losses and profits generated by the positions are realized at \(t + 1, \ldots, t + \tau\).

Interbank loan: After doing investments, banks update their intra-period liquidity as well as their ‘Lender’ and ‘Borrower’ status. Given the updated status, lending and borrowing between banks commence. The amount of liquidity transfer between two banks is restricted to the minimum of the lendable and asked amount. If a bank’s asked amount exceeds the lendable amount of another bank, the borrowing bank may negotiate with other bank it is linked to for the remaining amount. In this case, the borrowing bank does not receive the actual cash, until it can find enough lenders to cover its debt certificates.

Iterations: All updates described above are within one iteration. In each time step \(t\) the algorithm will be executed for several iterations. In particular, the
algorithm iterates through ‘Debt certificates and liquidity update’, ‘Lenders and borrowers’, ‘Dividends’, ‘Investments’, and ‘Interbank loan’ until there are no further cash transactions within the interbank network.

Failure and liquidation: Those banks that are ultimately not able to redeem their debt certificates fail and are removed from the network without replacement. After liquidation, cash will be first paid out to their depositors. The remaining cash will be distributed to the creditors in order of the amount of the money they loaned.

4.2.3 Parameter setting

The interbank network is initialized with \( N = 32 \) banks. Each bank is assigned a random constant relative risk aversion, \( \frac{1}{k} \), from a uniform distribution \( U(1,2) \). The size of banks, \( s^k \), are sampled from a normal distribution, \( N(1000,20) \). To determine the new deposit at each time step in 4.1, we set \( \sigma_A \) to 0.5. The minimum required reserve is set to 3\%, \( \beta = 0.03 \). Banks have to pay depositors an interest rate of 0.3\% (\( r_a = 0.003 \)). Interest rate for interbank borrowing is 0.3\% (\( r_c = 0.003 \)).

To determine which bank has to pay dividends, we set the minimum capital/deposit ratio requirement, \( \lambda = 0.3 \). We set the maturity, \( \tau \), of both risky and risk-free investments to 3 time steps. The simulated ‘real-world’ probability, \( p \), that the risky investment is successful is set between 5\% and 95\%. The ‘real-world’ return of successful risky investments is set to 6\%, \( \rho^+ = 0.06 \), and the loss of unsuccessful investments is -5\%, \( \rho^- = -0.05 \). The return of risk-free investments is 0.3\%, \( r_b = 0.003 \).

To calculate the portfolio volume, \( J^k \), as in Equation (4.10), we set \( \sigma_{D_0} = 0.5 \). We run all simulations for 1,000 time steps. Within the simulation, the lifetime of a firm is defined as the amount of time steps until it goes bankrupt.

4.3 Network Estimation

To determine the structure of the Singaporean banking network, it is necessary to have a reliable algorithm that is capable of identifying eventual clusters of banks based on interbank lending reporting data. The aim of this section is therefore to identify the most effective network estimation algorithm. In this section we show a proof of principle by simulating the agent-based interbank lending model described in Section 4.2 over the artificial interbank network shown in Figure 4.2. We will estimate the interbank network from the borrowings and lendings time series, using the algorithms described below.
4.3.1 Algorithm 1

Consider two banks $i$ and $j$. At time step $t$, if the borrowing for $i$ and the lendings for $j$ increase, we connect bank $i$ and $j$. If a connection already exists between these two banks, we add one more unit of weight to the linkage to keep track of the number of times this link is reinforced. This algorithm runs through all timesteps and generates an adjacency matrix and a weight matrix.

Next, we neglect all connections with a weight equal to or smaller than a certain threshold. The aim of doing so, is to distinguish between real links and spurious links generated by this algorithm. To choose the threshold, we determine the natural cut-off bound for weighted links, by plotting the number of links as a function of weight $x$ and then perform an exponential fit of the form $a \exp(bx)$, where $a$ and $b$ are fitting parameters. This is shown in Figure 4.3.

![Figure 4.3](image)

Figure 4.3: Fitting the empirical number of estimated links obtained using algorithm 1 (solid blue curve) within 20 weight bands to a decaying exponential (dashed red curve) of the form $a \exp(-bx)$, where $a = 58 \pm 12$ and $b = 0.03 \pm 0.01$. The weight scale over which the number of links decay is $1/b = 33$, shown as the black dashed vertical line.
We then define the inverse of $b$ as the natural cut-off bound. Figure 4.4 shows the estimated interbank network using the natural cut-off bound. To evaluate the performance of the algorithm we count the proportions of correctly estimated linkages over linkages within and between clusters, and the proportion of correctly estimated linkages over the total number of estimated linkages, again within and between the clusters. We call the first proportion the coverage of the algorithm, and the second proportion the exactness of the algorithm. For algorithm 1, we find that a coverage and exactness of 47% and 100% respectively for the local banks cluster, a coverage and exactness of 41% and 73% respectively for the foreign banks cluster, and a coverage and exactness of 43% and 13% respectively of the local-foreign bank links.

Figure 4.4: Estimated network using algorithm 1 with natural cut-off at $\frac{1}{b} = 33$. Only the correctly estimated links are shown.

4.3.2 Algorithm 2

In this second algorithm, beside connecting all borrowers with all lenders, we further consider the amount of borrowed and lent money. First we connect each borrower with each lender as in algorithm 1. Since we know the exact amount of borrowings and lendings of each bank, we make use of this information to calculate the fraction borrowings/lendings for each pair of borrower and lender. If the fraction of a link is exactly 1 we assign 1 as the weight. If the fraction of a link is smaller than 1, we assign the fraction as the weight. If the fraction of a link is larger than 1, we assign the inverse of the fraction as the weight. Hence, we always assign weights smaller than 1 to links which do not have fraction equals to 1.
The idea behind this step in the algorithm is that the increase in lending of bank $i$ should be equal to the increasing in borrowing of bank $j$, if bank $j$ borrowed only from bank $i$, and bank $i$ lent only to bank $j$. Of course, bank $j$ can borrow from more than one bank, and bank $i$ can lend to more than one bank, but these scenarios are less likely compared to pairwise loans. Therefore it is very unlikely that a bank with a small increase in borrowings to have borrowed the money from a bank with a large increase in lendings.

![Figure 4.5: Fitting the empirical number of estimated links obtained using algorithm 2 (solid blue curve) within 20 weight bands to a decaying exponential (dashed red curve) of the form $a \exp(-bx)$, where $a = 80 \pm 10$ and $b = 0.05 \pm 0.01$. The weight scale over which the number of links decay is $1/b = 18$, shown as the black dashed vertical line.](image)

After all connections are drawn and weights are calculated, we again define the natural cut-off by fitting the frequency distribution of links as a function of weight to a decaying exponential (see Figure 4.5) and delete all connections that have a weight equal to or smaller than this cutoff. Figure 4.6 shows the estimated network of algorithm 2 with coverage and exactness of 53% and 100%
respectively for the local banks cluster, coverage and exactness of 39% and 91%
respectively for the foreign banks cluster, and coverage and exactness of 43% and
33% respectively for the local-foreign bank links.

Figure 4.6: Estimated network using algorithm 2 with natural cut-off at $t = 18$.
Only the correctly estimated links are shown.

4.3.3 Algorithm 3

As with algorithm 2, we assume we know the exact amount of borrowings and
lendings of each bank at every time step. First, we randomly chose a pair of
borrower $i$, with borrowing $B_i$, and lender $j$, with lending $L_j$. If $B_i > 0$ and
$L_j > 0$, we add a unit weight to the link between them, and update the borrowing
and lending from both banks as follows: $\tilde{B}_i = B_i - \min[B_i, L_j]$ and $\tilde{L}_j = L_j - \min[B_i, L_j]$. Further, if $B_i = 0$, bank $i$ will be removed, never to be picked again.
Similary, if $L_j = 0$, bank $j$ will be removed, never to be picked again. As these
steps are repeated, fewer and fewer borrowers and lenders will remain, and at
some point, we end up with either no borrowers or no lenders. The algorithm
then terminates.

In contrast to algorithms 1 and 2, we do not simply connect all borrowers
to all lenders, and thereafter weigh the links. Instead of getting a large number
of spurious links, we aim to approximate the realistic number of connections
at each time step. Lastly, we again define the natural cut-off by fitting the
frequency distribution of links as a function of weight to a decaying exponential
(see Figure 4.7), and delete all connections that have a weight equal to or smaller
than this cutoff. This algorithm gives us coverage and exactness of 53% and 100%
respectively for the local banks cluster, coverage and exactness of 27% and 74%
Figure 4.7: Fitting the empirical number of estimated links obtained using algorithm 3 (solid blue curve) within 20 weight bands to a decaying exponential (dashed red curve) of the form $a \exp(-bx)$, where $a = 74 \pm 7$ and $b = 0.16 \pm 0.03$. The weight scale over which the number of links decay is $1/b = 6$, shown as the black dashed vertical line.
respectively for the foreign banks cluster, and coverage and exactness of 43% and 14% respectively for the local-foreign bank links. The estimated network is shown in Figure 4.8.

Figure 4.8: Estimated network using algorithm 3 with natural cut-off at $\frac{1}{b} = 6$. Only the correctly estimated links are shown.

4.3.4 Algorithm 4

Algorithm 4 is an add-on and is used as a potential clean-up step for either algorithm 2 or 3. According to MAS Notice 755 ‘Weekly Report on S$ Transactions’ the Monetary Authority of Singapore is informed about the amount of borrowing and lending between Singaporean banks and foreign banks in Singapore. Therefore, besides the amount of money that was borrowed or lent by each bank at each time step, we can further distinguish between money local banks borrowed from and lent to foreign banks. This means we can categorize lenders into local lenders and foreign lenders. Similarly, borrowers can be divided into local borrowers and foreign borrowers. Algorithm 2 or 3 can then be run on the following data blocks individually: local lender and borrower, foreign lender and borrower, local lender and foreign borrower, foreign lender and local borrower. Thereafter, we combine the network estimations of all four data blocks to create the final estimated network. This additional information cuts down the amount of arbitrary linkages created by the previous algorithm and increases the exactness. Finally, we define the natural cut-off for algorithm 2 (see Figure 4.9) or 3 (see Figure 4.10) by fitting the empirical link frequency distributions as functions of weight, and delete all connections that have a weight equal to or smaller than these cutoffs.
Figure 4.9: Fitting the empirical number of estimated links obtained using the combination of algorithms 2 and 4 (solid blue curve) within 20 weight bands to a decaying exponential (dashed red curve) of the form $a \exp(-bx)$, where $a = 72 \pm 12$ and $b = 0.06 \pm 0.02$. The weight scale over which the number of links decay is $1/b = 15$, shown as the black dashed vertical line.
Figure 4.10: Fitting the empirical number of estimated links obtained using the combination of algorithms 3 and 4 (solid blue curve) to a decaying exponential (dashed red curve) of the form $a \exp(-bx)$, where $a = 78 \pm 6$ and $b = 0.14 \pm 0.02$. The weight scale over which the number of links decay is $1/b = 7$, shown as the black dashed vertical line.
Figure 4.11(a) shows the estimated network created by algorithm 2 with algorithm 4 as an add-on. The coverage and exactness within the local banks cluster are 60% and 100% respectively, the coverage and exactness within the foreign banks cluster are 56% and 85% respectively, and the coverage and exactness of local-foreign bank links are 57% and 100% respectively. When combining algorithm 4 with algorithm 3, the estimated network (Figure 4.11(b)) has coverage and exactness of 73% and 100% respectively within the local banks cluster, coverage and exactness of 30% and 76% respectively within the foreign banks cluster, and coverage and exactness of 29% and 50% respectively for local-foreign bank links.

Figure 4.11: Estimated networks using (a) algorithm 2 with algorithm 4 as add-on and natural cut-off at $\frac{1}{b} = 15$, and (b) algorithm 3 with algorithm 4 as add-on and natural cut-off at $\frac{1}{b} = 7$. Only the correctly estimated links are shown.
4.3.5 Estimation of real interbank networks

To evaluate and compare the performances of designed algorithms we introduce the parameters 'coverage' and 'exactness'. The former is obtained by counting the proportions of correctly estimated linkages over the real linkages within and between clusters of local and foreign banks. The latter is measured by counting the proportion of correctly estimated linkages over the total number of estimated linkages. Hence, a reliable algorithm should receive both high values in coverage and exactness.

The network to be estimated is divided into three parts. The largest and most dense part is the local-banks cluster followed by a much smaller and slightly sparser foreign-bank cluster. Both clusters are connected through very few linkages being the third part to be estimated. We evaluate the performance of coverage and exactness of the algorithms on all three parts separately.

The first algorithm we designed connects each borrower with each lender at every time step and weighs these links thereafter. The resulting estimated network then comprises of linkages with weights above a certain threshold. This strategy performs well in terms of coverage as it creates all possible interbank connections. However, given the limited input of data on lending activities of banks the amount of realistic linkages out of all possible ones can be low. The performance in exactness is especially low if the algorithm needs to estimate a small and relatively sparse network configuration.

For the case where additional information on the amount of borrowed and lent money of each bank are given, we designed the following two types of algorithms. Algorithm 2 identifies pairwise loans, where the lending of one bank equals the borrowing of the other. However, it assumes that it is very unlikely that a bank with small increase in borrowing to have borrowed money from a bank with large increase in lending. As a results, pairwise-loan linkages receive higher weight compared to other linkages. On the other hand, algorithm 3 approximates the feasible number of connections at every time step. Using the amount of borrowed and lent money of each bank as restrictions on possible cash transfer it determines feasible pairs of borrower and lender. Comparing algorithms 1,2 and 3 we find that the assumption of pairwise loans in combination with additional data produces a more exact estimation of the linkages between clusters, while the exactness of the same part using algorithm 3 still remains low.

Without further data on cash transactions, we find that the larger and denser a cluster is the easier it is to estimate even with minimal network information. In fact, all three algorithm show relatively similar good performances in estimating the local-bank cluster in terms of both coverage and exactness. The more sparse and smaller a network configuration is, we find, the more relevant data is needed to provide exact estimations.
Taking assumptions a step further we pretend to have access to data on ‘Weekly Report on \$S Transaction’ where the Monetary Authority of Singapore is informed about the amount of borrowing and lending between Singaporean banks and foreign banks in Singapore. With these additional information we can further increase the performances of the algorithms by cutting down the amount of arbitrary linkages created previously. Including this data into our algorithms, both coverage and exactness of estimations were largely improved. Out of all algorithm 2 achieved the best results in estimating both dense and sparse clusters.

Depending on the access to data on the Singapore interbank market we planed to use the introduced algorithms accordingly. Unfortunately, we did not manage to gain access to real-world borrowings and lending data collected by MAS, and therefore, have to continue the research program based on artificial interbank networks.

The original plan was to conduct a case study using an estimated Singapore interbank network based on real-world data. However, as mentioned earlier, as the project progressed we never got to the chance to analyse the necessary data. Instead of studying the interbank network topology of Singapore, we decided to continue our investigations theoretically on a random network. This type of network is very well understood and often used in studies on interbank networks [68, 71, 85] as it offers valuable insights into the relationships between network topologies [86] and is able to mimic properties of real-world networks [87]. Thus, results presented in the rest of this thesis will be based on a random graph network \( G(N, c) \), in which linkages between the \( N \) nodes appear with probability \( 0 \leq c \leq 1 \). A fully connected network is represented by \( c = 1 \), whereas \( c = 0 \) represents an unconnected network.

### 4.4 Portfolio Optimization and Systemic Risk

Using the simulation described in Section 4.2 we now try to understand the effects of the system performance if the structure of the network, such as the number of interbank linkages, or certain initial assumptions in the network, such as market information and returns of assets, changes. Lastly, we will provide insights into the efficacy of certain banking regulations and interventions in this Section.

In Subsection 4.4.1 we investigate the performance of the financial system if banks have incomplete market information of assets. While the calculation of the optimal investment requires an estimation of possible return and loss which in turn are dependent on the expectation of an investor (see Appendix A), investors are hardly fully informed about the market situation of assets and there are frequent cases were information is distorted in practice. Hence, it is likely that investors are underestimating or overestimating the real risks of investments. For
this reason, beside simulating an interbank network, where banks have complete information, we additionally run simulations where the network is initialized with cautious and eager banks respectively.

Subsection 4.4.2 investigates the effects on system performance if the composition of the interbank network changes. In particular, we address interbank linkages as they are known to be crucial for providing efficient capital redistribution within the interbank market, but also act as a medium for failure spreading across the banking network (see Subsubsection 4.4.2.1). In addition, we also investigate the effects of change in network size (see Subsubsection 4.4.2.2).

In Subsection 4.4.3 we investigate the performance of banks in the network when changing the returns of assets and the success probability of risky assets. This will give us more insight into the nature of the short selling investment strategy itself.

Lastly and most importantly in Subsection 4.4.4, we simulate and analyse regulations and interventions, to determine which measures are the most effective according to standard banking market performance indicators. In particular, we refer to the recently introduced Basel III. This regulatory standard calls for more stringent regulations on bank liquidity and leverage amongst other banking requirements. Since in our simplified model leverage only occurs when banks hold short positions, we compared the following three regulation scenarios to understand their impacts: (1) short sales are allowed and unregulated, (2) short sales are allowed but penalized, and (3) short sales are not allowed.

All data analysed in this section are collected from 250 individual 1000-time-step simulations. For the analysis we define a surviving bank as a bank that lives through an entire simulation, hence for 1000 time steps.

4.4.1 Cautious and eager banks

We started the simulation by assuming fully risk-aware banks who are informed about \( p \), the real-world success-probability of risky assets, and the real-world returns and loss of both risky and risk-free assets. In practice, however, banks can only estimate the probable outcome of investments. Hence, we extended our model to allow banks to calculate their investment strategies with a probability \( p_b \) that differs from \( p \). Ultimately we want to investigate how bank lifetimes change when we let banks be more cautious, \( p > p_b \), or more eager, \( p < p_b \).

One would expect cautious banks that overestimate market risks will suffer less from unsuccessful risky investments than eager banks that underestimate the risk. This intuition may indeed be true for markets without leverage. However, our results show that once leverage is allowed, both overestimation and underestimation of risks have severe consequences.

We ran simulations for different returns of risky and risk-free investment with
Figure 4.12: The average percentage, $\mu$, of surviving banks for $p^+ = 0.06$, $p^- = -0.05$, and $r_b = 0.003$. The solid blue curve shows the average percentage of surviving risk-aware banks to a given real-world success-probability of risk investment $p$, where $p$ ranges from 5% to 95%. The average percentage of surviving cautious banks to a given $p$ is shown by the dashed magenta curve, while the dashed black curve shows the average percentage of surviving eager banks to a given $p$. 
risk-aware, cautious and eager banks (see Figure 4.12 and 4.13). To simulate cautious banks, we set $p_b$ to be always smaller than $p$ by 5%. For eager banks we set $p_b$ to be larger by 5%.

Figure 4.12 shows the average percentage of surviving banks with the default parameter setting (see Section 4.2.3) for risk-aware, cautious, and eager banks. For each bank type, the survivability curve has a sharp peak. At this peak, banks hold almost no short positions, hence there is almost no leverage (see Section 4.4.3). The three peaks are also shifted relative to each other. This implies that a stable financial system is only guaranteed whenever banks avoid leverage. Almost all risk-aware banks survive by avoiding leverage at $p = 50\%$ while eager and cautious banks default. Later, in Section 4.4.3 we will see that the default is due to holding short positions.

Similarly, Figure 4.13(a) and 4.13(b) show the average percentage of surviving banks if we increase the risky asset return and risk-free return respectively. While Figure 4.12 and Figure 4.13(a) show that the number of survivors decreases sharply when banks miscalculate risks, Figure 4.13(b) shows that a divergence of $p_b$ from $p$ at the plateau still guarantees network stabilization. The plateau is the outcome of increased risk-free return. Section 4.4.3 shows that increasing the risk-free return also increases the affection for long positions, hence no leverage.

Overall, the results show that cautious banks do not necessarily survive better than eager banks. Independent of a bank’s outlook on the market, it is able to prevent default by avoiding leverage.

### 4.4.2 Network effects

An important question regarding our analysis is whether the default of banks can be limited by reducing counterparty risks, for example, through decreasing the number of linkages in the network. Investigating this question will help us answer whether the dominant impact of less linkages is the reduction in the ability of banks to help each other or a decrease in the level of contagion. Apart from this, we also study whether the size of the network has any notable effect. To do that we increase the number of banks in the interbank network to $N = 100$ and repeat the simulation using the default parameter setting (see Section 4.2.3). We then compare whether the number of surviving banks show any significant changes compared to the network with $N = 32$.

#### 4.4.2.1 Decreasing interbank linkages

In our model, the real-world success probability of risky investments is $p$. So with probability $1 - p$ investing banks experience a loss with the risky investment. In an interbank network where all banks do risky investments at the same time, a
(a) The average percentage, $\mu$, of surviving banks for the following asset returns: $\rho^+ = 0.15$, $\rho^- = -0.05$, $r_b = 0.003$.

(b) The average percentage, $\mu$, of surviving banks for the following asset returns: $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.03$.

Figure 4.13: The average percentage, $\mu$, of surviving banks for increased risky and risk-free returns respectively. The solid blue curve shows the average percentage of surviving risk-aware banks to a given real-world success-probability of risk investment $p$, where $p$ ranges from 5% to 95%. The average percentage of surviving cautious banks to a given $p$ is shown by the dashed magenta curve, while the dashed black curve shows the average percentage of surviving eager banks to a given $p$. 
random loss can be compared to a random shock. In this scenario we investigate whether a decrease of interbank lending channels is more likely to help moderate the level of contagion or further increase liquidity shortage in the network. To do so we run simulations for various probability of linkages from $c = 1$ to $c = 0.2$ with a decrement of 0.2.

Figure 4.14: The average percentage, $\mu$, of surviving banks as a function of the probability of interbank connections, $c$, for $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.003$. While the percentage of surviving banks remain relatively high between 80% and 92% for $c = 1$, 0.8 and 0.6, it drops to only 28.91% and 1.8% as $c$ reaches 0.4 and 0.2 respectively.

First, we ran simulations with the default parameter setting (see Section 4.2.3), before increasing the returns of risky and risk-free assets respectively. We observe that cutting interbank lending and borrowing channels further increase liquidity shortage, leading to more bank defaults. Under the default parameter setting, see Figure 4.14, the average percentage of survivors remains relatively high until $c \approx 0.5$. At the peak of surviving banks ($p = 50\%$), we find 92% of the banks surviving in a fully-connected interbank network with $c = 1$. When the connection probability falls to $c = 0.8$, we still have 91% survivability at $p = 50\%$. This survivability becomes 80% in a network with $c = 0.6$. Below this connection probability, the percentage of surviving banks plunged to 29% and 2% for $c = 0.4$.  

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and $c = 0.2$ respectively.

Similar outcomes are found in an interbank network where the return of risky asset was raised from $p^+ = 0.06$ to 0.15 (see Figure 4.15(a)). The peak at $p = 25\%$ in a complete network has 90\% surviving banks, 88\% when $c$ drops to 0.8 and 80\% when $c$ equals 0.6. The further decrease in interbank lending channels results in a sudden decrease of survivior to 30\% at $c = 0.4$ and 3\% at $c = 0.2$. However, in the case where risk-free return has been increased from $r_b = 0.003$ to 0.03, the average percentage of surviving banks only falls from 100\% at $c = 1$ to 76\% at $c = 0.2$ (see Figure 4.15(b)).

To investigate why the percentage of surviving bank is less sensitive to decreasing interbank linkages if the return of risk-free assets are high we look at the investment strategy and borrowing behaviour of banks.

As the strategy of investment only changes with bank’s risk aversion and the expected return of both assets, it is independent of the decrease in interbank linkages. Given fixed risk aversion and expected returns, banks always invest into the same fraction of risky and risk-free assets. In fact, when comparing $y^*$ the fraction of invested risky asset for all three return scenarios as $c = 20$, we have found $y^*$ to be always around zero if the percentage of surviving banks peaks.

Next, we compared the average amount of borrowings per time step when $c = 20$ for all three return scenarios. We have found borrowings to be very very low at the peak of percentage survival for all three cases. But as low borrowings can be either due to low demand for cash or due to the limited access to it or both, we also investigated the average amount of borrowings when $c = 100$. Comparing the amount of borrowings when $c = 100$ and $c = 20$, we have found borrowings to be in general relatively low if risk-free return is high. At $c = 100$ it is only 1.7 times higher than at $c = 20$. However, in the original return scenario and if risky return is high the demand for borrowings is 5.9 and 5.6 times higher at $c = 100$

Hence, the survival of banks is less sensitive to decreasing interbank linkages if demand for borrowings is low enough such that a limited access to cash becomes less crucial.

### 4.4.2.2 Increasing the network size

Of course, the Singapore interbank market is puny compared to the interbank markets in the US, Europe or Japan. Perhaps the poor survivability of banks that we have found is due to the small size of the network, and not so much because of short positions in their investments? Certainly, experts have found that larger networks offer more possibilities for risk sharing. To check whether this will also be the case for us, we compare the simulations of two networks with identical settings but with different sizes. That is, in addition to simulating interbank networks with $N = 32$ banks, we ran additional simulations with a
(a) The average percentage, $\mu$, of surviving banks for various $c$ and the following asset returns: $\rho^+ = 0.15$, $\rho^- = -0.05$, $r_b = 0.003$.

(b) The average percentage, $\mu$, of surviving banks for various $c$ and the following asset returns: $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.03$.

Figure 4.15: The average percentage, $\mu$, of surviving banks for increased risky and risk-free returns respectively. While the percentage of surviving banks remain relatively high for densely connected networks it drops radically as $c$ reaches 0.4 and 0.2 respectively.
larger interbank network of 100 banks. Other than the size, all settings follow the
default parameter setting (see Section 4.2.3). We also assumed a fully connected
network.

Analyzing the survivability results for a network of 100 banks, we find that
average percentage of surviving banks increases for very low and very large $p$, as
shown in Figure 4.16. Interestingly, Figure 4.17 shows that the amount of short
positions decreased relative to the 32-bank network for low and large success rates
$p$ (see Figure 4.19(a)). Since in our model surplus banks either do investments
or give interbank loans until liquidity demand is satisfied or cash reserves are
exploited, a decrease in overall investments implies an increase in overall interbank
lending. This means that illiquid banks are more likely to be able to satisfy their
demand for cash in a larger network than in a smaller network.

![Figure 4.16: The average percentage, $\mu$, of surviving banks in a fully connected
interbank network with 100 banks and the following asset returns: $\rho^+ = 0.06,$
$\rho^- = -0.05,$ $r_b = 0.003$. Apart from a pronounced peak at the probability
$p = 50\%$ the lifetime increases in the extreme limits of $p$, where risky investments
are very successful or very unsuccessful.](image)

4.4.3 Short Sales

In this subsection we will provide more insight into the nature of the short selling
investment strategy. By analyzing the lifetime of banks, the amount of short
positions and leverage, we investigate whether and how short positions affects the survivability of banks. This analysis is performed for various scenarios, where amongst others we change the asset returns and success probabilities.

For a complete network with 32 risk-aware banks simulated using the default parameter settings (see Section 4.2.3), we assume that short sales are allowed and risk-free borrowing equals lending. Figure 4.18 shows the average percentage of surviving banks, which strongly peaks at probability $p = 50\%$ where 92\% of banks survive. For probabilities $p < 40\%$ and $p > 55\%$ the number of survivors rapidly drops to zero.

When we compare the average percentage of surviving banks with the amount of short sales (see Figure 4.19(a)), we observe that the peak in Figure 4.18 coincides with a minimum in short sales. In fact, there are no short sales at all for $p = 50\%$ but the amount of short sales increases once $p \neq 50\%$. A trivial explanation for the increase in short sales at $p \neq 50\%$ is that once the probability for a successful risky investment, $p$, increases banks start to sell short risk-free assets. The return from such short sales is used to invest into additional risky assets. On the other hand the situation is reversed once the probability $p$ decreases. Banks then borrow risky assets and speculate to return a lower amount than initially borrowed. The return from risky short sales is invested into additional risk-free investments. However, at $p = 50\%$ the uncertainty of a risky investment to be
Figure 4.18: The average percentage, $\mu$, of surviving banks in a fully connected interbank network with the following asset returns: $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.003$. There is a pronounced peak at $p = 50\%$. It coincides with a minimum in short sales in Figure 4.19(a).

successful is equal to the uncertainty that the same investment is unsuccessful. At this value of $p$ banks find short positions unprofitable, which in turn results in a relatively stable network with more than 90% of banks surviving. The symmetry in the number of both risky and risk-free short positions seems to be characteristic for this parameter setting per se as we will shortly see in other scenario settings.

Besides the short positions we are also interested in the leverage, $L$, of a bank. Excessive leverage has been indentified as one possible source of banking failure [64]. The leverage is defined as

$$L = \frac{\text{value of total assets in the portfolio}}{\text{own money invested in the portfolio}}.$$  

So $L = 1$ means investments have been made by using own money only, hence no short positions has been taken. To provide an easy comparison of leverage between scenarios, we particularly look at the leverage per bank per time step.

Figure 4.19(b) shows the amount of leverage per bank per time step for the default parameter setting scenario. Leverage per bank per time step is shown to be directly proportional to short sales per bank per time step. This is true as banks take on leverage only through holding short positions. Again we observe a symmetry in the level of leverage across the range of $p$. It coincides with the symmetry in the number of short positions that seems to be characteristic of this specific parameter setting as discussed above.
Figure 4.19: Short sale and leverage per time step obtained from a fully connected interbank network with the following asset returns: $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.003$. The minimum in short sales coincides with the peak in average percentage of surviving banks in Figure 4.18. Leverage per bank per time step is directly proportional to short sales per bank per time step.
Figure 4.20: The average percentage, $\mu$, of surviving banks, short sale and leverage per bank per time step in a fully connected interbank network with the following asset returns: $\rho^+ = 0.15$, $\rho^- = -0.05$, $r_b = 0.003$. The distinct peak of average percentage of survivors is at $p = 25\%$ with $90.43\%$, where the number of short positions is not zero and leverage is slightly over 1.
We now move on to the next two scenarios where either the risky asset return or risk-free assets return is changed.

**Increasing the return of risky asset:** First, we increase the return of risky asset $\rho^+$ from 6% to 15%, while letting the remaining parameter settings as is. Figure 4.20(a) shows results on the average percentage of surviving banks with increased risky asset. Comparing the 'original' results on average percentage of surviving banks in Figure 4.18 to Figure 4.20(a) we observe a shift of the peak from $p = 50\%$ to the lower value $p = 25\%$. In fact, 90% of banks survive at $p = 25\%$ compared to 92% at $p = 50\%$ in the 'original' scenario.

Investigating the results on short positions in Figure 4.20(b), however, we find that with the new parameter setting an absence of short sales does not necessarily lead to a maximum percentage of surviving banks. In fact, banks are not short selling at all at $p = 30\%$ but only 53% survive, while the peak is at $p = 25\%$, where banks already hold risky short positions. Furthermore, we do not find the symmetry in the amount of short positions anymore. The number of risk-free short positions at very high $p$ is much larger than the number of risky short positions at very low $p$.

Overall, short positions per bank per time step decreased compared to the 'original' scenario in Figure 4.19(a). The number of risky short positions per bank per time step at $p = 5\%$ decreased from $2.12 \times 10^3$ in the 'original' scenario to 620. The number of risk-free short positions per bank per time step at $p = 95\%$ decreased slightly from $2.28 \times 10^3$ to $1.50 \times 10^3$. While the basic short sale strategy remained the same, with high $p$ leading to high number of risky short sales and low $p$ leading to high number of risk-free short sales, the increased return of risky asset seems to suppress the need for holding a lot of short positions. The dramatic decrease in risky short positions is due to the increased repayment in case short selling does not work out. A possible explanation for the slightly lower number of risk-free short positions is, that the higher a possible profit is, the less one needs to sell short to retain one's maximum utility. Equivalently, overall leverage is also lower than the leverage of the 'original' scenario.

Furthermore, Figure 4.20(b) shows that the number of risk-free short sales, $1.50 \times 10^3$ is much higher than the number of risky short sales with 620. As discussed earlier, the significantly low amount of risky short positions is due to the increased repayment in case short selling does not work out.

**Increasing the return of risk-free asset:** Now we increase the return of risk-free asset $r_b$ from 0.3% to 3%, while letting the remaining parameter setting stay as they are. Figure 4.21 shows the average percentage of surviving banks, the amount of short positions and leverage for this new scenario.
Figure 4.21: The average percentage, $\mu$, of surviving banks, short sale and leverage per bank per time step in a fully connected interbank network with the following asset returns: $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.03$. Every bank considers holding short positions. While banks are still holding a small amount of risky short positions at $p = 70\%$, at $p = 75\%$ they are already selling short risk-free assets.
By only increasing the risk-free return, we observe a shift of the peak in average percentage of surviving banks from originally $p = 50\%$ (see Figure 4.18) to $p = 70\%$ (see Figure 4.21(a)). Furthermore, the peak is now less sharp but changed into a plateau with slightly higher percentage of survivors of 100% at $p = 75\%$ and 100% at $p = 70\%$.

Interestingly, with increased risk-free return we find that every bank considers holding short positions. While banks are still holding a small amount of risky short positions at $p = 70\%$, at $p = 75\%$ they are already selling short risk-free assets.

Again high $p$ leads to high number of risky short sales and low $p$ leads to high number of riskfree short sales, the combination of higher risk-free return and low $p$ leads to an even higher number of risky short sales. Indeed, Figure 4.21(b) shows that the amount of risky short sales increased to $2.86 \times 10^3$ at $p = 5\%$ from originally $2.12 \times 10^3$. Another straightforward observation is that the number of risk-free short sales decreased, namely from $2.28 \times 10^3$ to $1.50 \times 10^3$ at $p = 95\%$. It is trivial to hold less risk-free short positions as the interest rate for borrowing risk-free asset increases to 3%, while the possible return for risky asset stays the same.

If we compare the amount of short sales per bank per time step in Figure 4.21(b) to the amount of short sales per bank per time step in Figure 4.20(b), we see that the number of risky short sales per bank per time step with $2.86 \times 10^3$ is far higher than 620 at $p = 5\%$. This is due to the fact that the return of risk-free asset of 3% in Figure 4.21(b) is much higher than 0.3% in Figure 4.20(b). Also the additional risky repayment of 6% in Figure 4.21(b) is much lower than 15% in Figure 4.20(b). As expected, the number of risk-free short sales are slightly lower, namely $1.06 \times 10^3$ (see Figure 4.21(b)) compared to $1.50 \times 10^3$ (see Figure 4.20(b)). This outcome is intuitive as in Figure 4.20(b) interest for borrowing risk-free assets was lower and expected return on risky assets was higher.

So far our results imply that high leverage has negative impact on the survivability of banks in general. However putting average survivability aside, is leverage advantageous for banks in terms of wealth? In particular, is wealth of banks increasing with leverage? To answer this question we have a look at the capital and leverage of each bank for different network performances.

As per definition of our model the capital of a bank comprises of a liquid part (cash) and an illiquid part (inaccessible money) (see Equation 4.7). Banks only go bankrupt if the liquid part becomes negative, i.e. bank has negative cash. Hence, while a bank can fail even with positive capital, it can survive with negative capital. Figure 4.22 shows leverage on the x-axis and capital on the y-axis for 32 banks of a single run. In this run roughly 60% of banks survive. The blue data points show surviving banks, the red ones show banks that failed. Comparing
all 32 banks, those with the highest leverage have lower capital than banks with lowest leverage. In fact, the latter tend to have the highest capital. However, we do not observe any correlation between both values as leverage becomes neither too high nor too.

In Figure 4.23 we change to a simulation scenario, where almost all banks (92%) survive and again compare leverage and capital of each bank. This time we observe that all banks have the same amount of leverage but various amount of capital, hence showing no correlation between both values at all. Similarly, we do not find any correlation between leverage and capital if the comparison is done for a scenario where all banks fail (see Figure 4.24).

However, looking at simulations individually may not give us full insight into the correlation between leverage and capital in general, as the scale of both in each simulation is very limited. Hence, in Figure 4.25 we combine the data of all three simulations into one. The red data points are taken from simulation 2, where all banks have leverage 1. In simulation 1, represented by blue data points, leverage is between 1.5 and 2.2. Lastly, leverage is between 50 and 90 in simulation 3 (black data points). This comparison of leverage and capital across simulations shows us that there is a trend for higher capital if leverage is significantly high - however this increased wealth comes with survivability as the trade-off.

4.4.4 Regulations

In this section we compare how the lifetime of banks changes if regulations on leverage are introduced. Because in our simplified model leverage only occurs when banks hold short positions, we compare the following three regulation scenarios to understand the impact of them: (1) short sales are allowed and un-
regulated, (2) short sales are allowed but penalized and (3) short sales are not allowed.

Risk-free borrowing > risk-free lending: Until now we have assumed that the risk-free lending rate is equal to the risk-free borrowing rate. Risk-free lending rate is the rate an investor gets when he holds long positions of the risk-free asset. If he holds short positions of it he has the risk-free borrowing rate. Setting risk-free borrowing rate higher than the lending rate, acts as a penalty for selling short risk-free assets. In this scenario we simulate exactly this by increasing the risk-free borrowing rate.

For the original parameter setting (see Subsection 4.2.3) we increase the risk-free borrowing rate from 0.3% to 1%. Figure 4.26 shows that this dramatically increases the average percentage of surviving banks at $p = 55\%$ and $p = 60\%$ from originally 0% to 74% and 42% respectively. The reason for the increased percentage of surviving banks is shown in Figure 4.27. As risk-free borrowing rate increases the amount of risk-free short sales decreases. However, increasing the risk-free borrowing rate is only enough to save banks from default at certain values of $p$, here 55% and 60%.

Next, we increase the return of risky asset from 6% to 15% and set the risk-free borrowing rate again at 1%. Here, we are able to observe the same outcome as above. Higher risk-free borrowing rate only increases the amount of survivors for certain values of $p$ around the peak (see Figure 4.28).

Then we increase risk-free return or lending rate from 0.3% to 3% and set risk-free borrowing rate to 5%. This relatively high risk-free borrowing rate makes banks practically avoiding risk-free short positions, except at $p = 95\%$. From $p = 90\%$ to $p = 0.75\%$ banks do not hold any short positions, while they originally would. Figure 4.29 shows the resulting plateau of maximum percentage of
Figure 4.26: Average percentage, $\mu$, of surviving banks from an interbank network with $N = 32$ banks with $p = p_b$, $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.003$, and $c = 1$. Risk-free lending rate is 0.03%, while borrowing rate is 0.1%.

Figure 4.27: Short sales per bank per time step from an interbank network with $N = 32$ banks with $p = p_b$, $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.003$, and $c = 1$. Risk-free lending rate is 0.3%, while borrowing rate is 1%.
surviving banks, which spans exactly over the values of $p$ where no short sales exist.

**Short sales are disallowed:** When short sales are disallowed, investors are only able to hold long positions. This is realized by constraining $y^*$ in chapter A.0.9. We change equation A.17 into

$$y^* = \min \left[ \frac{(E[r_A] - r_F) B}{\sigma_A^2}, 1 \right],$$

where $y^* \in [0, 1]$. While previously banks would have hold short positions in risky asset, indicated by $y^* < 0$, they now invest everything in risk-free asset and nothing in risky asset as $y^* = 0$ now. Equivalently, when previously $y^* > 1$ means banks would hold short positions in risk-free asset, now $y^* = 1$ having banks invest everything in risky asset and nothing in risk-free asset.

We simulate this scenario again for three different asset returns. First, we run simulations with the default parameter setting (see Subsection 4.2.3). Then we increase the return of risky asset from $\rho^+ = 0.06$ to $\rho^+ = 0.15$, and finally we increase the return of risk-free asset from $r_b = 0.003$ to $r_b = 0.03$.

Figure 4.30 shows the comparison of the average percentage of surviving banks between allowing short sales and not allowing them in the default parameter setting.
Figure 4.29: Average percentage, $\mu$, of surviving banks in an interbank network with $N = 32$ banks with $p = p_b$, $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.03$, and $c = 1$. Risk-free lending rate is 3%, while borrowing rate is 5%.

Figure 4.30: Average percentage, $\mu$, of surviving banks in an interbank network with $N = 32$ banks with $p = p_b$, $\rho^+ = 0.06$, $\rho^- = -0.05$, $r_b = 0.003$, and $c = 1$. When short sales are disallowed, average percentage of surviving banks increases.
setting scenario. In both cases at \( p = 50\% \) bank are not short selling, hence the average percentage of surviving banks are the same. As \( p < 50\% \), banks would start selling short risky assets if they were allowed. However, with the constraint of \( y^* \) to be at least zero, banks invest nothing into the risky asset but everything into the risk-free asset. Since the return of risk-free assets is a fail-safe investment, all banks survive. When \( p > 50\% \), banks would start selling short risk-free assets if they were allowed. With the new constraint of \( y^* \leq 1 \), however, banks are restricted to invest nothing into the risk-free asset but everything into the risky asset. Then the average percentage of surviving banks should increase with the increase of the success probability \( p \) of risky asset. Exactly this outcome is shown in Figure 4.30. At \( p = 55\% \) the average percentage of survivors is the lowest with 73%. As \( p \) increases the strategy of investing everything into risky assets becomes more profitable leading to more average surviving banks.

Figure 4.31 shows the comparison of the average percentage of surviving banks between allowing short sales and not allowing them when increasing the return of risky asset and risk-free asset respectively. When short sales are allowed and risky asset return increases from \( \rho^+ = 0.06 \) to 0.15, we have seen that banks do no hold short positions at \( p = 30\% \) (see Figure 4.20(b)). Hence, the average percentage of surviving banks when short sales are allowed and not allowed coincide at exactly \( p = 30\% \) (see Figure 4.31(a)). While the bank’s strategy for \( p < 30\% \) would be holding risky short positions when short sales are allowed, banks hold 100% long positions in risk-free asset and 0% in risky assets when short sales are not allowed. Hence, the average percentage of surviving banks for \( p < 30\% \) is at 100% when short sales are disallowed. While the strategy for \( p > 30\% \) would be holding risk-free short positions when short sales are allowed, when short sales are not allowed banks invest everything long into risky assets. Hence, the more \( p \) approaches 95% the less risky is the investment and the more banks survive.

If we increase the risk-free return from \( r_b = 0.003 \) to 0.003, we observed that banks do not consider having long sales regardless of the value of \( p \) (see Figure 4.21(b)). While banks are still holding a small amount of risky short positions at \( p = 70\% \), at \( p = 75\% \) they have started to sell risk-free assets short. Hence, when short sales are not allowed banks invest everything into the risk-free asset until \( p = 70\% \) and for \( p > 70\% \) banks invest everything into the risky asset. Interestingly, the increase in risk-free return and not allowing short sale prevent banks from failure.

4.4.5 Summary and Outlook

In our studies on a random network with 32 banks we have investigated (1) the consequences of risk-aware, cautious and eager banks, (2) Network size effects and systemic risk through interbank loans, (3) the effects of leverage through
Figure 4.31: Average percentage, $\mu$, of surviving banks in an interbank network with $N = 32$ banks with $p = p_b$. 

(a) $p^+ = 0.15, p^- = -0.05, r_b = 0.003$

(b) $p^+ = 0.06, p^- = -0.05, r_b = 0.03$
short sales and (4) the effectiveness of leverage regulations.

The results show that cautious banks do not necessarily survive longer than eager banks. Independent of the bank’s market estimation, it is able to prevent its default whenever it avoids leverage.

Also, we have observed no increased contagion for increased interbank lending. Instead we find that lifetimes of banks improve when the number of linkages between banks and therefore the access to interbank lendings increase. Furthermore, increasing the size of the network has a stabilizing effect as well. In a large network we observe more interbank lending activities accompanied by improved lifetimes especially in market situations when risky investments are highly successful or highly unsuccessful.

An important finding of our work is, that although banks in our model have complete market information they do not perform well if they invest with high leverage. The comparison of average percentage of surviving banks for regulated scenarios with the average percentage of surviving banks for the unregulated case reveals that it may be advantageous to introduce regulations for specific market situations. For a network consisting of 32 banks we find that if the system is left unregulated the amount of surviving banks are very low when banks chose to either strongly invest in risky or risk-free assets. When banks are more reluctant towards short sales we observe a sharp peak in the average percentage of surviving banks. This peak widens when we introduce regulations in form of penalties. If short sales are completely forbidden the network becomes very stable and banks go bankrupt rather seldomly.

Apart from the regulations on leverage discussed in this thesis it would be important to test possible interventions such as emergency interventions paid by public money, such as the injection of new capital or nationalizing the bankrupt bank. In addition we plan to analyze emergency interventions paid by private money, such as the acquisition of a bankrupt bank by others. Based on this complete scenario analysis we will be able to recommend the optimal combination of regulations and interventions for specific market situations.

Although our simulations on an artificial random network provide a good understanding of how lifetime of banks can be prolonged through different measures it would be desireable to use the actual Singapore interbank network to provide an even more realistic estimation of interbank market dynamics.
Chapter 5

Conclusion

In this thesis we have presented two projects on systemic risks in complex networks using agent-based modelling.

The model of the first project simulates a dynamical multi-layer supply chain network comprising of levels of suppliers, manufacturers and one level of retailers. Instead of looking at the supply chain as a static architecture, we have chosen a model that is able to incorporate realistic features that triggers equally realistic dynamics in the supply chain.

One of the features is price dispersion. It assigns different prices to the same goods, which is what we often find in real world. It makes consumers to always look out for the best price before deciding to buy a product at a certain retailer. Hence, manufacturers are constantly on the search for cheaper suppliers. As they change suppliers, the supply chain changes with it.

Another feature is production shortfall. Sometimes, production quota are not able to be met. In the model, we assume production shortfall to appear randomly to any manufacturer and then cascades through the supply chain propagating loss to connected customers.

Using these features, the model is able to reproduce a realistic network structure. It shows that in equilibrium only a few but large raw material suppliers and retailers are at both the bottom and top layer. Whereas there are many small and medium sized manufacturers on the middle layer of the supply chain.

In fact, it is well known that firms in the middle of the supply chain are in a more advantageous position as they face lower costs in terms of distance and volatility [75]. Moreover, it has been found that strategic alliances involving mid-level firms is also advantageous due to the creation of customer values [76]. To further investigate this phenomena we have incorporated different strategies to the management of mid-level firms and investigate the benefit of these once applied. In particular, we allow these firms to diversify by either buying over one
of its suppliers, or by buying over one of its customers or by merger. Our goal is to explore whether such management practices can enhance a mid-level firm’s chances of surviving an economic downturn.

In theory, the motives for merger and acquisitions are two-fold. First, there are the economic motivations to improve internal efficiencies through the economy of scale, economy of scope, and synergies, and the acquisition of resources such as raw material, technology, products and brands, distribution chains, and management skills. Second, there are strategic motivations on enhancing external relations through expanding markets by product growth, and geographical expansion. Through mergers, competing firms pool their capitals and market shares, to leap-frog over leading competitors [88, 89, 90].

The diversification we have modelled is more commonly referred to as vertical integration. On a supply chain, vertical integration means the expansion of the production program to products of the previous, next, both or all levels of the supply chain. The expansion of products to the next level is called forward vertical integration. A firm can achieve this by setting up the capabilities to make products at the next level, or by buying over its own customers. In contrast, backward vertical integration means that production capabilities are extended to the previous level. During the industrial age successful companies tried to own as much of their supply chain as possible. The driving force for this strategy was the gain in efficiency through economies of scale. Nowadays, companies in the oil industry apply this kind of diversification [91].

In fact, many of the world’s largest successful enterprises, for example, IBM, Hewlett-Packard, General Electric, Wesfarmers, Bidvest, or ITC Limited and Mitsubishi to name a few, are highly diversified. The strategy of General Electric, Wesfarmers, Bidvest and ITC Limited is called conglomerate diversification. These firms regularly market new products that are technologically and economical unrelated to their current products. This kind of diversification was very common after the Second World War, and today we understand it as diversification across multiple supply chains.

To identify the benefits of these management strategies we simulated the supply chain network under normal economic conditions and during a financial crisis, when the amount of demand by retailers is lower. Mid-level firms were only allowed to diversify during the crisis. As the survival of firms was our main concern in this project we have recorded the lifetimes of undiversified and diversified firms. In particular, we determined the cumulative distribution of lifetimes in each level of the supply chain for undiversified and diversified firms. Furthermore, we also observe how a specific mid-level firm performs if it either chooses to diversify or not to diversify while living through the start of a financial crisis.

In low demand times the number of long-lived firms in the lower to middle
level of the supply chain is lower, while the average lifetime of firms is shorter in the upper levels. However, by allowing mid-level firms to forward or backward vertical integrate during crisis not only increases average lifetime significantly but also the number of long-lived firms. While horizontal merger also increases average lifetime it is not at all as effective in saving long-lived firms than the other two strategies. Out of all three management strategies, in times of low demand forward vertical integration – having customers obliged to order only from their parent company – maintains the largest average lifetime and the highest number of long-lived firms. These firms that buy over customers during crisis need to compete less for the already lower order volume than firms using other strategies. This suggests, that securing orders when demand is low, in general, is safer than securing supplies or increasing working capital.

This picture however changes if demand increases. By testing all three strategies for normal times we have found that this time the favourable strategy is no longer forward vertical integration. In fact, it even performs slightly worse than during crisis times in terms of average lifetime and number of long-lived firms. Instead, the other two remaining strategies increased their performance with backward vertical integration being the most advantageous strategy. While some orders can be secured for the parent company by forward vertical integration, the bought-over subsidiary company loses the potential to order with the cheapest and also decreases its ability to hedge against possible production shortfalls. Then in times of higher demand it seems that the disadvantage of the subsidiary company out-weights the advantage of the parent company. This agrees with the fact that backward vertical integration becomes the better strategy. The greater the order volume is in our model the higher a production shortfall can be, if it occurs. Hence, hedging against shortfalls by buying over suppliers that become obliged to only supply their parent company becomes, in general, the better strategy.

Additional results on tagged comparisons suggest that backward vertical integration is also the preferable strategy if firms live through the transition of high demand to low demand times. Average lifetime can be slightly increased this way. In fact, our simulations suggest firms to always diversify. In times of higher demand or normal economic situation, firms should concentrate their efforts in hedging against production shortfalls by backward vertical integration. This should also be the main effort when the economic situation undergoes a turnaround. After the high demand time has settled into a lower demand time, firms should apply forward vertical integration instead. In this case the advantage of securing orders in times of lower order volume becomes more crucial instead.

As mentioned in Chapter 3, we are not simulating the supply chain of any specific industry. Therefore, we assume a linear increase of sales price mark-ups across the supply chain. When a linear mark-up function is used, the results
show that forward vertical integration is favoured during a financial crisis whereas backward vertical integration is preferable when firms live through sudden economic downturns. If we change the form of the mark-up function, we may end up favouring backward vertical integration during a crisis and forward vertical integration when facing a sudden downturn. However, we believe that a non-linear mark-up function will not affect our conclusion that overall as far as firms in the middle of a supply chain are concerned, the main benefit of diversification is not to increase profits or lower operating costs. Instead, diversification helps mid-level firms more effectively manage the systemic risks they are exposed to, and help them survive longer in a competitive and uncertain business world.

In the second project we used an agent-based model to simulate an interbank network, where banks are agents interacting with each other through interbank lendings and borrowings. In our work we were particularly interested to understand how failure of banks may be connected to short sales and network effects.

As discussed in Section 4.3 we developed algorithms capable to create an estimated structure of the original network based on the time series of lending and borrowing data. However, without the access to the required data by the Singaporean authorities, we had no possibilities to estimate the Singapore interbank network. The presented results have therefore been obtained from a random network.

In our model (see Section 4.2) we allow banks to do both risk-free and risky investments, with risk-free investments giving a 100% positive but low return while risky investments possibly yielding much higher positive returns but higher losses in case the investment is unsuccessful. We further assume that both investments originate from the outside of the financial sector and banks are offered an infinite number of such investments. Furthermore, banks that are opting for short positions will always find assets to borrow and buyers to sell their assets to.

To realize a simple investment behaviour we used portfolio theory (see Section 6). It uses, amongst others, the utility function and risk-aversion of a bank to determine the optimal amount to invest into a risky stock. We assume that all banks have a constant relative risk aversion utility function. This function basically states, that banks are generally risk averse i.e. even when their demand for risky asset rises as they get wealthier, the banks still invest the same fraction of wealth into risky assets.

In theoretical and empirical studies it is argued that subsidies and regulations are applied prior to every financial crisis [6, 7]. Such regulations may distort domestic supply or demand in the long run, which leads to mispricing of assets and miscalculation of risks. As a result of increasing asset prices it has been observed that prior to financial crises leverage increased significantly [81, 82, 83]. At the same time, underestimation of risks leads to higher ratings for risky
assets. This encourages investors to go in with a higher leverage [81]. Jickling [64] gives a comprehensive list of identified causes of the financial crisis of 2007, which points out the complexity behind financial crises. To take a step towards understanding the complexity of banking crises, we aimed to analyse the impact of different regulations where banks are (1) free (2) penalized or (3) forbidden to do speculations in form of short positions. Contagion and systemic risks were realized through interbank lendings and borrowings.

For the unregulated case we found that in market situations where banks avoid short sales, the lifetime of banks has a sharp peak. This finding is independent of the amount of chosen return for the risky and risk-free asset. With the introduction of penalties for short sales we observe that the sharp peak in lifetimes widens i.e. with a reduced amount of short sales the number of surviving banks increases. However, in the extreme limits where risky investments are always successful or unsuccessful the number of failing banks does not change. Finally, when we completely forbid short sales we observe that especially in the two extreme limits all banks survive. This result has to be expected when banks only invest into assets that are 95% successful. Interestingly we found that depending on the return for risky and risk-free assets there exists a particular success rate at which a noticeable number of banks fail. This is an important finding as it shows, that even if short sales are forbidden there is no guarantee that all banks survive. We believe that this drop in lifetimes is caused by banks with low risk aversion that are more vulnerable towards failure due to unsuccessful investments. Compared to the penalized and unregulated case, the scenario with forbidden short sales results in longer lifetimes for all market situations.

Our results indicate that the amount of short sales in our model has a significant influence on the lifetime of banks. From the study of the scenario without short sales we can see that short sales are not the only cause of bank failure. In specific market situations when banks with low risk aversion strongly invest into risky assets, there may occur significant increases in the number of failing banks. In our model the failure of banks is therefore a combined effect of large volumes of short sales and large volumes of risky long positions.

In addition we investigate the consequences of incomplete or wrong market information. To do that we let banks speculate based on slightly wrong investment probabilities. We identify that the over- and underestimation of probabilities for successful risky investments has similar severe consequences with a sharp drop in lifetimes.

To understand the importance of the network structure itself we have investigated how a reduced number of linkages affects the lifetime of banks. From our results we can clearly see that the number of surviving banks is proportional to the number of linkages. This implies that in our model the benefits of interbank lending outweighs the disadvantages of contagion. If all banks would be isolated
from each other the investment strategy of each bank would sooner or later result in failure of the bank. On the other hand, if the bank is connected to other banks and is able to borrow money if required, the lifetime sharply peaks in market situations where short sales are rare. This suggests that interbank lendings have a stabilizing effect in our interbank network.

Apart from network coverage we have also studied network size effects. When we compare our results for 32 and 100 banks we observe that especially in the extreme limits when risky investments are highly successful or unsuccessful the lifetime of banks increases. This intriguing result is accompanied by an overall reduction in the volume of short sales or equivalently an increased volume of interbank lendings. Therefore we conclude that banks in our network lend more money to other banks in need if they are connected to many banks. From a different perspective this means that banks that are close to failing profit from a large number of linkages i.e. well connected banks in our network have easier access to interbank lendings. However, it is important to note that the increase of lifetime for larger networks is small and only occurs close to the extreme cases where risky investments are either highly successful or highly unsuccessful. Overall the amount of short sales remains the dominating factor for bank lifetimes. Interbank lendings may only act as damping effect in large and highly connected networks.
Chapter 6

Appendix A: Portfolio analysis

In this chapter I will explain some portfolio theory that is used in this project. The methods described are used to simulate investment behavior of banks in the network. It is assumed that banks only invest in two types of assets, one risky and one risk-free. The mathematical methods introduced in this section consists of a small part of modern portfolio theory. In order to get a more comprehensive introduction into portfolio theory a more complete introduction is given in [92, 93].

A.0.6 Weights of assets in a portfolio

The weight of asset \( i \), \( y_i \), in a portfolio is

\[
y_i = \frac{\text{Value invested in } i}{\text{Total value of portfolio}}. \quad (A.1)
\]

The weights of all assets in a portfolio must sum to 1.

Example Consider the two-asset portfolio in this project, consisting of a risky asset \( A \) and risk-free asset \( F \). Assume that $200 is held in asset \( A \) and $600 is held in asset \( F \). The total value of this portfolio is $800. Using equation A.1 the weights for both assets are \( y_A = \frac{200}{800} = 0.25 \) and \( y_F = \frac{600}{800} = 0.75 \). Both weights sum up to 1.

In the above example both weights are positive. However, investors could invest with money they get by selling assets they do not own. In this case, the weight of the asset they sell is negative.
Example  Again consider a portfolio of a risky asset $A$ and risk-free asset $F$. Assume that an investor borrowed asset $A$ and sold them for $200. He also is holding $600$ in asset $F$. Borrowed money carries a negative sign, hence the total value of the portfolio is $400$. Again, using equation A.1 the weight for both assets are $y_A = \frac{-200}{400} = -0.5$ and $y_F = \frac{600}{400} = 1.5$. Again, both weights sum up to 1.

If an investor sells assets he does not own, this strategy is called short selling or borrowing. In this case the weight of this asset is negative. The opposite is called selling long or lending, where the weight of this asset is positive. The idea of short sales will be discussed below.

A.0.7 Short sales

Short selling is an investment strategy, that is used to profit from the decline of the price of assets. In combination with long sales, investors can boost the return of their investment. The next example shows how profits can be made by using short sales.

Example  Suppose the value for each share of company $A$ is currently $10$. An investor speculates that the share will drop in value. He borrows 100 shares and sell them for $1000$. Then at the time of close-out (the time when he need to return the borrowed shares), the worth of each share of company $A$ falls to $9$. The investor now buys 100 shares for $900$ and return it to the lender. Overall, the investor made a profit of $100$.

In the case above short sales obviously make sense, since the return on the share is negative. But it also makes sense to short sell, if the return is positive. The cash obtained from short sale can be used to invest into a share that produces a higher expected return.

Example  Suppose the expected return of each share of company $A$ is 14% and that of company $B$ is 8%. Further assume than an investor has only $100 to invest into shares of $A$ and $B$. If he invests $100$ into $A$, he receives a return of $14$. However, he could short sell shares of $B$ worth $1000$ and then use this cash and his own and invest $1100$ into $A$. This will give him an expected return of $154$. After close-out he needs to return the short sales and pay $80$. Overall, he will receive an expected profit of $74$.

The above example shows that through short sales expected return can be increased. However, in doing so risk also increases. Whether an investor will make
short positions and to what extend depends on this preference for risk relative to return. Investors with higher risk aversion are less willing to invest into risky investments than investors with lower risk aversion.

A.0.8 Portfolio of a risky and a risk-free asset

Again, consider a risky asset $A$ and a risk-free asset $F$. For asset $A$, assume that $r_{A}^{+}$ is the successful return and $r_{A}^{-}$ is the unsuccessful return. Further assume, that $r_{A}^{+}$ is realized with probability $p$ and $r_{A}^{-}$ with probability $1-p$. For the risk-free asset, return of $r_{F}$ is always realized.

The expected return and standard deviation of asset $A$ are

$$E[r_{A}] = pr_{A}^{+} + (1-p)r_{A}^{-}, \quad (A.2)$$

$$\sigma_{A} = \sqrt{p (r_{A}^{+} - E[r_{A}])^2 + (1-p) (r_{A}^{-} - E[r_{A}])^2}. \quad (A.3)$$

As for the risk-free asset $F$, expected return is $r_{F}$ and standard deviation is zero. Using this with the results of equations A.2 and A.3 the mean-standard
deviation diagram for portfolios can be constructed, as shown in figure A.1.

**Mean-standard deviation diagram** In the mean-standard deviation diagram the $x$-axis is the standard deviation of return, whereas the $y$-axis is the expected return. With standard deviation as a proxy for risk, portfolios with large standard deviations are riskier than those with smaller standard deviations.

Knowing the risk and expected return of the assets $A$ and $F$, the points $[0, r_F]$, $[\sigma_A, E[r_A]]$ are inserted into the diagram. In figure A.1 'risk-free asset investment 1' refers to $[0, r_F]$ and 'risky stock investment 2' refers to $[\sigma_A, E[r_A]]$.

At point $[0, r_F]$, wealth is fully invested into the risk-free asset $F$ and nothing is invested into the risky asset $A$. Whereas, at point $[\sigma_A, E[r_A]]$, nothing is invested into $F$ but everything is invested into $A$.

The line $AB$, drawn through the points is called the ‘Capital Market Line’, CML. Each point or portfolio on this line is a linear combination of both assets. Starting from point $[\sigma_A, E[r_A]]$, where nothing is invested into the risk-free asset, when moving to the left the proportion to be invested into the risk-free asset in a portfolio increases gradually. However, moving to the right further decreases the proportion invested into the risk-free asset, creating a negative weight equivalent to short sales of risk-free assets. The CML has the following linear equation

$$E[r] = r_F + \sigma \frac{E[r_A] - r_F}{\sigma_A},$$

(A.4)

where $r$ is the return of a portfolio and $\sigma$ is the standard deviation of $r$.

**Proof.** Let $x_0$ be the initial wealth invested into a portfolio and let $x$ be the final wealth of this portfolio. Then

$$x = x_0 (1 + r),$$

(A.5)

where

$$r = y r_A + (1 - y) r_F,$$

(A.6)

and $y$ is the weight of asset $A$ and $(1 - y)$ is the weight of asset $F$. Using equation A.6 the expected return of the portfolio is

$$E[r] = y E[r_A] + (1 - y) E[r_F]$$

$$= y E[r_A] + (1 - y) r_F$$

$$= r_F + y (E[r_A] - r_F).$$

(A.7)
Before calculating the variance of \( r \), we have to define the covariance, \( \text{Cov}(r_A, r_F) \), and correlation, \( \rho(r_A, r_F) \), between \( r_A \) and \( r_F \) first. Covariance and correlation show to which degree two return move together. Positive covariance and correlation show that two return tend to move together, whereas negative covariance or correlation show that both returns tend to move into opposite directions. The relation between covariance and correlation is the following

\[
\rho(r_A, r_F) = \frac{\text{Cov}(r_A, r_F)}{\sigma_A \sigma_F},
\]

where

\[
\text{Cov}(r_A, r_F) = E[(r_A - E[r_A])(r_F - E[r_F])].
\]

Since the term \( (r_F - E[r_F]) \) is zero, \( \text{Cov}(r_A, r_F) \) is zero in this case. The variance of \( r \) is equivalent to the variance of two random variables due to A.6

\[
\begin{align*}
\text{Var}(r) &= \text{Var}(y r_A + (1 - y)r_F) \\
&= y^2 \text{Var}(r_A) + (1 - y)^2 \text{Var}(r_F) + 2y(1 - y) \text{Cov}(r_A, r_F) \\
&= y^2 \text{Var}(r_A).
\end{align*}
\]

Rearranging A.10 results in

\[
y = \frac{\sqrt{\text{Var}(r)}}{\sqrt{\text{Var}(r_A)}} = \frac{\sigma}{\sigma_A}.
\]

Substituting A.11 into A.7 gives us A.4.

The AC line in figure A.1 represents portfolios with negative weights in risky asset \( A \). The linear equation is the following

\[
E[r] = r_F - \sigma \frac{E[r_A] - r_F}{\sigma_A},
\]

which is the CML (equation A.4) but with a negative slope. The more an investor moves to the right of the line - equivalent to increasing short positions of the risky asset - the lower is the expected return of the portfolio but the higher the risk.

**Risk-free lending and borrowing** The short sale of risk-free assets is also referred to as risk-free borrowing, whereas long sale is called risk-free lending. If both lending and borrowing rates are equal, the CML line is a continuous line as shown in figure A.1. However, in practice borrowing rates are generally higher than lending rates resulting in a ‘kinked’ CML line, see figure A.2.
Figure A.2: Mean-standard deviation diagram of portfolios of a risky and a risk-free asset showing a kink in the CML line at point $B$. This is due to the higher risk-free borrowing rate, $r_B$, compared to the risk-free lending rate, $r_F$.

The $BC$ line has a slope of $(E[r_A] - r_B)/\sigma_A$ and all points beyond point $B$ are feasible portfolios with negative weights in risk-free asset $F$.

A.0.9 Optimal portfolio

Given the CML, each investor has his own optimal proportion $y$ which he invests into the risky-asset $A$. His preference is represented by his utility function $U(x)$. To calculate the optimal portfolio we are interested in maximizing $U(x)$.

Utility function For this project, a power utility function is assumed. To be more precise, we use the well-known narrow power utility function from [94],

$$U(x) = \frac{B}{B-1}x^{1-\frac{1}{\theta}},$$

(A.13)

where wealth $x > 0$ and $B > 0$. This is a constant relative risk aversion (CRRA) utility function. It is characterized by the positive decreasing absolute risk aver-
and the constant relative risk aversion

\[ RRA(x) = x ARA(x) = \frac{1}{B} \]

The positive \( ARA(x) \) implies that investors are risk averse. At the same time a decreasing \( ARA(x) \) with increasing wealth \( x \) lets investors' demand for risky asset rise as their wealth increases. However, because of the constant \( RRA(x) \), investors invest the same fraction of wealth into risky assets.

**Optimal proportion of risky asset**  The preference of each investor is described by equation A.13 with different values of \( B \). To determine the optimal proportion, \( y^* \), of risky asset, we need to calculate the first order condition of the expected utility function, \( E[U(x)] \), with respect to \( y \).

Again, assume the wealth of a portfolio to be A.5 and the realized return of this portfolio to be A.6. Then as shown before, the expected portfolio return is A.7 and the portfolio variance is A.10. Let the expected final wealth of this portfolio to be

\[ E[x] = x_0 (1 + E[r]) \quad \text{(A.14)} \]

and the variance of the final wealth be

\[ \text{Var}[x] = x_0^2 \text{Var}[r] \quad \text{(A.15)} \]

The expected utility function is

\[ E[U(x)] = \frac{B}{B - 1} E \left[ x^{1 - \frac{1}{B}} \right] \quad \text{(A.16)} \]

Maximizing A.16 with respect to \( y \), \( \max_y E[U(x)] \), results in

\[ y^* = \frac{(E[r_A] - r_F) B}{\sigma_A^2} \quad \text{(A.17)} \]
References


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