DEVELOPMENT OF
FUZZY-APPROACH-BASED OPTIMIZATION
METHODOLOGIES FOR SUPPORTING WATER
SUPPLY-DEMAND MANAGEMENT

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DEVELOPMENT OF FUZZY-APPROACH-BASED OPTIMIZATION METHODOLOGIES FOR SUPPORTING WATER SUPPLY-DEMAND MANAGEMENT

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SUMMARY

Management of water supply-demand system is complicated with a variety of uncertainties which may be sourced from information collection, data measurement, and human judgment. The interaction between the declined water supply and increasing water demand would also influence the operation strategies of various system components. These challenges may lead to difficulties of formulating and solving the related management models and bring difficulties to decision making processes. Development of effective decision support methodologies becomes critical for water supply-demand management. This study aims to develop a set of fuzzy-approach-based optimization models for water supply-demand management under uncertainty.

In order to deal with general shaped fuzzy parameters in water supply-demand system, a combined genetic algorithm and fuzzy simulation approach (GAFSA) was developed through integrating fuzzy chance-constrained programming (FCCP) and genetic algorithm (GA) into a general optimization framework. The results showed that GAFSA allowed system constraints violation at specified possibilistic confidence levels, leading to model solutions with higher system benefits. The proposed model could help agricultural water managers analyze the balance of the overall system benefit and the failure risk of environmental compliance. A novel superiority-inferiority-based sequential fuzzy programming (SISFP) model was also proposed for dealing with general shaped fuzzy parameters in water supply-demand system. The fuzzy objective function and constraints with general-shaped fuzzy coefficients could be transformed into their crisp equivalent by using fuzzy superiority and inferiority measures. The results obtained from the study case in Tianjin Binhai New Area, China shows the proposed method has sought a reasonable balance among the water availability, water demand, adoption of water-saving measures, and benefit/cost of each water users.

Further, in water resources allocation problems, uncertainty could exist in many system components and their relationships. An extended fuzzy parametric
programming (EFPP) model was proposed for handling all possible fuzzy conditions in water supply-demand system. EFPP deals with flexible constraints by allowing violation of constraints at certain satisfaction degrees (i.e. $\alpha$ levels), and employs the fuzzy ranking method to handle trapezoidal-shaped fuzzy coefficients. The objective function is defuzzified by using $\beta$ cuts and weighting factors. The reliability of EFPP was tested by comparing its solutions with those from FCCP. A series of decision alternatives at various satisfaction degrees of a water resources allocation case were obtained. Generally, the higher the $\alpha$ level, the lower the system benefit. In comparison, the $\beta$ level in the objective function posed less sensitive impacts on both objective function and model solutions. The comparison results indicated that EFPP performed equally well with FCCP in addressing parameter uncertainties; but it demonstrated a wider applicability due to its extended capacity of handling fuzzy relations in the model constraints and fuzzy coefficients in the objective functions.

Moreover, the integration of decision making and optimization framework is investigated in this study. An integrated fuzzy programming and decision analysis (IFPDA) method was proposed for a multi-layer urban water distribution system management under uncertainty. The balance between the satisfaction degree of achieving system objective and feasibility level of meeting the related constraints were analyzed by fuzzy inference procedures. The results indicated that, IFPDA was advantageous in (i) dealing with fuzzy uncertainties in objective function and both sides of the model constraints and in the context of an urban water distribution system management; (ii) helping analyze trade-offs between minimization of operation cost and reliability of running the system, and (iii) linking optimization model outputs with decision analysis.

Finally, a robust fuzzy programming (RFP) model was proposed for a multi-reservoir system for seeking optimal release strategies under uncertainty and climate change. RFP owns advantages of both SMP and FMP models, which could deal with random inflow in model, balance the solution robustness and model robustness, and handle the economic parameters which are expressed as fuzzy sets. The climate change impact on water supply-demand management was also
considered. CaDENCE was used for precipitation downscaling, and SVM was used for temperature downscaling. BNN model was applied to simulate the monthly inflows of three reservoirs under future climate change conditions projected from HadCM3 A2 and B2 scenarios for providing the input to the optimization model. The results of study case in the area of GVRD, Canada indicated that, under future condition, the water releases would increase at spring and decrease at winter under A2 emission scenario compared to the current condition; and the dry period would extend under B2 emission scenario.
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LIST OF PUBLICATIONS

Journals:


Manuscript in preparation:

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CHAPTER 1 INTRODUCTION

1.1 Background

1.1.1 Water scarcity problem

Water is a critical resource for socio-economic development and for maintaining healthy ecosystems. About 96.5% of water on earth is from seas and oceans, only 2.5% of the planet’s water is freshwater, and about 98% of that freshwater is in groundwater and ice (Kaushik, 2012). Over the past decades, the increasing water scarcity problems have led to great tensions and conflicts among the competing domestic, agricultural and industrial sectors, and brought excessive pressure to the environment. Water scarcity problems are mainly caused by rapid urbanization, growth of the population and impact of extreme weather events.

Rapid urbanization is one of the major causes of water scarcity problems. Currently, approximate 50% of human beings live in cities now, and nearly 60% of the world’s population will be urban dwellers within two decades (UNHSP, 2008). Urban growth is more rapid in the developing countries than that in developed countries. It is predicted that the urban population in 2030 will be double that in 2000 in Asia and Africa (Meinzen-Dick and Appasamy, 2002). Due to such rapid urbanization, about 2/3 of urban population still lacks access to basic services, such as safe and clean water and sanitation. Rapid urbanization leads to an increase of urban water demand and consumption, and brings serious consequences on the environment. Moreover, a rapid increase of built-up areas reduces the capability of natural infiltration and produces rapid flows of peak storm water, subsequently disturbing the local hydrological cycle and environment (Porto et al., 2008).

Global climate change is also a severe environmental issue facing the world today. With the increase of the projected global temperature, it is suggested that there will be more frequent extreme events with a greater magnitude, such as droughts and floods, (Sivakumar, 2011). It is also anticipated that the proportion of the land surface suffering extreme drought may increase by about 10 times by the 2090s.
compared to the present day (Solomon et al., 2009). Generally, climate change would exacerbate the water scarcity problem by: (i) decreasing storage capacity of natural water by glacier or snowcap melting, leading to reduction of the long-term water availability in glacier- or snowmelt-fed river basins; (ii) changing precipitation intensity and patterns and prolonging droughts; (iii) concentrating precipitation and snowmelt into shorter time frames, making more extreme water releases and more sustained drought events; this would influence the reliability and capacity of water supply infrastructure (Pacific Institute, 2009).

1.1.2 Water supply-demand management

Major sources of water supply include surface water, groundwater, desalination, and frozen water. High water consumption rates could guarantee high standard of living. Domestic consumption of water includes drinking, washing, bathing, watering lawn or garden, and other household activities (Michelsen et al., 1998). Agricultural water use includes irrigation, livestock, fishery, and forest use, where irrigation consumes a majority of freshwater resources (Li et al., 2011). The industrial sector water use has been growing in recent years. The main use of industrial water use is for cooling purpose, which could take a quarter to a half of total amount of industrial water use (Economic and Asia, 2009).

The traditional management techniques are based on augmentation of water supply, which places emphasis on satisfying demands by building and developing new water storage and treatment facilities and structures using available sources to satisfy increasing water requirements (Vairavamoorthy et al., 2008). The water supply strategies include reduce water losses during water delivering by upgrading water supply distribution system, increase available water supply by improving cost-effective technologies (e.g. water reuse, recycling, groundwater pumping, desalination of seawater, and rainwater harvesting), and provide integrated water supply connections.

However, in some developing countries, because of the increasing growth of population and rapid urbanization, the capital for new water supply development
projects is failing to meet the increasing demand (Butler and Memon, 2006). Moreover, the supply based management approach may cause over-use of the resources, water pollution and social problems. Hence, it seems more rational to enhance the traditional supply augmentation method by incorporating demand management strategies. Water demand management is based on using new technologies, laws and regulations, and educational programs to promote water conservation and improve water-use efficiency. Demand management strategies are mainly comprised of non-structural measures, such as legal and economic incentives which could change the behavior of water users in order to decrease the water consumption (Butler and Memon, 2006). Examples include installing water efficient fixtures and water-wise landscaping for domestic sector, using non-potable water for industrial process use, replacing water-cooled equipment with air-cooled one for industrial sector, and installing sprinklers (and/or adjusting irrigation controller) with seasonal change for agricultural sector.

1.2 Objectives and Scopes

Optimization model has been deemed as an effective means of planning water supply-demand systems, because it could help determine optimal water allocations among competing sectors, balancing supply and demand of water resources, and dealing with complex interrelationships among various components in systems. However, water supply-demand management is complex with many uncertainties related to water demands and availability, environmental requirements and regulations. In the previous studies (as discussed in the literature review), various optimization methods (e.g. stochastic, fuzzy, interval programming) have been developed to address uncertainties existed in water resources management problems. Nevertheless, there are a number of research gaps in the previous studies: (i) the traditional approach for solving fuzzy programming is restricted to dealing with fuzzy parameters with only special shapes (e.g. triangular); (ii) there are limited tools to handle multiple fuzzy conditions (i.e. both fuzzy relationships and fuzzy coefficients in the model); (iii) there is a lack of studies on integrating decision making within an optimization framework; (iv) there are limited studies on
assessing impact of climate change on management of water supply-demand systems. The objective of this PhD research topic is to develop a set of fuzzy-approach-based optimization methodologies for water supply-demand management under uncertainty. In detail, the scopes of this study are:

(1) To develop a combined genetic algorithm and fuzzy simulation approach (GAFSA) through integrating fuzzy chance-constrained programming (FCCP) and genetic algorithm (GA) into a general optimization framework. GAFSA could tackle generally-shaped fuzzy membership functions on both sides of the model constraints, rather than handle single special forms like triangular or trapezoidal. An agricultural water resources management problem is used for demonstration. A comparison between GAFSA and FCCP in a same study case is also given. To mitigate the limitation of GAFSA in suboptimal solution and time-consuming solution process, a novel superiority-inferiority-based sequential fuzzy programming (SISFP) model is proposed, which could transform fuzzy objective function and constraints with general-shaped fuzzy coefficients into their crisp equivalent by using fuzzy superiority and inferiority measures. The water supply-demand management system in Tianjin Binhai New Area, China, consisting of five sources of water, five water users at three districts (i.e. Tanggu, Hanggu, and Dagang), is used for methodology demonstration. The advantage and necessity of SISFP in dealing with general-shaped fuzzy parameters is further verified by comparing to fuzzy models with both deterministic and specially-shaped fuzzy parameters.

(2) To develop an extended fuzzy parametric programming (EFPP) model for supporting allocation of water resources problems under uncertainty. EFPP could deal with flexible constraints (i.e. fuzzy relations) by allowing violation of constraints at certain satisfaction degrees (i.e. \( \alpha \) levels), and employs fuzzy ranking method to handle trapezoidal-shaped fuzzy coefficients. The objective function is defuzzified by using \( \beta \) cuts and weighting factors. A water resources allocation case is used for method demonstration. The applicability of EFPP is compared with FCCP using the same study case.
(3) To propose an integrated fuzzy programming and decision analysis (IFPDA) method which would deal with fuzzy uncertainty in both model constraints and objective functions, and link fuzzy programming with decision making. The fuzzy decision analysis criteria are enhanced to mitigate the influence of parameter scale problems in decision analysis. A multi-layer urban water distribution system management is used for demonstration. The uncertain information associated with water demands, treatment capacities, water transfer cost, and leakage rate was described as trapezoidal-shaped fuzzy sets and embedded into a fuzzy programming framework. The balances between the satisfaction degree of achieving system objective and feasibility level of meeting the related constraints are analyzed by fuzzy inference procedures.

(4) To develop a robust fuzzy programming (RFP) model for seeking optimal multiple reservoir operations and water demand operations under uncertainty. The approach is used for addressing random reservoir inflow and fuzzy economic variables existing in the study system, and obtaining solution which could balance the solution robustness and model robustness. The climate change impact on water supply-demand management is also examined. Conditional density estimation network creation and evaluation (CaDENCE) and support vector machine (SVM) are used for precipitation and temperature downscaling, respectively. The BNN (Bayesian Neural Network) model is applied to simulate the monthly reservoir inflows. A multi-reservoir system, located at the Greater Vancouver Regional District, Canada is used for methodology demonstration.

1.3 Structure of the Thesis

Figure 1.1 illustrates the overall structure of this study. The thesis is organized into 8 chapters. The background of water scarcity the major factors intensifying water scarcity (i.e. urbanization and climate change), and the aims and scopes of this study are introduced in Chapter 1. Chapter 2 reviews the problems encountered and solution methods in water supply-demand systems, and various optimization techniques which were widely used in dealing with uncertainties in water resources
management systems.

In Chapter 3, a combined genetic algorithm and fuzzy simulation approach (GAFSA) is developed through integrating fuzzy chance-constrained programming (FCCP) and genetic algorithm (GA) into a general optimization framework. An agricultural water resources management problem that has been investigated by a number of previous studies is used for demonstration. A comparison of GAFSA to FCCP is given, and the potential limitations of the proposed method were also discussed.

In Chapter 4, a novel superiority-inferiority-based sequential fuzzy programming (SISFP) model was proposed for supporting water supply-demand analysis under uncertainty. The water supply-demand management system in Tianjin Binhai New Area, China, consisting of five sources of water, five water users at three districts (i.e. Tanggu, Hanggu, and Dagang), is used for methodology demonstration. The advantage and necessity of SISFP in dealing with general-shaped fuzzy parameters is further verified by comparing to fuzzy models with both deterministic and specially-shaped fuzzy parameters.

In Chapter 5, an extended fuzzy parametric programming (EFPP) model was proposed for planning and management of water resources under uncertainty. The applicability of EFPP is demonstrated by a numerical example and a water resources allocation case. The reliability of EFPP is also tested by comparing its solutions with those from fuzzy chance-constrained programming (FCCP) using the same case.

Chapter 6 proposed an integrated fuzzy programming and decision analysis (IFPDA) method for urban water distribution system management under uncertainty. A modified real-world case is investigated, which is characterized by a network structure including water sources, treatment plants, reservoirs, and consumers. A enhanced fuzzy decision analysis is used to compare and screen the best solutions under various scenarios.

Chapter 7 presents a robust fuzzy programming (RFP) model for seeking optimal multiple reservoir operations and water demand operations under uncertainty.
Precipitation downscaling is done by CaDENCE, and temperature downscaling is carried out by using SVM. The BNN model is employed into the climate change model in order to simulate the monthly reservoir inflows. A multi-reservoir system, located at the Greater Vancouver Regional District, Canada, is used for methodology demonstration.

Chapter 8 draws general conclusion of this research work and gives brief recommendations for future works.

Figure 1.1 Structure of the thesis.
CHAPTER 2 LITERATURE REVIEW

Deterministic optimization tools have been developed and widely used in water resources planning and management for many decades. A variety of optimization techniques, including linear/nonlinear programming, multi-objective programming, and dynamic programming were proposed and applied to different fields of water supply management. Water demand management focuses on reducing water demand by water-saving measures, such as installation of water-saving devices. However, there are many uncertainties associated with water supply-demand systems. In order to deal with them, inexact optimization models based on and interval linear programming have been widely used.

2.1 Water Supply System Management

Water supply system management refers to allocation of limited water resources to growing water demand for ensuring healthy socio-economic development and eco-environmental protection. Water supply management mainly includes optimization of reservoir operation, water network distribution, and irrigation allocation (Cunha, 2003).

2.1.1 Irrigation system optimization

Irrigation system is complex due to many factors which will influence the irrigation strategies, such as crop types, growth stages, and soil profiles. The planning for irrigation water management consists of land distribution, irrigated water allocation to various crops, and water delivery schedules in terms of delivered water amounts according to the objectives. Many optimization techniques have been applied to deal with irrigation management problems, including linear programming, nonlinear programming, dynamic programming, etc. For instance, Afzal et al. (1992) proposed a linear programming model for seeking optimal use of water in different quality for irrigation and land distribution for different crops. Benli and Kodal (2003) developed a nonlinear optimization model for analyzing irrigated water
amount, planning of cropping pattern, and benefits of farms under limited and adequate water supply conditions. Tilahun and Raes (2002) used an optimization dynamic programming model to analyze the yield response factors, effects of irrigation levels, and initial soil moisture on irrigation scheduling.

Recently, a number of researchers tried to couple simulation and optimization models for planning of irrigation system. For example, Kuo and Liu (2003) applied a simulation-optimization model to Delta irrigation area in Utah, where the water demand was simulated and the economic benefit was optimized. Shang and Mao (2006) incorporated simulation models, including soil water balance calculation, soil water dynamics analysis and crop simulation, into an optimization framework to study the scheduling of irrigation for winter wheat in an irrigated area in Xiaohe, North China. García-Vila and Fereres (2012) developed an economic model integrated with AquaCrop to help make decisions on cropping patterns under different water scarcity levels. Hejazi et al. (2014) proposed a simulation-optimization framework incorporating reanalysis-based weather forecasts from a regional climate model (RCM) for an irrigation scheduling optimization problem in the Havana Lowlands region, Illinois.

### 2.1.2 Reservoir system optimization

Management of reservoir operation systems involve many factors such as reservoir capacity, inflows, and water demand requirements from agricultural, municipal and industrial sectors. In the past few decades, a variety of optimization models, ranging from simple optimization models to complex simulation-optimization approaches, have been developed for long-/short-term and real-time operation of single-/multiple-reservoir systems.

Among numerous optimization methods, linear programming (LP) is one of the most popular methods for optimizing reservoir systems. Windsor (1973) formulated a LP model that includes storage and release capacity constraints for the analysis of multi-reservoir systems. Moy et al. (1986) developed a multi-objective mixed-integer LP for the analysis of the resilience, reliability, and vulnerability of a
reservoir for water supply. The limitation of LP is that it is restricted to solving problems with linear constraints and objective functions.

However, several nonlinear relations, like evaporation losses and hydropower generation exist in a reservoir system; hence, nonlinear programming (NLP) was developed to solve these problems. A series of algorithms, including generalized reduced gradient (GRG) and sequential quadratic programming (SQP), are used for dealing with NLP. Sylla (1995) presented a deterministic NLP model for a large-scale reservoir system and solved it by using GRG techniques. Teegavarapu and Simonovic (2000) developed a mixed integer NLP model with binary variables for optimizing daily operation of four reservoirs. Barros et al. (2003) applied a NLP model to a multi-objective reservoir system. Tu et al. (2008) used a quadratic programming model for optimizing new hedging rules of a multi-reservoir system in southern Taiwan. The disadvantage of NLP is the intense computation requirements. Nowadays some software packages such as LINGO, MINOS, and LODO are widely used for solving NLP problems (Leyffer and Mahajan, 2010).

Dynamic programming (DP), due to its visualization of water allocation at different levels of decision, becomes an appealing method and has been widely used for handling reservoir operation problems (Baliarsingh, 2010). Opricović (1993) proposed a dynamic compromise programming for a multi-purpose water reservoir control problem. Homayoun-far et al. (2010) developed a continuous model of dynamic game for reservoir operation, and used collocation methods and Ricatti equations to solve the model. More recently, the simulation-based optimization approaches have been developed for planning reservoir systems, where the simulation models are usually used for representing performance of reservoirs. Neelakantan and Pundarikanthan (2000) presented a typical simulation-optimization methodology for optimizing the multiple hedging rules for the operation of drinking water reservoirs. Ngo et al. (2007) combined simulation and optimization models and applied them to a multi-objective reservoir for solving conflicts in reservoir operation. Suiadee and Tingsanchali (2007) proposed a simulation-optimization model to determine optimal reservoir operation rules for Nam Don Reservoir, where
genetic algorithm (GA) was selected for solving the optimization model. Kang and Park (2014) presented a simulation-optimization model, consisting of reservoir operation, water demand prediction, and optimization submodels for optimal water allocations from multiple reservoirs.

2.1.3 Distribution networks optimization

Distribution networks are essential parts of all water supply systems. For the past three decades, a large number of researches have made efforts in tackling optimal design problems of water distribution systems, leading to the development of many optimization techniques and their applications.

A number of algorithms were proposed for minimizing the water distribution cost, reducing leakage of water, and satisfying water demand requirement. They include linear/nonlinear programming, heuristic-based programming, and multi-objective optimization methods. Alperovits and Shamir (1977) made the first attempt to use a linear programming gradient method to optimize the design of a water distribution system. However, the distribution network design problem is by nature nonlinear and limits the applicability of a LP model (Vasan and Simonovic, 2010). Later on, several nonlinear optimization models were proposed. For example, Lansey and Mays (1989) developed a nonlinear programming model integrated with a network simulator to optimize water distribution system. Varma et al. (1997) developed a nonlinear quadratic programming model to optimize the design of the distribution network of a water supply system.

Many researchers also tried to use heuristic methods for the design of water distribution network. Heuristic methods could generate optimal or not necessarily optimal solutions for complicated problems (Reca and Martínez, 2006). Genetic Algorithm (GA) is one of the commonly used methods. Simpson et al. (1994) used GA technique for optimizing pipe networks. Nazif et al. (2010) developed a GA-based optimization model to minimize the amount of leakage in a water distribution networks by optimizing variation of hourly water level in a storage tank during different seasons. Other heuristic techniques were also applied in
optimization problems of water distribution systems, including harmony search method, simulated annealing, and ant colony optimization method (Cunha and Sousa, 1999; Geem et al., 2001; Maier et al., 2003).

More recently, how to deal with multi-objective tasks in improving reliability of water distribution systems, such as leakage reduction, economic design, and sensor positioning, has become one of the major concerns. For example, Farmani et al. (2005) allied population-based evolutionary algorithms (EAs) for handling multi-objective optimization problems for water distribution systems. Baños et al. (2010) introduced a multi-objective memeetic algorithm which simultaneously optimized the network reliability, and the total cost of investment at the demand nodes. Fattahi and Fayyaz (2010) presented a compromise programming model for Hamedan potable water network, considering three objectives including leakage water amount, social satisfaction level, and cost of water distribution cost. Kurek and Ostfeld (2013) developed a multi-objective optimization methodology for water distribution systems which considered the cost of tank sizes and two water quality objectives.

2.2 Water Demand management

In water demand sectors, agriculture is the largest water user throughout the world (Pereira et al., 2002). Irrigation becomes more and more important in an agricultural system due to the pressure of survival caused by the growing population. Water demand management for irrigation under water scarcity focuses on reducing the water demand at the farm and increasing the crop yields and income per unit water used (Pereira et al., 2002). The related strategies may include: (i) selecting low water demand crops and economical crop patterns to reduce irrigation requirements; (ii) improving irrigation system uniformity, reusing water spills and controlling evaporation from soil to save water in irrigation; (iii) improving techniques of fertilizing and disease/pest control to increase yields per unit water usage (Pereira et al., 2002).

Non-agricultural users include domestic consumers and industrial users. The
domestic consumption of water aims to meet the basic water requirements of the residents. The basic human needs for water may include drinking water for survival, water for sanitation services, and water for human hygiene etc. (Gleick, 1996). Water in industry is employed for transportation, cooling, and serving as an ingredient or a solvent of finished products. The dominant user is electric power generation, which needs a huge amount of cooling water (Shiklomanov, 2000). Other heavy industrial water users are related to chemistry, petroleum chemistry, metallurgy, paper manufacture, and machine building (Shiklomanov, 2000). Water demand management for non-agricultural users focuses on measures that make better and more efficient use of limited supplies. Techniques used in non-agricultural water demand management may include: water loss reduction, wastewater reuse, installation of water-saving devices, water-pricing adjustment, water-codes strengthening, and launch of educational campaigns.

Dube and van der Zaag (2003) analyzed water-use patterns in the city of Masvingo, Zimbabwe, and projected the future demand considering the water-saving measures including pressure management, leakage control, awareness campaigns, and technical advice to water users. Chen et al. (2005) proposed a demand-oriented water management approach, which required institutional reinforcement, reform of water pricing policy and adjustment of agricultural sector based on a virtual water trade analysis for the irrigated agricultural sector located in the Heihe River Bain, China, in response to the increase of water scarcity problem. Aulong et al. (2009) applied a cost-effectiveness analysis method for supporting decisions of optimal combination of water supply and demand management measures at the river basin scale. Lee et al. (2011) analyzed the impacts of incentives of water conservation on water demand (including unit exchange and rebates programs for clothes washers, toilets, and showerheads) and evaluated the water savings and use-trend shifts of the water users during the study period. Nikouei et al. (2012) examined alternative policies for encouraging farmers in adopting water conservation measures, which could reduce irrigated water use in the studied basin.
2.3 Uncertainties in Water Supply-Demand Management

Uncertainties associated with hydrological forecast, water demand, and regulatory requirement can significantly affect water resources management (Wilchfort and Lund, 1997). Uncertainties in water supply arise from the seasonal and annual variability in rainfall, runoff, evaporation, snowfall, and snowmelt (Lund et al., 1995; Singh et al., 2010). Surface water supply estimates may be influenced by the uncertainty associated with future states of climate conditions, such as temperature, precipitation, and wind. Groundwater resources may be affected significantly by uncertainty for quantifying the model's parameters of the physical characteristics of the aquifer (Singh et al., 2010).

Uncertainties in water demand are due to the projected demand for water, which is typically calculated separately for different users (e.g. irrigation, municipal, and industrial). Each of these water demands are influenced by a number of factors. The water-using technologies, changes in farming practices, crop types, and land-use regulations will influence irrigation demand. The growth in population and increase of per-capita usage of water will affect municipal demand. The economic growth and energy prices will have effect on industrial demand. Many of these factors are highly volatile and will result in large uncertainties (Singh et al., 2010).

There are a number of studies on the characterization of uncertainties in water resources management systems. Cai and Rosegrant (2004) studied the effect of hydrologic uncertainty, which was related to both water availability and requirements, on selection of field irrigation technologies. Etchells and Malano (2005) identified sources of uncertainty found in water allocation models for improving the robustness of decision-making. Ajami et al. (2008) used an integrated Bayesian uncertainty estimation framework to assess model structural and parameter uncertainties on reliability, vulnerability, and resilience of the water management systems.
2.4 Inexact Optimization Models

Since extensive uncertainties exist in water supply-demand systems discussed above, the deterministic methods may lead to infeasible or inapplicable solutions when they are applied to practical problems. In order to enhance applicability of the developed models, studies of inexact optimization techniques that can reflect uncertainties are imperative.

2.4.1 Fuzzy optimization models

Fuzzy mathematical programming (FMP) incorporates fuzzy set theory into an ordinary optimization framework, which could deal with vagueness in decision maker’s aspirations (or preference) and ambiguity in knowledge (or information) in an optimization model. FMP can be classified into two categories: mathematical programming with vagueness and mathematical programming with ambiguity. The first type is called fuzzy flexible programming (FFP) (e.g. fuzzy parametric programming), and the latter is fuzzy possibilistic programming (FPP) (e.g. fuzzy chance-constrained programming).

2.4.1.1 Fuzzy chance-constrained programming

Stochastic chance-constrained programming has been extended from stochastic to fuzzy environments (Liu and Iwamura 1998). As a novel FMP method, a fuzzy chance-constrained programming (F CCP) model incorporates several predetermined confidence levels to represent the satisfaction of fuzzy constraints. Similar to a SCCP model, the conventional technique of solving FCCP is to transform the fuzzy constraints into deterministic ones based on the predetermined confidence levels. A general FCCP model can be formulated as follows (Liu and Iwamura, 1998):

Minimize \( f(X, \theta) \)  

Subject to:

\( \text{Pos} \{ \theta \mid h_i(X, \theta) \leq 0, \ i = 1, 2, ..., k \} \geq \alpha \)
where $X$ and $\theta$ are decision vector and fuzzy vector, respectively; $\alpha$ is predetermined confidence level to the fuzzy constraints; $\text{Pos}\{\cdot\}$ represents the possibility of the events in $\{\cdot\}$; $h_i(X, \theta)$ are constraint functions; $i$ is index of constraints, and $k$ is number of constraints. The word ‘others’ denotes other deterministic constraints.

Previously, many applications of FCCP have been reported in the water resources management field (Guo and Huang 2009; Li et al., 2009). He et al. (2008) proposed a FCCP model integrated with fuzzy simulation, where the possibility of satisfying the related environmental standards was measured. The groundwater pumping schemes with optimal pumping rates was obtained from the model. Guo and Huang (2009) developed a FCCP model coupled with a two-stage stochastic model for solving a water allocation problem, where the water loss rates during water transferring were expressed as fuzzy sets under different confidence levels. Xu and Qin (2010) developed an inexact FCCP model for dealing with fuzzy parameters on both sides of the model constraints and applied the model to an agricultural water resources problem. A variety of decision alternatives under specified confidence levels were generated from the model. Guo et al. (2013) developed an inexact FCCP model for tackling a crop-water allocation problem with different planting stages and precipitation years.

2.4.1.2 Fuzzy flexible programming (FFP)

FFP is a mathematical programming model using fuzzy sets to represent the vague aspirations of decision makers and constraints. Its core concept is to use tolerance to allow a certain level of flexibility and elasticity be reflected in the objective function and constraints. The common way to solve FFP is to convert the fuzzy constraints into crisp ones by defining a control variable $\lambda$ which corresponds to the satisfaction degree of the fuzzy decision. A general FFP model can be formulated as follows (Herrera and Verdegay, 1995):
Maximize \[ z = \sum_{j=1}^{J} c_j x_j \] \hspace{1cm} (2.2a)

subject to:

\[ \sum_{j=1}^{J} a_{ij} x_j \leq b_i \hspace{1cm} \forall i \] \hspace{1cm} (2.2b)

\[ \sum_{j=1}^{J} d_{jk} x_j \leq e_k \hspace{1cm} \forall k \] \hspace{1cm} (2.2c)

\[ x_j \geq 0, \hspace{1cm} \forall j \] \hspace{1cm} (2.2d)

where \( x_j \) are deterministic decision variables; \( a_{ij}, b_i, d_{jk} \) and \( e_k \) are deterministic coefficients; \( j \) and \( J \) are the index and number of the decision variable, respectively; \( i \) is the fuzzy constraint index; \( k \) is the index of deterministic constraint; the symbols \( \equiv \) and \( \leq \) denote fuzzy equality and inequality, respectively.

Previously, the FFP models were successfully integrated with many other mathematical programming models and applied to a number of water resources management problems. (Maqsood et al., 2005) incorporated FFP into a two-stage stochastic optimization framework for dealing with a regional water resources management problem, where the model solutions with different risk levels of criteria-violation were obtained. Li et al. (2008) developed a model which integrated FFP into an interval multistage programming to analyze different policy scenarios under different levels of economic consequences. Regulwar and Gurav (2012) developed a multi-objective fuzzy programming model based on FFP for supporting sustainable irrigation planning and management.

2.4.1.3 Fuzzy robust programming

Fuzzy robust programming (FRP), firstly proposed by Liu et al. (2003) was developed based on fuzzy set theory for addressing complexities in uncertainty characterization, data unavailability, and computational difficulty (Leung, 1988;
Watkins and Mckinney, 1995; Vassiadou-Zeniou et al., 1996; Kunjur and Krishnamurty, 1997). FRP model allows fuzzy uncertainties on both sides of constraints represented by fuzzy membership functions (Zhu et al., 2009). During process of solving FRP, a fuzzy decision space is delimited by dimensional enlarging of the original fuzzy constraints, which leads to enhancement of optimization robustness (Nie et al., 2014). A general FRP can be formulated as follows (Liu et al., 2003):

\[
\text{Min } z = \sum_{j=1}^{J} c_j x_j
\]  

subject to:

\[
\sum_{j=1}^{J} \tilde{a}_{ij} x_j \leq \tilde{b}_i \quad \forall i
\]  

\[
\sum_{j=1}^{J} \tilde{d}_{jk} x_j \leq e_k \quad \forall k
\]  

\[
x_j \geq 0, \quad \forall j
\]

where \( \tilde{a}_{ij} \) and \( \tilde{b}_i \) are fuzzy coefficients (or parameters); \( x_j \) are deterministic decision variables, \( c_j, \tilde{d}_{jk} \) and \( e_k \) are deterministic coefficients; \( j \) and \( J \) are the index and number of decision variables, respectively; \( i \) is the fuzzy constraint index; \( k \) is the index of deterministic constraint; the decision space for FRP problems can be delimited by deterministic constraints shown as follows (Liu et al., 2003):

\[
\sum_{j=1}^{J} \bar{a}_{ij}^s x_j \leq \bar{b}_i^s, \quad \forall i, s
\]  

\[
\sum_{j=1}^{J} \bar{a}_{ij}^l x_j \leq \bar{b}_i^l \quad \forall i, s
\]

where \( \bar{a}_{ij} = \sup(a_{ij}) \), \( \bar{b}_i^s = \sup(b_i^s) \), \( a_{ij}^l = \inf(a_{ij}) \), and \( b_i^l = \inf(b_i^l) \); \( \sup\{\bullet\} \) and

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\( \text{inf}\{\cdot\} \) denotes the superior limiting value and the inferior limiting value among set \( \{\cdot\} \), respectively; and \( s \) represents the levels of \( \alpha \)-cut.

In the field of water resources management, Nie et al. (2008) proposed a FRP model for dealing with an agricultural water quality management problem. The obtained solutions reflected a tradeoff between the stability and optimality of the study system. Nie et al. (2008) also applied the FRP model to a stream water quality management problem, where the parameters associated with environmental requirement and safety coefficient were represented by fuzzy boundary intervals.

### 2.4.1.4 Other extensions of FMP

Over the past years, many integrations of fuzzy set theory with other mathematical programming methods have been developed, like fuzzy dynamic programming (FDP), fuzzy-stochastic programming (FSP), and fuzzy integer programming (FIP). However, there are relating few studies and applications of FIP due to the difficulty of model solutions. Li et al. (2007) developed a mixed interval-fuzzy integer programming to solve a flood control problem. FDP is usually used to reflect the tradeoff between the optimization goals and constraints within a dynamic optimization framework (Kickert, 1978). Chen and Fu (2005) developed a FDP model which was derived by applying the fuzzy iteration model to classic dynamic programming for dealing with water resources allocation problems. Safavi and Aliljanian (2010) used FDP for sustainable management of water resources in optimal crop planning. FSP is developed to handle fuzzy random variables in a mathematical programming model. Guo et al. (2010) developed a fuzzy-stochastic two-stage programming approach for planning irrigation in an agriculture system. Xu et al. (2013) developed a nonlinear bi-level programming model to deal with fuzzy random variables in a regional water resources allocation problem. The fuzzy random variables were transformed into fuzzy numbers, and then solved by an interactive fuzzy programming technique.

Generally, the major advantage of a FCCP model is that it could reflect the tradeoff among the objective function values (e.g. total system benefit and cost), tolerance
values of the constraint, and the predetermined confidence level, which could be valuable to decision makers. However, it also makes the process of decision making more difficult in choosing the most suitable decision alternative. FCCP normally generates a set of solutions under different confidence levels that could lead to overwhelmed number of scenarios when coupled with other mathematical programming models (when dealing with problems under multiple uncertainties). FFP model allows flexibility and elasticity be reflected in the model. Similar to FCCP, it could also reflect the tradeoff among the model objective, violation of model constraint, and the satisfaction level. However, it is relatively weak in dealing with ambiguous coefficients and is more applicable to deal with fuzzy relationships. FRP is designed for handling highly uncertain variables (i.e. dual uncertainties) which are expressed as fuzzy boundary intervals (Nie et al., 2008; Zhu et al., 2009). However, it is generally not suitable to be used for reflecting vague relationships or objective functions (Xu and Qin, 2010).

2.4.2 Stochastic optimization models

Stochastic mathematical programming (SMP) is a type of optimization model with uncertain parameters in objective or constraints being represented by probability distributions. For model solution, the stochastic programming model needs to be converted to its deterministic equivalent. The SMP method is advantageous in dealing with hydrological parameters which has random features, such as inflow of reservoirs.

2.4.2.1 Stochastic chance-constrained programming

Stochastic chance-constrained programming (SCCP) allows decision makers to consider the constraints in terms of the probability of their satisfaction (Charnes and Cooper, 1959). SCCP assumes that the constraints could remain satisfaction at a predetermined confidence level. In most of the previous studies, the chance constraints had to be converted to their crisp equivalents through a series of mathematical manipulations (Huang, 1998). A general SCCP model can be formulated as follows (Cao et al., 2009):
Minimize $f(X, \eta)$ \hspace{1cm} (2.5a)

Subject to:

$$\Pr \{ \eta \mid h_i(X, \eta) \leq 0, \ i = 1, 2, \ldots, k \} \geq \alpha$$ \hspace{1cm} (2.5b)

others \hspace{1cm} (2.5c)

where $X$ is a decision variable vector; $\eta$ is a random vector with a continuous or discrete joint probability distribution function; $\alpha$ is predefined confidence level of the stochastic constraints; $\Pr \{ \ast \}$ represents the probability of the events in $\{ \ast \}$; the ‘others’ denotes other deterministic and non-zero constraints.

The SCCP models were successfully applied in a number of water resources management problems (Paudyal and Dasgupta, 1990; Feiring et al., 1998; Huang, 1998). For instance, Azaiez et al. (2005) proposed a SCCP model for optimal operation of a multiple reservoir system under multiple periods, where the random inflows of the main reservoir were treated by chance constraints. Yong et al. (2008) proposed an inexact SCCP model and applied it to the Lake Qionghai watershed, China, for seeking optimal strategies for water quality improvement. Zhang et al. (2009) proposed an inexact SCCP model and applied it to a water supply management problem where the objective was to maximize economic return with constraints including water availability and social/environmental policy and regulations. Xie et al. (2011) developed a SCCP model integrated with interval linear programming for handling a water quality management problem in Binhai New Area, Tianjing. The solution obtained from the model could reflect the tradeoff among system reliability, economic benefits and pollutant discharges. Xu et al. (2012) proposed an inexact SCCP model for planning a multi-layer water supply system, and the results obtained could reflect the balance between system reliability and cost. Ji et al. (2014) proposed an inexact SCCP model, which is capable of dealing with the left-hand-side random parameters in the model for nonpoint sources water quality management.
2.4.2.2 Two-stage stochastic programming

Two-stage stochastic programming has been deemed as one of the major SMP approaches. The fundamental ideal of TSP is that decisions should be made dependent on available data rather than future observations. In TSP, the decision process is divided into two stages. In the first stage, a set of decisions are taken based on probable future events. Then, in the second stage, the corrective action can be taken when full information is received. A general model of TSP can be written as (Li et al., 2010):

\[ \text{Maximize } f = CX - E[Q(X, \xi)] \quad (2.6a) \]

Subject to:

\[ AX \leq B \quad (2.6b) \]
\[ X \geq 0 \quad (2.6c) \]

where \( X \) is the first-stage decision made before the random variable is observed; \( \xi \) is the random variable.; \( Q(X, \xi) \) is the optimal value of the following programming model (Li et al., 2010):

\[ \text{Maximize } Q(Y, \xi) \quad (2.7a) \]

Subject to:

\[ W(\xi)Y = h(\xi) - T(\xi)X \quad (2.7b) \]
\[ Y \geq 0 \quad (2.7c) \]

where \( Y \) is adaptive decision in the second-stage, which relies on the realization of the random variable; \( Q(Y, \xi) \) denotes the second-stage function; \( \{T(\xi), W(\xi), h(\xi)\} \) are functions of the random variable \( \xi \). 

Over the past decades, TSP methods have been widely explored (Mobasheri and Harboe, 1970; Wang and Adams, 1986). Cai and Rosegrant (2004) developed a scenario-based TSP model for planning an irrigation system, incorporating
hydrological uncertainties associated with both availability and requirements of water. Li and Huang (2008) developed an interval TSP model for allocation of water resources among multiple users within multiple reservoir systems. Li et al. (2010) developed a TSP model for agricultural irrigation planning, where the solutions could reflect the balance between system benefits and the penalties caused by the violation of predefined policies. Xie et al. (2013) developed an inexact TSP model for a multi-regional water resources management problem and obtained different water allocation schemes under various inflow levels.

2.4.2.3 Stochastic robust optimization

Stochastic robust optimization (SRO) model is firstly developed by Mulvey et al. (1995). As one of the SMP approaches, SRO can not only deal with influence of uncertain parameter existing in optimization models, but also control risk of model failure caused by violations of constraints (Mulvey et al., 1995). The constraints in a SRO model can be categorized into two types: (i) structural constraint which is free of uncertainty, similar to the deterministic constraint, and (ii) control constraint which is subject to uncertainty. Correspondingly, the model has two types of decision variables: (i) structural variables, which could determine the system composition and size, and (ii) control variables, which are selected after uncertainties are observed (Mulvey et al., 1995). A typical SRO model can be written as (Mulvey et al., 1995):

\[
\text{Minimize } z = \sigma (X, Y_1, Y_2, \ldots, Y_s) + \gamma p (\delta_1, \delta_2, \ldots, \delta_s) \tag{2.8a}
\]

Subject to:

\[
AX \geq B \tag{2.8b}
\]

\[
C_X X + D_Y Y_s + \delta_s \geq E_s \quad \text{for all } s \in \Omega \tag{2.8c}
\]

\[
X \geq 0, \ Y_s \geq 0, \ \delta_s \geq 0 \quad \text{for all } s \in \Omega \tag{2.8d}
\]

where \( \gamma \) is weight coefficient; \( s \) is index of scenario; \( X \) and \( Y \) are vectors of design
variables and control variables, respectively; \( A \) and \( B \) are deterministic coefficients or parameters; \( \delta \) represents the infeasibility of the model under scenario \( s \). Equation (2.8b) is the structural constraint with fixed coefficients which are free of noise, and Equation (2.8c) is the control constraint with coefficients which are subject to noise (Xu et al., 2009). Equation (2.8d) denotes the non-negative constraints. The first term of objective function \( \sigma(\cdot) \) represents robustness of the solution that captures the degree of risk aversion, and the second term \( \rho(\cdot) \) represents the robustness of model which penalizes violation of the constraints.

Previously, a number of applications of SRO were reported in water resources management. Watkins and McKinney (1997) proposed a SRO model and applied it to two water resources management problems: urban water transfers and groundwater quality management. Xu et al. (2009) proposed an inexact two-stage SRO model, which could handle uncertainties in water resources allocation problems and generate optimal and balanced water allocation schemes. Xu et al. (2013) developed a hybrid interval-robust optimization model which coupled SRO and ILP for a water quality management problem located in New Binhai District of Tianjin, China.

### 2.4.2.4 Other extensions of SMP

There have been a number of extensions of SMP models, such as stochastic integer programming (SIP), stochastic dynamic programming (SDP), and stochastic multiobjective programming (SMOP). In SIP, random elements are introduced to integer programming frameworks to account for probabilistic uncertainties in model parameters. Li et al. (2008) developed an inexact multistage stochastic integer programming model for dealing with uncertainty in water resources management. SDP has been widely used in reservoir systems management. Ben Alaya et al. (2003) used SDP for the design of optimal rules for the water resources management of the Nebhana dam, Tunisia. Galelli and Soncini-Sessa (2010) proposed a new approach which combined metamodelling and SDP for the design of reservoir release policies for irrigation. SMOP is an important extension of SMP, which could be used to deal with multiple objectives in water resources management systems. Tilmant and
Kelman (2007) used multi-objective optimization techniques for planning multi-reservoir systems taking into account the stochastic characteristic of the inflows.

### 2.4.3 Interval optimization models

Interval linear programming (ILP) is also a popular type of optimization model and has been widely explored and applied to deal with uncertainties in water resources and environmental management problems (Huang, 1996; Huang and Loucks, 2000; Li et al., 2006). ILP could deal with uncertainties approximated by only the lower and upper boundaries, which are generated when the available data are insufficient to get the related distribution information (either as a membership function or a probability distribution function). A general model of ILP can be given by (Huang, 1996):

\[
\text{Maximize } f^+ = \sum_{j=1}^{J} c_j^+ x_j \quad (2.9a)
\]

subject to:

\[
\sum_{j=1}^{J} a_{ij}^+ x_j \leq b_i^+ \quad \forall i \quad (2.9b)
\]

\[
\sum_{j=1}^{J} d_{jk} x_j \leq e_k \quad \forall k \quad (2.9c)
\]

\[
x_j \geq 0 \quad \forall j \quad (2.9d)
\]

where \( a_{ij}^+ \), \( b_i^+ \) and \( c_j^+ \) denote a set of interval numbers; \( x_j \) are deterministic decision variables; \( d_{jk} \) and \( e_k \) are deterministic coefficients; \( j \) and \( J \) are the index and number of the decision variable, respectively; \( i \) is the fuzzy constraint index; \( k \) is the index of deterministic constraint.

In terms of ILP applications, Jansson (1988) proposed a self-validating method to solve an interval linear programming problem. Matloka (1992) studied the
generalization of ILP methods and provided an effective solution algorithm. Huang et al. (1994) and Huang (1996, 1998) developed and applied several interval mathematical programming methods for water resources planning and management problems. For instance, Huang (1996) developed an interval linear programming model integrated with water quality management model for dealing with a planning problem of agricultural water pollution control. More recently, Zhang et al. (2009) presented an interval de Novo programming approach for optimizing water resources management systems. Qin and Xu (2011) proposed an acceptability-index-based ILP model for the analysis of water supply systems in an urban area.

2.5 Summary of Literature Review

In this chapter, the optimization problems in water supply systems, including optimizations of irrigation system, water distribution system, and reservoir system were reviewed. The water demand management for agricultural and nonagricultural users was also mentioned. Finally, the inexact optimization models, which generally include FMP, SMP, and IMP, for water resources management under uncertainties were discussed. From literature review, several research gaps in the field of water resources management are identified:

(1) SMP, IMP, and FMP have their own advantages and limitations in handling uncertainties. SMP owns capability of effectively dealing with various probabilistic uncertainties in decision making. Previously, many applications of SMP methods in the field of water resources management have been reported. However, the high computational requirements for specifying the probability distribution of parameters may affect their applicability (Xu and Qin, 2010). IMP method is capable of handling uncertain parameters which are expressed as interval numbers. It has a lower data requirement. However, IMP may encounter difficulties in tackling highly uncertain parameters in the right-hand sides of the model constraints (Maqsood et al., 2005). FMP method has a good balance between data requirement and information accuracy, and is more flexible to be used in water resources management. However, most of the
previous studies dealt with fuzzy distributions as triangular or trapezoidal shapes (Liu and Iwamura, 1998). Novel fuzzy programming approaches which could handle general-shaped fuzzy parameters are desired.

(2) Fuzzy flexible (FF) programming allows flexibility and elasticity be reflected in the objective function and constraints but is relatively weak in dealing with ambiguous coefficients. Fuzzy possibilistic (FP) programming tackles fuzzy coefficients in objective function and/or constraints, but is less capable of dealing with fuzzy relationships. In water resources allocation problems, uncertainty could be associated with many system components and their relationships, which signifies that both vagueness and ambiguity would exist in a mathematical programming model. To benefit general-purpose applications, it is desired that a sophisticated model that could handle both fuzzy conditions be advanced.

(3) Most of the previous optimization efforts focused on obtaining solutions without considering decision analysis. For example, FCCP could offer a set of solutions under various confidence levels. However, it is still difficult for decision makers to choose a reasonable decision alternative as they either lack the in-depth knowledge of interpreting the modeling outputs or do not have the related engineering background. Hence there is a need to incorporate decision making methods (e.g. multi-criteria decision analysis technique) into optimization framework to help decision makers evaluate and rank the potential solution options.

(4) In many previous studies, demand and supply uncertainties were considered to be independent. However, some factors that affect supply uncertainty may also influence demand. For example, climate change may alter precipitation pattern and would influence the prediction of water supplies. It could also impact demand projections by influencing usage rates, crop yields, and population growth. Rapid urbanization would not only disturb the local hydrological cycle, but also increase urban water demand and consumption. Hence, it is desired that the complex interactions among multiple factors in water supply-demand management systems be investigated.
3.1 Introduction

For many decades, agriculture has focused on adopting new technologies or improving existing farming practices for boosting production which leads to many environmental problems, such as the soil erosion, non-point sources pollution, and water shortage (Huang, 1996). These problems have been related to both water quantity and water quality, and urged planners to develop a comprehensive management strategy. However, the agricultural water resources management system is complex with various uncertainties derived from many factors such as economic objective, environmental requirement, and policy regulation (Huang and Xia, 2001). For example, the economical parameters (e.g. benefit of crops) are usually changing dynamically. The environmental loading capacities are normally estimated based on empirical experience. The estimate of these parameters may be subject to human judgments. Therefore, effective decision-support tools with the capability of tackling uncertainties are desired to be developed.

Over the past years, a lot of inexact optimization methodologies have been applied to handle uncertainties existing in water resources management systems. Most of these techniques were based on stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP), and interval linear programming (ILP) (Beck, 1987; Cardwell and Ellis, 1993; Nie et al., 2007; Rong and Lahdelma, 2008; Rehana and Mujumdar, 2009; Aviso et al., 2010; Lv et al., 2010; Fan and Huang, 2012). Stochastic chance-constrained programming (SCCP) is one of the major methods of SMP, which offers a means of allowing the decision makers to consider objectives based on the probability of their achievement (Charnes and Cooper, 1959). The SCCP models were successfully applied in a number of environmental management problems (Wagner and Gorelick, 1987; Singh and Chakrabarty, 2011). Recently, chance-constrained programming has been extended to fuzzy
environments (Liu and Iwamura, 1998). Fuzzy chance-constrained programming (FCCP) model is one kind of FMP method, which incorporates some predetermined confidence levels of the satisfaction of fuzzy constraints into the models. Similar to SCCP model, the conventional technique of solving FCCP is to transform the fuzzy constraints to their crisp equivalents based on the predetermined confidence levels. Previously, FCCP has been applied in many fields, including solid waste management (Huang et al., 1992; Li et al., 2007) and water resources management (Guo and Huang, 2009; Li et al., 2009). However, in water quality management field, the related studies are very limited. Zhang et al. (2009) made the first attempt in applying a robust chance-constrained fuzzy possibilistic programming (RCFPP) model to agricultural water quality management problem for providing decisions for various agricultural activities.

Nevertheless, Most of the previous study assumed that the fuzzy parameters are with triangular or trapezoidal-shaped fuzzy membership functions. It is easy to reformulate the model with fuzzy parameters of special distributions to their equivalent crisp counterparts. However, in many real-world applications, the fuzzy parameters may have membership functions in various shapes. For example, in agricultural activities, the soil loss rate is affected by many factors such as crop type, climate condition, and soil profile; the on-site measurement data may lead to the generation of membership functions in complex shapes. Use triangular-shaped fuzzy parameters will lose some information and lead to infeasible solutions. Since it is difficult to convert the fuzzy constraints with complex fuzzy parameters, a genetic algorithm (GA) aided with fuzzy simulation approach, which is proposed by Liu and Iwamura (1998), will be employed to deal with such a problem. GA could be used to generate the model solution and fuzzy simulation will be used to check the feasibility of a solution.

Thus, the objective of this study is to develop a combined genetic algorithm and fuzzy simulation approach (GAFSA) for solving the fuzzy chance-constrained programming (FCCP) model for agricultural water resources management problem. Genetic algorithm (GA) is a search heuristic inspired by the evolutionary ideas of
natural selection and genetic. It will be used to solve a FCCP model with both sides being associated with fuzzy coefficients in varied shapes. GAFSA will be used to an agricultural water resources management problem. Advantages and limitations of the proposed method will be further discussed.

3.2 General Methodology

3.2.1 Fuzzy chance-constrained programming

Fuzzy chance-constrained programming (FCCP) was thoroughly discussed by Liu and Iwamura (1998). It allows incorporation of a series of preset confidence levels to the optimization framework, which reflects the degree of constraints satisfaction due to impacts of possibilistic uncertainties. A general FCCP model is expressed as follows:

Minimize \( f = CX \) \hspace{1cm} (3.1a)

Subject to:

\[ Pos\{\tilde{A},\tilde{B} | \tilde{A}X \leq \tilde{B}\} \geq \alpha \] \hspace{1cm} (3.1b)

\[ DX \leq E \] \hspace{1cm} (3.1c)

\[ C, A \neq 0 \] \hspace{1cm} (3.1d)

\[ X \geq 0 \] \hspace{1cm} (3.1e)

where \( X \) is a deterministic decision variables vector; \( \tilde{A} \) and \( \tilde{B} \) are vector of which fuzzy variables with membership functions \( \mu(\tilde{A}) \) and \( \mu(\tilde{B}) \), respectively; \( C, D \) and \( E \) are vectors of deterministic auxiliary variables; \( pos\{\cdot\} \) denotes possibility of events in \( \{\cdot\}\); \( \alpha \) is a predetermined confidence level. In model (3.1), Equation (3.1b) represents the fuzzy chance constraints with both side items being described as fuzzy sets.
3.2.2 Combined genetic algorithm and fuzzy simulation approach (GAFSA)

To solve model (3.1), the fuzzy chance-constraints need be converted to their respective crisp equivalents. In many real-world applications, the fuzzy membership functions usually have specific shapes, such as exponential, Gaussian, triangular, and trapezoidal. Hence it is difficult to convert them to their respective deterministic equivalents unless they are all expressed in some special forms like triangular or trapezoidal (Liu and Iwamura, 1998). To solve this problem, we incorporate GA into a general FCCP model, and try to seek exact or approximate solution. Comparing to the conventional linear programming algorithms, GA relies on a penalty-based evaluation procedure and a number of directional searching algorithms (inspired by natural evolutions, such as initiation, mutation, selection, and crossover) to seek optimal model solutions.

Based on the concepts and techniques of possibility theory established by Zadeh (1978), Equation (3.1b) can be transformed into:

\[
\text{Pos}\{\tilde{A}, \tilde{B} \mid \tilde{A}X \leq \tilde{B}\} = \text{Sup}\{\min_{a \in R} \mu_a(a), \mu_b(b) \mid a, b \in R, a \leq b\}
\]  

(3.2)

Based on model (3.1), a general GAFSA model could be written as follows (Liu and Iwamura, 1998; Qin et al., 2010):

\[
\text{Minimize } f = CX + CPF + IPF
\]  

(3.3a)

Subject to:

\[
\text{CPF} = \begin{cases} 
0 & \text{if } \text{pos}\{\cdot\} \geq \alpha \text{ is met} \\
\lambda_1 & \text{if } \text{pos}\{\cdot\} \geq \alpha \text{ is not met}
\end{cases}
\]  

(3.3b)

\[
\text{IPF} = \begin{cases} 
0 & \text{if } DX \leq E \text{ is met} \\
\lambda_2 & \text{if } DX \leq E \text{ is not met}
\end{cases}
\]  

(3.3c)

\[C, A \neq 0\]  

(3.3d)
where $f$ is a transformed objective function based on Equation (3.1a), which is convenient for GA to seek optimal solutions of model (3). According to Poojari and Varghese (2008) and Qin et al. (2010), $CPF$ in Equation (3.3b) is a penalty factor for describing the influence of fuzzy chance-constraints violation at a predetermined confidence level $\alpha$; $\lambda_1$ is a large real number and can be used to quantitatively reflect the violation of fuzzy chance constraints. Similarly, $IPF$ in Equation (3.3c) is a penalty factor for reflecting the deterministic constraints violation where $\lambda_2$ is a large real number. The values of $\lambda_1$ and $\lambda_2$ are much higher than the objective function values in order to get applicable solutions. As shown in model (3.3), it is important to calculate the possibilistic value of event $\{\cdot\}$. According to Liu and Iwamura (1998), a fuzzy-simulation-based iteration process can be used for such a purpose. The iteration process is described as follows:

(i) Take the hypercubes A and B containing $\alpha$-cut for both fuzzy numbers $\tilde{a}$ and $\tilde{b}$, and generate two crisp numbers $a$ and $b$. If $aX b \leq b$, calculate $\mu_A(a)$ and $\mu_B(b)$ through the fuzzy membership functions and set $p = \min[\mu_A(a), \mu_B(b)]$.

(ii) Generate $a'$ and $b'$ satisfying $a'X b' \leq b'$ and obtain the corresponding $\mu_A(a')$ and $\mu_B(b')$. If $p < \min[\mu_A(a'), \mu_B(b')]$, then set $p = \min[\mu_A(a'), \mu_B(b')]$; otherwise, $p$ remains unchanged.

(iii) Repeat the above processes until a given number of cycle $n$ is reached. Finally, the maximum value $p$ will be regarded as the desired possibility to be determined. Finally, the model solution at a specific confidence level can be obtained. Figure 3.1 shows some common fuzzy membership functions on both sides of the constraints. Generally, the fuzzy simulation approach is applicable as long as the fuzzy membership functions are given.

According to Equations (3.3b) and (3.3c), a compliance check is needed for both the uncertain and deterministic constraints for each potential solution (i.e. a chromosome)
generated by GA iteration. Obviously, the best solution should fall within the solution pool without triggering any penalty of constraints violation. As the fuzzy simulation process is flexible on the shape of fuzzy variables, the proposed GAFSA framework is able to tackle complicated forms of fuzzy membership functions. As shown in Figure 3.2, the detailed procedures of GACCP model can be summarized as follows:

![Common fuzzy membership functions](image)

* (a) Trapezoidal vs. exponential shape; (b) trapezoidal vs. triangular shape; (c) triangular vs. exponential shape; (d) exponential vs. exponential shape

Figure 3.1 Common fuzzy membership functions on both sides of the constraints.

Step 1: Identify fuzzy uncertainties and obtain their corresponding fuzzy possibility distributions;

Step 2: Formulate a FCCP model;
Step 3: Define the penalty factors of constraints satisfaction and build the fitness function based on model (3.3);

Step 4: Initiate the GA searching process, with each chromosome representing a potential model solution;

Step 5: Calculate the confidence levels of satisfaction of fuzzy chance constraint through the iteration-based fuzzy simulation;

Step 6: Evaluate the value of fitness function according to a predetermined confidence level;

Step 7: Check the stop criteria; if satisfied, generate the final solutions of $f_{opt}$ and $X_{opt}$, if not, proceed with standard GA operations of selection, crossover, and mutation, and go back to Step 4.

Figure 3.2 General framework of a GAFSA model.
3.2.3 Example of testing effectiveness of GAFSA

In order to test the effectiveness of GAFSA, a standard approach for solving triangular-shaped FCCP model proposed by Liu and Iwamura (1998) was used for comparison. A small numerical example is given as follows:

Maximize \[ f = 3x_1 + x_2 \]  \hspace{1cm} (3.4a)

Subject to:

\[ (0.1, 0.15, 0.2)x_1 \leq (0.95, 1, 1.05) \]  \hspace{1cm} (3.4b)

\[ (0.1, 0.25, 0.3)x_2 \leq (0.175, 0.2, 0.225) \]  \hspace{1cm} (3.4c)

\[ 0.25x_1 - 0.3x_2 \leq 1 \]  \hspace{1cm} (3.4d)

\[ x_1, x_2 \geq 0 \]  \hspace{1cm} (3.4e)

where the coefficient, like \((0.1, 0.15, 0.2)\) in Equation (3.4b), represents a triangular fuzzy set, where 0.1 and 0.2 are the lower and upper bounds of the fuzzy set, and 0.15 is the most likely value. Table 3.1 shows the comparison of solutions between GAFSA and FCCP. It shows that, at the same confidence level, the solutions obtained from GAFSA are close to those obtained from FCCP. It is also found that the GAFSA can only reach suboptimal solutions. For example, at a confidence level of 0.6, the objective value obtained from GAFSA (i.e. 17.02) is lower than that from FCCP (i.e. 17.08). Generally, GAFSA is capable of solving CCP model with fuzzy parameters on both sides of constraints and obtaining reasonable solutions.

Table 3.1 Comparison of solutions between GAFSA and FCCP

<table>
<thead>
<tr>
<th>Confidence level</th>
<th>GAFSA</th>
<th>FCCP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(x_1)</td>
<td>(x_2)</td>
</tr>
<tr>
<td>0.6</td>
<td>5.31</td>
<td>1.09</td>
</tr>
<tr>
<td>0.7</td>
<td>5.21</td>
<td>1.01</td>
</tr>
<tr>
<td>0.8</td>
<td>5.11</td>
<td>0.92</td>
</tr>
<tr>
<td>0.9</td>
<td>5.03</td>
<td>0.85</td>
</tr>
</tbody>
</table>
In order to illustrate the applicability of GAFS for solving a general-shaped FCCP model, a numerical example is provided as follows:

\[
\text{Maximize } f = 3x_1 + x_2 \\
\text{Subject to:} \\
\tilde{a}x_1 \leq \tilde{b} \tag{3.5b} \\
\tilde{c}x_2 \leq \tilde{d} \tag{3.5c} \\
0.25x_1 - 0.3x_2 \leq 1 \tag{3.5d} \\
x_1, x_2 \geq 0 \tag{3.5e}
\]

where \( \tilde{a} \) is a triangular fuzzy number \((0.1, 0.15, 0.2)\), \( \tilde{c} \) is a trapezoidal fuzzy number \((0.1, 0.23, 0.26, 0.3)\), \( \tilde{b} \) and \( \tilde{d} \) are exponential fuzzy numbers, defined as

\[
\mu_b(\xi) = \exp\left(-|\xi - 1|/0.05\right), \text{ and } \mu_d(\xi) = \exp\left(-|\xi - 0.2|/0.025\right).
\]

Table 3.2 shows the solutions at various confidence levels obtained through GAFS. As the confidence level increases from 0.7 to 0.9, the values of \( x_1, x_2 \), and objective value would decrease. It means that a lower confidence level would lead to a higher risk of constraint violation. Conversely, at a higher confidence level, a lower risk of system failure could be obtained. Consequently, the solution of this numerical example shows that the GAFS approach could deal with generally shaped fuzzy membership functions on both sides of the constraints.

Table 3.2 Solutions of the numerical example

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Confidence Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p = 0.7 )</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>5.1325</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.9421</td>
</tr>
<tr>
<td>Objective Values</td>
<td>16.3397</td>
</tr>
</tbody>
</table>
### 3.3 Application in agricultural water resources management

#### 3.3.1 Study case

A hypothetical agricultural water resources management problem, which is adapted from Huang (1996) and Nie et al. (2008), is used for demonstration. The model parameters are referred to the data from literatures (Tisdale and Nelson, 1966; Haith, 1982; Huang, 1996; Nie et al., 2008). The total tillable land of the study area is 96.4 ha. Vegetables, wheat, and potato are the main crops, and the main livestock are pig, cattle, and poultry. The entire area was divided into three subareas, with the tillable land being 34.1, 23.9, and 40.4 ha, respectively. Water for irrigation is withdrawn from three branches of a river flowing through the area. In this rural system, the two main ways to bring economic benefit are the cultivation of crops and breeding of the livestock. For detailed background information, readers are referred to Huang (1996) and Nie et al. (2008).

The hypothetical case is highly simplified and considers limited crops, livestock and nonpoint source losses. A real-world case is normally more complicated, involving more elements of soil pollution (e.g. chemical, biological and physical elements), governmental policies (e.g. labour requirement, land use restriction, product quality, and environmental standards), and agricultural activities (e.g. spraying, weeding and fertilizing). This could lead to the growth of model sizes and complexity of constraint considerations.

#### 3.3.2 Formulation of a GAFSA model

In this studied system, the nonpoint source losses of nitrogen, phosphorus, and soil from manure and fertilizer have led to the pollution problems in the canal (Huang, 1996). The water contamination problem could decrease the available water resources amount, and aggravate the water shortage problems. This, in turn, would influence the crop cultivation and agricultural income. The decision makers are responsible for generating optimal schemes of water usages and agricultural activities according to the given restrictions and objectives, in order to maximize the total system benefit. Due to the features of vagueness and imprecision, parameters
associated with environmental loading capacities could be expressed by fuzz sets with various membership functions (Xu and Qin, 2010). This is deemed reasonable as, in practical applications, these data are normally difficult to be accurately defined or measured, and some empirical judgment or statistical analysis has to be made in order to quantify the related uncertainties. The distributions of these data could have large variations due to various reasons like spatial-temporal changes, climate impact, and data shortage; if fuzzy set theory is used to describe such uncertainties, the corresponding membership functions could hardly be fitted by simple forms like triangular shapes. For demonstration purpose, in this study, the soil loss rate from the land planted with crops and the maximum allowable soil loss rate are assumed as fuzzy sets with triangular and trapezoidal shapes, respectively. The maximum allowable soil-phase nitrogen and phosphorus loss rates are expressed as fuzzy sets in exponential forms (as shown in Table 3.2 and Figure 3.3). In reality, the procurement of the related information should rely on site survey, literature review, and expert consultations. Other deterministic model parameters are referred to Nie et al. (2008) and Xu and Qin (2010) (shown in Tables 3.2 and 3.3).
Table 3.3 Model parameters related to crops and livestock

<table>
<thead>
<tr>
<th>Parameters related to crops</th>
<th>Wheat $(i = 1)$</th>
<th>Vegetabl $(i = 2)$</th>
<th>Potato $(i = 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit yield of crop $i$ (kg/ha)</td>
<td>6000</td>
<td>21500</td>
<td>15000</td>
</tr>
<tr>
<td>Dissolved nitrogen concentration in wet season runoff from the land planted with crop $i$ (mg/l)</td>
<td>1.5</td>
<td>2.7</td>
<td>2.2</td>
</tr>
<tr>
<td>Dissolved nitrogen concentration in dry season runoff from the land planted with crop $i$ (mg/l)</td>
<td>0.8</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Dissolved phosphorus concentration in wet season runoff from the land planted with crop $i$ (mg/l)</td>
<td>0.15</td>
<td>0.26</td>
<td>0.23</td>
</tr>
<tr>
<td>Dissolved phosphorus concentration in dry season runoff from the land planted with crop $i$ (mg/l)</td>
<td>0.08</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>Unit nitrogen requirement of crop $i$ (kg/ha)</td>
<td>105</td>
<td>155</td>
<td>100</td>
</tr>
<tr>
<td>Wet season runoff from the land planted with crop $i$ (mm)</td>
<td>65</td>
<td>82</td>
<td>86</td>
</tr>
<tr>
<td>Dry season runoff from the land planted with crop $i$ (mm)</td>
<td>45</td>
<td>51</td>
<td>55</td>
</tr>
<tr>
<td>Net energy potential of crop $i$ (Mcal/kg)</td>
<td>3.3</td>
<td>0.22</td>
<td>0.8</td>
</tr>
<tr>
<td>Digestible protein content of crop $i$ (%)</td>
<td>0.108</td>
<td>0.01</td>
<td>0.018</td>
</tr>
<tr>
<td>Flow rate of irrigation water required by crop $i$ in subarea $1$ $[(m^3/s)/ha]$</td>
<td>0.032</td>
<td>0.051</td>
<td>0.043</td>
</tr>
<tr>
<td>Flow rate of irrigation water required by crop $i$ in subarea $2$ $[(m^3/s)/ha]$</td>
<td>0.031</td>
<td>0.05</td>
<td>0.042</td>
</tr>
<tr>
<td>Flow rate of irrigation water required by crop $i$ in subarea $3$ $[(m^3/s)/ha]$</td>
<td>0.033</td>
<td>0.052</td>
<td>0.044</td>
</tr>
<tr>
<td>Soil loss rate from the land planted with crop $i$ (kg/ha)</td>
<td>5100</td>
<td>17500</td>
<td>7600</td>
</tr>
<tr>
<td>Unit price of crop $i$ ($/kg$)</td>
<td>0.45</td>
<td>0.97</td>
<td>0.72</td>
</tr>
<tr>
<td>Unit farming cost of crop $i$ ($/ha$)</td>
<td>1650</td>
<td>10500</td>
<td>2750</td>
</tr>
<tr>
<td>Unit cost to deliver water to $S_u$ in Subarea $1$ $$/((m^3/s))$</td>
<td>2450</td>
<td>2800</td>
<td>2600</td>
</tr>
<tr>
<td>Unit cost to deliver water to $S_u$ in Subarea $2$ $$/((m^3/s))$</td>
<td>4000</td>
<td>4400</td>
<td>4200</td>
</tr>
<tr>
<td>Unit cost to deliver water to $S_u$ in Subarea $3$ $$/((m^3/s))$</td>
<td>5200</td>
<td>5600</td>
<td>5400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters related to livestock</th>
<th>Cattle $j = 1$</th>
<th>Swine $j = 2$</th>
<th>Poultry $j = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit manure amount generated by livestock $j$ (t/unit)</td>
<td>18</td>
<td>1.8</td>
<td>0.04</td>
</tr>
<tr>
<td>Unit digestible protein requirement of livestock $j$ (kg/unit)</td>
<td>350</td>
<td>35</td>
<td>1.8</td>
</tr>
<tr>
<td>Unit net energy requirement of livestock $j$ (Mcal/unit)</td>
<td>5200</td>
<td>515</td>
<td>188</td>
</tr>
<tr>
<td>Unit average return from livestock $j$ ($/unit$)</td>
<td>900</td>
<td>75</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Note: The related data are adapted from Huang (1996) Xu and Qin (2010), and Nie et al. (2008).
Table 3.4 General model parameters

<table>
<thead>
<tr>
<th>Deterministic parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum allowable total nitrogen loss rate (kg/ha)</td>
<td>38</td>
</tr>
<tr>
<td>Nitrogen concentration of manure (kg/t)</td>
<td>13</td>
</tr>
<tr>
<td>Nitrogen content of soil (%)</td>
<td>0.02</td>
</tr>
<tr>
<td>Phosphorus content of soil (%)</td>
<td>0.0009</td>
</tr>
<tr>
<td>Nitrogen volatilization/denitrification rate of manure (%)</td>
<td>0.3</td>
</tr>
<tr>
<td>Phosphorus volatilization/denitrification rate of manure (%)</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum loss rate of dissolved nitrogen by runoff (kg/ha)</td>
<td>2.3</td>
</tr>
<tr>
<td>Maximum loss rate of dissolved phosphorus by runoff (kg/ha)</td>
<td>0.02</td>
</tr>
<tr>
<td>Unit cost of fertilizer application ($/kg)</td>
<td>1.5</td>
</tr>
<tr>
<td>Unit cost of manure collection/disposal ($/t)</td>
<td>7.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fuzzy parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum allowable soil loss rate (kg/ha)</td>
<td>(3700, 5000, 6500, 9000)</td>
</tr>
<tr>
<td>Maximum allowable soil-phase nitrogen loss rate (kg/ha)</td>
<td>Exp[-</td>
</tr>
<tr>
<td>Maximum allowable soil-phase phosphorus loss rate (kg/ha)</td>
<td>Exp[-</td>
</tr>
<tr>
<td>Soil loss rate from the land planted with wheat (i = 1)</td>
<td>(2600, 5100, 7700)*</td>
</tr>
<tr>
<td>Soil loss rate from the land planted with vegetable (i = 2)</td>
<td>(9000, 17500, 26500)</td>
</tr>
<tr>
<td>Soil loss rate from the land planted with potato (i = 3)</td>
<td>(3870, 7600, 11400)</td>
</tr>
</tbody>
</table>

Note: Data are adapted from Huang (1996), Xu and Qin (2010), and Nie et al. (2008); (a, b, c, d)* represents a trapezoidal-shape fuzzy set with a, b, c, and d being the four sequential parameters from left to right. (d, e, f)* represents a triangular fuzzy set, where d and f are the minimum and maximum possible values and e is the most likely value. Exp[·] represents an exponential fuzzy set.

Hence, the model for agricultural water resources problem could be formulated as follows (Nie et al., 2008; Xu and Qin, 2010):

\[
\text{Maximize } f = \sum_{i=1}^{T} \sum_{j=1}^{I} \delta Y_{ij} S_u + \sum_{j=1}^{J} \eta_j Z_j - \sum_{i=1}^{T} \sum_{t=1}^{I} G_i S_u \\
- G_M \sum_{i=1}^{I} M_i - G_F \sum_{i=1}^{I} F_i - \sum_{t=1}^{T} \sum_{i=1}^{I} v_i W_i S_u + CPF + IPF
\]  
(3.6a)

Subject to:
\[ CPF = \begin{cases} 0 & \text{if } G(X, \xi) \text{ is met} \\ \lambda_1 & \text{if } G(X, \xi) \text{ is not met} \end{cases} \quad (3.6b) \]

\[ IPF = \begin{cases} 0 & \text{if } H(X, \eta) \text{ is met} \\ \lambda_2 & \text{if } H(X, \eta) \text{ is not met} \end{cases} \quad (3.6c) \]

\[
G(X, \xi) = \begin{cases} \text{Pos}\left\{ \bar{L}_i, \bar{b} \left| \sum_{i=1}^{T} \sum_{j=1}^{I} \bar{L}_i S_{ij} \leq \bar{b} \sum_{j=1}^{T} K_j \right\} \geq \alpha \\
\end{cases}
\]

\[
H(X, \eta) = \begin{cases} \text{Pos}\left\{ \bar{L}_i, \bar{c}_2 \left| \sum_{i=1}^{T} \sum_{j=1}^{I} h_2 \bar{L}_i S_{ij} \leq \bar{c}_2 \sum_{j=1}^{T} K_j \right\} \geq \alpha \\
\end{cases}
\]

\[
\sum_{i=1}^{I} M_i - \sum_{j=1}^{J} B_j Z_j = 0
\]

\[
(1 - p_i)gM_i + (1 - p_2)F_i - \sum_{j=1}^{T} q_j S_{ij} \geq 0, \quad \forall i
\]

\[
\sum_{i=1}^{T} \sum_{j=1}^{I} Y_{ij} \beta_{ij} S_{ij} - \sum_{j=1}^{J} E_j Z_j \geq 0
\]

\[
\sum_{i=1}^{T} \sum_{j=1}^{I} Y_{ij} \gamma_{ij} S_{ij} - \sum_{j=1}^{J} D_j Z_j \geq 0
\]

\[
H(X, \eta) = \begin{cases} \sum_{i=1}^{I} \left( M_i g + F_i - q_i \sum_{j=1}^{T} S_{ij} \right) \leq a \sum_{j=1}^{T} K_j \\
\sum_{i=1}^{I} S_{ij} \leq K_i, \quad \forall t \\
\sum_{i=1}^{I} \sum_{j=1}^{I} (R_{ij} N_{ij} + R_2 N_{2j}) S_{ij} \leq u_1 \sum_{j=1}^{T} K_j \\
\sum_{i=1}^{I} \sum_{j=1}^{I} (R_{ij} P_{ij} + R_2 P_{2j}) S_{ij} \leq u_2 \sum_{j=1}^{T} K_j \\
\sum_{i=1}^{I} W_i S_{ij} \leq Q_i, \quad \forall t \\
\end{cases} \quad (3.6e)
\]

\[
S_{in}, M_{in}, F_{in}, Z_j \geq 0, \quad \forall i, j, n, t \quad (3.6f)
\]

where
\( f \) = net system income ($);

\( t, i \) and \( j \) (\( t = 1, 2, ..., T; i = 1, 2, ..., I; j = 1, 2, ..., J \)) are indexes of subarea, crops and livestock, respectively;

\( T, I \) and \( J \) are numbers of subarea, crops and livestock, respectively;

\( Y_i \) = unit yield of crop \( i \) (kg/ha);

\( S_{it} \) = area of crop \( i \) in subarea \( t \) (ha);

\( Z_j \) = number of livestock \( j \) in the study area;

\( \delta_i \) = unit price of crop \( i \) ($/kg);

\( \eta_j \) = unit average return from livestock \( j \) ($/unit);

\( G_i \) = unit farming cost for crop \( i \) ($/ha);

\( G_M \) = unit cost of manure collection and disposal ($/t);

\( G_H \) = unit cost of fertilizer application ($/kg);

\( \nu_{it} \) = unit cost to deliver water to \( S_{it} \) ($/m^3/s);

\( M_i \) = amount of manure applied to crop \( i \) (t);

\( F_i \) = amount of fertilizer nitrogen applied to crop \( i \) (kg);

\( W_{it} \) = flow rate of irrigation water required by crop \( i \) in subarea \( t \) (m\(^3\)/s/ha);

\( B_j \) = unit amount of manure generated by livestock \( j \) that needs to be disposed (t/unit);

\( p_i \) = nitrogen volatilization/denitrification rate of manure (%);

\( g \) = nitrogen concentration of manure (kg/t);
$p_2 =$ nitrogen volatilization/denitrification rate of fertilizer ($\%$);

$q_i =$ unit nitrogen requirement of crop $i$ (kg/ha);

$\beta_i =$ net energy potential of crop $i$ (Mcal/kg);

$E_j =$ unit net energy requirement of livestock $j$ (Mcal/unit);

$\gamma_i =$ digestible protein content of crop $i$ ($\%$);

$D_j =$ unit digestible protein requirement of livestock $j$ (kg/unit);

$K_t =$ tillable area in subarea $t$ (ha);

$L_i =$ soil loss rate from land planted with crop $i$ (kg/ha);

$b =$ maximum allowable soil loss rate (kg/ha);

$c_i =$ maximum allowable solid-phase nitrogen loss rate (kg/ha);

$c_2 =$ maximum allowable solid-phase phosphorus loss rate (kg/ha);

$a =$ maximum allowable total nitrogen loss rate (kg/ha);

$h_1 =$ nitrogen content of soil ($\%$);

$h_2 =$ phosphorus content of soil ($\%$);

$N_{1i} =$ dissolved nitrogen concentration in wet season runoff from land planted to crop $i$ (mg/l);

$N_{2i} =$ dissolved nitrogen concentration in dry season runoff from land planted to crop $i$ (mg/l);

$R_{1i} =$ wet season runoff from land planted to crop $i$ (mm);

$R_{2i} =$ dry season runoff from land planted to crop $i$ (mm);
\( u_1 \) = maximum allowable loss rate of dissolved nitrogen by runoff (kg/ha);

\( u_2 \) = maximum allowable loss rate of dissolved phosphorus by runoff (kg/ha);

\( P_{1i} \) = dissolved phosphorus concentration in wet season runoff from land planted to crop \( i \) (mg/l);

\( P_{2i} \) = dissolved phosphorus concentration in dry season runoff from land planted to crop \( i \) (mg/l);

\( Q_t \) = maximum canal flow within subarea \( t \) (m\(^3\)/s); \( \alpha \) = prescribed confidence level.

The objective of the system is to maximize the net economic benefit of the crop cultivation and livestock breeding. Constraints 3.6(b) and 3.6(c) define the penalty factors of reflecting violation of fuzzy constraints and deterministic constraints, respectively. Constraint 3.6(d) denotes fuzzy constraints, including (i) the soil loss amounts being lower than the allowable soil loss amounts, and (ii) the pollutant loss amounts being lower than the allowable pollutants discharge amounts. Constraint 3.6(e) denotes the deterministic constraints, including (i) the total amount of manure generated by livestock being equal to the amount of manure applied to crop, (ii) the nutrient offered by manure and fertilizer satisfying the nutrient requirement of crop, (iii) the total yield of crops meeting the energy and digestive protein requirements, and (iv) the irrigation water requirement not exceeding the maximum canal water amounts. Constraint 3.6(f) stipulates that the decision variables are non-negative.

The GAFSA model is implemented in the platform of MATLAB 2008a. The hardware settings are: (1) CPU: AMD Phenom(tm) X4 B95 Processor 3.00GHz; (2) SIMM: 4GB (DDR3 1333MHZ). The parameters of using genetic algorithm are: (1) pop sizes = 100; (2) maximum generations = 2000; (3) single-point crossover rate = 0.8; (4) the mutation rate = 0.01; (5) termination tolerance on fitness function value = \( 1 \times 10^{-6} \); (6) the penalty factor for fuzzy violation \( (\lambda_1) = 2 \times 10^6 \); (7) the penalty factor for infeasibility \( (\lambda_2) = 2 \times 10^6 \). Generally, the average computational time of solving the agricultural water resources management model took about 50 minutes with the circle number \( n = 3000 \).
3.3.3 Result Analysis

Table 3.4 shows the results obtained from the GAFSA model, including the system benefit and the amounts of crops, livestock, manure and fertilizer under different confidence levels. It indicates that the majority of the area in subarea 1 would be used for planting crops, while the subareas 2 and 3 would not be utilized completely and the land left over in subarea 3 would be larger. For example, under the confidence level of 0.7, the cropping areas are determined to be 34.02, 18.56, and 32.46 ha for subareas 1 to 3, respectively. This is mainly due to the stricter irrigation requirement on water quantity and the higher cost of distributing water in subarea 3. From Table 3.4, the number of cattle and swine would be relatively lower than that of the poultry. This may be caused by multiple factors, including the related protein and energy demands, market prices and manure. It is also shown that the number of poultry would increase as the confidence level increases. For example, the number of poultry is 1912, 3000, 3045, and 3728 at the confidence levels of 0.7, 0.8, 0.9, and 0.95. This is because the potato and wheat have higher energy potential and digestible protein for breeding poultry. When the confidence level increases, the cropping areas for both of them would increase, leading to the growth of poultry size.

Figure 3.4 shows the optimized cropping areas under different confidence levels. It is indicated that the vegetable and potato would occupy most of the areas, while the wheat would only cover a small portion. For example, under the confidence level of 0.8, the crop areas of wheat, vegetable, and potato is 7.46, 39.91, and 41.06 ha, respectively. Figure 3.4 also shows that the cropping area would have large variations under different confidence levels. The crop area of wheat and potato under the confidence levels of 0.7 to 0.95 increases from 2.99 to 12.27 ha, and from 24.40 to 45.64 ha, respectively; while the crop area of vegetable under the confidence levels of 0.7 to 0.95 decreases from 67.65 to 26.55 ha. The decrease of crop area of vegetable is due to the fact that the vegetable has a good market price which could contribute to the net income; however, the pollution generated by the vegetable is higher than those by other crops. Conversely, the cropping areas for potato would increase when the confidence level increases, which is because, when the confidence level and system reliability become higher, the environmental loading capacities
and the standard of pollutant discharge would be stricter; the system prefers to choose a crop with a relatively lower environmental impact and a higher yielding capacity (and thus a higher benefit). The potato, with moderate market price, levels of yield, and pollutant discharge, could not only contribute to the system benefit, but also satisfy the water quality requirement; it is more desirable under a stricter environmental standard.

Table 3.5 Solutions of GAFSA model based on generally-shaped fuzzy membership functions of uncertain parameters.

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Confidence Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.7</td>
</tr>
<tr>
<td>$S_{11}$ Wheat</td>
<td>2.28</td>
</tr>
<tr>
<td>$S_{12}$ Wheat</td>
<td>0.01</td>
</tr>
<tr>
<td>$S_{13}$ Wheat</td>
<td>0.70</td>
</tr>
<tr>
<td>$S_{21}$ Vegetable</td>
<td>20.10</td>
</tr>
<tr>
<td>$S_{22}$ Vegetable</td>
<td>15.15</td>
</tr>
<tr>
<td>$S_{23}$ Vegetable</td>
<td>22.40</td>
</tr>
<tr>
<td>$S_{31}$ Potato</td>
<td>11.64</td>
</tr>
<tr>
<td>$S_{32}$ Potato</td>
<td>3.40</td>
</tr>
<tr>
<td>$S_{33}$ Potato</td>
<td>9.36</td>
</tr>
<tr>
<td>$Z_{1}$ Cattle</td>
<td>43</td>
</tr>
<tr>
<td>$Z_{2}$ Pig</td>
<td>3</td>
</tr>
<tr>
<td>$Z_{3}$ Poultry</td>
<td>1912</td>
</tr>
<tr>
<td>$M_{1}$ Wheat</td>
<td>0.14</td>
</tr>
<tr>
<td>$M_{2}$ Vegetable</td>
<td>766.87</td>
</tr>
<tr>
<td>$M_{3}$ Potato</td>
<td>82.79</td>
</tr>
<tr>
<td>$F_{1}$ Wheat</td>
<td>352.20</td>
</tr>
<tr>
<td>$F_{2}$ Vegetable</td>
<td>2207.30</td>
</tr>
<tr>
<td>$F_{3}$ Potato</td>
<td>1875.00</td>
</tr>
</tbody>
</table>

Net Income ($10^5$) 8.06 7.58 7.06 6.65

Note: $i =$ type of crop; $S_{it} =$ cropping area (ha), where $t =$ index of subarea; $M_{it} =$ applied manure amount ($t$); $F_{ij} =$ amount of applied nitrogen fertilizer (kg); $Z_{ij} =$ size of livestock husbandry (unit), where $j =$ livestock.
Figure 3.4 Optimized cropping areas under various confidence levels.

Figure 3.5 and Figure 3.6 indicate that, with the change in the cropping areas, the manure/fertilizer requirement for the vegetable would be lower than that under a higher confidence level; while, the amount of the wheat and potato has the opposite trend. It is also found that the amount of manure is much higher than that of fertilizer by orders of magnitude. Meanwhile, with the increase of the confidence level, the total amount of fertilizer would decrease, while the total amount of manure does not have an obvious trend. The reason is that the manure and fertilizer not only provide nutrient and energy to the crops but also cause the nonpoint sources pollution problems; their usage should then be controlled to meet the environmental requirements. Manure is associated with livestock husbandry and produced locally, which could contribute to the net income; while fertilizer is bought from external sources, which is much more expensive than manure. Therefore, the total amount of fertilizer would be lower under a higher confidence level.

The net income of the system would decrease with the increase of the confidence level. For example, at the confidence levels of 0.7, 0.8, 0.9, 0.95, the net incomes would be 8.06, 7.58, 7.06, and 6.65 ($\times 10^5$), respectively. This implies that the
system benefit would become higher when the water quality requirement is less strict. Thus, a conservative plan may lead to more reliable system; conversely, planning for a higher system benefit may result in a higher risk of failure.

![Figure 3.5 Applied manure amounts for different crops.](image)

![Figure 3.6 Optimized total manure/fertilizer amounts under various confidence levels.](image)
Generally, the proposed GAFSA is capable of handling fuzzy parameters existing in agricultural water quality management system with generally shaped membership functions. The results obtained from GAFSA model could reflect the various trend of system benefit under different system reliabilities. The higher the system reliability, the less the net income the system would get. The results would provide useful information for water quality managers to make or select a more profitable decision at a reasonable reliability level.

The cycle index \( n \) for calculating the possibility \( p \) not only influences the accuracy of the model results, but also determines the required time for solving the model. Therefore, several tests were performed to search for a suitable number with lower computational requirement but acceptable accuracy. The following equation is recommended to check the relative errors using various cycle numbers:

\[
RE_n = \frac{p_n - p_N}{p_N} \times 100\% \tag{3.6}
\]

where \( p_n \) is the possibility calculated using \( n \) cycles in fuzzy simulation and \( n \) is the number of cycle index; \( p_N \) is the possibility under a sufficiently large number of cycles (i.e. \( 5 \times 10^5 \)); \( RE_n \) is the relative error under the cycle index of \( n \). Figure 3.7 shows the number of cycle index vs. the accuracy of the value of possibility. Obviously, the number of cycle index would greatly influence the accuracy of the value of possibility. An increase of the cycle index would lead to the reduction of relative errors. When the number of cycle index increases from 100 to 5000, the relative error would decrease from 42% to 4.5%; however it would take a longer time to finish the fuzzy simulation process and thus make the GA-based optimization process more time-consuming. Therefore, a trade-off should be made between the computational requirement and simulation accuracy. It is recommended that the decision makers define an acceptable criterion for relative error (such as 5%) before launching the fuzzy simulation.
Figure 3.7 Relative errors under various cycle indexes.

3.4 Discussion

3.4.1 Comparison with FCCP Method

In order to demonstrate the applicability of GAFSA, we also applied the traditional FCCP method to solve the same problem. The parameters expressed as generally-shaped fuzzy sets were all replaced with triangular-shaped fuzzy sets in order to make sure the model can be solved using the algorithm proposed by Liu and Iwamura (1998). Table 3.5 lists the results obtained from traditional FCCP method. The table shows that the system benefits obtained from FCCP at confidence level of 0.7, 0.8, 0.9, 0.95 are 8.61, 7.75, 6.97, and 6.61 ($ \times 10^5$) are close to the results obtained from GAFSA. The table also shows that there exist deviations in solutions of decision variables. This is due to the fact that (i) the shapes of fuzzy distributions would influence the optimization results, and (ii) GA could only reach suboptimal solutions, especially when the number of decision variables is large. Generally, GAFSA is advantageous than FCCP in terms of its capability in dealing with generally-shaped fuzzy variables, but inferior in its weakness of achieving less optimal solutions.
Table 3.6 Solutions from FCCP method based on triangular fuzzy membership functions of uncertain parameters

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Confidence Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.7</td>
</tr>
<tr>
<td>$S_{11}$ Wheat</td>
<td>0</td>
</tr>
<tr>
<td>$S_{12}$ Wheat</td>
<td>0</td>
</tr>
<tr>
<td>$S_{13}$ Wheat</td>
<td>0</td>
</tr>
<tr>
<td>$S_{21}$ Vegetable</td>
<td>29.64</td>
</tr>
<tr>
<td>$S_{22}$ Vegetable</td>
<td>0</td>
</tr>
<tr>
<td>$S_{23}$ Vegetable</td>
<td>0</td>
</tr>
<tr>
<td>$S_{31}$ Potato</td>
<td>4.46</td>
</tr>
<tr>
<td>$S_{32}$ Potato</td>
<td>23.90</td>
</tr>
<tr>
<td>$S_{33}$ Potato</td>
<td>2.72</td>
</tr>
<tr>
<td>$Z_{1}$ Cattle</td>
<td>33</td>
</tr>
<tr>
<td>$Z_{2}$ Pig</td>
<td>0</td>
</tr>
<tr>
<td>$Z_{3}$ Poultry</td>
<td>1785</td>
</tr>
<tr>
<td>$M_{1}$ Wheat</td>
<td>0</td>
</tr>
<tr>
<td>$M_{2}$ Vegetable</td>
<td>589.00</td>
</tr>
<tr>
<td>$M_{3}$ Potato</td>
<td>76.40</td>
</tr>
<tr>
<td>$F_{1}$ Wheat</td>
<td>0</td>
</tr>
<tr>
<td>$F_{2}$ Vegetable</td>
<td>0</td>
</tr>
<tr>
<td>$F_{3}$ Potato</td>
<td>2790.72</td>
</tr>
<tr>
<td>Net Income ($10^5$)</td>
<td>8.61</td>
</tr>
</tbody>
</table>

Note: $i =$ type of crop; $S_{it}$ = cropping area (ha), where $t =$ index of subarea; $M_{it}$ = applied manure amount ($t$); $F_{it}$ = amount of applied nitrogen fertilizer (kg); $Z_{ij}$ = size of livestock husbandry (unit), where $j =$ livestock.

### 3.4.2 Further discussions on GAFSA

(1) In real world application, the generation of membership functions is a complicated task. Some membership functions are based on the engineer’s experience and are generated intuitively; others are determined by statistical techniques, such as direct rating, polling, and reverse rating (Mazloumzadeh et al., 2008). However, in our study, some parameters are considered to have specific
shapes of fuzzy membership functions (such as triangular, trapezoidal, and exponential form) in order to simplify the fuzzy simulation process and better demonstrate the applicability of GAFSA.

(2) Parameters associated with environmental loading capacities (e.g. soil loss amounts) are normally estimated based on empirical experience and subject to human judgments. Hence they could be expressed by fuzzy sets with various membership functions, and other parameters are deemed as deterministic. The parameters in objective function (e.g. economical parameters) may also be subjected to uncertainties. To simplify the fuzzy simulation process for an easier demonstration, we assume these parameters are deterministic. However, in real-world applications, identifying the uncertainty of a specific parameter is not an easy task. We need to analyze them according to history record, experiment data, or survey information. If too many uncertain inputs are encountered, we can conduct a sensitive analysis first and find out the most sensitive parameters for uncertainty assessment.

(3) GA could give better optimal solution than that of conventional algorithm when the amount of decision variables is not so many. However, it is generally time-consuming to obtain convergent solutions when there are hundreds of decision variables. Also, the accuracy of solutions is also affected by local optima that cannot be easily solved by GA. Therefore, it is recommended that GA is more suitable for small scale problems. According to our study, it is suggested that the number of decision variables is better controlled within 20. Further test is required for larger-scale applications.

(4) GA was used by Qin et al. (2010) to solve a stochastic chance constrained model. Monte Carlo simulation technique was proposed to estimate the probability of meeting the stochastic constraint and check the feasibility of potential solutions. However, Monte Carlo technique can only handle stochastic parameters. The fuzzy-simulation-based iteration process, as applied in this study, can be used to estimate the possibilistic value of fuzzy constraint satisfaction.
3.5 Summary

A combined genetic algorithm and fuzzy simulation approach (GAFSA) was developed in this study and applied to an agricultural water resources system. GAFSA incorporated genetic algorithm (GA) into a fuzzy chance-constrained programming framework, allowing some constraints with fuzzy variables to be satisfied at specified confidence levels. Moreover, with the assistance of GA, GAFSA was capable of tackling fuzzy parameters with generally shaped membership functions. The results indicated that higher confidence level would result in lower system benefit. A conservative planning scheme could bring a more reliable system, but would be less economically attractive. Conversely, planning towards a higher system benefit would lead to a higher risk of system failure. Moreover, the results could also assist agricultural water managers to make a trade-off between the overall system benefit and the failure risk of environmental compliance.

This study made a first attempt to apply the GAFSA to solve FCCP model in an agricultural water resources management system. The results implied that the proposed method could also be further applied to many other water resources management problems. However, GAFSA also showed some limitations: (i) the evaluation process in GA may lead to a suboptimal solution; (ii) the solution process is time-consuming when number of cycle index is large. Further study is needed to tackle these limitations.
CHAPTER 4 A SEQUENTIAL FUZZY MODEL WITH GENERAL-SHAPED PARAMETERS FOR WATER SUPPLY-DEMAND ANALYSIS

4.1 Introduction

The increasing water scarcity problems have led to great tensions and conflicts among competing water users, and brought enormous pressure to the environment. The techniques of water supply management rely on the development of new water storage and treatment facilities and structures (Vairavamoorthy et al., 2008). Demand management strategies mainly consist of conservation measures, such as efficient irrigation and water fixture retrofits (Wilchfort and Lund, 1997). Incorporating water demand with supply (namely supply-demand management) would be more efficient to solve water shortage problems. However, management of such a system is complicated due to interactions among the water supply and demand components, and a variety of uncertainties associated with system inputs, such as availability of water sources, demand of water users, and leakage during transportation. Therefore, it is critical to develop an efficient decision support methodologies for water supply-demand management in consideration of uncertainty treatment.

To deal with uncertainties which could be expressed as fuzzy, random and interval variables, different mathematical tools (e.g. fuzzy set theory, probability theory, and interval analysis) have been explored (Chang et al., 1997; Feiring et al., 1998; Li et al., 2006; Han et al., 2011). Among these tools, fuzzy mathematical programming (FMP) is deemed an efficient alternative since the fuzzy distribution is relatively easy to obtain and has less strict requirement in computation. Previously, there have been a large number of applications of FMP in water resources management field (Slowinski, 1986; Kindler, 1992; Wang and Huang, 2012). For instance, Jairaj and Vedula (2000) applied a fuzzy mathematical programming model to a multi-reservoir system of Cauvery River Basin, Karnataka State, India, where the reservoir inflows is considered as fuzzy sets. Simonovic and Nirupama (2005) developed a spatial fuzzy compromise programming to address different uncertainties in spatial water
resources decision-making and all uncertain variables are modeled by fuzzy sets. Guo and Huang (2009) applied a two-stage fuzzy chance-constrained programming to a water allocation problem, where uncertainties in the objective function and the left-hand sides of constraints were presented as fuzzy sets. Peng and Zhou (2011) proposed a fuzzy-dependent chance programming for planning of water allocation in a coastal city, considering fuzzy objectives of maximizing the possibilities of satisfying the water requirements for six users. Wang and Huang (2011) developed an interactive multi-stage fuzzy programming for determining the allocation of water resources, which could obtain a range of solutions under various degrees of feasibility and tradeoff between constraint-violation risk and economic efficiency. Lv et al. (2012) used a superiority and inferiority measures developed by Van Hop (2007) for managing water resources system, where water resources demands are presented as fuzzy sets.

The previous studies made valuable attempts in solving water resources management problems with uncertain parameters expressed as fuzzy sets. Generally, the traditional way to solve a fuzzy programming model is to transform it into one or more deterministic forms (Liu and Iwamura, 1998). However, the membership functions of fuzzy parameters in most of the previous studies are limited to triangular and/or trapezoidal shapes. In fact, a real-world water supply-demand management system is normally subject to uncertainties with complex natures and they hardly present in special shapes like triangular. Fuzzy membership functions are normally generated based on data surveys, experience of engineer, statistical analysis and human judgment. For example, some parameters are usually evaluated by measurement and the membership function may follow the shape of a normal distribution (e.g. water loss rate); and some parameters may have clear boundary and usually determined by decision makers, the membership function is preferably triangular-shape (e.g. allowable water amount). The simplification of a complex fuzzy membership function (such as exponential, monotonous and normal distribution) into triangular or trapezoidal could ease the solution of the problem, but may lead to loss of valuable distribution information and bring potential risk of reduced accuracy and rationality of solutions. Previously, very few studies were
devoted to tackling this issue. Some attempts were made in other fields. For example, Liu and Iwamura (Liu and Iwamura, 1998) proposed to use heuristic searching method (i.e. genetic algorithm) to solve fuzzy chance-constrained programming problems; however, the method were difficult to obtain global optima, especially for large scale problems (Qin et al., 2010).

Therefore, this study aims to develop a novel superiority-inferiority-based sequential fuzzy programming (SISFP) model for supporting water supply-demand management under uncertainty. The method is based on the concept of superiority and inferiority measures, proposed by Van Hop (2007), in dealing with triangular fuzzy membership functions. We extended such method to address multiple shapes of fuzzy membership functions (i.e. so called general shaped fuzzy sets) that could be linked with parameters existing in both objective function and model constraints. The applicability of SISFP and its advantages in dealing with water supply-demand management problems will be demonstrated though a real-world case in Tianjin Binhai New Area, China. The region is suffering from serious water shortage problem and need sound long-term management strategies in balancing its water supply and demand. The general methodology will be introduced first, followed by a simple numerical example. Then the study case will be presented and the related results will be analyzed and discussed.

4.2 General Methodology

A fuzzy linear programming can be given by (Delgado et al., 1989):

\[
\text{Maximum } f = \tilde{C}X
\]  

subject to:

\[
\tilde{A}X \leq \tilde{B} \tag{4.1b}
\]

\[
DX \leq E \tag{4.1c}
\]

\[
\tilde{A} \geq 0, \tilde{C} \geq 0, D \geq 0 \tag{4.1d}
\]
\[ X \geq 0 \] 

where, \( X = (x_1, x_2, \ldots, x_j) \) is vector of decision variables, 
\( \tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_j) \) and \( \tilde{B} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_j) \) are fuzzy vectors with general-shaped fuzzy membership functions (such as trapezoidal, exponential, and Gaussian shapes).

### 4.2.1 Fuzzy sequential method

In order to solve the model (4.1), the fuzzy programming needs to be converted into its deterministic form by the sequential method (Van Hop, 2007). Hence, the superiority and inferiority measures will be introduced for comparison between two fuzzy sets. Let two fuzzy numbers be denoted as \( \tilde{R} \) and \( \tilde{S} \) (as shown in Figure 4.1). Two intervals can be obtained under a specific \( \alpha \)-cut level: \( \tilde{R}_\alpha = (R^L_\alpha, R^U_\alpha) \) and \( \tilde{S}_\alpha = (S^L_\alpha, S^U_\alpha) \).

According to Van Hop (Van Hop, 2007), if \( \tilde{R}_\alpha \geq \tilde{S}_\alpha \), we have:

\[
SP_\alpha (\tilde{R}, \tilde{S}) = R^U_\alpha - S^U_\alpha \quad IF_\alpha (\tilde{S}, \tilde{R}) = (R^L_\alpha - S^L_\alpha)
\]

(4.2)

where, \( SP_\alpha \) and \( IF_\alpha \) are defined as the superiority of \( \tilde{R} \) over \( \tilde{S} \), and the inferiority of \( \tilde{S} \) to \( \tilde{R} \) under an \( \alpha \)-cut level. Then, the total superiority and inferiority can be calculated as (Van Hop, 2007):

\[
SP (\tilde{R}, \tilde{S}) = \int_0^1 SP_\alpha (\tilde{R}, \tilde{S}) d\alpha = \int_0^1 (R^U_\alpha - S^U_\alpha) d\alpha
\]

(4.3a)

\[
IF (\tilde{S}, \tilde{R}) = \int_0^1 IF_\alpha (\tilde{S}, \tilde{R}) d\alpha = \int_0^1 (R^L_\alpha - S^L_\alpha) d\alpha
\]

(4.3b)
Figure 4.1 Superiority and inferiority of $\tilde{R}$ and $\tilde{S}$.

The solution of Equation (4.3) needs to be derived in light of particular shapes of fuzzy numbers. Triangular or trapezoidal is the simplest. To demonstrate the general way of derivation, we consider two fuzzy numbers $\tilde{R}(a, b)$ and $\tilde{S}(c, d)$ with Gaussian and exponential-shaped membership functions, respectively (as Figure 4.2 showed). Their membership functions can be written as:

$$
\mu_R(x) = \exp\left(-\left(\frac{x-a}{b}\right)^2\right), \quad \mu_S(x) = \exp\left(-\left|\frac{x-c}{d}\right|\right)
$$

The interval of $\tilde{R}$ and $\tilde{S}$ under $\alpha$-cut of $\alpha$ is $(a-b\sqrt{\ln \alpha^{-1}}, a+b\sqrt{\ln \alpha^{-1}})$ and $(c-d\ln \alpha, c+d\ln \alpha)$, respectively (Van Hop, 2007). Then the superiority and
inferiority of \( \tilde{R} \) and \( \tilde{S} \) under \( \alpha \) can be calculated as:

\[
SP_\alpha (\tilde{R}, \tilde{S}) = (a + b\sqrt{\ln \alpha^{-1}}) - (c + d \ln \alpha)
\] (4.5a)

\[
IF_\alpha (\tilde{S}, \tilde{R}) = (a - b\sqrt{\ln \alpha^{-1}}) - (c - d \ln \alpha)
\] (4.5b)

The total superiority \( \tilde{R} \) over \( \tilde{S} \), and the inferiority of \( \tilde{R} \) to \( \tilde{S} \) can be obtained using Equations (4.3a) and (4.3b). To calculate the definite integral in Equations (4.3a) and (4.3b), we can use other numerical integration method, such as the Adaptive Simpson’s rule, which is proposed by Kuncir (Kuncir, 1962). A definite integral on the interval \([a, b]\) using adaptive Simpson’s rule can be written as follows (Cheney and Kincaid, 2012):

\[
I = \int_a^b f(x)dx = S(a, c) + S(c, b) + E
\] (4.6)

where \( c \) is the middle point of interval \([a, b]\); \( S(a, c) \) and \( S(c, b) \) are given based on Basic Simpson’s rule on the intervals \([a, c]\) and \([c, b]\), respectively (Larson and
Edwards, 2010):

\[
S(a, c) = \frac{c - a}{6} \left[ f(a) + 4f\left(\frac{a + c}{2}\right) + f(c) \right] \tag{4.7a}
\]

\[
S(c, b) = \frac{b - c}{6} \left[ f(c) + 4f\left(\frac{b + c}{2}\right) + f(b) \right] \tag{4.7b}
\]

The error \( E \) can be written as (Cheney and Kincaid, 2012):

\[
E = \frac{1}{15} \left[ S(a, c) + S(c, b) - S(a, b) \right] \tag{4.8}
\]

Given the desired error tolerance \( \epsilon \), the recursion can be terminated when (Cheney and Kincaid, 2012)

\[
\frac{1}{15} \left| S(a, c) + S(c, b) - S(a, b) \right| < \epsilon \tag{4.9}
\]

Therefore, the total superiority \( \tilde{R} \) over \( \tilde{S} \), and the inferiority of \( \tilde{R} \) to \( \tilde{S} \) can be written as follows:

\[
SP(\tilde{R}, \tilde{S}) = \int_0^1 SP_\alpha(\tilde{R}, \tilde{S}) \, d\alpha = \int_0^1 \left[ (a + b\sqrt{\alpha^{-1}}) - (c + d \ln \alpha) \right] \, d\alpha
\]

\[
= (a - c) + (d + \frac{\sqrt{\pi}}{4} b) \tag{4.10a}
\]

\[
IF(\tilde{S}, \tilde{R}) = \int_0^1 IF_\alpha(\tilde{S}, \tilde{R}) \, d\alpha = \int_0^1 \left[ (a - b\sqrt{\alpha^{-1}}) - (c - d \ln \alpha) \right] \, d\alpha
\]

\[
= (a - c) - (b + \frac{\sqrt{\pi}}{4} d) \tag{4.10b}
\]

The superiority and inferiority of other fuzzy distribution shapes can also be derived in a similar manner. Figure 4.3 shows some common types of fuzzy numbers and their corresponding superiority and inferiority, where \( \tilde{R} \) is on the right-hand side (RHS) and \( \tilde{S} \) is on the left-hand side (LHS). It should be noted that if the sides of the fuzzy parameters are changed, a minus sign can be added. For example, if \( \tilde{R} \) is on
the LHS and \( \tilde{S} \) is on the RHS, the superiority and inferiority would be 
\[
SP(\tilde{S}, \tilde{R}) = -SP(\tilde{R}, \tilde{S}) \quad \text{and} \quad IF(\tilde{R}, \tilde{S}) = -IF(\tilde{S}, \tilde{R})
\]
respectively.

<table>
<thead>
<tr>
<th>Fuzzy numbers</th>
<th>Superiority and inferiority</th>
<th>Membership functions</th>
</tr>
</thead>
</table>
| Trapezoidal vs          | \( SP(\tilde{R}, \tilde{S}) = \frac{a_1 + a_4}{2} - \frac{b_1 + b_4}{2} \) | ![Graph of trapezoidal]
| Trapezoidal:            | \( IF(\tilde{S}, \tilde{R}) = \frac{a_1 + a_4}{2} - \frac{b_1 + b_4}{2} \) | ![Graph of trapezoidal]
| \( \tilde{R} = (a_1, a_2, a_3, a_4) \) | \( \tilde{S} = (b_1, b_2, b_3, b_4) \)                      |
| Gaussian vs             | \( SP(\tilde{R}, \tilde{S}) = \frac{a_1 + a_4}{2} - \left( \frac{b + c\sqrt{\pi}}{2} \right) \) | ![Graph of Gaussian]
| Trapezoidal:            | \( IF(\tilde{S}, \tilde{R}) = \frac{a_1 + a_4}{2} - \left( \frac{b - c\sqrt{\pi}}{2} \right) \) | ![Graph of Gaussian]
| \( \tilde{R} = (a_1, a_2, a_3, a_4) \) | \( \tilde{S} = (b, c) \)                                      |
| Exponential vs          | \( SP(\tilde{R}, \tilde{S}) = \frac{a_1 + a_4}{2} - (b - c) \) | ![Graph of exponential]
| Trapezoidal:            | \( IF(\tilde{S}, \tilde{R}) = \frac{a_1 + a_4}{2} - (b + c) \) | ![Graph of exponential]
| \( \tilde{R} = (a_1, a_2, a_3, a_4) \) | \( \tilde{S} = (b, c) \)                                      |
| Gaussian vs.            | \( SP(\tilde{R}, \tilde{S}) = a_1 - a_4 + \frac{\sqrt{\pi}}{2} (b_1 - b_4) \) | ![Graph of Gaussian]
| Gaussian:               | \( IF(\tilde{S}, \tilde{R}) = a_1 - a_4 - \frac{\sqrt{\pi}}{2} (b_1 - b_4) \) | ![Graph of Gaussian]
| \( \tilde{R} = (a_1, b_1) \) | \( \tilde{S} = (a_2, b_2) \)                                  |
| Exponential vs          | \( SP(\tilde{R}, \tilde{S}) = (a_1 - a_2) - (b_1 - b_2) \) | ![Graph of exponential]
| Exponential:            | \( IF(\tilde{S}, \tilde{R}) = (a_1 - a_2) + (b_1 - b_2) \) | ![Graph of exponential]
| \( \tilde{R} = (a_1, b_1) \) | \( \tilde{S} = (a_2, b_2) \)                                  |

Notes: (a1, a2, a3, a4)* represents trapezoidal-shaped fuzzy number; (b, c)* represents Gaussian-shaped fuzzy number with distribution function of \( \text{Exp}\!\{-\{(x-b)/c\}^2\} \); \( <b, c> \)* represents exponential-shaped fuzzy number with distribution function of \( \text{Exp}\!\{-\{(x-b)/c\}^\} \); \( SP(\tilde{R}, \tilde{S}) = -SP(\tilde{S}, \tilde{R}) \), and \( IF(\tilde{S}, \tilde{R}) = -IF(\tilde{R}, \tilde{S}) \).

Figure 4.3 The relationship between different shapes of fuzzy numbers.

### 4.2.2 Superiority-inferiority-based fuzzy sequential programming

Based on the sequential method, the model (4.1) can be transformed to (Van Hop, 2007):
Maximum $f = F - P_{To} \cdot (\lambda_s + \lambda_i) - P_{Te} \cdot (U + V)$  \hspace{1cm} (4.11a)

subject to:

$\lambda_s = SP\left(F, \tilde{C}X\right)$  \hspace{1cm} (4.11b)

$\lambda_i = IF\left(\tilde{C}X, F\right)$  \hspace{1cm} (4.11c)

$U = SP\left(\tilde{A}X, \tilde{B}\right)$  \hspace{1cm} (4.11d)

$V = IF\left(\tilde{B}, \tilde{A}X\right)$  \hspace{1cm} (4.11e)

$D \cdot X \leq E$  \hspace{1cm} (4.11f)

$\lambda_s, \lambda_i, U, V \geq 0, X \geq 0$  \hspace{1cm} (4.11g)

where $P_{To}$ and $P_{Te}$ are sets of penalties for objective function and constraints, respectively. $P_{To}$ and $P_{Te}$ are real number which are larger than the objective function value by orders of magnitude or more, which can force the model to reject the infeasible solutions. As indicated from model (4.11), if the superiority of LHS is larger than that of RHS, or the inferiority of RHS is lower than that of LHS, the objective function would subject to penalty.

Figure 4.4 shows the possible conditions for the LHS and RHS of constraints assuming the LHS is in triangular shape and the RHS is in Gaussian one. The model (4.11), as proposed by Van Hop (Van Hop, 2007), only considers the condition that there is an intersection between RHS and LHS of the constraints (as L1 and R1 shown in Figure 3). If there is no intersection between RHS and LHS (as L2 and R1 shown in Figure 3), the value of $U$ and $V$ would be lower than zero, which may lead to infeasible solutions.

Therefore, a modification is made to model (4.11) as follows:

$Maximum \quad f = F - P_{To} \cdot (\lambda_s + \lambda_i) - P_{Te} \cdot (U + V)$  \hspace{1cm} (4.12a)
subject to:

\[ \lambda_i = \max \left[ 0, \ SP\left( F, \ \tilde{C}X \right) \right] \]  

(4.12b)

\[ \lambda_i = \max \left[ 0, \ IF\left( \tilde{C}X, \ F \right) \right] \]  

(4.12c)

\[ U = \max \left[ 0, \ SP\left( \tilde{A}X, \tilde{B} \right) \right] \]  

(4.12d)

\[ V = \max \left[ 0, \ IF\left( \tilde{B}, \tilde{A}X \right) \right] \]  

(4.12e)

\[ DX \leq E \]  

(4.12f)

\[ X \geq 0 \]  

(4.12g)

Figure 4.5 shows the overall framework of the proposed SISFP. The detailed operation procedures are: (i) acquire the fuzzy possibility distribution and formulate the fuzzy mathematic programming model; (ii) convert the constraints and objective function into their superiority and inferiority measures; (iii) define the penalty factors...
for the objective function and constraints; (iv) formulate the superiority-inferiority-based fuzzy programming model; (v) generate the final optimal solutions.

![Diagram of SISFP approach]

Figure 4.5 General framework of SISFP approach.

### 4.2.3 Numerical example

A numerical example is used to demonstrate how the proposed method works. A fuzzy optimization problem with multiple shapes of fuzzy variables is written as follows:

$$\text{Maximize } f = (2.5, 3, 3.5)x_1 + (0.5, 1, 1.5)x_2$$  \hspace{1cm} (4.13a)

Subject to:
\[(0.1, 0.15, 0.2)x_1 \leq \langle 1, 0.05 \rangle \quad (4.13b)\]

\[0.25x_1 - 0.3x_2 \leq 1 \quad (4.13d)\]

\[x_1, x_2 \geq 0 \quad (4.13e)\]

where the \((a, b, c)\) represents a triangular-shaped fuzzy set, \((a, b, c, d)\) represents a trapezoidal-shaped fuzzy set, \(<a, b>\) represents an exponential-shaped fuzzy set with fuzzy membership function as \(\exp\left(-\frac{1}{b}|x-a|\right)\), and \((a, b)\) represents a Gaussian-shaped fuzzy set with fuzzy membership function as \(\exp\left\{-\left[\frac{(x-a)}{b}\right]^{2}\right\}\). The fuzzy programming model can be converted to its deterministic form as follows:

Maximize \(f = F - 10(\lambda_i + \lambda_j) - 10(u_i + v_i + u_z + v_z)\) \hspace{1cm} (4.14a)

Subject to:

\[\lambda_i = \max \left[0, \lambda - (3x_1 + x_2) - 0.25(x_1 + x_2) \right] \quad (4.14b)\]

\[\lambda_j = \max \left[0, \lambda - (3x_1 + x_2) + 0.25(x_1 + x_2) \right] \quad (4.14c)\]

\[u_i = \max \left[0, 0.175x_1 - 0.95 \right] \quad (4.14d)\]

\[v_i = \max \left[0, 0.125x_1 - 1.05 \right] \quad (4.14e)\]

\[u_z = \max \left[0, 0.165x_2 - 0.16455 \right] \quad (4.14f)\]

\[v_z = \max \left[0, 0.28x_2 - 0.23545 \right] \quad (4.14g)\]

\[0.25x_1 - 0.3x_2 \leq 1 \quad (4.14h)\]
The linear programming model (4.14) is solved in LINGO 9.0 (Schrage, 1999), and the optimal solution is: \( x_1 = 5.197, \quad x_2 = 0.997, \) and \( f = 16.588. \)

4.3 Case Study

4.3.1 Overview of the study system

Tianjin is situated in the northeast part of North China Plain, bordering Beijing Municipality and Hebei Province, and it is the largest coastal city in northern China (Liu, 2011). The Binhai New Area (the study area, denoted as BNA) is located in the eastern part of Tianjin’s main urban area, and close to the Bohai Sea (see Figure 4.6). It consists of three districts, namely Tangu, Hangu, and Dagang, with an area of 2,270 km² and a coastline of 153 km (Liu, 2009). Its population was approximately 1.13 million in 2011 with a growth rate at 1.82%. BNA is the most dynamic economic growth center in Tianjin. In 2010, the gross output is 350 billion Yuan, in which 230 billion Yuan is from industry, and in 2020, the total output value is expected to reach 1,000 billion Yuan (Xie et al., 2011). The annual rainfall of BNA is about 500 to 700 mm, which is evenly distributed throughout the year. The annual mean evaporation is about 1,910 mm. The quantity of water resource per capita is 160 m³, which is \( \frac{1}{15} \) of the average level in China (Liu, 2009; Xie et al., 2011).

Figure 4.7 shows the framework of the water supply-demand system in BNA. The main water supply sources consist of surface water, ground water, seawater, transferred water (from Luan River and Yellow River), and recycled water. Water resources are mainly allocated for satisfying agricultural, industrial (including the tertiary industry), domestic, and eco-environmental water needs. Due to rapid growth of industry in BNA, the traditional water sources have failed to meet the rising need. Moreover, the over-extraction of ground water in BNA is leading to land subsidence and other environmental and geological problems (Liu, 2011). Currently, water shortage restricts the development and expansion of industry and becomes a serious issue in this region. Unconventional water resources, including seawater desalination
and recycled water, have been used to fill the gap between water supply and demand. Seawater desalination is deemed an effective measure to mitigate the water shortage problem. In the past years, several projects have been carried out in BNA, such as Beijiang power plant desalination project, Xinquan desalination plant in Dagang District, and desalination project in harbor industrial zone (Liu, 2009). Water recycling is considered as an effective way to provide new source of water and control water pollution. The sources of wastewater for recycling are mainly from industry, the tertiary industry and domestic sector. Moreover, as many Chinese cities are facing serious water shortage problems, Tianjin has become a paradigm of promoting water-saving technologies. Water conservation measures are considered effective in reducing water demand directly rather than adding additional waters-supply infrastructures. In this study, a number of water-saving measures, including drip irrigation and spray irrigation for agricultural sector, installation of water-saving valves and sanitary wares for industry, the tertiary industry and domestic sector, and Xeriscaping for eco-environmental sector, are adopted for water users.

Figure 4.6 Map of the study area (Binhai New Area, Tianjin, China).
To properly allocate water resources to different users, an effective water supply-demand management system is desired. The objective is to allocate water from various water sources to five competing water users in three districts over two periods with a maximized system benefit. In such a problem, uncertainties are associated with many system components such as water availability, water loss rate, and benefit/cost coefficients, and would significantly affect the related model solutions and decision making. For example, hydrological system is generally subject to various weather patterns and spatial complexity, and is dynamic and random in nature; the water loss rate during transferring is affected by the transferring condition, such as pump losses and pipe defects, and its value may also be affected by measurement error or human judgment. Using fuzzy set theory to describe these parameters is a viable alternative in handling uncertainties, but the related membership functions should be specifically defined based on the nature of the
parameters.

### 4.3.2 Model formulation

Putting the related uncertain parameters in fuzzy formats, the following SISFP model can be established to address the water supply-demand management problem at BNA:

\[
\text{Maximum } f = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} \left( G\tilde{D}_{ij} - C\tilde{T}_{it} \right) \cdot XW_{ijkt} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} \tilde{Y}_{ijt} \cdot \tilde{S}_{it} \quad (4.15a)
\]

subject to:

1. **Constraints of water availability**

\[
\sum_{j=1}^{J} \tilde{\gamma}_{it} \cdot XW_{ijkt} \leq A\tilde{S}_{jk} \quad \forall i, j, t; k = 1, 2 \quad (4.15b)
\]

\[
\sum_{j=1}^{J} \sum_{k=1}^{K} \tilde{\gamma}_{it} \cdot XW_{ijkt} \leq A\tilde{T}_{it} \quad \forall t; k = 3, 4 \quad (4.15c)
\]

\[
\tilde{\gamma}_{it} \cdot XW_{ijkt} \leq \tilde{\mu}_{it} \cdot \sum_{k=1}^{K} XW_{ijkt} \quad \forall i, j, t \quad (4.15d)
\]

2. **Constraints of water demand**

\[
\sum_{k=1}^{K} XW_{ijkt} \leq DM_{ij} \cdot \left( 1 - Y_{ijt} \tilde{\eta}_{it} \right) \quad \forall i, j, t \quad (4.15e)
\]

\[
\frac{DM_{ij} \cdot \left( 1 - Y_{ijt} \tilde{\eta}_{it} \right) - \sum_{k=1}^{K} XW_{ijkt}}{DM_{ij} \cdot \left( 1 - Y_{ijt} \tilde{\eta}_{it} \right) - \sum_{k=1}^{K} XW_{ijkt}} \leq SI_{it} \quad \forall i, j, t \quad (4.15f)
\]

3. **Non-zero constraint**

\[
XW_{ijkt} \geq 0 \quad (4.15g)
\]

where
\( i \) = index of water users (\( i = 1 \) to 5 represent agriculture, industry, tertiary industry, domestic sector, and eco-environment, respectively);

\( j \) = index of consuming zones (\( j = 1 \) to 3 denote Tanggu, Hangu, and Dagang District, respectively);

\( k \) = index of water sources (\( k = 1 \) to 5 represent surface water, groundwater, desalination water, transferred water, and recycled water, respectively);

\( t \) = index of periods (\( t = 1 \) and 2 represents period 2015-2020 and period 2020-2025, respectively);

\( X_{i j k t} \) = water amount from water source \( k \) for water user \( i \) at district \( j \) during period \( t \) (\( \text{Mm}^3/\text{Year} \));

\( GDP_{i j t} \) = gross domestic product (GDP) of water user \( i \) at district \( j \) during period \( t \) (Yuan/m\(^3\));

\( CT_{k t} \) = unit cost of water source \( k \) during period \( t \) (Yuan/m\(^3\));

\( ST_{i t} \) = unit cost of conservation measure of water user \( i \) during period \( t \) (Yuan/m\(^3\));

\( ASG_{j k t} \) = available water amount at district \( j \) from water source \( k (= 1 \text{ or } 2) \) during period \( t \);

\( ATS_{k t} \) = available water amount from water source \( k (= 3 \text{ or } 4) \) during period \( t \);

\( DM_{i j t} \) = maximum demand of water user \( i \) at district \( j \) during period \( t \) (\( \text{Mm}^3/\text{Year} \));

\( SI_{i t} \) = maximum allowable shortage index of water user \( i \);

\( \mu_{i t} \) = reclamation rate of waste water from water user \( i \) at period \( t \);

\( \gamma_{i t} \) = water-transfer safety factor (approximately equaling to \( 1/(1-\text{loss rate}) \)) of water user \( i \) at period \( t \);

\( \eta_{i t} \) = water-saving rate of water user \( i \) at period \( t \);
$Y_{ijt} = $ binary variable, where $Y_{ijt} = 1$ if a water-saving measure is adopted and $Y_{ijt} = 0$ otherwise.

The system considers a 10-year planning horizon, which is divided into 2015-2020 and 2020-2025. The objective of the system is to maximum the total system benefit of allocating water from five sources to three districts. Constraints (4.15b) to (4.15d) denote that the total amount of distributed water cannot exceed the available water amount of water sources. Constraint (4.15e) means that the distributed water of each user should be lower than its maximum demand, avoiding overly distributing water to the user which could generate high GDP. Constraint (4.15f) means that the shortage degree of each user should be lower than the allowable shortage index of each user. Constraint (4.15g) restricts the decision variables to be nonzero.

The fuzzy membership functions of the involved uncertain parameters should be identified based on characteristic of information, effort in data acquisition, method of statistical analysis, and/or judgment of decision makers. Based on the availability of information, the shape of membership functions could be largely different. In this study, the water-transfer safety factor (i.e. $\gamma_{\mu}$) and reclamation rate of waste water (i.e. $\mu_{\mu}$) are expressed as Gaussian-shaped fuzzy sets as these parameters are considered subject to measurement errors (with Gaussian-shape distribution). The Gaussian shape is similar to the shape of normal distribution in stochastic theory and could be obtained by statistical analysis as well; however, the peak value (highest membership degree) should be 1 and the summation of the area under the membership function may not necessarily be 1 (Oh and Pedrycz, 2000). The available water amounts (i.e. $A\tilde{G}_{jkt}$ and $A\tilde{T}_{k}\mu$) are expressed as trapezoidal-shaped fuzzy sets; the water-saving rate (i.e. $\tilde{\eta}$) from a specific user is expressed as a triangular-shaped fuzzy set. This is based on the assumption that these parameters are subject to data scarcity and could not be described by a “statistically sound” distribution (like Gaussian). For a parameter of interest, it is relatively easier for decision makers to choose the lowest value, the highest value, and the most possible value (or the range of the most possible value) from all available
Table 4.1 Available water amounts from different sources (Mm³/Year)

<table>
<thead>
<tr>
<th>District</th>
<th>Sources</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tanggu</td>
<td>$k = 1$</td>
<td>(5.85, 8.45, 17.55, 20.15)*</td>
<td>(5.85, 8.45, 17.55, 20.15)</td>
</tr>
<tr>
<td></td>
<td>$k = 2$</td>
<td>(8.55, 12.35, 25.65, 29.45)</td>
<td>(8.55, 12.35, 25.65, 29.45)</td>
</tr>
<tr>
<td>Hangu</td>
<td>$k = 1$</td>
<td>(5.4, 7.8, 16.2, 18.6)</td>
<td>(4.5, 6.5, 13.5, 15.5)</td>
</tr>
<tr>
<td></td>
<td>$k = 2$</td>
<td>(4.5, 6.5, 13.5, 15.5)</td>
<td>(4.5, 6.5, 13.5, 15.5)</td>
</tr>
<tr>
<td>Dagang</td>
<td>$k = 1$</td>
<td>(7.65, 11.05, 22.95, 26.35)</td>
<td>(7.2, 10.4, 21.6, 24.8)</td>
</tr>
<tr>
<td></td>
<td>$k = 2$</td>
<td>(11.7, 16.9, 35.1, 40.3)</td>
<td>(11.7, 16.9, 35.1, 40.3)</td>
</tr>
<tr>
<td></td>
<td>$k = 3$</td>
<td>(75, 105, 195, 225)</td>
<td>(125, 175, 325, 375)</td>
</tr>
<tr>
<td></td>
<td>$k = 4$</td>
<td>(207.5, 290.5, 539.5, 622.5)</td>
<td>(285, 399, 741, 855)</td>
</tr>
</tbody>
</table>

Note: $(a, b, c, d)^*$ represents a trapezoidal-shaped fuzzy set; $k$ = index of water sources ($k = 1$ to 5 represent surface water, groundwater, desalination water, transferred water, and recycled water, respectively) and $t$ = index of periods ($t = 1$ and 2 represents period 2015-2020 and period 2020-2025, respectively).

Table 4.2 The maximum water demand of each user (Mm³/Year)

<table>
<thead>
<tr>
<th>Water users</th>
<th>Tanggu</th>
<th>Hangu</th>
<th>Dagang</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t = 1$</td>
<td>$t = 2$</td>
<td>$t = 1$</td>
</tr>
<tr>
<td>Agriculture</td>
<td>30.30</td>
<td>32.00</td>
<td>17.27</td>
</tr>
<tr>
<td>Industry</td>
<td>160.80</td>
<td>173.35</td>
<td>32.12</td>
</tr>
<tr>
<td>The tertiary industry</td>
<td>26.00</td>
<td>34.35</td>
<td>4.69</td>
</tr>
<tr>
<td>Domestic</td>
<td>1.75</td>
<td>1.86</td>
<td>1.46</td>
</tr>
<tr>
<td>Eco-environment</td>
<td>20.50</td>
<td>29.82</td>
<td>4.97</td>
</tr>
</tbody>
</table>

Note: $t$ = index of periods ($t = 1$ and 2 represents period 2015-2020 and period 2020-2025, respectively).

Table 4.3 GDP and costs of each user at each district (Yuan/m³)

<table>
<thead>
<tr>
<th>Periods</th>
<th>Districts</th>
<th>Agriculture</th>
<th>Industry</th>
<th>The tertiary industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>$j = 1$</td>
<td>(8.8, 11, 13.2)*</td>
<td>(903.2, 1129, 1354.8)</td>
<td>(2704.8, 3381, 4057.2)</td>
</tr>
<tr>
<td></td>
<td>$j = 2$</td>
<td>(26.4, 33, 39.6)</td>
<td>(183.2, 229, 274.8)</td>
<td>(924, 1155, 1386)</td>
</tr>
<tr>
<td></td>
<td>$j = 3$</td>
<td>(5.6, 7, 8.4)</td>
<td>(276, 345, 414)</td>
<td>(1526.4, 1908, 2289.6)</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>$j = 1$</td>
<td>(10.4, 13, 15.6)</td>
<td>(1540.8, 1926, 2311.2)</td>
<td>(2924, 3655, 4386)</td>
</tr>
<tr>
<td></td>
<td>$j = 2$</td>
<td>(32, 40, 48)</td>
<td>(287.2, 359, 430.8)</td>
<td>(1421.6, 1777, 2132.4)</td>
</tr>
<tr>
<td></td>
<td>$j = 3$</td>
<td>(7.2, 9, 10.8)</td>
<td>(458.4, 573, 687.6)</td>
<td>(2031.2, 2539, 3046.8)</td>
</tr>
</tbody>
</table>

Note: $(a, b, c)^*$ represents a triangular-shaped fuzzy set; $j$ = index of districts ($j = 1$ to 3 denote Tanggu, Hangu, and Dagang, respectively) and $t$ = index of periods ($t = 1$ and 2 represents period 2015-2020 and period 2020-2025, respectively).
Table 4.4 Unit costs of different water sources and water-saving measures

<table>
<thead>
<tr>
<th>Sources</th>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface water</td>
<td>(1.28, 1.6, 1.92)*</td>
<td>(1.92, 2.4, 2.88)</td>
</tr>
<tr>
<td>Groundwater</td>
<td>(3.2, 4, 4.8)</td>
<td>(6.4, 8, 9.6)</td>
</tr>
<tr>
<td>Desalination plant</td>
<td>(6, 7.5, 9)</td>
<td>(4.4, 5.5, 6.6)</td>
</tr>
<tr>
<td>Transferred water</td>
<td>(6.56, 8.2, 9.84)</td>
<td>(4.96, 6.2, 7.44)</td>
</tr>
<tr>
<td>Recycled water</td>
<td>(2.4, 3, 3.6)</td>
<td>(2.25, 3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost of water-saving measures (M Yuan/Year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
</tr>
<tr>
<td>Industry</td>
</tr>
<tr>
<td>The tertiary industry</td>
</tr>
<tr>
<td>Domestic</td>
</tr>
<tr>
<td>Environment</td>
</tr>
</tbody>
</table>

Note: \((a, b, c)\)* represents a triangular-shaped fuzzy set; \( t \) = index of periods (\( t = 1 \) and \( t = 2 \) represent period 2015-2020 and period 2020-2025, respectively).

Table 4.5 Utilization coefficient, reclamation rate, water-saving rate and shortage index

<table>
<thead>
<tr>
<th>Water users</th>
<th>Period</th>
<th>Utilization coefficient</th>
<th>Reclamation rate</th>
<th>Water-saving rate</th>
<th>Shortage index</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 )</td>
<td>( t = 1 )</td>
<td>(1.31, 0.07)*</td>
<td>0</td>
<td>(0.4, 0.5, 0.6)**</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>( t = 2 )</td>
<td>(1.25, 0.07)</td>
<td>0</td>
<td>(0.44, 0.55, 0.66)</td>
<td>0.3</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>( t = 1 )</td>
<td>(1.19, 0.07)</td>
<td>(0.3, 0.08)</td>
<td>(0.16, 0.2, 0.24)</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>( t = 2 )</td>
<td>(1.17, 0.07)</td>
<td>(0.35, 0.08)</td>
<td>(0.2, 0.25, 0.3)</td>
<td>0.2</td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>( t = 1 )</td>
<td>(1.22, 0.07)</td>
<td>(0.2, 0.08)</td>
<td>(0.24, 0.3, 0.36)</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>( t = 2 )</td>
<td>(1.2, 0.07)</td>
<td>(0.25, 0.08)</td>
<td>(0.28, 0.35, 0.42)</td>
<td>0.2</td>
</tr>
<tr>
<td>( i = 4 )</td>
<td>( t = 1 )</td>
<td>(1.23, 0.07)</td>
<td>(0.15, 0.08)</td>
<td>(0.12, 0.15, 0.18)</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>( t = 2 )</td>
<td>(1.21, 0.07)</td>
<td>(0.2, 0.08)</td>
<td>(0.16, 0.2, 0.24)</td>
<td>0.1</td>
</tr>
<tr>
<td>( i = 5 )</td>
<td>( t = 1 )</td>
<td>(1.33, 0.07)</td>
<td>0</td>
<td>(0.04, 0.05, 0.06)</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>( t = 2 )</td>
<td>(1.3, 0.07)</td>
<td>0</td>
<td>(0.08, 0.1, 0.12)</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note: \((\mu, \sigma)\)* represents a Gaussian-shaped fuzzy set with membership function \( \exp\left[-\left((x-\mu)/\sigma\right)^2\right] \); \((a, b, c)\)** represents a triangular-shaped fuzzy set; \( i \) = index of water users (\( i = 1 \) to 5 represent agriculture, industry, the tertiary industry, domestic sector, and eco-environment, respectively).

information, and then generate its membership function as a triangular (or trapezoidal) shape. It should be noted that the main objective of this study is to explore an effective way to solve the inexact optimization problem with multiple
shapes of fuzzy parameters, rather than to investigate how uncertain information should be accurately obtained. Therefore, the definition of parameters for this study case is somewhat empirical but this should cause no harm to demonstration of the general methodology as the real distributions could be more complicated. Tables 4.1 to 4.5 list all parameters related to the study system; most of them are referred to Liu (2011) with some modifications.

4.4 Results and Discussions

Table 4.6 shows the solution of water allocations from SISFP. The decision variables are in deterministic forms but the objective function value is fuzzy. The distributed water amounts would be affected by the GDP and water demand from water users,

Table 4.6 Solution of water allocations from SISFP

<table>
<thead>
<tr>
<th></th>
<th>$t = 1$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>$t = 2$</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$j = 1$</td>
<td>$i = 1$</td>
<td>0</td>
<td>6.34</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$i = 2$</td>
<td>0</td>
<td>9.26</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8.44</td>
</tr>
<tr>
<td></td>
<td>$i = 3$</td>
<td>0</td>
<td>63.2</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td>$i = 4$</td>
<td>9.38</td>
<td>0</td>
<td>11.8</td>
<td>1.34</td>
<td>12.6</td>
<td>11.3</td>
<td>0</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>$i = 5$</td>
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<td>20.1</td>
<td>1.48</td>
<td>0.10</td>
<td>0</td>
<td>43.7</td>
<td>5.41</td>
<td>0.16</td>
</tr>
<tr>
<td>$j = 2$</td>
<td>$i = 1$</td>
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<td>5.85</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4.96</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$i = 2$</td>
<td>0</td>
<td>4.88</td>
<td>0</td>
<td>0</td>
<td>0.26</td>
<td>0</td>
<td>0</td>
<td>4.20</td>
</tr>
<tr>
<td></td>
<td>$i = 3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$i = 4$</td>
<td>6.17</td>
<td>5.01</td>
<td>2.13</td>
<td>1.12</td>
<td>3.05</td>
<td>6.13</td>
<td>12.6</td>
<td>4.83</td>
</tr>
<tr>
<td></td>
<td>$i = 5$</td>
<td>0</td>
<td>4.01</td>
<td>0.27</td>
<td>0.08</td>
<td>0</td>
<td>5.90</td>
<td>0.90</td>
<td>0.14</td>
</tr>
<tr>
<td>$j = 3$</td>
<td>$i = 1$</td>
<td>0</td>
<td>8.29</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.94</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$i = 2$</td>
<td>0</td>
<td>12.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>$i = 3$</td>
<td>0</td>
<td>16.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$i = 4$</td>
<td>14.2</td>
<td>47.7</td>
<td>4.11</td>
<td>2.46</td>
<td>83.5</td>
<td>17.3</td>
<td>78.4</td>
<td>8.83</td>
</tr>
<tr>
<td></td>
<td>$i = 5$</td>
<td>0</td>
<td>21.7</td>
<td>0.52</td>
<td>0.18</td>
<td>0</td>
<td>33.2</td>
<td>1.65</td>
<td>0.30</td>
</tr>
</tbody>
</table>

System benefit: (642.29, 806.97, 971.65) Billion Yuan

Note: $i =$ index of water users ($i =$ 1 to 5 represent agriculture, industry, the tertiary industry, domestic sector, and eco-environment, respectively), $j =$ index of districts ($j =$ 1 to 3 denote Tanggu, Hangu, and Dagang, respectively), $k =$ index of water sources ($k =$ 1 to 5 represent surface water, groundwater, desalination water, transferred water, and recycled water, respectively), and $t =$ index of periods ($t =$ 1 and 2 represents period 2015-2020 and period 2020-2025, respectively).
and the cost of water sources. For example, the surface water and seawater would be distributed to industrial sector only, due to the fact that the demand of industrial sector is the highest and the GDP generated from industry is relatively high. It is also indicated that, the agricultural sector would rely heavily on the transferred water, due to its high availability. Moreover, the allocated amount of groundwater would tend to decrease during period 2, for example, the allocated amounts of groundwater at Tanggu District during periods 1 and 2 are 9.26 and 8.44 (Mm$^3$), respectively, which is due to the rise of groundwater cost. Table 4.7 shows the binary variables obtained from SISFP. During period 1, all water-saving measures would prefer to be adopted; while during period 2, the saving measures of industry and the tertiary industry at Tanggu District would not be recommended. This may because the industry and the tertiary industry could generate high GDP values and the water demand from the tertiary industry is relatively low. Moreover, the available water amounts from desalination and transfer and the reclamation rate of wastewater would both increase, leading to the satisfaction of the demand of industry and the tertiary industry.

Table 4.7 Solution of binary variables from SISFP

<p>| Water users |</p>
<table>
<thead>
<tr>
<th>i = 1</th>
<th>i = 2</th>
<th>i = 3</th>
<th>i = 4</th>
<th>i = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>t = 2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: $i$ = index of water users ($i$ = 1 to 5 represent agriculture, industry, the tertiary industry, domestic sector, and eco-environment, respectively), $k$ = index of water sources ($k$ = 1 to 5 represent surface water, groundwater, desalination water, transferred water, and recycled water, respectively), and $t$ = index of periods ($t$ = 1 and 2 represents period 2015-2020 and period 2020-2025, respectively).

Figure 4.8 shows the water supply and demand amounts of various water users at different districts and periods. The water amounts allocated to Dagang district would rank the highest, followed by Tanggu and Hangu Districts; this is consistent with the demand of each district. It is found that the water demands of the tertiary industry and domestic sector are well satisfied, especially during period 2 where the water
shortage of the tertiary industry is zero; while the water demands of agriculture and eco-environment are the least satisfied. This is because the tertiary industry of BNA would bring the highest benefit to the system, with relatively insignificant water demand; while, the benefit from agriculture and eco-environment is the lowest. Figure 4.8 also shows that water amounts saved from agricultural irrigation are the highest. This is due to the low unit cost and high water-saving potential of the water-saving strategies (e.g. drip irrigation and spray irrigation).

Figure 4.9 illustrates the water supplies from various sources at different districts and periods. From Figure 4.9(a), as the total amount of transferred water is the highest, the supply from water transfer would be the highest. Similarly, as the surface water has the lowest availability of water sources, the supply from surface water would be the lowest. For instance, the amounts of surface water, groundwater, desalination plant, transferred water and recycled water to Tanggu District at period 2 are 6.45, 8.44, 44.86, 130.46, and 49.24 (Mm3), respectively. The water amounts from period 1. For example, the water amounts from desalination plants at Dagang District

![Figure 4.8 Water supply and demand amounts of various users.](image-url)
during periods 1 and 2 are 11.57 and 90.52 (Mm³), respectively; and the recycled water amounts at Tanggu District during periods 1 and 2 are 21.66 and 49.24 (Mm³), respectively. This is because the available water from desalination and recycled water, and the reclamation rate would all increase, and the unit costs of seawater desalination and waste water treatment are dropping. However, the available water amounts of surface and ground water shows little change. Figure 4.9(b) shows that all the water sources would be preferred to transfer to the industrial sector (i.e. \( k = 2 \)), while the domestic sector would receive the lowest water amount. The nonconventional water sources (i.e. desalination seawater and recycled water) are mainly used for industrial sector. This is because the industrial sector has the highest demand and a relative high GDP generation, and the domestic sector has the lowest demand with no GDP contribution. The demand of the eco-environmental sector is satisfied mainly by transferred water. This is because the eco-environmental sector generates no GDP and other sources are not motivated to distribute water to this sector, except for the transferred water which has adequate amount.

In general, the proposed model could effectively address the complex nature of fuzzy characterization of water-transfer safety factor, wastewater reclamation rate, the net benefits derived from water, and water-saving rate of the system, and also take demand management measures into consideration. The obtained solutions have sought a well balance among the water availability, water demand, adoption of water-saving measures, and benefit/cost of each water users.

In order to further demonstrate the advantage of SISFP, two models, including the model with deterministic parameters (DM) and the one with only triangular-shaped fuzzy parameters (ASM), are used to solve the same problem. The deterministic parameters are set to be the most possible values of the fuzzy sets. The triangular-shaped fuzzy numbers are transformed from Gaussian-shaped fuzzy numbers by taking \( \mu \pm 3\sigma \) as the upper and lower bounds, and the mean value as the most possible value. This is because the triangular information can only take the
Figure 4.9 Comparison of model solutions: (a) water supply amounts from various sources at different districts; (b) water supply amounts from various sources to different water users.

Note: S = Surface water, G = Groundwater, D = Desalination water, T = Transferred water, R = Recycled water, A = Agriculture, I = Industry, T = Tertiary industry, DM = Domestic, and E = Eco-environment.

extreme upper and lower boundaries, but the boundaries are difficult to be determined. Three standard deviations from the mean can account for more possibility of the event than one standard deviation from the mean. So the upper and
lower boundaries of the triangular are assumed to be approximated by \( \mu \pm 3\sigma \), avoiding loss of information. Figure 4.10 shows the comparison results. Line DL denotes the left-hand side curve when substituting the solution obtained from DM into the constraint of SISFP. AL and AR denote the right-hand side and left-hand side of ASM constraints. GL is LHS of SISFP constraint, and GR and TR are RHS of SISFP constraint.

Figure 4.10(a) shows the Constraint (4.15c) of DM, ASM, and SISFP when \( k = 5 \) and \( t = 2 \). It is found that the solutions obtained from DM and ASM are larger than that obtained from SISFP. For DM, the solution substituting into deterministic constraint (as Line DT shows) could satisfy the RHS, which is a deterministic value (i.e. 150 Mm\(^3\)). However, it may cause constraint violation if the RHS is trapezoidal-shaped fuzzy number (i.e. Line TR), where the RHS has a possibility of being less than the mean value (i.e. 150 Mm\(^3\)). For ASM, the most possible value is 150 (Mm\(^3\)), however, for trapezoidal-shaped condition, the possible value ranges from 105 to 195 (Mm\(^3\)). Hence, the solution under triangular-shaped condition also has a risk of failure caused by the underestimate of the possibility of the value less than 150 Mm\(^3\). Figure 4.10(b) shows the Constraint (4.15d) of DM, ASM, and SISFP when \( k = 3 \) and \( t = 1 \). It is observed that the solution obtained from DM is higher than that obtained from SISFP, while the solution from ASM is the lowest. This is due to the fact that the triangular-shaped membership function tends to overestimate the possibility of RHS’s value (i.e. the wastewater reclamation capability). For example, the possibility of RHS’s value equaling to 10 is 0.57 for Gaussian-shaped membership function, which is lower than that for triangular-shaped membership function (i.e. 0.78).

Generally, Compared to the DM and ASM (which may result in overestimation or underestimation), the general-shaped fuzzy parameters (which are expected to have a more accurate description of the real parameter distribution) could better reflect the situation of the study system and SISFP could lead to more reasonable solutions. From the above comparison, it is believed that the proposed model is suitable for many real-world water resources management problems where the related input parameters are normally complicated with uncertainty due to various reasons (like data shortage, measurement errors, decision prejudice and/or unsound statistical
Figure 4.10 Comparisons among general-shaped condition, triangular-shaped condition, and deterministic condition: (a) constraint (15c) with $k = 5$ and $t = 2$; (b) constraint (15d) with $k = 3$ and $t = 1$.

assumptions), and the related management model could be faced with issues of losing accuracy of solutions, if such uncertain information is highly simplified. Moreover, although this study only attempted Gaussian, trapezoidal, and triangular shapes of fuzzy parameters, the algorithm of numerical integral method could theoretically
tackle any kind of shapes of membership functions as long as the membership functions are explicitly provided, which could be obtained by curve fitting (e.g. polynomial curve fitting).

FMP can be generally categorized into fuzzy flexible programming and fuzzy possibilistic programming. Fuzzy flexible programming allows flexibility and elasticity be reflected in the constraints, such as fuzzy parametric programming (FPP) (Mula et al., 2006). Each constraint of FPP can be considered to be a fuzzy event with membership degree indicating the satisfaction degree of the constraint (Chanas, 1983). Fuzzy possibilistic programming tackles fuzzy coefficients in constraints and can be solved either by $\alpha$-cut-based approach or fuzzy sequential approach. A typical $\alpha$-cut-based approach is fuzzy chance constraint programming, where the $\alpha$-cut level indicates the confidence level of the model. Fuzzy sequential approach is based on extracting features from fuzzy sets, such as an area under the membership function, a center of gravity, or various intersection points between fuzzy sets (Rao and Shankar, 2013). In this study, the proposed SISFP uses fuzzy sequential approach based on the feature of area under the membership function. SISFP only generates a single decision alternative and the objective function could also be treated as fuzzy to reflect uncertainty associated with cost information. The handling scheme behind SISFP is different from FPP and FCCP which could obtain a set of solutions under different scenarios (e.g. at various satisfaction levels or confidence levels). Compared to FPP and FCCP, the SISFP approach tends to seek a balanced result considering constraint satisfaction under a fuzzy environment, instead of discretizing the decision domain into various tradeoffs. Selection of different methods depends on the preference of decision makers; SISFP provides a more straightforward and direct solution and other methods may require further comparison and evaluation in order to come up with the best solution.

4.5 Summary

A superiority-inferiority-based sequential fuzzy programming (SISFP) method was developed for supporting management of water supply-demand systems under
uncertainty. The proposed method incorporates a sequential method into the fuzzy optimization framework. The sequential method based on superiority and inferiority measures could reflect the relative relationship between general-shaped fuzzy sets on the right- and left-hand sides of model constraints. The water supply-demand system of Binhai New Area in Tianjin, China, was used to demonstrate the proposed methodology. A number of components of the study case, such as available inflow, water safety factor, net benefit of water usage, unit cost and water-saving rate of water-saving measures, and reclamation rate are expressed as fuzzy sets in specific shapes of membership functions (e.g. trapezoidal and Gaussian shape). The results showed that the proposed method was capable of generating the solutions of water allocation amounts and the adoption of water-saving measures. Moreover, a comparison among the DM, ASM, and SISFP showed that the model with general-shaped fuzzy sets could obtain more reasonable results, while the DM and ASM would lead to either overestimated or underestimated solutions.

The SISFP method has advantages of dealing with general-shaped fuzzy parameters in objective function and both sides of the constraints. However, some limitations are also found and further studies are needed. For example, the superiority and inferiority measures may be nonlinear if the fuzzy membership function is complicated, which would increase the computational burden. Moreover, the result obtained from SISFP cannot reflect the trade-off between the system benefit and risk failure. This study is the first attempt to apply SISFP to solve urban water supply-demand problem, and the proposed method is expected to be also applicable to other similar water resources management problems, where the involved uncertain parameters have complex fuzzy distributions.
CHAPTER 5 APPLYING AN EXTENDED FUZZY PARAMETRIC APPROACH TO THE PROBLEM OF WATER ALLOCATIONS

5.1 Introduction

Water resources allocation is an important task for distributing water resources to various users for ensuring healthy socio-economic development and eco-environmental protection. The task is especially critical for areas that are currently suffering from water scarcity problems and facing even greater challenges under future climate change. The conflict among different water users is hardly avoidable. But, application of management models, that fully consider the uneven spatial and temporal distributions of water resources, the interactions between water supply and demand, and the regulatory requirement of local authorities, will surely benefit the related allocation and planning processes. In recent years, it has been recognized that the intrinsic uncertainties linking with many system components in water resources allocation could also affect the effectiveness of management strategies that are normally made based on deterministic conditions. These uncertainties could be related to water availability (e.g. fluctuating hydrological condition), water demand (e.g. growing population and changing weather), transportation/storage loss, water prices, and even human judgment (e.g. regulatory policies).

The previous research efforts relied heavily on stochastic, fuzzy, and interval techniques in tackling uncertainties (Huang, 1996; 1998; Abolpour and Javan, 2007; Chen and Chang, 2010; Xu and Qin, 2010; Nikoo et al., 2012). Among various alternatives, fuzzy mathematical programming (FMP) was found effective in dealing with uncertainties caused by measurement errors, implicit knowledge, and ambiguous human judgment. The definition of the fuzzy parameters in FMP has less strict data requirement than that of stochastic ones, and the fuzzy parameters contain richer distribution information than interval numbers (Inuiguchi and Ramik, 2000; Baudrit et al., 2005). For decades, many types of FMP models were proposed for
solving water resources management problems (Slowinski, 1986; Kindler, 1992; Lee and Chang, 2005). Depending on the way of handling uncertainties, FMP can be categorized into fuzzy flexible (e.g. fuzzy parametric programming) (FF) (Faye et al., 2005; Mula et al., 2006; Tan, 2011), fuzzy possibilistic (FP) (e.g. fuzzy chance constrained programming) (Julien, 1994; Tanaka et al., 2000; Riverol et al., 2006), and fuzzy robust (FR) programming models (Xu and Goulter, 1999; Zhu et al., 2009). Maqsood et al. (2005) incorporated fuzzy flexible programming into a two-stage stochastic optimization framework, which embedded risk information into the constraints and objective function. Nie et al. (2008) advanced a water management model based on fuzzy robust programming approach for water quality problem, where the model could reflect a compromise between system stability and optimality. Xu and Qin (2010) proposed a double-sided FCCP model for an agricultural water quality system. The proposed model could handle uncertainties expressed as possibilistic distributions in the constraints and allow system violations at predetermined confidence levels.

The above-mentioned fuzzy approaches have specific scopes of applicability, in the sense of handling (i) fuzziness in objective function and/or constraints; (ii) fuzziness of relationship and/or model parameters. FF programming allows flexibility and elasticity be reflected in the objective function and constraints but is relatively weak in dealing with ambiguous coefficients (Inuiguchi and Ramik, 2000; Torabi and Hassini, 2008). FP programming tackles fuzzy coefficients in objective function and/or constraints, but is less capable of dealing with fuzzy relationships (Inuiguchi and Ramik, 2000; Torabi and Hassini, 2008). FR programming is designed for handling highly uncertain variables (i.e. dual uncertainties) which are expressed as fuzzy boundary intervals (Nie et al., 2008; Zhu et al., 2009); it is generally not suitable to be used for reflecting vague relationships or objective functions (Xu and Qin, 2010). In water resources allocation problems, uncertainty could exist in many system components and their relationships. To benefit general-purpose applications, it is desired that a sophisticated model that could handle all possible fuzzy conditions mentioned above be available.
Herrera and Verdegay (1995) gave a general introduction of three models of fuzzy parametric linear programming. These models have showed advantages in dealing with fuzzy-relation-based constraints, fuzzy coefficients in system objective, and fuzzy parameters in model constraints. Although the different types of fuzziness were treated individually, the parametric models did show a potential to be coupled together for handling more complicated cases. This further topic was not discussed in the previous studies, and the applicability of such a method in engineering problems has yet to be explored. In addition, the models were developed for triangular-shaped fuzzy sets, and incapable of reflecting more general cases. Thus, an extended fuzzy parametric programming (EFPP) method, which is based on the models proposed by Herrera and Verdegay (1995), will be developed in this study. A numerical example and a water resources allocation problem will be used for demonstration.

5.2 Methodology

5.2.1 Extended Fuzzy Parametric Programming

A fuzzy linear programming (FLP) problem in consideration of fuzziness under a general condition can be formulated as (Delgado et al., 1989):

\[
\text{Max } z = \sum_{j=1}^{J} \tilde{c}_j x_j
\]  
\hspace{1cm} (5.1a)

subject to:

\[
\sum_{j=1}^{J} \tilde{a}_{ij} x_j \leq \tilde{b}_i \quad \forall i
\]  
\hspace{1cm} (5.1b)

\[
\sum_{j=1}^{J} d_{jk} x_j \leq e_k \quad \forall k
\]  
\hspace{1cm} (5.1c)

\[
x_j \geq 0 , \quad \forall j
\]  
\hspace{1cm} (5.1d)

where, \( \tilde{c}_j \), \( \tilde{a}_{ij} \), and \( \tilde{b}_i \) are fuzzy coefficients (or parameters); \( x_j \) are deterministic
decision variables, \(d_{jk}\) and \(e_k\) are deterministic coefficients; \(j\) and \(J\) are the index and number of the decision variable, respectively; \(k\) is the index of deterministic constraint; the symbols \(\preceq\) denotes fuzzy inequality. The fuzzy coefficients for trapezoidal fuzzy sets could be expressed as \(\tilde{c}_j = (c_{j1}^1, c_{j2}^2, c_{j3}^3, c_{j4}^4)\), \(\tilde{a}_j = (a_{j1}^1, a_{j2}^2, a_{j3}^3, a_{j4}^4)\), and \(\tilde{b}_j = (b_{j1}^1, b_{j2}^2, b_{j3}^3, b_{j4}^4)\), respectively. In this study, we consider trapezoidal shape a relatively general shape of a fuzzy membership function. It could address the minimum, maximum, and most possible range of an uncertain variable. The triangular-shaped fuzzy membership function is a special case of trapezoidal ones when the most possible range converges to a single point.

Assume the flexibility of the constraints could be represented by fuzzy sets. When the constraints are fully satisfied, the membership degree of the constraints would be 1; when the constraints are totally violated, the membership degree of the constraints would be 0. Let a fuzzy number \(\tilde{\theta}_i\) represent the allowable maximum violation of the constraints that is to be determined by decision makers. The membership degree of constraints \(\mu_i(x)\) would linearly decrease over the interval \((\tilde{b}_i, \tilde{b}_i + \tilde{\theta}_i)\), and could be expressed as (Herrera and Verdegay, 1995):

\[
\mu_i(x) = \begin{cases} 
1 & \text{if } \sum \tilde{a}_j x_j \leq \tilde{b}_i \\
\frac{[\tilde{b}_i + \tilde{\theta}_i] - \sum \tilde{a}_j x_j}{\tilde{\theta}_i} & \text{if } \tilde{b}_i \leq \sum \tilde{a}_j x_j \leq \tilde{b}_i + \tilde{\theta}_i \\
0 & \text{if } \sum \tilde{a}_j x_j \geq \tilde{b}_i + \tilde{\theta}_i
\end{cases}
\]

(5.2)

where the fuzziness could exist in both the fulfillment of the constraints and the coefficients in constraints. For simplicity, \(\sum_{j=1}^{J'}()\) is represented by \(\sum()\). To deal with fuzzy relationship, the fuzzy ranking method (FRM), studied by many researchers (Yager, 1978; 1981; Yager, 1988), will be employed. FRM has the following definitions (Herrera and Verdegay, 1995):
\[ FR(\tilde{A}) \geq FR(\tilde{B}) \Rightarrow \tilde{A} \geq \tilde{B} \]  \hspace{1cm} (5.3a)

\[ FR(\tilde{A} + \tilde{B}) = FR(\tilde{A}) + FR(\tilde{B}) \]  \hspace{1cm} (5.3b)

\[ FR(r\tilde{A}) = r \cdot FR(\tilde{A}) \]  \hspace{1cm} (5.3c)

where \( FR(\cdot) \) is defined as a fuzzy ranking function; \( \tilde{A} \) and \( \tilde{B} \) are fuzzy numbers; \( r \) is a deterministic coefficient. Examples of fuzzy ranking functions include Yager’s first, second, and third indexes (Yager, 1978; 1981). In this study, we use Yager’s first index to deal with fuzzy coefficients. Therefore, \( \mu_i(x) \) in Equation (5.2) can be transformed to (Cadenas and Verdegay, 1997):

\[
\mu_i(x) = \frac{\left(\tilde{\alpha}_i + \tilde{\theta}_i\right) - \sum \tilde{\alpha}_g x_j}{\tilde{\theta}_i} \Rightarrow \mu_i(x) = \frac{FR\left(\tilde{\alpha}_i + \tilde{\theta}_i\right) - \sum \tilde{\alpha}_g x_j}{FR\left(\tilde{\theta}_i\right)}
\]  \hspace{1cm} (5.4)

To handle fuzzy parameters, a satisfaction degree (i.e. \( \alpha \)) is introduced. If the decision makers prefer a confidence level of constraint satisfaction to be \( \alpha \), the membership degree of the constraints \( \mu_i(x) \) should be higher than \( \alpha \), where \( \alpha \in [0,1] \). Then the constraint (5.1b) could be further transformed to (Cadenas and Verdegay, 1997):

\[
\frac{FR\left(\tilde{\alpha}_i + \tilde{\theta}_i - \sum \tilde{\alpha}_g x_j\right)}{FR\left(\tilde{\theta}_i\right)} \geq \alpha \Rightarrow \frac{FR\left(\tilde{\alpha}_i\right) + FR\left(\tilde{\theta}_i\right) - FR\left(\sum \tilde{\alpha}_g x_j\right)}{FR\left(\tilde{\theta}_i\right)} \geq \alpha
\]  \hspace{1cm} (5.5)

\[
FR\left(\sum \tilde{\alpha}_g x_j\right) \leq FR\left(\tilde{\alpha}_i\right) + (1-\alpha) \cdot FR\left(\tilde{\theta}_i\right)
\]

Consider the fuzziness in the objective function, the membership degree of the objective function \( \mu(z) \) could be expressed in the following trapezoidal form:
\[ \mu(z) = \begin{cases} 
0 & \text{if } z \leq c_j^1 x_j, \ z \geq c_j^4 x_j \\
\frac{z - c_j^1 x_j}{c_j^2 x_j - c_j^1 x_j} & \text{if } c_j^1 x_j \leq z \leq c_j^2 x_j \\
1 & \text{if } c_j^2 x_j \leq z \leq c_j^3 x_j \\
\frac{c_j^3 x_j - z}{c_j^4 x_j - c_j^3 x_j} & \text{if } c_j^3 x_j \leq z \leq c_j^4 x_j 
\end{cases} \]  
(5.6)

Equation (5.6) can be converted into crisp sets using \( \beta \)-cut, where the range under \( \beta \)-cut represents the aspiration range of the objective function values that the decision makers would accept. The membership degree \( \mu(z) \) should be higher than \( \beta \), then we can obtain the following relationship:

\[ \mu(z) \geq \beta \Rightarrow L(\beta) \leq \sum c_j x_j \leq U(\beta) \]  
(5.7)

where, \( L(\beta) = (1-\beta)\sum c_j^1 x_j + \beta\sum c_j^2 x_j \), and \( U(\beta) = \beta\sum c_j^3 x_j + (1-\beta)\sum c_j^4 x_j \).

Then it turns into a problem with interval-type objective function, which can be written as (Ishibuchi and Tanaka, 1990; Herrera and Verdegay, 1995):

\[ \text{Max } \{ z \mid z \in [L(\beta), U(\beta)], \ \beta \in [0, 1] \} \]  
(5.8)

A simple way to handle the interval objective function is to assign a weight vector \( \varepsilon \) to both \( L(\beta) \) and \( U(\beta) \). In this study, we assume an equal importance of \( L(\beta) \) and \( U(\beta) \) and use their average as the objective function:

\[ \text{Max } \frac{1}{2}[L(\beta) + U(\beta)] \]  
(5.9)

Then, the fuzzy parametric model can be expressed as:

\[ \text{Max } z = \frac{1}{2} \left[ (1-\beta) \left( \sum_{j=1}^{j'} c_j^1 x_j + \sum_{j=1}^{j'} c_j^4 x_j \right) + \beta \left( \sum_{j=1}^{j'} c_j^2 x_j + \sum_{j=1}^{j'} c_j^3 x_j \right) \right] \]  
(5.10a)

subject to:
where $\alpha$ reflect the satisfaction degree of constraints. The selection of $\alpha$ depends on
the preference of decision makers. The higher the values of $\alpha$, the higher the
satisfaction degree of constraints would be. If decision makers are willing to make a
conservative plan, higher values of $\alpha$ should be selected; conversely, if they prefer a
higher objective function value (normally this may lead to higher risk of system
violation), lower values of $\alpha$ should be chosen. The parameter $\beta$ represents the level
of the aspiration range of objective function. The higher the level of $\beta$, the narrower
the range of $[L(\beta), U(\beta)]$. To examine the influence of $\beta$ on the objective function, we
could assume $\beta$ has an increment of $\Delta \beta$. Then, the objective function becomes:

$$
Max \ z' = \frac{1}{2} (\beta + \Delta \beta) \cdot [(\sum c^2_j x_j + \sum c^3_j x_j) - (\sum c^2_j x_j + \sum c^3_j x_j)] \\
+ \frac{1}{2} (\sum c^4_j x_j + \sum c^5_j x_j)
$$

(5.11)

The equation shows that the influence of $\beta$ on the objective function value relies
heavily on the distribution of the fuzzy coefficients. The trend is generally linear
provided that the decision variables do not have significant variations.

Figure 5.1 shows the EFPP’s general framework. The steps of using EFPP are: (i)
identify fuzzy uncertain parameters and obtain the fuzzy membership function of
each variable; (ii) determine the maximum violation of constraints that the decision
maker would accept; (iii) establish the aspiration level to the objective function and
assign a weight vector to objective function (or use average); (iv) establish the
satisfaction degree to the constraints and transform constraints using FRM; (v)
formulate the fuzzy parametric linear programming model and generate the final
optimal solutions.

Figure 5.1 General framework of EFPP.

5.2.2 Numerical Example

Consider the following numerical problem:

\[ \text{Max } f = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 \quad (5.12\text{a}) \]

subjective to:

\[ \tilde{a}_{11} x_1 - \tilde{a}_{12} x_2 \geq \tilde{b}_1 \quad (5.12\text{b}) \]

\[ \tilde{a}_{21} x_1 + \tilde{a}_{22} x_2 \leq \tilde{b}_2 \quad (5.12\text{c}) \]

\[ x_j \geq 0 \quad (5.12\text{d}) \]
where \( c_1 = (1, 2, 3, 11), c_2 = (3, 7, 8, 9), a_{11} = (1, 2, 2.5, 3), a_{12} = (1, 2.5, 3.5, 4), b_1 = (2, 3, 5, 6), a_{21} = (1.5, 3.5, 4, 6), a_{22} = (3, 4, 6, 7.5), b_2 = (29, 35, 36, 40). \\

According to equation (5.7),

\[
l_1(\beta) = 1 + \beta, \quad u_1(\beta) = 11 - 8\beta, \quad l_2(\beta) = 3 + 4\beta, \quad u_2(\beta) = 9 - \beta. \tag{5.13}
\]

The objective function could be written as:

\[
Max \ f = [(1 + \beta)x_1 + (3 + 4\beta)x_2, (11 - 8\beta)x_1 + (9 - \beta)x_2] \tag{5.14}
\]

Assume that \( \tilde{\theta}_1 = (1, 1.5, 3, 4.5) \) and \( \tilde{\theta}_2 = (3, 5, 5.5, 7) \) are the admissible violations, and a linear ranking function based on the first index of Yager (Yager, 1981) is used. Then the fuzzy parametric model can be written as:

\[
Max \ f = \frac{1}{2}[(12 - 7\beta)x_1 + (12 + 3\beta)x_2] \tag{5.15a}
\]

subject to:

\[
2.125x_1 - 2.75x_2 \geq 4 - 2.5(1 - \alpha) \tag{5.15b}
\]

\[
3.75x_1 + 5.125x_2 \leq 35 + 5.125(1 - \alpha) \tag{5.15c}
\]

\[x_j \geq 0 \tag{5.15d}
\]

\[\alpha, \beta \in [0,1] \tag{5.15e}
\]

where \( \alpha \) and \( \beta \) are the satisfaction degree of the constraints and the aspiration level of the objective function, respectively. At a \( \beta \) level of 0.9, the results of \((f, x_1, x_2)\) obtained from EFPP under \( \alpha \) levels of 0.9, 0.6, and 0.3 are (36.99, 5.51, 2.89), (39.15, 5.53, 3.18), and (41.31, 5.55, 3.47), respectively. At an \( \alpha \) level of 0.9, the results of \((f, x_1, x_2)\) at \( \beta \) levels of 0.9, 0.6, 0.3 are (36.99, 5.51, 2.89), (41.48, 5.51, 2.89), and (46.88, 9.47, 0.315), respectively. The results indicate that, as \( \alpha \) and \( \beta \) levels decrease, the objective function value would both increase. The decision variables would not
change until the $\beta$ level drops below 0.3, which implies that, $\beta$ may only show notable impact on the model solutions when its influence on the objective function reaches a certain threshold.

5.3 Application in Water Resources Allocation Problem

5.3.1 Case background and model formulation

The same method will be used to a hypothetical water allocation problem (Maqsood et al., 2005; Li et al., 2006), where two reservoirs are serving as water sources for three users, including municipality, agriculture, and industry. A target water allocation amount to each consuming sector is assigned for each reservoir. Generally, an excessively high water target could lead to water shortage problems when the water availability is low; the corresponding penalties could also be high. Conversely, a too low target may cause waste of water resources during high-flow seasons. Therefore an optimal water allocation scheme from reservoirs to water users is desired. Thus, the problem need to be considered is how to allocate water from various water sources (i.e. reservoirs) to three competing users over three periods so that the overall system benefit can be maximized; while at the same time, the restrictions of water availability and regulatory requirement should be met.

In such a water resources allocation system, the available water amount is influenced by many factors such as the annual and seasonal variation of rainfall, runoff, and evaporation, as well as groundwater interaction; the value of net benefit and the penalty rely on the market condition and human judgment; the water loss rate is affected by transferring condition (e.g. soil absorption) and infrastructure reliability. Problems of data procurement, survey methods, equipment failure and human judgment could cause large errors of these parameters. In real-world applications, efforts should be made to ensure an accurate quantification of these uncertainties. In this study, we assume the uncertain parameters be expressed as trapezoidal-shaped fuzzy sets (listed in Tables 5.1 and 5.2).
Table 5.1 Water demand of users, available flow of reservoirs, and loss rate of water

<table>
<thead>
<tr>
<th>Time period</th>
<th>Water demand (× 10^6 m³)</th>
<th>Available flow (× 10^6 m³)</th>
<th>Loss rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Municipality</td>
<td>Agriculture</td>
<td>Industry</td>
</tr>
<tr>
<td>t = 1</td>
<td>9.7</td>
<td>13</td>
<td>11.5</td>
</tr>
<tr>
<td>t = 2</td>
<td>10</td>
<td>13.5</td>
<td>12</td>
</tr>
<tr>
<td>t = 3</td>
<td>11</td>
<td>14.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Table 5.2 Net benefits and penalties

<table>
<thead>
<tr>
<th>Time period</th>
<th>Net benefit (× 10^6 $)</th>
<th>Penalty (× 10^6 $)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reservoir 1</td>
<td>Reservoir 2</td>
</tr>
<tr>
<td>t = 1</td>
<td>Municipality (8, 10, 11, 12)</td>
<td>(7, 9, 9.5, 11)</td>
</tr>
<tr>
<td></td>
<td>Agriculture (3, 4.2, 4.3, 5)</td>
<td>(2, 3.5, 3.7, 4)</td>
</tr>
<tr>
<td></td>
<td>Industry (6, 8, 8.3, 10)</td>
<td>(5, 7, 7.5, 9)</td>
</tr>
<tr>
<td>t = 2</td>
<td>Municipality (8.5, 11, 11.5, 12.5)</td>
<td>(6.5, 8, 8.5, 10)</td>
</tr>
<tr>
<td></td>
<td>Agriculture (3.5, 4.3, 4.5, 5.5)</td>
<td>(2, 3.5, 4, 4.5)</td>
</tr>
<tr>
<td></td>
<td>Industry (7, 8, 9, 10)</td>
<td>(6.75, 8, 9.5)</td>
</tr>
<tr>
<td>t = 3</td>
<td>Municipality (10, 11, 12, 14)</td>
<td>(9, 105, 11, 13)</td>
</tr>
<tr>
<td></td>
<td>Agriculture (3.5, 4.7, 5, 6.5)</td>
<td>(3, 4, 4.5, 5)</td>
</tr>
<tr>
<td></td>
<td>Industry (7, 8.5, 9, 11)</td>
<td>(6, 8, 8.5, 10)</td>
</tr>
</tbody>
</table>

The model for this water allocation problem can then be formulated as follows:

\[
\text{Maximize } f = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} \tilde{NB}_{ijt} \cdot TG_{ijt} - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{j=1}^{J} \tilde{CT}_{i} \cdot DL_{ijt}
\]  

(5.16a)

subject to:
\[
\sum_{j=1}^{J} TG_{ijt} = TTG_{it} \quad \forall i, t \quad (5.16b)
\]

\[
\sum_{i=1}^{I} (1 + \tilde{\eta}_j) \cdot (TG_{ijt} - DL_{ijt}) \leq T\tilde{N}F_{jt} \quad \forall j, t \quad (5.16c)
\]

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} \tilde{\eta}_{ij} \cdot (TG_{ijt} - DL_{ijt}) \leq T\tilde{W}L \quad \forall t \quad (5.16d)
\]

\[
DL_{ijt} \leq TG_{ijt} \quad \forall i, j, t \quad (5.16e)
\]

\[
TG_{ijt} \geq 0, \quad DL_{ijt} \geq 0 \quad \forall i, j, t \quad (5.16f)
\]

where

\( i \) = index of water users, and \( i = 1, 2, \ldots, I (I = 3); \)

\( j \) = index of reservoirs, and \( j = 1, 2, \ldots, J (J = 2); \)

\( t \) = index of periods, and \( t = 1, 2, \ldots, T (T = 3); \) \( \eta_{ij} \) = water loss rate from reservoir \( j \) to user \( i; \)

\( NB_{ijt} \) = unit net benefit of water delivered from reservoir \( j \) to user \( i \) at period \( t \) ($/m^3$);

\( TG_{ijt} \) = water allocation target of reservoir \( j \) to user \( i \) at period \( t \) ($10^6$ m$^3$);

\( CT_{it} \) = unit penalty of not satisfying the target of user \( i \) at period \( t \) ($$/m^3$$);

\( DL_{ijt} \) = water amount failed to be delivered from reservoir \( j \) to user \( i \) at period \( t \) ($10^6$ m$^3$);

\( TTG_{it} \) = water allocation target for user \( i \) at period \( t \) ($10^6$ m$^3$);

\( TWL \) = total allowable water loss ($10^6$ m$^3$);

\( INF_{jt} \) = available water amount for reservoir \( j \) at period \( t \) ($10^6$ m$^3$).

The objective function (5.16a) is to obtain the maximum system benefit of allocating
water from two reservoirs to three consumers. Constraint (5.16b) means that the total allocation target of reservoirs to each water user should be equal to the demand of each user; constraint (5.16c) denotes that the total allocated water from each reservoir should be smaller than its available capacity; constraint (5.16d) means that the total amount of water loss should be lower than the allowable water loss; constraint (5.16e) denotes that the shortage should not exceed a predefined target; constraint (5.16f) stipulates the non-negativity of all decision variables. The fuzzy parameters are associated with unit benefits, unit penalties, water loss rates, allowable water losses and available water amounts. The fuzzy relations in constraints (5.16c) and (5.16d) mean that the total allocation amounts and water loss amounts are not strictly restricted by the policy regulations and violations are allowable to a certain degree (i.e. satisfaction degree). Constraints (5.16c) and (5.16d) can be transformed to:

$$\sum_{j=1}^{J} (1 + \tilde{h}_{ij}) \cdot (TG_{ij} - DL_{ij}) \leq I\tilde{NF}_{j} + (1 - \alpha) \cdot \tilde{\theta}^{inf} \quad (5.17a)$$

$$\sum_{j=1}^{J} \sum_{j=1}^{J} \tilde{h}_{ij} \cdot (TG_{ij} - DL_{ij}) \leq TWL + (1 - \alpha) \cdot \tilde{\theta}^{inf} \quad (5.17b)$$

where $\tilde{\theta}^{inf}$ and $\tilde{\theta}^{inf}$ is the acceptable tolerances of water availability and total water loss amount, respectively. Constraint (5.17a) shows that if the total allocated water from reservoir is higher than its total water availability, the satisfaction degree would decrease. It would not be acceptable if the total allocated water amount exceeds the tolerance of water availability (i.e. $I\tilde{NF}_{j} + \tilde{\theta}^{inf}$). Constraint (5.17b) shows a similar treatment for water losses.

5.3.2 Results

For simplicity of demonstration, we firstly assume the aspiration level of the objective function is identical to the satisfaction degree of constraints. The fuzzy parameters in constraints are dealt with by the first index of Yager [26, 27]. Table 5.3 lists the solution of the allocated target and shortage under various satisfaction degrees. For reservoir 1, the water target assigned to agriculture is the highest,
followed by industry and municipality. For example, at period 1, under satisfaction degree of 0.9, the target amounts from reservoir 1 to municipality, agriculture, and industry are 5.36, 8.77, and 7.44 ($\times 10^6$ m$^3$), respectively. This is because the demand of agriculture is the highest and the available flow of reservoir 1 is higher than that of reservoir 2; consequently, more water would be allocated to satisfy the demand of agriculture from reservoir 1. Table 5.3 also shows that, as the satisfaction degree decreases, the target amount from reservoir 1 to municipality would increase. For example, at period 2, as the satisfaction decreases from 0.95 to 0.7, the target allocation amount from reservoir 1 to municipality would increase from 5.32 to 6.13 ($\times 10^6$ m$^3$). This is due to the fact that a lower satisfaction level corresponds to a higher violation degree; reservoir 1, which has a higher net benefit, would be preferred to supply more water.

Table 5.3 Allocated solutions obtained through EFPP method

<table>
<thead>
<tr>
<th>Target from reservoirs to users ($\times 10^6$ m$^3$)</th>
<th>$\alpha, \beta = 0.95$</th>
<th>$\alpha, \beta = 0.9$</th>
<th>$\alpha, \beta = 0.8$</th>
<th>$\alpha, \beta = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>$t = 2$</td>
<td>$t = 3$</td>
<td>$t = 1$</td>
<td>$t = 2$</td>
</tr>
<tr>
<td>M</td>
<td>R1</td>
<td>5.32</td>
<td>5.59</td>
<td>7.30</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>4.38</td>
<td>4.41</td>
<td>3.70</td>
</tr>
<tr>
<td>A</td>
<td>R1</td>
<td>8.91</td>
<td>9.10</td>
<td>11.1</td>
</tr>
<tr>
<td>I</td>
<td>R1</td>
<td>7.58</td>
<td>7.78</td>
<td>9.19</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>3.92</td>
<td>4.22</td>
<td>3.31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shortage existing in users ($\times 10^6$ m$^3$)</th>
<th>$\alpha, \beta = 0.95$</th>
<th>$\alpha, \beta = 0.9$</th>
<th>$\alpha, \beta = 0.8$</th>
<th>$\alpha, \beta = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>$t = 2$</td>
<td>$t = 3$</td>
<td>$t = 1$</td>
<td>$t = 2$</td>
</tr>
<tr>
<td>M</td>
<td>R1</td>
<td>0.15</td>
<td>2.90</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>R1</td>
<td>4.12</td>
<td>3.89</td>
<td>6.96</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>I</td>
<td>R1</td>
<td>3.07</td>
<td>2.89</td>
<td>5.34</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: M = municipality; A = agriculture; I = industry; R1 = reservoir 1; R2 = reservoir 2.

The total shortages of municipality, agriculture, and industry over periods 1 to 3 under various satisfaction degrees have been plotted in Figure 5.2. The total shortage amount of municipality is the lowest and that of agriculture is the highest. It means that the target of municipality would be satisfied first, and that of agriculture would
be of least priority. This is due to the highest net benefit the municipality could bring in and also the highest penalty if the target of municipality could not be satisfied. It also shows that as $\alpha$ level decreases, the shortage amount of the three users would decrease. For example, at period 3, under satisfaction degrees of 0.95, 0.9, 0.8 and 0.7, the total water shortage amount of agriculture are 6.96, 6.63, 5.97, and 5.31 ($\times 10^6$ m$^3$), respectively. This implies that, under a lower satisfaction degree, the decision makers would be more optimistic about the water availability and prefer to allocate more water and avoid more shortage problem.

![Figure 5.2 Water shortage of municipality, agriculture and industry.](image)

Figure 5.2 shows the total target and water allocation amount from reservoirs 1 and 2. It demonstrates that, the total target amount from reservoir 2 is lower than that from reservoir 1; correspondingly, the total allocated amount would be lower. For example, at period 1, under satisfaction degree of 0.9, the total target from reservoirs 1 and 2 is 21.56 and 12.64 ($\times 10^6$ m$^3$), respectively; and the total allocation is 15.02 and 12.63 ($\times 10^6$ m$^3$), respectively. This is because reservoir 1 has higher net benefit and capacity. It also shows that, as $\alpha$ decreases, the total target from reservoir 1 would
decrease and the total allocation amount from reservoir 1 would increase. For example, at period 2, as the satisfaction degree drops from 0.95 to 0.7, the total target of reservoir 1 would decrease from 22.5 to 22.0 ($\times 10^6$ m$^3$), and the allocation from reservoir 1 would increase from 15.7 to 18.4 ($\times 10^6$ m$^3$). This is because, as the satisfaction degree decreases, the allowable violation of the constraints (i.e. constraints of water available and loss) would increase, and the discrepancy between water target and allocation (i.e. shortage amount) would reduce; this could lead to a higher system benefit.

Figure 5.3 Total target and allocation amounts from reservoir 1 and 2.

Figure 5.4 presents the net benefits of the system at various satisfaction degrees. A higher satisfaction degree has an obvious lower system benefit but a lower risk of system reliability, and vice versa. Generally, the results obtained through EFPP demonstrates that the approach is capable of (i) tackling uncertainties in water allocation problems as fuzzy sets; (ii) dealing with fuzzy parameters in both objective function and constraints; (iii) handling fuzzy relationship by allowing violation of the accomplishment of the constraints; (vi) helping decision makers understand the
balance between system benefit and reliability.

5.3.3 Comparison with FCCP

To verify the solutions from EFPP, we have tried to use fuzzy chance constrained programming (FCCP) model, proposed by Liu and Iwamura (1998), to solve the same problem after a few modifications. As FCCP could only tackle fuzzy coefficients in model constraints, the treatment of fuzzy relationship is not considered. Also, the fuzzy objective function has been converted to its deterministic version by averaging the lower and upper bounds of the fuzzy coefficients. The fuzzy variables in the constraints are still expressed as trapezoidal-shaped fuzzy sets. We use confidence levels of 0.95, 0.9, 0.8, and 0.7 for the FCCP model; they are deemed equivalent to the satisfaction degrees used in EFPP. The objective-function values obtained from both FCCP and EFPP are shown in Figure 5.4, and decision variables are shown in Figure 5.5. At a high satisfaction degree or confidence level (i.e. 0.95), the system benefit obtained from EFPP is slightly lower than that from FCCP. As the satisfaction degree decreases, the system benefits from EFPP would outstrip the results at the same confidence level from FCCP. For example, under satisfaction
degrees of 0.9, 0.8, and 0.7, the system benefits from EFPP would be 498.26, 551.79, and 605.18 ($\times 10^6$ $\), respectively; those from FCCP at confidence levels of 0.9, 0.8, and 0.7 are 488.90, 502.13, and 515.48 ($\times 10^6$ $\), respectively.

![Figure 5.5 Comparison between FCCP and EFPP.](image)

Notes: $\alpha$ represents satisfaction degree of EFPP or confidence level of FCCP.
Figure 5.5 shows the values of decision variables obtained from FCCP and EFPP. It is obvious that the solutions from both models are fairly close to each other at the same levels of satisfaction or confidence. For instance, at period 3, at a satisfaction degree of 0.7, the target amounts from reservoir 1 to municipality, agriculture, and industry from EFPP are 6.50, 10.3, and 8.48 ($\times 10^6$ m$^3$), respectively; and at a confidence level of 0.7, the target amounts obtained from FCCP are 6.99, 10.7, and 8.87 ($\times 10^6$ m$^3$), respectively. The solutions of allocation from EFPP are somewhat higher than those obtained from FCCP when $\alpha$ level is lower than 0.8; this explains the higher benefits of EFPP shown in Figure 5.4. The comparable solutions under the same model settings show that both FCCP and EFPP are capable of reflecting the balance of system benefit and reliability. However, EFPP could deal with both fuzzy coefficients and fuzzy relationships, while FCCP cannot. In water resources allocation problems, uncertainty could exist in many system components and their relationships. Therefore, EFPP has a wider applicability in dealing with models with fully fuzzy coefficients in constraints and objective functions, as well as fuzzy inequality in constraints.

5.3.4 Solutions under different $\alpha$ and $\beta$ levels

Figure 5.6 shows the net benefits under various $\alpha$ and $\beta$ levels. Obviously, the higher the $\alpha$ value, the more reliable the system and the lower the system benefit. For instance, when $\beta = 0.9$, the net benefit would increase from 498.26 to 793.68 ($\times 10^6$ $\$$) when $\alpha$ level decreases from 0.9 to 0.1. Figure 6 also shows that the objective function is more sensitive to $\alpha$ than $\beta$. For example, when $\alpha = 0.9$, the difference of net benefits between $\beta = 0.9$ and 0.1 is 9.23 ($\times 10^6$ $\$$); when $\beta = 0.9$, the difference of net benefits between $\alpha = 0.9$ and 0.1 is 295.42 ($\times 10^6$ $\$$). It is also found that the values of decision variables would not vary with the change of $\beta$ for this study case. This is due to the fact that the distributions of fuzzy coefficients in the objective function only cause negligible influence on model solutions, as we have explained in the numerical example section.

5.3.5 Applicability of EFPP to other engineering management problems

The proposed method has a potential to be applied to many other engineering
management problems, where uncertainties could be described by fuzzy sets. For example, water quality management is also complicated with uncertainties existed among many socio-economic, environmental, and technical factors, as well as their interactions. The optimization model for water quality management may include environmental constraints, such as the pollutant loading restrictions. The estimation of pollutant loads and maximum allowable discharge amounts involves experience of experts, model estimation errors and data shortage, and the related parameter uncertainties could be described by fuzzy sets. The decision makers may also accept a certain level of exceedance of environmental constraints, and the unit cost associated with the water quality treatment may also vary with market conditions and subject to uncertainties. EFPP would be most suitable for such type of problems, provided that the model structures and parameters be specifically designed and estimated.

![Figure 5.6 Net benefits under different values of $\alpha$ and $\beta$.](image)

5.4 Summary

An extended fuzzy parametric programming (EFPP) approach was developed in this study and applied to a water resource allocation problem. The proposed method could
deal with fuzzy parameters with trapezoidal-shaped distribution functions in the model, and also fuzzy relations in the constraints. The results obtained from a water allocation problem showed that EFPP was capable of tackling a wide range of fuzziness in the management model and allow water managers analyze the balance of system benefit and risk of failure. Compared with conventional FCCP method, EFPP was flexible in handling the fuzzy relationship and fuzzy parameters in both objective function and constraints and had a more general applicability.

The EFPP method also showed several limitations. Firstly, EFPP was restricted to fuzzy variables with triangular or trapezoidal shaped membership functions. For more general-shape fuzzy variables, heuristic techniques may be employed (Liu and Iwamura, 1998). Secondly, the value of using EFPP should also be compared with methods with multiple uncertainty-analysis techniques such as coupled fuzzy-stochastic theory (Maqsood et al., 2005; Li et al., 2006) as some particular forms of uncertainty may be better described by other algorithms like interval or stochastic. Nevertheless, the proposed method was proved effective in simple water allocation problems, and further applications and verifications in a wider range of engineering fields with more complicated conditions are expected.
CHAPTER 6 INTEGRATING DECISION ANALYSIS WITH FUZZY PROGRAMMING: APPLICATION IN URBAN WATER DISTRIBUTION SYSTEM OPERATION

6.1 Introduction

Water is a critical resource for socio-economic development and for maintaining healthy ecosystems. Over the past decades, increasing water scarcity problems have led to great tensions and conflicts among the competing domestic, agricultural and industrial sectors, and brought excessive pressure to the environment. For urban areas, water shortage problems are mainly caused by the rapid urbanization, the growth of the population and the impact of extreme weather events. Other causes may be associated with inefficient management or unscientific designs that exacerbate water shortage troubles. Hence, when there is limited room to increase water supply, it becomes essentially important to develop effective management schemes in order to properly balance the satisfaction of water demands, especially in a cost-efficient manner.

As one of the components of a water supply system, the water distribution network is important for ensuring the delivery of high-quality water to satisfy the demand of consumers. It generally involves water sources, treatment plants, storage provisions, and consuming zones, as well as distribution pipelines connecting various facilities. However, an urban water distribution network is intrinsically complex, involving many components and considerations such as water demands, transport paths, environmental regulations, and available resources (Wilchfort and Lund, 1997; Mousavi et al., 2004). In many cases, much information related to these components is difficult to obtain and subject to uncertainties. This would lead to the difficulty of formulating and solving the related management models.

Over the past years, many inexact optimization approaches were developed for tackling the uncertainties involved in the water supply management problems. Kindler (1992) developed a water resources allocation model based on concepts of
fuzzy set and satisficing theory. Huang and Loucks (2000) proposed an interval TSP model for dealing with uncertainties in water resources system. Babayan et al. (2005) formulated a stochastic constrained single-objective optimization model for the least cost design of water distribution networks. Maqsood et al. (2005) presented an inexact fuzzy two-stage program for tackling water resources management problems, where uncertainties were expressed not only as discrete intervals but also possibility and probability distributions. Li et al. (2008) developed an interval-parameter fuzzy multistage programming method, and applied it to water resources management system where uncertainties were described in multiple forms. More recently, Lu et al. (2009) advanced a rough-interval two-stage programming method for a conjunctive water-allocation system. Chen and Chang (2010) proposed a fuzzy multi-objective evaluation method to water resources redistribution under uncertainty, where four types of fuzzy operators were employed to address the complexities associated with decision making. Qin and Xu (2011) developed an improved interval programming method, named acceptability-index based interval approach, for analyzing urban water supply problems under uncertainty.

Generally, the above-mentioned methods show advantages in dealing with uncertainties in water resources management problems in one way or another. From a methodology point of view, interval programming may encounter difficulties in tackling highly uncertain parameters in the right-hand sides of the model constraints (Maqsood et al., 2005). Stochastic chance-constrained programming (SCCP) can deal with uncertainty embedded within the objective function and both sides of model constraints (Singh and Minsker, 2008); but it is relatively cumbersome to convert the objective function and constraints with uncertainty to its crisp equivalents and may have high computational requirement. Fuzzy chance-constrained programming (FCCP) may be a viable alternative (Liu and Iwamura, 1998). Compared with other types of fuzzy programming, FCCP also involves conversion of constraints to their deterministic equivalents; but the converting process has a relatively low requirement of computation and the model could offer more flexible solutions which aim at achieving low system costs at predetermined confidence levels. More importantly, results obtained from FCCP reflect the balance of system
cost and reliability in a fuzzy environment, which has an advantage to be linked with fuzzy inference for decision analysis (Jiménez et al., 2007). From the application point of view, there is a lack of methods that could tackle uncertainty and decision-making analysis simultaneously, in the context of urban water distribution system operation.

Thus, as an extension to the previous works, this paper aims to present an integrated fuzzy programming and decision analysis (IFPDA) approach for tackling complex uncertainties associated with the management of urban water distribution systems. A modified real-world case will be investigated, which is characterized by a network structure including water sources, treatment plants, reservoirs, and consumers. The water demand, capacities of facilities, leakage rate and transferring costs are considered uncertain and handled as trapezoidal-shaped fuzzy sets. The management model allows violations of system constraints at specified confidence levels; after obtaining the model solutions, fuzzy ranking will be used to compare and screen the best solutions under various scenarios. The study is useful for helping urban water managers optimize water allocation strategies, in consideration of complexities of system uncertainty and interactions of multiple system components.

6.2 Urban Water Distribution Network

6.2.1 System characterization

Water distribution network is an essential part of an urban water supply system, typically involving water sources (such as rivers, lakes and groundwater), water treatment/storage facilities, and water demand sectors. The raw water collected from rivers (or groundwater) should be delivered to treatment facilities for purification, transferred to reservoirs for storage, and finally pumped to water demand sectors (e.g. agricultural, domestic and industrial users) for usage. Water leakage from a distribution system is a general problem and needs extensive efforts for control (Arreguín Cortes and Ochoa Alejo, 1997). A cost-efficient water distribution system should be able to transfer adequate amount of water to water consumers, and at the same time ensure a limited loss and minimal operation cost. Many considerations
need to be addressed in order to achieve such a goal, such as minimizing water
distribution cost, selecting appropriate transport path, reducing leakage rate,
satisfying capacities of facilities, and meeting water demand requirement. Moreover,
water distribution system management involves decisions regarding the tradeoff
between the transferred water amount and the system failure risk. If water demand
satisfaction is in priority and the transferred water amount is large, the water loss
and system cost could also be high; conversely, if it is desired to save cost, the risk
of system failures would be high.

The structure and components of a water distribution network are determined by
many factors such as the hydrogeological/climatic conditions, socio-economic
development status, and political concerns. It has spatial variations all over the world.
For example, the water supply in Singapore comprises local catchment water,
imported water from Malaysia, recycled water, and desalinated water. The raw water
from these sources is transferred to treatment plants for filtering, disinfecting and
chemically treating (Luan, 2010). The distribution network system in New York City (NYC)
consists of dams, reservoirs, tunnels and distribution pipes to deliver water to
the residents. NYC is supplied by surface water without treatment. The system also
includes sewers and treatment plants for collecting and purifying the wastewater of
the city (National Research Council 2000). In London, the water source mainly
comes from the River Thames and River Lea, and the rest is from groundwater. The
water from rivers and shallow wells is suffering from pollution and needs to be
treated (Keane and Kerslake, 1988). However, a common feature of these water
distribution networks is that: (i) most of the water distribution systems are
characterized by water sources, treatment plants, storage facilities, and consuming
distribution networks; (ii) the water managers are working towards the direction of
having a high-quality, reliable, and low-cost water delivery system.

6.2.2 The complexity of uncertainty

Water distribution system management requires a lot of data collection and
information gathering, and it is always complicated with uncertainty problems that
may be originated from measurement errors, human judgment, and data shortage (Grayman, 2005). For example, identification of water leakage rates may be influenced by pipe defects, connector failures, metering errors and pump losses; the operation costs, including water transfer, treatment, and system maintenance, may be affected by market condition and local weather. The related information may be highly inexact and could bring complexity to decision making. Applications of deterministic optimization models may generate inapplicable or infeasible solutions, which could easily lead to waste of water resources or system failures (not satisfying the water demands).

6.2.3 The suitability of algorithms for tackling uncertainty

Methodologies for reflecting uncertainties will offer more flexibility than deterministic ones. Different alternatives, such as stochastic, interval and fuzzy approaches can be used for handling uncertainty based on data quality and availability. For example, stochastic theory is useful in reflecting characteristics of random information; fuzzy set theory is advantageous in representing features of imprecision and vagueness; interval theory is relatively straightforward to describe the extreme conditions of an uncertain data, but it has the lowest quality of information. Random variables require information of probabilistic distribution functions (PDFs), which could be generated based on historical records. Upon a lack of data, PDFs can be obtained by using the maximum entropy approach or simplified assumptions (Shen and Perloff, 2001). Fuzzy set theory is a useful alternative for dealing with uncertain parameters. Its possibilistic distribution can be defined based on public survey or expert experience using limited data. For example, we could simply use the minimum possible value, the maximum possible value, and the most likely value to generate the distribution of a fuzzy parameter using engineer’s experience or available data points.

In this study, we attempt to use fuzzy set theory for tackling uncertain information (e.g. facilities capacity and water demands) in water distribution system management. This is based on the facts that: (i) a fuzzy parameter is relatively flexible to define; (ii)
a fuzzy programming model requires less computational effort to transform into deterministic forms using defuzzification techniques (Liu and Iwamura 1998); (iii) the fuzzy programming outputs are intrinsically straightforward to link with fuzzy inference or reasoning operations for supporting decision analysis. Stochastic theory may also be a viable option for dealing with similar problems and it will be discussed in future studies.

6.3 General Framework of IFPDA

Figure 6.1 shows the general framework of the IFPDA approach. Based on FCCP, a set of solutions at various predetermined confidence levels could be obtained first. The fuzzy decision analysis will then be used to evaluate the tradeoff between the feasibility degree of objective function value and the satisfaction degree of constraints, and rank the solutions at various confidence levels. The final decision can then be the one that has the best score. In the context of urban water distribution network management, the framework can be understood as: (i) analysing the uncertain information related to water distribution system, such as the capacities of water treatment and storage facilities, water leakage rate, unit transfer cost and water demand, and obtaining the possibilistic distribution of these uncertain data; (ii) formulating and solving the IFPDA model, which aims to minimize the overall system cost subject to the satisfaction of water demand of consuming zones, water balance constraints, and capacity restrictions; (iii) defining the aspiration range of confidence levels, which represents the range of violation degrees the water managers and the related stakeholders would accept; (iv) applying fuzzy decision analysis to obtain the best decision alternative, considering the balance between system cost and risk.

6.3.1 Fuzzy programming

Fuzzy programming is used to optimize a problem under a fuzzy environment. It can generally be categorized into (i) fuzzy compromise (Bender and Simonovic 2000), (ii) fuzzy chance-constrained (He et al. 2008), and (iii) fuzzy robust (Nie et
al., 2014) programming approaches. The common feature of a fuzzy chance-constraint model is that it could incorporate confidence levels (or possibility levels) into an optimization framework to reflect the degree of constraints satisfaction and generate a set of solutions under various conditions.

![General framework of IFPDA model.](image)

Among many alternatives, the trapezoidal-shaped fuzzy chance-constrained programming (TFCCP), as introduced by Liu and Iwamura (1998), has proved successful in dealing with model constraints with trapezoidal fuzzy coefficients. It has a good potential to be applied in water supply management, as many of the uncertain parameters are easily definable as trapezoidal fuzzy sets. For example, water demand at a specific consuming area may be influenced by many factors, such as water price, weather condition, and local population density, and shows considerable temporal fluctuations. In many cases, we may only have limited information regarding its extreme value (i.e. upper or lower-bound level) and the most likely range of the value. To describe such a parameter with the uncertain feature, a trapezoidal-shaped fuzzy set \((a, b, c, d)\) can be used. The most likely values (ranging from \(b\) to \(c\)) will have a membership degree of 1. For the boundary
values \((a \text{ and } d)\), the membership degree will correspond to 0. The values between \(a \text{ and } b\), and between \(c \text{ and } d\) will have membership degrees within 0 and 1 and can be obtained by linear interpolation. A general FCCP model, based on Liu and Iwamura (1998), is formulated as:

\[
\begin{align*}
\text{Maximize} & \quad \tilde{f} = \sum_{j=1}^{J} \tilde{c}_j x_j \\
\text{Subject to:} & \quad \text{pos} \left( \bar{a}_j, \bar{b}_j \left| \sum_{j=1}^{J} \bar{a}_j x_j \leq \bar{b}_j \right. \right) \geq \alpha \quad \forall i \\
& \quad \sum_{j=1}^{J} d_j x_j \leq e_k \quad \forall k \\
& \quad \tilde{c}_j, \bar{a}_j \neq 0 \quad \forall i, j \\
& \quad x_j \geq 0 \quad \forall j
\end{align*}
\]

where \(j\) is decision variables (DVs) index and \(J\) is the DV number; \(i\) is the fuzzy constraint index, and \(I\) is the total number of such constraints; \(k\) is the index of deterministic constraints and \(K\) is the total number; \(x\) are deterministic DVs; \(\bar{a}_j, \bar{b}_j, \text{ and } \tilde{c}_j\) are trapezoidal fuzzy sets, e.g. \(\bar{a}_j = (a^1_j, a^2_j, a^3_j, a^4_j)\), \(\bar{b}_j = (b^1_j, b^2_j, b^3_j, b^4_j)\), and \(\tilde{c}_j = (\bar{c}_j^1, \bar{c}_j^2, \bar{c}_j^3, \bar{c}_j^4)\) with membership functions \(\mu(\bar{a}_j)\), \(\mu(\bar{b}_j)\) and \(\mu(\tilde{c}_j)\), respectively; \(\text{pos}\{\cdot\}\) means the possibility of event occurrence in \(\{\cdot\}\); \(\alpha\) is a predefined confidence level. In model (6.1), Equation (6.1b) denotes the fuzzy chance constraints where both sides are associated with parameters expressed as fuzzy sets. Other formulas are similar to what should be included in a deterministic model, including equations (6.1a), (6.1c), (1d) and (6.1e).
Let a trapezoidal fuzzy set be $\tilde{r} = (r_1, r_2, r_3, r_4)$. According to Liu and Iwamura (1998),
for a confidence level $\alpha (0 \leq \alpha \leq 1)$, $\text{Pos}\{\tilde{r} \leq 0\} \geq \alpha$ if and only if $(1-\alpha)r_1 + \alpha r_4 \leq 0$.
Hence, at a confidence level of $\alpha$, the fuzzy chance constraints (6.1b) can be
converted to the deterministic equivalents as:

$$
(1-\alpha)\left(\sum_{j=1}^{J} a_{ij}x_j - b_i^4\right) + \alpha \left(\sum_{j=1}^{J} a_{ij}^2x_j - b_i^3\right) \leq 0 \quad \forall i
$$

(6.2)

However, Eq. (6.2) only applies to model constraints and is incapable of dealing with
the objective functions which may also contain uncertain parameters. As suggested
by Iskander (2002), the membership function of the objective function $\tilde{f}$ can be
expressed as follows:

$$
\mu_f(z) = \left\{
\begin{array}{ll}
\frac{z - \sum_{j=1}^{J} c_{ij}^1x_j}{\sum c_{ij}^2x_j - \sum_{j=1}^{J} c_{ij}^1x_j} & \sum_{j=1}^{J} c_{ij}^1x_j \leq z \leq \sum_{j=1}^{J} c_{ij}^2x_j \\
1 & \sum_{j=1}^{J} c_{ij}^1x_j \leq z \leq \sum_{j=1}^{J} c_{ij}^3x_j \\
\frac{\sum_{j=1}^{J} c_{ij}^4x_j - z}{\sum_{j=1}^{J} c_{ij}^3x_j - \sum_{j=1}^{J} c_{ij}^4x_j} & \sum_{j=1}^{J} c_{ij}^3x_j \leq z \leq \sum_{j=1}^{J} c_{ij}^4x_j
\end{array}
\right.
$$

(6.3)

where $z$ is the objective function value; $c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4$ represent four sequential
parameters of a trapezoidal-shape fuzzy set. According to the predetermined
confidence level $\alpha$, Equation (6.3) can be rewritten to the equations as follows:

$$
\mu_f(z) \geq \alpha \Rightarrow \left\{
\begin{array}{ll}
\sum_{j=1}^{J} c_{ij}^1x_j \leq z \leq \sum_{j=1}^{J} c_{ij}^3x_j \\
z - \sum_{j=1}^{J} c_{ij}^1x_j & \Rightarrow \left\{
\begin{array}{ll}
z \geq \alpha \sum_{j=1}^{J} c_{ij}^1x_j + (1-\alpha)\sum_{j=1}^{J} c_{ij}^2x_j \\
\sum_{j=1}^{J} c_{ij}^2x_j - \sum_{j=1}^{J} c_{ij}^1x_j & \Rightarrow z \leq \sum_{j=1}^{J} c_{ij}^3x_j
\end{array}
\right.
\right.
$$

(6.4a)
As the objective is to maximize the function \( f \), we are only interested in \( z \leq (1 - \alpha) \sum_{j=1}^{J} c_j^4 x_j + \alpha \sum_{j=1}^{J} c_j^3 x_j \). Thus, the objective function can be written as:

\[
\text{Maximize } \bar{f} = (1 - \alpha) \sum_{j=1}^{J} c_j^4 x_j + \alpha \sum_{j=1}^{J} c_j^3 x_j
\] (6.5)

The general TFCCP model as described by Equation (6.1) can be converted to the crisp linear programming problem as follows (Liu and Iwamura 1998; Iskander 2002):

\[
\text{Maximize } \bar{f} = (1 - \alpha) \sum_{j=1}^{J} c_j^4 x_j + \alpha \sum_{j=1}^{J} c_j^3 x_j
\] (6.6a)

Subject to:

\[
(1 - \alpha) \left( \sum_{j=1}^{J} a_j^4 x_j - b_i^4 \right) + \alpha \left( \sum_{j=1}^{J} a_j^3 x_j - b_i^3 \right) \leq 0 \quad \forall i
\] (6.6b)

\[
\sum_{j=1}^{J} d_j x_j \leq e_k \quad \forall k
\] (6.6c)

\[
\tilde{c}_{ij}, \tilde{a}_{ij} \neq 0 \quad \forall i, j
\] (6.6d)
Solving Equation (6.6) by the conventional simplex approach, the fuzzy objective function value \( \tilde{f}_{\text{opt}} \) and decision variables \( x_{j,\text{opt}} \) can be obtained. The proof of the lemma regarding the model transformations are shown in the Supporting Document.

### 6.3.2 Fuzzy decision analysis (FDA)

From fuzzy programming, a set of solutions can be obtained at various confidence levels. However, it is still difficult for decision makers to reach a decision among many alternatives, as the tradeoff analysis requires professional experience and subjective judgment. To tackle this problem, a fuzzy decision analysis (FDA) method will be used to aid decision makers choosing the best solution. FDA is modified from the framework proposed by Jiménez et al. (2007). The detailed procedures are listed as follows:

**Step 1: Definition of Fuzzy Goal.**

Define the range of the confidence levels, i.e. \( M = [\alpha_1, \alpha_2, \ldots, \alpha_p, \ldots, \alpha_P] \), where \( \alpha_p \) is the confidence level of interest \( (p = 1, 2, \ldots, P) \) and \( P \) is the total number of concerned confidence levels. According to \( \tilde{f}_{\text{opt}} \) at different confidence levels, define the membership function of the fuzzy goal \( \tilde{G} \) as follows (Jiménez et al., 2007):

\[
\mu_{\tilde{G}}(f) = \begin{cases} 
1 & f \leq \underline{G} \\
\frac{\underline{G} - f}{\underline{G} - \overline{G}} & \underline{G} \leq f \leq \overline{G} \\
0 & f \geq \overline{G}
\end{cases}
\]

(6.7)

where \( \underline{G} \) and \( \overline{G} \) are lower and upper bounds of the tolerance threshold of the goal, respectively; \( f \) is a specific value that fuzzy set \( \tilde{G} \) would achieve at the membership degree of \( \mu_{\tilde{G}}(f) \). \( \underline{G} \) and \( \overline{G} \) represent the solutions with the highest and lowest satisfaction degrees of objectives, respectively, that decision
makers want to reach. As Equation (7) indicates, if \( f \leq G \), the objective function value is totally satisfactory, and the value of \( \mu_\tilde{G}(f) \) will be 1; conversely, \( f \geq \tilde{G} \) means that the objective function value is unacceptable, and the value of the \( \mu_\tilde{G}(f) \) will be 0.

Step 2: Satisfaction Degree of Fuzzy Objectives.

Compute the satisfaction degree of each fuzzy number \( \tilde{f}_{opt}(\alpha_i) \) to the fuzzy goal \( \tilde{G} \).

An index proposed by (Yager, 1978) could be used for such a purpose:

\[
K_{\tilde{G}}[\tilde{f}_{opt}(\alpha)] = \frac{\int_{-\infty}^{\infty} \mu_{\tilde{f}_{opt}}(f) \cdot \mu_{\tilde{G}}(f) \, df}{\int_{-\infty}^{\infty} \mu_{\tilde{f}_{opt}}(f) \, df} \quad (6.8)
\]

where the denominator is the area under \( \mu_{\tilde{f}_{opt}}(f) \), and the numerator is the possibility of occurrence \( \mu_{\tilde{f}_{opt}}(f) \) of each crisp value \( f \) weighted by \( \mu_{\tilde{G}}(f) \).

Equation (6.8) is an extension of the centre of gravity (COG) defuzzification method (Yager, 1978), where \( \mu_{\tilde{G}}(f) \) is deemed as a weighting value. COG could produce a crisp value \( K_{\tilde{G}} \) that represents the possibility distribution of \( \mu_{\tilde{f}_{opt}} \) weighted by \( \mu_{\tilde{G}}(f) \).

Step 3: Non-Normalized Decision

Define two fuzzy sets \( \tilde{P} \) and \( \tilde{M} \) with membership functions \( \alpha_i \) and \( K_{\tilde{G}}[\tilde{f}_{opt}(\alpha)] \), respectively. These two sets represent two criteria: the feasible degree of the constraints and the satisfaction degree of the objective. The fuzzy decision \( \tilde{D} \) is defined as the intersection of the two sets (Bellman and Zadeh, 1970), which can be computed based on the following fuzzy aggregation operation (Jiménez et al., 2007):

\[
\mu_{\tilde{D}}[\tilde{x}_{opt}(\alpha)] = \alpha \cdot K_{\tilde{G}}[\tilde{f}_{opt}(\alpha)] \quad (6.9)
\]
where * represents algebraic product. Equation (6.9) is based on real-scale and it is possible that the non-commensurable formats may pose considerable influences on the evaluation results. The best solution is the one with the highest membership degree, i.e. \( \max \{ \alpha K_{\tilde{G}} \tilde{f}_{\text{opt}}(\alpha) \} \).

Step 4: Normalized Decision.

Normalize two criteria (i.e. \( \alpha \) and \( K_{\tilde{G}} \)) in order to easily compare values measured using different scales, a min-max normalization method could be used:

\[
\bar{\alpha} = \frac{\alpha - \alpha_{\text{min}}}{\alpha_{\text{max}} - \alpha_{\text{min}}} \quad (6.10a)
\]

\[
\bar{K}_{\tilde{G}} = \frac{K_{\tilde{G}} - K_{\tilde{G},\text{min}}}{K_{\tilde{G},\text{max}} - K_{\tilde{G},\text{min}}} \quad (6.10b)
\]

\[
\mu_{\tilde{G}}(x_{\text{opt}}(\alpha)) = \bar{\alpha} \cdot \bar{K}_{\tilde{G}} \tilde{f}_{\text{opt}}(\alpha) \quad (6.10c)
\]

where \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) are the possible lowest and highest confidence levels, respectively; \( K_{\tilde{G},\text{min}} \) and \( K_{\tilde{G},\text{max}} \) are the lowest and highest satisfaction degrees, respectively. After normalization, \( \bar{\alpha} \) and \( \bar{K}_{\tilde{G}} \) will fall within the range \([0, 1]\). The best solution is the one with the highest membership degree, i.e. \( \max \{ \bar{\alpha} \cdot \bar{K}_{\tilde{G}} \tilde{f}_{\text{opt}}(\alpha) \} \).

### 6.4 Application in Urban Water distribution operation

#### 6.4.1 Overview of the study case

The proposed method will be used to an urban water distribution network, which is thoroughly discussed by Fattahi and Fayyaz (2010) and Qin and Xu (2011). The study case is closely similar to the urban water distribution system of Hamedan city in Iran (Fattahi and Fayyaz 2010), except for a reduced complexity in terms of the number of water sources, reservoirs and consuming zones. The data used in this study
is referred to the work of Fattahi and Fayyaz (2010) and shall not deviate significantly from a real-world condition. This is for the benefit of methodology demonstration and easiness of system characterization and description. The concerned multi-layer urban water distribution system is showed in Figure 6.2. Potable water collected from surface and ground water was transferred to the water treatment facilities for purification. Then the treated water was transferred to the reservoirs and distributed to the responding consuming zones. In this urban water distribution system, the water demand of the consuming zones must be satisfied and the capacity of treatment plants and reservoirs should not be exceeded. The unit water transfer cost and the leakage rate play important roles in identifying cost-efficient transfer paths. Also, the inventory of each treatment plant at each month will be influenced by the inventory at the previous month, the water amount received from water sources, and the water amount transferred to reservoirs. Optimization model is desired to help identify suitable water flow paths among water sources, treatment and storage facilities and water consumers, with the goal of achieving the lowest system cost.

Figure 6.2 The studied urban water supply system (after Qin and Xu, 2011).
6.4.2 Formulation of an IFPDA model

The input parameters for the water sources inventories, the transfer costs and the leakage rates of the prescribed distribution network and the capacities of the treatment plants and the reservoirs are listed in the Tables 6.1 and 6.2. The water consuming zones’ demands are listed in Appendix A (i.e. Table A.10). Most of these parameters are modified from the previous studies (Fattahi and Fayyaz 2010; Qin and Xu 2011). In a water distribution system, much system information may subject to uncertainty due to measurement errors, implicit knowledge, and ambiguous and unquantifiable human judgment. For example, the water demand of a consuming zone has large temporal and spatial variations due to change of population and fluctuation of water price. Selection of a safety factor may cause uncertainty associated with capacities of water storage and treatment facilities. Moreover, the unit transferring and purchasing cost may be influenced by technology and weather and changing dynamically. These uncertain parameters could be described by trapezoidal-shaped fuzzy membership functions, as they are relatively easy to define and do not have a strict requirement on data availability (Li et al. 2008). The modification of the parameters is as follows. A trapezoidal fuzzy number is defined as $A = (a, b, c, d)$, where the support of fuzzy number, $\text{supp}(A) = [a, d]$, and the core of fuzzy number (where the membership value equals one), $\text{core}(A) = [b, c]$. We use the interval value in Qin and Xu (2011) as the boundary values of fuzzy support $[a, d]$; $b$ and $c$ are assumed equaling to $(2a+d)/3$ and $(a+2d)/3$, respectively. Such a treatment is by no means accurate for real application. As the main focus of this study is to demonstrate the proposed methodology, we believe this assumption will not affect the validity of demonstration.

A TFCCP model for the urban water distribution management case can then be written as (Fattahi and Fayyaz, 2010; Qin and Xu, 2011; Xu et al, 2012):

$$
\text{Minimize } f = \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{k=1}^{K} X_{JT} X_{jt} PR_{jk} + \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{k=1}^{K} X_{JT} X_{jt} CJT_{jt} + \sum_{t=1}^{T} \sum_{r=1}^{R} \sum_{k=1}^{K} XTR_{rk} CTR_{r} + \sum_{r=1}^{R} \sum_{z=1}^{Z} \sum_{k=1}^{K} XRZ_{rzk} CRZ_{rz}$$

(6.11a)
Table 6.1 General parameters related to reservoirs, treatment plants and water sources ($\times 10^3 \text{ m}^3$)

<table>
<thead>
<tr>
<th>Treatment plants</th>
<th>Beginning inventory</th>
<th>Maximum capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP 1</td>
<td>(5, 5.83, 6.33, 7.5)*</td>
<td>(1300, 1500, 1700, 1900)</td>
</tr>
<tr>
<td>TP 2</td>
<td>(10, 11, 12, 13)</td>
<td>(2100, 2600, 3000, 3500)</td>
</tr>
<tr>
<td>TP 3</td>
<td>0</td>
<td>∞</td>
</tr>
<tr>
<td>TP 4</td>
<td>0</td>
<td>∞</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reservoirs</th>
<th>Beginning inventory</th>
<th>Maximum capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV 1</td>
<td>(16, 19, 23, 26)</td>
<td>(3150, 3750, 3950, 4350)</td>
</tr>
<tr>
<td>RV 2</td>
<td>(6.5, 9, 11, 13.5)</td>
<td>(580, 650, 690, 740)</td>
</tr>
<tr>
<td>RV 3</td>
<td>(1, 1.8, 2.7, 3.5)</td>
<td>(185, 200, 220, 235)</td>
</tr>
<tr>
<td>RV 4</td>
<td>(6.5, 9, 11, 13.5)</td>
<td>(345, 400, 410, 435)</td>
</tr>
<tr>
<td>RV 5</td>
<td>(2, 2.8, 3.7, 4.5)</td>
<td>(185, 200, 220, 235)</td>
</tr>
<tr>
<td>RV 6</td>
<td>(22, 27, 33, 38)</td>
<td>(560, 620, 650, 700)</td>
</tr>
<tr>
<td>RV 7</td>
<td>(4, 4.8, 5.7, 6.5)</td>
<td>(345, 380, 400, 435)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water sources</th>
<th>Beginning inventory</th>
<th>Maximum exited amounts</th>
<th>Unit purchasing cost ($/10^3 \text{ m}^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS 1</td>
<td>15000</td>
<td>4000</td>
<td>(20, 25, 29, 35)</td>
</tr>
<tr>
<td>WS 2</td>
<td>2050</td>
<td>3300</td>
<td>(450, 530, 550, 600)</td>
</tr>
</tbody>
</table>

Beginning inventory maximum exited amounts

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>4172</td>
<td>14143</td>
<td>18159</td>
<td>12292</td>
<td>17067</td>
<td>20582</td>
<td>19255</td>
<td>15582</td>
<td>3006</td>
<td>275</td>
<td>309</td>
<td>84</td>
</tr>
<tr>
<td>8929</td>
<td>3059</td>
<td>3914</td>
<td>2648</td>
<td>3676</td>
<td>4432</td>
<td>4169</td>
<td>3387</td>
<td>691</td>
<td>115</td>
<td>123</td>
<td>66</td>
<td></td>
</tr>
</tbody>
</table>

Note: $k$ is the index number of months; $(a, b, c, d)*$ represents a trapezoidal-shape fuzzy set with $a$, $b$, $c$, and $d$ being the four sequential parameters; WS = water sources; TP = treatment plant; RV = reservoirs; data are adapted from Fattahi and Fayyaz [17] and Qin and Xu [8]
Table 6.2 Unit transfer cost and leakage rate

<table>
<thead>
<tr>
<th>Path</th>
<th>Starting node</th>
<th>Ending node</th>
<th>Unit transfer cost($/10^3 m^3)</th>
<th>Leakage rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS 1</td>
<td>TP 1</td>
<td>(2.5, 3.5, 4, 5)*</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>WS 1</td>
<td>TP 2</td>
<td>(280, 320, 400, 430)</td>
<td>(0.01, 0.019, 0.021, 0.03)</td>
<td></td>
</tr>
<tr>
<td>TP 1</td>
<td>RV 2</td>
<td>(320, 350, 430, 450)</td>
<td>(0.05, 0.07, 0.09, 0.12)</td>
<td></td>
</tr>
<tr>
<td>TP 1</td>
<td>RV 3</td>
<td>(420, 440, 640, 660)</td>
<td>(0.05, 0.07, 0.09, 0.12)</td>
<td></td>
</tr>
<tr>
<td>TP 1</td>
<td>RV 5</td>
<td>(418, 430, 640, 660)</td>
<td>(0.01, 0.014, 0.016, 0.02)</td>
<td></td>
</tr>
<tr>
<td>TP 2</td>
<td>RV 2</td>
<td>(220, 300, 330, 350)</td>
<td>(0.05, 0.07, 0.09, 0.12)</td>
<td></td>
</tr>
<tr>
<td>TP 2</td>
<td>RV 4</td>
<td>(220, 240, 300, 325)</td>
<td>(0.03, 0.04, 0.05, 0.07)</td>
<td></td>
</tr>
<tr>
<td>TP 2</td>
<td>RV 5</td>
<td>(315, 400, 420, 445)</td>
<td>(0.03, 0.04, 0.05, 0.07)</td>
<td></td>
</tr>
<tr>
<td>TP 2</td>
<td>RV 6</td>
<td>(350, 450, 570, 590)</td>
<td>(0.07, 0.09, 0.12, 0.15)</td>
<td></td>
</tr>
<tr>
<td>TP 2</td>
<td>RV 7</td>
<td>(110, 120, 210, 230)</td>
<td>(0.01, 0.014, 0.016, 0.02)</td>
<td></td>
</tr>
<tr>
<td>TP 3</td>
<td>RV 1</td>
<td>(4400, 4800, 6300, 6500)</td>
<td>(0.51, 0.56, 0.6, 0.67)</td>
<td></td>
</tr>
<tr>
<td>TP 3</td>
<td>RV 2</td>
<td>(1400, 1600, 2200, 2500)</td>
<td>(0.25, 0.3, 0.4, 0.5)</td>
<td></td>
</tr>
<tr>
<td>TP 3</td>
<td>RV 3</td>
<td>(1350, 1500, 2200, 2400)</td>
<td>(0.25, 0.3, 0.4, 0.5)</td>
<td></td>
</tr>
<tr>
<td>TP 3</td>
<td>RV 4</td>
<td>(530, 570, 800, 830)</td>
<td>(0.08, 0.09, 0.11, 0.13)</td>
<td></td>
</tr>
<tr>
<td>TP 4</td>
<td>RV 1</td>
<td>(330, 400, 430, 470)</td>
<td>(0.01, 0.016, 0.02, 0.03)</td>
<td></td>
</tr>
<tr>
<td>TP 4</td>
<td>RV 4</td>
<td>(386, 400, 570, 590)</td>
<td>(0.03, 0.04, 0.05, 0.06)</td>
<td></td>
</tr>
<tr>
<td>TP 4</td>
<td>RV 7</td>
<td>(110, 140, 210, 230)</td>
<td>(0.06, 0.08, 0.11, 0.14)</td>
<td></td>
</tr>
<tr>
<td>RV 1</td>
<td>CZ 1</td>
<td>(580, 600, 710, 750)</td>
<td>(0.45, 0.47, 0.48, 0.5)</td>
<td></td>
</tr>
<tr>
<td>RV 2</td>
<td>CZ 2</td>
<td>(180, 200, 240, 260)</td>
<td>(0.15, 0.2, 0.22, 0.25)</td>
<td></td>
</tr>
<tr>
<td>RV 3</td>
<td>CZ 3</td>
<td>(180, 200, 240, 260)</td>
<td>(0.1, 0.12, 0.13, 0.15)</td>
<td></td>
</tr>
<tr>
<td>RV 4</td>
<td>CZ 4</td>
<td>(180, 200, 240, 260)</td>
<td>(0.38, 0.4, 0.41, 0.43)</td>
<td></td>
</tr>
<tr>
<td>RV 5</td>
<td>CZ 5</td>
<td>(180, 200, 240, 260)</td>
<td>(0.15, 0.2, 0.22, 0.25)</td>
<td></td>
</tr>
<tr>
<td>RV 6</td>
<td>CZ 6</td>
<td>(340, 360, 500, 520)</td>
<td>(0.42, 0.46, 0.47, 0.5)</td>
<td></td>
</tr>
<tr>
<td>RV 7</td>
<td>CZ 7</td>
<td>(180, 200, 240, 260)</td>
<td>(0.05, 0.06, 0.08, 0.09)</td>
<td></td>
</tr>
</tbody>
</table>

Note: \((a, b, c, d)^*\) represents a trapezoidal-shape fuzzy set with \(a, b, c,\) and \(d\) being the four sequential parameters; WS = water sources; CZ = consuming zone; TP = treatment plant; RV = reservoirs; data are adapted from Fattahi and Fayyaz [17] and Qin and Xu [8].
Subject to:

Constraints of consuming zones

\[
p_{os}\left\{ s \sum_{r=1}^{g} (1 - L\tilde{Z}_{rz}^r) \cdot X\tilde{R}_{rk} \geq \tilde{D}_{zh}(s) \right\} \geq \alpha \quad \forall k \tag{6.11b}
\]

\[X\tilde{R}_{rk} \leq Z\tilde{R}_{rz}^r \cdot U \quad \forall r, z, k \tag{6.11c}\]

Constraints of reservoirs

\[
\tilde{I}_R^r = I_{r,k-1}^r + \sum_{t=1}^{T} (1 - L\tilde{T}_{tr}) \cdot XTR_{rk} - \sum_{z=1}^{Z} X\tilde{R}_{rz}^r \quad \forall r, k = 2, \ldots, K \tag{6.11d}
\]

\[
\tilde{I}_R^1 = I_{r,1}^r \quad \forall r \tag{6.11e}
\]

\[XTR_{rk} \leq ZTR_{tr} \cdot U \quad \forall t, r, k \tag{6.11f}\]

\[p_{os}\left\{ s | I_{rk} \leq V\tilde{R}_{rk}(s) \right\} \geq \alpha \quad \forall r, k \tag{6.11g}\]

Constraints of treatment plants

\[
\tilde{I}_T^t = I_{t,k-1}^t + \sum_{j=1}^{J} (1 - L\tilde{X}_{j}^r) \cdot XJT_{jk} - \sum_{r=1}^{R} XTR_{rk} \quad \forall t, k = 2, \ldots, K \tag{6.11h}
\]

\[
\tilde{I}_T^1 = I_{t,1}^t \quad \forall t \tag{6.11i}
\]

\[XJT_{jk} \leq ZJT_{j} \cdot U \quad \forall j, t, k \tag{6.11j}\]

\[p_{os}\left\{ s | I_{tk} \leq V\tilde{T}_{tk}(s) \right\} \geq \alpha \quad \forall t, k \tag{6.11k}\]
Constraints of water sources

\[ IJ_{jk} = IJ_{j,k-1} - \sum_{t=1}^{T} XJT_{jt,jk} + BJ_{jk}, \quad \forall j, k = 2, \ldots, K \]  
\[ (6.11) \]

\[ IJ_{j1} = IRO_{j} - \sum_{t=1}^{T} XJT_{jt,j1} + BJ_{j1}, \quad \forall j \]  
\[ (6.11m) \]

\[ \sum_{t=1}^{T} XJT_{jtk} \leq MJ_{jk}(s) \quad \forall j, k \]  
\[ (6.11n) \]

Constraints of leakage

\[
\text{pos} \left( \sum_{j=1}^{J} \sum_{r=1}^{R} \sum_{k=1}^{K} XJT_{jk} \cdot L\tilde{X}J_{j} + \sum_{r=1}^{R} \sum_{z=1}^{Z} \sum_{k=1}^{K} XTR_{rk} \cdot L\tilde{X}T_{r} \\
+ \sum_{r=1}^{R} \sum_{z=1}^{Z} \sum_{k=1}^{K} XZR_{rz} \cdot L\tilde{X}Z_{rz} \leq \tilde{T}L \right) \geq \alpha
\]  
\[ (6.11o) \]

(6) Technical constraints

\[ XJT_{jtk} \geq 0, XTR_{rk} \geq 0, XZR_{rz} \geq 0, \quad \forall j, t, r, z, k \]  
\[ (6.11p) \]

where

\[ f = \text{net system cost (\$)}; \]

\[ k = \text{index of time periods, and } k = 1, 2, \ldots, K; \]

\[ j = \text{index of specific water resources, and } j = 1, 2, \ldots, J; \]

\[ t = \text{index of treatment plants, and } t = 1, 2, \ldots, T; \]

\[ r = \text{index of reservoirs, and } r = 1, 2, \ldots, R; \]

\[ z = \text{index of consuming zones, and } z = 1, 2, \ldots, Z; \]
\( \alpha \) = acceptable level of constraints-satisfaction from treatment plant \( t \) to reservoirs \( r \);

\( BJ_{jk} \) = recovered water for each water resource \( j \) in each season \( k \) \((\times 10^3 \text{ m}^3)\);

\( CJT_{jt} \) = unit transfer cost of water from water resource \( j \) to treatment plant \( t \) \(($/10^3 \text{ m}^3)\);

\( CTR_{tr} \) = unit transfer cost of water from treatment plants \( t \) to reservoirs \( r \) \(($/10^3 \text{ m}^3)\);

\( CRZ_{rz} \) = unit transfer cost of water from reservoirs \( r \) to consuming zones \( z \) \(($/10^3 \text{ m}^3)\);

\( D_{zk} \) = amount of water required for consuming zone \( z \) in season \( k \) \((\times 10^3 \text{ m}^3)\);

\( IRO_r \) = inventory of reservoir \( r \) at the first of the planning horizon \((\times 10^3 \text{ m}^3)\);

\( IR_{rk} \) = inventory of reservoir \( r \) at the end of each season \( k \) \((\times 10^3 \text{ m}^3)\);

\( ITO_t \) = inventory of treatment plant \( t \) at the first of the planning horizon \((\times 10^3 \text{ m}^3)\);

\( IT_{tk} \) = inventory of treatment plant \( t \) at the end of each season \( k \) \((\times 10^3 \text{ m}^3)\);

\( IRO_j \) = inventory of water resource \( j \) at the first of the planning horizon \((\times 10^3 \text{ m}^3)\);

\( IR_{jk} \) = inventory of water resource \( j \) at the end of each season \( k \) \((\times 10^3 \text{ m}^3)\);

\( LXJ_{jt} \) = leakage rate of water in network from water resource \( j \) to treatment plant \( t \) (%);

\( LXT_{tr} \) = leakage rate of water in network from treatment plant \( t \) to reservoir \( r \) (%);

\( LXZ_{rz} \) = leakage rate of water in network from reservoirs \( r \) to consuming zone \( z \) (%);

\( MJ_{jk} \) = maximum amount of water can be extracted from water resource \( j \) at each season \( k \);
$PR_{jk} =$ unit purchasing cost of water from water resources $j$ at each season $k$ ($$/10^3 \text{ m}^3$);

$TL =$ allowed maximum leakage amount ($\times 10^3 \text{ m}^3$);

$VR_{rk} =$ capacity of reservoir $r$ at each season $k$ ($\times 10^3 \text{ m}^3$);

$VT_{tk} =$ capacity of treatment plant $t$ at each season $k$ ($\times 10^3 \text{ m}^3$);

$XJT_{jtk} =$ amount of water transferred from water resource $j$ to treatment plant $t$ at each season $k$;

$XTR_{trk} =$ amount of water transferred from treatment plant $t$ to reservoir $r$ at each season $k$;

$XRZ_{rzk} =$ amount of water transferred from reservoir $r$ to consuming zone $z$ at each season $k$;

$ZRZ_{rz} =$ binary variable used to define path from reservoir $r$ to consuming zone $z$;

$ZJT_{jt} =$ binary variable used to define path from water resource $j$ to treatment plant $t$;

$ZTR_{tr} =$ binary variable used to define path from treatment plant $t$ to reservoir $r$.

The objective function (6.11a) aims to minimize the cost for purchasing and transferring the water from sources to water consumers through the treatment plants and reservoirs. Constraints (6.11b) and (6.11c) denote that the water distributed to each consuming zone must satisfy their water demand requirement; constraints (6.11d) to (6.11n) calculate the inventory of each reservoir, treatment plant, and water source, denoting that the inventory of each facility must not exceed their capacities; constraints (6.11o) means that the total leakage amounts must not exceed the allowed maximum leakage amounts; constraint (6.11n) stipulates that the decision variables are non-negative. Moreover, constraints (6.11b), (6.11g), (6.11k),
(6.11o) are treated as fuzzy chance constraints, since the parameters in right-hand side and left-hand side of these constraints are expressed as trapezoidal fuzzy sets.

### 6.5 Result Analysis

Part of the allocated solutions at confidence levels of 0.5, 0.7 and 0.9 obtained from the IFPDA model is presented in the Table 6.3 and Table 6.4. Full solutions can be found in the Appendix A (Tables A1 to A9). In general, the planning of water supply is mainly influenced by the water demand at each consuming zone as well as the interaction among the system components. Table 6.3 shows the allocated water amounts transferred to the consuming zones 2, 4, and 6 at each month. The consuming zone 4 is allocated the highest water amount, which is because the water demand of the zone 4 is the greatest. Table 6.4 shows the water delivered to the reservoirs 4, 6 and 7 from the treatment plant 2. It demonstrates that the amounts stored by reservoirs at each month are influenced by the water demand at their corresponding consuming zones. For instance, at the confidence level of 0.9, the water delivered to the reservoir 7 from the treatment plant 2 is 619.41 ($\times 10^3$ m$^3$) at month 2, which is much higher than the water demand of the consuming zone 7 at the same month. Correspondingly, the water delivered to the reservoir 7 from the treatment plant 2 at month 3 dropped to zero. Table 6.5 shows the water purchased from water sources at various months. It can be found that the amounts of water transported from the water sources to the treatment plant is affected by both the demand of water users, and the inventory of the treatment plants. For instance, at the confidence level of 0.9, the water amount delivered to the treatment plant 1 from the water source 1 at month 6 is 2377.72 ($\times 10^3$ m$^3$), which is much higher than the total demand of the consuming zones 2, 3 and 5 at month 6. Since the inventory of the treatment plant is sufficient for supplying water to reservoirs 2, 3 and 5 in the next month, the water delivered to the treatment plant 1 from the water source 1 is zero at month 7.

The solution also shows that the water allocations would be influenced by the leakage
rate, the unit water transfer and the purchasing cost. For example, the treatment plants 2, 3 and 4 are expected to transfer the water to the reservoir 4; whereas, only plant 2 would supply water as its corresponding unit transfer cost is the lowest. The treatment plants 2 and 4 are designated to provide water to the reservoir 7. While, only the treatment plant 2 is selected according to the allocation solutions. This is because the treatment plant 2 has a lower unit purchasing cost from the water source 1 and a lower leakage rate for delivering water to the reservoir 7. Moreover, the leakage rate also affects the water allocation result. For example, from Table 6.3, at month 4, confidence level of 0.5, the water delivered to the consuming zone 2 (i.e. 225.68×10³ m³) is lower than that delivered to the consuming zone 6 (i.e. 281.70×10³ m³), albeit the demand of water at the consuming zone 2 is somewhat higher. This is because the leakage rate from the reservoir 2 to the consuming zone 2 is somewhat lower. Moreover, from Table 6.3, the water delivered from reservoirs to consuming zones at each month would increase as the confidence level increases. For instance, the water at month 6 provided by the reservoir 2 to the consuming zone 2 at the confidence levels of 0.5, 0.7, 0.9 are 266.22, 280.44, and 295.17 (×10³ m³), respectively.

Table 6.3 Solutions of the allocated water amounts from reservoirs to consuming zones (×10³ m³) under the confidence levels of 0.5, 0.7, and 0.9.

<table>
<thead>
<tr>
<th></th>
<th>α = 0.5</th>
<th></th>
<th>α = 0.7</th>
<th></th>
<th>α = 0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r = 2</td>
<td>r = 4</td>
<td>r = 6</td>
<td></td>
<td>r = 2</td>
</tr>
<tr>
<td>r = 2</td>
<td>239.43</td>
<td>504.03</td>
<td>297.56</td>
<td>253.19</td>
<td>528.55</td>
</tr>
<tr>
<td>r = 4</td>
<td>225.68</td>
<td>474.77</td>
<td>281.70</td>
<td>239.19</td>
<td>499.00</td>
</tr>
<tr>
<td>r = 6</td>
<td>229.30</td>
<td>482.57</td>
<td>285.95</td>
<td>242.88</td>
<td>506.88</td>
</tr>
<tr>
<td>z = 2</td>
<td>225.68</td>
<td>474.77</td>
<td>281.70</td>
<td>239.19</td>
<td>499.00</td>
</tr>
<tr>
<td>z = 4</td>
<td>235.81</td>
<td>496.23</td>
<td>293.39</td>
<td>249.50</td>
<td>520.67</td>
</tr>
<tr>
<td>z = 6</td>
<td>266.22</td>
<td>560.59</td>
<td>328.47</td>
<td>280.44</td>
<td>585.68</td>
</tr>
<tr>
<td>k = 1</td>
<td>195.27</td>
<td>410.41</td>
<td>245.56</td>
<td>208.26</td>
<td>434.00</td>
</tr>
<tr>
<td>k = 2</td>
<td>229.30</td>
<td>482.57</td>
<td>285.95</td>
<td>242.88</td>
<td>506.88</td>
</tr>
<tr>
<td>k = 3</td>
<td>252.46</td>
<td>532.31</td>
<td>312.53</td>
<td>266.45</td>
<td>557.12</td>
</tr>
<tr>
<td>k = 4</td>
<td>286.49</td>
<td>604.47</td>
<td>352.92</td>
<td>301.07</td>
<td>630.01</td>
</tr>
<tr>
<td>k = 5</td>
<td>276.35</td>
<td>582.04</td>
<td>341.23</td>
<td>290.75</td>
<td>607.35</td>
</tr>
<tr>
<td>k = 6</td>
<td>262.60</td>
<td>553.76</td>
<td>325.29</td>
<td>276.76</td>
<td>578.79</td>
</tr>
</tbody>
</table>

Note: α = confidence level; z = index of consuming zones; k = index of months; r = index of reservoirs.
Table 6.4 Solutions of the allocated water amounts from treatment plants to reservoirs \((\times 10^3 \text{ m}^3)\) under the confidence levels of 0.5, 0.7, and 0.9.

<table>
<thead>
<tr>
<th>(k)</th>
<th>(\alpha = 0.5)</th>
<th>(\alpha = 0.7)</th>
<th>(\alpha = 0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t = 1)</td>
<td>(t = 2)</td>
<td>(t = 1)</td>
<td>(t = 2)</td>
</tr>
<tr>
<td>(r = 2)</td>
<td>(r = 4)</td>
<td>(r = 7)</td>
<td>(r = 2)</td>
</tr>
<tr>
<td>1</td>
<td>264.69</td>
<td>534.98</td>
<td>186.80</td>
</tr>
<tr>
<td>2</td>
<td>517.02</td>
<td>510.51</td>
<td>178.04</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>518.90</td>
<td>181.25</td>
</tr>
<tr>
<td>4</td>
<td>524.42</td>
<td>510.51</td>
<td>178.04</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>533.58</td>
<td>187.67</td>
</tr>
<tr>
<td>6</td>
<td>524.42</td>
<td>602.78</td>
<td>366.35</td>
</tr>
<tr>
<td>7</td>
<td>0.00</td>
<td>872.30</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>260.56</td>
<td>511.62</td>
<td>181.25</td>
</tr>
<tr>
<td>9</td>
<td>255.06</td>
<td>144.15</td>
<td>275.99</td>
</tr>
<tr>
<td>10</td>
<td>255.06</td>
<td>1071.66</td>
<td>234.71</td>
</tr>
<tr>
<td>11</td>
<td>314.04</td>
<td>204.16</td>
<td>18.68</td>
</tr>
<tr>
<td>12</td>
<td>298.41</td>
<td>595.44</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: \(\alpha\) = confidence level; \(r\) = index of reservoirs; \(t\) = index of treatment plants; \(k\) = index of months.

Table 6.5 Solutions of the allocated water amounts from water sources to treatment plants \((\times 10^3 \text{ m}^3)\) under the confidence levels of 0.5, 0.7, and 0.9

<table>
<thead>
<tr>
<th>(k)</th>
<th>(\alpha = 0.5)</th>
<th>(\alpha = 0.7)</th>
<th>(\alpha = 0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(j = 1)</td>
<td>(j = 2)</td>
<td>(j = 2)</td>
<td>(j = 2)</td>
</tr>
<tr>
<td>(t = 1)</td>
<td>(t = 2)</td>
<td>(t = 4)</td>
<td>(t = 1)</td>
</tr>
<tr>
<td>1</td>
<td>568.92</td>
<td>1850.79</td>
<td>2979.00</td>
</tr>
<tr>
<td>2</td>
<td>992.80</td>
<td>709.84</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>119.89</td>
<td>969.15</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>819.65</td>
<td>709.84</td>
<td>135.12</td>
</tr>
<tr>
<td>5</td>
<td>307.91</td>
<td>1099.40</td>
<td>1591.44</td>
</tr>
<tr>
<td>6</td>
<td>870.41</td>
<td>1397.50</td>
<td>3500.00</td>
</tr>
<tr>
<td>7</td>
<td>438.47</td>
<td>1197.10</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>380.45</td>
<td>1065.76</td>
<td>1160.64</td>
</tr>
<tr>
<td>9</td>
<td>615.71</td>
<td>1172.07</td>
<td>1701.72</td>
</tr>
<tr>
<td>10</td>
<td>696.91</td>
<td>1774.83</td>
<td>2049.71</td>
</tr>
<tr>
<td>11</td>
<td>672.71</td>
<td>1038.13</td>
<td>3500.00</td>
</tr>
<tr>
<td>12</td>
<td>639.91</td>
<td>613.86</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: \(\alpha\) = confidence level; \(j\) = index of water sources; \(t\) = index of treatment plants; \(k\) = index of months.
Table 6.6 represents the system cost at various confidence levels from IFPDA. The minimum values of the total cost would increase when the confidence levels increase. For example, as the confidence level increases from 0.1 to 0.9, the minimum values of the system cost $\bar{f}$ would increase from 2.871 to 4.833 ($\times 10^7$ $\text{\$}$). This implies that, a lower confidence level would lead to a lower system cost and a higher risk of system failure (e.g. the allocated water amount cannot satisfy the demand of the consuming zones). Conversely, a higher confidence level would bring a more stable system, but face with a higher system cost.

In order to choose the best decision from various alternatives of solutions, FDA is used. Generally, the balance of two conflicts, i.e. to minimize the overall system cost and to satisfy the feasibility degree of constraints, will be analyzed. It is assumed that decision makers consider the range of confidence level as $M = [0.1, 0.2, \ldots, 1]$. According to the system costs at various confidence levels obtained from FCCP model, the upper and lower bounds of the tolerance threshold are determined as 6.383 and 3.896 ($\times 10^7$ $\text{\$}$), respectively. The membership function of the fuzzy set $\tilde{G}$ is as follows:

$$
\mu_{\tilde{G}}(f) = \begin{cases} 
1 & f \leq 3.896 \times 10^7 \\
\frac{3.896 \times 10^7 - f}{6.383 \times 10^7 - 3.896 \times 10^7} & 3.896 \times 10^7 \leq f \leq 6.383 \times 10^7 \\
0 & f \geq 6.383 \times 10^7 
\end{cases} \quad (6.12)
$$

Table 6.6 shows the result obtained from the FDA. The membership degrees at confidence levels of 0.1 to 0.9 are 0, 0.095, 0.216, 0.275, 0.333, 0.335, 0.302, 0.237, and 0.136, respectively, and the system costs $\bar{f}$ are 2.871, 3.115, 3.363, 3.596, 3.837, 4.074, 4.319, 4.572, 4.833, and 5.102 ($\times 10^7$ $\text{\$}$), respectively. Apparently, the membership degree at the confidence level of 0.6 ranks the first. At confidence level of 0.1, the transferred water amount is low, and the system cost is the lowest, i.e. (3.896, 4.389, 5.082, 5.442) $\times 10^7$ $\text{\$}$. But the risk of failure is too high and cannot satisfy the water demand. Conversely, at confidence level of 1, the transferred water
amount is high, which would lead to more serious waste of water and much higher system cost, i.e. \( (4.530, 5.102, 5.949, 6.383) \times 10^7 \) $, Hence the membership at the confidence level of 0.1 and 1 is zero. The solution at the confidence level of 0.6 could not only satisfy the water demand of the consuming zone and guarantee a stable system, but also save the system cost. Therefore the most optimal alternative should be the solution at the confidence level of 0.6.

Table 6.6 System costs and ranking results from IFPDA method

<table>
<thead>
<tr>
<th>Confidence level ( \alpha )</th>
<th>( \hat{J}_{opt} \left( \times 10^7 $ \right)</th>
<th>( \bar{J} \left( \times 10^7 $ \right)</th>
<th>\bar{a}</th>
<th>\bar{K}_G(f)</th>
<th>\bar{\mu}_O(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>(3.896, 4.389, 5.082, 5.442) *</td>
<td>2.871</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.2</td>
<td>(3.993, 4.499, 5.209, 5.579)</td>
<td>3.115</td>
<td>0.111</td>
<td>0.851</td>
<td>0.095</td>
</tr>
<tr>
<td>0.3</td>
<td>(3.904, 4.405, 5.108, 5.481)</td>
<td>3.363</td>
<td>0.222</td>
<td>0.972</td>
<td>0.216</td>
</tr>
<tr>
<td>0.4</td>
<td>(3.999, 4.513, 5.232, 5.515)</td>
<td>3.596</td>
<td>0.333</td>
<td>0.826</td>
<td>0.275</td>
</tr>
<tr>
<td>0.5</td>
<td>(4.044, 4.554, 5.308, 5.695)</td>
<td>3.837</td>
<td>0.444</td>
<td>0.748</td>
<td>0.333</td>
</tr>
<tr>
<td>0.6</td>
<td>(4.139, 4.661, 5.433, 5.829)</td>
<td>4.074</td>
<td>0.556</td>
<td>0.602</td>
<td>0.335</td>
</tr>
<tr>
<td>0.7</td>
<td>(4.235, 4.770, 5.560, 5.965)</td>
<td>4.319</td>
<td>0.667</td>
<td>0.454</td>
<td>0.303</td>
</tr>
<tr>
<td>0.8</td>
<td>(4.332, 4.879, 5.688, 6.103)</td>
<td>4.572</td>
<td>0.778</td>
<td>0.305</td>
<td>0.237</td>
</tr>
<tr>
<td>0.9</td>
<td>(4.431, 4.990, 5.818, 6.242)</td>
<td>4.833</td>
<td>0.889</td>
<td>0.153</td>
<td>0.136</td>
</tr>
<tr>
<td>1</td>
<td>(4.530, 5.102, 5.949, 6.383)</td>
<td>5.102</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: \((a, b, c, d)\)* represents a trapezoidal-shape fuzzy set where \(a, b, c,\) and \(d\) are four sequential parameters.

From the study results, it is demonstrated that the IFPDA method is capable of (i) dealing with trapezoidal-shaped fuzzy parameters and coefficients existing in both the model constraints and the objective function; (ii) analyzing the trade-off between the failure risk and system cost, and offering useful information for decision makers to identify the upper and lower bounds of the tolerance threshold; (iii) evaluating the tradeoff between the satisfaction degree of the objective value and the feasibility degree of constraint; (iv) rank the solutions at various confidence levels and help decision makers choose the most cost-effective decision alternative.
6.6 Comparing TFCCP with Interval Methods

The inexact optimization part of the concerned urban water supply case can also be addressed by the acceptability-index-based two-step interval programming (AITIP) approach (proposed by Qin and Xu, 2011) and two-step interval programming (TIP) method (proposed by Huang and Loucks, 2000) by replacing the fuzzy parameters in objective function and both sides of the constraints by interval numbers. The lowest and the highest possible values of a fuzzy parameter (i.e. boundary values of the fuzzy support) are set as the upper and lower bounds of the corresponding interval number. The comparisons among IFPDA, AITIP and TIP are shown in Figure 6.3 and Figure 6.4. Compared with IFPDA, AITIP allows the violation of part of model constraints at a range of acceptability indexes (Qin and Xu 2011). A higher acceptability index means less strict constraints of the model, which means that the value of the acceptability index is equal to $1 - \alpha$, where $\alpha$ is the confidence level in IFPDA. From Figure 6.3, the total amounts of water delivered from reservoirs to consumers obtained from AITIP model are close to that based on the IFPDA model at the confidence levels of 0.9, 0.8 and 0.7. For instance, at month 6, confidence level of 0.9, the total amount of water delivered from reservoirs to consuming zones based on IFPDA model is 3739.269 ($\times 10^3$ m$^3$), while, the allocated solution from AITIP is 3847.259 ($\times 10^3$ m$^3$).

Figure 6.3 also shows that the solutions obtained from both IFPDA and AITIP would fall within the solution intervals from TIP. For instance, at month 8, the total amounts of water received by consuming zones are 3258.330, 3180.552, and 3103.669 ($\times 10^3$ m$^3$) at the confidence levels of 0.9, 0.8, and 0.7, respectively; while, the solution from TIP is [2589.190, 4419.117] ($\times 10^3$ m$^3$). Figure 6.4 shows the comparison of the system costs obtained from the three methods. It is indicated that the upper and lower bounds of the system cost from AITIP model is close to that obtained from the IFPDA model; however, the interval width of their solutions is narrower than that from TIP. Generally, both IFPDA and AITIP own the ability of assisting decision makers to make an analysis of the trade-off between the risk of system failure and the system
cost and. TIP generates too wide solutions which are difficult for decision makers to use. More importantly, IFPDA can link with decision analysis using fuzzy operation, while any of the interval approaches cannot.

![Figure 6.3 Comparison of solutions among IFPDA, TIP and AITIP.](image)

### 6.7 Further Discussions on FDA

The lower bound of the tolerance threshold $G$ is the solution with the highest satisfaction degree which decision makers want to obtain. The upper bound of tolerance threshold $\bar{G}$ is the solution with the highest fulfilment level of model constraints. When $f \leq G$, the objective value is totally satisfactory, conversely, if $f \geq \bar{G}$, the factory degree will be 0. To define the lower and upper bounds of the tolerance threshold depends on the wishes of the decision makers. In some case, the
decision makers may not be willing to admit a high risk of failure or high system cost according to the real-world condition. Hence, when using the FDA, the definitions of the lower and upper bounds of the tolerance threshold could influence the final decision outputs. Figure 6.5 shows the results based on different ranges of the confidence level which decision makers may interest. It is found that when the lower bound of tolerance threshold increases, the confidence level of the final result would also increase. However, if the upper bound of the tolerance threshold decreases, the confidence level of the final results would decrease. For example, when the upper bound of the tolerance threshold is 1, if the lower bound of the tolerance threshold rises from 0.1 to 0.5, the confidence levels of the best solution are 0.6, 0.7 and 0.8, respectively. Conversely, when the lower bound of the tolerance threshold is 0.1, if the upper bound of the tolerance threshold decreases from 1 to 0.6, the confidence levels...
levels of the best solution are 0.6, 0.5, and 0.3, respectively. Moreover, the result based on the non-normalization method proposed by Jimenez et al. (2007) can also be used to calculate the membership degrees. The results are plotted by dash line in Figure 6.5. It is shown that the satisfaction solution based on non-normalization method is at the confidence level of 0.9, which is different from the result obtained from the normalization one (i.e. 0.6). However, at the confidence level of 0.9, the transferred water amounts are relatively higher, which results in more serious waste of water resources and higher system cost. The solution at the confidence level of 0.9 may not be a satisfaction decision alternative. The unreasonable result is caused by the difference of the scales in the evaluation criteria. Therefore, using the normalization method could mitigate the influence by the different scales and make the result more reasonable.

Figure 6.5 Results under different ranges of confidence levels.
6.8 Summary

In this study, an integrated fuzzy programming and decision analysis (IFPDA) approach was proposed for solving the problem of urban water distribution network operation under uncertainty. The uncertain parameters associated with the objective function and both sides of the model constraints were expressed as trapezoidal-shaped fuzzy sets. Based on an urban water distribution network operation case adapted from literature, it was found that IFPDA could offer planning schemes at different confidence levels, and help find out the best solution based on fuzzy decision analysis.

This study made a number of contributions to the related circle. From the methodology point of view, the proposed IFPDA method is novel in: (i) dealing with fuzzy uncertainty in both model constraints and objective functions; (ii) linking fuzzy programming with decision making; (iii) having enhanced FDA criteria to mitigate the influence of parameter scale problems in decision analysis. From the application point of view, IFPDA (i) makes one step forward in linking decision analysis to optimization model outputs, and projecting the uncertain information associated with urban water distribution systems into planning process; (ii) could be applied to other similar planning problems where the management model can be formulated in a risk-based framework, the related uncertain information be described as fuzzy sets and the cost-risk trade-off analysis needs to be conducted.

To incorporate IFPDA in real applications, a number of concerns should be paid special attention by various stakeholders. Firstly, the model must be accurately defined to approximate the real system which may have many local features. Secondly, efforts should be made to collect the related data and characterization of the related uncertainties. Thirdly, post-optimization efforts are imperative to be conducted, in order to properly consider other factors that may potentially influence the management decisions, such as policy issues, availability of funds, and maintenance practices.
IFPDA is only applicable to address fuzzy parameters in trapezoidal- or triangular-shaped membership functions. For general cases (like Gaussian or Bell distribution), it is still a challenging problem to transform the fuzzy numbers into their crisp forms. Further works need to be done to improve the method, such as using genetic algorithm in a FCCP framework. The FDA process is also relatively simple as only two criteria are involved in decision making. It is difficult to solve multiobjective problem with multiple criteria. Multi-criteria decision analysis could help decision makers to evaluate alternatives against multiple conflicting criteria and would be a potential alternative to planning complex problems. However, how to incorporate this into a FCCP framework needs further investigations.
7.1 Introduction

Due to rapid urbanization, growing population and escalated extreme weather events, how to effectively manage water supply and demand becomes a challenging issue for many cities around the world. Among various components of the management system, planning of reservoir operation is essential to allocate water from multiple sources in the system to satisfy water demand over the planning horizon. However, reservoir operation is one of the most complicated problems within an integrated water resources planning and management framework, since the inflow estimates are subjected to randomness caused by the seasonal and annual variability in rainfall, runoff, evaporation, snowfall, and snowmelt (Wilchfort and Lund, 1997) and water demand is also subject to various uncertainties like fluctuation of cost and efficiency of non-structural measures (e.g. legal and economic incentives) (Ng and Kuczera, 1993). Furthermore, the climate change impact has become an increasingly important factor in planning and managing water resources, as it could potentially influence the regional precipitation pattern and river runoffs, and in turn, bring further complexities to the water supply-demand management system (Milly et al., 2008).

For tackling uncertain inflow, stochastic mathematical programming (SMP) has been widely used in planning of reservoir operation, such as chance-constrained programming (Houck, 1979; Ouarda and Labadie, 2001; Azaiez et al., 2005), two-stage stochastic linear programming (Seifi and Hipel, 2001), multistage stochastic programming (Watkins and McKinney, 1997; Lee et al., 2008), and stochastic dynamic programming (Loucks et al., 1981; Braga et al., 1991; Fayaed et al., 2013). Among various alternatives, the stochastic robust optimization (SRO)
model is capable of not only addressing risk-averse behaviors, but also allowing the evaluation of tradeoffs between the expected value of the objective function and risk of model infeasibility (Mulvey et al., 1995). Previously, a number of applications of SRO have been reported in water resources management. Watkins and McKinney (1997) applied SRO to planning of urban water transfer and management of ground-water quality. Ricciardi et al. (2007) utilized a SRO model to groundwater remediation system design, where the model uncertainty attributable to hydraulic conductivity was accounted for. Xu et al. (2009) developed an inexact two-stage SRO model for dealing with both random and interval variables in planning of water resources allocation. Xu and Qin (2013) applied SRO to the planning of a multiple reservoir operation, where the reservoir inflows were considered as random variables.

Although SRO can be used to deal with random uncertainties, it is challenging to handle uncertainties in water demand management, which are better represented by fuzzy sets. In fact, many parameters in water demand components show characteristics of imprecision and vagueness. For example, the cost of water-saving measure may be influenced by government policy and the determination is subjected to the human feelings and judgment. So it is more suitable to be represented by fuzzy sets. It is thus desired that fuzzy mathematical programming (FMP) be integrated into an SRO optimization framework for handling complex forms of uncertainties in water supply-demand management. Previously, many FMP methods have been used in the field of water resources management (Maqsood et al., 2005; Guo and Huang, 2009; Regulwar and Gurav, 2012; Wang and Huang, 2012). Some researchers have attempted to couple fuzzy programming with SRO in solid waste management. For example, Xu et al. (2009) developed a stochastic robust fuzzy interval linear programming, which integrated SRO and FMP into a general framework, and applied it to a problem of municipal solid waste management.

Climate change may pose significant impacts on water supply systems in the coming decades, leading to higher risks of water shortage problems. For the regional climate study, General Circulation Model (GCM) is a powerful tool to investigate the
changing climate under different levels of greenhouse gas emissions (Mailhot et al., 2007). Dynamical and statistical downscaling methods are the two fundamental approaches to bridge the gap between GCM and local weather information (Fowler et al., 2007). Compared with dynamical method, statistical downscaling is computationally efficient and easy to implement in different regions. The statistical downscaling approach could be classified into three types, including stochastic weather generator, regression model and weather types model (Fowler et al., 2007). In recent years, the potential impact of climate change on water resources management has been investigated by a number of researchers. Eum and Simonovic (2010) investigated optimal reservoir operations taking into account the potential impact of climate change by using an integrated water resources management model consisting of a K-nearest neighbor (KNN) weather generator, a streamflow synthesis and reservoir regulation SSARR hydrological model, and a differential evolution optimization model. Islam and Gan (2014) assessed the future outlook of water resources management of the South Saskatchewan River Basin of Alberta under the effects of climate change by using modified interaction soil biosphere atmosphere (MISBA), irrigation district model (IDM), and water resources management model (WRMM) models with the climate scenarios projected by four general circulation models forced by emissions reported by the Intergovernmental Panel on Climate Change (IPCC).

Based on the above-mentioned review, it is found that there are relatively limited studies on application of integrated optimization methods for water supply-demand management under complex uncertainties. In practical applications, the release of water from reservoirs (as the main water supply source) needs to consider the balance between stochastic water supply (that is influenced by hydrological process) and deviation from the release target (that is affected by demand changes and regulation policies). From the methodology point of view, there is a need to have a more efficient method that could address both fuzzy and stochastic types of uncertainties in the context of water supply-demand management. Although some types of FMP methods are available to be used, such as fuzzy chance-constrained
programming and fuzzy flexible programming; they normally generate solutions under different confidence/satisfaction levels that could lead to overwhelmed number of scenarios when coupled with a SRO model. Van Hop (2007) proposed a new fuzzy sequential approach based on fuzzy attainment values. The method can transform fuzzy constraints into deterministic equivalents by using attainment values of fuzzy variables. It could effectively avoid generating too many additional variables and constraints comparing to other sequential approach, and seems to be more suitable to be integrated into a SRO model. There is currently no study that looks into the coupling of these two models. Finally, the study in the field of climate change impact assessment on water supply-demand management is relatively scarce. There is limited research on the incorporation of climate change study and optimization of water supply-demand system.

Thus, this study aims to develop a robust fuzzy programming (RFP) model, coupling SRO and fuzzy attainment sequential method, to water supply-demand management under climate-change conditions. The study case is adapted from the water management system in the Greater Vancouver Regional District (GVRD), Canada, which is receiving water from multiple reservoirs. The region is suffering from water shortage problem and need effective management strategies in balancing its water supply and demand, especially under the threat of climate change. The model could help determine the optimal reservoir water release and identify appropriate long-term and short-term water-saving measures under various uncertainties. The climate change impact on the water supply is investigated with the aid of downscaling and black-box hydrological forecasting techniques, and could help provide a view on the changes of possible adaptation strategies in future conditions.

7.2 Study Area

The studied water supply-demand management system (i.e. GVRD) is adapted from a real-world case located in the south part of the Georgia Basin, Canada. The GVRD
currently has a population over 2 million, which is a major water consumption region in the Georgia Basin (Huang et al., 2006). The major sources of water for the GVRD are Capilano, Seymour, and Coquitlam reservoirs (as shown in Figure 7.1). The Capilano River Watershed has a drainage area of 212 km² with approximately 7% of its area below the Cleveland Dam. The total length of Capilano River is 33 km, among which 6 km is below the dam. The Capilano reservoir has a usable storage capacity of 55 Mm³ (Compass, 2012). The Seymour watershed has a drainage area of 186 km² with about 32% of its area below the Seymour Falls Dam. The total length of Seymour River is 39 km with 19 km below the dam. The Seymour reservoir has an effective storage capacity at about 27 Mm³ (Compass, 2012). The Coquitlam watersheds drainage area is 193 km² and Coquitlam Lake is approximately 13 km long with an available storage of 175 Mm³ (BC Hydro, 2005). There are a variety of hydrological factors affecting the reservoir inflows, such as precipitation, snowpack and temperature. It should be noted that, other reservoirs, such as Burwell Lake and Loch Lomond located in Seymour watershed, and Palisade Lake located in Capilano watershed are not considered as supply sources in this study due to their limited contributions (Huang et al., 2006).

Due to rapid change in population, the water supply system in GVRD may not be able to meet the demands based on current supply capacities. Water demand management could be an effective way to mitigate the problem. It emphasizes on using existing resources more efficiently, and seeking opportunities to decrease water demand rather than investing on additional infrastructures. In this study, several water-saving measures, including long-term and short-term types are considered. The short-term conservation measures include: (i) replacement of water-saving showers, toilet, faucet, laundry and dishwasher; (ii) installation of outdoor water-saving kits, sprinkling, and rainbarrel collector (Millock and Nauges, 2010; Shuster et al., 2013). GVRD provides about 60% of supplied drinking water for residents, and the major use of water indoor is from toilets, clothes washers, faucets and showers. Switch to water-saving devices can effectively save indoor water usage. For example, a low flush toilet can save 9 to 18 L per flush over a
standard toilet (Ashmore, 1989). Outdoor water-saving kits and sprinkling can water lawn more efficiently. Rain barrel, which makes rain as a resource, can collect rainwater from rooftops for reuse, such as watering lawn. Long-term measures include education, metering, and leak detection (Millock and Nauges, 2010). Education program can build water conservation awareness and understanding to save water and money on operational and production costs. Metering and leak detection can reduce waste of water. The cost and efficiency of each water-saving measure (in a fuzzy triangular form), which is mainly referred to BC Ministry of Community & Rural Development (2009), have been listed in Table 7.1.

Figure 7.1 Map of the study area.

In a water supply-demand system, many parameters could subject to uncertainties. The reservoir inflow has seasonal and annual variability due to random natures of weather factors like rainfall and snow. Meanwhile, many parameters in water demand components show characteristics of imprecision and vagueness. The cost of water-saving measure may be influenced by government policy; and the efficiency of measures may suffer from imprecision measurement. Such parameters are
Table 7.1 Unit costs and efficiencies of long-term and short-term conservation measures

<table>
<thead>
<tr>
<th>Measures</th>
<th>Unit cost of water saved (CAD $/m³)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long-term</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education ($k = 1$)</td>
<td>(0.95, 1.06, 1.16)</td>
<td>(18%, 20%, 22%)</td>
</tr>
<tr>
<td>Metering ($k = 2$)</td>
<td>(0.61, 0.68, 0.75)</td>
<td>(18%, 20%, 22%)</td>
</tr>
<tr>
<td>Leak detection ($k = 3$)</td>
<td>(1.14, 1.27, 1.40)</td>
<td>(9%, 10%, 11%)</td>
</tr>
<tr>
<td><strong>Short-term</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Showers</td>
<td>(2.14, 2.38, 2.62)</td>
<td>(2.7%, 3%, 3.3%)</td>
</tr>
<tr>
<td>Toilet ($j = 1$)</td>
<td>(1.67, 1.85, 2.04)</td>
<td>(9%, 10%, 11%)</td>
</tr>
<tr>
<td>Faucet ($j = 2$)</td>
<td>(1.30, 1.44, 1.59)</td>
<td>(2.7%, 3%, 3.3%)</td>
</tr>
<tr>
<td>Laundry ($j = 3$)</td>
<td>(11.67, 12.97, 14.27)</td>
<td>(2.25%, 2.5%, 2.75%)</td>
</tr>
<tr>
<td>Dishwasher ($j = 4$)</td>
<td>(13.15, 14.61, 16.07)</td>
<td>(0.14%, 0.15%, 0.17%)</td>
</tr>
<tr>
<td>Outdoor water kits ($j = 5$)</td>
<td>(0.052, 0.058, 0.063)</td>
<td>(8.6%, 9.5%, 10.5%)</td>
</tr>
<tr>
<td>Sprinkling bylaw ($j = 6$)</td>
<td>(0.028, 0.031, 0.034)</td>
<td>(2.7%, 3%, 3.3%)</td>
</tr>
<tr>
<td>Rainbarrel program ($j = 7$)</td>
<td>(1.27, 1.42, 1.56)</td>
<td>(7.2%, 8%, 8.8%)</td>
</tr>
</tbody>
</table>

Note: ($a$, $b$, $c$) represents a triangular-shaped fuzzy set; source: BC Ministry of Community & Rural Development (2009).

suitable to be expressed as fuzzy sets due to easiness of parameter definitions. For example, a triangular fuzzy membership function can be defined by the lowest value, the highest value, and the most possible value based on the available information of an uncertain variable. Another complexity is that the climate change in the future may exacerbate the drought severity and duration, and in turn, affect the supply-demand management system. At the same time, there could be a continuous increase in water demand due to population growth and economic development. Thus, it is important to develop an effective tool to help identify optimal water supply schemes and demand management strategies under complexities of uncertainty and climate change. It should be noted that, due to data shortage in developing the followed mathematical models, the structure of study system has been simplified and the required data are referred to various sources. However, we believe this simplification won’t affect the demonstration of the proposed methodology.
7.3 Methodology

7.3.1 General framework

Figure 7.2 shows a general framework of the proposed methodology. For water supply management, the observed historical inflow of each reservoir is discretized based on probability levels. Then the discretized inflows with a joint probability distribution are used in the stochastic robust programming. For future conditions, GCM model output is used to obtain the local meteorological data, including precipitation (PCP), maximum temperature ($T_{\text{max}}$), and minimum temperature ($T_{\text{min}}$). The observed weather and runoff data (i.e. inflow, PCP, $T_{\text{max}}$, and $T_{\text{min}}$) are used to train a Bayesian Neural Network (BNN) at monthly scale; and then the BNN model is used for projecting future inflows. The future inflow is discretized similarly for providing input to the management model. For demand management, the fuzzy programming, with the cost and efficiency of water-saving measures expressed as fuzzy sets, are incorporated into the stochastic robust programming model, leading to the formulation of a RFP model. After solution, the monthly reservoir release and water-saving measures under various probabilistic scenarios can be obtained. The detailed description of the individual method will be given in the followed sections.

7.3.2 Robust fuzzy programming (RFP) model

Stochastic robust optimization (SRO) was firstly proposed by Mulvey et al. (1995), which incorporates risk of model infeasibility and non-optimality into an optimization framework and seeks robust solutions. A SRO model consists of structural constraints which are free of any noise, and control constraints which are subject to uncertainty in input data (Mulvey et al., 1995). A general SRO model can be formulated as follows (Xu et al., 2010):

\[
\text{Minimize } \sum_{s} p_s \xi_s + \lambda \sum_{s} p_s \left[ \left( \xi_s - \sum_{s} p_s \xi_s \right) + 2\theta_s \right] + \omega \sum_{s} p_s \delta_s
\]  

(7.1a)

Subject to:
\[ \xi_s - \sum_{i=1}^{s} p_i \xi_i + \theta_s \geq 0 \quad (7.1b) \]

\[ AX \geq B \quad (7.1c) \]

\[ C_sX + D Y_s + \delta_s = E_s \quad s \in \Omega \quad (7.1d) \]

\[ X, Y_s, \delta_s, \theta_s \geq 0 \quad s \in \Omega \quad (7.1e) \]

where \( X \) is structural variables and \( Y_s \) is control variables; \( A \) and \( B \) are deterministic coefficients or parameters; \( C_s, D_s, \) and \( E_s \) are uncertain parameters with each scenario \( s \); \( p_s \) is probability level of uncertain variables, and \( \sum_{s=1}^{S} p_s = 1 \) (\( s \in \Omega \)), where \( \Omega = (1, 2, \ldots, S) \) is the set of scenarios; \( \xi_s = f(X, Y_s) \) is the random objective function value under scenario \( s \); \( \lambda \) and \( \omega \) are weighting factors; \( \theta_s \) is a slack variable; \( \delta_s \) represents the infeasibility of the model under scenario \( s \). Equations (7.1c) and (7.1d) are the structural constraint and control constraint, respectively. A SRO model
for the studied supply-demand management case can be formulated as follows (Huang et al., 2006; Xu and Qin, 2013):

Maximize \( f = \sum_{i=1}^{I} \sum_{s=1}^{S} \sum_{t=1}^{T} p_{st} X_{ist} - \lambda \sum_{s=1}^{S} \sum_{t=1}^{T} \sum_{i=1}^{I} X_{ist} - \sum_{s=1}^{S} \sum_{t=1}^{T} p_{st} \left( \sum_{i=1}^{I} X_{ist} - \sum_{s=1}^{S} \sum_{t=1}^{T} p_{st} \cdot X\_{ist} + 2\theta_{ist} \right) \)

\[-\omega \sum_{s=1}^{S} \sum_{t=1}^{T} \sum_{k=1}^{K} p_{st} \left( DM_{t} - \sum_{i=1}^{I} X_{ist} + 2\theta_{2st} \right) \]

(7.2a)

Subject to:

\[ \sum_{i=1}^{I} X_{ist} - \sum_{i=1}^{I} \sum_{s=1}^{S} p_{st} \cdot X_{ist} + \theta_{ist} \geq 0 \quad \forall s, t \]

(7.2b)

\[ DM_{t} - \sum_{i=1}^{I} X_{ist} + \theta_{ist} \geq 0 \quad \forall s, t \]

(7.2c)

\[ ST_{st} = ST_{st-1} + Q_{ist} - X_{ist} - W_{st} \quad \forall i, s, t \]

(7.2d)

\[ ST_{st} \leq ST_{st}^{\text{max}} \quad \forall i, t \]

(7.2e)

\[ \delta_{st} = \max \left( 0, DM_{t} - \sum_{i=1}^{I} X_{ist} + \theta_{2st} \right) \quad \forall s, t \]

(7.2f)

\[ DM_{t} \left( \sum_{k=1}^{K} Y_{ik} \cdot \eta_{ik} + \sum_{j=1}^{J} Y_{2jst} \cdot \eta_{2j} \right) \geq \delta_{st} \quad \forall s, t \]

(7.2g)

\[ \sum_{k=1}^{K} LCT_{k} \cdot Y_{ik} \cdot LW_{k} + \sum_{j=1}^{J} \sum_{t=1}^{T} SCT_{j} \cdot Y_{2jst} \cdot SW_{jt} \leq BGT \]

(7.2h)

\[ X_{ist} \geq 0 \quad \forall s, t, k \]

(7.2i)

where,

\( i, s, t \) = index of reservoirs, scenarios, and months, respectively;

\( I, S, T \) = numbers of reservoirs, scenarios, and months, respectively;
\( j, k \) = index of short-term and long-term water-saving measures, respectively;

\( J, K \) = numbers of short-term and long-term water-saving measures, respectively;

\( X_{ist} \) = water release from reservoir \( i \) under scenario \( s \) at month \( t \) (Mm³);

\( Q_{ist} \) = reservoir inflow of reservoir \( i \) under scenario \( s \) at month \( t \) (Mm³);

\( ST_{it} \) = final storage of reservoir \( i \) at month \( t \) (Mm³);

\( W_{it} \) = monthly water volume released for fishery use (Mm³);

\( DM_{it} \) = water demand at each month \( t \);

\( LW_{k} \) = water amount saved from long-term conservation measure \( k \) under scenario \( s \) (Mm³);

\( SW_{jt} \) = water amount saved from short-term conservation measure \( j \) under scenario \( s \) at month \( t \);

\( LCT_{k} \) = cost of long-term water conservation measure \( k \) ($/Mm³);

\( SCT_{j} \) = cost of short-term water conservation measure \( j \) ($/Mm³);

\( BGT \) = allowed budget for water conservation measures;

\( p_{st} \) = probability under scenario \( s \) at month \( t \);

\( \eta_{1k} \) = water saving rate of long-term measure \( k \);

\( \eta_{2j} \) = water saving rate of short-term measure \( j \);

\( \lambda, \omega \) = weight coefficient;

\( BGT \) = allowed maximum budget;

\( \theta_{s} \) = slack variable under scenario \( s \);
\( \delta_{st} \) = amount of water deficit under scenario \( s \) at month \( t \) (Mm\(^3\));

\( Y_{1k} \) = binary variable, where \( Y_{1k} = 1 \) if long-term measures are adopted; \( Y_{1k} = 0 \) if otherwise;

\( Y_{2jst} \) = binary variable, where \( Y_{2jst} = 1 \) if short-term measures are adopted; \( Y_{2jst} = 0 \) if otherwise.

The first term of Equation (7.2a) denotes the expected value of water release from reservoirs; the second term denotes the variation of expected water release value; and the third term means the amount of under-achievements or overachievements of water demand. Equations (7.2b) and (7.2c) are used to avoid handling of the absolute item in the objective function. Equations (7.2d) and (7.2e) are reservoir water balance equations. Equation (7.2f) denotes the water deficit amount; Equation (7.2g) means the water saved from long and short-term water-saving measures should be higher than water deficit amount. Equation (7.2h) denotes the cost of adopting long and short-term measures should not exceed the allowable budget.

In this model, the cost and efficiency of long/short-term water-saving measures and allowable budget in Equation (7.2g) and (7.2h) are expressed as fuzzy parameters. In order to deal with these fuzzy parameters in constraints, a FMP proposed by Van Hop (2007) is employed. The fuzzy parameters are expressed as triangular-shaped fuzzy sets. The general fuzzy programming model is shown as follows:

Maximize \( f = \sum_{j=1}^{J} c_j x_j \) \hspace{1cm} (7.3a)

subject to:

\[ \sum_{j=1}^{J} \bar{a}_{mj} x_m \leq \tilde{b}_m \] \hspace{1cm} (7.3b)

\( \text{others,} \) \hspace{1cm} (7.3c)
where \( \tilde{a}_{mj} \) and \( \tilde{b}_m \) are triangular-shaped fuzzy coefficients, i.e. \( \tilde{a}_{mj} = (a_{mj}^1, a_{mj}^2, a_{mj}^3) \), and \( \tilde{b}_m = (b_m^1, b_m^2, b_m^3) \), respectively; \( x_j \) are deterministic decision variables; \( j \) and \( J \) are the index and number of the decision variable, respectively; \( m \) is the index of fuzzy constraint; the word ‘others’ denotes other deterministic constraints.

The fuzzy attainment method can be written as follows (Van Hop, 2007):

Maximize \( f = \sum_{j=1}^{J} c_j x_j - \sum_{i=1}^{i} \psi_m \)  \hspace{1cm} (7.4a)

subject to:

\( \psi_m = D_m \left[ \sum_{j=1}^{J} \tilde{a}_{mj} x_m, \tilde{b}_m \right] \forall m \)  \hspace{1cm} (7.4b)

\( \psi_m \geq 0 \forall m \)  \hspace{1cm} (7.4c)

\( \text{others,} \)  \hspace{1cm} (7.4d)

where \( \psi_m = D_m \left[ \sum_{j=1}^{J} \tilde{a}_{mj} x_m, \tilde{b}_m \right] \) denotes the average lower-side attainment index of fuzzy variables and can be calculated as follows:

\( D_m \left[ \sum_{j=1}^{J} \tilde{a}_{mj} x_m, \tilde{b}_m \right] = \text{Max} \left[ 0, \frac{1}{2} \left( \sum_{j=1}^{J} a_{mj}^3 x - b_m^1 \right) \right] \)  \hspace{1cm} (7.5)

Equation (7.5) can be transformed to:

\( D_l \left[ \sum_{j=1}^{J} \tilde{a}_{ij} x, \tilde{b}_l \right] = \frac{1}{2} \left( \sum_{j=1}^{J} a_{ij}^3 x - b_l^1 + SL \right) \)  \hspace{1cm} (7.6a)

\( \sum_{j=1}^{J} a_{ij}^3 x - b_l^1 + SL \geq 0 \)  \hspace{1cm} (7.6b)
where $SL_i$ is a slack variable which is used to relax the constraint. If $\sum_{j=1}^{l} a_j^i x - b_i^l \geq 0$, then $SL_i = 0$; while if $\sum_{j=1}^{l} a_j^i x - b_i^l \leq 0$, then $SL_i = b_i^l - \sum_{j=1}^{l} a_j^i x$. The detailed proof can be referred to (Van Hop, 2007; Chou et al., 2009; Li and Chen, 2011). The final fuzzy robust programming (FRP) model can be rewritten as follows:

Maximize $f = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{t=1}^{T} p_{st} X_{ist} - \lambda \sum_{j=1}^{S} \sum_{t=1}^{T} p_{st} \left( \sum_{j=1}^{S} \sum_{s=1}^{T} p_{st} \cdot X_{ist} + b_i^1 \right) + 2\theta_{ist}$

$$\sum_{t=1}^{T} \sum_{s=1}^{S} X_{ist} - \sum_{t=1}^{T} \sum_{s=1}^{S} p_{st} \cdot X_{ist} + \theta_{ist} \geq 0 \ \forall s,t$$

$$DM_t - \sum_{j=1}^{l} X_{ist} + \theta_{2st} \geq 0 \ \forall s,t$$

$$ST_{it} = ST_{it-1} + Q_{ist} - X_{ist} - W_{it} \ \forall i,s,t$$

$$ST_{it}^{min} \leq ST_{it} \leq ST_{it}^{max} \ \forall i,t$$

$$\delta_{it} = DM_t - \sum_{j=1}^{l} X_{ist} + \theta_{2st} \ \forall s,t$$

$$\psi_{1st} = D_{1st} \left[ \delta_{it} , DM_t , \left( \sum_{k=1}^{K} LCT_k \cdot Y_{ik} \cdot LW_k + \sum_{j=1}^{J} Y_{2jst} \cdot \tilde{Y}_{2j} \right) \right] \ \forall s,t$$

$$\psi_{2s} = D_{2s} \left[ \sum_{k=1}^{K} LCT_k \cdot Y_{ik} \cdot LW_k + \sum_{j=1}^{J} Y_{2jst} \cdot \tilde{Y}_{2j} , BGT \right] \ \forall s$$

$$X_{ist} \geq 0, \ \psi_{1st} \geq 0, \ \psi_{2s} \geq 0 \ \forall s,t,k$$
where $D_{1}[.]$ and $D_{2}[.]$ are average lower-side attainment index of constraints (7.7g) and (7.7h), respectively.

### 7.3.3 Climate change impact assessment

The climate change impact assessment is carried out following a similar framework proposed by (Lu et al., 2014), and the related outputs are incorporated into the optimization model for seeking cost-effective adaptation strategies. Figure 7.3 shows the flow chart of the methodology, and the detail procedures of the methodology are introduced as follows:

![Flow chart of methodology](image)

**Figure 7.3 Flow chart of methodology.**

**Step 1: Statistical downscaling** Firstly, the predictor selection is based on the Spearman correlation coefficient between monthly precipitation and large scale predictors. The Conditional Density Estimation Network Creation and Evaluation (CaDENCE) method is used for precipitation downscaling. It is developed by Cannon (2012) which provides routines for creating and evaluating conditional density estimation network (CDEN) in the R programming. The predicted value
from CDEN model can be given by Cannon (2012):

\[
y_{kt} = \sum_{j=1}^{J} h \left( \sum_{i=1}^{I} X_{it} W_{ij}^{(1)} + b_{j}^{(1)} \right) W_{jk}^{(2)} + b_{k}^{(2)}
\]  

(7.8)

where \(y_{kt}\) and \(X_{it}\) is the \(k\)th output and observed value of the \(i\)th predictor for case \(t\), \(h(\bullet)\) is the transfer function of the hidden layer, \(i\) and \(I\) are index and number of predictors, respectively, \(j\) and \(J\) are the index and number of hidden nodes, respectively, \(k\) is index of outputs, \(W_{ij}^{(1)}\) and \(W_{ij}^{(2)}\) are weights of input-hidden layer and hidden-output layer, respectively, and \(b_{j}^{(1)}\) and \(b_{k}^{(2)}\) are bias of input-hidden layer and hidden-output layer, respectively. Other details about CaDENCE could refer to the study of Cannon (2012). In this study, 20 ensembles are used for projecting future conditions, which could mitigate the data shortage problem and make sure the predicted results from model could cover the observed values. It is noted that the precipitation of the first station is treated firstly; then, the second station is downscaled upon the output of first station and so on. This procedure could guarantee the spatial correlation of the three stations.

Then, the temperature downscaling is condition upon the precipitation using support vector machine (SVM) (Tripathi et al., 2006). The estimation of nonlinear function can be given by (Tripathi et al., 2006):

\[
y = f(x) = \sum_{i=1}^{I} \left( \alpha_{i} - \alpha_{i}^{*} \right) K(x_{i}, x) + b
\]  

(7.9)

where \(\alpha_{i}\) and \(\alpha_{i}^{*}\) are the Lagrange multipliers. The support vectors are defined as the data points corresponding to nonzero values for \((\alpha_{i} - \alpha_{i}^{*})\). \(K(x_{i}, x)\) is a kernel function which meets the Mercer requirement, \(b\) is adjustable parameter in model. Other details about SVM can be referred to (Tripathi et al., 2006). Similar to the precipitation downscaling, the temperature downscaling is also condition upon the downscaled precipitation of the three stations using SVM. This methodology framework could keep the inter-site spatial correlations and inter-variable cross
correlations. NCEP reanalysis data (i.e. 1997-2003) (Kalnay et al., 1996) is applied for calibrating the downscaling models and the HadCM3 A2 and B2 scenarios (Gordon et al., 2000; Pope et al., 2000) are used for generating climate scenarios for future periods (i.e. 2011-2099). The performance of the downscaling results is showed in Figures B.1 to B.6 in Appendix B.

**Step 2: Hydrological modeling** The Bayesian Neural Network (BNN) model is applied for the monthly flow data simulation. The posterior probability density calculated by Bayes theorem can be written as (Khan and Coulibaly, 2006):

$$p(w|D) = \frac{p(D|w)p(w)}{p(D)}$$

(7.10)

where $p(D|w)$ is the likelihood function, $p(D)$ is the denominator, which is a normalization factor, $p(w)$ is the prior probability distribution of weights $w$, and in this study Gaussian prior distribution is used for weights and biases of the network. The likelihood function can be expressed as (Khan and Coulibaly 2006):

$$p(D|w) = \frac{1}{Z_D(\beta)} \exp(-\beta E_D)$$

(7.11)

where $Z_D(\beta)$ is the normalization factor, $E_D$ is the error function, and $\beta$ is the hyperparameter that controls the variance of the noise. The details of BNN could refer to the study of Khan and Coulibaly (2006). The procedures could be summarized as follows. Firstly, observed weather data (Precipitation, $T_{\text{min}}$ and $T_{\text{max}}$) is used for training BNN model firstly. The input variables are chosen by the Spearman correlation coefficients and listed in Table 7.2. Secondly, apply the downscaled weather data to BNN model. Finally, generate the current (1997-2006) and future scenarios (2011-2099).
Table 7.2 The selected variables for modelling monthly runoff using BNN method.

<table>
<thead>
<tr>
<th></th>
<th>Capilano</th>
<th>Coquitlam</th>
<th>Seymour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PCP</td>
<td>Tmax</td>
<td>Tmin</td>
</tr>
<tr>
<td>Flow 1</td>
<td>Lag-0</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lag-3</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lag-4</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Flow 2</td>
<td>Lag-0</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lag-1</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lag-2</td>
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<tr>
<td></td>
<td>Lag-3</td>
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<tr>
<td></td>
<td>Lag-4</td>
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<td></td>
</tr>
<tr>
<td>Flow 3</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Lag-4</td>
<td>√</td>
<td></td>
</tr>
</tbody>
</table>

Note: PCP represents the precipitation; Tmax represents the maximum temperature; Tmin represents the minimum temperature.

7.3.4 Analysis of joint probabilistic distribution of inflows

To obtain optimal water release schemes from the SRO model, it is important to discretize the stochastic inflows of the three reservoirs at different levels of probabilities. For simplicity, we choose three probability levels for the inflows, including low (L), medium (M), and high (H). The general procedures for the discretization of inflows are listed as follows:

(i) Plot the histograms of the monthly inflows for different reservoirs;

(ii) Based on the histogram, categorize the monthly inflows of each reservoir into three categories, including low (L), medium (M) and high (H). For example, the inflow of Capilano at January under current condition can be categorized into L (9.30-18.09 m³/s), M (18.09-26.85 m³/s), and H (26.85-34.40 m³/s) conditions;

(iii) Take the middle value of the inflow interval as the average magnitude of the inflow, which represent the inflow level (i.e. $Q_{st}$) under that scenario. For example, the inflows of Capilano at January under L, M and H conditions are 13.70, 22.47, and
(iv) Since each reservoir has three scenarios (L, M and H), there are 27 scenarios in total for the inflow conditions. For example, LLL means all of the inflow of three reservoirs are under the low condition. The joint probabilities can be obtained based on the frequency of occurrence for each scenario. For example, if the LLL scenario in January exists 5 years in 16-years period, the probability is 5/16. Table 7.3 lists the joint probabilities under different scenarios. It is noted that the scenarios with low probabilities (less than 0.15) are ignored and only 9 scenarios listed in the table are selected.

Table 7.3 The joint probabilities under various scenarios

<table>
<thead>
<tr>
<th>Month</th>
<th>LLL</th>
<th>MMM</th>
<th>HHH</th>
<th>MLM</th>
<th>LML</th>
<th>MML</th>
<th>LLM</th>
<th>HMH</th>
<th>HHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
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<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
</tr>
<tr>
<td>Feb</td>
<td>0.57</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0.29</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.14</td>
<td>0.29</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
</tr>
<tr>
<td>Apr</td>
<td>0.17</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
</tr>
<tr>
<td>May</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0.29</td>
<td>0</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>Jun</td>
<td>0.5</td>
<td>0</td>
<td>0.33</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
</tr>
<tr>
<td>Jul</td>
<td>0.57</td>
<td>0.14</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Aug</td>
<td>0.71</td>
<td>0</td>
<td>0.29</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sep</td>
<td>0.57</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.29</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oct</td>
<td>0.14</td>
<td>0.14</td>
<td>0.29</td>
<td>0.29</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Nov</td>
<td>0.5</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>Dec</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: L = low inflow; M = Medium inflow; H = high inflow.

Figure 7.4 shows the comparison of joint probabilities among observed inflow, simulated flow under HadCM3 A2 emission scenario, and simulated flow under HadCM3 B2 Scenario during 1997-2006. Because LLL, MMM, and HHH are major scenarios with higher probabilities than other scenarios, only these three scenarios are used for comparison. It is found that the maximum and minimum probabilities based on the results of 20 ensembles under both A2 and B2 emission scenarios could generally cover the probability levels of the observed inflow. Figure 7.5 shows the variation of the probability of LLL scenario under A2 and B2 scenarios for the future
period from 2011-2099. It is found that, under A2 scenario, the probability of low flow would decrease from January to August, but increase from September to December. Under B2 scenario, the probability of low flow would increase at most of the months, except for February, March, June and August. The simulated flows of each reservoir under HadCM3 A2 and B2 emission scenarios for the future period are plotted in Appendix B (i.e. Figures B.7 to B.9).

Figure 7.4 Probabilities of the simulated flows under HadCM3 A2 and B2 emission scenarios for the current period (i.e. 1997-2006). Note: a1, a2, and a3 represent the LLL, MMM, and HHH scenarios under HadCM3 A2 scenario, respectively; b1, b2, b3 represent the LLL, MMM, and HHH scenarios under HadCM3 B2 scenario, respectively.
Figure 7.5 Probabilities of LLL Scenario under HadCM3 A2 and B2 Scenarios in the future. Note: a1, a2, and a3 represent LLL probability levels during periods of 2011-2040, 2041-2070, and 2071-2099, respectively; b1, b2, and b3 represent LLL probability levels during periods of 2011-2040, 2041-2070, and 2071-2099, respectively.

7.4 Result Analyses

7.4.1 Planning under current condition

The solutions of the releases of three reservoirs at different months under three major scenarios, namely LLL, MMM, and HHH scenarios, under current condition are
listed in Table 7.4. Obviously, the releases would alter under different scenarios, low inflow generally corresponds to low release and high inflow leads to high release. It is indicated that the release of each reservoir in August is relatively higher due to higher demand in the summer period. The balance among these three reservoirs is also obvious. In low inflow condition (i.e. LLL), the release of Coquitlam is zero except for July and August, and the release of Coquitlam at August (i.e. 56.81 Mm\(^3\)) is much higher than Capilano and Seymour reservoir. It may indicate that the Coquitlam reservoir with large storage (175 Mm\(^3\)) would store inflow and release more water when severe drought occurs. It is also found that the Capilano reservoir which has a medium-size storage would supply more water to satisfy the demand. Moreover, according to the results obtained, the shortage would only exist at October at LLL scenario under current condition. The long-term water-saving measures would not be adopted. The short-term measures of installation of water-saving showers, water-saving facet, and outdoor water kits would be recommended.

Table 7.4 Allocated release from different reservoirs under LLL, MMM, and HHH scenarios (Mm\(^3\))

<table>
<thead>
<tr>
<th>Month</th>
<th>Capilano</th>
<th>Seymour</th>
<th>Coquitlam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LLL</td>
<td>MMM</td>
<td>HHH</td>
</tr>
<tr>
<td>Jan</td>
<td>0.00</td>
<td>35.74</td>
<td>71.47</td>
</tr>
<tr>
<td>Feb</td>
<td>74.48</td>
<td>96.96</td>
<td>119.5</td>
</tr>
<tr>
<td>Mar</td>
<td>27.78</td>
<td>61.76</td>
<td>95.74</td>
</tr>
<tr>
<td>Apr</td>
<td>30.26</td>
<td>48.84</td>
<td>67.41</td>
</tr>
<tr>
<td>May</td>
<td>5.75</td>
<td>29.16</td>
<td>52.57</td>
</tr>
<tr>
<td>Jun</td>
<td>28.17</td>
<td>57.26</td>
<td>86.35</td>
</tr>
<tr>
<td>Jul</td>
<td>24.24</td>
<td>52.47</td>
<td>80.71</td>
</tr>
<tr>
<td>Aug</td>
<td>21.32</td>
<td>37.09</td>
<td>52.87</td>
</tr>
<tr>
<td>Sep</td>
<td>50.24</td>
<td>63.44</td>
<td>76.64</td>
</tr>
<tr>
<td>Oct</td>
<td>17.13</td>
<td>54.34</td>
<td>91.55</td>
</tr>
<tr>
<td>Nov</td>
<td>24.28</td>
<td>69.30</td>
<td>114.1</td>
</tr>
<tr>
<td>Dec</td>
<td>28.20</td>
<td>54.38</td>
<td>80.56</td>
</tr>
</tbody>
</table>

The trade-off between the expected water deficit amounts and expected water release amount is plotted in Figure 7.6. The value of $\lambda$ is taken as 10. It is demonstrated that,
as the $\omega$ value increases, the expected water release amount would increase, while the expected amounts of water deficit would show opposite trend. For example, when $\omega$ value increases from 0 to 10, the expected water deficit would decrease from 73.86 to 0.70 (Mm$^3$), while the expected water release amount would increase from 606.20 to 629.97 (Mm$^3$). This is because the increase of $\omega$ means the level of system reliability and the model feasibility becomes higher, then the water deficit would decrease and the expected water release amount would increase.

Figure 7.6 Trade-off between the expected water deficit amounts and expected water release amount.

Comparing to deterministic model, the fuzzy sequential model could deal with fuzzy parameters existing in the studied system. The deterministic methods may lead to infeasible and inapplicable solutions. For example, the solution obtained from deterministic model may easily cause violation of model constraints with low possibility in model constraints. For example, for the constraint (7.2h), the cost of the water-saving measures obtained from the deterministic model may exceed the lower bound of the allowable budget. Comparing to traditional fuzzy chance-constraint approaches, the fuzzy sequential method embedded in FRP model
would only lead to a single balanced solution, and has a much reduced computational complexity.

### 7.4.2 Management under future conditions

Based on the inflow results simulated by hydrological model, the average inflow amount under A2 Scenario would increase from February to July, but decrease from August to January. For example, compared to current inflow, the average inflow of Capilano reservoir at March during period 1 to 3 would increase by 64.9%, 56.8%, and 71.4% respectively, and at October would decrease by 46.7%, 53.7%, and 33.9%, respectively. The average inflow amount under B2 Scenario would decrease through the whole year.

Figure 7.7 illustrates the water release of each reservoir under A2 emission scenario during three periods, namely 2011-2040, 2041-2070, and 2071-2099, based on the solutions of 20 ensembles. It is found that, at the early stage of this century (2011-2040), the median of the release in Capilano reservoir during spring period would increase by more than 20%, and the highest increase rate could reach up to 59% at May. The releases at 75th percentile of ensemble range from March to May are higher than the current release. Similarly, at the same period, the release in Seymour and Coquitlam reservoirs during spring period would also increase significantly, and the highest increase rates of the median releases would occur in April and February, respectively. During summer period, the release at June is still higher than the current level but would decrease from July, and the releases of all three reservoirs would decrease significantly in August. During winter period, the releases in October and November would decrease notably (more than 20% for the median values). In the late winter (December and January), the release would not change significantly. This may because, A2 scenario corresponds to a high CO\textsubscript{2} emission and this could aggregate the snowmelt from spring to early summer and the precipitation is also higher, and thus the inflow during this period would increase. However, the inflow during late summer and early winter would decrease.
Figure 7.7 Expected water release of each reservoir under A2 Scenario during different periods. a1, a2 and a3 denote water release of Capilano reservoir during period 2011 to 2040, 2041 to 2070, and 2071-2099 respectively; b1, b2, and b3 denote water release of Seymour reservoir during period 2011 to 2040, 2041 to 2070, and 2071-2099 respectively; c1, c2, and c3 denote water release of Coquitlam reservoir during period 2011 to 2040, 2041 to 2070, and 2071-2099 respectively.

At the middle of this century (2041-2070), the releases would increase dramatically in March during spring period. For other months in the spring period, the releases vary less significantly compared to the first period (2011-2040). During summer period, the releases in August are still low, but the release in September becomes somewhat higher. During winter period, the decrease rates in October and November
Figure 7.8 Expected water release of each reservoir under B2 Scenario during different periods. a1, a2 and a3 denote water release of Capilano reservoir during period 2011 to 2040, 2041 to 2070, and 2071-2099 respectively; b1, b2, and b3 denote water release of Seymour reservoir during period 2011 to 2040, 2041 to 2070, and 2071-2099 respectively; c1, c2, and c3 denote water release of Coquitlam reservoir during period 2011 to 2040, 2041 to 2070, and 2071-2099 respectively.

are similar to those of the first period. At the end of this century (2071-2099), it is suggested that the release would continually decreasing from late August to early November for both Capilano and Seymour reservoirs, especially in October and November; while releases at the rest months would increase. It is noted that the release in September for Coquitlam reservoir has an increase rate up to 150% compared with those in current condition; this is to compensate the decrease for the
other reservoirs at the summer season. It is expected that the climate impact may lead to a shortened dry period.

Figure 7.8 illustrates the water release of each reservoir under B2 emission scenario. Similar to A2 emission Scenario, the release at August would decrease the most. However, during spring season, the releases of Capilano and Seymour reservoir would decrease in February, March and April. This is because the CO₂ emission under B2 scenario is less than that under A2 scenario (Parry et al., 2004). The snowmelt process would not be influenced significantly under B2 scenario compared to that in A2. The decrease of release at spring season would lead to a water shortage during that period, and also the dry period would be expected to extend.

Table 7.5 lists the probabilities of adopting long-term water-saving measures under LLL scenario. The probability is calculated based on the frequency of occurrence of the measures in 20 groups of solutions. The long-term water-saving measures would be mainly adopted under B2 emission scenario during the third period (2071-2099), since the shortage is the most serious during that period. The probabilities of adopting long-term water-saving measures under A2 emission scenario are all zero. This is because, compared to B2 emission scenario, the shortage problem under A2 emission scenario is less serious and the shortage period would only occur during summer. Hence the short-term water-saving measures are more preferred than long-term ones. The probability of adopting metering is the highest for its high efficiency and low cost. Table 7.6 shows the probabilities of adopting short-term water-saving measures under both A2 and B2 scenarios. The table shows that, under A2 emission scenario, the shortage problem is most serious in July during periods 1 and 2, and June in period 3; the water deficit would barely occur in September.

Almost all the measures need to be adopted in July at period 2, where the probabilities would range from 0.95 to 1; no measures would be taken in September. Table 7.6 also shows that the water deficit is generally more serious under B2 scenario during summer period. The shortage problem would be more serious in June, August and September, and July is wetter than that under A2 scenario in
Table 7.5 The probabilities of adopting long-term water-saving measures under LLL scenario

<table>
<thead>
<tr>
<th>Measures</th>
<th>A2 Emission Scenario</th>
<th>B2 Emission Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Period 2</td>
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<td>0</td>
</tr>
<tr>
<td>$k = 3$</td>
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<td>0</td>
</tr>
</tbody>
</table>

Note: $k = $ index of long-term measures

Table 7.6 The probabilities of adopting short-term water-saving measures under LLL scenario

<table>
<thead>
<tr>
<th>Period</th>
<th>Measures</th>
<th>A2 Emission Scenario</th>
<th>B2 Emission Scenario</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>JUN</td>
<td>JUL</td>
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<td>0.9</td>
</tr>
<tr>
<td></td>
<td>$j = 3$</td>
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<td>0.85</td>
</tr>
<tr>
<td></td>
<td>$j = 4$</td>
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</tr>
<tr>
<td></td>
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<td>0.85</td>
</tr>
<tr>
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<td></td>
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<tr>
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<td></td>
<td>$j = 4$</td>
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<td>0.95</td>
</tr>
<tr>
<td></td>
<td>$j = 7$</td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td>$j = 8$</td>
<td>0.25</td>
<td>0.95</td>
</tr>
<tr>
<td>Period 3</td>
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<td>0.6</td>
</tr>
<tr>
<td></td>
<td>$j = 2$</td>
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<td>0.9</td>
</tr>
<tr>
<td></td>
<td>$j = 3$</td>
<td>0.85</td>
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<tr>
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<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$j = 7$</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$j = 8$</td>
<td>0.9</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Note: $j = $ index of short-term measures.
periods 1 and 2. In period 3, September becomes drier. So the probabilities of using different measures are above 0.85 in June at period 1; the probability of adopting measures would increase in September from period 1 to period 3 and some probabilities would reach up to 0.75.

7.4.3 Further discussions

Generally, the proposed FRP model has a number of advantages. Firstly, it embeds fuzzy sequential method into a robust optimization framework and could effectively handle both random inflow in water supply side and fuzzy parameters associated with economic coefficient and water saving strategies in water demand side. Through case study, the balance among the expected water release, deviation of water release, and risk of constraint violation is effectively reflected by the robust algorithm, and the size of the model solutions and computational complexities are well controlled by using fuzzy attainment values. Secondly, the model could effectively address future climate-change impact by considering changes on water supply patterns caused by future climate conditions. In this study, the projection of future inflows was based on 20 ensembles from downscaling and hydrological forecasting techniques. Possible long-term/short-term management measures are also obtained to help decision makers in making appropriate adaptation strategies. Thirdly, this study shed some light on how to address complexities on water supply-demand management problems caused by both uncertainty and climate change. The methodology framework is applicable to many areas upon modification of the optimization model format and collection of specific data.

However, there are also a number of limitations in this study. One of the main problems is the lack of data on the climate change study. The quality and quantity of meteorological data in this region are both not satisfactory. If more data were available, the result would be more reliable. Besides, other data, such as the cost and efficiency water-saving measures, were referred to different sources. We believe it is acceptable for the purpose of methodology demonstration. Secondly, the climate change study suffers from many uncertainties and relies on the results of
GCM. In this study, we only considered one GCM model and two emission scenarios. Other scenarios may lead to different results and need to be considered in future studies (Fowler et al., 2007). Thirdly, the process of calculating the joint probability distribution was simplified, where only three scenarios (i.e. high, medium, and low flow) were accounted for. This is for the benefit of reducing the number of scenarios in robust optimization. More scenarios or water shortage levels could be taken into consideration in further studies. Finally, this study only addressed a number of water-saving measures. More long-term and short-term measures including both supply enhancement and demand management (such as water reuse and water transfer), and the relationship between the long-term and short-term measures (e.g. the effectiveness of some short-term measures may be limited by the adoption of long-term measures) could be considered.

7.5 Summary

A robust fuzzy programming (RFP) model was proposed and applied to a multi-reservoir system for seeking optimal release strategies under uncertainty and climate change. RFP owns advantages of both SMP and FMP models, which could deal with random inflow in model, balance the solution robustness and model robustness, and handle the economic parameters which are expressed as fuzzy sets. A multi-reservoir system in the area of GVRD was investigated for demonstration the proposed RFP model. The climate change impact on water supply-demand management was also considered. CaDENCE was used for precipitation downscaling, and SVM was used for temperature downscaling. BNN model was applied to simulate the monthly inflows of three reservoirs under future climate change conditions projected from HadCM3 A2 and B2 scenarios for providing the input to the optimization model. The study results indicated that, under future condition, the water releases would increase at spring and decrease at winter under A2 emission scenario compared to the current condition; and the dry period would extend under B2 emission scenario. This study is useful in helping decision makers seek optimal water supply-demand management or adaptation strategies under
various complexities that caused by uncertainties and/or impact of climate change.
CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

This study aims to develop a set of fuzzy-approach-based optimization methodologies for water supply-demand management under uncertainty. The following conclusions can be drawn:

(1) A combined genetic algorithm and fuzzy simulation approach (GAFSA) was developed in this study, incorporating genetic algorithm (GA) into a FCCP framework. GAFSA allows some constraints with fuzzy variables to be satisfied at specified confidence levels, and was capable of tackling fuzzy parameters with generally shaped membership functions with the assistance of GA. A study case of water resources management within an agricultural system was used for demonstration. The results indicated that the results could also assist agricultural water managers to make a trade-off between the overall system benefit and the failure risk of environmental compliance.

(2) A superiority-inferiority-based sequential fuzzy programming (SISFP) method was developed, which incorporates a sequential method into the fuzzy optimization framework for dealing with general-shaped fuzzy sets on the right- and left-hand sides of model constraints. The results obtained from a water supply-demand system of Binhai New Area in Tianjin, China showed that the proposed method has advantages of dealing with general-shaped fuzzy parameters in objective function and both sides of the constraints and could generate the solutions of water allocation amounts and the adoption of water-saving measures. A comparison among different models (e.g. DM, ASM, and SISFP) showed that the model with general-shaped fuzzy sets could obtain more reasonable results and avoid overestimated or underestimated solutions.
(3) An extended fuzzy parametric programming (EFPP) approach was developed with the purpose of dealing with fuzzy parameters with trapezoidal-shaped distribution functions in the model, and also fuzzy relations in the constraints. The results obtained from a case of water resource allocation problem indicated that EFPP was capable of tackling a wide range of fuzziness in the management model and allow water managers analyze the balance of system benefit and risk of failure. The EFPP was superior to the conventional FCCP method in flexibly handling the fuzzy relationship and fuzzy parameters in both objective function and constraints and had a more general applicability.

(4) An integrated fuzzy programming and decision analysis (IFPDA) approach was proposed which links fuzzy programming with decision making in this study. The proposed IFPDA method is capable of dealing with fuzzy uncertainty in both model constraints and objective functions and linking decision analysis to optimization model outputs. The FDA criteria were also enhanced to mitigate the influence of parameter scale problems in decision analysis. The results of the study case based on an urban water distribution network operation case showed that the IFPDA could generate various decision alternatives at different confidence levels, and assist decision makers to determine the best planning scheme by FDA.

(5) A robust fuzzy programming (RFP) model was proposed and applied to a multi-reservoir system for seeking optimal release strategy under inflow stochasticity and demand management options. SRF owns advantages of both SMP and FMP model, which could deal with random inflow in model and balance the solution robustness and model robustness, and handle the economic parameters which expressed as fuzzy sets. A multi-reservoir system of GVRD was investigated to demonstrate the applicability of SRF model. The climate change impact on water supply-demand management is also considered. CaDENCE was used for precipitation downscaling, and SVM was used for temperature downscaling. BNN model was applied to simulate the monthly inflows of three reservoirs under future climate change impact under HadCM3
A2 and B2 scenarios for the input of the optimization model. The results obtained from the optimization model shows that under future condition the water releases would increase at spring and decrease at winter under A2 emission scenario compared to current condition; and the dry period would be expected to extend under B2 emission scenario. This study is useful in generating optimal water supply and demand patterns under complex uncertainties and addressing impact of climate change on water supply-demand system.

The originality of the work is summarized as follows:

(1) The main contribution of this research was the development of a set of fuzzy-approach-based optimization methodologies for dealing with different fuzzy conditions in water supply-demand systems. For example, the GAFSA and SISFP can deal with general-shaped fuzzy parameters in an inexact optimization model; the EFPP model can handle both fuzzy coefficients and fuzzy relationship; and the FRP model can tackle both random input related with water supply and fuzzy parameters associated with water demand.

(2) The decision analysis method was integrated into an optimization framework, which could help decision makers evaluate and rank the possible solution options. While most of the previous optimization efforts focused on obtaining solutions without considering decision analysis.

(3) As a new attempt, the complex interaction amongst water availability, water demand, and climate change were analyzed based on an integration of climate-change impact assessment and the optimization framework. The impacts of precipitation and temperature variations on reservoir operation and water demand management could be quantified.

(4) The developed methodologies were applied to a number of real-world study cases, including Binhai New Area in Tianjing, China, the urban water distribution system in Hamedan city, Iran, and the Greater Vancouver Regional District (GVRD) in Georgia Basin, Canada. The related discoveries and results
are useful for supporting decisions of water supply-demand management under uncertainty.

8.2 Recommendations

This work still has some limitations that need to be improved in future studies. Firstly, the problem under investigation is assumed as linear and has a single objective. However, water supply-demand systems are complex with characteristics of multi-objectivity and interactions among multiple system components. Secondly, each developed method has its own advantages and disadvantages, thus it could lead to different solutions. Hence, risks may be associated with the decision alternatives generated from the modelling results. Thirdly, the fuzzy decision analysis process proposed in this study is relatively simple as only two criteria are involved in decision making. It is difficult to solve a multi-objective problem with multiple criteria. Finally, only climate change impact was investigated in this study; other factors, such as land use, were not involved. Therefore, future studies are necessary to address:

(1) Development of nonlinear programming models as well as involvement of multiple objectives could better reflect the relationships among system components and help extend the application ranges of the developed methodologies. Multi-criteria decision analysis (MCDA) could help decision makers evaluate alternatives against multiple conflicting criteria and would be a potential alternative to planning complex problems. How to incorporate MCDA into an optimization framework needs further investigations.

(2) There is a number of different GCMs available (e.g. BCM, CSIRO-MK, ECHAM5, and HADCM3), and they give different results. The weather simulated by these models also depends in part on the assumed atmospheric concentration of greenhouse gasses. Hence, the projected weather for a given period in the future depends on the model and the emission scenario used. All these would result in a variety of uncertainties associated with the inputs of the water resources...
management system. It is desired to investigate the water resources management under the impact of climate change as well as the uncertainty generated from the climate prediction.

(3) Land use can affect the evapotranspiration process by infiltration, the redistribution process of soil water, velocity of the overland flow, and flow rates of floodplain. The water balance of catchment can be influenced by land use and planning of land use need to be coped with droughts, water scarcity and flooding. How to link land use planning and water management is desired to investigate in the future study.
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## APPENDIX A

Table A.1 Allocated solutions of IFPDA at confidence level of 0.1 \(\times 10^3\) m³

### Water amounts transferred from water sources to treatment plants

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Note: \(j\) = index of water sources; \(t\) = index of treatment plants; \(r\) = index of reservoirs; \(z\) = index of consuming zones; \(k\) = index of months
Table A.2 Allocated solutions of IFPDA at confidence level of 0.2 ($\times 10^3$ m$^3$)

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**Water amounts transferred from reservoirs to consuming zones**

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Note: $j =$ index of water sources; $t =$ index of treatment plants; $r =$ index of reservoirs; $z =$ index of consuming zones; $k =$ index of months
Table A.3 Allocated solutions of IFPDA at confidence level of 0.3 ($\times 10^3$ m$^3$)

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Water amounts transferred from treatment plants to reservoirs

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Water amounts transferred from consuming zones

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Note: $j$ = index of water sources; $t$ = index of treatment plants; $r$ = index of reservoirs; $z$ = index of consuming zones; $k$ = index of months.
Table A.4 Allocated solutions of IFPDA at confidence level of 0.4 \((\times 10^3\ m^3)\)

Water amounts transferred from water sources to treatment plants

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Water amounts transferred from treatment plants to reservoirs

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Water amounts transferred from reservoirs to consuming zones

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Note: \(j\) = index of water sources; \(t\) = index of treatment plants; \(r\) = index of reservoirs; \(z\) = index of consuming zones; \(k\) = index of months
Table A.5 Allocated solutions of IFPDA at confidence level of 0.5 ($\times 10^3$ m$^3$)

**Water amounts transferred from water sources to treatment plants**

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<td>696.91</td>
<td>672.71</td>
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**Water amounts transferred from treatment plants to reservoirs**

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<td>524.42</td>
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**Water amounts transferred from reservoirs to consuming zones**

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<td>162.89</td>
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Note: $j$ = index of water sources; $t$ = index of treatment plants; $r$ = index of reservoirs; $z$ = index of consuming zones; $k$ = index of months
Table A.6 Allocated solutions of IFPDA at confidence level of 0.6 ($\times 10^3$ m$^3$)

Water amounts transferred from water sources to treatment plants

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Water amounts transferred from treatment plants to reservoirs

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Water amounts transferred from reservoirs to consuming zones

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Note: $j$ = index of water sources; $t$ = index of treatment plants; $r$ = index of reservoirs; $z$ = index of consuming zones; $k$ = index of months
Table A.7 Allocated solutions of IFPDA at confidence level of 0.7 ($\times 10^3$ m$^3$)

### Water amounts transferred from water sources to treatment plants

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### Water amounts transferred from treatment plants to reservoirs

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### Water amounts transferred from reservoirs to consuming zones

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Note: $j$ = index of water sources; $t$ = index of treatment plants; $r$ = index of reservoirs; $z$ = index of consuming zones; $k$ = index of months
Table A.8 Allocated solutions of IFPDA at confidence level of 0.8 ($\times 10^3$ m$^3$)

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Water amounts transferred from treatment plants to reservoirs

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Note: $j$ = index of water sources; $t$ = index of treatment plants; $r$ = index of reservoirs; $z$ = index of consuming zones; $k$ = index of months
Table A.9 Allocated solutions of IFPDA at confidence level of 0.9 ($\times 10^3$ m$^3$)

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Water amounts transferred from reservoirs to consuming zones

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Note: $j$ = index of water sources; $t$ = index of treatment plants; $r$ = index of reservoirs; $z$ = index of consuming zones; $k$ = index of months.
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Note: $z$ is the index number of consuming zones; $k$ is the index number of months; $(a, b, c, d)*$ represents a trapezoidal-shape fuzzy set with $a$, $b$, $c$, and $d$ being the four sequential parameters from left to right; data are adapted from Fattahi and Fayyaz (2010) and Qin and Xu (2011).
APPENDIX B

Figure B.1 Comparison of observed precipitation with that simulated using downscaling model under A2 scenario.
Figure B.2 Comparison of observed precipitation with that simulated using downscaling model under B2 scenario.
Figure B.3 Comparison of observed Tmax with that simulated using downscaling model under A2 scenario.
Figure B.4 Comparison of observed Tmin with that simulated using downscaling model under A2 scenario.
Figure B.5 Comparison of observed Tmax with that simulated using downscaling model under B2 scenario.
Figure B.6 Comparison of observed Tmin with that simulated using downscaling model under B2 scenario.
Figure B.7 Comparison of flow under low flow conditions at Capilano Reservoir: (a1) A2 Scenario during period 1; (a2) A2 Scenario during period 2; (a3) A2 Scenario during period 3; (b1) B2 Scenario during period 1; (b2) B2 Scenario during period 2; (b3) B2 Scenario during period 3.
Figure B.8 Comparison of flow under low flow conditions at Seymour Reservoir: (a1) A2 Scenario during period 1; (a2) A2 Scenario during period 2; (a3) A2 Scenario during period 3; (b1) B2 Scenario during period 1; (b2) B2 Scenario during period 2; (b3) B2 Scenario during period 3.
Figure B.9 Comparison of flow under low flow conditions at Coquitlam Reservoir: (a1) A2 Scenario during period 1; (a2) A2 Scenario during period 2; (a3) A2 Scenario during period 3; (b1) B2 Scenario during period 1; (b2) B2 Scenario during period 2; (b3) B2 Scenario during period 3.