REFERENCE-FREE BEAM-SAMPLING (RFBS) METHODOLOGY AND SYSTEM FOR OPTICAL FREEFORM SURFACE MEASUREMENTS

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Abstract

Freeform surfaces of optical quality are of huge demand in the precision and optical industry. Although six degree-of-freedom manufacturing technologies are available to fabricate such surfaces, the metrology infrastructure cannot simultaneously perform accurate, universal and non-contact large area measurement in a short time. The main challenges are the varying curvature across the surface together with the large dynamic range. The Shack-Hartmann wavefront sensing (SHWS) technique samples the wavefront by capturing the focal spot image of the surface, which is compared with the reference image to measure the wavefront slope. The wavefront is then revealed through reconstruction. SHWS technique has merits of area-based, non-contact, high accuracy, and insensitivity to vibration. It is used for flatness and aberration measurements, optical alignment, and adaptive optics. If applying the SHWS technique to measure freeform surfaces, the sampling is an issue as the focal spot image obtained is of poor quality, which degrades the measurement performance and many of the times even make it impossible to reveal the surface profile. Also, the reference image from a flat mirror is no more valid due to out of measurement range. Furthermore, the lateral resolution is too limited to detect the varying curvatures. Last but not least, the wavefront slope data is difficult, or even impossible, to be extracted because of image distortions.

This dissertation designed and developed the reference-free beam-sampling (RFBS) methodology and system for freeform surface measurements. In the RFBS, the beam-sampling and the reference-free techniques have been devised and developed as the sensing mechanism. The method uses the beam itself as the optical probe to sample the surface. Through introducing a lateral disturbance to the modulated beam, the second order derivative of the surface is measured. The digital scanning technique is proposed and integrated into the sensing mechanism to enhance the lateral resolution. The dynamic windowing and the adaptive centroiding algorithms have been devised to extract the measurement raw data from the captured images, which is then reconstructed into the surface through the proposed 3D reconstruction technique. Freeform surfaces of various forms were measured. It is shown that tens nanometer height accuracy is achieved when measuring surfaces with over 1mm peak-to-valley value. In addition to the advantages of SHWS, no reference image is needed, no mechanical scanning is involved, and no pre-knowledge of the surface is required for the proposed method. Therefore, the RFBS system shows high potential for in-situ nm-level freeform surface measurements.
List of Symbols

$Z$ The form of the surface

$\varphi$ The form of the wavefront. In this dissertation, the wavefront is assumed to be of the same mathematical description as that of the surface.

$\Delta x$ The displacement in the $x$-direction

$\Delta y$ The displacement in the $y$-direction

$I_{x,y}$ The intensity of the image pixel at position $(x,y)$

$C_x$ The matrix of centroid position data in the $x$-direction

$C_y$ The matrix of centroid position data in the $y$-direction

$X$ The matrix of position data in the $x$-direction

$Y$ The matrix of position data in the $y$-direction

$f$ The focal length

$S$ The slopes, or the first order derivatives

$\Delta S$ The slope changes, or the second order derivatives

$\Delta d$ The small disturbance value

$p$ The center distance between two neighboring lenslets, or pitch
\( r_x \) The radius in the \( x \)-direction

\( r_y \) The radius in the \( y \)-direction

\( \theta \) The diverging angle

\( l \) The distance between the image detector and the surface

In the symbols, matrixes are represented by the uppercased letter \( A \). single values are represented by the standard letter \( a \).
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<td>Reference-Free Beam Sampling</td>
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<td>SHWS</td>
<td>Shack-Hartmann Wavefront Sensing</td>
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<td>RMSE</td>
<td>Root-Mean-Square Error</td>
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<td>PV</td>
<td>Peak-to-Valley</td>
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<td>FMM</td>
<td>First Moment Method</td>
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<td>CCD</td>
<td>Charge Coupled Device</td>
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<td>Signal to Noise Ratio</td>
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Chapter 1  Introduction

1.1  Motivation

Freeform surfaces are generally defined as surfaces without rotational symmetry (Evans and Bryan, 1999). There are many ways to classify freeform surfaces (Forbes, 2012). This dissertation follows Clayor’s (Clayor et al., 2004) classification and focuses on the first category: continuous smooth surfaces modeled using a mathematic formula, such as a spiral mirror (Forbes, 2007). The other three categories are: discontinuous surfaces with steps, such as a Fresnel lens; structured surfaces, such as a microlens array; and multiple surfaces on a single substrate, such as the prism used in the head mounted display (Cheng et al., 2009). A special case of freeform surfaces would be aspherics, where there is still an axis of rotational invariance (Henselmans, 2009).

1.1.1  Evolution of freeform surfaces

In the 130-year-old history of optical design, researchers have been dealing with rotationally symmetrical surfaces. It is only around a decade ago that the industry started to shift toward freeform surfaces thanks to the advancements of the five-axis diamond turning, micro injection moulding, and the simultaneous evolution of
computer-controlled small-lap polishing (Kevin et al., 2012, Michaeli et al., 2007, Owari et al., 2006, Seok-min and Shinill, 2003).

In the imaging field, the transition was triggered and accelerated by the desire of compensating three different aberrations simultaneously: spherical, coma, and astigmatism. As these three are correlated to each other, it is impossible to control each parameter individually to achieve full compensation. However, with freeform surfaces, it is possible to correct the three aberrations at one time. In the non-imaging field, the popularity of freeform surfaces was mainly attributed to the fact that the spline-based modeling, which provides local control of a surface for imaging or illumination optimization, is very often used as the basis function for describing the surface shape in the computer-aided design (CAD) (Hughes et al., 2005, Kevin et al., 2012).

The main advantage of freeform surfaces in high precision optical systems is their excellent performance in reduction, or even elimination, of geometrical aberrations. For example, as shown in Figure 1.1, when a spherical lens is applied to focus a collimating beam, a sharp focal spot cannot be obtained due to aberration effect. This so-called spherical aberration is universally present in all spherical lenses, regardless of alignment or manufacturing errors (Abdul, 2006). The further away the ray from the optical axis, the larger the deviation of the intersection point as compared to the focal point. Spherical aberrations result in poor image quality. To reduce aberrations, multiple spherical elements are applied in series.
A more elegant way to reduce aberrations is by applying aspherical or freeform optics. With a proper setting of the conic constant and aspheric coefficients, the profile of the lens is tuned in such a way that even rays far away from the optical axis intersect precisely at the focal point. In this way, the beam can be well-focused through the implementation of a single element.
Because of the complex surface profile design capability, freeform optics offer a higher number of parameters for simplifying systems while optimizing their performance (Rocktaschel and Tiziani, 2002, Savio et al., 2007). This can lead to systems with fewer surfaces (Kubala et al., 2003), less weight, smaller dimensions and higher states of correction (Plummer, 2005), to mention only a few advantages.

For example, the left drawing of Figure 1.2 is a camera objective with the mere employment of spherical lenses; the one on the right is a camera objective involves aspherical lenses. While being of similar performance, the one on the right is only half the size and half the weight.

Nowadays, freeform optics play a particularly important role in fields where a high number of optical surfaces are not practical, like in astronomical telescopes. In the next generation of optical ground-based telescopes, the European Extremely Large Telescope (E-ELT) as shown in Figure 1.3, makes use of 798 freeform mirrors to
enable the world-largest telescope gather more light, correct for atmospheric distortions, and provide much sharper images than the Hubble Space Telescope (Walker, 2013).

Freeform surfaces’ high design flexibility makes a single element capable of having the ideal shape to cater for various angles of view. As such, freeform surfaces are especially effective in wide-angle lenses. One of the successful applications in our daily life would be the utilization of freeform lenses in the camera module of mobile phones (Jiang et al., 2007). Without freeform lenses, mobile phones would not be as powerful and slim as they are today (Takahashi, 2011). In addition, freeform lenses also provide the camera with better clarity and field of depth (Sherif et al., 2004, Marks et al., 1999), decreased aberrations, and even enlarged field of view (Li and Yi, 2012, Jeong et al., 2006, Dowski and Cathey, 1995). Nowadays, almost every high-end camera makes use of freeform lenses to ensure high image quality, so as to guarantee users with best vision.

Moreover, efficiency of airplane engines, drag reduction for automotive bodies and increased lifespan of prosthetics are just some examples of the gains potentially achievable with suitable advances in freeform manufacturing (Clayor et al., 2004, Ohl Iv et al., 2004, Zhang et al., 2012, Krishnasamy et al., 2004).

As discussed above, freeform elements with optical quality have become more and more important today (Garrard et al., 2005). They have a wide application across many industries, such as light-emitting diode (LED) illumination systems, display
technologies, medical devices, opto-electronic and communication devices, digital cameras, visible sighting devices, and night vision devices (Li et al., 2014, Qiu and Gui, 2013, Forbes, 2013). Currently, three challenges that hinder freeform surfaces to expand the applications are the description or characterization methods, the fitting mechanisms, and the design strategies in various scenarios (Qin et al., 2014, Zhu et al., 2013, Yang et al., 2014). Nevertheless, the trend of freeform optics are to become more personalized and diversified (Yu et al., 2014).

1.1.2 Examples of freeform surface applications

The very first significant implementation of freeform surfaces was the progressive lens forms in eyeglasses (Yu et al., 2014). Progressive lenses are designed for elderly with ametropia. The freeform shapes provide more than two refractive powers on the whole surface, so as to ensure continuous visions at all distances. Zeiss commercialized this technology as early as 1960s, and in the 1990s it has become ubiquitous.

Intraocular lenses (IOLs) came to the market in the 1980s. They greatly ease life of cataracts-removed patients. The physical design of the IOLs is one important factor to ensure low risk and good vision after cataract surgery. Traditional IOLs are spherical. Aspherical IOLs was launched in 2004, which provide patients with reduced visual aberrations. The improved contrast sensitivity benefits especially younger people greatly (Nanavaty et al., 2009).
In the semiconductor industry, LEDs are gradually replacing the traditional lighting source for their low cost and easy adaption to modular applications. However, the traditional LED devices have a Lambertian light distribution. The rigid design could not meet practical requirements in various application circumstances. Freeform designs give much more freedom in reshaping the beam while maintaining a compact design with excellent optical performance (Wu et al., 2013, Feng et al., 2013, Wang et al., 2013, Doskolovich et al., 2013). As such, freeform optical systems are developed to form prescribed illumination distribution on the target area.

Microscopes have more than 400 years of history (Rochow and Tucker, 1994). One of the limitations of the traditional microscopes is the single view angle, which makes it difficult to discern the object shapes with complex spatial structures. Three-dimensional (3D) vision microscopies are then developed to tackle this issue. The key element of the system is freeform optics, which generate multiple views of the object from various directions simultaneously (Li and Yi, 2010).

Recently, the desire for optical see-through head-mounted display (OST-HMD) is rapidly emerging (Hua and Javidi, 2014). This device allows optical superposition of digital information onto the direct view of the physical world, without obstructing the user from seeing the real world. The key limitation of OST-HMD technology is visual discomfort caused by the image source as a two-dimensional (2D) flat surface located at a fixed distance from the eye. This is sometimes
referred to as the accommodation-convergence discrepancy. Another limiting factor for HMDs is the packaging constraints. It is necessary that the devices are lightweight and compact, which means that the number of optical elements deployed should be minimum (Bauer and Rolland, 2014). Due to these concerns, augmented reality (AR) systems are yet to be embraced in most practical applications. However, much research work has already demonstrated the possibility of overcoming these limitations by using freeform surfaces (Bauer and Rolland, 2014, Hua and Javidi, 2014, Hu and Hua, 2014, Pan et al., 2014). There is high potential to design a lightweight and compact HMD with an aesthetically pleasing and unobtrusive system, while maintaining high optical performances.

A special kind of freeform surfaces is of toroidal shape. Toroidal surfaces, as shown in Figure 1.4, are generally described by Equation (1-1). $r_x, r_y$ are the radius measured in the two orthogonal directions.

$$Z = \frac{1}{r_x} x^2 + \frac{1}{r_y} y^2$$

While being non-rotationally symmetric, toroidal surfaces have symmetry in the $x$- and $y$-direction respectively.
Because of the characteristic of having two different radii at two orthogonal axes, toroidal mirrors are frequently used to correct for astigmatism, which describes the phenomenon of rays propagating in two perpendicular planes focus at two different distances. A sharp focal spot can be formed at the imaging plane through a toroidal mirror, where a spherical mirror fails to achieve this purpose.

Toroidal surfaces have wide applications in optics and manufacturing industry (Menchaca and Malacara, 1986). They are employed in the motion picture wide-screen projector, where an extra-large horizontal field is employed (Dewald, 1999). Also, they are frequently seen in focusing and/or aberration correction devices for spectroscopes, in the anamorphic systems with different magnifications in two orthogonal directions, in laser applications, in solar energy collector applications,
and many other optical instrument applications. In this dissertation, toroidal mirrors are used as one of the examples to demonstrate the capability of measuring freeform surfaces.

1.1.3 Global challenges of freeform surface measurements

Even with such a huge market needs, freeform surfaces currently still have limited implementation in commercial applications. This is mainly due to the poor metrology infrastructure (Qiu and Gui, 2013). As the performance of the optical elements is profoundly affected by the geometrical shape, it is therefore of high importance to improve the measurement accuracy (Henselmans, 2009). Many companies, although actively implementing freeform optics, are eagerly searching for efficient inspection methods, such as II-VI Incorporated, and Tinsley Precision Instruments, who brought the Hubble Space Telescope up to the full intended performance by using freeform surfaces.

Freeform surface measurement technologies are divided into two categories: contact and non-contact. Currently, contact-probe based profilers are widely used for freeform surface measurements (Whitehouse, 1994, Albert et al., 2012, Weckenmann et al., 2006, Weckenmann and Schuler, 2011, Weckenmann et al., 2004, Widdershoven et al., 2011, Petz et al., 2012, Manske et al., 2007, Liebrich and Knapp, 2012). It is achieved by mechanically scanning the freeform surface under test, and therefore, is slow. Slow techniques are very susceptible to measurement errors due to temperature variations during the measurement. In
addition, optical elements risk scratching or damaging in contact testing, especially for plastic or polymer elements. Although coordinate measuring machines (CMMs) are the most commonly used contact-based tool for their large measurement range, the micrometer level uncertainty (Schellekens et al., 1998) and long testing time (Qiu and Gui, 2013) make the technology unsatisfactory for measuring freeform surfaces. Therefore, non-contact measurement techniques are desired. Moreover, in an optical system, each optical element modifies the wavefront according to the optical design of the whole system (Wyant and Creath, 1992). If a spherical wave or a plane wave enters such an optical element with a freeform surface, a wavefront with a very high departure from the best fitting sphere is produced depending on the conjugates used in the particular test configuration. Hence, even the fundamental single element with either aspherical or freeform surfaces can only be tested properly if one can deal with freeform wavefronts in a test set-up. Furthermore, it is very important to test wavefront transmitted through optical elements because inhomogeneity of the lens material or coating can deteriorate the wavefront even when the surfaces are free of error. However, the contact-probe based profile measurement cannot provide a total reflection of how the wavefront (Geary, 1995) is modulated after reflected from or refracted through the optical element. This is because that not only the physical surface of the optical element, but also the material purity, homogeneity, coating uniformity, and thickness will play a role in adjusting the coming wavefront. As such, non-contact measurement techniques to measure wavefront are highly demanded and provide additional
benefits in evaluating the optical system performance, such as alignment (Henselmans, 2009). Generally speaking, the resolution of the non-contact sensors is not satisfactory. High resolution sensors usually come with a short measuring range. It is unlikely to use a short range sensor to measure an unknown freeform surface.

Besides the specific challenges of contact and non-contact techniques, the common challenges are difficulty in precise alignment, lack of a reference standard for uncertainty and traceability analysis, and trade-off between accuracy and speed. To summarize, with manufacturing processes becoming increasingly flexible, the measuring machines also need to have higher flexibility. Although various techniques and methods of 3D surface measurements have been attempted in industries over the years, the problem of precise, fast, and capable to measure freeform surfaces is yet to be solved (Cioboata et al., 2013). Because of the challenges in freeform surface measurements described above, so far, this is still an open research topic. According to National Institute of Standards and Technology (NIST), “no single, widely recognized, general, validated way exists for calibrating or measuring complex surfaces with nm-level uncertainties” (Soons et al., 2010).

1.2 Objective and scope of work

As the current hindering factor for the popularization of freeform optics in commercial products is the poor metrology infrastructure, this dissertation aims to devise and develop a methodology and system for measuring freeform surfaces
with \( nm \)-level accuracy. In order to achieve this main objective, the following requirements need to be fulfilled: non-contact and area measurement; able to measure surfaces of various forms; achieve \( nm \)-level accuracy; and automatically perform data processing.

Chapter 2 aims to investigate the state-of-the-art ultra-precision freeform surface measurement technologies. The study reveals that the trend of freeform surface measurements is using non-contact sensor heads for acquiring surface information; hence, non-contact techniques are further examined. After analyzing the performance of the well-known non-contact techniques, Shack-Hartmann wavefront sensor is found to have high potential to perform freeform surface measurements. Therefore, intensive studies are conducted on the Shack-Hartmann wavefront sensing system to identify and analyze the issues when applying it to measure freeform surfaces.

As discovered in the literature review, one main issue of the Shack-Hartmann wavefront sensor is the rigid setting of the sampling plane. Chapter 3 aims to devise a sampling technique for acquiring surface information without a physical plane. At the same time, this sampling technique should work in a non-contact mode, and able to cover an area at one shot for fast measurement speed. The devised sampling technique thus addresses the requirements of non-contact measurement, short measurement time, and able to measure surfaces of various
forms. Chapter 3 will also present an overview and demonstration of the devised and developed methodology and system for freeform surface measurements.

Chapter 4 aims to tackle the issue of no proper references, which prohibits many techniques, including Shack-Hartmann wavefront sensor, from measuring freeform surfaces. In those systems, the reference is required to compare with the test piece. The calculated difference represents the surface slope and can be further processed to reveal the form to be measured. This chapter proposes a technique to measure the second order derivatives by comparing the test piece with itself. Since no reference is involved in the proposed technique, surfaces of various forms can therefore be measured.

The objective of Chapter 5 is to devise a technique to enhance the lateral resolution of the sampled data. The technique shall not involve mechanical scanning, or require additional hardware. Measurement accuracy can then be improved as the obtained data has a better lateral resolution. Chapter 5 will also study the optimization strategy, so as to make the technique work the most effectively. When integrated with the techniques proposed in the previous two chapters, the sensing mechanism for freeform surface measurements is complete. The data is acquired in a non-contact mode and the whole surface is measured within a short time. Surfaces of various forms can be measured, yet with a high lateral resolution to facilitate achieving the objective of \( nm \)-level accuracy.
Chapter 6 aims to extract the signal from the measured raw data effectively. As the surfaces are freeform, the raw data most likely has varying characteristics, such as signal to noise ratio. The proposed data extraction techniques should be able to identify and locate all the signals automatically and at one go. Furthermore, the proposed techniques should analyze each signal independently, and extract the useful information based on its particular condition. By doing so, Chapter 6 fulfills the requirement of automatic data processing, and facilitates the system to achieve nm-level accuracy.

The objective of Chapter 7 is to devise a technique that reveals the form of the surface from the extracted signals. The chapter firstly compares the possible approaches. The study shows that those involving the Southwell algorithm have better performance in practical conditions. However, its direct implementation is not applicable for freeform surface measurements, due to the algorithm’s incapability to handle missing data, and not applicable for second order derivatives measurements. This chapter then devises the reconstruction technique to address this issue. The technique reveals the surface form from the second order derivatives, and minimum effect is observed in the presence of missing data.

The data processing mechanism of the freeform surface measurement system is mainly consisted of the data extraction techniques and the reconstruction technique. It enhances the system performance, as well as enables the system to automatically process the measurement data and reveal the surface, where least user-input is
required. When integrated together with the sensing mechanism, the devised and developed methodology and system is able to achieve the main objective: freeform surface measurements with \( nm \)-level accuracy.

### 1.3 Organization of thesis

The scope and organization of this dissertation is shown in Figure 1.5. Chapter 2 provides the literature review on non-contact measurement techniques. The devised and developed reference-free beam-sampling methodology and system is described in Chapter 3. Its key sensing technique, the beam-sampling technique, is described in the same chapter as well. This technique uses the light beam itself to sample the surface, so as to achieve highly flexible sampling. Chapter 4 describes the devised reference-free technique to obtain the raw data through measuring the second order derivative of the surface. The proposed digital scanning technique to enhance the lateral resolution without mechanical movement is explained in Chapter 5. Chapter 6 presents the proposed dynamic windowing technique and the adaptive centroiding technique to automatically extract data from measurements. The 3D reconstruction technique is developed to reconstruct freeform surfaces with improved accuracy, which is discussed in Chapter 7. Chapter 8 presents the system error analysis and recommends optimization strategy in practical application. Finally, Chapter 9 concludes the dissertation and comments on the future work.
Figure 1.5 Organization of the dissertation.
Chapter 2 Literature Review

The state-of-the-art ultra-precision freeform surface measurement confirms the trend of using non-contact sensor heads for acquiring surface information, as discussed in the first part of the chapter. Non-contact techniques either measure a surface point by point, or measure the entire surface in one single shot. The well-known point-based techniques are laser triangulation and confocal microscopy. The well-known area-based techniques are interferometry, Shack-Hartmann wavefront sensing (SHWS), moiré fringe projection, and focus variation. Their working principles limit the techniques’ feasibility in freeform surface measurements as explained in the second part of the chapter. Compared to other techniques, the SHWS stands out because of certain merits. Therefore, intensive literature review is conducted on the SHWS system to identify and analyze the issues when applying it for freeform surface measurements in the third part.

2.1 State-of-the-art ultra-precision freeform surface measurement

Contact-based systems, such as UA3P from Panasonic or the Talysurf PGI series from Taylor Hobson, have been the dedicated metrology systems for freeform surface measurements for many years (Pinto et al., 2011). State-of-the-art CMM machines, such as Isara 400 from IBS Precision Engineering (Widdershoven et al., 2011) and NMM from SIOS Gmbh have also been proposed as ultra-precision
tools for measuring optical surfaces. The state-of-the-art non-contact measurement systems would be the NANOMEFOS developed by TNO together with Zeeko and Verifire series from Zygo.

Recently, large research interest is shown on integrating an optical sensor head with a high precision moving stage. For example, (Steinkopf et al., 2014) proposed to integrate a chromatic sensor with a diamond turning machine for non-contact measurements of freeform surfaces within the manufacturing process. The sensor measures the distance to the test surface at different positions and a point cloud is generated which describes the 3D map of the surface. The measurement accuracy is at sub-µm level, and the accuracy decreases with increasing surface slope. The system performance is also affected by vibration of the machine axes.

UltraSurf, a commercial product from OptiPro Systems, integrates a sub-micrometer non-contact point sensor with a precision motion control to collect surface information. Various sensors are interchangeable in this system, such as a white light confocal chromatic sensor and a laser interferometry head (DeFisher et al., 2011). The motion system keeps the optical sensor normal to the surface, and moves it to different positions to obtain a point cloud of the surface to be measured. As it is essential to keep the sensor normal to the surface and within the depth-of-field, it is therefore of crucial importance to provide the motion system with a precise description of the surface, in terms of point clouds, mathematical equations, or CAD models.
PLu APEX from Sensofar, another successful commercial product, mainly consists of a confocal sensor head with a XY translation stage. The confocal tracking device can achieve nm-level accuracy with a typical acquisition speed of 1mm/s for freeform surface measurements (Pinto et al., 2011).

There is also research work done to combine an optical vision sensor with a tactile probe. The tactile probe locates the nominal positions of area of interest, and the optical vision sensor follows to conduct measurement. By reading feedback from the tactile probe, the optical sensor goes to the required focusing position to capture a clear image for measurements (Chen et al., 2014).

Another dominating trend is to modify or improve the interferometry-based system setup for measuring freeform surfaces with large deviation, due to its high performance in measuring flat and spherical surfaces. For example, (Fuerschbach et al., 2014) proposed to use a series of adaptive subsystems in an interferometer. Each system acts as a null lens to compensate for a particular aberration type present in the freeform surface. This design makes the system capable of adapting and measuring a wide variety of surface shapes by simply adjusting the individual parameter of each nulling system.

(Li et al., 2014) proposed to reduce the surface’s optical power by immersing the object in a wet cell containing an optical liquid with controlled refractive index that is close to the surface optical power. The deviation of the wavefront, which is a
small variation from plane, is then to be measured. The complete information of the surface is obtained by adding the measurand to the design model.

(Rhee et al., 2013, Ghim et al., 2014) have explored the potential of using lateral shearing interferometry for freeform surface measurement. This is due to the rationale that the test wavefront is to be interfered with the sheared vision of itself. As such, no reference wavefront is needed. However, there are still many critical issues to be solved before it can be implemented, such as the orthogonality problem and increased error (tens-\(\mu m\) level) in measuring surfaces with large range (sub-\(mm\) level).

To summarize, state-of-the-art freeform measurement technologies are either contact-based, which has been a common practice in industry because of no other suitable substitutes, or a non-contact optical sensor mounted on a moving stage. Recent research interest has been shown towards the latter case as an elegant tool for measuring freeform surfaces of optical quality. In this kind of systems, the overall performance is intrinsically affected by the optical sensor. Therefore, it is necessary to review various non-contact measurement techniques, and evaluate each technique’s capability in measuring freeform surfaces.

2.2 Non-contact measurement techniques

2.2.1 Challenges in non-contact freeform surface measurements
Non-contact measurement techniques generally can be divided into two categories: point-based and area-based. Well-known point-based techniques include laser triangulation and confocal microscopy. Well-known area-based techniques include interferometry, Shack-Hartmann wavefront sensing (SHWS), moiré fringe projection, and focus variation. The detailed literature review of these techniques is addressed in the next chapter. When applying for freeform surface measurements, they all have certain disadvantages or limitations.

(i) **Difficulty in obtaining the measurement raw data**

This challenge is particularly true in area-based techniques. When the surface to be measured has big dynamic range, the captured image either has poor signal to noise ratio (SNR), or the correct correspondence between the image and the surface cannot be established. Hence, the real signals cannot be registered and the measurement raw data cannot be obtained.

In addition, some techniques need the reference wavefront to obtain the measurement raw data, such as interferometry. Some techniques measure the first order differentiation of the wavefront by comparing with the reference image, such as SHWS. The reference piece needs to be of similar form as that of the surface to be measured. In most cases, it is not practical, or even impossible, to have such kind of a reference piece.

(ii) **Measurement quality has poor consistency**
As freeform surfaces have varying curvatures, the resolution in the lateral direction, which refers to the plane perpendicular to the optical axis, of the area-based techniques is too limited to detect details of the surface. Also, while measurement data of high SNR may be obtained at some surface area, the measurement data from other areas have poor SNR. Therefore, the measurement quality has poor consistency that makes further processing of the data a big issue.

(iii) Long measurement time

Point-based techniques need lateral scanning to reconstruct the surface. Besides lateral scanning, some techniques also require vertical scanning to obtain the measurement data, such as confocal microscopy and focus variation. The mechanical scanning not only brings in errors, but also results in long measurement time. In addition, some area-based techniques also require mechanical scanning to achieve nanometer (nm) level accuracy, such as phase shifting interferometry. Mechanical scanning not only increases the measurement time, but also complicates the system (Hongtao et al., 2013).

2.2.2 Overview of non-contact measurement techniques

(i) Laser triangulation

Laser triangulation is primarily applied for 3D surface measurements when the speed is not the main concern (Dorsch et al., 1994). Its fundamental working principle is best illustrated in Figure 2.1.
A light beam from the laser source is focused onto the surface to be measured. The spot is imaged through a lens. The position of the spot image is determined and recorded by a position detector. The height information, $z$, is related to the spot position through Equation (2-1),

$$\Delta z = \frac{\Delta x}{\sin \theta} \quad (2-1)$$

where $\Delta x$ is the lateral shift value measured at the detector plane; $\theta$ is the angle of triangulation between the axis of illumination and the axis of observation. In this way, the height information is revealed through measuring the position of the spot in the position sensor. Although being a simple and robust sensing mechanism, besides the speed issue due to scanning, the shadow effect may occur when measuring freeform surfaces. Shadow effect refers to the situation where the image sensor fails to see the focal spot (Zeng et al., 1997).
Furthermore, to reduce the physical size of the measurement instrument, the distance between the laser source and the sensor is to be kept small. However, the distance is inherently limited by non-linearity to maintain the system’s accuracy, especially when measuring freeform surfaces (Clarke et al., 1991).

(ii) **Confocal microscopy**

Confocal microscopy can effectively measure surfaces at micro-scale (Hibbs, 2000, Hocken et al., 2005). Its fundamental working principle is best illustrated in Figure 2.2: A collimated illuminating beam, as formed by passing through a pinhole and a lens, is focused on the object surface through a microscopic objective lens. The
reflected rays are directed to another optical path by interacting with a beamsplitter along the way. In the redirected path, a pinhole placed after a lens system only let pass light from the plane of focus, which subsequently falls onto the detector, while blocking the rest. The microscopic objective lens is normally piezoelectric-driven to scan in the vertical direction. Images of different intensity are observed when the objective lens is of different vertical distance from the object surface. The brightest image is captured when the point of interest is at the focal plane.

By reading the vertical position feedback from the microscopic objective lens, the height information is revealed. In this way, the 3D profile is measured through scanning both vertically the piezoelectric-driven objective lens, and laterally the motorized stage. Systems built based on this principle for surface profile measurement can achieve near nanometer vertical resolution at each sampling position (Hocken et al., 2005) and sub-micron lateral resolution (Doia et al., 2007). Alternative methods have been developed to increase the acquisition speed such as the chromatic confocal microscopy. The main disadvantage of the confocal technique is the mechanical movement involved. Besides time consuming, the shear on the specimen brings errors in the measurement process (Claxton et al., 2006).

(iii) Interferometry

Interferometry is a well-known technology to measure wavefront. When two waves are coherent, they can interfere with each other.

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The resulting interference pattern, or termed as interferogram, reflects the optical path difference between the two waves, and the phase difference is, therefore, measured. Among various configurations, the Fiezau interferometer is the mostly used for its least number of components involved (Fang et al., 2013). Its fundamental working principle is best illustrated in Figure 2.3: A coherent light passes the reference flat surface and gets reflected at the test surface. The reflected light combines with the light portion reflected at the reference surface. When two coherent wavefront superimpose on each other, interference occurs and a fringe pattern is observed in the imaging system. The fringe pattern, or the interferogram, shows fringes of a constant phase difference between the two waves. Therefore, the interferogram is essentially a contour map of the deviation of the test surface as compared to the reference. The optical path difference (OPD), which is two times the height deviation, is related to the intensity of the interferogram according to Equation (2-2) (Hariharan, 2006).
\[ I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\alpha_1 - \alpha_2) \]  

(2-2)

where \( I_1 \) and \( I_2 \) represent the intensities of the two waves, \( \alpha_1 \) and \( \alpha_2 \) represent the phase of the two waves. The height information as relative to the reference surface is calculated by a phase-unwrapping process on the obtained interferogram.

The interferometry technique alone has limited accuracy and dynamic range (Leach, 2012). Complemented with the phase-shifting technique, the vertical accuracy of the so-called phase-shifting interferometer (PSI) is greatly increased. Today’s commercial product can easily achieve 3D topography surface measurement with 0.1\( \text{nm} \) vertical resolution, e.g., MicroXAM-100 from KLA Tencor and Verifire Series from Zygo. However, PSI can only measure surface flatness or sphericity by comparing with ideal reference lens. At the same time, it has random noise and is sensitive to environment condition because series of phase-shifted interferograms are taken at different times, which makes measurement vulnerable to vibration. Through the years, a lot of improvements have been made to widen its application. Some prominent advances include incorporating the computer generated holography (CGH) in the system, integrating with vertical scanning, and so on (Wyant, 2006). Although these advancements increase the measurement range and make testing of aspheric surfaces possible, they come with other drawbacks. CGH is expensive as the null lens has to be specially designed and fabricated for each specific surface shape (Wyant, 2006).
Also, the accurate alignment of CGH into the system is critically important (Zhao and Burge, 2013, Millerd et al., 2004). The vertical scanning interferometer (VSI) requires mechanical scanning in the vertical direction. Errors caused by mechanical movement are introduced. In addition, VSI measures different zones of a surface at different vertical positions, and the whole wavefront is only revealed after stitching. The more zones need to be stitched; the measurement results have larger error.

In the last decades, there are some new commercial instruments that are vibration insensitive, or are able to measure freeform surfaces without a physical CGH. The simultaneous PSI technique detects different phase-shifted interferograms at the
same time, so as to avoid temporal phase scanning (Brock et al., 2005). As illustrated in Figure 2.4, the beam, combined with the one from the reference and the one from the test mirror, passes through a holographic element. During this process, the combined beam is split into four separate beams, which then passes through a phase mask at four different segments. The four segments of the phase mask, which is placed just in front of the camera, introduces different phase shifts between the reference and test beams: 0°, 90°, 180°, and 270°. In this way, all four phase-shifted interferograms are captured at the same time (Brock et al., 2005). Since no phase scanning is involved, the system is insensitive to vibration (Kimbrough et al., 2006). However, there is a limit to the number of interferograms that can be split into, as the bigger the number is, the poorer the image contrast is. Furthermore, the lateral resolution is compromised as the four phase-shifted images are to be captured by one image detector. Consequently, the accuracy is limited.

In some interferometry instruments, a computer simulated virtual CGH is used to replace the physical one. The virtual CGH, which acts as a reference interferogram, is simulated based on the mathematical description of the surface form. After subtracting the reference from the captured interferogram, non-rotationally symmetrical surface measurements are transformed into flat surface measurements (Burge, 1995). While the system is more flexible in measuring surfaces of various forms, the pre-knowledge of the surface is needed. In addition, the surface can only have limited deviation compared to its mathematical description, as the subtracted interferogram is assumed as from flatness measurements.


Figure 2.5 Principle of the sensing mechanism of Shack-Hartmann wavefront sensor.

(iv) **Shack-Hartmann wavefront sensing (SHWS)**

SHWS has been widely adopted for aberration measurements and adaptive optics to facilitate system alignment. As illustrated in Figure 2.5, it uses a lenslet array to focus the incoming wavefront, either reflected or transmitted from the specimen. A detector captures the spot image at the focal plane. The centroid locations of the focal spots are compared with those of the reference spots, which are sampled from a plane wavefront. The centroid displacements are related to the wavefront slope, and the wavefront is revealed after reconstruction.

SHWS is comparable to interferometry in terms of accuracy, and has a much wider potential application because the sensitivity and measurement range are able to be adjusted in the system (Hartmann, 1900). In addition, SHWS is insensitive to vibration as all the environment-related noise is removed in the measurement process.
Furthermore, from its working principle, it is clear that the requirement of a coherent light is not necessary. However, if applying SHWS to measure freeform surfaces, there are issues such as sampling and referencing that make it incapable of carrying out the desired task. The detailed analysis is provided in Section 2.3.2.

(v) Moiré fringe projection

Since the past few decades, moiré fringe projection (Malacara, 2007, Reid et al., 1984) has been extensively used for surface form measurement in industry (Malacara, 2007). Its fundamental working principle is best illustrated in Figure 2.6: A projection unit projects a structured pattern, usually a sinusoidal fringe pattern, onto the object surface. The grating is phase modulated by the object height distribution, which then forms an image at the reference grating plane. The
reference grating and the phase-modulated grating interfere with each other. A number of moiré fringes are thus observed. Through a phase-unwrapping process on the moiré fringe image (such as Fourier transform method, phase stepping method, and spatial phase detection method) (Sai, 2010), the phase-modulated grating pattern at the object surface can be calculated. The surface shape is revealed after mapping the unwrapped phase distribution to surface coordinates. This technique is mainly used to measure surfaces with low reflectivity. Alternative methods have been developed to measure mirror-like surfaces by detecting the fringe reflection (Ritter and Hahn, 1983). Still, it is difficult for measuring surfaces with high reflectivity as image data of some areas are missing or unreliable. The accuracy depends on the density of fringes used. The higher the density is, the better the result will be. Nevertheless, the current accuracy in the vertical direction is at the level of tens micron meters.

(vi) Focus variation

Focus variation is a new technology developed in the last decade for surface metrology (Danzl et al., 2009). Its working principle is best illustrated in Figure 2.7: A focused light illuminates the object surface. The rays are then reflected back and received by an image sensor. At each lateral position, the captured images are of varying sharpness/contrast when the vertical distances between the objective lens and the specimen are different.
The height of the corresponding position on the surface is related to the vertical value when the observed image has the highest sharpness. 3D profile is measured by calculating the vertical value of best focus for all the positions through analyzing images taken at different planes (Danzl et al., 2009). Similar to confocal microscopy, vertical scanning by making use of piezoelectric elements is needed; although lateral scanning is not necessary. The measurement accuracy decreases with highly reflective surfaces, where on-focus evaluation is difficult as the sharpness has minimum variation within a wide range. This makes it difficult to be applied for reflective optical surface measurements.

2.2.3 Comparison of non-contact measurement techniques
A summary of the comparison for the techniques described above is shown in Table 2.1. As it is difficult to quantify the dynamic range of each technique due to its large dependency on the particular system configuration, the table firstly compares this value of each technique as determined by its intrinsic working principle (in the row of “Dynamic range”), and then gives the value for the listed product as an example (in the row of “Measurable range”). Some products do not specify their measurable range. This is under the assumption that the surface to be measured can be tilted, or the instrument can be shifted, in different orientations until the area of interest can be captured. In this case, “Nil” is indicated instead of a particular number. In Table 2.1, three stars indicate fully satisfactory; two stars indicate partially satisfactory; one star indicates least satisfactory.

To summarize, the area-based techniques measure the surface at one shot and an image is the raw data. Thus, they can measure a surface faster. However, they come with disadvantages such as the limited dynamic range, or the measurable slope. This means that only surfaces close to flat can be measured. Otherwise, they are beyond the system’s measuring capability. In addition, a reference, which acts like a datum to be compared with the surface to be measured, is required. Furthermore, the resolution in the lateral direction is limited as the surface is to be analyzed based on one image. The point-based techniques scan the whole surface to acquire sufficient data. This mechanism is much more flexible. Thus, when compared with the area-based techniques, they have a larger measurement range and lateral resolution.
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<th>Laser triangulation</th>
<th>Confocal microscopy</th>
<th>PSI</th>
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<td>nm-level accuracy</td>
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<td>Vertical scanning</td>
<td>No</td>
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<td>No</td>
<td>Yes</td>
<td>No</td>
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<td></td>
<td></td>
<td>Area</td>
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<td>Dynamic range</td>
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<td>Suitability for reflective</td>
<td>*</td>
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<tr>
<td>surface measurements</td>
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<td>Absolute reference</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
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<tr>
<td>Possibility for in-situ</td>
<td>***</td>
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<tr>
<td>measurement</td>
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</tr>
<tr>
<td>Example of commercial product</td>
<td>MICROTRAK 3 from MTI Instruments</td>
<td>LSM 710 from Carl Zeiss</td>
<td>Verifire Asphere from Zygo</td>
<td>CCI from Taylor Hobson</td>
<td>HASO3 128-GE from Imagine Optic</td>
<td>Quartz scanner 1200 DBE from Phase Vision</td>
<td>InfiniteFocus from Alicona</td>
</tr>
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<td>10µm</td>
<td>33.3°</td>
<td>±3°</td>
<td>Nil</td>
<td>85°</td>
</tr>
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However, they are slow because they measure only one or a few data points at a time. Slow techniques are very susceptible to measurement errors due to temperature variations and vibration during the measurement.

From the detailed comparison, SHWS technique stands out because of its simple system design and potential for in-situ measurement, while maintaining a high performance. There are still great challenges when use it for measuring freeform surfaces. The next section further investigates the working principle of SHWS, so as to guide the design and development of a new method and system that overcomes SHWS’s challenges while keeps its advantages.

2.3 SHWS: State of the art

2.3.1 Working principle of SHWS

2.3.1.1 Sensing mechanism

SHWS consists of a lenslet array and an image sensor, usually a charge coupled device (CCD). The lenslet array is usually an array of lenses of the same focal length as shown in Figure 2.8. As illustrated in Figure 2.9, the lenslet array samples the incoming wavefront, reflected or transmitted, onto a CCD. Reference focal spots are generated when the wavefront is reflected from a flat mirror. If the surface to be measured is not flat, tilt in the reflected or transmitted wavefront will happen, and this leads to a shift of focal spots’ location at the detector plane.
Figure 2.8 Example of a lenslet array used in the SHWS system (Jenoptik, Lyncee).

Figure 2.9 Principle and signal flow of SHWS.
The centroid location \((C_x, C_y)\) of each focal spot is calculated by Equation (2-3), where \(n\) denotes the total number of pixels within the window of interest, and \(I_{x,y}\) denotes the intensity value at pixel \((x, y)\).

\[
C_x = \frac{\sum_{x=0}^{n} \sum_{y=0}^{n} I_{x,y} X}{\sum_{x=0}^{n} \sum_{y=0}^{n} I_{x,y}}, \quad C_y = \frac{\sum_{x=0}^{n} \sum_{y=0}^{n} I_{x,y} Y}{\sum_{x=0}^{n} \sum_{y=0}^{n} I_{x,y}}
\]  

(2-3)

The centroid displacements between the reference focal spots and the test focal spots are measured. From Figure 2.10, the displacement in the \(x\)-direction divided by the focal length is equal to the average of the wavefront slope measured at the respective lenslet. This process is expressed by Equation (2-4), where \(\phi(x,y)\)
denotes the wavefront measured at each lenslet position, $f$ represents the focal length, and $\Delta x$ represents the centroid displacement in the $x$-direction.

\[
\frac{d\varphi(x, y)}{dx} = \frac{\Delta x}{f}
\]  

(2-4)

The left hand side (LHS) of the equation gives the first order derivative of the wavefront, or the wavefront slope. The relationship in the $y$-direction is likewise. Thus, through comparing the reference image and the test image, the wavefront slope matrix at the sampling plane is obtained. After a reconstruction process, the wavefront is revealed (Shack, 1971).

From the working principle, it is clear that the raw data needed for the wavefront sensing is the focal spot images of both the reference flat wavefront and the aberrated wavefront from the surface to be measured. After obtaining the two focal spot images, as indicated in Figure 2.9, each focal spot is registered into a respective window, within which the centroid position is to be calculated. Through a reconstruction step, the wavefront is revealed. The whole measurement process is illustrated in the flowchart, as shown in Figure 2.11.

2.3.1.2 Focal spots windowing

The first step to process the captured image is to separate focal spots and register them into respective windows.
Figure 2.11 Flowchart of the measurement process of the SHWS system.

Figure 2.12 Focal spots image processed by the uniform windowing method.
The process of separation and registration of windows is referred to as windowing. The basic criteria for windowing is to enclose the complete focal spot, but excluding pixels belong to neighboring focal spots. As the distribution of focal spots is correlated to the surface sampling condition, the distribution of the windows on the image is correlated to the surface sampling condition as well.

The focal spot windowing is conducted by defining a uniform rectangular grid. This uniform windowing method is straightforward, but needs the estimation of the offset, the pitch between neighboring windows, and the size of the window, as indicated in Figure 2.12. The estimation process may involve several rounds of trial-and-error. Also, the uniform windowing method is rigid as each window has a fixed shape and size. In addition, the focal spots cannot deviate much from a uniform distribution. Otherwise, the uniform windowing method is difficult or even impossible to be applied, which frequently happens when analyzing images from freeform surfaces. So far, there is no research work on this topic reported.

2.3.1.3 Centroid finding

After focal spots are windowed, centroid locations are to be calculated for obtaining the wavefront slopes at the sampling aperture. This process is referred to as centroiding. Because the centroid displacement is the raw data for further processing, the system performance is significantly dependent on the technique used to determine the centroid positions (Xia and Ma, 2010). The most commonly applied method is to use the first moment algorithm as described by Equation (2-3).
The centroid accuracy calculated by the first moment algorithm is not satisfactory for practical measurements because it is severely affected by background and photon noise (Vyas et al., 2009a, Vyas et al., 2009b), especially if the signal spot is smaller than the sub-aperture area (Xia and Ma, 2010, Vyas et al., 2009a, Vyas et al., 2009b). Supplementary techniques have been introduced to significantly improve the first moment algorithm’s detection accuracy, such as thresholding, high powering, sub-windowing, and iteration (Vyas et al., 2009a, Vyas et al., 2009b, Jiang et al., 2005, Yin et al., 2010). Thresholding excludes the contribution of low-intensity pixels, which are identified as noise. High powering is elaborated by Equation (2-5), which increases the SNR. $\alpha$ indicates the power increased to.

$$I_{\text{new}} = I_{x,y}^{\alpha}$$ (2-5)

As illustrated in Figure 2.13, sub-windowing is conducted in such a way that after the basic window is defined, shift the window to make the calculated centroid $P_1$ at
the new window’s center position. The centroid is re-calculated inside the new window, which is denoted as \( P_2 \). *Iteration* is normally implemented together with sub-windowing, as illustrated in Figure 2.13. It is applied until the difference between the centroid positions calculated in the window before and after shifting is less than a certain value, which is set as **two pixels** in this dissertation.

These techniques were mostly studied on computer simulations of flat or spherical surfaces, and used a global threshold value. Therefore, the conclusions derived may not be valid for freeform surface measurements. Moreover, an appropriate threshold value is dependent on the image characteristics, such as the brightness and contrast, and the characteristics of an image from a freeform surface usually vary over the entire area. In this case, a pre-determined global threshold value is nearly impossible to identify.

2.3.1.4 Wavefront reconstruction

With the measured centroid displacements between the reference focal spots and the test focal spots, wavefront reconstruction is to be performed. From the mathematical point of view, with known first order derivatives, the original form can be obtained through integration. Similarly, in SHWS, the wavefront slope, which is in analogy to the first order derivative, is reconstructed to the wavefront, which is in analogy to the original surface function (Southwell, 1980).
There are many reconstruction algorithms available. Some of the most commonly used methods include Southwell algorithm (Southwell, 1980) and polynomial fitting (Malacara, 2007). Southwell (Southwell, 1980) assumes that the wavefront slopes are linear and the wavefront form is continuous. Take the $x$-direction as an example. The wavefront dependence between grid points is represented by Equation (2-6).

$$\varphi = a_0 + a_1 x + a_2 x^2$$  \hspace{1cm} (2-6)

where $a_0$, $a_1$, and $a_2$ are the coefficients. As illustrated in Figure 2.14, the wavefront value at each grid point, $\varphi_{i,j}$, is related to the slope and wavefront values at its four neighboring grids, $\varphi_{i-1,j}$, $\varphi_{i,j-1}$, $\varphi_{i+1,j}$, and $\varphi_{i,j+1}$, as Equation (2-7) (Southwell, 1980), where $S_{i,j}^x$ denotes the slope in the $x$-direction and $S_{i,j}^y$ denotes

Figure 2.14 Wavefront reconstruction model by using Southwell algorithm.
the slope in the $y$-direction, $p$ denotes the pitch distance between the neighboring two grid points.

$$\frac{S_{i+1,j}^x + S_{i,j}^x}{2} \phi_{i+1,j} - \phi_{i,j}$$
$$\frac{S_{i-1,j}^x + S_{i,j}^x}{2} \phi_{i-1,j} - \phi_{i,j}$$
$$\frac{S_{i,j+1}^y + S_{i,j}^y}{2} \phi_{i,j+1} - \phi_{i,j}$$
$$\frac{S_{i,j-1}^y + S_{i,j}^y}{2} \phi_{i,j-1} - \phi_{i,j}$$

(2-7)

To solve for $\phi$, the matrix iterative method can be applied to Equation (2-7).

Equation (2-8) is obtained to describe $\phi$:

$$\phi_{i,j} = \overline{\phi}_{i,j} + b_{i,j} / g_{i,j}$$

(2-8)

where $\overline{\phi}_{i,j}$ is averaging wavefront value of the neighboring grid points calculated as

$$\overline{\phi}_{i,j} = (\phi_{i-1,j} + \phi_{i+1,j} + \phi_{i,j-1} + \phi_{i,j+1}) / g_{i,j}$$

and

$$b_{i,j} = (S_{i-1,j}^x - S_{i+1,j}^x + S_{i,j-1}^y - S_{i,j+1}^y) / g_{i,j}$$

$\phi_{i,j}$ is a variable that equals to 2 if the grid point of interest is at four corners; equals to 3 if the grid point of interest is at edges; equals to 4 if elsewhere. Then the successive over relaxation (SOR) method as the iterative technique is applied to Equation (2-8) due to its high convergence speed (Young, 1972), and Equation (2-9) is obtained as following.
\[
\phi_{i,j}^{m+1} = \phi_{i,j}^m + \omega \left( \phi_{i,j}^m + \frac{f_{i,j}}{g_{i,j}} - \phi_{i,j}^m \right)
\] (2-9)

where \( \omega = \frac{2}{1 + \sin \left( \frac{\pi}{N+1} \right)} \), \( m \) is the index number of iterations, and \( N \) refers to the total number of grid points.

The basic procedure of polynomial fitting is to use a polynomial fit to express the measured wavefront slopes. Thus, rather than integrating over the discrete data points to retrieve the wavefront, this method integrates over a polynomial (Comejo and Malacara, 1976). For example, assumes the wavefront is represented by a linear combination of a polynomials as Equation (2-10),

\[
\varphi = \sum_{n=0}^{N} A_n f_n
\] (2-10)

where \( N \) is the number of polynomials, \( f_n \), used and \( A_n \) are the coefficients. Normally, the polynomials are configured in such a way that the coefficients are directly related to optical aberration terms, such as coma and astigmatism (Goodwin, 2006). The wavefront slopes are related to the coefficients as

\[
S_{i,j}^x = \sum_{n=0}^{N} A_n \frac{\partial f_n}{\partial x} \quad S_{i,j}^y = \sum_{n=0}^{N} A_n \frac{\partial f_n}{\partial y}
\] (2-11)
The least square fitting of these analytical functions to the measured wavefront is then conducted on Equation (2-11).

The two methods have their own advantages and disadvantages. Basically, Southwell algorithm can accurately retrieve the wavefront to be measured, although numerical integration involved in this process introduces an accumulation of inherent errors. Polynomial fitting does not involve numerical integration. Therefore, the relevant errors are excluded. However, the fitting itself brings in errors when smoothing the data or introducing oscillations. Thus, this method generally is less reliable compared to Southwell algorithm, especially at the surface edges (Malacara, 2007). Furthermore, the result obtained through Southwell algorithm directly describes the form of the wavefront, while the result obtained through polynomial fitting reflects the optical features.

There are other methods to satisfy specific requirements in various application scenarios. For example, two-dimensional Fast Fourier Transform (FFT) algorithm (Freischlad and Koliopoulos, 1986, Poyneer et al., 2002, Poyneer et al., 2003) is implemented when low sensitivity to noise in the data is crucial. Two-dimensional cubic spline function (Ahlberg et al., 1967, Groening et al., 2000) is recommended for measuring a strongly aspheric wavefront as it assumes curves instead of straight lines between two neighboring points. Multigrid methods are proposed when real time is needed, or the size of the measured data is large, due to their fast convergence speed (Gilles et al., 2002, Ellerbroek et al., 2003, Vogel and Yang,
2006). However, these methods only have satisfactory performance in certain scenarios. Their accuracy would downgrade dramatically if several conditions are mixed together. For example, if the data of the aspheric wavefront has low SNR, cubic spline fitting has poor performance (Ares and Royo, 2006). Also, these methods are targeted for particular applications. For example, the two-dimensional cubic spline only works when the centroid displacement is measured with the ideal aspheric wavefront as the reference (Groening et al., 2000). Hence, these methods generally lack of universality. Therefore, integration and fitting are still the dominant wavefront reconstruction approaches.

2.3.2 Issues in freeform surface measurements

SHWS is used for flatness and aberration measurements, optical alignment, and adaptive optics where the wavefront to be measured has limited deviation from flat. If applied on freeform surface measurements, issues of sampling, referencing, and lateral resolution will occur.

2.3.2.1 Sampling

Sampling in the SHWS refers to use the lenslet array to focus the wavefront and generates the focal spot image. First of all, when the wavefront has large dynamic range and varying curvature, no good focal spots image can be obtained for the whole field of view. Even some spots can be well-focused; others are of bad quality due to the fixed focal length of the sampling aperture.
If wavefront is not properly focused, the measurement will carry large error, which is studied and demonstrated in the next chapter.

Secondly, the physical presence of the lenslet array makes the setup rigid, which brings in the risk of crosstalk, as illustrated in Figure 2.15. Crosstalk makes the correct focal spots registration difficult, and consequently brings error in the wavefront slope calculation.

Thirdly, to have a high lateral resolution, the configuration of the array needs to be large. For a fixed area to be measured, a large number of lenslets is desired. However, a large configuration indicates that the allowable displacement of each focal spot at CCD is small. As a result, the measurable range is limited. In other words, there is a trade-off between the measurement range and sensitivity.
Lastly, the focal length acts like a lever, which amplifies the displacement at the sampling aperture and makes it identifiable at CCD. If the wavefront slope is large, the focal length has to be short so that the focal spot displacement will not exceed the allowable area. However, this implies that small wavefront slope under this circumstance will not be detected at the CCD. As a consequence, the sensitivity is poor.

In a word, for freeform surface measurements, the lenslet array as the sampling aperture makes the system configuration complicated to achieve the measurement performance. In most applications, it is hard to make a suitable choice.

Wavefront sensing with a random screen proposed by Loktev, M. et al. (Loktev et al., 2007) is a good attempt to tackle with the sampling issue. As illustrated in Figure 2.16, the lenslet array is replaced with a screen of arbitrarily modulated intensity and/or phase. The wavefront is then developed into a random intensity

Figure 2.16 Wavefront sensing with a random screen (Loktev et al., 2007).
pattern through the screen, and then recorded by an image sensor. Once the screen is chosen and its structure is known, the corresponding phase modulation and intensity modulation can be calculated. Hence, ideally, the wavefront to be measured can be directly traced back based on the intensity distribution captured by the image sensor. Except this, it works similar as SHWS: two images are taken; one from the reference flat surface and the other from the specimen. By comparing the two images, wavefront slopes are obtained.

Although this system does not involve the lenslet array as the sampling aperture, the screen cannot provide a big measurement range because of its limited intensity levels and phase levels. Therefore, this method could not solve the sampling issue completely.

2.3.2.2 Reference

From the working principle of SHWS, a reference image is a necessary component for the wavefront slope calculation. The reference image is captured by sampling the reference wavefront, which is reflected from a flat mirror, in the system. The need for a reference image causes problems in many application scenarios.

First of all, the surface to be measured needs to be of similar form to the reference. For surfaces that only have a small discrepancy from flat, or the so-called near-flat shape, a flat mirror is a proper reference piece. However, for surfaces that come with a shape far beyond flat, such as aspheric or freeform, it is very difficult, or even impossible to use the flat mirror as a reference, as a well-focused reference
image cannot be obtained by the sampling aperture designed and configured for sampling the freeform surface. Hence, this requirement limits the application of the SHWS system.

Secondly, the centroid position is an intensity-dependent data, which is affected not only by the surface form, but also by the optical properties of the surface, such as reflectivity. If two surfaces with the same profile but different reflectivity distribution, the measured data through wavefront sensing will be the reflectivity difference between the two surfaces, which is part of the form measurement error. Therefore, the difference of the optical properties between the reference and test surface should be kept minimum, so that what is measured is only the surface profile. As a result, this requirement limits the availability of a suitable reference.

Thirdly, the repositioning of the two pieces exactly at the same spatial position, including the angular orientation, is important and still difficult to achieve. In cases of measuring non-near-flat surfaces, the accuracy relies highly on the exact matching of the areas corresponding to each sampling aperture. Mismatch will greatly degrade the system’s performance.

Fourthly, although normally the reference image only needs to be taken periodically (depend on the stability of the light source (Zhong et al., 2008)), if there are some system changes, the reference image must be taken again. The maintenance and operation with an updated and valid reference is troublesome.
Last but not least, for some applications, a suitable reference is not available or even applicable (Jobling and Kwiat, 2010).

There are some new developments and extensions of the SHWS system to tackle with this reference issue. One of the successful attempts was done by Zou et al. (Zou and Rolland, 2005, Zou and Rolland, 24 June 2008, Zou et al., 2008). In the system they developed, which is called as “Differential Shack-Hartmann Curvature Sensor” (DSHCS), the aperture is moved a known differential distance in the x- and y- directions. The slopes at each grid point before and after the differential movement are measured. In this way, the slope differential, or curvatures of the wavefront in the x- and y- directions are obtained, as illustrated in Figure 2.17.

Figure 2.17 Illustration of the working principle of DSHCS (Zou and Rolland, 2005).
Figure 2.18 (a) Schematic illustration of the system setup of DSHCS. (b) Actual experiment setup of DSHCS (Zou and Rolland, 2005).
During the measurement, as shown in Figure 2.18, the wavefront is split into three channels: z-direction, x-direction, and y-direction. Lenslet array A, B and C are put in the same paths of the three channels separately. By moving A a differential distance in the x-direction, and B a differential distance in the y-direction, the coordinates of the grid points generated by A, B and C reveal the wavefront curvatures in the x- and y- directions respectively.

DSHCS aims at obtaining the absolute curvature of the wavefront at each grid point. It needs three channels, three sampling apertures and three CCDs in the system so as to generate the desired measurement data. An ideal flat is needed for marking the reference centroid positions. The absolute curvature is determined by measuring the centroid positions before and after the differential movement, with respect to the previously marked reference positions.

Another outstanding attempt is done by Jobling and Kwiat (Jobling and Kwiat, 2010). They developed a system named as “Single-Arm Virtual-Interferometer” (SAVI), in which a lateral displacement is mechanically introduced to the surface to be measured, and in this way two images are obtained. The displacement is equal to the pitch of the aperture. The sensor mechanism can be illustrated by Figure 2.19. The operating steps are summarized in Figure 2.20. The centroid displacement matrix obtained from SAVI is the residual surface gradient, i.e., the slope of the residual shape after subtracting a basis shape, which is the largest magnitude feature of the surface.
The residual shape is thus a small variation around flat. The Southwell algorithm is then applied to these residual surface gradients, which gives the residual shape. The final shape is the summation of the basis shape and the reconstructed residual shape. From the working principle, it is clear that the surfaces can be measured should not deviate much from the pre-determined basis shapes. Otherwise, the system cannot work effectively, or even is not feasible. Therefore, although no reference image is used to calculate the centroid displacement matrix, the system
cannot be applied when there is no knowledge of the accurate mathematical description of the surfaces to be measured.

2.3.2.3 Lateral resolution

Within a certain area to be measured, the more sampling points, the better the lateral resolution. In the SHWS system, each local sampling aperture acts like an optical “probe”. The probe array samples the surface and each probe carries the information of the surface’s respective local area. In order to generate more sampling points so as to increase the lateral resolution, more probes are desired to sample a surface. However, due to the following reasons, there is a limitation to the maximum number of lenslets that can be used in the system.

First of all, an increase in the number of probes indicates a decrease in the diameter of each lenslet. However, the size of the lenslet cannot be reduced infinitely due to engineering capability. Currently, the smallest lenslet commercially available has a pitch size of 100µm, manufactured by OKO Optical.

Secondly, the accuracy of SHWS is affected by the focal spot centroid finding process (López and Ríos, 2010). Generally speaking, for a fixed focal length, bigger the lenslet diameter, smaller the focal spot, and thus higher the detect-ability (Ríos and López, 2009). As such, it is desirable to increase the diameter of the lenslet, so as to reduce the spot size (Rocktaschel and Tiziani, 2002).
Thirdly, large focal spots have higher chance of overlapping with each other, which makes correlation of each spot with its corresponding lenslet ambiguous (Díaz-Uribe et al., 2009). Therefore, it is also desirable to increase the diameter of the lenslet for a fixed focal length, so as to increase the measurement range of the system.

Last but not least, when the SHWS is applied to measure highly aberrated wavefront, the number of lenslets is further limited so as to provide a large measurement range (Schmitt et al., 2009). In other words, crosstalk is more likely to happen when the number of focal spots is larger (Vasyl, 2004), which makes correct registration of each focal spot impossible.

Therefore, as described above, there is a trade-off between the high lateral resolution achieved by increasing the number of lenslets and applying of the SHWS system to measure surfaces with large dynamic range. Scanning was hence proposed to increase the lateral resolution without increasing the number of lenslets. Generally, they can be categorized as sub-aperture scanning and whole-aperture scanning.

Sub-aperture scanning refers to the situation that not all probes are activated at a time. Some are switched on while others are switched off. Through scanning, each probe is selectively activated in a predesigned manner. This method is firstly applied in Hartmann Wavefront Sensing (HWS) system (Platt and Shack, 2001), based on which SHWS was developed.
Figure 2.21 Sub-aperture scanning in HWS by making use of a moveable aperture (Wells and Myrick, 2003).

HWS uses holes as probes, rather than lenslets. Other than that, the two systems are the same. The extreme way of applying sub-scanning is to scan a screen with only a single aperture, or a single hole, as illustrated in Figure 2.21. The aperture goes around the surface by making use of a motorized XY-stage (Wells and Myrick, 2003). The improvements of the system compared with the SHWS include the followings.

a) The total number of sampling points can be tailored to satisfy the specific requirement of different applications.

b) The measurement range is large, as in each image the focal spot can go to anywhere at the detector plane without causing any registration problem.

c) The measurement range is increased without any trade-off in sensitivity.

The main drawback of the system besides mechanical vibration that increases measurement error, is time consuming, as one aperture is required to scan the whole area of the surface.
Another system which avoids mechanical vibration was developed. The main idea is to use liquid crystal, or any other programmable device, to act as the sampling aperture (Olivier et al., 2000). The small hole is programmed to scan around the whole area of the surface. Besides the improvements elaborated in the previous system, this setup also features flexible probe design. The focal length and size of the hole can be easily modified according to the requirements of different applications.

Sub-aperture scanning has also been applied in the SHWS system, but with a different mechanism. One typical way is to add a mask directly in front of the lenslet array. This mask is usually non-transparent with holes of the size of the lenslets, and arranged in the same pattern with that of the lenslet array. The pitch of the holes is several times bigger than the lenslet array. Hence, while some lenslets
actively sample the corresponding areas with the light beam passing through, others remain inactivated as the light beam is blocked by the mask. Several consecutive positions of the mask achieved through scanning give the complete information of all the lenslets. Some systems implement this method by making use of a translatable plate as shown in Figure 2.22 (Yoon et al., 2006), and some make use of a programmable mask, such as a spatial light modulator (SLM) (Lindlein et al., 2001). Both kinds of systems have the advantage of increased measurement range with no trade-off in sensitivity or lateral resolution. Systems with a SLM also feature flexible probe design and no mechanical moving parts. However, both kinds of systems have limited lateral resolution as the total number of sampling points is constrained by the lenslet array configuration.

Whole-aperture scanning refers to the situation that all probes are switched on, and scanning is achieved through varying the lateral position of the sampling plane. This method has already been applied in HWS. Basically, there are two modes of scanning discussed: one is rotating a screen with holes arranged along radial lines, as shown in Figure 2.23 (Díaz-Uribe et al., 2009, Liu et al., 2006); the other is linearly translating a screen with holes arranged in a square array, as shown in Figure 2.24 (Díaz-Uribe et al., 2009). Through scanning, sampling points are easily increased to some hundreds or even to more than one thousand. The improvement in lateral resolution is proven in experiments. Besides the long test time induced by the mechanical movement, the main drawback is the error generated in assigning the hole positions introduced additionally in the process of scanning.
As the accuracy of the system relies highly on the identification of the sampling points to their corresponding area in each scanning step, the performance of the system is downgraded. So far, little research work has been reported that address how to apply whole-aperture scanning in the SHWS system.

2.3.2.4 Signal to noise ratio (SNR)

If the wavefront is freeform, the sampled focal spot image would have a big deviation from uniform distribution. Also, the focal spots are of inconsistent SNR. These two characteristics make further processing of the raw images difficult, or even impossible. The uniform windowing method cannot register every focal spots in one step. The centroid finding algorithms with a global threshold have changing effectiveness depending on the SNR of each particular focal spot.
2.4 Summary

Compared to other non-contact measurement techniques, SHWS stands out for its simple system design and potential for in-situ measurement, while maintaining a high performance. With a further investigation of its working principle, three main challenges in applying the system for freeform surface measurements are identified, being sampling, reference, and lateral resolution. Besides these three main issues, the data processing algorithms involved, being focal spot windowing, centroid finding, and reconstruction, either increase error or not feasible in dealing with captured raw images, due to the poor and varying SNR. The next chapter describes the devised and developed Reference-Free Beam-Sampling methodology and system, through which freeform surfaces can be measured with nm-level accuracy.
Chapter 3 Reference-Free Beam-Sampling Methodology and System

To achieve area-based nm-level freeform surface measurements, this dissertation devised and developed the novel Reference-Free Beam-Sampling (RFBS) methodology and system. In the system, the beam-sampling technique is invented (Chapter 3) to sample the surface. Through the designed reference-free technique (Chapter 4), second order derivatives of the surface are obtained, which are calculated from the raw images with the proposed dynamic windowing technique and adaptive centroiding technique (Chapter 6). The digital scanning technique (Chapter 5) is developed and applied in the sensing mechanism to achieve high accuracy. Through applying the designed 3D reconstruction technique (Chapter 7) on the second order derivatives, freeform surfaces are measured. This chapter firstly demonstrates the SHWS system’s deficiency in measuring freeform surfaces. Then, an overview of the RFBS methodology and system for freeform surface measurements is provided. Both simulation and experiment studies are presented to prove its feasibility and correctness.

3.1 Digital SHWS system

From the literature review, the SHWS system cannot be applied for freeform surface measurements.
This is mainly because of the sampling issue as no well-focused spot images can be obtained. To demonstrate this observation, a digital SHWS system was developed in the lab. The digital SHWS system uses the spatial light modulator (SLM), a programmable liquid crystal device as diffractive optical lenses (DOLs). Each pixel is assigned a specific gray level according to the phase function of a diffractive lens. A pattern as shown in Figure 3.1 displays on the device to add an additional phase distribution to the beam passing through. As such, the phase of the incoming beam is modulated, and the outgoing beam is diffracted as if passed by a DOL array (Zhao et al., 2006). Compared to the SHWS system where a physical lenslet array is used, the digital SHWS system has flexible sampling aperture design, without any lead time. After passing through SLM, the phase of the light beam is modulated spatially (has the similar effect as adding a refractive index), which then reaches the imaging plane and captured by CCD (Zhao et al., 2006).
Figure 3.2 Schematic of the experiment setup of the digital SHWS system.

Figure 3.3 Experiment setup of the digital SHWS system.

The schematic drawing of the system setup is shown in Figure 3.2, and Figure 3.3 shows the actual experiment setup. The specifications of the key components are listed in Table 3.1.
Table 3.1 Specification of key components used in the digital SHWS system.

<table>
<thead>
<tr>
<th>Spatial Light Modulator (SLM)</th>
<th>LC2002 from Holoeye; 600×800 pixels; Pixel size of 32µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCD</td>
<td>AM-1600CL; 4872×3248 pixels; Pixel size of 7.4µm</td>
</tr>
</tbody>
</table>

A toroidal mirror as described by Equation (1-1) was measured in the digital SHWS system. The radius in the two orthogonal directions is 137 mm and 112.5 mm respectively. The focal length of the lenslet array is designed to be 50 mm. The captured focal spot image is shown in Figure 3.4. From the zoomed-in view of one focal spot, the low focusing quality is observed. Since no reference image is available for the slope measurement, the designed reference-free technique was applied to obtain the second order derivatives. The detailed discussion of the reference-free technique is addressed in Chapter 4. The final reconstructed surface is shown in Figure 3.5. The benchmarked peak-to-valley (PV) value of the measured area is calculated to be 0.214 mm. The root-mean-square error (RMSE) of the reconstructed surface as compared to the nominal one is 0.2062 µm. Sub-µm level accuracy can only be achieved when using the digital SHWS system to measure toroidal mirrors of sub-mm PV with the reference-free technique. The limitation of the SHWS in freeform surface measurements is thus demonstrated.
Figure 3.4 The focal spot image captured in the digital SHWS system.

Figure 3.5 (a) Reconstructed surface of the toroidal mirror in the digital SHWS system (b) The corresponding error map as compared to the nominal surface.
3.2 RFBS methodology and system

The key sensing mechanism of the RFBS methodology is the beam-sampling technique: the lighting beam is used as the optical “probe” to sample the surface to be measured. When incident on the surface, the probe responds to the profile of the sampled area and the propagation direction of the reflected beam is changed accordingly. The reflected beam therefore carries the surface profile information and is subsequently received by an image sensor. The distorted spot received on the image sensor is a sampled result of the surface profile, which is extracted by analyzing the distorted spot’s intensity distribution. From the working principle, it is clear that the surface profile changes the propagation direction of the beam, which is represented by the intensity distribution of the received image. Therefore, proper sampling is not related to the focusing quality. Hence, the sampling issue is addressed by the beam-sampling technique.

As illustrated in Figure 3.6, to achieve area-based measurement, a modulated beam, which consists of a matrix of beamlets, is designed. The received image would then be a matrix of distorted spots. Each spot is registered by the dynamic windowing technique, and its centroid position is calculated by the adaptive centroiding technique, which is related to the averaged slope of the sampled area. A small lateral disturbance is then introduced to the modulated beam. The centroid position of the newly received spot at the image sensor is also calculated.
Through measuring the centroid displacements of the spots received at the image sensor before and after the lateral disturbance, the second order derivatives of the sampled area in the shifting direction can be obtained (Zhao et al., 2011). The surface is calculated by applying the 3D reconstruction technique.

The key element of the RFBS methodology is the modulated beam. The simplest design of the modulated beam is a matrix of circular beamlets.
Figure 3.8 Setup of the RFBS system.

This is achieved through letting a collimated beam pass through a mask, which consists of a matrix of circular holes, to allow light pass through, while blocking the remaining areas, as illustrated in Figure 3.7.

The configuration of the developed RFBS system is as shown in Figure 3.8. The beam modulation is achieved through adding a light modulator and a polarizer after the collimated beam. Same as SLM, this particular model of light modulator (Figure 3.9) is intrinsically a liquid crystal device (HOLOEYE), which can be programmed to modulate the beam. Also, it is flexible as various settings of modulation can be programmed. However, it only provides phase modulation.
A polarizer is therefore placed directly after the light modulator to convert phase modulation to intensity modulation. This is because the polarizer only allows light wave of a certain phase to pass through, while blocking the others. This means that the output from the polarizer would be light wave of the same phase, but varying amplitude depending on the polarizer angle. In this way, phase modulation is converted to intensity modulation.

3.3 Surfaces measured in the RFBS system

To demonstrate the correctness of the RFBS methodology and its feasibility in measuring freeform surfaces, three kinds of surfaces are measured in the RFBS system for different purposes:

1) A spherical mirror with focal length of 203.2\(\text{mm}\) and diameter 76.2\(\text{mm}\), as shown in Figure 3.10(a). This surface has been used as a benchmark to demonstrate the RFBS system’s feasibility in measuring traditional spherical surfaces with \(\text{nm}\) level accuracy.
1) A freeform surface with polynomial coefficients up to 10th order and having tens-μm level PV, as shown in Figure 3.10(b). Although being of varying curvatures, a flat mirror is still applicable as the reference piece in this situation. This surface has also been measured in the SHWS system to compare the performance of the SHWS system and the RFBS system.

2) A toroidal mirror with radius of 112.5mm in the x-direction, and 137mm in the y-direction, as shown in Figure 1.4. As currently there is no well-recognized standard freeform piece, toroidal mirrors are chosen in this dissertation to demonstrate the RFBS system’s capability in measuring freeform surfaces for the following four reasons:

   a. Toroidal mirrors are non-rotationally symmetric due to different radius in the two orthogonal directions. They are a special kind of
freeform surface while having symmetry in the $x$- and $y$-direction respectively.

b. Large errors resulted when measure toroidal mirrors in the SHWS system, as demonstrated in Section 3.1.

c. A universal issue in freeform surface measurement is the exact matching of the surface coordinate and the measurement system’s coordinate. It is difficult to assess a system’s performance as the final error is a mixture of coordinate matching and the system’s intrinsic accuracy. The coordinate matching error is minimized when measuring toroidal mirrors because of the symmetry in the $x$- and $y$-direction respectively.

d. Toroidal surfaces have wide applications in optics and manufacturing fields. It is of significant industrial values to measure their profiles.

For the spherical mirror and the toroidal mirror, the theoretically calculated profile is used as the nominal surface. The freeform surface, which was manufactured by the diamond turning technology with Equation (3-1) as its designed formula, was measured using Ultra Accuracy 3-D Profilometer (UA3P) from Panasonic, a probing-based commercial product. The measurement result as shown in Figure 3.11(a) is used as the nominal surface for comparison in experiment.
Figure 3.11 (a) UA3P direct measurement result. (b) An alternative view of the measurement data.
As can be seen from Figure 3.11(b), the sample form is changing in the $x$-direction, so the average of the results in the $y$-direction is also plotted. It is referred to as a freeform polynomial in this dissertation.

$$\varphi = 0.016 - 4.5372 \times 10^{-4} x^2 + 6.181 \times 10^{-6} x^4 - 3.018 \times 10^{-8} x^6$$
$$+ 5.9494 \times 10^{-11} x^8 - 4.0408 \times 10^{-14} x^{10}$$

(3-1)

### 3.4 Simulation verification of the RFBS methodology and system

The simulation platform of the RFBS system as shown in Figure 3.12 was developed in the Matlab environment. The modulated beam incidents on the surface and then reflected back, which subsequently reaches onto the CCD plane. The light propagation of each ray in the platform till intersection with the CCD plane is calculated and simulated by optical ray tracing.
Table 3.2 System settings in measuring three surfaces.

<table>
<thead>
<tr>
<th></th>
<th>Spherical mirror</th>
<th>Freeform polynomial</th>
<th>Toroidal mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (mm)</td>
<td>1.28</td>
<td>1.28</td>
<td>1.152</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>0.448</td>
<td>0.448</td>
<td>0.448</td>
</tr>
<tr>
<td>CCD position (mm)</td>
<td>130</td>
<td>145</td>
<td>145</td>
</tr>
<tr>
<td>Digital scanning step</td>
<td>4</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Sampling points matrix</td>
<td>78×50</td>
<td>103×20</td>
<td>100×88</td>
</tr>
<tr>
<td>Measurement area (mm²)</td>
<td>24.64×15.68</td>
<td>16.31×11.52</td>
<td>19.008×16.704</td>
</tr>
</tbody>
</table>

The modulated beam has the same design as Figure 3.7. Various parameters of the system in measuring the three surfaces are listed in Table 3.2. Pitch refers to the centre distance between the neighboring beamlets. Diameter refers to the size of the beamlet. CCD position refers to the distance between the CCD plane and the surface. The reconstructed surfaces are shown in Figure 3.13, and the measurement results are summarized in Table 3.3. From the results, the correctness and feasibility of the RFBS methodology and system has been proven. nm-level accuracy is achieved in measuring freeform surfaces of mm-level PV.
Figure 3.13 (a) Simulation result of the spherical mirror. (b) Simulation result of the freeform polynomial. (c) Simulation result of the toroidal mirror.
Table 3.3 Simulation results of measuring three surfaces in the RFBS system.

<table>
<thead>
<tr>
<th></th>
<th>Spherical mirror</th>
<th>Freeform polynomial</th>
<th>Toroidal mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmarked PV((mm))</td>
<td>1.0533</td>
<td>0.0101</td>
<td>1.3227</td>
</tr>
<tr>
<td>Measured PV((mm))</td>
<td>1.0533</td>
<td>0.0101</td>
<td>1.3227</td>
</tr>
<tr>
<td>RMSE ((nm))</td>
<td>2.9</td>
<td>8.1</td>
<td>4.6</td>
</tr>
</tbody>
</table>

3.5 Experiment verification of the RFBS methodology and system

In the RFBS system, as shown in Figure 3.8, the system configurations in measuring the three surfaces are the same as their respective simulation setting summarized in Table 3.2. The received images are shown in Figure 3.14. The reconstructed surfaces are shown in Figure 3.15. The measurement results are summarized in Table 3.4. The experiment results also demonstrate the correctness and feasibility of the RFBS methodology and system in measuring freeform surfaces of \(mm\)-level PV, while achieving \(nm\)-level accuracy.

3.6 Summary

This chapter describes the reference-free beam-sampling methodology and system for measuring freeform surfaces. By making use of a light modulator to generate a specially modulated lighting, the beam itself samples the surface. Second order derivatives are measured through the reference-free technique. Digital scanning is applied to enhance measurement accuracy.
Figure 3.14 (a) Image of the spherical mirror received in the RFBS system. (b) Image of the freeform polynomial. The rectangular box indicates area of interest. (c) Image of the toroidal mirror.
Figure 3.15 (a) Experiment result of the spherical mirror measured in the RFBS system. (b) Experiment result of the freeform polynomial. (c) Experiment result of the toroidal mirror.
Table 3.4 Experiment results of measuring three surfaces in the RFBS system.

<table>
<thead>
<tr>
<th></th>
<th>Spherical mirror</th>
<th>Freeform polynomial</th>
<th>Toroidal mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmarked PV (mm)</td>
<td>1.0533</td>
<td>0.0155</td>
<td>1.3227</td>
</tr>
<tr>
<td>Measured PV (mm)</td>
<td>1.0523</td>
<td>0.0152</td>
<td>1.3223</td>
</tr>
<tr>
<td>RMSE (nm)</td>
<td>24.4</td>
<td>56.7</td>
<td>50.7</td>
</tr>
</tbody>
</table>

In the RFBS methodology, no physical sampling aperture is required. This makes the system highly flexible and feasible for freeform surface measurements. Three different kinds of surfaces were measured in both the simulation platform and experiment setup. Both studies demonstrate that nm-level accuracy is achieved in measuring freeform surfaces with mm-level PV value.

In the RFBS system, while proper sampling is realized through the beam-sampling technique, the reference issue in freeform surface measurements is addressed by the invented reference-free technique. In the next chapter, the reference-free technique is discussed in detail.
Chapter 4 Reference-Free Technique for Measurements without a Reference

In freeform surface measurements, a proper reference is most of the time unavailable. The reference-free technique is proposed in the RFBS system to measure the second order derivatives of the surface without the need of a reference. This is achieved by introducing a lateral disturbance digitally to the modulated beam. The data obtained through the reference-free technique is the second order derivatives, which can be further processed into the surface form.

4.1 Algorithm of the reference-free technique

Figure 4.1 illustrates the modelling of the reference-free technique. When a collimated beam incidents on the surface, the propagation direction of the reflected beam is changed by the profile of the sampled area. The intersection position of the reflected beam at the detector plane is related to the surface slope. If a flat reference is available, the displacement of the intersection point compared with the reference reveals the absolute surface slope. For example, as shown in Figure 4.1, $A'$ and $B'$ are the intersection points of two sampled positions on the surface, $A$ and $B$, respectively. $A^o$ and $B^o$ correspond to the intersection positions of a flat reference.
The slope at these two positions, $s_A$ and $s_B$, can be calculated from Equation (4-1).

\[
s_A = \frac{x_{A'} - x_{A^o}}{l} \quad s_B = \frac{x_{B'} - x_{B^o}}{l}
\]  

(4-1)

Without the flat reference, the position of $A^o$ and $B^o$ cannot be obtained. However, from Equation (4-1), Equation (4-2) is easily derived.

\[
s_B - s_A = \frac{x_{B'} - x_{A'} - \Delta d}{l}
\]  

(4-2)

The left-hand-side (LHS) of Equation (4-2) measures the slope change at point $A$. The right-hand-side (RHS) can be calculated without the reference positions. The second order derivative at point $A$ is defined as the slope change over a small distance, as described by Equation (4-3).
\[
\Delta s_A = \frac{s_B - s_A}{\Delta d}
\]  \hspace{1cm} (4-3)

Combining Equation (4-2) and (4-3), and take into consideration the lateral disturbance value \(\Delta d\), the second order derivative at point A can thus be obtained through Equation (4-4).

\[
\Delta s_A = \frac{x_B' - x_A' - \Delta d}{\Delta d} = \frac{x_B' - x_A' - \Delta d}{l \cdot \Delta d}
\]  \hspace{1cm} (4-4)

\[
= \frac{\Delta x - \Delta d}{l \cdot \Delta d}
\]

The smaller the \(\Delta d\) is, the closer the calculated \(\Delta s_A\) approaches the directly differentiated value. In this way, the second order derivatives of the surface can be obtained without the reference. The main steps of implementing the reference-free technique in the RFBS system are summarized in the flowchart as shown in Figure 4.2. The CCD receives two images: before and after the lateral disturbance. The centroid displacement of the distorted pattern reveals the second order derivative at each sampled position through Equation (4-4).

### 4.2 Non-linear model

As the second order derivatives are obtained through introducing a lateral disturbance to the modulated beam, the data obtaining mode can either be linear or non-linear.
Figure 4.2 Flowchart illustrating the algorithms of the reference-free technique.

In the linear model, lateral disturbance is introduced in one direction. Take point $P_0$ shown in Figure 4.3 as an example. Point $P_1$ is the sampling point after lateral disturbance $\Delta d$. $x_0$ and $x_1$ are the centroid positions corresponding to $P_0$ and $P_1$. The second order derivative at $P_0$ is thus approximated from Equation (4-5).

$$\Delta s_{P_0}^{linear} = \frac{x_1 - x_0 - \Delta d}{l \cdot \Delta d}$$  (4-5)

In the non-linear model, lateral disturbance is introduced in two directions. As shown in Figure 4.4, $P_1$ and $P_2$ are the sampling points after lateral disturbance introduced in the opposite directions. $x_0$, $x_1$, and $x_2$ are the centroid positions. The second order derivative at $P_0$ is thus approximated from Equation (4-6).
When $\Delta d$ infinitely approaches to zero, $\Delta s_{p_0}^{\text{linear}} = \Delta s_{p_0}^{\text{non-linear}}$. However, this is impossible to be achieved in practical applications due to hardware limitations.
Figure 4.5 (a) The second order derivatives measured when use the linear model. (b) The error map of the reconstructed surface.
Figure 4.6 (a) The second order derivatives measured when use the non-linear model. (b) The error map of the reconstructed surface.

For freeform surface measurements, the non-linear model is desired as the curvature is varying across the entire surface area. Non-linear modelling provides a
better representation of the surface profile as it considers the possibility of different slope changes in the opposite directions.

Take the toroidal mirror of radius 112.5mm and 137mm in the x- and y- direction as an example. Figure 4.5(a) shows the second order derivatives measured with the linear model in the RFBS simulation platform. The RMSE over 12.544mm×12.544mm surface area is measured to be 14.9nm. The error map is shown in Figure 4.5(b). Figure 4.6(a) shows the second order derivatives measured with the non-linear model in the RFBS simulation platform. The RMSE over the same surface area is measured to be 4.9nm. The error map is shown in Figure 4.6(b).

Through comparing the two results, not only the significant drop in RMSE is observed when the non-linear model is implemented, but also the corresponding error map reflects the surface profile. Theoretically, deep slope regions should induce large errors. As such, the error map from the non-linear model aligns well with the theoretical trend, while the error map from the linear model is not related to the surface profile.

### 4.3 Determination of the lateral disturbance value $\Delta d$

The key element in the reference-free technique is the lateral disturbance value $\Delta d$. Theoretically, as explained in Section 4.1, when $\Delta d$ approaches to zero, $\Delta S$ obtained through Equation (4-3) approaches the mathematically calculated second order derivatives at the corresponding sampling positions. As a result, $\Delta d$ should be
as small as possible to yield more accurate second order derivative measurements. However, due to the following reasons, there does not always have the smallest disturbance value.

a) When $\Delta d$ goes too small, the CCD may not be able to detect the centroid displacement before and after the lateral disturbance because the centroid location is the average information of the sampled surface profile. This sensitivity is determined by the size of the beamlet. The smaller the beamlet size, the smaller the effective disturbance value. In the RFBS system, when the beamlet is 0.96 mm in diameter, the smallest effective disturbance value was experimentally tested to be 0.032 mm.

b) As $\Delta d$ decreases, the capability of the calculated $\Delta S$ in representing the second order derivatives of the surface increases, and consequently the accuracy increases. However, this improvement in accuracy slows down with decreasing disturbance values.

For illustration, we set the beamlet size, $D$, as 0.96 mm, and the surface is described by Equation (3-1). The RMSE of the calculated $\Delta S$ when $\Delta d$ is of various values are plotted in Figure 4.7. From Figure 4.7, the slowing down of the improvement in accuracy with the decreasing $\Delta d$ can be observed. The highest achievable accuracy is defined as the RMSE value when $\Delta d = 0.032 mm$ and the optimal accuracy is defined as when RMSE is 10% bigger than the minimum achievable RMSE, which is also referred to as the threshold error.
Figure 4.7 RMSE of the second order derivatives at different $\Delta d$ values.

It can be calculated from Figure 4.7 that the threshold error intersects with the RMSE curve when $\Delta d = 1/7 \Delta$. In addition, through simulation studies by setting the beamlet of various sizes, it has been demonstrated that this factor is not affected by the beamlet size. Therefore, set $\Delta d$ at one-seventh of the beamlet size is recommended to achieve the optimum accuracy.

Experiment verification has been conducted on the surface as shown in Figure 3.11(b). The system configuration is listed in Table 3.2. With this configuration, the recommended disturbance value is calculated to be 5.7 pixels. Due to the limitation of the light modulator, 5 pixels are considered as the optimal value for $\Delta d$ instead. The reconstructed surfaces with different lateral disturbance values are plotted in Figure 4.8. The RMSE are summarized in Table 4.1.
Figure 4.8 Reconstructed result of the freeform polynomial when $\Delta d$ equals to 5 pixels, 10 pixels, 15 pixels, and 20 pixels. The measurement result from UA3P is plotted for comparison.

Table 4.1 RMSE at different lateral disturbance values.

<table>
<thead>
<tr>
<th>$\Delta d$ (pixels)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE ($\mu m$)</td>
<td>0.0567</td>
<td>0.5068</td>
<td>0.6645</td>
<td>0.7312</td>
</tr>
</tbody>
</table>

From the experiment, the importance of properly setting the lateral disturbance value has been demonstrated. When $\Delta d$ is at its optimal value, the surface can be accurately measured with nm-level accuracy.

4.4 Comparison of the reference-free technique and the referenced wavefront measurement technique

4.4.1 Measurement mechanism
Figure 4.9 (a) Illustration of the working mechanism of the referenced measurement. (a) Illustration of the working mechanism of the reference-free technique.

Mathematically, the referenced measurement takes two sets of data, one from the reference image and the other from the test image. The centroid displacement corresponds to the slope of the wavefront to be measured, as illustrated in Figure 4.9(a). The reference-free technique also takes two sets of data: both from the wavefront to be measured. The centroid displacement corresponds to the second order derivative of the wavefront, as illustrated in Figure 4.9(b).

Both techniques can measure surfaces after further processing the obtained data. However, to measure the slope, a flat reference is needed. In freeform surface measurements, where large slopes and varying curvatures present, a flat reference cannot be applied due to out of dynamic range. Also, even the flat reference is still valid; the repositioning of the two pieces is an issue as it is difficult to ensure the same spatial position when mounting the two pieces in the system. Since the reference-free technique only measures one surface and the lateral disturbance can
be precisely controlled in a digital way, it is simpler in operation and has higher accuracy as compared to the referenced technique.

4.4.2 Measurement performance

Simulation

The freeform polynomial as described by Equation (3-1) has been measured in simulation with both the reference-free technique and the referenced technique. The PV within the field of view is calculated to be 10.1\(\mu m\). The lateral disturbance is set at the optimal value as studied in the previous section. The reconstructed surfaces are shown in Figure 4.10. The RMSE of the measured surface when using the reference-free technique is 4.8\(nm\), and 2.4\(nm\) when using the referenced technique.

The RMSE resulted when using the reference-free technique is double that of the referenced technique is used. This is brought in during the reconstruction process, as explained in Chapter 7. In both cases, large errors happen at deep slope regions. In the ideal simulation platform, where the measurements are not affected by factors such as repositioning and material differences between the reference and the test pieces, the referenced technique yields better result than the reference-free technique.
Figure 4.10 (a) Simulation results of the freeform polynomial with the implementation of the reference-free technique. (b) Simulation results with the implementation of the referenced technique.

However, when the dynamic range of the surface increases, the referenced technique gradually loses its advantages due to the different leveling effect: the CCD distance acts as a lever that determines the beam intersection position at the detector plane and amplifies the slope to the centroid displacement. The corresponding sampling position on the reference piece and on the surface to be measured should have the same distance away from the CCD for the same leveling
effect. Deep slope regions are more vulnerable to have a different leveling effect as compared to the reference flat. Comparatively, the reference-free technique is less affected at the deep slope regions as the data is captured by comparing with the neighboring areas, where slope differences are small and the leveling effect is approximately the same.

For example, the toroidal mirror shown in Figure 1.4 has also been measured in the simulation platform. Within the field of view, the PV is calculated to be $1.3227\text{mm}$. The RMSE resulted when using the reference-free technique is $9.2\text{nm}$, and $9.3\text{nm}$ when using the referenced technique, as shown in Figure 4.11. As such, from this simulation studies, both techniques can measure surfaces with $\text{nm}$-level accuracy, and the reference-free technique has better performance when surfaces have large dynamic range.

**Experiment**

The freeform polynomial as shown in Figure 3.11(b) has been measured with both the reference-free technique and the referenced technique in the experiment setup. The reference is a $1/4\lambda$ precision optical flat mirror. The reconstructed surfaces are shown in Figure 4.12. The RMSE of the measured surface when using the reference-free technique is calculated to be $56.7\text{nm}$, and $58.9\text{nm}$ when using the referenced technique. Thus, the RMSE from the reference-free technique is $2.2\text{nm}$ smaller than that from the referenced technique.
Figure 4.11 (a) Simulation results of the toroidal mirror with the implementation of the reference-free technique. (b) Simulation results with the implementation of the referenced technique.

From the RMSE value, the reference-free technique yields better result and the error maps show that the referenced technique has poor performance at deep slope regions, while the reference-free technique has improved performance at the deep slope regions.
Figure 4.12 (a) Experiment results with the implementation of the reference-free technique. (b) Results from the implementation of the referenced technique.

Figure 4.13 Differences of the measurement results when using the reference-free technique and the referenced technique.
This is mainly because that error related to external factors such as repositioning and material differences are excluded when using the reference-free technique, while these factors are a significant error source when a reference surface is to be used. From the comparison of the two measurements, as shown in Figure 4.13, it is observed that relatively bigger differences occur at deep slope regions, which again proves the degraded accuracy of the referenced technique in measuring freeform surfaces.

4.4.3 Measurement range

Under the constraint of no crosstalk of neighboring centroid positions at CCD, both techniques can measure the same amount of centroid displacement, with different physical meanings. For the referenced technique, the centroid displacement represents the surface slope, as expressed by Equation (4-7), where \( l \) denotes the distance between the CCD and the surface; for the reference-free technique, the centroid displacement represents the surface second order derivative, as expressed by Equation (4-8). For a continuous surface profile, the second order derivatives are always smaller than the slopes (Malacara, 2007). Hence, with the same measurable centroid displacement, the corresponding dynamic range of the surfaces measured with the reference-free technique is thus larger than the dynamic range of the surfaces measured with the referenced technique. Furthermore, as described in Section 4.4.2, the dynamic range of the surfaces measured with the referenced technique is further limited for the same leveling effect.
\[ \Delta X = S \cdot l \] (4-7)

\[ \Delta X = \Delta S \cdot l + \Delta d \] (4-8)

To summarize, the reference-free technique has larger measurement range as compared to the referenced technique. Furthermore, in the ideal simulation platform, the reference-free technique has higher accuracy when measuring surfaces of large dynamic range. In addition, in experiment, where factors such as repositioning and material differences between the reference and the test pieces affect the measurement, the reference-free technique always generates smaller error. Therefore, it is concluded that the reference-free technique has wider applicability, higher performance, and simpler operation than where a reference is used. The reference-free technique successfully addresses the referencing issue in freeform surface measurements, and provides higher performance at the same time.

4.5 Summary

In the RFBS system, second order derivatives of the freeform surface are measured with the reference-free technique. The reference-free technique introduces a lateral disturbance digitally to the modulated beam. After reconstruction, the surface is measured without the need of a reference.

With the beam-sampling technique and the reference-free technique, freeform surfaces can be measured. However, the lateral resolution is limited by the pitch of the modulated beam. To achieve \( nm \)-level accuracy, the digital scanning technique
is devised and implemented in the sensing mechanism of the RFBS system to enhance the measurement performance. The next chapter discusses the digital scanning technique and presents its optimization strategy.
Chapter 5 Digital Scanning Technique for Enhanced Lateral Resolution

To make the RFBS system achieve nm-level accuracy, the devised digital scanning technique plays an important role through enhancing the lateral resolution. This chapter discusses the design of the digital scanning technique and its optimization strategy. With this technique, the lateral resolution is no longer limited by the pitch of the modulated beam. In addition, no mechanical movement is involved. In the RFBS system, with the implementation of the digital scanning technique, the lateral resolution is increased to 32µm (96µm without the digital scanning). Simulation and experiment have demonstrated that nm-level accuracy is achievable with the implementation of the digital scanning technique. The optimization strategy for practical application is also numerically studied and experimentally proved.

5.1 Digital scanning technique

The RFBS system uses a light modulator to generate the modulated beam as the optical “probe” to sample the surface. As shown in Figure 3.7, the design of the light modulator is a matrix of circular holes, to allow light pass through, while blocking the remaining areas. The circular holes then digitally shifted to sample the areas that are previously blocked. In this way, all sub-apertures are active in each
image, which makes full use of the modulated beam (Li et al., June 2006). After the scanning is done, the images are superimposed in the scanning direction. As positions between the neighboring beamlets are also sampled, the lateral resolution is greatly enhanced.

The advantages of the digital scanning technique are as follows.

a) Hundreds or even thousands of sampling points are possible, and hence the lateral resolution is greatly increased.

b) The increase in the lateral resolution does not sacrifice the measurement range of the RFBS system. This is because that the measurable centroid displacement is not reduced in each image.

c) No mechanical moving parts are presented in the system.

The freeform polynomial as described by Equation (3-1) is measured in the simulation platform of the RFBS system. The pitch of the modulated beam is set as 1.28\textit{mm}. Without applying the digital scanning technique, the number of sampling points in the \textit{x}-direction is 13, and the lateral resolution is 1.28\textit{mm}. The reconstructed surface is shown in Figure 5.1(a). The resulted RMSE is calculated to be 268.8\textit{nm}. With the implementation of the digital scanning technique and the scanning interval is set as 0.16\textit{mm} in the \textit{x}-direction, the total number of sampling points is 104. As the scanning interval is one-eighth of the pitch, it is referred to as an eight-step scanning. The lateral resolution is increased to 0.16\textit{mm}. 

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Figure 5.1 (a) Reconstructed results with no scanning. (b) Reconstructed results with digital scanning.

The reconstructed surface is shown in Figure 5.1(b). The resulted RMSE is calculated to be 8.1 nm.

From Figure 5.1, the importance of the digital scanning in achieving nm-level accuracy is demonstrated. Large errors occur at deep slope regions. To improve performance, more sampling points are desired at those regions.
Figure 5.2 (a) Experiment results with no scanning. (b) Experiment results with digital scanning.

Experiment has also been conducted on the freeform polynomial shown in Figure 3.11(b). The modulated beam has the same design as used in the simulation. Without scanning, the RMSE is calculated to be $435.2 \text{nm}$ and the result is shown in Figure 5.2(a). With the implementation of the digital scanning technique, the RMSE reduces to $56.7 \text{nm}$ and the result is shown in Figure 5.2(b).
Figure 5.3 Comparison of the surfaces measured with and without digital scanning.

Without scanning, the reconstructed surface has large deviation from the true profile, as can be observed from the averaged 1D plot shown in Figure 5.3. With an eight-step scanning, the reconstructed surface agrees well with the true surface profile.

5.2 Scanning strategy optimization

From mathematical point of view, more sampling points within a fixed surface area yield higher lateral resolution. Due to the reasons listed below, it may not be necessary to have as many sampling points as possible for surface form measurements.

a) The Nyquist sampling theorem indicates that when sampling points are spaced within $\frac{1}{2\omega}$ apart from each other, shapes of the surface frequency
\( \omega \) can be well reconstructed. In other words, a certain number of sampling points is good enough to fit out the desired surface form.

b) Each sampling point gives information of a local sampled surface area determined by the size of the beamlet. Even if plenty of points are added in, the size of the local sampled area that each sampling points represents remains the same. From this point of view, above a certain number of points, the improvement in system performance through enhancing the lateral resolution is very limited. Hence, more points may not necessarily result in a more accurate surface form as the performance is eventually determined by the beamlet setting. In short, height sensitivity and measurement accuracy is fundamentally related to the beamlet size, rather than the configuration of the sampling array.

c) It is not practical to always have a large number of sampling points as this conduct requires long operation time and makes the result vulnerable to vibration. Because of the concern in acquisition time needed, it is important to find an optimal number of sampling points, where the form accuracy achieved at this scanning step is within error tolerance, and beyond which the accuracy improvement is insignificant.

In the RFBS system, many factors contribute to the measurement accuracy, such as the centroid finding technique and the reconstruction algorithm. Especially when measuring freeform surfaces, where the surface profile is
complicated, it is difficult or even impossible to establish the direct relationship between the lateral resolution and the system performance. Therefore, simulation has been conducted on surfaces with periodic sine waves but different spatial frequency $\omega$ to verify the existence of the optimal scanning step, and to determine its value in various surface measurements if any. The PV of the surfaces within the field of view is the same as 10$\mu$m. Five different $\omega$ values are chosen for study: 0.125$mm^{-1}$, 0.25$mm^{-1}$, 0.5$mm^{-1}$, 0.75$mm^{-1}$, and 1$mm^{-1}$. The optimal scanning step is defined as the value at which the RMSE obtained deviates within 10% of the value achievable at the maximum number of sampling points, which has a 0.032$\mu$m lateral resolution as determined by the constraint of the light modulator. In other words, when the RMSE decreases to 1.1 times of the minimum RMSE, the corresponding scanning step is considered as optimal. The beamlets with the following diameters have been chosen for study to align with the hardware capability: 0.96$mm$, 1.6$mm$, 2.4$mm$, 2.88$mm$ and 4.8$mm$. The pitch of the modulated beam is the same as the beamlet diameter. The field of view is 20$mm \times 20 mm$.

During the simulation, the scanning steps where the corresponding RMSE is 1.1 times of the minimum RMSE are recorded. The average value for each modulated beam design and the corresponding standard deviation when measuring surfaces with different spatial frequencies are summarized in Table 5.1.
Table 5.1 Summary of the optimum scanning steps for various modulated beam settings.

<table>
<thead>
<tr>
<th>Lenslet diameter (mm)</th>
<th>0.96</th>
<th>1.6</th>
<th>2.4</th>
<th>2.88</th>
<th>4.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average value of the</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total number of</td>
<td>65</td>
<td>36</td>
<td>25</td>
<td>19</td>
<td>10</td>
</tr>
<tr>
<td>sampling points after</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scanning to result in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1 of the minimum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.3</td>
<td>0.5</td>
<td>1.0</td>
<td>0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>Optimal scanning step</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

The maximum standard deviation is 1.3. The average value is thus adopted to be the optimal scanning step for that particular modulated beam setting. From Table 5.1, it is observed that after a four-step scanning, the reconstructed surface is very much the same as the result obtained from the largest lateral resolution possible. More scanning conducted beyond this optimal value will have little improvement in the RMSE.

Although the optimal scanning steps when the modulated beam is of various settings are the same, the accuracy is different, which is limited by the beamlet size. For example, when the beamlet diameter is 4.8 mm and the spatial frequency of the surface is 1 mm⁻¹, the RMSE resulted at its corresponding optimal scanning step is 7.55 µm. If the beamlet diameter reduces to 0.96 mm, the RMSE resulted thereafter decreases to 0.24 µm with the same scanning step.
Table 5.2 Summary of the RMSE at the optimal scanning step for different modulated beam settings in various surface form measurements.

<table>
<thead>
<tr>
<th>Spatial frequency ($mm^{-1}$)</th>
<th>0.125</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beamlet Diameter 0.96mm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE at optimal scanning step ($\mu m$)</td>
<td>3.04E-03</td>
<td>1.27E-02</td>
<td>5.93E-02</td>
<td>0.125</td>
<td>0.238</td>
</tr>
<tr>
<td>Minimum RMSE ($\mu m$)</td>
<td>2.85E-03</td>
<td>1.10E-02</td>
<td>5.38E-02</td>
<td>0.113</td>
<td>0.228</td>
</tr>
<tr>
<td>Percentage increment</td>
<td>6.67%</td>
<td>15.45%</td>
<td>10.22%</td>
<td>10.62%</td>
<td>4.39%</td>
</tr>
<tr>
<td><strong>Beamlet Diameter 1.6mm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE at optimal scanning step ($\mu m$)</td>
<td>8.01E-03</td>
<td>3.22E-02</td>
<td>0.173</td>
<td>0.362</td>
<td>0.788</td>
</tr>
<tr>
<td>Minimum RMSE ($\mu m$)</td>
<td>7.46E-03</td>
<td>2.86E-02</td>
<td>0.157</td>
<td>0.328</td>
<td>0.726</td>
</tr>
<tr>
<td>Percentage increment</td>
<td>7.41%</td>
<td>12.5%</td>
<td>9.85%</td>
<td>10.2%</td>
<td>8.57%</td>
</tr>
<tr>
<td><strong>Beamlet Diameter 2.4mm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE at optimal scanning step ($\mu m$)</td>
<td>1.77E-02</td>
<td>6.53E-02</td>
<td>0.428</td>
<td>0.903</td>
<td>2.060</td>
</tr>
<tr>
<td>Minimum RMSE ($\mu m$)</td>
<td>1.61E-02</td>
<td>5.74E-02</td>
<td>0.383</td>
<td>0.810</td>
<td>1.875</td>
</tr>
<tr>
<td>Percentage increment</td>
<td>9.99%</td>
<td>1.36%</td>
<td>11.6%</td>
<td>11.6%</td>
<td>9.66%</td>
</tr>
<tr>
<td><strong>Beamlet Diameter 2.88mm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE at optimal scanning step ($\mu m$)</td>
<td>2.45E-02</td>
<td>8.52E-02</td>
<td>0.634</td>
<td>1.340</td>
<td>3.050</td>
</tr>
<tr>
<td>Minimum RMSE ($\mu m$)</td>
<td>2.28E-02</td>
<td>7.68E-02</td>
<td>0.580</td>
<td>1.235</td>
<td>2.835</td>
</tr>
<tr>
<td>Percentage increment</td>
<td>7.79%</td>
<td>10.9%</td>
<td>9.40%</td>
<td>8.85%</td>
<td>7.45%</td>
</tr>
<tr>
<td><strong>Beamlet Diameter 4.8mm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE at optimal scanning step ($\mu m$)</td>
<td>6.19E-02</td>
<td>0.174</td>
<td>2.060</td>
<td>4.080</td>
<td>7.550</td>
</tr>
<tr>
<td>Minimum RMSE ($\mu m$)</td>
<td>5.81E-02</td>
<td>0.165</td>
<td>1.896</td>
<td>3.910</td>
<td>7.176</td>
</tr>
<tr>
<td>Percentage increment</td>
<td>6.59%</td>
<td>5.33%</td>
<td>8.65%</td>
<td>4.37%</td>
<td>5.26%</td>
</tr>
</tbody>
</table>
The accuracies achieved at the optimal scanning step for various modulated beam settings, and their corresponding minimum RMSE are listed in Table 5.2.

d) As the number of the optimal scanning step is a fixed value once the modulated beam design has been chosen, it indicates that in various surface form measurements, once the beamlet size is defined, the system performance improves through scanning is determined.

To summarize, it is necessary to choose a suitable beamlet size as the highest level of accuracy the RFBS system can reach is intrinsically determined by the modulated beam setting. Scanning can help to improve the lateral resolution. However, the improvement in measurement accuracy that can be realized through scanning is limited to a certain level, beyond which it is necessary to reduce the beamlet size. In the scenarios that have been studied, a four-step scanning is sufficient to achieve the optimal performance for practical reasons.

5.3 Experiment verification

Experiment has been conducted on the freeform polynomial as shown in Figure 3.11(b) to verify the concept of the optimal scanning step. The modulated beam design is the same as listed in Table 3.2. From the simulation studies, the optimal scanning step is four. The measurement results with an eight-step scanning, a four-step scanning, and without scanning are shown in Figure 5.4.
Figure 5.4 Experiment results of the freeform polynomial measured in the RFBS system with an eight-step scanning, four-step scanning, and with no scanning.

From Figure 5.4, it is observed that when no scanning is conducted, the reconstructed plots deviate significantly from the nominal curve. The RMSE is measured to be 435.2nm. After conducting a four-step scanning, the reconstructed plots align well with the nominal curve. The RMSE is measured to be 60.3nm. If the scanning step increases to eight, the RMSE is measured to be 56.7nm.

From the four-step scanning to the eight-step scanning, the RMSE improves by 3.6nm. However, the number of images need to be taken is twice the number at the optimal scanning step. More images indicate longer measurement time and thus make the system vulnerable to vibration. More images also lead to a longer data processing time. Therefore, the four-step scanning is considered optimal from both the aspect of accuracy, and practical concerns.
5.4 Summary

In the RFBS system, the devised digital scanning technique plays an important role to enable the system for nm-level accuracy. This is achieved by digitally scanning the modulated beam so as to enhance the lateral resolution. Through simulation, the scanning step is optimized to make practical application much easier, yet reach a satisfactory level of accuracy: nm-level accuracy is achieved with the four-step scanning. Experiment on a freeform surface has demonstrated the effectiveness of the optimization strategy.

After acquiring all the images, the centroid positions of the spots are to be determined. The techniques for centroiding of images from a freeform surface are discussed in the next chapter.
Chapter 6 Windowing and Centroiding Algorithms for High Robustness

In the RFBS system, after obtaining distorted spots images, windowing and centroiding are conducted to extract the centroid positions. The centroid displacements correspond to the second order derivatives of the surface. As the distorted pattern is the sampled result of the surface, the image characteristics, such as spot distribution and SNR, are correlated to the surface profile. As freeform surfaces have large dynamic range with varying curvatures, the uniform windowing technique is not applicable. Furthermore, after defining the windows for each spot, the centroiding algorithm needs to be revisited as the image usually suffers from various image degradations. This chapter designed and proposed the dynamic windowing technique to automatically analyze the image, identify and register every spot at one go. After windows are defined, the devised adaptive centroiding technique is to be performed, which can effectively locate the centroid position for each spot accurately.

6.1 Dynamic windowing technique

6.1.1 Challenges of windowing in freeform surface measurements
In the RFBS system, the light modulator may bring in noisy spots due to high order diffraction as shown in Figure 6.1. The image may also suffer from background noise that muddles the distinguishability of the spots. Therefore, besides being able to window spots of varying shapes and sizes which are related to the surface profile, the windowing technique must also be capable of differentiating noisy spots from real signals, while being insensitive to the background noise.
Figure 6.3 Illustration of applying the uniform windowing technique on the image of the freeform polynomial.

Figure 6.2 shows an image of the freeform polynomial, as presented in Figure 3.11(b), received in the RFBS system. To accurately window each spot, the uniform windowing technique needs to be conducted area by area to manually adjust the window size and the pitch between different columns, as illustrated in Figure 6.3. This tedious process is not only time consuming but also vulnerable to operational error. For freeform surface measurements, it is desirable to have an intelligent windowing technique that can automatically identify and register each spot for every sampled area on the surface at one go.
6.1.2 Algorithms of dynamic windowing technique

To tackle with the background noise, the intensity at each pixel is re-calculated by averaging with its surrounding pixels. This averaging step helps removing noise inside each spot as shown in Figure 6.4, so that pixels with discontinuous intensities do not affect the identification of the spot as a whole, rather than as several separate spots.

The main difference between the noisy spots and the real signals is the intensity level. As shown in Figure 6.1, the noisy spots are usually dimmer while the real signals are much brighter. Therefore, set a proper threshold intensity level helps to differentiate the noisy spots from the real signals. However, because of the large dynamic range of freeform surfaces, it is difficult to set a fixed threshold to apply on every image. To address this issue, the dynamic thresholding is implemented. Firstly, a high threshold, such as 254, is applied. Pixels with intensity level bigger than the threshold are registered as signals.
If the total number of registered signals is less than desired, a lower threshold is applied and this process is repeated until all the signals are registered.

Even with the dynamic thresholding, the noisy spots may still be registered as real spots because of varying SNR at different positions. Two criteria are useful to filter out those noisy spots. The distance between the neighboring spots must exceed a
minimum distance. This value can be approximated from the image. The other
criterion is that each spot must be larger than a minimum size. This value can also
be approximated from the image.

Based on the above described rational, the dynamic windowing technique is
designed in the RFBS system. The steps are illustrated in the flowchart shown in
Figure 6.5.

The first step averages each pixel to remove background noise inside each spot.
Then, a high threshold value is set, such as 254, followed by digitizing the image.
Only clusters larger than the minimum spot size are registered. This step removes
random noise outside each cluster, such as abnormal pixels caused by
contamination. If the number of clusters is less than desired, the threshold value is
reduced and the digitization and cluster registration steps are repeated until the
number of clusters is sufficient. Each cluster then forms an area with a unique size
based on the number of “1” pixels presented. A further filtering step based on the
distance between neighboring clusters is conducted. If the distance is smaller than
the minimum approximated value, one of them will be identified as a noisy spot
depending on the distances with other neighboring spots. After these steps, valid
spots of varying shapes and sizes can be identified and registered into their
respective windows.

Four parameters need to be defined when implementing the dynamic windowing
technique.
**Total number of spots to register.** This parameter affects the final number of windows. The exact number is also affected by the thresholding step.

**Threshold reducing gradient.** This parameter determines the speed of the technique and how close the final number of windows approaches the initially defined value.

Small gradient results in slow speed and a closer number of windows as desired; large gradient results in fast speed but a number of windows with a relatively large difference as desired. Nevertheless, it has no effect on the shape and size of each window once the spot has been correctly identified.

**Minimum spot size and minimum distance between neighboring spots.** These two parameters are used for differentiating noisy signals from real signals and make the dynamic windowing technique feasible in the presence of various noises.

### 6.1.3 Experiment verification
The image as shown in Figure 6.2 is tested with the dynamic windowing technique. From visual examination of the image, the total number of spots is set as 150; the threshold gradient is 0.01; the minimum spot size is estimated as 3 pixels; and the minimum distance between neighboring spots is estimated as 35 pixels. The windowed image is shown in Figure 6.6.

Through the experiment, the feasibility and effectiveness of the proposed dynamic windowing technique is shown. As demonstrated in Figure 6.6, each window is isolated and has a unique size, which best accommodates the enclosed spot. Therefore, images of freeform surfaces can be windowed automatically by implementing the dynamic windowing technique, which is experimentally demonstrated to be of high robustness.

6.2 Centroiding methods for freeform surface measurements

6.2.1 Issues of centroiding in measuring freeform surfaces

After the windowing process, the centroid position of each spot is to be determined. Compared to flat surfaces, the following issues are encountered in the centroiding process when measuring freeform surfaces.

Noise

As described in the previous section, the image has background noise that muddles the distinguishability of the spots. The noise does not follow a pattern and is a result of many factors, including the system arrangement and alignment.
Figure 6.7 Image captured in the RFBS system with a mixture of real signals and noises.

For example, Figure 6.7 shows part of the image captured in the RFBS system. Besides the real signal spot, various kinds of noises, such as interference pattern and overlapping of higher order diffractions, can be observed.

**Low SNR**
Freeform surfaces can have large slopes. Because the image sensor is a plane, the spot intensity and contrast would drop amidst the background noise, and detection of the spot would be hampered. For example, Figure 6.8 shows part of the image of a freeform surface captured in the RFBS system. The spots of low SNR, resulted from deep slope regions, are observed.

**Variation of image characteristics**

Because the measured surface is freeform, image characteristics, such as intensity distributions, SNR, and spots shape, can vary for each window. For example, as shown in Figure 6.8, the inconsistence of SNR is clearly observed for each spot. For images have constant SNR, a suitable global threshold value of intensity, which acts as a noise filter, can be applied. For images of varying SNR, a global threshold is inaccurate to be applied to all windows.

### 6.2.2 Centroiding methods in freeform surface measurements

From the literature study, four techniques – thresholding, high powering, sub-windowing, and iteration – work well in complementing the first moment method (FMM) to calculate centroid positions. However, their effectiveness in freeform surface measurements in the RFBS system is unknown. Therefore, they were tested on the images captured in the RFBS system.

The surface is the freeform polynomial shown in Figure 3.11(b). The spot image received is shown in Figure 6.9.
Figure 6.9 Spot image of the freeform polynomial received in the RFBS system.

This image is taken where the beamlet diameter is set as 0.32 mm and the pitch is set as 1.28 mm. The CCD distance is measured to be 145 mm. Different threshold values and varying powers, with the inclusion/exclusion of sub-windowing and iteration, were experimentally investigated on Figure 6.9. The RMSE of the reconstructed surface with different settings is summarized in Table 6.1.
Table 6.1 RMSE from various centroiding techniques.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Power</th>
<th>RMSE (nm)</th>
<th>Without Sub-windowing</th>
<th>With sub-windowing</th>
<th>With sub-windowing and iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>168.6</td>
<td>150.8</td>
<td>145.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>69.2</td>
<td>66.1</td>
<td>60.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>56.9</td>
<td>56.7</td>
<td>56.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>55.4</td>
<td>55.4</td>
<td>55.4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>60.3</td>
<td>58.1</td>
<td>58.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>55.9</td>
<td>55.8</td>
<td>55.8</td>
<td></td>
</tr>
<tr>
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<td>5</td>
<td>55.4</td>
<td>55.4</td>
<td>55.4</td>
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<tr>
<td></td>
<td>10</td>
<td>56.0</td>
<td>56.0</td>
<td>56.0</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>55.2</td>
<td>55.1</td>
<td>55.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>55.4</td>
<td>55.4</td>
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<tr>
<td></td>
<td>5</td>
<td>56.0</td>
<td>56.0</td>
<td>56.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>57.2</td>
<td>57.2</td>
<td>57.2</td>
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<tr>
<td>150</td>
<td>1</td>
<td>54.1</td>
<td>54.1</td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>54.0</td>
<td>54.0</td>
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<tr>
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<td>54.1</td>
<td>54.1</td>
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<tr>
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<td>10</td>
<td>55.3</td>
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<td>200</td>
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<td>117.1</td>
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<td>10</td>
<td>116.1</td>
<td>116.1</td>
<td>116.1</td>
<td></td>
</tr>
</tbody>
</table>
From Table 6.1, it is shown that the smallest RMSE is obtained at a threshold value of 150, a power of 3, without sub-windowing or iteration. The corresponding reconstructed result is shown in Figure 6.10. This setting, though optimal, is hardware dependent, i.e., changing a component could require a different setting for optimal performance.

Among the four techniques, thresholding has the greatest influence on the reconstructed surface. A satisfactory RMSE can be obtained with a correct threshold value alone. If the threshold value is set to have a big gap from the satisfactory one, a high RMSE will be induced. The increment in RMSE cannot be rectified by the other techniques. This is true especially for deep slope regions, where the corresponding image has spots with low intensity and weak contrast. Figure 6.11(a), which is the result of wrong thresholding, has significant errors where the slopes are very large as the centroiding algorithm fails to accurately detect the centroid positions. Figure 6.11(b) shows that even after incorporating power, sub-windowing and iteration techniques with the same thresholding, no improvement is seen in the result.

Table 6.1 also shows that high powering is effective in improving the reconstruction accuracy only at lower threshold values. For example, without thresholding, a power of 10 can decrease the RMSE from 168.6nm to 55.4nm. The accuracy improvement increases diminishingly with increased power and eventually plateaus beyond a certain power value.
Figure 6.11 (a) Reconstructed surface with a threshold value of 0, no sub-windowing or iteration. (b) Reconstructed surface with a threshold value of 0, power 15, with sub-windowing and iteration.

It is also observed from Table 6.1 that the sub-windowing and iteration techniques do not significantly improve accuracy, while requires more computation time.
6.3 Adaptive centroiding technique for freeform surface measurements

6.3.1 Algorithms of the adaptive centroiding technique

The importance of thresholding has been demonstrated in the previous experiment; indeed, it is necessary to define an appropriate threshold value. As an appropriate threshold value is dependent on the image characteristics, it is impractical to apply a global threshold value for all windows in freeform surface measurements. Therefore, local thresholding is proposed to address this issue. Instead of using a global threshold, a local threshold is used for each window, which is defined individually by the local average intensity. Based on the local thresholding concept, the adaptive centroiding technique is designed with the following steps.

Firstly, the average intensity within each window is calculated through Equation (6-1), where $M$ and $N$ defines number of pixels in the $x$- and $y$-direction of each window.

$$I_{\text{avg}} = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} I(x_i, y_j)}{MN} \quad (6-1)$$

Then, the local threshold value is applied to all the pixels of each respective window, and a new intensity map is obtained through Equation (6-2).
Table 6.2 RMSE values for various settings of adaptive centroid finding.

<table>
<thead>
<tr>
<th>Power (nm)</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without iteration</td>
<td>64.3</td>
<td>57.3</td>
<td>56.6</td>
<td>56.9</td>
</tr>
<tr>
<td>With iteration</td>
<td>55.6</td>
<td>55.1</td>
<td>55.3</td>
<td>56.3</td>
</tr>
</tbody>
</table>

\[ I_{\text{new}}(x_i, y_j) = \begin{cases} 
  0 & \text{if } I(x_i, y_j) \leq I_{\text{avg}} \\
  I(x_i, y_j) - I_{\text{avg}} & \text{if } I(x_i, y_j) > I_{\text{avg}} 
\end{cases} \quad (6-2) \]

Lastly, the centroid position of each window is calculated from FMM.

6.3.2 Experiment verification

The adaptive centroiding technique with varying power values and the inclusion/exclusion of iteration were tested on the images used in the previous section. The RMSE results are listed in Table 6.2.

From Table 6.2, it is clear that the adaptive centroiding technique is capable of attaining a satisfactory degree of accuracy. Even at deep slope regions, the algorithm is capable of identifying the centroid positions accurately as shown in Figure 6.12.

Table 6.2 also shows that increasing the power and iteration contribute little in improving the accuracy of the result.
Figure 6.12 Centroid positions calculated with the adaptive centroiding technique and FMM with a global threshold and power 1. The red circles indicate centroid positions calculated with the adaptive centroiding technique at power 1. The blue plus signs indicate centroid positions calculated with FMM.

Figure 6.13 Spot image of the toroidal mirror received in the RFBS system.
Figure 6.14 Error of the reconstructed surface of the toroidal mirror from global threshold centroiding technique, and the adaptive centroiding technique.

To further verify the adaptive centroiding technique, the toroidal mirror with the radius of $100\text{mm}$ in the $x$-direction and $107.2\text{mm}$ in the $y$-direction was measured in the RFBS system. The captured image is shown in Figure 6.13. The error of the reconstructed surface is shown in Figure 6.14 after fitting.

When an optimal global threshold is applied, the RMSE is calculated to be $37\text{nm}$. With the implementation of the adaptive centroiding technique, the RMSE reduces to $10\text{nm}$. The effectiveness and improved performance of the devised adaptive centroiding technique is thus proven experimentally. Especially for surfaces with large slopes, the technique is highly robust that better performance is achieved.
Hence, the adaptive centroiding technique is desirable for freeform surface measurements.

6.4 Summary

In the RFBS system, the second order derivatives are obtained by comparing the centroid displacement before and after the lateral disturbance. It is therefore of crucial importance to determine the centroid positions accurately for high performance. The centroid positions are calculated from the intensity distribution of the spots, which are the sampled result of the surface profile. Windowing is a necessary step before calculating the centroids. The images of freeform surfaces have inconsistent characteristics, such as spots distribution and size of each spot, due to the large dynamic range and varying curvature of the surface. The uniform windowing is insufficient to define the window for every sampled area at one go. Its rigid working principle also makes the registration process tedious and troublesome. The dynamic windowing technique is proposed in the RFBS system to window every sampled area independently and at one go. Experiment demonstrated that the shape and size of each individual window through the dynamic windowing technique best suits its enclosed spot.

Even with the accurately defined windows, the centroiding techniques with global thresholding cannot determine the centroid positions properly. An appropriate threshold value is dependent on the image characteristics, which are usually varying over the entire image. On this basis, the adaptive centroiding technique is
devised. The proposed technique calculates the local threshold value, defined by the average intensity, for every window independently. Hence, the adaptive centroiding technique does not need the estimation of the global threshold, while is able to determine the centroid positions within every window accurately. Experiment has demonstrated the feasibility and effectiveness of the adaptive centroiding technique: if a global threshold is used, the RMSE is 37\textit{nm}; when the adaptive centroiding technique is applied, the RMSE is reduced to 10\textit{nm}.

Because of the high robustness of the dynamic windowing technique and the adaptive centroiding technique, the second order derivatives can be accurately calculated in the RFBS system, which are reconstructed into the surface form. The next chapter discusses the reconstruction techniques for freeform surface measurements.
Chapter 7 Surface Reconstruction Techniques for Freeform Measurements

The centroid displacements calculated in the RFBS system are related to the second order derivatives of the surface. In many other wavefront sensing systems, such as the SHWS, the slopes are the obtained data. The surface can be reconstructed from either the second order derivatives or the slope matrix. For freeform surface measurements, the reconstruction techniques must be re-studied for high performance in practical application. This chapter designs, analyzes and compares five different techniques to reconstruct wavefront, especially for practical measurements with noises and missing data, which is most likely to happen in the received images due to low and varying SNR. Simulation and experiment results show that in ideal condition without any noise in the system, direct fitting yields perfect results with zero error; the techniques including the Southwell algorithm have better performance in practical conditions, where there exist real issues such as noise, abnormal data, etc. Southwell reconstruction is therefore an essential step as it significantly improves the measurement stability and reliability, and enables the system feasible for practical applications. Based on this study, 3D reconstruction technique is devised for calculating the freeform surface with high accuracy. Besides the merits of the Southwell algorithm, 3D reconstruction can be
extended for applications with missing data, which usually happens when the sampled area has large dynamic range.

7.1 Wavefront reconstruction methods

Generally, the processing algorithms can include fitting, integration, Southwell reconstruction, and combinations of them as well. In this chapter, fitting is conducted by the curve fitting toolbox provided in Matlab; mathematical integration refers to calculate the coefficients through those of the lower order equation. While all fitting, integration, and Southwell reconstruction related methods work well in spherical surface measurement, their performances in freeform surface measurement have significant differences. In this section, the second order derivatives are referred to as curvatures. Five different methods were designed and investigated to suggest the most suitable approach for freeform surface measurement under practical condition. They are:

**SRF (Slope-Southwell Reconstruction-Fitting):** with the slope matrix as the raw data for further processing, Southwell reconstruction is conducted, followed by fitting the discrete height data obtained from the previous step. Southwell reconstruction is conducted once in this method.

**SFI (Slope-Fitting-Integration):** with the slope matrix as the raw data, directly fit out the shape. The surface form is then mathematically integrated from the coefficients obtained in the fitting step. No Southwell reconstruction is conducted in this method.
CRRF (Curvature-Southwell Reconstruction-Southwell Reconstruction-Fitting): with the curvature matrix as the raw data, conduct Southwell reconstruction to obtain the slope matrix, followed by another Southwell reconstruction step to obtain the discrete height data. Fitting is then conducted, which finally yields the form information. Southwell reconstruction is conducted twice in this method.

CFII (Curvature-Fitting-Integration-Integration): with the curvature matrix as the raw data, directly fit out the shape. The form is then integrated twice from the coefficients obtained. No Southwell reconstruction is conducted in this method.

CRFI (Curvature-Southwell Reconstruction-Fitting-Integration): with the curvature matrix as the raw data, conduct Southwell reconstruction to obtain the slope matrix, followed by a fitting step. The surface form is then integrated from the coefficients obtained in the previous step. Southwell reconstruction is conducted once in this method.

The above five methods mainly differ with each in whether Southwell reconstruction is conducted or direct fitting is applied. In order to suggest the most suitable one for freeform surface measurement, their performances in different scenarios are studied in the following sections.

7.1.1 Simulation parameters
Figure 7.1 RMSE resulted from applying the five methods on spherical surfaces.

To analyze the performance of wavefront reconstruction methods independently, the input raw data, either slope matrix or curvature matrix, is differentiated directly from the surface mathematical description. The surface has a size of $100\text{mm} \times 100\text{mm}$. The distance between neighboring points in the two orthogonal directions is 1mm. As such, the size of the input matrix is 100×100. The number of coefficients for fitting depends on the highest order of the surface form, $\alpha$. If fitting is conducted on the slope data, the highest order to fit is $\alpha$; if fitting is conducted on the second order derivative, the highest order to fit is $\alpha-1$.

### 7.1.2 Measurement of spherical surfaces

All methods work well in spherical surface measurement. This can be demonstrated in measuring the spherical surface as described by Equation (7-1).
\[ \varphi = 0.001r^2 \]  

(7-1)

where \( r \) represents the radial distance from the optical axis. Figure 7.1 shows the corresponding RMSE resulted when each method is applied, and the value is at 10^{-12} \( \mu m \) level, or say is zero.

### 7.1.3 Measurement of freeform surfaces

In order to compare different methods’ capability in measuring freeform surfaces, the measurements of the surface as described by Equation (7-2) are simulated.

\[ \varphi = 0.001r^2 + 10^{-5}r^3 - 10^{-7}r^4 \]  

(7-2)

Figure 7.2 shows the corresponding RMSE resulted when each method is applied. It is observed that both SRF and CRFI generate 0.31 \( \mu m \) RMSE, while CRRF generates 0.62 \( \mu m \) RMSE. No error occurred through SFI or CFII. This is due to the linear interpolation nature of the Southwell algorithm.

The linear progression algorithm works well for spherical shapes. However, for freeform surfaces, a non-linear relationship between neighboring grid point is present. Therefore, high-order terms usually cannot be accurately reconstructed. The thus induced error is termed as non-linear error. The RMSE corresponding to one-time Southwell reconstruction methods (SRF and CRFI) and two-times Southwell reconstruction methods (CRRF) are 0.31 \( \mu m \) and 0.62 \( \mu m \) respectively.
Figure 7.2 RMSE resulted from applying the five methods on freeform surfaces.

Since no Southwell reconstruction is conducted in SFI or CFII, the non-linear error is avoided.

7.1.4 Noise

In practical measurement, noise is inevitable. As such, the sensitivity to noise of the five methods is also studied with the same surfaces used in the previous section. A white noise with SNR 50 is added onto the directly simulated raw data, and repeated 1000 times. Figure 7.3 shows how the different methods react to the white noise, in terms of RMSE. It is observed that SRF and CRRF perform much better than CFII. This finding implies that the methods with direct fitting are significantly affected by white noise than the methods with Southwell reconstruction.
Figure 7.3 (a) RMSE resulted in 1000 testing times when applying the five methods on spherical surfaces with the presence of white noise. (b) RMSE resulted in 1000 testing times when applying the five methods on freeform surfaces with the presence of white noise.
The main reason is attributed to the zero-values error in the raw data that eventually accumulates into profile evaluation integration (Wei Gao, 2002). Thus, Southwell reconstruction helps to stable the measurement results and makes system robust. It also can be seen from Figure 7.3 that RMSE generated by CFII, SFI, and CRFI can reach as high as several microns. This indicates that these three methods can result in surfaces far away from the real shapes, and hence are not applicable in the real measurement. As the RMSE generated from SRF and CRRF is within $1\mu m$, the two methods are feasible in presence of noise.

### 7.1.5 Abnormal points

One possible scenario in the captured data when measuring freeform surfaces is the existence of abnormal data point. This means that some surface areas are poorly sampled compared to others, due to large dynamic range or contamination at the surface, such as dust, finger print, etc. As such, the response and sensitivity of the five methods to the abnormal point at different locations is studied. The same surfaces, both spherical and freeform, used in the previous studies are simulated. Since the surfaces are rotationally symmetric, a 10% error is added to the point which is moving in the $x$-direction consecutively. Figure 7.4 shows how different methods react to the abnormal point, in terms of RMSE. For SFI and CFII, the measured surface contains large error if the abnormal point is at edges. For the rest three methods, where Southwell reconstruction is conducted at least once, the accuracy when the abnormal point is located at edges is greatly improved.
Figure 7.4 (a) RMSE resulted when applying the five methods on spherical surfaces with the presence of an abnormal point. (b) RMSE resulted when applying the five methods on freeform surfaces with the presence of an abnormal point.
The nearer the abnormal point approaches the center, the higher the accuracy of the measured surface. The more steps of Southwell reconstruction conducted, the smaller the standard deviation of the RMSE resulted. This finding implies that Southwell reconstruction helps to improve measurement stability and makes the system robust and feasible in practical applications, where there are possibly some sampling points containing large errors.

7.1.6 Missing point

In practical measurement of freeform surfaces, sometimes it is not able to obtain data at certain locations, due to surface contamination, failure in receiving the beam at deep slope regions, dents, or irregularity of the surface. Multiple neighboring points are required in Southwell reconstruction as shown in Figure 7.5:
for points at corners, two neighboring points are needed; for points at edges, three neighboring points are needed; for points elsewhere, four neighboring points are needed. This requirement limits Southwell’s application to rectangular matrix. As such, the algorithm cannot be applied when there are missing data in the matrix for reconstruction.

7.1.7 Experiment verification

To verify the above findings, experiment has been conducted on the freeform polynomial as shown in Figure 3.11(b). The slope matrix is obtained by using a 1/4λ precision optical flat mirror as the reference. The curvature matrix is obtained through the reference-free technique. Experiment results show that surfaces measured through SRF and CRRF agree with the UA3P measurement result as shown in Figure 7.6. Other three methods – SFI, CFII, and CRFI – contains very large errors as shown in Figure 7.6, which indicates that they are not feasible for real measurements. This agrees well with the simulation findings, where SRF and CRRF are least affected in presence of noise, and the other three methods are all dramatically affected. From this experiment, the robustness of Southwell reconstruction is demonstrated. If fitting is conducted before Southwell reconstruction, or conducted on the intermediate data, the possibility to obtain the accurate wavefront information is very low.
7.2 Extended Southwell reconstruction for measurements with missing points

7.2.1 Extended Southwell reconstruction

As discussed above, when there are missing data in the slope matrix or curvature matrix, Southwell reconstruction cannot be applied. The following modification and process is devised to extend Southwell reconstruction for higher flexibility.

**Step 1:** Firstly, each grid point searches for its available neighboring points within one pitch distance.

**Step 2:** Then wavefront is calculated as
\[ \varphi_o = \frac{1}{n} \sum_{i=1}^{n} \left[ \varphi_i - \frac{P_x}{2} \left( S_i^x + S_o^x \right) - \frac{P_y}{2} \left( S_i^y + S_o^y \right) \right] \]  \hspace{1cm} (7-3)

where \( n \) is the number of neighboring points detected for \( \varphi_o \), \( P_x \) is the pitch value in the \( x \)-direction and \( P_y \) is the pitch value in the \( y \)-direction.

**Step 3:** Apply SOR on Equation (7-3); the wavefront can be well reconstructed.

To demonstrate the effectiveness of the extended reconstruction algorithm, the freeform surface described by Equation (7-2) is simulated. The simulated surface is \( 100mm \times 100mm \), with the missing data covering a circular area of \( 10mm \) in radius. To study the effect when the missing data is at different locations, the simulation is conducted for each position when the circular area is moving around the surface, as demonstrated in Figure 7.7. The missing area is 3.14\% of the overall surface size. When reconstruction is conducted on the complete slope matrix, the RMSE is \( 0.3099\mu m \), and is used as the benchmarked RMSE.

The RMSE resulted from the data matrix with the missing area at different locations is shown in Figure 7.8. It is observed that RMSE varies with locations. The largest RMSE happens when the missing area is close to edges, and is \( 0.3156\mu m \), 2.097\% larger than the benchmarked RMSE.

Therefore, the extended reconstruction algorithm devised for making measurement feasible in the presence of missing data is demonstrated, with a small increment in the RMSE.
Figure 7.7 Illustration of the movement direction of the missing area across a surface.

Figure 7.8 RMS errors resulted when the missing areas are at different positions of the surface.

7.2.2 Experiment verification

The same slope matrix obtained in the previous experiment is used to verify the effectiveness of the extended reconstruction algorithm. A circular area of 3mm in diameter sits at three different positions on the surface, as shown in Figure 7.9(a).
Figure 7.9 (a) Illustration of three different locations of missing data. (b) The reconstructed results when the missing data is at three different locations.

The missing data covers 4% of the entire surface area. The RMSE resulted when applying the extended reconstruction algorithm at three different locations are 0.2312μm, 0.2823μm, and 0.2383μm respectively, as shown in Figure 7.9(b).
In this experiment, with a 4% missing area, the largest RMSE increment is 78\textit{nm}.

The slope matrix of a toroidal mirror of radius in the two orthogonal directions being 100\textit{mm} and 107.2\textit{mm} respectively is processed by the extended reconstruction algorithm to verify its effectiveness. The PV of the measured area is 0.5236\textit{mm}. For this experiment study, the error of the radius measurement in the two directions, $\varepsilon_{Rx}$ and $\varepsilon_{Ry}$, is evaluated.

In the first step, the effect of the missing data’s location is studied. A missing area of 5\textit{mm} in diameter sits at two different positions on the surface, as shown in Figure 7.10. The missing data covers 10\% of the entire surface area. In the next step, the effect of the missing data’s shape is studied. While the size of the missing area keeps the same, it is designed to be of rectangular shape of 7.5\textit{mm}×2.5\textit{mm}. The center of the missing area is the same as position 1.
Table 7.1 Effect of location, shape, and distribution of missing data. $\varepsilon_{Rx}$ and $\varepsilon_{Ry}$ denotes the error of the measured radius in the two orthogonal directions respectively.

<table>
<thead>
<tr>
<th></th>
<th>No missing data</th>
<th>Missing data at position 1, circular shape</th>
<th>Missing data at position 2, circular shape</th>
<th>Missing data at position 1, rectangular shape</th>
<th>Missing data at position 1 and 2, circular shape*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{Rx}$ (%)</td>
<td>0.0737</td>
<td>0.1974</td>
<td>0.2866</td>
<td>0.1974</td>
<td>0.2120</td>
</tr>
<tr>
<td>$\varepsilon_{Ry}$ (%)</td>
<td>0.0644</td>
<td>0.1050</td>
<td>0.2442</td>
<td>0.1050</td>
<td>0.3080</td>
</tr>
</tbody>
</table>

*The total size of the missing data is the same as other conditions.

Lastly, the effect of the missing data’s distribution is studied. Instead of only one position, the missing data spread over two positions. The total size keeps the same as 10% of the entire surface area. The two missing area is of diameter 3.56mm, and concentric with position 1 and 2 respectively. The radius error $\varepsilon_{Rx}$ and $\varepsilon_{Ry}$ resulted from these conditions are summarized in Table 7.1.

From the experiment results, it is observed that the accuracy of the reconstructed result in the presence of missing data is affected by location, shape, and distribution. Generally, if the missing data is at small slope regions, the result suffers less error than if the missing data at deep slope regions. Compared to other factors, varying the shape makes no significant difference in the measurement accuracy in this experiment. However, the final result is a combined effect caused by the size of missing data, its location, shape, and distribution.
7.3 3D reconstruction technique

Based on the previous studies, a 3D reconstruction technique is developed for freeform surface measurements with the second order derivatives as the raw data.

In the RFBS system, the raw data obtained after centroiding is the centroid positions before the lateral disturbance represented as \((X,Y)\); the centroid positions after the lateral disturbance introduced in the positive \(x\)-direction represented as \((X_{x+},Y_{x+})\); the centroid positions after the lateral disturbance introduced in the negative \(x\)-direction represented as \((X_{x-},Y_{x-})\); the centroid positions after the lateral disturbance introduced in the positive \(y\)-direction represented as \((X_{y+},Y_{y+})\); and the centroid positions after the lateral disturbance introduced in the negative \(y\)-direction represented as \((X_{y-},Y_{y-})\). Built on the non-linear model of the reference-free technique as described in Section 4.2, the 3D reconstruction technique calculates four matrices: \(\Delta S^x\), \(\Delta S^y\), \(\Delta S^{xy}\), and \(\Delta S^{yx}\) by using Equation (7-4), (7-5), (7-6), and (7-7) respectively.

\[
\Delta S^x = \frac{(X_{x+} - X - \Delta d_x) - (X_{x-} - X + \Delta d_x)}{2l \cdot \Delta d_x}
\]  \hspace{1cm} (7-4)

\[
\Delta S^y = \frac{(Y_{y+} - Y - \Delta d_y) - (Y_{y-} - Y + \Delta d_y)}{2l \cdot \Delta d_y}
\]  \hspace{1cm} (7-5)
\[ \Delta S^y = \frac{(Y_{x^+} - Y) - (Y_{x^-} - Y)}{2L \cdot \Delta d_x} \]  

(7-6)

\[ \Delta S^{yx} = \frac{(X_{y^+} - X) - (X_{y^-} - X)}{2L \cdot \Delta d_y} \]  

(7-7)

where \( \Delta d_x \) and \( \Delta d_y \) represent the lateral disturbance in the \( x \)- and \( y \)-direction respectively. With these four matrices, SOR is firstly applied on Equation (7-8) and (7-9) to get the slope data.

\[ S_{o}^x = \frac{1}{n} \sum_{i=1}^{n} \phi_i - \frac{p_x}{2} \left( \Delta S_i^x + \Delta S_o^x \right) - \frac{p_x}{2} \left( \Delta S_i^{yx} + \Delta S_o^{yx} \right) \]  

(7-8)

\[ S_{o}^y = \frac{1}{n} \sum_{i=1}^{n} \phi_i - \frac{p_y}{2} \left( \Delta S_i^{yx} + \Delta S_o^{yx} \right) - \frac{p_y}{2} \left( \Delta S_i^y + \Delta S_o^y \right) \]  

(7-9)

where \( p_x \) and \( p_y \) denote the pitch in the \( x \)- and \( y \)-direction respectively, \( n \) denotes the number of neighboring points. Then, from the study of the extended Southwell reconstruction, SOR is applied on Equation (7-3) with the slope matrix obtained in the previous step.

The cross terms \( \Delta S^{yx} \) and \( \Delta S^{yx} \) represents the slope changes measured in the \( xy \)-direction and the \( yx \)-direction respectively. Although these two terms can be neglected in spherical and flat surface measurements, they play an important role in accurate reconstruction of freeform surfaces. Take the toroidal mirror used in the previous section as an example.
Figure 7.11 (a) Error map of the reconstructed surface without considering cross terms. (b) Error map of the reconstructed surface by applying the 3D reconstruction technique.

Without considering the cross terms, the RMSE is as large as 191.4 nm, and the corresponding error map is shown in Figure 7.11(a). With the 3D reconstruction technique, the RMSE reduces to 14.3 nm, and the corresponding error map is shown in Figure 7.11(b).

**7.4 Summary**

In this chapter, five different surface reconstruction methods were designed, analyzed, and compared for measuring the surface with either the slope matrix, or the curvature matrix. For freeform surface measurement, direct fitting methods are more accurate theoretically, as they avoid the non-linear error introduced in the reconstruction process. However, in practical applications, where issues such as noise, abnormal data, or even missing data exist, reconstruction significantly
improves the stability and reliability of the measurement performance. Therefore, SRF method is suggested when the slope is the raw data, and CRRF method is suggested when the second order derivative is the raw data. Moreover, the extended reconstruction algorithm is designed to obtain the 3D surface with missing data points, which most likely happens at deep slope regions of the surface. Experiments have demonstrated that this new reconstruction algorithm can reinforce the system with stable and robust results under practical conditions, and enable the wavefront-based methods applicable for measurements of freeform surfaces. Based on this algorithm and with the non-linear modeling, a 3D reconstruction technique has been designed. The 3D reconstruction technique addresses the challenges in freeform surface measurements, being large dynamic range and varying curvatures, by calculating the second order derivatives in the $xy$- and $yx$-direction. In this way, freeform surfaces can be well reconstructed with $nm$-level accuracy, even when the raw data carries various noises.
Chapter 8 Error Analysis and System Optimization

Strategy

In this chapter, the error analysis of the system parameters is presented. The optimization strategy of the RFBS system is discussed and proposed.

8.1 Modulated beam design

The modulated beam, as the optical probe to sample the surface, can have different designs in measuring surfaces of various forms. And due to the flexibility of the light modulator in the system, different designs can be easily achieved. Toroidal mirrors were measured in the simulation platform to investigate the error contribution of each parameter, being pitch, fill ratio (FR), and the beamlet shape. As described in Chapter 3, pitch refers to the centre distance between the neighboring beamlets. FR is defined as the ratio of the beamlet size over the pitch. To guide the system design, the pitch and beamlet size are multiples of 0.032 mm, which is the pixel size of the light modulator.

Take the modulated beam of a design shown in Figure 3.7 as an example. The benchmarked PV of a toroidal surface over an area of 16 mm × 16 mm is calculated to be 1 mm. The RMSE of the reconstructed surface as compared to the nominal one is drawn in Figure 8.1.
As observed from Figure 8.1, while varying the pitch for 0.32\(mm\) results in variation of the RMSE of 7.0\(nm\), varying the diameter for 0.576\(mm\) results in variation of the RMSE of 3.1\(nm\). Therefore, the pitch needs to be optimized first for its larger effect on the system performance.

### 8.1.1 Pitch design of the modulated beam

When the pitch gets larger, the number of sampling points decreases. This will cause an increased error of the reconstructed surface due to insufficient sampling, as can be observed from Figure 8.2. As such, it is desirable to have a small pitch for high lateral resolution. Figure 8.2 shows that if 5\(nm\) is the cut-off RMSE, the maximum pitch is 0.448\(mm\), which is 14 pixels of the light modulator. If 10\(nm\) is the cut-off RMSE, the maximum pitch is 0.64\(mm\), which is 20 pixels of the light modulator.
Figure 8.2 RMSE of the reconstructed surface at different pitch settings. Different black lines represent constant pitch but varying FR.

Figure 8.3 Illustration of crosstalk due to insufficient pitch setting.

In the system design, the pitch cannot be too small. Otherwise, the distorted spots received at CCD may be too dense to window each spot correctly. Thus, the neighboring beamlets should be sufficiently apart to ensure a clear windowing without causing the spots crosstalk or overlap with each other as illustrated in Figure 8.3.
8.1.2 Fill ratio (FR) design of the modulated beam

For a fixed pitch, the smaller the beamlet size, which leads to smaller FR, the higher the accuracy is, as demonstrated in Figure 8.4. This is because of the sampling effect: with a larger beamlet, each probe covers more area and hence the averaging effect is larger, which will lead to lower lateral resolution and eventually poorer performance. Hence, it is desirable to have a small beamlet for least averaging effect.

It is observed in Figure 8.4 that if $5\,nm$ is the cut-off RMSE and the pitch is $0.448\,mm$ as determined in the previous section, the maximum FR would be 0.4286. When the pitch becomes smaller than $0.384\,mm$, which is 12 pixels, FR can be as large as one. If $10\,nm$ is the cut-off RMSE and the pitch is $0.64\,mm$ as determined in
the previous section, the maximum FR would be 0.25. When the pitch is smaller than 0.576\textit{mm}, which is 18 pixels, FR can be as large as one.

In the real situation, where the light modulator and CCD are both pixelated devices, there is a limit to how small the beamlet can reach. For stability and detectability, at least three pixels are required to form one beamlet. Therefore, the smallest beamlet would have a diameter of 0.096\textit{mm}.

The modulated beam as shown in Figure 3.7, incidents on a toroidal mirror of 100\textit{mm} radius in the \textit{x}-direction and 107.2\textit{mm} radius in the \textit{y}-direction. The pitch is 1.28\textit{mm}, and the scanning interval is 0.16\textit{mm}. The measured area is 18.8\textit{mm}×16.32\textit{mm}. The distance between the toroidal mirror and the CCD is 135\textit{mm}. The PV of the sampled area is thus calculated as 1.5014\textit{mm}. With these settings, beamlets of diameter 0.64\textit{mm} and 1.024\textit{mm} were measured. The received images are shown in Figure 8.5. The final results are summarized in Table 8.1.

Both simulation and experiment yields larger RMSE with larger beamlet size. Although the RMSE increment in the simulation is 13.7\%, the increment in the experiment is 73.7\%. The significant accuracy drop in the experiment is mainly because that crosstalk has already happened when the beamlet diameter is 1.024\textit{mm}, as can be observed when closely investigate the distorted spots, as shown in Figure 8.6. In addition, the distorted spots at the CCD show obvious diffraction ring, which has less effect when the beamlet diameter is 0.64\textit{mm}. 

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Figure 8.5 (a) Image received by the CCD when the beamlet diameter is 0.64\textit{mm}. (b) Image received by the CCD when the beamlet diameter is 1.024\textit{mm}.

The diffraction ring causes loss of signals, and this further increases error besides the deterioration resulted from increasing beamlet size and crosstalk.
Figure 8.6 (a) Illustration of the distorted pattern with beamlet diameter of 0.64 mm. (b) Illustration of the distorted pattern with beamlet diameter of 1.024 mm.

<table>
<thead>
<tr>
<th>Beamlet diameter (mm)</th>
<th>RMSE (nm)</th>
<th>Measured PV (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Experiment</td>
</tr>
<tr>
<td>0.64</td>
<td>20.5</td>
<td>98.2</td>
</tr>
<tr>
<td>1.024</td>
<td>23.3</td>
<td>170.6</td>
</tr>
</tbody>
</table>

8.1.3 Beamlet shape design

Beamlet shape corresponds directly to how the surface is to be sampled. Besides circular shape, many other designs are possible, such as diamond shape, ring shape, and square shape, as shown in Figure 8.7. From the study in the previous section, beamlet shape covers the smallest area is desirable.
Therefore, among the four designs, using modulated beam of diamond shape should generate the smallest error. However, in practical measurement, diamond shape should be avoided due to the phenomenon that diffraction is easily induced at four sharp edges. Also, the ring shape design is less preferable as compared to the circular shape design due to incomplete sampling.

The modulated beam design of circular shape and square shape were tested in both simulation and experiment on a toroidal mirror of radius $112.5\,mm$ in the $x$-direction and $137\,mm$ in the $y$-direction. The pitch is $0.96\,mm$. The scanning step is six. The beamlet diameter is $0.384\,mm$. The benchmarked PV calculated over a sampled area of $13.92\,mm \times 12\,mm$ is $1.0045\,mm$. The images received by the CCD with the two different beamlet shape designs are shown in Figure 8.8. The results are summarized in Table 8.2. From Table 8.2, it is observed that beamlet of circular shape generates more accurate result than that of square shape in both simulation and experiment. This is because circular beamlet covers smaller area, which has similar effect as smaller beamlet size.
Figure 8.8 (a) Image received by the CCD, when the beamlet is of circular shape. (b) Image received by the CCD, when the beamlet is of square shape.
Table 8.2 Results when beamlet is of different shape.

<table>
<thead>
<tr>
<th>Beamlet shape</th>
<th>RMSE (nm)</th>
<th>Measured PV (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Experiment</td>
</tr>
<tr>
<td>Circular</td>
<td>6.8</td>
<td>55.2</td>
</tr>
<tr>
<td>Square</td>
<td>9.0</td>
<td>56.4</td>
</tr>
</tbody>
</table>

It is also observed that while RMSE improvement of 24.4% is achieved in the simulation, only 2.13% improvement is achieved in the experiment. This means that while other settings keep the same, the shape of the beamlet design is not a significant error source in practical measurement. Hence, it is not necessary to adjust the beamlet shape design in measuring various surface profiles once other parameters are fixed. Pre-knowledge of the surface to be measured is not necessary.

8.1.4 Combination of scanning and lateral disturbance

As both the reference-free technique and the digital scanning technique require a digital shifting of the modulated beam, the scanning step can be optimized to achieve the two purposes simultaneously. From Chapter 5, an optimal scanning interval is $\frac{1}{4}$ of the pitch, and a recommended lateral disturbance value is $\frac{1}{7}$ of the pitch as studied in Chapter 4. As such, a scanning step of six is suggested.

8.2 Diverging beam compensation

8.2.1 Systems with a well-collimated modulated beam
When the modulated light beam is well-collimated, the CCD position has no effect on the measurement accuracy. This is because in the ideal simulation platform, rays coming in all directions can be perfectly detected and will contribute to the centroid calculation. As such, the CCD position does not affect the quality of the acquired image. A constant RMSE is observed when varying the CCD position in simulation.

### 8.2.2 Systems with non-collimating modulated beam and its compensation

In the real setup, it is difficult to ensure a well-collimated light source. To accurately measure the surface, it is therefore important to calibrate for the effect caused by the non-collimating beam. Generally, there are two methods to do the compensation.

The first method calibrates the pitch value at the surface, and compensates for the second order derivative measurements at the CCD plane. As shown in Figure 3.8, with a known diverging angle $\theta$, pitch $p$, and the lateral disturbance value $\Delta d$ at the light modulator plane, the pitch at the sample $p_{\text{sample}}$ is calculated from Equation (8-1). $h_1$ denotes the distance between the light modulator and the beam splitter; $h_2$ denotes the distance between the beam splitter and the surface to be measured; $h_3$ denotes the distance between the CCD and the beam splitter. The compensation value for the measured second order derivative matrix is calculated from Equation (8-2), where $N$ denotes the number of beamlets in the lateral disturbance direction. The second order derivatives of the surface $\Delta S$ are consequently calculated from Equation (8-3):
\[
p_{\text{sample}} = p \left( 1 + \frac{2(h_1 + h_2) \tan \theta}{N-1} \right) \quad (8-1)
\]

\[
\Delta d_{\text{CCD}} = \Delta d \left( 1 + \frac{2(h_2 + h_3 + h_1) \tan \theta}{N-1} \right) \quad (8-2)
\]

\[
\Delta S = \frac{C_{\text{after}} - C_{\text{before}} - \Delta d_{\text{CCD}}}{(h_2 + h_3) p_{\text{sample}}} \quad (8-3)
\]

where \( C_{\text{before}} \) and \( C_{\text{after}} \) denote the centroid position before and after the lateral disturbance respectively. The second method calibrates the calculated slope matrix after one-time reconstruction of the measured second order derivatives. Take the \( x \)-direction as an example. With a known diverging angle \( \theta \), the slope matrix resulted from the beam itself can be calculated as

\[
S_{\text{beam}} = \frac{\theta}{2} - \frac{\theta}{N-1} (i-1) \quad (8-4)
\]

where \( i \) is the index of the sampling point of interest. The second order derivatives are calculated as

\[
\Delta S = \frac{C_{\text{after}} - C_{\text{before}} - \Delta d}{(h_2 + h_3) p_{\text{sample}}} \quad (8-5)
\]
Figure 8.9 RMSE of the reconstructed wavefront when the modulated beam has various diverging angles.

And the slope matrix, $S$, is obtained after the one-time reconstruction. Then, the slope matrix of the surface, $S_{\text{surface}}$, is calibrated as

$$S_{\text{surface}} = S_{\text{beam}} + S \frac{1}{1 - S_{\text{beam}}S}$$  \hspace{1cm} (8-6)

The slope in the $y$-direction is calculated likewise.

Basically, the first method compensates for the diverging beam effect before reconstruction, while the second method compensates for the diverging beam effect after reconstruction. Simulation has been conducted to investigate the performance of the two methods. The sampled area is $16\text{mm} \times 16\text{mm}$, which results in a PV of $1.0213\text{mm}$. The pitch is $0.352\text{mm}$, with a beamlet diameter of $0.096\text{mm}$. The CCD is placed $150\text{mm}$ away from the surface. Under these conditions, the RMSE resulted at different diverging angle is drawn in Figure 8.9.
Figure 8.10 Illustration of the compensation modeling.

From Figure 8.9, it is observed that with the increment in the diverging angle, the error resulted from both methods increases. Also, the second method performs better than the first method. For both methods, without the pre-knowledge of the surface profile, the diverging angle effect cannot be fully compensated. This is because of the assumption that the three points are collinear: the centroid of the distorted spot at the CCD plane when the modulated beam is well-collimated (point A as shown in Figure 8.10), the centroid of the distorted spot at the CCD plane when the modulated beam is diverging (point B), and the intersection point on the CCD plane with the starting point at the corresponding sampled position and a direction vector parallel to the optical axis (point D). The accuracy of this assumption drops at deep slope regions, and also decreases with the increment in the diverging angle.
Figure 8.11 Example of two sampled points, with the corresponding centroid positions at the CCD plane when the modulated beam is well-collimated and has a diverging angle of $3^\circ$.

For example, Figure 8.11 shows two sampled positions on the surface ($D_1$ and $D_2$), with the corresponding centroid positions on the CCD plane when the modulated beam is well-collimated ($A_1$ and $A_2$) and has a diverging angle of $3^\circ$ ($B_1$ and $B_2$). The slope values at these two sampled positions are shown in Figure 8.12. At point $D_1$, the slope is much larger than that of point $D_2$. Therefore, the three points have a larger deviation from collinear at $D_1$. Consequently, the error of the slope after reconstruction is larger, as can be observed in Figure 8.13.

The increment in the RMSE along with the diverging angle, as observed from Figure 8.9, is due to the same reason: the collinear assumption becomes less accurate when the diverging angle increases.
Figure 8.12 The slope values at two sampled positions.

Figure 8.13 The error of the slope values at two sampled positions.

For example, at the same sampling positions, $D_1$ and $D_2$, the corresponding centroid positions when the modulated beam has a diverging angle of 1° ($B_1'$ and $B_2'$) are calculated and shown in Figure 8.14. It is clearly seen that $B_1'$ is closer to $\overrightarrow{D_1A_1}$, and $B_2'$ is closer to $\overrightarrow{D_2A_2}$. 
Figure 8.14 Example of two sampled points, with the corresponding centroid locations at the CCD plane when the modulated beam is well-collimated and has a diverging angle of 1° and 3° respectively.

Although the diverging angle effect cannot be fully compensated, the pre-knowledge of the surface specifications is not necessary in the compensation. If the information about the surface specifications is available, the diverging angle effect can be fully compensated by adding $S_{\text{comp}}$ to the slope matrix calculated from Equation (8-6).

$$
S_{\text{comp}} = S_{\text{spec}} - \frac{S_{\text{beam}} + S_{\text{spec-div}}}{1 - S_{\text{beam}}S_{\text{spec-div}}}
$$

(8-7)

where $S_{\text{spec}}$ is the theoretically calculated slope of the surface from its mathematical description; $S_{\text{beam}}$ is calculated from Equation (8-4); $S_{\text{spec-div}}$ is the theoretically calculated measured slope with the diverging angle effect. For example, under the same simulation condition as that of Figure 8.9, when the diverging angle is 3°,
with no pre-knowledge of the surface, the resulted RMSE is 98.2\textit{nm} by applying the second compensation method. With the pre-knowledge of the surface, the resulted RMSE is 3.2\textit{nm}. If the modulated beam is well-collimated, the corresponding RMSE would be 3.1\textit{nm}.

It is worthwhile to note that the second compensation method where reconstruction is conducted afterwards, although cannot fully compensate for the diverging angle effect in terms of RMSE; the form measurement error is 0\% for \(r_x\), and 0.068\% for \(r_y\). Hence, in the system, the second compensation method is recommended.

### 8.2.3 Experiment verification

With a non-collimating modulated beam, the CCD position also has no effect on the measurement accuracy. A constant RMSE is observed when varying the CCD position in the simulation platform.

The modulated beam in the developed RFBS system was measured to have a diverging angle of 0.49\textdegree. The beam as shown in Figure 3.7 samples a toroidal mirror, with radius of 137\textit{mm} in the \textit{x}-direction and 112.5\textit{mm} in the \textit{y}-direction. The pitch is 1.024\textit{mm}, and the scanning step is four.

The beamlet diameter is 0.4096\textit{mm}. The benchmarked PV over the sampled area of 15.616\textit{mm}×6.4\textit{mm} is 0.5374\textit{mm}. With these settings, CCD position of 195\textit{mm} and 215\textit{mm} were tested.
Figure 8.15 (a) Image received by the CCD at distance of 195 mm away from the surface. (b) Image received by the CCD at distance of 215 mm away from the surface.

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>RMSE (nm)</th>
<th>Measured PV (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Experiment</td>
</tr>
<tr>
<td>195</td>
<td>8.37</td>
<td>30.8</td>
</tr>
<tr>
<td>215</td>
<td>8.37</td>
<td>17.7</td>
</tr>
</tbody>
</table>

The images received by the CCD at the two distances are shown in Figure 8.15, with the red dashed rectangular box indicates the area of interest. The results are summarized in Table 8.3.

Different from the simulation conclusion, in the experiment, the further the CCD position, the higher the accuracy. This is because the CCD position has similar effect as that of the focal length in the SHWS system with lenslet array: the larger the distance, the higher the sensitivity, and thus the higher the accuracy. Besides
this levelling effect, in actual experiment, there is a threshold of the incident angle that can be detected by the camera pixel. This results in better image quality when the CCD is further away as the beam falls onto the same pixel would have smaller incident angle.

Figure 8.16 shows two distorted spots corresponding to the same sampling positions on the surface, but different CCD position. It is clearly seen that when the CCD is placed further away, the spots are clearer and thus larger SNR. When the CCD is placed closer, the spots are dimmer and blurrier. Consequently, higher performance is achieved with a larger distance between the CCD and the surface. Yet, when the distance is large, the measurable area is small. This means that there is a trade-off between the accuracy and the size of the surface area that can be sampled.

8.3 Optimization strategy of the RFBS system

From the simulation studies and experiment verifications, the first step to optimize the RFBS system is to set the pitch. Under the condition that there is no overlapping or crosstalk of distorted spots at the CCD, the pitch should be set small. If the neighboring beamlets are of different shape, or carries different colour coding, etc., the sampling points can be accurately registered even if crosstalk happens. Then the pitch can be set even smaller for high lateral resolution.
Figure 8.16 (a) Distorted spots when the CCD is 195mm away from the surface. (b) Distorted spots corresponding to the same sampling positions when the CCD is 215mm away from the surface.
The second step is to set the beamlet size. A small beamlet is desired for minimum averaging effect. Due to the hardware limitation, at least three pixels are required to form one beamlet. This value also depends on the light intensity, the robustness of the camera, and the reflectivity of the surface. As such, a proper beamlet size is set based on the image of the distorted spots. While as small as possible, the spots should not have identification issues and are stable.

The third step is to design the beamlet shape, which is not a big error source. Therefore, a circular shape is a universal and proper design for measuring surfaces of various profiles.

Lastly, as the modulated beam is not well-collimated, the CCD position affects the measurement performance in real applications. Generally, a large distance is desired for obtaining images of large SNR, which leads to high accuracy. However, the further the distance, the less number of sampling points can be captured. Therefore, under the condition that enough sampling points are obtained to cover the surface area of interest, the CCD distance should be as large as possible.

8.4 Uncertainty of the RFBS system

Same as all other equipment, the RFBS system has uncertainties in its measurement. The main uncertainty factors are discussed as follows.

1) Coordinate matching. The RMSE values are calculated by comparing the reconstructed surface with its mathematical equation. As such, it is of
significant importance to make sure of exact matching of the coordinate systems between the surface and the measurement system. In this dissertation, when measuring the freeform polynomial, this is achieved by transforming the coordinates of the reconstructed surface from the RFBS measurement and the UA3P scanned result. The transformation matrix is defined when the least square difference between the two surfaces are minimum. When measuring the toroidal mirrors, exact matching is realized by identifying the machining track of the surfaces from the captured images. The uncertainty in the coordinate matching process greatly affects the accuracy. However, the effect can be reduced by making a physical mark on the surface, which is a widely-adopted practice in freeform surface measurements.

2) Sampling positions. When calculating the RMSE value by comparing with the scanned data from another measurement system, it is essential to make sure of the same sampling positions in both systems. When measuring the freeform polynomial, this uncertainty is minimized by increasing the sampling frequency in the UA3P system. When measuring the toroidal mirrors, the uncertainty is minimized by comparing the fitted profile with the surface specifications.

3) Uniformity of the light beam. The uncertainty in the uniformity of the light beam brings error in the second order derivative measurements. The thus
induced zero-values error in the raw data eventually accumulates into profile calculation. In this dissertation, the effect of this uncertainty factor is minimized by using a new laser source and keeping a small distance between the laser source and the surface to be measured.

4) Alignment of components. In the RFBS system, components such as the CCD are to be aligned perpendicularly to the optical axis. The uncertainty of alignment introduces tilt in the reconstructed profile. In this project, this factor is minimized by a tilt removal process in both the reconstructed surface, and the profile to be compared with.

5) Surface fitting. Uncertainties in fitting of the reconstructed height data affect the accurate evaluation of the system performance, especially when measuring toroidal surfaces as the corresponding RMSE values are calculated after fitting. This factor can be minimized by ensuring a large sampling point matrix. In this dissertation, the matrix size is 100×88 to be used for the fitting process.

Uncertainties in the above described five factors affect the performance of the RFBS system. Some are unique, such as uniformity of the light beam; some are generic when measuring freeform surfaces, such as coordinate matching. While being inevitable, the uncertainties of these factors can be reduced with certain measures. In this dissertation, as discussed above, the effect of these factors has been minimized so to facilitate the accurate evaluation of the system performance.
8.5 Limitations of the RFBS system

Although being robust, the RFBS system still has certain limitations.

8.5.1 Type of freeform surfaces

As in all systems that measure the wavefront, it is essential that the surfaces are continuous and smooth. As such, discontinuous surfaces with steps such as Fresnel lens, structured surfaces such as microlens array shown in Figure 2.8, and multiple surfaces on a single substrate are beyond the system’s measurement capability.

8.5.2 Measurement range

When measuring convex surfaces, the measurement range is limited by the field of view of the CCD.

From simple geometry, the relationship between the radius of the surface and the required CCD size is established as Equation (8-8).

\[
D_{\text{CCD}} = D_{\text{spec}} + \frac{2D_{\text{spec}}l \sqrt{4r^2 - D_{\text{spec}}^2}}{2r^2 - D_{\text{spec}}^2}
\]  

(8-8)

where \(D_{\text{CCD}}\) indicates the diameter of the required CCD field of view, \(D_{\text{spec}}\) indicates the diameter of the surface, \(l\) indicates the distance between the surface and CCD, and \(r\) indicates the radius of the surface to be measured. For example, in the current setup, \(D_{\text{CCD}} = 24.035mm\). Assume the distance between the surface
and CCD is 50
mm, and \( D_{\text{spec}} = 15 \)mm, the maximum measurable radius is calculated to be 166
mm.

When measuring concave surfaces, the measurement range has less limitation as the CCD can be placed after the focal plane where the beam starts to diverge. Nevertheless, the limitation when measuring convex surfaces can be reduced by using a converging beam, instead of a collimated beam.

### 8.5.3 Reflectivity

The RFBS system is ideal for mirror surfaces with high reflectivity. The lower limit of the reflectance tested in this dissertation is around 70\% at wavelength of 632.8
mm.

### 8.6 Summary

This chapter analyses the error contribution of various system parameters and studies the optimization strategy of the RFBS system for measuring freeform surfaces. Three key design parameters were studied in both the simulation platform and the lab setup. Generally, in practical measurement, large detecting distance together with small beamlet size and small pitch value yield higher system performance. In addition, the compensation methods when the modulated beam in the system is non-collimating are discussed. With pre-knowledge of the surface form, the diverging angle effect can be fully compensated. Without pre-knowledge of the surface form, the RMSE will increase, depending on the divergence of the
modulated beam. For example, if the diverging angle is 3°, the RMSE will increase by 95.1 nm after compensation when measuring a toroidal mirror of 1.0213 mm PV. Nevertheless, the radius measurement is fully compensated even there is no information of the surface profile.

With this proposed optimization strategy, the measurement performance as presented in Chapter 3 is achieved: nm-level accuracy is achieved in the RFBS system when measuring freeform surfaces of mm-level PV.
Chapter 9 Conclusion

Utilizing more and more freeform surfaces are the emerging trend in many fields, such as the optical industry. The high flexibility of the surface forms brings advantages in the application, while imposes difficulties in the dimensional measurements. As the performance of the freeform surfaces is limited by its form accuracy, it is important to measure freeform surfaces accurately. This dissertation aims to devise and develop an area-based non-contact methodology and system for measuring freeform surfaces of optical quality with nm-level accuracy.

From literature review, it is found that well-known non-contact measurement techniques, such as interferometry, require mechanical scanning, additional components, or the accurate prescription of the surface to be measured if applied on freeform surface measurements. These requirements bring in limitations such as increased measurement time, limited applications, and degraded performance.

Compared to other techniques, the Shack-Hartmann wavefront sensing (SHWS) technique stands out for its insensitivity to vibration, no need of a coherent light source, while being an area-based technique and having the same accuracy as interferometry. However, SHWS is only for flatness and aberration measurements. If apply it for measuring freeform surfaces, sampling and referencing become a big issue, together with limited lateral resolution and difficulty in data processing. To
tackle with these issues, the reference-free beam sampling (RFBS) methodology and system has been devised and developed.

The RFBS system uses the modulated light beam to sample the surface to be measured. The CCD receives the distorted spots, the centroid positions of which carry the surface profile information. The challenge of sampling is addressed as the measurement performance does not rely on the quality of focusing of the received image. Also, the methodology is non-contact and area-based. A wide range of surfaces can be sampled with minimum modification to the system. The challenge of referencing is addressed by measuring the second order derivatives through introducing a lateral disturbance to the modulated beam. Surfaces of various forms can thus be measured as the methodology does not need a reference piece of similar form and material properties to obtain the raw data for further processing.

The digital scanning technique is proposed in the sensing mechanism to increase the lateral resolution without sacrificing the sensitivity and measurement range, and also play an important role in nm-level measurement.

With the received images, the dynamic windowing technique is applied to facilitate register freely-distorted spots, and their centroid positions are detected through the adaptive centroiding technique. By applying the 3D reconstruction technique, surface form is revealed. The three data processing techniques automatically process the obtained raw data and reveal the surface. They are highly robust and
reinforce the system with \textit{nm}-level measurement accuracy in presence of noise and varying image distortions.

All the proposed techniques in the RFBS system has been theoretically studied and verified in the simulation platform and experiment setup. After studying the influences of various parameters on system measurement performance theoretically and experimentally, the settings have been optimized. Freeform surfaces of various forms were measured in the RFBS system. 50\textit{nm} accuracy is achieved when the PV of the surface to be measured is 1.5mm.

The RFBS system does not need an absolute reference piece for measuring the surface form. There is no need of mechanical scanning to achieve high lateral resolution, no need of additional components to compensate for the large dynamic range of freeform surfaces to be measured, no need of coherent light source or the prescription of the surface profile. Therefore, the RFBS system can be integrated into a compact device for in-situ measurement with a white light source. It has a promising future toward the realization of large area freeform surface in-situ measurement.

\section*{9.1 Contributions}

A non-contact freeform surface measurement methodology with \textit{nm}-level accuracy
In this dissertation, a non-contact freeform surface measurement methodology is invented. This method uses the light beam itself to sample the surface, and the reflected beam is received by an image detector. Through a lateral disturbance of the light beam, the method measures the surface second order derivatives. A further digital scanning of the light beam increases the lateral resolution.

The proposed method does not need additional physical sampling aperture, which makes the method highly flexible. Also, since the measurement does not involve a reference piece, many complicated surface profile can be measured. In addition, the method can reach nm-level accuracy as no mechanical movement is involved, no repositioning of surfaces is required, and various parameters can be customized for specific application requirement. Therefore, freeform surfaces can be measured, and also with less cost, short preparation time, and high accuracy.

**Data processing algorithms for freeform surface measurements**

As freeform surfaces have varying curvatures, the measurement raw data normally has poor consistency and changing signal to noise ratio. This dissertation proposed the dynamic windowing algorithm and adaptive centroiding algorithm. These two algorithms are least affected by the inconsistency of the data to be processed, while capable of automatically identify the real signals and register them accordingly. The high robustness of the two algorithms not only makes them feasible in presence of varying image degradations, but also extracts signals with high accuracy. Furthermore, a 3D reconstruction algorithm is developed to more
accurately calculate the surface profile from the second order derivatives. The 3D reconstruction algorithm can even be applied when some data is missing caused by the large dynamic range of freeform surfaces.

Not only in the RFBS system, the proposed data processing algorithms have better performance in other differential measurement systems as well. With these algorithms, the profile of freeform surfaces can be obtained with less human input and higher accuracy.

A lab-system for freeform surface measurements

A lab-system of the RFBS methodology is developed. Through demonstration on freeform surfaces of various forms, the capability of the RFBS system in freeform surface measurements is shown. The RFBS system is able to measure freeform surfaces with over 1 mm PV value in a short time and can achieve tens nm-vertical accuracy after optimization. Therefore, freeform surface measurements are realized in the proposed RFBS system.

9.2 Future study

There are two directions for future work: stitching of measurement data and prototyping of a compact and portable device for in-situ measurements.

9.2.1 Data stitching
The field of view of the RFBS system is limited by the specifications of the light modulator and the CCD. The light modulator determines the surface area that can be sampled. The active area of CCD determines the size of the reflected beam that can be captured. The final measurable area is also affected by the system configuration and the surface profile. To achieve large area measurement, it is thus desirable to measure different surface areas and then stitch the data together.

The unique feature of the RFBS system is that the raw data obtained is the second order derivatives. Therefore, the stitching process can be conducted directly on the raw data, or on the intermediate reconstructed slope data, or on the surface height data. Each approach would have its advantages and disadvantages.

Preliminary studies through simulation have shown that if the stitching is conducted on the second order derivatives, the transformation matrix can be accurately calculated, and the whole surface area can be reconstructed with the smallest error as compared to other approaches. In addition, stitching on the second order derivatives is less sensitive to noise as abrupt changes are smoothed out at the reconstruction step. However, this approach results in a large matrix for reconstruction. As a consequence, the processing time is greatly increased.

More studies needed to be carried out for a stitching algorithm that best suits the RFBS system. This algorithm should be able to work together with the reconstruction process, so that while noises are filtered out, the surface information
is not lost. Furthermore, the algorithm should require least computation time for real-time measurements.

9.2.2 Prototyping of a compact and portable device

The RFBS is a lab system for the time being. As analyzed in the previous section, it has high potential for large area freeform surface in-situ measurements. In the future, the laser is to be replaced by a white light source. With the synchronization of digital scanning of the modulated light beam and the capturing of the image by the CCD, the device can automatically do the measurement. When two lenses (4f) are added into the system, the measurable area of the system can be enlarged. If the two lenses are added in an inverse direction, small features of the surface can be examined, which makes waviness, and even roughness measurement possible. The prototyping of a compact and portable device is of great value for in-situ measurements, while can be customized for different application requirements.
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Journal


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