REDUCING TRANSACTIONAL RISK VIA
PROTOTYPING IN DESIGN OUTSOURCING

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ABSTRACT

Design outsourcing is gaining popularity in many industries, such as electronics, aircraft, and automobile etc. Due to its innovative and unpredictable nature, design outsourcing bears more risks in transaction than traditional manufacturing and service outsourcing. In this context, transactional risk is defined as the risks resulted from inherent uncertainty regarding the design outcome faced by both buyers and sellers. Prototypes are commonly used in design outsourcing to elicit customers’ needs and convey designers’ capabilities, such that the information asymmetry and transactional risk would be reduced. However, in capital intensive industries, such as pharmaceutical, and aerospace, etc., prototypes are usually very expensive. Three quantitative models are developed in this research to analyze the trade-off between the value and cost of prototyping in three different, but progressive design outsourcing scenarios: (1) one buyer vs. one designer through single prototyping phase, (2) one buyer vs. one designer through multiple prototyping phases, and (3) one buyer vs. multiple designers through multiple prototyping phases.

The single-step prototype model refers to a scenario with bilateral contracting, in which a buyer needs to decide whether to buy the design from a seller. However, the buyer faces uncertainty regarding the value of the design, while the seller faces uncertainty regarding the cost of the design. These uncertainties drive the buyer and the seller to ask for extra premiums to hedge their risks, which could result in no deal even though a mutually beneficial transaction is possible. This model combines utility function and Bayesian updating to quantify the tradeoff between the risk reduction effect and cost of prototyping in design outsourcing. The results show that the prototype indeed reduces the transactional risk and improves customer’s utility.
The development of complex systems usually consists of multiple phases. A multiple-step prototyping model is subsequently developed to investigate transactional risk reduction over multiple phases. Instead of making a lump sum investment at front end, the customer can make multiple investment decisions along the development phase with more flexibility. A Bayesian-based real option model is developed to quantify the value of the prototypes. The result suggests that the traditional real option model overestimates the design value, which may lead to excessive investment on unnecessary prototypes.

The third model proposed in this research aims to study a multi-lateral scenario with one customer and multiple competing designers, which is more representative of design outsourcing in industries. A systematic quantification of transactional risk based on real option theory is investigated, in conjunction with a study on the customer’s screening decisions along the prototyping process. Numerical study is conducted to verify the validity and performance of the proposed models. The results show that this model is able to locate the optimal prototype value threshold to screen out incompetent design and maximize the customer’s expected payoff.

This research extends the research frontier in design outsourcing through introducing three innovative models to quantify the risk reduction through prototyping, to value the prototype in multi-phase development scenarios, and to optimize the buyer’s expected payoff in design contest. Practically, this research is able to assist the management to make informed and cost-effective prototyping decisions in design outsourcing.
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CHAPTER 1: INTRODUCTION

This chapter provides an overview of the background knowledge leading to this research. Through discussing the research context and motivation, the research problem is identified as reducing transactional risk via prototyping in design outsourcing. Accordingly, research objectives and scopes are defined, along with an outline of the research report.

1.1 Design Outsourcing

The source of innovation is increasingly coming from outside of traditional corporation boundaries instead of from within. According to a study on BusinessWeek (Engardio, B. Einhorn et al. 2005), 20% of the designs for mobile phones, 30% for digital cameras, 65% for notebook PCs, and 70% for PDAs are outsourced by original equipment manufacturers (OEM) to suppliers or specialized design firms. Design outsourcing is not constrained to electronics industry, but is spreading to other industries like commercial airplanes, as showcased by Boeing’s revolutionary development of Boeing 787 Dreamliner (Economist 2006). In the automobile industry, manufacturers outsource the designs of complete systems, such as dashboards, seats and safety systems, to suppliers that have the ability to provide entire systems (Zirpoli and Becker 2011).

However, unlike production, design is essentially a type of service with high density of knowledge content and creativity. Thus, the design outcome is highly unpredictable due to its innovation nature. The uncertainty of design outcome embeds risk in the design outsourcing transaction, which can result in significant losses for the design buyer.
1.2 Transactional Risk in Design Outsourcing

Design is about solving an innovative problem that requires integration of need information and solution information (von Hippel 2005). The innovative essence of design results in information asymmetrically distributed between the buyer and the designer in the context of design outsourcing. On the one hand, the buyer of a design may not be technically savvy or have the competence to precisely specify the design requirements (Zipkin. 2001). In other terms, the need information is ambiguous and uncertain to the designer. On the other hand, the designer’s capability cannot be accurately assessed by the buyer (Anton and Yao 2002), which distorts the solution information. In this research, the risk resulted from the inherent uncertainty regarding the design outcome and the information asymmetry between the buyer and seller when they are engaged in contracting for potential design outsourcing is defined as transactional risk, which is a major risk in design outsourcing process (Terwiesch and Loch 2004).

In the presence of transactional risk, the buyer and the designer may forgo the transaction opportunity due to their uncertainties over design value and design cost, respectively. These uncertainties over the value and cost can be formulated in terms of normal distributions, with mean value indicates expected value and cost, and variance represents the degree of uncertainty. Further, it is reasonable to assume that people are risk averse, and under the circumstances that there’s risk involved, both parties will ask for risk premiums to hedge their exposure to uncertainties, which result in lowering the buyer’s expected value and raising the designer’s expected cost. In particular, the higher the transactional risks, the higher the risk premiums will be demanded. Thus, a potential deal will possibly be impaired due to the high transactional risk.
The transactional risk can be reduced through elicitation of the buyer’s needs, and at the same time, conveying designer’s capabilities to his counterpart. Reaching an agreement over design outsourcing is often an iterative and lengthy process with laborious back-and-forth negotiations; and during this process, miscommunication and misunderstanding do exist between the buyer and the designer. Thus, the buyer is keen to see the details of each phase design prior to committing in purchasing the final design or mass production, such that they are able to better estimate the design value.

1.3 Prototyping as a Means to Reduce Transactional Risk

Prototyping is a good way to elicit the buyer’s needs, and to introduce the designer’s competency at the same time (Ulrich and Eppinger 2011). Prototyping is commonly deployed in nearly every stage of product development process. The types of prototypes vary in a diverse spectrum according to the stage of development of the product in fidelity detail, medium of construction, and manufacturing process, which all play important roles but address different aspects of the product development, with different purposes and intents.

Despite the fact that prototypes are practical in risk reduction (Barkan and Iansiti 1993), the high prototyping cost in capital intensive industry is a considerable issue. For instance, the development and prototyping process of a commercial aircraft could easily cost hundreds of millions of U.S. dollars (Thomas 2001). The investment decision in prototyping should be primarily made based on three parameters, (1) the present risk in the transaction, (2) the potential risk mitigation after prototyping, and (3) the prototyping cost. A quantitative model to examine the design buyer’s payoff in terms of these three parameters is much welcomed by the decision makers.
1.3.1 An illustration of prototyping process in aircraft design outsourcing

Prototyping is crucial in capital-intensive product development in terms of detecting unanticipated phenomena and reducing the risk of costly iterations (Ulrich and Eppinger 2011). In design outsourcing context, the prototyping is no longer an internal process, but a collaborative work between both of buyer and designer. For illustration purpose, the collaborative prototyping process between the U.S. Department of Defense and its contractors, in a new aircrafts development, is presented in the following.

The U.S. Department of Defense (DoD) is the largest single consumer in the world of purchased equipment. The Department buys about $45-90 billion worth of equipment and R&D services per year, within which $5-7 billion on air force equipment (Brown and Corzine 1998). Military aircraft projects have large risks due to the use of fledgling technologies, long development duration and huge investment (Kundu 2010). The prototyping process is crucial in reducing the risks but very expensive. Figure 1 illustrates a typical aircraft development process which is consisted of five decision-makings by the DoD and four phases of prototyping by the contractor.

Initially, DoD would issue a Request for Information (RFI) to the aerospace industry for new aircraft equipment concept. This request includes the basic requirements for the new vehicle, such as stealth necessity, maneuverability, supercruise and etc.

DoD’s RFI is then followed by the responding contractors’ phase 1 prototyping, conceptual design. Contractor conducts feasibility study and proposes new conceptual vehicle design in-line with DoD’s request and his own design capability. The design specification is accomplished through aircraft sizing, engine matching, preliminary weight estimation, and evolution of a family of aircraft with payload and range combinations.
### Figure 1.1 Military aircraft development process (Cosden 2002; Kundu 2010)

After receiving contractors’ proposals, DoD would review and then choose the preferred designs, and further fund developments of the chosen design concepts. In the case of F-22 “Raptor” development, the DoD provided $1 million each to Lockheed and other 5 companies for next phase of concept exploration (Cosden 2002). Figure 1-2 illustrates the procurement process in terms of the designs submitted by each design bidding firm.

<table>
<thead>
<tr>
<th>Department of Defense</th>
<th>Contractor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Timeline</strong></td>
<td><strong>Phase 1: Conceptual design</strong></td>
</tr>
<tr>
<td>Requirement definition</td>
<td>Generate aircraft specifications from DoD’s requirement, assess competition, set technology level, aircraft sizing, engine matching, airworthiness, and resource budget appropriation.</td>
</tr>
<tr>
<td>Review the proposal</td>
<td><strong>Phase 2: Project definition</strong></td>
</tr>
<tr>
<td>Choosing the preferred design and awarding funding to expand the concept.</td>
<td>Analysis and tests, performance guarantees, structural layout and stressing, system architecture, risk analysis, wind-tunnel and ground tests, and so on.</td>
</tr>
<tr>
<td>Design validation</td>
<td><strong>Phase 3: Detailed design</strong></td>
</tr>
<tr>
<td>Investigating the methods, procedures and technologies proposed by contractor.</td>
<td>Detailed parts design finished, parts fabrication, tests completed, and so on.</td>
</tr>
<tr>
<td>Accept the design</td>
<td><strong>Phase 4: Certification</strong></td>
</tr>
<tr>
<td>Mass production</td>
<td>Aircraft assembled, first flight and tests completed, compliance with standards, and so on.</td>
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</table>

Figure 1-1 Military aircraft development process (Cosden 2002; Kundu 2010)

After receiving contractors’ proposals, DoD would review and then choose the preferred designs, and further fund developments of the chosen design concepts. In the case of F-22 “Raptor” development, the DoD provided $1 million each to Lockheed and other 5 companies for next phase of concept exploration (Cosden 2002). Figure 1-2 illustrates the procurement process in terms of the designs submitted by each design bidding firm.
Upon receiving funding, the contractors subsequently resume phase 2 prototyping, *Project Definition*. The contractors are not asked to provide detailed blueprints in this phase; rather to expand upon the initial concept and develop a plan for processes to be used in the design and manufacture of the aircraft.

DoD subsequently investigated the methods, procedures and the technologies proposed by the contractors. If the design is approved, DoD will follow up investment on the next phase prototype, *Detailed Design*. In the F-22 project, Lockheed and Northrop each received $691 million from DoD for detailed design.

In phase 3 prototyping, peak manpower is deployed to push the project to completion. At the end of this phase, the aircrafts assembly and tests were near completion. In the case of F-22 development, DoD was favorable to Lockheed’s design in the end. The *risk and cost* were the determining factors in the final decision making, which quoted from one of air force office documents, "Advanced Tactical Fighter to F-22 Raptor” (Aronstein, Piccirillo et al. 1998),
“Lockheed ... was rated higher in their technical proposals and their plans for managing the development program ... the main factor in each of these ratings were the Air Force's assessment of risk.

Lockheed ... was considered more likely to accomplish what they proposed, and to manage the development program successfully ... the Air Force also concluded that the Lockheed aircraft would cost less ... The difference was not great ... but a few percentage points in as high-cost a program as the ATF involve large amounts of money.”

The duration of design development varies due to the degrees of complexity of the project. Generally, a conventional civil aviation development normally takes 2 to 3 years (Kundu 2010), however, the F-22 development took 16 years from the Request of Information to the first flight (Cosden 2002). The longer development duration, the more expensive each prototyping stage is, and the higher risk undertaken in the transaction. A quantitative analysis of the trade-off between risk reduction and cost burden of prototyping is very much expected and critical. However, the relevant literature is rare to be found.

1.4 Research Objectives and Scope

Regarding the investment decision upon prototyping, especially in capital-intensive industries, a number of significant but intricate questions are raised, including whether it is cost-effective to build a prototype? Who should pay for the prototype? How much the final product should be priced? How many prototypes should be built? And how to optimize a design contest mechanism to maximize the buyer’s expected payoff?
In order to tackle these questions, this research aims to build quantitative models to assist decision making in prototyping to reduce transactional risk in design outsourcing. Three quantitative models are developed in sequence to cater different transaction scenarios. The first model is constructed under a simplified setting of single contractor and single-phase prototyping. The second model relaxes the previous model into multi-phase prototyping. The third model further extends the second model into a more practical multi-contractor and multi-phase prototyping scenario.

1.4.1 Single designer, single-step prototype model

In this simplified scenario, the design buyer deals only with a single designer; and all the prototyping stages are summed and simplified to a single step. The buyer needs to decide whether to buy the design from the seller. However, the buyer faces uncertainty regarding the value of the design, while the seller faces uncertainty regarding the cost of the design. The decision to be made here is whether the buyer and seller should engage in prototyping before making the contracting decision, and if so, who should pay for it.

This research takes the perspective from the customer, who concerns the uncertain value of the design. Exponential utility function is employed in this single-step prototype model to gauge the decision maker’s willingness to tap into the outsourcing contract on account of his risk attitude, risk level, prototyping cost, expected value and etc. To the customer, the value of the prospective prototype is assumed to follow a normal distribution, of which the variance indicates the transactional risk level.

A distinct feature of this research is that a Bayesian updating process is used to assess the risk reduction through prototyping (Chen and Sun 2011). A prototype can feed ad-
ditional information to the prior estimation of design value, which is then updated to a posterior distribution. A decision models subsequently developed to weigh the prototyping effects of risk reduction and cost burden, and then to assist the customer’s decision making upon prototyping or not. A simulation based numerical analysis is conducted to investigate the prototyping decision with respect to a number of factors, such as decision maker’s risk attitude, design fee quoted by designer, and etc. This model thus provides a framework for investment decision upon prototyping in design outsourcing.

1.4.2 Single designer, multiple-step prototypes model

In reality, the development of complex products and systems, such as designing aircraft or architecture, often involve a series of prototypes in multiple stages. Instead of summing all the development phases into one, the multiple-step prototypes model provides a more flexible decision-making process, which assumes that the customer invests and collaboratively works with designer at every individual stage, and has the right to cancel the deal after each prototyping phase. The risk reduction effect, cost of prototyping, and development duration are examined at every stage.

In order to evaluate the decision-making at each phase of product development, real option analysis is employed to the multiple-step prototyping model. The idea of real option is originated from the option instrument in financial market. The option is a right but not obligation to purchase (call option) or sell (put option) the underlying asset (e.g. stocks or real assets) at a pre-determined price at (European option) or before (American option) a specific date (Datar and Mathews 2004). In the context of multiple-phase design outsourcing, prototypes give the customer the right, but not obliga-
tion, to purchase the final design from the designer. This is equivalent to buying a call option on the final design.

The traditional real option analysis assumes a constant risk level (volatility) throughout the whole development process. However, in this research, an original method that combines both real option valuation and Bayesian updating is developed to take the learning effects of each design phase into consideration.

1.4.3 Multiple designer, multi-prototyping phases

The third model is constructed under a more realistic design outsourcing setting, where the buyer purchases a single design out of multiple designers through multiple phases of design contest, in which the prospective designers compete with each other in dimensions of both proposed design values and asked prices. A design contest mechanism is built based on real option analysis and auction theory.

Design contest is a reversed auction form, where the buyer is the auctioneer and the designers are the contestants. In a design contest with multiple phases of competitions, it is crucial for the auctioneer to only select the competent designer(s) to the next phase based on the prior and current information to maximize his expected economic payoff.

The investment on prototype is similar to purchasing a call option. An alternative option model is developed to serve as a tool for the auctioneer to locate the optimal prototype value threshold and screen out the incompetent designer(s). A numerical study is then illustrated to validate the optimality of the calculated threshold.
Figure 1-3 Research framework

Figure 1-3 illustrates the research framework in two dimensions, number of prototypes and number of players. The three models proposed in this research are progressively developed from a simplified case to more realistic and more complicated scenarios.

1.5 Organization of the Report

The remainder of this report is organized as follows: Chapter 2 is literature review which broadly surveys the past relevant literatures on design outsourcing and prototyping in design development. The basic concept and recent development in each of these research streams are critically reviewed, and the gap in the existing literature is summarized. In Chapter 3, a single-step prototype model is developed. The Bayesian
updating method is introduced in this chapter to quantify the risk reduction through prototyping. A utility model is then developed to quantify the tradeoff between benefit and cost of the prototype. In Chapter 4, the single-step model is extended to a more detailed multi-step one. A Bayesian-based real option method is employed to calculate the prototype value in the presence of information updating. In Chapter 5, a multi-player and multi-phase design contest is developed to take competition among the designers into consideration. An optimal prototype value threshold is located to maximize the buyer’s expected payoff in the design contest.
CHAPTER 2 : LITERATURE REVIEW

design outsourcing is gaining popularity in both industry application and academic research. Although papers specifically addressing risk modeling and reduction in design outsourcing are still relatively scarce, there has been a large and growing body of literature on related topics in both engineering and management disciplines. This chapter reviews literature that is most relevant to design outsourcing and its associated risks, design risk modeling especially in a supply chain context, and the use of prototyping for design risk reduction.

2.1 Design Outsourcing

Design outsourcing can be viewed as a relatively new and more sophisticated form of outsourcing, which has been pervasive in modern economy. The most reported form of outsourcing has been production outsourcing. A lot of research has looked into various aspects of outsourcing in general. The literature review of this research starts by summarizing key research findings regarding the benefits, costs, and risks associated with outsourcing in general, highlighting the key differences between design outsourcing and production outsourcing.

2.1.1 Outsourcing in general

Outsourcing activities have been widespread in the past several decades with the rise of globalization and advancement of technologies, especially telecommunication and the Internet (Jennings, 1997; Quelin and Duhamel, 2003). On the macro level, Grossman and Helpman (2005) provide an economic model that explains the phenomenon of out-
sourcing in the global economy. On the micro level, the benefits that companies can derive from outsourcing have been well documented, ranging from the reduction of operational costs (Lacity and Hirschheim 1993), the ability to transform fixed costs into variable costs (Alexander and Young 1996), the ability to transfer operational and financial risks (Alexander and Young 1996), to the ability to focus on core competencies (Quinn and Hillmer 1995) while having access to the industry-leading external competencies and expertise (Kakabadse, 2002).

There is, however, also a large stream of literature that discusses the risks associated with outsourcing arrangements, particularly where non-peripheral business functions are concerned (Kremic, Tukel et al. 2006). Alexander and Young (1996) highlight the risk of becoming dependent on a supplier, Barthelemy (2001) draws attention to the hidden costs of outsourcing, and researchers such as Doig et al. (2001) and Quinn and Hilmer (1994) identify the possibility of a loss of vital know-how with respect to core competencies as a major risk factor in outsourcing. There is also the problem of selecting the most suited service provider and their longer-term ability to offer the capabilities that are needed in particular business environments with rapid technology change (Quinn 2000).

Problems can also occur when the customer is dependent on the contractor’s capabilities without alternative suppliers. As in the case of Japan tsunami in 2011, Apple Inc. along with many other consumer electronics companies lost their key chip suppliers in Japan, which resulted in under production and supply shortage (Neville 2011). Risk of data leakage and chances of fraud by third party suppliers can stem from the cases when the company has less control over the services that have been outsourced and also the people who are rendering these services. Hoecht and Trott (1999) demonstrat-
ed that there are trade-offs between access to cutting-edge knowledge via collaborative research and technology development in knowledge-intensive industries and the risk of losing commercially sensitive knowledge to competitors.

Weighing the benefits against the costs and risks, outsourcing has been traditionally concentrated in peripheral or none-strategic business functions like labor-intensive manufacturing and general administrative activities (Leavy 2004). Production outsourcing in the context of a supply chain has been extensively studied in particular. The decision concerning whether to outsource production or not is usually framed as a “make or buy” decision (Ulrich and Ellison 2005). The risks involved in production outsourcing have been generally studied under supply chain risk management (Jüttner, Peck et al. 2003). The primary risks involved in production outsourcing have been generally associated with uncertainty in capacity, lead time, demand, and costs etc. (Tang 2006). Many quantitative models have been developed to measure and manage risks involved in production outsourcing. For example, Schniederjans and Zuckweiler (2004) develop a quantitative approach for outsourcing/insourcing decision making in an international context. Liu et al. (2008) propose a heuristic approach based on genetic algorithms for production planning with outsourcing. Strategies and practical guidance have also been proposed to address various operational issues and risks in outsourcing (Amaral, Billington et al. 2006).

2.1.2 Design outsourcing vs. Production outsourcing

Recently, outsourcing has moved beyond production to much more strategic and vital services which include design, research and development. In an industry report, BusinessWeek reported the increasing breadth and depth of design outsourcing in the consumer electronics industry that includes PDAs, mobile phones, telecom equipment etc.
(Engardio, B. Einhorn et al. 2005). Chesbrough (2003a, 2003b) coined the term “open innovation” to describe the phenomenon of companies’ increasing reliance on external resources for better and faster design and innovation in an increasingly open and connected economy. Companies not only outsource design tasks to their supply chain partners and specialized design firms, but also to the public through 3rd party online platforms like innocentive.com by means of design contests (Terwiesch and Xu 2008). Von Hippel (2005) succinctly summarizes this new trend towards increasing design outsourcing as the democratizing of innovation. Outsourcing of innovation has been cited as the new engine of growth in developed economies by Quinn (2000).

Although much of the research findings on production outsourcing can be carried over to design outsourcing, there are some significant differences between these two forms of outsourcing. Fundamentally, production is more a “labor-based” activity, while design is more a “knowledge-based” activity. According to Simon (1996), design is essentially a process of problem solving with the objective to create an artifact, which does not exist. In contrast, production is mainly a process to realize and replicate a design with high quality and efficiency (Ulrich and Ellison 2005). As a result, there is inherent uncertainty regarding the outcome of design, while the outcome of production can be checked and verified unambiguously against the design of a product. It has been generally recognized that design outsourcing involves higher risks and is more difficult to manage relative to production outsourcing (Tasy 2010). Recent high profile cases like Boeing 787 Dreamliner have highlighted the pitfalls and potential disastrous consequences of mismanaged design outsourcing projects (Tang and Zimmerman 2009). How to effectively deal with risks in a distributed design environment, especially in a supply chain context, has been identified as a critical research issue in design outsourcing (Gassmann 2006, Hsuan and Mahnke 2011).
2.1.3 Transactional risk in design outsourcing

One particular risk in design outsourcing is the transactional risk during the contracting stage, which is the primary focus of this research as defined in Chapter 1. Terwiesch and Loch (2004) give a similar definition by referring transactional risk as the possibility that the contractor (i.e. designer) does not provide what the customer expects. The source of transactional risk in design outsourcing can be traced to the uncertainty regarding the design outcome during the contracting stage. In other words, a customer and a designer need to commit to a design project, typically in a form of a contract, before they actually engage in design activities. There is a time lag between contracting of design outsourcing and realization of the design outcome. The decisions to outsource a design have to be made with partial information or expected result of the design outcome.

According to von Hippel (2005), any successful design requires inputs of both need information and solution information, which, however, are usually distributed asymmetrically with customers (or users) and designers (or manufacturers). Besides the information asymmetry, such information is also “sticky” in the sense that it is very difficult for customers and designers to convey need information and solution information respectively to the counterparty even if they are willing to, especially when the product to be designed is very complex. For instance, in aircraft design and development, aircraft manufacturers (designers) usually are not able to meet airlines’ initial requirements due to technical complexities and unforeseen technical changes (Fielding 1999).

More specifically, for customers, the major source of risk stems from his inability to accurately articulate needs in terms of concrete and clear requirements, particularly when the product is complex and the customer is not technically savvy (Jia and Dyer 2008).
Although it is a common practice for customers to issue a detailed set of requirements before they outsource a design, it is not uncommon that such requirements contain ambiguities and errors, which misrepresent the true need information (Almefelt, Berglund et al. 2006). The customer’s distorted need information will mislead the designer in design problem solving and could lead to costly design changes or disputes. Another source of risk for customer is his inability to accurately evaluate a design solution. Because of the lack of technical understanding, the customer could often get ‘confused’ by the large variety and possibilities of design solutions (Huffman and Kahn 2000). Also, the real value of a design might takes a long time of actual usage before it can be accurately assessed (von Hippel 2005). Thus, there is an inherent uncertainty regarding the value of a design to a customer when he commits to outsource the design.

For designers, the main source of risk comes from the uncertainty concerning resources that may be required to fulfill the design task. Coinciding with the customer’s inability to accurately articulate needs, the designer is often unable to accurately utter his capabilities. It is often hard, if not impossible, to represent or describe a design or innovative solution in sufficient details without confusing customers. Furthermore, the designers are often exposed to the risks of requirement changes from the customers. Even though the customers could be contractually responsible for customer-initiated design changes, it is difficult to anticipate all possible changes ex ante and the contracts are usually incomplete. It is often the case in practice that designers need to modify the solutions to cope with customer’s modified requirements (Williams, Eden et al. 1995). Such design changes, especially during the later stage in the design process, are not only complicated but also expensive, which could lead to redesigns and delays in product launches (Zirpoli and Becker 2011).
The presence of transactional risks implies transaction costs (Williamson 1989), which create barriers in design outsourcing. When faced with uncertainty, both customers and designers engaged in a potential design outsourcing would demand risk premiums to compensate for the additional risks they are taking on (Chen, Zhang et al. 2009). Higher uncertainty regarding the outcome of design leads to less willingness to commit to outsource the design by the customer, and reluctance to accept a design task by the designer.

2.2 Design Risk Modeling

Any serious attempt to address the transactional risk in design outsourcing requires a conceptual framework to model design and its risks at the first place. Unlike production which has clearly defined goals, tasks and metrics, design is generally an open-ended process that is not conducive to quantitative modeling. This section surveys major efforts in academic research concerning the modeling of design risk.

2.2.1 Individual’s attitude towards design risk

Indifferent to any risk classification, there is great need to quantify individual differences in attitude towards risk. People make different decisions based on their individual risk attitude and the risk levels they face. In economics, individual’s risk attitude describes the shape of his or her utility function for the outcomes in question (Weber, Blais et al. 2002).

Prevailing consensus on risk qualification based on individual’s perception of the riskiness is classified into: (1) risk aversion, (2) risk neutral, and (3) risk seeking. A common used metric of risk attitude is defined as: $u''(x)/u'(x)$, where $u'(x)$ and $u''(x)$
denote the first and second derivatives of the utility function \( u(x) \) (Arrow 1973, Rabin 2000, Rieger, Wang et al. 2011). In simple words, a risk averse person is reluctant to accept a bargain with uncertain payoff rather than another bargain with more certain payoff, even though with lower expected payoff. A risk neutral person is indifferent with any bargain with the same expected payoff regardless the uncertainty. And a risk seeking person favors the risky one given two bargains with the same expected payoff.

In design context, an outsourcing decision a person made is also biased due to his or her inherent risk attitude. However, empirical research shows that, in practice, a rational decision maker can only be risk averse (Holt and Laury 2002) to a certain extent. This argument leads this research to construct utility model only under the assumption that the decision maker is risk averse in Chapter 3.

### 2.2.2 Design as projects

There has been a lot of research and there is ongoing debate regarding what design really is and how to (or whether it is possible to) quantitatively model a design process. Simon (1996) conceptualizes design as essentially consisting of two parts, namely alternative generation and alternative selection. For very specific engineering design problems, design can be formulated as an optimization problem when both the design objective and the solution space can be clearly defined (Finger and Dixon 1989, Finger and Dixon 1989, Wang and Shan 2007). However, in a general design scenario, the solution space for feasible alternatives is generally unknown, and the criteria for alternative selection cannot be accurately articulated.

Traditionally, design is viewed as a semi-structured, iterative, and trial-and-error process (Ulrich and Eppinger 2011). Techniques like Quality Functional Deployment
(QFD) are used to collect user information and translate them into a set of requirements, based on which design is carried out to find an alternative that fulfills the requirements. The function of design has been traditionally limited to the department of engineering, and the task of design is to realize the requirements typically given by the department of marketing. Although such a view can generally describe design practices in industries, it is difficult to quantitatively model a trial-and-error design process and use it to assist design task planning or execution.

Given the one-off nature of design, design is usually considered as projects (Mikkelsen, 1990). The majority of study on design risk modeling and management has been carried out under the framework of project risk management, focusing on the time it takes to complete the design tasks (Oehmen, Ben-Daya et al. 2010). There has been extensive research on concurrent engineering, which aims to shorten product development time by optimally overlapping design tasks in parallel (reference concurrent). For development of large engineering systems, Eppinger et al. (1994) develop a method based on design structure matrix (DSM) for organizing design tasks. As design risk is concerned, the main focus in this stream of literature is on the interdependencies among different design tasks. The solution can be generally summarized as to maximally decouple design tasks and minimize the interactions among different functional departments in a design team.

Another risk that has been explicitly considered in design project management is concerning the uncertainty in the duration of a design task and potential propagation of delays to the overall design project (Williams 1995). Many project management techniques like Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) have found applications in assisting the planning, sequencing and sche-
duling of design tasks in order to minimize the risks of project overrun (Krishnan and Ulrich 2001). Given the stochastic nature of the duration and outcome of design tasks, Ha and Porteus (1995) derive optimal policies for the timing of design reviews. The optimal policy balances the benefits of improved design quality in terms of reduced redesigns, and the costs of design reviews in terms of the time and cost of the design reviews. Similarly, Ahmadi and Wang (1999) propose methodologies to optimally structure the design review process by strategically placing design reviews along the design process and allocating engineering resources for each design phase. Luh et al. (1999) develop scheduling algorithms to minimize project tardiness and risk penalties by taking into consideration of the uncertain number of design iterations.

There has also been some literature that looks into design project management in a distributed design environment. Krause, et al. (1994) propose a project management system to enable and support effective distributed product design. Bames et al. (2006) discusses the risks involved in managing collaborative R&D projects based on an industry survey. Hameri and Nihtilä (1997) advocate the use of Internet and World-Wide Web as a technological means to support distributed product development. The main focus of this stream of research is concerning effective communication and decision making in a distributed environment, in contrast to co-located design teams.

In general, there has been extensive research on design risk management under the project management framework. The primary risks that have been considered are the associated with the uncertain duration, iterations, and dependencies of different design tasks. The solutions to address the risks can be generally summarized as effective information provision and communication, strategic design reviews, optimal design task sequencing and coordination. However, the focus of research has been placed
mainly with the development risk, i.e. the risk to realize a design, which implicitly assumes that the design project has been set up (or agreed upon if in a distributed environment). Little attention is paid to the transactional risk in a supply chain context. In other words, there is inadequate research that addresses the question of whether a design project should be started at the first place. Another limitation is concerning design risk modeling under the project management framework. By treating design as projects, it is generally assumed that the design tasks are known in advance and the uncertainty is concerning the duration or iterations of each design task. The uncertainty regarding the design outcome, however, is not adequately addressed.

### 2.2.3 Decision-based design

Design is essentially a process of problem solving that entails an uncertain outcome. The project management framework provides methodologies that mainly model and address uncertainties and risks in design processes, especially the stochastic durations of design tasks. However, design is generally viewed as a trial-and-error process with uncertain outcome. Recent development in Decision Based Design provides a theoretical framework that explicitly models and tackles uncertainty in design outcome. Although it has long been recognized that design essentially involves decision making, it is only until recently that design has been framed and modeled as decisions.

According to Hazelrigg (Hazelrigg 1998), design can be modeled as a decision-making process that seeks to maximize the value of a designed artifact under uncertainty and risk. More specifically, uncertainty is defined as “a lack of precise knowledge regarding the inputs to a model or process, or the model or process itself, or about future events that will influence the outcome of a decision”; and risk is defined as “the result of uncertainty on the outcome of a decision, namely the variability in the ultimate outcome.
Decision-Based Design (DBD) is a normative approach that prescribes a methodology to make unambiguous design alternative selections under uncertainty and risk wherein the design is optimized in terms of the expected utility Hazelrigg (1998). The foundation of decision-based design lies in a set of axioms, which include deterministic decision making, ordering of alternatives, reduction of compound lotteries, continuity, substitutability, transitivity, monotonicity, and reality of engineering design etc. (Hazelrigg 1999).

The key difference of decision-based design, relative to traditional model-based design, resides in the explicit modeling of uncertainty and risk in design decision making (Hazelrigg 1999b). Most of the engineering design models are deterministic, which are accurate in terms of calculating interim variables in specific domains like mechanical, electrical, or structural engineering. However, they are inaccurate when the ultimate goal of design in terms of profitability is concerned. In a decision-based design framework, designers’ preference is formulated as a utility function, and the objective of design is to maximize the expected utility (Hazelrigg 1998). In contrast to traditional design methodologies, decision-based design takes into consideration of a product’s total life cycle in meeting the needs from both the consumers and the producers. Explicit utility analysis can contribute to the design process by providing a formal, structured way in which to model subjective tradeoffs, particularly those that are nonlinear and/or that must be made under uncertainty (Thurston 2001).

Decision-based design has found wide applications in addressing many complex design problems. For example, Mistree et al. (1990) apply decision-based design as a paradigm for the design of naval ships. Nakao et al. (2002) develop decision-based process design to shorten the lead time for model design and production. Wassenaar, et al.

Decision-based design has also been applied for collaborative design in a distributed environment. Gu et al. (2002) develop a computational framework to support decision-based collaborative optimization. On top of a decision-based design framework, the method of collaborative optimization is used to determine the optimal design, where an optimizer on the system level is integrated within each subsystem or discipline. The resulting decision-based collaborative optimization framework simulates the existing relationship between business and engineering in multidisciplinary systems design. Most of the research on collaborative design assumes free sharing of information, which might not be true in many cases in a multidisciplinary design team or within a supply chain context. Xiao et al (2005) propose to use game theory as a mechanism to solve conflicts among engineering teams in collaborative multidisciplinary product realization.

In general, decision-base design provides a theory foundation and conceptual framework to quantitatively model design as a set of decision making. By formulating the objective of a design as a utility function, decision-based design is particularly useful as a modeling framework to study uncertainty and risks associated with the design outcome, which has been generally ignored in the design project risk management literature. Although there have been a few papers attempt to address distributed and colla-
borative design decision making in decision-based design literature, the main focus has been placed upon collaboration within a multidisciplinary design team. There is little research on the transactional risk on design outsourcing in a supply chain context.

2.3 Prototyping for Design Risk Reduction

Prototyping has been widely used as a means for risk reduction both in engineering design and in supplier selection in design outsourcing. This section reviews literature that concerns the general property of prototypes and their use in contract negotiation in particular. Special attention is on recent development in literature which treats prototypes as a means for information elicitation in a supply chain context.

2.3.1 Prototyping in engineering design

Prototyping is involved in almost every stage of product innovative process. The types of prototypes vary in a wide spectrum according to the stage of development of the product, medium of construction, and manufacturing process, which all play important roles but address different aspects of the product development. Based on the level of sophistication and the sequencing in product development, prototypes can be categorized into three kinds, as Table 2-1 shows.

In the primary stage of product design, various ideas are examined and alternatives are explored. Prototypes are created as a form of working models for demonstration purpose, but their specification is largely left to the customer and designer. There is no commitment by either parties to take them further in this form (Bergwerk 1989). The preliminary prototypes at this stage are generally inexpensive. Computer simulation and modeling are extensively used in the preliminary design to provide early visualiza-
tion of a concept and to predict performance of the proposed product (Medler 1991). For instance, in automotive development, clay models were traditionally used as the medium for designing the body styling and its precise shape. Nowadays, the computational prototyping and modeling are used interactively with the clay models to reduce time and improve quantitative accuracy.

Table 2-1 Application of prototypes (adapted from Barkan and Iansiti (1993))

<table>
<thead>
<tr>
<th>Preliminary prototypes</th>
<th>Subsystem prototypes</th>
<th>Full-system prototypes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project planning and</td>
<td>Concept selection</td>
<td>Drive product schedule</td>
</tr>
<tr>
<td>product definition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concept clarification</td>
<td>Process selection</td>
<td>Coordinate multi-functional activities</td>
</tr>
<tr>
<td>Product assessment by</td>
<td>Feasibility of component or subsystem</td>
<td>Predict ultimate product quality</td>
</tr>
<tr>
<td>focus groups</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost estimating and</td>
<td>Early problem identification</td>
<td></td>
</tr>
<tr>
<td>supplier bids</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Life testing, environmental robustness</td>
<td></td>
</tr>
</tbody>
</table>

One step further from the preliminary model is the subsystem prototype, which represents a portion of a system and can be tested for certain limited functions. The specification for the new product will be drawn up and agreed upon by both the customer and the designer. The ideas and proposals are then converted to a more detailed prototype to conduct further experimentation and test work. Such subsystem prototypes are essential during research and development for focused studies while avoiding the distractions and complexities of a complete system. These prototypes are built before all the final decisions have been made, and help identify possible interface problems or unexpected interactions between separate subsystems. A failure to discover a
subsystem level problem has severe repercussions on the cost, complexity, and quality of the resulting product (Barkan and Iansiti 1993).

Full engineering prototypes are necessary in the final stage of development to demonstrate design feasibility, to check on part-fit and assembly compatibility, and to reveal the high representativeness and quality that are expected and demanded (Bebb 1991). In this stage of product design, prototypes will be built as close as possible to the design of the product, and undergo a number of evaluations. Such final prototypes are employed as a confirmation of product and process quality. The quality as measured by the scope and character of needed changes is used as the basis for assessing the designer’s effectiveness. And the prototype quality is used to predict the ultimate product quality.

In a broader sense, prototyping can be viewed as a means of experimentation and testing in product design and development. There has extensive research on the sequencing and resource allocation for design testing. For example, Thomke (1998) discusses the effect of new experimentation methods like computer simulation, mass screening, and rapid prototyping upon product design performance. The general finding is that extensive utilization of prototypes can greatly improve design performance. However, despite the increasing availability and decreasing cost of prototyping technologies, prototypes are still costly in many cases. Dahan and Mendelson (1998) investigate the tradeoff between the expected profit of improving design and the cost and time requirements for prototyping. More specifically, they model prototyping as a probabilistic search and considers parallel, sequential, and a hybrid scenario for prototyping. In a similar paper, Loch et al. (2001) study the extent to which testing activities should be carried out in parallel or in series in product design. They find that parallel testing has
the advantage of proceeding more rapidly than serial testing but does not take advantage of the potential for learning between tests, thus resulting in a larger number of tests. They further derive the optimal testing strategy as a function of testing cost, prior knowledge, and testing lead time. However, most of the study on prototyping (or experimentation or testing) has implicitly assumed product design within a single enterprise. The primary focus in risk reduction is on engineering risks.

2.3.2 Prototyping for contract negotiation

Design has been traditionally viewed as an internal activity. In the context of design outsourcing, design tasks need to be carried out by the designer but need the inputs from the customer. As a result, collaboration is required in distributed design. Most of research on collaborative design has been focused on task allocation, information sharing, and project coordination etc. (Bames et al. 2006). In a supply chain context, especially during the contracting stage, there is an inherent conflict of interest between buyers and sellers. The buyer might contact multiple designers for the design job. The competition among the designers might give incentives for each designer to misrepresent or overstate their design capabilities. Also, the buyer might withhold certain information, e.g. the exact formula of how he evaluate design proposals and the willingness to pay for a particular design.

Prototypes provide a means to reduce the transactional risk in the context of design outsourcing. Essentially, prototypes add value in product development by creating additional information, which reduces uncertainty and risks (Browning, Deyst et al. 2002). Early tests with even rough prototypes can help reduce such risk by checking out key aspects of the concept, reducing development time, and minimizing adverse effects of design change (Barkan and Iansiti 1993). In the context of design outsourcing,
a prototype can help the buyer better assess the capabilities of a designer and estimate the value of the final design outcome; and it also helps the designer to better elicit the need information of the customer as well as better estimate the cost and time required to carry out the design.

Although prototypes have been widely used in contract negotiation in procurement (e.g. Brown and Corzine 1998), systematic study on the role of prototyping in design outsourcing has been rare. The paper that is most relevant to the study on the value of prototyping in design outsourcing is by Terwiesch and Loch (2004). Their study assumes the perspective of a designer, and the main use of the prototype is to help the designer in elicitation of customer needs, which customers generally are unable to accurately articulate. By creating one or several prototypes and presenting them to the customer, both the designer and the customer become better able to anticipate the outcome of the design process. Thus, such a collaborative prototyping process helps the customer to evaluate the unknown customized product, and it guides both parties in the search for the ideal product specification. They develop a game-theoretical model to study the optimal decisions of a designer and a customer in a collaborative prototyping scenario. Depending on whether the design problem is structured or not and the market characteristics, their model assists the designer in making decisions regarding whether prototypes should be offered at a profit, at cost, or even for free.

In general, prototyping has been widely used and studied in engineering design for reduction of technical risks concerning design realization. Little research has been devoted to the study of prototyping as a means for reduction of transactional risk in design outsourcing. Among the few papers that address the use of prototyping in supply chain contracting, most notably by Terwiesch and Loch (2004), it is assumed from a
designer’s perspective and the focus has been placed upon the pricing of prototypes. The value of prototyping in design outsourcing has not been systematically quantified.

2.4 Design Economics and Flexibility Valuation Using Real Options

The prototyping process in product design can be extended to multiple phases, i.e. multiple prototyping. From another perspective, the multiple prototypes can be seen as the different phases of the design development process. This stretches out this research from studying single-phase prototyping to the extent of multi-phase product design development, of which the decision making in each phase could have great impact on the success of the outsourcing transaction.

Discounted cash flow and real options analysis are the two methods that management commonly relies on to value corporate projects. Discounted cash flow (DCF) starts with an estimate of the expected changes in the company’s cash flows. The present value of the cash flows, determined by using a risk-adjusted discount rate, is compared with the investment cost to calculate a net present value (NPV). If it’s greater than zero, the project gets a go-ahead. A problem of this method is that it encourages the management to reduce the investment costs as much as possible, as the lower the cost, the higher the NPV. The naive structure is usually inflexible. In a highly volatile and capital-intensive industry, the flexibility to adapt, remain or quit from a project has value that is not captured by DCF (Ferreira, Kar et al. 2009).

2.4.1.1 Introduction to real options

Real options analysis can put a value on such management flexibility. This methodology allows the decision maker to create a decision tree that charts possible decision
points, with a value and possibility to each of those points. By summing up the values of the various contingent outcomes, the real options approach yields a valuation that incorporates the flexibility in the corporate project. Put differently, this approach is a tool that can be used to hedge against undesired outcomes, and are also a means to exploit the possibilities of upside that are created in uncertain situations (Miller and Clarke 2008).

Real options concept is an extension of the financial options to the application on industry investment and risk management. The financial option gives the investor a right, but not an obligation, to purchase the underlying asset at a given price during a set time frame. The feature distinguishes the real option from its financial double is that the underlying asset is an industry project instead of financial or commodity products.

Real options concept was first proposed by Myers (1977), who pointed out that when the underlying project has significant uncertainty, the project’s value should be equal to the Net Present Value (NPV) of the project plus the value of the real option. Ross (1978) analyzed the inherent potential investment opportunities of the risky underlying project, and further discussed the theory of real option valuation. The real options models were systematically categorized into seven fields by Trigeorgis (1993): option to defer, staged investment option, option to alter, option to abandon, option to switch, growth option, and interacting option. The real option pricing theory was then integrated with financial market rules by Amram and Kulatilaka (1999) to help the management make decisions under uncertainties through evaluating the non-trading assets in areas such as strategic investment, R&D project selection, design project valuation, and etc.
In a design economics context, real options method has various applications in product platform flexibility planning, product development management, engineering systems design, and etc. Jiao (2012) proposed a real options model to optimally combine product technologies in product platform planning. Focusing on planning flexibility, his research specifically deals with how to define, measure, and evaluate the flexibility associated with product platforms under uncertainty. Kalligeros (2004) provided a real options model formulation for engineering systems design. A traditional approach to systems design and architecture follows a path from demand identification and estimation, optimizations of system architecture, detailed design, prototyping, production and deployment. He proposed a real options-based framework for design optimization of engineering systems, which provides a very powerful tool for the evaluation of flexibility in a system.

2.4.1.2 Traditional decision-making theory vs. real options theory

Myers (1984) pointed out that discounted cash flow (DCF) theory has limitation in analyzing a company’s strategic capital budgeting process. Instead, he promoted that the options pricing should be used in such situations rather than the DCF method. The same idea was repercussively supported by Hodder and Riggs (Hodder and Riggs 1985), who pointed out that the project risk would subsequently decrease throughout the ongoing project. Furthermore, the management flexibility has the same impact of reducing risk in project. It is not the best practice to use a single discount rate (DCF) in evaluating a project’s process.

Trigeorgis and Mason (1987) further extended Hodder and Riggs’s idea, and pointed out that the traditional NPV or DCF theories assume the future cash flow from a project is perfectly predictable without uncertainty. However, the uncertainty of future
cash flow does exist in practice, thus the NPV or DCF are not able to include the management flexibility into the investment decision-making. Therefore, the traditional NPV or DCF approaches produce biased result in analyzing a risk project where the management has flexibility to influence the course of development.

Dixit and Pindyck (1995) pointed out another limitation of traditional investment decision-making, which assumes that the investment in strategic corporate planning cannot be deferred. The management is assumed to make only one time lump sum investment at a particular point within the investment time frame. This unreasonable assumption ignores the value created by the delay of investment decisions. In practice, the management can delay the decision-making until at certain point of time when more relevant information is gathered, then make a wiser and calculated investment decision.

Another limitation of NPV method is that it assumes a situation excludes follow-up investment (Ross 1995). For example, NPV cannot assess multiple-phase investment process, which is very common in large scale engineering system design planning, product line optimization, and other industrial related projects. A standard option model can be extended to compounded option model to accommodate such multi-phase investment situation.

2.5 Research Gap

Design outsourcing is a relatively new form of outsourcing and has attracted enormous attention from both the industry and academia. Although research findings on general outsourcing, production outsourcing in particular, can be carried over to the study of design outsourcing, design outsourcing poses specific challenges that have not been
well addressed in existing literature on outsourcing in general. One particular challenge lies in the transactional risk during the contracting stage in design outsourcing due to uncertain design outcome, and the difficulty to rigorously model a design problem. Existing literature on design risk management has primarily focused on the risks of design projects, especially the risks posed by the uncertain duration and potential iterations in design tasks.

Recent development in decision-based design promises a sound theoretical foundation and conceptual framework to explicitly model design and its associated risks. However, the research on design risk in the decision-based design community has been focused on how designers should make informed decisions under uncertainty and risks in a multidisciplinary team within a single enterprise. Little research in decision-based design has been devoted to the study of distributed design and transactional risk in a supply chain context, which generally involves conflicting incentives.

In practice, prototypes are widely adopted as a means for reducing transactional risks in design outsourcing. There has been extensive study on prototyping for risk reduction in engineering design. Again, the focus has been placed on reduction of engineering risks instead of transactional risk. A few papers have looked into the issue of collaborative prototyping and pricing of prototypes in a supply chain context. However, systematic study on the role of prototyping for reduction of transactional risks in the context of design outsourcing is still lacking.

In summary, there has been multidisciplinary research that addresses different aspects of transactional risks in design outsourcing, but there lacks an integrated approach that systematically study the role of prototyping for risk reduction in design outsourcing. More specifically, in the operations research and management literature, there still
lacks a rigorous method to model a design problem; while in the engineering design literature, the focus has been predominantly on the risks of design realization. Ample opportunity lies in combining these two disciplines of study to develop quantitative models and methods to systematically study and provide decision support regarding the use of prototyping for risk reduction in design outsourcing.
CHAPTER 3: SINGLE-STEP PROTOTYPE MODEL

The single-step prototype model, which generalizes the prototyping process into a single stage, refers to a scenario of one buyer vs. one designer; the buyer needs to decide whether to buy the design from the designer or not. This research takes the perspective from the customer, who concerns the uncertain value of the design. The customer would like to make a prototype so that he is able to better estimate the final design value. If he thinks prototyping is worthwhile, he will invest in it, otherwise he will cancel the deal due to the high risk and high prototyping cost. After prototyping, if the result is good, the customer will subsequently purchase the design, otherwise he will cancel the deal. Figure 3-1 shows the buyer’s decision process.

![Figure 3-1 Single-step prototype model](image)

3.1 The Impact of Transactional Risk

The buyer faces uncertainty regarding the value of the design, while the seller faces uncertainty regarding the cost of the design. These uncertainties drive the customer and the designer to ask for extra risk premiums to hedge their risks. In order to represent
the transactional risk, this research uses mean-variance model, which assumes that the customer’s estimated value and designer’s estimated cost follow normal distribution, with mean indicating expected value/cost, and variance indicating the risk level. Figure 9 illustrates a scenario that the customer’s initial expected value is higher than the designer’s expected cost; however, due to the existence of risk premiums, designer’s expected cost results to higher than customer’s expected value. In simple words, a deal goes to no deal.

![Diagram of customer's estimated value, designer's estimated cost, risk premium, deal, and no deal.]

**Figure 3-2 A deal to no deal**

In order to avoid the above situation, prototypes are often used in reality to reduce both customer’s and designer’s transactional risk over valuation and cost (Terwiesch and Loch 2004, Ulrich and Eppinger 2011). A single-step prototype model is developed in this chapter to generalize the prototyping process and quantify the tradeoff between...
the risk reduction and cost of prototyping. Eventually, this model aims (or serves) to answer questions like, is it cost effective to produce the prototype? Who should pay for the prototype?

### 3.2 Risk Attitude and Utility Function

Both the customer and the designer are exposed to certain degrees of transaction risk in design outsourcing due to the information and communication barrier. The realization of the transaction between both agents would depend on the level of risks and their attitudes toward the risk. Thus, decision maker’s risk attitude is a crucial factor in the risk model.

In general, decision makers can be differentiated into three kinds, *risk-averse, risk-neutral* and *risk-seeking*, depending on their attitudes towards risk. In this chapter, both customer and designer are assumed to be *risk averse* without loss of generality. The levels of risk averse can be reflected from the changes of decision maker’s utilities to a certain evaluation measure. Utility is a measure of relative satisfaction. In this model, the customer’s and designer’s utilities gauge their satisfactions towards the design outsourcing outcome in terms of economic benefits. Decision maker’s risk attitude is considered along with his utility in an *exponential utility function* (Kirkwood 2004), of which the mathematical representation is:

\[
u(x) = 1 - e^{-\frac{x}{R}}, \tag{3.1}\]

where \(u(x)\) represents the utility function, \(x\) is the evaluation measure, and \(R\), risk tolerance, indicates decision maker’s risk attitude. This exponential form of utility
function is most commonly applied in economics to describe one’s satisfaction with consideration of his risk aversion. The risk tolerance, $R$, is a positive real value, where higher $R$ implies less risk aversion. This study takes the perspective from the design buyer, therefore the evaluation measure, $x$, in the exponential utility function represents the customer’s *economic surplus*, indicated as $\pi = v - p$. $v$ is the value of final design for the customer and $p$ is the price that the design contractor quotes. The true value of $v$ remains unknown until the final design is observed. Before that, the buyer may have certain amount of prior knowledge or experience of the expecting design, according to the degree of difficulty of the project and the expertise of the designer. We assume that the buyer estimates the final design value to be, $v_0$, without observing any prototype or final design. $v_0$ is assumed to be a random variable which follows a normal distribution with mean, $\mu_0$, and variance, $Q_0$,

$$v_0 \sim N(\mu_0, Q_0).$$  

(3.2)

The design price quoted by the contractor without making prototype is given as $p_0$. To the customer, the goal is to maximize his economic surplus, $\pi_0 = v_0 - p_0$, in the condition that he only consents with the transaction given his payoff $\pi_0 \geq 0$. Without observing any information from prototype, the design buyer faces a puzzle between a stochastic payoff $\pi_0$ (deal) and a certain 0 (no deal). Certainty equivalent is introduced in the next section to facilitate this decision making.
3.3 Certainty Equivalent

Certainty equivalent is a mathematical concept that transforms a set of random outputs to a certain value taking into the decision maker’s risk attitude (Keeney and Raiffa 1993). For instance, given the customer’s certainty equivalent of his uncertain payoff $\pi_o$ is $\hat{\pi}_o$, he is indifferent between the random draw of $\pi_o$ and the certain payoff $\hat{\pi}_o$. In definition, the utility of the certainty equivalent equals to the expected utility of the evaluation measure. The mathematical relationship between $\pi_o$ and $\hat{\pi}_o$ is expressed as,

$$u(\hat{\pi}_o) = E[u(\pi_o)],$$  \hspace{1cm} (3.3)

$$\hat{\pi}_o = u^{-1}\left[E[u(\pi_o)]\right].$$  \hspace{1cm} (3.4)

Given that the design value, $v_o$, follows a normal distribution, $v_o \sim N(\mu_o, Q_o)$, the probability density function (PDF) of $v_o$ is,

$$f(v_o) = \frac{1}{\sqrt{2\pi Q_o}} e^{-\frac{(v_o - \mu_o)^2}{2Q_o}}.$$  \hspace{1cm} (3.5)

Following the certainty equivalent equation, $u(\hat{\pi}_o) = E[u(\pi_o)]$, the buyer’s certainty equivalent of payoff, $\hat{\pi}_o$, can be derived:
\[ u(\hat{\pi}_0) = E[u(\pi_0)] = \int u(\pi_0) f(\pi_0) d\pi_0 \]
\[ = \int_{-\infty}^{\infty} \left(1 - e^{-\frac{\pi_0 - p_0}{R}} \right) \left(1 + e^{-\frac{(\pi_0 - \mu_0)^2}{2Q_0}}\right) d\pi_0. \]  

(3.6)

After some mathematical manipulation, \( \hat{\pi}_0 \) can be simplified to,

\[ \hat{\pi}_0 = u^{-1}(E[u(\pi_0)]) = \mu_0 - p_0 - \frac{Q_0}{2R}. \]  

(3.7)

The certainty equivalent equation, \( \hat{\pi}_0 = \mu_0 - p_0 - \frac{Q_0}{2R} \), could be divided into two terms, \( (\mu_0 - p_0) \) and \( (-Q_0/R) \); where the latter term is the risk premium that the design buyer asks for. Either higher transactional risk, \( Q_0 \), or lower risk tolerance, \( R \), could reduce the buyer's certainty equivalent of payoff, \( \hat{\pi}_0 \). It's only reasonable for him to buy the design when \( \hat{\pi}_0 \) is positive.

Under a condition with minimum transactional risk, \( Q_0 \to 0 \), the second term, risk premium \( (-Q_0/R \to 0) \), such that the certainty equivalent simply equals to the expected design value minus the design price, \( \hat{\pi}_0 (Q_0 \to 0) = \mu_0 - p_0 \). This means that if there's no transactional risk involved \( (Q_0 \to 0) \), as long as the buyer's expected valuation is higher than the design price \( (\mu_0 > p_0) \), he is willing to make a deal \( (\hat{\pi}_0 > 0) \). On the contrary, when the customer faces high uncertainty in valuation or has lower risk tolerance, he would ask for higher risk premium, which reduces his willingness to pay.

Therefore, it’s feasible to increase the chance of successful transaction through reducing risk.

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3.4 Reducing Transaction Risk via Prototyping

3.4.1 Prototyping as sampling

In order to quantitatively value the risk reduction effect of prototyping, it is assumed that the customer could estimate the final design value reflected from prototype, $v_p$, based on his past experience and expertise. For instance, before launching the new designed car to the market, the automobile maker is not able to accurately evaluate how the market would react, which means that the true design value $v$ remains unknown. However, after observing the prototype, he may have an informed estimation of the market response ($v_p$) based on historical information and prior years of experience in automobile industry.

$v_p$ can be taken as a sample of actual design value, $v$, distorted by a noise factor $\varepsilon$:

$$v_p = v + \varepsilon,$$

where $\varepsilon$ is assumed to be a non-biased normal random variable with variance $\Sigma$, given $\varepsilon \sim N(0, \Sigma)$.

The variance of the noise variable, $\Sigma$, indicates the degree of fidelity of the prototype, with lower variance implying higher fidelity rate, and vice versa. For instance, the late stage prototype has higher fidelity rate than that closer to the early conceptual design stage. Without prototyping, the customer’s best estimate of the value of $v$ is $v_0$, so $v_p$ can be instead modeled as,

$$v_p = v_0 + \varepsilon.$$

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Chapter 3: Single-step Prototype Model

Given that \( v_0 \sim N(\mu_0, Q_0) \) and \( \varepsilon \sim N(0, \Sigma) \), \( v_p \) follows a normal distribution:

\[
v_p \sim N(\mu_0, Q_0 + \Sigma),
\]

with mean \( \mu_0 \) and variance \( (Q_0 + \Sigma) \). \( \Sigma \) is the noise carried from the prototype to the final design valuation.

### 3.4.2 Bayesian updating

Conditional on the outcome of prototyping, the customer can update his estimated value of final design. \( v_0 \) could be seen as his initial estimation of the final design value without making prototype, and \( v_p \) as the new observation. The probability function of the updated design value, \( v_1 \), can be expressed as the function of \( v_0 \) conditional on \( v_p \) through Bayesian inference theorem:

\[
P(v_1) = P(v_0 | v_p) = \frac{P(v_p | v_0) \cdot P(v_0)}{\int P(v_p | v_0) \cdot P(v_0) dv_0} = \frac{P(v_p | v_0) \cdot P(v_0)}{P(v_p)}. \tag{3.11}
\]

Reached by calculation (Appendix I), \( v_1 \) also follows a normal distribution, \( v_1 \sim N(\mu_1, Q_1) \), the mean and variance,

\[
\mu_1 = \frac{\Sigma}{Q_0 + \Sigma} \cdot \mu_0 + \frac{Q_0}{Q_0 + \Sigma} \cdot v_p, \tag{3.12}
\]

\[
Q_1 = \frac{\Sigma}{Q_0 + \Sigma} \cdot Q_0. \tag{3.13}
\]
The updated mean, $\mu_1$, can be seen as the combination of two components $\mu_0$ and $v_p$ with coefficients $\frac{\Sigma}{Q_0+\Sigma}$ and $\frac{Q_0}{Q_0+\Sigma}$, respectively. This function clearly shows that the designer’s expected valuation after prototyping is affected by both his prior estimation $\mu_0$ and the new observation $v_p$ from prototype. In another perspective, the updated mean $\mu_1$ will shift towards the observation $v_p$ from the original mean $\mu_0$. A very representative prototype with low noise, $\Sigma \rightarrow 0 \ (\frac{\Sigma}{Q_0+\Sigma} \rightarrow 0 \text{ and } \frac{Q_0}{Q_0+\Sigma} \rightarrow 1)$, would minimize the effect of prior valuation and maximize the observed information in future valuation.

Given that $\Sigma$ and $Q_0$ are both positive value, the updated variance, $Q_1$, contracts from the prior estimation with coefficient $\frac{\Sigma}{Q_0+\Sigma}$. This feature is one of the key points of this research, which quantitatively show the risk reduction after prototyping. As explained earlier, a no deal transaction could be transformed into a deal with the coup of reduced transactional risk.

### 3.5 Optimal Prototyping Decision

The Bayesian updating process described in the previous section provides a quantitative interpretation of the use of prototypes for transactional risk reduction. However, it is important to analyze the tradeoff between the risk reduction effect and the additional cost of prototyping, since the prototype can be very expensive in capital-intensive industry.
Suppose that the cost of prototype is $d$, which in general would be fully bore by the designer in design outsourcing transaction. However, if $d$ exceeds the designer’s maximum bearable cost, the designer is reasonable to withdraw from the transaction. In such case, the customer’s willingness to share a proportion of $d$ would bring the designer back to the table, and potentially benefit to both parties. A variable $\omega$ ($0 \leq \omega \leq 100\%$) is introduced to indicate the proportion of $d$ shared by the buyer. Therefore the buyer and the designer pay $\omega \cdot d$ and $(1-\omega) \cdot d$ in prototyping, respectively.

After prototyping, the designer could ask for a new price $P_1$, different from the $P_0$ without prototyping, with the consideration of cost $(1-\omega) \cdot d$ and design outcome.

The buyer has the option to choose deal or no deal after observing prototype. However, question remains in terms of favoring prototyping or not. Similar to the definition of certainty equivalent $\hat{\pi}_0$, $\hat{\pi}_1$ is defined to be the buyer’s certainty equivalent of payoff if he chooses prototyping. Mathematically speaking, he is only reasonable to prototype when $\hat{\pi}_1 > \hat{\pi}_0$.

Figure 3-3 illustrates the design buyer’s decision making process. The decision making points and outcomes are represented by the double circles and bold circles, respectively. The payoff functions of respective outcomes are also illustrated in the figure. $\hat{\pi}_1$ is calculated backward from nodes $\pi_{11}$ and $\pi_{12}$, where $\pi_{11}$ is a function of $u_1$. Therefore, the certainty equivalent of $\pi_{11}$, $\hat{\pi}_{11}$, needs to be described before deriving the function of $\hat{\pi}_1$. 

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Given that \( v_1 \sim N(\mu_1, Q_1) \), \( \mu_1 = \frac{\Sigma}{Q_0 + \Sigma} \cdot \mu_0 + \frac{Q_0}{Q_0 + \Sigma} \cdot v_p \), and \( Q_1 = \frac{\Sigma}{Q_0 + \Sigma} \cdot Q_0 \), the utility function of \( \hat{\pi}_{11}(v_1) \) could be described as,

\[
u(\hat{\pi}_{11}) = \int_{-\infty}^{\infty} u(\pi_{11}) f(v_1) dv_1
\]

\[
= \int_{-\infty}^{\infty} \left( 1 - e^{-\frac{v_1 - \mu_1}{\sqrt{2\pi}Q_1}} \right) \left( \frac{1}{\sqrt{2\pi}Q_1} e^{-\frac{(v_1 - \mu_1)^2}{2Q_1^2}} \right) dv_1
\]

After some mathematical manipulation, \( \hat{\pi}_{11} \) is derived to be,

\[
\hat{\pi}_{11} = \mu_1 - P_1 - \omega d \frac{Q_1}{2R},
\]

where \( \mu_1 = \frac{\Sigma}{Q_0 + \Sigma} \cdot \mu_0 + \frac{Q_0}{Q_0 + \Sigma} \cdot v_p \) and \( Q_1 = \frac{\Sigma}{Q_0 + \Sigma} \cdot Q_0 \).

\( \hat{\pi}_{11} \) represents the certainty equivalent of buyer’s payoff after prototyping and pur-chasing the final design, which is different from \( \hat{\pi}_0 = \mu_0 - p_0 - Q_0 / 2R \) with a linear prototyping cost term \( -\omega \cdot d \).
After prototyping, the design buyer has the options of, (1) purchasing the design $v_1$ with payoff $\hat{\pi}_{11}$, and (2) not purchasing, but bearing a sunk cost of $\pi_{12} = -\omega \cdot d$. The choice between $\hat{\pi}_{11}$ and $\pi_{12}$ depends on several parameters, those include:

**Table 3-1 Key parameters in design outsourcing model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>Initial expected valuation</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>Initial estimated transactional risk</td>
</tr>
<tr>
<td>$v_p$</td>
<td>Prototype value</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>Noise from prototype</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Design price</td>
</tr>
<tr>
<td>$d$</td>
<td>Total prototype cost</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Proportion of prototype cost shared by the buyer</td>
</tr>
</tbody>
</table>

All these variables could be assumed as given, except $v_p$, which is a random variable following a normal distribution of $v_p \sim N(\mu_0, Q_0 + \Sigma)$ (Equation 3.10). Thus, the deci-
tion making with prototyping could be seen as a lottery with possible outcomes \( \hat{\pi}_{11} \) or \( \pi_{12} \) depending on the result of prototyping, \( v_p \). This means that the buyer’s decision-making is dependent on his estimation of prototype quality.

It is assumed that \( v_p^* \) is a threshold in the spectrum of \( v_p \), where the buyer would choose to make the deal when observing \( v_p > v_p^* \), otherwise he would choose no deal. \( v_p^* \) is located at \( \hat{\pi}_{11} = \pi_{12} \). Given that,

\[
\hat{\pi}_{11} = \mu_0 + \frac{(v_p^* - \mu_0)Q_0}{Q_0 + \Sigma} - P_1 - \omega \cdot d - \frac{Q_0 \cdot \Sigma}{2R(Q_0 + \Sigma)} = \pi_{12} = -\omega \cdot d , \tag{3.16}
\]

the prototype value threshold is found at,

\[
v_p^* = \frac{\Sigma}{2R + \frac{Q_0 + \Sigma}{Q_0} \cdot P_1 - \frac{\Sigma}{Q_0} \cdot \mu_0} . \tag{3.17}
\]

Given that \( \hat{\pi}_1 \) is the certainty equivalent of the uncertain outcomes of \( \hat{\pi}_{11} \) and \( \pi_{12} \), the utility function of \( \hat{\pi}_1 \) is mathematically expressed as:

\[
u(\hat{\pi}_1) = \int_{-\infty}^{v_p^*} u(\pi_{12}) f(v_p) dv_p + \int_{v_p^*}^{\infty} u(\hat{\pi}_{11}) f(v_p) dv_p . \tag{3.18}
\]

If the prototype value \( v_p \) is in the range of \(( -\infty, v_p^* )\), the buyer would choose no deal, and \( v_p \) falls into the first term \( \int_{-\infty}^{v_p^*} u(\pi_{12}) f(v_p) dv_p \) of the Equation 3.18.
Given a draw of $v_p$ in the range of $(v_p^*, \infty)$, it will fall into the second term

$$\int_{v_p^*}^{\infty} u(\hat{\pi}_1) f(v_p) dv_p.$$ 

We are able to analytically solve the integral Equation 3.18 into,

$$u(\hat{\pi}_1) = \frac{1}{2}(1-e^{-R})(1+\text{erf}\left[\frac{P_1-\mu_0}{\sqrt{2(\Sigma+Q_0)}} + \frac{a\cdot \Sigma}{b}\right]) + \frac{1}{2}(1-\text{erf}\left[\frac{Q_0^2}{b}\right]) - \frac{1}{2} e^{-\frac{2R}{2R^2}} (1-\text{erf}\left[\frac{Q_0^2}{b}\right]).$$

where $a = Q_0 + 2P_1R - 2\mu_0 R$, $b = 2Q_0R\sqrt{(\Sigma+Q_0)}$, $c = \frac{a}{b}(Q_0 + \Sigma)$, and erf(·) is error function. The detail of the transformation is presented in Appendix I.

$\hat{\pi}_1$ can be derived from the inverse function of $u(\hat{\pi}_1)$,

$$\hat{\pi}_1 = -R \log\left\{1 - \frac{1}{2}(1-e^{-R})(1+\text{erf}\left[\frac{P_1-\mu_0}{\sqrt{2(\Sigma+Q_0)}} + \frac{a\cdot \Sigma}{b}\right]) + \frac{1}{2}(1-\text{erf}\left[\frac{Q_0^2}{b}\right]) - \frac{1}{2} e^{-\frac{2R}{2R^2}} (1-\text{erf}\left[\frac{Q_0^2}{b}\right])\right\}.$$  

Now we have the analytical solutions of both $\hat{\pi}_0$ (Equation 3.7) and $\hat{\pi}_1$ (Equation 3.20), which represent the certainty equivalents of the buyer’s payoff without and with prototyping, respectively. After inputting the respective parameters, the buyer could calculate and compare these two values and make an informed decision on prototyping.
3.6 Characteristics of Different Variables of $\hat{\pi}_i$

Unlike $\hat{\pi}_0$, the relationships between which and whose dependent variables ($\mu_0, P_0, Q_0, R$) can be easily interpreted to be positive linear, negative linear and reciprocal; the analytical solution of $\hat{\pi}_i$ is rather complex to understand its relationship with each dependent variable. This section aims to illustrate the characteristics of different variables of $\hat{\pi}_i$ in four settings, of which the details are presented in Table 3-2.

<table>
<thead>
<tr>
<th>Settings</th>
<th>Characteristics variables</th>
<th>Controllable variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected value, $\mu_0$</td>
<td>Transactional Risk, $Q_0$</td>
</tr>
<tr>
<td>1</td>
<td>0-100</td>
<td>0-150</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0-100</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>0-100</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>20</td>
</tr>
</tbody>
</table>

The eight dependent variables of $\hat{\pi}_i$ are categorized into two groups, characteristics and controllable. Given that $\mu_0$ and $Q_0$ are derived from the initial estimated $v_0$, and $R$ is the buyer's inherent characteristics, these three variables cannot be altered in the prototyping process. Contrarily, the rest variables are controllable by the buyer and the designer with the intention to influence buyer’s certainty equivalent of economical surplus $\hat{\pi}_i$ to certain extend. The values of each setting are designed to illustrate the cha-
characteristics of each dependent variable in the most presentable manner. In practice, the choices of variables are not constrained to the given settings.

### 3.6.1 Characteristic variables

**Setting 1: \( \hat{\pi}_1 \) in dimensions of \( \mu_0 \) and \( Q_0 \)**

The three-dimension Figure 3-4 illustrates the buyer’s certainty equivalent of payoff \( \hat{\pi}_1 \) on the vertical axis, and his initial estimation of design value \( \mu_0 \) and transactional risk \( Q_0 \) on horizontal axes in the range of \([0-100]\) and \((0-150]\) respectively. The surface color represents the fourth dimension of the gradient of \( \hat{\pi}_1 \) along the axis of \( Q_0 \).

![Figure 3-4 Setting 1: \( \hat{\pi}_1 \) in dimensions of characteristic variables, \( \mu_0 \) and \( Q_0 \)](image)

*Figure 3-4 Setting 1: \( \hat{\pi}_1 \) in dimensions of characteristic variables, \( \mu_0 \) and \( Q_0 \)*
It is observed that \( \hat{\pi}_i \) is positively related to \( \mu_0 \), but not in a linear fashion within full range of \( Q_0 \). Intuitively speaking, the buyer is more willing to make the deal, given higher expected value \( \mu_0 \).

\( \hat{\pi}_i \) is generally positively related to the risk level \( Q_0 \), due to the embedded optionality of the prototyping process, which limits the downside risk to the prototyping cost \( \omega \cdot d \) and captures the potential upside gain. This observation is just opposite from the payoff function of \( \hat{\pi}_o \). Recalling Equation 3-7, \( \hat{\pi}_o \) is negatively related to \( Q_0 \) due to the buyer’s risk adverse attitude. These contrastive results coincide with the distinct feature of prototyping in the design development. With prototyping, higher uncertainty provides more opportunity value to the buyer, and results his expected payoff to be higher.

From Figure 3-4 an interesting result is observed at the range of low risk level and high expected value, where \( \hat{\pi}_i \) reduces along the dimension of \( Q_0 \). This observation seems contradictory but indeed makes sense; as the buyer already satisfies with a design with low risk and high value prospect, he does not require more risk level to enjoy the potential upside gain.

**Setting 2: \( \hat{\pi}_i \) in dimensions of \( Q_0 \) and \( R \)**

Figure 3-5 illustrates \( \hat{\pi}_i \) in a three-dimension surface with two horizontal axes indicating the buyer’s estimated transactional risk \( Q_0 \) and his risk tolerance \( R \) in the range of \((0,100]\) and \((0,100]\), respectively. The surface color indicates the gradient of \( \hat{\pi}_i \) along the axis of \( Q_0 \).
Figure 3-5 $\hat{\pi}_1$ in dimensions of characteristic variables, $Q_0$ and $R$

The buyer’s certainty equivalent of payoff $\hat{\pi}_1$ increases as his risk adverse attitude relaxes from risk averse to risk neutral. This figure shows that when the buyer is extremely risk adverse, $R \to 0$, $\hat{\pi}_1$ decreases along $Q_0$ even in the presence of prototyping. In the other extreme, given $R = 100$, $\hat{\pi}_1$ is almost positive linear with the risk level $Q_0$.

An interesting observation can be found along axis $Q_0$ at $R = 15$, where $\hat{\pi}_1$ behaves in a convex fashion, which indicates that in the lower range of $Q_0$, the buyer is averse to the uncertainty more than the opportunity value which brings along. Further, as the risk level increases, the buyer values the uncertainty more than his aversion against risk.
3.6.2 Controllable variables

The buyer and the designer are able to manipulate and negotiate the controllable variables, such as the prototype fidelity rate, to influence the buyer’s decision making in transaction.

Setting 3: \( \hat{\pi}_i \) in dimensions of \( Q_0 \) and \( \Sigma \)

Figure 3-6 illustrates \( \hat{\pi}_i \) in three-dimension with two horizontal axes indicating \( Q_0 \) and prototyping fidelity \( \Sigma \); the vertical axis denotes \( \hat{\pi}_i \); the surface color represents the gradient of \( \hat{\pi}_i \) along axis \( Q_0 \).
The graph illustrates that the buyer’s certainty equivalent is reduced with the increase of $\Sigma$ (poorer prototype fidelity), which makes sense that the poor quality prototype conveys less information and is less effective in risk reduction. Furthermore, Figure 3-6 shows that slopes downward along in the range of high $\Sigma$. This interprets that given a poor fidelity prototype (high $\Sigma$), the buyer is less inclined to the high transactional risk deal, as the prototyping process does not bring enough useful information in his decision making.

3.7 Decision-making along Controllable Variables

Three settings of variables are given in Table 3-3 to exemplify how this model can be used in decision-making along different controllable variables. The buyer has three options at the initial stage of contracting, (1) making deal without prototyping, payoff $\hat{\pi}$, (2) no deal, payoff 0, and (3) making the prototype, payoff $\hat{\pi}$. The buyer is optimally to choose one of the options which maximizes his payoff, $\max(0, \hat{\pi}, \hat{\pi})$.

<table>
<thead>
<tr>
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<th>Characteristics variables</th>
<th>Controllable variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected value, $\mu_0$</td>
<td>Transactional Risk, $Q_0$</td>
</tr>
<tr>
<td>4</td>
<td>95</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>95</td>
<td>50</td>
</tr>
</tbody>
</table>

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**Setting 4: Optimal decision-making in dimension of Σ**

Figure 3-7 illustrates the buyer’s decision along the prototype fidelity rate, Σ. The dash line indicates \( \hat{\pi}_0 \), the dotted line represents no deal with payoff 0, the line with marker denotes \( \hat{\pi}_1 \), and the gray bold line represents the buyer’s optimal decision among no deal (0), deal without prototyping (\( \hat{\pi}_0 \)), and prototyping (\( \hat{\pi}_1 \)), with payoff, \( \max(0, \hat{\pi}_0, \hat{\pi}_1) \).

![Diagram](image)

**Figure 3-7 Setting 4: Optimal decision-making in dimension of Σ**

In Figure 3-7, is illustrated as a horizontal line, as is not a function of Σ; is downward sloping curve, which means that the buyer is less willing to make the prototype if
it’s a poor representation of the final design. This graph shows that under the given setting, the buyer is only willing to pay for the prototype if its fidelity rate $\Sigma < 21$. Otherwise, the buyer is optimal to choose to make the deal without prototyping.

**Setting 5: Optimal decision-making in dimension of $P_0$ and $P_1$**

Figure 3-8 illustrates buyer’s decision among the three options along the dimension of design price, where $P_0$ is assumed to be equal to $P_1$ for the sake of simplicity.

$\hat{\pi}_0$ and $\hat{\pi}_1$ are both downward sloping lines in linear and convex fashions, respectively.

Given the design price $P < 55.4$, the buyer is better off making deal without prototyping.
as the design is relatively cheap. However, in the price range of [55.4 77.9], the buyer is optimal to make the prototype first to reduce the transactional risk. The figure also shows that the buyer's maximum price to pay is at $P = 77.9$. If the design price is higher than 77.9, the buyer is not interested in neither paying for the prototype or making the deal.

**Setting 6: Optimal decision-making in dimension of $\omega$**

Figure 3-9 illustrates the buyer's optimal decision making in dimension of $\omega$, his shared proportion of prototype cost. By using this model, the buyer is able to locate his maximum prototype cost proportion, and then to negotiate with the designer in his favor. Figure 3-9 shows that the buyer is only able to bear the prototype cost up to 50.4%; otherwise, his optimal option is to make deal without prototyping.
Figure 3-9 Setting 6: Optimal decision-making in dimension of \( \omega \)

### 3.8 Summary

The risk model developed in this chapter shows that the inherent uncertainties and the consequent risks concerning the value and cost of product create barriers for the buyer and the designer to engage in making a deal. Unlike the prevailing research considering series of prototypes in product development, this study provides a quantitative guide for the design buyer in making the prototype-initiating decision.

The use of prototypes in design outsourcing is interpreted as a means for risk reduction. Information updating via prototyping is modeled as a Bayesian estimation process.
Chapter 3: Single-step Prototype Model

which succinctly captures the dynamics of risk evolution. A prototyping decision model based on risk analysis is also developed in this chapter, taking into consideration of a number of factors including the buyer’s risk attitude, his initial estimation accuracies, and the fidelity and cost of the prototype, as well as the proportion of prototype cost shared by manufacturers. Numerical study based on simulation reveals that an informed decision upon prototyping requires an intricate balance among the fidelity and cost of the prototype, sharing cost and price of the final product.

To fully understand the risks associated with design outsourcing contracting process, this study can be extended and enriched in a number of directions. First, this model can be developed to investigate multiple prototypes in product development. A second direction to extend this research is to consider multiple buyers and designers in a more general sales or procurement scenario concerning design bidding. Although the competition among multiple parties makes decisions more complex, it provides incentives for truthful information sharing as well as a pricing mechanism. This research makes contribution towards this end by providing a general risk model.
CHAPTER 4: MULTIPLE-STEP PROTOTYPE MODEL

Single-step prototype model, discussed in Chapter 3, is created to quantify the tradeoff between the risk reduction through prototyping and the corresponding cost in design outsourcing. The single-step model simplifies the entire design outsourcing process into an integrated phase. In the development of complex systems and products, e.g. aircraft design, pharmaceutical R&D, real estate development and etc., multiple development phases with idiosyncrasy costs and time durations (Copeland and Antikarov 2001) are often employed. In this chapter, a more practical and detailed multiple-step prototyping model is developed.

In a multi-phase design outsourcing process (Figure 4-1), the customer’s transactional risk can be continuously reduced throughout the prototyping process. Furthermore, the customer is able to invest in a series of prototypes, instead of making lump sum investment at the front end. Thus, the customer has the option to make a serial investment decisions (continue or stop) throughout the prototyping phases at his own flexibility. This multi-phase development framework implies that the customer has the right but not obligation to move on to the next phase of development after observing each phase of prototype. This feature coincides with the essential concept of real options analysis, a multi-step valuation method.

In this chapter, a Bayesian-based real options model is developed to help the design buyer in making the investment decisions in a multi-phase prototyping scenario. In the following sections, the background of real options analysis is introduced first. It is followed by the generalization of Bayesian updating method, which has been introduced
in Chapter 3. Combining these two models, a novel Bayesian-based real options model is then developed, and illustrated in a numerical simulation.

Figure 4-1 Multiple-step prototype model

4.1 Traditional Real Options Model

4.1.1 Introduction to real options

*Real options analysis* can put a value on management flexibility. This methodology allows the decision maker to create a decision tree that charts possible decision points, with a value and possibility to each of those points. By summing up the values of the various contingent outcomes, the real options approach yields a valuation that incorporates the flexibility in the corporate project. Put differently, this approach is a tool that can be used to hedge against undesired outcomes, and are also a means to exploit the possibilities of upside that are created in uncertain situations.
Real options concept is an extension of the financial options to the application on industry investment and risk management. The financial option gives the investor a right, but not an obligation, to purchase the underlying asset at a given price during a set time frame. This feature distinguishes the real option from its financial double that the underlying asset is an industry project instead of financial or commodity products. The comparison between financial and real option is illustrated in Table 4-1.

**Table 4-1 Comparison between financial and real option**

<table>
<thead>
<tr>
<th>Financial Option</th>
<th>Real Option on Design Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current value of stock</td>
<td>Current estimation of design value</td>
</tr>
<tr>
<td>Exercise price</td>
<td>Design price</td>
</tr>
<tr>
<td>Time to expiration</td>
<td>Time to final design delivery</td>
</tr>
<tr>
<td>Stock value uncertainty</td>
<td>Project value uncertainty</td>
</tr>
<tr>
<td>Riskless interest rate</td>
<td>Riskless interest rate</td>
</tr>
</tbody>
</table>

Trigeorgis (1993) systematically categorized the real options models and their respective applications into several distinct fields, such as option to defer, option to abandon, and etc. (Table 4-2). In the multi-phase design outsourcing process, an amount of early investment is prerequisite to design the early phase prototype; thus, this research context belongs to the *option to grow* category.
### Table 4.2 Categories of real options (Trigeorgis 1993)

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option to grow</td>
<td>An early investment is a prerequisite.</td>
<td>All infrastructure-based or strategic industries – esp. high tech, R&amp;D</td>
</tr>
<tr>
<td>Option to defer</td>
<td>Management holds a lease on valuable resources.</td>
<td>Real-estate development</td>
</tr>
<tr>
<td>Option to abandon</td>
<td>If market condition decline severely, management can abandon current operations.</td>
<td>Capital-intensive industries; financial services; new product introduction</td>
</tr>
<tr>
<td>Option to switch</td>
<td>Switch the mix of output/input, according to the market conditions.</td>
<td>Toys; machine parts; electric power</td>
</tr>
</tbody>
</table>

#### 4.1.2 Binomial tree real option model

There are three classic real option valuation methods, which are appropriate for different scenarios respectively (Trigeorgis 1993). Even though these solution methods are different from each other, they would give the same option value when the inputs and application framework are appropriately structured. The three real options valuation methods are:

1. The partial differential equation (PDE) approach
2. The dynamic programming approach
3. The simulation approach

Among the three approaches of option valuation, the dynamic programming one is the most flexible and popularly used. One of the important applications of the dynamic programming approach is the *binomial tree option valuation model*, which is based on a simple representation of the evolution of the underlying asset value. The binomial real option model was first proposed by Cox, Ross and Rubinstein (Cox, Ross et al. 1979). Essentially, the model uses a *discrete-time* framework to map the varying price...
of the underlying asset over time. The basic parameters in binomial tree option valuation are listed in Table 4-3.

<table>
<thead>
<tr>
<th>Table 4-3 Basic parameters in option valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of underlying asset</td>
</tr>
<tr>
<td>Strike price</td>
</tr>
<tr>
<td>Volatility of return in underlying value</td>
</tr>
<tr>
<td>Time to maturity</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
</tr>
<tr>
<td>Time interval on a binomial tree</td>
</tr>
</tbody>
</table>

The underlying asset, $v$, is what the holder of the option will receive if he eventually exercises the option, i.e. the final design in the design outsourcing context. In the multiplicative binomial model of uncertainty, the underlying asset is given an initial value, $v$, and moves up to $v \cdot u$ or down to $v \cdot d$ within a period of time of $\Delta t$ with probability $q$ and $(1 - q)$, where $u$ and $d$ indicate the coefficients of moving up and down respectively. Cox, Ross and Rubinstein (1979) derived the up and down coefficients, $u$ and $d$, as well as the relative probabilities, $q$ and $(1 - q)$, in a time interval $\Delta t$ to be,

$$u = e^{\sigma \sqrt{\Delta t}},$$  \hspace{1cm} (4.1)

$$d = \frac{1}{u},$$  \hspace{1cm} (4.2)

$$q = \frac{e^{r \Delta t} - d}{u - d},$$  \hspace{1cm} (4.3)

$$1 - q = \frac{u - e^{r \Delta t}}{u - d}.$$  \hspace{1cm} (4.4)
At the next binomial step, the possible asset values become $v \cdot u^2$, $v \cdot u \cdot d$ or $v \cdot d^2$. Such recursive binomial mesh can be extended into future steps (Figure 4-2).

In any option analysis and pricing problem, the strike price, $K$, is the amount of money that is paid on the date of exercise in the nominal amount. In the design outsourcing context, the strike price is the final design price asked by the designer.

Figure 4-2 Multiplicative binomial model

The volatility, $\sigma$, of the underlying asset represents the degree of uncertainty of the design outcome. In simple words, the volatility indicates how uncertain the final design value could turn out to be, with higher volatility indicates higher potential transactional risk.

Suppose that $v \cdot u = v_{up}$ and $v \cdot d = v_{down}$, the corresponding call option values, $C_{up}$ and $C_{down}$, can be indicated as,

$$C_{up} = \max(v \cdot u - K, 0)$$

(4.5)
Chapter 4: Multi-step Prototype Model

\[ C_{down} = \max(\nu \cdot d - K, 0) \]  
(4.6)

\[ v = \frac{q \times v_{up} + (1-q) \times v_{down}}{e^{rt}}, \]  
(4.7)

the corresponding option value in Figure 4-3 can be calculated with the function,

\[ C = \frac{q \cdot C_{up} + (1-q) \cdot C_{down}}{e^{rt}}. \]  
(4.8)

Figure 4-4 illustrates a very simple example of a double compound option valuation process. It is assumed that the underlying asset value starts from 100 with volatility \( \sigma = 100\% \) and risk-free rate of \( r = 5\% \). The binomial tree is mapped with the interval of \( \Delta t = 0.5 \). The up coefficient, \( u \), and its risk-neutral probability, \( q \), are then calculated to be,
$u = 2.028$ and $q = 34.68\%$. Three investments $I_1, I_2$ and $I_3$ will be respectively deployed at $t_0$, $t_1$, and $t_2$, of which $I_2$ and $I_3$ are regarded as two strike prices.

Table 4-4 Parameters in the simplified two-phase prototyping model

<table>
<thead>
<tr>
<th>Initial Inputs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated design value</td>
<td>$v$</td>
</tr>
<tr>
<td>Estimated transactional risk</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Time interval of binomial tree</td>
<td>$\Delta t$</td>
</tr>
<tr>
<td>Investment $I_1$ and $I_2$</td>
<td></td>
</tr>
<tr>
<td>Design price $P$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficients of Binomial Tree</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward coefficient</td>
<td>$u = \exp(\sigma \sqrt{\Delta t})$</td>
</tr>
<tr>
<td>Downward coefficient</td>
<td>$d = 1/u$</td>
</tr>
<tr>
<td>Probability of moving upward</td>
<td>$q = \frac{\exp(r\Delta t) - d}{u - d}$</td>
</tr>
</tbody>
</table>

Figure 4-4 Recombinant and non-recombinant binomial tree
The option valuation process starts from $t_2$ with the payoff function, $C(t_2) = \max(v(t_2) - P, 0)$. The option map is then calculated backwards to $t_0$. The result shows that $C(t_0) = 41.6 < I_1 = 50$, which suggests that it is not profitable to invest in this scenario, as the option cost $(I_1)$ is higher than its value $(C)$.

This example maps the binomial tree with an interval of $\Delta t = 0.5$. We are able to get a more accurate result through a finer mesh with smaller time interval. Interestingly, as $\Delta t \to 0$, this discrete-time binomial process will eventually converge to the continuous-time Black-Scholes formula (Black and Scholes 1973),

$$C = vN(x) - E(1+r)^{-\tau}N(x-\sigma\sqrt{T})$$

where

$$x = \frac{\ln\left(\frac{v}{E(1+r)^{-\tau}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}.$$  \hspace{1cm} (4.10)

The traditional real options valuation models assume constant risk level throughout the underlying project development. However, in the context of multi-phase design outsourcing, each phase of prototype serves as a sampling process, which provides valuable information about that final design value to the customer. As showed in Chapter 3, the transactional risk level will be reduced after the customer observes the prototype outcome. Thus, the traditional options valuation approach needs to be altered to adapt to the dynamic risk scenario.
4.2 Generalized Bayesian Updating

In Chapter 3, a single-step Bayesian updating methodology is developed. However, a generalized Bayesian process is needed to serve the multi-phase prototype scenario.

The design buyer is able to update his estimated value of the final design after observing the prototyping outcome. Let $v_0$ be the customer’s prior knowledge of final design value; in other words, the customer expects the design outcome to worth $v_0$ before observing any prototype. Suppose that the first prototype observation, $v_{p1}$, is drawn from a distribution $N(\mu_0, Q_0 + \Sigma_1)$ (as explained in Chapter 3); and the updated design value after observing the first prototype is $v_1$. The probability density function $f(v_1)$ is equivalent to the conditional probability, $f(v_0|v_{p1})$,

$$f(v_1) = f(v_0|v_{p1}) = \frac{f(v_{p1}|v_0) \cdot f(v_0)}{f(v_{p1})} \quad (4.11)$$

The PDF of the second prototype, $v_2$, could be similarly shown as a Bayesian updating from $v_1$,

$$f(v_2) = f(v_1|v_{p2}) = \frac{f(v_{p2}|v_1) \cdot p(v_1)}{f(v_{p2})}, \quad (4.12)$$

where, $v_{p2}$ is the observation from the second prototype. In Chapter 3, the updated mean and variance of $v_1$ are derived to be,
Chapter 4: Multi-step Prototype Model

\[
\mu_i = \frac{Q_0 v_p + \mu_0 \Sigma_1}{Q_0 + \Sigma_1} \quad \text{and} \quad Q_1 = \frac{Q_0 \Sigma_1}{Q_0 + \Sigma_1}.
\]

Given that \( v_1 \) follows normal distribution, the probability density function (PDF) of \( v_1 \) can be written as,

\[
f(v_1) = \frac{1}{\sqrt{2\pi Q_1}} \exp\left[-\frac{(v_1 - \mu_1)^2}{2Q_1}\right]. \quad (4.13)
\]

The PDF of \( v_p \) is,

\[
f(v_p) = \frac{1}{\sqrt{2\pi(Q_0 + \Sigma_1)}} \exp\left[-\frac{(v_p - \mu_0)^2}{2(Q_0 + \Sigma_1)}\right]. \quad (4.14)
\]

It is assumed that \( v_{p1} = v_0 + \Sigma_1 \), where \( v_0 \sim N(\mu_0, Q_0) \) and \( \Sigma_1 \sim N(0, \Sigma_1) \), the conditional probability \( f(v_{p1}|v_1) \) can be derived to,

\[
f(v_{p1}|v_1) = \frac{1}{\sqrt{2\pi \Sigma_1}} \exp\left[-\frac{(v_{p1} - v_1)^2}{2\Sigma_1}\right] \quad (4.15)
\]

So the PDF of \( v_2 \) is calculated to be,

\[
f(v_2) = f(v_1|v_{p2}) = \frac{f(v_{p2}|v_1)f(v_1)}{f(v_{p2})} \quad (4.16)
\]

The PDF of \( f(v_{p2}|v_1) \cdot f(v_1) \) is,

\[
f(v_{p2}|v_1) \cdot f(v_1) = \frac{1}{2\pi \sqrt{\Sigma_2 Q_1}} \exp\left[-\frac{(v_{p2} - v_1)^2}{2\Sigma_2} - \frac{(v_1 - \mu_1)^2}{2Q_1}\right] \quad (4.17)
\]
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The mean and variance of $v_2$ are,

$$
\mu_2 = \frac{Q_1 v_{p2} + \mu_1 \Sigma_2}{Q_1 + \Sigma_2},
$$

(4.18)

$$
Q_2 = \frac{Q_1 \Sigma_2}{Q_1 + \Sigma_2},
$$

(4.19)

where $\mu_1 = (Q_0 v_{p1} + \mu_0 \Sigma_1) / (Q_0 + \Sigma_1)$ and $Q_1 = Q_0 \Sigma_1 / (Q_0 + \Sigma_1)$.

So the properties of $(n+1)$th prototyping can be represented with the $n$th’s properties.

$$
\mu_{n+1} = \frac{Q_n v_{p_{n+1}} + \mu_n \Sigma_{n+1}}{Q_n + \Sigma_{n+1}}
$$

(4.20)

$$
Q_{n+1} = \frac{Q_n \Sigma_{n+1}}{Q_n + \Sigma_{n+1}}
$$

(4.21)

This is a recursive relationship, with decreasing variance $Q$. Here, $Q_0$ and $\Sigma$ are pre-determined parameters, so the values of $Q_1$ to $Q_{n+1}$ follow the above formula.

### 4.3 A Bayesian-based Real Option Model

The Bayesian updating method serves as a tool to take account of incoming information and update the prior knowledge of project. Real option method takes account of the flexibility value. The idea of combining both Bayesian method and real options serves the need of modeling the learning effects into multiple prototyping.
4.3.1 Risk reduction through prototype sampling

The traditional real option valuation method assumes a constant volatility throughout the whole valuation process. The single volatility assumption would only be appropriate if the volatility estimation is independent of the prototyping process, in other words, the prototyping outcome does not affect the initial estimated volatility, $\sigma$. Unfortunately, this assumption is not reasonable in practice.

Equation 4.21 indicates that the volatility will be continuously reduced throughout multiple prototyping process,

$$Q_{n+1} = \frac{Q_n \cdot \Sigma_{n+1}}{Q_n + \Sigma_{n+1}}.$$
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This means that new information collected from the previous prototype will contribute in reducing the previously estimated volatilities. This risk reduction effect is illustrated in Figure 4-5 where the initial variance is assumed to be $Q_0 = 100\%$, and the fidelity rate at each development phase is assumed to be $\Sigma = 50\%$.

As indicated in the Figure 4-5, the transactional risk is continuously decreased from the initial 100% to as low as 5% after 10 iterations of Bayesian updating. Such dynamic volatility effect should not be excluded in the option valuation model.
4.3.2 Non-recombinant binomial tree model

Traditionally, the real option can be valued by using recombinant binomial tree with a constant risk level. However, the recombinant tree does not fit to the dynamic risk level case, as the updated risk level will affect the up and down coefficient, given the up coefficient, \( u \), in the binomial tree is a function of volatility level \( \sigma \), \( u = e^{\sigma \sqrt{t}} \); such that the binomial tree does not recombine in the future steps, as shown in the Figure 4-6.

A non-recombinant binomial tree model is applied to adapt to the dynamic risk level situation. However, unlike the recombinant binomial model, which has \( i \) end nodes at step \( i \); the non-recombinant one has \( 2^{i-1} \) end nodes instead, which requires substantially more computation than its recombinant counterpart. For example, given a 20 steps binomial tree, the non-recombinant model will generate \( 2^{19} = 524,288 \) end nodes, comparing to the recombinant one only 20 end nodes.
Fortunately, it is not necessary to have a large number of steps to achieve a relative accurate result. Figure 4-7 illustrates the sensitivity of option value to the number of binomial steps. The option value swings, but quickly converges to a steady value around 22.8 given the number of binomial steps above 20.
4.3.3 Bayesian-based option valuation process

The process to value the Bayesian-based option is similar to traditional method, which stems the underlying assets in a binomial mesh, maps the end nodes to the option tree, and computes backwards to reach the option value.

Table 4-5 and Figure 4-8 illustrate a simple example of a two-phase Bayesian-based option value process, which is adapted from the example in Section 4.1.2. In this example, two phases of investment in prototyping are made at $t_0$ and $t_1$, with fidelity rate
50%. The buyer purchases the final design with price, $P$, at $t_2$, which is equivalent to the strike price in option valuation.

### Table 4-5 Parameters in the Bayesian-based option valuation

<table>
<thead>
<tr>
<th>Basic Inputs</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial estimated design value</td>
<td>$v$</td>
<td>100</td>
</tr>
<tr>
<td>Initial estimated transactional risk</td>
<td>$\sigma_0$</td>
<td>100%</td>
</tr>
<tr>
<td>Time interval of binomial tree</td>
<td>$\Delta t$</td>
<td>0.5</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r$</td>
<td>5%</td>
</tr>
<tr>
<td>Design price</td>
<td>$P$</td>
<td>50%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prototyping Information</th>
<th>Phase 1</th>
<th>Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototyping cost</td>
<td>$I_1 = 50$</td>
<td>$I_2 = 50$</td>
</tr>
<tr>
<td>Prototyping fidelity</td>
<td>$\Sigma_1 = 50%$</td>
<td>$\Sigma_2 = 50%$</td>
</tr>
<tr>
<td>Transactional risk level</td>
<td>$\sigma_1 = \sqrt{\frac{\sigma_0^2 \cdot \Sigma_1^2}{\sigma_0^2 + \Sigma_1^2}} = 44.72%$</td>
<td>$\sigma_2 = \sqrt{\frac{\sigma_1^2 \cdot \Sigma_2^2}{\sigma_1^2 + \Sigma_2^2}} = 33.33%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficients of Binomial Tree</th>
<th>Phase 1</th>
<th>Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward coefficient</td>
<td>$u_1 = \exp(\sigma_1 \sqrt{\Delta t}) = 1.37$</td>
<td>$u_2 = \exp(\sigma_2 \sqrt{\Delta t}) = 1.27$</td>
</tr>
<tr>
<td>Downward coefficient</td>
<td>$d_1 = 1 / u_1 = 0.73$</td>
<td>$d_2 = 1 / u_2 = 0.79$</td>
</tr>
<tr>
<td>Probability of moving upward</td>
<td>$q_1 = \frac{\exp(r\Delta t) - d_1}{u_1 - d_1} = 46.14%$</td>
<td>$q_2 = \frac{\exp(r\Delta t) - d_2}{u_2 - d_2} = 49.02%$</td>
</tr>
</tbody>
</table>

Given that the transactional risk level is reduced from the original $\sigma_0 = 100\%$ to $\sigma_1 = 44.72\%$ and $\sigma_2 = 33.33\%$, the non-recombinant binomial tree is illustrated in Figure 4-8. The option value is calculated backward in time from maturity. For instance, the top right node in the option valuation tree is calculated to be, $\max(302.8 - 50, 0)$, where 50 is the design price. The rest nodes at $t_2$ are then calculated in the similar fashion. Discounted back on time interval, 188.4 is calculated as the present value of the two forward values 252.8 and 137.7,
Chapter 4: Multi-step Prototype Model

\[ 188.4 = \frac{252.8 \times 49\% + 137.7 \times 51\%}{e^{rM}}. \] (4.22)

Further discounting back one step, the node 88.4 is calculated as the present value with the consideration of Phase 2 investment,

\[ 88.4 = \max \left( \frac{188.4 \times 49\% + 98.6 \times 51\%}{e^{rM}} - 50, 0 \right). \] (4.23)

![Diagram](image-url)

**Figure 4-8** Bayesian-based option valuation model
Discounting the payoff in time to $t_0$, the option value is calculated to be $18.5$, which is significantly smaller than the option value $41.6$, calculated in the similar classic option valuation example in Section 6.1.2. This result coincides with one of the properties of the option theory, the lower the risk, the lower the option value; as the two prototypes significantly reduce the risk level phase by phase.

### 4.4 Numerical Illustration

A numerical illustration is discussed in this section to demonstrate the feasibility of the Bayesian-based real option model in the application level. The sensitivity of some key parameters are also illustrated and discussed.

Airlines are usually not able to commit to purchase a new customized plane without detailed understanding of payload capabilities, maximum range, operating costs per seat mile, and price per plane; and aircraft manufacturers often could not provide these data to the airlines without a thorough conceptual or preliminary design.

After analyzing the requirements from an airline (customer), an aircraft manufacturer (designer) has estimated that the total cost of design process is about $100$ million, which can be disassembled into four phases: with duration 1 year in each phase. The number here is simplified to serve for illustration of the model mechanism.

- **Phase 1.** Conceptual study, 1 year, 10 million,
- **Phase 2.** Project definition, 1 year, 10 million,
- **Phase 3.** Detailed design, 1 year, 70 million,
- **Phase 4.** Certification, 1 year, 10 million.
The airline is concerned with the uncertain value of the final design, and decides to invest $10 million in phase 1 first, which can be regarded as creating an option to expand. The investment in the conceptual study gives the customer an opportunity to learn the value of the eventual design before committing a large investment on the following detailed design and prototyping.

If the outcome is satisfactory after phase 1, the customer will proceed to design and prototyping in the following phases. If the phase 1 result is not favorable, the customer will cancel the deal before potentially losing up to $100 million investment. The $10 million preliminary design creates an option to reduce the customers’ risk exposure. The basic setting of this deal is illustrated in Table 4-6, and the buyer’s decision-making process is shown in Figure 4-9 with time frame.

**Table 4-6 Basic setting**

<table>
<thead>
<tr>
<th>Initial estimated design value</th>
<th>$v_0 = 90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial estimated risk level</td>
<td>$\sigma_0 = 50%$</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r = 5%$</td>
</tr>
<tr>
<td>Investment</td>
<td>$\Sigma I = 100$</td>
</tr>
</tbody>
</table>
Chapter 4: Multi-step Prototype Model

Figure 4-9 Decision making of aircraft design development

On top of the basic setting, we assume that the prototype fidelity at four phases are $\Sigma_1 = 100\%$, $\Sigma_2 = 50\%$, $\Sigma_3 = 30\%$, and $\Sigma_4 = 10\%$ respectively, as the later phase prototypes have better fidelity and more detailed to represent to final design.

The Bayesian updating equation can be rewritten in terms of volatility,

$$
\sigma_{n+1}^2 = \frac{\sigma_n^2 \cdot \Sigma_{n+1}^2}{\sigma_n^2 + \Sigma_{n+1}^2}.
$$

(4.23)

The initial estimated volatility at phase 1 is $\sigma_0 = 50\%$ then it’s updated to $\sigma_1 = 44.72\%$ at phase 2, at phase 3 and $\sigma_3 = 31.11\%$ at phase 4, respectively.
Table 4-7 Prototyping information

<table>
<thead>
<tr>
<th>Phase</th>
<th>Cost</th>
<th>Development duration</th>
<th>Estimated fidelity</th>
<th>Initial and updated risk level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>$10 million</td>
<td>1 year</td>
<td>$\Sigma_1 = 100%$</td>
<td>$\sigma_0 = 50%$ (initial estimation)</td>
</tr>
<tr>
<td>Phase 2</td>
<td>$10 million</td>
<td>1 year</td>
<td>$\Sigma_2 = 50%$</td>
<td>$\sigma_1 = 44.72%$ (updated estimation)</td>
</tr>
<tr>
<td>Phase 3</td>
<td>$70 million</td>
<td>1 year</td>
<td>$\Sigma_3 = 30%$</td>
<td>$\sigma_2 = 37.80%$ (updated estimation)</td>
</tr>
<tr>
<td>Phase 4</td>
<td>$10 million</td>
<td>1 year</td>
<td>$\Sigma_4 = 10%$</td>
<td>$\sigma_3 = 31.11%$ (updated estimation)</td>
</tr>
</tbody>
</table>

4.4.1 Decision analysis

The decision analysis starts from the very last decision the customer makes, i.e. whether he should invest in phase 4 or not. The choice is between either (1) spending $10 million to certify the design, and then launch the product, or (2) walking away from the deal.

![Decision making at Phase 3](image)

Figure 4-10 Decision making at Phase 3
Figure 4-11 illustrates the customer’s payoff curve given his payoff function, \( \max(v - 10, 0) \):

![Payoff Curve Diagram](image)

**Figure 4-11 Payoff of call option**

Moving backwards in time, the decision to make in phase 3 is to either (1) spend $70 million on detailed design, or (2) walk away. The choice of making the detailed design creates an option in future. The time to maturity is 1 year. The strike price is $70 million. The underlying asset is what will be received in exchange for the strike price, which is the certificated aircraft. The volatility is the uncertainty in phase 3 estimate of the value of design in phase 4. The risk-free rate is assumed to be 5%.
Similarly, in phase 2, the customer need to decide to either (1) spend $70 million and 2 years of time on detailed design, or (2) walk away from the project. The investment in phase 2 means that the management purchases an option on option on a certified design. Here, the time to duration is 1 year. The strike price is $10 million. The volatility is the customer’s uncertainty of the design value in phase 3. Similar analysis can be repeated until the phase 1 is reached. The MATLAB code of calculating this Bayesian-based option value can be found in Appendix II.

After inputting all the above settings into the Bayesian-based real option model, the option value at phase 1 is calculated to be **24.06**. Given that the investment at phase 1 is 10, the result shows that it’s potentially profitable to invest on the first phase proto-
type, even though the total investment $\sum I = 100$ is higher than his initial expected value $v_0 = 90$.

### 4.4.2 Comparing traditional vs. Bayesian-based real option model and sensitivity analysis

Based on the above settings, this research calculates the option values based on both traditional and Bayesian-based real option model by varying selected variables while keeping others fixed.

Figure 4-13 shows the comparisons between the two models when the initial estimated value is given in the range $v_0 = [50, 90]$, within which the Bayesian-based model constantly gives a lower value than the traditional one. The black bold line indicates the investment threshold, as the investment at phase 1 development is 10, which means that only option values larger than 10 are favorable. This chart gives a break-even point at $v_0 = 67$ for Bayesian model. In other words, given the above settings, only when the initial estimated value of proposed project is larger than 67, the management can potentially profit from investing in phase 1 prototyping.

Given a range of estimated volatility $\sigma_0 = [10\%, 100\%]$, the option values of the two models are plotted in Figure 4-14, which indicates that the volatility level and option value are positively correlated. When $\sigma_0$ increases, the difference between the results of two models is amplified as well.
Figure 4-13 Sensitivity analysis of underlying design value

Figure 4-14 Sensitivity analysis of volatility level
Chapter 4: Multi-step Prototype Model

4.5 Summary

In this chapter, a multiple-step prototypes model is developed based on real option analysis. In this model, investing in each prototyping stage is seen as purchasing an option for the future development. The traditional real option analysis assumes constant risk level throughout the product design process. However, it is not applicable in design outsourcing with prototyping as the risk level is dynamic with each succession of prototypes. This research incorporates a Bayesian updating process into a real option model to measure the reduced risk level through prototyping. The Bayesian-based option valuation model takes the amount of investment and development duration at each development phase into consideration, and calculates an option value to assist the customer in making decisions regarding whether to prototype or not and how to allocate the investment.

A case study based on aircraft design outsourcing is subsequently presented to illustrate the multiple-step prototypes model. The results show that the Bayesian-based option valuation model gives a lower option value than the traditional model does. This result suggests that the traditional model overestimates the design value, which may lead to excessive investment. The main limitation of the model is the assumption of a bi-lateral contracting scenario with only one customer and one designer. In reality, it is often the case that one customer sources a design from multiple competing designers. The multi-lateral scenario will complicate the decision making in design outsourcing, as competition among designers could force truth telling and reduce prices but the customer needs to deal with multiple designers who might have different design capabilities. This scenario will be studied in future work.
CHAPTER 5: MULTI-DESIGNER AND MULTI-PHASE DESIGN CONTEST

We have discussed two scenarios of design outsourcing in last two chapters. In Chapter Three, we integrated the course of prototyping into a single phase; and in Chapter Four, we extended the previous model to multiple phases with the consideration of updated information from prototypes. These two scenarios are both based on the assumption of single source of outsourcing, which is unfortunately not very realistic for practical reasons. In real life, the buyer often sources a single design out from a group of designers. For instance, U.S. Department of Defense (DoD) sourced the F-22 design out from six different suppliers (Cosden 2002). In this chapter, the single designer assumption is relaxed to include multi-designer into outsourcing decision making.

An intuitive analogy to such multi-phase and multi-supplier suppliers setting is an interview process, where the employer (buyer) intends to hire one out of a list of job seekers (suppliers). The candidates’ capabilities (designer’s capabilities) are unclear to the employer, who needs to conduct several rounds of interviews (prototypes) to gather more information from the candidates. As interviewing job candidate is a costly and time consuming process, the employer needs to figure out an optimal screening solution at each round (prototyping phase) to hire the best candidate to maximize his expected payoff, \( \max (E[\text{value-cost}]) \).

In a contest of sophisticated product or system design, the buyer often provides the firms selected to participate with substantial subsidies (Che and Gale 2003), which are strategically allocated to a string of prototyping phases. The financial subsidy could
also be seen as an investment in prototyping. The question worth asked here is, which designer(s) should be invested in order to maximize the design buyer’s expected payoff? In other words, the buyer needs to form an optimal strategy to screen out the incompetent designer(s) after observing the submitted prototypes to avoid unnecessary following costs, and at the same time, select the most qualified potential designer, based on (1) his estimation of the designers’ capabilities, (2) investment amount in prototyping, (3) designers’ pricing. Such multi-player design sourcing setting is often referred as design contest, which is a particular type of reverse auction.

In the following sections, the background of auction theory and design contest is introduced first. It is followed by the construction of a multi-designer and multi-phase design contest model to locate the optimal prototype value threshold, which is used for selecting competent designers through prototyping phases. A numerical simulation is subsequently constructed, and the result shows that the design value threshold indeed implies the optimal designer-selection strategy in terms of maximizing the buyer’s expected payoff.

5.1 Introduction to Auction Theory and Design Contest

5.1.1 Auction theory

An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants. One party to an exchange often knows something relevant to the transaction that the other party does not know (McAfee and McMillan 1987). This sort of information barrier is referred as asymmetric information; where one party (e.g. designer) knows more information (e.g. his design capability) than the other party (e.g. design buyer).
The asymmetry of information is the essence of the auction problem. If the information is perfect, the auctioneer could gain all the profit from the trade. In fact, the auctioneer does not know the bidder’s valuations; thus, he needs an auction mechanism to extract that piece of information.

Generally, there are four basic types of auction mechanisms when a unique item is to be bought or sold: the English auction (also called ascending-bid auction), the Dutch auction (also called descending-bid auction), first-price sealed-bid auction, and the second-price sealed-bid auction (also called the Vickrey auction).

### 5.1.2 Design contest

Design contest is type of reverse auction, where the buyer is the auctioneer, and the sellers are the contestants. Design contests have played an important role in the design and innovation procurement in history. In 1714, the British Parliament held a contest for a method of determining longitude at sea to within 1.5 degrees with a prize of £20,000 (Che and Gale 2003). In 1829, the Liverpool and Manchester Railway held a design contest for the best design for engine to provide passenger service between the two cities with offering a prize of £500 (Taylor 1995; Fullerton and McAfee 1999). Recently, a prize was awarded for newly developed vaccines (Kremer 2001), and for a manned flight to Mars (Zubrin 1996). The U.S. Department of Defense (DoD) awards billions of dollars annually to newly developed weapon and defense systems through design contest (Fullerton, Linster et al. 2002).

Generally, there are two types of design contests, (1) fixed-prize tournament, where the winner’s prize is specified up front, and (2) auction style contest, where *ex ante* investments are important; for example, defense contractors often make R&D investments to
produce prototypes and then bid prices for the production contract (Che and Gale 2003). Fullerton et al. (2002) analyzed these two prize structures, first-price auctions and fixed prizes, and concluded that auctions are superior to a range of fixed prizes in terms of expected revenues for the auctioneer. Fullerton and McAfee (1999) further extended the theory of auction form design contest by showing that the optimal number of competitors is two. In line with the arguments of previous literatures, this research limits the design contest format into: auction style design contest with two suppliers.

The optimal auction formats discussed in previous literatures were focusing on how to set the auction rule ex ante the contest. However, in a design contest with multiple rounds of competitions, it is also very crucial for the auctioneer to know which designer(s) should be selected to the next round of competition based on current and past information.

Extending the work of Fullerton et al. (2002), who proved the auction is superior to the fixed prizes tournament, this research integrates the concept of real options into the auction form design contest framework, and develops a tool for the design buyer (auctioneer) to optimally select qualified contestant(s) in each phase of contest in terms of maximizing his expected payoff.

5.2 Multi-phase Design Contest Model

Suppose that a buyer wishes to procure a design from two symmetric and risk neutral designers, a and b, using a K-phase, $K \geq 2$, design contest. At phase $k$, $k = 1, 2, \ldots, K$,
designer $i$, $i = a$ or $b$, submitted a prototype valued $v_k^i$. The phase $k$ prototyping duration is denoted as $t_k$, and $\sum_{k=1}^{K} t_k = T$.

It is assumed that the designer’s prototype value, $v(t)$, follows a Wiener process, where only the current value is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant (Hull 2009).

$$\frac{dv}{v} = \mu dt + \sigma dz$$

(5.1)

$\mu$ denotes the drift rate, $\sigma$ indicates the volatility rate, $dz = \varepsilon \sqrt{dt}$, and $\varepsilon \sim N(0,1)$. At phase $k$, $0 < k < K$, if designer $i$’s prototype $v_k^i$ is qualified (the qualifying criteria will be discussed later), he will be awarded a subsidy $I_k$ for producing a more detailed prototype in phase $(k+1)$. At the end of last design phase $K$, the outlasted designer $i$ could submit an offer $(v_T^i, p^i)$ to the buyer, with idiosyncratic design valued $v_T^i$ and asked price $p^i$.

It is assumed that the bidding process is constructed in a sealed first price auction fashion, thus each designer’s bidding, i.e. design value $v_T$ and asked price $p$, is private information, which is only available for the buyer to assess. Fullerton et al. (2002) proved that, the designer’s pricing is a function of his final design value, $p^i = P(v_T^i)$, under such bidding structure.
5.2.1 Designer's pricing function

Given that the design contest model is constructed in a sealed first price auction fashion, and is based on the four basic assumptions of auction model (McAfee and McMillan 1987).

A1. The bidders are risk neutral.
A2. The independent-private-values assumption applies.
A3. The bidders are symmetric.
A4. Payment is a function of bids alone.

Fullerton et al. proved that the designer's optimal pricing is a function of his final design value (Fullerton, Linster et al. 2002),

\[ P(v_T) = \frac{\int F(\xi) d\xi}{F(v_T)} + MC, \]  \hspace{1cm} (5.2)

where, \( v_T \) indicates the value of the design submitted at the final stage of the contest, \( F(v_T) \) denotes the cumulative distribution function (CDF) of \( v_T \), and \( MC \) indicates designer's manufacturing cost. The manufacturing cost is assumed to be known. The CDF of \( v_T, F(v_T) \), could be derived from Itô's lemma given that \( v(t) \) follows a Wiener process, \( dv/v = \mu dt + \sigma dz \).

Suppose that a variable \( x(t) \) follows the Itô's process,
where $dz$ is a basic Wiener process, and $a$ and $b$ are functions of $x$ and $t$. Itô’s lemma shows that a function $G$ of $x$ and $t$ follows the process,

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} a(x,t) + \frac{\partial G}{\partial x} b(x,t) + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2,$$

(5.4)

Given that $dv/v = \mu dt + \sigma dz$, from Itô’s lemma, a function $G$ of $v(t)$ follows,

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} \mu v + \frac{\partial G}{\partial v} \frac{1}{2} \frac{\partial^2 G}{\partial v^2} \sigma^2 v^2 dt + \frac{\partial G}{\partial v} \sigma v dz.$$

(5.5)

Define that,

$$G = \ln v$$

(5.6)

We can derive the partial derivatives of $G$,

$$\frac{\partial G}{\partial v} = \frac{1}{v}, \quad \frac{\partial^2 G}{\partial v^2} = -\frac{1}{v^2}, \quad \frac{\partial G}{\partial t} = 0$$

Itô’s lemma shows that,

$$dG = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dz$$

(5.7)
Such that $G = \ln v$ follows a generalized Wiener process, with constant drift rate

$$\mu - \frac{\sigma^2}{2},$$

and constant volatility $\sigma$. Such that the change of $G = \ln v$ from time 0 to $T$ is normally distributed with mean $\left(\mu - \frac{\sigma^2}{2}\right)T$ and variance $\sigma^2 T$. Such that,

$$\ln v_T - \ln v_0 \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right)$$

(5.8)

or,

$$\ln v_T \sim N\left(\ln v_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right)$$

(5.9)

So the CDF of $v$ at maturity $T$, $v(T)$, is indicated as,

$$F(v_T) = \frac{1}{2} + \frac{1}{2} \text{erf}\left[\frac{\ln v_T - \text{mean}}{\sqrt{2 \text{var}}}\right],$$

(5.10)

where mean = $\log(v_0) + \left(\mu - \frac{\sigma^2}{2}\right)t$, and var = $\sigma^2 T$. The designer’s pricing function,

$$P\left(v_T\right) = \frac{\int_{0}^{v_T} F(\xi)d\xi}{F(v_T)} + MC,$$

(5.11)

is completed after plugging $F(v_T)$ and $MC$ into Equation 5.2.
### 5.2.2 Locating the optimal prototype value threshold

The selected design $v_k$ will be awarded $I_k$ at phase $k$, and the buyer has the right but not obligation to purchase the final design with price $P(v_T)$. This process is similar to investing $I_k$ on a project with a potential payoff $\max(v_T - P(v_T), 0)$. This is equivalent to purchasing a call option with time duration $T$, underlying value $v_k$, and strike price $P(v_T)$. Option theory is derived from the financial world. A call (put) option gives its holder the right but not obligation to purchase (sell) the underlying asset at the pre-defined strike price at the maturity.

According to the option theory, the buyer is only potentially profitable to purchase the option if the option value $C$ is worth more than the payment to it. Equivalently, the real option to the designer buyer is the option to go through the prototyping phases and then decide whether to purchase the final design; and the payment to purchase this option is to invest $I_k$ to the designer to continue his prototyping. Such that, the buyer would only invest $I_k$ to a prototype $v_k$ given that $C(v_k) \geq I_k$, where $C(v_k)$ is the function to price the option given design value $v_k$. The option value, $C(v_k)$ is a strictly increasing and absolutely continuous function for $v_k \geq 0$. There exists a threshold value, $v_k^*$, such that only a higher value is attractive. Given that $C(v^*) = I$, $v^* = C^{-1}(I)$.

And in the final phase $K$, the buyer simply chooses the one provides highest payoff to him, $\max[v_T^i - P(v_T^i), 0]$. Figure 5-1 illustrates the buyer's decision making process at the presence of two designers, $a$ and $b$. 
Chapter 5: Multi-designer and Multi-phase Design Contest

Figure 5-1 Multi-phase prototyping selection
5.3 Alternative Option Pricing

In order to find the optimal prototype value threshold $v_k^*$ at prototyping phase $k$, we need to price the option with the payoff, $\max[v_T - P(v_T), 0]$. Unlike the traditional call option with a fixed strike price, the strike price of this option is a function of the underlying value at maturity $T$, $P(v_T)$. This unconventional option valuation structure is referred as alternative option in this research.

The Black-Scholes partial differential equation can be applied to price such alternative option, however in this case, there is no analytical solution. A numerical method, finite difference method, is applied to solve the Black-Scholes PDE and calculate the option value due to its fast convergence.

5.3.1 Black-Scholes partial differential equation

Given the prototyping process follows $dv/v = \mu dt + \sigma dz$, the option value $C(v_t)$ can be illustrated in a Black-Scholes partial differential equation (Black and Scholes 1973),

$$ rC = \frac{\partial C}{\partial t} + rv \frac{\partial C}{\partial v} + \frac{1}{2} \sigma^2 v^2 \frac{\partial^2 C}{\partial v^2} $$  \hspace{1cm} (5.12)

The above PDE solution is unique to its boundary conditions. In this case, the PDE is bounded by three conditions, (1) $C(v_t, t = T) = \max[v_T - P(v_T), 0]$, (2) $C(v_t = 0, t) = 0$, and (3) $C(v \to \infty, t) = v - P(v)$. A numerical method, finite difference methods (FDM), is used to approximately price the option value at $C(v_t, t = 0)$. 
Notably, the Black-Scholes options pricing model has a number of drawbacks, such as the assumption of random walks, constant volatility level, and etc. Fortunately, in the design contest model constructed in this chapter, these two limitations can be neglected, as the design value is also assumed to follow a stochastic process, which can be simulated using random walk. Further, the dynamic volatility assumption is dropped due to the complexity of the existing model. Such that a modified Black-Scholes PDE model can be a fit to this research context.

5.3.2 Finite difference method

Finite difference method is a numerical way to approximate the partial differential equation by a set of difference equations. Essentially, the idea is to substitute the partial derivatives with approximate finite differences, which are solved in a backward recursive fashion. The original research of finite difference method can be traced back to 1928, conducted by Courant, Friedrichs et al. (1928). Thomée gave a comprehensive summation of the development of finite difference method of past 70 years (Thomée 2001).

The life of the option $T$ could be divided into $M$ equally spaced intervals of length $\Delta t = T / M$, which results a total of $M+1$ time nodes,

$$0, \Delta t, 2\Delta t, ..., T.$$ 

Given that $v_{\text{max}}$ is a sufficiently high design value, we could divide the interval $[0, v_{\text{max}}]$ into $N$ segments, with $N+1$ equally spaced nodes $\Delta v = v_{\text{max}} / N$,

$$0, \Delta v, 2\Delta v, ..., v_{\text{max}}.$$
A \((M+1)(N+1)\) grid is constructed with \((M+1)\) time nodes and \((N+1)\) value nodes. A point \((i, j)\) on the grid represents the option value at time \(i\Delta t\) and design value \(j\Delta v\). Correspondingly, the option value at point \((i, j)\) is indicated as \(C_{i,j}\). The \((M+1)(N+1)\) grid is illustrated in Figure 5-2, where the boundary conditions are emphasized in the “bold lines”, and the “×” represents the solution points we are looking for.

\[V_{max} = (N+1)\Delta v\]

\[0 = C_{i,0}\]

\[C_{M+1,j} = \max(v_j - p(v_j), 0)\]

Figure 5-2 Grid for finite difference method
5.3.3 Implicit finite difference method

The *Implicit* finite difference method is an always convergent and numerically stable FDM scheme (Brandimarte 2004). Each of the partial derivatives in the Black-Scholes partial differential equation (Equation 5.12) can be approximated as,

\[
\frac{\partial C}{\partial v} = \frac{C_{i,j+1} - C_{i,j-1}}{2\Delta v}, \tag{5.13}
\]

\[
\frac{\partial C}{\partial t} = \frac{C_{i+1,j} - C_{i,j}}{\Delta t}, \tag{5.14}
\]

\[
\frac{\partial^2 C}{\partial S^2} = \left(\frac{C_{i,j+1} - C_{i,j}}{\Delta v} - \frac{C_{i,j} - C_{i,j-1}}{\Delta v}\right)/\Delta v = \frac{C_{i,j+1} + C_{i,j-1} - 2C_{i,j}}{\Delta v^2}. \tag{5.15}
\]

By substituting the above three equations back into Black-Scholes equation, we could rewrite the it into,

\[
a_j C_{i,j-1} + b_j C_{i,j} + c_j C_{i,j+1} = C_{i+1,j} \tag{5.16}
\]

where,

\[
a_j = \frac{1}{2} \Delta t \left(-\sigma^2 j^2 + r j \right), \quad b_j = 1 + \Delta t \left(\sigma^2 j^2 + r \right), \quad \text{and} \quad c_j = -\frac{1}{2} \Delta t \left(\sigma^2 j^2 + r j \right).
\]

Such that the node value at \((i+1,j)\), \(C_{i+1,j}\), could be interpreted from the three nodes at its previous time \(i\) with three coefficients, \(a_j\), \(b_j\), and \(c_j\) (Figure 5-3).
The above equation could be rewritten in a more general matrix form as:

\[ \mathbf{M} \cdot \mathbf{C}_i = \mathbf{C}_{i+1} + \mathbf{b}_i \]  \hspace{1cm} (5.17)

where \( \mathbf{C}_i \) and \( \mathbf{b}_i \) are \( (N-1) \) dimensional vectors,

\[ \mathbf{C}_i = \begin{bmatrix} C_{i,1} & C_{i,2} & C_{i,3} & \cdots & C_{i,N-1} \end{bmatrix}^T \]  \hspace{1cm} (5.18)

\[ \mathbf{b}_i = \begin{bmatrix} -a_i C_{i,0} & 0 & 0 & \cdots & -c_{N-1} C_{i,N} \end{bmatrix}^T \]  \hspace{1cm} (5.19)

And \( \mathbf{M} \) is a \( (N-1) \times (N-1) \) symmetric matrix,

\[ \mathbf{M} = \begin{bmatrix} b_1 & c_1 & 0 & \cdots & 0 \\ a_2 & b_2 & c_2 & \cdots & 0 \\ 0 & a_3 & b_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & c_{N-2} \\ 0 & \cdots & 0 & a_{N-1} & b_{N-1} \end{bmatrix} \]  \hspace{1cm} (5.20)

We can’t solve this linear system by simply inverting the tridiagonal matrix, instead, we could use an iterative method, called successive over relaxation (SOR).
5.3.4 Solving the linear system using successive over relaxation

Iterative method is efficient to solve a large system of linear equation, \( \mathbf{M} \cdot \mathbf{C}_i = \mathbf{C}_{i+1} + \mathbf{b}_i \), which characterized by a sparse matrix \( \mathbf{M} \). A variant of Jacobi iteration and Gauss-Seidel iteration, called successive over relaxation or SOR iteration, is applied in this research to solve the linear equations due to its fast convergency.

We could generalize the linear equation \( \mathbf{M} \cdot \mathbf{C}_i = \mathbf{C}_{i+1} + \mathbf{b}_i \) into a \((N-1) \times (N-1)\) symmetric matrix \( \mathbf{x}_{i+1} = \mathbf{A} \cdot \mathbf{x}_i + \mathbf{d}_i \), where

\[
\mathbf{x}_i = \begin{bmatrix} x_{i,1} & x_{i,2} & x_{i,3} & \ldots & x_{i,N-1} \end{bmatrix}^T,
\]
\[
\mathbf{b}_i = \begin{bmatrix} b_{i,1} & b_{i,2} & b_{i,3} & \ldots & b_{i,N-1} \end{bmatrix}^T,
\]

\[
\mathbf{A} = \begin{pmatrix}
  a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,N-1} \\
  a_{2,1} & a_{2,2} & a_{2,3} & \cdots & \vdots \\
  a_{3,1} & a_{3,2} & a_{3,3} & \cdots & \vdots \\
  \vdots & \vdots & \vdots & \ddots & a_{N-2,N-1} \\
  a_{N-1,1} & \cdots & \cdots & \cdots & a_{N-1,N-1}
\end{pmatrix}.
\]

(5.21)
(5.22)
(5.23)

We could interpret the above linear equation as moving from the current point \( \mathbf{x}_i \) to the updated point \( \mathbf{x}_{i+1} \) with an additional displacement,

\[
\mathbf{x}_{i+1} = \mathbf{x}_i + \mathbf{r}_i,
\]

(5.24)

where \( \mathbf{r}_i \) is the displacement vector. We could further modify the iteration to,

\[
\mathbf{x}_{i+1} = \mathbf{x}_i + \omega \mathbf{r}_i = \omega \mathbf{x}_{i+1} + (1 - \omega) \mathbf{x}_i,
\]

(5.25)
where $\omega$ is called the *relaxation parameter*. The above equation could be finally described in a modified Gauss Seidel method,

$$
\begin{align*}
    z_i^{k+1} &= \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^k \right), \\
    x_i^{k+1} &= \omega z_i^{k+1} + (1-\omega) x_i^k.
\end{align*}
$$

(5.26)

(5.27)

### 5.4 Numerical Illustration

A simulated study is presented in this section to investigate and test the model proposed earlier, where we argued that we could find an optimal threshold $v^*$ to guide the buyer’s contestant-selection decision in the design contest.

This simulated study simplifies the design contest into two symmetric players and two prototyping phases. At first, the buyer invites two designer $a$ and $b$ to propose an initial prototype. At the end of the first prototyping phase, each of them would submit a design valued $v_i^a$ and $v_i^b$, which are derived from the same initial value $v_0$, and follow the same Wiener process, $dv/v = \mu dt + \sigma dz$, due to the symmetric designers assumption. If the value is higher than the pre-calculated optimal prototype value threshold, $v^*$, the design is qualified to the second round of competition, and will be simultaneously awarded an investment $I$ for the follow-up prototype. At the end of second prototyping, the designer would quote a price $P(v_2)$ according to its second prototype value $v_2$. The one who provides higher payoff $v - P(v)$ to the buyer would be awarded the contract. The basic inputs into the model are (1) subsidy to the designer $I$, (2) manufacturing cost $MC$, (3) risk free rate $r$, (4) time duration $T$, (5) initial estimated design
value $v_0$, and (6) volatility rate $\sigma$. The drift rate $\mu$ is assumed to be 0. Table 5-1 shows details of the settings and Figure 5-4 illustrates the design contest process.

**Table 5-1 Basic inputs**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidy to the designer</td>
<td>$I$</td>
<td>50</td>
</tr>
<tr>
<td>Manufacturing cost</td>
<td>$MC$</td>
<td>70</td>
</tr>
<tr>
<td>Risk free rate</td>
<td>$r$</td>
<td>5%</td>
</tr>
<tr>
<td>Time duration</td>
<td>$T$</td>
<td>2, $t_1=t_2=1$</td>
</tr>
<tr>
<td>Value of initial underlying</td>
<td>$v_0$</td>
<td>120</td>
</tr>
<tr>
<td>Volatility rate</td>
<td>$\sigma$</td>
<td>30%</td>
</tr>
</tbody>
</table>

**Figure 5-4** Simulated design contest process
5.4.1 Locating the optimal prototype value threshold

As Figure 5-4 illustrated, the buyer phases out the incompetent design(s) based on the threshold $v^*$, which is inversely located from the option value $C^{-1}(I)$. In order to use the finite difference method to approximate the option value, we need to setup a finite difference mesh in $v$ and $t$ dimensions. Table 5-2 shows the detailed mesh setup. The solution to this finite difference method is unique to its boundary conditions, (1) $C(v_t = 0, t) = 0$, (2) $C(v_t, t = T) = \max[v_T - p(v_T), 0]$, and (3) $C(v_t \rightarrow \infty, t) = v - P(v)$, where,

$$P(v) = \frac{o}{F(v)} + MC.$$

<table>
<thead>
<tr>
<th>Mesh Setup</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of time mesh point</td>
<td>$M$</td>
<td>100</td>
</tr>
<tr>
<td>Number of design value mesh point</td>
<td>$N$</td>
<td>400</td>
</tr>
<tr>
<td>Asset value interval</td>
<td>$[v_{\text{min}}, v_{\text{max}}]$</td>
<td>$[0, 150]$</td>
</tr>
<tr>
<td>Design value interval</td>
<td>$[t_1, t_2]$</td>
<td>$[1, 2]$</td>
</tr>
<tr>
<td>Asset step</td>
<td>$\Delta t = (t_2 - t_1) / M$</td>
<td>$0.01$</td>
</tr>
<tr>
<td>Time step</td>
<td>$\Delta v = (v_{\text{max}} - v_{\text{min}}) / N$</td>
<td>$0.375$</td>
</tr>
</tbody>
</table>
Figure 5-5 illustrates the boundary condition of $v - P(v)$ at the given setting, which uniformly increases with underlying value $v$, and later keeps constant when $v > 120$. Boundary conditions (2) and (3) are thus determined.

The option value $C(v)$ in the dimension of $v$ is illustrated in Figure 5-6. Given that the investment at the end of phase 1 is $I=50$, we could locate the threshold $v^* = C^{-1}(50)=125$. Next, a simulation is constructed to check this threshold's optimality.
5.4.2 Validating the optimal threshold with Monte Carlo simulation

A Monte Carlo simulation of a stochastic process is a procedure for sampling random outcomes for the process. It is assumed that the two designers’ prototyping process follows the same Wiener process, $\partial v / v = \mu dt + \sigma dz$. A Monte Carlo simulation is constructed to check whether the threshold located by the inverse option, $v^*$, is the optimal point. The random process of $\nu^a(t)$ and $\nu^b(t)$ are generated by the same pseudo code below:

![Figure 5-6 Option value](image-url)
where \( v_i \) is the design value delivered at phase 1. Incompetent \( v_i \) are phased out using \( v^* \), and the \textit{payoff to the buyer} by each designer, payoff\textsuperscript{a} or payoff\textsuperscript{b}, are subsequently calculated using pseudo code:

```matlab
if v1 ≥ v*
    v = v1;
    for i = 1 to M
        Δv(i) = vμΔt + σΔz ;
        v = v + Δv;
    end
    v2 = v;
    payoff to buyer = P(v2) - v2 - I;
else
    payoff to buyer = 0;
end
```

Further the buyer’s final surplus is calculated from,

\[
\text{Buyer’s surplus} = \max\left(\text{payoff}\textsuperscript{a}, \text{payoff}\textsuperscript{b}, 0\right)
\]  \hspace{1cm} (5.28)

As each of this process is a random process, in order to generalize the result, the process is repeated 10,000 times. And the calculated \textit{mean} and \textit{volatility} of the buyer’s payoffs in the dimension of \( v^* = [0, 200] \) is illustrated in Figure 5-7. The related MATLAB code can be found in the Appendix III.
The result shows that the buyer’s expected surplus maximizes around the prototype value threshold 125, which coincides with the result from alternative option theory model developed earlier.

As the threshold value is raised, the volatility of buyer’s surplus uniformly decreases. This makes sense that as the buyer increases the “standard of prototype quality”, less “low valued projects” are qualified into the subsequent phase of competition, thus the payoff risk reduces. An interesting observation from Figure 5-7 is in the range of $v^* = \ldots$
[0, 125], where raising the quality threshold results in increasing the buyer’s expected surplus and reducing his risk simultaneously.

5.5 Summary

Design contest is widely adopted in various industries by the auctioneers to source for good designs. Single phase design contest has been popularly researched; however, its multi-phase format was merely studied. This chapter introduced a multi-designer and multi-phase design contest which allows the buyer to optimally select the competent designers through each phase of prototyping using option theory. The simulation study shows that the threshold point (used for selecting competent contestants), which located by the model proposed earlier, indeed maximize the buyer's expected payoff.
three quantitative models have been developed in this research to assist the management to make informed and calculated decisions on prototyping in different design outsourcing scenarios. This chapter concludes the contribution of this research in the academic and layouts the limitations, and proposes some extended research for future work.

6.1 Summary

6.1.1 Contribution to academic research

This research extends the research frontier in design outsourcing through introducing some innovative models to quantify the risk reduction through prototyping, to value the prototype in multi-phase development scenario, to optimize the buyer’s expected payoff in design contest, and etc.

One of the main contributions of this research is the development of a modeling framework for risk analysis of design outsourcing. The majority of literature on risk management in outsourcing has been focused on production outsourcing, and the risks that have been addressed focus on uncertainties in capacity, lead time, demand and supply etc. This research takes an interdisciplinary approach and introduces recent development in design theory, decision-based design in particular, as a theoretical foundation to model design decisions in a supply chain context. By modeling design as a set of decisions subject to uncertainties and risks, rigorous decision analysis theories and tools like utility theory can be applied to study transactional risk in design outsourcing.
Another key novelty of this research is the modeling of prototyping as a “sampling” process, which has not been reported in literature. Unlike production outsourcing, the design outcome is uncertain, so as the prototyping outcome. The information brought by a prototype is very useful in terms of reducing the buyer’s transactional risk. This research also develops a Bayesian updating method to quantify the risk reduction through successive prototyping. More generally, the Bayesian updating method is essentially a procedure that takes into account the value of additional information based on the prototyping outcome.

A third area of contribution is the development of a set of mathematical models based on utility theory and real options that quantitatively study the risk reduction potential under different design outsourcing scenarios. More specifically, a utility model is developed to calculate the tradeoff between the benefit and the cost of a prototype with the consideration of decision maker’s risk attitude in a single phase prototyping scenario. This model serves as a tool to assist the management to make informed decision in prototyping. The simplified single phase prototyping model is then extended to a multi-phase one, where a modified real options method is employed to value the prototype with the consideration of updated prototype information. Distinct from the past literature, this model applies dynamic, instead of constant, risk level. Thus the prototype value calculated from this model is more conservative and more realistic than the result from the traditional real option model. A multi-phase and multi-player design contest is built to fit a more realistic design outsourcing scenario. Different from the optimal design contest formats discussed in previous literatures which focus on how to set the auction rule ex ante the contest, this model proposes an alternative real option method to locate the optimal prototype value threshold for the design buyer to select.
the competent design(s). Through controlling the prototype value threshold, the design buyer is able to maximize his expected payoff and reduce the associated risk.

6.1.2 Managerial insight

From the management point of view, there are some very interesting, even counter-intuitive, observations from the analysis based on the three models developed in this research. The transactional risk is generally considered as a “deal breaker”, thus people are always trying to avoid or reduce the risk. However, with the presence of prototyping serving as a screening process, the risk could bring more potential upside value to the design buyer. The utility model in this research shows that more transactional risk could increase the buyer’s willingness to make deal under certain circumstances. And the real option model shows that higher transactional risk level results higher prototype value.

Another interesting observation is that when the design buyer expects a prototype with good fidelity rate, he would prefer to invest in the prototype even though his expected valuation of the design is not higher than the price he pays for it, as he would expect the design could turn out to be good after prototyping.

Even though the risk reduction effect of the prototyping process is beneficial to the design buyer, the prototype value calculated from the Bayesian-based real option model is less than the result from its traditional counterpart. This observation seems to be paradoxical; however, it makes sense that the result from the Bayesian-based model is actually more realistic to reflect the risk reduction through prototyping, which is omitted in the traditional real option model.
The design contest model in this research introduces *prototype value threshold* to screen out the incompetent design(s). The buyer is able to control the expected value and the volatility level of his payoff through manipulating this threshold. By increasing this threshold, the buyer becomes “stricter” in the design contest that he is more likely to screen out the bad design(s), but at the same time he may overlook the good design(s) with a bad prototype. By using an alternative option model in this research, the buyer is able to locate the optimal prototype value threshold, which maximizes his expected payoff.

### 6.1.3 Practical decision support

The models developed in this research can help practitioners make operational decision, such as whether to make prototype based on its fidelity rate, design price, level of transactional risk, and etc.

In capital intensive industries, the prototyping process is a significant investment. And the fidelity rate of the prototype is also one of the key factors in the prototyping decision making. The model developed in this research is able to quantify the tradeoff between the fidelity rate and the cost of a prototype. The result shows that the better the prototype fidelity rate, the more willing the buyer wants to invest in prototyping.

This research also helps practitioners to quantify his maximum price willing to pay for the prototyping process and the final design. Thus, the management is able to make informed decisions in bargaining with his counterpart on final design pricing and prototyping cost sharing.
Last but not the least, the model is able to help management screen out the incompetent design(s) in the earlier prototyping phases in the design contest scenario. Thus, the management is able to save a significant amount unnecessary investment on the poor designs, and to maximize his expected payoff and lower the associated level of risk.

### 6.2 Limitation

The three models in this thesis are constructed to be practical in usage, however, an investment-intense decision is made based on many more factors beyond the scope of mathematical formulation. For example, the manufacturing process of F-22 project was simultaneously carried on by most of states in America. This decision was not made entirely based on economic consideration, rather than a political one. This category of limitations which cannot be mathematically modeled can also be extended to other factors, such as macro economical condition, management's preferences, technology infeasibility, and etc. There is little room left to improve the existing models in terms of quantitative along this line. However, the models in the research can be improved through relaxing the underlying assumptions.

The limitation of the research also lies on the straightforward underlying assumptions of the models. In the first model, the buyer's preference is assumed to follow an exponential utility function, which though is prevalent in economics research but not a universally accurate model for each individual decision maker. In the second model, the decision making is assumed to be multiple and serial. However, a more realistic scenario can be iterative. A failed prototype would result a deal to be cancelled, as a result of the second model. This is an improvement from the first model, but a more realistic
scenario would consist an option to iteratively redo the previous state of prototyping with small amount of investment, rather than jeopardizing the entire project due one failure. In the third model, the designers in the auction process are assumed to be risk neutral and symmetric. This assumption cannot be universally applied to all participants in auction. Further, the number of designers is limited to two in the research. The relaxation on this assumption and optimization of the number of designers at different conditions can improve the research to be more academically solid.

6.3 Future Work

6.3.1 Design contest with asymmetric and risk averse designers

The auction form that designed to maximize the expected revenue of the auctioneer when the players are risk neutral and symmetric has been intensely studied. However, the symmetric and risk neutral assumptions do not conform well to observations from the practice. Future work can be carried out to relax these two assumptions, and consider first-price design contest with asymmetric independent privation values and risk averse bidders. There has already been some literature on relaxing these two assumptions in auction theory, but not in the context of design contest. For example, Maskin and Riley (1984) proposed an optimal auction model with risk averse bidders. Bhalgat, Chakraborty and Khanna (2012) discussed the mechanism design for a risk averse auctioneer. Skaperdas and Gan (1995) examined the winning chance for a risk averse bidder in auctions. In the context of auction, the bidding function is in the dimension of price only; however, the bidding in the design contest is in two dimensions of both price and design quality.
6.3.2 Real option game in competition under uncertainty

Another prospective research area is to apply both real option analysis and game theory in valuing the buyer’s prototyping investment in a competitive environment under uncertainty. In a competitive environment, there are complicated and strategic decision dynamics among designers and buyers, which will affect the option value brought by prototypes and henceforth the decision making upon prototyping investment (Ferreira, Kar et al. 2009). Furthermore, participants engaged in design outsourcing contracting could strategically hide or misrepresent private information. In this sense, a prototype could serve as a “screening” tool for a buyer, while a “signaling” tool for a designer. By combining game theory and real option analysis, a hybrid model should be able to better model market uncertainty and decision-maker’s flexibility onto the game theory payoff matrices that capture competitive interaction and potential equilibrium conditions, which can help to design contracts that ensure the interests for both parties in design outsourcing.

6.3.3 Case studies

As the models developed in this research are mathematical and abstract in nature, future research is needed to validate the models regarding its performance and applicability. As performance is concerned, this research has relied on numerical simulation to evaluate the result of the proposed model due to lack of practical data. To test the viability of the models developed in this research, future research in terms of empirical study or case study is needed to investigate whether the assumptions underlying the models are valid or reasonable in different industry settings. Also, case studies can be also carried out in terms of estimating some of the key parameters, e.g. fidelity of a
prototype, which are necessary in implementing the risk models to assist practical decision making in actual design outsourcing situations. This will shed some light on when these models are applicable and when they are not. A case study based on actual industry data would be very useful to study the performance of the proposed model for actual industry implementations.

In summary, design outsourcing is still an emerging problem, which is becoming more prevalent as the focus of industry competition is increasingly moving from production and general services towards design and innovation. This research focuses on the transactional risk during the contracting stage of design outsourcing, makes some contribution towards developing rigorous models for risk quantification and reduction by means of informed prototyping decisions. However, it is a very complex problem and we have just scratched the surface. Future research needs to be carried out so that risks in design outsourcing could be better understood, more rigorously modeled, and more effectively reduced and managed, so that design outsourcing could become a source of competitive advantage for participating companies and innovation become the source of value creation in more industries.
REFERENCES


References


APPENDIX I: BAYESIAN UPDATING ESTIMATION

\[ u(\hat{\pi}_1) = \int_{-\infty}^{\hat{v}_p} u(\hat{\pi}_{12}) f(v_p) dv_p + \int_{\hat{v}_p}^{\infty} u(\hat{\pi}_{11}) f(v_p) dv_p \]

\( v_p \) is assumed to follow a normal distribution \( N(\mu_0, Q_o + \Sigma) \), so the density function of \( v_p \), \( f(v_p) \) is expressed as,

\[ f(v_p) = \frac{1}{\sqrt{2\pi(Q_o + \Sigma)}} e^{-\frac{(v_p - \mu_0)^2}{2(Q_o + \Sigma)}} \]

\( \hat{\pi}_{11} = \mu_0 + \frac{(v_p^* - \mu_0)Q_o}{Q_o + \Sigma} - P_1 - \omega \cdot d - \frac{Q_o \cdot \Sigma}{2R(Q_o + \Sigma)} = \hat{\pi}_{12} = -\omega \cdot d \)

\( v_p^* = P_1 + \frac{(P_1 - \mu_0) \Sigma}{Q_o} + \frac{\Sigma}{2R} \)

\[ u(\hat{\pi}_{12}) = 1 - e^{-\omega \cdot d \over R} \]

So the first part, \( \int_{-\infty}^{\hat{v}_p} u(\hat{\pi}_{12}) f(v_p) dv_p \) is calculated to,

\[ \int_{-\infty}^{\hat{v}_p} u(\hat{\pi}_{12}) f(v_p) dv_p = \frac{1}{2} \left[ 1 - e^{-\omega \cdot d \over R} \right] + \text{Erf} \left( \frac{P_1 - \mu_0}{\sqrt{2(\Sigma + Q_o)}} + \frac{a \cdot \Sigma}{b} \right) \]

where, \( a = Q_o + 2P_1R - 2\mu_0R \), and \( b = 2Q_oR\sqrt{2(\Sigma + Q_o)} \).
And the second part, \( \int_{v_p}^{\infty} u(\hat{x}_{11}) f(v_p) dv_p \) is calculated to,

\[
\int_{v_p}^{\infty} u(\hat{x}_{11}) f(v_p) dv_p = \frac{1}{2} (1 - \text{Erf}[c - \frac{Q_0^2}{b}]) - \frac{1}{2} e^{-\frac{a+2R_{Q_0d}}{2R}} (1 - \text{Erf}[c + \frac{Q_0^2}{b}])
\]

where, \( a = Q_0 + 2P_1R - 2\mu_0R \), \( b = 2Q_0R\sqrt{2(\Sigma + Q_0)} \), and \( c = \frac{a}{b}(Q_0 + \Sigma) \). Integrate both part,

\[
u_a(\hat{x}_1) = \frac{1}{2} (1 - e^{-\frac{P_1 - \mu_0}{\sqrt{2(\Sigma + Q_0)}}} + \frac{a \cdot \Sigma}{b}) + \frac{1}{2} (1 - \text{Erf}[c - \frac{Q_0^2}{b}]) - \frac{1}{2} e^{-\frac{a+2R_{Q_0d}}{2R}} (1 - \text{Erf}[c + \frac{Q_0^2}{b}])
\]

, where \( a = Q_0 + 2P_1R - 2\mu_0R \), \( b = 2Q_0R\sqrt{2(\Sigma + Q_0)} \), and \( c = \frac{a}{b}(Q_0 + \Sigma) \).
APPENDIX II: MATLAB CODE OF BAYESIAN-BASED REAL OPTION VALUATION

Now we try to combine phase 1 to phase 4, so the model can be easier to manipulate. First we need to find out which variables will affect the eventual option value that we want. S0, T1, T2, deltaT, r, sigma1, E1, E2, I1, I2 are the most essential inputs. Other variables, such as u1, d1, q1, etc are all derived from the essential inputs.

I will segregate this model into different modules by order. clear all previous data.

clear;

1st Module: Essential Inputs.

The essential inputs here are based on the case study in confirmation report.

S0 = 90; %S0 is the current value.
X = 100; %X is the strike price.
r = 0.05; %r is the risk-free rate for one year.
sigma1 = 1; %sigma1 is the volatility at phase 1.
E1 = 1; %E1 is the uncertainty of 1st prototype.
E2 = 0.5; %E2 is the uncertainty of 2nd prototype.
E3 = 0.3; %E3 is the uncertainty of 3rd prototype.
E4 = 0.1; %E4 is the uncertainty of 4th prototype.
I1 = 10; %I1 is the investment at the beginning of phase 1.
I2 = 10; %I2 is the investment at the beginning of phase 2.
I3 = 70; %I3 is the investment at the beginning of phase 3.
I4 = 10; %I4 is the investment at the beginning of phase 4.
T1 = 1; %T1 is time length of phase 1.
T2 = 1; %T2 is time length of phase 2.
T3 = 1; %T3 is time length of phase 3.
T4 = 1; %T4 is time length of phase 4.
deltaT = 1/20; %deltaT is the time interval.
Appendix II: MATLAB Code of Bayesian-based Real Option Valuation

% 2nd Module: Bayesian Updating.
%---------------------------------------------------------------
% sigma2 is the volatility at phase 2.
sigma2 = sqrt(sigma1*sigma1*E1/(sigma1*sigma1+E1));
% sigma3 is the volatility at phase 3.
sigma3 = sqrt(sigma2*sigma2*E2/(sigma2*sigma2+E2));
% sigma4 is the volatility at phase 4.
sigma4 = sqrt(sigma3*sigma3*E3/(sigma3*sigma3+E3));

%---------------------------------------------------------------
% 3rd Module: Number of Step in Binomial Tree
%---------------------------------------------------------------
n1 = T1/deltaT;    % n1 steps in phase 1.
n2 = T2/deltaT;    % n2 steps in phase 2.
n3 = T3/deltaT;    % n3 steps in phase 3.
n4 = T4/deltaT;    % n4 steps in phase 4.

%---------------------------------------------------------------
% 4th Module: Up and Down Factors in Binomial Tree
%---------------------------------------------------------------
% u1 is the up factor in phase 1.
% u2 is the up factor in phase 2.
% u3 is the up factor in phase 3.
% u4 is the up factor in phase 4.
% d1 is the down factor in phase 1.
% d2 is the down factor in phase 2.
% d3 is the down factor in phase 3.
% d4 is the down factor in phase 4.

%---------------------------------------------------------------
% 5th Module: Risk Neutral Probability
%---------------------------------------------------------------
q1 = (exp(r*deltaT) - d1)/(u1 - d1);
q2 = (exp(r*deltaT) - d2)/(u2 - d2);
q3 = (exp(r*deltaT) - d3)/(u3 - d3);
q4 = (exp(r*deltaT) - d4)/(u4 - d4);

%---------------------------------------------------------------
% 6th Module: Risk-Free Rate
%---------------------------------------------------------------
Appendix II: MATLAB Code of Bayesian-based Real Option Valuation

r1 = exp(r*deltaT); % r1 is the discounted rate at each step in phase 1.

r2 = exp(r*deltaT); % r2 is the discounted rate at each step in phase 2.

r3 = exp(r*deltaT); % r3 is the discounted rate at each step in phase 3.

r4 = exp(r*deltaT); % r4 is the discounted rate at each step in phase 4.

% ------------------------------------------

7th Module: Values of S1
% j1 is the index of S1. S1(j1=1) means the first value of S1.

% In order to make the programme more efficient, we prelocate the size of the S1 to (n1+1).
S1 = zeros(1,(n1+1));

for j1 = 1 : (n1+1)
    S1(j1) = S0*(u1^(j1-1))*(d1^(n1-(j1-1)));
end

% ------------------------------------------

8th Module: Values of S2
% j2 is the index of S2. S2(j2=2) means the second value of S2.
% k is the number of end nodes stems from each S1.

% In order to make the programme more efficient, we prelocate the size of the S2 to (n1+1)*(n2+1).
S2 = zeros(1,(n1+1)*(n2+1));

j2 = 1;
for j1 = 1:(n1+1)
    for k = 0 : n2
        S2(j2) = S1(j1)*(u2^k)*(d2^(n2-k));
        j2 = j2 + 1;
    end
end
j2 = j2 - 1;

% ------------------------------------------

9th Module: Values of S3
% Values of S3 are stemmed from the S2 value.
% j3 is the index of S3. S3(j3=3) means the third value of S3.
% k2 is the number of end nodes stems from each S2.
% In order to make the programme more efficient, we prelocate the size of the S3 to (n1+1)*(n2+1)*(n3+1).
S3 = zeros(1,(n1+1)*(n2+1)*(n3+1));

j3 = 1;
for j2 = 1 : (n1+1)*(n2+1) % number of S2 is (n1+1)*(n2+1)
    for k2 = 0 : n3
        S3(j3) = S2(j2)*(u3^k2)*(d3^(n3-k2));
        j3 = j3 + 1;
    end
end
j3 = j3 - 1;

% 10th Module: Values of S4
% Values of S4 are stemmed from the S3 values.
% j4 is the index of S4.
% k3 is the number of end nodes stemmed from each S3, so k3 = n4+1

% In order to make the programme more efficient, we prelocate the size of the S4 to (n1+1)*(n2+1)*(n3+1)*(n4+1).
S4 = zeros(1,(n1+1)*(n2+1)*(n3+1)*(n4+1));

j4 = 1;
for j3 = 1 : (n1+1)*(n2+1)*(n3+1)
    for k3 = 0 : n4
        S4(j4) = S3(j3)*(u4^k3)*(d4^(n4-k3));
        j4 = j4 + 1;
    end
end
j4 = j4 - 1;

% 11th Module: Option Value at Phase 4
% i4 is the number of middle_option_value generated from phase 4, which is expressed in Equation 6 in Cox, Ross and Rubinstein's model.
% i4 = (n1+1)*(n2+1)*(n3+1)*(n4+1)
i4 = 1;

% In order to make the programme more efficient, we prelocate the size of the middle_option_value3 to (n1+1)*(n2+1)*(n3+1)*(n4+1).
middle_option_value4 = zeros(1,(n1+1)*(n2+1)*(n3+1)*(n4+1));

% Assign every middle_option_value4 a value.
for j3 = 0 : (n1+1)*(n2+1)*(n3+1)-1
Appendix II: MATLAB Code of Bayesian-based Real Option Valuation

```matlab
%for j4 = 0 : n4
%middle_option_value4(i4) =
(factorial(n4)/(factorial(j4)*factorial(n4-j4))*(q4^j4)*((1-q4)^(n4-j4))*max(0,(S4(j4+1+j3*(n4+1))-I4)))/(r4^n4);
%i4 = i4 + 1;
%end
%end
%i4 = i4 - 1;

%The option_value_phase4 are summation of the above middle_option_value4.
%The number of option_value_phase4 is (n1+1)*(n2+1)*(n3+1)
%Assign every option_value_phase4 a value.

%In order to make the programme more efficient, we prelocate the size of
%the option_value_phase4 to (n1+1)*(n2+1)*(n3+1).
%option_value_phase4 = zeros(1,(n1+1)*(n2+1)*(n3+1));

%for j3 = 1 : (n1+1)*(n2+1)*(n3+1)
%option_value_phase4(j3) = sum_of_variable(1+(j3-1)*(n4+1),n4+1,middle_option_value4);
%end

%---------------------------------------------------------------
%12th Module: Option Value at Phase 3
%---------------------------------------------------------------
%Option value at phase 3.
%i3 is the number of middle_option_value generated from phase 3, which is
%expressed in Equation 6 in Cox, Ross and Rubinstein's model.
%i3=(n1+1)*(n2+1)*(n3+1)
i3 = 1;
%In order to make the programming more efficient, we prelocate the size of
%middle_option_value3.
middle_option_value3 = zeros(1,(n1+1)*(n2+1)*(n3+1));

%Assign every middle_option_value3 a value.
for j2 = 0 : (n2+1)*(n1+1)-1
for j3 = 0 : n3
  middle_option_value3(i3) =
  (factorial(n3)/(factorial(j3)*factorial(n3-j3))*(q3^j3)*((1-q3)^(n3-j3))*max(0,S3(1+j3+j2*(1+n2))-I4))/(r3^n3);
  i3 = i3 + 1;
end
end
i3 = i3 - 1;
```
%The option_value_phase3 is the summation of the above
%middle_option_value3.
%Assign every option_value_phase4 a value.

%In order to make the programme more efficient, we prelocate the size of
%the option_value_phase3 to (n1+1)*(n2+1).
option_value_phase3 = zeros(1,(n1+1)*(n2+1));

for j2 = 1 : (n2+1)*(n1+1)
    option_value_phase3(j2) = sum_of_variable(1+(j2-1)*(n3+1), n3+1, middle_option_value3);
end

%13th Module: Option Value at Phase 2
%---------------------------------------------------------------
%i is the number of middle_option_value generated from phase 2, which is
%expressed in Equation 6 in Cox, Ross and Rubinstein's model.
%i=(n1+1)*(n2+1).
i = 1;
%In order to make the programming more efficient, we prelocate the size of
%middle_option_value2.
middle_option_value2 = zeros(1,(n1+1)*(n2+1));

for j1 = 0 : n1
    for j2 = 0 : n2
        middle_option_value2(i) =
            (factorial(n2)/(factorial(j2)*factorial(n2-j2))*(q2^j2)*((1-q2)^(n2-j2))*max(0,option_value_phase3(1+j2+j1*(1+n1))-I3))/r2^n2;
        i = i + 1;
    end
end
i = i - 1;

%The option_value_phase2 is the summation of the above
%middle_option_value2.

%In order to make the programme more efficient, we prelocate the size of
%the option_value_phase2 to (n1+1).
option_value_phase2 = zeros(1,(n1+1));

for j1 = 1 : n1+1
    option_value_phase2(j1) = sum_of_variable((j1-1)*(n2+1)+1, n2+1, middle_option_value2);
end
%% 13th Module: Option Value at Phase 1

% Middle option on phase 1 has (n1+1) values.

% In order to make the programme more efficient, we prelocate the size of
% the middle_option_value1 to (n1+1).
middle_option_value1 = zeros(1,(n1+1));

for j1 = 0:n1
    middle_option_value1(j1+1) = (factorial(n1)/(factorial(j1)*factorial(n1-j1))*(q1^j1)*((1-q1)^(n1-j1))*max(0,option_value_phase2(j1+1)-I2))/((r1)^n1);
end

% Option value at phase only has one value.
option_value_phase1 = sum_of_variable(1,n1+1,middle_option_value1)
%% Option parameters
clear;

S = 115; % Value of the underlying
K = 50; % Strike (exercise price)
r = 0.05; % Risk free interest rate
sigma = 0.3; % Volatility
T = 1; % Time to expiry
MC=70;

%% Method

% Number of asset mesh points
M = 100;
% Number of time mesh points
N = 200;
% Specify extremes
Szero = 0;
Smax= 205;

% Create mesh
solution_mesh=zeros(N+1,M+1);
Smesh=0:(Smax/M):Smax;
Tmesh=T:-(T/N):0;

% Specify timestep, dt
dt=T/N;

dS=Smax/M;

% Pricing function
m = log(Smesh)+(r-sigma^2/2)*T;
volatility = sigma^2*T;

Pricing=zeros(1,M+1);
Fa=zeros(1,M+1);
intFa=zeros(1,M+1);
for i=1:length(Smesh)
    if Smesh(i)==0
        Pricing(i)=MC;
    else
        Fa(i)= 0.5+0.5*erf((log(Smesh(i))-m(i))/sqrt(2*volatility^2));
        Xa=0:0.01:Smesh(i);
        Ya=0.5+0.5*erf((log(Xa)-m(i))/sqrt(2*volatility^2));
        intFa(i)=trapz(Xa,Ya);
    end
end

Pricing(i)=MC+(intFa(i)/Fa(i)); % Pricing acts like strike price in this exotic option setting
end
temp=isnan(Pricing);

for i=1:length(Smesh)
    if temp(i) == 1
        Pricing(i)=MC;
    end
end

%%%%%%%%%%%%%%%% Boundary %%%%%%%%%%%%%%%%%

solution_mesh(1,:) = max(Smesh-Pricing,0);  % Option value = 0 @t=0;
solution_mesh(:,1) = 0;                       % Option value = 0 @S=0;
solution_mesh(:,M+1) = Smesh(M+1) - Pricing(M+1);  % Option value @S=Smax

A = @(i) 0.5*dt*(r*i-sigma^2*i^2);
B = @(i) 1 + (sigma^2*i^2 + r)*dt;  % Define the functions A, B & C
C = @(i) -0.5*dt*(sigma^2*i^2+r*i);

%%%%%%%%%%%%%%%% Construct Tridiagonal Matrix, Tri %%%%%%%%%%%%%%%%%

Acoeffs = zeros(M+1,1); Bcoeffs = zeros(M+1,1); Ccoeffs = zeros(M+1,1);
for i=1:M+1
    Acoeffs(i) = A(i-1);  Bcoeffs(i) = B(i-1);  Ccoeffs(i) = C(i-1);
end
Tri = diag(Acoeffs(2:end),-1) + diag(Bcoeffs) + diag(Ccoeffs(1:end-1),+1);
Tri_Inv = inv(Tri);  % Compute inverse

%%%%%%%%%%%%%%%% Implicit Euler Iteration %%%%%%%%%%%%%%%%%

for j=1:N
    temp=zeros(M+1,1);
    temp(1)=A(0)*solution_mesh(j+1,1);
    temp(end)=C(M)*solution_mesh(j+1,M+1);  % Boundary terms
    RHS=solution_mesh(j,:)'-temp;
    temp=Tri_Inv*RHS;
    solution_mesh(j+1,(2:end-1))=temp(2:end-1);
end

plot(Smesh,solution_mesh(N+1,:));
xlabel('Underlying'); ylabel('V(S,t=0)');

%%%%%%%%%%%%%%%% Extract Desired Values Using Interpolation %%%%%%%%%%%%%%%%%

interp1(Smesh,solution_mesh(N+1,:),S)
clear;
d = 50;
MC = 70;
r = 0.05;
T = 1;

K = 70;

v0 = 110;

N = 10000;

% Rate of return, ror, follows normal distribution N(0,50%)
% First design
m = 0; sigma = 0.3;

ror1 = normrnd(m, sigma, 1, N);
v1 = v0*exp(ror1);
rorb1 = normrnd(m, sigma, 1, N);
vb1 = v0*exp(rorb1);

ror2 = zeros(1, N);
rorb2 = zeros(1, N);
v2 = zeros(1, N);
vb2 = zeros(1, N);
Fa = zeros(1, N);
Fb = zeros(1, N);
intFa = zeros(1, N);
intFb = zeros(1, N);
Pa = zeros(1, N);
Pb = zeros(1, N);
surplus_a = zeros(1, N);
surplus_b = zeros(1, N);

vstar = 0:1:200;

for j = 1:length(vstar)

% Locate vstar
% find two numbers a and b, with f(a)f(b)<0

% Second design
for i = 1:N
if v1(i) > vstar(j)
    ror2(i) = normrnd(m, sigma);
    v2(i) = exp(ror2(i))*v1(i);
else
    v2(i) = 0;
end
end
for i=1:N
    if vb1(i) > vstar(j)
        rorb2(i)=normrnd(m,sigma);
        vb2(i) = exp(rorb2(i))*vb1(i);
    else
        vb2(i)=0;
    end
end

%Calculate pricing of designer a and b
%F(v2) follows lognormal distribution with mean 4.4552, and
%volatility 0.5.
%refer to the 16 April 2013 presentation
%CDF of F(v) = 0.5+0.5*erf((log(x)-mean)/sqrt(2*volatility^2))
%p(v)=MC+(int(F(v))/F(v))

for i=1:N
    me = log(v1(i))+(r-sigma^2/2)*T;
    volatility = sigma^2*T;
    if v2(i)==0
        Pa(i)=MC;
    else
        Fa(i)= 0.5+0.5*erf((log(v2(i))-
        me)/sqrt(2*volatility^2));
        %Calculate int(F(v))
        Xa=0:0.1:v2(i);
        Ya=0.5+0.5*erf((log(Xa)-me)/sqrt(2*volatility^2));
        intFa(i)=trapz(Xa,Ya);
        %Calculate Pa
        Pa(i)=MC+(intFa(i)/Fa(i));
    end
end

for i=1:N
    meb = log(vb1(i))+(r-sigma^2/2)*T;
    volatility = sigma^2*T;
    if vb2(i)==0
        Pb(i)=MC;
    else
        Fb(i)= 0.5+0.5*erf((log(vb2(i))-
        meb)/sqrt(2*volatility^2));
        %Calculate int(F(v))
        Xb=0:0.1:vb2(i);
        Yb=0.5+0.5*erf((log(Xb)-meb)/sqrt(2*volatility^2));
        intFb(i)=trapz(Xb,Yb);
        %Calculate Pa
        Pa(i)=MC+(intFb(i)/Fb(i));
end
\begin{verbatim}
Pb(i)=MC+(intFb(i)/Fb(i));

end

end

%Calculate surplus of the buyer
for i=1:N
  if v1(i) > vstar(j)
    surplus_a(i)=max(v2(i)-Pa(i),0)-d;
  else
    surplus_a(i) = 0;
  end
end

for i=1:N
  if vb1(i) > vstar(j)
    surplus_b(i)=max(vb2(i)-Pb(i),0)-d;
  else
    surplus_b(i) = 0;
  end
end

for i=1:N
  surplus_buyer(i)=max(surplus_a(i),surplus_b(i));
end

mean_surplus_a(j) = mean(surplus_a)
mean_surplus_b(j) = mean(surplus_b)
mean_surplus_buyer(j)=mean(surplus_buyer);
end

plot(vstar,mean_surplus_a)
hold all
plot(vstar,mean_surplus_b)
hold all
plot(vstar,mean_surplus_buyer)
\end{verbatim}