A THEORETICAL STUDY OF AN ELECTRIC-LESS PORTABLE COOLER

YUAN YU

SCHOOL OF MECHANICAL & PRODUCTION ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY

2005
A Theoretical Study of an Electric-less Portable Cooler

Yuan Yu

School of Mechanical & Production Engineering

A thesis submitted to the Nanyang Technological University in fulfilment of the requirement for the degree of Master of Engineering

2005
Acknowledgements

The author will like to express her deepest appreciation to her supervisors, Prof. Lye Sun Woh and A/P Ooi Kim Tiow, for their kind guidance, advice and encouragement, extend throughout the project.

She would also like to thank all the technicians in Thermal and Fluids Research Lab, Design Research Center for their help.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgement</td>
<td>I</td>
</tr>
<tr>
<td>List of Figures</td>
<td>VI</td>
</tr>
<tr>
<td>List of Tables</td>
<td>X</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>XI</td>
</tr>
<tr>
<td>Summary</td>
<td>XIII</td>
</tr>
<tr>
<td><strong>Chapter 1 Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Project Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Objectives</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Scope</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Thesis Organization</td>
<td>5</td>
</tr>
<tr>
<td><strong>Chapter 2 Literature Review</strong></td>
<td>6</td>
</tr>
<tr>
<td>2.1 The Evolution and Problems of Portable Coolers</td>
<td>6</td>
</tr>
<tr>
<td>2.2 A Review on Phase Change Energy Storage Technology</td>
<td>9</td>
</tr>
<tr>
<td>2.2.1 Phase Change Materials (PCM) and its Selection</td>
<td>9</td>
</tr>
<tr>
<td>2.2.1.1 PCM Properties and Characteristics</td>
<td>10</td>
</tr>
<tr>
<td>2.2.1.2 Chemical Stability</td>
<td>12</td>
</tr>
<tr>
<td>2.2.1.3 Heat Transfer Enhancement Methods</td>
<td>12</td>
</tr>
<tr>
<td>2.2.2 Major Applications of Phase Change Materials</td>
<td>13</td>
</tr>
<tr>
<td>2.3 Modeling Heat Transfer Mechanism for Phase Change Materials</td>
<td>14</td>
</tr>
<tr>
<td>2.3.1 Modeling PCM Formation</td>
<td>15</td>
</tr>
<tr>
<td>2.3.2 Modeling Phase Change Behavior</td>
<td>15</td>
</tr>
<tr>
<td>2.3.2.1 Phase Change around Cylindrical Geometry</td>
<td>15</td>
</tr>
<tr>
<td>2.3.2.2 Solidification of PCM inside a Spherical Capsule</td>
<td>16</td>
</tr>
<tr>
<td>2.3.2.3 Conventional Distinct Interface Model</td>
<td>17</td>
</tr>
<tr>
<td>2.3.2.4 Mushy Zone Model</td>
<td>19</td>
</tr>
<tr>
<td>2.3.3 Plane Finned Geometries For Latent Heat Storage Applications</td>
<td>20</td>
</tr>
<tr>
<td>2.3.3.1 Parallel Plate Finned and Finless Geometries</td>
<td>20</td>
</tr>
<tr>
<td>2.3.3.2 Radial finned cylinders</td>
<td>20</td>
</tr>
</tbody>
</table>
2.4 Summary of Findings 21

Chapter 3 Design and Development of a Portable Cooler 22

3.1 Design Consideration 22
  3.1.1 Refrigerant 22
  3.1.2 Phase Change Material and Container 26
  3.1.3 Thermal Conductivity Enhancer 28
  3.1.4 Heat Source 29
  3.1.5 Insulator 29
  3.1.6 Box Material Property 31

3.2 Conceptual Design 32

3.3 Experimental Setup and Procedure 39

Chapter 4 One-dimensional Mathematical Model and Simulation 43

4.1 One-dimensional Mathematical Model 44
  4.1.1 Assumptions 44
  4.1.2 Dimensions and Physical Properties of the Model 47
  4.1.3 Physical Model 48
  4.1.4 Numerical Analysis 55

4.2 Computer Simulation of 1-D Model 57
  4.2.1 Modeling Stage Description 57
  4.2.2 Initial and Boundary Conditions 58
  4.2.3 Results and Discussion 58

4.3 Limitations of One-dimensional Simulation 62

Chapter 5 Numerical Simulation and Validation 63

5.1 Simulation Model 63
  5.1.1 Governing Differential Equations 63
  5.1.2 Modeling Basic Fluid Flow 67
  5.1.3 Modeling Solidification/Melting 68
  5.1.4 Modeling Heat Transfer 69
  5.1.5 Modeling Vaporization of Liquid Nitrogen 71
  5.1.6 Boundary Conditions 72

5.2 Numerical Simulation 73
## Contents

5.3 Grid Dependency 74

5.4 Simulating the Experimental Conditions 77
   5.4.1 Modeling Stage Description 77
   5.4.2 Initial Modeling Objectives 78

5.5 Simulation Results 79
   5.5.1 Stage 1 79
      5.5.1.1 PCM Solidification 79
      5.5.1.2 Liquid Nitrogen Vaporization 83
      5.5.1.3 Air Space Temperature Distribution 85
   5.5.2 Stage 2 86

5.6 Validation of the Simulation Model 88
   5.6.1 Stage 1 89
   5.6.2 Stage 2 91

### Chapter 6 Parametric Studies 94

6.1 Effects of Design Parameters on the PCM Solidification 94
   6.1.1 Fin Number Effect 94
   6.1.2 Thermal Conductivity Enhancer Effect 98
   6.1.3 PCM Container Shape Effect 105
   6.1.4 Pre-cooling Process Effect 107

6.2 Heat Transfer in the Cooler with 4 Fins 109
   6.2.1 Heat Transfer in the Cooling Air Space 114
   6.2.2 Cooler Box Wall 116
   6.2.3 Fins 118
   6.2.4 PCM Container 121
   6.2.5 Cooling Rate and Operation Cost 122

6.3 Effects of Design Parameters on the Cooling Rate 123
   6.3.1 Fin Number Effect 123
   6.3.2 Fin Length Effect 128
   6.3.3 Heat Source Location Effect 129
   6.3.4 PCM Container Shape Effect 132

### Chapter 7 Conclusions and Future Work 135

7.1 Contributions and Conclusions 135

7.2 Future Work 136
Contents

References 137

Appendix I  Drawings of Experimental Model
Appendix II  One-dimensional Mathematical Model
Appendix III  FORTRAN Program Flowchart
Appendix IV  User Defined Function
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>A Thermoelectric Refrigerator Based on the Peltier Effect</td>
<td>8</td>
</tr>
<tr>
<td>3.1</td>
<td>Design of door</td>
<td>29</td>
</tr>
<tr>
<td>3.2</td>
<td>Parts and Assembly Drawing of the New Portable Cooler</td>
<td>34</td>
</tr>
<tr>
<td>3.3</td>
<td>The Fabricated Cooler Box</td>
<td>35</td>
</tr>
<tr>
<td>3.4</td>
<td>Liquid Nitrogen Input Through the Network of Pipes</td>
<td>35</td>
</tr>
<tr>
<td>3.5</td>
<td>Liquid Nitrogen Input Through Sub-cooling Pipes</td>
<td>36</td>
</tr>
<tr>
<td>3.6</td>
<td>Liquid Nitrogen Input Through Cooling Coil</td>
<td>37</td>
</tr>
<tr>
<td>3.7</td>
<td>Drawing of Experimental Setup</td>
<td>38</td>
</tr>
<tr>
<td>3.8</td>
<td>Identical Two Segments of the Experimental Setup</td>
<td>41</td>
</tr>
<tr>
<td>3.9</td>
<td>Thermocouple Location Layout to Measure Inner Box Wall Temperature</td>
<td>42</td>
</tr>
<tr>
<td>4.1</td>
<td>One-dimensional Model Setup</td>
<td>43</td>
</tr>
<tr>
<td>4.2</td>
<td>One-dimensional Model: Assumption 1</td>
<td>44</td>
</tr>
<tr>
<td>4.3</td>
<td>One-dimensional Model: Assumption 2</td>
<td>45</td>
</tr>
<tr>
<td>4.4</td>
<td>One-dimensional Model: Assumption 3</td>
<td>45</td>
</tr>
<tr>
<td>4.5</td>
<td>Flux Balance over Control Volume 1</td>
<td>49</td>
</tr>
<tr>
<td>4.6</td>
<td>Flux Balance over Control Volume 2</td>
<td>51</td>
</tr>
<tr>
<td>4.7</td>
<td>Simplification Calculation of Thermal Resistance of PCM</td>
<td>52</td>
</tr>
<tr>
<td>4.8</td>
<td>Flux Balance over Control Volume 3</td>
<td>54</td>
</tr>
<tr>
<td>4.9</td>
<td>One-dimensional Simulation Results Illustration</td>
<td>59</td>
</tr>
<tr>
<td>4.10</td>
<td>One-dimensional Simulation Results of $U_x$ in Stage 2</td>
<td>61</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.11</td>
<td>One-dimensional Simulation Results of $T_x$ and $T_b$ in Stage 2</td>
<td>62</td>
</tr>
<tr>
<td>5.1</td>
<td>Flux balance over a control volume</td>
<td>64</td>
</tr>
<tr>
<td>5.2</td>
<td>Schematics of Model for Boundary Conditions</td>
<td>72</td>
</tr>
<tr>
<td>5.3</td>
<td>Three-dimensional Simulation Model</td>
<td>74</td>
</tr>
<tr>
<td>5.4</td>
<td>Meshing of the Liquid Nitrogen flow region</td>
<td>74</td>
</tr>
<tr>
<td>5.5</td>
<td>Meshing of PCM region</td>
<td>75</td>
</tr>
<tr>
<td>5.6</td>
<td>Mesh of Heat Source</td>
<td>75</td>
</tr>
<tr>
<td>5.7</td>
<td>Mesh of Air</td>
<td>76</td>
</tr>
<tr>
<td>5.8</td>
<td>Symmetric surfaces of $Y = 0$ and $Z = 150mm$</td>
<td>79</td>
</tr>
<tr>
<td>5.9</td>
<td>PCM solidification in stage 1</td>
<td>82</td>
</tr>
<tr>
<td>5.10</td>
<td>Liquid nitrogen vaporization in stage 1</td>
<td>85</td>
</tr>
<tr>
<td>5.11</td>
<td>Temperature distribution of the air space ($t = 10$ minutes)</td>
<td>86</td>
</tr>
<tr>
<td>5.12</td>
<td>Temperature distribution and velocity in air space at $t = 1$ hour</td>
<td>88</td>
</tr>
<tr>
<td>5.13</td>
<td>Temperatures are measured at these three locations</td>
<td>89</td>
</tr>
<tr>
<td>5.14</td>
<td>Temperature variation at point 1</td>
<td>90</td>
</tr>
<tr>
<td>5.15</td>
<td>Temperature variation at point 2</td>
<td>90</td>
</tr>
<tr>
<td>5.16</td>
<td>Temperature variation at point 3</td>
<td>90</td>
</tr>
<tr>
<td>5.17</td>
<td>Temperature measurement location for melting process</td>
<td>91</td>
</tr>
<tr>
<td>5.18</td>
<td>Temperature variation at point 1</td>
<td>91</td>
</tr>
<tr>
<td>5.19</td>
<td>Temperature variation at point 2</td>
<td>92</td>
</tr>
<tr>
<td>5.20</td>
<td>Temperature variation at point 3</td>
<td>92</td>
</tr>
<tr>
<td>6.1</td>
<td>Cooler boxes with different fin numbers</td>
<td>95</td>
</tr>
<tr>
<td>6.2</td>
<td>Heat transfer rate between PCM and nitrogen</td>
<td>96</td>
</tr>
<tr>
<td>Figure 6.3</td>
<td>Net heat transfer rate of PCM with time</td>
<td>97</td>
</tr>
<tr>
<td>Figure 6.4</td>
<td>Heat transfer rate of fins</td>
<td>97</td>
</tr>
<tr>
<td>Figure 6.5</td>
<td>Net Heat transfer of air in the air space</td>
<td>98</td>
</tr>
<tr>
<td>Figure 6.6</td>
<td>Two arrangements of thermal conductivity enhancer</td>
<td>99</td>
</tr>
<tr>
<td>Figure 6.7</td>
<td>Temperature profiles of PCM with 4 straight fins inserted (thickness=5mm)</td>
<td>102</td>
</tr>
<tr>
<td>Figure 6.8</td>
<td>Temperature profiles of PCM with 10 straight fins inserted (thickness=2mm)</td>
<td>104</td>
</tr>
<tr>
<td>Figure 6.9</td>
<td>Different PCM container shapes</td>
<td>105</td>
</tr>
<tr>
<td>Figure 6.10</td>
<td>PCM solidification at t = 2 hours</td>
<td>107</td>
</tr>
<tr>
<td>Figure 6.11</td>
<td>Heat transfer rate at PCM container wall</td>
<td>108</td>
</tr>
<tr>
<td>Figure 6.12</td>
<td>Net heat transfer of PCM with different initial temperature</td>
<td>108</td>
</tr>
<tr>
<td>Figure 6.13</td>
<td>One identical part of the 4-fin cooler</td>
<td>109</td>
</tr>
<tr>
<td>Figure 6.14</td>
<td>Temperature distribution in the cooler box with 4 fins at t =1 hour</td>
<td>111</td>
</tr>
<tr>
<td>Figure 6.15</td>
<td>Velocity of air flow in the cooler box with 4 fins at t =1 hour</td>
<td>114</td>
</tr>
<tr>
<td>Figure 6.16</td>
<td>Cooling air space</td>
<td>114</td>
</tr>
<tr>
<td>Figure 6.17</td>
<td>Heat transfer of air in the cooling space</td>
<td>115</td>
</tr>
<tr>
<td>Figure 6.18</td>
<td>Total and radiation heat transfer in the air space</td>
<td>116</td>
</tr>
<tr>
<td>Figure 6.19</td>
<td>Individual box wall</td>
<td>117</td>
</tr>
<tr>
<td>Figure 6.20</td>
<td>Heat transfer of box wall</td>
<td>117</td>
</tr>
<tr>
<td>Figure 6.21</td>
<td>Fin 1 and fin 2</td>
<td>118</td>
</tr>
<tr>
<td>Figure 6.22</td>
<td>Wall name of identical fin and PCM container</td>
<td>118</td>
</tr>
<tr>
<td>Figure 6.23</td>
<td>Heat transfer of fin sidewalls</td>
<td>119</td>
</tr>
<tr>
<td>Figure 6.24</td>
<td>Total and radiation heat transfer of fin 1 sidewalls</td>
<td>119</td>
</tr>
<tr>
<td>Figure 6.25</td>
<td>Total and radiation heat transfer of fin 2 sidewalls</td>
<td>119</td>
</tr>
<tr>
<td>Figure 6.26</td>
<td>Heat transfer of fin tips</td>
<td>120</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Figure 6.27</td>
<td>Total and radiation heat transfer of fin 1 tip</td>
<td>120</td>
</tr>
<tr>
<td>Figure 6.28</td>
<td>Total and radiation heat transfer of fin 2 tip</td>
<td>120</td>
</tr>
<tr>
<td>Figure 6.29</td>
<td>Heat transfer of PCM container</td>
<td>121</td>
</tr>
<tr>
<td>Figure 6.30</td>
<td>Total and radiation heat transfer of PCM container</td>
<td>121</td>
</tr>
<tr>
<td>Figure 6.31</td>
<td>Total and radiation heat transfer of heat source</td>
<td>122</td>
</tr>
<tr>
<td>Figure 6.32</td>
<td>Heat source cooling process in 4-fin cooler</td>
<td>123</td>
</tr>
<tr>
<td>Figure 6.33</td>
<td>PCM containers with 50, 10, 4, 2 and 0 fin</td>
<td>124</td>
</tr>
<tr>
<td>Figure 6.34</td>
<td>Cooling rate comparison of heat source</td>
<td>125</td>
</tr>
<tr>
<td>Figure 6.35</td>
<td>Instantaneous heat transfer rate of fins</td>
<td>126</td>
</tr>
<tr>
<td>Figure 6.36</td>
<td>Temperature distribution at surface of Z = 150mm in the coolers</td>
<td>128</td>
</tr>
<tr>
<td>Figure 6.37</td>
<td>Two different fin lengths</td>
<td>128</td>
</tr>
<tr>
<td>Figure 6.38</td>
<td>Cooling process for cooler fitted with different fin lengths</td>
<td>129</td>
</tr>
<tr>
<td>Figure 6.39</td>
<td>Fin heat transfer rate</td>
<td>129</td>
</tr>
<tr>
<td>Figure 6.40</td>
<td>Three different locations of the heat source</td>
<td>130</td>
</tr>
<tr>
<td>Figure 6.41</td>
<td>Temperature variations of heat source during the cooling process for three different heat source locations</td>
<td>131</td>
</tr>
<tr>
<td>Figure 6.42</td>
<td>Heat transfer of fins</td>
<td>131</td>
</tr>
<tr>
<td>Figure 6.43</td>
<td>Heat source cooling rate with different PCM containers</td>
<td>132</td>
</tr>
<tr>
<td>Figure 6.44</td>
<td>Minimum air velocity in the air space</td>
<td>132</td>
</tr>
<tr>
<td>Figure 6.45</td>
<td>Temperature distributions of heat source and air space at surface of Z=150mm in coolers with different PCM containers</td>
<td>133</td>
</tr>
<tr>
<td>Figure 6.46</td>
<td>Maximum air velocity in the air space</td>
<td>134</td>
</tr>
<tr>
<td>Figure 6.47</td>
<td>PCM container wall heat transfer rate</td>
<td>134</td>
</tr>
</tbody>
</table>
List of Tables

Table 1.1  Market Survey on Existing Domestic and Portable Mini Refrigerators 2
Table 2.1  Comparison of Organic and Non-organic PCM 12
Table 3.1  Composition of air by % volume 24
Table 3.2  Selected refrigerants and their properties 25
Table 3.3  Thermal conductivity of various metals at 0°C 28
Table 3.4  Thermal conductivity of common insulating materials 30
Table 3.5  Several selected plastics and their property 31
Table 4.1  One-dimensional Simulation Results in Stage 1 60
Table 5.1  Fluid average properties 73
Table 5.2  Solid average properties 73
Table 5.3  Simulation results of the models with different mesh size 77
Table 6.1  Solidification time and average temperature for different fin numbers 95
Table 6.2  Results of different arrangements of thermal conductivity enhancer 105
Table 6.3  Solidification time for PCM at different initial temperatures 107
List of Symbols

Cdg — specific heat capacity of vapor nitrogen, J/kg·K
CXL — specific heat capacity of liquid PCM, J/kg·K
CXS — specific heat capacity of solid PCM, J/kg·K
Dc — diameter of each coil of the cooling tube, m
Dx — average density of PCM, kg/m³
Gr — Grashof number
H — height of the cooling space, m
Hd — enthalpy of vaporization of liquid nitrogen, J/kg
Hx — enthalpy of fusion of the solid PCM, J/kg
kj — thermal conductivity of the cooler box wall, W/m·K
kf — thermal conductivity of copper of fin, W/m·K
kx — thermal conductivity of PCM, W/m·K
La — length of the cooling space, m
Ld — total length of the coiling tube, m
Lj — thickness of the cooler box wall, m
Lf — length of the fin, m
n — number of coils of the coiling tube
Pr — Prandtl number
rd — radius of the coiling tube, m
rx — radius of the cylindrical PCM container, m
t — time, s
tf — thickness of the fin, m
List of Symbols

$T_b$ — temperature of the air in the cooling space, K

$T_d$ — temperature of liquid nitrogen, K

$T_d^*$ — vaporization temperature of liquid nitrogen, K

$T_k$ — temperature of the exterior surface of the cooler box wall, K

$T_x$ — temperature of the PCM, K

$T_x^*$ — solidification/melting temperature of the PCM, K

$U_d$ — ratio of the weight of vapor nitrogen to the total weight of nitrogen

$U_x$ — ratio of the weight of liquid PCM to the total weight of PCM

$W_d$ — mass flow rate of liquid nitrogen, kg/s

$W_x$ — average mass of the PCM per unit volume of the PCM container, kg/m$^3$

$x$ — distance along the $x$-coordinate axis, m
Presently, the vapor compression and the absorption systems are the two most widely used systems in the portable refrigerators available in the market. In spite of its widespread acceptance and use, recent attempts have been made to consider more environmentally refrigerant such as nitrogen and carbon dioxide, other energy source substitution, better performance enhancement systems, lower operation cost, maintainability and portability. To cater for these demands, an investigative study was made in this project to develop a new environmental friendly electric-less portable cooler.

The new refrigeration system employs an open system concept that uses the phase change material (PCM) for thermal energy storage and a completely environmentally friendly refrigerant to solidify the PCM. To prove the working principle of the system, a cooler box has been designed, fabricated and instrumented. Tests were carried out to measure and establish the performance of the system. The experimental results suggested that the new system, under its initial design stage, is capable of achieving performance level that is comparable, if not better than the current vapor compression system, at a much lower cost.

A three-dimensional simulation has been carried out for the cooler box to analyze and predict the performance of the system. The numerical prediction showed a good agreement between the numerical and experimental results. The simulation results can also contribute to the parametric studies.

In terms of environmental issue and economical factor the new refrigeration concept is definitely viable. However, further study and development are still necessary to increase the efficiency.
Chapter 1

Introduction

1.1 Project Background

Refrigeration has been defined as “the science of providing and maintaining temperatures below that of surrounding atmosphere” [1]. The process involves the study of thermodynamics and mechanics phenomena to enable one to draw heat from substances thereby lowering its temperatures. A lot work has been carried out to better understand the heat transfer phenomena characteristics, enhance its process performances and technologies as well as generate new ideas and uses. This has led to refrigeration becoming an essential part of modern living where it is used for food preservation, goods transportation and storage, and air conditioning.

Presently, several refrigeration systems have been developed and are commercially available in the market [2-11]. These systems include the vapor compression system, absorption refrigeration system, thermoelectric system, steam jet system, air cycle system, magnetic system and cryogenic system. Of these, the vapor compression and the absorption systems are the two most widely used systems. Table 1.1 shows a list of refrigeration systems employed in current domestic and portable refrigerators.

In spite of its widespread acceptance and use, recent attempts have been made to consider alternative more environmentally refrigerant gases such as nitrogen and carbon dioxide, other energy source substitution, better performance enhancement systems and maintainability, and portability. This is mainly owing to environmental and economic concerns raised by lobby groups and competition for greater share in a “matured” market.

On environmental concerns, the general public has become aware of the damaging effects caused by refrigerants used in previous systems had on the environment. These refrigerants consist of chlorofluorocarbons (CFCs) and hydrochlorofluorocarbons (HCFCs) that have high ozone depletion potential. Destruction of the ozone layer would increase the likelihood of skin cancer, crop damage, disruption to the ecological water systems and increase in the carbon dioxide in the air.
<table>
<thead>
<tr>
<th>Major Class</th>
<th>Refrigeration Method</th>
<th>Refrigerants</th>
<th>Company</th>
<th>Model</th>
<th>Cost ($)/Unit</th>
<th>Dimension (mm) WxHxD</th>
<th>Volume (L)</th>
<th>Insulation Material</th>
<th>Temp Range</th>
<th>Energy</th>
<th>Power (W/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>Vapor Compression</td>
<td>R134a</td>
<td>LG</td>
<td>GR-051SF</td>
<td>-</td>
<td>442x501x420</td>
<td>46</td>
<td>Polyurethane Foam</td>
<td>-</td>
<td>AC</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>SANYO</td>
<td>SR-172W</td>
<td>$200</td>
<td>452x460x470</td>
<td>49</td>
<td>Polyurethane Foam</td>
<td>-</td>
<td>AC</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>GE</td>
<td>TAX2SNBWH</td>
<td>$119</td>
<td>480x462x477</td>
<td>48</td>
<td>Polyurethane Foam</td>
<td>-</td>
<td>AC</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Samsung</td>
<td>SR-058</td>
<td>$229</td>
<td>449x497x439</td>
<td>52</td>
<td>-</td>
<td>-0.20</td>
<td>AC</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>SANYO</td>
<td>SR-2404</td>
<td>-</td>
<td>460x618x475</td>
<td>24 bottles</td>
<td>-</td>
<td>-</td>
<td>AC</td>
<td>-</td>
</tr>
<tr>
<td>Portable</td>
<td>Vapor Compression</td>
<td>R134a</td>
<td>Minus 40</td>
<td>5580f</td>
<td>$1,800</td>
<td>530x357x383</td>
<td>80</td>
<td>Polyurethane Foam</td>
<td>-0.57</td>
<td>SolarDC</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Explorer</td>
<td>RV250</td>
<td>$299</td>
<td>502x373x478</td>
<td>59</td>
<td>-</td>
<td>-</td>
<td>AC/DC</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Dongfa</td>
<td>V12/24.50</td>
<td>$890</td>
<td>612x416x367</td>
<td>50</td>
<td>Polyurethane Foam</td>
<td>-0.20</td>
<td>DC 12/24</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Dongfa</td>
<td>V12/24.20</td>
<td>$810</td>
<td>477x392x257</td>
<td>20</td>
<td>Polyurethane Foam</td>
<td>-0.23</td>
<td>DC 12/24</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Frigemate</td>
<td>MF30</td>
<td>$550</td>
<td>560x398x404</td>
<td>90</td>
<td>-</td>
<td>-</td>
<td>DC 12/24</td>
<td>36-72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Frigemate</td>
<td>MF40</td>
<td>$690</td>
<td>563x398x490</td>
<td>135</td>
<td>-</td>
<td>-</td>
<td>DC 12/24</td>
<td>36-73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Explorer</td>
<td>FP40</td>
<td>$351</td>
<td>560x450x430</td>
<td>38</td>
<td>-</td>
<td>-</td>
<td>AC,AD,LPG</td>
<td>72-110</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R134a</td>
<td>Explorer</td>
<td>FP50</td>
<td>$399</td>
<td>560x480x430</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td>AC,AD,LPG</td>
<td>84-110</td>
</tr>
<tr>
<td></td>
<td>Absorption cycle</td>
<td>Ammonia</td>
<td>RV Mobile</td>
<td>RC2000</td>
<td>$399</td>
<td>546x436x443</td>
<td>33</td>
<td>ABS</td>
<td>-</td>
<td>AC,AD,LPG</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Theromoelectric Cooler</td>
<td>-</td>
<td>Igloo</td>
<td>Kool Mate 40</td>
<td>$147</td>
<td>450x450x500</td>
<td>38</td>
<td>14 below ambient</td>
<td>AC/DC</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1.1 Market Survey on Existing Domestic and Portable Mini Refrigerators [2-11]
Although manufacturers of such refrigeration systems have switched to the use of R-134a, hydrofluorocarbon (HFC) that is CFC free, it gives rise to another concern that is global warming. This concern relates to an increase in the earth temperature resulting in “greenhouse effect” caused by the process and use of fossil fuels to produce the HFCs. Elevated global temperatures could result in coastal floods and shift in weather patterns. Koelet [12] remarked that the new R-134a still possesses a high degree of global warming potential. Some engineers have turned to the use of ammonia as a refrigerant. Although ammonia is environmentally friendly, it is nevertheless toxic, potentially flammable and has a high leakage rate.

On energy utilization, electricity remains the main energy source to operate present refrigeration systems. Electricity is obtained via burning of fossil fuels. It is estimated that the world would experience a severe shortage in 50 - 100 years time resulting in high fuel prices. Research is being carried out to develop alternative energy sources such as fuel cells or cell batteries for use in such systems.

Present refrigeration systems involve active motion parts such as compressors to drive the refrigeration cycle. This requires regular maintenance to ensure consistent performance as well as minimize wear. Newer designs have since been developed that makes use of more durable materials, incorporate ease of assembly and maintenance features having fewer part count aimed to prolong system life as well as achieve better performance yield. The search to develop a more efficient cost effective, quiet and maintenance free refrigeration system will still continue.

Besides inherent mechanical inefficiencies, the current types of refrigerants used particularly in a vapor compression refrigeration system are thermodynamically restrictive having a limited cooling rate. Noting that most refrigeration systems, nowadays, are used for food and organic preservation and storage, the ability to keep the product freshness is an important issue. Refrigerant like R134a used in present vapor compression refrigerator does not have a fast enough cooling rate to maintain food freshness. According to a study by Sandra J Ley D.C. [13], a typical vapor compression refrigerator would require at least 14 hours to cool a piece of beef weighing 90 kg from 60 °C to 7 °C whereas the recommended cooling rate by the food service industry for food to remain fresh is 4 hours. This works out to be about 0.2
°C/min. Otherwise, the food could be subjected to microbiological contamination. One could refine the cooling flow pattern through modeling but improvements are expected to be incremental.

On portability, various portable refrigerators exist which cater either to remote regions where conventional electrical supply is unavailable or as mobile units in vehicles and coaches, mobile hospitals, and commercial and pleasure crafts. Such systems tend to be electrically powered deriving their energy source from generators or batteries. When coupled with generators, the system can be quite cumbersome, whereas, rechargeable batteries like nickel-cadmium or small sealed lead acid types are used. Such batteries contain toxic heavy metals such as cadmium, mercury and lead which are not environmentally friendly. Solar power panels have also been used as an alternative energy source for refrigeration systems but such options tend to be expensive. This aspect would continue to be of interest to researchers.

1.2 Objectives

Based on the above limitations, an alternative approach that could serve as a non-electrical portable cooler requiring minimum or free maintenance and operates on an environmental friendly refrigerant would be of interest. The project was therefore conceived with the following objectives in mind:

(a) To develop an alternative non-electrical portable refrigeration system that can operate well below 0°C having a steep cooling rate of at least 0.2 °C/min.

(b) To evaluate the parameter effect on the developed system design and performance.

(c) To understand the thermal and flow characteristics of the system.

1.3 Scope

The project scope would involve a study into the theoretical and experimental aspects and analysis in adopting the alternative approach for cooling. A physical system has been designed, fabricated and instrumented. Experimental tests would be conducted to assess the system viability and performance.
In the theoretical study, a 3-D model would be formulated to further understand the fluid flow and heat transfer characteristics of the system. A commercially computational code would be used to assist in the analysis of this model. Conservation of mass, momentum and energy has been evaluated to assess the flow and heat transfer characteristics of the system. The model prediction is to be validated with experimental results.

1.4 Thesis Organization

The thesis consists of seven main chapters. A brief description of each chapter is as follows:

Chapter 2 reviews the work related to portable refrigeration technology. An overview of the problems encountered in existing portable coolers, the power storage technology for portable coolers including electricity energy storage and thermal energy storage are also made. This chapter also outlines the research on solidification/melting modeling.

Chapter 3 analyzes design approaches suggested in this work. It addresses the design considerations of the new portable cooler. Also, the experimental is setup to conduct test feasibility of the design. Data gathered from experiment will be used to test the validation of the results obtained from 3-D model simulations (Chapter 5).

Chapter 4 describes the one-dimensional numerical FORTRAN model to quickly see the workability of the empty new cooler.

Chapter 5 presents the three-dimensional numerical study of the new portable cooler with a heat source to be cooled in the cooling space to verify the effectiveness of the model.

Chapter 6 studies the important design parameters and their effects on the thermal performance of the new cooler.

Chapter 7 summaries the main conclusions and contributions achieved in this work. Some suggestions for future work are also provided.
Chapter 2

Literature Review

2.1 The Evolution and Problems of Portable Coolers

A portable cooler is a refrigerator designed for easy mobility and transportation. It is widely used in vehicles, mobile hospitals, vaccine storage, yachts, pleasure crafts, commercial shipping, caravans, buses, coaches and trains. These mechanical portable refrigeration systems tend to be small in size.

Presently, portable refrigeration systems adopt similar vapor compression and absorption refrigeration systems to provide the “refrigeration” effect. The key difference being that these portable systems are relatively more compact and smaller in size. Additional electrical power sources, circuitry and components such as batteries, charges and transformers, are required to be installed in the portable refrigerator for mobility. Cooling is induced when the refrigerant undergoes through a liquid – gaseous transformation. This phase transformation occurs when heat is transferred from the “warm” objects and surroundings to the cool refrigerant. Typically, the system experiences large volume expansion during the liquid-gas phase and volume compression at gas-liquid phase. Complicated compressors, pressure vessels or special design features, such as internal bellows [14], would be incorporated to deal with the pressure variances. To achieve consistent operating efficiency, regular maintenance to the moving parts would be needed. Besides this, vapor compression systems generally make use of R134a refrigerant. Such refrigerant has a high global-warming potential [15] leading to an increase in the earth’s temperature owing to the use of fossil fuels. In excessive unmonitored or uncontrolled usage, elevated global temperatures could result in coastal flooding and the shifting of major climatic zones and may have serious implications for agricultural productivity. It is also possible that this could lead to buildup of “greenhouse effect.”

In absorption systems, water and ammonia are substances that have been successfully used as absorbent and refrigerant respectively. There is no doubt that ammonia is very toxic and has an unpleasant smell. Refrigerant leaks do occur at any time and at any point due to poor quality workmanship, age of equipment and vibration. Refrigerant
when contacted with food products would be disastrous. A mixture of dry air and ammonia of about 16-20% at ignition temperature can result in explosion when exposed to an electric spark or an open flame [16].

Another limitation with current portable refrigeration systems is the need to provide for portable electricity or power to drive the system especially in outdoors and rural areas where there are no electrical outlets such as beaches, sports events, secluded spots, natural parks and forests. One possible means is to make use of rechargeable batteries. Batteries can however incur physical accidental damage owing to vibration or collision and harsh environments such as marine and swamp applications. The physical damage and exposure caused by dampness or humidity can permanently invalidate the batteries [17]. Another portable electrical source is solar energy. Although such energy is free and can be used to supply the required direct current, the solar panel used to concentrate the sun's energy is rather expensive and often very relatively large. The high cost and space considerations deter its usage hence it is seldom adopted in the portable coolers [18].

Another portable refrigeration system is the portable thermo-electric cooler based on solid-state refrigeration that cools without coils, compressors and pumps or makes use of harmful gases [19]. It is called solid-state refrigeration because the thermoelectric elements are in a totally solid state, whereas those devices used in the vapor cycle have moving mechanical parts that require a working fluid. The thermoelectric system exploits the use of the Peltier effect [20] and inherently more reliable and require little or no maintenance. The lack of refrigerants offers convenience, environmental friendly and safety benefits. It also allows one to manufacture tiny thermoelectric coolers making them the most suitable choice for microelectronics applications [21].

The Peltier effect, discovered by the French scientist, Jean Peltier, occurs when an electric current passes through the junction of two dissimilar metals. The temperature difference between the dissimilar metals result in either a cooling or heating effect to take place. By regulating the direction of the heat flow, one can therefore control the heat dissipation or transfer by altering the direction of current flows [22]. To create a thermoelectric refrigerator on the bases of the Peltier effect (Figure 2.1), heat is absorbed from a refrigerated space and then dissipated to a warmer environment. The
difference between these two quantities is the net electrical work that needs to be supplied.

Nevertheless, these refrigerators are not overly popular because they have a low efficiency of cooling performance. The cooling efficiency, $ZT$, of a thermoelectric cooler could be evaluated based on the formula as follows:

$$ZT = S^2 \sigma T/K$$

where $S$ is the Seebeck coefficient

$\sigma$ is the electrical conductivity

$T$ is the cold side temperature and $K$ is the thermal conductivity [23].

Even with a $ZT$ value of 1 commonly found in present thermoelectric coolers, it is only a third of the effectiveness of a normal compression cycle refrigerator [24].

Thermoelectric coolers have another inherent drawback in that they can operate only over a limited temperature range. It is theoretically possible to have the hot plate operating at a temperature range of about 75°C against the cold side of 35 °C. This can only happen if there is no thermal load, which will not happen in a real system. Typical applications yield about half of the theoretical temperature difference. Since thermoelectric modules will not perform as well in colder temperatures, their temperature range becomes much smaller [26]. Usually, refrigeration in the existing thermoelectric coolers can be lower than normal atmospheric temperature 22-25 °C [27]. That is to say, when the environmental temperature is 25°C, the refrigerated
space temperature can be only as low as 0°C. This temperature is still quite high for storing many perishable foods such as meat and diary products.

Much research work is still needed to improve the current thermoelectric cooler to a full-fledged one [28,29,30,31]. Currently, research work is focused on improving the efficiency of thermoelectric coolers, reducing the cost of producing them and increasing their applications [32, 33, 34]. Also, researchers are trying to maximize the electricity output for a given heat source by changing the materials used [35] as well as better predict their reliability and long-term behavior [36].

From the above review, the search for a reliable, small, compact, lightweight, minimal maintenance, quiet and environmentally friendly portable cooler that can operate at low refrigeration temperature of -18°C remains elusive.

2.2 A Review on Phase Change Energy Storage Technology

Energy storage can take the form of latent heat storage via phase change or sensible heat storage through temperature change. In latent heat storage, a material acts as a heat sink in extracting heat from the environment. The larger the latent heat capacity of the material, the greater is its ability to remove heat from its surroundings. The latent heat storage method has been exploited in a large number of applications [37] because it provides much higher storage density with a smaller temperature difference between heat storage and release.

In this review, the focus would be on the latent heat of fusion (solid to liquid or vice versa) of phase changing materials (PCM), their methods and applications would be discussed. This is for other phase changes, such as liquid to gas or evaporation, is not appropriate as the latent heat capacity of evaporation tends to be low and the volumetric phase change is very substantial that can pose serious safety problems. To overcome the large volumetric changes, current practice is to store the liquid material in high-pressure tanks with relief valves to prevent explosion caused by pressure built in the tank as the liquid changes to gaseous form.

2.2.1 Phase Change Materials (PCM) and its Selection
Presently, there are more than 500 PCMs available in the market that could serve as latent heat storage covering a wide range of temperatures. They exist in the form of hydrated salts, paraffin waxes, fatty acids and eutectics of organic and non-organic compounds. PCM properties and information are available in the PCM handbook [38], published literature such as Lane [39] and Humphries and Griggs [40]. For PCM to be used as phase change thermal energy storage in this project, the temperature difference between fusion and the desired operating or application temperature is to be small to maximize thermal efficiency. Important considerations for PCM selection include high latent heat of fusion, chemically stable, cheap, non-toxic and non-corrosive.

2.2.1.1 PCM Properties and Characteristics

PCM hydrated salts typically possess a high volumetric storage density (350 MJ/m$^3$), relatively high thermal conductivity (0.5W/130m°C) at moderate cost. One common example is Glauber salt (Na$_2$SO$_4$·H$_2$O), having a composition of 44%Na$_2$SO$_4$ and 56%H$_2$O by weight [41,42]. It is one of the cheapest PCMs that can be used for thermal energy storage having a high latent heat of 254 kJ/kg (377MJ/m$^3$) with a melting temperature of about 32.4°C. Like most hydrated salts, it encounters phase segregation and sub-cooling restrictions that limit its application. Attempts such as Biswas [41] to add extra portion of water to facilitate dilution and prevent the formation of heavy anhydrous salt was made. Although this makes the system stable and can be reused, the storage density is reduced and the system experiences significantly larger temperature fluctuations. Bentonite clay, a thickening agent, with the Glauber salt was suggested [41] to deal with the problem of phase segregation. This too has its drawback in reducing the rates of crystallization and heat transfer to the salt due to the lower thermal conductivity of the mixture. As for dealing with sub-cooling problems, Borax, as suggested by Telkes [42], was considered as a nucleating agent. This would require a thickening agent to prevent the high-density borax from settling to the bottom of the container.

Feldman and Shapiro [43] had performed analysis on the thermal properties of fatty acids (capric, lauric, palmitic and stearic acids) and their binary mixtures. The melting range of these fatty acids was found to vary between 30 and 65°C, whereas their latent heat of transition was observed to be from 153 to 182 kJ/kg. The reasonable high
latent heat values make them attractive candidates for thermal energy storage in space heating applications.

Another experimental investigation relates to palmitic acid [44] as a PCM for energy storage. The parametric study of phase change transition included transition time, temperature range and propagation of the solid liquid interface as well as the heat flow rate characteristics of the employed circular tube storage system. Dimaano and Escoto [45] have evaluated a mixture of capric and lauric fatty acids for low temperature storage. The melting point of the mixture was about 14°C and its latent heat of fusion/melting ranges between 113 and 133 kJ/kg, depending on composition. Their latent heat values are therefore lower than those of hydrated salts and fatty acids.

Pure paraffin waxes are very expensive. Commercial paraffin waxes are cheap with moderate thermal storage densities (200 kJ/kg or 150 MJ/m³) and a wide range of melting temperatures operating from 20°C to 70°C. They undergo negligible subcooling and are chemically inert and stable with no phase segregation. However, they have low thermal conductivity (0.2W/m°C), which limits their applications. Metallic fillers, metal matrix structures, finned tubes and aluminum shavings were used to improve their thermal conductivity. P-116 is a commercial paraffin wax that has been extensively investigated and documented. It has a melting temperature of about 47°C and a latent heat of melting of about 210 kJ/kg. More recent works include those of Himran et al.[46], Faith [47] and Hasnain [48].

The above findings reveal that inorganic compounds, such as hydrated salts, have a volumetric thermal storage density that is higher than most organic compounds due to their higher latent heat of fusion and density as illustrated in table 1. This is consistent to the studies highlighted in published literature by Abhat [49] and Marshall [50].
### Table 2.1 Comparison of Organic and Non-organic PCM

<table>
<thead>
<tr>
<th>Property</th>
<th>Organic PCM</th>
<th>Inorganic PCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, kg/m</td>
<td>800</td>
<td>1600</td>
</tr>
<tr>
<td>Specific heat, kJ/kg</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Latent heat, kJ/kg</td>
<td>190</td>
<td>230</td>
</tr>
<tr>
<td>Latent heat, kJ/m³</td>
<td>152</td>
<td>368</td>
</tr>
<tr>
<td>Storage mass for $10^6$ J, kg</td>
<td>5300</td>
<td>4350</td>
</tr>
<tr>
<td>Storage volume for $10^6$J,m³</td>
<td>6.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Relative storage mass</td>
<td>1.25</td>
<td>1.0</td>
</tr>
<tr>
<td>Relative storage volume</td>
<td>2.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

#### 2.2.1.2 Chemical Stability

Another key consideration for selection concerns the chemical stability of PCM. PCM needs to be chemically stable for it to be appropriate for extended use and recycling. The useful life of PCMs container systems could also be prolonged. Material degradation occurs when the material properties undergo chemical changes owing to operating conditions or the presence of contaminants or material corrosion [51].

Kimura and Kai [52] have used NaCl to improve the stability of CaCl$_2$.6H$_2$O, containing slightly more water than the stoichiometric composition. The salt was found to be very stable even after undergoing through more than a thousand of heating-cooling cycles. Gibbs and Hasnain [53] has also conducted thermal cycling tests and confirmed that paraffin exhibits excellent thermal stability as its thermal behaviour did not degrade during operation or contaminated when contact with metals. On corrosion, tests using PCMs were performed with salt hydrates [54,55]. Thermal cycling tests on paraffin PCM used in wallboard revealed that there was no observable deterioration in the thermal energy storage capacity [56]. The study seemed to indicate that PCM in the form of hydrated salts is chemically stable.

#### 2.2.1.3 Heat Transfer Enhancement Methods
As a heat storage and transfer medium, phase change materials have unacceptable low thermal conductivity that makes heat charging and discharging slow during the PCM melting and fusion phase. Heat transfer enhancement methods are therefore required if PCM is to be used for any latent heat thermal storage applications [57]. Investigative works to use metallic fillers, metal matrix structures, finned tubes and aluminum shavings to enhance the PCM thermal conductivity such as paraffins [48] have been made. For finned tubes in thermal storage systems, Morcos [58], Sadasuke and Naokatsu [59], Costa et al [60] and Padmanabhan and Murthy [61] have reported the thermal performance for various fin configurations. Bauer and Wirtz [62], on the other hand, studied into the use of thin aluminum plates filled with PCM. Another method is to embed the PCM in a metal matrix structure [63] whereas, Mehling et al.[64], Fukai et al. [65] and Py et al.[66] proposed that the PCM be embedded inside a graphite matrix to increase the thermal heat conductivity in the PCM without much reduction in energy storage.

### 2.2.2 Major Applications of Phase Change Materials

Presently, phase change materials have been used in a wide range of thermal management applications such as moderating heat load demand and storage in domestic building applications, providing cooling and thermal management of electronic components. For storage, PCM has been used as a medium to store solar energy during the day. The stored heat is then released at night to heat up rooms or water for washing, bathing, cooking, steam generation and distillation. Typically, PCM is incorporated into conventional building materials to form high heat capacity building materials where the energy storage capacity owing to the latent heat of fusion of the combined materials is enhanced. One such application involves mixing paraffins with n-octadecane as the primary ingredient having a melting point of 73/F (22.8/C) [67,68] being used for passive solar heating of a building. The PCM wallboard absorbs solar energy and subsequently used to supply a larger amount of heat to the building heating than is otherwise practical. Aside from thermal storage and heat supply, these walls also serve multiple functions acting as support structures, space divider and an acoustical or fire separator with no or minimal additional space or expense requirements.
Another thermal storage application is making use of cheaper off-peak electricity to produce chilled water or ice to cool areas during peak-hours. Lower off-peak utility rates give the customer the benefit of reduced operating costs when heating, ventilating and air-conditioning (HVAC) systems operate during off-peak hours. Studies conducted on commercial buildings have found that operating based on time-of-use utility rates, the energy cost savings could range from 10% to 50% [69]. This shift in peak loads is expected to be larger in residential buildings as there would be fewer internal heat loads from lights and other equipment compared with commercial buildings.

Finally, PCM has successfully been used to improve the thermal management of electronic devices such as notebook computers and wearable electronic devices. Esam M., et. al [70] investigates the effectiveness of a thermal control unit (TCU) for portable electronic devices to prolong their operating time. It is composed of an organic phase change material named Eicosane and a thermal conductivity enhancer. The primary function of the TCU is to absorb the excessive heat that is generated in the heat source component during operating time by allowing the PCM to melt and then release the heat to the environment when the portable computer is not in use while keeping the electronic outer surface temperature relatively low. This approach was found to be capable in extending the operational period of the computer.

2.3 Modeling Heat Transfer Mechanism for Phase Change Materials

Cool thermal storage using the latent heat concept offers a good option because of its high density and near constant temperature heat removal characteristics during the discharge cycle. Published solutions to problems relating to phase change with or without thermal free convection and in finned or finless geometry is vast and extremely rich. Some of these results and applications can be found in Rao and Sastri [71], Sparrow and Chuck [72], Voller [73], Voller and Prakash [74], Kim and Kaviany [75], Schneider [76], Date [77] and Clavier [78]. Others like Ismail [79,80], Castro [81], Ismail and Alves [82], Neto [83], Bellecci [84] and Choi [85] have worked on experimental and theoretical studies on latent heat thermal storage, phase change problems and applications in thermal insulation, thermal windows, PCM walls and
roofs. Resulting from these works, various mathematical models have been developed to better model the PCM behavior under various thermal conditions and geometric configurations.

### 2.3.1 Modeling PCM Formation

Somasundaram [86] investigated into ice formation and ice harvesting. In ice formation, chilled water leaving the chiller is passed through a large number of coiled tubes in the tank and effectively transfers heat from the storage medium (ice/water) to the transport fluid (brine). During the course of the charging, ice is built up on the outside of the coiled tubes and melts when relatively warm brine is circulated through the same tubes. Jekel et al. [87] developed a model to simulate the time-dependent performance of a static ice-on-coil storage tank. Basic heat transfer and thermodynamic relations were employed to solve for the rate of heat transfer from the brine to the tank during both charging and discharging. The performance predicted by the model was found to compare well with the manufacturer's data. Through the findings, ways to better control the ice formation on pipe surfaces can be made.

In ice harvesting, the work focuses on the means to remove ice by local instantaneous fusion. Vargas and Bejan [88] showed that the production of ice by convection cooling followed by contact melting could be maximized by properly selecting the frequency of the intermittent freezing and removal cycle. There were other studies conducted on ice-on-coil thermal energy storage systems by Lee and Jones [89], Vick [90,91] and Neto and Krarti [92]. These studies serve as a useful framework and reference to better understand and model PCM formation.

### 2.3.2 Modeling Phase Change Behavior

#### 2.3.2.1 Phase Change around Cylindrical Geometry

Landau [93] proposed the boundary immobilization method where solutions to the moving interface are realized in the fixed domain. The model assumes to be axisymmetric and that the phase-change material is initially in the liquid phase and at a temperature slightly above or equal to the phase-change temperature. The solidification around a cylinder submersed in a phase change material is said to observe the conservation equation of energy with the external bounding surfaces
exhibiting adiabatic behaviour. The working fluid is assumed to have a uniform and specified temperature at entry. The approach of immobilization was presented and applied to the PCM phase change in an annular space representing the main section of a latent heat thermal energy storage system. It was reported that the results obtained are quite precise that could be used in validating and sizing some of the important geometrical parameters like annular radii, thickness in the design of latent heat thermal storage systems.

Ismail and Goncalves [79] developed another two-dimensional model based on conduction for solving the phase change heat transfer problem around a vertical cylinder submersed in PCM (water). The energy equation is coupled to the flow problem by an energy balance. The system of equations was solved numerically by using an average control volume technique. The results show the effects of the variation of the Biot number, the Stefan number, the inlet fluid temperature have on the ratio of the outer to the inner tube radius on the solidified mass fraction and the time for complete solidification.

### 2.3.2.2 Solidification of PCM inside a Spherical Capsule

The study of solidification of PCM inside a spherical capsule attracted many scientists. Chou [94] developed an analytical solution for freezing a saturated liquid inside or outside spheres; Pedroso and Domoto [95] created perturbation solutions for spherical solidification of saturated liquids; Riley et al. [96] studied on the inward solidification of spheres and circular cylinders; Kern and Wells [97] gave simple analysis and working equations for the solidification of cylinders and spheres; Moore and Bayazitoglu [98] researched on the melting within a spherical enclosure; Hill and Kucera [99] studied freezing a saturated liquid inside a sphere; Milanez and Ismail [100] made theoretical and experimental investigations on solidification in spheres; Prud'homme et al. [101] made a heat transfer analysis for solidification of slabs, cylinders, and spheres; Gillessen et al. [102] observed glass formation by containerless solidification of metallic droplets in drop tube experiments; Gibbs and Hasnain [103] studied technical grade phase change heat storage materials for solar heating applications; Feng et al. [104] developed the energy equation of a sphere in an unsteady and non-uniform temperature field; Folio and Lacour [105] focused on heat
transfer by conduction and convection between a spherical droplet particle and a two-phase fluid.

Bedecarrats et al. [106] studied the fusion in spherical geometries, Tao [107] and Ismail [80] presented approaches for the analysis of solidification in cylindrical and spherical geometries. Their approaches analyze the solidification process in cylinders and spheres considering an average heat transfer coefficient between the external surface of the solidified mass and the cooling fluid. A fixed grid approach to solve the mathematical models was used. The model was validated by comparisons with available results. Design parameters relating latent heat storage systems such as the diameter of the spherical shell, material of the shell, and the Biot number were evaluated. Others works that conducted numerical and experimental studies on the process of phase change within spherical shell include Cho and Sunderland [108], Shih presented a theoretical and experimental study on a commercial system, which uses phase change material encapsulated in spherical containers stacked in the storage tank. They developed a simulation program that considers aspects of both the surrounding heat transfer fluid and the phase change material during the charging and discharging periods. The simulation results were found to compare well with the experimental observations.

2.3.2.3 Conventional Distinct Interface Model

Another approach is the conventional distinct interface model that considers the boundary phase change boundaries to be moving as well as taking into account the internal absorbing, emitting and scattering of energy. Abrams and Viskanta [109] are amongst the earliest researchers who studied the Stefan problem including combined radiative and conductive transfer. They investigated the effects of internal radiation on the melting and solidification of emitting-absorbing semi-transparent crystals in a one-dimensional planar region. Another researcher, Habib [110,111] employed the heat balance integral method to study the solidification of semi-transparent planar and cylindrical media. The temperature profile within the solid phase was approximated as a combination of polynomial and logarithmic functions and the temperature profile in the liquid phase was assumed uniformed. These studies all demonstrated that internal radiative heat transfer has significant effects on the temperature distributions and the
interface moving velocities. However, these studies did not consider the effects of scattering.

On scattering, Webb and Viskantant [112] studied the buoyancy effects in the liquid phase whereas Seki et al. [113] used a finite difference method to study back melting of a cloudy ice layer immediately adjacent to a surface that is heated because of the absorption of short-wave radiation transmitted through the ice. The ice was irradiated from above and thus the model contained two liquid layers separated by a solid layer. Scattering was considered but emission by the ice was neglected. Cho and Ozisik [114] examined the one-dimensional solidification of a semi-infinite medium whose surface temperature was kept at a temperature lower than the melting temperature. The medium was assumed to experience isotropic scatter. This study was extended into a multi-dimensional problem of phase change in semi-transparent medium by Kim and Yimer [115]. In their studies, they investigated the effects of thermal radiation on solidification of an absorbing, emitting with isotropic scattering gray medium bounded between two concentric cylinders of finite length. The differential ($P_1$) approximation was used to deal with the radiation part in this analysis.

Nishimura et al [116] examined the phase change of planar layer of semi-transparent phase change materials under solar irradiation. Internal radiation effects during the Bridgeman crystal growth process was studied by Matsushima and Viskanta [117]. They considered an ideal vertical Bridgeman process consisting of a rectangular domain including the melting portion at the bottom of the geometry and the crystal at the top. The semi-transparent states in both crystal and melt are taken into account using the $P_1$ differential approximation. It was found that internal radiation has a great influence on the temperature distribution close to the solid-liquid interface.

In the above investigations, works by Adams and Viskanta [118] assumes that a distinct interface between liquid and solid zones exists and the phase change problem has been treated in a similar framework to the conventional melting or solidification problem. The model is able to register the effects of internal radiation resulting in overheating of the solid during the melting process or under-cooling of the melting portion during the solidification process.
2.3.2.4 Mushy Zone Model

Chan et al. [119] postulates that there exists a two-phase mushy zone between the pure liquid and the pure solid regions. The mushy zone is formed owing to internal radiation transfer and is different from the conventional dendritic interface formed during alloy processing. The mushy region is isothermal having a local equilibrium melting temperature of the material thereby avoiding the inconsistency encountered in the previous distinct interface model. The model was later re-evaluated by Chan and Hsu [120] that employed the enthalpy method. In that method, the phase change analysis in the mushy zone can be determined naturally without postulating its existence. The mushy zone model is further supported by the observations by Dorsey, [121] and Knight, [122] that internal melting possibly occur within ice.

The mushy zone model has since been accepted by other researchers in their studies of phase change of semi-transparent materials. Oruma et al. [123] investigated the solidification of a semi-infinite body with surface temperature kept below the freezing point of the material. The medium assumes to have isotropic scatter. Based on the mushy zone model, Khatarni [124] examined into the melting and ablation behaviour of a square of Teflon under irradiation of a laser beam.

Siegel [125] studied the radiative cooling of a solidifying droplet layer as a way of heat dissipation from space equipment. Conduction and scattering in the medium were ignored in his study. His results showed that, under the condition of pure radiative heat transfer, the solid fraction varies continuously from a completely solidified outer layer to the pure liquid in the center. The study also supports the concept of mushy zone solidification.

It is found that most of the studies in this area have considered a one-dimensional geometry, typically a semi-infinite body with the bulk materials at the equilibrium melting temperature, or a finite plate with the surface at specified temperatures. This restriction has limited the application of these studies in practice.

From the above review, the use of PCM for thermal cooling could offer an alternative solution in the design of a new portable cooler. The experimental and theoretical
studies on the heat transfer and energy analysis of phase change materials in a cooler box are therefore worth exploring.

2.3.3 Plane Finned Geometries For Latent Heat Storage Applications

2.3.3.1 Parallel Plate Finned and Finless Geometries

A series of numerical and experimental studies were realized on parallel plate finned and finless geometries with the objective of developing thermal models to use in energy storage systems, and particularly ice storage systems for refrigeration and air conditioning applications. Ismail (1999) [80] had developed a model based on pure conduction formulation and solved numerically by a fixed grid scheme. The second model allows for natural convection in the PCM liquid phase, and was formulated and solved by the finite element approach. The parameters analyzed numerically include the fin length, fin thickness, number of fins, the aspect ratio of the annular space and the temperature difference between the phase change temperature and the tube wall temperature. The results indicate that these parameters have significant effects on the time for complete solidification. The study highlighted that the fin thickness has a relatively small influence on the solidification time and that the fin length affects strongly the time for complete solidification and the solidification rate. It was also reported that the number of fins has a strong influence on the time for complete solidification and the rate of mass solidification. The aspect ratio of the annular space has a strong effect on the time for solidification of a certain mass, and increasing this ratio leads to increasing the time for complete solidification. The temperature difference has an opposite effect on the solidification of a certain mass of PCM, and the time for the complete solidification seems to decrease with the increase of this temperature difference.

2.3.3.2 Radial finned cylinders

Based on the above approach, a similar study was made with the radial finned tube submerged in PCM. A model for the solidification around a radially finned tube with constant wall temperature is developed and solved numerically. As the limiting boundary for phase change around the tube is symmetrically arranged, the work focuses the phase change analysis of only one tube. The model is based upon pure heat conduction formulation, and the enthalpy method by Bonacina et al. (1973) [126] was adopted. The finite difference approach and the Alternating Direction Implicit scheme
were used to discretize the system of equations and the associated boundary, initial
and final conditions. The study highlights the analysis is able to predict the thermal
performance of a radial finned tube. The study shows that the number of fins, fin
length, fin thickness and the aspect ratio of the finned tube arrangement have a strong
influence on the time for complete solidification and the solidification rate. The tube
material and the tube wall temperature seem to exert a strong effect on the time for
complete solidification. The thermal performance under different operational
conditions can be predicted from an established set of influenced equations.

2.4 Summary of Findings

The solid-liquid transformation PCM could offer a good viable alternative for portable
thermal storage. It has a large energy to weight storage capacity ratio enabling the
system to store and release large amounts of thermal energy at near isothermal
conditions operating at low pressures. The study would therefore focus on a thermal
storage system where an appropriate type of PCM is kept in a radial finned cylinder
located inside a highly thermal insulated box.

Useful experimental and numerical studies will be conducted to look into
solidification/melting behavior of PCM, its thermal performance as well as free
convection in the cooling space of the portable cooler. The theoretical study that leads
to a better understanding of the solidification/melting of PCM using the mushy zone
model taking into consideration the combined radiative and conductive transfer as well
as the emitting and scattering effects will be made. Simulated flow visualization in the
cooling space based on the CFD software would be made.
Chapter 3

Design and Development of a Portable Cooler

This section discusses the design and development of an investigative portable cooler using PCM as a means of refrigeration or cooling. The key design considerations for this portable cooler are refrigeration or cooling being carried out via passive rather than electrical means, weighs less than 23 kg, a recommended weight for lifting of compact objects [127], only makes use of environmental friendly refrigerants or substances, has high cooling rate that operates at a recommended temperature of -18 °C to minimize microbiological activities [13] requiring minimal maintenance.

3.1 Design Consideration

Refrigeration of the new electric-less portable cooler is greatly affected by the property of refrigerants and phase change material, amount of heat load or heat source subjected, the quality of insulation and the property of material used.

3.1.1 Refrigerant

From the earlier discussion, besides environmental friendly requirement such as ozone depletion and global warming effect, some other considerations in selecting the refrigerant include refrigerating effect, non-toxic, non-corrosive, non-flammable, inexpensive and readily available.

i. Refrigerating effect

The heat-carrying ability of the refrigerant is called the refrigerating effect. It consists of two components: latent and sensible heats. The amount of latent heat is usually much higher than the amount of sensible heat because more energy is required to change phase.

In general, the amount of refrigerating effect depends greatly on the latent heat of vaporization. Besides latent heat of vaporization, specific heat of vapor between the boiling temperature and the temperature at which the vapor no longer performs useful
refrigeration is also important. A higher specific heat in the vapor phase will absorb more heat and produce better refrigeration. Therefore, the refrigerant to be used in the new freezing device must have sufficiently high latent heat of vaporization and high specific heat in order to produce a better refrigerating effect.

ii. **Non-Toxic, Non-corrosive and Non-Flammable**

The refrigerant is to produce the refrigerating effect in order to preserve the food. Therefore, regardless of whether the refrigerant is being concealed or have no direct contact with the food, it must be non-toxic.

Majority of the refrigerants are highly hazardous chemical. At low temperatures, current refrigerants consisting of chlorofluorocarbons (CFCs) and hydrochlorofluorocarbons (HCFCs) can cause burns to human skin and even eye tissue on contact. At high temperatures, they produce phosgene, a lethal gas which had been used in chemical welfare [12]. A concentration of 0.02 - 0.05 % is lethal. As they are odorless, they are difficult to detect. In the selection of refrigerants for the new cooler box, all HCFCs and HFCs including ammonia that are toxic and with unpleasant odor will therefore not be considered.

Besides toxicity, the refrigerant must be non-corrosive and non-flammable, in order to allow the refrigerator to be used in any environment and any locations without hazards.

iii. **Availability and Cost**

Cryogenic liquids are one of the alternatives used other than conventional refrigerants. They are produced by air liquefaction and they are environmental friendly. Table 3.1 shows the composition of air from which the cryogenic liquids are abstracted from.
In air liquefaction, air is collected and compressed under preset pressure and temperature whereby a gas reaching its saturation point will condense into liquid. Different types of gases have different vapor pressures and temperatures.

If a gas is abundantly available in the air, the process of air liquefaction will be easier and faster. This means that the cryogenic system will be less complex. For example, liquid nitrogen is the easiest to obtain from the air by a simple liquid distillation technique. Helium is the most difficult due to its extremely low fraction in air and low temperature, resulting in the development of a complex refrigeration system. Another consideration is gas abundance. The more abundant a gas is, the easier it is to produce leading to lesser cost. In recent times, due to an increase in oxygen production, the availability of liquid nitrogen has become abundant as it is a by-product of the oxygen production.
Refrigerant (Coldest to warmest) | Boiling Temperature 1 atm (°C) | Latent heat of vaporization at boiling point, 1atm (kJ/kg) | Vapor Specific heat at boiling point \( C_p, 1 \text{atm} \) (kJ/kg) | Physical property | Unit Cost (S$/kg)
--- | --- | --- | --- | --- | ---
Liquid Helium | -268.7 | 20.9 | 7.068 | Colorless | 0.7
Liquid Hydrogen | -252.73 | 443 | 10.844 | Colorless, Explosive | NA
Liquid Nitrogen | -195.6 | 199 | 1.084 | Colorless, Inert, Non-toxic | 0.16
Liquid Argon | -185.72 | 161.9 | 0.546 | Colorless, Inert, Non-toxic | 0.45
Liquid Oxygen | -182.8 | 213 | 1.084 | Blue, Flammable | 0.25
Liquid CO\(_2\) | -78.5 | 572 (sublimation) | 0.5 | Colorless, Pungent smell, Inert | 0.1
Dry Ice | -78.5 | | | | 1.9

Table 3.2 Selected refrigerants and their properties [129]

From Table 3.2, most of the other refrigerants are either very expensive or unsuitable to be used due to their toxicity and flammability. Liquid nitrogen and liquid carbon dioxide are the cheapest refrigerants. In terms of latent heat energy per unit cost comparison, the computations are as follows:

\[
1 \text{ kg of liquid carbon dioxide} = S\$0.1 \\
\text{Liquid carbon dioxide: } 572 \text{ kJ/kg} \times \text{kg} / S\$0.1 = 5720 \text{ kJ/S$}
\]

\[
1 \text{ kg of liquid nitrogen} = S\$0.16 \\
\text{Liquid nitrogen: } 199 \text{ kJ/kg} \times \text{kg} / S\$0.16 = 1244 \text{ kJ/S$}
\]

From the above calculation, liquid carbon dioxide is 460 % cheaper than liquid nitrogen.

As mentioned earlier, a good refrigerant must have high latent heat of vaporization in its liquid state and high specific heat in its vapor state in order to produce better refrigeration. In terms of specific heat, dry ice has 0.5 kJ/kg.K and liquid nitrogen vapor has 1.084 kJ/kg.K. Therefore liquid nitrogen is 200 % better than liquid carbon dioxide.
Although liquid carbon dioxide is cheaper, a liquid carbon dioxide spray has to be kept under a pressure of about 5.2 bar as at atmospheric pressure, otherwise, carbon dioxide can only exist in a gaseous state rather than a liquid. Spray nozzles for liquid carbon dioxide should, therefore, be designed to avoid blockage due to snow formation during pressure drop. This will make the mechanical design and maintenance of the new refrigeration system more complex. In addition, carbon dioxide is one of the global warming gases. From the above comparison, liquid nitrogen was selected as the refrigerant.

3.1.2 Phase Change Material and Container

Liquid nitrogen is selected as the refrigerant because it is totally environmentally friendly, non-toxic, non-corrosive, non-flammable, has a good refrigerating effect and it is cheap to produce. Nevertheless, liquid nitrogen evaporates under atmospheric pressure at –196 °C, which is too low for most of the required refrigeration. At this temperature, the texture and quality of food products would be affected. To avoid operating at extreme low temperatures, it is proposed that cooling can be carried out indirectly and in stages instead of direct cooling. The concept of using the phase change medium is introduced into the evaporator and it acts like a “cool buffer” for heat exchange between the space to be cooled and the liquid nitrogen. The introduction of this phase change medium, in fact, brings about many other advantages for the system too. It can reduce the heat transfer driving force by decreasing the temperature difference between the space to be cooled and the surroundings. Also, it eliminates the temperature fluctuation in the cooling space making the cooler workable without any power source because the phase change medium can store and release a large amount of thermal energy at near isothermal conditions. More importantly, it allows the cooler to operate at a derived temperature.

There are hundreds of potential PCM candidates that are reported in published literatures. However, few materials have actually been tested and suitably used for PCM refrigeration applications. The PCMs are selected on the basis of their heat of fusion and melting temperatures. However, many PCMs have a high latent heat of fusion and a convenient melting temperature but are hazardous or corrosive. Therefore,
the PCM must exhibit certain desirable thermodynamic and chemical properties. In addition, economic considerations and availability at large scales must also be considered.

The various criteria that control the PCM selection for cooling units depend on the application. In general, the PCM should possess the following criteria: the PCM melting temperature should be lower than -18°C. The latent heat of fusion must be high, allowing a small amount of PCM to store a large amount of energy. High specific heat is desirable, which will provide additional sensible heat storage capacity. It must have high thermal conductivity, which makes the PCM melting and solidification homogeneous and could also prevent potential PCM overheating. It must have chemical stability, so that the PCM will not be changed periodically. The PCM must be non-poisonous, non-flammable, and non-explosive.

The most critical properties are PCM melting temperature and latent heat of fusion. Sodium nitrite has been used as a PCM in this research because its melting temperature can be low as -20 °C, which is lower than the desired operating temperature of the portable cooler. However, its properties are different in each phase. For the numerical simulation, different properties in solid and liquid phases are used as well as during PCM melting.

The PCM container should use the materials with very high conductivity because this will result in good heat transfer. Thermal conductivity for some substances is shown in Table 3.3.
Chapter 3 Design and Development of a Portable Cooler

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity, $k/WmK$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver (pure)</td>
<td>410</td>
</tr>
<tr>
<td>Copper (pure)</td>
<td>385</td>
</tr>
<tr>
<td>Aluminum (pure)</td>
<td>202</td>
</tr>
<tr>
<td>Nickel (pure)</td>
<td>93</td>
</tr>
<tr>
<td>Iron (pure)</td>
<td>73</td>
</tr>
<tr>
<td>Carbon steel, 1% C</td>
<td>43</td>
</tr>
<tr>
<td>Lead (pure)</td>
<td>35</td>
</tr>
<tr>
<td>Chrome-nickel steel (18% Cr, 8% Ni)</td>
<td>16.3</td>
</tr>
</tbody>
</table>

Table 3.3 Thermal conductivity of various metals at 0°C [130]

In the above table, silver is the material with the highest thermal conductivity. Unfortunately, it is too soft to withstand the stress resulting from the expansion during phase change of PCM. Also, pure silver is very expensive. Copper also possesses a very high thermal conductivity, much harder and cheaper than silver. Copper is therefore used to construct the container.

An important issue that needs to be addressed is natural convection, the major heat transfer mode in the cooling space. Since there is no electrical fan used to circulate the air, the heat exchange between the air inside the box and the melting PCM will be slow. An increase in the heat transfer between them would be needed via introducing highly conductive metal fins.

3.1.3 Thermal Conductivity Enhancer

All phase change materials with high latent heat storage capacity have unacceptable low thermal conductivity, which makes heat charging and discharging slow. An effective method to improve heat transfer is using a thermal conductivity enhancer. The enhancer is designed as metal fins inserting into the PCM. The fins are dispersed in the PCM so that the mixture behaves thermally like a homogenous material with both high thermal conductivity and high heat capacity.
Aluminum, titanium and stainless steel are the metals used for the enhancer. The thermal conductivity of these metals is high. They are corrosion resistant and high in strength to weight ratio. Aluminum is widely used because of its low density, high thermal conductivity, and corrosion resistance. It can be soldered, brazed, and welded. In addition, aluminum-aluminum joints are generally superior in strength to joints between aluminum and dissimilar metals.

### 3.1.4 Heat Source

Objects with high initial temperature to be cooled is the heat source or heat load for the cooler. However, the surrounding is a large heat source that is unnecessary to the cooler box. Since heat transfer always readily takes the least resistance path, opening of cooler door under room temperature is one of the main heat sources as it breaks the shield of insulation against intensive heat transfer. To avoid experiencing unnecessary heat source during opening the door, the door should be designed to open from the top as shown in Fig. 3.1. As such, the cold gas from within the internal place of the refrigerator will not flow out easily. At the same time, the warm air from the surrounding being lower in density, will not easily flow inwards.

![Figure 3.1 Design of door](image)

### 3.1.5 Insulator

The surrounding is a large heat source which may be considered as an envelope of space where all matters from within dissipate and absorb heat energy to achieve thermal equilibrium. The second law of thermodynamics shows that heat transfer occurs in the direction of decreasing temperature. A cooler operating under room temperature is a heat sink absorbing all heat energy from the surrounding due to the large temperature difference across its boundary. The material selected as the box wall should therefore have a high thermal insulation property because this will reduce heat gains thereby reducing the energy use required in the refrigeration system.
Chapter 3 Design and Development of a Portable Cooler

i. **Insulation against Conduction**

Effective insulation against conduction can be achieved through proper selection of materials. Such materials usually possess low thermal conductivity property. Table 3.4 shows several common insulating materials in which vacuum is considered the best in insulation against conduction.

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity k, (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiberglass (broken flake), unevacuated</td>
<td>0.05</td>
</tr>
<tr>
<td>Polyurethane Form (used in typical refrigerator)</td>
<td>0.05</td>
</tr>
<tr>
<td>Polystyrene, expanded Extruded (R-12)</td>
<td>0.027</td>
</tr>
<tr>
<td>Molded beads</td>
<td>0.04</td>
</tr>
<tr>
<td>Cork</td>
<td>0.039</td>
</tr>
<tr>
<td>Low vacuum (range of vacua: $10^2$-$10^5$Pa)</td>
<td>0.001-0.009</td>
</tr>
</tbody>
</table>

**Table 3.4 Thermal conductivity of common insulating materials [131,132]**

ii. **Insulation against Convection**

Insulation against convection can be achieved through stagnant air gap or reducing the air gap into many smaller spaces. Air that is not in motion is a very good insulator because air itself is a bad conductor of heat. For example, cork is a good insulator of heat as the material contains trapped air within it. With more and smaller air spaces, this limits the convection currents, which enhances the insulating effects.

However, the difficulty in using air as the insulator is because air cannot be kept motionless. Once it is heated up, it will vibrate and create convection current. However, in small gaps of 2 mm, convection current is minimum. Therefore, for effective insulation against convection, vacuum would be the best choice.

iii. **Insulation against Radiation**

Reflective insulation like aluminum or lead-coated sheet is effective to retard radiant energy waves. The reflection prevents much of heat from entering the refrigerated space.
3.1.6 Box Material Property
As the temperature decreases inside a cooler, the air initially occupying the enclosure cools and contracts, leading to a pressure drop. The cooler with lower inner pressure will be physically exposed to atmospheric pressure and may rupture. Since cryogenic fluid is used as the refrigerant, pressure will be built up within the enclosure due to the vaporization of refrigerant. Hence, the mechanical property of material used to construct the framework for the cooler is important.

Metals are ideal choice but due to its high thermal conductivity, it cannot be used. Metals should be used to reinforce components only. Plastic will be more suitable in this case. However, most polymeric materials become hard at low temperature and may even exhibit brittle behavior. The change in the property is due to the glass transition temperature of the polymeric materials, where at this temperature, they experience the transition from rubbery to rigid state. Glass transition temperature differs significantly among all polymeric material due to the difference in their chemical composition. The ultimate working temperature of the portable cooler -20 °C should be used as a guide for selecting the appropriate plastic to be used. Table 4.5 shows several sets of material properties that can be used as main body for the cooler.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Material</th>
<th>Code/Trade Mark</th>
<th>Thermal Conductivity (W/mK)</th>
<th>Glass Transition Temperature (°C)</th>
<th>Modulus of Elasticity E (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[133]</td>
<td>Polyethylene</td>
<td>LDPE</td>
<td>0.33</td>
<td>-110</td>
<td>0.225</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HDPE</td>
<td>0.48</td>
<td>-90</td>
<td>1.08</td>
</tr>
<tr>
<td>[133]</td>
<td>Polytetrafluoroethylene</td>
<td>PTFE, Teflon®</td>
<td>0.25</td>
<td>-97</td>
<td>0.95</td>
</tr>
<tr>
<td>[134]</td>
<td>Fiber</td>
<td>Mylar®</td>
<td>0.15</td>
<td>-70</td>
<td>0.49</td>
</tr>
<tr>
<td>[135]</td>
<td>Chlorinated Polypropylene</td>
<td>PPC</td>
<td>0.15</td>
<td>-20</td>
<td>1.345</td>
</tr>
<tr>
<td>[133]</td>
<td>Polyvinyl Chloride rigid</td>
<td>PVC</td>
<td>0.18</td>
<td>-87</td>
<td>3.25</td>
</tr>
<tr>
<td>[135]</td>
<td>Polymethyl Methacrylate</td>
<td>PMMA</td>
<td>0.2</td>
<td>-95</td>
<td>2.74</td>
</tr>
</tbody>
</table>

Table 3.5 Several selected plastics and their property
From Table 3.5, the materials will be compared in terms of their thermal conductivity, glass transition and modulus of elasticity. PVC and Perspex are included for comparison.

In general, a good material for the body of the cooler should have low thermal conductivity as the priority, followed by low glass transition temperature and high modulus of elasticity.

In terms of thermal conductivity, Mylar\textsuperscript{R} and PPC are appropriate materials to use with thermal conductivity of 0.15 W/mK. Mylar\textsuperscript{R} has a poorer mechanical property but it has a lower working temperature. PPC has a better mechanical property having an ultimate working temperature of the portable cooler of -20 °C.

In terms of low transition temperature, LDPE, HDPE and Teflon\textsuperscript{R} are suitable materials. However, LDPE and HDPE do not have good mechanical strength and are poor heat insulators. Teflon\textsuperscript{R} is best in providing good thermal conductivity and mechanical properties.

For PVC and PMMA, they have good mechanical properties but PMMA is not considered as they are not suitable for use in low temperature.

From the comparison, it can be concluded that Teflon\textsuperscript{R} is the most suitable material for use in the new cooler operating at low temperature.

3.2 Conceptual Design

Fig. 3.2 and Fig 3.3 show the design of the new cooler box of 0.6 m × 0.6 m × 0.4 m with a cooling volume of 0.108 m\textsuperscript{3} (108 liters) constructed for this project. The cooler consists of an airtight rubber seal, cooling coil with liquid nitrogen inlet and outlet, three directional valve, sub-cooling pipe, air flow guide, PCM container with fins and pressure relief valve. It weighs about 16 kg.

As shown in Fig. 3.4, a cylinder containing liquid nitrogen will be placed outside the box. Liquid nitrogen will be supplied from the nozzle of the cylinder and introduced
into the cooler via the *Liquid Nitrogen Inlet*. A *Three Directional Valve* is used to adjust and distribute the amount of liquid nitrogen into various regions of the empty cooling space as shown in Fig. 3.5.
Figure 3.2 Parts and Assembly Drawing of the New Portable Cooler
Chapter 3 Design and Development of a Portable Cooler

Figure 3.3 The Fabricated Cooler Box

Figure 3.4 Liquid Nitrogen Input Through the Network of Pipes
Figure 3.5 Liquid Nitrogen Input Through Sub-cooling Pipes

After the heat load being added to the cooler, a *Three Directional Valve* is used to control and adjust the flow of liquid nitrogen into the *PCM Container*. As shown in Fig. 3.6, liquid nitrogen absorbs heat from the surrounding PCM and then evaporates. Since the presence of nitrogen vapor displaces any oxygen and helps preserve the food, nitrogen vapor after being used for refrigeration will be circulated within the box before discharging from the *Pressure Relief Valve*. An *Airflow Guide* is designed to navigate the flow in the box to minimize temperature variations. This is shown in Fig. 3.6.
Figure 3.6 Liquid Nitrogen Input Through Cooling Coil
Figure 3.7 Drawing of Experimental Setup

- LN2 Flow Rate Meter
- LN2 Input Valve
- Pressure Regulator
- LN2 Volume Meter
3.3 Experimental Setup and Procedure

In this experiment, various temperatures at different locations inside the box are to be measured using thermocouples. The thermocouples are glued onto the inner wall of the box and the cold accumulator as well as the coiling tubes at different locations by using conductive thermal glue. Since the liquid nitrogen is supplied from the center, the box can be conveniently divided into 2 symmetrical segments as shown in Fig. 3.8. The thermocouple layout and locations can be found in Fig. 3.9. The experimental readings at the different locations would be used to compare with the simulated results on the cooling behavior within the box.

In this project, two sets of experiments are conducted. The first set of experiments records the temperatures under no heat load conditions. The second set of experiments involves placing a heat load at different locations in the box.

**Experiment 1 : No heat load condition**

This set of experiments consists of eight main steps namely:

a) Prior to the experiment, the cover of the box must be closed tightly.

b) The initial temperatures of the thermocouples are recorded.

c) Liquid nitrogen will be piped through the sub-cooling tube into the air space of the cooler box until the thermocouples read 255 K (-18 °C).

d) The time interval to record the temperature is once every 30 seconds and the recording time starts immediately after liquid nitrogen is piped fully into the box. The amount of liquid nitrogen consumed in step (c) is also recorded.

e) The liquid nitrogen will be introduced into the cooling coil at the flow rate of 2 kg/h until the thermocouples 4 and 5 indicate -22 °C. Since the melting point of PCM is -20 °C, cooling to -22 °C ensures all the PCM solidification completes.

f) In step (e), the time interval to record the temperature is once every minute and the recording time starts immediately after liquid nitrogen enters the cooling tube.

g) The liquid nitrogen input is stopped and the cooler box is then left to see how fast it reaches ambient.

h) Thereafter the pressure relief valve will discharge unwanted vapor nitrogen from the cooler box once the set pressure is reached.
Experiment 2: Heat load condition at specified location

In this experiment, a heat load is to be placed inside the cooler box at different locations. The experimental procedure is as follows:

i) The cooler box cover is open. The aluminum block with initial temperature of 300 K (27 °C) is then lowered into the cooling space at a designated location. The experiment is conducted at ambient room temperature (i.e., 300 K (27 °C)).

j) The cover of the box must be closed tightly after the aluminum block is lowered into the cooler box.

k) The experiment ends when all the PCM has been melted. This would mean that thermocouples 4 and 5 register a value of about 255 K (-18 °C).

l) In step (k), the time interval to record the temperature is 1 minute and the recording time starts immediately after the aluminum block is set into the cooler box.
Figure 3.8 Identical Two Segments of the Experimental Setup
Figure 3.9 Thermocouple Location Layout to Measure Inner Box Wall Temperature
Chapter 4

One-dimensional Mathematical Model and Simulation

A transient 1-D model is formulated to quickly examine the initial idea and the workability of the design. An empty cooler box is studied in this 1-D model, in other words, no heat source is loaded in the cooling space.

![Figure 4.1 One-dimensional Model Setup](image)

In Fig. 4.1, the physical model is approximately symmetrical in all the x, y, z directions. Since the bottom of the box is a square, heat transfer in y-direction is assumed to be the same as that in z-direction. Since the walls of the cooler box are well insulated and the liquid nitrogen flows along the x-direction, it is anticipated that temperature variation along the x direction is more significant than the y-direction. Hence only the time-dependent heat transfer along the x-direction is studied in this simplified transient 1-D model.
4.1 One-dimensional Mathematical Model

4.1.1 Assumptions

Assumption 1:

In this 1-D model, the model is simplified to schematic shown in Fig. 4.2.

Noted that the pre-cooling tube in Fig. 4.2 is the tube network for initial cooling. In the modeling study, it is simplified to the schematic shown in Fig 4.2(b)

Assumption 2:

As shown in Fig. 4.3, the cooling tube consists of two parts, which are coiling part (blue color) and straight part (red color). Since normally all the liquid nitrogen vaporizes in the coiling part and the straight part only serves as the passage to exhaust vapor nitrogen from the tube, the straight part can be neglected in the one-dimensional model analysis.

Assumption 3:

Since the sealing rubber greatly reduces the air leakage between the cover and the body of the cooler box, the whole cooler box can be assumed to behave as hermetically sealed as shown in Fig. 4.4. Heat transfer from the outside to the inside of the cooler box is merely by conduction through the box wall which is made up of a good thermal insulation material.
Chapter 4 One-dimensional Mathematical Model and Simulation

Figure 4.3 One-dimensional Model: Assumption 2

Figure 4.4 One-dimensional Model: Assumption 3
Assumption 4:
The temperature distribution of air in the cooling space \((T_b)\) varies only with time \((t)\), that is

\[ T_b = f_1(t) \]

In fact, due to the large cooling space in the cooler box, \(T_b\) is strongly dependent on both location and time. In order to obtain an even temperature distribution, a small fan with low power consumption will be suggested in reality. If the design was found to be workable by the one-dimensional model with the assumption of \(T_b = f_1(t)\), this could be extended to include an installed small fan in the cooler.

Assumption 5:
The temperature distribution of phase change material in the cylindrical container varies only with time, that is

\[ T_x = f_2(t) \]

For PCM assembly, according to the Lumped Capacitance Method,

\[ Bi = \frac{h_b r_x}{k_x} \]

where \(h_b\) is the free convection heat transfer coefficient for air outside the PCM assembly, \(r_x\) is the radius of the PCM container and \(k_x\) is the thermal conductivity of the PCM.

Therefore, for PCM assembly,

\[ Bi = \frac{h_b r_x}{k_x} = \frac{3.22 \text{ W/m}^2 \text{K} \times 0.05 \text{ m}}{1.74 \text{ W/mK}} = 0.09 < 0.1 \]

The assumption of \(T_x\) (temperature distribution of PCM) being varied only with time is therefore valid \((B_i = 0.09 < 0.1)\).

The assumption of \(T_x = f_2(t)\) will be only used in the analysis of the simulation results of the one-dimensional model.

In the one-dimensional model deduction, \(T_x\) refers to the temperature of PCM contacting the cylindrical container wall. Since the thermal resistance of PCM will be considered, the temperature distribution of PCM will depend on both location and time.

Assumption 6:
Temperature distribution of nitrogen in the cooling coil tube \((T_d)\) varies only with x-direction \((x)\), that is
Chapter 4 One-dimensional Mathematical Model and Simulation

\[ T_d = f_3(x) \]

**Assumption 7:**
The ratio of the weight of the liquid phase for the phase change material to the total weight of the phase change material \( U_x \) varies only with time \( t \), that is
\[ U_x = f_4(t) \]

**Assumption 8:**
The ratio of the weight for the vapor nitrogen to the total weight of nitrogen \( U_d \) is a function of \( x \) only, that is
\[ U_d = f_5(x) \]

**Assumption 9:**
Thermal resistance of the PCM container wall and the coiling tube wall that are made of copper will be neglected.

### 4.1.2 Dimensions and Physical Properties of the Model

(1) Dimensions
- \( L_j \) – thickness of the cooler box wall, 0.1m
- \( L_a \) – length of the cooling space in the box, 0.6m
- \( H \) – height of the cooling space in the box, 0.3m
- \( t_f \) – thickness of each fin, 0.001m
- \( L_f \) – length of each fin, 0.1m
- \( r_x \) – radius of the cylindrical PCM container, 0.05m
- \( D_c \) – diameter of each coil of the cooling coil tube, 0.03m
- \( n \) – number of coils of the cooling coil tube, 15
- \( r_d \) – radius of the cooling coil tube, 0.003m

(2) Physical Properties
In this research, average properties based on the model component’s volume are used in the numerical model. This approximate method is proposed by Chow [137], who examined the validity of modeling the thermal control unit (TCU) with average material properties by comparing its results with a TCU with multimaterial composition. A strong agreement was fund between the two cases.
Chapter 4 One-dimensional Mathematical Model and Simulation

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Air</strong></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho = 1.225\text{kg/m}^3$</td>
</tr>
<tr>
<td>Specific Heat Capacity</td>
<td>$C_p = 1006.43\text{J/kg.K}$</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$k = 0.0242\text{W/m.K}$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu = 1.7894\times 10^{-5}\text{kg/m.s}$</td>
</tr>
<tr>
<td>Expansion Coefficient</td>
<td>$\beta = 0.00395/°\text{C}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PCM</strong></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$D_x = 1300\text{kg/m}^3$</td>
</tr>
<tr>
<td>Specific Heat Capacity Liquid</td>
<td>$C_{XL} = 3765.69\text{J/kg.K}$</td>
</tr>
<tr>
<td>Solid</td>
<td>$C_{XS} = 1882.85\text{J/kg.K}$</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$k_x = 1.74\text{W/m.K}$</td>
</tr>
<tr>
<td>Melting Heat</td>
<td>$H_x = 300000\text{J/kg}$</td>
</tr>
<tr>
<td>Melting Temperature</td>
<td>$T_x^* = 253\text{K}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vapor Nitrogen</strong></td>
<td></td>
</tr>
<tr>
<td>Specific Heat Capacity</td>
<td>$C = 1046.03\text{J/kg.K}$</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$k = 0.0256\text{W/m.K}$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu = 1.67\times 10^{-5}\text{kg/m.s}$</td>
</tr>
<tr>
<td>Vaporization Heat</td>
<td>$H_d = 198000\text{J/kg}$</td>
</tr>
<tr>
<td>Vaporization Temperature</td>
<td>$T_d^* = 77\text{K}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Insulation Wall</strong></td>
<td></td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$k_j = 0.02\text{W/m-K}$</td>
</tr>
<tr>
<td><strong>Copper Fin</strong></td>
<td></td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$k_f = 387.6\text{W/m-K}$</td>
</tr>
</tbody>
</table>

The Initial and Boundary Conditions are

Initially, $T_b$ (temperature of the air in the cooler box) is 255K; $T_x$ (temperature of the PCM) is 255K.

$T_k$ (temperature of the exterior surface of the cooler box wall) is constantly 300K and $W_d$ (mass flow rate of liquid nitrogen) is 2kg/h.

### 4.1.3 Physical Model

(1) Equation of $T_b = f_1(t)$
Figure 4.5 Flux Balance over Control Volume 1

Control volume 1 in fig. 4.5 is enclosed by the interior surfaces of the box wall (purple color) and the finned exterior surface of the cylindrical PCM container wall (yellow and green color).

The circuit must account for the conduction in the cooler box wall ($R_{kb}$), convection from the exposed base ($R_{t,base}$), and conduction/convection from the fins ($R_{t,fin(50)}$).

Thermal resistance for the cooler box wall is
Chapter 4 One-dimensional Mathematical Model and Simulation

\[ R_{lb} = \frac{L_j}{k_j \times A_{box, exterior}} = \frac{L_j}{k_j \times [4(L_a + 2L_j)(H + 2L_j) + 2(L_a + 2L_j)^2]} = 1.4 K/W \]

For a single fin,

\[ R_{t,\text{fin}(1)} = 5.48 K/W \]

Hence, for 50 fins,

\[ R_{t,\text{fin}(50)} = \frac{R_{t,\text{fin}(1)}}{50} = \frac{5.48}{50} = 0.11 K/W \]

For the exposed base,

\[ R_{t,\text{base}} = \frac{1}{h_b \times (2\pi r - 50t_f) \times H} = 3.89 K/W \]

With

\[ R_{bx} = R_{\text{equiv.}} = [(0.11)^{-1} + (3.89)^{-1}]^{-1} = 0.11 K/W \]

Without the fins, the convection resistance of the PCM is

\[ R_{\text{tran}} = (2\pi r_s H h_b)^{-1} = 3.3 K/W \]

Hence there is considerable advantage to using the fins.

\[ q_{in} = \frac{T_t - T_b}{R_{lb}} = \frac{300 K - T_b}{1.4 K/W} \]

\[ q_{out} = \frac{T_b - T_x}{R_{bx}} = \frac{T_b - T_x}{0.11 K/W} \]

At an instant, energy terms include the rate at which thermal energy enters and leaves through the control surface, \( q_{in} \) and \( q_{out} \). The rate of change of energy stored within the control volume is designated as \( q_{st} \). A general form of the energy conservation requirement may then expressed as

\[ q_{in} - q_{out} = q_{st} \]
\[
\frac{300K - T_b}{1.4K/W} - \frac{T_b - T_x}{0.11K/W} = V_b D_b C_b \frac{dT_b}{dt}
\]

\[
\frac{300K - T_b}{1.4K/W} - \frac{T_b - T_x}{0.11K/W} = [(L_a^2 - \pi r_s^2 - 50t_f L_f)H]D_b C_b \frac{dT_b}{dt}
\]

So

\[
\frac{dT_b}{dt} + 0.08T_b - 0.07T_x - 1.67 = 0 \tag{4.1}
\]

(2) Equation of \(T_x = f_2(t)\) and \(U_x = f_4(t)\)

Control volume 2 in fig. 4.6 is enclosed by the finned surface of the PCM cylindrical container (yellow and green color) and the surface of the coiling tube wall (blue color).
The circuit must account for the overall thermal resistance between the air in the cooler box and the PCM ($R_{bx}$), conduction in the cooler box wall ($R_{kx}$), thermal resistance between PCM and the exterior surface of the coiling tube wall ($R_{x}$), and convection form the interior surface of the coiling tube wall ($R_{d}$).

PCM in fig. 4.7 (a) can be approximately viewed as a hollow cylindrical system shown in (b).

$$R_{x} = \frac{\ln[r_{x}/(D_{c}/2)]}{2\pi k_{x}H} = 0.37 K/W$$

$$R_{ad} = R_{x} + R_{d} = R_{x} + \frac{1}{h_{d} \times A_{nbe}} = R_{x} + \frac{1}{h_{d} \times (2\pi r_{d}) \times (\pi D_{c} \times n)} = 0.6 K/W$$
\[
R_{kX} = \frac{L_j}{k_j \times 2 \times A_{PCM, top}} = \frac{L_j}{k_j \times 2 \times \pi r_x^2} = 318.47 K/W
\]

\[
q_{in} = \frac{T_b - T_x}{0.11K/W} + \frac{300 - T_x}{318.47K/W}
\]

\[
q_{out} = \frac{T_x - T_d}{R_{sd}} = \frac{T_x - T_d}{0.6K/W}
\]

\[
q_{in} - q_{out} = \frac{T_b - T_x}{0.11K/W} + \frac{300 - T_x}{318.47K/W} - \frac{T_x - T_d}{0.6K/W} = q_{st}
\]

\[
q_{st} = \begin{cases} 
W_x \cdot C_{XL} \cdot \frac{dT_x}{dt} & (T_x > T_x^*, U_x=1) \\
H_x \cdot W_x \cdot \frac{dU_x}{dt} & (T_x=T_x^*, 0 < U_x <1) \\
W_x \cdot C_{XS} \cdot \frac{dT_x}{dt} & (T_x < T_x^*, U_x=0)
\end{cases}
\]

So
\[
\frac{dT_x}{dt} - 8.02E^{-4}T_x + 9.49E^{-4}T_x - 1.47E^{-4}T_d - 8.29E^{-5} = 0 \quad (T_x > T_x^*, U_x=1) \quad (4.2)
\]

\[
\frac{dU_x}{dt} - 1.01E^{-5}T_x + 1.19E^{-5}T_x - 1.85E^{-6}T_d - 1.04E^{-6} = 0 \quad (T_x=T_x^*, 0 < U_x <1) \quad (4.3)
\]

\[
\frac{dT_x}{dt} - 1.6E^{-3}T_x + 1.9E^{-3}T_x - 2.95E^{-4}T_d - 1.66E^{-4} = 0 \quad (T_x < T_x^*, U_x=0) \quad (4.4)
\]

(3) Equation of \( T_d = f_3(x) \) and \( U_d = f_5(x) \)
Control volume 3 in fig. 4.8 is enclosed by the interior surface of the coiling tube wall (blue color). The length of the coiling tube per unit height along the x-direction is viewed as a constant $C_0$ for simplification and $C_0$ is defined as follows:

$$C_0 = \frac{L_d}{H} = \frac{\pi D_c n}{H} \approx 4.71$$

![Figure 4.8 Flux Balance over Control Volume 3](image)

Since liquid nitrogen begins vaporization as soon as introduced into the coiling tube at inlet, it will experience liquid-vapor phase change as flowing inside the coiling tube and leave the tube at the outlet in vapor as shown in the figure below.
So

\[ q_{st} = W_d \times H_d \times \frac{dT_d}{dx} \times \delta_x = 110 \frac{dT_d}{dx} \delta_x \]

The heat entering the control volume 3 evaporates liquid nitrogen as

\[ q_{in} = q_{st} \]

\[ \frac{dT_d}{dx} - 0.05T_d + 3.89 = 0 \quad (T_d = T_d^*, \ 0 < U_d < 1) \quad (4.5) \]

Then the cold vapor nitrogen whose temperature is \( T_d^* \) will further absorb heat from the surrounding phase change material until it leaves from the outlet of the coiling tube.

So

\[ q_{st} = W_d \times C_{dg} \times \frac{dT_d}{dx} \times \delta_x = 0.59 \frac{dT_d}{dx} \delta_x \]

The heat entering the control volume 3 increases the temperature of the vapor nitrogen as

\[ q_{in} = q_{st} \]

\[ \frac{dT_d}{dx} - 9.42(T_d - T_d^*) = 0 \quad (T_d \geq T_d^*, \ U_d=1) \quad (4.6) \]
4.1.4. Numerical Analysis

(1) Equation of $T_b$

\[
\frac{dT_b}{dt} + 0.08T_b - 0.07T_x - 1.67 = 0
\]

Therefore

\[(T_b)_{m+1} = (1 - 0.08\Delta t)(T_b)_m + 0.07\Delta t(T_x)_m + 1.67\Delta t \quad (4.7)\]

(2) Equation of $T_x$ and $U_x$

\[\begin{align*}
(1) \quad & \frac{dT_x}{dt} - 8.02E^{-4}T_b + 9.49E^{-4}T_x - 1.47E^{-4}T_d - 8.29E^{-5} = 0 \quad (T_x > T_x^*, \ U_x=1) \\
(2) \quad & \frac{dU_x}{dt} - 1.01E^{-5}T_b + 1.19E^{-5}T_x - 1.85E^{-5}T_d - 1.04E^{-6} = 0 \quad (T_x=T_x^*, \ 0 < U_x <1) \\
(3) \quad & \frac{dT_x}{dt} - 1.6E^{-5}T_b + 1.9E^{-3}T_x - 2.95E^{-4}T_d - 1.66E^{-5} = 0 \quad (T_x < T_x^*, \ U_x=0)
\end{align*}\]

Therefore

\[\begin{align*}
(1) \quad & (T_x)_{m+1} = 8.02E^{-4}\Delta t(T_b)_m + (1 - 9.49E^{-4}\Delta t)(T_x)_m + 1.47E^{-4}\Delta t[(T_d)_1 + (T_d)_{N_{max}}]/2 + 8.29E^{-5}\Delta t \\
& \quad (T_x > T_x^*, \ U_x=1) \\
(2) \quad & (U_x)_{m+1} = (U_x)_m + 1.01E^{-5}\Delta t(T_b)_m + 1.85E^{-6}\Delta t[(T_d)_1 + (T_d)_{N_{max}}]/2 - 3.01E^{-3}\Delta t \\
& \quad (T_x=T_x^*, \ 0 < U_x <1) \\
(3) \quad & (T_x)_{m+1} = 1.6E^{-5}\Delta t(T_b)_m + (1 - 1.9E^{-3}\Delta t)(T_x)_m + 2.95E^{-4}\Delta t[(T_d)_1 + (T_d)_{N_{max}}]/2 + 1.66E^{-5}\Delta t \\
& \quad (T_x < T_x^*, \ U_x=0)
\end{align*}\]

(3) Equation of $T_d$ and $U_d$

\[\begin{align*}
(1) \quad & \frac{dU_d}{dx} - 0.05T_x + 3.89 = 0 \quad (T_d = T_d^*, \ 0<U_d<1) \\
(2) \quad & \frac{dT_d}{dx} - 9.42(T_x - T_d) = 0 \quad (T_d \geq T_d^*, \ U_d=1)
\end{align*}\]
Chapter 4 One-dimensional Mathematical Model and Simulation

Therefore

\[ (U_d)_{n+1} = (U_d)_n + 0.05\Delta x(T_x)_m - 3.89\Delta t \]
\[ (T_d)_{n+1} = (1 - 9.42\Delta x)(T_d)_n + 9.42\Delta x(T_x)_m \]

(4.11)

(4.12)

The deduction of the equations 4.1 to 4.12 is shown in Appendix II.

4.2 Computer Simulation of 1-D Model

A computer simulation based on the one-dimensional mathematical model was formulated to quickly assess the feasibility of the new portable cooler. Fortran programming language was used to formulate the computer model.

4.2.1 Modeling Stage Description

Following the operational procedures of the cooler box, the simulation process is divided into two stages. These stages are described as follows:

**Stage 1: PCM Solidification**

At time zero, the liquid nitrogen is input into the cooler box through the central coiling tube at a constant mass flow rate of 2 kg/h until all the PCM is solidified. The initial temperature of the model is set at 255 K (-18 °C). In the experiment, this low temperature of 255 K (-18 °C) is obtained by spraying liquid nitrogen directly into the cooler box through pre-cooling tubes. The temperature of the exterior cooler box wall is set constant at 300 K (27 °C), which is the ambient temperature under which the portable cooler operates.

**Stage 2: PCM Melting**

In this stage, the liquid nitrogen supply is stopped and the cooler box is left to experience a temperature rising period for the next 24 hours (1 day).

The ForTran programming flowchart showing the 1-D model is given in Appendix III.
4.2.2 Initial and Boundary Conditions

In stage 1, which is dealing with the solidification of the PCM, the initial temperatures for both the air in the cooling space, $T_b(1)$ and PCM, $T_x(1)$ are set at 255 K. All the PCM is taken as at the liquid phase initially, hence $U_x(1) = 1$. The initial temperature of liquid nitrogen $T_d(1)$ is 77 K and at the coiling tube inlet, the nitrogen is assumed to entirely exist in liquid phase, therefore $U_d(1) = 0$.

In stage 2, which is dealing with the melting of the PCM, the initial temperature of the air in the cooling space, $T_b(1)$ is taken as 255 K. The initial temperature of the PCM, $T_x(1)$ is set at 253 K, which is lower than its solidification point by 1 K. At time zero, all the PCM in stage 2 is assumed to be solid, hence $U_x(1) = 0$.

**Stage 1: PCM Solidification**

The Initial and Boundary Conditions are,

\[ T_k = 300\text{K}, \quad T_b(1) = 255\text{K}, \quad T_x(1) = 255\text{K}, \quad U_x(1) = 1, \quad T_d(1) = 77\text{K}, \quad U_d(1) = 0, \quad W_d = 2\text{kg/h}. \]

**Stage 2: PCM Melting**

The Initial and Boundary Conditions for stage 2 are,

\[ T_k = 300\text{K}, \quad T_b(1) = 255\text{K}, \quad T_x(1) = 253\text{K}, \quad U_x(1) = 0, \quad W_d = 0. \]

4.2.3 Results and Discussion

In this section, results for stage 1 simulation will first be shown and discussed and it then follows by those of stage 2.

**Results of Stage 1: PCM Solidification**

Simulation results of $T_d, U_d, T_x, U_x, T_b$ for stage 1, as modeled by schematic shown in Fig. 4.9 and summarized in Table 4.1.

$T_d$ — temperature of liquid nitrogen, K

$U_d$ — ratio of the weight of vapor nitrogen to the total weight of nitrogen

$T_x$ — temperature of the PCM, K

$U_x$ — ratio of the weight of liquid PCM to the total weight of PCM
$T_b$ — temperature of the air in the cooling space, K

Figure 4.9 One-dimensional Simulation Results Illustration
### Table 4.1 One-dimensional Simulation Results in Stage 1

#### t = 50 minutes

<table>
<thead>
<tr>
<th>X (m)</th>
<th>$T_d$ (°C)</th>
<th>$U_d$ (%)</th>
<th>$U_s$ (%)</th>
<th>$T_x$ (°C)</th>
<th>$T_b$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-196</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-196</td>
<td>0.139</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>-196</td>
<td>0.278</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-196</td>
<td>0.417</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>-196</td>
<td>0.556</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>-196</td>
<td>0.695</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>-196</td>
<td>0.834</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>-196</td>
<td>0.973</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>-172.1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>-146.8</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>-125.8</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>170</td>
<td>-108.2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>-93.6</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-81.3</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>210</td>
<td>-71.2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>230</td>
<td>-62.7</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>-55.6</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>-49.7</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>-44.7</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>290</td>
<td>-40.6</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>-37.2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### t = 100 minutes

<table>
<thead>
<tr>
<th>X (m)</th>
<th>$T_d$ (°C)</th>
<th>$U_d$ (%)</th>
<th>$U_s$ (%)</th>
<th>$T_x$ (°C)</th>
<th>$T_b$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-196</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-196</td>
<td>0.124</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>-196</td>
<td>0.248</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-196</td>
<td>0.372</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>-196</td>
<td>0.496</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>-196</td>
<td>0.620</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>-196</td>
<td>0.744</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>-196</td>
<td>0.868</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>-196</td>
<td>0.992</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>-171.4</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>-149.4</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>170</td>
<td>-131.0</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>-115.7</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-103.0</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>210</td>
<td>-92.3</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>230</td>
<td>-83.5</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>-76.1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>-69.9</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>-64.7</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>290</td>
<td>-60.5</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>-56.9</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 0.546 | -20        | -16.8     |

| 0.057 | -20        | -18       |

*Table 4.1 One-dimensional Simulation Results in Stage 1*
From the results shown in Table 4.1, the following observations may be made,

(i) An acceptable short PCM solidification time

In the practical use of the cooler box, the objects to be cooled will be added into the box only after all the PCM have been solidified. The function of PCM solidification is similar to that of recharging the batteries. Therefore, the solidification time must be minimized for the convenience of the user.

From Table 4.1, it can be seen that nearly all the PCM (1 - 0.057 % = 99.943 %) has been solidified after 100 minutes. The solidification time of 100 minutes is an acceptable one in practice even though the shorter the better.

(ii) A low temperature of -18 °C can be obtained in the cooling space

It can be seen from Table 4.1 that the cooling space can be cooled down to -18 °C when all the PCM is solidified.

**Results of Stage 2: PCM Melting**

The design of the new cooler box is feasible based on the analysis of the results shown in Figures 4.6 and 4.7. Fig. 4.7 indicates that the low temperature of -18 °C can be maintained in the cooling space for 17 hours or more during the PCM melting period (Fig. 4.6). This period is long enough for a short journey usage such as one-day excursion.

![Figure 4.6 One-dimensional Simulation Result of Ux in Stage 2](image)
4.3 Limitations of One-dimensional Simulation

The following limitations of the one-dimensional analysis call for the establishment of three-dimensional simulation:

- The variations of temperatures of $T_b$ and $T_x$ at different locations across the space cannot be known so far.

- Both $U_x$ and $U_d$ are obtained as the ratios (%) from the one-dimensional mathematical model. From these data, one cannot predict the exact locations of the liquid-solid fronts. Also, $U_x$ and $U_d$ are only average data. In reality, $U_x$ and $U_d$ should vary.

- For heat transfer analysis of a cooler box with a heat source inside, a three-dimensional model must be used.

Therefore, a three-dimensional mathematical model will be used to carry out heat performance and parametric studies study in the following part of this project.
Chapter 5
Numerical Simulation and Validation

With the advent of the computer technology, the computer simulation has been used extensively in research and development studies. Today, the computer simulation is an indispensable tool in many areas of research and development work. In this project, simulation studies have been carried out to study the natural convection flow and heat transfer in a portable cooler with considerable complex geometrical configuration. The interaction between the three different media in the cooler and possibly with different phases: air, liquid and solid, are studied. The mass, momentum and energy equations were solved using a commercially available computational code. The heat transfer among the three different flow fields, which are liquid nitrogen, phase change material (PCM) and air within the cooler box, was studied. The study includes the effects of vaporization, solidification/melting and natural convection.

In this study, a commercially available computational fluid dynamics code Fluent was employed. The computation was carried out on an Intel Pentium IV personal computer with a 1.80GHz CPU and 1GB of RAM.

5.1 Simulation Model

As the flow field is always under pressure which is the atmospheric pressure, the three-dimensional incompressible flow equations can be used to model the flow fields in the portable cooler. In this section, the details of setting up of the computation will be discussed and presented.

5.1.1 Governing Differential Equations

The numerical solution of heat transfer, fluid flow, and other related processes begin with the setting up of the computational model that follows the laws governing these processes. The following sections present the setting up of the mathematical model for all these processes.
i. Conservation of a Chemical Species

Consider Fig. 5.1, let \( m_l \) denotes the mass fraction of a chemical species. In the presence of a velocity field \( u \), the conservation of \( m_l \) is expressed as

\[
\frac{\partial}{\partial t}(\rho m_l) + \text{div}(\rho u m_l + J_l) = R_l
\]  

(5.1)

Here \( \frac{\partial}{\partial t}(\rho m_l) \) denotes the rate of change of the mass of the chemical species per unit volume. The quantity \( \rho u m_l \) is the convection flux of the species, i.e., the flux carried by the general flow field \( \rho u \). The symbol \( J_l \) stands for the diffusion flux, which is normally caused by the gradients of \( m_l \). The divergence of the two fluxes (convection and suffusion) forms the second term of the differential equation. The quantity \( R_l \) on the right-hand side is the rate of generation of the chemical species per unit volume. The generation is caused by chemical reaction. Of course, \( R_l \) can have a positive or negative value depending on whether the reaction actually produces or destroys the chemical species, and \( R_l \) is zero for a nonreacting species.

If the diffusion flux \( J_l \) is expressed by the use of Fick’s law of diffusion, we can write

\[
J_l = -\Gamma_l \text{grad}m_l
\]  

(5.2)

where \( \Gamma_l \) is the diffusion coefficient. The substitution of Eq. (5.3) into (5.2) leads to

\[
\frac{\partial}{\partial t}(\rho m_l) + \text{div}(\rho u m_l) = \text{div}(\Gamma_l \text{grad}m_l) + R_l
\]  

(5.3)
Chapter 5 Numerical Simulation and Validation

ii. The Energy Equation

For a steady low-velocity flow with negligible viscous dissipation, the energy equation can be written as

\[ \text{div}(\rho u h) = \text{div}(k \text{grad}T) + S_h \]  \hspace{1cm} (5.4)

where \( h \) is the specific enthalpy, \( k \) is the thermal conductivity, \( T \) is the temperature, and \( S_h \) is the volumetric rate of heat generation. The term \( \text{div}(k \text{grad}T) \) represents the influence of conduction heat transfer within the fluid, according to the Fourier law of conduction.

For ideal gases, solids and liquids, we can write

\[ c \text{grad}T = \text{grad}h, \]  \hspace{1cm} (5.5)

where \( c \) is the constant-pressure specific heat. With this substitution, the energy equation becomes

\[ \text{div}(\rho u h) = \text{div}(\frac{k}{c} \text{grad}h) + S_h \]  \hspace{1cm} (5.6)

If \( c \) is constant, the \( h \sim T \) relation simplifies to

\[ h = cT \]  \hspace{1cm} (5.7)

which would lead to

\[ \text{div}(\rho u T) = \text{div}(\frac{k}{c} \text{grad}T) + \frac{S_h}{c} \]  \hspace{1cm} (5.8)

In this manner, either the enthalpy or the temperature can be chosen as the dependent variable.

The steady heat-conduction situation is obtained by setting the velocity \( u \) to zero; thus,

\[ \text{div}(k \text{grad}T) + S_h = 0 \]  \hspace{1cm} (5.9)

iii. A Momentum Equation

The differential equation governing the conservation of momentum in a given direction for a Newtonian fluid can be written along similar lines; however, the complication is greater because both shear and normal stresses must be considered and because the Stokes viscosity law is more complicated than Fick’s law or Fourier’s law.

With \( u \) denoting the x-direction velocity, we write the corresponding momentum equation as
\[
\frac{\partial}{\partial t} (\rho u) + \text{div}(\rho u \mu) = \text{div}(\mu \text{grad} u) - \frac{\partial p}{\partial x} + B_x + V_x \tag{5.10}
\]

where \( \mu \) is the viscosity, \( p \) is the pressure, \( B_x \) is the x-direction body force per unit volume, and \( V_x \) stands for the viscous terms that are in addition to those expressed by \( \text{div}(\mu \text{grad} u) \).

iv. The Turbulence-Kinetic-Energy Equation

The equation for the kinetic energy \( k \) of the fluctuating motion, which reads
\[
\frac{\partial}{\partial t} (\rho k) + \text{div}(\rho uk) = \text{div}(\Gamma_k \text{grad} k) + G - \rho \varepsilon \tag{5.11}
\]

where \( \Gamma_k \) is the diffusion coefficient for \( k \), \( G \) is the rate of generation of turbulent energy, and \( \varepsilon \) is the kinematic rate of dissipation. The quantity \( G - \rho \varepsilon \) is the net source term in the equation. A similar differential equation governs the variable \( \varepsilon \).

v. The General Differential Equation

This brief journey through some of the relevant differential equations has indicated that all the dependent variables of interest here seem to obey a generalized conservation principle. If the dependent variable is denoted by \( \phi \), the general differential equation is
\[
\frac{\partial}{\partial t} (\rho \phi) + \text{div}(\rho u \phi) = \text{div}(\Gamma \text{grad} \phi) + S \tag{5.12}
\]

where \( \Gamma \) is the diffusion coefficient, and \( S \) is the source term. The quantities \( \Gamma \) and \( S \) are specific to a particular meaning of \( \phi \).

The four terms in the general differential equation are the unsteady term, the convection term, the diffusion term, and the source term respectively. The dependent variable \( \phi \) can stand for a variety of different quantities, such as the mass fraction of a chemical species, the enthalpy or the temperature, a velocity component, the turbulent kinetic energy, or a turbulence length scale. Accordingly, for each of these variables, an appropriate meaning will have to be given to the diffusion coefficient \( \Gamma \) and the source term \( S \).
Even in the construction of a computer program, it is sufficient to write a general sequence of instructions for solving Eq. (5.12), which can be repeatedly used for different meanings of \( \phi \) along with appropriate expression for \( \Gamma \) and \( S \), and, of course, with appropriate initial and boundary conditions. Thus, the concept of the general \( \phi \) equation enables us to formulate a general numerical method and to prepare general-purpose computer program.

The following sections of 5.1.2, 5.1.3 and 5.1.4 will present the detailed equations and considerations used by FLUENT that are coherent to the widely accepted governing differential equations discussed above.

### 5.1.2 Modeling Basic Fluid Flow

For the present study, the velocity and the temperature fields are important and hence the conservation of mass, conservation of momentum and energy must all be fulfilled. Moreover these have to be specified for the media that involved: liquid nitrogen, phase change material (PCM) and air.

**i. The Mass Conservation Equation**

The equation for conservation of mass, or continuity equation, can be written as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = S_m
\]

Eq. (5.13) is the general form of the mass conservation equation and is valid for incompressible as well as compressible flows. The source \( S_m \) is the source term, which has to be considered in the case of evaporation of liquid nitrogen. This can be incorporated using the user-defined function.

**ii. Momentum Conservation Equations**

Conservation of momentum in an inertial (non-accelerating) reference frame is described by

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot \mathbf{\tau} + \rho \mathbf{g} + \mathbf{F}
\]
where $p$ is the static pressure, $\tau$ is the stress tensor (described below), and $\rho \ddot{g}$ and $\vec{F}$ are the gravitational body force and external body forces (e.g., that arise from interaction with the dispersed phase), respectively. $\vec{F}$ also contains other model-dependent source terms such as porous-media and user-defined sources.

The stress tensor $\tau$ is given by

$$\tau = \mu(\nabla \vec{v} + \nabla \vec{v}^T) - \frac{2}{3} \nabla \cdot \vec{v} \, I$$

(5.15)

where $\mu$ is the molecular viscosity, $I$ is the unit tensor, and the second term on the right hand side is the effect of volume dilation.

### 5.1.3 Modeling Solidification/Melting

In the portable cooler, the PCM would undergo solidification/melting phase change.

An enthalpy-porosity technique is used in FLUENT for modeling the solidification/melting process. In this technique, the melt interface is not tracked explicitly. Instead, a quantity called the liquid fraction, which indicates the fraction of the cell volume that is in liquid form, is associated with each cell in the domain. The liquid fraction is computed at each iteration based on an enthalpy balance.

The mushy zone is a region in which the liquid fraction lies between 0 and 1. The mushy zone is modeled as a “pseudo” porous medium in which the porosity decreases from 1 to 0 as the material solidifies. When the material has fully solidified in a cell, the porosity becomes zero and hence the velocities also drop to zero.

**i. Energy consideration for melting and solidification**

The enthalpy of the material is computed as the sum of the sensible enthalpy, $h$, and the latent heat, $\Delta H$:

$$H = h + \Delta H$$

(5.16)

where

$$h = h_{ref} + \int_{T_{ref}}^{T} C_p dT$$

(5.17)

and $h_{ref}$ = reference enthalpy
Chapter 5 Numerical Simulation and Validation

\[ T_{\text{ref}} = \text{reference temperature} \]
\[ C_p = \text{specific heat at constant pressure} \]

The liquid fraction, \( \beta \), can be defined as

\[
\beta = \begin{cases} 
0 & \text{if } T < T_{\text{solidus}} \\
1 & \text{if } T > T_{\text{liquidus}} \\
\frac{T - T_{\text{solidus}}}{T_{\text{liquidus}} - T_{\text{solidus}}} & \text{if } T_{\text{solidus}} < T < T_{\text{liquidus}}
\end{cases}
\]  

(5.18)

Eq. (5.18) is referred to as the lever rule. Other relationships between the liquid fraction and temperature (and species concentrations) are possible, but are not considered here.

The latent heat content can now be written in terms of the latent heat of the material, \( L \):

\[ \Delta H = \beta L \]  

(5.19)

The latent heat content can vary between zero (for a solid) and \( L \) (for a liquid).

For solidification/melting problems, the energy equation is written as

\[
\frac{\partial}{\partial t}(\rho H) + \nabla \cdot (\rho \bar{v} H) = \nabla \cdot (k \nabla T) + S
\]  

(5.20)

where \( H = \text{enthalpy (see Eq. 5.16)} \)
\( \rho = \text{density} \)
\( \bar{v} = \text{fluid velocity} \)
\( S = \text{source term} \)

The solution for temperature is essentially an iteration between the energy equation (Eq.5.20) and the liquid fraction equation (Eq. 5.18). Directly using equation 6.18 to update the liquid fraction usually results in poor convergence of the energy equation. The method suggested by Voller and Swaminathan [136] is used to update the liquid fraction. For pure metals, where \( T_{\text{solidus}} \) and \( T_{\text{liquidus}} \) are equal, a method based on specific heat is used instead.

5.1.4 Modeling Heat Transfer

All the walls of the cooler including nitrogen coils wall, PCM container wall and thermal insulation box wall, aluminum block used as heat source and the 50 copper fins are defined as the solid regions. The air region within the cooler box is considered as natural convection flow.

i. Energy Equation in Solid Regions
In the solid regions, the energy transport equation has the following form:

\[
\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\rho v h) = \nabla \cdot (k \nabla T) + S_h
\]  

(5.21)

where

- \(\rho\) = density
- \(h\) = sensible enthalpy, \(\int_{T_{ref}}^{T} c_p dT\)
- \(k\) = conductivity
- \(T\) = temperature
- \(S_h\) = volumetric heat source

The second term on the left-hand side of Eq. (5.21) represents convective energy transfer. The terms on the right-hand side of Eq. (5.21) are the heat flux due to conduction and volumetric heat sources within the solid, respectively.

ii. Natural convection Flows

In pure natural convection, the strength of the buoyancy-induced flow is measured by the Rayleigh number:

\[
Ra = \frac{g \beta \Delta T L^4 \rho}{\mu \alpha}
\]  

(5.22)

where \(\beta\) is the thermal expansion coefficient:

\[
\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_{\rho}
\]  

(5.23)

and \(\alpha\) is the thermal diffusivity:

\[
\alpha = \frac{k}{\rho c_p}
\]  

(5.24)

iii. The Boussinesq Model

For many natural-convection flows, faster convergence can be obtained with the Boussinesq model than by setting up the problem with fluid density as a function of temperature. This model treats density as a constant value in all solved equations, except for the buoyancy term in the momentum equation:

\[
(\rho - \rho_0) g \approx -\rho_0 \beta (T - T_0) g
\]  

(5.25)
where $\rho_0$ is the (constant) density of the flow, $T_0$ is the operating temperature, and $\beta$ is the thermal expansion coefficient. Eq. (5.25) is obtained by using the Boussinesq approximation $\rho = \rho_0(1 - \beta \Delta T)$ to eliminate $\rho$ from the buoyancy term. This approximation is accurate as long as changes in actual density are small; specifically, the Boussinesq approximation is valid when $\beta(T - T_0) << 1$.

### 5.1.5 Modeling Vaporization of Liquid Nitrogen

There is no ready model for the simulation of the vaporization of liquid nitrogen in the coils combining the solidification/melting of PCM and free convection of air in the cooling space. However, the use of user-defined functions (UDFs) can customize the FLUENT code to fit this particular modeling need.

Liquid nitrogen is considered as a heat sink here. The heat $Q$ that is needed to evaporate 1-kg liquid nitrogen can be calculated as

$$Q = Q_{\text{latent}} + Q_{\text{sensible}}$$
$$= H + C_p(T_1 - T_2)$$

where $H$ is the latent heat of liquid nitrogen and $C_p$ is the specific heat of vapor nitrogen. $T_1$ is the temperature of vapor nitrogen at the outlet of the cooling tube and $T_2$ is the vaporization point of liquid nitrogen.

The more liquid nitrogen introduced into the cooler box, the more heat needed to vaporize it. The relation can be shown as follows:

$$\frac{m_{\text{introduced}}}{1\text{kg}} = \frac{Q_{\text{needed}}}{Q}$$

where $m_{\text{introduced}}$ is the mass of liquid nitrogen enters the cooler box and $Q_{\text{needed}}$ is the heat needed to vaporize them.

In FLUENT, both energy source term and mass source term are defined by UDF. That is when a certain amount of liquid nitrogen ($m_{\text{introduced}}$) enters the model, the heat of $Q_{\text{needed}}$ will be extracted from the sounding PCM, and then the mass of this amount of nitrogen ($m_{\text{introduced}}$) will vanish from the model.

The UDF coding is shown in Appendix IV.
5.1.6 Boundary Conditions

The boundary conditions imposed in the simulation model are presented below. These are modeled after the experimental model. As shown in Fig. 5.2, these conditions are described as:

(a) Inlet: Constant mass flow rate of liquid nitrogen normal to the inlet ‘a’.
(b) Outlet: Out flow of vapor nitrogen normal to the outlet ‘b’.
(c) Solid Wall: Constant room temperature at the external sides of the thermal insulation box wall ‘c’.

Figure 5.2 Schematics of Model for Boundary Conditions

The time-average physical properties for solid and fluids for various media are shown in Tables 5.1 and 5.2. The structure of the whole model is complex, containing liquid nitrogen, PCM, thermal conductivity enhancer, heat source, and air. Detailed modeling of this complex model is computationally challenging. Therefore, approximating the model thermo-physical properties is essential for modeling and performing parametric studies of the cooler box. In this research, average properties based on the model component’s volume are used in the numerical model. This approximate method is proposed by Chow [137], who examined the validity of modeling the thermal control unit (TCU) with average material properties by comparing its results with a TCU with multimaterial composition. A strong agreement was found between the two cases.
Table 5.1: Fluid average properties

<table>
<thead>
<tr>
<th></th>
<th>PCM</th>
<th>Liquid Nitrogen</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>1300</td>
<td>808</td>
<td>1.225</td>
</tr>
<tr>
<td>C_p (J/kg-K) Liquid:</td>
<td>3765.69 / Solid: 1882.85</td>
<td>149.5</td>
<td>1006.43</td>
</tr>
<tr>
<td>Thermal Conductivity (W/m-K) Solid:</td>
<td>1.74 / Liquid: 2.32</td>
<td>0.14535</td>
<td>0.0242</td>
</tr>
<tr>
<td>Viscosity (kg/m-s)</td>
<td>0.00553</td>
<td>0.00019</td>
<td>1.7894e-05</td>
</tr>
<tr>
<td>Melting Heat (J/kg)</td>
<td>300000</td>
<td>198000</td>
<td></td>
</tr>
<tr>
<td>Solidus Temperature (K)</td>
<td>253</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>Liquidus Temperature (K)</td>
<td>253</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Solid average properties

<table>
<thead>
<tr>
<th></th>
<th>Insulation Wall</th>
<th>Aluminum</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>40</td>
<td>2719</td>
<td>8978</td>
</tr>
<tr>
<td>C_p (J/kg-K)</td>
<td>2310</td>
<td>871</td>
<td>381</td>
</tr>
<tr>
<td>Thermal Conductivity (W/m-K)</td>
<td>0.02</td>
<td>202.4</td>
<td>387.6</td>
</tr>
</tbody>
</table>

5.2 Numerical Simulation

Fig. 5.3 shows the three dimensional model of the portable cooler and it also represents the solution domain in the present study. The objective of performing this study is to investigate the performance of the cooler. In the numerical simulations, different zones of PCM during solidification/melting including liquid, solid and mushy zones can be easily examined. The airflow by natural convection in the enclosure can also be clearly visualized whereas it is difficult or expensive to perform the physical experiment. In addition, changes in the design parameters and their effects on the performance of the cooler can also be carried out with relative ease in the simulation.
Moreover, the numerical solution enables us to examine the flow and temperature distribution and to locate the dead zone or hot spots.

![Figure 5.3 Three-dimensional Simulation Model](image)

Second-order Upwind scheme had been used to discretise the equations using the finite-volume method. SIMPLE algorithm has been adopted to solve the equations. Since the heat transfer and phase change taking place inside the cooler box are inherently transient, it is reasonable to simulate them directly through time-dependent model.

### 5.3 Grid Dependency

The simulation model consists of four domains, which are liquid nitrogen, PCM, aluminum block and air. The meshes of those three fluids are shown in Fig. 5.4 to Fig. 5.7.

![Figure 5.4 Meshing of the Liquid Nitrogen flow region](image)
The PCM region were modeled by dividing it into two parts as 1 and 2 as shown in Fig. 5.5. Part 1 is the portion of PCM that is near the liquid nitrogen domain shown in Fig. 5.4. Due to the complex shape and the narrow space of the PCM in Part 1, it is meshed more finely than Part 2.
Because of the complicated shape of the air space in the cooler box, the air region was meshed separately in two parts as shown in Fig. 5.7. Part 1 is the air space within the space of the fins and Part 2 is the air space outside the fin region in the box.

To ensure that the results are mesh independent, the results of models with 105533, 158300, 211066, 263832 and 316599 elements are compared in terms of temperature at specific locations at the steady state condition.

Table 5.3 below shows the results from the models with different mesh sizes.
Chapter 5 Numerical Simulation and Validation

<table>
<thead>
<tr>
<th>Discrepancy</th>
<th>Model Number</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Applicable</td>
<td>1</td>
<td>105,533</td>
</tr>
<tr>
<td>Discrepancy Between Model 1 and Model 2 (%)</td>
<td>3.7</td>
<td>2</td>
</tr>
<tr>
<td>Discrepancy Between Model 2 and Model 3 (%)</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>Discrepancy Between Model 3 and Model 4 (%)</td>
<td>0.41</td>
<td>4</td>
</tr>
<tr>
<td>Discrepancy Between Model 4 and Model 5 (%)</td>
<td>0.39</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.3 Simulation results of the models with different mesh size

The results shows that the discrepancies obtained from models having 211066, 263832 and 316599 elements were less than 0.5%, so the model with 211,066 elements is selected for performing the remaining simulation as it gives the best compromise between the computational time required and the simulation accuracy.

5.4 Simulating the Experimental Conditions

5.4.1 Modeling Stage Description

Following the experimental procedure, the simulation process is divided into two time stages for comparison and verification of the simulation model. These stages are described as follows:

1. At time equals zero, liquid nitrogen is directly supplied into the nitrogen coil at the rate of 2 kg/h until all the PCM is solidified. The initial temperature of the model is taken as 255 K (-18 °C). During the experiment, this low temperature is obtained by spraying liquid nitrogen into the box directly using the pre-cooling tube for a quick initial cooling to achieve 255 K. The exterior wall temperature of the cooler box wall is taken as constant at 300 K (27 °C), that is the room temperature under which the cooler operates. No heat source is added at this stage.

2. At the beginning of the second stage, a 300 K aluminum cube of 50 mm x 50 mm x 50 mm is introduced into the cooling space of the cooler box. It is assumed that the initial temperature of the air in the cooler box is 255 K and the temperature of
PCM is 252 K uniformly. The solidification temperature of the PCM materials is 253 K, therefore, the initial temperature of PCM is set as 252 K to assure a complete solidification occurs. In one-dimensional simulation and in the experiment, the PCM took more than 16 hours to completely liquify. The temperature of the entire cooler drops quickly at first, then it decreases slowly and it keeps almost constant at every location there after. Due to the time step of less than 0.1 second in FLUENT calculation, the entire PCM melting process of more than 16 hours in experiment will demand a great effort and will take almost indefinitely to complete. Therefore, in the simulation study, the stage 2 is simulated only for the first hour. Since the main purpose of simulation in stage 2 is to study the design parameters of the cooler box to enhance the cooling rate of heat source, the 1-hour simulation instead of more than 16 hours’ simulation is thus acceptable.

5.4.2 Initial Modeling Objectives

The simulation study will be verified against the measured results to ensure that a proper simulation study has been carried out. After which simulation studies can be used to further study and design the cooler box. Thus, the initial modeling objectives are:

a) To compare the measured and the predicted results of temperature distributions in the cooler box for validation of the simulation model. The close agreement or otherwise between the experimental and numerical results will demonstrate the validity or otherwise of the model.

b) To visualize and examine the solidification/melting of PCM and natural convection of the air space in the box. The phase change of PCM and the airflow in the box cannot be visualized in experiment, and also cannot be accurately obtained by one-dimensional simulation model.

Two typical symmetric surfaces of Y = 0 and Z = 150 mm are selected for examination. The symmetric surface of Y = 0 is shown in Fig. 5.8 (a). It is the sectional horizontal-view of the cooler box. The surface Y = 0 is selected for comparison because it divides the whole model into two identical domains: front and back. It also cuts across most significant portions of the cooler box and these are the
heat source, the air space around the heat source, the air space between the two adjacent fins, the PCM and the liquid nitrogen region. The surface \( Z = 150 \text{ mm} \) in Fig. 5.8 (b) is the sectional vertical-view of the cooler box. It divides the whole model into two identical regions: the upper and lower parts. It also cuts across the heat source, the air around the heat source, 50 fins, the PCM and the liquid nitrogen region.

(a) Symmetric Surface of \( Y = 0 \)  
(b) Symmetric Surface of \( Z = 150 \text{ mm} \)

Figure 5.8 Symmetric surfaces of \( Y = 0 \) and \( Z = 150 \text{ mm} \)

5.5 Simulation Results

In this section, some important FLUENT simulation results at surface of \( Y = 0 \) will be shown in pictures and explained.

5.5.1 Stage 1

The results obtained in stage 1 simulation as described in 5.4.1 will be presented here.

5.5.1.1 PCM Solidification

Results from FLUENT simulation shows that all the PCM could be solidified in 115 minutes. Fig. 5.9 illustrate the phase change and temperature distribution of PCM at \( t = 10 \text{ minutes} \), \( t = 50 \text{ minutes} \) and \( t = 115 \text{ minutes} \).
Chapter 5 Numerical Simulation and Validation

i. Phase Change

ii. Temperature distribution

(a) \( t = 10 \text{ minutes} \)
Chapter 5 Numerical Simulation and Validation

i. Phase Change

ii. Temperature distribution

(b) \( t = 50 \) minutes
Chapter 5 Numerical Simulation and Validation

i. Phase change

ii. Temperature distribution

(c) t = 115 minutes

Figure 5.9 PCM solidification in stage 1
From Fig. 5.9, it can be seen that the PCM in the vicinity of the coiling tube in which liquid nitrogen passes through solidified first, then the part of PCM adjacent to the PCM container wall. Since there is no thermal conductivity enhancer in the PCM in this simulation, the PCM far away from the coiling tube can be fully solidified after 115 minutes due to the relatively low thermal conductivity of the PCM. When all the PCM is solidified, the temperature distribution of PCM is not uniform. The lowest temperature part is around the coiling tube at 77.3 K, which is approximately equal to the vaporization point of the liquid nitrogen of 77 K. The highest temperature of the PCM is at locations close to the container wall, 252 K, which is slightly lower than the solidification point of the PCM at 253 K.

5.5.1.2 Liquid Nitrogen Vaporization

Fig. 5.10 presents the temperature distribution of liquid nitrogen at the coiling tube wall and the vent of the tube at \( t = 0.5 \) minute and \( t = 3 \) minutes.
Chapter 5 Numerical Simulation and Validation

i. Temperature distribution of liquid nitrogen at the coiling tube wall

ii. Temperature distribution of liquid nitrogen at the vent of the tube

(a) $t = 0.5$ minute

i. Temperature distribution of liquid nitrogen at the coiling tube wall
ii. Temperature distribution of liquid nitrogen at the vent of the tube

(b) $t = 3$ minutes

Figure 5.10 Liquid nitrogen vaporization in stage 1

It is noted that the vent of the tube is meshed in a round shape. However, after being enlarged by about 10 times, the shape is somewhat distorted. From the results of temperature distribution of the liquid nitrogen at the coiling tube wall, it can be seen that the temperature increases gradually from the tube inlet to the outlet. It is because the liquid nitrogen absorbs heat from the surrounding PCM as it flows along the coiling tube. The results of temperature distribution of the liquid nitrogen at the vent of the tube show that the liquid nitrogen in the vicinity of the tube wall extracts more heat from the PCM and hence it is always at a higher temperature than the PCM at the center of the tube. And with time, the lowest temperature of the nitrogen leaves the tube vent becomes lower and lower, from 115 K at $t = 0.5$ minute to 82.5 K at $t = 3$ minutes. After 3 minutes, this temperature change with time gets much slower, and after about 7 minutes, nitrogen leaves the tube vent at 77 K and the nitrogen at the tube wall is always 77 K. Since the vaporization point of liquid nitrogen is 77 K, the temperature of nitrogen at the tube vent shows that all the liquid nitrogen experienced liquid-vapor phase change and leaves the tube in vapor phase in the first 3 minutes; then only part of liquid nitrogen experience the phase change and it leaves the tube vent in the mixture of liquid and vapor.

5.5.1.3 Air Space Temperature Distribution
Fig. 5.11 shows the temperature distribution of the air space and PCM at surface $Y = 0$ at $t = 10$ minutes.

i. Temperature distribution of the air space and PCM

Fig. 5.11 shows that during the PCM solidification, the heat transfer from the air space to the PCM is slower than that from the outside to the air space so that the temperature of air space continually increases. The initial temperature of air space is 255 K, after 10 minutes, the temperature increases to 257 K. When all PCM is solidified, the temperature of air space increases to 268 K. Since the temperature at the end of stage 1 is the initial temperature of stage 2 for heat source cools down, 268 K is too high and a design is required. The enlarged part shows that at the end of stage 1, only the part of air nearby the PCM is cooled down to a low temperature.

ii. Temperature distribution of the air space

Figure 5.11 Temperature distribution of the air space ($t = 10$ minutes)

5.5.2 Stage 2

The results obtained in stage 2 simulation as described in 5.4.1 will be presented here.
Chapter 5 Numerical Simulation and Validation

(a) Temperature distribution of air space and heat source

(b) Velocity of air flow by natural convection in the cooler box
Fig. 5.12 (a) shows that although the air space at the vicinity of the PCM container is cooled down from 255 K initially to 253 K after 1 hour, the heat source with initial temperature of 300 K could hardly be cooled down. To find the reason for this, the velocity of the airflow in the cooler box is also shown. From Fig. 5.12 (b) and (c), it is seen that the velocity of airflow in the narrow space among 50 fins is nearly zero. It is obvious that the narrow space restrains the movement of the air by natural convection, thus the air trapped among the fins becomes heat transfer barrier between the heat source and the PCM. In spite of this negative role of the fins, the existence of fins can enhance heat transfer by enlarging the surface area of PCM container wall. Therefore, parametric study will be needed to find out the optimal fin number and fin length.

5.6 Validation of the simulation model

In this section, the prediction from the simulation model will be compared with measured results. Two stages as described in 5.4.1 of comparison will be made. The first stage is to compare the temperature variation during the charging process. In this process the liquid nitrogen flows through the feeding coil with the intention to solidify the PCM material and hence to provide the cooler box with the ability to absorb heat and maintain the inside temperature of the air space. The second stage is to compare the temperature during the melting of the PCM material. During the second case, there is no feeding of liquid nitrogen.
5.6.1 Stage 1

In this stage, a flow rate of 2 kg/h of liquid nitrogen is allowed to flow into the cooling coil with the intention to solidify the PCM material. The nitrogen liquid/vapor exits through the vent hole at the end of the coil. The temperature measurement is carried out at three locations for comparison purposes, as shown in Fig. 5.13. These three locations are labeled as follow: location 1 is at the interior wall of the cooler box, 2 is at the exterior wall of the PCM cylindrical container, and location 3 is at the vent of the coiling tube wall.

![Figure 5.13 Temperatures are measured at these three locations](image)

The results of the comparison for the temperature-time variation at these three locations 1, 2, and 3 are shown in Figs. 5.14, 5.15 and 5.16 respectively. Results were recorded 10 minutes after the commencement of the experiment. The results show that a little discrepancy (maximum 4.46 %) occurs for results at point 1 and 2 and good agreement was obtained for results at point 3. The results of point 1 and 2 are calculated from FLUENT. And the result of point 3 is obtained from UDF coding. Point 1 is located in the inner wall of the cooler box. It’s initial temperature is 255 K after pre-cooling. Since the outer wall of the cooler box is constantly 300 K, heat enters the cooler box from the exterior wall to the inner wall by conduction. Also, there is a distance between point 1 and the PCM, heat is abstracted by the PCM slower than the heat building up at point 1. Therefore, the temperature of point 1 increases with time as shown in Fig. 5.14. Point 2 and 3 are close to the PCM, so their temperatures decrease with time.
Figure 5.14 Temperature variation at point 1

Figure 5.15 Temperature variation at point 2

Figure 5.16 Temperature variation at point 3
5.6.2 Stage 2

During this process, there is no input of the liquid nitrogen into the system. The whole system is left to warm up by a heat source which is essentially an aluminum block which was initially at 300 K. The placement of the aluminum block is schematically shown in Fig. 5.17. Again in this case, the temperatures of three locations are measured. These are location 1; at the interior box wall, location 2; at the top surface of the aluminum block and location 3; at the exterior wall of the PCM container. In the experiment, the aluminum block is hanging in the middle of the cooler box by a thin string, to reduce the possible heat conduction through the string.

![Figure 5.17 Temperature measurement location for melting process](image)

The comparison between the predicted and measured temperatures with time is shown in Figs. 5.18, 5.19, and 5.20.

![Figure 5.18 Temperature variation at point 1](image)
From the comparison, the followings can be observed:

1. From Fig.5.14 and 5.18, it can be seen that the temperature at location 1, i.e. the temperature of the interior wall of the cooler box, is always slightly under predicted. This could be due to the difference in the value of the thermal conductivity of the box wall material used in the simulation and that in the experiment may not be exactly the same, which could be due to the manufacturing variations. A higher thermal conductivity value in practice could lead to slightly more heat being transferred into the cooler box by conduction than that in the simulation.

2. Location 2 in Fig.5.15 and location 3 in Fig.5.20 are both at the exterior wall of the PCM container. At this point, the simulation data are always lower than the experimental ones. That maybe due to the neglecting of thermal resistance of the
PCM container wall that is made of copper. The results in Fig.5.20 show that there appears to have no difference between the measured and the predicted results apart from the temperature rise at location 3, after 3000 seconds. The temperature rise in the experiment is caused by the completely melting of the PCM.

3. Fig 5.16 shows the temperature comparison at the vent hole at the end of the coil. At this point, the simulation data are almost the same as the experimental values. The results show that the model predicts well the vaporization of liquid nitrogen in the cooling coil tube. Due to the limited capacity of the melting/solidification model in FLUNET, the vaporization process is simulated by a UDF. The agreement of experimental and numerical data here proves that the feasibility of the UDF.

4. Fig.5.19 shows the temperature comparison at the top surface of the aluminum block that is used as a heat source. The results in Fig.5.19 show that the heat source with initial temperature of 300 K was almost not cooled down in as long as 1 hour. Therefore, parameter studies are very necessary to be carried to enhance the thermal performance of the current design.

The comparison between the measured data and prediction shows that the model can be used to predict the performance of the cooler with accuracy within 5 % of the temperature range. It was calculated that the maximum temperature error is 4.46 % and the average error is 1.27 %. The close agreement between the experimental and numerical results indicates that the three dimensional model is reasonably accurate and is suitable to be used to further study the performance of the cooler box.
Chapter 6 Parametric Studies

In this chapter, effects of design parameters on the performance of the cooler box will be studied. These parameters are the fins used in the air space, the thermal conductivity enhancer (TCE) in the PCM container, the shape of the PCM container and the necessity of pre-cooling process before the heat source is placed into the cooler box. Details on the heat transfer in the cooler as well as economic issues will also be looking into.

6.1 Effects of Design Parameters on the PCM Solidification

In this section, the effects of design parameters on the PCM solidification are studied. These design parameters are the fin number, thermal conductivity enhancer (TCE), PCM container shape and the necessity of pre-cooling process. Except the study of pre-cooling process in 6.1.4, initial temperature for PCM solidification is 255 K.

6.1.1 Effects of Fin Number

In this section the effects of fin number on the PCM solidification are studied. Simulation studies will be conducted for three cases with fins number 50, 4 and 0 as shown in Fig. 6.1. The studies will help to find out possibly the shortest PCM solidification time and the lowest temperature distribution in the air space when all the PCM is solidified.

(a) 50 Fins
The process of PCM to reach the solid state for three different fin numbers has been simulated by FLUENT and recorded. The PCM solidification time and the final average temperature of air space for cooler box equipped with different fin numbers are shown in Table 6.1. In the actual application of the cooler box, the time taken for the PCM to completely solidify is very crucial as it defines the so-called “recharging time” for the cooler box between the applications. The final average temperature at the end of this stage should be as low as the PCM solidification temperature and this temperature will then be used as the initial temperature for the stage 2 as described in 5.4.1, which then a heat source will be placed in the cooler box, to observe the performance of the cooler box.

<table>
<thead>
<tr>
<th>Fin Number</th>
<th>PCM Solidification Time</th>
<th>Final Average Temperature of Air Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>115 minutes</td>
<td>268 K</td>
</tr>
<tr>
<td>4</td>
<td>111 minutes</td>
<td>259 K</td>
</tr>
<tr>
<td>0</td>
<td>110 minutes</td>
<td>259 K</td>
</tr>
</tbody>
</table>

Table 6.1: Solidification time and average temperature for different fin numbers

From the table above, it is concluded that although their solidification times are almost the same, the cooler with 4 fins or without fin seems to show a slightly faster PCM solidification process and a lower average temperature in the air space when all the PCM is solidified.
Fig. 6.2 shows the heat transfer rate at the nitrogen tube wall during the cooling process. The negative heat flux indicates that heat is transferred from PCM to the nitrogen coil.

The result shows that the initial heat transfer rate was a high 411W and it drops to 96W about 100 minutes later.

![Heat Transfer Rate Between PCM and Nitrogen](image)

*Figure 6.2 Heat transfer rate between PCM and nitrogen*

This high initial heat transfer rate is due to not only the large initial temperature difference between the PCM, which is at 255 K and the liquid nitrogen, which is at 77 K, but also the evaporation cooling given by the evaporation of the liquid nitrogen. This high cooling rate will result in solidification of the PCM near the nitrogen tube wall in a very short time. Once the PCM material close to the wall solidifies, the temperature between the solidified PCM and the liquid nitrogen is significantly reduced, the heat transfer rate from PCM to nitrogen coil reduces, hence the reduction in the heat transfer rate with time. The table also shows that there is virtually no difference for cases with different fin number.

Fig. 6.3 shows the variation of net heat transfer rate with time for PCM. The net heat transfer is defined as the heat transfer into PCM through the PCM container wall minus the heat transfer away from PCM through the nitrogen tube wall.
Chapter 6 Parametric Studies

The minus sign in the above figure implies that heat is extracted from PCM. Since the curve in Fig. 6.3 is almost same as that in Fig. 6.2, it can be seen that the heat transfer away from the PCM through the nitrogen tube wall is much higher than that into the PCM through the PCM container wall, which results in the solidification of PCM in a short time. Also, from the figure above, it can be concluded that the heat transfer is independent of the number of the fin.

Figure 6.4 shows that the heat transfer rate of the fins in the cooler box for two cases: 50 fins and 4 fins. The minus sign indicates that the fins are absorbing heat from the air space in the cooler box and vice versa. It can be seen from the figure that for the
In the case of 50 fins, heat is transferred from the fins to the air, opposite to which they are expected to function. This reverse function of fins in the case of 50 fins can be explained as follows: in the simulation model, the top and the bottom portion of the fins are in close contact with the top cover and the base of the cooler. Because of this contact, the fins form the heat conduction path from the cover and the base into the air space through fins. The situation is exemplified by the fact that these fins are made from copper, a very good heat conductor.

![Net Heat Transfer of Air](image)

Figure 6.5 Net Heat transfer of air in the air space

Therefore, in Fig 6.5, air in the cooler box with 50 fins rejects much more heat into the air space than that in the cooler box with 4 fins or no fin, which causes the temperature of the air in the cooler box with 50 fins to be as high as 268 K when all the PCM is solidified. However, in the case for 4 fins or no fin, the temperature of air in the air space is only at 259 K.

### 6.1.2 Thermal Conductivity Enhancer Effect

One major issue that needs to be addressed is that all phase change materials (PCM) with high latent heat storage capacity have unacceptably low thermal conductivity, which makes heat charging and discharging process slow during the PCM solidification and melting. The thermal conductivity of the PCM used in this study (NaNO₃) solution is only 1.74 W/m-K, which is about 1/200 of that of the copper. Therefore in order to have a shorter charging and recharging time, the effective thermal conductivity of the PCM must be improved.
In this study, the thermal conductivity enhancer (TCE) will be introduced in the PCM space to enhance the heat transfer within the PCM. In previous studies, in section 2.2.1.3, on heat transfer enhancement methods, an increase in the heat transfer during PCM phase change is demonstrated by inserting a high conductive metal matrix into the PCM. The matrix is dispersed in the PCM so that the combination of PCM and the metal matrix provides a material with a higher thermal conductivity and still maintain a high heat capacity. Aluminum, titanium, copper and stainless steel are among the metals used for the TCE. They are corrosion resistant and high in thermal conductivity. The metal used as TCE in this study is copper. For simplicity, straight fins are used in computational studies for preliminary analyzes and performance predictions, since modeling a metal honeycomb or foam matrix demands too much effort.

However, the insertion of the metal matrix reduces the volume available for the PCM. Therefore, the employment of this thermal conductivity enhancer (TCE) should be compromised between the improvement in the heat transfer characteristics and reduction in the heat carrying capacity. Two cases were studied. In the first case, four fins of 5 mm thick each were inserted into the PCM space and in the other, ten 2 mm thick fins were used. In the study, the initial temperature of the PCM was taken as 255 K and no fin at the PCM container wall. Fig. 6.6 shows the schematic diagrams of the fin arrangement for the two cases mentioned above.

(a) 4 Fins (thickness=5mm)          (b) 10 Fins (thickness=2mm)

Figure 6.6 Two arrangements of thermal conductivity enhancer
Figures 6.7 and 6.8 show the temperature profiles of PCM with 4 and 10 straight fins inserted respectively at symmetric surface of \( Z = 150 \text{ mm} \).
Chapter 6 Parametric Studies

$t = 30$ minutes

$t = 40$ minutes
Figure 6.7 Temperature profiles of PCM with 4 straight fins inserted (thickness=5mm)

t = 50 minutes

t = 5 minutes
Chapter 6 Parametric Studies

$t = 10$ minutes

$t = 15$ minutes
Figure 6.8 Temperature profiles of PCM with 10 straight fins inserted (thickness=2mm)
The solidification times are shown in Table 6.2. The results show that the solidification time has reduced from more than 100 minutes to only 50 minutes for the case of 4 fins and 25 minutes for the case of 10 fins. The results also show that the heat transfer is more uniform in the case of more fins.

<table>
<thead>
<tr>
<th>Fins</th>
<th>Solidification Time</th>
<th>Homogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>50 minutes</td>
<td>good</td>
</tr>
<tr>
<td>10</td>
<td>25 minutes</td>
<td>better</td>
</tr>
</tbody>
</table>

Table 6.2 Results of different arrangements of thermal conductivity enhancer

The shorter solidification time observed are the results of the improvement in the “effective” thermal conductivity of the PCM with TCE in the PCM container.

6.1.3 PCM Container Shape Effect

In this section, effects of the container shape on the solidification of PCM were studied. Two cases were considered and are shown in Fig. 6.9. These are bare cylindrical and rectangular containers. The initial air space temperature for both cases were taken as 255 K.

(a) Cylindrical PCM container  
(b) Rectangular flat PCM container

Figure 6.9 Different PCM container shapes

The application of fins in the air space imposes a size limitation in the cooler box, and hence the fins reduce the volume available for the heat source. Therefore, in order to increase the surface areas of the PCM container, one may consider using a rectangular
flat container with the same volume as the cylindrical container, as this latter container has larger surface area thus, it is expected to promote heat transfer better than the cylindrical container, when both are without fins. Simulation studies have been carried out to study the effects of the rectangular container as compared to the cylindrical container when both have the same PCM volume and both are without fins.

For the same volume containers, a simple calculation below shows that the surface area of the rectangular container is twice as large as the surface area of the cylindrical container.

Volume of the cylindrical PCM container without fin is

\[ V_1 = \pi \times \text{radius}^2 \times \text{height} = \pi r^2 h = 3.14 \times 0.05^2 \times 0.3 = 0.002355 \text{m}^3 \]

Its surface area is,\( S_1 = 2\pi rh = 2 \times 3.14 \times 0.05 \times 0.3 = 0.0942 \text{m}^2 \)

Surface area of the rectangular flat PCM container with the same volume is,

\[ S_2 = \text{Length} \times \text{Width} \times 2 + \text{Length} \times \text{Height} \times 2 = 0.3 \times 0.3 \times 2 + 0.3 \times 0.26 \times 2 = 0.1956 \text{m}^2 \]

Fig 6.10 shows the temperature profile of the rectangular PCM container at surface of \( Y = 0 \) with time. Simulation results show that even with the insertion of the TCE, the PCM cannot be solidified quickly and uniformly, except at the small neighborhood regions near the container wall and the fins, after 2 hours of continuously supplying the liquid nitrogen into the system. It is because the PCM in the rectangular flat PCM container disperses widely, so the part far from the fins and tube can hardly be solidified.
6.1.4 Pre-cooling Process Effect

In this section the effects of the initial temperature of the PCM on its solidification time will be investigated. Two cases were considered, there are both with a cylindrical PCM container without TCE and with 50 fins in the air space. The initial temperatures of 255 K and 300 K have been applied to the entire cooler box.

Table 6.3 shows the solidification time for both cases. The result shows that the solidification for the case that the initial temperature is 300 K takes 183 minutes and that for the case with initial temperature of 255 K, 115 minutes. The result also shows that for the first case, the average temperature in the air space when all the PCM is solidified is still 300 K, which is too high for any useful applications of the cooler box. This also shows that the design of the cooler box without the pre-cooling process is ineffective in its function for cooling purpose.

<table>
<thead>
<tr>
<th>Initial Temperature</th>
<th>Solidification Time</th>
<th>Final Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>255K</td>
<td>115 minutes</td>
<td>268 K</td>
</tr>
<tr>
<td>300K</td>
<td>183 minutes</td>
<td>300 K</td>
</tr>
</tbody>
</table>

Table 6.3 Solidification time for PCM at different initial temperatures
Fig. 6.11 and 6.12 show that heat transfer through PCM container wall and the net PCM heat transfer rate, respectively.

![Heat Transfer Rate at PCM Container Wall with Different Initial Temperature](image)

**Figure 6.11 Heat transfer rate at PCM container wall**

It can be seen from Fig. 6.11 that the heat transfer rate between the air in the cooler box and the PCM in the cylindrical container under the initial temperature of 300 K is much higher than that under the initial temperature of 255 K. That is due to a large temperature difference of the former between liquid nitrogen and the other parts of the cooler box, initially. From Fig. 6.12, it can be seen that the heat removal rates for both initial temperatures are nearly same. However, more heat needs to be removed for solidification of the PCM with initial temperature of 300 K, thus results in a longer PCM solidification time.

![Net Heat Transfer of PCM with Different Initial Temperature](image)

**Figure 6.12 Net heat transfer of PCM with different initial temperature**
6.2 Heat Transfer in the Cooler with 4 Fins

Simulation and experimental results in Fig. 5.19 show that the heat source temperature remain virtually the same after it was put into the cooler box for 1 hour. From the velocity of airflow pictures in 5.12, it can be seen that the velocity of the air that is trapped into the narrow space among the 50 fins is nearly zero. That is to say that the natural convection failed to take place between the lower temperature PCM and the heat source, which results in the heat source temperature remains virtually the same. The original purpose of adding 50 fins to the PCM container is to accelerate the natural convection by enlarging the heat transfer surface of the PCM container. However, this prove to be too many fins and the fins are arranged too closely to each other, the air that is trapped in the narrow space between two fins is almost stationary, which becomes the obstacle of heat exchange between the PCM and the heat source.

Therefore, at this juncture, it is reasonable to reduce the fins number in order to promote the convection flow within the fins. Simulation studies for a cooler with 4 fins has been carried out to observe the convection flow and heat transfer in the air space. To minimize the simulation time and to take advantage of the symmetrical nature of the set-up, only half of the model is needed to be simulated and the solution domain is shown in Fig. 6.13.

![Figure 6.13 One identical part of the 4-fin cooler](image)
Fig. 6.14 shows that the heat source with an initial temperature of 373K can be cooled down to 258 K in an hour. Fig. 6.15 shows the velocity vector indicating the convection current in the air space near the heat source and the PCM container. The figure shows that the hot air near the heat sources rises and is replaced by the lower temperature air and hence heat transfer occurs. Note that the magnitude of the velocity is very small. It can also be seen from the figure that, at the opposite side of the cooler, the flow is different as it is far away from the heat source. The only convection heat transfer at this side is due to the heat transfer across the wall of the cooler box from outside.
Temperature distribution at surface of $Z = 150$ mm

Temperature distribution at surface of $Y = 0$ mm

Figure 6.14 Temperature distribution in the cooler box with 4 fins at $t = 1$ hour
Chapter 6 Parametric Studies

Velocity of air flow at surface of Y = 0
Chapter 6 Parametric Studies

Enlarged part of left region

Right region
Chapter 6 Parametric Studies

6.2.1 Heat Transfer in the Cooling Air Space

The air space or cooling space inside the cooler box is enclosed by three boundaries and these are the box wall, the fin wall and the PCM container wall as shown in Fig. 6.16. Since the air in the cooling space separates the heat source and the heat sink (PCM container), the convection heat transfer by air plays an important role in the heat...
transfer inside this space. The following discussion is thus focused on the studies of heat transfer of the air in the cooling space.

Fig. 6.17 shows the heat transfer rates across these surfaces. In the figure, the positive value of heat transfer rate refers to the heat that is transferred out of the surface and vice versa. For the heat source, the heat is transferred out of it surfaces into the air space. The heat transfer rate decreases to about 1 watt after 1500 seconds later, as the temperature difference between the heat source and the air in the cooler is very small. The results also shows that, due to the good thermal insulation material used for the cooler box wall, heat transfer through the box wall is almost negligible. The negative values of the heat transfer by fins, implying that the fins adsorbing heat.

The results also show that the external wall of the PCM transfers more heat than the fins, which is mainly due to its large temperature difference between the air and the cylinder surface temperature. This cylindrical surfaces, include the fin base, and thus the heat transfer through the exposed cylindrical surface is only around 3.5 to 4 watts, which is 12 watts minus away the 8.5 watt through the base of the fin.

Figure 6.17 Heat transfer of air in the cooling space
Fig. 6.17 also shows that a total of about 12 watts absorbs by the PCM, which is the heat passes through the cylindrical wall, which already includes the heat through the fin base. It is obvious that this amount of heat mainly comes from the air in the cooling space rather than the heat source.

Besides convection, radiation between the heat source wall and the PCM container wall in the cooler box has also been considered since the initial temperature of the heat source is as high as 300 K and the initial temperature of PCM is only 252 K. Fig. 6.18 demonstrates the relationship between the radiation heat transfer and total heat transfer rates.

![Total and Radiation Heat Transfer](image)

**Figure 6.18 Total and radiation heat transfer in the air space**

From the figure, it can be seen that the magnitude of the free convection is in close magnitude with the radiation heat transfer.

### 6.2.2 Cooler Box Wall

Next, consider the heat transfer rate of the individual wall of the cooler box as shown in Fig. 6.19.
Fig. 6.20 shows the heat transfer rate of the four inner walls of the cooler box. In fact, there are six walls altogether. Due to the symmetry of the model, the top and bottom, the front and back walls are identical, respectively. Therefore, in Fig. 6.20, only 4 walls are shown. Even though the walls are made of the same materials and operates at the same initial conditions, but due the placement of the heat source and the location of the PCM, the heat transfer rate is different for each of the wall. The figure shows that the heat transfer rate through the top or bottom walls is nearly double that through other walls. It is because the PCM is contacting the top and bottom walls, the large temperature difference (the temperature of the exterior box wall is kept constant at 300
K and the initial temperature of PCM is set at 252 K) has led to a large heat transfer rate. That is to say, much heat enters the cooler box through the top and bottom walls. Heat transfer rates of the other 4 vertical walls are almost the same.

### 6.2.3 Fins

Next, consider the fin itself. The fin walls consist of fin tips and fin side surfaces. Due to the symmetrical nature of the model, only half the cooler has been modeled. Thus, only two fins are shown in the figure, instead of four. Fin 1 is located near the heat source and fin 2 is further ways from the heat source as shown in Fig. 6.21 below.

![Figure 6.21 Fin 1 and fin 2](image)

The details of the walls for the fins are shown in Fig. 6.22.

![Figure 6.22 Wall name of identical fin and PCM container](image)
The results for the heat transfer rate for the side surfaces of the fins are shown in Fig 6.23. The result shows that the fins absorb heat from the air and thus the heat transfer rates depicted as negative values. Fin 1 absorbs more heat from the vicinity air, as expected as it is located near the heat source.

![Heat Transfer of Fin Sidewalls](image)

**Fig.6.23 Heat transfer of fin sidewalls**

![Total and Radiation Heat Transfer of Fin 1 Side Walls](image)

**Fig. 6.24 Total and radiation heat transfer of fin 1 sidewalls**

![Total and Radiation Heat Transfer of Fin 2 Side Walls](image)

**Fig. 6.25 Total and radiation heat transfer of fin 2 sidewalls**
Chapter 6 Parametric Studies

Fig. 6.24 and 6.25 show the variation of the radiation and the total heat transfer rates. The results show that the radiation heat transfer mode is equally important in natural convection consideration, as would be expected.

![Heat Transfer of Fin Tips](image)

**Fig. 6.26 Heat transfer of fin tips**

Fig. 6.26 shows that the heat transfer through the fin tip is a lot smaller than other surfaces of the fin and its variation mirrors the variation of the heat transfer at the side surface.

![Total and Radiation Heat Transfer of Fin 1 Tip](image)

**Fig. 6.27 Total and radiation heat transfer of fin 1 tip**

![Total and Radiation Heat Transfer of Fin 2 Tip](image)

**Fig. 6.28 Total and radiation heat transfer of fin 2 tip**
Chapter 6 Parametric Studies

The corresponding radiation heat transfer for the fin tip are also shown in Fig. 6.27 and 6.28, again, the magnitude of radiation heat transfer is always comparable to that of the natural convection.

6.2.4 PCM Container

Fig. 6.29 shows the heat transfer through the exposed cylindrical wall of the PCM container and the fin base. Obviously, the fins have a very good thermal conductivity and closer to the heat source, hence resulting in a higher heat transfer rate at the fin base.

![Heat Transfer of PCM Container](image1)

**Fig. 6.29 Heat transfer of PCM container**

![Total and Radiation Heat Transfer of PCM Container](image2)

**Fig. 6.30 Total and radiation heat transfer of PCM container**

Fig. 6.30 shows the variation of the radiation heat transfer rate of the PCM container, in this case it is only for the exposed cylindrical surface.
Fig. 6.31 Total and radiation heat transfer of heat source

Fig. 6.31 shows that the radiation heat transfer rate of the heat source is similar in magnitude to that of the convection.

### 6.2.5 Cooling Rate and Operation Cost

The 4-fin cooler with a TCE that has 10 straight fins and 2mm each fin is studied here.

The initial temperature of the heat source is set at 300 K (27 °C). After 40 minutes, the heat source in the 4-fin cooler is cooled down to 258 K (-15 °C). Therefore, cooling rate of this 4-fin cooler is estimated as flows:

\[
\frac{(27 \, ^{\circ}C -15 \, ^{\circ}C)}{40 \, \text{minutes}} = 0.3 \, ^{\circ}C /\text{minute}
\]

As mentioned in chapter 1, the recommended cooling rate by the food service industry is about 0.2 °C/min. Otherwise, the food could be subjected to microbiological contamination. Therefore, the 4-fin cooler meets this standard.

Experimental results show that the liquid nitrogen used for the pre-cooling process (cooling the empty cooler from 300 K to 255 K) is 0.5 kg. The simulation shows that the complete solidification of the PCM with an initial temperature of 255 K needs 25 minutes when the liquid nitrogen flows at 2 kg/h continuously, hence it can be calculated that the total liquid nitrogen consumed is:

\[
0.5 \, \text{kg} + 2 \, \text{kg/h} \times (25/60)\text{h} = 1.3 \, \text{kg}
\]
Hence the cost for 4.2 kg of liquid nitrogen is:
\[ \text{S}\$0.16/\text{kg} \times 1.3 \text{ kg} = 0.21 \text{ S}\$ \]
Further simulation was carried out to examine how long all the PCM can be melted and Fig. 6.32 below is obtained.

![Heat Source Cooling Process in 4-fin Cooler](image)

**Figure 6.32 Heat source cooling process in 4-fin cooler**

From the above figure, it can be seen that the lowest temperature the heat source can reach is 254 K and the PCM takes 28 hours for a complete melting. Therefore, the operation cost of the 4-fin cooler is estimated as:

\[ \text{Cost of operation per hour} = \frac{0.21 \text{ S}\$/28\text{hours}} = 0.75 \text{ cent} \]

The cost calculated above shows that the new 4-fin cooler is economical to operate.

### 6.3 Effects of Design Parameters on the Cooling Rate

In this section, the effects of design parameters on the cooling rate of the heat source are studied. These design parameters are fin number, fin length, heat source location and PCM container shape. Simulation studies have been carried out and the temperature of the heat source was monitored over time with the same initial heat source temperature of 300 K and the initial temperature of the air in the cooler box is 255 K.

#### 6.3.1 Fin Number Effect
In this section, the effects of fin number of the PCM container on the cooling rate of the heat source are shown. The PCM containers with 5 different fin numbers have been studied and these are PCM containers with 50, 10, 4, 2 and 0 fin as shown in Fig. 6.33.

![PCM containers with 50, 10, 4, 2 and 0 fin](image)

Figure 6.33 PCM containers with 50, 10, 4, 2 and 0 fin
Chapter 6 Parametric Studies

Fig. 6.34 shows various cooling rates of the heat source when fitted with different numbers of fin.

![Cooling Rate Comparison of Heat Source](image)

**Figure 6.34 Cooling rate comparison of heat source**

The results show that the cooling rate of the heat source for the cooler box fitted with 4 fins is capable of reaching the lowest temperature at the shortest time. It reached the temperature of 258 K 1400 seconds later, when others are at still above 261 K.

Fig. 6.35 shows the instantaneous heat transfer rate for fins (only fins) in the coolers fitted with different numbers of fin. In the figure, the positive values refer to heat transfer into the fin and vice versa. The results show that the fins in the 10-fin cooler absorbed the most heat from the air space whereas the fins in the 50-fin cooler actually rejects a small amount of heat into the air space. The rejection of heat from the fins of the 50-fins cooler was due to the conduction of heat through the tip of the fins which is in close contact with the top and the bottom walls of the cooler box. However, in the later part of the cooling process, after 270 seconds, the reverse is true. This latter effect is the results of the higher air temperature in the cooling space that eventually reverses the heat flow: heat is now flowing from the air space into the fin.
Event though the results in Fig. 6.35 show that the fins in the cooler fitted with 10 fins transfer the most heat, but results in Fig. 6.34 suggests that it is the cooler fitted with 4 fins that transfer the most heat which results in the lower temperature of the heat source. Fig. 6.36 is given below to explain this phenomenon. Initially, the temperatures of the air around the heat source and fins/PCM container are both 255 K, hence, no heat transfer occurs. After a certain period of time, the air surrounding the heat source is heated up and at the mean while, the air around the fins and the PCM container wall is cooled down. Due to this temperature difference, heat transfer by convection between the heat source and the fins/PCM container takes place. From Fig. 6.36, it can be seen that since the amount of air around the fins, which is firstly cooled down by fins, in the cooler with 10 fins, is much larger than those in other three coolers with 4, 2 and no fin respectively. Therefore, the fins in the cooler fitted with 10 fins transfer the most heat as shown in Fig. 6.35. However, since the airflow in the cooler with 10 fins is more restricted as compared to that in the other three coolers, Fig. 6.34 suggests that it is the cooler fitted with 4 fins and not 10 fins that results in the lowest temperature of the heat source.
(a) 10-fin cooler

(b) 4-fin cooler

(c) 2-fin cooler
The studies show that the fins help to promote heat transfer in the case of the cooler box, but too many fins results in the difficulties in promoting airflow and hence results in high resistance for convection heat transfer, such as the case in the 50-fin cooler box. For the cooler studied, the cooler fitted with 4 fins has shown to give better cooling effects than others.

### 6.3.2 Fin Length Effect

The simulation studies in previous sections show that the cooler box fitted with 4 fins on the PCM container wall is a better design. In this section, the effects of fins length will be looked into. Simulation studies have been carried out for two coolers fitted with 4 fins. All other conditions are the same as those used in previous simulations, excepts now one cooler is fitted with 50 mm long fins and the other is 100 mm. Their top views are shown in Fig.6.37.
The result in Fig. 6.38 shows that though the initial cooling rate is the same for both designs, the cooler fitted with 100 mm long fins cools better. This is obviously shown in the figure for the cooling period after 1000 seconds. Fig. 6.39 reconfirms this point.

**Figure 6.38 Cooling process for cooler fitted with different fin lengths**

![Cooling Process for Cooler Fitted with Different Fin Lengths](image)

**Figure 6.39 Fin heat transfer rate**

![Fin Heat Transfer Rate Comparison](image)

### 6.3.3 Heat Source Location Effect

In this section, simulation studies were carried out for coolers fitted with 4 fins of 100 mm length with heat source placed in three different locations, as shown in Fig 6.40:
Location 1 (Besides Two Fins): The heat source lies in the middle of the neighboring two fins; the distance between the center of the heat source and the central PCM container is 150 mm.

Location 2 (Between Two Fins): The heat source lies in the middle of the neighboring two fins; the distance between the center of the heat source and the central PCM container is 75 mm.

Location 3 (Besides One Fin Tip): The heat source lies opposite to the tip of one fin; the distance between the center of the heat source and the tip of fin is 50 mm.
Chapter 6 Parametric Studies

Temperature Variations of Heat Source during the Cooling Process for Three Different Heat Source Locations

![Graph of Temperature Variations](image)

**Fig. 6.41** Temperature variations of heat source during the cooling process for three different heat source locations

From Fig. 6.41, it can be seen that the heat source in location 1 can be cooled down to a temperature that is 4 K lower than that in the heat source in location 2. However, the heat source with initial temperature of 300 K in location 3 can only be cooled down to 299.9332 K after one hour.

Even though location 1 yields the lowest heat source temperature, however with the heat source at the location 2, the PCM can absorb more heat than that in location 1 as shown in fig.6.42, but the natural convection is much more restricted in location 2. Therefore, at location 2, the air around the PCM is cold but the cold air can hardly flow to the heat source, which explains the lower heat source temperature at location 1.

![Graph of Heat Transfer of Fins](image)

**Figure 6.42** Heat transfer of fins
6.3.4 PCM Container Shape Effect

In this section, coolers with different shape PCM containers as shown in Fig. 6.9 will be studied.

![Heat Source Cooling Rate with Different PCM Containers](image1)

**Fig. 6.43 Heat source cooling rate with different PCM containers**

From the above figure, it can be seen that the cooling rates of the heat source in the cooler box with either no-fin cylindrical container or rectangular flat container are almost the same.

![Minimum Air Velocity in the Air Space](image2)

**Fig. 6.44 Minimum air velocity in the air space**
From Fig. 6.44, it can be seen that since the large surface area of the rectangular PCM container and it transverses the whole cooling space, the cooler box is easier to get an even temperature compared with the one with a cylindrical PCM container as shown in Fig. 6.45.

**Figure 6.45** Temperature distributions of heat source and air space at surface of $Z=150\text{mm}$ in coolers with different PCM containers
Fig. 6.46 Maximum air velocity in the air space

Fig. 6.46 reflects that generally, the natural convection current is stronger in the rectangular PCM container, as it has a larger heat transfer area.

Fig. 6.47 PCM container wall heat transfer rate

Fig. 6.47 shows that heat transfer rate at the rectangular PCM container wall is much larger than that at the cylindrical PCM container wall. It is because the surface area of the rectangular PCM container wall is much larger than that of the cylindrical PCM container wall.
Chapter 7

Conclusions and Future Work

7.1 Contributions and Conclusions

In this project, the conceptual design for a new cooler was developed. Liquid nitrogen was used as a cooling medium. A phase change material (PCM) was used as the thermal storage medium to take full advantage of the cooling ability of the liquid nitrogen and at the same time to provide portability for the cooler box. The PCM also serves as an effective cooling buffer between the liquid nitrogen with an extremely low vaporization temperature and the food requiring to be cooled at a moderately low temperature. The PCM chosen was the Na$_3$OH with solidification temperature of -20 °C.

Following the conceptual design, a physical model for a cooler has been designed, built, instrumented and tested. Comparison between the measured and the predicted data were made and generally good agreements were obtained. A discrepancy of less than 5% was observed.

The 3-D simulation model was then used to investigate the various effects of design parameters on the heat transfer performance of the portable cooler. The studies include time-dependent temperature distributions in the cooling air space, heat source cooling rate, natural convection in the box and PCM melting/solidification.

The study shows that heat transfer enhancer could greatly improve the thermal conductivity of PCM. With the assistance of heat transfer enhancer, PCM could be solidified in 50 minutes instead of more than 100 minutes. The study also shows that PCM takes a shorter time to melt completely and the temperature of it is also more uniform.

Moreover, it was found from the simulation that the existence of 50 fins in the small air space become the barrier to the natural convection. Results showed that the cooler box with 4 fins is the best design as it gives the fast cooling rate of heat source.
Observation in temperature distributions shows that 4 fins are more effective than 50 or 10 fins. Results also show that pre-cooling may be essential to improve the performance of the cooler.

7.2 Future Work

The verified three dimensional FLUENT mathematical model will be used to carry out computerized optimization. A multi variable, constrained direct search optimization technique should be linked to the mathematical model to search for optimum combination of design variable for the cooler box of various capacities and design. i.e. To provide more cooling space for bigger heat source rather than in the center of the box.

More efforts will be focused on the metal matrix with high thermal conductivity dispersed into the PCM. Mathematical model will be established to specify the effective thermal conductivities for the heterogeneous PCM matrix. For simplicity, straight fins were used in computational studies for preliminary analyzes and performance predictions in this project. Modeling metal matrix with much more complicated shape such as honeycomb or foam matrix will be carried out to optimize the design of the metal matrix.

The UDFs used in this project needs to be improved by taking account of the liquid-vapor flow pattern rather than only the amount of liquid nitrogen introduced into the model. It is because in the two-phase mixture, heat absorbed by liquid nitrogen is strongly dependent on the liquid-vapor flow pattern.

It is suggested that in order to improve the heat transfer in the air space and to promote a better cooling rate of the heat source, a small fan with low power consumption could be added in the air space to induce force convection heat transfer rather than via natural convection. This small fan can be driven by small dry cells in order to ensure the portability of the cooler.

Application performances of the newly designed cooler box such as heat load capacity, storage time will be studied and meaningful comparison method will be devised to compare the new portable cooler with the existing similar products in the market.
References


17. United States Environmental Protection Agency Office of Enforcement, The Battery Act, 2002


22. An Introduction to Thermoelectrics. Tellurex Corporation. February 8, 2001


27. http://www.supercool.se/


42. Telkes M. Nucleation of super saturated inorganic salt solution. Indust Eng Chem 1952; 44:1308


References


136. Fluent 6.1 User’s Guide


Appendix I
Horizontal View of Assembly Drawing
Vertical View of the Assembly Drawing (no box cover)
Tech. Requirements:

(1) All fins are made of copper.
(2) 50 fins totally.

Vertical View of PCM Container
Cooling Coiling Tube (1 Coil)

\[ d_i = 5 \text{ mm} \]
\[ d_0 = 6 \text{ mm} \]

\[ \theta = 30 \text{ mm} \]
Toch. Requirements:

1. The cooling tube has 15 coils.
2. All pipes are made of copper.

Cooling Coiling Tube
Tech. Requirement:
The body is made of copper.

PCM Container Body
Appendix II
1. Physical Model

(1) Equation of $T_b = f_1(t)$

Control volume 1 in fig. Appendix II.1 is enclosed by the interior surfaces of the box wall (purple color) and the finned exterior surface of the cylindrical PCM container wall (yellow and green color).
The circuit must account for the conduction in the cooler box wall (R_{kb}), convection from the exposed base (R_{t,base}), and conduction/convection from the fins (R_{t,fin(50)}).

h_b is the convection coefficient between the air in the cooler box and the finned PCM container wall. According to the approximate method proposed by Chow [137], the average physical properties of air in the cooler box are

\[
\begin{align*}
\text{viscosity} & \quad \mu = 1.7894 \times 10^{-5} \text{kg/m.s} \\
\text{density} & \quad \rho = 1.225 \text{kg/m}^3 \\
\text{specific heat capacity} & \quad C_p = 1006.43 \text{J/kg.K} \\
\text{thermal conductivity} & \quad k = 0.0242 \text{W/m.K} \\
\text{volumetric thermal expansion coefficient} & \quad \beta = 0.00395/\degree C \\
\text{local acceleration} & \quad g = 9.807 \text{m/s}^2
\end{align*}
\]

\[
Gr = \left( L_j^3 \times \rho^2 \times g \times \beta \times \Delta t \right) / \mu^2
\]

\[
= (0.1^3 \times 1.225^2 \times 9.807 \times 0.00395 \times [-18 - (-20)] / (1.7894 \times 10^{-5})^2
\]

\[
= 3.63 \times 10^5
\]

Pr = \( C_p \times \mu / k = 1006.43 \times 1.7894 \times 10^{-5} / 0.0242 = 7444 \)

\[
Gr \times Pr = 3.63 \times 10^5 \times 0.7444 = 2.7 \times 10^5
\]

\[
\begin{align*}
\left\{ 
\begin{array}{ll}
h_b \times L_j / k = 1.36 \times (Gr \times Pr)^{0.1667} & \quad Gr \times Pr < 10^4 \\
h_b \times L_j / k = 0.59 \times (Gr \times Pr)^{0.25} & \quad 10^4 < Gr \times Pr < 10^9
\end{array}
\right.
\end{align*}
\]

Since \( 10^4 < Gr \times Pr = 2.7 \times 10^5 < 10^9 \)

\[
h_b \times L_j / k = 0.59 \times (Gr \times Pr)^{0.25}
\]

\[
h_b = 0.59 \times (Gr \times Pr)^{0.25} \times k / L_j = 0.59 \times (2.7 \times 10^5)^{0.25} \times 0.0242 / 0.1 = 3.25 \text{W/m}^2.K
\]

Thermal resistance for the cooler box wall is

\[
R_{kb} = \frac{L_j}{k \times A_{box,exterior}} = \frac{L_j}{k_j \times [4(L_u + 2L_j)(H + 2L_j) + 2(L_u + 2L_j)^2]}
\]

\[
= \frac{0.1}{0.0242 \times [4(0.6 + 2 \times 0.1)(0.3 + 2 \times 0.1) + 2(0.6 + 2 \times 0.1)^2]} = 1.4 \text{K/W}
\]

For a single fin, \( R_{t,fin(1)} = \theta_b / q_f \), where from Table 3.4 [138],
\[ q_f = (h_b P k_f A_c)^{1/2} \sinh mL_f + (h_b / mk_f) \cosh mL_f \cosh mL_f + (h_b / mk_f) \sinh mL_f \]

With \[ P = 2(H + t_f) = 2(0.3 + 0.001) = 0.602m \]
and \[ A_c = t_f \times H = 0.001 \times 0.3 = 0.0003m^2, \]
\[ m = \left( \frac{h_b P}{k_f A_c} \right)^{1/2} = \left( \frac{3.25 \times 0.602}{387.6 \times 0.0003} \right)^{1/2} = 4.1m^{-1} \]
\[ mL_f = 4.1 \times 0.1 = 0.41 \approx 0.4 \]
\[ \frac{h_b}{mk_f} = \frac{3.25}{4.1 \times 387.6} = 0.002 \]
and
\[ (h_b P k_f A_c)^{1/2} = (3.25 \times 0.602 \times 387.6 \times 0.0003)^{1/2} = 0.477W / K \]
use of Table B.1[138] yields, for a single fin,
\[ R_{r, \text{fin}(1)} = \frac{1.0811 + 0.002 \times 0.4108}{0.477 \times (0.4108 + 0.002 \times 1.0811)} = 5.48K / W \]
Hence, for 50 fins,
\[ R_{r, \text{fin}(50)} = \frac{R_{r, \text{fin}(1)}}{50} = \frac{5.48}{50} = 0.11K / W \]
For the exposed base,
\[ R_{r, \text{base}} = \frac{1}{h_b \times (2\pi r_x - 50t_f) \times H} = \frac{1}{3.25 \times (2\pi \times 0.05 - 50 \times 0.001) \times 0.3} = 3.89K / W \]
With
\[ R_{bx} = R_{\text{equiv.}} = [(0.11)^{-1} + (3.89)^{-1}]^{-1} = 0.11K / W \]
Without the fins, the convection resistance of the PCM is
\[ R_{\text{tran}} = (2\pi r_x H h_b)^{-1} = (2\pi \times 0.05 \times 0.3 \times 3.25)^{-1} = 3.3K / W \]
Hence there is considerable advantage to using the fins.

\[
\begin{align*}
\mathcal{J}_{\theta_7} & \rightarrow \mathcal{T}_k \rightarrow \mathcal{R}_{k_{ib}} \rightarrow \mathcal{T}_{ib} \\
q_{in} &= \frac{T_e - T_a}{R_{ib}} = \frac{300K - T_b}{1.4K / W}
\end{align*}
\]
At an instant, energy terms include the rate at which thermal energy enters and leaves through the control surface, \( q_{in} \) and \( q_{out} \). The rate of change of energy stored within the control volume is designated as \( q_{st} \). A general form of the energy conservation requirement may then expressed as

\[
q_{in} - q_{out} = q_{st}
\]

\[
\frac{300K - T_b}{1.4K/W} - \frac{T_b - T_x}{0.11K/W} = V_b D_b C_b \frac{dT_b}{dt}
\]

\[
\frac{300K - T_b}{1.4K/W} - \frac{T_b - T_x}{0.11K/W} = [(L_a^2 - \pi r_b^2 - 50t_f L_f )H] D_b C_b \frac{dT_b}{dt} = \left\{ \left[ (0.6m)^2 - \pi \times (0.05m)^2 - 50 \times 0.001m \times 0.1m \right] \times 0.3m \right\} \times 1.225 \text{kg/m}^3 \times 1006.43 \text{J/kg.K} \frac{dT_b}{dt}
\]

So

\[
\frac{dT_b}{dt} + 0.08 T_b - 0.07 T_x - 1.67 = 0
\]

(2) Equation of \( T_x = f_2(t) \) and \( U_x = f_4(t) \)
Control volume 2 in fig. Appendix II.2 is enclosed by the finned surface of the PCM cylindrical container (yellow and green color) and the surface of the coiling tube wall (blue color).

The circuit must account for the overall thermal resistance between the air in the cooler box and the PCM ($R_{bx}$), conduction in the cooler box top & bottom walls ($R_{kx}$), thermal resistance between PCM and the exterior surface of the coiling tube wall ($R_x$), and convection form the interior surface of the coiling tube wall ($R_d$).
PCM will experience liquid-solid phase change, so

\[ R_x = \begin{cases} R_{x,\text{solid}} & \text{PCM is totally solid} \\ R_{x,\text{liquid-solid}} & \text{PCM is the mixture of liquid and solid} \\ R_{x,\text{liquid}} & \text{PCM is totally liquid} \end{cases} \]

To obtain the accurate values of \( R_{x,\text{solid}}, R_{x,\text{liquid-solid}} \) and \( R_{x,\text{liquid}} \) demands great experimental work. Therefore, in this research, average properties based on the component's volume are used. This approximate method is proposed by Chow [137].

The effective thermal conductivities of PCM are shown in the table below.

<table>
<thead>
<tr>
<th>PCM</th>
<th>k (W/m.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>1.74</td>
</tr>
<tr>
<td>Melting</td>
<td>2.13</td>
</tr>
<tr>
<td>Liquid</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Since the main purpose of the one-dimensional model is to examine the workability of the initial design, in order to simplify the modeling process, the author considered the worst situation only instead of considering all the possible situations during the cooler's application. If the design of cooler could act under the worst working situation, it must be workable under other better situations. Since \( R_{x,\text{solid}} \) is larger than both \( R_{x,\text{liquid-solid}} \) and \( R_{x,\text{liquid}} \), it is the worst situation for heat transfer in the cooler box. Therefore, the author used \( R_x = R_{x,\text{solid}} \) to test the validity of the initial design.

PCM in fig. Appendix II.3 (a) can be approximately viewed as a hollow cylindrical system shown in (b).
Figure Appendix II.3 Simplification Calculation of Thermal Resistance of PCM

\[ R_x = R_{x, \text{solid}} = \frac{\ln[r_x/(D_c/2)]}{2\pi k x H} = \frac{\ln[0.05/(0.03/2)]}{2\pi \times 1.74 \times 0.3} = 0.37 \text{ K/W} \]

Liquid nitrogen will experience liquid-vapor phase change, so

\[
R_d = \begin{cases} 
R_{d, \text{liquid-vapor}} & \text{Nitrogen is the mixture of liquid and vapor} \\
R_{d, \text{vapor}} & \text{Nitrogen is totally vapor} 
\end{cases}
\]

Since liquid nitrogen flashes into vapor once it is introduced into the coiling tube, the author used \( h_d = h_{d, \text{vapor}} \) in the one dimensional modeling.

If \( \Pr = (C \times \mu) / k \approx 1 \) and \( R_c \times (2r_d / D_c)^2 > 0.1 \)

\[
(h_d \times 2r_d) / k = \Pr\left[26.2 \times (\Pr^{2/3} - 0.074)\right] \times (R_c^{0.8} \times (2r_d / D_c)^{0.1} \times \left\{ 1 + 0.098/[R_c \times (2r_d / D_c)^2]^{1/5} \right\} \\
\]

Where

\( \Pr = (C \times \mu) / k \)

\( \Re = (2r_d \times G) / \mu \)
G: mass flux density of the nitrogen flowing into the coiling tube
Wd: mass flow rate of liquid nitrogen
C: specific heat capacity of vapor nitrogen
µ: viscosity of vapor nitrogen
k: thermal conductivity of vapor nitrogen

According to the approximate method proposed by Chow [137], the average physical properties of vapor nitrogen are

\[ C = 1046.03 \text{J/kg.K} \]
\[ \mu = 1.67 \times 10^{-5} \text{kg/m.s} \]
\[ k = 0.0256 \text{W/m.K} \]

\[ G = W_d / (\pi \times r_d^2) = 2 / (\pi \times 0.003^2) = 70771 \text{kg/m}^2 \cdot \text{s} = 19.66 \text{kg/m}^2 \cdot \text{s} \]

\[ \Pr = (1046.03 \times 1.67 \times 10^{-5}) / 0.0256 = 0.7 \approx 1 \]

\[ R_e = (2 \times 0.003 \times 19.66) / 1.67 \times 10^{-5} = 7063 \]

\[ R_e \times (2 r_d / D_c)^2 = 7063 \times [(2 \times 0.003) / 0.03]^2 = 282.52 > 0.1 \]

So

\[ h_d = \Pr / [26.2 \times (\Pr^{2/3} - 0.074)] \times (R_e^{0.8}) \times (2 r_d / D_c)^{0.1} \times \left[ 1 + 0.098 \times (R_e \times (2 r_d / D_c)^2)^{1/5} \times (k / 2 r_d) \right] \]

\[ = 0.7 / [26.2 \times (0.7^{2/3} - 0.074)] \times (7063^{0.8}) \times (0.006 / 0.03)^{0.1} \times \left[ 1 + 0.098 \times (7063 \times (0.006 / 0.03)^2)^{1/5} \right] \]

\[ \times (0.0256 / 0.006) = 166 \text{W/m}^2 \cdot \text{K} \]

\[ R_{sd} = R_x + R_d = R_x + \frac{1}{h_d \times A_{tube}} = R_x + \frac{1}{h_d \times (2 \pi r_d) \times (\pi D_c n)} = 0.37 + \frac{1}{166 \times 0.0266} = 0.6 K/W \]

\[ R_{ks} = \frac{L_j}{k_j \times 2 \times A_{PCM.top}} = \frac{L_j}{k_j \times 2 \times \pi r_s^2} = \frac{0.1}{0.02 \times 2 \pi \times 0.05^2} = 318.47 K/W \]
\[
q_{in} = \frac{T_b - T_x}{0.11K/W} + \frac{300K - T_x}{318.47K/W}
\]

\[
q_{out} = \frac{T_x - T_d}{R_{sd}} = \frac{T_x - T_d}{0.6K/W}
\]

\[
q_{in} - q_{out} = \frac{T_b - T_x}{0.11K/W} + \frac{300K - T_x}{318.47K/W} - \frac{T_x - T_d}{0.6K/W} = q_{st}
\]

\[q_{st} = \begin{cases} 
W_x \cdot C_{XL} \cdot \frac{dT_x}{dt} & (T_x > T_x^*, U_x=1) \\
H_x \cdot W_x \cdot \frac{dU_x}{dt} & (T_x=T_x^*, 0 < U_x <1) \\
W_x \cdot C_{XS} \cdot \frac{dT_x}{dt} & (T_x < T_x^*, U_x=0) 
\end{cases} \]

According to the approximate method proposed by Chow [137], the average physical properties of PCM are:

\[C_{XL} = 3765.69 \text{J/kg.K} \]

\[C_{XS} = 1882.85 \text{J/kg.K} \]

\[H_x = 300000 \text{J/kg} \]

\[D_x = 1300 \text{kg/m}^3 \]

\[W_x = D_x V_x = D_x (\pi r_x^2 H - \pi r_d^2 \pi D_c n) = 1300 \times (\pi \times 0.05^2 \times 0.3 - \pi \times 0.003^2 \times \pi \times 0.03 \times 15) = 3.01 \text{kg} \]

So

\[\frac{dT_x}{dt} - 8.02E^{-4}T_b + 9.49E^{-4}T_x - 1.47E^{-4}T_d - 8.29E^{-5} = 0 \quad (T_x > T_x^*, U_x=1) \]

\[\frac{dU_x}{dt} - 1.01E^{-5}T_b + 1.19E^{-5}T_x - 1.85E^{-6}T_d - 1.04E^{-6} = 0 \quad (T_x=T_x^*, 0 < U_x <1) \]

\[\frac{dT_x}{dt} - 1.6E^{-3}T_b + 1.9E^{-3}T_x - 2.95E^{-4}T_d - 1.66E^{-4} = 0 \quad (T_x < T_x^*, U_x=0) \]
(3) Equation of $T_d = f_3(x)$ and $U_d = f_5(x)$

Control volume 3 in fig. Appendix II.4 is enclosed by the interior surface of the coiling tube wall (blue color). The length of the coiling tube per unit height along the x-direction is viewed as a constant $C_0$ for simplification and $C_0$ is defined as follows:

$$C_0 = \frac{L_d}{H} = \frac{\pi D_n}{H} = \frac{\pi \times 0.03 \times 15}{0.3} = 4.71$$

![Figure Appendix II.4 Flux Balance over Control Volume 3](image)

From the deduction in (2) mentioned above, it can be seen that heat is transferred from the surrounding PCM into the coiling tube with the height of $H$ is $\frac{T_x - T_d}{0.6K/W}$. Therefore, heat is transferred into the volume with the height of $\delta_x$ as shown in Fig. Appendix II.4 is calculated as

$$q_m = \frac{T_x - T_d}{0.6K/W} \times \delta_x = \frac{T_x - T_d}{0.6K/W} \times \delta_x = \frac{5.56(T_x - T_d)}{0.3}\delta_x$$
Since liquid nitrogen begins vaporization as soon as introduced into the coiling tube at inlet, it will experience liquid-vapor phase change as flowing inside the coiling tube and leave the tube at the outlet in vapor as shown in the figure below.

At point \( x \) on the \( x \)-coordinate axis, the enthalpy of liquid nitrogen is

\[
W_d \times U_d \times H_d
\]

At point \( x + \delta x \) on the \( x \)-coordinate axis, the enthalpy of liquid nitrogen is

\[
W_d \times U_d \times H_d + \frac{d(W_d \times U_d \times H_d)}{dx} \times \delta x
\]

So

\[
q_u = W_d \times U_d \times H_d + \frac{d(W_d \times U_d \times H_d)}{dx} \times \delta x - W_d \times U_d \times H_d = W_d \times H_d \times \frac{dU_d}{dx} \times \delta x
\]

\[
= 0.00056 \text{kg/s} \times 198000 \text{J/kg} \times \frac{dU_d}{dx} \times \delta x = 110 \frac{dU_d}{dx} \delta x
\]

The heat entering the control volume 3 evaporates liquid nitrogen as

\[
q_{in} = q_u
\]

\[
5.56(T_x - T_a) \delta x = 110 \frac{dU_d}{dx} \delta x
\]

\[
\frac{dU_d}{dx} - 0.05T_x + 3.89 = 0 \quad (T_d = T_d^*, 0 < U_d < 1)
\]
Then the cold vapor nitrogen whose temperature is $T_d^*$ will further absorb heat from the surrounding phase change material until it leaves from the outlet of the coiling tube.

At point $x$ on the $x$-coordinate axis, the enthalpy of vapor nitrogen is

$$W_d \times C_{dg} \times (T_d - T_d^*)$$

At point $x + \delta x$ on the $x$-coordinate axis, the enthalpy of vapor nitrogen is

$$W_d \times C_{dg} \times (T_d - T_d^*) + \frac{d[W_d \times C_{dg} \times (T_d - T_d^*)]}{dx} \times \delta_x$$

So

$$q_{st} = W_d \times C_{dg} \times (T_d - T_d^*) + \frac{d[W_d \times C_{dg} \times (T_d - T_d^*)]}{dx} \times \delta_x - W_d \times C_{dg} \times (T_d - T_d^*)$$

$$= W_d \times C_{dg} \times \frac{dT_d}{dx} \times \delta_x = 0.00056 \text{ kg/s} \times 1046.03 \text{ J/kg.K} \times \frac{dT_d}{dx} \times \delta_x = 0.59 \frac{dT_d}{dx} \delta_x$$

The heat entering the control volume 3 increases the temperature of the vapor nitrogen as

$$q_{in} = q_{st}$$

$$5.56(T_x - T_d) \delta_x = 0.59 \frac{dT_d}{dx} \delta_x$$

$$\frac{dT_d}{dx} - 9.42(T_x - T_d) = 0 \quad (T_d \geq T_d^*, U_d=1)$$

2. Numerical Analysis

The total refrigeration time is divided into $M_{\text{max}}$ segments as shown in figure below.
The total height of the box is divided into \( N_{\text{max}} \) segments as shown in figure below.

(1) Equation of \( T_b \)

\[
\frac{dT_b}{dt} + 0.08T_b - 0.07T_x - 1.67 = 0
\]

\[
\frac{(T_b)_{m+1} - (T_b)_m}{\Delta t} = -0.08(T_b)_m + 0.07(T_x)_m + 1.67
\]

Therefore

\[
(T_b)_{m+1} = (1 - 0.08\Delta t)(T_b)_m + 0.07\Delta t(T_x)_m + 1.67\Delta t
\]

(2) Equation of \( T_x \) and \( U_x \)

1. \( \frac{dT_x}{dt} - 8.02E^{-4}T_b + 9.49E^{-4}T_x - 1.47E^{-5}T_d - 8.29E^{-5} = 0 \) \( (T_x > T_x^*, U_x=1) \)

2. \( \frac{dU_x}{dt} - 1.01E^{-3}T_b + 1.19E^{-5}T_x - 1.85E^{-6}T_d - 1.04E^{-6} = 0 \) \( (T_x=T_x^*, 0 < U_x <1) \)

3. \( \frac{dT_x}{dt} - 1.6E^{-3}T_b + 1.9E^{-3}T_x - 2.95E^{-4}T_d - 1.66E^{-4} = 0 \) \( (T_x < T_x^*, U_x=0) \)

So

1. \( \frac{(T_x)_{m+1} - (T_x)_m}{\Delta t} = 8.02E^{-4}(T_b)_m - 9.49E^{-4}(T_x)_m + 1.47E^{-4}(T_d)_m + 8.29E^{-5} \)

\( (T_x > T_x^*, U_x=1) \)
\( (U_x)_{m+1} - (U_x)_m \over \Delta t = 1.01E^{-5} (T_b)_m - 1.19E^{-5} T_x^* + 1.85E^{-6} (T_d)_m + 1.04E^{-6} \) \\
\( \text{for} \ (T_x = T_x^*, 0 < U_x < 1) \)

\( (T_x)_{m+1} - (T_x)_m \over \Delta t = 1.6E^{-3} (T_b)_m - 1.9E^{-3} (T_x)_m + 2.95E^{-4} (T_d)_m + 1.66E^{-4} \) \\
\( \text{for} \ (T_x < T_x^*, U_x = 0) \)

Therefore

1
\( (T_x)_{m+1} = 8.02E^{-4} \Delta t (T_b)_m + (1 - 9.49E^{-4} \Delta t)(T_x)_m + 1.47E^{-4} \Delta t [(T_d)_1 + (T_d)_{N_{max}}] / 2 + 8.29E^{-5} \Delta t \) \\
\( \text{for} \ (T_x > T_x^*, U_x = 1) \)

2
\( (U_x)_{m+1} = (U_x)_m + 1.01E^{-5} \Delta t (T_b)_m + 1.85E^{-6} \Delta t [(T_d)_1 + (T_d)_{N_{max}}] / 2 - 3.01E^{-3} \Delta t \) \\
\( \text{for} \ (T_x = T_x^*, 0 < U_x < 1) \)

3
\( (T_x)_{m+1} = 1.6E^{-3} \Delta t (T_b)_m + (1 - 1.9E^{-3} \Delta t)(T_x)_m + 2.95E^{-4} \Delta t [(T_d)_1 + (T_d)_{N_{max}}] / 2 + 1.66E^{-4} \Delta t \) \\
\( \text{for} \ (T_x < T_x^*, U_x = 0) \)

(3) Equation of \( T_d \) and \( U_d \)

1
\( {dU_d \over dx} - 0.05T_x + 3.89 = 0 \) \( \text{for} \ (T_d = T_d^*, 0 < U_d < 1) \)

2
\( {dT_d \over dx} - 9.42(T_x - T_d) = 0 \) \( \text{for} \ (T_d \geq T_d^*, U_d = 1) \)

So

1
\( (U_d)_{n+1} - (U_d)_n \over \Delta x = 0.05(T_x)_m - 3.89 \) \( \text{for} \ (T_d = T_d^*, 0 < U_d < 1) \)

2
\( (T_d)_{n+1} - (T_d)_n \over \Delta x = 9.42(T_x)_m - 9.42(T_d)_n \) \( \text{for} \ (T_d \geq T_d^*, U_d = 1) \)

Therefore

1
\( (U_d)_{n+1} = (U_d)_n + 0.05\Delta x(T_x)_m - 3.89\Delta t \) \( \text{for} \ (T_d = T_d^*, 0 < U_d < 1) \)

2
\( (T_d)_{n+1} = (1 - 9.42\Delta x)(T_d)_n + 9.42\Delta x(T_x)_m \) \( \text{for} \ (T_d \geq T_d^*, U_d = 1) \)
Appendix III
Appendix IV
#include "udf.h"

DEFINE_SOURCE(mass_source, c, t, dS, eqn)
{
    int ID=3;
    real tempture,source;
    int curr_ts;

    curr_ts=RP_Get_Integer("time-step");
    tempture=C_T(c,t);

    if(tempture>77)
        source =-800;
    else
        source=0;
    return source;
}

DEFINE_SOURCE(energy_source, c, t, dS, eqn)
{
    int ID=3;
    real tempture,source;
    int curr_ts;

    curr_ts=RP_Get_Integer("time-step");
    tempture=0;

    tempture=C_T(c,t);

    if(tempture>77)
        source =-20100*800;
    else
        source=0;
    return source;
}