Improved techniques for detection and localization of underwater acoustic sources using acoustic vector sensors

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Statement of Originality

I hereby certify that the work embodied in this thesis is original work done by me and has not been submitted for a higher degree to any other University or Institute.

_________________________________________  _______________________________________
Date                                            Hari Vishnu
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Contents

Chapter 1 ................................................................................................................................. 1
Introduction............................................................................................................................... 1
1.1. Background and Motivation ......................................................................................... 1
1.2. Challenges ....................................................................................................................... 2
  1.2.1. Low signal-to-noise ratio (SNR) ........................................................................... 4
  1.2.2. Characteristics of noise in the ocean .................................................................... 5
  1.2.3. Signal field in the ocean ......................................................................................... 7
1.3. Research focus ................................................................................................................ 10
  1.3.1. Source detection ................................................................................................... 10
  1.3.2. Source localization ............................................................................................... 11
1.4. Contributions .................................................................................................................. 11
1.5 Thesis outline ................................................................................................................... 13
Chapter 2 ............................................................................................................................... 17
Literature Review..................................................................................................................... 17
  2.1. Ambient noise in the ocean ......................................................................................... 19
  2.2. Acoustic Vector Sensors ............................................................................................ 21
  2.3. Signal source detection .............................................................................................. 23
    2.3.1. Detection in non-Gaussian noise ........................................................................ 24
    2.3.2. Detection using a sensor array ........................................................................... 24
    2.3.3. Detection using an AVS array ............................................................................ 25
  2.4. Source localization ....................................................................................................... 26
    2.4.1. Localization using AVS ...................................................................................... 27
    2.4.2. Near-field source localization ............................................................................ 27
    2.4.3. Shallow water source localization ................................................................... 28
    2.4.4. Source localization in non-Gaussian noise ....................................................... 29
  2.5. Stochastic resonance and suprathreshold stochastic resonance ............................... 30
    2.5.1. SR and SSR based detection .............................................................................. 32
  2.6. Summary ....................................................................................................................... 33
Chapter 3 ............................................................................................................................... 36
Data models ............................................................................................................................ 36
  3.1. Definition of Shallow Ocean ...................................................................................... 36
  3.2. Comparison between models ..................................................................................... 37
  3.3. Signal model ................................................................................................................ 40
    3.3.1. Signal model for an array of APS ...................................................................... 44
    3.3.2. Signal model for an array of AVS .................................................................... 46
    3.3.3. Signal model for an AVS located in the near-field of a source ....................... 49
  3.4. Noise model ................................................................................................................ 51
    3.4.1. Probability density function of noise ................................................................. 52
    3.4.2. Spatial correlation in shallow ocean noise for APS ........................................... 59
    3.4.3. Spatial correlation in shallow ocean noise for AVS ......................................... 62
  3.4. Summary ....................................................................................................................... 63
Chapter 4 ............................................................................................................................... 65
Detection in shallow ocean in presence of Gaussian noise ........................................65
  4.1. Introduction ........................................................................................................65
  4.2. Formulation of detectors ..................................................................................67
    4.2.1. Matched Filter Detector (MFD) .................................................................71
    4.2.2. Energy Detector (ED) ...............................................................................71
    4.2.3. Subspace Detector (SD) ............................................................................72
    4.2.4. Truncated Subspace detector (TSD) .........................................................75
    4.2.5. Approximate signal form detector (ASFD) ..............................................84
  4.3. Performance analysis .........................................................................................87
    4.3.1. Matched filter detector (MFD) .................................................................87
    4.3.2. Energy detector .........................................................................................88
    4.3.3. Subspace detector ......................................................................................89
    4.3.4. Truncated subspace detector (TSD) .........................................................91
    4.3.5. Approximate signal form detector (ASFD) ..............................................92
    4.3.6. Asymptotic performance analysis ..............................................................93
  4.4. Simulation results ..............................................................................................100
  4.5. Summary ........................................................................................................113
Chapter 5 .................................................................................................................116
Detection in shallow ocean in presence of non-Gaussian noise ..........................116
  5.1. Introduction ......................................................................................................116
  5.2. Single sensor detector based on suprathreshold stochastic resonance ...........118
    5.2.1. SSR detector ..............................................................................................121
    5.2.2. Optimization of quantizer noise variance for weak signal detection ......125
    5.2.3. Relation between SSR detector and locally optimal detector ...............127
    5.2.4. Performance analysis of SSR detector ...................................................131
    5.2.5. Optimization of quantizer noise pdf ......................................................139
    5.2.6. SSR detection of non-weak signals ........................................................144
  5.3. Detection using array of acoustic vector sensors ...........................................145
    5.3.1. Optimal detector (OD) ..............................................................................147
    5.3.2. GLRT detectors .........................................................................................149
    5.3.3. SSR detectors .............................................................................................156
    5.3.4. Simulation Results .....................................................................................159
  5.4. Summary ........................................................................................................169
    5.4.1. Single-sensor SSR detector ......................................................................169
    5.4.2. Detection using AVS array ........................................................................171
Chapter 6 ..................................................................................................................172
Source localization in ocean using acoustic vector sensors ...............................172
  6.1. Introduction ......................................................................................................172
  6.2. Localization of sources in shallow water with non-Gaussian noise ...............174
    6.2.1. Azimuth estimation methods .....................................................................175
    6.2.2. SSR denoiser design based on correlation gain ......................................177
    6.2.3. Simulation results .....................................................................................190
  6.3. Near-field localization using a single AVS .....................................................195
    6.3.1. Existing methods .......................................................................................195
    6.3.2. Drawbacks of existing methods and motivation for U-MUSIC ..............197
    6.3.3. The uni-AVS MUSIC method .................................................................200
    6.3.4. A performance measure for comparison ..................................................209
List of abbreviations

3D : Three Dimensional
APS : Acoustic Pressure Sensors
ASFD : Approximate Signal vector Form Detector
AVS : Acoustic Vector Sensors
cdf : cumulative distribution function
CGM : Cauchy Gaussian Mixture
CLT : Central Limit Theorem
CRB : Cramer Rao Bound
DIFAR : Directional Frequency Analysis and Recording
DOA : Direction Of Arrival
ED : Energy Detector
ENR : Signal energy to noise power Ratio
ESPRIT : Estimation of Signal Parameters via Rotational Invariance
GLRT : Generalized Likelihood Ratio Testing
GM : Gaussian Mixture
GG : Generalized Gaussian
HCA : Horizontal Circular Array
HLA : Horizontal Linear Array
i.i.d : independent and identically distributed
LO : Locally Optimal
LRT : Likelihood Ratio Test
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFD</td>
<td>Matched Filter Detector</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimate</td>
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<tr>
<td>MSD</td>
<td>Mean Square Difference</td>
</tr>
<tr>
<td>MUSIC</td>
<td>MUltiple SIgnal Classification</td>
</tr>
<tr>
<td>NMSE</td>
<td>Normalized Mean-square Signal estimation Error</td>
</tr>
<tr>
<td>NP</td>
<td>Neyman Pearson</td>
</tr>
<tr>
<td>NQ</td>
<td>Noisy Quantizer</td>
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<td>OD</td>
<td>Optimal Detector</td>
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<tr>
<td>pdf</td>
<td>probability density function</td>
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<tr>
<td>R-MUSIC</td>
<td>Rayleigh-MUSIC</td>
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<tr>
<td>RLEE</td>
<td>Relative Location Estimation Error</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
</tr>
<tr>
<td>ROCs</td>
<td>Receiver Operating Characteristics</td>
</tr>
<tr>
<td>RSSI</td>
<td>Received Signal Strength Indication</td>
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<tr>
<td>$\alpha S$</td>
<td>Symmetric $\alpha$-stable</td>
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<tr>
<td>SD</td>
<td>Subspace Detector</td>
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<tr>
<td>SIM</td>
<td>Subspace Intersection Method</td>
</tr>
<tr>
<td>SR</td>
<td>Stochastic Resonance</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SSR</td>
<td>Suprathreshold Stochastic Resonance</td>
</tr>
<tr>
<td>SSR-MUSIC</td>
<td>SSR enhanced MUSIC</td>
</tr>
<tr>
<td>SSR-SIM</td>
<td>SSR enhanced SIM</td>
</tr>
<tr>
<td>SSR-UD</td>
<td>SSR</td>
</tr>
<tr>
<td>ST</td>
<td>Students t- distribution</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<td>--------------</td>
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<tr>
<td>STD</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>TSD</td>
<td>Truncated Subspace Detector</td>
</tr>
<tr>
<td>UD</td>
<td>Unconstrained Detector</td>
</tr>
<tr>
<td>U-MUSIC</td>
<td>uni-AVS MUSIC</td>
</tr>
<tr>
<td>UMP</td>
<td>Uniformly Most Powerful</td>
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<tr>
<td>VLA</td>
<td>Vertical Linear Array</td>
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List of mathematical symbols

\( f \) : frequency of signal
\( \omega \) : angular frequency
\( \lambda \) : wavelength of signal
\( k \) : fundamental wave number
\( r, r_s \) : range of source from reference (subscript \( s \) added for chapter 6)
\( z_s \) : depth of source in channel
\( \phi, \phi_s \) : azimuth of source with respect to reference (subscript \( s \) added for chapter 6)
\( \hat{r} \) : estimate of range
\( \hat{\phi} \) : estimate of azimuth
\( \omega_s \) : source parameter vector
\( \omega \) : vector of parameters being estimated
\( a \) : array manifold/steering vector computed for the source location parameters
\( a_{near} \) : 4×1 AVS array manifold for source in near-field
\( R \) : number of significant energy-carrying rays
\( h \) : depth of ocean channel
\( M \) : number of propagating normal modes in shallow ocean channel
\( k_m \) : modal wave number (real part of the eigenvalue) of \( m^{th} \) normal mode
\( \zeta_m \) : attenuation coefficient (imaginary part of the eigenvalue) of \( m^{th} \) normal mode
\( \delta \) : attenuation in ocean bottom
\( \psi_m(z) \) : normalized eigenfunction of the \( m^{th} \) normal mode at depth \( z \)
\( \psi_s \) : elevation angle of source with respect to \( x-y \) plane (chapter 6)
\( \hat{\psi} \) : estimate of elevation
\( c \) : velocity of sound in water
\( \rho \) : density of water column
\( c_b \) : velocity of sound in sediment bottom
\( \rho_b \) : density of sediment bottom
\( \lambda_b \) : wavelength in ocean bottom
\( \alpha_c \) : critical angle of sediment bottom boundary
\( N \) : number of sensor elements in sensor array
\( z_a \) : depth of HLA
\( d \) : deflection coefficient
\( d \) : inter-element separation in array
\( \tau \) : extension of pressure sensor from velocity sensor triad (section 6.3)
\( T \) : number of data snapshots collected
\( s(t) \): \( t \)th snapshot of signal vector

\( x(t) \): \( t \)th snapshot of data vector

\( w(t) \): \( t \)th snapshot of noise vector

\( S \): aggregate signal vector (all snapshots of signal vector)

\( X \): aggregate data vector (all snapshots of data vector)

\( W \): aggregate noise vector (all snapshots of noise vector)

\( W \): matrix of noise eigenvectors

\( s_n(t) \): \( n \)th channel measurement in \( t \)th snapshot of signal vector

\( p_m(t) \): complex amplitude of acoustic pressure at the \( n \)th sensor in \( t \)th snapshot

\( p_{\text{comp}} \): complex amplitude of acoustic pressure for compact AVS for source in near-field

\( p_{\text{ext}} \): complex amplitude of acoustic pressure for extended AVS for source in near-field

\( p_{mn}(t) \): contribution of \( m \)th mode to the acoustic pressure at \( n \)th sensor

\( z_n \): depth of the \( n \)th sensor

\( B(t) \): slowly varying complex quantity whose magnitude is proportional to the strength of the source

\( B(\hat{r}, \hat{\phi}, \hat{\psi}) \): normalized MUSIC search spectrum for range search

\( \Omega_{mn}(\phi) \): phase shift with respect to reference sensor, of \( m \)th mode’s contribution at \( n \)th sensor

\( x_m \): real roots of equation (3.11)

\( v_{xn}(t) \): complex amplitude of the \( x \) component of particle velocity at the \( n \)th sensor

\( v_{yn}(t) \): complex amplitude of the \( y \) component of particle velocity at the \( n \)th sensor

\( A(\phi) \): modal steering matrix

\( a_m(\phi) \): modal steering vector for \( m \)th mode

\( b(t) \): mode amplitude vector

\( \zeta \): a random variable

\( \sigma^2 \): variance (of environmental/input noise)

\( u \): parameter of GM pdf

\( e \): parameter of GG pdf

\( \nu \): parameter of ST pdf

\( \alpha \): parameter of CGM pdf

\( \kappa \): angle of line connecting two vertically separated points with vertical

\( J_n \): Bessel function of the first kind of \( n \)th order

\( R_0 \): noise correlation matrix for array of sensors (subscript \( H \) for HLA, \( V \) for VLA)

\( R \): data covariance matrix

\( \hat{R} \): estimate of \( R \)
\( H_j \): hypothesis that \( j \) sources are present

\( \tilde{x}(t) \): pre-whitened version of \( x(t) \)

\( \tilde{s}(t) \): pre-whitened version of \( s(t) \)

\( \tilde{w}(t) \): pre-whitened version of \( w(t) \)

\( P_D \): probability of detection

\( P_{FA} \): probability of false alarm

\( L \): number of quantizers/quantizer-pairs in SSR pre-processor

\( L(.) \): likelihood ratio

\( L_G(.) \): generalized likelihood ratio

\( \hat{s}(t) \): estimate of \( \bar{s}(t) \)

\( \gamma \): test statistic

\( \eta \): detection threshold

: unknown phase (section 6.3)

\( E_s \): total energy of the signal summed over all the snapshots

\( E_s' \): total energy of the truncated signal vectors over all snapshots

\( \lambda \): signal energy to noise power ratio

\( \lambda' \): truncated signal’s energy to noise power ratio

\( \lambda'' \): approximate signal’s energy to noise power ratio

\( \varepsilon \): normalized mean square error

: signal energy (chapter 5)

\( V(\phi) \): \( M \)-dimensional modal subspace spanned by the columns of \( R_0^{-1/2} A(\phi) \)

\( u_m(\phi) \): \( m^{th} \) orthonormal basis vector of \( V(\phi) \)

\( f_c \): upper cut-off frequency

\( \tilde{s}'(t, \phi) \): truncated signal vector

\( M' \): truncated number of modes

\( V'(\phi) \): \( M' \)-dimensional truncated modal subspace

\( M'_opt \): optimal value of \( M' \)

\( P_m a_m(\phi) \): projection of \( a_m(\phi) \) on the truncated modal subspace \( V'(\phi) \)

\( E_{M,w}(\phi) \): \( L_2 \) norm of the projection error vector

\( K_M(\phi) \): \( L_2 \) norm of the modeling error vector

\( g(\phi) \): approximation to \( g_m(\phi) \)

\( V''(\phi) \): \( N \)-dimensional approximate signal subspace

\( Q(.) \): right-tail probability function of the standard normal distribution

\( \chi^2_D \): chi square distribution with \( D \) degrees of freedom

\( \chi^2_D(\lambda) \): non-central chi square distribution with \( D \) degrees of freedom, non-centrality parameter \( \lambda \)

\( F_{D1,D2} \): \( F \)-distribution with \((D1, D2)\) degrees of freedom

\( F'_{D1,D2}(\lambda) \): noncentral \( F \)-distribution with \((D1, D2)\) degrees of freedom, non-centrality parameter \( \lambda \)
$F_{D1,D2}(\lambda_1, \lambda_2)$: doubly noncentral F-distribution with ($D_1$, $D_2$) degrees of freedom, non-centrality parameters ($\lambda_1$, $\lambda_2$)

$\tau$: standard deviation ratio

$\mu_j$: mean of $\gamma$ under $H_j$

$\nu_j$: standard deviation of $\gamma$ under $H_j$

$A$: signal RMS amplitude

$f_w(\xi)$: pdf of environmental noise

$F_w(\xi)$: cdf of environmental noise

$f_q(\xi)$: pdf of quantizer noise

$F_q(\xi)$: cdf of quantizer noise

$q_l$: quantizer noise input to $l^{th}$ quantizer

$\sigma$: SD of quantizer noise (section 5.2)

: SD of environmental noise (chapter 4, section 5.3)

$\sigma_q$: SD of quantizer noise (section 5.3)

$\sigma_{q-opt}$: $\sigma_q$ that maximizes $P_D$ or $G$

$y_l$: output of $l^{th}$ quantizer (single sensor case)

$y_n(l)$: output of $l^{th}$ quantizer pair for $n^{th}$ sensor

$\bar{y}(l)$: output of SSR preprocessor in $l^{th}$ snapshot (single sensor case)

$\bar{y}_n$: output of SSR preprocessor for $n^{th}$ sensor

$y$: denoised array data vector

$Y$: aggregate preprocessor output data vector (all snapshots of $y$)

$T(X)$: test statistic involving of matched filter with $X$. (subscripts SSR, LO, OD designate the test statistics of these detectors).

$m_j$: means of $T_{SSR}(X)$ under $H_j$

$\lambda_j^2$: variance of $T_{SSR}(X)$ under $H_j$

$G_{wq,L}$: processing gain for finite $L$

$G_{wq}$: processing gain for $L \rightarrow \infty$

$\sigma_{opt}$: $\sigma$ that maximizes $G_{wq}(\sigma)$

(5.2)

$G_{wq,SSR}$: processing gain for $L \rightarrow \infty$, $\sigma = \sigma_{opt}$

$J_{wq}$: mean-square difference between transform

$N_q$: NQ transform

$S_{wq}$: SR transform

$\hat{e}_j$: estimate of $e$ under $H_j$

$u$: signal eigenvector

$A_{MUSIC}$: MUSIC ambiguity function

$A_{SIM}$: SIM ambiguity function

$D(\phi)$: $(3N \times (M+1))$ matrix formed by concatenation of $A(\phi)$ and $u$

$C_{SX}$: noisy correlation

$C_{SY}$: denoised correlation

$G$: correlation gain

$u_\phi, u_\psi$: truncated signal eigenvectors

$a_\phi, a_\psi$: truncated steering vectors
\( u_i \): \( i \)th element of \( \mathbf{u} \)

\( c_1, c_2 \): constants

\( l, m, n \): expressions defined in section 6.3

\( S_{\text{comp}} \): range sensitivity of compact AVS

\( S_{\text{ext}} \): range sensitivity of extended AVS

\( d_c \): critical separation

\( d_c' \): predicted value of critical separation

\( (x_s, y_s, z_s) \): \((x, y, z)\) coordinates of source

\( \Delta \phi \): error in azimuth estimate
### Table of Figures

Fig. 3.1: Array geometry: (a) top view of HLA, (b) side view of HLA/HCA......43
Fig. 3.2: Array geometry: (a) top view of 6-sensor HCA, (b) side view of VLA......43
Fig. 3.3: Variation of kurtosis of (a) GM pdf vs. parameter $u$, and (b) GG pdf vs. parameter $e$.................................................................55
Fig. 3.4: Probability density functions (zero mean unit variance) for (a) GM pdf for different values of parameter $u$, and (b) GG pdf for different values of parameter $e$.....55

Fig. 4.1: Plots of NMSE $e_{TSD}$ and its components ($e_{TSD}^{(1)}(M')$ and $e_{TSD}^{(2)}(M',\phi)$) vs. $M'$ for HLA. (a) SNR = 0 dB, (b) SNR = 10 dB, (c) SNR = 30 dB........................................78
Fig. 4.2: Plots of NMSE $e_{TSD}$ and its components ($e_{TSD}^{(1)}(M')$ and $e_{TSD}^{(2)}(M',\phi)$) vs. $M'$ for VLA. (a) SNR -10 dB, (b) SNR 0 dB, (c) SNR 10 dB.................................................79
Fig. 4.3: $E_{M,m}^{\prime}(0)$ versus $m$ for different values of $M'$ for HLA. $f = 350$ Hz, $M = 15$. 82
Fig. 4.4: $K_M^{\prime}(0)$ versus $M'$ for (a) HLA and (b) VLA, $f = 350$ Hz, $M = 15$.........................83
Fig. 4.5: Variation of (a) standard deviation ratio and (b) deflection coefficient of TSD vs. number of retained modes $M'$, at different values of SNR, for HLA (solid lines) and VLA (dashed lines), $f = 350$ Hz, array length $N = 6$, source azimuth $\phi = 20^\circ$.................................................................97
Fig. 4.6: Variation of (a) standard deviation ratio and (b) deflection coefficient vs. SNR for $f = 350$ Hz, array length $N = 6$, source azimuth $\phi = 20^\circ$........................................99
Fig. 4.7: Comparison of non-asymptotic and asymptotic theoretical results for the case of known noise variance. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) HLA, (b) VLA .................101
Fig. 4.8: Comparison of theoretical (non-asymptotic) and simulation results for the case of known noise variance. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) HLA, (b) VLA .........103
Fig. 4.9: Bearing estimation errors of SD, TSD and ASFD. (a) Bias vs. SNR. (b) Root-mean-square error vs. SNR.................................................................103
Fig. 4.10: Comparison of theoretical (non-asymptotic) and simulation results for the case of unknown noise variance. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) HLA, (b) VLA ....105
Fig. 4.11: Variation of $P_D$ (at $P_{FA} = 0.001$) with frequency for HLA with 6 sensors. $\text{SNR} = -9$ dB.................................................................107
Fig. 4.12: Comparison of performance of 4-channel AVS ($N = 6$) and 3-channel AVS ($N = 8$). $P_D$ vs. SNR at $P_{FA} = 0.001$ for HLA.................................................................108
Fig. 4. 13: Comparison of performance of HCA (dashed lines) and HLA (solid lines) for the case of known noise variance. \( P_D \) vs. SNR at \( P_{FA} = 0.001 \). ..........................110

Fig. 4. 14: Comparison of simulated performance of AVS and APS HLAs. \( P_D \) vs. SNR at \( P_{FA} = 0.001 \). (a) TSD and ASFD. (b) ED and ASFD .................................111

Fig. 5. 1: Schematic diagram of SSR preprocessor .................................................................120
Fig. 5. 2: (a) Linear matched filter detector. (b) Noisy quantizer (NQ) detector. NQ detector becomes SSR detector if \( \sigma = \sigma_{opt} \) .................................................................122
Fig. 5. 3: Nonlinear characteristics of NQ transform with different non optimal values of \( \sigma \), SR transform and LO transform. Input noise: GM \((u = 0.1)\), quantizer noise:
Gaussian ........................................................130
Fig. 5. 4: Plots of \( G_{eq} \) vs. \( \sigma \) for (a) GG \((e = 0.5)\) and GM \((u = 0.01)\) input noise and (b) CGM \((\alpha = 0.7)\) and ST \((v = 3)\) input noise; and theoretical and experimental plots of \( P_D \) vs. \( \sigma \), for (c) GG \((e = 0.5)\) and GM \((u = 0.01)\) input noise and (d) CGM \((\alpha = 0.7)\) and ST \((v = 3)\) input noise. \( P_D \) is plotted for \( P_{FA} = 0.1, T = 300 \). ..........................132
Fig. 5. 5: Plot of \( G_{eq} \) vs. number of quantizers \( L \). Input noise: GM \((u = 0.01)) \).........134
Fig. 5. 6: ROCs of LO detector, SSR detector and matched filter. Input noise: (a) GG \((e = 0.5)\), (b) GM \((u = 0.01)\), (c) CGM \((\alpha = 0.7)\), (d) ST \((v = 3)\). \( A = 0.05, T = 300 \). 134
Fig. 5. 7: Plots of \( P_D(P_{FA} = 0.1, T = 80) \) vs. (a) parameter \( e \) of GG input noise, (b) parameter \( u \) of GM input noise, (c) parameter \( \alpha \) of CGM input noise and (d) parameter \( v \) of ST input noise, for 5 different detectors. ............................................................136
Fig. 5. 8: Plots of (a) optimal \( P_q \) vs. \( e \) and (b) \( \sigma_{opt} \) vs. \( e \), for GG input noise, (c) optimal \( e_q \) vs. \( u \) and (d) \( \sigma_{opt} \) vs. \( u \), for GM input noise, (e) optimal \( e_q \) vs. \( \alpha \) and (f) \( \sigma_{opt} \) vs. \( \alpha \), for CGM input noise, (g) optimal \( e_q \) vs. \( v \) and (h) \( \sigma_{opt} \) vs. \( v \), for ST input noise. Quantizer noise is GG .........................................................140
Fig. 5. 9: Plots of \( G_{eq} \) vs. \( \sigma \) for different values of \( e_q \), (a) GM \((u = 0.01)\), (b) CGM \((\alpha = 0.7)\) and (c) ST \((v = 3)\) input noise. Quantizer noise is GG ..........................................................141
Fig. 5. 10: Plots of \( \sigma_{opt} \) vs. \( A \) for GG \((e = 0.5)\), GM \((u = 0.01)\), CGM \((\alpha = 0)\) and ST \((v = 3)\) input noise. \( P_{FA} = 10^{-4}, T = 80 \) ........................................141
Fig. 5. 11: Plots of \( P_D \left(P_{FA} = 10^{-4}, T = 80 \right) \) vs. \( A \) for SSR detector, OD and matched filter. Input noise: (a) GG \((e = 0.5)\), (b) GM \((u = 0.01)\), (c) CGM \((\alpha = 0)\), (d) ST \((v = 3)\) .........................................................143
Fig. 5. 12: \( P_D \) vs. \( P_{FA} \) at SNR = -10 dB. GG \((e = 0.5)\) noise. (a) \( f = 50 \) Hz, (b) \( f = 350 \) Hz. All detectors are plotted for detection case (a)-when \( \sigma^2 \) and \( e \) are known ........................160
Fig. 5. 13: \( P_D \) vs. parameter \( e \) of GG noise at \( P_{FA} = 0.1 \), SNR = -5 dB, for GLRT and SSR detectors in case (a) ........................................................................162
Fig. 5. 14: \( \sigma_{q,opt} \) of SSR detectors vs. parameter \( e \) of GG noise at \( P_{FA} = 0.1 \), SNR = -5 dB ........................................................162
Fig. 5. 15: \( P_D \) vs. parameter \( e \) of GG noise at \( P_{FA} = 0.1 \), SNR = -5 dB, for GLRT and SSR detectors in case (b) .........................................................166

xvii
Fig. 5. 16: $P_D$ vs. parameter $e$ of GG noise at $P_{FA} = 0.1$, SNR = -5 dB, for GLRT and SSR detectors in case (c)........................................................................................................167

Fig. 6. 1: $\sigma_{q-opt}$ vs. input RMS amplitude $A$ for different GG environmental noises...183
Fig. 6. 2: Correlation gain $G$ vs. input SNR for different GG environmental noises ($e = 0.1$, 0.5 and 1) for $\sigma_q = \sigma_{q-opt}$........................................................................................................183
Fig. 6. 3: Correlation gain vs. input SNR for different values of $\sigma$ for 10-element AVS array in GG ($e = 0.5$) noise ........................................................................................................185
Fig. 6. 4: Correlation gain vs. input SNR for different quantizer noise pdfs for 10-element array in GG ($e = 0.5$) noise for $\sigma_q = \sigma_{q-opt}$.................................................................186
Fig. 6. 5: $\sigma_{q-opt}$ vs. input RMS amplitude $A$ for different quantizer noise pdfs for 10-element array in GG ($e = 0.5$) noise..........................................................................................186
Fig. 6. 6: Correlation gain vs. number of quantizers, at input SNR -5 dB for 10-element AVS array in GG ($e = 0.5$) noise for $\sigma_q = \sigma_{q-opt}$.................................................................188
Fig. 6. 7: a) Bias and b) RMSE of MUSIC estimator vs. SNR (in dB) ..............................189
Fig. 6. 8: Ambiguity function of MUSIC estimator at SNR 4 dB .................................189
Fig. 6. 9: a) Bias and b) RMSE of SIM estimator vs. input SNR (in dB), environmental noise is GG ($e = 0.5$) distributed.................................................................191
Fig. 6. 10: Ambiguity function of SIM estimator at input SNR 4 dB ..............................192
Fig. 6. 11: Probability of resolution of two sources at 43° and 50°.................................192
Fig. 6. 12: (a) Ambiguity function of MUSIC estimator at input SNR 7 dB, showing resolution of sources at 43° and 50°, (b) magnified version of (a)..............................193
Fig. 6. 13: (a) RMSE of azimuth estimates, (b) magnified version of (a) with logarithmic scale, comparing Eigen-RSSI, ESPRIT and U-MUSIC methods. $N = 800$, $(x_o, y_o, z_o) = (63.3, 75.4, 17.4)$ m..........................................................................................212
Fig. 6. 14: (a) RMSE of elevation estimates, (b) magnified version of (a) with logarithmic scale, comparing Eigen-RSSI, ESPRIT and U-MUSIC methods. $N = 800$, $(x_o, y_o, z_o) = (63.3, 75.4, 17.4)$ m..........................................................................................213
Fig. 6. 15: (a) RMSE of range estimates (logarithmic scale), and (b) RLEE of source localization (logarithmic scale) vs. SNR, comparing Eigen-RSSI and U-MUSIC methods. $N = 800$, $(x_o, y_o, z_o) = (63.3, 75.4, 17.4)$ m..........................................................................................213
Fig. 6. 16: Sensitivity of elevation estimate to the azimuth estimate vs. error in azimuth angle estimation, $(x_o, y_o, z_o) = (63.3, 75.4, 17.4)$ m, SNR = -4 dB. ........................................215
Fig. 6. 17: RMSE of range estimates by Eigen-RSSI and U-MUSIC methods vs. separation $d$ of pressure sensors from velocity sensors. $N = 800$, $(x_o, y_o, z_o) = (63.3, 75.4, 17.4)$ m, SNR = 20 dB. ................................................217
Fig. 6. 18: RMSE of range estimates by Eigen-RSSI and U-MUSIC methods vs. range $r_s$ of source from velocity sensors. $N = 800$, $(x_o, y_o, z_o) = (63.3, 75.4, 17.4)$ m, SNR = 20 dB, $d = 5$ m........................217

xviii
Abstract

Our work deals with the detection and localization of acoustic sources in shallow underwater channels. These are signal processing problems of paramount importance in many underwater acoustic applications. The performance of these applications is limited by the high ambient noise in the ocean which is often non-Gaussian. Moreover, the signal field measured at a sensor array, due to an acoustic source in shallow ocean, is generally unknown. We focus on algorithms that employ acoustic vector sensors (AVS), a new type of sensors that are superior to the conventional pressure sensors. This allows the development of improved algorithms without a need for increase in the size of the sensor array to be deployed in the shallow ocean.

The problem of detection is dealt with in three parts – (i) detection using an array of AVS in Gaussian noise, (ii) detection using a single sensor in non-Gaussian noise and (iii) detection using an array of AVS in non-Gaussian noise. In a shallow-ocean detection scenario, implementation of the optimal detector requires complete knowledge of the signal field at the array and is hence impractical. Using generalized likelihood ratio testing, we design suboptimal detectors that utilize partial knowledge of the signal field for such a case. We first formulate these detectors for the simpler case of Gaussian environmental noise and then extend this to the more general and practical case of non-
Gaussian noise. We explore how detection performance in non-Gaussian noise can be enhanced by using an SSR preprocessor and show that this provides a simple and near-optimal solution to detection in a wide range of non-Gaussian noise pdfs.

Source localization algorithms in an ocean environment are severely affected by low SNR and perform poorly because they do not exploit the known fact that the environmental noise is non-Gaussian in nature. We present a method to improve the performance of azimuth estimation algorithms by combining the SSR phenomenon with conventional azimuth estimation methods. This offers better performance with relatively no increase in complexity. We also develop a MUSIC-based approach for three-dimensional localization of a single source in the near-field using a single AVS. This method yields closed-form expressions for the location estimates thus allowing 3D source localization with low computational complexity.
Chapter 1

Introduction

"The formulation of a problem is often more essential than its solution. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advances in science."

- Albert Einstein

1.1. Background and Motivation

Signal processing techniques for ocean environments are of importance in a number of applications [1–4] such as port and maritime security, naval and defense applications, marine shipping and transportation systems, underwater surveying and tracking of marine fauna and geo-acoustic characterization. Underwater acoustic signal processing techniques have been prevalent since the beginning of the twentieth century. The development and efficient implementation of these techniques have been recognized as essential elements to sensing and mastering the oceans. Underwater acoustic applications often employ a variety of signal processing algorithms such as signal
detection, localization, tracking, estimation or characterization of sources and source parameters, estimation of ocean and geological parameters and underwater communication.

Shallow water signal processing is a subset of underwater signal processing which is particularly complex in nature [4]. The reasons for this will be discussed later in this chapter. When the depth of the ocean floor is on the order of a few hundred meters [5], [6], the ocean is referred to as being ‘shallow’. The thrust for research in shallow-water signal processing is increasing because of the wide range of applications centered on shallow-water areas. However, as a medium of wave-propagation the shallow ocean is quite different from air or free-space and offers more challenges from a signal processing perspective. Hence, there is a growing need for signal processing algorithms that are suitable to be used in a shallow-ocean environment and are simple, powerful, efficient and cost-effective.

1.2. Challenges

An underwater channel such as an ocean, lake or river is a dynamic, inhomogeneous and constantly fluctuating medium. One of the characteristics of water, especially salt water, is that it exhibits strong conductivity to electromagnetic waves. Hence, electromagnetic waves that are effective in air and vacuum are easily dissipated in an underwater medium and cannot be used effectively [3], thus rendering the ocean opaque to electromagnetic waves. It is more practical to use acoustic waves which consist of mechanical vibrations, for signal processing applications in the ocean [4].
Acoustic waves undergo less attenuation and thus can propagate over large distances in the ocean. While electromagnetic wave-propagation ranges under current aerial conditions hardly exceed a few kilometers, sound propagation in the ocean can currently be observed at ranges of up to thousands of kilometers.

Acoustic waves play the same role in oceans as that played by electromagnetic waves (such as radio waves) in the atmosphere and in space. However, signal processing in underwater acoustic environments is often hampered by a high level of ambient noise [3], [5], [7]. The transmitted signals are also deformed due to repeated absorption into the medium and attenuation. The shallow ocean is a more complex medium for wave-propagation as compared to the deep sea and therefore signal processing problems in shallow water channels are more complicated in nature. The development of shallow ocean models and signal processing applications based on these models has been a focal point of research in the last century. Over time, researchers have developed detailed mathematical modeling techniques for the shallow ocean waveguide which incorporate many aspects of shallow ocean wave propagation and allow an excellent characterization [5–8]. These have paved the way for superior performance of shallow water signal processing applications. Our work deals with signal processing techniques that are suited to wave-propagation in a shallow ocean environment. These techniques aim to overcome the existing challenges associated with signal processing in a shallow ocean environment and to exploit the known nature of the shallow ocean for improved performance. Our work focusses on two aspects of signal processing: detection and localization.
1.2.1. Low signal-to-noise ratio (SNR)

The performance of source detection and localization algorithms in underwater environments is often limited by the low SNR in the ocean [3], [5], [7]. Signal processing applications in an ocean environment have conventionally utilized arrays of acoustic pressure sensors (APS), also referred to as hydrophones. In low SNR environments such as that encountered in the ocean, it is necessary to collect a larger amount of data so that these signal processing methods perform satisfactorily. This can be done by obtaining more snapshots of data (increasing observation time) or increasing the number of sensors in the array. However, the cost of deployment of an array is usually related to the number of sensors [9]. Increasing the number of sensors entails an increase in the cost, complexity and difficulty of deployment of the arrays. In the case of towed arrays, this also leads to an increase in the drag force on the array elements [10]. Thus, there is a limitation in the usable size of the array. In this work, we employ a new type of sensor known as acoustic vector sensor (AVS) which allows signal processing algorithms to perform better without requiring an increase in the array size.

AVS are sensors that measure the components of particle velocity in addition to the acoustic pressure. AVS technology has been shown in recent years to be more effective than the conventional APS in signal processing applications such as detection, localization, communication, tracking, and inversion [9], [11–19]. The superiority of AVS over APS stems from the fact that velocity measurements provide
additional information about the acoustic field, viz., the direction of the impinging acoustic waves. While an APS array can extract directional information by measuring the propagation delay between sensors, the AVS can obtain this information directly from the velocity measurements. These measurements enable unambiguous localization of underwater sources using shorter and/or sparser AVS arrays, in contrast to APS arrays which also suffer from other drawbacks such as location ambiguities\(^1\) [9]. Since the cost and complexity of deployment of an array is related to the number of sensors [9], [10], it follows that AVS arrays can be deployed more cheaply and effectively than APS arrays to obtain better performance in the low SNR environments encountered in oceans. AVS arrays can also be used in confined spaces since fewer sensors are required. Hence, the AVS is being explored as a superior alternative to APS in signal processing applications. Our work gives special emphasis on development of methods suitable for acoustic vector sensors.

1.2.2. Characteristics of noise in the ocean

Acoustic signals in the ocean are often contaminated by noise and interferences that are different from those encountered in wave-propagation problems in air or free space [4]. Unlike the case of wave-propagation above water, noise in underwater acoustic propagation cannot be adequately modeled by a Gaussian probability distribution [20–22]. Contributions from sources such as geological activity, human and marine biological sources and shipping cause the noise to be impulsive and highly non-

\(^1\) The ‘ambiguity’ refers to the inability of some sensors or sensor arrays to distinguish between waves coming from two different directions, such as right and left (referred to as the right-left ambiguity).
Gaussian in nature. This noise is often better modeled by probability density functions (pdf) with heavier tails than the Gaussian pdf [20–22], especially in the case of shallow ocean noise [23]. The noise in shallow ocean is also spatially correlated because noise from all sources travels through the same oceanic waveguide. Hence the noise has a spatial correlation structure that is determined by the waveguide geometry [24–28].

Classical signal processing algorithms are based on the assumption that the signal is contaminated by Gaussian noise. This assumption is often justified on the basis of the celebrated central limit theorem (CLT) [29]. It also allows development of simple algorithms, and derivation of simple and mathematically tractable expressions describing the performance for most signal processing problems. However, as the noise in the ocean is often non-Gaussian in nature, these algorithms perform poorly when employed in an ocean environment. Moreover, the optimal detection and localization algorithms for non-Gaussian noise are impractical to implement since they are computationally too complex and require prior knowledge of the noise pdf.

In the case of detection, for example, the problem of signal detection is highly simplified by the assumption of Gaussian environmental noise. If this assumption is made, the optimal choice becomes the matched filter detector that may be implemented with ease if the signal is known. However, the performance of the matched filter is sub-optimal in non-Gaussian noise environments. The optimal
detector for such an environment is too complex to implement if the hardware resources are limited.

The formulations of most underwater source localization algorithms also do not take into account the fact that the environmental noise is impulsive in nature. Thus, conventional source localization algorithms exhibit a performance that is inferior to the optimal performance when employed in an ocean environment. Hence it is necessary to explore other near-optimal, simple and robust solutions to detection and localization in non-Gaussian noise. By combining the conventional methods effectively with the nonlinear phenomenon of suprathreshold stochastic resonance (SSR) a considerable improvement in detection and localization performance can be achieved in ocean environments with relatively no increase in complexity.

1.2.3. Signal field in the ocean

A distinctive characteristic of shallow ocean as a wave-medium that sets it apart from the deep sea medium is the phenomenon of multi-mode propagation [5–8]. The deep sea can be treated as a non-dispersive wave-propagation medium. Acoustic waves from each source may be assumed to reach the sensors directly with plane wave-fronts if the source is significantly far from the sensor array or with spherical wave-fronts if the source is close to the sensor array. If the source or receiver is located at shallow depths in the ocean, there is an additional acoustic path due to reflection of the acoustic waves from the sea surface, which can be easily taken into account [6].
However, if the ocean channel itself is shallow, these simple models of propagation are no longer accurate.

Signal propagation in the shallow ocean is characterized by its dispersive and multimodal nature [4]. Propagation takes place through an inhomogeneous and randomly time-varying shallow ocean waveguide which consists of a rough and irregular sea surface at the top and horizontally stratified sediment at the bottom. Acoustic waves propagating in shallow water often undergo multiple reflections from the ocean boundaries (sea surface and sea bottom). The interaction of the waves with the ocean floor results in their attenuation, and detection ranges in shallow water are often limited by this factor. Due to the presence of top and bottom ocean boundaries, the acoustic waves emitted by a signal source arrive at a receiver through several paths that may involve multiple reflections and refractions from these boundaries. These multiple paths have different arrival times. Thus the overall field at a receiver due to an acoustic source is formed by a superposition of contributions from all these multiple paths.

Source detectors in the ocean generally employ a network or an array of sensors rather than a single sensor. Thus it is necessary to incorporate the multi-modal nature of propagation of waves in shallow ocean into the algorithm in order to effectively fuse together the information from different sensors for effective detection of the signal. In this scenario, implementation of the optimal detector (OD) requires complete knowledge of the signal field at the array [1]. However, the signal field at the
array is generally unknown due to the unknown location or time-varying nature of the source, due to which the optimal detector is impractical to implement even if the environmental noise is Gaussian. In such a case, it is necessary to design suboptimal detectors based on generalized likelihood ratio testing (GLRT) [1] that utilize partial knowledge of the signal field in the shallow ocean [13]. Improved detection performance can be achieved if AVS arrays are used in place of APS arrays due to the advantage offered by the inherent directionality of an AVS [9]. Our work aims to develop effective algorithms using the GLRT approach for AVS array based detection in shallow ocean.

In the case of a source in the near-field, the signal field due to this source at an AVS can be used to localize the source completely in terms of its range, azimuth and elevation with respect to the sensor. However, an inherent challenge in performing this localization is the computational complexity involved with the three-dimensional (3D) search required to obtain the source location estimates. In this work, we aim to obtain a low-complexity source localization algorithm that utilizes a single compact AVS.

To summarize, signal processing techniques developed for shallow water channels must take into account the characteristics of the noise and signal field of these channels. Our work presents improved techniques that use acoustic vector sensors for source detection and localization which are two fundamental aspects of signal processing. The methods presented in this work are able to perform better than conventional methods in shallow ocean channels as they effectively incorporate the
properties of shallow ocean signal propagation and noise into their respective algorithms. They also use the advantages of AVS to yield performance superior to that obtained from APS.

1.3. Research focus

1.3.1. Source detection

Signal detection is a fundamental problem in signal processing and there is a large volume of literature devoted to it. Detection is employed in a broad spectrum of applications such as sonar [2], [20], [21], radar [30], watermarking and digital security [31], communication, bio-medicine, speech processing, seismology, economics and control [1]. In the context of shallow-water applications, source detection techniques are used in sonar systems which are an integral part of coastal surveillance and monitoring systems. Sonar is broadly classified into active and passive sonars. The active sonar is different from passive sonar in that the former sends out beams of acoustic waves into a region of observation and detects presence of targets from their reflection of these waves [2], [3]. The passive sonar, on the other hand, simply listens for presence of acoustic sources in the ocean. Detection algorithms are employed in both active and passive sonars. In this work, we focus on the more general research problem of passive detection of acoustic sources using an AVS array.
1.3.2. Source localization

Source localization refers to acquisition of information on the location of a target object. It involves estimation of one or more location parameters of a source of acoustic or electromagnetic waves. Localization algorithms are integral to several fields such as security, astronomy, geology, seismology, defense and transportation [3], [32]. Consequently, the last century has seen the development of a large number of methods for localization of sources. Source localization algorithms are also often extended to source tracking algorithms. We focus on two specific cases of localization: the problem of localization of a source in impulsive noise, and the problem of localization of a near-field source using a single AVS.

1.4. Contributions

This work focuses on methods for improved signal detection and localization in shallow ocean. We focus on algorithms utilizing AVS that are more powerful and effective than those using conventional hydrophones.

A significant contribution in this work is the study of a simple, robust and near-optimal detector that uses the phenomenon of SSR [33], [34]. This detector is able to obtain superior performance of detection in heavy-tailed noise environments with a relatively simple implementation. We present a detailed performance study of this SSR detector and show that it can provide a simple and near-optimal solution to
detection in a wide range of non-Gaussian noise pdfs. We present details of its design and how its performance can be maximized. We also demonstrate that this detector is robust to uncertainty in modeling the noise pdf and is thus an attractive option for improving detection performance in an impulsive noise environment. This detector is developed with the intention of improving detection in underwater acoustics. However the results of this study may also be used for any application in which the signal is contaminated by heavy-tailed noise, such as in seismology, low-frequency atmospherics, digital watermarking [35], [36] and economics [37].

Detection using an array of sensors requires the use of suboptimal GLRT detectors that utilize partial knowledge of the signal field in the shallow ocean [12], [13]. In this work, we present several novel algorithms and strategies to perform effective array-based detection using an array of AVS. These combine inherent advantages of an AVS to obtain improved performance of detection over APS arrays. We first formulate these detectors using the GLRT approach for the simple case of Gaussian environmental noise. We then extend this to the more general and practical case of non-Gaussian environmental noise using the GLRT approach and by extending our results on SSR denoising [33]. The treatment of AVS-array based detection constitutes one of the main contributions in this work.

Source localization is a well explored problem, and the literature contains several effective techniques for source localization. When employed in an ocean environment, the performance of standard source localization methods is inferior to
the optimal performance because they do not exploit the known fact that the environmental noise is impulsive in nature. We present a method to obtain improved performance of azimuth estimation in ocean environments by combining the benefits of the SSR phenomenon with conventional methods of localization. These methods offer better performance with relatively no increase in complexity.

Further we also develop a uni-AVS MUSIC (U-MUSIC) approach for 3D (i.e., azimuth, elevation and range) localization of a single source in the near-field using a single compact AVS [38]. We decouple the 3D localization problem into step by step estimation of azimuth, elevation and range and derive closed form solutions for these parameter estimates, by which a complex 3D search for the parameters can be avoided. We show that the proposed approach outperforms the existing method presented in [39] that uses an extended sensor system consisting of two nodes, when the sensor system is required to be mounted in a confined space. Thus the method developed by us allows 3D localization of a source, with low computational complexity and use of a single compact AVS.

1.5 Thesis outline

This thesis has been organized as follows.

In chapter two, we broadly review the existing literature related to the methods of detection and localization. We touch upon some of the existing methods
for optimal signal processing in the shallow ocean and the challenges associated with these methods. We then discuss the SSR phenomenon that has been fairly well explored in the literature as a means to obtain near-optimal and simple-to-implement algorithms for detection and localization in shallow ocean. We also discuss advances made in the emerging AVS technology and the literature that precedes our contribution to the field of AVS array-based detection.

In chapter three, we present mathematical models of data and assumptions that will be used throughout this work. This includes a detailed discussion on the shallow ocean signal models that were earlier developed for APS and recently extended to AVS models. The different noise pdfs and pdf families used to describe the noise in a shallow ocean environment will be described. Some models describing the spatial correlation of noise in the ocean will also be discussed.

Chapter four deals with detection of acoustic sources in shallow ocean using an array of AVS. It will be shown that just as in other applications, the inherent directionality advantages of an AVS lead to more powerful detection performance than that achieved by an APS array of equal number of elements. The environmental noise is assumed to be Gaussian, so that straightforward and mathematically tractable algorithms based on GLRT testing may be formulated and a detailed analysis of the detection algorithms can be done. This also serves as the basis for the problem of detection in a more challenging non-Gaussian noise environment which will be dealt with in the next chapter.
Chapter five explores the problem of detection of sources in a shallow ocean dominated by non-Gaussian impulsive noise. This problem is more challenging than the one explored in chapter four. In the first subsection, this chapter discusses in detail an SSR-based detection strategy to improve the detection performance of a single sensor in impulsive noise. The single sensor detection problem is considered first so as to gain an in-depth understanding of the SSR preprocessor before applying it to array based detectors. The SSR-detector is shown to provide robust near-optimal performance over a wide range of non-Gaussian noise environments with a relatively simple implementation. This chapter further discusses the problem of AVS array-based detection in non-Gaussian noise. We use the results and insights gained from chapter four on GLRT based detection methods, as well as valuable discussions on SSR detection from the first half of this chapter. By combining our knowledge on SSR denoising with the conventional GLRT approach, we develop methods for detection in shallow ocean dominated by impulsive non-Gaussian noise using an array of AVS.

Chapter six deals with the localization of acoustic sources in the ocean using AVS. The first part of this chapter explores the use of the SSR phenomenon to boost the performance of azimuth estimation methods in the presence of impulsive environmental noise. A preprocessor based on SSR is shown to enhance the performance of azimuth estimation in a shallow ocean environment, and the design of this preprocessor is presented. The second part of this chapter presents a near-field source localization method called U-MUSIC. This method aims to obtain closed-form
expressions for estimates of source location parameters, thus avoiding computationally complex searching algorithms.

In chapter seven, we summarize the major contributions of this work, and describe possible avenues for future research in the direction outlined in the thesis.
In chapter one, we briefly introduced the contents of this work and its contributions towards the vast field of underwater acoustics. Before we discuss our contributions in detail, however, it is essential to take a look at the existing literature on this topic. This chapter discusses existing results, methods, discussions and conclusions that provide the starting point for our contributions.

Oceans make up more than seventy per cent of earth’s surface, and it has been identified for quite some time that sound waves are the key to exploring their vast expanse. This observation has been made by Leonardo Da Vinci as early as the 15th century.
fifteenth century [4]. Early theoretical advances in acoustics followed from advances in physics and mathematics through contributions from scientists such as Galileo, Newton, Rayleigh, Euler, Lagrang, d’Alembert etc. Newton is credited with the first theoretical attempt to describe acoustic propagation in a fluid [6].

However, practical realizations of underwater acoustic applications were possible only by the start of the twentieth century [3] due to availability of sufficient technological know-how. The development of these techniques received a major boost due to world events such as the world wars, sinking of the Titanic and the cold war [3], [5], [40]. English meteorologist Lewis Richardson is said to have filed the first patent for a sonar a month after the sinking of the Titanic [41]. The need to detect German submarines underwater became pressing during World War I when the Allies suffered heavy losses at sea from their attacks. This stimulated the search for technical means of detecting submerged vessels, which led to further development of the sonar. World War II once more enforced the necessity of underwater acoustic techniques to fight submarines and led to a development of new devices for more tasks such as mine control, torpedo guidance systems, etc. Other factors that led to the development of underwater acoustic technology include a pressing need to improve the effectiveness of fishing fleets which arose during the 1940-1950s period [40]. This led to the development of an independent branch of engineering called fish echolocation. In the 1970s, world economics called for shifting the areas of exploration of oil and ore deposits towards the deep-water shelf of coastal seas. As a result, new systems of underwater position finding based on acoustic techniques were added to the traditional
suite of equipment. Further development was spurred by requirements of applications such as guarding of ports, helping of divers and swimmers, underwater drilling etc.

Thus, the need for superiority in military, security and trade spurred the development of several technologies for the detection and localization of sources in an ocean. Nowadays, underwater acoustic processing algorithms are being continuously upgraded and developed for their vast range of applications in exploring the ocean, and terms such as sonar and submarines have become very common indeed. However, one of the main challenges faced in this field is the ambient noise in the ocean.

2.1. Ambient noise in the ocean

In conventional signal processing problems, the environmental noise is considered to be Gaussian distributed. This assumption is justified in the case of wireless communications owing to the central limit theorem (CLT) and also because it makes the detection and localization problems mathematically tractable and simpler to implement. However, this assumption is often violated in underwater acoustics where the noise is impulsive in nature owing to contributions from man-made, natural and biological activities such as snapping shrimp, croaking fish and whales [5], [20–22]. The probability density function (pdf) of this noise is better modeled by pdfs with a heavier tail than that of the Gaussian pdf, especially in the case of shallow ocean acoustics. In the next chapter, we will study in greater detail these mathematical models of the noise in underwater acoustic applications.
Since the ocean acoustic noise is not Gaussian, most of the conventional methods of detection and localization are no longer optimal in nature and sometimes even show degradation in performance owing to the spiky nature of the noise. Development of optimal techniques suited for non-Gaussian noise is too complex and leads to algorithms that are hard to implement owing to their non-linear nature. Indeed, a clear challenge for research in underwater acoustic signal processing is the development of optimal or near-optimal techniques that are feasible, simple to implement and provide fairly good performance. In this work, our focus is on two main signal processing problems of detection and localization. Our aim is to develop efficient detection and localization techniques with particular emphasis on application to non-Gaussian ocean noise environments.

A primary hurdle faced in the effective implementation of detection and localization techniques in underwater environments is the low SNR levels encountered in the ocean due to high ambient noise. To combat the effect of low SNR, signal processing algorithms must use larger arrays of sensors in order to collect more data, which is infeasible and leads to higher costs of implementation. In recent years, the AVS technology has emerged as a superior alternative to the APS, allowing signal processing applications to achieve higher performance without any increase in the array size. In our work, we focus on algorithms that utilize the AVS for detection and localization and aim to achieve better performance than what is possible by using APS. Hence, prior to a discussion on the use of AVS arrays for our algorithms, it is
important to take a look at the AVS technology that has generated a huge amount of research interest in recent years.

### 2.2. Acoustic Vector Sensors

AVS are different from traditional APS in that they measure the components of particle velocity in addition to the acoustic pressure. The AVS technology has advanced rapidly due to the established superiority of AVS over APS in signal processing applications. This superiority stems from the fact that velocity measurements provide additional information about the acoustic field, viz. the direction of the impinging acoustic waves. While an APS array can extract directional information only by measuring the propagation delay between sensors, the AVS can additionally obtain this information directly from the velocity measurements. These measurements also enable unambiguous bearing estimation of underwater sources using shorter and/or sparser AVS arrays in contrast to APS arrays which suffer from several ambiguities [9], [11], [15], [42].

Velocity sensors per se are not a new technology. They have been available, implemented and studied for quite some time. However, the importance of simultaneous measurement of velocity and acoustic pressure was first demonstrated experimentally by D'Spain et al [42] only in 1991, sparking interest in the development of AVS. Since then, several experimental setups such as the DIFAR (Directional Frequency Analysis and Recording) array [14] and the recently conducted Makai experiment [43] have demonstrated the effectiveness of AVS in signal
processing applications. The impressive performance of the vector sensors has been demonstrated in numerous signal processing problems such as source localization [9], [17], [39], [44–63], tracking [19], [64], [65], communication [66–70] and inversion problems [71], [72]. Recently, Krishna and Anand [73] demonstrated that AVS are effective for detection of sources as well. The compactness and improved performance of AVS arrays make them ideal for use in fields such as underwater acoustic surveillance, port security [74] and underwater imaging [75].

The initial theoretical framework for signal processing applications using AVS was introduced by Nehorai and Paldi [15] who presented the measurement model for an AVS for plane-waves propagating in a homogeneous medium. Several theoretical treatments of the AVS then followed, and numerous algorithms for localization of sources employing AVS have been developed. In a series of papers, Hawkes and Nehorai dealt with theoretical aspects of AVS signal processing, such as characterization of the AVS array performance in terms of error measures and the Cramer-Rao bound (CRB) [11], [76], and characterization of intra-sensor and inter-sensor correlation of noise in an AVS array in a homogeneous medium [77]. Hawkes and Nehorai also illustrated how a distributed AVS array can function as a sensor network for localization [78]. In this setup, each AVS can function as a separate node of the network. Zhong et al [19], [64] explored the possibility of two-dimensional (2D) DOA tracking of acoustic sources with AVS using particle filtering and extended it to multi-modality likelihood based tracking. The problem of tracking sources using AVS has also been studied by Santos et al [65]. The feasibility of using AVS in underwater
acoustic communications was demonstrated by Song et al [79], [80], and also Abdi, Chen and their coworkers [66–70]. The results show that AVS provide an effective solution for acoustic communication in underwater platforms, especially when the amount of space available is limited. The AVS may also be employed as an effective measurement system for the estimation of geo-acoustic parameters [72].

The superiority of AVS over APS is thus evident in many applications. We aim to use this advantage of AVS in obtaining improved algorithms for source detection and localization.

2.3. Signal source detection

One of the problems considered in this work is that of signal detection in an ocean environment. The term detection, in general, refers to the process of making a decision about the presence or absence of a source such as an intruding enemy submarine or a marine animal. Signal detection has been studied by Bayes as early as eighteenth century, and classical detection theory has since been furthered by contributions from Legendre, Gauss, Fisher, Neyman and Pearson [81]. Some excellent references on detection theory include books by Kay [1] and Van Trees [32], [81], [82]. The current literature on detection theory covers several aspects pertaining to underwater acoustics, such as sonar/radar signal processing, detection of unknown signals, array signal processing and non-linear detection algorithms.
2.3.1. Detection in non-Gaussian noise

Much of the existing literature on detection makes the simplifying assumption that the environmental noise is Gaussian in nature. If the environmental noise is non-Gaussian in nature, however, optimal detectors (OD) as per the Neyman-Pearson (NP) detection criterion have to be implemented using complex non-linear transformations of the data [1]. The implementation of these transformations is a computationally difficult task. These detectors also require prior knowledge of the noise pdf, and hence their performance is sensitive to errors in modeling the noise pdf. It is therefore of interest to design near-optimal detectors which are easy to implement and are robust with respect to noise modeling errors. One way to do this is to use non-linear preprocessing techniques such as SSR [34] or non-linear wavelet denoising [83] to obtain near-optimal performance. An approach to near-optimal detection in non-Gaussian noise using the phenomenon of SSR will be taken up in a forthcoming chapter.

2.3.2. Detection using a sensor array

In practical scenarios, target detection involves the use of an array of sensors rather than just a single sensor. Array-based detection techniques have been explored in the context of passive radar and sonar in the literature. Beginning from results by Bryn [84], works presented by Vanderkulk [85], Van Trees, Kelly [86], Cox [87] and others [88–94] are a few of the works on array-based detection. Most of these works focus on the approach of optimum array-based detection, and detection using matched subspace detectors [88] and their formulation as a GLRT strategy. Concise reviews of these methods are given in the review paper by Scharf [89] as well as in the text by Van.
Trees [32]. The GLRT is a strategy used for detection when some of the parameters in the detection problem are unknown, by estimating these unknown parameters beforehand through maximum likelihood estimation. The GLRT has been shown to be the uniformly most powerful (UMP) test among the detectors that follow the invariance condition [1], [32], [89] and is a practical choice for detection. This is because very often the source parameters such as the source bearing, strength, range, etc. are unknown during the time of detection. In the context of acoustic signal processing, the existing GLRT methods may be applied well for detection problem using an array of APS. The problem of adapting matched subspace detectors for robust detection in non-Gaussian noise using an APS array has been dealt with by Desai et al [92], [93] previously.

2.3.3. Detection using an AVS array

The problem of signal detection in shallow ocean using an AVS array has received attention only recently beginning with the work of Krishna and Anand [73] who presented modified versions of the subspace detection algorithms for an array of AVS. They showed that it is possible to tap into the inherent directionality power of AVS to achieve much better detection performance than that achievable by an APS array with equal number of elements. However, the methods presented by them suffer from drawbacks such as degradation in performance with an increase in the frequency of the signal, and thus have limited application. We present some methods of detection for AVS array-based detection in underwater acoustic environments, which are simpler and more efficient than those in [73] and are based on a GLRT approach. These also
aim to overcome the drawbacks in the existing work. We commence our treatment of AVS array based detection with the simple assumption that the environmental noise is Gaussian and then navigate to the more practical case when the noise is impulsive in nature [95]. We attempt the problem of detection in non-Gaussian noise by extending the detectors formulated for Gaussian noise, and also attempt to synergize the SSR denoising approach with our newly developed methods for AVS array based detection in impulsive noise.

2.4. Source localization

Source localization refers in general to acquisition of information on the location of the target objects in the ocean. Localization and its associated research problems such as beamforming and array calibration have been studied quite well in the literature. A complete 3D (three dimensional) localization of a source in terms of its cylindrical coordinates would include estimation of the azimuth (or bearing) of the source, its range from a reference point, and its depth in the ocean. Methods which have been developed for general source localization include maximum likelihood estimate (MLE) and maximum a-posteriori estimate [32] methods, quadratic beamforming approaches such as Bartlett [96] and Capon (or minimum variance distortionless response) [97], subspace based methods such as multiple signal classification (MUSIC) [98] and estimation of signal parameters via rotational invariance (ESPRIT) [99], the minimum-norm method [100], maximal entropy method

\[ ^2 \text{Alternatively a source may also be localized in the azimuth-elevation-range (spherical coordinates) space, or Cartesian (x-y-z) space.} \]
Direction-of-arrival (DOA) estimation using arrays has been discussed almost exhaustively in the text by Van Trees [32].

### 2.4.1. Localization using AVS

In the recent past, array signal processing approaches such as the Capon [9], ESPRIT [39], [56–61], [102], ROOT-MUSIC [55], MUSIC [54], SIM [17], [63], MLE [44], [45] and other [46], [103] algorithms have been employed for AVS signal based DOA estimation. Zha and Qiu [58] illustrated how AVS could be employed for bearing estimation when the environment is contaminated by impulsive noise.

### 2.4.2. Near-field source localization

Most of the above mentioned methods make the far-field assumption, viz. that the source is located at a sufficiently far distance so that the wave-fronts of the acoustic waves emanating from the source are considered to be planar in nature. However the localization of near-field sources differs from that of far-field sources because the wave-fronts can no longer be considered planar, and their curvature is a nonlinear function of range.

The problem of near-field localization has found attention in several papers such as [104–107]. Near-field localization has a broad range of applications such as sonar [108], seismic exploration [109] and electronic surveillance [110], and recent research has attempted to elevate the performance of near-field localization using the advantages of an AVS [111], [112]. Tichavsky et al [61] developed an ESPRIT
method using a single vector hydrophone for 2D azimuth and elevation angle estimation for sources in the near-field. Xu et al [57], [102] presented an analysis of a conjugate multiple-invariance ESPRIT method which can be used for direction-finding of non-circular signals using a single AVS. Wu and Wong [39] presented an approach by which complete 3D localization of a near-field source can be done. Their method employs a ‘spatially extended’ AVS which is constructed by using a co-located triad of velocity sensors and a pressure sensor placed at a certain distance away from the triad. This method, which is a combination of eigen-structure based DOA estimation and RSSI based range estimation, will be referred to as the Eigen-RSSI method in our work. Even though Eigen-RSSI provides good performance in localization of sources, its limitation is that it requires the pressure sensor to be located at a specific distance and direction from the velocity sensors. Thus the sensor system loses out on its compactness.

Recently, Wu, Wong and Lau [113] presented the array manifold of a compact AVS (which contains co-located pressure and velocity sensors [15]), that lies in the near-field of a source, thus opening doors for a 3D near-field source localization method using a single AVS.

2.4.3. Shallow water source localization

When the ocean is shallow, the problem of localization of a source becomes more complicated than that in a deep ocean. This is because the signal field at an array of sensors is formed from the contribution of multiple plane waves that emanate from the
source and travel through different paths undergoing reflections from the ocean boundaries. In such a case, estimation of the azimuth of the source using conventional methods is impractical because it requires either (i) knowledge of two other location parameters of the source (eg. range and depth), (ii) a complex multi-dimensional search for unknown source location parameters, or (iii) using the plane-wave assumption which leads to biased estimates of the bearing. To eliminate these difficulties, two techniques for high-resolution bearing estimation of sources in shallow ocean have been developed, namely the subspace intersection method (SIM) [114], [115] and Rayleigh-MUSIC (R-MUSIC) [116], [117]. Shallow water localization has also been studied in [44], [118–122].

2.4.4. Source localization in non-Gaussian noise

In the case of source localization in non-Gaussian noise, implementation of the asymptotically optimal maximum likelihood estimator is often computationally complex and hence practically difficult. Conventional low-complexity beamforming methods do not exploit the fact that the noise pdf is non-Gaussian in nature and they are therefore sub-optimal. In ocean acoustics, since the signals are often distorted and the ambient noise levels are high, a primary hurdle faced in source localization is the low SNR of the signals from sources. In some cases, the performance of source localization methods can even degrade due to the impulsive nature of noise, which is true especially in noise environments that are alpha stable distributed [58]. Hence it is necessary to develop new methods or adapt the existing methods of localization to work in low SNR environments. One method of doing this involves the use of
fractional lower order statistics (FLOS) to adapt the localization or tracking methods to impulsive noise. This is effective when the noise can be modeled as a symmetric $\alpha$-stable ($\alpha$S) process [58], [123].

Multi-sensor denoising of sensor array data is generally used to improve the performance of localization algorithms. A limited improvement in localization performance is possible when the signals are embedded in Gaussian noise [124], [125]. However, it is possible to achieve high SNR gain when the noise is non-Gaussian in nature, by data-denoising using a SSR denoiser [126], and in this thesis we aim to further explore this prospect [119].

2.5. Stochastic resonance and suprathreshold stochastic resonance

A reasonable approach to working in impulsive noise is to discard the impulsive outlier values associated with underwater acoustic noise. This warrants the need for a preprocessor before the detection/localization algorithm with nonlinear ‘limiter’ characteristics which allows the signal to pass through fairly undistorted but limits the outlier values of the noise. Kassam [5] proposed quantization of data for mitigating the effects of impulsive noise for improved signal detection, and then followed some optimum quantization schemes for improved signal processing in non-Gaussian noise. Recently, it has been shown that the phenomena of stochastic resonance (SR) and
suprathreshold stochastic resonance (SSR) may be used to implement low-complexity nonlinear processors for non-Gaussian noise.

The phenomenon of stochastic resonance, discovered in 1981 by Benzi et al [127], is defined as a non-monotonic variation of a system performance measure with respect to input noise intensity. SR manifests itself in several system performance measures such as probability of detection [128–141], output SNR [127], [142], SNR gain [139], [141], [143], mutual (Shannon) information [144], Kullback entropy [145], Fisher information [146], and cross-correlation [147]. SR is exhibited by several classes of nonlinearities, including dynamic bistable systems, static (memoryless) systems with a threshold nonlinearity [142], and also locally linear systems with saturation nonlinearities [148]. A two-level quantizer is the simplest example of the systems of the second class. SR may be realized either by adding noise or by tuning the quantizer threshold [149].

The realization of SR occurs in single nonlinearities wherein the signal alone is ill-positioned to provide an optimal output for some given informational task with zero noise. This may occur in several ways. One example is in the case of threshold nonlinearities, where the signal itself is too weak to trigger a transition from one state to another or to advertise its presence in some other fashion. Another example is in the case of nonlinearities that are locally linear (in the weak signal range), but saturate at higher signal amplitude regions [148]. In both these cases, the beneficial effect of SR arises from the ability of the added noise to displace the operating zone of the
nonlinearity into a region more favorable to the signal. A phenomenon related to SR, called SSR, is similar but works with arrays of nonlinearities. Stocks [150] showed that if a single quantizer is replaced by an array of quantizers and the primary input is supplemented by an additional independent noise at each quantizer, the phenomenon of SSR ensues. The additional noise samples at the quantizer inputs are independent and identically distributed (i.i.d), and the output of the array is obtained by averaging the outputs from all quantizers. This extended version of SR which exhibits itself in arrays of nonlinearities is called suprathreshold stochastic resonance [34], [36], [151]. It has been observed in arrays of saturating systems as well [152]. Preprocessors based on a noisy array of common device nonlinearities such as quantizers and saturating systems, can retain the inherent simplicity of denoising using these simple devices and achieve better performance by utilizing the diversity provided by the injected noise. This injected noise provides an additional degree of flexibility and allows the SSR based preprocessor to achieve an enhancement in the SNR at its output.

2.5.1. SR and SSR based detection

Using optimal quantization (SR) to approximate the nonlinear transformation associated with the optimal detector is a strategy that has been widely used in the design of simple suboptimal detectors in non-Gaussian noise [128–131]. This approach is prompted by the fact that the optimal transform for weak-signal detection reduces to a simple one-bit quantization if the noise is Laplacian. The constructive use of noise to aid detection has also been at the center of research attention in recent years [34], [36], [139–141], [151], [153]. The use of SSR for detection of deterministic
signals in non-Gaussian noise was first proposed by Rousseau et al [34]. They proposed a simple test statistic which is obtained by correlating a replica of the signal with the output of the SSR system. Chen et al [137] have derived sufficient conditions for improving of the performance of a fixed detector by adding noise. Chen and Varshney [138] have considered the SR noise optimization problem for detectors with variable quantizer threshold. Patel and Kosko [151] have derived necessary and sufficient conditions for the existence of NP-optimal SR noise. In a subsequent paper [36] they have also determined the necessary and sufficient conditions for noise-enhanced detection of deterministic signals in non-Gaussian noise using quantizer arrays. Guerriero et al [136] have studied SR effect in the context of sequential detection for shift-in-mean problems. The benefits of SR also extend to noisy quantizer array-based linear mean-squared error estimation [154]. SR based detectors have also been practically implemented to detect watermarks in audio signals [35].

2.6. Summary

This chapter gave a review of the existing literature on topics that we will explore in greater detail in forthcoming chapters. In this chapter, we first reviewed the growth of underwater acoustic signal processing technology over the years. Further, the literature on the signal processing problems of source detection and localization was reviewed. These two fundamental problems have been studied for several decades, hence there is a huge body of literature devoted to these topics and their specific applications such as
shallow water detection/localization, array based detection/localization, algorithms for non-Gaussian noise and near-field localization.

The high level of ambient noise in the ocean and the non-Gaussian nature of this noise have been hurdles in achieving effective performance in detection and localization algorithms in the past. Some approaches have attempted to tackle the problem of low SNR and adapt the algorithms to the case of non-Gaussian noise. These include using preprocessors to denoise the data and provide an SNR gain. One such preprocessor that is of interest in our work is a denoiser based on the phenomenon of suprathreshold stochastic resonance. This phenomenon has been studied in detail in the literature and some work has been done on using SSR to boost the performance of detection. This preprocessor will be used in forthcoming chapters to improve the performance of our algorithms in impulsive noise that is found in ocean environments.

The performance of underwater signal processing algorithms can be improved without increasing the array size or data observed by using a superior kind of sensor known as an acoustic vector sensor. The literature on the AVS has established that it is a good alternative to the APS in many signal processing applications. We focus on development of algorithms that use this sensor for detection and localization, and in forthcoming chapters we will show that these provide effective performance.
Thus this chapter highlights existing standards, benchmarks and methods in the underwater acoustic signal processing literature, and the limitations associated with them. It elucidates the motivation for an advancement of the research in these fields. In general our study of the vast literature on detection and localization indicates that though these methods have been explored in detail in the past, the challenges and shortcomings that exist in their implementation are still considerable. These challenges leave vast space for research on improvement of these techniques to make them more efficient, robust, and cost-effective for underwater environments.
Chapter 3
Data models

Most of the fundamental ideas of science are essentially simple and may, as a rule, be expressed in a language comprehensible to everyone.
-Albert Einstein

In this chapter, we describe the physical and mathematical models of data that are used in our work. The symbols used will be preserved throughout the thesis unless otherwise mentioned. These models which describe the signal field and noise at an array of APS/AVS in a shallow ocean environment are used in the simulation results presented henceforth.

3.1. Definition of Shallow Ocean

In underwater acoustic applications, the concept of ‘shallow’ water has two definitions: hypsometric and acoustic [5]. The hypsometric definition is based on the fact that continents have continental shelves of depths on the order of a few hundred
meters, beyond which the ocean depth rapidly increases. Therefore, a shallow ocean region is often taken to mean continental shelf waters shallower than about 200 m [5], [6]. Acoustically, the ocean is said to be shallow whenever the propagation of sound is characterized by multiple encounters with the sea surface and the sea floor. By this definition, some hypsometrically shallow-water areas may be acoustically deep, and vice-versa. In this thesis, we follow the acoustic definition of shallowness. Hence, shallow-water regions are distinguished from deep-water regions by the relatively greater role played in shallow-water by the reflecting and scattering boundaries. Thus, besides the water depth, the sea floor is a very important part of the marine environment that distinguishes shallow-water propagation from deep-water propagation.

3.2. Comparison between models

Acoustic propagation models can be broadly classified into five categories based on the assumptions used in the models and based how they are formulated from different solutions to the wave equation [5]. These categories include the ray theoretic models [6], [155–159], normal mode models [160–162], multipath expansion models [163], fast field models [164–166] and parabolic equation models [167].

The simplest among these are the ray theoretic and normal mode models. The fast field (or wavenumber integration) models are accurate, but involve computation of the complete spectral form of the wavefield numerically, which is often computationally expensive. These models reduce to the simpler normal mode models
in the case of a far-field source [161]. Parabolic equation models are useful over other methods when range-dependent environments are to be taken into account [161]. Ray theoretic models assume that the wave amplitude varies slowly with position when compared to the phase. These models compute the signal field at a point using ray-tracing, and they are relatively simple at short ranges when the number of rays to be computed is fewer [4]. Normal-mode solutions are derived from an integral representation of the wave equation, and assume that the energy propagation through the shallow ocean channel can be considered as a sum of propagating normal modes. One of the advantages of normal mode methods is their better accuracy as compared to simple ray theoretic methods because they inherently take into account properties of wave-propagation such as dispersion [5]. They also have an advantage over ray theoretic models in that the sound field can be easily calculated for any given combination of frequency, source depth and receiver depth. Ray models, on the other hand, must be sequentially executed for each change in source or receiver depth.

Since the number of modes increases with increase in the frequency, normal mode solutions are very complex when the frequency of the signal is high. Some normal mode models require assumptions like cylindrical symmetry of the medium to obtain simple and practical solutions in such a case. Thus one of the disadvantages of normal mode methods as compared to ray-theoretic approaches is the high computational burden when the frequency is high [4]. The degree of information required concerning the structure of the sea floor is also high for a normal mode computation [5]. The complexity of the ray theoretic and normal mode models can be
compared in terms of the number of rays and modes to be computed by these two approaches respectively [8]. Let a narrowband source with center frequency \( f \) be located at a depth \( z_s \) in a channel of depth \( h \). The number of significant energy-carrying rays in the ray model is approximately given by

\[
R \approx \frac{2r}{h} \tag{3.1}
\]

where \( r \) represents the range of the source from a reference sensor, and \( h \) represents the depth of the ocean channel. In the normal mode approach, the number of modes is approximately computed as

\[
M \approx \frac{2h\lambda}{\lambda} = \frac{2hf}{c} \tag{3.2}
\]

where \( c \) is the velocity of sound in water, and \( \lambda \) is the wavelength of the propagating signal. Hence the number of modes is more than the number of significant rays when \( M \gg R \), i.e. when

\[
f \gg \frac{cr}{h^2} \tag{3.3}
\]

The above condition represents a case where the ray model can be considered significantly less complex as compared to the normal mode model. It can be seen that for low signal frequency or large source range, the normal mode model offers a more computationally feasible and accurate solution. In the case of near-field sources, however, the normal mode description of the acoustic field is inaccurate. This is because when the source is in the near-field, a component of the acoustic field called the continuous spectrum becomes prominent, whereas it is discarded in the normal mode model. Hence, a thumb rule to assess the applicability of normal mode models is
that they can be considered valid for source ranges which are greater than ten source wavelengths [161].

### 3.3. Signal model

This section describes the signal models used for the simulation results presented henceforth. In all sections except section 6.3, we consider the signal field in a shallow ocean due to a narrowband source at an array of sensors. The signal field at an AVS in deep ocean due to a source located in the near-field is presented separately in subsection 3.3.3. The rest of this section describes the shallow ocean model for an array of sensors. Due to the higher accuracy of the normal mode methods in the far-field case, we will consider the normal mode model [4], [7]. This is done for the sake of accuracy, and for practical considerations one may also choose to employ the ray model if computational complexity is a concern at high frequencies or short source ranges.

We consider range-independent propagation that assumes a horizontally stratified ocean. In this case, properties of the ocean such as sound speed, bathymetry and sea bottom composition are considered invariant in range and azimuth with respect to a sensor array, and may vary only with depth. We will use the widely used Pekeris model [7]. This is a simple model that incorporates all characteristic features of underwater acoustic propagation in shallow ocean including multipath propagation. This model is found to be adequate to illustrate the performance of the processing techniques that we will be considering in our work for shallow ocean applications.
The Pekeris model comprises a homogeneous water layer of constant depth over a fluid half-space of sediment. The radiation from a source at a given frequency can be decomposed into an angular spectrum of plane waves. Each of these plane wave components undergoes multiple reflections at the ocean boundaries. The components which undergo a phase shift of an integral multiple of $2\pi$ after each cycle (one surface and one bottom reflection) interfere constructively. The components that do not interfere constructively become progressively smaller and are negligible at far distances from the source. Each of the surviving components is called a normal mode. Each normal mode also has the property of dispersion which implies that its phase velocity is a function of frequency.

For a channel with given depth the number of normal modes increases with frequency. When a pulse of finite duration is transmitted through a shallow water channel it splits into several modal pulses during propagation. Each modal pulse is composed of a band of frequencies and since different frequency components travel with different velocities, every modal pulse gets stretched as it propagates with higher frequencies coming in at the front end and lower frequencies trailing behind. Thus the signal field due to the pulse is composed of a partially overlapping train of pulses. Some works in the literature suggest that dynamically tracking the normal modes and channel parameters of the ocean prior to signal processing applications can help improve the performance of these applications [168–170].
We consider a narrowband source to be located at a depth $z_s$ in a shallow Pekeris channel of depth $h$. Consider an $N$-element sensor array that collects $T$ number of snapshots of data. The Pekeris channel consists of a water column of density $\rho$ over sediment bottom of density $\rho_b$ and a sound speed $c_b$. The source is located at an azimuth (or bearing) $\phi$, a depth $z_s$ and a range $r$ with respect to the reference element of the array\(^3\). The sensor array is assumed to be in the far-field region with respect to the source. We consider a uniform horizontal linear array (HLA), uniform horizontal circular array (HCA) and uniform vertical linear array (VLA) configurations. The HLA and VLA configurations are often considered in underwater acoustic monitoring systems because they lead to simpler mathematical expressions in signal processing algorithms. This is due to the fact that the inter-sensor spacing (and hence the phase difference of waves incident on these sensors) is constant [32]. For example, using the uniform HLA leads to a Van der Monde structure of the array manifold that makes it more convenient to formulate many algorithms. Moreover, these arrays can be easily implemented as towed line arrays [171] or as arrays mounted along a ship [172], as has been demonstrated in multiple examples in the literature [14], [43]. The HCA is considered in our study so as to observe the effect of array geometry on the performance of signal processing methods. This was considered especially because some earlier work has shown that a circular geometry has some advantages over a linear geometry in the case of an APS array [173].

\(^3\) Note that in chapter 6 the subscript ‘s’ is added to $\phi$ to denote source azimuth, unlike other chapters.
In the case of an HLA the sensor closest to the source is considered to be the reference sensor, whereas in the case of a VLA the topmost sensor is considered to be the reference. In the case of an HCA, the center of the array is considered to be the reference point. The elements of the array are separated by a uniform distance of \( d \) meters. The source azimuth angle is measured with respect to the end fire direction in case of the HLA, with respect to a diameter of the sensor array circle in the case of an HCA, and an arbitrary direction for the VLA. Fig. 3.1 shows the (a) top view of the geometry of the HLA, (b) side view of an HLA/HCA, and Fig 3.2 shows (a) top view of the geometry of a 6-sensor HCA and (b) side view of a VLA. Note that in the case
of horizontal arrays (HLA or HCA), all the sensors lie at the same depth $z_1$, which is clear from Fig. 3.1 (b), whereas in the case of a vertical array the depths of the sensors vary from $z_1$ to $z_N$ (Fig. 3.2 (b)). The difference in geometry between an HLA and HCA can be seen in their top views: an HLA will be seen as a straight line of sensors (Fig. 3.1 (a)) while an HCA will be seen as an array of sensors lying in a circle (Fig. 3.2 (a)).

### 3.3.1. Signal model for an array of APS

We first consider the signal model for an array of APS (also called hydrophones) in the far-field case. The far-field scenario refers to the cases where $kr >> 1$ [113] where $k = \frac{2\pi f}{c_w}$ is the wave number. Let $x(t)$, $s(t)$ and $w(t)$ denote, respectively, the $i^{th}$ snapshot of the $N \times 1$ data vector, signal vector and noise vector at the $N$-sensor APS array. The signal vector $s(t)$ can be represented as

$$ s(t) = [s_1(t), \ldots, s_N(t)]^T, \quad (3.4) $$

$$ s_n(t) = p_n(t), \; n = 1, \ldots, N \quad (3.5) $$

where $p_n(t)$ denotes the complex amplitude of acoustic pressure at the $n^{th}$ sensor.

According to the normal mode propagation model, in the far-field scenario, the contribution of the so-called ‘continuous spectrum’ of the acoustic field can be neglected, and the pressure $p_n(t)$ can be approximated by a sum of $M$ discrete normal modes [7] which can be represented as

$$ p_n(t) = \sum_{m=1}^{M} p_{mn}(t), \quad (3.6) $$

where $p_{mn}(t)$ denotes the complex amplitude of the acoustic pressure at the $n^{th}$ sensor.
where $p_{mn}(t)$ refers to the contribution of the $m^{th}$ mode to the acoustic pressure at the $n^{th}$ sensor given by [7]

$$p_{mn}(t) = b_{mn}(t) \Omega_{mn}(\phi),$$  \hspace{1cm} (3.7)

where $\Omega_{mn}(\phi)$ denotes the phase shift with respect to the reference sensor, of the $m^{th}$ mode’s contribution at the $n^{th}$ sensor. The term $b_{mn}(t)$ is given as

$$b_{mn}(t) = B(t) \psi_m(z_n) \psi_m(z_n) \exp\left(i k_m r - \zeta_m r\right),$$  \hspace{1cm} (3.8)

where $k_m$ and $\zeta_m$ are the modal wave number (real part of the eigenvalue) and attenuation coefficient (imaginary part of the eigenvalue) of the $m^{th}$ normal mode, $\psi_m(z)$ is the normalized eigenfunction of the $m^{th}$ normal mode, $z_n$ is the depth of the $n^{th}$ sensor. $B(t)$ is a slowly varying complex quantity whose magnitude is proportional to the strength of the source. For a given channel, the number of propagating modes $M$ is, in general, an increasing function of the signal frequency. Note that

(i) for a VLA, $\Omega_{mn}(\phi) = 1$,

(ii) for an HLA, $\Omega_{mn}(\phi) = \exp\left(i(n-1)k_m d \cos(\phi)\right)$, and

(iii) for an HCA, $\Omega_{mn}(\phi) = \exp\left(i k_m \left(x_n \cos(\phi) + y_n \cos(\phi)\right)\right)$, where $(x_n, y_n)$ denote the $x$-$y$ coordinates of the $n^{th}$ sensor with respect to the reference point (center of array).

For a Pekeris channel, the expression for $b_{mn}(t)$ is given as [7]
Chapter 3  Data models

\[ b_{nm}(t) = B(t) \frac{\sin\left(x_m x_n / h\right) \sin\left(x_m x_i / h\right)}{1 - \left(\tan(x_m) / x_m\right) \left(\rho \zeta \sin(x_m) / \rho_b x_m, \right)^2 \exp\left(i k_m r - \zeta_n r \right) / \sqrt{k_m r}}, \]  
(3.9)

where the variable \( \zeta \) is defined as

\[ \zeta = kh \sqrt{1 - \left(c / c_b\right)^2}, \]  
(3.10)

where \( k = 2\pi/\lambda \) is the fundamental wave number. The quantities \( x_m \) for \( m = 0, 1, \ldots, M-1 \) are the real roots of the equation

\[ \cot(x) + (mx)^{-1} \sqrt{\zeta^2 - x^2} = 0, \]  
(3.11)

and \( x_m < \zeta \). The horizontal wave numbers \( k_m \) of the normal modes are given by

\[ k_m = \sqrt{k^2 - (x_m / h)^2} \]  
(3.12)

The number of normal modes \( M \) is given by

\[ M = \left\lfloor 2fh \sqrt{1/c^2 - 1/c_b^2} + 0.5 \right\rfloor \]  
(3.13)

where \( \lfloor x \rfloor \) denotes the largest integer smaller than \( x \). It is worth mentioning here that the number of modes increases with frequency. The above expressions (3.4)-(3.13) describe the Pekeris model of the shallow ocean for an array of APS.

3.3.2. Signal model for an array of AVS

In this subsection, the signal field model for an array of AVS is considered. The normal mode model can be extended to an AVS array. We assume that each AVS in
the array has three outputs, viz. the acoustic pressure and two orthogonal horizontal components of particle velocity. The vertical component of particle velocity is not considered since in the far-field shallow ocean scenario, the inclusion of this additional measurement is found to increase the complexity of the detection and localization algorithms without yielding any significant improvement in performance. This will also be demonstrated in chapter 4. This is because the energy of the z-velocity measurement is low in the case of a far-field shallow ocean scenario. We shall scale the velocity components by the factor $\sqrt{2} \rho c$. As we will see later, this helps maintain dimensional uniformity of the measured quantities and also ensures that all the noise components have equal variance [17].

Let $x(t), s(t)$ and $w(t)$ denote, respectively, the $t^{th}$ snapshot of the $3N \times 1$ data vector, signal vector and noise vector at the $N$-sensor AVS array. The aggregate data vector and aggregate signal vectors of dimension $3NT \times 1$ can be represented as $X = [x^T(1), \ldots x^T(T)]^T$ and $S = [s^T(1), \ldots s^T(T)]^T$. The signal vector $s(t)$ in the $t^{th}$ snapshot can be represented as [17]

$$s(t) = [s_1(t) \ldots s_{3N}(t)]^T,$$

where

$$s_{3n-2}(t) = p_n(t), s_{3n-1}(t) = \sqrt{2} \rho c v_{xn}(t), s_{3n}(t) = \sqrt{2} \rho c v_{yn}(t), n = 1 \ldots N$$

where $p_n(t)$ denotes the complex amplitude of acoustic pressure at the $n^{th}$ sensor and $(v_{xn}(t), v_{yn}(t))$ denote the corresponding complex amplitudes of the horizontal $(x, y)$ components of particle velocity. The relation between acoustic pressure $p$ and particle
velocity $v$ at a point $r = (x, y, z)$ at time $t$ is governed by the law of conservation of momentum

$$
\rho \frac{dv}{dt} + \nabla p = 0.
$$

(3.16)

where $\nabla$ denotes the spatial gradient operator. Using (3.14), (3.15) and (3.16) and invoking the far-field condition ($k_m r >> 1$ for all $m$) we can write [17]

$$
s_{3m-1}(t) = \sqrt{2} \rho c v_m(t) = \sqrt{2} \cos(\phi) \sum_{m=1}^{M} \frac{k_m}{k} p_m(t),
$$

(3.17)

$$
s_{3n}(t) = \sqrt{2} \rho c v_n(t) = \sqrt{2} \sin(\phi) \sum_{m=1}^{M} \frac{k_m}{k} p_m(t),
$$

(3.18)

Hence, the $i^{th}$ snapshot of the signal vector can be expressed as

$$
s(t) = A(\phi) b(t),
$$

(3.19)

where $A(\phi)$ is a $3N \times M$ modal steering matrix and $b(t)$ is the mode amplitude vector. In the case of a horizontal array (denoted by the subscript H), $b(t)$ and $A(\phi)$ are defined as

$$
b_H(t) = B(t) \left[ \psi_1(z_a) \psi_1(z_r) \frac{\exp(ik_r - \zeta_r r)}{\sqrt{k_r r}} ... \psi_M(z_a) \psi_M(z_r) \frac{\exp(ik_M r - \zeta_M r)}{\sqrt{k_M r}} \right]^T,
$$

(3.20)

$$
A_H(\phi) = [a_{1,H}(\phi) \ldots a_{M,H}(\phi)],
$$

(3.21)

and $\{a_{m,H}(\phi); m = 1, \ldots M \}$ are the modal steering vectors for the HLA, defined as

$$
a_{m,H}(\phi) = d_m(\phi) \otimes g_m(\phi),
$$

(3.22)
\[ g_m(\phi) = [1 \sqrt{\frac{k_m}{k}} \cos(\phi) \sqrt{\frac{k_m}{k}} \sin(\phi)]^T, \quad (3.23) \]

\[ d_m(\phi) = [1 \exp(i k_m d \cos(\phi)) \ldots \exp(i(N-1)k_m d \cos(\phi)) ]^T, \quad (3.24) \]

for an HLA, and

\[ d_m(\phi) = [\exp(i k_m (x_1 \cos(\phi) + y_1 \sin(\phi))) \ldots \exp(i k_m (x_N \cos(\phi) + y_N \sin(\phi))) ]^T, \quad (3.25) \]

for an HCA, where the symbol \( \otimes \) denotes the Kronecker product.

In the case of a VLA (denoted by the subscript V), \( b(t) \) and \( A(\phi) \) are defined as

\[ b_V(t) = B(t) \left[ \psi_1(z_i) \frac{\exp(i k_i r - \zeta_i r)}{\sqrt{k_i r}} \ldots \psi_M(z_i) \frac{\exp(i k_M r - \zeta_M r)}{\sqrt{k_M r}} \right]^T, \quad (3.26) \]

\[ A_V(\phi) = [a_{1,V}(\phi) \ldots a_{M,V}(\phi)], \quad (3.27) \]

and the modal steering vectors for a VLA are defined as

\[ a_{m,V}(\phi) = [\psi_m(z_i) g_{m1}^T(\phi) \ldots \psi_m(z_N) g_{mN}^T(\phi)]^T. \quad (3.28) \]

The shallow ocean propagation model based on the normal mode approach detailed in this subsection is used in all chapters of this thesis to model the signal field incident at an array of APS or AVS, with the exception of chapter 6.3.

### 3.3.3. Signal model for an AVS located in the near-field of a source

This subsection presents the signal model for an AVS located in the near-field of a source in a deep ocean environment. This model is used in section 6.3 for the problem of near-field source localization. Assume that a single acoustic signal source is present...
in an isotropic homogeneous ocean medium at spherical coordinates \((r_s, \phi_s, \psi_s)\) with respect to the AVS, where \(r_s\) is the range of the source from the sensor, \(0 < \phi_s < 2\pi\) is the azimuth angle, and \(-\pi/2 < \psi_s < \pi/2\) is the elevation angle measured with respect to the \(x\)-\(y\) plane\(^4\). The wave-fronts of the waves emanating from the source can thus be considered as either spherical for a receiver in the near-field or planar when the receiver is in the far-field.

Let \(x(t), t = 1, \ldots, T\) represent the \(t\)th snapshot of the 4×1 measurement vector at the output of an AVS, which measures the \(x\), \(y\) and \(z\) components of particle velocity and the acoustic pressure. The measurement vector can be written as [113]

\[
    x(t) = a_{\text{near}} s(t) + w(t)
\]

(3.29)

where \(s(t)\) refers to the \(t\)th snapshot of the source signal, \(a_{\text{near}}\) refers to the 4×1 complex array manifold of the signal field measured by an AVS when it is in the near-field of an acoustic source, and \(w(t)\) denotes a 4×1 vector of the additive zero-mean white environmental noise in the measurement of the \(t\)th snapshot. We assume that the velocity components are scaled by the factor \(\rho c\) for the sake of dimensional homogenity. The array manifold of a compact AVS with the scaled \(x\), \(y\) and \(z\) measurements of particle velocities \(v_x\), \(v_y\) and \(v_z\), and the acoustic pressure \(p_{\text{comp}}\) are given by [113]

\(^4\) Note that in section 6.3 the signal model considers spherical coordinates and an AVS with 4 outputs. The \(z\)-velocity is considered here as it is found to be effective when used in the deep ocean scenario.
Since the pressure measurement in the near-field manifold is a function of range, it can be used for estimation of range in addition to azimuth and elevation.

### 3.4. Noise model

Ambient noise in the ocean arises from a number of sources such as wind, currents, ice-cracking, shipping, geological activity and biologically generated noise. In most signal processing algorithms, the noise is generally assumed to be isotropic, Gaussian and temporally and spatially uncorrelated. These assumptions are often made so as to make it simpler and convenient to formulate signal processing algorithms in underwater acoustic applications [29]. These also simplify the theoretical analysis of the algorithms and hence allow a deeper study of these signal processing problems.

However, none of these assumptions regarding the noise is always true. The noise is anisotropic due to the fact that a major contribution to ambient noise comes from wave action that gives rise to a dense random distribution of noise sources in a thin layer of water beneath the ocean surface [25]. The spatial correlation is due to the fact that noise from all sources travels through the same oceanic waveguide as a signal. Hence, the noise has a spatial correlation structure that is determined by the
waveguide geometry. Due to the presence of several impulsive noise sources such as distant shipping, geological activity and snapping shrimp, underwater acoustic noise distribution is generally heavy-tailed in nature [20–22], [29]. Hence, the conventional assumption of Gaussian noise is not valid and it is necessary to consider other distributions to model ambient noise in the ocean.

3.4.1. Probability density function of noise

3.4.1.1. Gaussian and non-Gaussian noise models

The modus operandi followed in our work is to first develop algorithms based on the assumption that the environmental noise is Gaussian distributed. This leads to convenient theoretical analysis of the problems and allows us to do a deeper study into the working of the algorithms. Later on we generalize these algorithms to the practical case of non-Gaussian environmental noise by extending or adapting the algorithms developed for Gaussian environmental noise.

In sections 4 and 6.4, the $N\times1$ environmental noise vector $\mathbf{w}$ at an array of sensors has a complex circular Gaussian pdf given by

$$f_{\text{Gauss}}(\mathbf{w}) = \frac{1}{(\pi^{N} |\sigma^{2}\mathbf{R}_0|)} \exp \left[ -\frac{1}{\sigma^2} \mathbf{w}^H \mathbf{R}_0^{-1} \mathbf{w} \right]$$

where $\mathbf{R}_0$ denotes the spatial correlation matrix of the noise at the elements of the array, and $\sigma^2$ denotes the variance of the noise.
In chapter 5 and section 6.2, noise is assumed to be non-Gaussian in nature. Several noise models are used to model such non-Gaussian noise pdfs. In section 5.2, theoretical and simulation results have been obtained for four different heavy-tailed environmental noise distributions, viz. Gaussian mixture (GM), generalized Gaussian (GG), Cauchy-Gaussian mixture (CGM) and the Students $t$-distribution (ST). The GG and GM models are standard models that have been used commonly in the literature to describe non-Gaussian noise processes [29]. They have been shown to model underwater acoustic noise fairly well [21], and possess several advantages such as simplicity and flexibility. Another family of noise pdfs that has been shown experimentally to model underwater acoustic noise effectively, is the family of $S\alpha S$ distributions [23]. We use the CGM model to capture the characteristics of data corrupted by $S\alpha S$ noise. The ST distribution is considered a natural choice when data that has been modeled using the Gaussian pdf is remodeled to take the impulsiveness of collected data into account. This distribution finds widespread application in a variety of fields including underwater acoustics [37], [174].

Prior to a description of the noise models used, we take a look at a measure of non-Gaussianity of the noise pdf known as kurtosis. Kurtosis is a fourth order statistic of the noise pdf which is a measure of its impulsiveness. The kurtosis of a zero-mean random variable $\xi$ is defined as

$$
\kappa_\xi = \frac{E[\xi^4]}{(E[\xi^2])^2} - 3.
$$

(3.32)
The kurtosis of a Gaussian random variable is zero. Probability density functions with heavier tails than the Gaussian are more impulsive in nature and have a positive value of kurtosis (leptokurtic). Pdfs with lighter tails than Gaussian have a negative value of kurtosis (mesokurtic).

3.4.1.2. Gaussian mixture (GM) mode

One approach to model a non-Gaussian processes is to start from the Gaussian model and account for the appearance of inappropriate or outlier samples. The GM model follows this approach. We consider the following single-parameter family of two-component GM pdfs of a random variable $\xi$ with unit variance

$$f_{GM}(\xi, u) = uf_{Gauss}\left(\xi, \frac{1}{2u}\right) + (1-u)f_{Gauss}\left(\xi, \frac{1}{2(1-u)}\right), \quad 0 < u \leq 0.5.$$ (3.33)

where $f_{Gauss}(\xi, \sigma^2)$ denotes the Gaussian pdf of a zero-mean real random variable $\xi$ with variance $\sigma^2$, given by the expression

$$f_{Gauss}(\xi, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\xi^2}{2\sigma^2}\right).$$ (3.34)

The pdf $f_{GM}(\xi, u)$ maximizes the kurtosis for a given ratio of variances of the components. For $u = 0.5$, the pdf in (3.33) is Gaussian with zero kurtosis, and as $u$ is reduced the kurtosis increases monotonically.
Fig. 3. 3: Variation of kurtosis of (a) GM pdf vs. parameter $u$, and (b) GG pdf vs. parameter $e$

Fig. 3. 4: Probability density functions (zero mean unit variance) for (a) GM pdf for different values of parameter $u$, and (b) GG pdf for different values of parameter $e$
3.4.1.3. \textit{Generalized Gaussian (GG) model}

The GG distribution is another popular model used for non-Gaussian noise [29], with references dating back to 1923 by Subbotin [175]. The pdf of a GG random variable of unit variance with exponential parameter $e$ is given by

$$f_{GG}(\xi, e) = A(e) \exp(-B(e) |\xi|^e), e > 0,$$

where $A(e) = e^{-\sqrt{\Gamma(3/e)} / 2(\Gamma(1/e))^{1/2}}$, $B(e) = e^{\left( \Gamma(3/e) \right)^{1/2}}$, and $\Gamma(.)$ is the Gamma function. The GG distribution is Gaussian for $e = 2$, leptokurtic (heavy-tailed) for $e < 2$, and mesokurtic (light-tailed) for $e > 2$. Impulsiveness increases monotonically as $e$ is decreased.

Figure 3.3 shows the variation of the kurtosis of the (a) GM and (b) GG pdfs, with the corresponding pdf parameters $u$ and $e$ respectively. It can be seen that the kurtosis of the pdfs are zero at $u = 0.5$ and $e = 2$ for the GM and GG pdfs respectively. For values of $e$ or $u$ lower than these values, the pdfs have positive values of kurtosis.

Figure 3.4 shows several instances of the (a) GM pdf and (b) GG pdf at different values of parameters $u$ and $e$ respectively. When $e = 2$ or $u = 0.5$, the pdfs reduce to Gaussian pdf. It can be seen that for any values of the parameters $u$ or $e$ less than these values, the pdfs have heavier tails than the Gaussian pdf.
Section 5.2 deals with complex array data contaminated by complex non-Gaussian environmental noise modeled by the GG pdf. However, in this case the pdf of the array noise vector is different from that described in (3.35) for a single sensor with real data. The pdf of an \( N \times 1 \) noise vector \( w \) at an array with \( N \) elements is modeled by the following circular complex GG pdf with variance \( \sigma^2 \)

\[
f_{GG}(w) = \left( \frac{J(e)}{\sigma^2} \right)^{3N} \exp\left( -\frac{K(e)}{\sigma^2} \sum_{n=1}^{N} |w(n)|^e \right), \quad 0 < e < 2, \quad (3.36)
\]

\[
J(e) = \frac{e \Gamma(4/e)}{2 \pi \Gamma(2/e)} \quad K(e) = \left[ \frac{\Gamma(4/e)}{2 \Gamma(2/e)} \right]^{e/2}.
\]

3.4.1.4. Students t-distribution (ST) model

The pdf of a ST random variable with unit variance and degrees of freedom \( \nu \) is given by

\[
f_{ST}(\xi, \nu) = \frac{((\nu+1)/2)^{\nu/2}}{\sqrt{(\nu-2) \pi}} \left( 1 + \frac{\xi^2}{\nu-2} \right)^{-(\nu+1)/2}, \quad \nu > 2, \quad (3.37)
\]

For a finite value of \( \nu \), the ST distribution represents a class of heavy-tailed leptokurtic distributions with algebraic tails. The distribution approaches the Gaussian distribution with unit variance as the parameter \( \nu \rightarrow \infty \). The ST distribution is often used as an alternative to the normal distribution as a model for data, especially when the observed real data has heavier tails than the normal distribution can account for [37]. It is a natural choice to model data that is corrupted by impulsive noise. It has been widely employed in several applications such as statistical analysis of blood flow, genomics,
Chapter 3  Data models

economics, traffic, etc. [37] and has been shown to be effective in modeling the data in underwater acoustics as well [174].

3.4.1.5.  Cauchy Gaussian mixture (CGM) model

We consider the following family of Cauchy Gaussian mixture (CGM) pdfs given in [176]

\[
\begin{align*}
f_{\text{CGM}}(\xi, \alpha) &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \xi^2 \right) + \frac{\sqrt{2}}{\pi(1+2\xi^2)} - (1-\alpha) - \frac{\sqrt{2}}{\pi(1+2\xi^2)}, \\
&\quad 0 \leq \alpha \leq 1. 
\end{align*}
\]  

The pdf in (3.38) is an approximation to the pdf of the SαS distribution with scale factor \(1/\sqrt{2}\). As the value of mixture parameter \(\alpha\) is decreased, the impulsiveness of the pdf increases. In our work with impulsive noise environments it is interesting to consider the class of SαS distributions [22], [29], [176], [177] which can model a wide range of impulsive noises. They are sometimes more effective than other pdf families such as the GG or GM in modeling noise in waters where snapping shrimp contribute a dominant part of the ambient noise. Their effectiveness in modeling underwater acoustic data has also been shown experimentally [23].

The SαS distributions are a generalization of the Gaussian pdf but are not same as the GG pdfs. The primary disadvantage of the SαS distributions is that most of them do not have closed form expressions for the pdfs. Instead, they can be described only by their characteristic functions. So far only the Cauchy, Gaussian and one-sided Pearson distributions have been found to have closed form expressions. Thus the CGM
pdf aims to encapsulate the properties of SaS distributions into a pdf with a single closed form expression by incorporating a mixture of two of these distributions (Cauchy and Gaussian).

The four families of input noise pdfs mentioned above cover a wide range and variety of symmetric uni-modal heavy-tailed distributions. These pdfs can be used to model ambient noise in underwater acoustic applications [29], [176].

3.4.2. Spatial correlation in shallow ocean noise for APS

The noise at a point in a medium consists of contributions from numerous sources around the point. Since the noise from all sources travels through the same oceanic waveguide, the noise at two spatially separated points in the medium may be correlated. A simple and fairly accurate theoretical model for the correlation in noise in underwater shallow ocean has been provided by Buckingham [25], [26]. Using this model it is possible obtain the spectral correlation between noise samples at vertically [25] and horizontally [26] separated points. Buckingham’s model has been shown to be virtually indistinguishable from the corresponding result for a similar environmental situation predicted from a more general computer simulated model [25], [26]. The model has also been shown to agree with results from measured data taken from sites such as Eureka, North California and Jellicoe Channel, New Zealand [28]. Both these channels were shallow-water channels with fluid, sedimentary seabeds.
Chapter 3 | Data models

The above model assumes that losses due to the bottom are sufficiently small so that continuous radiation from near-field sources may be neglected and the only significant contribution to the noise field is in the form of modal energy from more distant sources. It is assumed that the noise is produced by independent sources located at same depth below the surface. This mathematical assumption is associated with the noise produced by sources such as wind or wave-action on the surface of the ocean. These sources produce sound field which can be represented as a sum of normal modes. If a sufficient number of modes can propagate in the channel, the noise field is substantially homogenous over a large proportion of the water away from the boundaries (ocean surface and floor). The shallow water model used in this work in formulating the correlation model assumes a uniform channel of depth $h$ with the sound speed $c$ which is independent of depth. The water column overlies a semi-infinite, low-loss, sedimentary bottom with the sound speed $c_b > c$.

Consider two points in a homogeneous and azimuthally uniform noise field, and let the angle between the line connecting the points and the vertical be $\kappa$. i.e, when $\kappa = 0$ and $\pi/2$ the points are vertically and horizontally separated respectively. The coherence function (normalized cross-spectral density function) between the noise fluctuations at the two points can be represented as a function of $\kappa$ as [25]

$$\Gamma(d, \omega, \kappa) = \frac{1}{M} \sum_{m=1}^{M} J_0(kd) \left(1 - \left(\frac{m}{M}\right)^2 \sin^2(\alpha_\kappa) \sin(\kappa)\right) \cos \left[\frac{mkd \cos(\kappa) \sin(\alpha_\kappa)}{M}\right]$$ (3.39)
where $J_0$ is the Bessel function of the first kind of zero order, $d$ is the separation of the sensors, $\omega = 2\pi f$ is the angular frequency, $M$ is the total number of modes that the channel can support, and $\sin(\alpha_c) = \sqrt{1 - \left(\frac{c}{c_p}\right)^2}$, where $\alpha_c$ is the critical angle of the bottom boundary. When $\kappa = 0$ (vertically separated points), the Bessel function in (3.39) is unity and the expression for the coherence function of noise at the two points at depths $z_i$ and $z_j$ is obtained as

$$
\Gamma(d,\omega,0) = \frac{\sin\left((L+.5)\frac{\pi d}{h}\right) - \sin\left((L+.5)\frac{\pi (z_i + z_j)}{h}\right)}{\sqrt{2L+1} - \frac{\sin\left((2L+1)\frac{\pi z_i}{h}\right)}{\sin\left(\frac{\pi z_i}{h}\right)}} \frac{\sin\left(\frac{(2L+1)\pi z_i}{h}\right)}{\sin\left(\frac{\pi z_i}{h}\right)} \frac{\sin\left(\frac{(2L+1)\pi z_j}{h}\right)}{\sin\left(\frac{\pi z_j}{h}\right)}
$$

(3.40)

Note that in (3.40), the vertical spatial coherence function is dependent not only on the separation $d = z_j - z_i$, but also on the mean depth of the two observation points given by $0.5(z_i + z_j)$. The coherence is independent of the mean depth when the sensors are located near the center of the channel provided that the total number of modes supported by the channel is greater than ten. This is the condition of quasi-homogeneity. In this case the vertical spatial coherence function can be used to obtain a simpler expression for the coherence function. Under the assumption of quasi-homogeneity and large number of modes ($M > 10$), coherence function is obtained to be

$$
\Gamma(d,\omega,0) = \frac{1}{2M} \left[ \sin\left(\frac{(M+.5)(\pi d)}{h}\right) - 1 \right].
$$

(3.41)

When $\kappa = \pi/2$ (horizontally separated points), (3.39) becomes
\[ \Gamma (d, \omega, \pi / 2) = \frac{1}{M} \sum_{m=1}^{M} J_0 \left( \frac{k d}{1 - \left( \frac{m}{M} \right)^2 \sin^2 \alpha_c} \right)^{1/2} \]  

which is the horizontal coherence function we have used in our simulations.

### 3.4.3. Spatial correlation in shallow ocean noise for AVS

A simple model for the spatial correlation of noise within the measurements of a single AVS has been proposed by Nagananda and Anand [17]. It may be extended to the case of correlation in the noise in the measurements of two AVS. Consider two AVS at depths \(z_i\) and \(z_j\) in a Pekeris channel, each consisting of three sensors which measure the acoustic pressure, and the \(x\) and \(y\) particle velocities scaled by a factor \(\sqrt{\frac{2}{\rho_c}}\). The 3x3 noise correlation matrix for measurements at the two AVS assumes the form [12]

\[ R(z_i, z_j) = \sigma^2 r(z_i, z_j) I_N \]  

where \(I_N\) denotes the \(N\timesN\) identity matrix, \(r(z_i, z_j)\) is defined as

\[ r(z_i, z_j) = \frac{\sum_{m=1}^{M} \sin(\gamma_m z_i) \sin(\gamma_m z_j)}{\sum_{m=1}^{M} \sin^2(\gamma_m z_1)} = r(z_j, z_i) \]  

where \(\gamma_m = \sqrt{k^2 - k_n^2}\) and the parameter \(\sigma^2\) is a measure of noise intensity. Now, if an AVS array of \(N\) elements is considered, the array noise vector \(w\) is of dimension \(3N\times1\). The \(3N\times3N\) correlation matrix of \(w\) can be written as

\[ R_0 = \sigma^2 R \]
The spatial correlation decays rapidly in the horizontal direction, and it can be ignored when the distance is λ/2 (half-wavelength) or larger. Hence, for a horizontal array (denoted by subscript $H$), the matrix $R_0$ is obtained as

$$R_{0,H} = \sigma^2 I_{3N} \quad \text{(3.46)}$$

For a vertical array (denoted by subscript $V$), $R_0$ is obtained as

$$R_{0,V} = \begin{bmatrix} R(z_1, z_1) & R(z_1, z_2) & \cdots & R(z_1, z_N) \\ \vdots & \ddots & \ddots & \vdots \\ R(z_N, z_1) & \cdots & \cdots & R(z_N, z_N) \end{bmatrix} \quad \text{(3.47)}$$

The correlation matrix $R_{0,H}$ is an identity matrix, which implies that the noise at the sensors is considered uncorrelated in case of a horizontal array (i.e., as long as there is no vertical separation of sensors).

### 3.4. Summary

This chapter presented the mathematical models of the signal field at an array of sensors (APS and AVS) due to a narrowband source in a shallow ocean channel. The normal mode model is used to model the signal field due to its accuracy. The probability distributions used in our work to model the ambient noise include Gaussian as well as non-Gaussian noise pdfs. The Gaussian distribution is used in the initial treatment of detection problems in chapter 4. The non-Gaussian noise environment describes a more realistic scenario, and the detection algorithms formulated in chapter 5 as well as the work on localization in section 6.2 assume that acoustic signal sources are located in such an environment. Different probability density functions are used to
describe this non-Gaussian noise which is normally encountered in shallow water acoustic signal processing applications. The correlation of noise at two spatially separated points on the ocean which affects the measurements of a sensor array in a shallow ocean channel, is modeled by the Buckingham model.
Chapter 4
Detection in shallow ocean in presence of Gaussian noise

“Does sound have rhythm? Does it rise and fall like the ocean? Does sound come and go like wind?”
- Myron Uhlberg

4.1. Introduction

Signal detection using an array of sensors finds a wide range of applications such as sonar [2], [20], [21], radar [30], communication, bio-medicine and seismology [32]. This chapter formulates array based detectors for narrowband acoustic sources in shallow underwater channels. These detectors use arrays of AVS whose superiority over APS in localization applications has been established. This superiority of the AVS arises due to its directional measurements of particle velocities. In our work, the objective is to formulate detection methods that use this directionality feature to obtain improved detection performance. Further, this chapter undertakes a detailed study of these detectors and gauges their relative merits and demerits.
In a practical scenario the location of the source and/or environmental parameters are unknown or time-varying due to which an optimal detector cannot be implemented. Hence, we will formulate alternative detectors for the detection of an acoustic source based on the generalized likelihood ratio testing (GLRT) approach. The GLRT detectors will be formulated by making suitable assumptions on the signal and noise models. By using this approach, we formulate four detection strategies with different requirements and performance. Two of these strategies, namely the energy detector (ED) and the subspace detector (SD) are formulated using conventional approaches. Two other detectors, namely the truncated subspace detector (TSD) and approximate signal form detector (ASFD) are formulated from alternative signal models with assumptions made to improve the performance and at the same time reduce the complexity of detection. Theoretical expressions describing the performance of all these detectors will be derived in this chapter. The performance of the detectors and its dependence on signal and environmental parameters (such as the noise level), available prior information (on the signal, environment or noise) and array geometry will be investigated through theoretical analysis and simulations. A performance measure named the normalized mean square estimation error (NMSE) will be introduced to serve as a simple performance indicator. The relationship between the performance of the detectors and the NMSE will be shown.

Through detailed investigations and performance analysis, we will highlight the relative merits and demerits of each detector. We will show that the methods formulated in this chapter are able to exploit the strengths of an AVS array to yield
performance superior to that of an APS array. The TSD provides near-optimal and robust detection performance and has a relatively low requirement on prior information that makes it a good choice for AVS array based detection in the ocean. The ASFD is a good alternative for detection in scenarios where no prior information on the signal is available.

The environmental noise is assumed to be Gaussian in this chapter to provide a simple starting point for the treatment of detection. The chapter is organized as follows. Various detection strategies are formulated in section 4.2. The issue of optimization of the TSD is also discussed in this section. In section 4.3, the theoretical performance analysis of the detectors is presented and relative merits and demerits of the different detection strategies are discussed. Simulation results in support of the theoretical predictions are presented in section 4.4. A summary and conclusions are presented in section 4.5. The work done in this chapter also serves as a springboard for the work on detection in non-Gaussian noise that is discussed in chapter 5 which deals with a more general framework of detection.

### 4.2. Formulation of detectors

The problem of detection of a narrowband source can be cast in the form of the following binary hypothesis testing problem:

$$H_0: x(t) = w(t), \ t = 1, \ldots, T$$
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

\[ H_1 : x(t) = s(t) + w(t), \]  \hspace{1cm} (4.1)

where \( H_0 \) denotes the hypothesis that no sources are present, \( H_1 \) denotes the hypothesis that a single source is present, and \( x(t) \), \( s(t) \) and \( w(t) \) denote the \( t^{th} \) snapshot of the data vector, signal vector and noise vector respectively. If the noise vector \( w(t) \) in the detection problem being considered is correlated, it is known that the data vector is to be ‘prewhitened’ prior to being used in the detection algorithm [1]. Hence for the sake of simplicity we represent the problem as

\[ H_0 : \bar{x}(t) = \bar{w}(t), \hspace{0.5cm} t = 1, \ldots, T \]

\[ H_1 : \bar{x}(t) = \bar{s}(t) + \bar{w}(t), \]  \hspace{1cm} (4.2)

where

\[ \bar{x}(t) = R_0^{-1/2} x(t), \bar{s}(t) = R_0^{-1/2} s(t), \bar{w}(t) = R_0^{-1/2} w(t), \]  \hspace{1cm} (4.3)

are the pre-whitened versions of the \( t^{th} \) snapshot of \( x(t) \), \( s(t) \) and \( w(t) \) respectively. It is assumed that the noise correlation matrix \( R_0 \), defined in (3.46) for HLA/HCA and in (3.47) for VLA is known. The joint likelihood functions of \( T \) snapshots of the data vector under hypotheses \( H_0 \) and \( H_1 \) are given by

\[ f(\bar{X} | \sigma^2; H_0) = \frac{1}{(\pi\sigma^2)^{3NT}} \exp \left[ -\frac{1}{\sigma^2} \sum_{t=1}^{T} \bar{x}^H(t) \bar{x}(t) \right] \]  \hspace{1cm} (4.4)

\[ f(\bar{X} | \sigma^2, \bar{S}; H_1) = \frac{1}{(\pi\sigma^2)^{3NT}} \exp \left[ -\frac{1}{\sigma^2} \sum_{t=1}^{T} (\bar{x}(t) - \bar{s}(t))^H (\bar{x}(t) - \bar{s}(t)) \right] \]  \hspace{1cm} (4.5)
where $\bar{X} = [\bar{x}^T (1),...,\bar{x}^T (T)]^T$, $\bar{S} = [\bar{s}^T (1),...,\bar{s}^T (T)]^T$. The Neyman Pearson (NP) detection theorem states that in order to maximize the probability of detection $P_D$ at a given probability of false alarm $P_{FA}$, the ratio of (4.5) and (4.4) that is known as the likelihood ratio $L(\bar{X})$, or its equivalent, must be used as the test statistic [1]. The logarithm of $L(\bar{X})$ is given by

$$
\log L(\bar{X}) = \frac{1}{\sigma^2} \sum_{t=1}^{T} \left[ 2 \text{Re} \left( \bar{x}^H (t) \bar{s} (t) \right) - \bar{s}^H (t) \bar{s} (t) \right] 
$$

(4.6)

The primary hurdle faced in the detection of a signal in the ocean is that the signal vectors $s(t), t = 1,..., T$ at the sensor array are unknown due to lack of knowledge of the source location and/or uncertainties in source, receiver or environment parameters. Hence, we perform a GLRT after replacing each vector $\bar{s}(t)$ in (4.5) by its maximum likelihood (ML) estimate $\hat{s}(t)$. The ML estimate may use any prior information that is available. Different detectors presented in this chapter use different types of prior information. The noise variance $\sigma^2$ is not known in some cases as such data may not be available or because the variance is a time-varying quantity. If noise-only data is available it may be possible to obtain a good estimate of $\sigma^2$. However, if a reliable a priori estimate of $\sigma^2$ is not available it becomes an additional unknown parameter in the detection problem. Hence, we shall consider the detection problem for two different cases, viz. (a) the noise variance $\sigma^2$ is known, and (b) $\sigma^2$ is unknown.
For case (a), the logarithm of the generalized likelihood ratio (GLR) is obtained from (4.6) on replacing \( \tilde{s}(t) \) by \( \hat{s}(t) \). Hence, the test statistic for this case can be written as

\[
\gamma_{\text{Case(a)}}(\mathbf{X}) = \sum_{t=1}^{T} \left[ 2 \text{Re}(\mathbf{x}^H(t)\hat{s}(t)) - \mathbf{x}^H(t)\hat{s}(t) \right],
\]

where the signal-vector estimates \( \hat{s}(t) \) are different for different detectors, and \( \text{Re}(x) \) denotes the ‘real part of \( x \).

For case (b), the following estimates of \( \sigma^2 \) under \( H_0 \) and \( H_1 \) are readily obtained by maximizing the likelihood functions in (4.4) and (4.5) respectively:

\[
\hat{\sigma}_0^2 = \frac{1}{3NT} \sum_{t=1}^{T} \mathbf{x}^H(t)\bar{x}(t),
\]

\[
\hat{\sigma}_1^2 = \frac{1}{3NT} \sum_{t=1}^{T} [\hat{x}(t) - \hat{s}(t)]^H [\bar{x}(t) - \hat{s}(t)].
\]

Substituting (4.8) into (4.4) and (4.5) respectively, we get the following expression for GLR for case (b):

\[
L_{G,\text{Case(b)}}(\mathbf{X}) = \left[ \frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2} \right]^{3NT},
\]

and further simplification yields the following expression for the test statistic:

\[
\gamma_{\text{Case(b)}}(\mathbf{X}) = \frac{\sum_{t=1}^{T} \mathbf{x}^H(t)\bar{x}(t)}{\sum_{t=1}^{T} [\hat{x}(t) - \hat{s}(t)]^H [\bar{x}(t) - \hat{s}(t)]}.\]
We shall now introduce the different detectors and discuss their performance in detail.

4.2.1. Matched Filter Detector (MFD)

It is well-known that if $s(t)$, $t = 1, \ldots, T$ and $\sigma^2$ are known, the optimal detector in the NP sense is the replica-correlator, otherwise known as a matched filter detector (MFD) [1]. Even though this detector cannot be implemented in practice, we shall consider it solely for the purpose of comparison since the performance of the MFD provides an upper bound on the performance of other realizable detectors. In this case, the log-likelihood ratio in (4.6) leads to the following likelihood ratio test (LRT): ‘Decide $H_1$ if $\gamma_{MFD}(\mathbf{X}) > \eta$’, where

$$
\gamma_{MFD}(\mathbf{X}) = \sum_{t=1}^{T} \text{Re}(\mathbf{x}^H(t)\hat{s}(t))
$$

(4.11)

is the test statistic of the MFD and $\eta$ is the detection threshold that is generally chosen based on the $P_{FA}$ that the detector is designed for.

4.2.2. Energy Detector (ED)

If $s(t)$, $t = 1, \ldots, T$ are completely unknown, the ML estimate of $\hat{s}(t)$ is given by

$$
\hat{s}_{\text{ED}}(t) = \mathbf{x}(t) \text{ for all } t,
$$

(4.12)

If the noise variance $\sigma^2$ is known, the test statistic is given by

$$
\gamma_{\text{ED}}(\mathbf{X}) = \sum_{t=1}^{T} \mathbf{x}^H(t)\mathbf{x}(t)
$$

(4.13)
This is the well-known ED [1] which is presented here only for the sake of comparison.

The normalized mean square signal estimation error (NMSE) of the ED, defined as

$$\varepsilon_{ED} = \frac{\sum_{t=1}^{T} E[\left( \hat{s}_{ED}(t) - \bar{s}(t) \right)^{H} (\hat{s}_{ED}(t) - \bar{s}(t))]}{\sum_{t=1}^{T} E[\bar{s}^{H}(t)\bar{s}(t)]},$$

is given by

$$\varepsilon_{ED} = \frac{3NT}{\lambda}, \quad (4.14)$$

where $\lambda$ is the signal energy to noise power ratio (ENR).

The GLRT does not provide a meaningful solution if the signal is completely unknown and the noise variance is also unknown.

### 4.2.3. Subspace Detector (SD)

The ED described in Section 4.2.2 assumes no prior information about the structure of the signal vector. However, the performance of a detector can often be improved by incorporating more prior information about the signal that may be available, in the detection process. One such detector is the SD [12], [73]. This detector is based on the fact that the $3N$-dimensional signal vector belongs to an $M$-dimensional modal subspace. Thus, instead of measuring the energy of the data vector as ED does, SD measures the projection of the data onto the $M$-dimensional modal subspace. We shall
present here a simpler formulation of the subspace detector and its theoretical performance analysis.

We know that the pre-whitened signal vector can be expressed as
\[ \tilde{s}(t) = R_0^{-1/2} s(t) = R_0^{-1/2} A(\phi)b(t), \]
where \( A(\phi) = [a_1(\phi) \ldots a_M(\phi)] \) is the \( 3N \times M \) modal steering matrix and \( \{a_m(\phi); m = 1, \ldots, M\} \) are modal steering vectors defined in (3.21) and (3.22) or (3.27) and (3.28) for an HLA or VLA respectively. Here and henceforth in the chapter, we omit the subscripts \( H \) and \( V \) for the sake of brevity. The columns of \( A(\phi) \) are linearly independent if \( 3N > M \). Hence the \( 3N \)-dimensional signal vector \( \tilde{s}(t) \) belongs to the \( M \)-dimensional modal subspace \( V(\phi) \) spanned by the linearly independent columns of \( R_0^{-1/2} A(\phi) \), i.e.

\[ V(\phi) = \text{span}\{R_0^{-1/2}a_1(\phi) \ldots R_0^{-1/2}a_M(\phi)\} = \text{span}\{u_1(\phi) \ldots u_M(\phi)\} \]

where \( \{u_1(\phi) \ldots u_M(\phi)\} \) is the orthonormal basis of \( V(\phi) \) obtained through a Gram-Schmidt transformation process. We note that \( \tilde{s}(t) \) depends on \( \phi \) though this dependence is generally suppressed for the sake of brevity. The signal vector \( \tilde{s}(t) \) may therefore be expressed as

\[ \tilde{s}(t; \phi) = U(\phi)\beta(t), \quad U(\phi) = [u_1(\phi) \ldots u_M(\phi)]. \]

The vector \( \beta(t) \) and the azimuth \( \phi \) are unknown. For a given \( \phi \), the subspace \( V(\phi) \) is known if the modal wave numbers \( \{k_m; m = 1, \ldots, M\} \) are known in the case of an HLA/HCA, and if the modal wave numbers as well as the mode functions \( \{\psi_m(z); m = 1, \ldots, M\} \) are known in the case of a VLA. Assuming that this information is available,
the problem of estimating the $3N$-dimensional signal vector $\tilde{s}(t)$ is reduced to the problem of estimating the $M$-dimensional vector $\beta(t)$ and the azimuth $\phi$. The conditional ML estimator of $\beta(t)$ is given by

$$\hat{\beta}(t \mid \phi) = [U^H(\phi)U(\phi)]^{-1}U^H(\phi)\tilde{x}(t).$$

(4.18)

Hence, for a given $\phi$ the estimate of $\tilde{s}(t;\phi)$ can be written as

$$\hat{s}_{SD}(t \mid \phi) = U(\phi)\hat{\beta}(t \mid \phi) = D(\phi)\tilde{x}(t),$$

(4.19)

where

$$D(\phi) = U(\phi)U^H(\phi)$$

(4.20)

is the projection matrix onto the modal subspace $V(\phi)$. On replacing $\tilde{s}(t)$ by $\hat{s}_{SD}(t \mid \phi)$ in (4.5) and maximizing with respect to $\phi$, we get the following estimate of $\phi$:

$$\hat{\phi}_{SD} = \arg \max \left[ \sum_{t=1}^{T} \tilde{x}^H(t)D(\phi)\tilde{x}(t) \right].$$

(4.21)

From (4.19) the unconditional estimate of $\tilde{s}(t)$ can be written as

$$\hat{s}_{SD}(t) = D(\hat{\phi}_{SD})\tilde{x}(t).$$

(4.22)

On substituting (4.22) into (4.7) and (4.10), we obtain the following expressions for the test statistic for case (a) ($\sigma^2$ known) and case (b) ($\sigma^2$ unknown)

$$\gamma_{SD,\text{Case(a)}}(\tilde{X}) = \sum_{t=1}^{T} \tilde{x}^H(t)D(\hat{\phi}_{SD})\tilde{x}(t)$$

(4.23)
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

\[ \gamma_{SD, \text{Case } b}(\tilde{X}) = \frac{\sum_{t=1}^{T} \tilde{X}^H(t)D(\hat{\phi}_{SD}) \tilde{X}(t)}{\sum_{t=1}^{T} \tilde{X}^H(t)(I_{3N} - D(\hat{\phi}_{SD})) \tilde{X}(t)} \]  

(4.24)

4.2.4. Truncated Subspace detector (TSD)

The SD method presented in Section 4.2.3 can be employed only if the columns of \( \mathbf{A}(\phi) \) are linearly independent, i.e. if \( M \leq 3N \). Since the number of modes \( M \) increases as the frequency is increased [7], the applicability of SD is limited by an upper cut-off frequency \( f_c \) that increases with the number of sensors \( N \); a longer array (larger \( N \)) is required for detection of signals of higher frequency. Moreover it is observed that for a given array length \( N \), the performance of SD suffers degradation as the signal frequency \( f \) is increased even if \( f < f_c \). This progressive degradation can be explained by considering the NMSE \( \varepsilon_{SD} \)

\[ \varepsilon_{SD} = \frac{\sum_{t=1}^{T} E[(\hat{s}_{SD}(t) - \bar{s}(t))^H(\hat{s}_{SD}(t) - \bar{s}(t))]}{\sum_{t=1}^{T} E[\bar{s}^H(t)\bar{s}(t)]}. \]  

(4.25)

It can be readily shown that

\[ \varepsilon_{SD} = \frac{MT\sigma^2 + E\left[ \sum_{t=1}^{T} \bar{s}^H(t;\phi) \left(I_{3N} - D(\hat{\phi}_{SD})\right) \bar{s}(t;\phi) \right]}{E_s}. \]  

(4.26)

where \( E_s \) is the total signal energy and \( \hat{\phi}_{SD} \) is defined in (4.21). The quantity \( \bar{s}^H(t;\phi) \left(I_{3N} - D(\hat{\phi}_{SD})\right) \bar{s}(t;\phi) \) is equal to zero if the estimate \( \hat{\phi}_{SD} \) is equal to the true
value $\phi$. We may therefore assume that

$$E\left[\sum_{t=1}^{T} s^H(t;\phi)\left(I_{3\times N} - D(\hat{\phi}_{SD})\right)s(t;\phi)\right]$$

is small compared to $MT\sigma^2$ to arrive at the following result

$$\varepsilon_{SD} = \frac{MT}{\lambda}, \quad (4.27)$$

where $\lambda$ is the ENR defined in (4.15). The NMSE increases linearly with increasing $M$ and hence it increases with increasing frequency.

In order to extend the applicability of the SD to shorter arrays/higher frequencies and to arrest the degradation associated with increasing frequency, we propose a detector called TSD that uses a truncated model of the signal vector obtained by projecting $\tilde{s}(t,\phi)$ onto a truncated modal subspace defined as

$$V'(\phi) = \text{span}\left\{ R_0^{-1/2}a_1(\phi), \ldots, R_0^{-1/2}a_M(\phi) \right\} = \text{span}\{u_1(\phi), \ldots, u_M(\phi)\}, M' < M. \quad (4.28)$$

The set of spanning vectors of $V'(\phi)$ is a subset of the set of spanning vectors of the full modal subspace $V(\phi)$ defined in (4.16). The truncated signal vector $\tilde{s}'(t,\phi)$ can be written as

$$\tilde{s}'(t,\phi) = R_0^{-1/2}A'(\phi)b'(t) = U'(\phi)\beta'(t), \quad (4.29)$$

where $\beta'(t)$ is an $M'$ dimensional vector, and

$$A'(\phi) = [a_1(\phi), \ldots, a_M(\phi)], \quad U'(\phi) = [u_1(\phi), \ldots, u_M(\phi)] \quad (4.30)$$

The conditional ML estimator of $\beta'(t)$ is given by

76
\[ \hat{\beta}'(t \mid \phi) = U'(\phi)^H \tilde{x}(t) . \]  

(4.31)

Hence, for a given \( \phi \) the estimate of \( \tilde{s}'(t, \phi) \) can be written as

\[ \hat{s}_{TSD}(t \mid \phi) = U'(\phi)\hat{\beta}'(t \mid \phi) = D'(\phi)\tilde{x}(t) , \]

(4.32)

where

\[ D'(\phi) = U'(\phi)U'(\phi)^H \]

(4.33)

is the projection matrix onto the truncated modal subspace \( V'(\phi) \). Expressions for the estimate of \( \phi \) and the unconditional estimate of \( \tilde{s}(t) \) using the TSD algorithm can be obtained using a procedure analogous to that for SD described in Section 4.2.3. Thus we get

\[ \hat{\phi}_{TSD} = \arg \max \left[ \sum_{t=1}^{T} \tilde{x}^H(t) D'(\phi) \tilde{x}(t) \right] . \]

(4.34)

\[ \hat{s}_{TSD}(t) = D(\hat{\phi}_{TSD})\tilde{x}(t) . \]

(4.35)

It can be readily shown that the NMSE of the signal estimator defined in (4.35) is given by

\[ \varepsilon_{TSD} \triangleq \sum_{t=1}^{T} \frac{E[(\hat{s}_{TSD}(t) - \bar{s}(t))^H(\hat{s}_{TSD}(t) - \bar{s}(t))]}{\sum_{t=1}^{T} E[\tilde{s}^H(t)\tilde{s}(t)]]} = \varepsilon_{TSD}^{(1)}(M'') + \varepsilon_{TSD}^{(2)}(M', \phi) \]

(4.36)

where

\[ \varepsilon_{TSD}^{(1)}(M'') = \frac{M' T}{\lambda} , \text{ and} \]

(4.37)
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

Fig. 4.1: Plots of NMSE $\varepsilon_{TSD}$ and its components $(\varepsilon_{TSD}^{(1)}(M')$ and $\varepsilon_{TSD}^{(2)}(M', \phi)$ vs. $M'$ for HLA. (a) SNR = 0 dB, (b) SNR = 10 dB, (c) SNR = 30 dB

\[
\varepsilon_{TSD}^{(1)}(M', \phi) = \frac{E \left[ \sum_{t=1}^{T} \bar{s}^H(t; \phi) (I_{3N} - D^T(\hat{\phi}_{TSD})) \bar{s}(t; \phi) \right]}{E_s}
\]

\[
\varepsilon_{TSD}^{(2)}(M', \phi) = 1 - \frac{1}{E_s} \left[ \sum_{t=1}^{T} \bar{s}^H(t) D(\phi) \bar{s}(t) \right]
\]

where

\[
\lambda' = \frac{E_s}{\sigma_x^2}, E_s = \sum_{t=1}^{T} E[\bar{s}'(t)^H \bar{s}'(t)].
\]

In (4.39), $E_s$ is the total energy of the truncated signal vectors over all snapshots and
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

Fig. 4. 2: Plots of NMSE $\varepsilon_{TSD}$ and its components ($\varepsilon_{TSD}^{(1)}(M')$ and $\varepsilon_{TSD}^{(2)}(M',\phi)$) vs. $M'$ for VLA. (a) SNR -10 dB, (b) SNR 0 dB, (c) SNR 10 dB

$\lambda'$ is the ENR for the truncated signal. The NMSE $\varepsilon_{TSD}(M',\phi)$ has two components $\varepsilon_{TSD}^{(1)}(M')$ and $\varepsilon_{TSD}^{(2)}(M',\phi)$. The first component $\varepsilon_{TSD}^{(1)}(M')$ is the error due to noise which is analogous to the MSE $\varepsilon_{SD}$ defined in (4.25). Equation (4.37) indicates that $\varepsilon_{TSD}^{(1)}(M')$ decreases linearly as the number of retained modes $M'$ is reduced. The second component $\varepsilon_{TSD}^{(2)}(M',\phi)$ is the error due to truncation of the normal mode expansion of the signal vector. It is seen from (4.38) that $\varepsilon_{TSD}^{(2)}(M',\phi) = 0$ when $M' = M$. As $M'$ is reduced, $\varepsilon_{TSD}^{(2)}(M',\phi)$ increases; but this increase is very slow.
Illustrative plots of the total MSE and its components versus the number of retained modes $M'$ are shown in Fig. 4.1 for a 6-sensor HLA and three different values of SNR viz. 0, 10 and 30 dB. Similar plots for a 6-sensor VLA are shown in Fig. 4.2 for SNR values -10 dB, 0 dB and 10 dB. The signal frequency is 350 Hz and the number of modes is $M = 15$. The channel parameters, array parameters and source position for these figures are the same as those listed in section 4.4. In Figs. 4.1 and 4.2, the component $\varepsilon_{TSD}^{(1)}(M')$ (red dashed line) increases linearly with the number of retained modes $M'$, as predicted by (4.37). The component $\varepsilon_{TSD}^{(2)}(M',\phi)$ (green dotted line) decreases with increase in $M'$ due to reduction in signal modelling error due to truncation. The blue solid line denotes the sum total of the red and green lines. It is seen from Figs. 4.1 and 4.2 that as $M'$ is reduced from $M$ to 1, the total MSE $\varepsilon_{TSD}(M',\phi)$ reduces and reaches a minimum at an optimal value of $M'$ that is quite small (especially in the case of the HLA) and may even be equal to 1. Let the optimal value of $M'$ be denoted by $M'_\text{opt}$. We note that $\varepsilon_{TSD}(M'_\text{opt},\phi)$ is significantly smaller than $\varepsilon_{SD}$. We can therefore expect the performance of TSD to be significantly better than that of SD. We also note from a comparison of the plots (a), (b) and (c) of Figs. 4.1 and 4.2 that the value of $M'_\text{opt}$ increases with an increase in SNR. The use of a truncated signal model in TSD also has additional advantages of (i) reducing the need for channel information to modal wave numbers of the first $M'$ modes only, and (ii) reducing the computational complexity.
The value of $M'_{opt}$ depends primarily on the signal-to-noise ratio, the degree of correlation among the modal steering vectors $\{a_1(\phi)\ldots a_M(\phi)\}$, the total number of modes $M$ and to a lesser extent on the channel parameters and the location of the source. In general, the value of $M'_{opt}$ for the HLA is smaller than that for the VLA since the modal steering vectors of the HLA are more highly correlated than those of the VLA. It is seen from (4.37) that the rate of reduction of $\varepsilon_T^{(1)}(M')$ with decreasing $M'$ becomes higher if $\lambda$ is reduced. Consequently the value of $M'_{opt}$ decreases as $\lambda$ is reduced. Figures 4.1 and 4.2 illustrate the dependence of $M'_{opt}$ on the more commonly used measure of SNR defined in (4.87). It is seen that $M'_{opt}$ has a larger value at higher SNR, and also that the values of $M'_{opt}$ for an HLA are smaller than those for a VLA.

The error $\varepsilon_T^{(2)}(M',\phi)$ due to truncation of the modal expansion is small because (i) the modal vectors $\{a_1(\phi)\ldots a_M(\phi)\}$ in the expansion of $\tilde{s}(t,\phi)$ are highly correlated and (ii) amplitudes of the discarded higher order modes are generally quite small due to faster attenuation of the higher order modes. Let $P_M a_m(\phi)$ denote the projection of $a_m(\phi)$ on the truncated modal subspace $V(\phi)$ for $m>M'$. The $L_2$ norm of the projection error vector is given by

$$E_{M',m}(\phi) = \|a_m(\phi) - P_M a_m(\phi)\|^2.$$  

(4.40)

It can be readily shown that $E_{M',m}(\phi)$ increases as $|\phi-\pi/2|$ is increased. Hence,
Fig. 4.3: $E_{M',m}(0)$ versus $m$ for different values of $M'$ for HLA. $f = 350$ Hz, $M = 15$.

$E_{M',m}(\phi)$ is maximum at $\phi = 0$. The proximity between $V(\phi)$ and $V'(\phi)$ is illustrated in Fig. 4.3 that shows plots of $E_{M',m}(0)$ versus $m$ at frequency of 350 Hz for a 6-sensor HLA and four different values of $M'$, viz. $M' = 1, 2, 3, 4$. The channel parameters, signal frequency and source range and depth have the same values as in Section 4.4. It is evident that the difference between the full modal subspace $V(\phi)$ and the truncated modal subspace $V'(\phi)$ is negligible for all $m$ and for all $M'>2$. It follows that the signal vector may be modeled using a drastically truncated modal subspace without causing a significant modeling error. This conclusion can be tested by considering the modeling error due to approximation of the signal vector $\tilde{s}(t,\phi)$ by its truncated version $\tilde{s}'(t,\phi)$ defined in (4.29). The $L_2$ norm of the modeling error vector is given by

$$K_M(\phi) = \| \tilde{s}(\phi) - \tilde{s}'(\phi) \|^2.$$  (4.41)
Fig. 4.4: $K_M(0)$ versus $M'$ for (a) HLA and (b) VLA, $f = 350$ Hz, $M = 15$.

In (4.41), the dependence of the signal vector on $t$ is suppressed. Figure 4.4 shows the plots of $K_M(0)$ versus $M'$ for a 6-sensor HLA and also for a 6-sensor VLA for 3 different values of SNR. All the other parameters have the same values as in Fig. 4.3.

It is seen from Fig. 4.4 that the signal modeling error due to modal subspace truncation is negligible for $M' \geq 3$ for the HLA and $M' \geq 7$ for the VLA. Consequently, the optimal value of $M'$ for minimizing the mean square signal estimation error $\varepsilon_{TSD}(M', \phi)$ is very small for an HLA and somewhat larger for a VLA.

Expressions for the test statistics for TSD can be obtained using a procedure analogous to that for SD described in Section 4.2.3. Thus we get
\begin{align}
\gamma_{TSD,\text{Case}(a)}(\tilde{X}) &= \sum_{t=1}^{T} \tilde{x}^H(t)D'(\hat{\phi}_{TSD})\tilde{x}(t), \quad (4.42) \\
\gamma_{TSD,\text{Case}(b)}(\tilde{X}) &= \frac{\sum_{t=1}^{T} \tilde{x}^H(t)D'(\hat{\phi}_{TSD})\tilde{x}(t)}{\sum_{t=1}^{T} \tilde{x}^H(t)(I_{3N} - D'(\hat{\phi}_{TSD}))\tilde{x}(t)}. \quad (4.43)
\end{align}

### 4.2.5. Approximate signal form detector (ASFD)

The SD and TSD seek to achieve better performance than the ED by exploiting the knowledge of the modal wave numbers. If this information is not available, it is still possible to achieve better detection than the ED by using the knowledge of the structure of the vector \( g_m(\phi) \) (see (3.23)) associated with each AVS. Since the modal wave numbers are subject to fairly tight bounds, viz. \( 1 > k_1/k > \ldots > k_M/k > c/c_b \), where \( c = \) sound speed in water, \( c_b = \) sound speed in the ocean bottom and \( k = 2\pi f/c \), we may use the approximation \( k_m/k \approx 1 \) for all \( m \) and hence approximate \( g_m(\phi) \) as

\[
g_m(\phi) \approx g(\phi) = [1 \ \sqrt{2} \cos(\phi) \ \sqrt{2} \sin(\phi)]^T \quad (4.44)
\]

This approximation considerably simplifies the detection problem and leads to a much simplified detection algorithm that we refer to as the approximate signal form detector [12]. The signal vector \( \tilde{s}(t,\phi) = R_0^{-1/2}s(t,\phi) \) can now be approximated by the approximate signal vector defined as

\[
\tilde{s}^a(t,\phi) = R_0^{-1/2}H(\phi)p(t), \quad (4.45)
\]

where
\[ \mathbf{H}(\phi) = \mathbf{I}_N \otimes \mathbf{g}(\phi) = [\mathbf{h}_1(\phi), \ldots, \mathbf{h}_N(\phi)] \]  
(4.46)

is a $3N \times N$ matrix with linearly independent columns, and $\mathbf{p}(t) = [p_1(t), \ldots, p_N(t)]^\top$ where $p_n(t)$ denotes the acoustic pressure at the $n$th sensor. The approximate signal vector $\tilde{s}''(t, \phi)$ belongs to the $N$-dimensional subspace $V''(\phi)$ defined as

\[ V''(\phi) = \text{span}\{\mathbf{R}_0^{-1/2} \mathbf{h}_1(\phi), \ldots, \mathbf{R}_0^{-1/2} \mathbf{h}_N(\phi)\} = \text{span}\{\mathbf{u}_1''(\phi), \ldots, \mathbf{u}_N''(\phi)\} , \]  
(4.47)

where $\{\mathbf{u}_1''(\phi), \ldots, \mathbf{u}_N''(\phi)\}$ is the orthonormal basis of $V''(\phi)$. We can therefore rewrite (4.45) as

\[ \tilde{s}''(t, \phi) = \mathbf{U}''(\phi) \mathbf{\beta}''(t) , \text{ where } \mathbf{U}''(\phi) = [\mathbf{u}_1''(\phi), \ldots, \mathbf{u}_N''(\phi)] \]  
(4.48)

and $\mathbf{\beta}''(t)$ is an $N$-dimensional vector. The signal vector approximation defined in (4.45), (4.46) and (4.48) is qualitatively different from that defined in (4.29). The approximation in (4.29) involves truncation of the normal mode expansion whereas (4.45) and (4.46) involve an approximation of the relation among the components of the signal vector measured by an AVS. In (4.29), the unknown vector $\mathbf{\beta}'(t)$ is $M'$-dimensional whereas the unknown vector $\mathbf{\beta}''(t)$ in (4.48) is $N$-dimensional. The conditional ML estimate of $\mathbf{\beta}''(t)$ and the corresponding estimate of $\tilde{s}(t, \phi)$ are given by

\[ \hat{\mathbf{\beta}}''(t \mid \phi) = \mathbf{U}''(\phi)^H \hat{\mathbf{x}}(t) \text{ and } \]  
(4.49)

\[ \hat{s}_{\text{ASFD}}(t \mid \phi) = \mathbf{D}''(\phi) \hat{\mathbf{x}}(t) , \]  
(4.50)

where
\[ \mathbf{D}''(\phi) = \mathbf{U}''(\phi) \mathbf{U}''(\phi)^H. \]  

(4.51)

Expressions for the estimate of \( \phi \) and the unconditional estimate of \( \tilde{s}(t) \) can be obtained using a procedure analogous to that for SD and TSD. Thus we get

\[
\hat{\phi}_{\text{ASFD}} = \arg \max \left[ \sum_{t=1}^{T} \tilde{x}^H(t) \mathbf{D}''(\phi) \tilde{x}(t) \right], \quad \text{and} \\
\hat{s}_{\text{ASFD}}(t) = \mathbf{D}''(\hat{\phi}_{\text{ASFD}}) \tilde{x}(t).
\]

(4.52)

(4.53)

The NMSE of the signal estimator defined in (4.53) is given by

\[
\mathcal{E}_{\text{ASFD}}(\phi) \triangleq \sum_{t=1}^{T} \mathbb{E}[(\hat{s}_{\text{ASFD}}(t) - \tilde{s}(t))^H (\hat{s}_{\text{ASFD}}(t) - \tilde{s}(t))]
\]

\[
= \mathcal{E}_{\text{ASFD}}^{(1)} + \mathcal{E}_{\text{ASFD}}^{(2)}(\phi),
\]

(4.54)

where

\[
\mathcal{E}_{\text{ASFD}}^{(1)} = \frac{NT}{\lambda},
\]

(4.55)

\[
\mathcal{E}_{\text{ASFD}}^{(2)}(\phi) = \frac{\mathbb{E}\left[ \sum_{t=1}^{T} s''(t; \phi) \left( 1 - \mathbf{D}''(\hat{\phi}_{\text{ASFD}}) \right) \tilde{s}(t; \phi) \right]}{E_s} = \frac{\mathbb{E}\left[ \sum_{t=1}^{T} s''(t) \mathbf{D}''(\phi) \tilde{s}(t) \right]}{E_s} = 1 - \frac{\lambda''}{\lambda}
\]

(4.56)

\[
\lambda'' = \frac{E_{\epsilon}}{\sigma_s^2}, E_{\epsilon} = \sum_{t=1}^{T} \mathbb{E}[s''(t)^H \tilde{s}''(t)].
\]

(4.57)

The NMSE \( \mathcal{E}_{\text{ASFD}} \) also has two components \( \mathcal{E}_{\text{ASFD}}^{(1)} \) and \( \mathcal{E}_{\text{ASFD}}^{(2)}(\phi) \). The first component \( \mathcal{E}_{\text{ASFD}}^{(1)} \) is the error due to noise and it is analogous to the NMSE component \( \mathcal{E}_{\text{TSD}}^{(1)} \) defined in (4.37). The second component \( \mathcal{E}_{\text{ASFD}}^{(2)}(\phi) \) arises from the signal modeling
error due to the approximation (4.44). We have $\varepsilon_{ASFD}^{(1)} = \frac{1}{3} \varepsilon_{ED}$ while the other component $\varepsilon_{ASFD}^{(2)}(\phi)$ is quite small in comparison because the approximation in (4.44) introduces only a small error. It follows that $\varepsilon_{ASFD} \approx \frac{1}{3} \varepsilon_{ED}$. Hence the ASFD is expected to perform better than the ED. This prediction is confirmed by the asymptotic analysis in Section 4.3.6 and the simulation results presented in Section 4.4.

Expressions for the test statistics of ASFD which are analogous to the corresponding expressions for SD and TSD are given by

$$\gamma_{ASFD,\text{Case}(a)}(\tilde{X}) = \sum_{t=1}^{T} \tilde{x}^H(t) D''(\hat{\phi}_{ASFD}) \tilde{x}(t),$$

(4.58)

$$\gamma_{ASFD,\text{Case}(b)}(\tilde{X}) = \frac{\sum_{t=1}^{T} \tilde{x}^H(t) D''(\hat{\phi}_{ASFD}) \tilde{x}(t)}{\sum_{t=1}^{T} \tilde{x}^H(t) (I_N - D''(\hat{\phi}_{ASFD})) \tilde{x}(t)}.$$ 

(4.59)

It may be seen from (4.58) that the ASFD measures the projection of the data vector into the $N$-dimensional approximate signal subspace $V''(\phi)$.

### 4.3. Performance analysis

#### 4.3.1. Matched filter detector (MFD)

For the MFD, the probability of detection $P_D$ and probability of false alarm $P_{FA}$ are given by [1]
\[ P_{FA} = \frac{\eta}{\sqrt{\frac{E_s \sigma^2}{2}}}, \quad P_D = \frac{Q^{-1}(P_{FA}) - \sqrt{2\lambda}}{Q^{-1}(P_{FA})}, \]  

where \( E_s \) is the energy of the pre-whitened signal summed over all snapshots and \( \lambda \) is the ENR defined in (4.15), \( Q(.) \) denotes the right-tail probability function of the standard normal distribution and \( Q^{-1}(.) \) denotes the inverse of this function.

### 4.3.2. Energy detector

The test statistic \( \gamma_{ED} \) of the ED defined in (4.13) is the total energy of the pre-whitened data vectors. It is the sum of squares of \( 6NT \) independent Gaussian random variables with variance \( \sigma^2/2 \) under both hypotheses. Under \( H_0 \) the means of these random variables are zero; and under \( H_1 \), the sum of their means is equal to

\[
E_s = \sum_{i=1}^{T} E{[\hat{s}^H(t)\hat{s}(t)].}
\]

It follows that

(i) under hypothesis \( H_0 \) we have \( \frac{2}{\sigma^2} \gamma_{ED} \sim \chi^2_{6NT} \) which denotes that the random variable \( \frac{2}{\sigma^2} \gamma_{ED} \) has chi-squared distribution with \( 6NT \) degrees of freedom, and

(ii) under hypothesis \( H_1 \) we have \( \frac{2}{\sigma^2} \gamma_{ED} \sim \chi^2_{6NT} (2\lambda) \), which denotes the non-central chi-squared distribution with \( 6NT \) degrees of freedom and noncentrality parameter \( 2\lambda \).

Thus we have

\[
P_{FA,ED} = Q_{\chi^2_{6NT}} \left( \frac{2\eta}{\sigma^2} \right), \quad (4.61)
\]
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

\[ P_{D,ED} = Q_{\chi^2_{\text{SD}}(2A)} \left( \frac{2\eta}{\sigma^2} \right), \]  \tag{4.62}

where the right hand sides of (4.61) and (4.62) denote the right-tail probability functions of the respective distributions.

4.3.3. Subspace detector

Deriving expressions for \( P_D \) and \( P_{FA} \) requires determination of the pdfs of the test statistic under both hypotheses. The test statistic is defined in (4.23) for case (a) (noise variance known) and in (4.24) for case (b) (noise variance unknown). It is difficult to determine the pdfs since the test statistics are highly nonlinear functions of the data. But, we can determine approximate expressions for the pdfs by assuming that the estimate \( \hat{\phi}_{SD} \) is equal to the true value \( \phi \). This assumption is validated by the simulation results presented in Section 4.4. We note that \( \tilde{x}^H(t)D(\phi)\tilde{x}(t) \) is a quadratic form involving a complex circular Gaussian random vector \( \tilde{x}(t) \), \( D(\phi) \) is an idempotent matrix of rank \( M \) and

\[ \tilde{s}^H(t)D(\phi)\tilde{s}(t) = \tilde{s}^H(t)\tilde{s}(t) = E_S. \] \tag{4.63}

We can apply Graybill’s theorem [178], [179] to find the distribution of the quadratic form \( \tilde{x}^H(t)D(\phi)\tilde{x}(t) \). Thus it can be readily shown that \( \frac{2}{\sigma^2} \gamma_{SD,\text{Cas}(a)} \sim \chi^2_{2MT} \) under \( H_0 \) and

\[ \frac{2}{\sigma^2} \gamma_{SD,\text{Cas}(a)} \sim \chi^2_{2MT} \left( 2\lambda \right) \] under \( H_1 \). Therefore, the expressions for \( P_{FA} \) and \( P_D \) under case (a) can be written as
In case (b), the matrices $\mathbf{D}(\hat{\phi}_{SD})$ and $\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})$ in the numerator and denominator of (4.24) are idempotent matrices of rank $M$ and $3N-M$ respectively. Hence, on applying Graybill’s theorem [178], [179] we find that

(i) The numerator of (4.24) scaled by the factor $(2/\sigma^2)$ has chi-squared distribution with $2MT$ degrees of freedom under hypothesis $H_0$, and

(ii) The numerator has noncentral chi-squared distribution with $2MT$ degrees of freedom and noncentrality parameter $\lambda$ under $H_1$.

(iii) The denominator of (4.24) scaled by the factor $(2/\sigma^2)$ has chi-squared distribution with $(6N-2M)T$ degrees of freedom under both hypotheses since

$$\bar{s}^H(t)(\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})){\bar{s}}(t) = 0 \text{ in view of (4.63).}$$

The numerator and denominator of (4.24) are statistically independent since they represent the squares of the norms of projections of $\hat{x}(t)$ onto mutually orthogonal subspaces. It follows that under hypothesis $H_0$ the test statistic defined in (4.24) has $F$ distribution, denoted by $F_{2MT,(6N-2M)T}$, and under hypothesis $H_1$ it has noncentral $F$ distribution with noncentrality parameter $2\lambda$, denoted by $F'_{2MT,(6N-2M)T}(2\lambda)$. Hence the expressions for $P_{FA}$ and $P_D$ can be written as

$$P_{FA,SD,Case(b)} = Q_{F_{2MT,(6N-2M)T}}(\eta),$$

$$P_{D,SD,Case(b)} = Q_{F'_{2MT,(6N-2M)T}(2\lambda)},$$

(4.64)

(4.65)
Detection in shallow ocean in presence of Gaussian noise

\[ P_{D,SD,\text{Case}(b)} = Q_{F, 2M'T(MN-2M'T)}(2\lambda)(\eta), \]  

(4.67)

where \( Q_{F}(\cdot) \) and \( Q_{F}(\cdot) \) denote the right-tail probability functions of the respective distributions.

### 4.3.4. Truncated subspace detector (TSD)

As in the case of TSD, we can once again determine the approximate distributions of the test statistics defined in (4.42) and (4.43) under the assumption that \( \hat{\phi}_{\text{TSD}} = \phi \).

Before proceeding further we note some important differences between SD and TSD.

It can be shown that

\[ \tilde{s}^H(t)D'(\phi)\tilde{s}(t) = \tilde{s}'^H(t)\tilde{s}'(t), \]  

(4.68)

where \( \tilde{s}'(t) = U'\beta'(t) \) is the truncated signal vector defined in (4.28). It follows that \( \tilde{s}^H(t)D'(\phi)\tilde{s}(t) \neq \tilde{s}'^H(t)\tilde{s}'(t) \), whereas \( \tilde{s}^H(t)D(\phi)\tilde{s}(t) = \tilde{s}'^H(t)\tilde{s}'(t) \). Also, \( \text{rank}(D'(\phi)) = M' \) whereas \( \text{rank}(D(\phi)) = M \). Hence, we have \( \frac{2}{\sigma^2} \gamma_{TSD,\text{Case}(a)} \sim \chi_{2M'T}^2 \) under \( H_0 \) and

\[ \frac{2}{\sigma^2} \gamma_{TSD,\text{Case}(a)} \sim \chi_{2M'T}^2(2\lambda') \] under \( H_1 \), and

\[ P_{\text{FA,TSD,Case}(a)} = Q_{\chi_{2M'T}^2} \left( \frac{2\eta}{2^2} \right), \]  

(4.69)

\[ P_{D,TSD,\text{Case}(a)} = Q_{\chi_{2M'T}^2(2\lambda)} \left( \frac{2\eta}{\sigma^2} \right), \]  

(4.70)

where \( \lambda' \) is the ENR for the truncated signal defined in (4.38). For case (b), the numerator of (4.43) scaled by the factor \( (2/\sigma^2) \) has chi-squared distribution with \( 2M'T \)
degrees of freedom under hypothesis $H_0$ and noncentral chi-squared distribution with $2M'T$ degrees of freedom and noncentrality parameter $2\lambda'$ under $H_1$. The denominator of (4.43) scaled by the factor $(2/\sigma^2)$ has chi-squared distribution with $(6N-2M')T$ degrees of freedom under hypothesis $H_0$ and noncentral chi-squared distribution with $(6N-2M')T$ degrees of freedom and noncentrality parameter $2(\lambda-\lambda')$ under $H_1$. The numerator and denominator are statistically independent. It follows that under hypothesis $H_0$ the test statistic defined in (4.43) has $F$ distribution, denoted by $F_{2M'T,(6N-2M')T}$, and under hypothesis $H_1$ it has a doubly noncentral $F$ distribution denoted by $F'_{2M'T,(6N-2M')T(2\lambda', 2\lambda-2\lambda')}$. Hence the expressions for $P_{FA}$ and $P_D$ can be written as

$$P_{FA,\text{TSD,Case}(b)} = \frac{\chi^2_{2M'T,(6N-2M')T}(\eta)}{\eta},$$

$$P_{D,\text{TSD,Case}(b)} = \frac{\chi^2_{2M'T,(6N-2M')T(2\lambda', 2\lambda-2\lambda')}(\eta)}{\eta},$$

where $Q_F(.)$ and $Q_{F'}(.)$ denote the right-tail probability functions of the respective distributions.

### 4.3.5. Approximate signal form detector (ASFD)

The test statistics of ASFD for cases (a) and (b) given by (4.58) and (4.59) respectively, are analogous to the corresponding test statistics for the TSD, with $D'(\phi)$ replaced by $D''(\phi)$. Since $D''(\phi)$ is an idempotent matrix of rank $N$, the expressions for $P_{FA}$ and $P_D$ are analogous to those for the TSD. Thus we have

$$P_{FA,\text{ASFD,Case}(a)} = \frac{\chi^2_{2M'T(6N-2M')T}(\eta)}{\eta}$$

where $Q_{\chi^2_{2M'T(6N-2M')T}(\eta)}(\frac{2\eta}{\sigma^2})$.
Detection in shallow ocean in presence of Gaussian noise

\[ P_{D,ASFD,C\text{ase}(a)} = Q_{F_{3,NT}^{2\lambda^*}} \left( \frac{2\eta}{\sigma^2} \right) \]  \hspace{1cm} (4.74)

\[ P_{FA,ASFD,C\text{ase}(b)} = Q_{F_{2,NT}^{\lambda^*}} (\eta), \]  \hspace{1cm} (4.75)

\[ P_{D,ASFD,C\text{ase}(b)} = Q_{F_{2,NT}^{\lambda^*}} (\eta), \]  \hspace{1cm} (4.76)

where \( \lambda'' \) is defined in (4.57).

4.3.6. Asymptotic performance analysis

The performance analysis in the preceding subsections is based on 3NT data samples provided by an AVS array of \( N \) sensors over \( T \) snapshots. It is of interest to also consider an asymptotic (3NT→∞) performance analysis since such an analysis facilitates an easy comparison of the performance of different detectors and provides a better insight into their relative merits. For case (a) and for a given \( \phi \), the test statistic of the detectors defined in (4.13), (4.23), (4.42), and (4.58) are asymptotically Gaussian as 3NT→∞. Hence, if the noise variance is known the asymptotic expression for the receiver operating characteristic (ROC) of each detector can be written as [1]

\[ P_{D,Case(b)} \doteq O \left( \frac{v_0}{v_1} Q^{-1}(P_{F}\lambda) \frac{\mu_h - \mu_0}{v_1} \right) = O \left( \frac{Q^{-1}(P_{F}\lambda) - d}{\tau} \right), \]  \hspace{1cm} (4.77)

where \( \doteq \) denotes asymptotic equality, and

\[ \tau = \frac{v_1}{v_0}, d = \frac{\mu_h - \mu_0}{v_0}, \mu_j = E[\gamma; H_j], v_j^2 = \text{var}(\gamma; H_j), j = 0,1. \]  \hspace{1cm} (4.78)
In (4.78), \( \mu_j \) and \( \nu_j \) are respectively the mean and standard deviation of the test statistic \( \gamma \) under hypothesis \( H_j \); \( \tau \) is the ratio of standard deviations and \( d \) is the parameter known as the deflection coefficient. It is seen from (4.77) that for a given \( P_{FA} \), the \( P_D \) increases when \( d \) and/or \( \tau \) increases. Hence, the relative magnitudes of \( d \) and the relative magnitudes of \( \tau \) provide measures for comparing the performance of the different detectors under consideration.

For the case of known noise variance, each detector has a test statistic of the form defined in (4.7) that is reproduced below:

\[
\gamma_{\text{Case}(a)}(\tilde{X}) = \sum_{t=1}^{T} \left[ 2 \text{Re}\left( \tilde{x}^H(t)\hat{s}(t) - \tilde{s}^H(t)\hat{s}(t) \right) \right].
\]

By using the expressions for \( \hat{s}(t) \) for different detectors defined in (4.12), (4.22), (4.35) and (4.53) and noting that \( \tilde{x}(t) = \tilde{w}(t) \) under \( H_0 \) and \( \tilde{x}(t) = \tilde{s}(t) + \tilde{w}(t) \) under \( H_1 \), we can obtain the following results

\[
\mu_{0,ED} = 3NT\sigma^2, \quad \mu_{1,ED} = E_s + 3NT\sigma^2, \quad \nu_{0,ED}^2 = 3NT\sigma^4, \quad \nu_{0,SD}^2 = 2\sigma^2 E_s + 3NT\sigma^4, \quad (4.79)
\]

\[
\tau_{ED} = \sqrt{1 + \frac{2\lambda}{3NT}}, \quad d_{ED} = \frac{\lambda}{\sqrt{3NT}}, \quad (4.80)
\]

\[
\mu_{0,SD} = MT\sigma^2, \quad \mu_{1,SD} = E_s + MT\sigma^2, \quad \nu_{0,SD}^2 = MT\sigma^4, \quad \nu_{0,SD}^2 = 2\sigma^2 E_s + MT\sigma^4, \quad (4.81)
\]

\[
\tau_{SD} = \sqrt{1 + \frac{2\lambda}{MT}}, \quad d_{SD} = \frac{\lambda}{\sqrt{MT}}, \quad (4.82)
\]
\[ \mu_{0,TSD} = M' T \sigma^2, \quad \mu_{1,TSD} = E_s' + M' T \sigma^2, \quad \nu_{0,TSD}^2 = M' T \sigma^4, \quad \nu_{0,TSD}^2 = 2\sigma^2 E_s' + M' T \sigma^4 \] (4.83)

\[ \tau_{TSD} = \sqrt{1 + \frac{2\lambda'}{M' T}}, \quad d_{TSD} = \frac{\lambda'}{\sqrt{M' T}}, \] (4.84)

\[ \mu_{0,ASFD} = N T \sigma^2, \quad \mu_{1,ASFD} = E_s' + N T \sigma^2, \quad \nu_{0,ASFD}^2 = N T \sigma^4, \quad \nu_{0,ASFD}^2 = 2\sigma^2 E_s' + N T \sigma^4, \] (4.85)

\[ \tau_{ASFD} = \sqrt{1 + \frac{2\lambda''}{N T}}, \quad d_{ASFD} = \frac{\lambda''}{\sqrt{N T}}. \] (4.86)

We can draw some useful conclusions from equations (4.79)-(4.86). We recall that \(3N > M > M'\). It follows from (4.80) and (4.82) that \(d_{SD} > d_{ED}\) and \(\tau_{SD} > \tau_{ED}\). It can be readily verified that the signal energy-to-noise power ratios (ENRs) \(\lambda\) (for the signal vector \(\tilde{s}\)) and \(\lambda''\) (for the approximate signal vector \(\tilde{s}'\)) are very close to each other. It can also be verified that the value of \(\lambda'\) (ENR of the truncated signal vector \(\tilde{s}^*\)) decreases very slowly as \(M'\) is reduced, and the rate of reduction of \(\lambda'\) becomes large only at very small values of \(M'\). Therefore, it follows from (4.84) that the values of \(d_{TSD}\) and \(\tau_{TSD}\) keep increasing as \(M'\) is reduced until an optimal value of \(M'\) is reached, as shown in Fig. 4.5. (We recall that a different criterion of optimality, viz. minimization of the normalized mean square signal estimation error \(\varepsilon_{TSD}\), was considered previously. The two criteria may lead to slightly different optimal values of \(M'\).) It is seen from Fig. 4.5 that \(d_{TSD} > d_{SD}\) and \(\tau_{TSD} > \tau_{SD}\) as long as the truncation of the signal vector in TSD is not too severe. We may therefore make the following predictions: (1) TSD performs better than SD, which in turn performs better than ED, and (2) the performance of TSD keeps improving as \(M'\) is reduced till it reaches its optimal value. In the case of ASFD, it is obvious from (4.80) and (4.86) that \(d_{ASFD} > \)
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

d_{ED} and \tau_{ASFD} > \tau_{ED}, and therefore we can predict that ASFD performs better than ED. The performance of ASFD relative to that of the SD and TSD depends on the relative magnitudes of \( N, M \) and \( M' \). It is seen from (4.82) and (4.86) that \( d_{ASFD} > d_{SD} \) and \( \tau_{ASFD} > \tau_{SD} \) if \( M > N \). Consequently ASFD performs better than SD if \( 3N > M > N \), and worse than SD if \( M < N \). It is seen from (4.84) and (4.86) that \( d_{TSD} > d_{ASFD} \) and \( \tau_{TSD} > \tau_{ASFD} \) if \( M' < N \). For HLA, optimal value of \( M' \) is very close to 1 and therefore optimal \( M' \) is almost always less than \( N \). Hence, for HLA, TSD with optimal \( M' \) performs better than ASFD almost always. For VLA, it turns out that optimal \( M' \cong N \) and consequently, for optimal \( M' \), \( d_{TSD} \cong d_{ASFD} \) and \( \tau_{TSD} \cong \tau_{ASFD} \). Therefore, for VLA, the performance of ASFD is very close to that of TSD with optimal \( M' \).

In order to illustrate the conclusions and predictions of the preceding paragraph, we shall consider an example with \( N = 6, T = 20, f = 350 \text{ Hz}, M = 15, \) and \( \text{SNR} = -10 \text{ dB} \). SNR (in dB) is defined as

\[
\text{SNR} = 10 \log_{10} \left( \frac{1}{N} \sum_{n=1}^{N} \frac{|p_n|^2}{\sigma_n^2} \right),
\]

(4.87)

where \( p_n \) and \( \sigma_n^2 \) are respectively the signal component of acoustic pressure and the variance of noise at the \( n^{\text{th}} \) sensor. Only the pressure components of signal and noise are considered in the definition of SNR as per normal convention [17], to facilitate a fair comparison between the performance of AVS and APS (acoustic pressure sensor) arrays. Values of the channel parameters and source coordinates used in this example are the same as those listed in section 4.4. For TSD, we have chosen the optimal
Fig. 4.5: Variation of (a) standard deviation ratio and (b) deflection coefficient of TSD vs. number of retained modes $M'$, at different values of SNR, for HLA (solid lines) and VLA (dashed lines), $f = 350$ Hz, array length $N = 6$, source azimuth $\phi = 20^\circ$.

values of $M'$, which turn out to be $M' = 1$ for the HLA and $M' = 5$ for the VLA as shown in Fig. 4.5. For this example, we have the following values of ENRs: $\lambda = 35.03$, $\lambda' = 34.44$ (for $M' = 1$), and $\lambda'' = 35.02$ for the HLA, and $\lambda = 29.02$, $\lambda' = 24.86$ (for $M' = 5$), and $\lambda'' = 28.96$ for the VLA. Values of $d$, $\tau$, the normalized mean square signal estimation error $\varepsilon$, and the asymptotic probability of detection $P_D$ (for false alarm probability $P_{FA} = 0.001$) for different detectors are tabulated in Table 4.1. The values of $d$, $\tau$, and $P_D$ confirm the predictions of the preceding paragraph. We also note that a reduction in the value of $\varepsilon$ is almost always accompanied by an increase in the value of $P_D$ as expected. For VLA, we have $d_{TSD} = 2.486 < d_{ASFD} = 2.644$ and $\tau_{TSD} = 1.224 > \tau_{ASFD} = 1.218$ in the present example. On substituting these values of in (4.77), we see
Table 4.1: Values of deflection coefficient $d$, standard deviation ratio $\tau$ and NMSE $\varepsilon$ for different detectors, at SNR -10 dB, $T = 20$ snapshots, 6-sensor AVS array

<table>
<thead>
<tr>
<th>Detectors</th>
<th>Deflection coefficient $d$</th>
<th>Standard deviation ratio $\tau$</th>
<th>NMSE $\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSD (HLA)</td>
<td>7.698</td>
<td>2.108</td>
<td>0.561</td>
</tr>
<tr>
<td>ASFD (HLA)</td>
<td>3.196</td>
<td>1.258</td>
<td>3.366</td>
</tr>
<tr>
<td>SD (HLA)</td>
<td>2.022</td>
<td>1.111</td>
<td>8.57</td>
</tr>
<tr>
<td>ED (HLA)</td>
<td>1.846</td>
<td>1.093</td>
<td>10.28</td>
</tr>
<tr>
<td>TSD (VLA)</td>
<td>2.486</td>
<td>1.224</td>
<td>3.401</td>
</tr>
<tr>
<td>ASFD (VLA)</td>
<td>2.644</td>
<td>1.218</td>
<td>3.811</td>
</tr>
<tr>
<td>SD (VLA)</td>
<td>1.675</td>
<td>1.093</td>
<td>10.34</td>
</tr>
<tr>
<td>ED (VLA)</td>
<td>1.529</td>
<td>1.078</td>
<td>12.41</td>
</tr>
</tbody>
</table>

$P_D$ of ASFD is larger than $P_D$ of TSD if $\{Q^{-1}(P_{FA}) - 2.486\}/1.224 > Q^{-1}(P_{FA}) - 2.644\}/1.218$, i.e. if $P_{FA} > Q(34.72)$. Since $Q(34.72)<<1$, it follows that $P_D$ of ASFD is larger than $P_D$ of TSD for almost all values of $P_{FA}$. It is however emphasized that, under different conditions, $P_D$ of TSD-VLA with optimal $M'$ may be higher than that of ASFD-VLA, but the difference in performance is always small.

Additional results based on the same example are provided in Figs. 4.5 and 4.6. Figure 4.5 shows the variation of $d_{TSD}$ and $\tau_{TSD}$ with $M'$ for different values of SNR. For an HLA, the values of $d_{TSD}$ and $\tau_{TSD}$ are maximized at $M' = 1$, whereas for a VLA, they are maximized at $M' = 5$. It is also seen from Fig. 4.5 that the maximum values of $d_{TSD}$ and $\tau_{TSD}$ for the VLA are consistently smaller than those for the HLA at all values
of SNR. Hence, for the TSD, the performance of an HLA may be expected to be consistently better than that of a VLA. The variation of $d$ and $\tau$ with SNR for different detectors is shown in Fig. 4.6. In this figure, plots for TSD are shown for the optimal values of $M'$, viz. $M'_{\text{opt}} = 1$ for the HLA and $M'_{\text{opt}} = 5$ for the VLA. Figure 4.6 confirms once again that, if $N < M < 3N$ and $M'$ is optimal, we have $d_{\text{TSD}} > d_{\text{ASFD}} > d_{\text{SD}} > d_{\text{ED}}$ and $\tau_{\text{TSD}} > \tau_{\text{ASFD}} > \tau_{\text{SD}} > \tau_{\text{ED}}$ for HLA, and $d_{\text{ASFD}} \approx d_{\text{TSD}} > d_{\text{SD}} > d_{\text{ED}}$ and $\tau_{\text{ASFD}} \approx \tau_{\text{TSD}} > \tau_{\text{SD}} > \tau_{\text{ED}}$ for VLA.

It is known that the number of modes $M$ increases as the frequency is increased. It follows from (4.82) that both $d_{\text{SD}}$ and $\tau_{\text{SD}}$ reduce with increasing
frequency. We may therefore expect the performance of the SD to degrade as the frequency is increased till $M$ reaches the threshold $3N$. For all the other detectors, the performance is expected to be independent of frequency. All the predictions stated above will be verified through simulation results presented in Section 4.4.

If noise variance is not known (case (b)), the test statistics defined in (4.24), (4.42) and (4.59) converge asymptotically to ratios of independent Gaussian random variables with non-zero means. The asymptotic pdfs of the test statistics have complicated expressions that do not provide any useful insights. Therefore, we shall not pursue the asymptotic analysis of case (b).

4.4. Simulation results

This section presents a detailed study of the performance of the detectors presented in Section 4.2 in detecting a single source in a shallow ocean using an array of AVS. The ocean is modeled as a Pekeris channel [7] comprising a homogeneous water layer of constant depth over a fluid half-space. The following values of channel, source and array parameters have been assumed unless otherwise stated: ocean depth $h = 70$ m, sound speed in water $c = 1500$ m/s, sound speed in ocean bottom $c_b = 1700$ m/s, density of water $\rho = 1000$ kg/m$^3$, density of ocean bottom $\rho_b = 1500$ kg/m$^3$, attenuation at ocean bottom $\delta = 0.2$ dB/$\lambda_b$, where $\lambda_b = c_b/\nu$ is the wavelength in the ocean bottom. The sea-bottom parameters considered correspond to typical values for a shallow ocean environment with a sandy bottom [7]. The number of sensors $N = 6$ (unless otherwise mentioned), array depth $z_1 = 40$ m for HLA/HCA and $z_1 = 15$ m for the
Detection in shallow ocean in presence of Gaussian noise

Fig. 4.7: Comparison of non-asymptotic and asymptotic theoretical results for the case of known noise variance. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) HLA, (b) VLA
topmost sensor in the VLA, inter-sensor spacing $d = \frac{\lambda}{2} = 15/7$ m, where $\lambda = \frac{c}{f}$ is the wavelength in water, source at range $r = 5000$ m, depth $z_s = 40$ m, azimuth $\phi = 20^0$
with respect to the endfire direction of the HLA or the reference axis of the HCA, and frequency $f = 350$ Hz. At this frequency, the number of normal modes in the channel is $M = 15$. Detection is done using $T = 20$ snapshots of data (unless otherwise mentioned) and the probability of false alarm is fixed at $P_{FA} = 0.001$.

Before presenting the results, we recall that ED and ASFD do not require any
prior information about the channel. The TSD requires the knowledge of wave numbers of $M'$ lowest order modes and the SD requires the knowledge of all the modal wave numbers if an HLA/HCA is deployed. If a VLA is used, the SD and TSD require the knowledge of the modal eigenfunctions also in addition to the modal wave numbers.

Figure 4.7 shows the theoretical plots of $P_D$ versus SNR at $P_{FA} = 0.001$ for different detectors for the case of known noise variance (case (a)). The non-asymptotic results (from (4.60)-(4.62), (4.64)-(4.65), (4.69)-(4.70) and (4.73)-(4.74)) are shown as dashed lines and the asymptotic results (from (4.77) in conjunction with (4.79)-(4.86)) are shown as solid lines. The TSD results have been obtained using values of $M'$ that minimize the NMSE $\varepsilon_{TSD}$. Performance of the unrealizable MFD is also shown to indicate the upper bound on $P_D$ for any realizable detector. It is seen that the asymptotic and non-asymptotic results exhibit the same trend, even though there are some quantitative differences between the two sets of results. Among the realizable detectors, the ranking of HLA-based detectors in decreasing order of performance is TSD, followed by ASFD, SD, and ED. For VLA-based detectors, the order of TSD and ASFD is reversed. These results are consistent with the values of the deflection coefficient $d$ shown in Table 4.1 and Fig. 4.5, and are in conformity with the predictions in Section 4.3. It is noteworthy that the ASFD that does not require any channel information, compares favorably with the TSD that requires the knowledge of wave numbers/eigenfunctions of the lowest $M'$ modes of the channel. The ASFD compensates for the lack of channel information by exploiting the relation among
Chapter 4  Detection in shallow ocean in presence of Gaussian noise

Fig. 4. 8: Comparison of theoretical (non-asymptotic) and simulation results for the case of known noise variance. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) HLA, (b) VLA

Fig. 4. 9: Bearing estimation errors of SD, TSD and ASFD. (a) Bias vs. SNR. (b) Root-mean-square error vs. SNR.
different components of the signal measured by an AVS. The performance of the SD is poor even though it uses the modal wave number/eigenfunction information of all the $M$ modes. This is so because the number of modes $M$ is large and hence the NMSE of the SD is high, which leads to a degradation in performance. The performance of the ED is the poorest because it does not use any prior information.

In Fig. 4.8 the theoretical (non-asymptotic) results (dashed lines) are compared with the simulation results (solid lines) for both the HLA and the VLA. For, SD, TSD and ASFD, two types of simulation results are considered, viz. simulations of realizable detectors that do not assume prior knowledge of the bearing $\phi$ (solid lines), and simulations of unrealizable detectors which assume that $\phi$ is known (dotted lines). The variance of noise is assumed to be known (case (a)). The theoretical results are based on the expressions for $P_{FA}$ and $P_{D}$ given in Sections 4.3.1 to 4.3.5. The following observations can be made from Fig. 4.8. For MFD and ED, the simulation results match the theoretical predictions very closely. In the case of SD, TSD and ASFD, there is very good agreement between theoretical results and the simulation results that are based on the assumption that $\phi$ is known. But the $P_{D}$ of the realizable versions of these detectors are significantly lower than the theoretical predictions at low SNR. The difference between theoretical predictions and actual performance can be explained as follows. The theoretical results are based on the assumption that the bearing estimates $\hat{\phi}_{SD}$, $\hat{\phi}_{TSD}$ and $\hat{\phi}_{ASFD}$ may be approximated by the true value $\phi$. This assumption has been made to simplify the theoretical analysis. But the means and standard deviations of the bearing estimation errors keep increasing as SNR is reduced.
Fig. 4.10: Comparison of theoretical (non-asymptotic) and simulation results for the case of unknown noise variance. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) HLA, (b) VLA

as illustrated in Fig. 4.9. Consequently, for SD, TSD and ASFD, the gap between theoretical predictions and actual performance also keeps increasing as SNR is reduced. Even though the theoretical results tend to overestimate the actual performance, the former provide a useful insight into the comparative performance of different detectors.

It is also of interest to compare the performance of HLA-based detectors (Figs. 4.7 (a) and 4.8 (a)) and VLA-based detectors (Figs. 4.7 (b) and 4.8 (b)). It is seen that the performance of an HLA-based detector is better than that of a similar VLA-based
One reason for this difference is that noise at the sensors of the HLA is i.i.d., whereas noise at the sensors of the VLA is correlated and it has spatially varying variance. The whitening transformation leads to a reduction of the energy of the VLA signal vector and a consequent reduction in the performance of all VLA-based detectors. In the case of the TSD, there is an additional reason for the better performance of HLA. The optimal value of $M'$ for the HLA is lower than that for the VLA due to the higher degree of correlation among the modal steering vectors of the HLA. Therefore the NMSE $\varepsilon_{TSD}$ for the VLA is higher than that for the HLA, and the higher NMSE translates into a lower detector performance. Finally, it is seen from Fig. 4.9 that (1) for SD and TSD, the bearing estimation error of a VLA-based detector is larger than that of a similar HLA-based detector, and (2) for ASFD, the bearing estimation errors of HLA and VLA are almost the same. Therefore, for SD and TSD, the actual differences between the performance of HLA-based and VLA-based detectors (shown in Figs. 4.8(a) and 4.8(b)) are larger than the theoretically predicted differences (shown in Figs. 4.7(a) and 4.7(b)).

Figure 4.10 shows a comparison of theoretical (non-asymptotic) and simulation performances of TSD, ASFD and SD for the case of unknown noise variance $\sigma^2$ (case (b)). A comparison of Figs. 4.8 and 4.10 shows that lack of knowledge of $\sigma^2$ causes degradation in performance of all the detectors. But the degradation is less severe in the case of TSD. It is well known that this degradation can be arrested by employing secondary data vectors that are statistically identical to the noise-only data vectors [180].
We shall now study the variation of detector performance with frequency. Figure 4.11 shows simulation results for the variation of probability of detection $P_D$ with frequency $f$ when $P_{FA} = 0.001$ and SNR = -9 dB. All other parameters have the values mentioned at the beginning of this section. When $f = 30$ Hz, the number of normal modes $M$ is equal to 1 and thus the SD and the TSD are equivalent. As $f$ is increased, $M$ increases and consequently the performance of the SD suffers a progressive degradation. For $f > 120$ Hz, we have $M > N = 6$ and hence the performance of the SD dips below that of the ASFD. At $f = 398$ Hz, $M = 3N = 18$, and the performance of SD is the same as that of ED. At still higher frequencies, $M > 3N$ and hence the SD cannot be used for detection. The performance of the other detectors does not vary with frequency. These results are in agreement with the predictions at the end of Section 4.3.6.
Note that throughout our work (with the exception of section 6.3) we have assumed that we use a 3-channel AVS that measures the acoustic pressure, x-velocity and y-velocity at a point. The use of the z-velocity measurement is avoided as it adds to the complexity without making any significant contribution to the performance of detection. To demonstrate this, in Fig. 4.11 we compare the performance of two AVS HLAs that use a 3-channel AVS (which does not measure the z-velocity) and a 4-channel AVS. The number of measured channels of both the arrays is kept equal at 24.
so as to compare arrays with equal computational complexities. i.e., we assume that the 3-channel array uses \( N = 8 \) sensors, and the 4-channel array uses \( N = 6 \) sensors for detection. The measurement model for the 4-channel AVS is given in [17] and it has not been mentioned in chapter 3 for the sake of brevity. The detection performance of the TSD and ED methods are used for comparison.

Figure 4.11 plots the variation of \( P_D \) with SNR for the two types of AVS arrays. It shows that a 3-channel AVS array (solid lines) outperforms a 4-channel AVS array (dashed lines) with equal number of measurement channels. The reason for this is that the energy of the z-velocity measurement is low in the case of a source in the far-field in a shallow ocean channel. This result is applicable in the case of far-field shallow-ocean localization as well [17]. However, if we were to compare arrays with equal number of sensors, the 4-channel AVS array would outperform the 3-channel AVS array due to availability of a larger amount of data. Also, the z-velocity is useful for signal processing applications in the case of a deep ocean environment as its energy content is significant enough to contribute to the performance. An observation that contrasts with the observation drawn here by us is that in geo-acoustic parameter estimation the z-velocity component is seen to be more useful than other measurements [72].

The discussion so far was limited to the study of the performance of detectors using linear arrays (HLA and VLA). It is also of interest to study the variation in performance with change in array geometry. This problem is an extensive one and
there are several possible array geometries that may be investigated, though the linear array is commonly used as it is simple to deploy. We investigate this problem to a limited degree by observing the performance when a horizontal circular array (HCA) is used for detection. The performance of the HCA is compared against the performance of an HLA in Fig. 4.12, when the TSD and ED methods are used and all other simulation parameters are the same as Fig. 4.8. The HCA has 6 sensors and a radius equal to half-wavelengths of the signal (hence, the inter-element spacing is also equal to half-wavelength). The figure, that shows the variation of $P_D$ with variation in SNR, shows that the performance of an HCA is the same as that of an HLA when the

![Graph showing comparison of HCA and HLA performance](image-url)
Fig. 4. 14: Comparison of simulated performance of AVS and APS HLAs. $P_D$ vs. SNR at $P_{FA} = 0.001$. (a) TSD and ASFD. (b) ED and ASFD

ED is used. This is expected since the energy of the signal is kept the same in both the cases of the HLA and HCA. However, the performance of the TSD with the HCA is not as good as its performance with an HLA. Hence we conclude that keeping the array elements in a circular geometry does not yield an improved performance as compared to its linear configuration.

In the context of source localization applications, the superiority of AVS arrays over acoustic pressure sensor (APS) arrays has been well-established [9], [17], [39],
It is therefore interesting to compare the detection performance of AVS and APS arrays. Such a comparison is shown in Fig. 4.14. It is seen from Fig. 4.14 (a) that for the ED, the performance of the 6-sensor AVS HLA is equal to that of the 18-sensor APS HLA. This result can be explained by noting that the performance of the ED depends only on the ENR $\lambda$, and that the ENR at an $N$-sensor AVS array is equal to the ENR at a $3N$-sensor APS array. It is also seen from Fig. 4.13 (a) and Fig. 4.8 (a) that the performance of the ASFD with a 6-sensor AVS array is better than that of the ED with an 18-sensor APS array. This observation is significant because neither the ED nor the ASFD requires any information about the channel. In the case of the TSD, the performance of the 6-sensor AVS HLA is slightly better than that of the 18-sensor APS HLA. This difference is attributed to the superior bearing estimation capability of the AVS array due to additional directivity provided by the velocity measurements of the AVS [181]. Better bearing estimation translates into a better estimation of the signal vector and thus a better detection performance.

These twin advantages of an AVS array, viz. higher SNR and directivity, over an APS array of the same size $N$ and aperture (= $(N - 1)d$ for a uniform linear array) are recognized and documented in the literature on AVS array signal processing [9], [181]. The ability of an $N$-sensor AVS array to attain the same level of performance as a $3N$-sensor APS array is a result of great practical interest. In an array-based measurement system, a considerable portion of the cost is related to the construction, deployment and location calibration of sensor packages, and these costs depend on the number of sensor packages deployed [9]. Therefore, a reduction from $3N$ sensor packages in an APS array to $N$ sensor packages in an AVS array will lead to a
significant cost reduction. Moreover, in a towed array system, the drag on the towed array depends on the length of the array [10]. The drag can be reduced considerably if an APS array is replaced by a shorter AVS array.

4.5. Summary

A detailed investigation of four methods, viz. ED, SD, TSD and ASFD, for narrowband detection of an acoustic source in a range-independent shallow-ocean using an AVS array was presented in this chapter. Expressions for probability of false alarm and probability of detection of all the detectors were derived for the asymptotic case and the finite-data case, and the theoretical predictions were compared with simulation results. During detection, the signal vector at the sensor array is not known due to the unknown location of the source. Hence, all the detectors employ a GLRT that involves a maximum likelihood estimation of the signal vector. The ED does not use any model for the array signal vector. The SD and TSD employ models based on the normal mode theory. The ASFD employs a model that exploits the relation between the acoustic pressure and particle velocity at each sensor. Different models lead to different signal-vector estimation errors. Expressions for the NMSE were derived for each detector and it was shown that there exists a strong negative correlation between the NMSE and the detector performance.

The detection strategies of the four detectors can be compared in the following manner in terms of their computation of test statistics:
(i) The ED measures the energy of the data vector and uses it as its test statistic.

(ii) The SD measures the projection of the data vector into the $M$-dimensional modal subspace defined by the modal steering vectors.

(iii) The TSD measures the projection of the data vector into an $M'$-dimensional subspace defined by a small number $M'$ of modal steering vectors.

(iv) The ASFD measures the projection into an $N$-dimensional approximate signal subspace defined by an approximate signal vector.

If an HLA is employed, the signal model used by the SD requires the knowledge of all the modal wave numbers while the TSD requires the knowledge of wave numbers of a small number of the lowest order modes. If a VLA is employed, the knowledge of the corresponding mode functions is also required. No channel information is required by the ED and the ASFD.

Theoretical predictions as well as simulation results indicate that in the case of an HLA, the best performance is achieved by TSD, followed by ASFD, SD and ED in that order. These results are consistent with the NMSEs associated with these detectors. In the case of a VLA, the order of TSD and ASFD is reversed. For all the detectors, the performance of the HLA is significantly better than that of the VLA because (i) the HLA provides a better estimate of the azimuth of the source, and (ii) noise at the VLA is considered spatially correlated whereas it is assumed to be uncorrelated in the case of an HLA. TSD, SD and ED are detection strategies that can be used by both APS and AVS arrays. For a given detector and array size, the
performance of an AVS array is significantly better than that of the APS array. The ASFD is a detection strategy that is unique to an AVS array and its performance is significantly better than that of the ED. This observation is noteworthy because the ASFD does not require any knowledge of the channel. The above discussion clearly establishes that using an array of AVS along with the TSD or ASFD methods is advantageous over using an array of APS, even if an equal amount of prior information is used when either array is employed. Thus the improvement offered in detection due to an array of AVS is evident.

The analysis presented in this chapter is based on the assumption that the environmental noise is Gaussian. It is known that the assumption of Gaussianity is not valid in all environments. Therefore, it would be of interest to enlarge the scope of the analysis to include non-Gaussian/impulsive noise.
Chapter 5

Detection in shallow ocean in presence of non-Gaussian noise

“The inventor looks upon the world and is not contented with things as they are. He wants to improve whatever he sees, he wants to benefit the world. The spirit of invention possesses him, seeking materialization.”
- Alexander Graham Bell

5.1. Introduction

In chapter 4, the problem of detection using an AVS array was investigated in detail under the assumption that the environmental noise follows a Gaussian pdf. Several detection strategies and algorithms were formulated under this assumption that allowed simpler mathematical treatment in detection and led to mathematically tractable expressions in the theoretical analysis.

This assumption is not always valid in practice. As described in chapters 1 to 3, the ambient noise in the ocean is often impulsive and non-Gaussian in nature owing to
contributions from shipping, geological, human and marine biological sources. In many cases it is more suitable to model this noise using pdfs with heavier tails [20–22]. The detectors presented in chapter 4 cannot provide optimal detection performance in such an environment. Hence, it becomes necessary to modify the detection problem and detectors to tackle a non-Gaussian noise environment. The work in this chapter aims to develop a more general and practical solution for detection in shallow ocean in which the environmental noise is treated as being non-Gaussian in nature.

Detection of signal sources can be done using a single sensor or an array of sensors. The work in this chapter consists of two parts: detection of deterministic signals using (i) a single sensor, and (ii) using an array of sensors. The intention behind approaching the detection problem in this manner is as follows. We firstly gain an in-depth understanding of formulation of the detection problem and detection methods for non-Gaussian noise using a single sensor. These results are then applied to AVS array-based detectors to obtain improved detection algorithms using AVS array data.

In the first part of this chapter, we consider a single-sensor detector. We adapt the well-known matched-filter detector to perform improved detection in non-Gaussian noise using a nonlinear processor based on suprathreshold stochastic resonance (SSR). This SSR-based detector is analyzed in detail and its design, optimization and performance are discussed. The aim of this work in this section is
twofold. The first is to obtain a near-optimal detector for non-Gaussian noise with no additional requirement on the computational complexity. The second is to obtain a deeper understanding of the contribution of an SSR preprocessor in detection so as to be able to apply it better to array-based detectors that have been explored in chapter 4.

In the second part of this chapter, we consider detection using a sensor array. We build upon the methods explored in chapter 4, extending these results to formulate detectors for non-Gaussian noise environments. These detectors are formulated using the GLRT approach, as well as by using results on SSR described in the first part of the chapter. The performance of these detectors is evaluated and their optimization is investigated.

5.2. Single sensor detector based on suprathreshold stochastic resonance

This section discusses detection of signals buried in non-Gaussian noise, using a single sensor. Consider the detection of a deterministic signal $A s(t)$ with a known waveform $s(t)$ and a constant but unknown amplitude $A > 0$, embedded in additive white non-Gaussian noise $w(t)$. The data consists of $T$ samples/snapshots of $x(t)$. In the symbolic notations for signal, data and noise, the subscript denoting sensor number will be dropped in this section as only a single sensor is used. The detection problem is cast as the following hypothesis testing problem

$$H_0: X = W$$
Chapter 5  Detection in shallow ocean in presence of non-Gaussian noise

\[ H_1: X = AS + W \]  \hspace{1cm} (5.1)

where \( X = [x(1) \ldots x(T)]^T \) denotes the aggregate data vector of measurements from a single sensor, \( S = [s(1) \ldots s(T)]^T \) denotes the aggregate signal vector (that is assumed to be known) and \( W = [w(1) \ldots w(T)]^T \) denotes the aggregate noise vector. The random variables \( w(1), \ldots, w(T) \) denote \( T \) samples of the zero-mean input noise that has a non-Gaussian pdf. Note that the framework for the detection problem is defined in a very general form here and is applicable to any scenario where a signal is buried in non-Gaussian noise. In the context of underwater acoustic signal detection which is considered in this thesis, the problem refers to the detection of acoustic signals, and the input noise samples in \( W \) correspond to the ambient environmental noise.

It is assumed that the input noise \( w(t) \) has unit variance with a pdf \( f_w(\xi) \) and cumulative distribution function (cdf) \( F_w(\xi) \), and that \( f_w(\xi) \) is an even function. The input noise that is impulsive in nature can be modeled by several heavy-tailed input noise distributions. Four such distributions, viz. Gaussian mixture (GM), generalized Gaussian (GG), Cauchy-Gaussian mixture (CGM) and the Students t-distribution (ST), are considered in the simulations in subsections 5.2.4 to 5.2.6. The definition of all these pdfs is given in subsection 3.4.1. We also assume the signal energy is normalized, i.e

\[ \frac{1}{T} \sum_{t=1}^{T} s^2(t) = 1, \]  \hspace{1cm} (5.2)

so that \( A^2 \) denotes the signal power.
Optimal detection of the deterministic signal $AS$ in additive white noise $W$ involves computation of the following test statistic [1]

$$T_{OD}(X) = \sum_{t=1}^{T} \ln \left( \frac{f_w(x(t) - As(t))}{f_w(x(t))} \right), \quad (5.3)$$

where $f_w(\xi)$ is the input noise pdf of any random variable by $\xi$. The test statistic of the locally optimum (LO) detector for the detection of a weak signal ($A \to 0$) is generated by the nonlinear correlator

$$T_{LO}(X) = \sum_{t=1}^{T} Z_w(x(t)) s(t), \quad (5.4)$$

where
\[ Z_w(x) = -\left(\frac{df_w(x)}{dx}\right) / f_w(x) \]  

is a memoryless transformation that will be referred to as the locally optimal (LO) transform. The subscript \( w \) denotes dependence of the LO transform on the environmental noise. Both the OD and LO detector reduce to a simple replica correlator or matched filter if the noise is Gaussian. But, if the noise is non-Gaussian, the transformations defined in (5.3) and (5.5) are nonlinear functions and their implementation becomes a computationally difficult task. These detectors also require the prior knowledge of the noise pdf and hence their performance is sensitive to errors in modeling the noise pdf. It is therefore of interest to design near-optimal detectors that are easy to implement and robust with respect to noise modeling errors.

In this section we present details of the design and performance analysis of such a near-optimal nonlinear correlator-detector based on SSR [34]. The motivation for this work stems from the need to avoid the computational complexity associated with the optimal and LO detectors. The SSR detector can be easily implemented using an array of noisy one-bit quantizers and a correlator. Design of the SSR detector involves optimization of the variance and the pdf of quantizer noise.

### 5.2.1. SSR detector

The SSR detector consists of an SSR preprocessor shown in Fig. 5.1, followed by a replica correlator or matched filter [33], [34]. The SSR preprocessor is a parallel array of \( L \) one-bit quantizers. The input to the \( l \)th quantizer is \( x(t) + \sigma q_l(t) \), where \( \{q_l(t); l = 1, 2, \ldots, L\} \) are i.i.d white noise processes with unit variance, pdf \( f_q(\xi) \) that is
assumed to be an even function, and cdf $F_q(\xi)$. The quantizer noise processes \{\sigma_q(t); l = 1, 2, \ldots, L\} are independent of $w(t)$. The output of the $l^{th}$ quantizer is

$$y_l(t) = \text{sgn} [x(t) + \sigma_q(t)],$$

where sgn(.) denotes the signum function. The output $\bar{y}(t)$ of the SSR preprocessor is the mean of the individual quantizer outputs

$$\bar{y}(t) = \frac{1}{L} \sum_{l=1}^{L} y_l(t).$$

The preprocessor output is correlated with a replica of the signal to derive the test statistic for the SSR detector given by

$$T_{SSR}(X) = T(Y) = \sum_{t=1}^{T} \bar{y}(t)s(t),$$

Fig. 5.2: (a) Linear matched filter detector. (b) Noisy quantizer (NQ) detector. NQ detector becomes SSR detector if $\sigma = \sigma_{opt}$
where $Y = [\bar{y}(1), \ldots, \bar{y}(T)]^T$ is the preprocessor output vector. Thus the test statistic of the SSR detector has the same form as that of the conventional linear matched filter given by:

$$T(X) = \sum_{t=1}^{T} x(t)s(t).$$

(5.9)

However, the SSR detector is a nonlinear detector since $\bar{y}(t)$ is a nonlinear function of $x(t)$. The schematic diagrams of the linear matched filter and the SSR detector are shown in Figs. 5.2(a) and 5.2(b) respectively. Since the test statistic $T_{SSR}(X)$ is a nonlinear function of $X$, it is not possible to derive analytical expressions for the pdf of $T_{SSR}(X)$ under the hypotheses $H_0$ and $H_1$. However, it is possible to do an asymptotic $(T \to \infty)$ performance analysis of the SSR detector by invoking the classical central limit theorem. Let

$$m_j = E[T_{SSR}(X); H_j],$$

(5.10)

and

$$\lambda^2_j = V[T_{SSR}(X); H_j],$$

(5.11)

denote the means and variances of $T_{SSR}(X)$ under the hypotheses $H_j$ ($j = 0, 1$) respectively. We then have the following asymptotic expressions for the probability of false alarm $P_{FA}$ and probability of detection $P_D$:

$$P_{FA} = Q\left(\eta - \frac{m_0}{\lambda_0}\right),$$

(5.12)
Chapter 5  Detection in shallow ocean in presence of non-Gaussian noise

\[ P_D = Q\left(\frac{\eta - m_1}{\lambda_1}\right) = Q\left(\frac{\lambda_0}{\lambda_1} Q^{-1}(P_{FA}) - d \right). \]  \hfill (5.13)

where \( \eta \) is the detector threshold that is usually selected based on the \( P_{FA} \) requirements of the detector, \( Q(.) \) is the complementary cdf of a standard normal distribution, \( Q^{-1}(.) \) denotes its inverse, and the quantity \( d \) called deflection coefficient is defined as

\[ d = \frac{m_1 - m_0}{\lambda_1}. \]  \hfill (5.14)

The quantities \( m_j \) and \( \lambda_j \) \((j = 0, 1)\) depend on the pdfs \( f_w(\xi) \) and \( f_q(\xi) \) and also on the standard deviation (STD) \( \sigma \) of the quantizer noise.

In subsection 5.2.2 we invoke the assumption that the signal is weak \((0 < A < 1)\) in order to simplify the analysis of the detection problem. Further, we investigate the performance of the SSR detector under the weak-signal assumption and later extend the investigation to non-weak signals. It will be shown in subsection 5.2.2 that \( \lambda_0 = \lambda_j \) and \( d = \sqrt{\varepsilon} G_{wq}(\sigma) \) where \( \varepsilon \) is the signal energy and the quantity \( G_{wq}(\sigma) \) is defined in (5.29). Hence, \( P_D \) is maximized for a given \( P_{FA} \) if \( \sigma \) is chosen to maximize \( G_{wq}(\sigma) \). The detector in Fig. 5.2(b) will be designated as the noisy quantizer (NQ) detector for arbitrary \( \sigma \). The NQ detector that uses the optimal value of \( \sigma \) (that maximizes \( G_{wq}(\sigma) \)) is designated as the SSR detector.
5.2.2. Optimization of quantizer noise variance for weak signal detection

It can be shown, by using (5.6) and (5.7) and invoking the assumption that \{q_l(t); l = 1, 2, \ldots, L\} are i.i.d white noises independent of the white noise \(w(t)\), that

\[
E\left[\bar{y}(t); H_j\right] = 1 - 2 \int_{-\infty}^{\infty} \{F_q(-\xi/\sigma)\} f_w(\xi - A_j s(t))d\xi,
\]

(5.15)

\[
E[\bar{y}^2(t); H_j] = 1 + 4(1 - 1/L) \int_{-\infty}^{\infty} F_q^2(-\xi/\sigma) - F_q(-\xi/\sigma) f_w(\xi - A_j s(t))d\xi, j = 0, 1
\]

(5.16)

where \(A_0 = 0, A_1 = A\). Assuming that the signal is weak, i.e, \(0 < A \ll 1\), we use the approximation

\[
f_w(\xi - A_j s(t)) = f_w(\xi) - A_j s(t)[df_w(\xi)/d\xi].
\]

(5.17)

On substituting (5.17) into (5.15) and (5.16) and using the symmetry assumptions \(f_w(\xi) = f_w(-\xi)\) and \(f_q(\xi) = f_q(-\xi)\), we obtain

\[
E[\bar{y}(t); H_j] = A_j s(t) K_{wq}(\sigma),
\]

(5.18)

\[
V(\bar{y}(t); H_j) = E[\bar{y}^2(t)] - \{E[\bar{y}(t)]\}^2 = 1 - (1 - 1/L) U_{wq}(\sigma).
\]

(5.19)

where

\[
K_{wq}(\sigma) = 4 \int_{0}^{\infty} f_q(\xi)f_w(\sigma\xi)d\xi,
\]

(5.20)

\[
U_{wq}(\sigma) = 8\sigma \int_{0}^{\infty} F_q(\xi)\{1 - F_q(\xi)\} f_w(\sigma\xi)d\xi.
\]

(5.21)

On combining (5.20) and (5.21) with (5.8), (5.10) and (5.11), we obtain
\[ m_0 = E[T_{SSR}(X); H_0] = 0, \quad (5.22) \]
\[ m_1 = E[T_{SSR}(X); H_1] = ANK_{wq}(\sigma), \quad (5.23) \]
\[ \lambda_0^2 = V(T_{SSR}(X); H_0) = N[1 - (1 - 1/L)U_{wq}(\sigma)], \quad (5.24) \]
\[ \lambda_1^2 = V(T_{SSR}(X); H_1) = \lambda_0^2, \quad (5.25) \]

On substituting (5.22)–(5.25) into (5.14) we obtain
\[ d = \sqrt{\varepsilon} G_{wq,L}(\sigma), \quad (5.26) \]

where
\[ \varepsilon = TA^2, \quad (5.27) \]
\[ G_{wq,L}(\sigma) = \frac{K_{wq}(\sigma)}{\sqrt{1 - (1 - 1/L)U_{wq}(\sigma)}}, \quad (5.28) \]

It is seen that as \( L \) is increased, \( G_{wq,L}(\sigma) \) increases monotonically and asymptotically attains the value
\[ G_{wq}(\sigma) = \frac{K_{wq}(\sigma)}{\sqrt{1 - U_{wq}(\sigma)}}. \quad (5.29) \]

One can make the approximations \( G_{wq,L}(\sigma) = G_{wq}(\sigma) \) and \( d = \sqrt{\varepsilon} G_{wq}(\sigma) \) if \( L \) is sufficiently large. Hence, substituting (5.25), (5.26) and (5.28) into (5.13) and approximating \( G_{wq,L}(\sigma) \) by \( G_{wq}(\sigma) \) we find that \( P_D \) is maximized for a given \( P_{FA}, \varepsilon, f_w(\xi), \) and \( f_q(\xi) \) by choosing \( \sigma \) so as to maximize \( G_{wq}(\sigma) \). Define
\[ \sigma_{opt} = \arg \max G_{wq}(\sigma), \quad (5.30) \]
\[ G_{wq,SSR} = G_{wq} (\sigma_{opt}). \quad (5.31) \]

For input noise of STD = 1, we have deflection coefficient \( d = \sqrt{\varepsilon} \) for the matched filter, \( d = \sqrt{\varepsilon} G_{wq}(\sigma) \) for the NQ detector, and \( d = \sqrt{\varepsilon} G_{wq,SSR} \) for the SSR detector. Hence, \( G_{wq}(\sigma) \) and \( G_{wq,SSR} \) can be considered as the processing gain of the NQ detector and the SSR preprocessor respectively. Usually, \( G_{wq,SSR} > 1 \) if the input noise is leptokurtic (kurtosis > 0), thereby indicating that the SSR detector performs better than the matched filter in leptokurtic noise. A noisy quantizer array with \( \sigma \neq \sigma_{opt} \) provides a lower processing gain \( G_{wq}(\sigma) < G_{wq,SSR} \). We also note that \( G_{wq}(\sigma) \) is the ideal processing gain for \( L \to \infty \), and that the realizable processing gain for finite \( L \) is \( G_{wq,L}(\sigma) \) defined in (5.28).

When \( L = 1 \), \( G_{wq,L}(\sigma) \) reduces to \( K_{wq}(\sigma) \) that is a decreasing function of \( \sigma \) with the maximum at \( \sigma = 0 \). This result implies that for a single quantizer, the SSR system shows no improvement with addition of quantizer noise, and that more than one quantizer is needed for the detector to benefit from addition of noise. This result has also been derived by Patel and Kosko [36] using a different approach.

**5.2.3. Relation between SSR detector and locally optimal detector**

In this subsection we shall consider the relationship between the SSR detector and the LO detector [1] for the detection of a weak signal in non-Gaussian noise. The test statistic of the SSR detector is defined in (5.8). Since \( \{q(l); l = 1, 2, ..., L\} \) are i.i.d
random variables for each \( t \), the sequence \( \{ \bar{y}(t); t = 1, 2, \ldots, T \} \) converges in probability to the conditional mean of the quantizer output, denoted as:

\[
\lim_{L \to \infty} \bar{y}(t) \xrightarrow{p} E\left[ \text{sgn}\left( x + \sigma q_t(t) \right) \mid x(t) \right] = 1 - 2F_q\left( \frac{x(t)}{\sigma} \right) = 2F_q\left( \frac{x(t)}{\sigma} \right) - 1 \quad (5.32)
\]

where \( \xrightarrow{p} \) denotes convergence in probability. The last equality in (5.32) follows from the assumption that \( f_q(\xi) \) is an even function. We refer to the transformation defined in (5.32) as the noisy quantization (NQ) transform and denote it by \( N_q(x(t); \sigma) \):

\[
N_q(x; \sigma) = 2F_q(x/\sigma) - 1. \quad (5.33)
\]

The LO transform (5.5) depends on the pdf \( f_w(\xi) \) of input noise while the NQ transform (5.33) depends on the pdf \( f_q(\xi) \) and the STD \( \sigma \) of quantizer noise. The output of the SSR system converges in probability to the NQ transform as the number of quantizers \( L \) tends to infinity. The mean-square difference (MSD) between the normalized LO transform and the normalized NQ transform under hypothesis \( H_j (j = 0,1) \) is given by

\[
J_{wq}(\sigma; H_j) = E\left[ \left\{ \frac{N_q(x(t); \sigma)}{\sqrt{E[N_q^2(x(t); \sigma); H_j]}} - \frac{Z_w(x(t))}{\sqrt{E[Z_w^2(x(t); H_j)]}} \right\}^2 \right] ; H_j
\]

\[
= 2 - 2 \int_{-\infty}^{\infty} N_q(\xi; \sigma) Z_w(\xi) f_w(\xi - A_j s(t)) d\xi
\]

\[
\sqrt{\int_{-\infty}^{\infty} N_q^2(\xi; \sigma) f_w(\xi - A_j s(t)) d\xi} \cdot \sqrt{E[Z_w^2(x(t); H_j)]}
\]

In (5.34) \( E[Z_w^2(x(t); H_j)] \) is independent of \( \sigma \). The remaining terms in (5.34) are independent of the hypothesis if the signal is weak. Hence, under the weak signal
approximation, $J_{wq}(\sigma; H_j)$ is independent of $H_j$ and minimizing $J_{wq}(\sigma; H_j)$ is equivalent to maximizing

$$
\tilde{G}_{wq}(\sigma) = \frac{2 \int_{-\infty}^{\infty} N_q(\xi; \sigma) Z_q(\xi) f_q(\xi) d\xi}{\sqrt{\int_{-\infty}^{\infty} N_q^2(\xi; \sigma) f_q(\xi) d\xi}}.
$$

(5.35)

On substituting (5.5) and (5.33) into (5.35) we obtain

$$
\tilde{G}_{wq}(\sigma) = \frac{K_{wq}(\sigma)}{\sqrt{1 - U_{wq}(\sigma)}} = G_{wq}(\sigma),
$$

(5.36)

where $G_{wq}(\sigma)$ is the processing gain defined in (5.29). Thus, the optimum value of $\sigma$ under both the hypotheses, given by

$$
\sigma_{opt} = \arg \min [J_{wq}(\sigma)] = \arg \max [G_{wq}(\sigma)],
$$

(5.37)

simultaneously maximizes the processing gain $G_{wq}(\sigma)$ and minimizes mean-square difference $J_{wq}(\sigma)$. We shall designate the NQ transform of $x(t)$ corresponding to the optimum value of $\sigma$ as the SR transform, and denote it by

$$
S_{wq}(x(t)); S_{wq} (x) = N_q(x; \sigma_{opt}).
$$

(5.38)

The SR transform depends on the pdfs of input noise as well as the quantizer noise. For given $f_u(\xi)$ and $f_q(\xi)$, the SR transform represents the closest that an SSR system consisting of a parallel array of one-bit quantizers can approach an LO transform.
The input-output characteristics of the normalized LO transform are compared with those of the normalized NQ transform for different values of $\sigma$ in Fig. 5.3. The input noise $w(t)$ has the GM pdf (defined in (3.30)) with $u = 0.1$, and the quantizer noise $q(t)$ is Gaussian. For $\sigma = 0$, the NQ transform reduces to one-bit quantization. The mean-square difference $J_{wq}(\sigma)$ between the LO transform and the NQ transform attains the minimum value of $J_{wq-min}$ for the value of $\sigma_{opt} = 0.74$, and this optimum NQ transform is the SR transform defined in (5.38). It may be noted from Fig. 5.3 that the difference between the outputs of LO transform and SR transform is small for small values of the input where most of the probability mass of the input noise pdf $f_w(\xi)$ is
It follows from the above analysis that the SSR detector can be visualized as a
two-stage approximation to the LO detector. The LO transform is approximated by the
SR transform, and the SR transform is asymptotically approached by the SSR system
as $L \to \infty$. The optimization of the SSR preprocessor is essentially a ‘shaping’ of its
input-output characteristics to become closer to non-linear characteristics of the LO
detector. This interpretation suggests the possibility of further improvement in the
performance of an SSR detector by other methods such as using a higher number of
quantization levels, using quantizers with different threshold levels, or using an array
of any other low-complexity SR devices (if available) in order to achieve a closer fit
between the SR transform and the LO transform.

5.2.4. Performance analysis of SSR detector

Theoretical and simulation results have been obtained for four different heavy-tailed
input noise distributions, viz. GM, GG, CGM and ST distributions. The quantizer
noise is assumed to be Gaussian throughout this subsection. All results and
conclusions are similar for other choices of quantizer noise pdf. The issue of
optimization of quantizer noise pdf is discussed in subsection 5.2.5.

Figures 5.4(a) and 5.4(b) show the variations of the processing gain $G_{wq}(\sigma)$ as
Fig. 5.4: Plots of $G_wq$ vs. $\sigma$ for (a) GG ($e = 0.5$) and GM ($u = 0.01$) input noise and (b) CGM ($\alpha = 0.7$) and ST ($v = 3$) input noise; and theoretical and experimental plots of $P_D$ vs. $\sigma$, for (c) GG ($e = 0.5$) and GM ($u = 0.01$) input noise and (d) CGM ($\alpha = 0.7$) and ST ($v = 3$) input noise. $P_D$ is plotted for $P_{FA} = 0.1$, $T = 300$.

the standard deviation $\sigma$ of the quantizer noise is varied, for input noise with (a) GG ($e = 0.5$) and GM ($u = 0.01$) pdfs with unit variance, and (b) ST ($v = 3$) pdf with unit variance and CGM ($\alpha = 0.7$) pdf respectively. The parameters of the pdfs are chosen so as to represent noise that is impulsive. Figures 5.4(c) and 5.4(d) show the
corresponding plots of probability of detection $P_D$ for probability of false alarm $P_{FA} = 0.1$. The asymptotic theoretical $P_D$ has been determined using (5.13), (5.26), (5.28), (5.20) and (5.21), and the experimental $P_D$ has been determined by averaging over 35000 Monte Carlo simulations. It is seen that both $G_{wq}$ and $P_D$ are maximized at $\sigma = 0.92$ when the input noise is GM ($\mu = 0.01$), at $\sigma = 0$ when the input noise is GG ($e = 0.5$), at $\sigma = 0.335$ when the input noise is ST ($\nu = 3$), and at $\sigma = 0.65$ when the input noise is CGM ($\alpha = 0.7$). Hence, for weak signals, maximizing $G_{wq}$ (and minimizing $J_{wq}$) is equivalent to maximizing $P_D$. It is also seen from Fig. 5.4(c) and (d) that the theoretical and experimental plots of $P_D$ are close to each other. A slight deviation of the theoretical predictions from the experimental observations is observed only in the case of GG ($e = 0.5$) input noise when the STD of quantizer noise is nearly equal to zero. In this case, the theoretically determined optimal STD of quantizer noise is zero, and the theoretical probability of detection is greater than the observed one. However, this deviation does not exist for less leptokurtic GG input noise for which the optimal value of STD of quantizer noise is greater than zero.

The relationship between the realizable processing gain $G_{wq,SSR-L} = G_{wq,L}(\sigma_{opt})$ of the SSR preprocessor and the number of quantizers $L$ is shown in Fig. 5.5 for GM ($\mu = 0.01$) input noise. It is seen from Fig. 5.5 that $G_{wq,SSR-L}$ increases monotonically and tends asymptotically to $G_{wq,SSR}$ as $L \to \infty$. This is similar to the result obtained by Patel and Kosko [36] that shows that the rate of initial SR effect increases when the number of quantizers is increased. It is also seen that, as $L$ is increased, $G_{wq,SSR-L}$ rises
Chapter 5  Detection in shallow ocean in presence of non-Gaussian noise

Fig. 5.5: Plot of $G_{wq}$ vs. number of quantizers $L$. Input noise: GM ($u = 0.01$)

Fig. 5.6: ROCs of LO detector, SSR detector and matched filter. Input noise: (a) GG ($e = 0.5$), (b) GM ($u = 0.01$), (c) CGM ($\alpha = 0.7$), (d) ST ($v = 3$). $A = 0.05$, $T = 300$
quite rapidly towards its asymptotic value $G_{wq,SSR}$. Similar behavior is observed for other choices of input noise pdf also. Therefore, henceforth we will approximate $G_{wq,SSR,L}$ by $G_{wq,SSR}$ assuming that $L$ is sufficiently large, and drop the distinction between the realizable and ideal processing gains. This is a valid approximation because a quantizer is relatively cheap and simple hardware and hence forming an array of large number of quantizers is practically possible.

In Figs. 5.6(a)-(d), the ROCs of the different detectors are plotted. The ROC of the LO detector, SSR detector and matched filter are plotted experimentally for GG, GM, CGM and ST input noise respectively. The theoretical ROC for the SSR detector are also plotted, and they match very well with the experimental plots (except in the case of GG ($e = 0.5$) input noise, when the quantizer noise has STD = 0). These figures indicate that the performance of the SSR detector is nearly optimal and much better than that of the matched filter. The improvement in performance of the SSR detector with respect to the matched filter is particularly striking for CGM input noise.

We will now explore the dependence of the performance the SSR detector on the impulsiveness of input noise. We recall that kurtosis, a measure of impulsiveness, increases when parameter $e$ of the GG pdf, $u$ of the GM pdf, $\alpha$ of the CGM pdf or $\upsilon$ of the ST pdf is decreased. The variation in probability of detection $P_D$ (for $P_{FA} = 0.1$) with respect to the input noise parameters $p, u, \alpha$ and $\upsilon$ is plotted in Figs. 5.7(a)-5.7(d). Each panel contains five plots corresponding to five different detectors, viz. (i) LO, (ii)
Fig. 5.7: Plots of $P_D (P_{FA} = 0.1, T = 80)$ vs. (a) parameter $e$ of GG input noise, (b) parameter $u$ of GM input noise, (c) parameter $\alpha$ of CGM input noise and (d) parameter $v$ of ST input noise, for 5 different detectors.

SSR, (iii) NQ with $\sigma = 1$, (iv) NQ with $\sigma = 0$ detectors, and (v) matched filter. Let the $P_D$ of these detectors be denoted as $P_{D,LO}$, $P_{D,SSR}$, $P_{D,NQ(1)}$, $P_{D,NQ(0)}$ and $P_{D,MF}$ respectively. All plots except the plot for the matched filter in Fig. 5.7(c) have been plotted using theoretical expressions. These figures provide the following observations and inferences.

5 There are no closed – form expressions available for the performance of a matched filter in CGM noise, which is considered in Fig. 5.7(c).
(i) In GG, GM or ST input noise, $P_{D,LO} \geq P_{D,MF}$, and the equality holds if and only if $e = 2$, $u = 0.5$ or as $\nu \to \infty$. The implication is that LO detector reduces to matched filter if the input noise is Gaussian, and it performs better than matched filter in all other cases. This result is well known. $P_{D,LO}$ increases monotonically as $p$, $u$ or $\nu$ is reduced (kurtosis of noise is increased); $P_{D,LO} \to 1$ as $p \to 0$.

(ii) For the SSR detector, $P_{D,SSR}$ is slightly lower than $P_{D,MF}$ for Gaussian input noise ($e = 2$, $u = 0.5$ or $\nu \to \infty$). This conclusion is in conformity with the fact that the matched filter is the optimal detector in Gaussian noise. $P_{D,SSR}$ increases monotonically as $e$, $u$, $\alpha$ or $\nu$ is decreased. Also, $P_{D,SSR} = P_{D,MF}$ when $e = 1.9$ (kurtosis = 0.1075), $u = 0.36$ (kurtosis = 0.26) or $\nu = 17$ (kurtosis = 0.4615). Hence, the SSR detector performs better than the matched filter in leptokurtic input noise irrespective of the shape of the noise pdf, unless the kurtosis is very close to 0; and the difference in performance between the two detectors increases monotonically as the kurtosis of input noise increases. Also $P_{D,SSR} < P_{D,LO}$ for all $e$, $u$, $\nu$ and $\alpha$. However, the asymptotic performance of SSR detector is fairly close to that of the LO detector in most cases.

(iii) The plots for the NQ detector ($\sigma = 1$) and NQ detector ($\sigma = 0$) in Figs. 5.7(a)-5.7(d) illustrate the reduction in preprocessor performance due to non-optimal choice of $\sigma$. However, even with non-optimal values of $\sigma$, the performance of the NQ detectors in heavy-tailed input noise is much better than that of matched filter. We therefore conclude that quantization is a critical step in SSR preprocessing, while optimization of $\sigma$ merely provides an incremental improvement in performance. This can be explained as follows.
The net denoising effect of SSR preprocessing can be explained by three simultaneous phenomena taking place in the preprocessor:

(i) Quantization removes the outlier values from the impulsive input noise that contaminates the signal. This is a beneficial effect and leads to SNR enhancement of signal.

(ii) However, this quantization causes distortion of the signal that leads to a reduction of its correlation with the original signal. This can lead to degradation of detector performance.

(iii) Addition of an optimal amount of quantizer noise helps in partially restoring the correlation. Thus it enhances the detector output by counter-acting phenomenon (ii).

When the input noise becomes more impulsive, the positive effect (SNR enhancement) due to the phenomenon (i) becomes more significant than the negative effect of phenomenon (ii). Therefore the need for phenomenon (iii) to counter-act phenomenon (ii) diminishes. In other words, as the input noise becomes more impulsive the need for addition of quantizer noise is lower because the quantization-induced loss of correlation is smaller. In very impulsive noise, quantization alone can provide a good improvement in performance.

It follows from the above discussion that the SSR detector is fairly robust in the presence of error in modeling the input noise pdf since quantization is a more critical step as compared to fine-tuning the intensity of quantizer noise. The knowledge of
input noise pdf is only needed to optimize the effect of phenomenon (iii). However if input noise pdf is not known, optimization of $\sigma$ based on an approximate model of input noise pdf is sufficient to ensure a performance that is much better than that of the matched filter.

### 5.2.5. Optimization of quantizer noise pdf

The design of the optimal SSR detector for a given input noise pdf $f_n(\xi)$ requires the determination of the optimal standard deviation and the optimal pdf $f_q(\xi)$ of the quantizer noise. One way of obtaining the optimal pdf of quantizer noise is to parameterize the quantizer noise pdf and maximize the processor gain $G_{wq}$ with respect to all the parameters of the quantizer noise including the standard deviation. This approach can be explored elaborately by modeling the quantizer noise using a pdf with multiple parameters. Here, we consider a restricted version of this approach by modeling the quantizer noise by a GG pdf with a single parameter $e_q > 0$. This choice is suggested by the fact that the entire range of values of kurtosis [-1.2, $\infty$) is included in the class of GG distributions. Now, the problem of optimizing the detector reduces to the readily solvable problem of maximizing $G_{wq}(\sigma, e_q)$ with respect to $\sigma$ and $e_q$. For example, it can be shown that for CGM ($\alpha = 0.7$), GM ($u = 0.01$), ST ($\nu = 3$) and GG ($e = 0.5$) input noises, the optimal values of $(\sigma, e_q)$ are (0.64, 2), (0.76, $\infty$), (0.31, 3.7), and (0, -) respectively.
Fig. 5.8: Plots of (a) optimal $p_q$ vs. $e$ and (b) $\sigma_{opt}$ vs. $e$, for GG input noise, (c) optimal $e_q$ vs. $u$ and (d) $\sigma_{opt}$ vs. $u$, for GM input noise, (e) optimal $e_q$ vs. $\alpha$ and (f) $\sigma_{opt}$ vs. $\alpha$, for CGM input noise, (g) optimal $e_q$ vs. $v$ and (h) $\sigma_{opt}$ vs. $v$, for ST input noise. Quantizer noise is GG.

In Fig. 5.8, the optimal values of $\sigma$ and $e_q$ are plotted against parameter $p$ of GG input noise, parameter $u$ of GM input noise, parameter $\alpha$ of CGM input noise and parameter $\nu$ of ST input noise respectively. Figures 5.8(a), (c), (e) and (g) indicate that the optimal value of $e_q$ exhibits a wide range of variation with respect to input.
Fig. 5.9: Plots of $G_{wq}$ vs. $\sigma$ for different values of $e_q$, (a) GM ($u = 0.01$), (b) CGM ($\alpha = 0.7$) and (c) ST ($v = 3$) input noise. Quantizer noise is GG.

Fig. 5.10: Plots of $\sigma_{opt}$ vs. $A$ for GG ($e = 0.5$), GM ($u = 0.01$), CGM ($\alpha = 0$) and ST ($v = 3$) input noise. $P_{FA} = 10^{-4}$, $T = 80$. 

Chapter 5 | Detection in shallow ocean in presence of non-Gaussian noise
noise pdf. The optimal value of $e_q \to \infty$ (uniform distribution) as the impulsiveness of the input noise pdf approaches that of Gaussian noise. Using a different optimality condition viz. maximal rate of initial SR effect, Patel and Kosko [36] have shown that uniform quantizer noise is optimal for a wide range of input noise pdfs.

Figures 5.8(b), 5.8(f) and 5.8(h) indicate that $\sigma_{opt}$ decreases as impulsiveness of GG or CGM input noise is increased. This trend is in conformity with the observation of Kosko and Mitaim [144] who showed that the SR effect fades as noise becomes more impulsive. This decrease in $\sigma_{opt}$ is because as the input noise becomes more impulsive, the benefit of quantization in removing large outlier values of noise becomes more significant compared to the signal distortion due to quantization. Therefore the importance of adding quantizer noise to enhance the signal quality diminishes as the impulsiveness of the noise increases. But, the plot for GM input noise in Fig. 5.8(d) does not fully follow this trend of monotonic variation. In Fig. 5.8(b) (GG input noise), $\sigma_{opt} = 0$ for $e < 1.05$. It follows that, for GG input noise with $e < 1.05$, injection of additional noise does not provide improvement in performance over that of a simple quantizer-correlator.

Figure 5.9 shows plots of $G_{wq}$ versus $\sigma$ for different values of $e_q$ for GM ($u = 0.01$), CGM ($\alpha = 0.7$) and ST ($\nu = 3$) input noises. These plots indicate that the change in the peak value of $G_{iq}$ due to variation in $e_q$ is very small. Hence, the choice of quantizer noise pdf does not have a significant impact on the performance of the SSR detector; a near-optimal performance can be obtained by choosing any value of $e_q$ in
Fig. 5.11: Plots of $P_D$ ($P_{FA} = 10^{-4}$, $T = 80$) vs. $A$ for SSR detector, OD and matched filter. Input noise: (a) GG ($e = 0.5$), (b) GM ($u = 0.01$), (c) CGM ($\alpha = 0$), (d) ST ($v = 3$).

the interval $[2, \infty)$. It is also seen that the peaks in Fig. 5.9 are quite broad, indicating low sensitivity to variation in the value of quantizer noise variance. Hence near-optimal performance can be achieved by choosing Gaussian ($e_q = 2$) or uniform ($e_q = 3$).
quantizer noise pdf and tuning its standard deviation with an approximate model of input noise. These noise pdfs are good choices as quantizer noise as they can be generated easily through low complexity and low power circuits as shown in [182–184].

5.2.6. SSR detection of non-weak signals

The discussion in subsections 5.2.2-5.2.5 so far has been confined to the problem of detection of weak signals with $0 < A << 1$ which allowed a simpler and more detailed analysis of the detectors. It is also of interest to examine the performance of the SSR detector when the weak-signal condition is violated. This requires the determination of the means $m_j$ and standard deviations $\lambda_j$ of the test statistic $T_{SSR}(X)$ without the weak-signal approximation, and then finding the value of $\sigma$ that maximizes $P_D$ for a given $P_{FA}$. When $A$ is not very small, the problem of optimization of the noisy quantizer detector gets modified due to the following reasons: (i) The approximation $\left(\frac{\lambda_0}{\lambda_1}\right) = 1$ breaks down, (ii) values of $\lambda_0/\lambda_1$ and the parameter $d$ in (5.13) depend on both $A$ and $\sigma$. Hence maximization of $G_{wq,L}(\sigma)$ or $G_{wq}(\sigma)$ does not lead to maximization of $P_D$. It is also evident from (5.13) that optimum $\sigma$ depends not only on $A$ but also on $P_{FA}$ in the case of non-weak signals.

The variation of optimum $\sigma$ (for maximization of $P_D$) with respect to $A$ is shown in Fig. 5.10 for GG, GM, CGM and ST input noise. The optimum $\sigma$ increases with $A$ in an almost linear fashion if the input noise is CGM, GG or ST, but the variation is non-monotonic in the case of GM input noise.
Plots of $P_D$ (at $P_{FA} = 10^{-4}$) versus $A$ for SSR detector, NP-optimal detector (OD) and matched filter are compared in Fig. 5.11. The test statistic for OD is given by (5.3). The performance of the SSR detector is substantially superior to that of the matched filter and very close to that of the OD, indicating the SSR detector is effective for detection of non-weak signals as well.

This concludes section 5.2 on SSR detection using a single sensor. It has been demonstrated that an SSR preprocessor significantly improves the performance of a matched filter detector that uses a single sensor. The detector can be tuned to maximize the probability of detection by choosing the optimal value of standard deviation $\sigma$ of the quantizer noise. The procedure for the design of the SSR preprocessor can be readily extended to the detection of non-weak signals using a single sensor as well. Thus SSR preprocessing can be employed as a strategy to make the array based detectors designed for Gaussian noise usable in an environment contaminated by non-Gaussian noise. In the next section, we will deal with such array-based detectors designed for this purpose.

**5.3. Detection using array of acoustic vector sensors**

This section tackles the problem of detection in an environment contaminated by non-Gaussian noise using an array of acoustic vector sensors. The objective is to formulate various detectors to be applied in such an environment and to evaluate their
performance. There are two strategies that are explored in this section for detection in non-Gaussian noise:

(i) The first strategy is to extend the GLRT formulation of detectors done in chapter 4, to the case of non-Gaussian environmental noise.

(ii) The second strategy is to employ SSR preprocessing to denoise the array data and boost the detection performance. The results of section 5.2 are extended to array-based detectors.

The framework that will be followed in this section for formulating the detection problem and the GLRT detectors is the same as that used in chapter 4. Here our focus is primarily on the formulation and performance of detectors in non-Gaussian noise, and we are less concerned with details of generalizing these results to different array geometries or more data snapshots. Hence we will simplify the problem by assuming that detection is done with an HLA using a single snapshot of data \( T = 1 \). The detection problem can be cast in the form of the following hypothesis testing problem:

\[
H_0 : x = w
\]
\[
H_1 : x = s + w.
\] (5.39)

where \( s \) represents the array signal vector, \( x \) the array data vector and \( w \) the vector of environmental noise at the sensor array. All these vectors are of dimension \( 3N \times 1 \). Note that unlike in chapter 4, the dependence of these vectors on the snapshot number \( t \) is dropped because only a single snapshot of data is considered. The joint likelihood functions of the array data vectors under hypotheses \( H_0 \) and \( H_1 \) are given by
Chapter 5  Detection in shallow ocean in presence of non-Gaussian noise

\[ f(x| e, \sigma^2; H_0) = f_{GG}(x), \quad (5.40) \]

\[ f(x| s, e, \sigma^2; H_1) = f_{GG}(x-s), \quad (5.41) \]

where \( f_{GG}(\cdot) \) is the generalized Gaussian pdf for an array vector defined in (3.36), \( \sigma^2 \) is the variance of noise and \( e \) is the exponential parameter of GG the noise pdf. When the value of \( e = 2 \), the GG pdf reduces to a circular complex Gaussian pdf and the detection problem is a simpler version of the problem dealt with in chapter 4. Thus the test statistics of detectors defined in this section reduce to those of the detectors of signals in Gaussian environmental noise when the parameter \( e = 2 \).

5.3.1. Optimal detector (OD)

When the signal vector \( s \) is known, it is possible to implement the NP-optimal detector [1] that maximizes the probability of detection at a constant probability of false alarm. For GG environmental noise, the ratio of likelihood functions (5.41) and (5.40) (i.e, likelihood ratio) is given by

\[ L(x) = \frac{f(x| s, e, \sigma^2; H_1)}{f(x| e, \sigma^2; H_0)} \quad (5.42) \]

Simplification of the above likelihood ratio yields the test statistic of the optimal detector as:

\[ T_{OD}(x) = \sum_{n=1}^{2N} \left[ |x_n| - |x_n - s_n| \right]. \quad (5.43) \]
where $x_n$ denotes the $n^{th}$ element of vector $x$. The optimal decision criterion is “Decide $H_1$ if $T_{OD}(x) > \eta$” where $\eta$ is the detection threshold. The vector $s$ is unknown because it depends on the unknown location of the source and/or on channel parameters that are unknown or time-varying. The variance of the environmental noise is also generally unknown because it is a time-varying parameter. This makes it impossible to realize the OD. However, the OD is used as a benchmark against which other detection methods can be compared. When $e = 2$, the OD reduces to the MFD described in subsection 4.2.1.

When the parameters of the signal $s$ or the noise $w$ are unknown, they are estimated as per the GLRT detection approach by maximizing the likelihood functions with respect to these unknown parameters [1]. The noise variance $\sigma^2$ may sometimes be unknown, and in some cases the shape of the pdf of the environmental noise (i.e. parameter $e$ of noise pdf) is also unknown. In this chapter, we shall consider the detection problem for three different cases:

(a) The noise pdf is completely known ($\sigma^2$ and $e$ are known),

(b) The noise pdf is partially known ($\sigma^2$ is unknown but the exponential parameter $e$ of noise is known), and

(c) The noise pdf is completely unknown (both $\sigma^2$ and $e$ are unknown).

The cases (a) and (b) mentioned above are the same as cases (a) and (b) considered in chapter 4 (where $e = 2$ is assumed).
We now discuss the formulation of the AVS array-based detectors for non-Gaussian noise. The detectors will be designed using two strategies, namely (i) formulation of detectors based on a purely GLRT approach, and (ii) combining SSR-preprocessing with AVS array-based detectors designed for Gaussian noise.

5.3.2. GLRT detectors

In a situation where the OD cannot be implemented, the GLRT based strategy for formulation of detectors [1] is to perform a likelihood ratio test for detection by replacing unknown parameters in the test by their MLE. It has been shown that even though the GLRT is sub-optimal, it is asymptotically the most powerful test among all tests that follow the condition of ‘invariance’ [88]. Hence in a detection problem where there are unknown parameters involved, the GLRT is often the best option.

We recollect from chapter 4 that for case (a), the logarithm of the GLR is obtained by replacing $s$ by its MLE $\hat{s}$. Hence the test statistic for case (a) can be written as

$$
\gamma_{\text{Case(a)}}(x) = \sum_{n=1}^{3N} \left[ |x_n|^r - |x_n - \hat{s}_n|^r \right],
$$

(5.44)

where the signal-vector estimate $\hat{s}$ is obtained in different ways for different detectors. For case (b), we recall from chapter 4 that the expression for the test statistic is obtained by replacing the parameters $\sigma^2$ and $s$ by their estimates. The test statistic is of the form
In case (c), the exponential parameter $e$ is also unknown in addition to $\sigma^2$ and $s$. The numerator and denominator terms in (5.45) are minimized with respect to $e$ to obtain the estimates $\hat{e}_1$ and $\hat{e}_0$ of this parameter under hypotheses $H_1$ and $H_0$ respectively. Substituting these in (5.45) yields the GLR as

$$
\gamma_{\text{Case (c)}}(x) = \frac{\min_e \sum_{n=1}^{3N} |x_n|^{\hat{e}}}{\sum_{n=1}^{3N} |x_n - \hat{s}_n|^{\hat{e}}}. 
$$

Note that the test statistic for case (c) is obtained by minimizing both the numerator and denominator of the expression in case (b) with respect to the unknown parameter $e$. Since the environmental noise in the ocean is known to be more impulsive in nature than Gaussian noise, $\hat{e}_0$ and $\hat{e}_1$ can be obtained by maximization within the range $(0, 2]$. However, this search is still very complex and increases the complexity of the GLRT detector when pdf is unknown.

Now we discuss the formulation of four different detectors that consider different signal models. Among these, the unconstrained detector (UD) and SD are formulated using conventional approaches to detection whereas the TSD and ASFD are novel approaches introduced in this work.
5.3.2.1. **Unconstrained detector**

A conventional approach to detection that is often found in the literature [1] is to consider the signal vector $s$ to be totally unknown. The likelihood function $f(x|s, e, \sigma^2; H_1)$ is maximized with respect to $s$ to obtain the unconstrained MLE $\hat{s} = x$. Substitution of this into (5.44) and simplification of the resulting likelihood ratio yields the test statistic of the UD for GG noise as

$$T_{UD, \text{Case(a)}}(x) = \sum_{n=1}^{3N} |x_n|^e.$$

(5.47)

When $e = 2$ (Gaussian noise), the UD reduces to an energy detector described in subsection 4.2.2. As in 4.2.2, the GLRT with no knowledge of signal or variance does not yield a meaningful test statistic under cases (b) and (c).

5.3.2.2. **Subspace detector**

It is known that if $3N > M$ where $N$ is the number of sensors in the array and $M$ is the number of normal modes, the columns of the modal steering matrix $A(\phi)$ are linearly independent and the $3N$-dimensional vector $s$ belongs to the $M$-dimensional modal subspace $V(\phi)$ spanned by the columns of $A(\phi)$. Based on this property of $s$, the SD was formulated for Gaussian noise in subsection 4.2.3 and we refer the reader to this subsection for a deeper discussion on the subspace detector. The formulation of the SD can be done in a similar manner for the case of GG environmental noise. As shown in (4.17), we can represent $s$ in an alternative form as
where $U(\phi)$ is a unitary matrix that can be obtained by QR decomposition of $A(\phi)$ as

$$A(\phi) = U(\phi) R.$$

In (5.48), $\beta = Rb$ is a transformed version of the unknown mode amplitude vector $b$. The vector set $\{u_1(\phi)\ldots u_M(\phi)\}$ is the orthonormal basis of $V(\phi)$ obtained through a Gram-Schmidt transformation process. The problem of estimating the $3N$-dimensional signal vector $s$ is reduced to the simpler problem of estimating the $M$-dimensional vector $\beta$. Maximization of the likelihood function $f(x;H_1) = f(x; e, \sigma^2, \beta, \phi, H_1)$ with respect to $\beta$ is equivalent to the minimization problem

$$\hat{\beta}(\phi) = \min_\beta \sum_{n=1}^{3N} |x_n - r_n(\phi) \beta(\phi)|^{\gamma - 2} \text{sgn}[x_n - r_n(\phi) \beta(\phi)],$$

that can be reduced to the following.

$$\sum_{n=1}^{3N} |x_n - r_n(\phi) \hat{\beta}(\phi)|^{\gamma - 2} [\hat{\beta}(\phi) - r_n^H(\phi)x_n] = 0,$$

where $r_n(\phi)$ is a row vector denoting the $n^{th}$ row of $U(\phi)$, and $\hat{\beta}(\phi)$ denotes the conditional MLE of $\beta$ for a given $\phi$. From (4.19), the following closed form solution of (5.51) is readily obtained if $e = 2$ (Gaussian):

$$\hat{\beta}(\phi) = U^H(\phi)x.$$
When $e \neq 2$, no closed form solution of (5.51) is available and it can only be solved by iterative methods such as Newton’s method which incur a large computational cost. Hence we use the estimate of $\beta$ in (5.52) as an approximation to the conditional MLE of $\beta$. The use of this estimate is justified by the simulation results in subsection 5.2.5 that show that the detector yields fairly good performance with this approximation.

Another possible estimate of $\beta$ is possible when $e = 1$ (Laplacian noise). In this case, the estimate of $\beta$ can be obtained by weighted median filtering of $x$ [29]. This suggests that median filtering can be used as an alternative method to implement the detectors. However, the weighted median estimate will not be considered in this work, and we continue the formulation of the using the Gaussian estimate in (5.52).

From (4.21) and (4.22), the approximate MLEs of $s$ can be written as

$$\hat{s}(\phi) = U(\phi)U^H(\phi)x,$$

(5.53)

The test statistics of the SD are thus given by

$$T_{SD,Case(a)}(x) = \min_{\phi} \sum_{n=1}^{3N} \left[ |x_n|^e - |x_n - r_n(\phi)U^H(\phi)x|^e \right],$$

(5.54)

and

$$T_{SD,Case(b)}(x) = \sum_{n=1}^{3N} |x_n|^e / \min_{\phi} \sum_{n=1}^{3N} |x_n - r_nU^H(\phi)x|^e$$

(5.55)

for cases (a) and (b) respectively.
5.3.2.3. **Truncated subspace detector**

The subspace detector has several limitations such as degradation of performance with increasing frequency and an upper limit on the frequency of operation. As discussed in subsection 4.2.4, the TSD is formulated by us to overcome these limitations by truncating the normal-mode expansion of the acoustic field to a small number $M'$ of the lowest order modes to obtain the approximation given below:

$$ s'(\phi) = A'(\phi)\beta' = U'(\phi)\beta', \quad (5.56) $$

where

$$ A'(\phi) = [a_1(\phi) \ldots a_{M'}(\phi)], \quad U'(\phi) = [u_1(\phi) \ldots u_{M'}(\phi)] \quad (5.57) $$

and $\beta'$ is a vector with $M'$ elements. As discussed in chapter 4, the optimum number of modes $M'$ is selected as the value that minimizes the NMSE associated with the TSD. For an HLA, the optimal value of $M'$ is quite small and generally equal to 1. The conditional ML estimator of $\beta'$ can be obtained by maximizing the likelihood function $f(x; e, \sigma^2, \beta', \phi, H_1)$ with respect to $\beta'$, yielding the equation

$$ \sum_{n=1}^{3N} [x_n - r'\phi(\phi)\hat{\beta}(\phi)]^{-2} \left[ \hat{\beta}(\phi) - r_n^{H}(\phi)x_n \right] = 0. \quad (5.58) $$

where $r'_n(\phi)$ is a row vector denoting the $n^{th}$ row of $U'(\phi)$, and $\hat{\beta}'(\phi)$ denotes the conditional MLE of $\beta'$ for a given $\phi$. From (4.19), the following closed form solution of (5.58) is obtained if $e = 2$:

$$ \hat{\beta}'(\phi) = U'(\phi)^H x. \quad (5.59) $$
The estimate in (5.59) is used as an approximation to the MLE of $\beta'$. The test statistic of the TSD is thus given by

$$T_{TSD, Case(a)}(x) = \min_\phi \left[ \sum_{n=1}^{3N} \left| x_n \right|^2 - \left| x_n - r_n(\phi)U^H(\phi)x \right|^2 \right].$$

(5.60)

for case (a), and as

$$T_{TSD, Case(b)}(x) = \sum_{n=1}^{3N} \left| x_n \right|^2 / \min_\phi \left[ \sum_{n=1}^{3N} \left| x_n - r_n(\phi)U^H(\phi)x \right|^2 \right].$$

(5.61)

for case (b), respectively.

5.3.2.4. Approximate signal form detector

The ASFD is a novel detector formulated by exploiting the fact that the modal wave numbers $\{k_m; m = 1, \ldots, M\}$ are very close to one another (see subsection 4.2.5). Using the approximation $k_m/k(z_a) \approx 1$, we get the following approximate expression for the array signal vector

$$s''(\phi) = H(\phi)p, \quad H(\phi) = I_N \otimes g(\phi),$$

(5.62)

where $g(\phi)$ is given in (4.44). The approximate signal vector $s''(\phi)$ belongs to the $N$-dimensional subspace $V''(\phi)$ defined as

$$V''(\phi) = \text{span}\{h_1(\phi), \ldots, h_N(\phi)\} = \text{span}\{u_1'(\phi), \ldots, u_N'(\phi)\},$$

(5.63)

where $\{u_1'(\phi), \ldots, u_N'(\phi)\}$ is the orthonormal basis of $V''(\phi)$. We can therefore rewrite (5.62) as
On replacing $s$ by $s''$ in (5.41) and maximizing the resultant likelihood function $f(x; \beta'', \phi, H_1)$ with respect to $\beta''$, we obtain the nonlinear equation

$$
\sum_{n=1}^{3N} | x_n - r''_n(\phi) \hat{\beta}''(\phi) |^{-2} [ \hat{\beta}''(\phi) - r''_n H(\phi) x_n ] = 0.
$$

(5.65)

where $r''_n(\phi)$ is a row vector denoting the $n^{th}$ row of $U''(\phi)$, and $\hat{\beta}''(\phi)$ denotes the conditional MLE of $\beta''$ for a given $\phi$. If $e = 2$, the solution of (5.65) is given by

$$
\hat{\beta}''(\phi) = U''(\phi)^H x
$$

(5.66)

When the estimate in (5.66) is used as an approximation to the MLE, the test statistics of the ASFD are given by

$$
T_{ASFD, Case(a)} (x) = \min_{\phi} \sum_{n=1}^{3N} | x_n | - | x_n - r''_n(\phi) U''(\phi) x |^{e}, \quad \text{and} \quad (5.67)
$$

$$
T_{ASFD, Case(b)} (x) = \sum_{n=1}^{3N} | x_n |^{e} / \min_{\phi} \sum_{n=1}^{3N} | x_n - r''_n U''(\phi) x |^{e}
$$

(5.68)

for cases (a) and (b) respectively.

### 5.3.3. SSR detectors

An alternative way to perform detection in non-Gaussian noise is to employ a detector designed for Gaussian noise preceded by an SSR preprocessor. Section 5.2 of this thesis demonstrated the effectiveness of this preprocessor in enhancing the...
performance of detection of a single sensor. Here, we attempt to apply the results described in section 5.2 to the case of detection using an array of sensors. It has been shown that when the ambient noise is impulsive in nature it is possible to obtain better detection performance than that obtained in Gaussian noise by using an SSR preprocessor. The preprocessor provides an SNR gain by removing the impulsive components of noise.

The schematic of the SSR preprocessor has been given in section 5.2 and we use a modified version of the same processor here for denoising array data. Since the AVS array data vector is assumed to be complex, each component of the AVS array output \( x_n = x_n^r + i x_n^i \); \( n = 1, \ldots, 3N \) is applied to the SSR preprocessor that is composed of a parallel array of \( L \) pairs of one-bit quantizers. Here, we refer to ‘quantizers pairs’ instead of separate quantizers as we are concerned with denoising of complex data. The inputs to the \( l \)th quantizer pair are \( x_n^r + \sigma_q q_n (2l-1), x_n^i + \sigma_q q_n (2l) \)

where \( \{q(1),\ldots,q(2L)\} \) are injected i.i.d white noise processes with unit variance which are collectively referred to as quantizer noise. Based on our observations in section 5.2 on the optimization of quantizer noise pdf, we consider the quantizer noise to be Gaussian distributed. The output of the \( l \)th quantizer pair is

\[
y_n(l) = \text{sgn}[x_n^r + \sigma_q q_n (2l-1)] + i \text{sgn}[x_n^i + \sigma_q q_n (2l)]
\] (5.69)

and the output of the SSR preprocessor is
\[
\bar{y}_n = \frac{1}{L} \sum_{l=1}^{L} y_n(l).
\] (5.70)

In this manner, we obtain the denoised array data vector \( y = [\bar{y}_1, \ldots, \bar{y}_N]^T \). Since the output \( \bar{y}_n \) is a sum of \( L \) i.i.d complex random variables, it is asymptotically \((L \to \infty)\) complex Gaussian distributed irrespective of the pdf of environmental noise. Hence detection may be done in any type of impulsive environmental noise by treating the denoised array vector \( y \) as a complex Gaussian random vector. The vector \( y \) can be used in place of the undenoised data vector \( x \) and used in the detection algorithms described in section 4.2 that are designed for a signal in Gaussian noise. This approach is similar to the single-sensor detector described in section 5.2 which consisted of an SSR preprocessor followed by a detector designed for Gaussian environmental noise (i.e matched filter). The combined system comprising of the SSR preprocessor followed by any of the GLRT detectors for Gaussian noise described in section 4.2 is referred to here as an SSR detector. The preprocessor is relatively simple to implement as it consists only of an array of one-bit quantizers.

It is also worth noting the following points regarding the SSR detector, derived from the previous discussion on SSR detection in section 5.2:

(i) The SNR gain of the output of the SSR preprocessor with respect to the input increases monotonically with the number of quantizer pairs \( L \) and approaches an asymptotic limit.
(ii) In detection case (a), the standard deviation $\sigma_q$ of the quantizer noise is chosen as the optimal value $\sigma_{q,\text{opt}}$ that maximizes the probability of detection $P_D$ unless otherwise mentioned.

(iii) The SSR detector is expected to be robust with respect to change in environmental noise pdf. This is because the denoised array data vector is Gaussian distributed irrespective of the input noise pdf. From the discussion in section 5.2, it is also known that the gain of an SSR preprocessor does not change much with change in environmental noise pdf. Hence if the noise pdf is unknown (case (c)), the SSR detector is expected to provide robust detector performance with an intermediate value of $\sigma_q$ chosen for the preprocessor. It will be seen in the results in 5.3.4 that the SSR detector does indeed provide robust detection performance with simple implementation but its performance in case (c) is not as effective as the GLRT detector which is more complex.

### 5.3.4. Simulation Results

In this subsection we compare the performance of the various array based detectors formulated in this section with the two different strategies (GLRT and SSR preprocessing), through simulations. The simulations assume a 10-sensor horizontal AVS array with half-wavelength spacing is used in a Pekeris channel [7] with the following parameters: ocean depth $h = 70$ m, sound speed in water $c = 1500$ m/s, bottom sound speed $c_b = 1700$ m/s, bottom attenuation $\delta = 0.5$ dB/wavelength, density
Fig. 5.12: $P_D$ vs. $P_{FA}$ at SNR = -10 dB. GG ($e = 0.5$) noise. (a) $f = 50$ Hz, (b) $f = 350$ Hz. All detectors are plotted for detection case (a)-when $\sigma^2$ and $e$ are known.

ratio $\rho_b/\rho = 1.5$. The array depth is $z_a = 40$ m, the source is at a range $r = 5$ km, depth $z_s = 40$ m, azimuth $\phi = 33^\circ$. The array SNR is -10 dB in Fig. 5.12 and -5 dB in Figs. 5.13-5.16 (for the definition of array SNR, refer to (4.87)). The probability of false alarm is chosen as $P_{FA} = 0.1$ for the Figs. 5.13-5.16. The results describing the performance of the TSD are shown for $M' = 1$ which is the value that maximizes the probability of detection $P_D$ of the TSD for the current simulation parameters. The SSR detectors use $L = 300$ quantizer pairs for denoising. The performance of detection of the detectors does not show any significant improvement on increasing $L$ beyond this value.

We first compare the performance of the detectors in terms of their ROC at two different values of signal frequency. The plots in Fig. 5.12 show the ROC at an array
SNR = -10 dB for the following nine detectors under the assumption that the environmental noise variance is known: OD, UD, SD, TSD, ASFD, SSR-UD, SSR-TSD, SSR-SD and SSR-ASFD. The plots are shown for signal frequency (a) 50 Hz (corresponding to $M = 2$ modes) and (b) 350 Hz ($M = 15$). The environmental noise is considered to be GG ($\epsilon = 0.5$) distributed which describes a noise environment that is more impulsive than Laplacian noise ($\epsilon = 1$). The $P_{FA}$ range plotted in the figure is $0 < P_{FA} < 0.1$ as higher false alarm rates are generally not considered in detection applications.

In the ROC plot in Fig. 5.12, a detector A may be considered superior to another detector B in performance if A achieves a larger $P_D$ at a particular value of $P_{FA}$. It can be seen from Fig. 5.12 that among the realizable detectors considered, the TSD provides the best performance. The performance of the SD degrades as the frequency increases due to increase in the signal estimation error incurred by it, and at a frequency of 350 Hz the SD can only yield a performance equivalent to that of the UD. The performance of the SSR-SD is better than that of SD at higher frequency; however, it is still inferior when compared with those of TSD/SSR-TSD even though it uses more prior information than the TSD/SSR-TSD. The ASFD and SSR-ASFD also outperform the SD at a higher frequency even though they use no prior information about the signal or the channel. Thus it is preferable to use the TSD/SSR-TSD over the SD/SSR-SD when the prior information on the modal wave numbers is available. When this prior information is not available it is preferable to use the
Chapter 5 | Detection in shallow ocean in presence of non-Gaussian noise

Fig. 5.13: $P_D$ vs. parameter $e$ of GG noise at $P_{FA} = 0.1$, SNR = -5 dB, for GLRT and SSR detectors in case (a).

Fig. 5.14: $\sigma_{q,opt}$ of SSR detectors vs. parameter $e$ of GG noise at $P_{FA} = 0.1$, SNR = -5 dB
ASFD/SSR-ASFD. As the applicability of the SD/SSR-SD can no longer be justified because it gives a poor performance with high requirements on prior information, and as the SD is ineffective for signals of high frequency, we will not consider the performance of the SD/SSR-SD in the figures that follow henceforth.

We now investigate the variation in performance of the detectors with variation in the impulsiveness of the environmental noise. Figs. 5.13, 5.15 and 5.16 plot the variation of the $P_D$ of the various detectors with the parameter $e$ of the environmental noise at $P_{FA} = 0.1$ and SNR = -5 dB for case (a) ($\sigma^2$ and $e$ known), case (b) ($\sigma^2$ unknown but $e$ known) and case (c) ($\sigma^2$ and $e$ unknown) respectively. Recall that the circular complex GG environmental noise pdf reduces to a circular complex Gaussian pdf for $e = 2$ and its impulsiveness decreases as the value of $e$ reduces.

Figure 5.13 plots the variation of $P_D$ with variation in the parameter $e$ for the case (a) detectors that assume that the pdf of noise is known during detection. This figure compares the six detectors presented in this section (illustrated by the solid lines for GLRT detectors and dashed lines for SSR detectors) as well as three detectors formulated for Gaussian noise in chapter 4 (illustrated by dotted lines). In order to avoid ambiguity we refer to the latter three detectors as Gaussian detectors (i.e Gaussian-TSD, Gaussian-ASFD and Gaussian-UD). These detectors use the test statistics defined in (4.42), (4.58) and (4.13) respectively. These detectors are GLRT detectors formulated for the case $e = 2$ (Gaussian noise). The Gaussian detectors are plotted in this figure to draw a comparison against the GLRT detectors formulated in
this chapter, when a non-Gaussian noise environment is considered. This tells us how
detectors formulated with a Gaussian noise assumption would work when the noise is
non-Gaussian in nature.

The SSR detectors whose performance is shown in Fig. 5.13 use the optimal
STD of quantizer variance denoted by $\sigma_{q,\text{opt}}$ that varies with the parameter $e$ of noise.
The variation of $\sigma_{q,\text{opt}}$ with $e$ corresponding to the SSR detectors in Fig. 5.13, is plotted
in Fig. 5.14. From Fig. 5.13 we observe:

(i) The performance of the six detectors formulated for non-Gaussian noise
    environments improves as $e$ is reduced (i.e. as impulsiveness of environmental
    noise increases). This can be explained as follows. For a given SNR, when the
    parameter $e$ is reduced, the energy of an impulsive noise pdf shifts from the
    central region of the pdf towards the tail regions which correspond to the
    outlier values of noise. The detectors are able to more effectively reject these
    outlier values. Hence when the noise is more impulsive, the detectors are able
to achieve better performance. A similar observation has been made in section
5.2 with respect to the effectiveness of an SSR preprocessor in non-Gaussian
noise, viz. that the SNR gain of an SSR preprocessor increases with increase in
impulsiveness (kurtosis) of the noise pdf.

(ii) The performance of the Gaussian detectors (dotted lines) deteriorates with
decreasing value of $e$. This shows that detectors formulated under the
assumption of Gaussian environmental noise are highly unsuitable for
environments when the noise is highly impulsive in nature. This emphasizes
the need for formulating detectors that are suitable for such environments, as done in this chapter.

(iii) Overall, the TSD is the most effective detection scheme over the whole range of heavy-tailed GG noise pdfs. The performance of the ASFD/SSR-ASFD in case (a) is better than the UD (note that both these detectors use no prior information on the signal).

(iv) SSR preprocessing provides a performance gain for the SSR detectors over the Gaussian detectors only for $e < 1.9$. This is on expected lines as SSR has been reported to be less effective for Gaussian or near-Gaussian environmental noise when the value of $e$ is close to 2 (refer section 5.2).

(v) In detection case (a) ($\sigma^2$ and $e$ are known), the best available option for detection is the TSD when the $M'$ modal wave numbers are known. If this information is unknown, it is preferable to use the SSR-ASFD in highly impulsive noise ($e < 1$) if an SSR processor is available, or an ASFD if the preprocessor is not available.

(vi) Using the SSR preprocessor is highly beneficial for the UD and provides an improvement in its performance for $e < 2$. It provides marginal improvement in the performance of the ASFD which is limited to the range of GG pdfs with $e < 1$. SSR preprocessing is not beneficial in case of the SD and TSD.

The reason for this difference in performance gains for different methods is as follows. The SSR preprocessor is a nonlinear system, which provides an SNR gain that is unequal for signals of different amplitudes. Since the signal amplitude varies from
Fig. 5.15: $P_D$ vs. parameter $e$ of GG noise at $P_{FA} = 0.1$, SNR = -5 dB, for GLRT and SSR detectors in case (b).

sensor to sensor, data from each element of the array goes through different SNR gains due to SSR denoising. This leads to some distortion of the signal structure due to changes in the relationship among various elements. The information in this signal structure is (i) crucial and important for the TSD/SD methods, (ii) moderately important for the ASFD, and (iii) not important for the UD (which measures only the signal content in the data and not the relationship between the data elements). Hence even though the signal undergoes SNR improvement, the performance of the TSD and ASFD do not improve considerably due to distortion in the signal structure. But SSR is beneficial in improving the performance of the UD that does not depend only on the SNR and not on the signal structure. The above discussion gives crucial insight into the applicability of SSR, viz. that SSR may not be very effective in algorithms that utilize the complex signal structure. It is also observed from Fig. 5.14 that the value of
Fig. 5.16: $P_D$ vs. parameter $e$ of GG noise at $P_{FA} = 0.1$, SNR = -5 dB, for GLRT and SSR detectors in case (c).

$\sigma_{q,opt}$ of the SSR detectors increases with increase in the value of $e$. This trend is in agreement with the observations made in section 5.2 on a single sensor SSR detector.

In Fig.5.15, the variation of $P_D$ with $e$ is plotted for case (b) detectors that assume that the variance $\sigma^2$ of environmental noise is unknown. Figure 5.15 yields the following inferences:

(i) There is degradation in the performance of the detectors when the variance $\sigma^2$ of environmental noise is unknown. The difference between the plots in Fig. 5.15 with the corresponding plots in Fig. 5.13 represents the degradation in detector performance due to lack of knowledge of $\sigma^2$. This degradation is lower in the case of the TSD as compared to the ASFD and the SD showing the
greater robustness of TSD. In a practical scenario, this degradation can be prevented if the noise pdf is estimated in advance using secondary noise-only data [11].

(ii) The SSR-ASFD cannot perform better than the ASFD in case (b) unlike it did in case (a). This shows that the effectiveness of SSR detectors is reduced when the noise parameters are unknown.

(iii) The TSD and ASFD offer a clear advantage over the UD viz. that they can be used even when the parameters of the environmental noise are unknown.

In Fig. 5.16, the value of $P_D$ is plotted against the value of $e$ for case (c) detectors that assume that the pdf of the environmental noise is completely unknown. i.e., in this case both the parameters $\sigma^2$ and $e$ are unknown. The GLRT detectors are required to perform an additional estimation of the parameter $e$ of the noise pdf which increases their complexity. The SSR detectors are implemented with a suboptimal value of quantizer noise standard deviation $\sigma_q \neq \sigma_{q,opt}$. $\sigma_q$ is chosen as an intermediate value in the range of variation observed from Fig. 5.14, since $\sigma_{q,opt}$ is not known at the time of detection. Figure 5.16 yields the following inferences:

(i) There is degradation in the performance of the detectors in case (c) as compared to case (b) due to lack of knowledge of the parameter $e$ of environmental noise. The difference between the plots in Fig. 5.16 with the corresponding plots in Fig. 5.15 represents the degradation.

(ii) The degradation in case (c) as compared to case (b) is low in the case of the TSD/SSR-TSD showing the greater robustness of the TSD method.
(iii) The SSR detectors are not as effective as the corresponding GLRT detectors in case (c).

This concludes the investigation on various AVS array-based detection techniques for signals contaminated by non-Gaussian noise, in different cases.

5.4. Summary

This chapter dealt with the problem of detection of signals in impulsive noise that cannot be modeled as Gaussian noise. The work was dealt with in two parts as sections 5.2 and 5.3. Section 5.2 discusses the design and performance of a single-sensor detector based on SSR for detection of deterministic signals in non-Gaussian noise. Section 5.3 deals with the formulation and performance investigation of AVS array-based detectors for signals in shallow ocean contaminated by non-Gaussian environmental noise.

5.4.1. Single-sensor SSR detector

The single sensor SSR detector consists of an SSR preprocessor and a matched filter. The SSR system is tuned to maximize the $P_D$ by choosing the optimal value of standard deviation $\sigma$ of the quantizer noise. In the case of weak signals, optimization of the preprocessor can be reduced to the maximization of a function of $\sigma$. Maximization of the $P_D$ also minimizes the mean-square difference between the output of the quantizer array and that of the optimum nonlinear transformation of the LO
detector. Thus the optimization of the SSR preprocessor involves modifying its input-output characteristics to become closer to the LO detector’s non-linear characteristics.

It is shown that the SSR preprocessor provides a significant improvement in performance over the matched filter. For weak signals, the performance of the SSR detector is very close to that of the LO detector. The performance improvement due to SSR stems primarily from quantization, while the addition of optimum amount of quantizer noise improves the performance further and brings it closer to that of the LO detector. For GG, ST and CGM input noise, the optimum value of $\sigma$ decreases as the impulsiveness of input noise increases. Further improvement in performance is possible through an optimal choice of quantizer noise pdf. The sensitivity of the SSR detector performance to non-optimal choice of quantizer noise pdf is quite low. The SSR detector is robust with respect to errors in modeling the input noise pdf. Hence near-optimal performance can be achieved by choosing Gaussian or uniform quantizer noise pdf and tuning its standard deviation with an approximate model of input noise.

The design of the SSR preprocessor can be readily extended to the detection of non-weak signals. In this case, the optimal value of $\sigma$ depends on the signal amplitude and false alarm probability. The SSR detector provides a significant improvement in performance over the matched filter in the case of non-weak signals also.
5.4.2. Detection using AVS array

In section 5.3, we discussed the formulation of AVS array-based detection schemes for sources in shallow ocean environments contaminated by impulsive noise. These include two new detection schemes called the TSD and ASFD that are extended from schemes discussed in chapter 4. We also propose AVS array-based detectors that perform SSR preprocessing prior to detection in order to achieve improved performance.

The results indicate that the conventional SD is not an effective option for detection as its performance degrades with increasing frequency due to an associated increase in the MSE. The TSD is the most effective and robust detection scheme for a wide range of noise pdfs if the modal wave numbers of the ocean channel are known. If the modal wave numbers of the channel are unknown, the ASFD provides an alternative option to detection. It is more effective than the conventional UD.

SSR preprocessing is effective in enhancing the performance of the UD, whereas its effectiveness when used with the ASFD is marginal and limited to the case when the environmental noise pdf is known and $e < 1$. SSR preprocessing is not beneficial in case of the SD and TSD because the SSR system is a nonlinear system. The applicability of SSR to array detection is limited to cases where the noise environment is well-known. As the noise becomes more impulsive, the performance of all the detectors improves because they are able to discard more effectively the outlier values arising from noise.
Chapter 6

Source localization in ocean using acoustic vector sensors

“Equations are more important to me, because politics is for the present, but an equation is something for eternity.”
- Albert Einstein

6.1. Introduction

The primary concern in underwater source localization is the low SNR of signals received in an ocean channel owing to the high level of ambient noise. The performance of DOA estimation algorithms degrades rapidly as the SNR reduces. For example, it has been shown that in the case of plane wave DOA estimation using MUSIC and MLE methods, the mean square error is inversely proportional to the SNR [185]. In a practical scenario the SNR encountered is often too low to obtain reliable estimates using DOA estimation methods. Algorithms that employ arrays of AVS in place of APS can achieve a better performance of source localization [9], [17], [39],
[44–63] thus enabling these algorithms to operate at a lower SNR. However, most of the existing AVS-based localization algorithms do not take into account the property that underwater acoustic noise is impulsive in nature [20], [23]. Moreover, much of the existing work assumes that the AVS array is located in the far-field of the source, and the problem of localizing near-field sources has received less attention.

In section 6.2, we show how the performance of AVS-based azimuth estimation in shallow water environments can be improved by using an SSR preprocessor. The SSR preprocessor makes use of the fact that the noise in such environments is impulsive. The benefits of SSR for APS-based azimuth estimation are known [186]. Other examples of such preprocessors used for APS arrays include those based on SR [187] and wavelet denoising [125]. An approach based on fractional lower order statistics has also been used to improve localization performance in alpha-stable noise environments [58]. The contribution of our work is the design of an SSR preprocessor [119] presented in section 6.2 for AVS-array based azimuth estimation under the assumption that the environmental noise is impulsive and of finite-variance.

In section 6.3, we aim to fill the gap in the AVS localization literature by considering the problem of near-field localization. Near-field localization has a broad range of applications such as sonar [108], seismic exploration [109] and electronic surveillance [110]. However, only limited work has been done on localizing near-field sources [39], [57], [61] and these methods have shortcomings such as requiring separation of sensors in an AVS or that they are limited to DOA estimation alone. We
overcome these shortcomings by proposing a uni-AVS MUSIC (U-MUSIC) approach for 3D location parameter estimation. We have chosen to develop the U-MUSIC method for a deep ocean near-field localization scenario and have not focused on the shallow ocean scenario as a near-field shallow ocean model of an AVS is not yet available in the literature. This method can be extended to the case of shallow ocean in the future.

6.2. Localization of sources in shallow water with non-Gaussian noise

Denoising techniques based on SSR exploit the fact that the environmental noise in the ocean has a heavy-tailed non-Gaussian distribution [20]. In this section we investigate the enhancement offered by an SSR based preprocessor for azimuth estimation methods in shallow-ocean and present a method for its design. We will show that the use of this preprocessor leads to a significant improvement in the azimuth estimation performance of the MUSIC and SIM algorithms at low SNR. The improved performance of the SSR-enhanced methods is demonstrated in terms of sharper peaks in the ambiguity function, better resolution of closely spaced sources and lower bias and lower RMS error in azimuth estimation.
6.2.1. Azimuth estimation methods

Consider the problem of estimating the azimuth \( \phi_s \) of an acoustic source located in a shallow ocean environment using an HLA of AVS. The geometry of the setup is as described in section 3.3. We consider azimuth estimation using the MUSIC and SIM processors. The source location is described by the source parameter vector \( \omega_s = [\phi_s, r_s, z_s]^T \), where \( \phi_s \) corresponds to the azimuth (or bearing), \( r_s \) corresponds to the range and \( z_s \) corresponds to the depth of the source. If \( r_s \) and \( z_s \) are unknown parameters in addition to the azimuth, the MUSIC beamformer requires the estimation of \( r_s \) and \( z_s \) along with the azimuth \( \phi_s \). In the case of SIM, this estimation of \( r_s \) and \( z_s \) can be avoided. We briefly review the MUSIC and SIM methods below.

6.2.1.1 Multiple signal classification (MUSIC)

MUSIC is a popular method of multiple source azimuth estimation based on the eigenvalue decomposition of the data covariance matrix \( \mathbf{R} \). In MUSIC, the estimation of the location of a single source is done by searching for the highest peak of the MUSIC ambiguity function, represented by

\[
\hat{\omega}_{\text{MUSIC}} = \arg \max_{\omega} \left[ \mathbf{A}_{\text{MUSIC}}(\omega) \right] = \arg \max_{\omega} \left[ \frac{1}{a^H(\omega) \mathbf{W} W^H a(\omega)} \right], \tag{6.1}
\]

where \( \omega = [\phi, r, z]^T \) denotes a certain source location being searched for presence of a source, \( a \) is the steering vector computed for a given \( \omega \) and \( \mathbf{W} \) is the matrix of noise eigenvectors that correspond to the 3N-1 smallest eigenvalues obtained from the
eigenvalue decomposition of $R$. Alternatively, the MUSIC estimate may also be obtained by searching for the highest peak of the ambiguity function

$$
\hat{\omega}_{\text{MUSIC}} = \arg \max_{\omega} \left[ a^H(\omega) u u^H a(\omega) \right].
$$

(6.2)

where $u$ represents the signal eigenvector obtained from the eigenvalue decomposition of $R$, corresponding to the largest eigenvalue. Both (6.1) and (6.2) yield equal estimates [188]. Generally, ambiguity function plots are normalized with respect to the maximum value (peak) of the plot in order to facilitate comparison with other ambiguity function plots. If $r_s$ and $z_s$ are known, the source azimuth $\phi_s$ can be estimated by maximizing the ambiguity function with respect to $\phi$ alone which is a 1D search. However, if $r_s$ and $z_s$ are unknown parameters, $A_{\text{MUSIC}}(\phi, r, z)$ has to be maximized with respect to all three arguments which is a computationally complex 3D search.

6.2.1.2 Subspace Intersection Method (SIM)

In a practical scenario it is often of interest to estimate the unknown source azimuth $\phi_s$ without estimating the unknown range and depth ($r_s$ and $z_s$) of the source. The SIM algorithm of azimuth estimation proposed by Lakshmipathi and Anand [114] and extended to AVS by Nagananda and Anand [17] is a method that allows azimuth estimation in ocean in such a scenario where there is no prior knowledge of the range or depth of the source. SIM is based on determining the intersection of two subspaces, namely the modal subspace $V(\phi)$ spanned by the modal steering vectors and the signal subspace $V_s(\phi)$ spanned by the signal eigenvectors of array covariance matrix $R$. It is
known that these subspaces intersect only at azimuth angles that correspond to source positions, i.e, at $\phi = \phi_s$. This fact is exploited in order to estimate the source positions. The SIM algorithm is briefly described below.

Define the $(3N \times (M+1))$ matrix $D(\phi)$ as

$$D(\phi) = [A(\phi) \ u],$$

(6.3)

where $A(\phi)$ is the $(3N \times M)$ modal steering matrix defined in chapter 3. Perform a QR decomposition to factorize the matrix $D(\phi)$ as

$$D(\phi) = U(\phi) \ Z(\phi),$$

(6.4)

where $U(\phi)$ is a matrix whose columns are orthonormal vectors and $Z(\phi)$ is an upper triangular matrix. The SIM estimate $\hat{\phi}_{SIM}$ of the azimuth is given as the location of the peak of the SIM ambiguity function which is expressed as

$$\hat{\phi}_{SIM} = \arg\max_\phi \left[ A_{SIM}(\phi) \right] = \arg\max_\phi \left[ \frac{1}{z_{M+1,M+1}(\phi)} \right],$$

(6.5)

where $z_{M+1,M+1}(\phi)$ denotes the $M+1^{th}$ diagonal element of $Z(\phi)$.

6.2.2. SSR denoiser design based on correlation gain

The performance of the azimuth estimation processors described above degrades with reduction in SNR. Below a certain critical value of SNR the performance of azimuth estimation methods degrades drastically. This value of SNR is referred to as the threshold SNR, and the azimuth estimation methods cannot be used below this threshold SNR. We aim to arrest the degradation in performance due to reduction in
SNR by using an SSR preprocessor. This preprocessor denoises the array data vector and thereby improves the performance of azimuth estimation. The SSR preprocessor has been previously described in section 5.3 in the context of application to detection in non-Gaussian noise. The preprocessor uses $L$ quantizer pairs for SSR denoising of the complex array data vector $x(t)$. In this subsection we describe the design of this SSR denoiser with a different objective in mind, viz. the performance enhancement of azimuth estimation.

All the afore-mentioned methods in subsection 6.2.2 require an estimate of the array covariance matrix $\mathbf{R}$ for the purpose of azimuth estimation. Hence, the performance of azimuth estimation depends on the accuracy of estimation of $\mathbf{R}$ that is estimated using a finite number of snapshots of the array data vector as

$$\hat{\mathbf{R}} = \frac{1}{T} \sum_{t=1}^{T} x(t)x^H(t),$$

(6.6)

where $x(t)$ denotes the $i^{th}$ data snapshot. Due to the presence of environmental noise and finite number of snapshots, the estimate $\hat{\mathbf{R}}$ has errors. This estimation error is due to the imperfect correlation between the noisy signal and the clean signal.

In this section, we aim to design an optimal SSR preprocessor to improve the performance of source localization. It may intuitively appear that the optimal SSR preprocessor should aim to optimize the SNR gain of its signal output with respect to its input. This strategy has been explored in [186]. However, we propose a simple measure called ‘correlation gain’ to facilitate easier design of the SSR preprocessor for
localization. We selected this measure for the design of the SSR preprocessor instead of SNR gain, due to the following reason.

Our work on SSR-aided detection done in chapter 5 shows us that optimizing SNR gain provided by an SSR preprocessor leads to optimal detection performance only for a single sensor detector, and that this is not true in the case of an array-based detector. For an array based detector, only the unconstrained detector benefits from the SNR improvement provided by an SSR preprocessor. However, the performance of the TSD and ASFD are not considerably improved by the use of SSR preprocessing. This is because the SSR preprocessor is a nonlinear processor, which leads to a degradation in the structure of the signal in the SSR denoised vector. This removes the correlation of the denoised data vector with the signal, which leads to errors in azimuth estimation. Thus, in order to improve the performance of azimuth estimation, enhancing the correlation between the data and the signal is more important than enhancing the SNR of the data itself.

Hence we propose a measure that tries to maximize this correlation in order to enhance the performance. Consider the noisy correlation $C_{sx}$ between the aggregate noisy array data vector $X = [x^T(1) \ldots x^T(T)]^T$ (that consists of all snapshots of the array data vector) and the aggregate signal vector $S = [s^T(1) \ldots s^T(T)]^T$ (that consists of all snapshots of the signal vector). The noisy correlation is defined as
Consider the conventional Bartlett (or delay-and-sum) beamformer, which consists of finding the peak of the normalized ambiguity function given by

$$\hat{\mathbf{a}}_{\text{Bartlett}}(\omega) = \arg \max_{\omega} \left[ \frac{\mathbf{a}^H(\omega) \mathbf{R}(\omega) \mathbf{a}(\omega)}{\mathbf{a}^H(\omega) \mathbf{a}(\omega)} \right]$$

(6.8)

The peak of the ambiguity function corresponds to the true source direction. Hence the steering vector computed for the true source direction is a scaled version of the signal vector. Therefore the expected peak value of the Bartlett ambiguity function is given by

$$\Lambda_{\text{Bartlett}} = E \left[ \frac{\mathbf{s}^H \hat{\mathbf{R}} \mathbf{s}}{\mathbf{S}^H \mathbf{S}} \right]$$

(6.9)

By substituting (6.6) into (6.8), we obtain

$$\Lambda_{\text{Bartlett}} = E \left[ \frac{\sum_{t=1}^{T} \mathbf{s}^H \mathbf{x}(t) \mathbf{x}^H(t) \mathbf{s}}{T \mathbf{S}^H \mathbf{S}} \right] = E \left[ \frac{\sum_{t=1}^{T} |\mathbf{s}^H \mathbf{x}(t)|^2}{T \mathbf{S}^H \mathbf{S}} \right]$$

$$= E \left[ \frac{\mathbf{S}^H \mathbf{X}^2}{\mathbf{S}^H \mathbf{S}} \right] = E \left[ \mathbf{X}^H \mathbf{X} \right] |C_{\mathbf{Sx}}|^2$$

(6.10)

The last equality in (6.10) follows from the definition of \(C_{\mathbf{Sx}}\) in (6.7). From (6.10), we see that for a given level of expected energy of the observed data (\(E[\mathbf{X}^H \mathbf{X}]\)), the peak of the noisy Bartlett ambiguity function is proportional to the square of the noisy correlation \(C_{\mathbf{Sx}}\). Thus, a boost in the value of \(C_{\mathbf{Sx}}\) implies an increase in the peak of the
Bartlett ambiguity function for a given amount of signal energy and noise energy in the observed data. An enhancement of the peak for a constant level of the noise floor leads to an improvement in the performance of localization using the Bartlett beamformer. Thus (6.10) explains why it is more important to improve the correlation of the data with the signal. We aim to do this by SSR denoising of the data.

After SSR denoising of all snapshots of the array data vector, we obtain the aggregate denoised data vector \( Y = [y^T(1) \ldots y^T(T)]^T \). The correlation of \( Y \) with \( S \) is given as the denoised correlation

\[
C_{SY} = \frac{E\left[S^H Y\right]}{\sqrt{E\left[S^H S Y^H Y\right]}}. \tag{6.11}
\]

We define the correlation gain \( G \) that gives the improvement in correlation of data vectors, as

\[
G = \frac{C_{SY}}{C_{SX}}. \tag{6.12}
\]

Maximizing the correlation gain \( G \) leads to a maximization of the correlation of the denoised data with the signal. This also effectively leads to optimization of the performance of the Bartlett beamformer from (6.10). Hence we choose the design objective of the SSR preprocessor as the maximization of the value of \( G \). We propose that in general, this denoising would also improve the performance of all other azimuth estimation methods that are based on the estimate \( \hat{R} \).
Chapter 6  |  Source localization in ocean using acoustic vector sensors

We show that good performance of azimuth estimation can be obtained using other beamformers also by designing the SSR preprocessor to achieve this objective. This is done by choosing the value of STD $\sigma_q$ of quantizer noise, pdf of quantizer noise, and the number of quantizer pairs $L$ in the SSR preprocessor to maximize $G$. Let the value of $\sigma_q$ that maximizes $G$ be referred to as $\sigma_{q,opt}$. In general, the value of $\sigma_{q,opt}$ depends on several factors, such as the pdf of the environmental noise $w$, quantizer noise $q$ and the input SNR.

In order to select the optimal value of $\sigma_q$ we investigate the variation of $G$ as well as $\sigma_{q,opt}$ with various factors such as the input array SNR, number of quantizer pairs $L$ and the pdf of quantizer noise. For this, we consider an example of a scenario where a 10-element uniform HLA of AVS is used for azimuth estimation in shallow ocean. The shallow ocean channel is modeled by a Pekeris model with the following parameters. The ocean channel has a constant depth of 100 m, the sound speed in water is 1500 m/s, sound speed in sediment layer is 1700 m/s, sediment density is 1500 kg/m$^3$ and the attenuation in the sediment layer is 0.5 dB/wavelength. The HLA is placed at a depth of 10 m in the ocean. A narrowband transmitting source is located at a range of 3000 m from the first sensor element of the HLA and at a depth of 30 m. The transmitting frequency of the source is 50 Hz and the azimuth of the source with respect to the array axis is 60 degrees. The elements of the HLA are separated by a distance of half-wavelength and azimuth estimation is done using $T = 50$ snapshots of data unless otherwise mentioned. The received signals are denoised by an SSR denoiser that uses quantizer noise that is Gaussian distributed unless otherwise menti-
Fig. 6.1: $\sigma_{q-opt}$ vs. input RMS amplitude $A$ for different GG environmental noises.

Fig. 6.2: Correlation gain $G$ vs. input SNR for different GG environmental noises ($e = 0.1, 0.5$ and $1$) for $\sigma_q = \sigma_{q-opt}$.
Chapter 6  Source localization in ocean using acoustic vector sensors

-aged. All the plots are obtained by averaging over 4000 Monte Carlo trials.

Consider the RMS signal amplitude $A$ defined as

$$A = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \frac{|p_n|^2}{\sigma_n^2}}, \quad (6.13)$$

where $p_n$ and $\sigma_n^2$ are respectively the signal component of acoustic pressure and the variance of noise at the $n^{th}$ sensor. The RMS amplitude $A$ is so defined because it gives the RMS amplitude of the signal at the array of sensors, when the environmental noise at all sensors is of unit variance ($\sigma^2 = 1$). From equation (4.87), the SNR can be expressed as a function of $A$ as

$$\text{SNR} = 10\log_{10} \left( \frac{A^2}{\sigma^2} \right), \quad (6.14)$$

Fig. 6.1 shows the variation of $\sigma_{q-opt}$ with the input RMS amplitude $A$. The corresponding variation of gain $G$ of the SSR denoiser with the input SNR is shown in Fig. 6.2. At each SNR, the value of $\sigma_{q-opt}$ is obtained as the value of $\sigma_q$ that maximizes the value of $G$. The number of quantizer pairs used in the SSR denoiser is 70 (the justification for choosing this number will be seen from the discussion on Fig. 6.6). The environmental noise is modeled as a complex circular GG pdf with parameter $e = 0.1, 0.5$ and 1.

From Fig. 6.1, we can see that the value of $\sigma_{q-opt}$ is a monotonically increasing and a nearly linear function of $A$. Hence it is required to add more quantizer noise as the SNR increases for better performance. From Fig. 6.2, we see the SSR preprocessor
Fig. 6. 3: Correlation gain vs. input SNR for different values of $\sigma$ for 10-element AVS array in GG ($e = 0.5$) noise provides a correlation gain greater than one when the environmental noise is leptokurtic, and the gain increases when the value of SNR is lower. The gain is higher when the value of parameter $e$ is lower, i.e, when the noise is more leptokurtic the preprocessor provides higher denoising performance. This observation concurs with the observations made in the case of the SSR preprocessor used for detection in chapter 5. The above observations show that the SSR preprocessor is able to effectively discard more noise energy as the noise becomes more heavy-tailed or the SNR reduces.

\footnote{Even though the correlation gain increases as the SNR reduces, the performance of azimuth estimation still suffers degradation as the input SNR is reduced.}
We have also investigated the variation of performance of the SSR denoiser at different values of quantizer noise STD $\sigma_q$. Fig. 6.3 shows the variation of correlation
gain $G$ plotted as a function of input SNR for different values of $\sigma_q$. The environmental noise pdf is GG with parameter $e = 0.5$. It is seen that at low SNR, $G$ decreases rapidly as $\sigma_q$ is increased; while at high SNR, $G$ decreases slowly as $\sigma_q$ is reduced. Hence for the purpose of SSR denoising, the value of $\sigma_q$ should be selected depending on the prior knowledge available regarding the input SNR. If no such information is available, moderate values of $\sigma_q$ (e.g. $\sigma_q = 0.4$) may be selected such that they provide a moderately high value of $G$ and a robust performance over a wide range of SNRs.

The pdf of the quantizer noise used is another factor that affects the performance of the preprocessor. For this, we model the quantizer noise pdf as GG distributed with parameter $e_q$, which is an approach similar to that used in section 5.2. Fig. 6.4 shows the variation of $G$ with input SNR when quantizer noise with different pdfs is added at the optimal STD. The corresponding values of optimal STDs of the different pdfs of quantizer noises added is plotted in Fig 6.5. The environmental noise is GG with $e = 0.5$. The quantizer noises added are complex circular Laplacian (GG with $e_q = 1$), Gaussian ($e_q = 2$) and uniform noise ($e_q = \infty$). It can be seen that as the kurtosis of the quantizer noise decreases, the SSR denoiser offers slightly higher performance in terms of $G$. However, the performance difference is found to be very marginal. Uniform quantizer noise is found to give the best performance for the simulation parameters considered. This is in agreement with the observations made for the case of detection by Patel and Kosko [36] that uniform distributed quantizer noise is optimum over a wide range of environmental noise pdfs. From Fig. 6.5, it can be observed that as the kurtosis of the quantizer noise increases (ie the parameter $e_q$
Fig. 6.6: Correlation gain vs. number of quantizers, at input SNR $-5$ dB for 10-element AVS array in GG ($e = 0.5$) noise for $\sigma_q = \sigma_{q,opt}$

decreases), the optimal STD $\sigma_{q,opt}$ of quantizer noise required to be added also increases. Hence we may accordingly choose the value of $\sigma_q$ depending on the pdf of quantizer noise pdf available. Gaussian noise and uniform noise are good choices to be used as quantizer noise as they may be generated easily through low complexity and low power implementations, as shown in [182–184], for example.

The performance of an SSR preprocessor is also directly affected by the number of quantizer pairs $L$. To investigate this variation, in Fig. 6.6 we plot $G$ versus $L$ at an input SNR of $-10$ dB. The environmental noise is GG with $e = 0.5$. It is observed that the gain increases monotonically with the number of quantizers used but seems to saturate at a value of around $L = 70$ quantizer pairs. This phenomenon has also been observed in section 5.2 for the case of detection. For the current simulation parameters, using more than 70 quantizer pairs does not seem to offer significant
Fig. 6. 7: a) Bias and b) RMSE of MUSIC estimator vs. SNR (in dB)

Fig. 6. 8: Ambiguity function of MUSIC estimator at SNR 4 dB
performance improvement in terms of gain $G$. Hence, for the rest of the simulations we will use $L = 70$ quantizer pairs in the SSR preprocessor for azimuth estimation.

The various aspects of design of the SSR preprocessor have been discussed. In the next section, we present some results to show the performance improvement offered by an SSR preprocessor for the problem of azimuth estimation in shallow ocean.

### 6.2.3. Simulation results

In this subsection, we present simulation results to show the performance improvement in azimuth estimates by the MUSIC and SIM processors when they are preceded by a SSR preprocessor. The bias and RMS error of the azimuth estimators, and the ambiguity function plots of the processors will be the performance measures considered. We consider the azimuth estimation of sources in a Pekeris channel with the same simulation parameters as considered in section 6.2.3.

The performance of DOA estimators are generally compared in terms of the bias, root mean square error (RMSE) and ambiguity function plots of the estimators [32]. Fig. 6.7 shows the variation of (a) bias and (b) RMSE of the MUSIC azimuth estimator with variation in the SNR in a shallow ocean environment with and without the aid of SSR denoising (referred to as MUSIC and SSR-MUSIC respectively). The method uses 50 data snapshots for azimuth estimation. The range and depth are assumed to be known in this case. It is observed from the plot that the dashed line
Fig. 6. 9: a) Bias and b) RMSE of SIM estimator vs. input SNR (in dB), environmental noise is GG ($e = 0.5$) distributed

representing the SSR aided MUSIC estimator (SSR-MUSIC) performs better than the solid line representing the normal MUSIC estimator, in terms of reduced bias and RMSE of azimuth estimation. This is noticeable especially at lower SNR. As the SNR reduces, the performance of the MUSIC estimator degrades progressively and finally breaks down at an SNR of 7 dB, but with SSR preprocessing the onset of the breakdown has been delayed to -1 dB. The normalized ambiguity function of the MUSIC estimator at an SNR of 4 dB has been plotted in Fig. 6.8. It shows that the SSR aided MUSIC has a sharper ambiguity function than normal MUSIC thus showing improvement in azimuth estimation.
Fig. 6. 10: Ambiguity function of SIM estimator at input SNR 4 dB

Fig. 6. 11: Probability of resolution of two sources at 43° and 50°
Fig. 6.12: (a) Ambiguity function of MUSIC estimator at input SNR 7 dB, showing resolution of sources at $43^\circ$ and $50^\circ$, (b) magnified version of (a)

Simulations performed with the SIM processor also show that SSR preprocessed SIM (SSR-SIM) yields better azimuth estimates. Thus SSR preprocessing is effective in enhancing azimuth estimation. The SIM processor has slightly lower performance than that of MUSIC, but the former has the advantage of lower computational complexity when there is no prior knowledge of the source range and depth. Fig. 6.9 shows the variation of (a) bias and (b) RMSE of a SIM processor with variation in the SNR. The errors in estimation are lower when SSR denoising is employed. The SIM ambiguity function is plotted in Fig. 6.10, and is seen to be less sharp than the MUSIC ambiguity function in Fig. 6.8. The SSR-SIM ambiguity function is still sharper than that of normal SIM and thus the performance is improved.
The resolution of an azimuth estimation method is another measure of its performance. Resolution is defined as the ability of an azimuth estimator to distinguish signals coming from two closely placed sources as separate ones. As the SNR reduces, two source signals that are close in terms of angular separation tend to become indistinguishable from one another leading to lower resolution. Using SSR preprocessing, however, it is possible to improve the resolution of the estimators. We will consider resolution of the MUSIC estimator in an environment similar to the one mentioned previously. In the simulation results shown it is assumed that there are two sources located at azimuth angles of 43 and 48 degrees with respect to the array axis. The azimuth estimation is done using an array of 15 AVS and 200 data snapshots. The rest of the parameters are as specified in the previous simulation. The two sources are considered to be resolved if the azimuth estimation method can detect two distinct peaks in the region surrounding the true source angle positions. Fig. 6.11 shows the probability of resolution of the two sources plotted as a function of the SNR. It can be seen that the resolution improves and approaches a value of 1 as the SNR increases. It is also seen that the probability of resolution of the SSR-MUSIC estimator is always higher than that of the normal MUSIC estimator.

In Fig. 6.12 (a) the ambiguity function of the MUSIC estimator at an SNR of 7 dB is shown. Fig. 6.12 (b) shows a magnified version of Fig. 6.12 (a) in which the peak region can be observed more clearly. It can be seen that at this low SNR, MUSIC
fails to resolve the two sources. But SSR-MUSIC is capable of resolving the two sources as can be seen from the two peaks in the ambiguity function. Thus the improvement in resolution offered by using the SSR preprocessor is clearly demonstrated.

6.3. Near-field localization using a single AVS

This section considers the problem of 3D (azimuth, elevation and range) localization of a single source in a deep ocean in the near-field using a single AVS. We propose a uni-AVS MUSIC (U-MUSIC) approach for 3D location parameter estimation based on a compact AVS structure. We decouple the 3D localization problem into step by step estimation of azimuth, elevation and range, and derive closed form solutions for the location parameter estimates by which a complex 3D search for the parameters can be avoided. It is shown that the proposed approach outperforms the existing methods when the sensor system is required to be mounted in a confined space.

6.3.1. Existing methods

Some contributions in the field of near-field source localization have been made by Wu and Wong [39], Tichavsky et al [61] and Xu et al [57]. Tichavsky et al [61] developed an ESPRIT method using a single vector hydrophone for 2D azimuth and elevation angle estimation. Xu et al [57] presented an analysis of a conjugate multiple-invariance ESPRIT method that can be used for direction-finding of non-circular signals using a single AVS. Both these methods are limited to direction-finding and do not discuss the estimation of the range of the source. The only work in the literature
that deals with complete 3D source localization is an approach by Wu and Wong [39] which is computationally simple and does not require any search algorithms. We refer to this method as the Eigen-RSSI method.

6.3.1.1 The Eigen-RSSI method

This method employs a ‘spatially extended’ AVS that is constructed by using a co-located triad of velocity sensors and a pressure sensor placed at a certain distance away from the triad. The extended AVS system consists of a velocity triad placed at a distance $r_s$ from the source, and a pressure sensor separated by a distance $d$ from the velocity sensors in a known direction. Eigen-RSSI [39] initially estimates azimuth and elevation using elements of the eigenvector obtained from eigenvalue decomposition of the data. It then uses the RSSI method to estimate the range of the source, which is described below.

The magnitude of the pressure field $p_{comp}$ at the location of the velocity sensors is estimated from the particle velocity measurements using the relation

$$
|p_{comp}| \approx \rho c \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{6.15}
$$

The magnitude of the ratio of this estimated pressure field with the pressure field $p_{ext}$ measured at the extended pressure sensor is known to be equal to

$$
|p_{comp}/p_{ext}| = (r_{ext}/r_s), \tag{6.16}
$$

---

7 This approximation discards the term $1/\sqrt{1+1/(kr_s)^2}$. 

where $r_{ext}$ is the range of the extended pressure sensor from the source. This relation assumes that the energy path loss model of the signal follows the inverse square law [39].

The extension $d$ of the pressure sensor from the velocity sensors is known. The direction of this extension is also known. For example, in the case when the extension $d$ is assumed to be along the $x$ axis (as assumed in [39]), we have the relation

$$r_{ext}^2 = (r_s \cos(\phi_s) \cos(\psi_s) \pm d)^2 + (r_s \sin(\phi_s) \cos(\psi_s))^2 + (r_s \sin(\psi_s))^2$$  \hspace{1cm} (6.17)

The relation in (6.17) can be easily obtained from vector resolution of $r_s$ along the $x$, $y$, and $z$ directions. In (6.17), the cases ‘$-d$’ and ‘$+d$’ refer to the two cases when the pressure sensor is closer to and further away than the velocity sensors from the source, respectively. From (6.15), (6.16) and (6.17), the value of $r_s$ is determined. This constitutes the RSSI method of range estimation.

The estimation of range using RSSI, in addition to the estimation of the azimuth and elevation comprises complete localization of the source.

6.3.2. **Drawbacks of existing methods and motivation for U-MUSIC**

The motivation behind developing the U-MUSIC algorithm is as follows:

(i) Though the Eigen-RSSI method provides good performance in localization of sources using two separated ‘anchor nodes’ (the pressure sensor and the velocity sensor triad), its limitation is that it requires the pressure sensor to be

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8 The Eigen-RSSI method allows incorporation of path loss models other than the inverse square law also.
located at a specific distance and direction from the velocity sensors. Thus the extended sensor system loses out on a significant advantage in using a single AVS for localization, namely, the compactness. Lack of compactness is a severe limitation in situations that require the sensor system to be accommodated in a restricted space for localization, such as autonomous underwater vehicles, room object tracking and hearing aid systems [189]. In a multiple-sensor-based measurement system, a considerable portion of the cost of deployment of the sensor nodes depends on the location calibration of the sensors and the number of sensors deployed [9]. Hence a less compact system with more nodes leads to a larger deployment cost and time.

Moreover, AVS are generally constructed using compact geometry in the sense that they contain co-located pressure and velocity sensors [15]. Some existing examples of such an AVS being used include the DIFAR array [14], Swallow floats [42] and the Wilcoxon array [43] that have been deployed successfully. The sensor arrangement for the Eigen-RSSI method also requires additional calibration of the separated sensor system and this constitutes additional complexity in its deployment. Lack of calibration or improper calibration of the sensor nodes could lead to serious degradation in location estimation [190–192]. Thus, in this work we aim to use a compact AVS for 3D localization.
(ii) Since the near-field manifold of an AVS is a function of range, it is clear that a single compact AVS can be used to estimate the range in addition to the source azimuth and elevation and thus it is capable of performing 3D source localization. However, using asymptotically efficient methods such as MLE or MUSIC for this involves a computationally expensive search for the location parameters in the azimuth-elevation-range space [185], [188].

(iii) Using the MLE in near-field localization has another disadvantage. The MLE has a mathematically tractable solution when the environmental noise is Gaussian in nature, but its implementation becomes more complex in the case of non-Gaussian environmental noise [185], [188]. This is an added disadvantage in real applications since the acoustic noise in ocean is often non-Gaussian in character.

Our current work aims to overcome disadvantages of the above mentioned approaches: we attempt to obtain a computationally simple 3D source localization method that preserves the compactness of the AVS and yet yields good performance of localization. Our approach employs the signal eigenvector of the data correlation matrix similar to MUSIC and yields closed form solutions for the estimation of location parameters. The method will be referred to as uni-AVS MUSIC (U-MUSIC). U-MUSIC decouples the 3D localization problem into step-by-step estimation of the location parameters and does not require a complex search for the location parameters as conventional MUSIC does. The decoupling of range-estimation from DOA
estimation is not new in itself. Some methods proposed previously in the literature such as those by Weiss and Friedlander [104] and Hung et al [106] have proven useful in reducing the complexity of MUSIC-based source localization using an array of pressure sensors. Weiss and Friedlander reduced the complexity of a 2D range-azimuth search by converting it into a 1D search for range combined with polynomial rooting procedure that replaces the azimuth search. Hung et al further extended this to 3D source localization by reducing the 3D search to a range search combined with polynomial rooting for azimuth and elevation. While these methods dealt with measurements obtained from arrays of APS, our method concerns the decoupling of range, azimuth and elevation estimates from measurements obtained from a single AVS. Note that U-MUSIC is also not be confused with Root-MUSIC [56] since the former does not require an array of AVS. The main advantage of U-MUSIC is that it avoids the need for a 3D search in the azimuth-elevation-range space and hence reduces the computational complexity. We will show that U-MUSIC yields asymptotically efficient performance. Furthermore, this method can be applied in a compact AVS system.

### 6.3.3. The uni-AVS MUSIC method

In this subsection, we elaborate on a novel method named U-MUSIC that is derived from MUSIC for near-field source localization using a compact AVS. MUSIC is an efficient method that has been shown to be asymptotically equivalent to the MLE [185]. It does not require the assumption that the environmental noise is Gaussian in nature [185]. Note, however, that the U-MUSIC method derived herein does not
perform a complex 3D search for the location as is usually done in MUSIC-based localization with an AVS. Rather, the emphasis here is to obviate the need for this 3D search by converting the search problem into closed form expressions for the location parameters.

The data model for the problem is presented in subsection 3.3.3. Wu, Wong and Lau [113] presented the near-field manifold of an AVS with a compact geometry in which they showed that the pressure measurement is dependent on the range of the source from the sensor. In this work, we perform range estimation using this pressure measurement.

6.3.3.1 Outline of the algorithm

At the outset, recall that conventional MUSIC [24] requires a computationally intensive 3D search for the largest peak in the beamforming spectrum to find the source location estimates $\hat{\phi}$, $\hat{\psi}$ and $\hat{\rho}$ of the azimuth, elevation and range, respectively. For the localization of a single source, MUSIC searches for the highest peak of the MUSIC spectrum given in (6.2) which is reproduced here

$$A_{\text{MUSIC}}(\omega) = a^H(\omega)uu^Ha(\omega),$$  \hspace{1cm} (6.18)

where $\omega$ represents the vector of parameters being estimated, $a$ represents a steering vector that is a function of $\omega$, and $u = [u_1 \ldots u_4]^T$ represents the signal eigenvector obtained from the eigenvalue decomposition of the data correlation matrix, corresponding to the largest eigenvalue. The vector $u$ is of dimension $4 \times 1$ in this case.
The unknown location parameter vector is $\omega = [r_s, \phi_s, \psi_s]^T$ in this case. MUSIC searches for the steering vector that is best contained in the signal subspace spanned by $u$. The signal eigenvector $u$ has the same form as that of the array manifold $a$ and can be represented by [39]

$$\mathbf{u} \approx \frac{e^{i\eta} \mathbf{a}}{||\mathbf{a}||}, \quad (6.19)$$

where $||.||$ stands for the Frobenius norm and $\eta$ symbolizes an unknown phase. The normalization term $||\mathbf{a}||$ has been introduced into the denominator because by definition of an eigenvector, $||u|| = 1$. The above approximation converges to equality under noiseless or asymptotic conditions.

In the formulation of the U-MUSIC algorithm, our aim is to obtain closed form expressions for estimates of the azimuth, elevation and range. These expressions that are derived from MUSIC eliminate the need for a complex 3D search in the MUSIC spectrum as required by conventional MUSIC. This is achieved using a twofold approach. Firstly, the problem of simultaneously estimating all three source location parameters is broken down into individual 1D MUSIC searches by selectively using elements of the signal eigenvector $u$. Then, the closed form expressions for the estimates of the location parameters are obtained by maximizing the MUSIC spectrums associated with each 1D search. The algorithm first obtains the azimuth estimate by decoupling it from the overall 3D search. This decoupling is facilitated by restricting the measurements used to the two horizontal velocity measurements. Secondly, it estimates the elevation that can now be estimated using the three velocity
sensor measurements and the previously obtained azimuth estimate. The range is estimated in the final step using measurements from all four channels as it requires use of the DOA estimates.

6.3.3.2 Estimation of Azimuth

From the expression for the near-fold manifold in (3.29), and from (6.19), we observe that the truncated signal eigenvector \( u_\phi \) that contains elements of \( u \) corresponding to the horizontal (\( x \) and \( y \)) velocities alone can be expressed as a function of the source azimuth angle \( \phi_s \) as

\[
\begin{align*}
  u_\phi &= [u_1 \ u_2]^T = c_1 \ a_\phi(\phi),
\end{align*}
\]

where

\[
\begin{align*}
  a_\phi(\phi) &= [\cos(\phi), \sin(\phi)]^T,
\end{align*}
\]

and \( c_1 \approx e^{j\eta} \cos(\psi)/||a|| \) is an unknown constant. The estimate \( \hat{\phi} \) of the azimuth angle may be found by searching for the angle \( \phi \) that maximizes the projection of the steering vector \( a_\phi(\phi) \) into the signal subspace spanned by the vector \( u_\phi \). This allows a decoupled search for the estimate of the azimuth alone which can be expressed as

\[
\begin{align*}
  \hat{\phi} &= \arg \max_{\phi} [a_\phi^H(\phi) u_\phi u_\phi^H a_\phi(\phi)] = \arg \max_{\phi} [||u_1 \cos(\phi) + u_2 \sin(\phi)||^2],
\end{align*}
\]

where \( || \cdot || \) denotes the absolute value. The estimate \( \hat{\phi} \) can be obtained by maximization of the term within brackets in by setting its derivative with respect to \( \phi \) to zero, which yields the quadratic equation:
where $x^*$ and $\text{Re}(x)$ refer to the conjugate and real part of $x$, respectively. Solving (6.23) yields the closed form estimate for the azimuth as:

$$\hat{\phi} = \tan^{-1}\left(l + \sqrt{l^2 + 1}\right),$$

(6.24)

where

$$l = 0.5 \left(|u_2|^2 - |u_1|^2\right)/\text{Re}(u_1^* u_2).$$

(6.25)

Note that the $\tan^{-1}(.)$ function in (6.24) yields two possible values for $\hat{\phi}$ that are separated by an angle of $\pi$. In other words, if $\hat{\phi}$ is an estimate of the azimuth, then $\hat{\phi} + \pi$ is also an equally possible estimate. This is an ambiguity that is inherent in DOA estimation using velocity sensors alone [61]. The ambiguity can be resolved using the pressure measurement also. This procedure of disambiguation will be explained at the end of this section. For now, we will continue with our algorithm by selecting the value of $\hat{\phi}$ that lies within the interval $[0, \pi]$, and we will resolve the direction ambiguity in the final step.

6.3.3.3 Estimation of Elevation

Now consider estimation of the elevation angle using a procedure similar to the above. We obtain the elevation estimate using all velocity measurements and the known azimuth estimate $\hat{\phi}$, from the truncated signal eigenvector $u_{\psi}$ defined as

$$u_{\psi} = [u_1 \ u_2 \ u_3]^T = c_2 a_\psi(\phi_s, \psi_s),$$

(6.26)
where

\[ a_\psi(\psi, \phi) = [\cos(\psi)\cos(\phi), \cos(\psi)\sin(\phi), \sin(\psi)]^T, \quad (6.27) \]

and \( c_2 \approx e^{j\eta}/|a| \) is an unknown constant. The vector \( u_\psi \) is hence a function of source azimuth \( \phi_s \) and elevation \( \psi_s \). If the estimate \( \hat{\phi} \) of azimuth has been obtained, we can perform a decoupled search for the estimate \( \hat{\psi} \) of the elevation \( \psi_s \). This is done by searching for the value of \( \psi \) that maximizes the projection of \( a_\psi(\psi, \hat{\phi}) \) into the subspace spanned by \( u_\psi \), expressed as

\[
\hat{\psi} = \arg \max_\psi [a_\psi^H(\psi, \hat{\phi})u_\psi u_\psi^H a_\psi(\psi, \hat{\phi})]
= \arg \max_\psi [||u_1 \cos(\psi) \cos(\phi) + u_2 \cos(\psi) \sin(\phi) + u_3 \sin(\psi)||^2].
\quad (6.28)
\]

The estimate \( \hat{\psi} \) is obtained by equating the derivative with respect to \( \psi \) of the term within brackets in (6.28) to zero. This procedure is similar to the previous case of azimuth estimation. Solving the resulting quadratic equation yields the closed form estimate for the elevation angle as

\[
\hat{\psi} = \tan^{-1}(m + \sqrt{m^2+1}),
\quad (6.29)
\]

where

\[
m = 0.5 \left( |u_3|^2 - |a_\phi^H(\hat{\phi})u_\phi|^2 \right) / \text{Re}(u_3^* a_\phi^H(\hat{\phi})u_\phi).
\quad (6.30)
\]

Thus we have obtained closed-form estimates of the DOA of the source through (6.24) and (6.29). Note that the DOA estimation algorithm described until now can be used for both near-field as well as far-field sources with no prior
information about the signal bandwidth and spectra. This is because the measurement
data used is independent of the range of the source as well as the frequency of the
source due to co-location of the velocity hydrophones. However, the assumption of the
signal being narrowband is necessary for the following range estimation step.

6.3.3.4 Estimation of Range

We now obtain the estimate \( \hat{r} \) of the range \( r_s \) of the source by maximization of the
projection of the steering vector \( a_{near}(r, \phi, \psi) \) into the signal subspace which is
expressed as

\[
\hat{r} = \arg \max_r [B(r, \phi, \psi)],
\]  
\[ (6.31) \]

where

\[
B(r, \phi, \psi) = \frac{a_{near}^H(r, \phi, \psi) uu^H a_{near}(r, \phi, \psi)}{a_{near}^H(r, \phi, \psi) a_{near}(r, \phi, \psi)}. 
\]  
\[ (6.32) \]

\( B(r, \phi, \psi) \) represents the normalized spectrum for the range search. Note that the term
\( (a_{near}^H(r, \phi, \psi) a_{near}(r, \phi, \psi)) \) is introduced in the denominator of \( B \) for the sake of
normalization. This term is absent in the expressions for the spectrums in equations
(6.22) and (6.28) as the values of \( (a^H_{\phi}(\phi) a_{\phi}(\phi)) \) and \( (a^H_{\psi}(\psi, \phi) a_{\psi}(\psi, \phi)) \) are numerical
constants. The reason for choosing to normalize the range search spectrum is as
follows [106]. Sensor arrays can sense nearby sources more easily than sources that
are far away because the range sensitivity decreases with the range of the source from
the sensor due to spherical spreading. Thus, if the un-normalized spectrum is used to estimate the range of the source, the estimates will be biased towards nearer peaks. This bias is corrected by scaling the null spectrum in accordance with the intensity variation caused by spherical spreading.

Maximization of the term $B(r, \hat{\phi}, \hat{\psi})$ defined in (6.32) is performed with respect to $r$ in a similar manner as done for the case of azimuth and elevation. Equating the derivative of $B(r, \hat{\phi}, \hat{\psi})$ with respect to $r$ to zero yields the closed form expression for the U-MUSIC estimate of the range as:

$$\hat{r}(\hat{\psi}, \hat{\phi}) = \frac{2 \text{Im}(u^*_M a^H_M (\hat{\psi}, \hat{\phi})u_\psi)}{k \left( n - \sqrt{n^2 + 8 \text{Im}^2(u^*_M a^H_M (\hat{\phi}, \hat{\psi})u_\psi)} \right)}$$

(6.33)

where $\text{Im}(x)$ refers to imaginary part of $x$, and

$$n = |u_4|^2 + 2 \text{Re}(u^*_M a^H_M (\hat{\psi}, \hat{\phi})u_\psi) - |a^H_M (\hat{\psi}, \hat{\phi})u_\psi|^2.$$  

(6.34)

Thus with the estimation of all three location parameters by (6.24), (6.29) and (6.33), 3D localization of the source has been achieved.

6.3.3.5 Resolution of ambiguity in source direction

When DOA estimation is performed using the data from velocity sensor data alone, there is an inherent ambiguity in the DOA estimates between the possible source directions $(\phi, \psi)$ and $(\phi+\pi, -\psi)$. This ambiguity arises because the span of the vector
\( a_{\psi}(\phi, \psi) \) is the same as that of the vector \(-a_{\psi}(\phi, \psi)\). In physical terms, this implies that the velocity sensors by themselves cannot distinguish between dilations and compressions of the wave, and the pressure measurement is required to resolve this ambiguity [61].

At this point, we can resolve the ambiguity in source direction using the data from the pressure sensor by doing a simple comparison. The ambiguity can be resolved by using those DOA estimates (either \((\hat{\phi}, \hat{\psi})\) or \((\hat{\phi} + \pi, -\hat{\psi})\)) that yield a larger value of the function \(B\) defined in (6.32). This means that we choose the location estimates at which a larger peak of the ambiguity function \(B\) occurs in the 3D range-azimuth-elevation space. i.e, if \(B(\hat{r}(\hat{\phi}, \hat{\psi}), \hat{\phi}, \hat{\psi}) > B(\hat{r}(\hat{\phi} + \pi, -\hat{\psi}), \hat{\phi} + \pi, -\hat{\psi})\), we choose the estimates \((\hat{r}(\hat{\phi}, \hat{\psi}), \hat{\phi}, \hat{\psi})\), and if the reverse is true, we choose \((\hat{r}(\hat{\phi} + \pi, -\hat{\psi}), \hat{\phi} + \pi, -\hat{\psi})\) as the estimate of the source location. Alternatively, we may also limit the search interval of the azimuth to \([0, \pi]\) as mentioned in [61] which leads to no ambiguity in the DOA. Also, in some cases prior information on the location of the source may be available such as whether it is located within the upper \((\psi > 0)\) or lower \((\psi < 0)\) hemisphere or whether it is located in the left \((\phi \in [0, \pi])\) or right \((\phi \in [\pi, 2\pi])\) hemisphere. If such information is available, it is enough to resolve the ambiguity and one does not have to resort to the disambiguation method described in this section.
6.3.4. A performance measure for comparison

We propose a simple measure to theoretically compare the accuracy of range estimation using the Eigen-RSSI and U-MUSIC methods which is the sensitivity of their range estimation methods to the range $r_s$. This range-sensitivity provides a measure of performance of range-estimation which is simpler to compute, thus alleviating the need to do time-consuming Monte-Carlo simulations to compare the methods.

Observe that both Eigen-RSSI and U-MUSIC essentially utilize the pressure measurement for range-estimation. Let $S_{\text{comp}}$ denote the range sensitivity of the pressure measurement $p_{\text{comp}}$ of the compact AVS, and let $S_{\text{ext}}$ denote the range sensitivity of the pressure measurement $p_{\text{ext}}$ of the pressure sensor used in the RSSI method with the extended AVS. Both $p_{\text{comp}}$ and $p_{\text{ext}}$ are complex quantities. From the expression for $|p_{\text{comp}}|$ in (6.15) the range sensitivity of the compact AVS can be found as

$$S_{\text{comp}}(r_s) = \left| \frac{1}{p_{\text{comp}}} \left( \frac{\partial p_{\text{comp}}}{\partial r_s} \right) \right| = \frac{1}{r_s \sqrt{r_s^2 k^2 + 1}}. \quad (6.35)$$

For the case of the extended AVS, from equations (6.16) and (6.17) the sensitivity of the pressure measurement of the extended AVS reduces to

$$S_{\text{ext}}(d, r_s) = \left| \frac{1}{p_{\text{ext}}} \left( \frac{\partial p_{\text{ext}}}{\partial r_s} \right) \right| = \frac{1}{r_s} - \frac{d \pm r_s \cos(\phi_s) \cos(\psi_s)}{r_s^2 + d^2 \pm 2dr_s \cos(\phi_s) \cos(\psi_s)}. \quad (6.36)$$
In the numerator and denominator in (6.36) the cases ‘-’ and ‘+’ refer to the two cases when the pressure sensor is closer to and further away than the velocity sensors from the source, respectively. Equation (6.36) shows that the sensitivity of the extended AVS to the range degrades as the separation \( d \) decreases. When the separation between sensors in the extended AVS is decreased below a certain threshold value, the performance of Eigen-RSSI deteriorates and is lower than that of U-MUSIC. We define this threshold value of separation as the critical separation \( d_c \). We can predict the critical separation \( d_c \) by noting that range estimate of U-MUSIC is expected to be better than Eigen-RSSI when the range sensitivity of the compact AVS exceeds that of the extended AVS \( (S_{\text{comp}} > S_{\text{ext}}) \). This gives us a relation for \( d_c' \), the predicted value of critical separation, as

\[
d_c' \approx \arg \min_d [ |S_{\text{comp}}(r_s) - S_{\text{ext}}(d, r_s)| ].
\]  

(6.37)

The value of predicted critical separation \( d_c' \) in (6.37) is thus obtained from the comparison of sensitivities of the two types of sensors. We expect that when the sensors in an extended AVS are separated by a distance less than that predicted by (6.37), the performance of the Eigen-RSSI method deteriorates compared to the U-MUSIC method. It will be seen in subsection 6.3.5 that this theoretically predicted value \( d_c' \) is close to the experimentally observed value \( d_c \).

### 6.3.5. Simulation Results

This subsection presents simulation results to demonstrate the performance of the U-MUSIC method. Recall that U-MUSIC uses the compact AVS to estimate the azimuth,
elevation and range. In contrast, Eigen-RSSI [39] employs an extended AVS in which
the pressure sensor is separated from the velocity sensors in the x direction (as used in
[39]) by a known distance d. The noise (in each snapshot) at the AVS is assumed to be
Gaussian in nature and has internal covariance matrix $R_0$ given by [77]

$$R_0 = \sigma^2 \begin{bmatrix} I_3 & \theta \\ \theta^T & 1 \end{bmatrix},$$  \hspace{1cm} (6.38)

where $\theta$ is a 3×1 vector of zeros. This model assumes that the noise at all measurement
channels is uncorrelated and spherically isotropic, and the variance $\sigma^2$ of the noise of
the pressure measurements is thrice that of noise of the velocity measurements. The
performance measure used to evaluate individual parameter estimates is the RMSE,
and overall 3D localization is evaluated using the relative location estimation error
(RLEE) [39] computed as

$$\text{RLEE} = \frac{1}{r_s} \sqrt{(x_s - \hat{x}_s)^2 + (y_s - \hat{y}_s)^2 + (z_s - \hat{z}_s)^2},$$  \hspace{1cm} (6.39)

where $(x_s, y_s, z_s)$ refer to the $(x, y, z)$ coordinates of the source, and $(\hat{x}_s, \hat{y}_s, \hat{z}_s)$ refer to
their estimates given by

$$\begin{align*}
\hat{x}_s &= \hat{r} \cos(\hat{\psi}) \cos(\hat{\phi}), \\
\hat{y}_s &= \hat{r} \cos(\hat{\psi}) \sin(\hat{\phi}), \\
\hat{z}_s &= \hat{r} \sin(\hat{\psi}).
\end{align*}$$  \hspace{1cm} (6.40)
Fig. 6.13: (a) RMSE of azimuth estimates, (b) magnified version of (a) with logarithmic scale, comparing Eigen-RSSI, ESPRIT and U-MUSIC methods. $N = 800$, $(x, y, z) = (63.3, 75.4, 17.4)$ m

The RMSE and RLEE are computed from 50000 Monte Carlo simulations. The expression for the Cramer Rao Lower Bound (CRB) of the location estimates using the velocity sensors and a co-located pressure sensor is given in [113].

In Figs. 6.13-6.15, the RMSE of the estimates of azimuth, elevation and range, and RLEE of overall source location estimates obtained using U-MUSIC are compared against those obtained using Eigen-RSSI [39], ESPRIT [61] (for DOA estimates) and the CRB. Figure 6.13 (a) shows the RMSE of the azimuth estimate and Fig. 6.13 (b) shows a magnified version of Fig. 6.13 (a) with a logarithmic scale to highlight the variations clearly. Figure 6.14 (a) shows the RMSE of the elevation estimate and Fig.
Fig. 6.14: (a) RMSE of elevation estimates, (b) magnified version of (a) with logarithmic scale, comparing Eigen-RSSI, ESPRIT and U-MUSIC methods. $N = 800$, $(x_s, y_s, z_s) = (63.3, 75.4, 17.4) \text{ m}$

Fig. 6.15: (a) RMSE of range estimates (logarithmic scale), and (b) RLEE of source localization (logarithmic scale) vs. SNR, comparing Eigen-RSSI and U-MUSIC methods. $N = 800$, $(x_s, y_s, z_s) = (63.3, 75.4, 17.4) \text{ m}$
6.14 (b) shows its magnified version on a logarithmic scale. Figure 6.15 (a) and Fig. 6.15 (b) show the RMSE of range estimate and the RLEE respectively.

In these figures, the narrowband source of 50 Hz located at $\phi_s = 50^0$, $\psi_s = 10^0$ and $r_s = 100$ m with respect to the velocity sensors is localized using $T = 800$ snapshots of data. The spatially extended AVS is assumed to be implemented with the pressure sensor separated by $d = 5$ m from the velocity sensor triad. The expressions for the CRB for the extended AVS model are given in [39] and the expressions for the CRB of the compact AVS model are given in [113]. The CRB for DOA estimation is the same for both models which implies that the placement of the pressure sensor does not affect the CRB of DOA estimation performance in either model.

Figs. 6.13-6.15 show that U-MUSIC outperforms Eigen-RSSI in terms of overall source localization for the simulation parameters considered. Its performance in the estimation of azimuth and elevation is equivalent to that of Eigen-RSSI and ESPRIT if the SNR is higher than the ‘threshold SNR’ and in the asymptotic region of performance. However, note that the U-MUSIC estimate of elevation is better than those in ESPRIT and Eigen-RSSI in the threshold SNR region, and U-MUSIC degrades slower than the Eigen-RSSI estimate as the SNR is reduced showing its increased robustness to low SNR. U-MUSIC also consistently outperforms Eigen-RSSI in estimation of range for the simulation parameters considered and this is the

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9 Threshold SNR is the SNR at which the performance of localization estimates shows a drastic reduction or ‘breakdown’.
primary reason for its better source localization performance.

U-MUSIC also outperforms ESPRIT in the estimation of elevation and its performance of azimuth estimation is comparable to that of ESPRIT. The performance of all the methods considered in Figs. 6.13-6.15 tends to the CRB asymptotically showing that these methods are efficient\(^\text{10}\). In Fig. 6.15 (a), the CRB of the spatially extended AVS for range estimation is much lower than the CRB of the compact AVS. However, for the simulation parameters considered, the Eigen-RSSI method is not very close to the CRB of the extended-AVS model. On the other hand, U-MUSIC is very close to the bound on range estimation for the compact AVS model. This shows

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\(^{10}\) Note that in Fig. 6.13 (a) and 6.14 (a), the CRB increases with a decrease in SNR; however this trend is hard to observe in the figure as the value of CRB is much lower than the RMSE of the DOA estimation methods.
that U-MUSIC efficiently utilizes the information in the measurements to arrive at the range estimate. Furthermore, the AVS setup used for U-MUSIC is more compact than the extended AVS model as it does not require separation of the sensors.

Note that U-MUSIC yields azimuth estimates at par with or better than those of ESPRIT and Eigen-RSSI despite the fact that the azimuth angle is estimated in (6.24) using only two channel measurements instead of all three velocity sensor channels. This reflects the effectiveness of U-MUSIC in discarding the effect of noise through eigenvalue decomposition. It is also noteworthy that the subsequent elevation estimate \( \hat{\psi} \) is based on the estimate \( \hat{\phi} \) of the azimuth. One may expect \( \hat{\psi} \) to be less accurate since errors in the azimuth estimate could propagate into the estimation of the elevation and lead to increase in the error in \( \hat{\psi} \). However, \( \hat{\psi} \) is seen to be still accurate despite its dependence on \( \hat{\phi} \). This is because \( \hat{\psi} \) is not very sensitive to errors in the azimuth estimation as long as \( \hat{\phi} \) is fairly close to the true value of azimuth. To demonstrate this in Fig. 6.16 we plot the variation of the sensitivity of the elevation estimate \( \hat{\psi} \) to the azimuth estimate \( \hat{\phi} \) with variation in the error \( \Delta \phi \) in the azimuth estimate. The sensitivity of \( \hat{\psi} \) to \( \hat{\phi} \) is given by the derivative \( (\hat{\psi}/d\hat{\phi}) \) that can be computed from the expression (6.29). The error in the azimuth estimate is given by

\[
\Delta \phi = \hat{\phi} - \phi. \tag{6.41}
\]

The plot in Fig. 6.16 assumes the source parameters as used in Figs. 6.13-6.15 and an SNR of -4 dB. Fig. 6.16 shows that the sensitivity of \( \hat{\psi} \) to the azimuth estimation error
Fig. 6.17: RMSE of range estimates by Eigen-RSSI and U-MUSIC methods vs. separation $d$ of pressure sensors from velocity sensors. $N = 800$, $(x_s, y_s, z_s) = (63.3, 75.4, 17.4)$ m, SNR = 20 dB.

Fig. 6.18: RMSE of range estimates by Eigen-RSSI and U-MUSIC methods vs. range $r_s$ of source from velocity sensors. $N = 800$, $(x_s, y_s, z_s) = (63.3, 75.4, 17.4)$ m, SNR = 20 dB, $d = 5$ m.
approaches zero as long as \( \hat{\phi} \) is fairly close to the true azimuth \( \phi \). i.e, \( \partial \hat{\psi} / \partial \hat{\phi} \to 0 \) as \( \Delta \phi \to 0 \). Hence the performance of estimation of elevation is more or less unaffected by the performance of azimuth estimation. This explains the accuracy in the estimation of the elevation seen in Fig. 6.14. We compare the RMSE in range estimation using the U-MUSIC and Eigen-RSSI methods when the separation \( d \) between pressure and velocity sensors in the Eigen-RSSI method is varied, in Fig. 6.17. The plot shows that the accuracy of range estimation of Eigen-RSSI improves as the separation increases. When the separation is smaller than 7.4 m, the performance of extended AVS-based Eigen-RSSI is worse than that of the compact AVS-based U-MUSIC in Fig. 6.17. Hence the value of the critical separation as determined from the simulation results in Fig. 6.17 is \( d_c = 7.4 \) m. When the separation is increased beyond \( d_c \), the performance of Eigen-RSSI can surpass the U-MUSIC method in range estimation. This is due to the better sensitivity of the spatially extended AVS to \( r_s \) at high value of \( d \) as compared to the compact AVS as seen from equations (6.35) and (6.36). Equation (6.37) yields a predicted value of \( d_c' = 7.4 \) m for simulation parameters assumed in Fig. 6.17 which is equal to the corresponding value observed from simulations.

The performance of range estimation of the U-MUSIC and Eigen-RSSI methods is good for near-field sources but it degrades as the range \( r_s \) of the source increases. This is because waves emanating from the source become more planar in nature as \( r_s \) increases. This degradation in range estimation is depicted in Fig. 6.18. Figure 6.18 shows a plot of the variation of the RMSE of range estimates of the U-MUSIC and Eigen-RSSI methods with variation in the range \( r_s \) of the source from
velocity sensors \((d = 5 \text{ m} \text{ for the Eigen-RSSI method})\). The signal is received at an SNR of 20 dB and the source location parameters are the same as those considered in Figs. 6.13-6.15. The plots in Fig. 6.18 show that as the range \(r_s\) increases, the performance of both the methods degrades. This observation is on expected lines and can also be theoretically predicted from equations (6.35) and (6.36) derived by us. These equations show that the range sensitivity of both AVS models decreases as the source moves farther away from the sensors.

If the environmental noise is impulsive in nature, the performance of U-MUSIC can be enhanced using approaches such as SSR-preprocessing as mentioned in section 6.2, or by using fractional order correlation to compute the data correlation matrix and signal eigenvectors [58]. We wish to point these out as possible enhancements to U-MUSIC.

We end the discussion in this section with a comparison of the Eigen-RSSI and U-MUSIC methods. The computational complexities of both the methods are equivalent from a signal processing perspective. This can be seen from the fact that both these methods provide closed form expressions for the estimates of the range, elevation and azimuth, and both methods use the same pre-processing step of eigen-value decomposition of the data. Hence any difference in the relative complexities of these methods would be apparent only at the implementation level, arising from the difference in the finer details of the implementation of these algorithms.
When we compare the two methods in terms of other practical issues, we note that since the extended AVS requires a large separation of the pressure sensor from the velocity sensors it no longer possesses the compactness associated with a single sensor. Hence it cannot be used in applications that require the AVS to be mounted in a confined space. The extended AVS also requires calibration of this known separation of sensors in a known direction which is an additional burden during its deployment. Hence the U-MUSIC method using the compact AVS is advantageous compared to the extended AVS for source localization when the compactness of the sensor system is a priority.

### 6.4. Summary

This chapter deals with the problem of source localization using AVS. The existing AVS literature assumes that the source is located in the far-field and that the environmental noise is Gaussian in nature. We aim to fill the gaps in the current literature by focusing on the under-explored aspects of this problem, namely (i) azimuth estimation in impulsive noise and (ii) near-field localization.

The first part of the chapter described in section 6.2 deals with azimuth estimation using AVS in impulsive non-Gaussian noise. The ability of processors to localize underwater acoustic sources is limited by the low SNR encountered in the ocean. Section 6.2 discusses the design of a preprocessor based on the phenomenon of SSR that can be used to improve the performance of azimuth estimation in such environments. The impulsive environmental noise is modeled as a complex GG pdf.
The SSR preprocessor can perform denoising of underwater acoustic data that is corrupted by noise that is leptokurtic in nature. The improvement in performance of azimuth estimation offered by an SSR preprocessor can be optimized by appropriate selection of the quantizer noise pdf, standard deviation and the number of quantizer pairs. We propose that the design of the denoiser can be done by using a simpler performance measure known as the correlation gain.

The performance of the azimuth estimation methods is shown to improve at lower input SNR and as the environmental noise pdf becomes more heavy-tailed in nature. The beneficial effect of the SSR preprocessor on the performance of the MUSIC and SIM azimuth estimators is shown. The performance is found to be better in terms of reduced bias and RMSE in the azimuth estimates. The ambiguity functions of the SSR enhanced processors are sharper. The SSR preprocessor is also shown to improve the resolution of two close sources by the azimuth estimators in a leptokurtic noise. Hence our work shows that the SSR denoiser can be effectively used as a preprocessor to azimuth estimators for application in underwater acoustic channels.

In the second part of the chapter described in section 6.3, we present a novel method called U-MUSIC to localize an acoustic source located in the near-field using a single AVS. The proposed method provides closed form expressions for the estimates of source azimuth, elevation and range. Hence it avoids a complex 3D search for the location parameters that is required in conventional localization methods.
The U-MUSIC method employs a compact AVS with co-located pressure and velocity sensors, and performs better than the existing method presented by Wu and Wong [39]. Thus U-MUSIC is able to offer better performance with a more compact AVS setup than that provided by the extended AVS. Furthermore, the compact AVS configuration does not require calibration of the separated pressure sensor from the velocity sensors. An accurate expression for the threshold value of separation below which the compact AVS yields better performance than the extended AVS is derived.

Our work on U-MUSIC presented in this chapter has been done for a deep ocean scenario. It has not been extended to the case of a shallow ocean environment because a simple near-field AVS signal model has not yet been developed for shallow ocean. In the future the U-MUSIC method may be extended to the case of shallow ocean environment.
Chapter 7
Conclusion and Future work

In this chapter, we conclude by highlighting the contribution of our work. We also outline and discuss some possible future research directions where the work can be extended.

7.1. Conclusions

In our work we investigated two problems in shallow water acoustic signal processing using AVS, namely source detection and source localization. The primary challenges in these fields include the low SNR levels encountered in the ocean, the non-Gaussian nature of this noise and the multi-modal nature of propagation of signal fields in the shallow ocean environment. The contributions of our work to these fields and the inferences drawn from our work are described below.
7.1.1. Source detection

The work on source detection using AVS was undertaken in three stages.

7.1.1.1 GLRT based detection using AVS array in Gaussian ambient noise

In the first stage we formulated four methods for detection of a narrowband acoustic source using an AVS array and investigated their performance in detail. A range-independent shallow-ocean environment with Gaussian ambient noise was assumed. Two of these methods (ED and SD) were formulated based on conventional approaches and two of these (TSD and ASFD) are novel approaches formulated by using alternative signal models. The contributions and findings of our investigation are summarized below

(i) Using different assumptions and signal models leads to the formulation of different detectors. When no prior assumption is made on the signal vector structure and it is treated as unknown, the conventional ED detection strategy is obtained. If the signal models based on the normal mode theory are employed, better algorithms such as the SD and TSD can be formulated. By exploiting the relation between the pressure and particle velocity at each AVS, we can formulate the ASFD strategy.

(ii) Theoretical expressions for \( P_{FA} \) and \( P_D \) of the detectors were derived for the asymptotic case and the finite-data case.
(iii) Incorporating prior information on the signal does not always lead to improvement in the performance of a detector. This is evident in the fact that the performance of the SD is poor compared to that of the TSD.

(iv) Expressions for the NMSE of all detectors which serve as indicators of their performance were derived. There exists a strong negative correlation between the NMSE and the detector performance which establishes that the performance of a detector depends on the accuracy of its signal vector estimate.

(v) The TSD is a prime choice for detection when lower order modal wavenumbers of the channel are known. When these are unknown, the ASFD may be used to obtain performance better than the conventional ED.

To summarize, this work provides several schemes for detection using AVS arrays and investigates their performance in detail. Theoretical expressions and measures describing the performance are provided. This work also serves as a springboard for an extended investigation on detection using AVS arrays in non-Gaussian noise.

7.1.1.2 Single-sensor detector based on SSR

In the second stage of our investigation on detection, we undertook a detailed analysis of a single sensor SSR detector for sources in impulsive noise which consists of a preprocessor and a matched filter. We also present methods for the design of this preprocessor. From this work the following were concluded:
(i) For weak signals, optimization of the SSR preprocessor involves maximization of a simple function of the SD $\sigma$ of the quantizer noise.

(ii) Maximizing the $P_D$ minimizes the mean-square difference between the output of the quantizer array and that of the optimum nonlinear transformation of the LO detector. Hence the optimization of the SSR preprocessor is essentially a ‘shaping’ of its input-output characteristics to become closer to optimal nonlinear characteristics.

(iii) The SSR preprocessor provides a significant improvement in performance over the matched filter and its performance is close to optimal for the noise models considered in this thesis.

(iv) The improvement due to SSR stems primarily from quantization while the addition of optimum amount of quantizer noise improves the performance further.

(v) The design of the SSR preprocessor can be readily extended to the detection of non-weak signals.

This work has presented a detailed investigation into the SSR phenomenon and has described the mechanism of its working in impulsive noise. The design of the SSR preprocessor for improving the performance of detection is presented. The variation of the performance of the SSR-based detection scheme with various parameters has been investigated. Our work shows that this scheme is a near-optimal, simple, robust and effective approach in impulsive noise modeled by standard non-Gaussian noise models. Through these investigations, this work gave insights into the challenges
associated with detection in a non-Gaussian noise environment and provides a good strategy to upgrade the AVS array based detectors for application in such environments.

7.1.1.3 Detection using AVS array in non-Gaussian ambient noise

In the final stage of our work on detection, we extended the formulation of AVS array-based detection schemes to shallow ocean environments contaminated by impulsive noise. The impulsive noise is modeled by the GG model. The two strategies used are (i) extension of the GLRT approach to the case of non-Gaussian noise and (ii) adaptation of the detectors for Gaussian noise to non-Gaussian environments using SSR preprocessing of the data. The results obtained indicate that

(i) The TSD is the most effective and robust detection scheme for a wide range of noise pdfs if the modal wavenumbers of the ocean channel are known. If these are unknown the ASFD provides a good alternative option to detection.

(ii) SSR preprocessing is effective in enhancing the performance of the UD and that of the ASFD when the noise is impulsive. SSR is not beneficial in the case of the SD and TSD. This is because it is a nonlinear system that leads to distortion of the signal structure.

(iii) The TSD and ASFD allow signal detection even when noise parameters are unknown, which is not possible using the conventional UD.

(iv) The beneficial effect of SSR denoising is low when the noise parameters are unknown.
As the noise becomes more impulsive the performance of all formulated detectors improves as they can more effectively discard the outlier values arising from noise.

To summarize, this work presented AVS array-based detection strategies for an impulsive noise environment. Two strategies to achieve good performance of detection were presented which were both shown to be effective. The GLRT based strategy outperforms the SSR preprocessor based strategy, and can also be used in environments where the level and impulsiveness of noise are uncertain. The performance of detectors formulated based on these strategies were investigated. These detectors show good performance in impulsive GG noise by effectively discarding the outlier values arising from this noise.

The investigation of the detection problem in non-Gaussian noise complements that done on detection in Gaussian noise. It re-affirms the effectiveness of the TSD in terms of robustness to noise. Our work also shows that detection using AVS arrays offers performance superior to that using APS arrays. Through these investigations and contributions, we aim to provide clear-cut strategies for detection in a shallow ocean environment.

7.1.2. Source localization

Our research on source localization using AVS was aimed at developing improved algorithms for localization in low SNR ocean environments. We investigated two
Chapter 7 Conclusions and Future work

problems in AVS-based localization. The contributions and observations of our investigation on each problem are summarized below.

7.1.2.1 Source azimuth estimation in shallow ocean with non-Gaussian noise

The first problem considered by us with respect to AVS-based localization was the improvement of the performance of azimuth estimation in impulsive noise. A preprocessor based on SSR was developed to deliver this improvement. The following conclusions were drawn from this work:

(i) The improvement in performance of azimuth estimation offered by an SSR preprocessor can be optimized by appropriate selection of the quantizer noise pdf, standard deviation and the number of quantizer pairs. This observation has also been made in the case of SSR-enhanced detection.

(ii) We proposed a simple performance measure known as the correlation gain with which the design of the SSR denoiser can be done. Thus a SSR denoiser for azimuth estimation can be designed with the objective of improving correlation of the data with the pure signal.

(iii) The performance gain in azimuth estimation due to the SSR preprocessor increases as the input SNR becomes lower and as the environmental noise pdf becomes more heavy-tailed in nature. This shows the SSR preprocessor is able to effectively discard more noise energy as the noise becomes more heavy-tailed or the SNR reduces.
The SSR preprocessor yields significant improvement in the performance of azimuth estimators such as MUSIC and SIM, in presence of impulsive noise modeled by standard models. The performance is better in terms of reduced bias, reduced RMSE in the azimuth estimates and sharper ambiguity functions of the SSR enhanced processors. The resolution of two close sources by azimuth estimators has also improved by using the SSR denoiser in a leptokurtic noise environment.

7.1.2.2 Source localization of near-field sources

The second AVS-based localization problem investigated by us considered near-field sources. We presented a novel method called U-MUSIC to localize a near-field source using a single AVS.

(i) The proposed U-MUSIC method provides closed form expressions for the estimates of source azimuth, elevation and range. Hence it avoids a complex 3D search for the location parameters that is required in conventional localization methods.

(ii) U-MUSIC employs a compact AVS with co-located pressure and velocity sensors and performs better than the existing method presented in [39]. This performance advantage is evident when large separation of sensors in the system is not possible.

(iii) Thus U-MUSIC is able to offer better performance with a more compact AVS setup than that provided by the extended AVS used in [39]. Furthermore, the
compact AVS configuration does not require calibration of the separated pressure sensor from the velocity sensors.

(iv) The performance of localization is related to the range sensitivity of the measurements which can be used as a simplified performance measure.

(v) There exists a threshold value of separation below which the compact AVS yields better performance than the extended AVS. An accurate expression for this threshold separation is derived.

Thus, the U-MUSIC method proposed by us is able to obtain source location estimates using a single AVS and overcomes the shortcomings of the existing methods for this problem.

7.2. Future work

In this section we discuss several possible extensions to the work that was done in this thesis.

(i) Experimental validation of signal processing algorithms

Several methods for signal detection and localization using AVS have been developed in this thesis. These have been developed based on standard theoretical models of signals and noise in ocean environments. However, due to lack of availability of AVS, we have not been able to test and validate these algorithms experimentally. Experimental validation gives a more realistic evaluation of the relative performance of several algorithms, and the challenges associated with their practical application.
Hence, testing these algorithms with real experiments is an exciting advancement of our work which can add a lot of insight into the findings made in this thesis.

We have started some work in this direction by using the real data from the Shallow water cell experiment (SwellEx 96) [193] to validate our algorithms for localization. This experiment was conducted using pressure sensor arrays, near San Diego, California in 1996 in a shallow ocean environment using signals at frequencies between 49 and 400 Hz. Even though this data was obtained using pressure sensors, we were able to validate a few of our results using this data. However, apart from the fact that the data was obtained from a pressure sensors array, there are some more challenges in using data from the experiment: (a) the environment for the experiment was range-varying (horizontally unstratified), (b) the sound-speed in water was depth-varying, and (c) a moving source was considered, which made the data and source time-varying. One possible way for us around these hurdles is to expand our models to be able to use the SwellEx 96 experimental data for experimental validation.

(ii) Detection in non-Gaussian noise using median filtering

Our investigations have shown that the performance of detectors can be enhanced by improving estimation of the signal vector. Since the MLE of the signal vector cannot be obtained easily as closed form expression in non-Gaussian environmental noise, in our formulation we have used the Gaussian ML estimate ($e = 2$) to obtain the detectors. Alternatively, it is possible to easily estimate the signal vector if the environmental noise is considered to be Laplacian noise (GG pdf with $e = 1$). In this
case, the MLE does not have a closed-form expression but it can be obtained by weighted median filtering of the data [29] that is more robust and accurate in impulsive environments than the Gaussian estimate. This opens the possibility of an alternative method to implement the detectors with possibly better performance by using a weighted median filtered estimate of the signal vector.

(iii) AVS-based localization in alpha-stable noise

In our research on localization using AVS, we have investigated the use of SSR preprocessing for improving localization in non-Gaussian noise modeled by the GG pdf. The GG pdf was used because (i) it has a finite variance which makes it easy to work with, and (ii) it can model a wide range of finite-variance impulsive noise by variation in a single parameter. However, in some scenarios it has been shown that $S\alpha S$ noise pdfs are a more appropriate choice to model ocean noise [22]. Hence an investigation into the effectiveness of SSR in $S\alpha S$ noise (which may also be of infinite variance) is of practical interest.

In $S\alpha S$ noise, it has been suggested that performance of AVS-based localization can be enhanced using fractional lower order statistics (FLOS) [58], [123]. An investigation on the relative merits and demerits of SSR-based localization as compared to FLOS methods could be undertaken. An initial investigation by us showed that both these methods have merits and demerits that are complementary to each other. Hence, another possible direction for research is to combine both these...
approaches effectively to boost the performance. The fusion of these two approaches is a promising direction that can be considered.

(iv) Source number detection
A very important problem in signal processing is the detection of the number of sources whose signals impinge on an array of sensors [194], [195]. By combining the detection methods using AVS array that are presented in chapter 4 of this thesis along with information theoretic criteria (such as minimum descriptor length [195]) or maximum likelihood estimation, it is possible to perform joint source number estimation and DOA estimation of the sources in a shallow ocean environment. This is a promising field of work with important applications and investigation into this topic has already been undertaken by us.

(v) Flow noise models for AVS
In cases where a towed array of AVS is used, the measurements of the AVS are prone to not only isotropic noise but also other forms of noise in the ocean. This includes flow noise due to the turbulent flow of water around the sensor which affects the particle velocity measurements at the sensors. This noise can severely affect these measurements as the flow velocity increases [196]. However, if this noise can be suitably modeled, the distortion caused due to this noise at the velocity sensors can be reduced.
Studies on the effect of flow noise already exist in the literature such as those by Lauchle et al [197], [198]. In [198] Lauchle et al present a model for flow-induced self-noise on moored and drifting velocity sensors that arise due to oceanic currents such as those caused due to wave motion and tides. In [197] the mechanism of flow-induced self noise on velocity sensors has been studied. The results show a strong correlation between the sensor output and the forces created by the unsteady flow over the sensor. The presence of flow noise may adversely affect the performance localization and detection algorithms utilizing AVS. Hence, an investigation into the effect of flow noise in AVS measurements and how it can be modeled could aid signal processing applications using data from towed AVS arrays.
Author’s publications

Journals


• V. N. Hari, G. V. Anand, and A. B. Premkumar, “Narrowband signal detection techniques in shallow ocean by acoustic vector sensor array,” Digital Signal Processing, June 2013, DOI: 10.1016/j.dsp.2013.06.010

Conferences


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