INVESTIGATION OF RAMAN EFFECT IN INTEGRATED OPTICAL WAVEGUIDE DEVICES

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**SUMMARY**

Raman Effect has been widely utilized in the silicon photonics technology for their applications in optical interconnect and all-optical wavelength conversion, motivated by their unique advantages of simple fabrication, high energy efficiency and wide range of operating wavelengths. Functional components such as integrated waveguide Raman amplifiers, lasers and wavelength converters have been realized over the past decade. This research work aims to gain in-depth physical insights of these Raman-based waveguide devices for their future engineering applications.

We propose a novel amplitude propagation method (APM) that takes into account all linear and nonlinear processes influencing electromagnetic waves along semiconductor waveguides. APM addresses both amplitude and phase evolutions of optical waves, and provides the unique capability to universally analyze different physical processes within these Raman-based waveguide devices.

Three methodologies are proposed to tackle the detrimental free-carrier absorption loss in silicon waveguide Raman amplifiers and lasers; including bi-directional pumping scheme, chalcogenide waveguide Raman laser and silicon-chalcogenide slot waveguide Raman amplifier. Intensive theoretical investigation illustrates that the proposed approaches provide the much greener alternatives to the well-established silicon counterpart, highlighted by their outstanding energy efficiency improvement.

We propose the first Raman-assisted wavelength converter in chalcogenide waveguide based on CARS process. Theoretical investigation shows that highly efficient wavelength conversion can be obtained, together with broad bandwidth and simplified dispersion engineering process compared to the existing silicon Raman waveguide
wavelength converter. CARS process in the weak pump regime is also explored, producing simultaneous signal depletion and anti-Stokes emission.

In addition to the superior energy efficiency, we show that chalcogenide waveguide Raman amplifiers and lasers suffer lesser pump-to-Stokes RIN transfer compared to silicon devices. Further RIN transfer reduction can be achieved through cavity optimization or application of the bi-directional pumping scheme.
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## A

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tr>
<td>AOWC</td>
<td>All Optical Wavelength Conversion</td>
</tr>
<tr>
<td>APM</td>
<td>Amplitude Propagation Method</td>
</tr>
<tr>
<td>AR</td>
<td>Anti-Reflection</td>
</tr>
<tr>
<td>As$_2$Se$_3$WRA</td>
<td>As$_2$Se$_3$ Waveguide Raman Amplifier</td>
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<td>As$_2$Se$_3$WRL</td>
<td>As$_2$Se$_3$ Waveguide Raman Laser</td>
</tr>
<tr>
<td>As$_2$Se$_3$WRWC</td>
<td>As$_2$Se$_3$ Waveguide Raman Wavelength Converter</td>
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## C

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<tr>
<th>Acronym</th>
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<tr>
<td>CARS</td>
<td>Coherent Anti-Stokes Raman Scattering</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide Semiconductor</td>
</tr>
<tr>
<td>CTE</td>
<td>Coefficient of Thermal Expansion</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous-Wave</td>
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## E

<table>
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<tr>
<td>EDFA</td>
<td>Erbium Doped Fiber Amplifier</td>
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<tr>
<td>E-H</td>
<td>Electron-Hole</td>
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## F

<table>
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<tr>
<th>Acronym</th>
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<tbody>
<tr>
<td>FCA</td>
<td>Free Carrier Absorption</td>
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<tr>
<td>FOM</td>
<td>Figure of Merit</td>
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<td>FP</td>
<td>Fabry-Perot</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<td>---------</td>
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<tr>
<td>FWM</td>
<td>Four-Wave-Mixing</td>
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<td>GVD</td>
<td>Group Velocity Dispersion</td>
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<td>HF</td>
<td>High Frequency</td>
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<tr>
<td>IRM</td>
<td>Iterative Resonator Method</td>
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<td>LPL</td>
<td>Linear Propagation Loss</td>
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<td>MZI</td>
<td>Mach-Zehnder Interferometer</td>
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<td>NR</td>
<td>Newton-Raphson</td>
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<td>ODE</td>
<td>Ordinary Differential Equations</td>
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<td>PPMWL</td>
<td>Perfect Phase Match Wavelength</td>
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<tr>
<td>RKF</td>
<td>Runge-Kutta-Fehlberg</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>SARS</td>
<td>Stimulated Anti-Stokes Raman Scattering</td>
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<tr>
<td>SCSWRA</td>
<td>Silicon Chalcogenide Slot Waveguide Raman Amplifier</td>
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<tr>
<td>SDR</td>
<td>Signal Depletion Ratio</td>
</tr>
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<td>SPM</td>
<td>Self-Phase Modulation</td>
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<tr>
<td>SRS</td>
<td>Stimulated Raman Scattering</td>
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<td>Silicon Waveguide Raman Laser</td>
</tr>
<tr>
<td>SWRWC</td>
<td>Silicon Waveguide Raman Wavelength Converter</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse Electric</td>
</tr>
<tr>
<td>TM</td>
<td>Transverse Magnetic</td>
</tr>
<tr>
<td>TPA</td>
<td>Two Photon Absorption</td>
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<tr>
<td>WC</td>
<td>Wavelength Conversion</td>
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<td>Waveguide Raman Wavelength Converter</td>
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<tr>
<td>XPM</td>
<td>Cross-Phase Modulation</td>
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<tr>
<td>ZDW</td>
<td>Zero-Dispersion Wavelength</td>
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Chapter 1 INTRODUCTION

1.1 Background and Motivation

Traditional optical components are made using a variety of different materials, e.g. III-V compounds for sources, lithium niobate (LiNbO₃) for modulators, silica for fibers, and rare-earth materials for amplifiers. The diverse material platforms and production technologies subsequently result in the high cost of these components, which hugely limit the application of photonics. On the other hand, electronics industry is dominated by silicon. Its fabrication technology, complementary metal-oxide-semiconductor (CMOS), represents the most spectacular convergence of technological sophistication and economics of scale. Integration of optical components onto the silicon platform, silicon photonics technology, provides the attractive mean to realize compact and cost-effective optical components [1].

The vision of silicon photonics technology can be traced back to the 1980s by R. Soref et al. [2]. The research effort was moderate then, mainly focusing on the passive devices. Rapid evolution of silicon photonics technology took places in the beginning of the 21st century, driven by the urgent need for the optical interconnect to carter the ever-increasing on-chip data rates as suggested by the Moore’s Law [3]. Bandwidth improvement of the electrical interconnect nowadays can only be accomplished at the expense of increased latency, power consumption and architecture complexity. This is mainly due to the copper wire’s large loss, dispersion and cross-talk at high date rate. The ultimate bandwidth from optical interconnect appears as the ideal replacement [4].
However, photonic integration is technically much more challenging than electronic integration. In particular, silicon lacks of a direct mechanism for light generation, hindered by the material’s indirect band-gap and low internal quantum efficiency for light emission. Stimulated Raman scattering (SRS) provides one of the leading solutions to generate optical gain in silicon, motivated by the unique advantages of wide operating wavelengths and simple fabrication process. In fact, the first integrated continuous-wave (CW) laser in silicon platform is demonstrated based on SRS in 2004 [5]. Such encouraging progress drives global research attention to the belief that Raman amplification would be the final answer for the missing silicon-based laser.

Another important application of Raman Effect in the silicon photonics technology lies in the area of integrated all-optical wavelength conversion (AOWC). Integrated wavelength converters enable seamless integration of optical components that are operating at distinct wavelengths. Furthermore, they provide the low-cost reconfigurable capability and wavelength flexibility to the future fast-speed wavelength-routed network. Active researches in the past decade have led to the demonstration of AOWC in the silicon platform using various third-order nonlinear properties, motivated by the compact size and low fabrication cost [6]. An alternative approach can be achieved through coherent anti-Stokes Raman scattering (CARS) process. Raman-assisted wavelength conversion has been demonstrated recently in silicon waveguide, providing wider tuning range and potentially higher efficiency [7].

Although producing all components from silicon would be elegant, it could turn out to be unrealistic because of the stringent performance and cost-effectiveness demand from the market. In particular, recent progress of the Raman-based waveguide devices has been hugely limited by the intrinsic narrow Raman gain bandwidth and high free-carrier absorption of the silicon material. A hybrid solution using chalcogenide glasses
would be more promising, analogous to the case in Si-Ge photo-detector [8] and wafer bonded III-V silicon laser [9]. These materials possess the desirable combination of large and wide Raman gain spectrum, negligible free carrier absorption and fast response time for Raman-based applications [10].

1.2 Objectives

Various Raman Effects have been explored in the silicon photonics technology to realize integrated waveguide devices such as waveguide Raman amplifier, laser and wavelength converter. This research work aims to gain in-depth physical insights of these Raman-based waveguide devices for their future engineering applications. A theoretical model amplitude propagation method (APM) is proposed to provide the universal analytical tool for the devices, which addresses all relevant physical processes occurring along the waveguide. Comprehensive characterizations of the devices are subsequently performed based on APM, leading to various innovations to enhance devices performance for both the energy efficiency and noise.

1.3 Major Contributions of the Thesis

This thesis is focused on the theoretical investigation of the various Raman effects in waveguide, in order to optimize the performance of these Raman-based waveguide devices. The original contributions include:

- A theoretical modeling APM is proposed to study the Raman Effect in integrated waveguide devices, providing an analytical tool that can be
universally applied to a wide range of Raman-based integrated waveguide devices across various material platforms. (Chapter 3)

- Three novel methodologies are proposed and investigated, in an attempt to solve the detrimental high free-carrier absorption (FCA) problem that currently limits the energy efficiency of silicon waveguide Raman amplifiers and lasers.
  - Bi-directional pumping scheme: effectively suppress FCA to enhance Stokes output (Chapter 4.2)
  - As$_2$Se$_3$ waveguide Raman amplifiers and lasers: achieve simultaneous output enhancement and lasing threshold reduction, as well as superior device miniaturization capability compared to silicon (Chapter 4.3)
  - Silicon chalcogenide slot waveguide Raman amplifier: possess high Raman on-off gain, wide linear amplification range and low-loss integration into the silicon platform. (Chapter 4.4)

- The influence of less dominant physical processes are investigated for Raman-based silicon waveguide devices, namely the Coherent anti-Stokes Raman scattering (CARS) process in silicon waveguide Raman laser and nonlinear electronic processes in wavelength. (Chapter 5.2)

- Wavelength conversion through CARS process is proposed and comprehensively investigated in chalcogenide waveguides. 5.5dB conversion efficiency is observed, more than 10 dB higher than its silicon counterpart. Nonlinear dynamic phase shift could cause significant fluctuation from perfect phase matching condition, highlighted by a 20 dB efficiency reduction. In addition, characteristics of CARS process are explored in the weak pump regime, for the first time according to our knowledge. (Chapter 5.3)
Chapter 1: Introduction

- The characteristics of the relative intensity noise (RIN) transfer in $\text{As}_2\text{Se}_3$ waveguide Raman amplifier are studied. Initial Stokes and pump fluctuation is found to be essential in determining RIN transfer. (Chapter 6.3)

- The RIN transfer characteristics for $\text{As}_2\text{Se}_3$ waveguide Raman laser are explored. Bi-directional pumping scheme is also proposed and verified as an effective configuration to suppress RIN transfer. In addition, the optimization of cavity length and reflectivity on RIN transfer is performed. (Chapter 6.4)

1.4 Thesis Organization

Chapter 1 starts with background introduction of silicon photonics technology, which elaborates the significant of this research work from the application point of view. The objectives of the thesis are clearly stated, and the major contributions are presented, which is followed by the organization of the thesis. Chapter 2 gives an overview about the previous research results in various Raman effects and their applications in integrated waveguide devices. Both experimental and analytical work progresses are reviewed to support our research direction.

Chapter 3 discusses the novel amplitude propagation method (APM) modeling, which is utilized throughout the thesis to investigate various Raman-based waveguide devices. Detailed derivations of the APM model are presented, involving the discussion of linear and non-linear losses, as well as various Raman and electronic nonlinear effects. The different applications of APM model to silicon and chalcogenide waveguide based Raman devices are shown. Verification of the derived model using existing experiment and simulation results are also performed.
Chapter 4 presents a comprehensive study for the waveguide Raman amplifier and laser, which is based on stimulated Raman scattering (SRS) process. Three innovative devices are proposed and justified theoretically to combat the severe free-carrier absorption (FCA) loss drawback of silicon waveguide Raman amplifiers and lasers. These approaches are bi-directionally pumped silicon waveguide Raman lasers, As$_2$Se$_3$ waveguide Raman amplifier and laser, as well as silicon-chalcogenide slot waveguide amplifiers. Extensive analysis is performed to optimize key performance parameters.

Chapter 5 focuses on the investigation of coherent anti-Stokes Raman scattering (CARS) process in waveguide devices. The chapter starts with the study of CARS process in silicon platform, in both the waveguide Raman laser and wavelength converter. We then go on to propose and analyze a novel Raman-assisted wavelength converter in chalcogenide waveguides, tackling the narrow bandwidth and low efficiency drawbacks of the existing silicon waveguide Raman wavelength converter.

Chapter 6 is devoted to explore the pump-to-Stokes RIN transfer characteristics for the As$_2$Se$_3$ waveguide Raman amplifier (As$_2$Se$_3$WRA) and laser (As$_2$Se$_3$WRL), for the first time to the best of our knowledge. Derivations of the mathematical tools to analyze RIN transfer are discussed, extended from the APM model. The RIN transfer characteristics of As$_2$Se$_3$WRA and As$_2$Se$_3$WRL are subsequently investigated based on the derived model, using their well-known silicon counterparts as benchmark. In addition, cavity optimization and bi-directional pumping scheme are proposed as the RIN transfer suppression solution for As$_2$Se$_3$WRL.

Chapter 7 concludes the whole thesis with summary of the main achievements and recommendations for future work.
Chapter 2  BACKGROUND LITERATURE REVIEW

Two major categories of Raman processes have been applied in the silicon photonics technology to realize functional waveguide devices, namely stimulated Raman scattering (SRS) and coherent anti-Stokes Raman scattering (CARS). SRS is mainly utilized in integrated amplifiers and lasers for optical interconnect, while CARS is explored for integrated wavelength converters to implement all-optical wavelength conversion. In this chapter, a comprehensive review of the background theory and progress will be presented for these Raman processes and their related devices.

2.1  Stimulated Raman Scattering and Integrated Amplifiers and Lasers

Raman scattering refers to the inelastic scattering of photons when it interacts with matters. It was discovered by C.V. Raman and K. S. Krishnan in liquid, and by G. Landsberg and L. Mandelstam in crystals [11]. Classical electrodynamics provides a simple and intuitive macroscopic description of the Raman scattering process. In the spontaneous scattering, thermal vibrations of a lattice at frequency $\omega_v$ produce a sinusoidal modulation of the susceptibility. The incident pump field induces an electric polarization that is given by the product of the susceptibility and the incident field. The beating of the incident field oscillation $\omega_p$ with oscillation of the susceptibility $\omega_v$ produces induced polarizations at the sum frequency $\omega_p + \omega_v$ and at the difference frequency $\omega_p - \omega_v$ [12]. The radiation produced by these two polarization components is referred to as anti-Stokes ($\omega_a = \omega_p + \omega_v$) and Stokes ($\omega_s = \omega_p - \omega_v$) waves, respectively. This concept can be illustrated in the following energy
diagrams in Fig. 2.1 [13]. At room temperature, quantum statistics indicate that the Stokes emission is an order of magnitude higher than the anti-Stokes emission. The conventional “Raman scattering” thus refers to this pump-to-Stokes conversion, which is widely utilized for optical amplification and lasing [14].

![Energy level diagrams for Raman scattering process](image)

**Fig. 2.1 Energy level diagrams for Raman scattering process [13]**

Raman scattering in silicon was first observed in 1965 by J.M. Ralston and R.K. Chang. Using a helium-neon laser with an output wavelength of 0.6328 µm, backward Raman scattering from silicon was measured, and it was found that the Raman scattering efficiency in silicon was 35 times larger than that for diamond [15]. More detailed experiments were then conducted in the early 1970s, resulting in accurate measurement of the spontaneous and stimulated Raman scattering efficiency [16] as well as the Raman shift frequency of 15.6 THz [17]. These early research work focused more on the characterization of the Raman scattering process, showing that silicon has a relatively strong Raman scattering efficiency [18, 19].

The rapid advancement of silicon photonics technology in the 21st century redirects the global research attentions into the study for Raman scattering in silicon, driven by need for the silicon-based amplifier and laser. It is generally undesirable to generate
light directly from silicon, which is an indirect band-gap material with low internal quantum efficiency for light emission [20]. Various approaches are therefore proposed to generate light on silicon indirectly, including light emission from silicon nanostructures [21], erbium-doped silicon [22], dislocation-engineered silicon [23], Si/SiGe super-lattices [24], hybrid silicon lasers [9] and silicon parametric amplifiers [25].

Stimulated Raman scattering (SRS) provides another attractive alternative to generate optical gains in silicon. Raman scattering has long been identified as a non-linear effect to generate optical gain in fibers [26]. The Raman gain measured in silicon is 20 cm/GW, which is $10^3 - 10^4$ times larger than that in the silica fiber (~$0.93 \times 10^{-2}$ cm/GW) [27]. This is mainly due to the crystalline nature of silicon waveguide, which has a narrow and sharp Raman gain peak as compared to the amorphous silica fiber. On the other hand, the modal area in a silicon waveguide is roughly 100 times smaller than that in a fiber, resulting in a proportional increase in optical intensity. The combination makes it possible to realize chip-scale Raman devices that normally require kilometers of fiber to operate [14].

R. Clap et al. started the research for integrated silicon waveguide Raman amplifiers and lasers in 2002, with the first experimental observation of Raman gain in silicon waveguides in the telecom wavelengths [28]. Despite its superior gain, Stokes conversion efficiency from silicon waveguides has been extremely low ever since its proposal [27, 29, 30]. This could be attributed to the several non-linear absorption processes in silicon waveguides, which compete with SRS to reduce the Raman gain in the presence of high pump power. To be precise, these losses are the two-photon absorption (TPA) and subsequently induced free-carrier absorption (FCA). TPA is a typical non-linear absorption process in semiconductors when the sum of the energy of
two incident photons exceeds the energy band-gap of the semiconductor, which is the case in silicon in the 1.5 µm communication windows. In the TPA process, two photons are absorbed simultaneously to create a free electron-hole (E-H) pairs in the silicon waveguides. The power absorbed by the TPA process is governed by Eq. 2.1 in the following, where \( \beta \) is the TPA coefficient and \( A_{\text{eff}} \) is the effective core area \([31]\). The severity of the TPA process is thus dominated by the TPA coefficient, which has been measured in the 1.5 µm wavelength ranges to be \(~0.45\) cm/GW for silicon \([32]\). This is relative small as compared to the typical Raman gain coefficient of 20 cm/GW, resulting in negligible pump depletion as demonstrated in \([33]\).

\[
P_{\text{TPA}}(z) = \frac{\beta}{A_{\text{eff}}} \int_0^z P_{\text{pump}}^2(z) \, dz
\]

(2.1)

The free E-H pairs generated during the TPA process will trigger another optical absorption process in silicon, namely free carrier absorption (FCA). In the FCA process, electric field of the incident optical radiation accelerates the free carriers in silicon waveguides, which are subsequently decelerated by collisions with the lattice. During the process, energy of the radiation field is converted into heat in the process, resulting in pump power depletion. The FCA absorption coefficient can be described from the classical Drude model as in Eq. 2.2 \([34]\), in which \( N_{\text{eff}} \) is the free carriers density generated from TPA process. \( \tau_{\text{eff}} \) is the effective carrier recombination time, \( h\nu \) is the photon energy and \( \sigma \) is the FCA cross section.

\[
N_{\text{eff}}(z) = \frac{\tau_{\text{eff}}}{2h\nu} P_{\text{TPA}}(z) = \frac{\tau_{\text{eff}}}{2h\nu A_{\text{eff}}} \int_0^z P_{\text{pump}}^2(z) \, dz
\]

(2.2 a)

\[
\alpha_{\text{FCA}} = \sigma N_{\text{eff}}(z)
\]

(2.2 b)
Research in the past decade for silicon waveguide Raman lasers has been largely devoted to combat this FCA loss, which has been widely accepted as the most dominant loss mechanism [30, 31]. One of the most successful innovations is through the introduction of a reverse-biased p-i-n junction across the silicon waveguide as shown in Fig. 2.2 [35]. The large reverse voltage in the p-i-n junction sweeps out the free carriers swiftly, resulting in the reduction of the effective carrier recombination time ($\tau_{eff}$). This set-up subsequently led to the demonstration of the first CW silicon Raman laser in 2005 using a 4.8cm-long silicon ridge waveguide [5]. This group from Intel further improved their laser performance by replacing the silicon ridge waveguide with a 3cm-long racetrack cavity. This silicon Raman laser has a record-low threshold at 20mW, with 28% slope efficiency and a maximum output of 50mW [36, 37].

Fig. 2.2 Illustration of a silicon waveguide with a reverse-biased p-i-n junction (a) schematic diagram (b) SEM image [35]

Another group from UCLA led by Prof. B. Jalali tackled the problem of severe FCA loss by utilizing low duty cycle pumping. It has been shown both theoretically and experimentally that pulsed-pumping could yield negligible free-carrier generation, provided the pulse width is much smaller than carrier recombination time and the
pulse period is much larger than the recombination time [38, 39]. Such a pumping scheme allows time for carrier recombination between each pulse to reduce the FCA effect. By utilizing this result, B. Jalali et al. demonstrated the first directly modulated silicon Raman laser pumped by a mode-locked pulsed laser. The pulsed silicon waveguide Raman laser has a 9W peak pulsed pumping threshold and a slope efficiency of 12.5% [40]. Based on the same principle, T.K. Liang et al. also demonstrated an efficient pulse-pumped silicon waveguide Raman amplifier around the same time independently [41].

Shifting the operation wavelength to the mid-infrared (2-6µm) range could provide another attractive solution to the problematic FCA loss, also proposed by Prof. B. Jalali et al. from University of California, Los Angeles (UCLA) [42]. At these longer wavelengths, the combined energy of two photons is much less than the band-gap of silicon. TPA process in these wavelength ranges is thus negligible, resulting in low free-carrier density to suppress the FCA loss [43]. Although no lasers has been demonstrated with this mechanism, the amplifier reported in 2007 (as shown in Fig 2.3) displays a superior gain of 12dB in the 3.39 µm emission with a 2.8cm-long silicon waveguide [44]. The renowned research group also went on to propose two more techniques to limit the free-carrier absorption, through the utilization of build-in electrical field of p-i-n junction [45] or scaling of the waveguide dimensions [46].
Ion implantation is a well-known process in the semiconductor industry to reduce carrier lifetime in silicon, using helium [47], argon [48] or oxygen [49]. In 2007, a group in Chinese University of Hong Kong reported the first net fiber-waveguide-fiber gain from SRS in silicon platform by employing a helium ion implanted silicon waveguide [50]. This set up enabled net optical gain from a CW-pump without the need of an externally applied voltage. In addition, M. Krause et al. proposed a cladding pump scheme to mitigate the FCA loss [51, 52]. As shown in Fig. 2.4 below, this scheme launches pump and Stokes power into the intermediate cladding and silicon core respectively, analogous to the double-cladding structure commonly utilized in erbium-doped fiber amplifiers [53]. This set-up resulted in the isolation of Stokes wave from the strong free-carriers generated by the strong pump power, reducing its attenuation. 34dB Raman gain could be achieved from this cladding pump structure, although the waveguide length required is 18cm long.

Fig. 2.4 Schematic diagram of the cladding pumped silicon Raman amplifier [52]

Slot waveguide, which was proposed by V. Almeida et al. in 2004, could provide another attractive platform for Raman-based applications [54]. By exploring the strong
optical field in the refractive index discontinuity point, we could obtain tight optical confinement in the low refractive index slot region [55]. In addition, the slot structure provides an additional degree of freedom for dispersion engineering, which is beneficial for nonlinear applications such as parametric and Raman wavelength conversion [56, 57]. However, the relatively large propagation loss of 6~7 dB/cm might prohibit any meaningful application [58].

The discussion for FCA suppression would be incomplete without including chalcogenide glasses, which has been famous of the negligible FCA loss in it [59]. Chalcogenide glasses are based on the chalcogen elements S, Se and Te, covalently bonding to the networking-forming elements like As and Ge, resulting in the amorphous semiconductor-like properties [60, 61]. These glasses are transparent far into the infrared with large refractive index for tight optical confinement. They also exhibit third order nonlinearities (Kerr, Raman and Brillouin) that is around three-orders-of-magnitude higher than silica fibers, which has led to a number of exciting nonlinear optics demonstrations [62-67]. Most importantly, chalcogenide glasses possess low TPA and negligible FCA loss, benefited from their low carrier mobility [10]. Given the low-loss integrated waveguides realized recently by several groups using CMOS technology, chalcogenide glasses provide an ideal material alternative to silicon for nonlinear integrated optics [68-71].

Raman-based research in chalcogenide glasses has been largely limited in the fiber form, leading to characterization [72] and various demonstration of Raman amplifier [73-75] and laser [76, 77]. R. Stegeman et al. investigated the Raman gain characteristics of various chalcogenide glasses and concluded that the Raman gain is much larger in Se and As than S and Ge [72]. This result is echoed by two independently groups, in which Raman gain coefficient of As$_2$Se$_3$ is measured at
2.3cm/GW [74] and 5.1cm/GW [66] using pulsed and CW pumping respectively. This is more than two orders of magnitude higher than the Raman gain reported in silica fiber. Follow on these encouraging characterizations, A. Tuniz et al. obtained a 22dB Raman gain in As$_2$Se$_3$ glass fiber using a pulsed pump [75]. In addition, chalcogenide glass Raman fiber laser has been realized with a slope efficiency of 66% [76].

To summarize, experimental works for the past decades have successful realized the potential advantages of silicon waveguide Raman laser proposed in the earlier research works. Operating wavelength ranges are wide, ranging from 0.6µm to the 3.39µm mid-infrared range. Pulsed and CW silicon Raman laser have been realized with simple fabrication process, monolithically integrated with the existing CMOS technology. However, the problems remained are also obvious and vital. FCA loss is still severe, resulting in low device energy efficiency. Existing loss suppression mechanisms proposed are either incurring excess energy consumption [35, 45], incompatible with other devices operating in telecom wavelengths [44] or involving complex fabrication processes and increased propagation loss [46, 50, 52]. Innovations are highly anticipated to further suppress the detrimental FCA loss for device energy efficiency enhancement. Furthermore, slot waveguides and chalcogenide glasses have not been explored for waveguide Raman amplifier and laser, despite their promising characteristics for Raman-based applications.
2.2 Coherent Anti-Stokes Raman Scattering and Integrated Wavelength Converter

The presence of strong pump and Stokes field could trigger another non-linear process named coherent anti-Stokes Raman scattering (CARS). CARS is a Raman assisted four-wave mixing (FWM) process in which difference of pump and Stokes frequency modulates the susceptibility of material, resulting in the generation of anti-Stokes wave. Early research of the CARS process starts in the 1990s in fiber, aiming to provide a theoretical models for the interaction between SRS and the parametric FWM process [78, 79]. Experimental demonstration of the CARS process is then followed in the 21st Century, providing direct evidence for the interplay between Raman and parametric processes [80, 81]. Two pump photons are annihilated in the CARS process to generate one Stokes and one anti-Stokes photon. As a typical non-linear parametric process involving multiple wavelengths, the efficiency of CARS process is strongly influenced by the phase-matching conditions \((\Delta k = 0)\) between the different parties as expressed in Eq. 2.3. The subscripts ‘p’, ‘s’, ‘a’ stand for pump, Stokes and anti-Stokes wavelengths respectively. \(k\) and \(\Delta k\) represents phase and phase difference.

\[
\Delta k = 2k_p - k_s - k_a
\]  

(2.3)

While conventional theory believes there is no energy exchange between the electromagnetic waves in the CARS process, N. Vermeulen et al. proposed a different school of thought in the opposite way [82]. Fig. 2.5 below clearly illustrates the difference between these two perspectives. According to them, a Stokes photon and a pump photon are converted to an anti-Stokes photon and a pump photon, while annihilating two phonons in the medium. The same phase matching conditions in Eq. 2.3 is still satisfied, because the phonons involved are coherent. They then extended
Chapter 2: Background Literature Review

this concept further to address the heat mitigation capability [83-85] and its possible application for the optical cooling of Raman laser [86]. However, no experimental proof of temperature change in CARS process has been reported so far to substantiate their claim.

![Schematic diagram of the phase matching process of CARS process](image)

**Fig. 2.5** Schematic diagram of the phase matching process of CARS process (a) involving no phonon generation; (b) involving two phonons generation [82]

The presence of CARS in silicon waveguide had been confirmed in the observation of anti-Stokes emission by two independent group in silicon Raman laser experiments [87, 88]. Pumping at 1540nm, simultaneous Stokes and anti-Stokes emission are observed at 1675nm and 1427nm, exactly 15.6 THz shifted from the pump wavelength. Stokes and anti-Stokes spectrum in the experiments are presented in Fig. 2.6. Due to the non-phase matched condition of the waveguides in these experiments, the anti-Stokes emission was extremely low.
The dominant application of CARS process in integrated waveguide devices lies in the area of integrated wavelength converter. Prof. B. Jalali et al. observed the first Raman-assisted wavelength conversion in silicon waveguide in 2003, with an impressive detuning range over 200nm [89]. The conversion efficiency however was limited to $10^{-5}$, mainly due to the non-phase matched condition in the waveguides [90]. This drawback was subsequently addressed in the following year. They showed that large material dispersion of silicon in the 1550nm wavelength could be compensated with the waveguide birefringence, through careful design of the waveguide aspect ratios [91]. The phase matched waveguide would improve the conversion efficiency to 2%, with a potential to reach 65% by further reduction of waveguide cross section [92].
Low conversion and narrow bandwidth remains as the most severe drawback for silicon waveguide Raman wavelength converter. Prof. B. Jalali *et al.* achieved tremendous improvement in both areas in their recent work in 2010, taking advantage of the pump pulse broadening through self phase modulated (SPM). As shown in Fig. 2.7, the conversion bandwidth was improved to more than three times of the intrinsic Raman line-width of silicon [7]. Conversion efficiency of 1% was also observed, representing two orders of magnitude enhancement from previous results.

For the completeness of this background review, we also present the progress of other wavelength conversion mechanism in integrated waveguides here. They could serve as the comparison benchmark for the waveguide Raman wavelength converter. Motivated by the advantages of compact size and potential low cost, integrated wavelength converter has been viewed as the ultimate solution for the wavelength flexibility to meet the ever-increasing bandwidth demand [93]. One possible approach is through cross phase modulation (XPM), which has been demonstrated in both chalcogenide and silicon waveguide. Successful XPM induced wavelength conversion.
has been achieved in chalcogenide waveguide, with a 10nm signal detuning range [62]. Based on similar principle, J. B. Driscoll et al. also realized 10 Gb/s wavelength conversion in silicon waveguides [94].

The detuning range can be substantially improved using FWM. The research team led by Prof. B. J. Eggleton showed that zero-dispersion can be achieved in the planar chalcogenide waveguide through engineering of the waveguide dimensions [95]. 45nm detuning range is subsequently achieved in these dispersion-engineered chalcogenide waveguides [96]. The group also demonstrated the highest reported conversion speed of 160 Gb/s in the chalcogenide waveguide wavelength converter.

The first wavelength conversion based on FWM in silicon waveguide was demonstrated by R. Espinola et al. in 2005 [97]. Their work is quickly followed by the groups from Intel [98] and NTT [99], hugely improving the conversion efficiency. In particular, W. Mathlouthi et al. from Intel achieved an impressive conversion efficient of -5.5dB [100]. Another famous group actively engaged in silicon wavelength conversion is led by Prof. M. Lipson from Cornell University. Through precise control of the waveguide dispersion, they achieved a broad conversion bandwidth of 150nm with peak conversion efficiency of -9.6dB [101]. The group subsequently built on this impressive work to demonstrate a conversion speed of 40 Gb/s [102], as well as ultra-low power conversion using micro-ring structure [103].

In conclusion, we have presented a detailed review of CARS process, from theoretical background to applications in integrated wavelength converter. While enjoying the large detuning span advantage, wavelength conversion efficiency from CARS process in integrated waveguide is around 15dB lower than that from FWM. Significant improvement of the energy efficiency is urgently needed. In addition, the narrow
bandwidth of hundreds of Giga hertz in silicon waveguide Raman wavelength converters is far from satisfactory for their applications in the future optical network, which is expecting to enter the Tera-bits region.

2.3 Numerical Modeling for Integrated Raman Devices

Besides experimental works mentioned above, extensive numerical researches have been conducted for Raman based devices in silicon waveguide. The first numerical modeling for silicon waveguide Raman laser (SWRL) is formulated by the group headed by Prof. E. Brinkmeyer in Germany, which in term analytically predicted the feasibility of such laser in 2004, one year before its actual demonstration in 2005 [104]. They utilized a simple space model to investigate the steady-state performance of SWRL, which is also employed by the Intel group [105]. The model only addresses the dominant SRS process and amplitude related terms, limiting its validity to only the continuous wave (CW) Raman lasers.

The theoretical modeling for pulsed silicon waveguide Raman amplifier and laser was derived by X. Chen et al. in 2006 [106]. The presented analysis incorporates both the time domain description and nonlinear electronic processes into the steady state modeling, providing a precise tool to stimulate the evolution of pump and Stokes pulses along silicon waveguide. A similar space-time model was also proposed shortly after by V. Passaro et al., monitoring pulse evolution in both space and time domain [107]. In addition, S. Roy et al. provided a simplified version in 2009 [108]. Their modeling was able to describe the propagation of optical pulse and the chirp effect in silicon waveguide, without the need for the time domain derivation.
R. Claps et al. started the first analysis for CARS process in silicon waveguide in 2003 [89], with a modeling derived from coupled-mode theory [109]. They successfully illustrated the sinusoidal variation of Stokes to anti-Stokes conversion efficiency and its strong dependent on the phase-matching condition. Suppression of SRS gain in the perfect phase matching by CARS process, which is a well-known fact in fiber Raman amplifier, was also confirmed. However, nonlinear losses such as TPA and FCA are excluded in the model, hugely limiting its accuracy due to the severity of these losses in silicon waveguide.

While previous modeling involved either SRS or CARS process, N. Vermeulen et al. derived the Stokes-anti-Stokes iterative resonator method (IRM) to address both in silicon waveguide [110]. The IRM model is capable to study both the transient and steady-state characteristics and covers the less dominant coherent anti-Stokes Raman scattering (CARS) process. Typical computation time for the IRM is much longer than the other methods, mainly due to the iterations to obtain transient features. Furthermore, the model failed to address the interplay between nonlinear electronic and Raman processes, resulting in an inaccurate account for the evolution of anti-Stokes wave. Q. Lin et al. did analyze both electronic and Raman nonlinear processes in silicon waveguide [111]. Their investigation however aimed to provide an overall theoretical background for nonlinear optics in silicon waveguide, and thus could not be formulated into a single modeling to perform numerical simulations. A summary of all the numerical methods mentioned so far is presented in Table 2.1 below.

<table>
<thead>
<tr>
<th>Origin</th>
<th>Advantages</th>
<th>Disadvantages</th>
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| Prof. E. Brinkmeyer et al. and Intel Group in ref. [104] and [105] | • First modeling for silicon Raman laser  
• Simple space model with fast computation | • Include dominant SRS only  
• Only for CW operation |
In addition to the energy efficiency analysis, pump-to-Stokes relative intensity noise (RIN) transfer presents another attractive direction for numerical analysis of the waveguide Raman amplifier and laser. RIN transfer for Raman amplifier was first investigated in the fiber form both theoretically and experimentally in 2001 by C. Fludger et al. [112]. They modeled RIN noise as a small perturbation from the noise-free intensity. Counter-pumped Raman amplifier was found to outperform co-pumped one with a much smaller corner frequency. Further characterization of RIN transfer in fiber Raman amplifier is also performed in the later years, in terms of polarization [113] or noise origin [114].

Following the similar mechanism, D. Dimitropoulos et al. formulated the numerical modeling for noise figures in silicon waveguide Raman amplifier [115]. Based on the proposed model, the group identified the insufficient “walk-off” effect between pump and Stokes waves in silicon Raman amplifiers due to their short device length. The short device length subsequently reduced the fluctuation and increased the corner
frequency [116]. I. Rukhlenko et al. recently improved this model with the elimination of instantaneous free carrier recombination assumption. Significant reduction of RIN transfer could be induced by the free carrier, provided that the noise frequency was lower than the inverse of effective carrier recombination time [117].

M. Krause et al. presented the only investigation of fiber Raman laser, through both experimental and numerical investigation [118]. Sinusoidal variation of the RIN transfer was predicted across the frequency spectrum, in good agreement with the experimental characterizations shown in Fig. 2.8. Minimum RIN transfer was observed in the integral multiples of inverse round trip time, in which off-resonance modulation of the Stokes wave occurred. In addition, they optimized the noise performance of fiber Raman laser when it is utilized as pump source for fiber Raman amplifier. X. Liu et al. extended this analysis to silicon waveguide Raman laser recently [119]. However, their work employed the inaccurate instantaneous free carrier recombination assumption and failed to point out any significant difference between the RIN transfer of Raman laser in fiber and silicon waveguide.
To summarize, a number of analytical models have been proposed to investigate the energy efficiency and RIN transfer of the Raman-based devices in silicon waveguide. Unlike the extensively investigated Raman amplifier and laser, investigation of the Raman wavelength converter has been hugely limited by the lack of accurate numerical models in waveguide devices. Most importantly, none of the existing models can be applied universally to all waveguide Raman devices, which are essentially influenced by identical physical processes in the waveguide. In addition, comprehensive characterization of RIN transfer of waveguide Raman laser is highly desirable to control its noise performance in the actual applications.
Chapter 3 NUMERICAL MODELING FOR RAMAN EFFECT IN INTEGRATED WAVEGUIDE DEVICES

3.1 Introduction

For integrated waveguide devices based on nonlinear processes, a numerical model refers to the solution of differential equation sets using numerical methods. Intensive global researches in the silicon photonics technology over the past decade have led to the proposal of a wide range of numerical models for silicon waveguide Raman amplifier (SWRA) [108], silicon waveguide Raman laser (SWRL) [104] and silicon waveguide Raman wavelength converter (SWRWC) [89]; both in pulsed [106] or continuous-wave (CW) operation [105]. While each of these models has their advantages in some specific areas, they generally target individual device and could not be applied universally to all Raman-based silicon waveguide devices. In principle, evolution of electromagnetic waves along the silicon waveguide are influenced by all the Raman processes such as stimulated Raman scattering (SRS), stimulated anti-Stokes Raman scattering (SARS) and coherent anti-Stokes Raman scattering (CARS) [111]. The different dominant processes then result in the different functionalities of the devices. A complete numerical model addresses all these Raman processes will provide a unified tool to analyze all these Raman-based waveguide devices. In particular, the unified model is capable to investigate the influence of the less dominant Raman processes in each device, which could not be realized using existing numerical models that only address the dominant Raman process. This chapter introduces such unified mathematical model, amplitude propagation method (APM), proposed by us to describe SWRA, SWRL and SWRWC throughout this report.
In addition, chalcogenide glasses have been identified as an alternative material platform for integrated nonlinear optics devices [120]. Unlike the extensive study of Raman-based devices in silicon waveguides [5, 104, 110, 121], investigation of the chalcogenide Raman amplifier and laser are mainly limited in the fiber form [76, 77]. These fiber models could not be directly applied to the chalcogenide waveguide devices, which contain nonlinear TPA loss that is not present in the fiber form. Given the recent rapid maturing of chalcogenide waveguide devices [70, 96], the need for a numerical model to simulate the performance of these chalcogenide waveguide-based devices becomes increasingly urgent. In this chapter, we also extend the application of APM to the Raman-based chalcogenide waveguide devices, which has not been numerically studied before to the best of our knowledge.

This chapter starts with an introduction of Raman scattering efficiency and silicon waveguide structure that is used in this thesis. This is followed by a detailed derivation of our APM modeling involving the discussion of linear and non-linear losses; various Raman processes; as well as electronic nonlinear processes such as self phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM) in silicon waveguides. The different applications of APM model to silicon and chalcogenide waveguide based Raman devices will be shown, followed by verification using existing experimental and numerical results.
3.2 Rib Waveguide Structure and Raman Scattering Efficiency

![Coordinate system and structure of silicon rib waveguides](image)

Fig. 3.1 Coordinate system and structure of silicon rib waveguides

Fig. 3.1 displays the schematic diagram of the waveguide structure used for analysis in this thesis. For compact integrated waveguide devices, single-mode operation is highly desirable to reduce various losses resulting from waveguide bends and polarization conversion [122]. Rib waveguide is chosen instead of the popular nano-wire waveguide utilized in most nonlinear silicon photonics components, mainly due to its relatively large dimensions for single-mode operation [123]. In the rib waveguide structure, higher-order modes are leaky, radiating power out through the adjacent slab waveguides. This enables a larger degree of freedom for waveguide dimensions variation, which is required for dispersion engineering of the waveguide to achieve perfect phase matching for CARS process [91]. Compared to nano-wire, the rib waveguide also has a larger mode size, facilitating easier coupling to fiber devices. In addition, rib waveguide supports the inclusion of doping region in the side slab region, which is essential for the application of reversed p-i-n to suppress the detrimental nonlinear free-carrier absorption loss (FCA) [5]. For silicon waveguide devices, the
two adjacent side ridge waveguides are fabricated long the [011] direction on the (100) surface of a silicon wafer. The fabrication direction is chosen such that the \( x \) coordinate of our waveguide coordinate system coincides with the crystallographic \( x \)-axis of silicon and the TM modes are oriented along this direction. Typical values of the rib width \( (W) \), rib height \( (H) \) and slab height \( (h) \) are 1.5 \( \mu \)m, 1.5 \( \mu \)m, and 0.8 \( \mu \)m respectively in the experiments for single-mode operation [121]. The rib waveguide cross section coordinates are labeled according to Fig. 3.1 and assumed to be invariant along the \( z \)-axis. The length of the rib waveguide is denoted as \( L \).

Spontaneous and Stimulated Raman scattering efficiency in silicon hugely depends on the polarization of pump and Stokes waves relative to the crystallographic axes. Efficiency of forward and backward Raman scattering are equal due to the degeneracy of the phonons modes at the \( \Gamma \) point. Spontaneous Raman scattering efficiency \( (S) \) can be computed from the following equation [16]:

\[
S = S_0 \sum_{i=1}^{3} [e_p R_i e_s]
\]

where \( e_p \) and \( e_s \) are unit vectors for polarization of pump and Stokes light, respectively. \( S_0 \) is the polarization and crystal-orientation independent Raman scattering efficiency and \( R_i \) \((i=1, 2, 3)\) are the tensors denoting the three degenerate lattice vibrations contributed to Raman scattering. They are conventionally expressed as in Eq. 3.2 in the Cartesian coordinate system, aligning with the crystallographic axes [16]:

\[
R_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]
Simple substitution of these $R_i$ values into Eq. 3.1 reveals that the efficiency of Raman scattering is influenced by the choice of polarization of pump and Stokes light ($e_p$ and $e_s$). Efficiency is maximized at $S_0$ for TE polarized pump-TE polarized Stokes, TE polarized pump-TM polarized Stokes and TM polarized pump-TE polarized Stokes configuration. On the other hand, the efficiency for TM polarized pump-TM polarized Stokes is zero. To facilitate our investigation in this thesis, the waveguide in our numerical analysis is assumed to operate in single-mode condition for pump, Stokes and anti-Stokes wavelengths with the above mentioned dimensions. The pump is in $TE_0$ mode while Stokes and anti-Stokes are in $TM_0$ mode to enable maximum scattering efficiency, unless otherwise stated.

### 3.3 Amplitude Propagation Method (APM) Derivation

This research work aims to provide the comprehensive analysis of a broad range of Raman based integrated optical waveguide devices. APM is thus chosen due to its simplicity. However the existing APM model only takes into account the dominant SRS process and is insufficient to simulate the CARS-based devices. In this section, we will derive a novel APM model that can be universally applied to all Raman based devices. For SRS-based waveguide devices, the starting point is always the famous
Raman rate equation derived from the nonlinear Schrödinger equation. The initial growth of Stokes wave in a Raman scattering process can be described as below under the CW or quasi-CW approximation[26]:

\[
\frac{dI_s}{dz} = g_r I_p I_s
\]

(3.3)

$I_p$ and $I_s$ denote the pump and Stokes intensity respectively. $g_r$ is the Raman gain coefficient. It is sometimes expressed as $g_r(\Omega)$ to represent the variation of whole Raman gain spectrum with frequency difference ($\Omega$) between pump and Stokes waves. From more fundamental point of view, $g_r$ is related to the imaginary part of the third order non-linear susceptibility $\chi^{(3)}$, associated with a response time of phonon dephasing time. The $g_r(\Omega)$ spectrum thus has a strong spectral dependence with a peak at $\Omega_0$ as shown in Fig. 3.2 below. $\Omega_0$ is the frequency of oscillation of zone-center optical phonons in silicon with a value of 15.6 THz [33].

![Fig. 3.2 Spontaneous Raman scattering spectrum in terms of $\Omega$ [33]](image)

Eq. 3.3 only takes into account the Stokes wave, which should be supplemented by a counter-description of the depletion of pump waves through SRS process. If we
assume a conservation of photons in the simple context of pump/Stokes photons only, we can expand Eq. 3.3 into the following coupled differential equations:

\[
\frac{dI_p}{dz} = -\frac{\lambda_s}{\lambda_p} g_r I_p I_s \tag{3.4a}
\]

\[
\frac{dI_s}{dz} = g_r I_p I_s \tag{3.4b}
\]

In the context of fiber, the differential equation will be completed with the inclusion of linear fiber loss [26]. The case in silicon waveguide is slightly more complicated, because non-linear losses such as TPA and FCA highlighted in Chapter 2 contribute significantly to the degradation of pump and Stokes powers. A complete description of the evolution of electromagnetic waves along silicon waveguides thus requires the inclusion of all these linear and nonlinear losses, as illustrated by Eq. 3.5:

\[
\frac{dI}{dz} = -\alpha I - \beta I^2 - \sigma N_{\text{eff}} I \tag{3.5a}
\]

\[
N_{\text{eff}} = \frac{2 \beta \lambda \tau_{\text{eff}}}{2 hc} I^2 \tag{3.5b}
\]

Here, \(\alpha\) is the linear propagation loss, \(\beta\) is the TPA coefficient, \(\sigma\) is the FCA cross section and \(\tau_{\text{eff}}\) is the effective carrier lifetime. \(N_{\text{eff}}\) is the effective photo-generated carrier density in the silicon waveguide; \(h\) and \(c\) follow their usual physical meaning of Plank’s constant and free-space speed of light, respectively [33]. Combining Eq. 3.4 and 3.5 completely describe the propagation of optical pulse under the stimulated Raman scattering (SRS) process in a silicon waveguide. In the case of silicon Raman lasers, these equations need to be resolved into forward- and backward-propagating amplitude or power terms in order to account for the cavity enhancement effect. This
can be done by defining an effective modal area \( (A_{\text{eff}}) \) of the waveguide as the following:

\[
A_{\text{eff}} = \left( \iint |\varphi(x, y)|^2 \, dx \, dy \right)^{1/2} \left( \iint |\varphi(x, y)|^4 \, dx \, dy \right)^{-1/2}
\]  
(3.6)

In the above equation, \( \varphi(x, y) \) is the electric field profile of the optical mode, which can be obtained from commercial mode-solving software such as Rsoft FemSIM or Lumerical Mode Solution. By assuming a constant effective core area \( (A_{\text{eff}}) \) and TPA coefficient \( (\beta) \) for pump and Stokes wavelength, we can combine Eq. 3.4 and 3.5 to obtain the following propagation model as:

\[
\pm \frac{dE_{f,b}^f}{dz} = \left[ -\frac{g_r}{2A_{\text{eff}}} \frac{\lambda_p}{\lambda_p} \left( |E_s^f|^2 + |E_b^b|^2 \right) - \frac{\alpha_p}{2} \right]
- \frac{\beta}{2A_{\text{eff}}} \left( |E_p^f|^2 + 2|E_s^f|^2 + 2|E_b^b|^2 + 2|E_{f,b}^f|^2 \right) - \frac{\sigma_p}{2} N_{\text{eff}} |E_p^f|
\]  
(3.7a)

\[
\pm \frac{dE_{s,b}^f}{dz} = \left[ -\frac{g_r}{2A_{\text{eff}}} \left( |E_p^f|^2 + |E_b^b|^2 \right) - \frac{\alpha_s}{2} \right]
- \frac{\beta}{2A_{\text{eff}}} \left( |E_s^b|^2 + 2|E_p^f|^2 + 2|E_b^b|^2 + 2|E_{s,b}^f|^2 \right) - \frac{\sigma_s}{2} N_{\text{eff}} |E_s^b|
\]  
(3.7b)

\[
N_{\text{eff}} = \frac{\beta \tau_{\text{eff}}}{2 \hbar f_p A_{\text{eff}}} \left[ |E_p^f|^4 + |E_b^b|^4 + |E_s^f|^4 + |E_b^b|^4 + 4(|E_p^f|^2 |E_b^b|^2 + |E_s^f|^2 |E_s^s|^2) \right]
+ 4(|E_p^f|^2 |E_s^s|^2 + |E_p^f|^2 |E_b^b|^2 + |E_b^b|^2 |E_s^s|^2 + |E_s^s|^2 |E_s^s|^2)
\]  
(3.7c)

In these equations, \( E \) denotes the normalized amplitude of the slow-varying electric field under steady state continuous-wave (CW) pumping condition. In the conventional definition of electromagnetic wave intensity by Poynting theorem, \( \hat{I} = |E|^2/2Z \) (\( Z \) is the impedance of the dielectric medium). To simplify the calculation
of power, A. Liu et al. redefined the electric field amplitude $E$ in their modeling for silicon Raman laser, such that it took into account the coefficient of $1/2Z$ [105]. This definition subsequently results in a direct calculation of optical power from electric field amplitude as $P = |E|^2$. Similar approach is adopted in our research work for the definition of electric field amplitude propagating along the waveguide. The subscripts ‘$p$’ and ‘$s$’ are used to represent pump and Stokes components, respectively. Superscripts ‘$f$’ and ‘$b$’ stand for the forward- and backward-propagating components.

The terms involving $\beta$ account for TPA losses, addressing both the degenerate TPA (two photons of the same frequency) and non-degenerate TPA loss (two photons of different frequency). Eq. 3.7 represents the widely utilized models today to describe silicon Raman laser [104, 105]. The only discrepancy in these models is the coefficient for power terms in the formula to compute effective free-carrier density ($N_{eff}$) as in Eq. 3.7c. The coefficient of 4 used by us is similar to that in [104], while the coefficient used in [105] is 2. A detailed analysis of this discrepancy has been presented in Appendix A to justify our choice of coefficients.

The numerical modeling presented in Eq. 3.7 is sufficient to describe the evolution of pump and Stokes wave in the cavity of silicon Raman laser; under the assumption that only stimulated Raman scattering occurs in the cavity. However, anti-Stokes wave are also observed in the silicon Raman laser as mentioned in Chapter 2 [87, 88]. The existing models are therefore insufficient to analyze anti-Stokes wave in silicon Raman laser, and is inaccurate in the case when efficient anti-Stokes emission occurs.

In this research work, we improve the above mentioned models by incorporating the description of other Raman processes (SARS, CARS) as well as the non-linear interaction between these three waves. One should note that this is not a simple modification of the existing model, but a re-derivation of it from the standard
Schrödinger non-linear equation. While the existing models for waveguide Raman laser focus only on the intensity-related processes at pump and Stokes wavelength, our proposed APM presents a more comprehensive description involving both intensity and phase related processes at pump, Stokes and anti-Stokes wavelengths. This is achieved with the additional incorporation of phase sensitive nonlinear processes such as self-phase modulation (SPM), cross-phase modulation (XPM), four-wave mixing (FWM), stimulated anti-Stokes Raman scattering (SARS) and CARS. By doing so, we obtain a more accurate analysis of the propagation of the electromagnetic waves. In non-linear optics, the evolution of Stokes and anti-Stokes emission in Raman scattering process can be modeled as the following, by matching the coefficients to the current model [12]:

\[
\frac{dE_s}{dz} = \frac{g_r}{2A_{eff}} |E_p|^2 E_s + \frac{g_r}{2A_{eff}} \left[ E_p \right]^2 E_s^* e^{i\Delta k} \quad (3.8a)
\]

\[
\frac{dE_a}{dz} = -\frac{g_r}{2A_{eff}} |E_p|^2 E_a - \frac{g_r}{2A_{eff}} \left[ E_p \right]^2 E_a^* e^{i\Delta k} \quad (3.8b)
\]

\[
\Delta k = 2k_p - k_s - k_a \quad (3.8c)
\]

Subscript ‘a’ denotes the anti-Stokes components, k is the wave vector or phase for respective wave and \( \Delta k \) is the phase mismatch factor. The terms involving \( \Delta k \) describe the CARS process. SARS process is accounted for using the first term in Eq. 3.8b, in which the anti-Stokes wave is converted back into pump waves in a similar way as SRS. Besides the CARS and SARS processes, the presence of three waves in the silicon Raman laser will also trigger other non-linear process such as self-phase modulation (SPM), cross phase modulation (XPM). Furthermore, CARS is a multi-photons Raman scattering process connected by FWM, FWM process involves pump,
Stokes and anti-Stokes photons is thus needed to incorporate in. Experiment in silicon waveguide has been conducted to confirm the occurrence of these nonlinear processes to influence the phase of the co-propagating optical pulses, as shown in Fig. 3.3 [124]. To describe the phase-related CARS process accurately, we need to take into account all these non-linear processes. Simple incorporation of CARS term alone into the existing model such as the IRM model [110] while assuming a constant phase along the waveguide will result in incomplete representation.

Fig. 3.3 Experimental results for the SPM, XPM and FWM processes in silicon waveguides [124]

Mutual influence of the Raman processes with the electronic nonlinear processes has been comprehensively investigated in fiber optics. These processes can be accounted for in the general equation as below [79]:

\[
\frac{dE_s}{dz} = \frac{i\gamma}{A_{\text{eff}}} \left| E_s \right|^2 + 2 \times \left( \left| E_p \right|^2 + \left| E_a \right|^2 \right) + \frac{2i\gamma}{A_{\text{eff}}} \left| E_p \right|^2 E_a^* e^{i\Delta k z} \quad (3.9a)
\]

\[
\frac{dE_a}{dz} = \frac{i\gamma}{A_{\text{eff}}} \left| E_a \right|^2 + 2 \times \left( \left| E_p \right|^2 + \left| E_s \right|^2 \right) + \frac{2i\gamma}{A_{\text{eff}}} \left| E_p \right|^2 E_s^* e^{i\Delta k z} \quad (3.9b)
\]
\[
\gamma = \frac{2\pi f_p n_2}{c}
\]  

(3.9c)

Here, \( \gamma \) is non-linear parameter computed from non-linear index coefficient \( n_2 \). We assume the same \( \gamma \) value according to pump’s value for all three waves. Incorporating Eq. 3.8 and 3.9 into Eq. 3.7, we can obtain the differential equation sets for our amplitude propagation method (APM) as following:

\[
\pm \frac{dE_{p}^{f,b}}{dz} = \left[ -\frac{\alpha_p}{2} - \frac{g_r}{2A_{eff}} \frac{\lambda_s}{\lambda_p} \left( |E_s^{f}|^2 + |E_s^{b}|^2 \right) + \frac{g_s}{2A_{eff}} \frac{\lambda_d}{\lambda_p} \left( |E_a^{f}|^2 + |E_a^{b}|^2 \right) \right] E_{p}^{f,b} - \frac{B}{2A_{eff}} \left( |E_{p}^{f,b}|^2 + 2|E_s^{f}|^2 + 2|E_s^{b}|^2 + 2|E_a^{f}|^2 + 2|E_a^{b}|^2 \right) - \frac{\sigma_p}{2} N_{eff}
\]  

(3.10a)

\[
- \frac{i\gamma}{A_{eff}} \left( |E_{p}^{f,b}|^2 + 2|E_{p}^{f,b}|^2 \right) |E_{p}^{f,b} + \frac{i\gamma}{A_{eff}} \times [E_s] \times [E_a] \times e^{i\Delta k_z}
\]

\[
\pm \frac{dE_{s}^{f,b}}{dz} = \left[ \frac{g_r}{2A_{eff}} \left( |E_p^{f}|^2 + |E_p^{b}|^2 \right) - \frac{\alpha_s}{2} - \frac{B}{2A_{eff}} \left( |E_s^{f}|^2 + 2|E_s^{f}|^2 + 2|E_s^{b}|^2 + 2|E_s^{f}|^2 + 2|E_s^{b}|^2 + 2|E_s^{f}|^2 \right) \right] E_{s}^{f,b} + \frac{\sigma_s}{2} N_{eff} \left( |E_p^{f}|^2 + |E_p^{b}|^2 \right) |E_{s}^{f,b} + \left( \frac{g_r}{2A_{eff}} - \frac{i\gamma}{A_{eff}} \right) \times [E_p^{f,b}] \times [E_p^{f,b}] \times e^{i\Delta k_z}
\]  

(3.10b)

\[
\pm \frac{dE_{a}^{f,b}}{dz} = \left[ \frac{g_r}{2A_{eff}} \left( |E_p^{f}|^2 + |E_p^{b}|^2 \right) - \frac{\alpha_s}{2} - \frac{B}{2A_{eff}} \left( |E_a^{f}|^2 + 2|E_a^{f}|^2 + 2|E_a^{b}|^2 + 2|E_a^{f}|^2 + 2|E_a^{b}|^2 \right) \right] E_{a}^{f,b} + \frac{\sigma_s}{2} N_{eff} \left( |E_p^{f}|^2 + |E_p^{b}|^2 \right) |E_{a}^{f,b} + \left( \frac{g_r}{2A_{eff}} - \frac{i\gamma}{A_{eff}} \right) \times [E_p^{f,b}] \times [E_p^{f,b}] \times e^{i\Delta k_z}
\]  

(3.10c)

\[
N_{eff} = \frac{B\tau_{eff}}{2h f_p A_{eff}^2} \left( |E_p^{f}|^4 + |E_p^{b}|^4 + |E_s^{f}|^4 + |E_s^{b}|^4 + |E_a^{f}|^4 + |E_a^{b}|^4 \right)
\]

\[
+ 4 \left( |E_p^{f}|^2 |E_p^{b}|^2 + |E_p^{f}|^2 |E_s^{b}|^2 + |E_p^{f}|^2 |E_a^{b}|^2 + |E_p^{f}|^2 |E_a^{b}|^2 \right) + 4 \left( |E_p^{b}|^2 |E_s^{f}|^2 + |E_p^{b}|^2 |E_a^{f}|^2 + |E_p^{b}|^2 |E_a^{b}|^2 + |E_s^{f}|^2 |E_a^{b}|^2 \right) + 4 \left( |E_s^{f}|^2 |E_a^{f}|^2 + |E_s^{f}|^2 |E_a^{b}|^2 + |E_s^{b}|^2 |E_a^{f}|^2 + |E_s^{b}|^2 |E_a^{b}|^2 \right)
\]  

(3.10d)
FCA absorption cross section ($\sigma$) for different waves can be computed using the formula as below [121]:

$$\sigma_i = 6 \times 10^{-10} \times \lambda_i^2 (m) \quad i = p, s, a$$

(3.11)

Only continuous-wave (CW) operation is considered and higher orders Stokes waves are neglected by reasonably assuming large phase mismatch and high cavity losses in these wavelengths. The APM theoretical model addresses both phase and amplitude evolution of complex pump, Stokes and anti-Stokes fields along silicon waveguide, with the incorporation of all phase-related nonlinear processes including SPM, XPM, FWM, CARS and SARS. We would like to remark that an analogous model has been presented for CARS process in silicon waveguides [89]. Our model differs from their model with the incorporation of linear, TPA and FCA losses; which give a more accurate description of the propagation of the three waves along the waveguide. Furthermore, we remove the assumption of strong pump power by adding in the terms for nonlinear contribution of Stokes and anti-Stokes waves. Such modification provides our model the unique capability to analyze the weak pump regime, which will be further elaborated in Chapter 5.
Fig. 3.4 Schematic diagram of the propagation of pump, Stokes and anti-Stokes waves inside a silicon waveguide cavity

Fig. 3.4 illustrates the schematic diagram of the evolution of these propagating waves inside a silicon resonant Fabry-Perot (FP) cavity, formed by two reflective mirrors at the end. The z-axis is drawn to have its positive direction pointing towards the right-hand side. \( R \) is the reflectivity of the mirror, the subscripts ‘\textit{front}’ and ‘\textit{back}’ denote the left- and right-hand side mirror, respectively. The boundary conditions are then given as below:

\[
E_p^f (0) = \sqrt{1 - R_{\text{front}, p}} E_{\text{inc}} + \sqrt{R_{\text{front}, p}} E_p^b (0) \tag{3.12a}
\]

\[
E_p^b (L) = \sqrt{R_{\text{back}, p}} E_p^f (L) \tag{3.12b}
\]

\[
E_s^f (0) = \sqrt{R_{\text{front}, s}} E_s^b (0) \tag{3.12c}
\]

\[
E_s^b (L) = \sqrt{R_{\text{back}, s}} E_s^f (L) \tag{3.12d}
\]

\[
E_a^f (0) = \sqrt{R_{\text{front}, a}} E_a^b (0) \tag{3.12e}
\]

\[
E_a^b (L) = \sqrt{R_{\text{back}, a}} E_a^f (L) \tag{3.12f}
\]

The phase difference between intra-cavity pump wave and the incident pump wave is difficult to quantify exactly in actual experiment because \( L \gg \lambda_p \), where \( L \) denotes the waveguide length. Thus we assume it to be zero for simplicity. Another novelty of our APM modeling is its loss analysis capability for different types of losses in silicon waveguides. This is achieved by explicitly defining an overall loss factor \((\Gamma)\) as in Eq. 3.13 to quantify the loss in silicon Raman lasers for analysis purpose. The outputs in
the absence of linear, TPA and FCA losses are computed by setting their respective control parameters ($\alpha$, $\beta$, $\tau_{\text{eff}}$) to zero.

$$\Gamma(dB) = 10 \times \log \left( \frac{\text{Stokes output in the absence of the loss}}{\text{Stokes output in the presence of the loss}} \right)$$  \hspace{1cm} (3.13)

The equation sets presented in Eq. 3.10, supplemented by the boundary conditions in Eq. 3.12, completely describes the propagation of pump, Stokes and anti-Stokes in silicon waveguides. Simulation results based on the APM can thus be applied universally to model the steady-state performance of SWRA, SWRL and SWRWC; which involve the interaction of these three waves. In addition, the validity of this APM model can be extended to other nonlinear materials such as chalcogenide glasses. This can be done by removing the FCA-related terms in Eq. 3.10, which is negligible in chalcogenide due to its low carrier mobility [10]. The evolution of pump, Stokes and anti-Stokes waves, subjecting to linear and TPA losses can then be described as Equation 3.14.

\[
\frac{dE_p}{dz} = \left[ - \frac{g_r}{2A_{\text{eff}}} \frac{\lambda_s}{\lambda_p} |E_s|^2 + \frac{g_r}{2A_{\text{eff}}} \frac{\lambda_a}{\lambda_p} (|E_a|^2) - \frac{\alpha_p}{2} \right. \\
\left. - \left( \frac{\beta}{2A_{\text{eff}}} + i\gamma \right) \times \left( |E_p|^2 + 2 |E_s|^2 + 2 |E_a|^2 \right) \right] E_p \\
- \frac{i\gamma}{A_{\text{eff}}} \times [E_p]^* \times [E_s] \times [E_a] \times e^{i\Delta k z}
\]

Equation 3.14a

\[
\frac{dE_s}{dz} = \left[ \frac{g_r}{2A_{\text{eff}}} |E_p|^2 - \frac{\alpha_s}{2} - \frac{\beta}{2A_{\text{eff}}} (|E_s|^2 + 2 |E_p|^2 + 2 |E_a|^2) \right] E_s \\
- i\gamma (|E_s|^2 + 2 |E_p|^2 + 2 |E_a|^2) E_s + \left( \frac{g_r}{2A_{\text{eff}}} - i\gamma \right) \times [E_p]^* \times [E_a] \times e^{-i\Delta k z}
\]

Equation 3.14b
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\[
\frac{dE_a}{dz} = [-\frac{g_r}{2A_{eff}} |E_a|^2 - \frac{\alpha_n}{2} - \frac{\beta}{2A_{eff}} (|E_a|^2 + 2|E_p|^2 + 2|E_s|^2)]E_a
\]
\[-i\gamma (|E_a|^2 + 2|E_p|^2 + 2|E_s|^2)E_a + (-\frac{g_r}{2A_{eff}} - i\gamma) \times \left[ E_p \times [E_s] \right] \times e^{-i\beta z} \tag{3.14c}\]

3.4 Numerical Solution and Verification of APM

In principle, solution of APM in the form of Eq. 3.10 or 3.14 yields precise description of Raman-based devices performance in silicon or chalcogenide waveguides, respectively. Due to the complexity of these equations, numerical methods need to be applied to solve them instead of obtaining the analytical solution directly. Two major categories of numerical methods are involved in the solution procedures, namely the methods to solve the differential equations set in Eq. 3.10 (Eq. 3.14) and the optimization techniques to match the boundary conditions in Eq. 3.12.

The most common numerical method for solving the nonlinear Schrödinger equations are the split-step Fourier method [125] and the finite-difference method [26]. When it comes to solving the steady-state time-independent Schrödinger equations in our APM, the most popular approach is the Runge-Kutta-Fehlberg (RKF) method [121]. The RKF approach belongs to the family of finite-difference numerical method to solve ordinary differential equations (ODE), modified from the initial Runge-Kutta method. RKF uses both the fourth- and fifth-order-accuracy solutions to provide two estimates of the next-step value. The difference in these results is subsequently related to the accuracy of the solution, and can be used to decrease the integration step length if the error exceeds a specified accuracy. Applying RKF to the context of silicon waveguide Raman lasers will yield the computation of the following six values:
\[k_1 = hf[z_j, (E^y_{z_j})_j], \quad x = p, s, a; \quad y = f, b\]  
(3.15a)

\[k_2 = hf[z_j + \frac{h}{4}, (E^y_{z_j})_j + \frac{k_1}{4}]\]  
(3.15b)

\[k_3 = hf[z_j + \frac{3h}{8}, (E^y_{z_j})_j + \frac{3 \times k_1}{32} + \frac{9 \times k_2}{32}]\]  
(3.15c)

\[k_4 = hf[z_j + \frac{12h}{13}, (E^y_{z_j})_j + \frac{1932 \times k_1}{2197} - \frac{7200 \times k_2}{2197} + \frac{7296 \times k_3}{2197}]\]  
(3.15d)

\[k_5 = hf[z_j + h, (E^y_{z_j})_j + \frac{439 \times k_1}{216} - 8 \times k_2 + \frac{3680 \times k_3}{513} + \frac{845 \times k_4}{40}]\]  
(3.15e)

\[k_6 = hf[z_j + \frac{h}{2}, (E^y_{z_j})_j - \frac{27 \times k_1}{8} + 2 \times k_2 - \frac{3544 \times k_3}{2565} + \frac{1859 \times k_4}{4104} - \frac{11 \times k_5}{40}]\]  
(3.15f)

Here, \(h\) is the default step size and \(f\) represents the function on the right-hand side of Eq. 3.10 (Eq. 3.14). Based on these six values, we can compute the 4th-order and 5th-order approximate solutions of the \((j+1)^{th}\) step for the differential equations as:

\[(E^y_{z_j})^{4}_{j+1} = (E^y_{z_j})_j + \frac{25}{216} k_1 + \frac{1408}{2565} k_3 + \frac{2197}{4104} k_4 - \frac{1}{5} k_5\]  
(3.15g)

\[(E^y_{z_j})^{5}_{j+1} = (E^y_{z_j})_j + \frac{16}{135} k_1 + \frac{6656}{12825} k_3 + \frac{28561}{56439} k_4 - \frac{9}{50} k_5 + \frac{2}{55} k_6\]  
(3.15h)

Error checking can be performed in each step by computing the error \((Error)\) between 4th- and 5th-order approximate solutions and adjust the step size if it exceeds a default error \(\varepsilon\). The formula to compute the \(Error\) and the adjusted step size \((h')\) are given below in Eq. 3.16 and 3.17, respectively.

\[Error = \left| (E^y_{z_j})^{5}_{j+1} - (E^y_{z_j})^{4}_{j+1} \right|\]  
(3.16)
\[ h' = 0.840896 \times h \left( \frac{\partial h}{\text{Error}} \right)^{1/4} \]  

(3.17)

Simplicity and error checking capability are the two major incentives for us to employ RKF method in this research work. Only six values are needed to evaluate in each step of the RKF method. In addition, the 4\textsuperscript{th}-order accuracy obtained is superior to majority of other numerical methods in this domain.

The optimization technique for boundary condition matching in this research work is shooting methods [126]. It starts with an initial guess at one boundary of the cavity and propagates towards the other boundary. At the opposite boundary, Newton-Raphson (NR) Method is then applied to obtain the solution. For our cases, six propagating wave amplitudes are involved. We thus use the vector NR method with Jacobian computations. Another popular optimization technique for boundary condition matching is the iterative resonator method (IRM) [110]. The advantage of IRM is its capability of obtaining transient characteristics of silicon waveguide Raman lasers that is not available from shooting algorithm. However, the computation time of IRM is about 10 times longer. In this research work, intensive numerical simulations are needed to perform comprehensive steady-state analysis of the Raman-based waveguide devices. We therefore choose shooting algorithm because the computation load is expected to be extremely heavy.

In the following section, we will justify the validity of our APM with the reported experimental and numerical results. Due to the limited data available in the chalcogenide waveguide-based Raman devices; we will only verify our model on the silicon waveguide-based devices in this section. However, it should be noted that the accuracy of the APM can still be extended to the chalcogenide waveguide simulation,
which is derived in an identical procedure as the silicon waveguide without the incorporation of FCA loss. We start with simulations of the silicon waveguide Raman amplifier (SWRA) set-up due to its simplicity in boundary conditions. Fig. 3.5 displays the matching of our APM simulation results with experimental results obtained by Intel group for SWRA [29]. Parameter values used for the simulation are listed in Table 3.1 below, according to the experimental characterization. Since it is an amplifier set-up, all reflectivity is thus set to 0%. From Eq. 3.11, we obtain FCA cross section for pump and Stokes waves as $\sigma_p=1.27\times10^{-17}\text{cm}^2$ and $\sigma_s=1.49\times10^{-17}\text{cm}^2$ respectively. According to the experiment, the initial Stokes input is set at 2.6mW [29].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>On-off gain simulation values</th>
<th>Input-Output simulation values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>0.22dB/cm</td>
<td>1dB/cm</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>0.22dB/cm</td>
<td>1dB/cm</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Not Applicable</td>
<td>1dB/cm</td>
</tr>
<tr>
<td>$A_{eff}$</td>
<td>1.57µm$^2$</td>
<td>5µm$^2$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5cm/GW</td>
<td>0.7cm/GW</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>1455nm</td>
<td>1427nm</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>1474.3nm</td>
<td>1542nm</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Not Applicable</td>
<td>1329nm</td>
</tr>
<tr>
<td>$g_s$</td>
<td>18cm/GW</td>
<td>20 cm/GW</td>
</tr>
<tr>
<td>$L$</td>
<td>4.8cm</td>
<td>55 mm</td>
</tr>
<tr>
<td>$\tau_{eff}$</td>
<td>23 ns</td>
<td>2.5ns</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>$1.27\times10^{-17}\text{cm}^2$</td>
<td>$1.222\times10^{-21}\text{m}^2$</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>$1.49\times10^{-17}\text{cm}^2$</td>
<td>$1.427\times10^{-21}\text{m}^2$</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Not Applicable</td>
<td>$1.060\times10^{-21}\text{m}^2$</td>
</tr>
</tbody>
</table>
Fig. 3.5 On-off gain simulation result (solid curve) matched with Intel’s measured results (marker) [30]

Fig. 3.5 clearly shows that simulation from our APM methods matches well with the experimental results from Intel. This justifies the correctness of our APM in the modeling of the SWRA. The experimental results are reproduced by manual measurement of Raman gain plot in ref. [30]. Due to the relatively large measurement uncertainty reported in ref. [30], this comparison and subsequent result matching are still valid despite the potentially introduced human error. On-off Raman gain first increases linearly with the input pump power as it stimulates more Stokes emission. However, gain increment starts to drop and eventually saturates after pump power increases to above 0.4W. This is caused by the rapid growth of FCA loss as pump power rises.
Fig. 3.6 displays the simulated Stokes output variation as input pump power changes for silicon waveguide Raman laser (SWRA). Down-triangles are the simulation result based on the widely accepted model proposed by M. Krause et al. [104], served as the accuracy benchmark. Up-triangles and circles in Fig. 3.6 are results from our APM under non-phase matching ($\Delta k = 4000 \text{m}^{-1}$) and phase matching ($\Delta k = 0 \text{m}^{-1}$) conditions respectively. The parameters used in our simulation in Fig. 3.6 are listed in Table 3.1 above, following the same values as those used in [104].

Wavelengths for anti-Stokes emission are computed using Eq. 3.18 below, given the Raman shift in silicon is 15.6 THz. Following the same setting in [104], we assume identical linear absorption coefficients for all three wavelengths at 1dB/cm. FCA cross section for all three wavelengths are computed from Eq. 3.11. Nonlinear index coefficient $n_2$ takes its measured value of $6 \times 10^{-18} \text{m}^2/\text{W}$ [32]. The reflectivity for pump and Stokes waves is set as: $R_{\text{front, p}} = R_{\text{back, p}} = R_{\text{front, s}} = R_{\text{back, s}} = 30\%$. Reflectivity for anti-Stokes wave are set as $R_{\text{front, a}} = R_{\text{back, a}} = 0\%$, to be consistent with the original work.
\begin{equation}
\hat{\lambda}_a = \frac{c}{\frac{c}{\lambda_p} + 15.6 \times 10^{15}}
\end{equation}

Under non-phase matching condition, the efficiency of CARS process is very low. Stokes output from silicon Raman lasers originates from the SRS process only. This results in simulation results from our model under non-phase matching condition (up triangle) being exactly the same as those from M. Krause’s model (down triangles). These exact matching of simulation results provides a direct justification for the accuracy of our proposed APM. When the phase condition is matched (circles), we observe a much higher Stokes output and a lower threshold. These differences can be explained by the occurrence of efficient CARS process under phase-matching condition, which is not addressed by M. Krause’s model. Further elaboration on this concept will be provided in Chapter 5.

### 3.5 Summary

We present a detailed mathematical derivation of our proposed APM in this chapter, from the well-known Raman rate equation and nonlinear Schrödinger equations. Theoretical background and underlying assumptions are clearly illustrated. APM presents the most complete and precise descriptions for the evolution of pump, Stokes and anti-Stokes waves in waveguides; by taking into account the various Raman processes (SRS, SARS and CARS), electronic nonlinear phase-related processes (SPM, XPM and FWM) as well as linear and nonlinear loses (TPA and FCA). Compared to the previous numerical models for Raman-based waveguide devices, APM provides a unified tool that can be applied to a wide range of integrated
waveguide devices such as silicon waveguide Raman waveguide amplifiers, lasers and wavelength converters. We note here that APM does not take into account the time-domain information and is thus only applicable for modeling CW operation of the devices. In addition, we show that the validity of APM can also be extended to the less-investigated chalcogenide waveguide platform and weak pump regime. Justification of APM model is also provided by matching our simulation results with the well established experimental and numerical results to illustrate its correctness. The theoretical model will be used as the foundations for our theoretical investigation for the Raman-based devices in silicon and chalcogenide waveguide.
Chapter 4 STIMULATED RAMAN SCATTERING IN INTEGRATED WAVEGUIDE DEVICES

4.1 Introduction

On-chip integrated amplifiers and lasers remain as the greatest challenges in the final conquer of the promising silicon photonics technology [20]. Since its proposal in 2002 [28], light amplification by stimulated Raman scattering (SRS) provides an attractive approach for silicon-based amplifiers and lasers due to its unique advantages of wide operating bandwidth, simple fabrication process and extensive tuning capability. However, after a decade long of extensive research of silicon waveguide Raman amplifier (SWRA) and silicon waveguide Raman lasers (SWRL), the actual application of these devices is still hugely limited by their low energy efficiency [20]. This is mainly due to the high nonlinear two-photon absorption (TPA) induced free-carrier absorption (FCA) loss in the near-infrared wavelength range, resulting in a low energy conversion efficiency between pump and Stokes waves. Various methodologies have been proposed to suppress the detrimental FCA loss as reviewed in Chapter 2, with the reverse-biased p-i-n junction turns out to be the best solution. Given the large excess energy dissipation to sweep out free carriers and the complexity of the fabrication process to implement the p-n junction into conventional waveguide, the solution is far from satisfactory. In this chapter, we propose three novel approaches and evaluate their effectiveness in loss reduction and energy efficiency enhancement in the context of integrated waveguide Raman amplifiers and lasers. They are the bi-directional pumping scheme, chalcogenide waveguide Raman laser and slot waveguide Raman amplifier. In addition, a novel loss analysis technique is presented in this chapter, in order to investigate the various losses in SWRL.
4.2 Bi-directionally Pumped Silicon Waveguide Raman Amplifier and Laser

Bi-directional pumping scheme is not a new concept, which has been widely used in Raman fiber laser systems to enhance output Stokes power and operating wavelength range [128]. In this part, we extend this study to the SWRL field, which is based on a very different working principle as those in the Raman fiber laser. While enhancement of Stokes output power in Raman fiber laser is due to the different efficiency in the forward- and backward-propagating pump, bi-directionally pumped SWRL utilizes the reduction of pump intensity in the cavity. The intensity reduction subsequently results in a suppression of FCA loss, which displays a quadratic increment with pump intensity [129]. Our focus is on the influence of bi-directional pumping scheme in key laser performance parameters such as lasing threshold, Stokes output and cavity losses. Fig. 4.1 presents the schematic setup of the bi-directionally pumped SWRL to be used in our simulation and analysis. Compared to one-way pumping; the initial incoming pump power \(P_0\) is split into half and coupled in from both ends of the waveguide by a 50/50 coupler. APM model with the following parameters will be utilized for both one-way and bi-directional pumping scheme: \(A_{\text{eff}}=1.57 \, \mu \text{m}^2\), \(\lambda_p=1550\) nm, \(\lambda_s=1686\) nm, \(\alpha_p=\alpha_s=0.39 \, \text{dB/cm}\), \(\beta=0.5\) cm/GW, \(\sigma_p=1.45\times10^{-17}\) cm\(^2\), \(\sigma_s=1.72\times10^{-17}\) cm\(^2\), and \(L=4.8\) cm. \(\tau_{\text{eff}}\) is set at a low value of 1ns to account for the FCA suppression effect, assuming the presence of a reverse-biased p-i-n junction [5]. Raman gain is chosen at a moderate value of \(g_r=15\) cm/GW [121]. The reflectivity for Stokes wave is set as \(R_{\text{front, s}}=100\%\) and \(R_{\text{back, s}}=70\%.\) To further simplify the analysis, we assume no end-facet reflection for pump waves such that \(R_{\text{front, p}} = R_{\text{back, p}}=0\%.\) The boundary conditions are thus modified as Eq. 4.1 below for bi-directional pumping schemes.
We start with analyzing the effect of bi-directional pumping scheme on the pump power in the cavity of SWRA. The pump is set at 1W (thus 500mW at each end for bi-directional pumping scheme). With bi-directional pumping scheme, the peak power along the cavity is reduced from 0.3 W to less than 0.25 W, as shown in Fig. 4.2. This could help to suppress the FCA, which increases sharply with peak pump power. On the other hand, the average pump power along the waveguide is increased from 0.2W to 0.24W by employing bi-directional pumping scheme. This means a higher Raman
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gain for the Stokes signal could be achieved, which is influenced by the average pump power in the cavity. Bi-directional pumping scheme thus could suppress the FCA loss while increasing the Raman gain, to achieve the optimum goal of energy efficiency enhancement. Furthermore, the employment of bi-directional pumping enables a simple set-up without introducing additional optical losses along the cavity [130].

![Graph](image)

**Fig. 4.2 Average and peak pump power distribution along the waveguide [130]**

In Fig. 4.3(a), we simulate the variation of Stokes output with pump reflectivity $R_{\text{front,p}}$ while keeping the rest of reflectivity constant. Stokes output reduces sharply as reflection for pump power ($R_{\text{front,p}}$) increases from 0%, mainly due to the reduced portion of pump power coupled into the cavity. In this case, pump power coupling becomes a dominant loss mechanism, which will hinder our analysis of other important losses such as TPA and FCA. In order to get a physical insight of the cavity losses, we thus isolate the effect of this loss mechanism by applying zero pump reflection in the front facet. One should note that Stokes output is maximized at 0% pump reflection, which is consistent with our aim of Stokes output enhancement in
this research work. Therefore the use of no reflection in the front mirror for pump is justified. The distribution of pump power along waveguide at a pumping level of 2W is illustrated in Fig. 4.3(b) for both one-way and bi-directional pumping. It is clearly shown that bi-directional pumping results in a reduction of peak pump power within the cavity of SWRL. This reduction of peak cavity pump power greatly reduces higher order non-linear losses that rise sharply with peak power. The pump depletion ratio is more than 98% for both pumping schemes. With such a high pump depletion ratio, the introduction of end-facet reflection for pump waves has insignificant influence on our final results but complicating the simulation process. This thus provides further justification for the negligible pump end-facet reflection assumption in the current analytical model.

![Fig. 4.3 Variation of Stokes output with $R_{front,p}$; (b) Variation of pump and Stokes power along the cavity[129]](image)

Linear absorption loss, TPA loss and FCA loss are the three major loss mechanisms in SWRL [14]. We thus analyze the variation of these losses under one-way and bi-directional pumping here, as shown in Fig. 4.4. In order to facilitate our loss analysis, we explicitly define an overall loss factor ($\Gamma$) as Eq. 4.2, to quantify the loss in SWRL.
for analysis purpose. The outputs in the absence of losses are computed by setting their respective control parameters ($\alpha, \beta, \tau_{\text{eff}}$) to zero [129].

$$\Gamma(dB) = 10 \times \log\left(\frac{\text{Stokes output in the absence of the loss}}{\text{Stokes output in the presence of the loss}}\right)$$  \hspace{1cm} (4.2)

The loss analysis starts from pumping level of 0.4W, because losses are negligible below this pump power. In quantum mechanics, two pump photons with total energy equivalent to the silicon band-gap are absorbed to generate free carriers in the cavity in TPA process. The TPA process then induces the more severe FCA loss, in which subsequent incident photons lose energy when they collide with the free-carriers and accelerate them. In the absence of TPA process, no free carrier is generated to cause FCA loss. We therefore analyze TPA loss together with the presence of FCA loss. However, we are able to analyze FCA loss alone by assuming free-carriers generated
from TPA process recombine instantaneously. All three losses are found to be always smaller under bi-directional pumping scheme, indicating its loss suppression capability. The dominant FCA absorption is influenced by the density of the free carriers, which are mainly generated by TPA process of the pump photons. When the peak pump intensity in the cavity is reduced by bi-directional pumping scheme, the occurrence of TPA process is limited. This subsequently results in the generation of less free carriers, suppressing the FCA loss. The occurrence of TPA process increases with raising pump intensity to generate more free carriers, resulting in the increment of FCA loss. The FCA loss suppression capability from bi-directional pumping scheme is thus more significant at high pumping levels. One should note that the overall loss factor $\Gamma$ measures the percentage of each type of loss in the overall losses. Although the linear absorption loss is independent of the pump power, its overall loss factor $\Gamma$ drops as pump power increases because the overall losses elevate due to the TPA and FCA loss increments.

![Graph](image)

**Fig. 4.5 Input-output conversion efficiencies for one-way and bi-directional pumping schemes in the silicon waveguide Raman laser [129]**
In Fig. 4.5, pump-to-Stokes conversion efficiencies for one-way and bi-directional pumping scheme are displayed. Under the same total input pump power, bi-directional pumping scheme provides a higher output Stokes power compared to one-way pumping scheme. Below 0.4W pump power, Stokes output in the bi-directional pumping scheme is almost the same as that of the one-way pumping. The main reason is that low FCA loss in this pump power range results in insignificant loss reduction effect. Bi-directionally pumped SWRL thus displays no superiority at low pump power range. Above 0.4 W pumping powers, the continuing growth of FCA loss with pumping level causes a significant reduction in gain. The reduction of peak pump power in the cavity by bi-directional pumping starts to suppress the loss significantly, resulting in superior output enhancement. The output enhancement effect of bi-directional pumping scheme is clearly justified. One should note that the enhancement effect originated from its loss suppression capability by lowering the peak cavity pump power. In the low pump power region where the loss is negligible, this enhancement effect is not displayed. As a result, bi-directionally pumping scheme has no improvement in lasing threshold and linear operating regions. Other techniques such as reverse-biased p-i-n junction and ring cavity are needed to be employed in conjunction with bi-directional pumping scheme in order to improve those areas. The conversion efficiency curve is observed to have a parabolic shape. It increases rapidly at the low pump power region. As pump power increases, output increment rate reduces, and eventually saturates at high pump power. This is a unique characteristic of SWRL, which suffers from strong non-linear losses. Rapid FCA loss increment at high pump power cancels out the gain from SRS, resulting in output saturation. From our simulation, loss suppression from bi-directional pumping scheme provides another benefit in slowing down this gain saturation process. As a result, we observe a saturated gain for bi-directional pumping scheme at the higher pump power values.
Lasing threshold and pump-to-Stokes conversion efficiency strongly depend on the cavity condition. Variations of end-facet reflectivity thus affect these key laser parameters. In this section, we explore these features under one-way and bi-directional pumping schemes by changing the right end-facet reflectivity ($R_{back,s}$). Simulations are performed for two chosen values of $R_{back,s}$ at 40% and 70%. For both reflectivities, the bi-directionally pumped SWRL encounters less overall FCA loss as shown in Fig. 4.6(a). Overall FCA loss in the cavity is larger for higher end-facet reflectivity cavity, due to the larger portion of power retained in the cavity. At the peak pumping level of 2W, the FCA loss suppression for both reflectivity are the same at around 0.8 dB. In Fig. 4.6(b), we illustrate the pump-to-Stokes conversion efficiency for these two end-facet reflectivities. As expected, bi-directional pumping consistently enhances the pump-to-Stokes conversion efficiency for different reflectivities. Output Stokes power is superior for low end-facet reflectivity. The larger portion of Stokes power coupled out and the less FCA loss suffered in the cavity account for this. The Stokes output power increases more quickly and saturates more slowly for a lower reflectivity. This is a unique characteristic of SWRL due to the presence of TPA and FCA losses.
Furthermore, output Stokes power enhancement of bi-directional pumping is observed to be more significant at low cavity reflectivity. This further justifies the use of bi-directional pumping as an effective mechanism to increase the maximum output power of Raman silicon laser. By using a low end-facet reflectivity cavity, both the Stokes output and the output enhancement from bi-directional pumping are simultaneously maximized, optimizing the deliverable Stokes output power from SWRL.

![Graphs showing variation of threshold and peak Stokes power with R_{back,s}](image)

**Fig. 4.7 Variation of (a) threshold and peak Stokes power (b) performance parameter (PR) with right end-facet reflectivity for Stokes wave (R_{back,s}) [129]**

In Fig. 4.7(a), we simulate the variation of lasing threshold and peak Stokes output with respect to the end-facet reflectivity. Low end-facet reflectivity involves a higher cavity loss, therefore a higher lasing threshold. The lasing threshold is almost the same for both one-way and bi-directional pumping, due to the negligible FCA loss reduction in the low pump power range. Lasing threshold eventually increases to a very high value after $R_{back,s}$ reduces to below 3%, causing the cavity to stop lasing. As a result, we observe a sharp turning in the peak Stokes output around $R_{back,s} \approx 3\%$, forming a boundary between the lasing and non-lasing regions of the cavity. In the lasing region, peak Stokes output increases with decreasing end-facet reflectivity, together with the output power enhancement from bi-directional pumping scheme. This could be
attributed to the larger portion of Stokes power coupled out and the less FCA loss encountered. Absolute maximum Stokes output occurs at $R_{back, s}$ value of 3%, corresponding to the point where the cavity stops lasing. Due to the limitation of power available from CW-pumping source and the high coupling loss into silicon waveguide, lasing threshold is another key parameter in the design of SWRL [37]. To optimize the SWRL output, one thus needs to ensure a high peak Stokes power with low threshold consideration taken into account. A clear trade-off between these two aspects is evident from Fig. 4.7 (a).

In this research work, we specifically define a performance parameter ($PR$) to describe the trade-off quantitatively between lasing threshold and output Stokes power. $PR$ is defined as the absolute ratio of peak output power over the threshold according to Eq. 4.3, such that a maximized $PR$ will ensure a high peak Stokes power with respect to a low threshold. The reason behind the definition of $PR$ ratio is for the consideration of silicon waveguide Raman laser’s actual applications in the experimental context. The common pumping power available nowadays at the 1550nm window is around 1W, which can be around 300mW after coupling to silicon waveguides (typical coupling loss is around 5 dB). Therefore, lasing threshold becomes another important constraint for the design of SWRL, together with the pump-to-Stokes conversion efficiency. SWRL with a high lasing threshold will lose its usefulness in actual applications and will become meaningless to discuss it. One key advantages of Raman lasers is its capability to operate in a wide range of wavelengths including the mid-infrared range. Cascaded SWRL to push the operating wavelength to the infra-red range also requires the Raman laser to operate in a low threshold. This is another important motivation for the low threshold consideration in SWRL. In the optimization of Stokes output by end facet reflectivity variation, there is always a trade-off in the increase of lasing
threshold. We therefore define this \( PR \) ratio to describe the trade-off quantitatively and ensure the optimization is meaningful by taking lasing threshold also into account. The variation of it with respect to end-facet reflectivity \( (R_{\text{back},s}) \) is then plotted in Fig. 4.7 (b). The optimized \( PR \) ratio occurs at \( R_{\text{back},s} = 30\% \), with a 322mW lasing threshold and 280mW peak output power. At this optimized \( R_{\text{back},s} \) value of 30\%, the output power enhancement is around 24\% compared to the one-way pumping scheme.

\[
\text{Performance Parameter (PR)} = \frac{\text{Peak Stokes Output}}{\text{Threshold}}
\]  

\( (4.3) \)

To extend the concept of pump power division further, we go on to proposed a bi-directionally pumped Mach-Zehnder interferometer SWRL as shown in Fig. 4.8 [131]. Mach-Zehnder interferometer (MZI) has been widely used in silicon as an effective mean in modulation [132]. In this research work, we propose a novel cavity design for the SWRL by applying two reflective mirrors to the end of the MZI structure to form the cavity. Similar to bi-directionally pumping scheme, MZI cavity helps to reduce peak cavity power to suppress FCA loss by dividing the incoming powers into its two arms. Furthermore, by introducing the concepts of silicon-based modulators into the cavity, we could design a self-modulated silicon Raman laser based on that [132]. Bi-directional pumping scheme is also employed into this cavity to further reduce the cavity losses to enhance Stokes output. A reverse-biased p-n junction technique is assumed to be incorporated into the waveguide design in order to reduce the FCA loss. The initial incoming pump power \( (P_0) \) is split into half and coupled in from both ends of the waveguide by a 50/50 coupler. This pump power is further divided into half by the Y-splinter, resulting in one-fourth of the initial pump power \( (P_0/4) \) entering from each end of the waveguides. To simplify the analysis process, no reflection is applied for the pump power at the cavity end-facet. Stokes power
reflectivity is set at 100% and 70% respectively. A perfect constructive interference is assumed at the output-end Y-junction for the Stokes waves. This could be achieved in experiments by an addition of a heater in one of the arms for phase matching [133].

![Diagram of bi-directional pumped silicon waveguide Raman laser with MZI cavity](image)

**Fig. 4.8 Schematic diagram of the bi-directional pumped silicon waveguide Raman laser with MZI cavity [131]**

Figure 4.9 below presents the pump and Stokes power variations along the waveguide at an input pump power of 2W. Here, we compare our proposed bi-directionally pumped MZI cavity SWRL with the normal one-way pumped Fabry-Perot (FP) cavity (one arm). For both cases, the pump depletion ratio is more than 98%. Similarly to the previous discussion, we set pump reflectivity to 0% for both front- and back-mirrors to simplify our analysis. As shown in Fig. 4.9, MZI cavity with bi-directional pumping scheme hugely reduces the peak pump power in the cavity. Reduction of input pump power subsequently suppresses the non-linear FCA loss that rises sharply with peak powers in the cavity. The loss suppression eventually leads to an output enhancement. Another interesting observation is the gentler fluctuation of Stokes power along the MZI cavity, as compared to one-way FP cavity. Such a feature is favorable from the cavity stability point of view. A small fluctuation of Stokes power within the cavity could ensure the operation of the devices within the optical damage threshold of the material.
Chapter 4: Stimulated Raman Scattering in Integrated Waveguide Devices

### Fig. 4.9
(a) Pump power variation along waveguide (b) Stokes power variation along waveguide

- **Fig. 4.9 (a)** Pump power (b) Stokes power variation along cavity length of one-way and bi-directionally pumped MZI cavity silicon waveguide Raman laser

### Fig. 4.10
(a) Input-output conversion efficiency (b) Overall FCA loss analysis

- **Fig. 4.10 (a)** Pump-to-Stokes conversion efficiency and (b) Overall FCA loss analysis for different cavity designs

Four cases of pump-to-Stokes conversion efficiency and overall FCA loss analysis are presented here for comparison in Fig 4.10. They are the one-way pumped one-arm FP cavity waveguide, bi-directionally pumped FP waveguides, one-way and bi-directionally pumped MZI cavities respectively. For both FP and MZI cavities, bi-directional pumping scheme displays its superior loss suppression capability. Although both bi-directional pumping scheme and MZI cavities have the same effect
of dividing incoming pump power equally; the use of MZI cavity is able to suppress the FCA loss more, primarily due to lower power involved in each arm. This subsequently leads to the superior Stokes output for one-way pumped MZI cavity than bi-directionally pumped FP cavity. By applying both the MZI cavity and the bi-directional pumping scheme, an output power enhancement of 100% is achieved at 4W pumping level. One should note that lasing threshold also increases twice for the MZI cavity, due to the requirement for sufficient pump power to trigger stimulated emission in each arm.

4.3 Energy Efficient Chalcogenide Waveguide Raman Laser

The detrimental FCA loss in SWRL originates from the intrinsic high carrier mobility and long carrier recombination time of the silicon material. While the existing FCA suppression methodologies are mainly based on using the silicon waveguides, a promising alternative could be replacing the silicon material platform to completely get rid of this drawback. In addition to the negligible FCA loss, the material alternative should also possess some desirable properties that are essential for waveguide Raman laser. These primary optical characteristics include large refractive index for optical confinement and transparency in the telecom windows, comparable electronic and Raman susceptibility to silicon, low linear and nonlinear losses as well as a fabrication process that is fully compatible with the complementary metal-oxide semiconductor (CMOS) technology. Chalcogenide glass presents the ideal material choice from this perspective. The glasses have a transmission spectrum far into the mid-infrared wavelengths (below 12μm) due to its large atomic mass and relatively
weak bond strengths [61]. The refractive index ranges from 2 to more than 3 [134].

The measured nonlinearity is around 930 times of that of silica fiber [66]. Low-loss chalcogenide waveguides have been fabricated using the CMOS comparable technology, with a reported linear propagation loss as low as 0.25dB/cm [71]. TPA is generally small in chalcogenide glasses, benefited from their large band-gap energy (Eg~1.78eV) [10]. In particular, these glasses display relatively low carrier mobility. As the severity of the FCA loss is proportional to the carrier mobility in solid, FCA loss is negligible in these glasses [135].

In the context of Raman amplifiers and lasers, the Stokes output and the efficiency of the devices is determined by the Raman gain coefficient through the rate equation[26]. The strength of the stimulated Raman scattering (SRS) process, and thus the Raman gain coefficient constitutes the essential criteria to select the material for our chalcogenide waveguide Raman lasers. Among all chalcogenide glasses, As$_2$Se$_3$ possesses the largest Raman gain [66]. We subsequently choose As$_2$Se$_3$ waveguides instead of the other chalcogenide glasses, assuming the optical phonon properties of the chalcogenide waveguide is identical to that in bulk materials. The measured Raman gain coefficient is 5.1cm/GW, which is 780 times the value in silica fiber. Furthermore, Raman shift for As$_2$Se$_3$ and silicon is 7THz [75] and 15.6THz [121] respectively, making As$_2$Se$_3$ a more energy efficient material choice than silicon for Raman applications with its larger Stokes photon energy. Over the past decades, Raman gain of As$_2$Se$_3$ chalcogenide glass has been characterized [72, 75] and investigated in fiber lasers both theoretically [77] and experimentally [76]. Chalcogenide are traditionally not considered for optical interconnects due to the fact that silicon possesses a slightly larger Raman gain and tighter optical confinement. In this part, we show As$_2$Se$_3$ waveguide Raman lasers (As$_2$Se$_3$WRL) out-perform SWRL
in spite of these drawbacks. \( \text{As}_2\text{Se}_3 \) WRL completely solves the detrimental FCA loss drawback in the SWRL and displays many desirable laser characteristics in terms of ultra-high conversion efficiency, low lasing threshold and wide linear operation region [136].

Fig. 4.11 Schematic diagrams of the \( \text{As}_2\text{Se}_3 \) waveguide Raman laser, (a) waveguide cross-section dimensions, (b) the simulated mode field pattern of the fundamental TE\(_0\) mode \( (A_{\text{eff}}=1.97 \mu m^2) \) and (c) the cavity structure [136]

Fig. 4.11 (a) presents a typical cross-sectional diagram of the \( \text{As}_2\text{Se}_3 \) rib waveguide used, with the same dimensions as the silicon waveguide discussed in the previous chapter for the ease of comparison. Such a waveguide has been demonstrated experimentally using complementary metal-oxide-semiconductor (CMOS) technology on SiO\(_2\) substrates [71]. Mode solving with commercial software Rsoft FemSIM yields an effective area \( A_{\text{eff}}=1.97 \mu m^2 \) and \( 1.7 \mu m^2 \) for \( \text{As}_2\text{Se}_3 \) and silicon waveguides respectively. The simulated fundamental TE\(_0\) mode field pattern for the \( \text{As}_2\text{Se}_3 \) waveguide is also shown in Fig. 4.11 (b). The slightly larger effective area of \( \text{As}_2\text{Se}_3 \) is due to its smaller refractive index, which results in a poorer optical confinement. However, our following analysis will illustrate that this confinement drawback does
not significantly influence the performance of the \( \text{As}_2\text{Se}_3\)WRL. Instead, the larger modal area facilitates an easier coupling to the fiber components, which could reduce the coupling losses. The cavity consists of an \( \text{As}_2\text{Se}_3\) waveguide of length \( L \), closed at two ends with coatings to form a Fabry-Perot (FP) cavity as illustrated in Fig. 4.11 (c). The axis along the length of the waveguide is denoted as \( z \)-axis, with the positive direction pointing towards the right.

In the following part of this chapter, comprehensive numerical investigation on the SWRL and the \( \text{As}_2\text{Se}_3\)WRL are performed based on the simulation results from APM for both silicon and chalcogenide waveguides. The well-established SWRL will be used as a benchmark for comparison for the proposed \( \text{As}_2\text{Se}_3\)WRL. Simulation parameters for the \( \text{As}_2\text{Se}_3\) waveguide are set as \( L = 4.8\text{cm} \), \( \alpha_p = \alpha_s = 0.25\text{dB/cm} \), \( \beta = 0.25\text{cm/GW} \) and \( g_r = 5.1\text{cm/GW} \) according to the experimental characterization. We are aware that a wide range of values have been measured for Raman gain coefficient in \( \text{As}_2\text{Se}_3 \) waveguide [66, 75]. We choose to utilize the Raman gain value of 5.1cm/GW measured under the continuous-wave (CW) pump condition in [66], which is more appropriate for our proposed device than that measured in [75] with a pulsed pump. Pump wavelength (\( \lambda_p \)) is set at 1550nm to take advantages of the high-power Erbium-doped fiber amplifier (EDFA) in this range [137]. Stokes wavelength (\( \lambda_s \)) is expected at 1608nm. Simulation parameters for the SWRL are identical to those utilized in section 4.2 to ensure consistency throughout this thesis.

We start with analyzing the single-pass Raman gain of silicon and \( \text{As}_2\text{Se}_3 \) waveguides by setting all reflectivity to 0, as shown in Fig. 4.12 [138]. No net gain is observed in the normal silicon waveguide. This observation has been confirmed experimentally, due to the high FCA loss originated from a long carrier recombination time [29]. The high loss in the cavity cancels out any Raman gain generated. When a reverse-bias of
25V is applied to sweep out free carrier, a net gain occurs and saturates at around 6dB, in good agreement with experimental measurement [29]. Raman gain of As$_2$Se$_3$ waveguides is similar to that of the reverse-biased silicon waveguide in low-pump-power region, but out-performs silicon beyond 1.5W pumping. It should be noted that such a high pump power value for As$_2$Se$_3$ waveguide Raman amplifiers (As$_2$Se$_3$WRA) to outperform SWRA is not an absolute case. Further reduction can be achieved when the waveguide length or cavity losses are optimized. The problematic gain saturation of SWRA is also not observed in As$_2$Se$_3$WRA, evidenced by a 16dB gain at 4W pumping. We believe the negligible FCA loss and low TPA loss account for this improvement. In addition, the bandwidth of As$_2$Se$_3$ stimulated Raman gain is 2 THz [66], which is much wider than the 105GHz of silicon. Such a wide bandwidth will enable on-chip amplification of ultra-high speed signals up to Tera-bit/s; which is not achievable by silicon. As$_2$Se$_3$ waveguides thus provide a promising platform for amplification, without the need for extra voltage supply [138].

![Fig. 4.12 Single-pass Raman gain for As$_2$Se$_3$ and silicon waveguides under normal ($\tau_{eff}=23$ns) and 25V-biased ($\tau_{eff}=1$ns) operation [121]](image-url)
We go on to investigate and compare both the SWRL and the As$_2$Se$_3$WRL by setting the front facet reflectivity as $R_{\text{front,p}} = R_{\text{front,s}} = 30\%$. The back facet is covered with broadband high-reflectivity coating (90%), following the same reflectivity as in the silicon waveguide Raman laser experiment [121]. No lasing is observed for normal silicon waveguide due to the absence of net gain. We thus focus on analyzing key Raman laser performance parameters of the As$_2$Se$_3$ waveguide and the 25V-biased silicon waveguide. When pump power increases beyond the lasing threshold, rapid increment of conversion efficiency with respect to pump power is observed due to enhanced SRS effect. As pump power grows further, nonlinear losses such as TPA and FCA come into play, resulting into rapid depletion of the pump power. For both Raman lasers, conversion efficiency maximizes at 1W pumping as shown in Fig. 4.13. This is due to the trade-off between cavity SRS enhancement effect and loss increment with respect to the growing intra-cavity power, which represents the most energy-efficient operation point of Raman lasers. One should note that this is well within the optical damage threshold of the material. To the best of our knowledge, no optical damage threshold under a continuous-wave (CW) pumping is found for As$_2$Se$_3$ materials. An optical damage threshold of 100MW/cm$^2$ is reported under a pulsed-pumping with the pulse-width of 10ns in reference [59]. If such a wide pump pulse-width is treated as a good approximation for the case of CW pumping in our investigation, the corresponding optical damage threshold is calculated to be around 2W for our chosen waveguide with an effective core area of 1.97µm$^2$, indicating that our simulation range is reasonable. Furthermore, we would like to point out that 1W pumping is not the prerequisite to validate the argument in the thesis. In fact according to our simulation, As$_2$Se$_3$ waveguide Raman lasers outperform silicon Raman lasers over a wide range of pump power starting from its lasing threshold of 299mW. 1W pumping is only needed to achieve the best conversion efficiency for the given
parameters presented in this thesis. We thus believe that optical damage threshold will not play a crucial role in our overall theoretical analysis.

As the authors in reference [59] pointed out, imperfections on the surface of the input facet and along the waveguide play an important role in determining the apparent optical damage threshold of a waveguide device. Given the short history of development for chalcogenide planar waveguides, future fabrication advancements such as better mode matching, anti-reflection (AR) coating and improvement of waveguide quality will further raise their optical damage threshold.

![Graph showing conversion efficiency](image)

**Fig. 4.13 Influence of pump power on conversion efficiency for As$_2$Se$_3$ under different measured $g_r$ [66, 75] for As$_2$Se$_3$WRL and SWRL**

According to our simulations, As$_2$Se$_3$WRL is much greener in terms of energy consumption, highlighted by the maximum conversion efficiency of 31%. This efficiency is around five times better than the 6% efficiency of SWRL, even without taking the energy dissipation of 25V bias voltage into account. This could be attributed to the lower TPA and negligible FCA loss in As$_2$Se$_3$ waveguides, as well as its larger Stokes photon energy compared to silicon. Considering the large
experimental uncertainty in its future exploration [75], we also include the conversion efficiency curve for a smaller Raman gain of 2.2 cm/GW in Fig. 4.13. The device still outperforms its silicon counterpart in high-pump region under such a low Raman gain.

![Graph showing conversion efficiency curves for different materials.](image)

**Fig. 4.14** Influence of the input pump power on Stokes output for As$_2$Se$_3$WRL and SWRL. Stokes output subjected to no TPA ($\beta=0$ cm/GW) or linear propagation ($\alpha=0$ dB/m) loss are also displayed.

The problematic gain reduction and output saturation of SWRL at high pumping level is also not observed in As$_2$Se$_3$WRL, as shown in Fig. 4.14. A linear operation region up to 1.5W pump power is subsequently obtained. The lasing threshold is another key parameter for integrated laser sources, mainly due to the raise in cost as the pump power increases. We evaluate the lasing threshold by locating the pump power in which the overall gain in cavity reaches unity. It was found to be 299mW and 443mW for As$_2$Se$_3$WRL and SWRL respectively. The lower lasing threshold of As$_2$Se$_3$WRL is mainly due to the absence of FCA loss in the cavity, which prevents the depletion of pump power in the build-up stage. To investigate the influences of various losses in the As$_2$Se$_3$WRL, two special cases are also shown in Fig. 4.14. We explicitly simulate
the Stokes output when TPA and linear propagation loss are assumed absence respectively. The results suggest that linear propagating loss improvement will have a greater impact in reducing lasing threshold and enhancing conversion efficiency due to the relatively low TPA loss in As$_2$Se$_3$ materials. Impurity absorption and side-wall scattering are the major contributions limiting linear propagation loss [70]; future waveguide fabrication advancement in these areas is thus highly recommended. Despite its relatively smaller Raman gain, As$_2$Se$_3$ out-performs silicon as the material candidate for on-chip Raman laser with the negligible FCA loss.

Due to the precious space on chip, footprint size (waveguide length in our case) is a very important consideration in optical interconnect component design. Long waveguides requires spiral bending on the chip, which occupies large chip area and incurs additional bending loss [5]. Increment of waveguide length incurs two counter-acting effects in As$_2$Se$_3$ WRL: enhancement of Stokes wave from longer interaction length and reduction of Stokes wave due to the increased loss. As waveguide length increases from 5mm, lasing threshold drops because enhancement effect outplays the loss at low pump power around threshold. This scenario reverses after the pump power is fully depleted at around 55mm waveguide length, forming a minimum threshold there at 296mW, as shown in Fig. 4.15 (a). The same explanation accounts for the variation of maximum conversion efficiency at 1W pumping. The turning point occurs at a shorter waveguide length of 17mm, mainly due to the higher losses incurred at this high pump power. Maximum conversion efficiency achieved is 40%. Miniaturization by waveguide length reduction is thus beneficial in the perspective of conversion efficiency.
Beyond the maximum conversion efficiency point, a clear trade-off between lasing threshold and conversion efficiency exists. In order to compromise this trade-off between lasing threshold and optimal conversion efficiency, we specifically define a figure of merit ($FOM$) factor for waveguide Raman lasers as the absolute ratio of Stokes output power (1W pumping) to the lasing threshold power. This is illustrated as in Eq. 4.4. Maximum $FOM$ value of 1.1 occurs at 30mm waveguide length for As$_2$Se$_3$WRL, as shown in Fig. 4.15 (b), which is the optimized operating point in terms of energy consumption with low threshold and high conversion efficiency. This is almost an order of magnitude higher than the maximized $FOM$ value for SWRL at 60mm waveguide length, indicating a huge advantage for As$_2$Se$_3$ waveguide Raman lasers in terms of energy efficiency and devices miniaturization capability.

$$FOM = \frac{Output\ Stokes\ Power\ (at\ 1W\ pumping)}{Threshold\ Power}$$ (4.4)
Further reduction of As$_2$Se$_3$WRL lasing threshold can be achieved by optimizing the end facet reflectivity, as illustrated in Fig. 4.16 (a). We use the optimized cavity length ($L$) of 30mm here. Front end-facet reflectivity for pump ($R_{\text{front,p}}$) is set at 0% to maximize the coupled-in pump power [129]. Lasing threshold reduces when we increase reflectivity for Stokes wave ($R_{\text{front,s}}$), mainly due to the lower cavity loss incurred. 100mW threshold is achieved when we increases $R_{\text{front,s}}$ to 90%, which is three times lower than the 310mW at 30% reflection. This value is about two times better than the lowest obtainable lasing threshold in SWRL [5], without the need of any external supply of voltage. The variation of conversion efficiency with respect to $R_{\text{front,s}}$ increment displays a parabolic shape, with maximum value of 35% occurring at $R_{\text{front,s}} \approx 24\%$. Trade-off between Stokes output growth from cavity enhancement and drop in out-coupling ratio accounts for that. As shown in Fig. 4.16 (b), a further FOM factor optimization to 1.4 can be achieved at 60% reflectivity. We remark here that the optimization of As$_2$Se$_3$WRL with respect to waveguide length or cavity reflectivity is highly dependent on the area of interest, as illustrated by the very different optimized values obtained for distinct consideration of lasing threshold, conversion efficiency or
FOM. The primary objective of the $\text{As}_2\text{Se}_3$WRL thus determines which optimized values should be utilized.

### 4.4 Silicon-Chalcogenide Slot Waveguide Raman Amplifier

In Chapter 4.3, we have shown that $\text{As}_2\text{Se}_3$WRA provides an attractive alternative for on-chip amplification with its high energy efficiency and wide gain bandwidth. However, $\text{As}_2\text{Se}_3$WRA bases its operation on $\text{As}_2\text{Se}_3$ waveguides, which might encounter some difficulties in integrations with other silicon photonic components that are mainly based on silicon waveguides. One noticeable area will be the additional loss from coupling between waveguides of different refractive indices ($\Delta n \sim 0.7$). In addition, the very large coefficient of thermal expansion (CTE) of $\text{As}_2\text{Se}_3$ material could also generate cracking and dislocation when connected to silicon waveguide [61]. In order to address this material incompatibility issue, we propose a novel device design to incorporate the $\text{As}_2\text{Se}_3$WRA into the silicon platform using the well-known slot waveguide structure [54]. Since its initial proposal in 2004 [54], slot waveguide has received considerable global research attention with its extremely strong light confinement in the low refractive index region [55], leading to possible applications in the areas of erbium-based amplification [139] and nonlinear optics [56, 57]. In this part of the work, we extend this ultra-strong optical confinement concept of slot waveguides into the on-chip Raman amplification domain. While previous works are mainly based on slot materials such as air or SiO$_2$, we utilize $\text{As}_2\text{Se}_3$ as our slot materials here, which possesses superior properties for integrated Raman amplification [136]. To the best our knowledge, this silicon chalcogenide slot waveguide Raman
amplifier (SCSWRA) is the first ever investigation of slot waveguide for Raman amplification in all material platform.

Fig. 4.17 (a) Schematic diagram (b) simulated quasi-TE and (c) TM mode of the proposed SCSRA waveguide ($W=0.5\mu m$, $H=0.22\mu m$, $h=0.08\mu m$)

Fig 4.17 (a) illustrates the schematic diagram of the proposed silicon-chalcogenide slot waveguide, which only supports one quasi-TE mode in the silicon cladding and one quasi-TM mode in the chalcogenide slot, as shown by Fig. 4.17 (b) and (c) respectively. Such structure could be fabricated through direct bonding technology for hetero-integration [9]. Compared to the air/SiO$_2$ slot, As$_2$Se$_3$ slot results in less confined slot mode as evidenced by the larger portion of TM mode electric fields extending out of the slot region. While this might increase the effective area, it enables better overlaps between the TE and TM modes that are essential for the proposed SCSWRA. In normal SWRA, FCA loss is mainly contributed by the free carriers generated by the strong pump power. By pumping the slot waveguide in the TM mode, we can hugely suppress this FCA loss since the TM mode mainly lies in the chalcogenide slot region with low carrier mobility. On the other hand, the Stokes waves are launched into the TE mode of the slot waveguide. The mode field diameter and effective index of the quasi-TE mode is very similar to that of the normal silicon waveguide with the same dimensions. Such mapping of the silicon chalcogenide slot
waveguide with the silicon waveguide will facilitate a lossless transition of the SCSWRA into the silicon platform, solving the material incompatibility issue of As$_2$Se$_3$WRA. The optimized width ($W$), clad height ($H$) and slot height ($h$) of the slot waveguide is 0.5μm, 0.22μm and 0.08μm respectively. These dimensions ensure maximum modal overlap between the pump and the Stokes field, while maintaining their respective effective modal areas small.

The interaction of pump and Stokes waves in the SCSWRA can be described by numerically solving the coupled mode equations as in Eq. 4.5, modified from our proposed APM [52, 109]. The symbols follow the same physical meaning as presented in Chapter 3.

\[
\frac{dE_p}{dz} = \left[ -\frac{g_r}{2A_{eff}^R} \frac{\lambda_p}{\lambda_p} |E_s|^2 - \frac{\alpha_p}{2} - \frac{\beta |E_p|^2}{2A_{eff}^{PP}} - \frac{2|E_p|^2}{2A_{eff}^{PS}} \right] E_p \tag{4.5a}
\]

\[
\frac{dE_s}{dz} = \left[ \frac{g_s |E_p|^2}{2A_{eff}^{SS}} - \frac{\alpha_s}{2} - \frac{\beta |E_p|^2}{2A_{eff}^{PS}} - \frac{2\beta |E_p|^2}{2A_{eff}^{PS}} \right] - \frac{\beta \tau_{eff} \sigma_s}{2hf_{p} A_{eff}^{SS} A_{eff}^{PS}} \left( \frac{|E_p|^4}{A_{eff}^{PS}} + \frac{|E_s|^4}{A_{eff}^{SS}} \right) E_s \tag{4.5b}
\]

In Eq. 4.5, $A_{eff}^R$, $A_{eff}^{PP}$, $A_{eff}^{SS}$ and $A_{eff}^{PS}$ are the effective areas for Raman and TPA respectively, which can be computed from the modal field distribution of the slot waveguide [52] using Eq. 4.6. $e_i^j$ is the electrical field components of the $i^{th}$ wave along $j$-axis, with $i=p$ or $s$ and $j=x$, $y$ or $z$. The effective areas of our proposed SCSWRA for Raman and TPA processes are computed as $A_{eff}^R = 0.816\mu m^2$, $A_{eff}^{PP} = 0.22\mu m^2$, $A_{eff}^{SS} = 0.275\mu m^2$ and $A_{eff}^{PS} = 0.711\mu m^2$ using Eq. 4.6.

\[
A_{eff}^R = \frac{\int |e_p|^2 dA \times \int |e_s|^2 dA}{\int |e_p|^2 dA + |e_p \cdot e_s|^2 - 2(e_p^2(e_s^2)^2 - [(e_s^2)^2 - (e_p^2)^2] \times [(e_p^2)^2 - (e_s^2)^2] dA} \tag{4.6a}
\]
The performance of SCSWRA will be numerically investigated in the following section, using the well-established SWRA as the benchmark. The simulation parameters for silicon and As$_2$Se$_3$ material are set similarly to those in Chapter 4.2 and 4.3. The only exception will be the linear propagation loss for the slot region, which is reported to be around 6dB/cm [58]. The slot material used in reference [58] is SiO$_2$. Due to the severe scattering loss in the material boundaries of slot waveguides, we believe such loss is a good approximation of our proposed slot waveguide structure despite the material difference. FCA loss is found to have negligible influence on the on-off gain for SCSWRA, as shown by the coincidence of the Raman gain curve under various effective carrier lifetimes in Fig. 4.18. A possible reason might be due to the low power intensity in the silicon clad region, benefited from pump power isolation in the low FCA-loss chalcogenide slot from the SCSWRA structure. This is very different from the case of SWRA, which suffers more than 10dB gain reduction when the FCA is high, justifying the capability of SCSWRA for FCA suppression. The problematic gain reduction and output saturation in SWRA is not observed for our proposed SCSWRA. On-off Raman gain grows almost linearly for the SCSWRA within our simulation range, yielding a high gain of 7dB at 2W pumping power. The on-off gain for SCSWRA is lower than SWRA below 1W pumping, mainly due to the low Raman gain coefficient of As$_2$Se$_3$ material and the smaller modal overlap between the pump and Stokes waves in the proposed structure. We note here that this can be significantly improved by reducing the slot waveguide linear propagation loss or using slot material with higher Raman gain. Furthermore, this comparison is done based with SWRA with a 25V reverse-bias voltage to suppress FCA loss. SCSWRA
outperforms normal SWRA in terms of on-off Raman gain across all pump power range according to our simulation.

![Graph showing on-off Raman gain (dB) for SWRA and SCSWRA under various effective carrier life times](a)

**Fig. 4.18 Variation of on-off Raman gain (dB) for SWRA and SCSWRA under various effective carrier life times**

In the following section, we will examine the proposed SCSWRA’s miniaturization capability, which is critical for integrated on-chip amplifiers. While both Raman amplifiers generate similar gain under 1W pumping, the device miniaturization capability for SCSWRA is better than SRA, as illustrated in Fig. 4.19 (a). The optimized waveguide length is 3.5cm for SCSWRA, around 1cm shorter than SWRA. This could be attributed to the tighter confinement and lower nonlinear loss in the chalcogenide slot, resulting in more efficient SRS process. The optimized length increases slightly to 4.7 cm even at a high pump power of 2W. We note here that these values are much shorter than the 13cm reported for the recently proposed cladding pumping SWRA design [52], making them a suitable candidate for future integrated amplifiers. Further improvement in the propagation loss of slot waveguides will yield an even shorter optimized device length. Linear amplification range is another key parameter for amplifier, which is conventionally defined by the 3dB input saturation.
power \( (P_{\text{sat}}) \). The mechanism for this on-off gain drop in SWRA is mainly due to the severe FCA loss. In the small-signal operation region of the SWRA, increment of the Stokes input power has negligible influence on FCA loss and thus the on-off Raman gain, because the much larger pump power dominates the FCA loss in this region. The scenario changes when the input Stokes power reaches a level comparable to the pump power, in which an increment of the Stokes input will elevate the FCA loss. The increased loss in the cavity subsequently results in the on-off Raman gain drop. Our proposed SCSWRA thus outplays traditional SWRA with the 6.7dB wider linear amplification range, as illustrated in Fig. 4.19 (b).

![Graph showing the variation of on-off gain (dB) with waveguide length and Stokes power for both SWRA and SCSWRA](image)

Fig. 4.19 Variation of on-off gain (dB) with (a) waveguide length and (b) Stokes power for both SWRA and SCSWRA
4.5 Summary

This chapter proposes three novel methodologies, in an attempt to solve the detrimental high FCA loss problem currently hindering the development of silicon waveguide Raman laser (SWRL). Intensive and comprehensive numerical simulations are performed to investigate their validity based on our proposed APM modeling. The contributions of these methodologies compared with the existing results are summarized in Table 4.1 below.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>On-off Raman gain enhancement</th>
<th>Lasing output enhancement</th>
<th>Lasing threshold enhancement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi-directional pumping scheme</td>
<td>Not Applicable</td>
<td>24%</td>
<td>No effect</td>
</tr>
<tr>
<td>As$_2$Se$_3$ waveguide based Raman amplifier and laser</td>
<td>10dB</td>
<td>7 times</td>
<td>33%</td>
</tr>
<tr>
<td>Silicon-chalcogenide slot waveguide Raman amplifier</td>
<td>7dB</td>
<td>Not Applicable</td>
<td>Not Applicable</td>
</tr>
</tbody>
</table>

The first methodology is bi-directional pumping scheme. The scheme is found to be able to reduce the effective pump intensity in the cavity to suppress FCA loss in SWRL. This loss reduction subsequently results in the improvement of Stokes output. We then extend this pump division concept by introducing the bi-directionally pumped MZI cavity, which results in further FCA suppression and output enhancement.
In order to completely get rid of the FCA loss, a second methodology is proposed by replacing the silicon waveguide with As$_2$Se$_3$ waveguide, which experiences negligible FCA loss due to its low carrier mobility. Numerical simulation demonstrates that As$_2$Se$_3$ waveguides are a more energy-efficient integrated platform for amplification and lasing than silicon despite their smaller Raman gain. Compared to SWRL, seven times efficiency enhancement and 33% threshold reduction are simultaneously achieved for As$_2$Se$_3$WRL, without the need for any external voltage supply. Through comparison with the defined FOM for energy efficiency quantification, we further illustrated the advantage of As$_2$Se$_3$WRL in terms of device miniaturization and energy consumption.

The third methodology involves the introduction of silicon chalcogenide slot waveguide into SRS-based integrated devices, facilitating low-loss integration with current silicon platform. The proposed SCSWRA displays superior FCA suppression capability, as well as higher Raman on-off gain, excellent miniaturization capability and wider linear amplification range when compared to the well-established SWRA.
Chapter 5  COHERENT ANTI-STOKES RAMAN SCATTERING IN INTEGRATED WAVEGUIDE DEVICES

5.1  Introduction

Besides the SRS process discussed in Chapter 4, coherent anti-Stokes Raman scattering (CARS) process is another Raman process that has found many useful applications in the integrated optical waveguide platform. In this chapter, we will investigate the CARS process in both silicon and chalcogenide optical waveguide devices. All-optical wavelength conversion (AOWC) plays a critical role in the realization of future fast-speed wavelength-routed networks by eliminating wavelength continuity constraint and generating reconfigurable network [96]. AOWC in integrated waveguide devices based on nonlinear processes such as cross-phase modulation (XPM) and four-wave mixing (FWM) provides an attractive solution to that, due to their compact size and low-cost fabrication processes [62, 101, 102, 140, 141]. In addition, successful realization of efficient integrated wavelength converters could provide an efficient mean to connect various silicon photonics components with different operation wavelengths. An alternative approach for integrated AOWC can be achieved through coherent anti-Stokes Raman scattering (CARS) process [90]. When the phase mismatch between the pump, Stokes and anti-Stokes field is minimized, coherent information transfer from Stokes to anti-Stokes wavelength across more than 100nm span at the 1550nm telecommunication window occurs in the presence of a strong pump light [142]. Over the past few years, CARS-based wavelength conversion has been extensively investigated in silicon platform, driven by its advantages of wide detuning range and high efficiency [7, 89, 110]. However, silicon waveguide Raman
wavelength converters suffer from intrinsically inherent narrow bandwidth (105GHz), which hugely hinders their actual application in the wavelength-division multiplexing (WDM) systems. Although self-phase modulation (SPM) induced pump-pulse broadening can partially solve this problem, the methodology is far from satisfying with the limited bandwidth enhancement [7].

Unlike the extensive exploration of CARS process in the silicon waveguide Raman wavelength converter (SWRWC), the influence of it in the silicon waveguide Raman laser (SWRL) has not been investigated before. This chapter starts with the study of CARS process in silicon waveguides, both in SWRL and SWRWC. In particular, we analyze the influence of CARS in SWRL, as well as the nonlinear electronic contribution to the CARS process in SWRWC. We then go on to propose a novel Raman-assisted wavelength converter in chalcogenide waveguides, tackling the narrow bandwidth and low efficiency drawbacks of the SWRWC.

5.2 Investigation of the CARS Process in Silicon Waveguides

In Chapter 4, stimulated Raman scattering (SRS) has been investigated in silicon waveguides, realizing functional devices such as silicon waveguide Raman amplifiers (SWRA) and lasers (SWRL). Besides the dominant SRS process, the presence of a strong pump and Stokes field in the cavity of these devices triggers another Raman-assisted four-wave-mixing (FWM) process, namely the coherent anti-Stokes Raman scattering (CARS). In fact, anti-Stokes emission has been reported by two independent groups [87, 88] experimentally to confirm the existence of such a phenomenon in
SWRL. While intensive researches have been conducted for the SRS process in SWRL both theoretically [104, 105] and experimentally [5, 38], the influence of CARS on the performance of these devices have not been discussed before. One possible reason could be due to the generally weak contribution of CARS in the cavity, as compared to the dominant SRS process in SWRL [88]. In the following part, we specifically investigate this less dominant CARS process in SWRL. In particular, we study the performance of the SWRL when the cavity waveguide is dispersion-engineered to achieve perfect phase-matching condition. Under such a scenario, efficiency of CARS process is hugely enhanced to influence key laser performance indicators such as free-carrier absorption (FCA) loss and Stokes output power [127].

Fig. 5.1 (a) Variation of Stokes output power with input pump power from Model I (down triangles), Model II with $\Delta k = 4000 \text{m}^{-1}$ (up triangles) and Model II with $\Delta k = 0 \text{m}^{-1}$
Chapter 5: Coherent Anti-Stokes Raman Scattering in Integrated Waveguide Devices

(b) FCA loss analysis for non-phase-match at $\Delta k = 4000\text{m}^{-1}$ (solid line) and phase-match condition at $\Delta k = 0\text{m}^{-1}$ (dashed line)

For this part of the analysis, the two models approach is utilized to highlight the significance of APM modeling in investigating the less dominant physical process. Model I refers to the well-established steady-state model for SWRL proposed by M. Krause et al. [104], which only takes into account the dominant SRS process. Model II is our APM model. We start with analyzing the Stokes output and the FCA loss in SWRL under the influence of no CARS (Model I), non-phase-matched CARS and phase-matched CARS process. The reflectivities for pump and Stokes waves are set as: $R_{\text{front},p} = R_{\text{back},p} = R_{\text{front},s} = R_{\text{back},s} = 30\%$. The value for $\Delta k$ is set at $4000\text{m}^{-1}$ and $0\text{m}^{-1}$ to account for the non-phase-matched and phase-matched condition respectively.

Fig. 5.1 (a) presents the pump-to-Stokes conversion efficiency curve. Under non-phase-matching condition, the efficiency of CARS process is very low. Stokes output from SWRL is mainly due to the SRS process only. This results in simulation results from Model I (down triangles) being exactly the same results as Model II under non-phase-matching condition (up triangles). Such an exact matching of simulation results provides a direct justification for the accuracy of our proposed APM. When the phase condition is matched (circles), we observe a much higher Stokes output and a lower threshold. These differences can be explained by the occurrence of efficient CARS process under phase-matching condition, which is not addressed by Model I. In the view of quantum mechanics, CARS process generates Stokes and anti-Stokes photons by annihilating two pump photons [110]. When the phase-match condition is satisfied for pump, Stokes and anti-Stokes waves in the SWRL, efficient CARS process occurs to generate extra Stokes photons. These additional Stokes photons from CARS process subsequently boost the energy build-up process in the cavity to reduce lasing
threshold. At the same time, Stokes photons also contribute to the laser output and thus enhance it. Another possible explanation for the Stokes output enhancement is the increased depletion of pump power by CARS process, which subsequently reduces the FCA loss in the cavity that rises sharply with pump power \([129]\). FCA loss is quantified as the ratio of output Stokes power in the absence \((\tau_{\text{eff}}=0\text{ns})\) and presence \((\tau_{\text{eff}}=2.5\text{ns})\) of FCA loss in dB units. A significant loss reduction of more than 3dB is observed, as shown in Fig. 5.1 (b). In [91], D. Dimitropoulos et al. has illustrated the possibility of obtaining perfect phase-matching condition in silicon waveguides through careful dispersion engineering of the waveguide. Therefore we will not illustrate this further here. Utilization of such a phase-matched silicon waveguide cavity in SWRL could provide another effective methodology to simultaneously reduce the threshold and increase the Stokes output.
We proceed to further investigate the influence of end-facet reflectivity to the Stokes output of SWRL, when efficient CARS process occurs in the laser cavity. Compared to the previous SWRL end-facet reflectivity analysis in Chapter 4, an additional design freedom occurs here, namely the reflectivity for anti-Stokes wave. In order to facilitate the analysis, we assume a symmetrical cavity design such that $R_{\text{front, s}} = R_{\text{back, s}} = r_1$, $R_{\text{front, a}} = R_{\text{back, a}} = r_2$ and $R_{\text{front, p}} = R_{\text{back, p}} = r_3$. Stokes power displays a parabolic relationship with the variation of end-facet reflectivity for Stokes wave ($r_1$), as suggested in Fig. 5.2 (a) and (b). This is a unique characteristic of SWRL, which suffers from a strong FCA loss increases sharply with rising power [129]. Initial increment of $r_1$ retains more Stokes power in the cavity, stimulating more SRS and CARS processes to enhance the Stokes output. On the other hand, this enhanced portion of power also suffers larger FCA loss as it oscillates in the cavity. The loss increment, together with a smaller out-coupling ratio, tends to reduce the Stokes output. As $r_1$ further increases, the loss increment as well as the output reduction due to smaller Stokes transmission outplays the cavity enhancement effect, causing the Stokes output to drop. The maximized Stokes power is obtained at the trade-off point when $r_1 = 30\%$. Stokes output power drops slightly as $r_2$ changes from 0% to 100%, mainly due to the increased FCA loss as more anti-Stokes power is retained in the cavity. In Fig 5.2 (b), Stokes output decreases from maximum to 0 as $r_3$ increases, originated from the reduction in the pump power coupled into the cavity. The optimized end-facet reflectivity configuration to maximize the output Stokes power thus occurs at $r_1 = 30\%$, $r_2 = 0\%$ and $r_3 = 0\%$. 

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Besides the SWRL discussed above, CARS has also been widely investigated in silicon waveguides for wavelength conversion [7, 89-91]. It is well-known that the third-order susceptibility also has an electronic non-resonance part ($\chi^{NR (3)}$), which accounts for processes such as self-phase modulation (SPM), XPM and FWM. While these processes have been investigated separately for various applications [32, 94, 101], the influence of them in SWRWC has not been discussed before. In the following part, we perform a comprehensive investigation of the influence of these electronic non-resonant nonlinear processes on wavelength conversion efficiency in silicon Raman wavelength converters.

![Graph (a)](image1.png)

![Graph (b)](image2.png)

Fig. 5.3 (a) Variation of anti-Stokes output with $\Delta k$ and (b) Variation of wavelength conversion efficiency (dB) with pump power under normal, no electronic-nonlinear process influence ($\gamma=0$) and no CARS process ($g_r=0$) influence operation.
For our simulations, we adopt $\alpha_p = \alpha_s = \alpha_a = 0.39 \text{dB/cm}$, $L = 4.8 \text{cm}$, $\tau_{\text{eff}} = 1 \text{ns}$, $\lambda_p = 1550 \text{nm}$, $\lambda_s = 1686 \text{nm}$, $\lambda_a = 1434 \text{nm}$, $\beta = 0.5 \text{cm/GW}$, $\sigma_p = 1.44 \times 10^{-21} \text{m}^2$, $\sigma_s = 1.71 \times 10^{-21} \text{m}^2$, $\sigma_a = 1.23 \times 10^{-21} \text{m}^2$ and $g_r = 9.5 \text{cm/GW}$ according to experimental characterization \[121\]. The value for $\gamma$ is $2.43 \text{cm/GW}$, computed from non-linear refractive index measured in \[32\].

We start with the analysis of variation of anti-Stokes emission with respect to phase-mismatch factor $\Delta k$ as shown in Fig. 5.3 (a). Input pump and probe powers are 1.5W and 1mW respectively. When electronic nonlinear process is not taken into account ($\gamma = 0$), wavelength conversion efficiency symmetrically displays a $\text{sinc}^2$ relationship with respect to $\Delta k$. Maximum conversion efficiency occurs on the condition of $\Delta k = 0$. This is consistent with the results reported in \[89\], which assumes a constant phase relationship between pump, Stokes and anti-Stokes waves. However, this assumption becomes invalid when the pump power is high, such as in our simulated case here. Under such high pump power, dynamic nonlinear phase shift occurs to alter the phase relationship between the pump, Stokes and anti-Stokes waves. We monitor the phase evolution along the waveguide here by incorporating these electronic nonlinear terms into the APM modeling. Such symmetry no longer exists when we include the effects of SPM, XPM and FWM. This is mainly due to the deviation from the initial linear phase-match condition when the phases of the electromagnetic waves evolve along the waveguide. The maximum conversion efficiency now occurs at the positive $\Delta k$ region, in which the exact position depends on the magnitude of the nonlinear phase shift, influenced by the pump and probe powers. Another interesting point to note is that the presence of these electronic nonlinear processes are actually beneficial to the wavelength conversion process, supported by the reduction of conversion efficiency when we assume these processes are absent ($\gamma = 0$). Wavelength conversion efficiency based solely on electronic nonlinear processes is also shown ($g_r = 0$). The efficiency is about an order of
magnitude lower than that based on CARS process, justifying the capability of CARS process for conversion efficiency enhancement. Conversion efficiency first increases linearly as pump power increases, then saturates and decreases as pump power grows further as shown in Fig. 5.3 (b). The detrimental saturation originates from the high FCA loss in the silicon waveguide when pumping level is high. It should be noted that the conversion efficiency illustrated here is obtained under the linear phase matched condition ($\Delta k=0$), which is not necessarily the absolute maximum conversion efficiency for SWRWC. Dynamic phase shifts could cause a shift of the maximum efficiency position, resulting in a drop of efficiency in the $\Delta k=0$ position as shown in Fig. 5.3 (a). Wavelength conversion based on CARS (solid curve) is 15dB better than that based on FWM processes ($g_r=0$), as shown in Fig. 5.3 (b).

Due to the precious space on chip, device miniaturization (in this case waveguide length reduction) is an important consideration in the design of silicon photonic components. We perform a comprehensive three-dimensional analysis for conversion
efficiency as illustrated in Fig. 5.4, when both pump power and waveguide length vary. Two counter-acting effects occur as we increase the waveguide length; conversion efficiency enhancement due to longer interaction length and efficiency reduction from higher cavity losses. While the enhancement effect dominates in the shorter waveguide region, the losses become severe as the waveguide length grows. The conversion efficiency subsequently saturates then drops as waveguide length increases. The maximum conversion efficiency point occurs at the trade-off point when these two effects exactly cancel out each other. The position of balancing point depends on the severity of cavity loss suffered, mainly due to the pump power generated FCA loss. Shifting of maximum conversion efficiency point to the short waveguide length is observed as we increase the pump power. Waveguide miniaturization can thus be achieved with a high pumping power, illustrating a clear trade-off between device size and energy consideration. Optimized conversion efficiency of -9.1dB occurs at 1.5W pumping level with a waveguide length of 4cm.

5.3 Investigation of the CARS Process in Chalcogenide Waveguides

In the previous section, severe FCA-loss generated conversion efficiency saturation of SWRWC has been clearly illustrated. Together with the narrow conversion bandwidth, these two factors hugely hinder the development of SWRWC nowadays [7]. A close examination of these drawbacks reviews that they both are originated from the intrinsic properties of the silicon material. Replacement of the material platform presents an intuitive approach to combat the problem. Our investigation in Chapter 4 has shown that chalcogenide is a more energy-efficient platform for Raman
amplification and lasing than silicon. These materials possess the desirable combinations of high nonlinearity and Raman gain, intrinsically fast response time, negligible FCA loss as well as broad Raman gain bandwidth [136]. From this perspective, chalcogenide presents an ideal material alternative for integrated waveguide Raman wavelength converters. Among the chalcogenide glasses, As$_2$Se$_3$ is particularly appealing for CARS-based applications with its largest reported Raman gain of 5.1cm/GW [66]. Raman shift and bandwidth are measured at 7THz and 2THz respectively. In the following section, comprehensive theoretical investigation of the proposed As$_2$Se$_3$ waveguide Raman wavelength converters (As$_2$Se$_3$WRWC) will be presented, justifying its capability for future integrated AOWC applications [142].

![Schematic diagram of the As$_2$Se$_3$ waveguide Raman wavelength converter](image)

**Fig. 5.5 Schematic diagram of the As$_2$Se$_3$ waveguide Raman wavelength converter**

The efficiency of Raman-assisted wavelength conversion strongly depends on the phase-matching condition among pump, Stokes and anti-Stokes waves, which span more than 100 nm in wavelengths for As$_2$Se$_3$. Dispersion engineering the waveguide dimensions to minimize the phase-mismatch thus becomes essential under such wide wavelength spacing. Fig. 5.5 presents the As$_2$Se$_3$ rib waveguide used in this work, which has been demonstrated experimentally using complementary metal-oxide-
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semiconductor (CMOS) technology on SiO₂ substrates [71]. Low refractive index polyamide-imide (PAI) is utilized as the cladding with a coefficient of thermal expansion (CTE) matched to As₂Se₃. Effective phase-mismatch (κ) of pump, Stokes and anti-Stokes waves can be further divided into its various contributions as material dispersion (Δk_{MAT}), waveguide dispersion (Δk_{WD}), birefringence (Δk_B) and nonlinear phase shift (Δk_{NL}), as illustrated in Eq. 5.1 [91]. Here, P is the pump power.

\[ \kappa = \Delta k + \Delta k_{NL} = \Delta k_{MAT} + \Delta k_{WD} + \Delta k_B - 2\gamma P \]  

(5.1)

Conventional dispersion engineering design for phase-matching always starts from the computation of the material refractive index and dispersion, which is independently of the geometry of the waveguides. The refractive index of chalcogenide can be modeled as Eq. 5.2 [143] using the model from S. Wemple and M. DiDomenico, where \( E_o = 26 eV \) is the electronic oscillator energy and \( E_s = 4.1 eV \) is the Sellmeier gap energy for As₂Se₃. \( E_p \) is the photon energy.

\[ n^2 = 1 + \frac{E_o E_s}{(E_o)^2 - (E_p)^2} \]  

(5.2)

\[ \Delta k_{MAT} \approx -k_2 \Omega^2 - (k_4/12)\Omega^4 \]  

(5.3a)

\[ k_m = \left( \frac{d^m k}{d\omega^m} \right)_{\omega=\omega_p} \]  

(5.3b)

Material dispersion can then be computed from Eq. 5.3, by expanding the mode propagation constant in Taylor series and retaining up to the fourth-order terms to ensure sufficient accuracy [26]. \( \Omega \) is the Raman-shift, which is 7 THz for As₂Se₃ material. Fig. 5.6 (a) shows the material dispersion with respect to the pump wavelength in the region from 1.3µm to 1.7µm. Material dispersion of silicon is also
presented in the figure for the purpose of comparison using Sellmeier model of Li [144]. Our computed dispersion for silicon is slightly different from the values reported in [91]. This may be due to the contribution of the fourth-order dispersion \( (k_4) \), which is not included in [91]. In order to ensure sufficient accuracy around the near zero-dispersion wavelength range, fourth-order dispersion should be included into the Taylor series expansion. Material dispersion of As\(_2\)Se\(_3\) is around an order of magnitude smaller than that of silicon. We note here that smaller material dispersion might not directly imply a simpler phase-matching process. The actual dispersion engineering process is strongly dependent on the relative magnitude of the material, waveguide dispersions and birefringence.
Fig. 5.6 (a) Material phase-mismatch for As$_2$Se$_3$ and silicon. (b) Material ($\Delta k_{\text{MAT}}$), waveguide ($\Delta k_{\text{WD}}$) and linear phase-mismatch ($\Delta k$) for dispersion-engineered As$_2$Se$_3$ waveguides with respect to pump wavelength. Inset is the quasi-TE mode-field pattern of the As$_2$Se$_3$ waveguide

In order to obtain maximum anti-Stokes emission, one need to ensure that the effective phase-mismatch is minimized ($\kappa=0$). This can be achieved by compensating the material dispersion with waveguide dispersion, birefringence and nonlinear phase-shift according to Eq. 5.1. $\gamma$ is around 40 m$^{-1}$W$^{-1}$ for As$_2$Se$_3$ waveguides in the micron size, implying that nonlinear phase-mismatch ($\Delta k_{\text{NL}}$) is only 0.4 cm$^{-1}$ at the pump power of 500mW. This is approximately 40 times smaller than the material dispersion shown in Fig. 5.6 (a). We thus neglect its influence in the current discussion of material dispersion compensation. Waveguide dispersion and birefringence can be computed from the propagation constant obtained from the commercial finite-element solver Rsoft FemSIM. The material dispersion that we need to compensate is around -16cm$^{-1}$ in the 1550nm wavelength. According to our simulation, this can be achieved using waveguide dispersion alone, as shown in Fig. 5.6 (b). The dispersion-engineered waveguide dimensions are $H=W=1.8\mu$m, $h=0.53H$. Total linear-phase mismatch factor reaches zero for fundamental transverse electric (TE) mode in the 1550nm wavelength. This is very different from the phase-matching technique for CARS process in silicon waveguides, in which both waveguide dispersion and birefringence are utilized to compensate the strong normal material dispersion [91]. The pump, Stokes and anti-Stokes waves can thus co-propagate in the quasi-TE mode, enhance their interaction with the larger modal overlap. The comparable amplitude of the material and waveguide dispersions of As$_2$Se$_3$WRWC thus grants it a simple dispersion engineering process, presenting another advantage over the well-established
SWRWC. The quasi-TE mode field pattern are also shown in the inset of Fig. 5.6 (b), with effective core area \( A_{\text{eff}} \) computed as 2.15\( \mu \text{m}^2 \).

The flatness of the dispersion slope is another key parameter in the dispersion engineering of As\(_2\)Se\(_3\) waveguides for four-wave mixing (FWM)-based wavelength conversion (WC). It has been reported that FWM bandwidth can be approximately given by 

\[
\frac{4\pi}{k_2 L}
\]

with loss being ignored, where \( L \) is the interaction length and \( k_2 \) is the group velocity dispersion parameter [101]. The conversion bandwidth is thus inversely proportional to the square root of \( k_2 \), corresponding to the linear dispersion slope. A flat dispersion curve will result in a broad operation for FWM-based WC, which is often desirable in all-optical wavelength conversion application (AOWC).

Coherent anti-Stokes Raman scattering (CARS), being a subclass of FWM, generally follows a similar trend with the exception that the maximum bandwidth is simultaneously limited by the Raman gain bandwidth of the material (2 THz in As\(_2\)Se\(_3\)). Further engineering of the dispersion slope is possible to enhance the conversion bandwidth, through the introduction of the porous core [145] or dielectric over layers with appropriate CTE and refractive index [146]. Addition of porous core structure into the As\(_2\)Se\(_3\) waveguide, similarly to that in As\(_2\)Se\(_3\) fiber [145], results in extra design freedom in effective refractive index that can be utilized for dispersion slope engineering. Alternatively, dispersion engineering can be achieved through a double cladding structure with an inclusion of thin over layer [146]. Tight optical confinement and small dispersion curvature can be achieved simultaneously by carefully selecting the over-layer material refractive index and dimensions. Although silicon nitride (Si\(_3\)N\(_4\)) has been shown to be an effective over-layer material for the dispersion slope engineering of silicon waveguides, its application into our As\(_2\)Se\(_3\) waveguide
could cause possible thermal cracking due to its relatively small coefficient of thermal expansion (CTE) \[71\]. Alternative materials with CTE comparable to that of As\(_2\)Se\(_3\) and appropriate refractive index (between As\(_2\)Se\(_3\) and polyamide-imide cladding) are needed when incorporating this conformal dielectric over-layer structure into our device.

Given the feasibility to dispersion-engineer As\(_2\)Se\(_3\) waveguides, we proceed to investigate the characteristics and performances of As\(_2\)Se\(_3\) waveguide Raman wavelength converter (As\(_2\)Se\(_3\)WRWC) in the strong pump regime, using the well-established silicon waveguide Raman wavelength converters (SWRWC) as the benchmark. The same sets of simulation parameters for As\(_2\)Se\(_3\) waveguides as in Chapter 4 are employed as \(\lambda_p=1550\text{nm}, \lambda_s=1608\text{nm}, \beta=0.25\text{cm/GW}, a_p=a_s=a_d=0.25\text{dB/cm}, g_r=5.1\text{cm/GW} \) and \(L=4.8\text{cm}\). The anti-Stokes wavelength is expected at 1496nm \((\lambda_a=1496\text{nm})\), given the 7THz Raman shift of As\(_2\)Se\(_3\) material. \(\gamma\) is computed as 45.26 cm\(^{-1}\)GW\(^{-1}\) using \(n^2=2.4\times10^{-17}\text{m}^2/\text{W}\) measured in \[66\]. For comparison purpose, the same waveguide length and effective core area are assumed for the silicon counterpart. Other parameters of the silicon waveguide are set similarly as those in Chapter 5.2. Effective carrier lifetime is set as 1ns, assuming the application of a 25V reverse bias voltage. For the clarity of presentation, a schematic diagram of the experimental setup for CARS-based chalcogenide wavelength converter is shown in Fig. 5.7 below. The continuous wave pump and probe sources are first aligned with the TE polarization of the As\(_2\)Se\(_3\) waveguide, which is assumed to be phase matched in the pump wavelength. The two electromagnetic waves are subsequently combined using a wavelength division multiplexing (WDM) coupler. The multiplexed signals are then sent through the As\(_2\)Se\(_3\) waveguide to generate the
anti-Stokes signal. At the receiver end, a band-pass filtered followed by a photo-detector is used to detect the Stokes and anti-Stokes signal.

Fig. 5.7 Schematic diagram of experimental set-up for the chalcogenide waveguide Raman wavelength converter

In order to illustrate the influence of phase mismatch in the anti-Stokes emission, we start with the investigation of Stokes-to-anti-Stokes conversion efficiency when pump wavelength sweeps from 1500nm to 1700nm for As$_2$Se$_3$ WRWC as shown in Fig. 5.8 (a). The conversion efficiency is defined conventionally as the ratio (in dB) of the generated anti-Stokes output power over the input Stokes power. Input Stokes power is fixed at 1mW. Sinusoidal variation of efficiency with respect to pump wavelength is observed, similar to that of silicon as presented in previous section. This is due to the change of linear phase-match as the pump wavelength shifts, which subsequently results in the change of efficiency of the CARS processes. The conversion efficiency curves experience an asymmetrical shift towards the longer wavelength side when the pump power rises. We believe this is due to the dynamic nonlinear phase shift ($\Delta k_{NL}$),
which increases with pump power. Such a nonlinear shift is more significant in the As$_2$Se$_3$WRWC, mainly due to comparable magnitude of the Raman and electronic susceptibility.

![Graph](image)

**Fig. 5.8 (a)** Influence of pump wavelength on conversion efficiency for the As$_2$Se$_3$ waveguide under various pumping levels (b) Shift of PPMWL from DCWL with respect to pump power

Fig. 5.8(a) clearly illustrates that the zero dispersion wavelength (ZDW) does not always produce the highest conversion efficiency, especially when the pump power is high. In order to quantitatively characterize this dynamic nonlinear phase shift, we explicitly put forward the definition of perfect phase-match wavelength (PPMWL) here, which is defined as the pump wavelength with the highest conversion efficiency.
PPMWL varies with pump power and is the optimum operating pump wavelengths to achieve maximum conversion efficiency. This definition is very different from the conventional zero dispersion wavelength (ZDW), which is independent of pump power and remains unchanged once the waveguide dimensions are fixed. In our case, ZDW is 1550nm, as illustrated by the zero-crossing of Fig. 5.6 (b). The shift of PPMWL from ZDW will account for the direct contribution of dynamic nonlinear phase shift in the CARS phase-matching process. According to our simulation, this shift could reach around 45nm when the pump power is high (1W), resulting in a conversion efficiency difference of 20dB between the two wavelengths. While nonlinear dynamic phase shift might be too small to compensate for the strong normal material dispersion, we show here that it can cause significant deviation from the perfect phase match point. Careful consideration of the nonlinear dynamic shift is thus highly recommended when optimizing the conversion efficiency for As$_2$Se$_3$WRWC. This is very different from the case in SWRWC, in which nonlinear dynamic shift is always ignored [91]. We further characterize this shift of PPMWL with respect to pump power as shown in Fig. 5.8 (b). The observed linear relationship between the shift of PPMWL and pump power provides a further justification of our claim that this shift is caused by nonlinear phase shift ($\Delta k_{NL}=2\gamma P$). Such a behavior could provide a useful means to finely tune the operation wavelength of the As$_2$Se$_3$WRWC after it is fabricated by careful control of the input pump power.

For both WRWCs, conversion efficiency increases sharply at low pump power as shown in Fig. 5.9 (a). The anti-Stokes growth rate decreases as pump power increases before eventual saturation occurs. This could be due to the pump depletion and loss increment when the pump power grows, generated by two photon absorption of the pump waves. For the entire simulation range, As$_2$Se$_3$WRWC is much greener than
SWRWC in terms of energy consumption, supported by the higher conversion efficiency observed. The conversion efficiency reaches around 5.5dB at 1W pumping. This efficiency is more than 10dB better as compared to SWRWC, even without taking the energy dissipation of 25V bias-voltage in silicon waveguides into account. This could be attributed to the lower TPA and negligible FCA loss in As$_2$Se$_3$ waveguides. Another possible reason could be due to the larger Stokes photon energy in As$_2$Se$_3$ material compared to silicon, yielding higher output power when the same number of anti-Stokes photons is generated. We further elaborate the influence of dynamic nonlinear phase shift here by explicitly simulating the conversion efficiency under two different phase-matching scenarios. When dynamic nonlinear phase shift is not taken into account ($\Delta k = 0$), sinusoidal fluctuation of efficiency is observed with respect to pump power. Efficiency reduction as high as 20dB is observed, which is typical for a non-phase-matched case. Perfect phase-match is only obtained when we match the linear phase mismatch factor to the nonlinear phase shift. The observation re-emphasize our claim in the previous section that dynamic nonlinear phase shift plays a crucial role in determining the conversion efficiency of As$_2$Se$_3$WRWC through phase matching process. As a side note, the claim is redundant for SWRWC, illustrated by the negligible difference of efficiency curve in both cases. Fig. 5.9 (b) displays the variation of conversion efficiency with respect to input Stokes power. According to our simulation, no anti-Stokes emission is observed when the Stokes input is set to 0. Stokes input power is found to have negligible influence on conversion efficiency under various pump powers simulated, evident by the nearly flat conversion efficiency curve with respect to Stokes input power. The efficiency of CARS process is solely determined by the dominant input pump power as long as the Stokes input is turned on, in good agreement with the same observation in silicon waveguides [89]. We note here that this feature is highly desirable for wavelength
conversion in the actual optical network, in which conversion efficiency is insensitive to the incoming signal powers.

![Graph showing conversion efficiency vs. pump power and Stokes power](image)

**Fig. 5.9** (a) Influence of pump power on conversion efficiency for As$_2$Se$_3$ and silicon WRWC under various phase-match conditions (b) Variation of conversion efficiency with respect to input Stokes power for As$_2$Se$_3$ WRWC

Fig. 5.10 illustrates the distribution of pump, Stokes and anti-Stokes powers along the As$_2$Se$_3$ WRWC when the input pump power is 0.5W. Pump power decreases almost linearly along the waveguide, generating Stokes and anti-Stokes emission along the way. Pump depletion is only around 20% in the given waveguide length, suggesting a possibility for anti-Stokes emission enhancement through waveguide length increment.
Fig. 5.10 Distribution of pump, Stokes (×100) and anti-Stokes (×1000) power along the As$_2$Se$_3$ waveguide

We thus go on investigating the variation of conversion efficiency with respect to WRWC waveguide length, as shown in Fig. 5.11. Due to the precious space on chip, miniaturization of the footprint size (waveguide length in our case) is highly desirable for integrated photonic devices. Increment of waveguide length incurs two counter-acting effects: enhancement of anti-Stokes wave from longer interaction length and reduction of anti-Stokes wave due to the increased loss. For both As$_2$Se$_3$ and silicon WRWC under 0.5W pumping, conversion efficiency grows as waveguide length increases from a small value. The strong enhancement due to the high pump power in this region outplays the additional loss incurred. Pump depletion grows with further waveguide length increment, resulting in the weakening of its enhancement effect. The net outcome is the decreasing rate of the conversion efficiency growth. Maximum conversion efficiency occurs at the waveguide length in which pump is fully depleted. Any additional increment of the waveguide length yields no gain but losses, causing the conversion efficiency to drop. The optimized waveguide length for maximum conversion efficiency of As$_2$Se$_3$WRWC is 20cm, while that for SWRWC is 6.5cm. The difference could be due to the smaller loss in the As$_2$Se$_3$ waveguides, resulting in slower pump depletion. Although such a long length of 20cm might be
disadvantageous from the device footprint miniaturization point of view, we would like to point out that the value is only true in the present setting. Reduction in operation pump power or effective cross-sectional areas of the waveguide could yield a shorter optimized length with faster pump depletion. As shown in Fig. 5.11, the optimized device length is 12cm when the pump power reduces to 0.1W. Furthermore, other advance mechanisms such as bi-directional pumping scheme [129], double-pass geometry [147] and S-bend waveguide [5] can be applied to further resolve this device footprint issue.

![Conversion Efficiency vs Waveguide Length](image)

**Fig. 5.11** Influence of waveguide length on conversion efficiency for As$_2$Se$_3$WRWC (under various pumping powers) and SWRWC

In order to realize ultra-fast optical networks, information processing in the optical domain without going through the optical-to-electrical conversion is essential. All-optical pulse erasure constitutes one of the important components for all-optical signal process, enabling functions such as NOT/XOR logic gates [148], de-multiplexing [149] and label swapping [150]. Existing investigations of the CARS process mainly focus on the utilization of a strong pump power and a weak Stokes power, resulting in information transfer from the Stokes wavelength to the anti-Stokes wavelength [7, 79, 89, 90]. There is another regime in the CARS process involving a weak pump and a
strong Stokes wave. In this regime, information transfer occurs from the pump wavelength (signal) to the anti-Stokes wavelength. The different wavelength conversion mechanisms for these two regimes are illustrated in Fig. 5.12. Although the conversion efficiency and gain are limited, such a scheme results in the depletion of the original signal that is critical for special applications such as all-optical signal processing through pulse erasure [151, 152]. In this section, we theoretically investigate CARS process in the weak pump regime, in which a different wavelength conversion scheme is achieved with the depletion of the original signal. In order to avoid ambiguity in the terminologies, the pump wavelength of the CARS process is named as signal in the weak pump scheme and the Stokes wavelength is accordingly called Stokes pump.

Fig. 5.12 Schematic diagrams of Raman-assisted wavelength conversion in As$_2$Se$_3$ for 
(a) strong pump regime (conventional scheme) and (b) weak pump regime

We start with the analysis of the anti-Stokes and Stokes emission from As$_2$Se$_3$WRWC for both regimes with respect to pump power, as shown in Fig. 5.13. The same As$_2$Se$_3$
waveguide as the previous section is utilized and the signal input power is fixed at 1mW. While signal amplification as high as 10 times are observed in the strong pump regime (Fig. 5.12 (a)), signal depletion is observed for the weak pump scheme. In the weak pump region, signal (pump in the CARS process) is depleted continuously along the interaction length. This signal depletion subsequently results in the much lower conversion efficiency and anti-Stokes emission saturation above 0.4 W Stokes pump.

The trend of the anti-Stokes emission curve is completely reversed in the strong pump region, in which rapid anti-Stokes waves growth is only observed above 0.4W pump power. From the view of energy efficiency, the two regimes thus experience very different optimized operation conditions. While high pump power is desirable for conversion efficiency enhancement in the strong pump scheme, increment of the Stokes pump will yield very limited anti-Stokes growth in the weak pump scheme.

Fig. 5.13 Variation of anti-Stokes and signal emission with respect to pump power for
(a) strong pump and (b) weak pump regime
Fig. 5.14 The influence of signal and Stokes pump power on (a) Conversion efficiency and (b) Signal depletion ratio for As$_2$Se$_3$-WRWC in weak pump regime

For specific applications such as intensity modulation through signal erasure, depletion of the original signal is critical in addition to the anti-Stokes conversion efficiency. We thus characterize both the signal depletion ratio and conversion efficiency with respect to Stokes pump and signal power here in Fig. 5.14. No conversion is observed when either Stokes pump or signal input power is zero, illustrating the importance of the presence of both waves for the occurrence of CARS process in this regime. Both signal and Stokes pump power influence conversion efficiency in a very similar way, as shown in Fig. 5.13 (a). Efficiency increases sharply when the power rises from zero, and then grows with a decreasing rate before eventual saturation takes place with the increasing power. This is very different from the case in the strong pump regime, where the signal has little influence in the
conversion efficiency. A possible explanation for the importance of signal input power in the weak pump region may be due to the depletion nature of signal along the interaction length.

We explicitly defined signal depletion ratio (SDR) here as the ratios (percentage) of the output signal power as compared to the input signal power. SDR increases with Stokes pump power increment, in a similar pattern as the conversion efficiency. On the other hand, SDR remains mostly constant with respect to the growing signal power. Signal depletion is thus dominated by the Stokes pump power. 78% signal depletion is observed when the Stokes pump power is 1W. We would like to point out that the SDR defined by us measures the overall signal depletion capability of the whole device rather than the CARS alone. Both linear and nonlinear losses of the waveguide contribute to this overall signal depletion, resulting in the non-zero depletion when both Stokes pump and signal power is zero. The SDR can be further reduced through the use of longer waveguide or high Stokes pump power.

5.4 Summary

In conclusion, this chapter presents an in-depth theoretical investigation of coherent anti-Stokes Raman scattering (CARS) process in both silicon and chalcogenide waveguides. The influences of CARS and electronic nonlinear processes are analyzed in SWRL and SWRWC respectively, which are considered as negligible in the previous studies. We find that simultaneous lasing threshold suppression and Stokes output enhancement can be achieved when CARS process occurs efficiently in the silicon waveguide Raman laser cavity. Dispersion-engineering the silicon waveguide
for phase matching thus presents another attractive approach to enhance the energy efficiency of SWRL. Electronic nonlinear processes such as SPM, XPM and FWM could induce nonlinear dynamic phase shifts between optical waves as they propagate along the silicon waveguide, resulting in a shift of the maximum conversion efficiency point to the positive $\Delta k$ region.

In addition, wavelength conversion through CARS process is proposed and comprehensively investigated for the first time in chalcogenide waveguide. We show that the normal material dispersion can be compensated by the engineered waveguide dispersion alone in As$_2$Se$_3$ platform. However, nonlinear dynamic phase shift can induce significant fluctuation from this linear phase-match condition, resulting in 20dB conversion efficiency reduction. We characterize this phenomenon using the shift of explicitly defined perfect phase-match wavelength (PPMWL) from ZDW. A linear shift is observed with respect to pump power. In spite of its smaller Raman gain, As$_2$Se$_3$ waveguides are found to be a greener platform for Raman-assisted wavelength conversion than silicon waveguides. Ultra-high conversion efficiency of 5.5dB can be achieved under 1W pumping, more than 10dB better than its silicon counterpart. While previous analysis for CARS process mainly focuses on the strong pump regime, we explore the characteristics of the less popular weak pump regime by using the pump wavelength as signal. Signal depletion up to 78% is observed, simultaneously with anti-Stokes generation in the weak pump regime. The conversion efficiency is generally low and saturates quickly when either signal or Stokes pump power is high. This is very different from the case in the strong pump regime, in which conversion efficiency is solely determined by the pump power.
Chapter 6  PUMP-TO-STOKES RELATIVE INTENSITY NOISE (RIN) TRANSFER

6.1 Introduction

In addition to the energy efficiency, pump-to-Stokes relative intensity noise (RIN) has long been identified as another key performance parameter for Raman amplifiers [112] and lasers [118]. The fast gain dynamics of the stimulated Raman scattering (SRS) process gives rise to noise figures of these Raman-based devices that are distinct from conventional erbium-doped fiber amplifiers (EDFA) [153] and fiber lasers [154]. Through SRS process, small amplitude perturbations residing on the pump waves characterized as RIN are transferred to the Stokes wave, degrading the noise figure of the devices [115]. Over the past decades, pump-to-Stokes RIN transfer characteristics have been extensively investigated in Raman amplifiers and lasers in both the fiber [114, 155] and silicon waveguide platforms [116, 119].

In this chapter, we extend the study of the pump-to-Stokes RIN transfer characteristics to our proposed As$_2$Se$_3$ waveguide Raman amplifiers (As$_2$Se$_3$WRA) and lasers (As$_2$Se$_3$WRL), for the first time to the best of our knowledge. We start from deriving the mathematical tools to analyze RIN transfer in chalcogenide waveguide Raman amplifiers and lasers from the APM modeling. Based on the derived numerical model, we investigate the RIN transfer of As$_2$Se$_3$WRA and As$_2$Se$_3$WRL, using their well-known silicon counterparts as benchmark. In addition, we propose two RIN transfer suppression solutions for As$_2$Se$_3$WRL, through the application of either bi-directional pumping scheme or cavity optimization.
6.2 Numerical Modeling for RIN Transfer in Waveguide

Raman Amplifiers and Lasers

The pump-to-Stokes RIN transfer is generally distinct for various noise frequency components, which can range from zero to several gigahertz in the typical pump RIN spectrum [117]. Assuming that the intensity fluctuation has sinusoidal time dependence and the modulation indexes can be separated, we can express the intensity of the pump and Stokes waves along the waveguide as Eq. 6.1 below [116]. RIN is modeled as small perturbations on the mean power of each light beam. Subscripts \( p \) or \( s \) denote pump and Stokes components, while superscripts ‘\( f \)’, ‘\( b \)’ represent the forward- and backward-propagating waves respectively. The axis along the waveguide length is denoted as the \( z \)-axis. \( \bar{I}(z) \) is the steady state intensity without any RIN noise. \( n^{f,b}(z) \) and \( m^{f,b}(z) \) are the spatial modulation indices for pump and Stokes waves respectively. \( \Omega \) is the angular frequency of the noise component (\( \Omega=2\pi f \)).

\[
I_p^{f,b}(z,t) = \bar{I}_p^{f,b}(z)[1 + n^{f,b}(z) \exp(i\Omega t)] \quad (6.1 \text{ a})
\]

\[
I_s^{f,b}(z,t) = \bar{I}_s^{f,b}(z)[1 + m^{f,b}(z) \exp(i\Omega t)] \quad (6.1 \text{ b})
\]

As the intensity terms in Eq. 6.1 contain time components, the steady-state APM modeling becomes insufficient to solve them. This issue can be easily resolved by including additional terms to address the group velocities into the steady-state modeling, as shown in Eq. 6.2 below [117]. \( v_i \) is the group velocity at wavelength \( \lambda_i \), and the terms containing it take into account the temporal evolution of the optical waves as well as the “walk-off” between the pump and Stokes pulses. \( g_r \) stands for Raman gain coefficient, \( \alpha \) is linear propagation loss and \( \beta \) is TPA coefficient.
Chapter 6: Pump-to-Stokes Relative Intensity Noise (RIN) Transfer

\[
\pm \frac{dI_{p}^{f,b}}{dz} + \frac{1}{v_p} \frac{dI_{p}^{f,b}}{dt} = [-g_r \lambda_p (I_{s}^{f} + I_{s}^{b}) - \alpha_p - \beta(I_{p}^{f,b} + 2I_{p}^{b,f} + 2I_{s}^{f} + 2I_{s}^{b})]I_{p}^{f,b} \tag{6.2 a}
\]

\[
\pm \frac{dI_{s}^{f,b}}{dz} + \frac{1}{v_s} \frac{dI_{s}^{f,b}}{dt} = [g_s (I_{p}^{f} + I_{p}^{b}) - \alpha_s - \beta(I_{s}^{f,b} + 2I_{s}^{b,f} + 2I_{p}^{f} + 2I_{p}^{b})]I_{s}^{f,b} \tag{6.2 b}
\]

By substituting Eq. 6.1 into Eq. 6.2, we can derive the following differential equation sets (Eq. 6.3) by separating the terms for the steady-state optical intensities and spatial modulation indices. Terms containing higher-order multiples of the modulation index are neglected, under the assumption that the intensity fluctuation is much smaller than the steady state intensity \(|m|^2, |n|^2, |m*n|^2 << 1\) [116].

\[
\pm \frac{d\bar{I}_{p}^{f,b}}{dz} = [-g_r \lambda_p (\bar{I}_{s}^{f} + \bar{I}_{s}^{b}) - \alpha_p - \beta(\bar{I}_{p}^{f,b} + 2\bar{I}_{p}^{b,f} + 2\bar{I}_{s}^{f} + 2\bar{I}_{s}^{b})]\bar{I}_{p}^{f,b} \tag{6.3a}
\]

\[
\pm \frac{d\bar{I}_{s}^{f,b}}{dz} = [g_s (\bar{I}_{p}^{f} + \bar{I}_{p}^{b}) - \alpha_s - \beta(\bar{I}_{s}^{f,b} + 2\bar{I}_{s}^{b,f} + 2\bar{I}_{p}^{f} + 2\bar{I}_{p}^{b})]\bar{I}_{s}^{f,b} \tag{6.3b}
\]

\[
\pm \frac{dn_{p}^{f,b}}{dz} = -\frac{1}{v_p} ((\Omega)n_{p}^{f,b} - g_r \lambda_p (\bar{I}_{p}^{f} m^{f} + \bar{I}_{p}^{b} m^{b}) - \beta(\bar{I}_{p}^{f,b} n_{p}^{f,b} + 2\bar{I}_{p}^{b,f} n_{p}^{b,f} + 2\bar{I}_{p}^{f} m^{f} + 2\bar{I}_{p}^{b} m^{b})) \tag{6.3c}
\]

\[
\pm \frac{dn_{s}^{f,b}}{dz} = -\frac{1}{v_s} ((i\Omega)m_{p}^{f,b} + g_s (\bar{I}_{p}^{f} n^{f} + \bar{I}_{p}^{b} n^{b}) - \beta(\bar{I}_{p}^{f,b} m_{p}^{f,b} + 2\bar{I}_{p}^{b,f} m_{p}^{b,f} + 2\bar{I}_{p}^{f} n^{f} + 2\bar{I}_{p}^{b} n^{b})) \tag{6.3d}
\]

One should note that Eq. 6.3a and 6.3b are identical to our APM modeling for chalcogenide waveguide Raman amplifier and laser (Eq. 3.7). We therefore employ a two-step approach to simulate the RIN transfer of these devices. The steady-state noise-free optical intensity is computed in advance from APM modeling using the appropriate boundary conditions. The computed steady-state intensities can be subsequently put into Eq. 6.3 (c) and (d) to obtain the modulation index evolutions.
The numerical modeling derived here thus is an extension of APM to account for relative intensity noise, rather than a completely new one. Boundary conditions for the modulation indices are shown in Eq. 6.4, obtained by separating out the steady-state boundary conditions as shown in Eq. 4.1. The boundary conditions can be applied to both one-way and bi-directional pumping schemes, as shown in Fig. 6.1. \( I_p^0 \) and \( I_p^L \) are the input pump intensities, while \( n^0 \) and \( n^L \) are the initial pump intensity fluctuations.

Pump-to-Stokes RIN transfer \( H(\Omega) \) for the angular frequency \( \Omega = 2\pi f \) can then be computed from its conventional definition in Eq. 6.5.

\[
n^f(0) = \frac{I_p^0}{I_p^0(0)} n^0 (1 - R_{p,0}) + \frac{I_p^0(0)}{I_p^0(0)} n^0(0) R_{p,0}, m^f(0) = m^0(0) \quad (6.4a)
\]

\[
n^b(L) = \frac{I_p^L}{I_p^L(L)} n^b (1 - R_{p,L}) + \frac{I_p^L(L)}{I_p^L(L)} n^b(L) R_{p,L}, m^b(L) = m^0(L) \quad (6.4b)
\]

\[ H(\Omega) = 10 \log_{10} \frac{|m(L, \Omega)|^2}{|n^0|^2} \quad (6.5) \]
6.3 RIN Transfer Characteristics for the Waveguide Raman Amplifier

Unlike the extensive investigation of the RIN transfer characteristics of silicon waveguide Raman amplifiers (SWRA) [115-117], no previous study of the RIN transfer of chalcogenide waveguide Raman amplifiers has been reported before. In this section, we present a comprehensive analysis for the pump-to-Stokes RIN transfer of As$_2$Se$_3$ waveguide Raman amplifiers (As$_2$Se$_3$WRA), using its well-established silicon counterpart as benchmark for comparison. All the reflectivity is thus set to 0% in this chapter. Fig. 6.2 illustrates the RIN transfer spectrum for both SWRA and As$_2$Se$_3$WRA, covering the frequency range from 1 MHz to 1THz. Initial pump fluctuation is set at 1% ($n^0=0.01$; $n^L=0$). Results for SWRA are based on the non-instantaneous FCA model described in [117], using parameters presented in our earlier work [129]. As normal SWRA does not produce any net optical gains [136], the SWRA referred to in the following sections is assumed to have a 25V reverse-bias applied across the waveguide with an effective carrier life time of 1ns [121]. Identical waveguide length and dimensions are utilized for both waveguide Raman amplifiers (WRA) for comparison purpose. For both WRAs, pump power is 1W. This is well within the optical damage threshold of the As$_2$Se$_3$ material, which can be further enhanced by anti-reflection (AR) coating and improvement of waveguide quality.

The RIN transfer spectrum for a co-pumped waveguide Raman amplifier can be clearly classified into two distinct regions, with the boundary positions at around 1GHz. In the high-frequency (HF) region ($f>1$GHz), As$_2$Se$_3$WRA outperforms SWRA with a 3dB lower RIN transfer. This could be attributed to the material’s lower Raman gain coefficient, limiting the SRS interaction between pump and Stokes waves. On the
other hand, the RIN transfer of SWRA is much lower in the low-frequency (LF) region ($f<1\text{GHz}$). This is mainly due to the free-carrier generated RIN suppression effect, which limits the RIN transfer for frequency components that is below the inverse of the free carrier recombination time ($1/\tau_{ef}=1/1\text{ns}=1\text{GHz}$) [117]. One should note that the free-carrier induced RIN suppression is strongly dependent on the free-carrier density, and thus the pump power. The RIN transfer of SWRA is thus not always better than $\text{As}_2\text{Se}_3\text{WRA}$. This point will be further elaborated in the later part of this section, when we discuss the influence of pump power in RIN transfer. The trend of the results for SWRA matches well with that reported in [117], providing a direct justification of the validity of our modeling.

Besides the absolute amplitude of the RIN transfer, corner frequency is another important characteristic when analyzing RIN transfer spectrum for waveguide Raman amplifiers [116]. When the noise frequency exceeds a cut-off frequency ($f_c$) as defined in Eq. 6.6, the RIN transfer starts to drop (with oscillations) due to the “walk-off” averaging effect of the noise [112, 116, 117]. The corner frequency is thus defined as
the frequency when the RIN transfer drops by 3dB from the constant plateau due to this averaging effect. From Eq. 6.6, two direct conclusions can also be drawn for the behavior of the corner frequency. Corner frequency decreases with waveguide length ($L$); and the co-pumped corner frequency is much higher than that of the counter-pumping case. Both co-pumped SWRA and As$_2$Se$_3$WRA thus have similar corner frequencies at around 100GHz, while that for the counter-pumped scheme is around 0.2 GHz. The large difference arises from the nature of the “walk-off”, which results in a very different critical frequency [116]. As$_2$Se$_3$WRA suffers from higher RIN transfer as compared to the SWRA, mainly due to the combination of low cut-off frequency in the counter-pumping scheme and the free-carrier induced RIN suppression of silicon waveguides in these low-frequency region. Counter-pumping scheme thus is not recommend for the As$_2$Se$_3$WRA from this perspective.

$$f_c = \frac{v_p v_s}{v_p + v_s}$$

(6.6)

![Graph showing RIN transfer vs. pump power for co-pumped SWRA and As$_2$Se$_3$WRA in the low-frequency (LF) region.](image)

**Fig. 6.3** Influence of pump power in the RIN transfer for co-pumped SWRA and As$_2$Se$_3$WRA in the low-frequency (LF) region
In the earlier discussion, RIN transfer of As$_2$Se$_3$WRA has been shown to be higher than SWRA in the low-frequency (LF) region. We would like to point out that this conclusion is drawn due to the high pump power (1W) utilized in the previous discussion, as shown in Fig. 6.3. Increment of pump power induces a larger RIN transfer from the pump to the Stokes wave through enhanced SRS process for co-pumped As$_2$Se$_3$WRA. This scenario is much more complex for SWRA. Two-counter acting effects influence the RIN transfer of SWRA when the pump power rises: RIN transfer increment from enhanced SRS effect and RIN transfer suppression due to the higher free-carrier concentration. Enhanced SRS effect dominates in the low-pump regime, causing the RIN transfer to elevate with increasing pump power. When the pump power increases further above 0.7W, free-carrier-induced RIN transfer becomes significant. The RIN transfer subsequently drops with pump power increment, forming a maximum RIN transfer for co-pumped SWRA at 0.7W pumping level point. Below 0.7W pumping, As$_2$Se$_3$WRA still outperforms SWRA with a lower RIN transfer in the LF region.

![Fig. 6.4 Variation of the RIN transfer with respect to input Stokes modulation index ($m^0$) for co-pumped As$_2$Se$_3$WRA](image-url)
Previous investigations on RIN transfer have assumed zero input Stokes modulation index ($m^0=0$). This might not be valid in the practical application, in which noisy signals occur after long distance of transmission. As shown in Fig. 6.4, rapid raise of RIN transfer occurs when the initial modulation depth exceeds 0.01%. RIN transfer grows by 20dB when the modulation depth increases to 1%. Simulation for initial Stokes modulation index above 1% is beyond the reach of our mathematical modeling, in which the small perturbation assumption is violated. When the initial intensity fluctuation is high in the Stokes signal, the RIN transferred from the pump waves could induce a multiplication effect on it. Input Stokes signal noise suppression is thus highly recommended to control the RIN transfer in As$_2$Se$_3$WRA. Average RIN transfer increases with waveguide length, mainly contributed by the elongated SRS interaction to enhance the noise transfer. 3dB cut-off input Stokes modulation index occurs at $m^0=0.04\%$ for all three waveguide length, representing a maximum tolerance value for the input signal noise. Furthermore, RIN transfer curves converge for three different waveguide lengths when the initial Stokes modulation depth exceeds 1%, suggesting negligible influence of device length for a noisy input signal.

![Variation of the RIN transfer with respect to input pump modulation index ($n^p$) for co-pumped As$_2$Se$_3$WRA](image)

Fig. 6.5 Variation of the RIN transfer with respect to input pump modulation index ($n^p$) for co-pumped As$_2$Se$_3$WRA

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Reversed trend occurs for the input pump modulation index, as shown in Fig. 6.5. RIN transfer growth is only observed when pump modulation drops below 0.01%. When the initial pump intensity fluctuation is high, the RIN transfer remains constant. The critical parameter that determined the RIN transfer is thus the ratio of the initial intensity fluctuation in the Stokes signal to the pump, rather than the absolute Stokes modulation index \(m^0\) itself. In order to obtain a comprehensive understanding of this concept, we explicitly defined the MN ratio (MNR) here as Eq. 6.7. Recall from Eq. 6.5, RIN transfer \(H(\Omega)\) is primarily computed from the forward Stokes modulation index at \(z=L\) \((m^f(L))\) for As\(_2\)Se\(_3\)WRA. When MNR is small \((m^0<<n^0)\), \(m^f(L)\) can be approximated using Eq. 6.8 below by analytically solving Eq. 6.3 (d). Substituting Eq. 6.8 into Eq. 6.5, one can draw the obvious conclusion that the RIN transfer is independent of \(m^0\) and \(n^0\). When the MNR is large, this analysis is no longer true and the RIN transfer becomes sensitive to the input modulation index. In order to control the RIN transfer in As\(_2\)Se\(_3\)WRA, the initial Stokes intensity fluctuation must be kept small relative to the pump intensity fluctuation. We note here that the validity of this analysis can also be applied to other Raman amplifiers in silicon or fiber platforms.

\[
MNR = \frac{m^0}{n^0} \quad (6.7)
\]

\[
m^f(L, \Omega) = (+g, \bar{I}_p^f n^f - 2\beta\bar{I}_p^f n^f)L \quad (6.8a)
\]

\[
n^f = n^0 L(\frac{i\Omega}{\nu_p} - \beta\bar{I}_p^f) \quad (6.8b)
\]

Three distinct physical processes occur in As\(_2\)Se\(_3\)WRA when pump and Stokes waves propagate along the As\(_2\)Se\(_3\) waveguide, namely linear propagation loss (LPL), two-photon absorption (TPA) and stimulated Raman scattering (SRS). It will be very
interesting to explore the various contributions of these processes to the RIN transfer between pump and Stokes waves, as shown in Fig. 6.6. SRS process is clearly illustrated as the dominant process for pump-to-Stokes RIN transfer, contributing 20dB higher RIN transfer compared to the TPA process. This could be attributed to the low TPA efficient in the chalcogenide glasses, limiting the pump-Stokes interaction through it. Although linear propagation loss (LPL) does not facilitate any pump-Stokes interaction, our simulation shows that LPL does play a part in the pump-to-Stokes RIN transfer. We believe this is due to the influence of LPL in the steady-state pump/Stokes intensity ($I_p/I_s$), which in turn affects the evolution of modulation index and RIN transfer.

![Fig. 6.6 Influences of various physical processes on RIN Transfer in As$_2$Se$_3$WRA](image.png)
6.4 RIN Transfer Characteristics for the Waveguide Raman Laser

As$_2$Se$_3$ waveguide Raman lasers (As$_2$Se$_3$WRL) have been proposed and demonstrated in Chapter 4.3 as an attractive alternative to silicon waveguide Raman lasers (SWRL) for on-chip lasers, highlighted by their higher conversion efficiency and lower lasing threshold [136]. Very limited research has been conducted for the RIN transfer of chip-scale Raman lasers up to today, which is expected to experience very different behavior compared to the fiber Raman laser [118] due to its short device length. X. Liu et al. did present an investigation of RIN transfer in SWRL [119]. However, their modeling employed the instantaneous free carrier absorption assumption, which has been pointed out by I. Rukhlenko et al. to be inaccurate [117]. In this section, we present a comprehensive investigation of the pump-to-Stokes RIN transfer for both the As$_2$Se$_3$WRL and SWRL [156]. In particular, we propose a novel RIN transfer suppression solution by using bi-directional pumping scheme.

We start with analyzing the RIN transfer frequency spectrum for both As$_2$Se$_3$WRL and SWRL, as shown in Fig. 6.7. Simulation parameters for As$_2$Se$_3$WRL are set as \( L=5.45\text{cm} \), \( \alpha_p=\alpha_s=0.25\text{dB/cm} \), \( \beta=0.25\text{cm/GW} \) and \( g_r=5.1\text{cm/GW} \), \( \lambda_p=1550\text{nm} \) and \( \lambda_s=1608\text{nm} \) [136]. Results for SWRL are based on the non-instantaneous free-carrier recombination model described in [117], using parameters presented in our earlier work [129]. Reflectivity for pump and Stokes waves are \( R_{p,0}=R_{p,L}=0\% \) and \( R_{s,0}=R_{s,L}=90\% \) respectively. Initial pump fluctuation is set at 1\% \( (n^h=0.01; n^l=0) \). Pump power for both WRLs is 1 W, which represents the most energy-efficient operating point for the As$_2$Se$_3$ waveguide Raman laser according to our previous investigation [136]. This is well-within the optical damage threshold of As$_2$Se$_3$ material, which can be further
enhanced by methodologies such as anti-reflection (AR) coating and improvement of waveguide quality [59]. Furthermore, we would like to point out that 1W pumping is not the prerequisite to validate the argument in this chapter. Other than the average RIN transfer and its contrast, the trend of RIN transfer spectrum is invariant under the variation of pump power [118]. We thus believe that the amplitude of the pump power will not play a crucial role in our overall theoretical analysis.

![RIN transfer spectra for SWRL, one-way and bi-directionally pumped As₂Se₃WRL](image)

**Fig. 6.7** RIN transfer spectra for SWRL, one-way and bi-directionally pumped As₂Se₃WRL

Maximum RIN transfer for As₂Se₃WRL occurs in the multiples of inverse of round-trip time ($f = \frac{v_g}{2L} = 1$ GHz), in which $v_g$ is the group velocity of light in the waveguide [118]. SWRL thus has a longer rough trip time, which subsequently results in more RIN transfer resonance peak and high noise. Compared to SWRL, pump-to-Stokes RIN transfer is around 5dB lower for As₂Se₃WRL across all frequencies simulated above 300MHz. This could be attributed to the smaller Raman gain coefficient of the As₂Se₃ material, which results in a lower noise transfer from pump to Stokes wave. The case becomes a little bit different below 300MHz, because
efficient FCA-induced RIN suppression occurs in this region for SWRL [117]. The suppression effect is dominant when the noise frequency falls well below the inverse of the effective free carrier life time (1ns) in the silicon waveguide. This observation is not presented in the analysis performed by X. Liu et al., which overlooked this finite recombination time of free carriers.

Bi-directional pumping scheme has been introduced in SWRL as an effective FCA suppression technique to enhance the Stokes output [129]. However, influence of the scheme on RIN transfer has not been investigated in the context of Raman laser before in any material platforms. We find that RIN transfer peak in the odd multiples of the inverse round-trip time can be significantly suppressed from this scheme, as shown in Fig. 6.7. This is mainly due to the reduction of the round-trip time from the scheme, which subsequently results in the doubling of resonance frequency or free-spectral range FSR ($f = v_g/L = 2 \text{ GHz}$). Maximum RIN reduction over 35dB occurs in the fundamental cavity resonance frequency of 1 GHz. For comparison purpose, equal amount of pump power is supplied to the WRL for both one-way and bi-directional pumping schemes. For the symmetrical splitting of pump power illustrated in Fig. 6.7, amplitude of the RIN transfer in the common resonance peaks is similar for both one-way and bi-directional pumping schemes. Further reduction of the common on-resonance RIN transfer peak can be achieved by employing an asymmetrical splitting of the pump power, as illustrated in Fig. 6.8. Although the RIN suppression effect is smaller in the odd multiples of the FSR, RIN reduction across all frequencies is achieved. We note here that validity of this analysis for bi-directionally pumping scheme can be extended to other material platforms such as fiber and silicon waveguide Raman lasers.
Cavity designs and optimizations in terms of waveguide length play critical roles in the interaction of pump and Stokes waves through SRS process in WRL, which can therefore influence the pump-to-Stokes RIN transfer. These factors require careful investigation to manage the RIN transfer in As$_2$Se$_3$WRL. Although it is obvious that increment of waveguide length reduces the FSR and resonance frequency (as shown in...
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Fig. 6.9), the characteristics of the RIN transfer in the on-resonance peak and off-resonance dip are much more complex. Two counter-acting physical processes occur when the waveguide length varies: reduction of RIN transfer from longer SRS interaction length and growth of RIN transfer due to the increased losses (both linear and nonlinear losses). In the SRS-dominated region (L<5cm), intensive pump-Stokes interaction through SRS occurs due to the high pump power in the cavity. Increment of waveguide length in this region thus results in enhanced SRS interaction and a rapid drop of the on-resonance RIN transfer peak, as shown in Fig. 6.10.

![Graph showing RIN transfer with respect to waveguide length](image)

**Fig. 6.10 Variation of the on-resonance (dotted line) and off-resonance (solid line) RIN transfer with respect to the waveguide length for As$_2$Se$_3$WRL.**

As waveguide length further increases into the loss-dominated region (L>5cm), pump power are nearly depleted and the influence of SRS process becomes negligible. On resonance RIN transfer peak subsequently decreases in a slower rate in the loss dominant region before eventually reaching the minimum value of 6.2dB at 8cm waveguide length, as shown in Fig. 6.10. Given the less than 5% on-resonance RIN transfer variation in the loss reduction region, 5cm waveguide length is thus
recommended to keep the device compact. We note here that this value is very different from the 3cm optimization length for energy efficient, suggesting a trade-off between energy efficiency and noise performance. The trend for off-resonance RIN transfer dip is slightly different, in which increment of waveguide length results in the reduction of the dip with a constant decreasing rate in both regions. This might be due to the out-of-phase weak modulation of Stokes waves in the off-resonance scenario, which moderates the RIN transfer due to SRS interaction. Overall RIN transfer in $\text{As}_2\text{Se}_3$ WRL decreases with increment in waveguide length.

![Graph showing RIN transfer and Stokes output vs pump power](image)

**Fig. 6.11 Influence of pump power in the off-resonance RIN transfer dip and Stokes output for $\text{As}_2\text{Se}_3$ WRL.**

Similar trend occurs for the variation of the off-resonance peak with respect to the pump power. Off resonance RIN transfer peak grows as pump power increases above threshold, mainly due to the enhanced SRS processes in the waveguide, as shown in Fig. 6.11. This actually corresponds to an overall RIN transfer reduction. The effectiveness of this RIN suppression from pump power growth decreases as we further increase the pump power. High pump power is thus beneficial for both RIN transfer suppression and Stokes output. This is very different from the case in fiber, in
which off-resonance RIN transfer dip decreases with pump power increment [118]. On the other hand, the on-resonance RIN transfer peak of As$_2$Se$_3$WRL decreases with pump power growth. As similar trend has been reported in the fiber [118] and silicon waveguide [119] platforms, we neglect the analysis here.

Linear propagation loss has been identified as the most dominant loss mechanism for chalcogenide waveguide Raman lasers [136]. We thus investigate its influence on the RIN transfer here for bi-directionally pumped As$_2$Se$_3$WRL, as shown in Fig. 6.12. Linear propagation loss is found to have limited influence on the pump-to-Stokes RIN transfer. RIN transfer spectrum displays similar pattern under various linear losses, with slightly different average values and contrast. This is much expected because the linear loss ($\alpha$) term is actually absent from the differential equation set describing the evolution of the modulation index along the waveguide (Eq. 6.3). Linear loss thus only influences the RIN transfer indirectly, through the steady-state pump and Stokes wave intensities. In addition, average RIN transfer increases with growing linear loss, for both the on-resonance peak and off-resonance dip.
SRS interaction along the As$_2$Se$_3$WRL cavity can also be influenced by the end-facet reflectivity. Further reduction of the RIN transfer can thus be achieved through the optimization of cavity reflectivity, as shown in Fig. 6.13. In the following analysis, the waveguide length and pump power are fixed at 5.45cm and 1W respectively. On-resonance peak RIN transfer grows as either pump reflectivity ($R_{p,L}$) or Stokes reflectivity ($R_{s,L}$) decreases. Maximum feedback for both Stokes and pump within the cavity is thus advantageous from the view of RIN reduction, highlighted by the minimum RIN transfer of 4.8 dB occurs when $R_{p,L} = R_{s,L} = 90\%$. This is quite different from the reflectivity design for energy efficiency figure of merit ($FOM$) optimization ($R_{s,L} = 60\%$) [136], further justifying the existence of a trade-off between energy efficiency and RIN performance. RIN suppression from cavity reflectivity increment is especially effective in the low-reflection region, evident by the decreasing slope of the curve towards the high reflectivity side. While pump reflectivity displays
negligible influence on the Stokes output for both SWRL [129] and As$_2$Se$_3$WRL [136], we show here that it could generate RIN suppression as high as 5 dB when the Stokes reflectivity is low ($R_{s,L} = 30\%$). This is particularly useful to improve the RIN performance of the As$_2$Se$_3$WRL that emphasizing on Stokes output, which is usually achieved by a low out-coupling Stokes reflectivity. On the other hand, on-resonance RIN transfer peak reduction can be achieved from Stokes reflectivity variation is more than 10dB. Stokes reflectivity variation is thus more effective compared to pump reflectivity for RIN reduction.

6.5 Summary

In this chapter, we present an extension of the APM modeling that can be utilized to analyze the pump-to-Stokes RIN transfer in waveguide Raman amplifiers and lasers. Compared to the well-established SWRA, co-pumped As$_2$Se$_3$WRA generally suffers from less RIN transfer due to its relatively smaller Raman gain coefficient. The only exception occurs when pump power is high and noise frequency is low, in which free-carrier induces efficient RIN suppression in SWRA. Initial Stokes intensity fluctuation to pump fluctuation ratio (MNR) is found to play a critical role in determining the RIN transfer, generating RIN transfer growth as high as 20 dB. Low MNR is thus highly desirable to control the RIN transfer. The different contributions of various physical processes to RIN transfer in As$_2$Se$_3$WRA are also explored.

Furthermore, we comprehensively investigate the pump-to-Stokes RIN transfer in As$_2$Se$_3$WRL. Besides the superior energy efficiency, As$_2$Se$_3$WRL also outplays SWRL in terms of noise performance, highlighted by the 5dB lower RIN transfer.
across all frequencies. Bi-directional pumping scheme is proposed and verified, for the first time, as a RIN transfer suppression mechanism. While RIN transfer peak in the odd multiples of resonance frequency can be hugely reduced by symmetrically splitting the input pump waves, asymmetrical splitting of the pump power induces RIN transfer reduction across all frequencies. We show that overall RIN transfer in As$_2$Se$_3$WRL decreases with increment in waveguide length. Two regions are subsequently identified, distinguished by the different dominant physical processes that have opposite influence on RIN transfer. Further RIN transfer reduction can be realized through either pump or Stokes reflectivity increment.
Chapter 7 CONCLUSION AND FUTURE WORK

7.1 Conclusion

In this thesis, we have presented a comprehensive investigation of various Raman-based integrated waveguide devices, from theory to applications. A theoretical model, amplitude propagation method (APM), is proposed from the fundamental Raman rate equation and nonlinear Schrödinger equations. In addition to the amplitude related terms appeared in the earlier models, APM takes into account of phase-related nonlinear processes such as SPM, XPM, FWM, SARS and CARS. Precise descriptions for the amplitude and phase evolutions of the optical waves along waveguides can thus be obtained by numerically simulating the APM, providing an efficient tool to universally analyze waveguide Raman amplifier, laser and wavelength converter. The validity of APM is also justified using existing results, in terms of accuracy and computational time. Furthermore, we show that the application of APM can be extended to the chalcogenide waveguide platform and weak pump regime, which is not covered by any numerical models before.

We have proposed three methodologies to combat the severe free-carrier absorption (FCA) drawback that limits energy efficiency in silicon waveguide Raman amplifier and lasers. The first approach, bi-directional pumping scheme, is found to be able to reduce the effective pump intensity in the cavity to suppress FCA loss. The loss suppression subsequently results in the improvement of Stokes output. Further efficiency enhancement can be obtained through the introduction of Mach-Zehnder interferometer (MZI) cavity or end-facet reflectivity optimization. The second approach utilizes chalcogenide waveguide to replace the current silicon platform,
taking advantages of the material’s negligible FCA loss. Compared to the silicon waveguide Raman laser (SWRL), \( \text{As}_2\text{Se}_3 \) waveguide Raman laser (\( \text{As}_2\text{Se}_3 \text{WRL} \)) displays the desirable combination of high Stokes output, low lasing threshold and compact device size. We further extend this concept in the third approach, combining both materials using the well-known slot waveguide. The resultant silicon chalcogenide slot waveguide Raman amplifier facilitates low loss coupling to other silicon-based components, while providing superior energy efficiency and wide amplification range.

We have theoretically analyzed CARS in both silicon and chalcogenide waveguides. Significant influences of the less dominant physical process such as CARS and nonlinear electronic processes on the performance of silicon waveguide Raman laser and wavelength converters respectively are observed. Wavelength conversion through CARS process is proposed and comprehensively investigated in chalcogenide waveguides. We show that the dispersion compensation can be done more easily in \( \text{As}_2\text{Se}_3 \) waveguides than in silicon waveguides, utilizing only the waveguide dispersion. However, nonlinear dynamic phase shift can induce significant fluctuation from the phase-match condition, resulting in 20dB conversion efficiency reduction. The perfect phase match wavelength (PPMWL) is subsequently defined to characterize this shift from zero dispersion wavelengths. Our overall analysis shows that the \( \text{As}_2\text{Se}_3 \) waveguide Raman wavelength converter (\( \text{As}_2\text{Se}_3 \text{WRWC} \)) outperforms its silicon counterpart with 10dB higher efficiency and wider bandwidth. The characteristics of Raman-assisted wavelength conversion in the weak pump regime is also explored, for the first time to the best of our knowledge. Signal depletion up to 78% is observed with anti-Stokes generation in the weak pump regime. This is very
Chapter 7 Conclusion and Future Work

different from the case in the strong pump regime, in which conversion efficiency is solely determined by the pump power.

Furthermore, we have presented an extensive study of the pump-to-Stokes RIN transfer characteristics for both the As$_2$Se$_3$WRA and As$_2$Se$_3$WRL. The extension of APM to analyze RIN transfer is discussed, through the introduction of intensity perturbation. Initial Stokes intensity fluctuation to pump fluctuation ratio (MNR) is found to play a critical role in determining the RIN transfer of co-pumped As$_2$Se$_3$WRA, generating RIN transfer growth as high as 20 dB. Besides the superior energy efficiency, As$_2$Se$_3$WRL also outplays SWRL in terms of noise performance, highlighted by the 5dB lower RIN transfer across all frequency. Bi-directional pumping scheme is proposed and justified as an effective mechanism to suppress RIN transfer. The optimizations of the RIN transfer in terms of waveguide length and end-facet reflectivity are also performed.

7.2 Recommendations for Future Work

Though extensive analysis for the Raman Effect in waveguide devices has been discussed in this thesis, further exploration in several areas can still be carried out in the future. In the following, some of these possibilities are elaborated.

In addition to the conversion efficiency discussed in this thesis, relative intensity noise (RIN) transfer characteristics is another critical performance indicator for the application of wavelength converter [157]. In fact RIN transfer in waveguide Raman wavelength converter would be particularly significant, because strong interaction occurs between pump, Stokes and anti-Stokes waves through Raman processes [158].
Both pump and Stokes to anti-Stokes RIN transfers should be characterized, to provide comprehensive noise control for its future applications. However, modification of the existing RIN transfer analysis model is needed to facilitate such study. In particular, evolution of the anti-Stokes wave and its modulation index should be incorporated.

Integration of the Raman amplifier and wavelength converter could be another interesting domain for exploration. This can be achieved by continuous scaling of silicon waveguide dimensions, resulting in a smooth transition from amplifier to wavelength converter in a single waveguide. The resultant pre-amplified wavelength converter is expected to produce higher conversion efficiency through careful design of waveguide dimensions.

There has been a strong interest in the special waveguide for nonlinear integrated optics recently, such as slot waveguides [57] and photonic crystal waveguides [159]. Nonlinearity, and thus Raman Effect, is enhanced in these waveguides due to their tight optical field confinement or large group refractive index. While the existing investigations are mainly focused on the electronic nonlinear processes, these waveguides could be explored for Raman-based applications. Silicon-chalcogenide slot waveguide Raman amplifier proposed in this thesis is a good example. Other possibilities could include slot waveguide Raman laser, photonic crystal waveguide Raman laser and wavelength converter, with great potential to outperform their existing counterpart in normal silicon waveguides. Furthermore, the flexibility in the design of these special waveguide should be studied, in terms of material choice and waveguide dimensions for Raman-based applications.
In the literature for the modeling of silicon waveguide Raman laser, there exists a discrepancy in the coefficient for power terms in the formula to compute effective free carrier density \((N_{\text{eff}})\) as in Eq. 3.7c. Both “4” [104] and “2” [105] have been utilized as the coefficient for the non-degenerate two photon process. In this appendix, we present a detailed derivation of the formula to justify that the coefficient should be 4 instead of 2. The depletion of pump and Stokes power due to TPA losses can be described by the following equation:

\[
\pm \frac{dE_{f,b}}{dz} = -\frac{\beta}{2A_{\text{eff}}} \left( |E_{f,b}^p|^2 + 2|E_{f,b}^s|^2 + 2|E_{f,b}^{b,f}|^2 \right) E_{f,b}^p
\]  

(1a)

\[
\pm \frac{dE_{f,b}}{dz} = -\frac{\beta}{2A_{\text{eff}}} \left( |E_{f,b}^s|^2 + 2|E_{f,b}^p|^2 + 2|E_{f,b}^{b,f}|^2 \right) E_{f,b}^s
\]  

(1b)

Photon flux \((F)\) is defined as below:

\[
F = \frac{P}{hf} = \frac{|E|^2}{hf}
\]  

(2)

\(P\) represents photon power, \(f\) is the photon frequency. \(E\) represents the normalized electric field amplitude as defined in reference [105]. If we assume that all photons have the same energy as the pump such that \(hf_p \approx hf_s\), the TPA photon flux differential equation can then be written as [160] Eq. 3. The assumption is justified in the context of waveguide Raman laser, in which the pump power is much higher than the Stokes.
Appendix A Derivation of the Effective Carrier Density Formula

\[ \pm \frac{dF_{p}^{f,b}}{dz} = - \frac{\hbar f_{p}B}{A_{\text{eff}}} \left( F_{p}^{f,b} + 2F_{s}^{f} + 2F_{p}^{b,f} \right) F_{p}^{f,b} \] (3a)

\[ \pm \frac{dF_{s}^{f,b}}{dz} = - \frac{\hbar f_{p}B}{A_{\text{eff}}} \left( F_{s}^{f,b} + 2F_{p}^{f} + 2F_{p}^{b,f} \right) F_{s}^{f,b} \] (3b)

Considering a slice of waveguide \( \Delta z \) along the longitudinal \( z \)-axes, we could obtain the following relation based on conservation of energy such that two photons created an electron-hole (EH) pair:

\[ \frac{1}{2} (\text{photon} \_ \text{flux} \_ \text{in} - \text{photon} \_ \text{flux} \_ \text{out}) / \text{sec} = \text{EH} \_ \text{pairs} \_ \text{generated} / \text{sec} \] (4)

The free carrier generation rate per second \( (G_{\text{EH}}) \) per unit length is then presented as Eq. 5, by substituting Eq. 4 into Eq. 3.

\[
G_{\text{EH}} = \frac{1}{2} \left( \frac{dF_{p}^{f}}{dz} + \frac{dF_{s}^{f}}{dz} - \frac{dF_{p}^{b}}{dz} - \frac{dF_{s}^{b}}{dz} \right)
\]

\[ = \frac{\hbar f_{p}B}{2A_{\text{eff}}} \left( \left[ F_{p}^{f} \right]^{2} + 2F_{p}^{f}F_{s}^{f} + 2F_{p}^{b}F_{p}^{b} + \left[ F_{s}^{f} \right]^{2} + 2F_{p}^{b}F_{p}^{f} + 2F_{s}^{b}F_{s}^{f} \right) 
\]

\[ + \left[ F_{p}^{b} \right]^{2} + 2F_{p}^{b}F_{s}^{b} + 2F_{s}^{b}F_{p}^{b} + \left[ F_{s}^{b} \right]^{2} + 2F_{p}^{b}F_{p}^{b} + 2F_{s}^{b}F_{s}^{b} \right) 
\]

\[ = \frac{\hbar f_{p}B}{2A_{\text{eff}}} \left( \left[ F_{p}^{f} \right]^{2} + \left[ F_{s}^{f} \right]^{2} + \left[ F_{p}^{b} \right]^{2} + \left[ F_{s}^{b} \right]^{2} + 4F_{p}^{f}F_{p}^{b}F_{p}^{f}F_{p}^{b} + F_{p}^{f}F_{s}^{b} + F_{p}^{b}F_{s}^{f} + F_{p}^{b}F_{s}^{b} + F_{s}^{b}F_{s}^{b} + F_{s}^{b}F_{s}^{f} \right) \]

This is the free-carrier generation rate in the x-y cross section; we therefore need to divide it by the area of this cross section to obtain the actual generation rate. By assuming a uniform distribution of free carrier inside the waveguide, we can obtain the following equation under steady states [31]:

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\[ N_{\text{eff}} = \tau_{\text{eff}} \times G_{\text{EH}} \]
\[ = \tau_{\text{eff}} \times \frac{\hbar \beta}{2 A_{\text{eff}}} \left( |F_p^f|^2 + |F_s^b|^2 + |F_p^b|^2 + |F_s^f|^2 \right) + 4 \left( F_p^f F_p^b F_s^b + F_p^f F_s^b F^b F_s^f + F_p^b F_s^b F^b F_s^f + F_s^f F_s^b \right) \]
\[ = \frac{\beta \tau_{\text{eff}}}{2 \hbar \beta A_{\text{eff}}^2} \left( |E_p^f|^4 + |E_p^b|^4 + |E_s^b|^4 + 4(|E_p^f|^2 |E_p^b|^2 + |E_s^b|^2 |E_s^f|^2) \right) + 4(|E_p^f|^2 |E_s^f|^2 + |E_p^b|^2 |E_s^b|^2 + |E_p^f|^2 |E_s^b|^2) \]

Here, we transform \( F \) to \( E \) by substituting in Eq. 2. It is clearly shown in Eq. 6 that the coefficient should be 4 instead of 2 for the effective photon-generated carrier density calculation. We thus employ a coefficient of 4 in our modeling throughout this thesis.
PUBLICATIONS

Journal papers:


2. **Ying Huang**, Perry Ping Shum, Feng Luan and Ming Tang, “Raman-assisted wavelength conversion in chalcogenide waveguides”, Accepted by *Journal of Selected Topics in Quantum Electronics*, v17, n6, 2011.


Conference papers:

1. **Ying Huang**, Hongtao Zhou, Perry Ping Shum, Feng Luan, Songnian Fu, Eng Leong Tan and Ming Tang “Pump-to-Stokes RIN transfer characteristics of As$_2$Se$_3$ waveguide Raman laser”, Accepted by *16th Opto-Electronics and Communication Conference (OECC 2011)*, 2011. (Awarded Best Paper)


REFERENCES

References


References


[68] X. Gai, S. Madden, D. Y. Choi, D. Bulla, and B. Luther-Davies, "Dispersion engineered Ge11.5As24Se64.5 nanowires with a nonlinear parameter of 136W-1m-1 at 1550nm," *Optics Express*, vol. 18, n18, pp. 18866-74, 2010.


[72] R. Stegeman, G. Stegeman, P. Delfyett, L. Petit, N. Carlie, K. Richardson, and M. Couzi, "Raman gain measurements and photo-induced transmission effects


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