STUDY OF FRICTION BEHAVIOR OF
MICROMACHINED SIDEWALL SURFACES OF
MEMS DEVICES

WU JIE

School of Mechanical and Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2009
Acknowledgements

The work contained in this thesis would not have been possible without the guidance and support of my professors, colleagues, friends, and family. First I would like to thank my supervisor, Dr. WANG Shao, for his support, guidance and encouragement in all the time of the research. The same appreciation also goes to my supervisor, Associate Professor MIAO Jianmin. His ideas, experiences, and fully support in the past few years have been a great help for both my research and my life.

I would like to thank my friends in NTU, WU Mingjie, CHE Jin, Dr. CHEN Bang-tao, Dr. SUN Jianbo, YUAN Yanhui, Dr. TANG Gongyue, LIU Yan, TAN Chee Wee, TANG Chong Wei, XU Ting, WANG Cheng, Dr. Pradeep Dixit, and Mohammed Ashraf for all the days we worked together, all the discussions we had, and also for their friendships. I would like to thank Dr. CHEN Longqing for his help and sharing of experience. I would like to thank the staffs working and worked in Micromachines Center, Mr. WONG Kim Chong, Mr. Pek Soo Siong, Mr. LAU Joo Kiang, Mr. HOONG Sin Poh, Mr. Ho Kar Kiat, Mr. HOONG Sin Poh, Mr. Win Maung Maung, Mr. TANG Kok Soo, Mr. Nordin Bin Abdul Kassim, who offered their kindly assistance to this project.

The thanks to family are always the last but sure not the least. I would like to thank my family for their constant love and support, especially to my wife Jing, the love of my life – I couldn’t have done this without you by my side.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>viii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xv</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>xvi</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Overview of the Research</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Conventional Tribology</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Micro-/Nano-Tribology</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Micro-Electro-Mechanical Systems (MEMS)</td>
<td>7</td>
</tr>
<tr>
<td>1.5 Objectives and Scope</td>
<td>8</td>
</tr>
<tr>
<td>1.6 Organization of the Thesis</td>
<td>9</td>
</tr>
<tr>
<td>2 Literature Review</td>
<td>11</td>
</tr>
<tr>
<td>2.1 Conventional Tribological Measurements</td>
<td>11</td>
</tr>
<tr>
<td>2.2 Tribological Measurements by Microscopes</td>
<td>15</td>
</tr>
</tbody>
</table>
# 2.3 Micro-Tribological Measurements

- 2.3.1 Measurements with General MEMS Devices ........................................ 17
- 2.3.2 Measurements with In-Plane Friction/Wear MEMS Devices .......................... 19
- 2.3.3 Measurement with Out-of-Plane Friction/Wear MEMS Devices ..................... 22

# 2.4 Fabrication Techniques in MEMS

- 2.4.1 Process with SOI Wafers ................................................................. 28
- 2.4.2 Process with SOG Wafers .................................................................. 30

# 2.5 Summary ............................................................................................... 34

# 3 Design of MEMS Friction Testing Devices .......................................................... 35

- 3.1 Design of the Linear Friction Testing Device ................................................. 35
  - 3.1.1 Operation Principles of the Linear Device ............................................. 36
  - 3.1.2 Actuating Mechanism of the Linear Actuators ...................................... 38
- 3.2 Design of the Rotary Friction Testing Device ............................................... 41
  - 3.2.1 Operation Principles of the Rotary Device ........................................... 42
  - 3.2.2 Actuating Mechanism of the Rotary Actuator ....................................... 44
- 3.3 Flexure Designs for Linear Motions ............................................................ 47
  - 3.3.1 Basic Folded Flexure ........................................................................ 48
  - 3.3.2 Three-Turn Folded Flexure ................................................................ 55
  - 3.3.3 S-Shape Folded Flexure .................................................................... 57
- 3.4 Flexure Design for Rotary Oscillation .......................................................... 62

# 3.5 Summary ................................................................................................. 66

# 4 Fabrication of MEMS Testing Devices .............................................................. 69

- 4.1 Fabrication Process ................................................................................... 69
4.2 Critical Processes in Fabrication .................................................. 71
  4.2.1 Deposition of Metal Layer ...................................................... 72
  4.2.2 Anodic Bonding ................................................................. 72
  4.2.3 Deep Reactive Ion Etching (DRIE) ............................................. 73
4.3 Challenges in Deep Reactive Ion Etching ...................................... 76
  4.3.1 Heat Accumulation .............................................................. 78
  4.3.2 RIE Lag Effect ................................................................. 79
  4.3.3 Microloading Effect ........................................................... 81
4.4 Setup of the Testing Platform .................................................... 82
4.5 Static Measurements ................................................................... 84
  4.5.1 Measurements of the Feature Dimensions ................................. 85
  4.5.2 Measurements of Displacement ............................................... 86
4.6 Correction of Measurement Results ............................................. 88
  4.6.1 Effect of the Profile Tolerance ............................................... 89
  4.6.2 Effect of the Undercut Tolerance ............................................ 92
4.7 Dimension Characterization ....................................................... 93
4.8 Summary ..................................................................................... 94

5 Measurement and Analysis of the Linear Friction Testing Device ...... 95
  5.1 Contact Mechanics Analysis ...................................................... 95
  5.2 Development of a Quasi-Static Stick-Slip Model ......................... 101
  5.3 Friction Behavior of the Linear Device ....................................... 107
  5.4 Summary ................................................................................... 113

6 Measurement and Analysis of the Rotary Friction Testing Device .... 115
  6.1 Investigation of the Stick-Slip Phenomenon ................................ 115


6.1.1 Quasi-Static Stick-Slip Model .............................................. 115
6.1.2 Displacement Measurements .................................................. 117

6.2 Analysis of Equivalent Stiffness and Contact Point ....................... 118
6.2.1 Observation of the Sensing Bush .......................................... 118
6.2.2 Bush-Flat Model ............................................................... 119
6.2.3 Bush-Shaft Model ............................................................. 123

6.3 Prediction of the Regions with Sliding Wear ................................. 127
6.3.1 Condition for Pure Rolling ............................................... 128
6.3.2 Tilting of the Sensing Bush ............................................... 131

6.4 Friction Behaviors of the Rotary Device .................................... 132
6.5 Summary .................................................................................. 135

7 Conclusions and Future Work ....................................................... 137
7.1 Conclusions ........................................................................... 137
7.2 Future Work ........................................................................... 139
7.2.1 Improvement of Calibration ................................................ 140
7.2.2 Improvement of Measurement .............................................. 142
7.2.3 Expansion of Research Objectives ....................................... 144

7.3 Summary .................................................................................. 146

Appendix A Analysis of Basic Folded Flexure .................................. 147

Appendix B Analysis of Three-Turn Folded Flexure ............................. 156

Appendix C Details of Micro-Fabrication Processes ............................ 161
C.1 Photolithography ........................................................................ 161
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.1.1 Photoresist AZ7220 (Clariant)</td>
<td>161</td>
</tr>
<tr>
<td>C.1.2 Photoresist type: AZ9260 (Clariant)</td>
<td>162</td>
</tr>
<tr>
<td>C.2 Deposition and patterning the electrodes</td>
<td>162</td>
</tr>
<tr>
<td>C.3 Eutectic bonding of silicon to glass wafers</td>
<td>163</td>
</tr>
<tr>
<td>C.4 Anodic bonding of silicon to glass wafers</td>
<td>163</td>
</tr>
<tr>
<td>C.5 DRIE of Silicon</td>
<td>163</td>
</tr>
</tbody>
</table>

Appendix D  List of Publications  165

References  166
Abstract

Devices on a micrometer scale integrated with mechanical elements, sensors, actuators, and electronics on a common silicon substrate have been developed in the recent decades owing to the advancement of the technology of micro-electro-mechanical systems (MEMS). However, MEMS devices with sliding contact such as stepper motors, gas bearings, micro-motors and associated components of microengines, are still limited to laboratory studies. The occurrence of failure of rubbing interfaces affects the performance and reliability of these MEMS devices, leading to major obstacles to commercialization. The friction behavior of MEMS devices in the contact regime has not been well understood so far although it is closely related a dominant failure mechanism. Well developed theories and models for friction measurement at macro-scale might not be directly applied to micro-scale measurements due to the scaling effects. For lightly loaded surfaces with small roughnesses, contact phenomena occur at micro- or even nano-scales and, thus, the inter-molecular forces may play an important role in determining the asperity contact, rendering some conventional theories about friction, such as Amontons’ law, unsuitable. Therefore, specially designed MEMS devices for friction measurements on micro-scale are necessary.

Despite the significant efforts devoted to the study of the friction behavior at different contacting interfaces, friction transitions between the static and kinetic states (stick-slip phenomenon) at the sidewall surfaces have not yet been adequately characterized in the existing tests. Moreover, all reported devices were designed for measuring the
friction at contact interfaces with linear motion. Measurement results of these devices cannot truly reflect the tribological phenomena occurring at curved sidewall surfaces, which are essential for MEMS devices with rotating or oscillating components.

In the present project, linear and circular MEMS devices aimed at studying the friction behavior of both the linear and curved sidewall surfaces, respectively, were developed. Prototypes have been fabricated via silicon micro-machining using the deep reactive ion etching (DRIE) process. A quasi-static stick-slip model was developed to interpret the experimental data obtained via video image analysis. Contact analysis indicated that the silicon sidewall contact interface is in an elastic regime at the scale of the contact region and the surface scallops (waviness) caused by the DRIE process. A saturation phenomenon of the kinetic friction at the single-crystal silicon sidewall surfaces was observed while the normal load was increased for the linear devices. The average coefficient of kinetic friction is 0.66 before saturation of kinetic friction. For the circular devices, a general formulation was developed to determine the equivalent tangential stiffness of the contact point, and the measured coefficient of static friction exhibits a non-linear dependence on the normal load. According to the fitted data based on the experimental results, the true coefficient of static friction is found to be 0.64.

This project makes substantial and novel contributions to progress in both understanding of the tribological behaviors at different profiles of contact surfaces, and application of that knowledge to MEMS.
List of Figures

1.1 Schematic a free-body diagram of a body sliding on a surface ............... 3
1.2 The sketch of friction testing devices by da Vinci: (a) a block placed on a flat plane in numerous different orientations; (b) a structure to drag the sliding block with constant force ........................................ 4
1.3 Schematic diagram of the model of friction for two interlocking surfaces. During sliding, the top surface must move up and down over the bottom surface [22]. ................................................................. 6
1.4 Schematic diagram of the friction model with small asperities touching each other during the contact: (a) the normal force $N = 0$; (b) the normal force $N \neq 0$ ................................................................. 6
1.5 Schematic diagram of tribological issues in a MEMS device .................. 8

2.1 Schematic diagram of typical geometries for friction and wear testing [18]. ......................................................................................... 12
2.2 Schematic diagram of typical tribometers: (a) tribometers based on pin-on-disk; (b) tribometers based on ball-on-plate .......... 13
2.3 Schematic diagram of the sliding test apparatus based on rotating disc [36]. ................................................................................. 14
2.4 Schematic diagram of the experimental set-up of the wear test rig based on rotating disc [37]. ................................................................. 14
2.5 Schematic diagram of the operation of a scanning tunneling microscope (STM): (a) STM working in a constant current mode; (b) STM working in a constant height mode. ................................................ 16
2.6 Schematic diagram of the working principle of an atomic force microscope (AFM) ................................................................. 16
2.7 Schematic diagram of the cross-section of a micromotor ..................... 17
2.8 Schematic diagram of a laser-based friction measurement system [49]. 18
2.9 Schematic diagram of the connected gear and linkage arms of a micro-engine [34]. 19
2.10 Schematic diagram of a single-dimple friction testing microstructure [4]. 20
2.11 Schematic diagram of the surface micro-machine of Lumbantobing and Komvopoulos [51]. 21
2.12 Schematic diagram of the nanotractor to achieve rightward motion [52]. 21
2.13 Schematic diagram of the clamp-with-beam instrument for friction measurement at sidewall surfaces. 23
2.14 Schematic diagram of the beam-on-post testing device for friction and wear study at sidewall surfaces [5]. 24
2.15 Schematic diagram of the micro-tribotester for measurement of static and kinetic friction coefficients of sidewall surfaces [54]. 25
2.16 Schematic diagram of the MEMS device for sidewall friction and adhesion experiments [55]. 25
2.17 Schematic diagram of the steps used in the surface micromachining process. 26
2.18 Schematic diagram of bulk micro-machined structure. 27
2.19 Schematic diagram of mold formation using LIGA technique [59]. 27
2.20 Schematic diagram of a silicon on insulator (SOI) wafer. 28
2.21 Schematic diagram of the cantilever beam stiction by capillary forces. 29
2.22 Schematic diagram of notching phenomenon on a SOI wafer after DRIE process. 29
2.23 Schematic diagram of the anodic bonding process. 31
2.24 Schematic diagram of the Bosch process with alternating etching and passivation steps. 32
2.25 Schematic diagram of the RIE lag effect. 32
3.1 Schematic diagram of the linear friction testing device. 37
3.2 Free body diagram of the working principle of the linear testing device. 37
3.3 Schematic diagram of a comb drive displaying the presentation of the capacitance. 39

- ix -
3.4 Schematic diagram of the design of the MEMS friction testing device for curved sidewall interface ........................................ 42

3.5 Free body diagram of the working principle of the rotary friction testing device ......................................................... 43

3.6 Schematic diagram of a rotary comb drive ........................................ 45

3.7 Schematic diagram of a basic folded flexure: (a) the basic folded flexure before bending; (b) deformation of a basic folded flexure (dashed line is the beam with deflection) ........................................ 49

3.8 Schematic diagram of the bending effect of the basic folded flexure: (a) free-body diagram of segment 12 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment 12 with deflection ........................................ 49

3.9 Schematic diagram of the influence of the value of the beam width ratio, $b/a$. ......................................................... 53

3.10 Schematic diagram of the finite element analysis result of the deflection of a basic folded flexure by using ANSYS: (a) under a tangential load; (b) under a normal load ........................................ 54

3.11 Schematic diagram of a folded supporting flexure consists of three complete-cycle units and a half-cycle unit ........................................ 56

3.12 Schematic diagram of the finite element analysis of a three-turn folded flexure by using ANSYS ........................................ 57

3.13 Schematic diagram of a folded supporting flexure consists of three complete-cycle units and a half-cycle unit ........................................ 58

3.14 Schematic diagram of the model of an S-shape flexure by using ANSYS: (a) deflection of the flexure; (b) generation of an equivalent moment. ........................................ 60

3.15 Schematic diagram of the influence of the beam length of the S-shape folded flexure on the tangential stiffness, $k_t$, and the stiffness ratio, $k_n/k_t$: (a) the influence of the length of the connecting beam, $L_c$, with $L_b = 250 \mu m$; (b) the influence of the length of the bending beam, $L_b$, with $L_c = 16 \mu m$. ........................................ 61

3.16 Schematic diagram of the supporting flexure of rotary friction testing device: (a) diagram of the bending of the flexure; (b) free body diagram of the supporting beam; (c) free body diagram of the driving ring. ........................................ 63
3.17 Schematic diagram of the deflection of the flexure combination C of the rotary friction testing device by using ANSYS ......................... 65
3.18 Schematic diagram of the influence of the length of supporting beam on the rotary stiffness of the rotary friction testing device .............. 66
4.1 Schematic diagram of the fabrication process flow of the MEMS devices based on anodic bonding .................................................. 70
4.2 Micrograph of a deposited Cr/Au pattern .......................................... 72
4.3 Micrograph of the result of the anodic/eutetic bonding between silicon to glass with electrodes ......................................................... 73
4.4 Schematic diagram of a Multiplex ICP process chamber [108] ............... 74
4.5 SEM image of a linear friction testing device ...................................... 75
4.6 SEM image of a rotary friction testing device ..................................... 75
4.7 SEM image of a partial view of the comb actuator of the linear friction testing device .......................................................... 76
4.8 SEM image of a partial view of the comb actuator of the rotary friction testing device .......................................................... 77
4.9 SEM image of the sensing bush of the rotary friction testing device .......... 77
4.10 SEM picture of the damage caused by accumulated heat during the DRIE process ........................................................................ 78
4.11 Micrograph of the RIE lag effect ........................................................ 80
4.12 SEM picture of etching result after the final DRIE process by using optimized etching steps .......................................................... 81
4.13 Schematic diagram of the nonuniform dimensions of open areas due to SEM image of the damage of the supporting flexure caused by over etching during the DRIE process ........................................... 82
4.14 SEM image of the damage of the supporting flexure caused by over etching during the DRIE process ........................................... 83
4.15 Micrograph of the microloading effect during the DRIE process ........... 83
4.16 SEM image of the beam proctors of the rotary friction testing device ... 84
4.17 Schematic diagram of the experimental setup used for friction testing under controlled environmental conditions ............................ 85
4.18 Schematic diagram of the principles of the measurement pitches and dimensions of (a) a folded flexure; (b) a rotary comb drive .......... 86
4.19 Micrograph of the zoomed view of the anchor and a part of the sensing plate of a fabricated device. ................................................................. 88
4.20 Schematic diagram of a positive and a negative profiles after the DRIE process. ................................................................. 89
4.21 Schematic diagram of a positive profile of a pair of comb fingers. ....... 91
4.22 Schematic diagram of a positive profile of the cross section of a flexure beam. ................................................................. 91
4.23 Schematic diagram of the undercut effect: (a) an isotropic undercut $L_u \approx H_u$; (b) an anisotropic undercut $L_u \ll H_u$. ........................................ 92
4.24 Micrograph of the etched comb fingers: (a) top view; (b) bottom view (horizontal flipped). ................................................................. 93

5.1 Scanning electron microscope (SEM) image of a sidewall surface with waviness. ................................................................. 96
5.2 Schematic diagram of the contact region: (a) profile of the peak-to-peak contact at the sidewall surfaces and one sinusoidal ridge as well as the simplified arc of the profile; (b) contact of a smooth bump with the ridges as equivalent radius $R^*_u$ applied. ................................................................. 97
5.3 Schematic diagram of the elliptical contact areas along the contact sidewall. ................................................................. 97
5.4 Tangential driving force, $F_t$, displacement of the driving plate, $x$, and displacement of the sensing plate, $s$, predicated by the quasi-static stick-slip model (thick line: static friction; thin line: kinetic friction). ................................................................. 102
5.5 Measured displacement of the driving plate without any contact for $V_{t,max} = 19.4$ V, and $V_n = 0$. ................................................................. 108
5.6 Measured displacements of the driving plate and the sensing plate of a device made from a 150-μm-thick silicon wafer: (a) $V_n = 23.5$ V and $V_{t,max} = 19.4$ V; (b) $V_n = 22.0$ V and $V_{t,max} = 19.4$ V. ................................................................. 109
5.7 Measured displacements of the driving plate with bumps and sensing plate of a device made from a 300-μm-thick silicon wafer at 0.1 Hz, $V_n = 33$ V and $V_{t,max} = 20$ V. ................................................................. 110
5.8 Effects of a ramping normal load: (a) variation of the displacement of the driving plate at instant $D$, (b) variations of the kinetic friction force and coefficient with $V_n^2$ (bottom axis) and the normal load $N$ (top axis). ................................................................. 112
6.1 Equivalent tangential driving force, $F_{ce}$, displacement of the driving plate, $x$, and displacement of the sensing plate, $s$, predicated by the quasi-static stick-slip model (thick line: static friction; thin line: kinetic friction). ................................................................. 117

6.2 Measured displacements of the driving ring and the sensing bush for $V_n = 27.1$ V and $V_{c,max} = 48.6$ V. ................................................................. 118

6.3 Observation of the variation of the gap between the sensing bush and the driving ring: (a) initial gap; (b) expanded gap after rotation. ................. 119

6.4 Schematic diagram of the bush-flat model: (a) free-body diagram of the model, (b) motion of the bush ring. ......................................................... 121

6.5 Schematic diagram of the rigid bush ring connected to a flexure: (a) loading on the rigid bush ring, (b) free body diagram of Flexure $S$ and the rigid bush ring. ................................................................. 122

6.6 Schematic diagram of the bush-shaft model: (a) status of the ring and shaft before motion; (b) status of the ring and shaft after motion occurs. 124

6.7 Schematic diagram of a part of the sensing ring and Flexure $S$: (a) forces acting at the contact region; (b) equivalent forces and moment at the material point close to the end of Flexure $S$. .............................. 125

6.8 Free-body diagram for analyzing the working principle of the friction testing device: (a) the left part of the testing device; (b) the driving ring. 129

6.9 Diagram of the boundaries for pure rolling (occurring above a particular line for a given $\mu_s$). ......................................................... 131

6.10 Schematic diagram of the dimension of the sensing bush. ................. 131

6.11 Averaged forward and backward displacements of the sensing bush at instant $B$ for $V_{c,max} = 48.6$ V. ................................................................. 132

6.12 Averaged coefficients of static friction for $V_{c,max} = 48.6$ V (solid line for the fitting curve under increasing normal load and dashed line for the fitting curve under decreasing normal load). .............................. 133

6.13 Averaged adhesion force for $V_{c,max} = 48.6$ V (solid line for fitting curve under increasing normal loads and dashed line for fitting curve under decreasing normal loads). ................................................................. 134
7.1 The principle of the calibration of the flexure stiffness: (a) setup of the calibration), (b) schematic diagram of the probe and the flexure (zoomed view of the sensing bush of the rotary testing device. .......... 141

7.2 Schematic diagram of a structure with embedded piezoresistive sensors: (a) zoomed piezoresistive sensors for the linear testing device; (b) zoomed piezoresistive sensor for the rotary testing devices; (c) the working principle of a Wheatstone bridge. ............................... 143

7.3 Schematic diagram of the concept of time window for on-line measurement. ......................................................... 144

7.4 Design idea of the structure for measuring adhesion force in the future work. ......................................................... 145

A.1 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment23 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment23 with deflection. 148

A.2 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment34 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment34 with deflection. 150

A.3 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment45 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment45 with deflection. 152

A.4 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment56 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment56 with deflection. 154

B.1 Schematic of a half-cycle flexure unit: (a) the half-cycle flexure unit before bending; (b) deformation of a half-cycle flexure unit (dashed line is the beam with deflection). ........................................... 157

B.2 Free body diagram of unit 1 of a three-turn folded flexure .......... 157

B.3 Free body diagram of unit 2 of a three-turn folded flexure .......... 158

B.4 Free body diagram of unit 3 of a three-turn folded flexure .......... 159
List of Tables

2.1 Some details of typical geometries for friction and wear testing .......... 13

3.1 Material properties of single crystal silicon........................................ 48
3.2 Stiffnesses of a three-turn folded flexure ........................................... 55
3.3 Characteristics of three-turn folded flexures adopted in this project ........ 56
3.4 Stiffnesses of a three-turn folded flexure ........................................... 57
3.5 Characteristics of the S-shape flexure ............................................... 59
3.6 Characteristics of the S-shape flexure ............................................... 59

4.1 Parameters of the optimized DRIE process ......................................... 81
4.2 Local load per contact peak and maximum contact pressure for different conditions ................................................................. 93

5.1 Technical data of the etched sidewall surfaces .................................... 99
5.2 Local load per contact peak and maximum contact pressure for different conditions ................................................................. 100
5.3 Stiffnesses of the supporting flexures adopted in the testing devices ....... 107

6.1 Characteristics of the Fabricated Testing Device .................................. 130
Nomenclature

$\mu$  Coefficient of friction of the contact interface

$\mu_k$  Coefficient of kinetic friction

$\mu_s$  Coefficient of static friction

$\phi$  Tapered angle of the profile of flexure after DRIE

$\varepsilon_0$  Vacuum permittivity, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\varphi$  Tapered angle of the profile of comb fingers after DRIE

$C_p$  Parasitic capacitance of each finger pair, F

$E$  Young’s modulus, Pa

$f$  Friction force on the contacted surfaces, N

$F_n$  Normal electrostatic driving force, N

$F_t$  Tangential electrostatic driving force, N

$f_k$  Kinetic friction force, N

$f_{sm}$  The maximum static friction force, N

$H$  Thickness of floating components of the MEMS testing devices, \( \mu m \)

$I$  Area moment of inertia, m\(^4\)

$k_n$  Normal stiffness of a flexure, N/m

$k_t$  Tangential stiffness of a flexure, N/m

$N$  Normal force acting on the contacted surfaces, N
$r_c$  Effective moment arm of the circular friction testing device, $\mu$m  

$T$  Driving torque of the circular friction testing device, Nm  

$V_n$  Normal driving voltage, V  

$V_t$  Tangential driving voltage, V  

$V_{nc}$  The value of $V_n$ corresponding to the first occurrence of contact, V  

DRIE  Deep reactive ion etching  

FEA  Finite element analysis  

ICP  Inductively coupled plasma  

MEMS  Microelectromechanical systems  

SOG  Silicon-on-Glass
Chapter 1

Introduction

1.1 Overview of the Research

For higher performance and lower power consumption, smaller and smaller feature sizes become the pursuit of electromechanical devices during the recent decades. From visible sizes to micro-meter or even nano-meter scales, micro-devices have been adopted for various purposes, from fundamental mechanical understanding to the latest biomedical applications. Meanwhile, the reliability issues of micro-devices turns into the major obstacle to commercialization of these devices. As the dominant failure mechanism affecting the reliability of micro-devices with sliding contact interfaces, friction and wear in the contact regions of sidewall surfaces have not been well understood so far. Tribological phenomena, such as friction transition between static and kinetic states as well as failure mechanisms at sidewall surfaces, have not been adequately characterized in existing research. To understand the tribology relative characteristics, both scanning electron microscope (SEM) and atomic force microscope (AFM) based measurements have been utilized to obtain the roughness of an etched sidewall profile [1–3].

To obtain the real time tribological information of the contact surfaces, MEMS devices aimed at friction and wear measurement of the contact interfaces have been de-
Chapter 1

Introduction

veloped in the past decades [4–8]. However, all these reported devices were designed for measuring the friction and/or wear at a linear contact interface, such as contact occurred as dimple-on-pad, clamp-on-beam and post-on-beam. Measured results of these kinds of linear motion cannot truly reflect the tribological behaviors occurred at curved sidewall surfaces, which are widely adopted in the MEMS devices having rotation or oscillation components, such as micro-bearings and micro-motors [9].

In this project, novel micro-devices for friction characterization and wear measurement have been developed to study friction behavior at both linear and curved sidewall surfaces. To ensure the material properties and morphological nature of the contact sidewall surfaces under test are similar to those of some actual devices, the testing devices were fabricated via common adopted micro-fabrication techniques. As necessary background knowledge, techniques for micro-fabrication, theories of tribology and methodology of tribological measurement were the major concerns of this project.

1.2 Conventional Tribology

The word “tribology” was derived from the Greek word *tribos*, which means rubbing. Being accurately, tribology is the study not only of rubbing, but also of the science and technology of the interaction between contacting surfaces that are in relative motion and of related subjects and practices. Generally, tribology includes the study of friction, wear, and lubrication [10]. As the main causes for failure of rubbing interfaces, which are even significant in the industrial field, friction behavior as well wear are studied in the present research.

Friction is the resistance to the relative motion or tendency toward such motion between two solid bodies in contact, as shown in Figure 1.1. When a normal load (force) \( N \) is applied, a friction force \( f \) is generated at the contacting surfaces. From idle state to sliding occurring, two kinds of friction are involved. Static friction, \( f_s \), occurs while no relative motion between the contacting surfaces, and the value of a
static friction force equals the tangential force required to initiate the motion. Kinetic friction, \( f_k \), occurs during sliding of the two solid bodies and the value of a kinetic friction force equals the tangential force required to maintain the relative motion.

![Schematic free-body diagram of a body sliding on a surface.](image)

Figure 1.1 Schematic a free-body diagram of a body sliding on a surface.

Leonardo da Vinci was the first one who made quantitative studies of friction from a scientific point of view. He discovered that friction is independent of apparent area of contact by measuring friction force using the different sides of the same block, as shown in Figure 1.2(a). By dragging a sliding block with constant force, as shown in Figure 1.2(b), he also deduced that the coefficient of friction was the quotient of the friction load and the normal load [11]. However, only static friction was measured and no distinction between static and kinetic friction was made in da Vinci’s experiments.

In 1699, Guillaume Amontons found the law of friction, which states that the friction force is directly proportional to the normal load as

\[
f = \mu N
\]

where \( f \) is the friction force, \( \mu \) is the coefficient of friction, and \( N \) is the normal load. He also found that the friction force is independent of the apparent area of contact, which is known as Amontons’ Law [12]. In 1781 Charles Augustin de Coulomb verified Amontons’ observations and further made a clear distinction between static friction and kinetic friction, and he found that kinetic friction is independent of the velocity, which
is also known as Coulomb’s Law [13].

Wear is a much younger subject compared to the development of friction, and it was initiated from the accumulation of experience from daily life. Wear is the erosion of material from one or both surfaces in a sliding, rolling, or impact motion of two solid bodies. Ragnar Holm made one of the earliest substantial contributions to the study of wear [14]. Since the beginning of the twentieth century, knowledge in understanding the mechanisms of wear has been expanded tremendously from the growth of industry [15–17].

The purpose of research in tribology is to achieve an understanding in minimizing, or even eliminating, the loss resulting from friction and wear. Except the effects to daily life, such as walking, writing, cleaning, etc., tribology is critical to industry where a lot of machines with contact surfaces are included. Estimated direct and consequential annual loss to industries in USA due to tribological issues is approximately 4% of the
Chapter 1

Introduction

gross national product of the United States [18]. Estimated by Ilmenau University of Technology, annual losses in Germany are estimated at around € 35 billion. Through friction reduction and wear control, both economic saving and reliability improving can be achieved.

1.3 Micro-/Nano-Tribology

Since the nineteenth century, the study of interfacial phenomena approached to micro- and even nano-scales for further understanding the fundamental mechanisms of tribology [17, 19]. Furthermore, development of magnetic storage systems gave another need to study the friction and wear processes of micro- and nano-structures and molecularly thick lubricant films [20, 21]. To meet these requirements and to provide a bridge between science and engineering, micro-/nano-tribology is derived from conventional tribology (macro tribology).

Considering the asperities on micro-scale during contact, two major friction theories, geometrical friction theory and molecular adhesion theory, have been developed. For the geometrical theory, contacting regions were treated as rough surfaces which contain micrometer scale asperities. Thus, Eq. (1.1) could be explained in terms of the climbing of the asperities of one surface over those of the other, as shown in Figure 1.3. Work is only done by the normal load and no energy is dissipated during sliding. The friction force is given by $f = \tan \theta \cdot N$, where $\tan \theta$ is the maximum mean slope of the asperities [22].

The geometrical theory followed Amontons’ Law very well, however, it can not explain the fact that friction is a dissipative process. Bowden and Tabor proposed that the friction force arises from the shear force to break contacting junctions of the asperities, as shown in Figure 1.4. As a result, the friction force is proportional to the real area of contact as that of an adhesion [15]. Since the energy loss in this adhesion theory is described as plastic deformation of the asperities, it is also called plastic junction model.
The understanding of friction at the micrometer scale, consequently, has been reduced to an understanding of two new quantities: shear strength and real area of contact.

According to the above description, it can be noticed that the major differences between macro tribology and micro-/nano-tribology are dimension, mass and load. Micro-/nano-tribology is concerned with the size ranging from atomic and molecular scales to micro-scales, and the mass upon this kind of dimension is very small, normally about a few micrograms to a few milligrams. For mechanical components used in micro-/nano-structures, light mass, which is on the order of microgram, and light loads, which are in the order of microgram to milligram, are concerned. When the length of devices decreases from 1 mm to 1 μm, the area will decrease by a factor of a million ($10^6$) and the volume will decrease by a factor of a billion ($10^9$). The resistive forces such as friction, viscous drag, and surface tension that are proportional to the area, will decrease by a
Chapter 1

Introduction

significant smaller factor compared to the forces proportional to the volume, such as inertial and electromagnetic forces.

Recently, technologies aiming at small scales of mechanical devices, such as micro-electro-mechanical systems (MEMS), has motivated better fundamental understanding of friction. Due to large surface-to-volume ratios, tribological issues play a key role in both fabrication and operation of MEMS devices. The increase of resistive forces becomes critical in the research of the micro-devices or micro-areas of a macro-device, and its effects can be found even in interfacial phenomena in macro-structures since friction and wear occurs at single asperity contacts.

1.4 Micro-Electro-Mechanical Systems (MEMS)

Normally, micro-electro-mechanical systems (MEMS) are treated as the integration of mechanical elements, sensors, actuators, and electronic circuits on a common silicon substrate through the utilization of micro-fabrication technology [23]. As derived from integrated circuits, MEMS is developed to make complex electromechanical systems using batch fabrication techniques similar to those used for integrated circuits, and to unit these micro-electromechanical elements together with electronics.

Nowadays, MEMS products are adopted in many commercial areas including entertainment, communication [24], automation [25], etc. Premature device failures, however, are often observed at the micro-scale due to the surface forces acting as adhesion, friction and wear [26, 27]. As a dominant failure mechanism, the friction behavior of MEMS devices in the contact regime has not been well understood so far [28–30]. MEMS devices with sliding contact such as stepper motors [31], gas bearings [32], micro-motors [33] and associated components of micro-engines [9, 34], are still limited to laboratory studies. A schematic diagram of failures occurred at micro-scale is shown in Figure 1.5. Insufficient knowledge of the failure mechanism affecting the performance and reliability of these MEMS devices is the major obstacle to commer-
cialization.

![Figure 1.5 Schematic diagram of tribological issues in a MEMS device.](image)

Specially designed MEMS devices with the knowledge of micro-/nano-tribology are necessary to perform friction measurements at the micro-scale for fundamental understanding of the mechanisms of friction and wear at micro-/nano-scale. With noting the poor tribological properties of polycrystalline silicon (polysilicon), high surface energies and high adhesion between MEMS parts [35], single crystal silicon and relative micro-fabrication techniques were chosen in this project.

### 1.5 Objectives and Scope

The objective of this research is to develop bulk micro-machined micro-devices with rubbing surfaces, both linear and curved surfaces involved, for study of the micro-/nano-tribological characteristics at sidewall surfaces in MEMS device.

Corresponding to the objectives, the scope of the thesis involves the following:

(i) To develop micro-devices with stable contact pressure distribution for understanding friction transitions between the static and kinetic states at the sidewall interfaces of MEMS devices;

(ii) To develop micro-devices with curved rubbing surfaces for understanding friction mechanism at the curved sidewall interfaces of MEMS devices;

(iii) To study mechanical flexures with desired stiffnesses for linear motion as well as for rotary motion in the developed MEMS testing devices;
(iv) To explore the fabrication process of high-aspect-ratio MEMS devices based on bulk micro-machining;

(v) To characterize the developed MEMS testing devices from both static and dynamic scopes;

(vi) To analyze the experimental data obtained from the developed testing devices and to study the micro-/nano-tribological characteristics at sidewall contact interfaces of MEMS devices.

1.6 Organization of the Thesis

Chapter 2 reviews the development of the technologies, instruments as well as testing devices for tribological measurements. The review focuses on the MEMS testing devices being developed with in-plane or out-of-plane configurations. The major fabrication techniques developed for MEMS are also introduced.

In Chapter 3, details of the designs of the linear and the rotary MEMS testing devices are given. Analysis of the stiffnesses of the supporting flexures for the testing devices based on beam bending theory and working conditions is provided. Finite element analysis adopted to verify the results from theoretical analysis is also presented in this chapter.

Chapter 4 describes the critical micro-fabrication techniques adopted in developing the MEMS testing devices for this project. Challenges encountered and approaches achieved are discussed in detail. Fabrication results of the MEMS testing devices are also presented.

In Chapter 5, the process of characterization and measurement of the developed MEMS testing devices is presented. Optical techniques adopted in this project are highlighted. Fabrication tolerances and the corresponding influence on the performance of the testing devices are also discussed.
Chapter 1

Introduction

Chapter 6 focuses on the measurement and analysis of the linear friction testing device. Analysis based on contact mechanics of the sidewall contact interfaces is presented. A quasi-static stick-slip model applicable to any friction-related dynamic system has been developed to predict the motions of the testing devices. Friction behaviors of the linear testing device are also discussed.

In Chapter 7, analysis and discussion for the rotary friction testing device is provided. Prediction of the regions with sliding wear of the rotary testing device based on the derived equivalent stiffnesses is presented. Analysis of the tribological phenomena of the rotary testing devices are given.

Chapter 8 presents the conclusions and contributions of this project. Future work could be done in the areas relative with this project is also proposed.
Chapter 2

Literature Review

In this chapter, tribological measurements, especially the measurement at micro-/nano-scale, are introduced first. For micro-/nano-tribological measurements, previous works for both microscopic measurement and in-situ measurement are, then, reviewed.

2.1 Conventional Tribological Measurements

Tribological measurements based on established friction laws for evaluation of friction and wear properties have been developed with the understanding of fundamental mechanisms of friction and wear. The most commonly used interface geometries for tribological measurement are shown in Figure 2.1. Contact types and sliding properties of these testing geometries are listed in Table 2.1 [18].

As the basic concepts of the friction and wear measurement, these geometries have been adopted in conventional tribological testing devices, such as pin-on-disk and ball-on-plate tribometers, as shown in Figure 2.2. Moreover, those concepts have also been used for micro-tribological measurements via macroscopic structures.

In a side-drive micromotor, riders were designed to obtain the same contact pressure by Suzuki et al. [36]. Using simple macroscopic riders sliding on a disc, a range of MEMS materials have been studied qualitatively under both lubricated and dry sliding
conditions, as shown in Figure 2.3. Their results indicated that the wear rate in MEMS is expected to be far less than the bulk case and that the wear rate depends on the load even under the same contact pressure [36].

Beerschewinger and colleagues developed specimen-on-disc samples with controlled
Chapter 2

Table 2.1 Some details of typical geometries for friction and wear testing

<table>
<thead>
<tr>
<th>Type</th>
<th>Contact</th>
<th>Sliding Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin-on-disk</td>
<td>Point / conformal</td>
<td>Nondirectional</td>
</tr>
<tr>
<td>Pin-on-flat</td>
<td>Point / conformal</td>
<td>Reciprocating</td>
</tr>
<tr>
<td>Pin-on-cylinder</td>
<td>Point / conformal</td>
<td>Nondirectional</td>
</tr>
<tr>
<td>Thrust washers</td>
<td>Conformal</td>
<td>Nondirectional</td>
</tr>
<tr>
<td>Pin-into-bushing</td>
<td>Conformal</td>
<td>Nondirectional</td>
</tr>
<tr>
<td>Flat-on-cylinder</td>
<td>Line</td>
<td>Nondirectional</td>
</tr>
<tr>
<td>Crossed cylinders</td>
<td>Elliptical</td>
<td>Nondirectional</td>
</tr>
<tr>
<td>Ball-on-plate</td>
<td>Point</td>
<td>Nondirectional</td>
</tr>
</tbody>
</table>

Figure 2.2 Schematic diagram of typical tribometers: (a) tribometers based on pin-on-disk; (b) tribometers based on ball-on-plate.

areas of contact in 1994, as shown in Figure 2.3. Various materials compatible with micromachining were coated on these samples for friction and wear properties measurement at the same pressures as those calculated in micromotors [37, 38]. They found that for low nominal contact pressures the wear rate settled into a constant value independent of pressure, but for higher pressures wear rates were similar to that predicted for macro-scale contacts. Although their motors did not function well, their calculations, based on measured friction values, indicated the addition of a bushing and a more symmetric stator design would result in working devices [39].

Friction and wear behaviors under macro-scopes can be well studied using tribome-
ters developed. However, the understanding of environmental conditions of friction physics was poor. It was the advent of scanning tunneling microscope that brought the possibility to perform friction experiments under very well defined conditions.

As described by Amontons’ law, the frictional force is proportional to the normal load, and the coefficient of friction (COF) is independent of the load and the contact area. However, some experiments in micro-scale including those using microstructures in MEMS showed that Amontons’ laws cannot be used successfully in those cases. It is realized that the surface properties strongly related to friction and wear become increasingly important when the structures are getting smaller and smaller. To characterize wear and friction at micro-scale, some special instruments and methods will be needed, such as microscopes based on probe-scanning techniques.
Chapter 2

Literature Review

2.2 Tribological Measurements by Microscopes

It was noticed that a current will flow inside a thin potential barrier between two metals when a potential difference was applied to the metals [40]. Using vacuum as the potential barrier, scanning tunneling microscope (STM) was developed by Heinrich Rohrer and Gerd Binnig in 1982 [41].

In an STM, a sharp metal tip is adopted as the electrode of the tunnel junction. During the lateral scanning, the tip is brought very close to the surface to be investigated, which also works as the second electrode. The tunnel current varies exponentially with the distance between the two electrodes. The distance between the tip and the surface is 0.3 to 1 nm, and the convenient operating voltage is 10 mV to 2 V. As a result, a tunneling current varying from 0.2 to 10 nA is produced and a high resolution surface image, laterally less than 1 nm and vertically less than 0.1 nm, is obtained.

The STM can be operated in either a constant-current mode or a constant-height mode. In the constant current mode, the height of the tip is set to be variable to maintain a constant tunneling current, as shown in Figure 2.5(a). In the constant height mode, the tip remains at a nearly constant height during the scanning and the tunneling current is imaged, as shown in Figure 2.5(b).

The fact that only the surface of electrically conductive material can be measured constrains the usage of STM. In 1985, Gerd Binnig and his colleagues developed an instrument, atomic force microscope (AFM), which is capable of investigating both conducting and insulating surfaces of scientific and engineering interest at an atomic scale [42]. Using an AFM, a surface is scanned line by line by a very small tip (a radius ranging from 10 to 100 nm) attached on a cantilever. Bending of the tip due to the force between the tip and the surface is detected with the laser beam deflection method, as shown in Figure 2.6. A laser beam is projected to a mirror put on the back of the cantilever near its free end. The reflected beam from the vertex of the cantilever
Chapter 2

Literature Review

Figure 2.5 Schematic diagram of the operation of a scanning tunneling microscope (STM): (a) STM working in a constant current mode; (b) STM working in a constant height mode.

is directed through the mirror onto a squad photodetector.

With the scanning probes of AFMs, it is possible to study interfacial phenomena at a small scale with a light load [20, 43–45]. Noting that contact occurring at asperities is most solid to solid interfaces, the tip of an AFM can simulate a sharp single asperity traveling over a surface. Thus, scratching and wear processes at different normal loads could be studied in AFMs by using a sharp diamond tip [46, 47].

Figure 2.6 Schematic diagram of the working principle of an atomic force microscope (AFM)

Both optical and microscopic measurement methods can supply high resolution and accuracy of tribological information on materials. However, the low scanning speed
and excessive sharpness of the microprobe tips used in microscopic measurement can not simulate contact conditions similar to those of MEMS interfaces. Moreover, these methods can only be used before and/or after tribological behaviors. They cannot perform real time measurement of working micro-devices. To reflect details of the actual loading conditions, to obtain insight into the origins of friction in MEMS, and to ensure that the material properties and morphological nature of the contact surfaces under test are similar to those of actual devices, special MEMS devices for tribological characterization testing under contact conditions typical of those in MEMS devices are desirable.

2.3 Micro-Tribological Measurements

As the development of the tribological measurement on micro-/nano-scale, non-specified MEMS devices were first adopted for tribological measurement and specified devices were brought out soon after.

2.3.1 Measurements with General MEMS Devices

Using encountered tribological issues, MEMS devices for actuation or motion transferring were first adopted in the tribological measurement.

![Figure 2.7 Schematic diagram of the cross-section of a micromotor.](image)

The coefficient of kinetic friction of polycrystalline silicon in contact with silicon nitride was measured by Tai and Muller based on an electrostatic micromotor in 1990 [48]. The micromotor was developed using processes derived from IC (integrated circuit) fabrication techniques. As shown in Figure 2.7, the rotor would have a contact
with the hub during rotation, i.e., the contact between polycrystalline silicon and silicon nitride was obtained. The rotating angular has a relationship between the driving torque and the frictional torque. Using developed motion equations and observed starting and ending rotation angles, the kinetic friction coefficient between polycrystalline silicon and silicon nitride was estimated to be 0.2 and 0.38.

![Figure 2.8 Schematic diagram of a laser-based friction measurement system [49].](image)

At the same time, Gabriel et al. brought out an in-situ friction and wear measurement method based on a microturbine [49]. As shown in Figure 2.8, a neurophysiological recording pipette was used as an air nozzle to drive the component of a microturbine spinning. A laser beam was adopted to form a spot focussed on the blades or teeth of the integrated components. A photodiode was then aligned to maximize the output signal seen on an HP network analyzer. The deceleration of the rotating component was estimated from the measured profiles of rotation speeds. With considering the torques due to fluid sheer stress and friction as well as the rotational inertia, the kinetic friction coefficient was obtained as 0.28.

Mehregany et al. also carried out qualitative studies for an electrostatic wobble micromotor in 1992. The wear in the wobble micromotor was measured by comparing its gear ratios before and after the wear test since wear would lead to a decrease in gear ratio shown as an increase in bearing clearance [50].

In a significant enhancement to the micromotor concept, Garcia and Sniegowski
Figure 2.9 Schematic diagram of the connected gear and linkage arms of a microengine [34].

reported a design and process for a microengine that allowed a small gear driven by orthogonal electrostatic comb drives to be rotated via a pin linkage and to couple to other gears [34]. The gear of the microengine is connected to the linkage arms via a pin point, as shown in Figure 2.9. After running a number of cycles, the gear hubs were worn down and the gears exhibited severe wobble during operation. The debris was typically thrown out from the hub and collected on the gear face and surrounding substrate. By observing cross sections of the worn places, the wear effect of this MEMS device were then obtained.

In all these devices with contact surfaces, friction and wear are detrimental effects and cannot be readily measured. It is difficult to know the pressure distribution in detail in these devices. Thus, MEMS devices specified for realtime tribological measurement are desirable.

2.3.2 Measurements with In-Plane Friction/Wear MEMS Devices

With integrated actuating and sensing components, MEMS devices for friction and wear measurements were first developed using in-plane sliding interfaces.

Lim et al. developed a friction test structure based on a single electrostatic comb drive and a friction foot to which electrostatic voltages could be applied to controllably
vary the load [4]. Similar in-plane testing device using sphere-on-flat geometry was
developed by Sandia Lab [5]. A schematic diagram of this in-plane testing device is
shown in Figure 2.10. A moveable shuttle comprising “dimple” or “bump” was sup­
ported by folded flexures above a test surface. By applying voltages to the electrode
pads, the dimple/bump on the shuttle is brought into contact with an underlying electri­
cally grounded test surface. Driving by electrostatic comb drives with varying later
voltage, both static friction and kinetic friction can be achieved.

![Figure 2.10 Schematic diagram of a single-dimple friction testing microstructure [4].](image)

Based on the device of Lim et al., Lumbantobing and Komvopoulos developed a
surface micro-machine to investigate the dependence of static friction in MEMS on
the external load, apparent contact area, and environmental conditions [51]. In this
design, the suspended dimple was pulled into contact with the substrate by a gap closing
actuator, and the friction force at the instant of lateral displacement was determined
from voltage-force relationships, as shown in Figure 2.11. Comparing to the device of
Lim et al., this modified device is able to achieve larger lateral displacements at the
contact interface and friction testing over wider ranges of normal and friction forces.

To study friction and wear at the MEMS level, de Boer et al. designed a nanotractor
capable of generating relative movement on nano-scale between the contacting surfaces [52, 53]. As shown in Figure 2.12, during the motion of the nanotractor, the central actuation plate is sequentially actuated and released while the trailing and leading clamps are alternately held in place using electrostatic potentials. To prevent electrical shorting of the actuation plate to the plate electrode, electrically grounded standoffs were adopted as motion stopper. As reported, the nanotractor can travel ±100 μm in 50 nm steps.
In the recent decades, more and more MEMS applications with sidewall contact as the major load-bearing component have been developed, such as stepper motors [31], gas bearings [32], micro-motors [33] and associated components of microengines [9, 34]. Noting that the topography of side walls is significantly different from that of lateral surfaces, MEMS devices for out-of-plan measurement are necessary to study friction characteristics of sidewall surfaces.

2.3.3 Measurement with Out-of-Plane Friction/Wear MEMS Devices

Prasad et al. first developed a lateral micro-instrument for direct measurement of static and kinetic friction between two sidewall surfaces based on single crystal reactive etching and metallization (SCREAM) process [6]. In the clamp-with-beam instrument, two opposite clamps were adopted to make contact with a movable beam placed in-between the sidewall surfaces, as shown in Figure 2.13. During operation, each clamp is first brought into contact with the beam by a set of comb actuator under a DC voltage $V_{DC}$. Subsequently, the movable beam is driven by an AC voltage and sliding at the sidewall surfaces can be achieved. Stiction of the sidewall surfaces in vacuum has been observed by using this micro-instrument, and the value of static coefficient of friction of silicon-dioxide coated side walls was found to be 0.5.

Similar configuration was also adopted in the silicon friction meter developed by Hwang et al. by using deep reactive ion etching (DRIE) in 2006 [8]. The friction meter was designed to have different contact width and number of contact points. It was reported that coefficients of friction increased with the normal load while contact shape with different contact width and number of contact points did not have a noticeable effect on the friction coefficients.

In 1997, Sandia National Laboratory brought out a beam-on-post testing device for study of friction and wear at sidewall surfaces [5]. The device consists of two sets of comb actuators which are perpendicularly placed to each other while a movable beam
was adopted to connect the two actuators, as shown in Figure 2.14. A post is anchored to the substrate and has a cylindrical geometry facing the beam. During operation, the movable beam is first brought into contact with the post by the normal actuator under a DC voltage. The lateral actuator is then driven by a waveform to generate a sliding between the beam and the post. Initial results of the coefficient of friction for coated and uncoated sidewall surfaces of polycrystalline silicon were given. The sidewall surfaces coated with octadecyltrichlorosilane (ODTS) were found to have a three times longer lifetime than that of the uncoated surfaces. Some results and analysis of wear mechanisms in MEMS reliability have also been obtained by using this test structure [7].

Based on the beam-on-post configuration, a micro-tribotester for measurement of static and kinetic friction coefficients of sidewall surfaces has been developed by Guo et
Figure 2.14 Schematic diagram of the beam-on-post testing device for friction and wear study at sidewall surfaces [5].

al. in 2007 [54]. As shown in Figure 2.15, two perpendicularly placed but disconnected comb actuators were adopted in this tribotester. The normal actuator was designed to bring the cross-tip into contact with the opposite movable beam under a normal force. The lateral actuator drives the beam sliding back-and-forth against the tip. Stick-slip motion has been achieved by this tribotester. Measured coefficient of kinetic friction is no longer a constant value when normal force on the contact interfaces is variable.

A recently developed polycrystalline silicon micromachine with flat contact surfaces fabricated by surface micromachining was used to study friction and adhesion in sidewall surfaces of MEMS devices [55]. This micromachine includes two comb-driven shuttles, namely the normal shuttle and the lateral shuttle, as shown in Figure 2.16. The normal shuttle was adopted to apply both pushing force and pulling force of the sidewall surfaces along the normal direction. The lateral shuttle was used to generate relative motion or tendency toward such motion of the sidewall contact interface. A nonlinear relationship between the coefficients of friction and the normal load was found, and the explanation based on adhesion force has been given. The true static coefficient of friction after considering the adhesion effect, such as van der Waals force
Chapter 2

Literature Review

Figure 2.15 Schematic diagram of the micro-tribotester for measurement of static and kinetic friction coefficients of sidewall surfaces [54].

and capillary forces, can be determined from the nonlinear relationship of the measured coefficient of friction.

Figure 2.16 Schematic diagram of the MEMS device for sidewall friction and adhesion experiments [55].
2.4 Fabrication Techniques in MEMS

For the micro-tribological testing devices reviewed above, they are all developed via micro-fabrication techniques. Among diversity of applications, three types of fabrication techniques are included in MEMS, which are surface micro-machining, bulk micro-machining and LIGA.

Surface micro-machining is a technique based on the deposition and etching of different structural layers on top of the substrate [56]. Micro-mechanical structures or devices are made entirely on the surface of the substrate via additive processes. Typically, the desired structure is built up by depositing and patterning thin films of structural and sacrificial material on the substrate surface, as shown in Figure 2.17.

![Schematic diagram of the steps used in the surface micromachining process.](image)

Unlike surface micromachining, silicon substrates are selectively etched to produce structures in bulk machining. Wet etching and dry etching are the mainly used ways, and they are selected dependent on the etching media. Wet etching is a purely chemical process that can be highly selective and often does not damage the substrate [57]. Dry etching involves the exposure of the substrate to an ionized gas. Etching occurs through chemical or physical interaction between the ions in the gas and the atoms of the sub-
Chapter 2

Literature Review

strate. Dry etching techniques can almost be suitable to any material used for MEMS. A typical bulk micro-machining process via dry etching is shown in Figure 2.18.

![Figure 2.18 Schematic diagram of bulk micro-machined structure.](image)

Besides these two silicon based fabrication techniques, LIGA provides another fabrication method for the structures required very high aspect ratio. LIGA is a German acronym for Lithographie (Lithography), Galvaniformung (electroforming) and Abformung (molding) [58]. As shown in Figure 2.19, the LIGA process begins with ultraviolet (UV) or deep X-ray lithography to transfer the desired patterns on a thick film of photoresist. By using the photoresist as the outline of the expected product, desired metal is electroplated on the base. Finally, the product with required wall thickness is done after the removal of photoresist material.

![Figure 2.19 Schematic diagram of mold formation using LIGA technique [59].](image)
For silicon based MEMS devices with high-aspect-ratio structure and movable components such as the friction testing devices developed in this project, generally there are two fabrication methods adopted in bulk micro-machining, process with silicon-on-insulator (SOI) wafers or process with silicon-on-glass (SOG) wafers.

2.4.1 Process with SOI Wafers

Silicon-on-insulator (SOI) wafer is widely adopted in MEMS devices with movable components, such as micro-actuators, switches, etc. [60, 61]. The role of an SOI wafer is to electronically insulate a fine layer of single crystalline silicon from the rest of the silicon substrate. As shown in Figure 2.20, three layers are included in an SOI wafer. The bottom one is a silicon substrate (handle layer). The middle layer is a layer of silicon dioxide acting as an insulator. The top one is another layer of silicon layer (device layer).

![Figure 2.20 Schematic diagram of a silicon on insulator (SOI) wafer.](image)

Silicon dioxide (SiO₂) based SOI wafers can be produced by several methods. Smart-Cut process based on wafer bonding process developed by SOITEC corporation [62] uses ion implantation followed by controlled exfoliation to determine the thickness of the uppermost silicon layer. SIMOX (Separation by Ion Implantation of Oxygen) is another fabrication technology which uses an oxygen ion beam implantation process followed by high temperature annealing to create a buried SiO₂ layer [62, 63]. For MEMS applications with different thickness requirements for the device layers (1-50 μm) as well as good uniformity, BESOI (Bonded and Etchback SOI) is an alternative process method. In BESOI wafers, an etch stop is introduced before wafer bonding,
Chapter 2

Literature Review

typically by implanting a high dose of boron to produce a buried layer [62, 64]. Although the cost of SOI wafers is well controlled in IC industries, it is much higher than that of normal single crystalline wafers for MEMS purposes due to the relative low volume and yield rate. Furthermore, two defects limit the usage of SOI wafers in MEMS: stiction and footing phenomenon.

![Schematic diagram of the cantilever beam stiction by capillary forces.](image)

Figure 2.21 Schematic diagram of the cantilever beam stiction by capillary forces.

To release micro-structures from the substrate via removing sacrificial layers, wet etching is a usual step adopted in the fabrication of MEMS devices. During drying of the etched devices, the rinse liquid trapped in the small space between the substrate and micro-structures creates strong capillary forces, as shown in Figure 2.21. These forces induce stiction of the micro-structures. The small sacrificial gap (the oxide layer) of the SOI wafers, usually two or three micrometers, makes this stiction problem even more serious [65].

![Schematic diagram of notching phenomenon on a SOI wafer after DRIE process.](image)

Figure 2.22 Schematic diagram of notching phenomenon on a SOI wafer after DRIE process.

The notching phenomenon occurs in SOI wafers when deep silicon reactive ion etching (DRIE) is adopted as the etching method, as shown in Figure 2.22 [66, 67].
This footing phenomenon results in unwanted etching of silicon at the interface of the oxide and the handle layer. It also affects the roughness and position of the bottom surfaces, i.e., notching. As a consequence of footing phenomenon, notching causes a non-uniform distribution of mass and stiffness. The silicon fragments separated from the device layer due to the footing phenomenon would stick to feature parts of the MEMS device and cause an electrical short between electrodes [66].

2.4.2 Process with SOG Wafers

With excellent electrical insulating property and good mechanical characteristics, glass is a common adopted substrate material for MEMS devices. The low cost of glass wafers make SOG process more competitive comparing to the process based on SOI wafers. For fabrication with SOG wafers, anodic bonding and DRIE are two critical techniques.

2.4.2.1 Anodic Bonding

The working principle of an anodic bonding process is shown in Figure 2.23. In the bonding process, the glass substrate, Pyrex glass, is placed on top of the silicon wafer. A constant negative bias is applied on the glass substrate with respect to the grounded silicon via a pin-contact. At elevated temperatures, the glass substrate becomes a conductive solid electrolyte and the migration of sodium ions (Pyrex glass contains about 3-5% Na) towards the cathode is generated. As the ions moving, a space charge (bound negative charge) in the region of the glass-silicon interface is formed. During this charging process, the electric field is high enough to allow a drift of oxygen to the positive electrode (silicon wafer) reacting with silicon and creating Si-O bonds. The primary variables that control this process are temperature, voltage, time and surface roughness [68, 69].
2.4.2.2 Deep Reactive Ion Etching (DRIE)

Deep reactive ion etching (DRIE) is a powerful technique with distinct advantages. By using the Bosch technique developed by Robert Bosch Gmbh, deep etching is achieved by alternating etching cycles (using SF₆ and O₂) and sidewall passivating cycles (using C₄F₈) to achieve anisotropic etching results [70]. After each passivating cycle, the coated Teflon-like film is preferentially removed from the bottom of the trenches while preventing etch of the sidewall surfaces in the subsequent etching cycle, as shown in Figure 2.24. With well set parameters, high selectivity to photoresist or silicon dioxide (50:1 to 200:1), and high aspect ratio are applicable in bulk micro-machining by using DRIE process [71–73].

For the DRIE process, the etch rate, etched trench profile, and anisotropy are greatly dependent on the chamber pressure, gas flow rate, cycle time, and platen power, etc. To achieve a high aspect ratio (larger than 20:1) with relative large etching depth (larger than 100 μm), RIE lag and microloading were the two major phenomena affecting the final yield of the MEMS testing devices.

RIE lag or aspect ratio dependent etching (ARDE) is the generalization that trenches (or holes) with a smaller spacing etch slower than features with larger spacing, as shown in Figure 2.25. Some methods have been brought out in the previous research work and experiments to reduce the RIE lag effects, such as raising the process pressure in both the etching and passivating cycles [74], decreasing the time ratio of the etching cycle to...
the passivating cycle, the process pressure, and the platen power [75], or using higher SF₆ flow rates [76]. Noting that most previous research in the RIE lag reduction in plasma etching processes were performed under different experimental conditions and some of the experimental findings are even contradictory, these results are adopted only as guidelines in the fabrication of the MEMS testing devices of this project.

Figure 2.25 Schematic diagram of the RIE lag effect.
Chapter 2  

For a silicon DRIE process, the ionization degree of the glow discharge, and the ion flux reaching the silicon surface for etching are determined by the coil power. As increasing of the coil power, the increasing rates of the ion flux in wider trenches are larger than that in narrower trenches. The platen power accelerates the etching/passivating radicals to bombard the silicon surface. Thus, etch rates of both the exposed silicon and the etching mask (photoresist or other hard mask) could be increased. The duration of the etching and passivating cycles mainly affect the profile of the etched trenches. As the duration of the etching cycle increased, the etching gas (SF$_6$) flies for longer duration, and the amount of ions and chemical radicals is larger. Thus, the etching profiles of the trenches turns negative. On the contrary, the etching profiles of the trenches becomes more positive and the etching of trench bottom becomes more difficult as the duration of the passivating cycle increased. As another parameter, the chamber pressure affects the etching profiles through the formation of chemical radicals [76–79].

Microloading is the term to describe the effect that etch rate of features close together is lower than that of an identical but isolated feature. The microloading effect is caused by local depletion of etchant on regions of the wafer with high pattern density [80, 81]. Hedlund et al. found that the microloading effect is very dependent on the chamber pressure and it is relatively small ( < 10%) compared to the RIE lag effect [82, 83]. Karttunen et al. claimed that the microloading effect could be alleviated to some extent by using clever designs [84].

To reduce the loading effect, a method using a nested photomask was brought out by Bayt et al. [85]. Before lithography of photoresist with the nested photomask, an oxide layer is first patterned with the original mask. The DRIE process will etch a depth of the denser areas first. After a certain depth is achieved, the photoresist is stripped and the patterned oxide is used as a hard mask to complete the etch. The success of this etching scheme is too constrained by the etch timing, and may cause overetching and...
Chapter 2  

Literature Review

feature distortion.

To sum up, bulk micro-machining with SOG wafers is the most expectative and cost-effective fabrication technique to achieve the desirable MEMS friction testing devices. By adopting suitable processing parameters, bulk devices with high aspect ratio can be achieved with wafer bonding and DRIE processes.

2.5 Summary

The literature review aimed to provide a holistic view of the research that has been done to date on friction and wear measurement and the critical MEMS fabrication techniques relative to those research work. From conventional measurement methods to in-situ micro-testing devices, the dimensions of the objects to be measured are scaling down while the scopes of research are increasing. To obtain the fundamental understanding of micro-/nano-tribological characteristics of MEMS devices, specified micro-testing devices are necessary. Various types of micro testing devices were discussed for their pros and cons. It can also be observed that studies so far focused on mono friction phenomena, static friction or kinetic friction. No attention has been paid to the transition between the static friction and the kinetic friction. Moreover, only linear motions of rubbing surfaces have been studied and no research work has been reported for studying of curved sidewall surfaces. The need for novel micro testing devices with fewer shortcomings, wider research scope and effective cost has been underscored.
Chapter 3

Design of MEMS Friction Testing Devices

In this chapter, two kinds of MEMS testing devices for study of friction behaviors on both linear and curved sidewall interfaces are introduced. For study of the linear sidewall interface, a MEMS device with a sensing plate for sliding contact and a driving plate with two bumps for the Hertzian contact are developed. For study of the curved sidewall interface, a rotary MEMS device in oscillating motion was developed. In this project, electrostatic comb drives are adopted as the actuators. The actuating mechanisms of the linear and rotary comb actuators are introduced. Both numerical derivation and finite element analysis of the supporting flexures for the testing devices are also presented.

3.1 Design of the Linear Friction Testing Device

As reviewed in Chapter 2, significant efforts have been devoted to study friction and wear phenomena at sidewall surfaces in MEMS devices. However, friction transitions between the static and kinetic states (stick-slip phenomenon) at the sidewall surfaces have not yet been adequately characterized in the existing research work. The location
of contact or the contact pressure distribution on the surface of the stationary component in the existing devices may change significantly due to the tilting or deformation of the moving component used. To ensure that the material properties and morphological nature of the contact sidewall surfaces under test are similar to those of some actual devices, a linear MEMS device for friction characterization at sidewall surfaces is desirable.

3.1.1 Operation Principles of the Linear Device

The linear sidewall friction testing device consists of two orthogonal comb actuators, a flat plate (sensing plate), and a plate with two bumps (driving plate), as shown in Figure 3.1. The normal comb actuator is suspended by four folded flexures, called flexure combination $N$. The sensing plate is, in turn, connected to the normal comb actuator by two folded flexures, flexure combination $S$. The driving plate with a pair of bumps has been integrated with the tangential actuator which is supported by the other two folded flexures, flexure combination $T$. To obtain an observable distance while the sensing plate stick with the driving plate, stiffness of flexure combination $S$ is designed to be comparable to that of flexure combination $T$. The dual bump design can ensure that friction and wear take place at the same region on the bumps because the orientation of the flat sensing plate surface is uniquely determined by its engagement with the two bumps.

During the operation, three different electric potentials, $V_G$, $V_t$, and $V_n$, were adopted for the friction testing, as shown in Figure 3.1. To both comb actuator sets, a given voltage, $V_G$, was applied to all the grounded area of the device. A DC bias, $V_n$, was applied to the normal comb actuator to provide a normal force pushing the sensing plate into contact with the bumps on the opposite driving plate. Subsequently, a trapezoidal voltage waveform $V_t(t)$, where $t$ being time, was applied to the tangential comb actuator to actuate the driving plate. In response, the tangential displacements of the driving plate
and the sensing plate varied periodically and, thus, static friction and kinetic friction could occur alternately.

The free body diagram of the testing device is shown in Figure 3.2. According to the force equilibrium conditions of a massless system, the normal force, $N$, acting on
Chapter 3  

Design of MEMS Friction Testing Devices

the contacting surfaces is presented by

\[ N = F_n - R_n \]  \hspace{1cm} (3.1)

where \( F_n \) the electrostatic force exerted by the normal comb actuator, \( R_n \) is the restoring force of flexure combination \( N \) (Figure 3.1). The friction force \( f \) acting on the contact interface is given by

\[ f = F_t - R_t \]  \hspace{1cm} (3.2)

where \( F_t \) is the electrostatic force exerted on the driving plate by the tangential comb actuator, \( R_t \) is the restoring force of flexure combination \( T \) (Figure 3.1). Since the sensing plate is only driven by friction, the friction force at the contact interface can also be expressed as, \( f = R_s \), where \( R_s \) is the restoring force of flexure combination \( S \) (Figure 3.1). Combining Eqs. (3.1), (3.2) and the foregoing relation, the coefficient of friction of the contact interface is found to be

\[ \mu = \frac{f}{N} = \frac{F_t - R_t}{F_n - R_n} = \frac{R_s}{F_n - R_n} \]  \hspace{1cm} (3.3)

3.1.2 Actuating Mechanism of the Linear Actuators

An electrostatic comb drive consists of two interdigitated finger structures, one finger part is fixed and the other one is movable [86]. By applying different voltages to the different comb drive parts, the movable finger would be driven to move towards the stationary finger. As varying of the position of the movable finger, the capacitance of the whole comb drive is also varying. Due to their inherent characteristics, comb drives have been widely adopted for different purposes, such as resonators [87–89], electromechanical filters [90], optical shutters [91, 92], micro-grippers [93] and voltmeters [94]. They have also been used as the driving element in vibromotors [95] and micromechan-
Figure 3.3 Schematic diagram of a comb drive displaying the presentation of the capacitance.

In this project, only the role of actuator in comb drives were considered and exploited [97].

To obtain a linear motion of the driving plate, linear comb drives were adopted for the linear testing devices (Figure 3.1). The working concept of a linear comb drive is derived from that of a parallel-plate capacitor. The schematic diagram of a comb drive is shown in Figure 3.3, where the overlapping length of the fixed and movable finger is $x_0$, the width of each finger is $w$, and the gap spacing between two adjacent fingers (a pair of fingers) is $g$. With a potential difference, $V$, between the stationary and the movable finger applied, each finger pair can be treated as a parallel-plate capacitor and a capacitance $C_0$ is achieved. Meanwhile, a parasitic capacitance, $C_p$, at the end of each movable finger is also generated due to the undesirable fringing electric field.

With considering the parasitic capacitance, the total capacitance of a pair of comb fingers is given by

$$C = C_p + C_0 \quad (3.4)$$

According to the theory of a parallel-plate capacitor, the capacitance $C_0$ and parasitic capacitance $C_p$ of each finger pair are given by $C_0 = \frac{H x_0 \varepsilon}{g}$, and $C_p = \frac{H w \varepsilon}{(D - x)}$, respectively, where $\varepsilon$ is the permittivity of air, and $H$ is the thickness of the comb fingers. Usually, finger gap is designed to be much smaller than the parasitic distance,
Chapter 3  

Design of MEMS Friction Testing Devices

i.e., \( g \ll (D - x) \). Thus, the parasitic capacitance \( C_p \) is ignorable comparing to the capacitance \( C_0 \) [98], and Eq. (3.4) is simplified to be

\[
C = \frac{\varepsilon H x_0}{g}
\]  

(3.5)

Based on Eq. (3.5), the changed capacitance of a finger pair as the overlapping length increasing \( \Delta x \) is found to be

\[
\Delta C = \frac{\varepsilon H \Delta x}{g}
\]  

(3.6)

The electrostatic force \( F_{cp} \) between two electrodes is given by [99]

\[
F_{cp} = \frac{1}{2} \frac{\partial C}{\partial x} V^2
\]  

(3.7)

Combining with Eq. (3.7) and (3.6) without considering the force direction, the electrostatic force generated by one pair of fingers is given by

\[
F_{x0} = 2F_{cp} = \frac{\varepsilon H}{g} V^2
\]  

(3.8)

Thus, for a comb drive with \( n \) pairs of comb fingers, the electrostatic force is given by

\[
F_x = \frac{n \varepsilon H}{g} V^2
\]  

(3.9)

The electrostatic force generated by a linear comb drive exhibits a good controllable range of motion, and it is independent of the linear position of the movable fingers. The lateral electrostatic force increases with a decrease of finger gap as well as with an increase of the number of comb fingers. According to Eq. (3.9), the normal and
tangential driving forces of the linear friction testing devices are found to be

\[ F_n = \frac{n_n \varepsilon H}{g_n} V_n^2 = c_n V_n^2 \]  

(3.10)

and

\[ F_t = \frac{n_t \varepsilon H}{g_t} V_t^2 = c_t V_t^2 \]  

(3.11)

respectively, where \( n_n \) and \( n_t \) are the number of normal and tangential comb finger pairs, respectively, \( \varepsilon \) is the permittivity of air \( (8.855 \times 10^{-12} \, \text{F/m}) \), \( H \) is the thickness of the comb fingers, \( g_n \) and \( g_t \) are the finger gap between the comb fingers for the normal and tangential actuators, respectively, and \( c_n \) and \( c_t \) are the electrostatic force coefficients for the normal and tangential actuators, respectively, defined as \( c_n = \frac{n_n \varepsilon H}{g_n} \) and \( c_t = \frac{n_t \varepsilon H}{g_t} \), respectively. Combining Eq. (3.1) and (3.10), the normal force can be expressed as

\[ N = c_n (V_n^2 - V_{nc}^2) \]  

(3.12)

where \( V_{nc} \) is the \( V_n \) value corresponding to the first occurrence of contact, given by \( V_{nc} = (R_n / c_n)^{1/2} \).

### 3.2 Design of the Rotary Friction Testing Device

It is noticed that more and more MEMS devices with rotary or oscillating rubbing surfaces have been developed in recent years [9]. Nevertheless, all the reported friction testing devices were designed for measuring friction and/or wear at linear contact interfaces, such as contact occurred as dimple-on-pad, clamp-on-beam and post-on-beam. Measured results of these kinds of linear motion can not truly reflect the tribological behaviors occurred at the curved sidewall surfaces. Therefore, a device aiming at friction testing for the micro devices with bush-shaft structures is desirable.
3.2.1 Operation Principles of the Rotary Device

The rotary friction testing device consists of two sets of comb actuators, a bush (sensing bush) and a ring (driving ring), as shown in Figure 3.4. The normal comb actuator is suspended by four folded flexures, called flexure combination N. The sensing bush is, in turn, connected to the normal actuator by flexure S. The driving ring is integrated with two diagonally placed circumferential comb actuator (rotary comb), which is supported by four cantilever beams, called flexure combination C. The two symmetrically placed sensing bushes can be performed simultaneously or separately during the operation. In this project, only one sensing bush was used.

Similar with the linear friction testing device, three different electric potentials, \( V_G \), \( V_l \) and \( V_c \) were applied during the operation. As shown in Figure 3.4, the ground voltage, \( V_G \), was applied to all the grounded components of the testing device. A DC bias, \( V_n \), was applied to the normal comb actuator to provide a normal force to push the sensing bush into contact with the driving ring. The free body diagram of the working principle of the testing device with one side of the normal comb actuator is shown.
in Figure 3.5. Using the same actuating method, the force equilibrium in the normal direction would be the same as that of the linear testing device when contact occurs, and Eqs. (3.1), (3.10) and (3.12) are also applicable. Considering the diagonally placed rotary actuators (Figure 3.4), the driving torque is given by

$$ T = 2r_c F_{cm} $$

(3.13)

where $r_c$ is the effective moment arm.

To set the driving ring in an oscillating motion, a trapezoidal voltage waveform, $V_c(t)$, with $t$ being time, was applied to the circular comb actuator to oscillate the driving ring. Thus, the tangential displacements of the driving ring and the sensing bush varied periodically. Pure rolling as well as rolling with sliding could occur according to the external loads and the friction force between the contacting surfaces, i.e., static friction and kinetic friction could occur alternately. Combining Eqs. (1.1) and (3.12), the maximum static friction force, $f_{sm}$, and the kinetic friction force, $f_k$, are respectively
Chapter 3  

Design of MEMS Friction Testing Devices

found to be

\[ f_{sm} = \mu_s N = \mu_s c_n \left( V_n^2 - V_{nc}^2 \right) \]  

and

\[ f_k = \mu_k N = \mu_k c_n \left( V_n^2 - V_{nc}^2 \right) \]  

where \( \mu_s \) is the coefficient of static friction and \( \mu_k \) is the coefficient of kinetic friction.

3.2.2 Actuating Mechanism of the Rotary Actuator

As the extension of linear comb drives, rotary comb drives (torsional comb) were adopted in the rotary testing devices (Figure 3.4). To provide angular displacement, the rotary comb drive is widely used in the application of angular rate sensor such as gyroscopic sensor [100].

Different from the linear comb drives, the comb fingers of a rotary comb (stator and rotor), lie on the arcs of concentric circles, as shown in Figure 3.6. When potential difference between the stator and the rotor generates, the rotor (movable finger) would be driven to rotate towards the stator (stationary finger). In a rotary comb drive, \( R_i \) is the inner radius of one stator finger, \( r \) is the outer radius of the adjacent rotor finger (Figure 3.6), and \( \theta \) is the initial overlapping angle of the fingers.

Based on the Gauss Flux theorem [101], the potential difference between the stator and the rotor is given by

\[ V = \int_{r_i}^{R_i} E_c dr \]  

where \( E_c \) is the electric field intensity. Considering

\[ E_c = \frac{Q}{\varepsilon A} \]  

and

\[ A = r \theta \cdot H \]
the potential is found to be

\[ V = \int_{r_i}^{R_i} \frac{Q}{\varepsilon H \theta} \, dr = \frac{Q}{\varepsilon H \theta} \ln \left( \frac{R_i}{r_i} \right) \]  

(3.19)

where \( Q \) is the electric charge and \( A \) is overlapped area.

Based on Eq. (3.19) with noting \( C = Q/V \), where \( C \) is the capacitance of the capacitor formed by a finger pairs, the capacitance is given by

\[ C = \frac{\varepsilon H \theta}{\ln \left( \frac{R_i}{r_i} \right)} \]  

(3.20)

Noting that the gap between two adjacent finger is \( g = R_i - r_i \), Eq. (3.20) can be expressed as

\[ C = \frac{\varepsilon H \theta}{\ln \left( \frac{g}{r_i} + 1 \right)} \]  

(3.21)

The potential energy of the capacitor and the circumferential electrostatic force \( F_{c0} \) is given by

\[ U = \frac{1}{2} CV^2 \]  

(3.22)
and
\[ F_{\theta} = \frac{\partial U}{\partial \rho} \]  
(3.23)
respectively, where \( \rho \) is the length of the curve, and \( \partial \rho \) is the distance the finger has moved in the circumferential direction. According to Eq. (3.21) and (3.22), the change of potential energy is given by
\[ \Delta U = \frac{1}{2} \frac{\varepsilon HV^2}{\ln \left( \frac{g}{r_i} + 1 \right)} \cdot \Delta \theta \]  
(3.24)
Combining Eq. (3.23) and (3.24) with noting \( \Delta \rho = r \Delta \theta \), the circumferential electrostatic force of one pair of fingers is found to be
\[ F_{c\theta} = \frac{1}{2} \frac{\varepsilon HV^2}{r \ln \left( 1 + \frac{g}{r_i} \right)} \]  
(3.25)
The negative sign indicating the direction of the electric force here has been neglected.
To the rotary comb drive with \( n \) pairs of fingers, the electrostatic force is found to be
\[ F_{cr} = \sum_{i=1}^{n} \frac{\varepsilon HV^2}{2r_i} \frac{1}{\ln \left( 1 + \frac{g}{r_i} \right)} \]  
(3.26)
The log term in the denominator of Eq. (3.26) can be expanded in a Taylor series as
\[ \ln \left( 1 + \frac{g}{r_i} \right) = \frac{g}{r_i} - \frac{1}{2} \left( \frac{g}{r_i} \right)^2 + \frac{1}{3} \left( \frac{g}{r_i} \right)^3 \]  
(3.27)
Generally, the gap between a finger pair is much smaller than the radius of a rotary comb finger, i.e., \( g \ll r_i \). Thus, the second and higher order terms in Eq. (3.27) can be ignored, and Eq. (3.27) is simplified to be
\[ \ln \left( 1 + \frac{g}{r_i} \right) = \frac{g}{r_i} \]  
(3.28)
Substituting Eq. (3.28) into Eq. (3.26), the electrostatic force of a rotary comb drive is found to be

\[ F_{eq} \approx \sum_{i=1}^{n} \frac{\varepsilon t V^2}{2g} = \frac{n \varepsilon HV^2}{2g} \]  

(3.29)

Noting that only one side of the movable fingers was counted in the above derivation and the same result would be achieved for the derivation of the other side of the movable fingers, the electrostatic force generated by a rotary comb drive is found to be

\[ F_x = 2F_{eq} = \frac{n \varepsilon H V^2}{g} \]  

(3.30)

which is equivalent to a linear comb drive force.

3.3 Flexure Designs for Linear Motions

In the linear testing device, flexures supporting the normal and tangential actuators as well as the sensing plate, are denoted as flexure combination \( N, T \) and \( S \), respectively (Figure 3.1). To achieve a linear motions of the movable components without obvious transverse variation, those supporting flexures are required to be flexible along the motion direction (tangential direction) while being stiff along the direction perpendicular to the motion direction (normal direction). Thus, a large stiffness ratio, \( k_n/k_t \), is desirable, where the tangential stiffness of a supporting flexure is denoted as \( k_t \) and the normal stiffness is \( k_n \). To obtain desirable \( k_n/k_t \), folded flexures were introduced as the supporting flexures for all the suspended components in the linear testing device.

In this project, finite element analysis of the supporting flexures was carried out by using a commercial finite element method package, ANSYS. As a powerful tool for the numerical solution of a wide range of engineering problems [102], the explicit analysis results of the flexures obtained by finite element analysis were used as the verification of the numerical analysis above. Noting that the friction testing devices were made of single crystal silicon, the material properties are listed in Table 3.1.
Table 3.1 Material properties of single crystal silicon

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$ (GPa)</td>
<td>165</td>
</tr>
<tr>
<td>Density, $\rho$ (Kg/m$^3$)</td>
<td>2330</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.22</td>
</tr>
</tbody>
</table>

3.3.1 Basic Folded Flexure

A basic folded flexure can be treated as a combination of segments and nodes which are the joints of two segments. One end of the flexure is fixed while loads are acting on its free end, as shown in Figure 3.7. The length of the vertical segments (bending beams) are denoted by $b$ or $2b$. Those short horizontal segments with a length, $a$, are treated as rigid connecting beams. In the following analysis, $u_i$ and $v_i$ are adopted to represent the absolute displacement of each node in the lateral and transverse direction, respectively, and $\theta_i$ represents the absolute tilting angle of each node, where $i$ represents the node number. Similarly, $u_{ib}$ and $v_{ib}$ are adopted to represent the rigid-body displacement, and $u_{ir}$ and $v_{ir}$ represent the relative displacement of each node in the lateral and transverse direction, respectively.

3.3.1.1 Numerical Derivation

For each beam segment in a basic folded flexure, it is treated as one-end fixed and one-end free cantilever beam. For segment 12, the bending effect under the exerted loads is shown in Figure 3.8. Based on the force and moment equilibrium conditions, relations are found to be

$$\sum F(x) = 0 : F_{x2} = F_x$$  \hspace{1cm} (3.31)

$$\sum F(y) = 0 : F_{y2} = F_y$$  \hspace{1cm} (3.32)

$$-48-$$
Chapter 3  
Design of MEMS Friction Testing Devices

Figure 3.7 Schematic diagram of a basic folded flexure: (a) the basic folded flexure before bending; (b) deformation of a basic folded flexure (dashed line is the beam with deflection).

Figure 3.8 Schematic diagram of the bending effect of the basic folded flexure: (a) free-body diagram of segment 12 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment 12 with deflection.
Combining the foregoing relations, the moment on node 2 is found to be

\[ M_2 = M - bF_x + 2aF_y \]  \hspace{1cm} (3.34)

The relative displacements of segment12 based on beam bending theory give

\[ u_{2r} = \frac{b^2 M_2}{2EI} + \frac{b^3 F_{x2}}{3EI} \]  \hspace{1cm} (3.35)

\[ v_{2r} = 0 \]  \hspace{1cm} (3.36)

\[ \theta_{2r} = \frac{bM_2}{EI} + \frac{b^2 F_{x2}}{2EI} \]  \hspace{1cm} (3.37)

where \( E \) is the Young's modulus, and \( I \) is the area moment of inertia given by \( I = H w^3/12 \). Combining Eqs. (3.31), (3.34), (3.35), (3.36) and (3.37), the relative displacements on node 2 are given by

\[ u_{2r} = \frac{3b^2 M - b^3 F_x + 6ab^2 F_y}{6EI} \]  \hspace{1cm} (3.38)

\[ \theta_{2r} = \frac{2bM - b^2 F_x + 4abF_y}{2EI} \]  \hspace{1cm} (3.39)

The rigid-body displacement of segment12 with noting beam tilting is given by

\[ u_{2b} = u_1 + b\theta_1 \]  \hspace{1cm} (3.40)

\[ v_{2b} = v_1 \]  \hspace{1cm} (3.41)

Noting that the node 1 is full constrained, displacements of node 1 are given by \( u_1 = 0 \), \( v_1 = 0 \) and \( \theta_1 = 0 \). Combining Eq. (3.40) and (3.41) with noting the foregoing
relations, the rigid body displacements of node 2 are found to be

\[ u_{2b} = 0 \quad (3.42) \]

\[ v_{2b} = 0 \quad (3.43) \]

Combining Eqs. (3.38) and (3.42) with noting \( u_2 = u_{2b} + u_{2r} \), the lateral displacement of segment 12 is found to be

\[ u_2 = \frac{3b^2M - b^3F_x + 6ab^2F_y}{6EI} \quad (3.44) \]

Combining Eqs. (3.36) and (3.43) with noting \( v_2 = v_{2b} + v_{2r} \), the transverse displacement of beam 12 is

\[ v_2 = 0 \quad (3.45) \]

Based on Eq. (3.39) with noting \( \theta_1 = 0 \) and \( \theta_2 = \theta_1 + \theta_{2r} \), the tilting angle of beam 12 is found to be

\[ \theta_2 = \frac{2bM - b^2F_x + 4abF_y}{2EI} \quad (3.46) \]

Similar derivation based on beam theory has been performed to each beam segment and the detailed process was included in Appendix A. As a result, displacements of node 6 are found to be

\[ u_6 = \frac{8b^3F_x - 6ab^2F_y}{6EI} \quad (3.47) \]

\[ v_6 = \frac{24abM - 6ab^2F_x + 36a^2bF_y}{6EI} \quad (3.48) \]

\[ \theta_6 = \frac{8bM + 8abF_y}{2EI} \quad (3.49) \]

Noting that the flexure is always connected with a rigid body, the tilting of node 6 is constrained, i.e., \( \theta_0 = 0 \). Substituting the foregoing relation into Eq. (3.49), the relation...
between the force and the moment is found to be

\[ M = -aF_y \]  

(3.50)

With \( F_y = 0 \), Eq. (3.47) gives the tangential stiffness of the basic folded flexure as

\[ K_t = \frac{F_x}{u_6} = \frac{6EI}{12a b^2 + 8b^3} \]  

(3.51)

Similarly, substitution Eq. (3.50) into Eq. (3.48) with \( F_x = 0 \) gives the normal stiffness of the basic folded flexure as

\[ K_n = \frac{F_y}{v_6} = \frac{6EI}{36a^2 b + 16a^3} \]  

(3.52)

When \( b \gg a \), simplification of the Eq. (3.51) gives

\[ k_t = \frac{3EI}{4b^3} \]  

(3.53)

Thus, the ratio of the simplified stiffness, \( k_t \), to the original stiffness, \( K_t \), is given by

\[ r_t = \frac{k_t}{K_t} = \frac{3ab^2 + 4b^3}{4b^3} = \frac{3\left(\frac{b}{a}\right)^2 + 4\left(\frac{b}{a}\right)^3}{4\left(\frac{b}{a}\right)^3} \]  

(3.54)

According to Eqs. (3.51) and (3.52), the stiffness ratio of the flexure is found to be

\[ r_k = \frac{k_n}{k_t} = \frac{3ab^2 + 2b^3}{9a^2 b + 4a^3} = \frac{3\left(\frac{b}{a}\right)^2 + 2\left(\frac{b}{a}\right)^3}{9\left(\frac{b}{a}\right)^2 + 4} \]  

(3.55)

The influence of the value of \( b/a \) to that of \( r_t \) and \( r_k \) is shown in Figure 3.9. The difference between the simplified stiffness and the original one could be neglected, and the ratio of tangential stiffness to the normal stiffness would be larger than 100 when \( b/a \) is larger than 20. Noting that the value of \( b/a \) was designed to be larger than 30.
in this project, the simplification of the stiffness is, then, applicable to the following calculations. Moreover, considering that the lateral driving force is the dominant force during the operation, transverse deviation of the flexure during the motion of the testing device can then be neglected in this project.

### 3.3.1.2 Finite Element Analysis

As a verification of the numerical analysis, finite element analysis of the basic folded flexures by using ANSYS was also performed in this project. In this project, the thin-width and high-thickness design of the beam segments of the supporting flexures, i.e., $h/w_f > 10$, were suitable to be simulated as two dimensional beams. In this project, BEAM3 element in ANSYS was chosen as the finite element. BEAM3 element is a uniaxial element equipped with tension, compression, and bending capabilities. This element has three degrees of freedom, i.e., translations in $x$- and $y$- directions as well as rotation about the $z$-axis.

The analysis results in ANSYS is shown in Figure 3.10, and the values of the tangential stiffness, $k_t$, and normal stiffness, $k_n$, were, hence, obtained. As listed in Table 3.2, the values of stiffness obtained by both numerical analysis and finite element anal-
Figure 3.10 Schematic diagram of the finite element analysis result of the deflection of a basic folded flexure by using ANSYS: (a) under a tangential load; (b) under a normal load.
ysis were very close, and deviations of the results from these two methods were limited in an acceptable range.

### 3.3.2 Three-Turn Folded Flexure

In this project, combination of basic folded flexures, i.e., three-turn folded flexure, were adopted to achieve large displacements of the driving and sensing components. As shown in Figure 3.11, a three-turn folded flexure consists of three basic (complete-cycle) flexure units and a half-cycle flexure unit. According to Hooke’s law, the equivalent stiffness of a multi-turn flexure satisfies

\[
\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_i}
\]

(3.56)

where \( k_1 \) to \( k_i \) are the stiffnesses of each flexure unit in the series, respectively. Thus, the equivalent stiffness \( k_{\text{eq}} \) is smaller comparing to that of each flexure unit included and larger linear displacement can then be achieved.

As derived in Appendix B, the bending analysis of a three-turn folded supporting flexure based on beam theory gives the lateral displacement of the three-turn folded flexure as

\[
\delta = \frac{193b^3}{42EI} F_x
\]

(3.57)
Figure 3.11 Schematic diagram of a folded supporting flexure consists of three complete-cycle units and a half-cycle unit.

Table 3.3 Characteristics of three-turn folded flexures adopted in this project

<table>
<thead>
<tr>
<th>Parameters</th>
<th>flexure combination N and T</th>
<th>flexure combination S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of connecting beam, a</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Length of bending beam, b</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>Length of bending beam, 2b</td>
<td>600</td>
<td>400</td>
</tr>
<tr>
<td>Width of beam segments, w_j</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Thus, the equivalent stiffness of this flexure is found to be

\[ k_t = \frac{42EI}{193b^3} \]  \hspace{1cm} (3.58)

As shown in Figure 3.1 and Figure 3.4, pairs of three-turn folded flexures were adopted in the flexure combinations in this project, such as flexure combinations N, T and S. These flexures pairs were symmetrical placed to eliminate tangential deviation of the moving components. Thus, deviation perpendicular to the moving direction during the motion of the moving parts of the testing device could be ignored. Characteristics of the three-turn folded flexures designed for this project were listed in Table 3.3.

As verification of the numerical analysis, finite element analysis was also carried out for a three-turn folded flexure, as shown in Figure 3.12. Dimensions of the flexure
Chapter 3  

Design of MEMS Friction Testing Devices

Figure 3.12 Schematic diagram of the finite element analysis of a three-turn folded flexure by using ANSYS.

<table>
<thead>
<tr>
<th>Table 3.4 Stiffnesses of a three-turn folded flexure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical Analysis(N/m)</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>4.23</td>
</tr>
</tbody>
</table>

followed that of flexure combination N, as listed in Table 3.3. The values of the stiffness of a three-turn folded flexure obtained via numerical analysis and finite element analysis (FEA) were listed in Table 3.4. The numerical results obtained based on Eqs. (3.58) is very close to the finite element analysis result, Thus, derivation based on combination of basic flexure unit is reliable and can be adopted as a general derivation method for folded flexures with similar structures.

3.3.3 S-Shape Folded Flexure

Although symmetrically placed multi-turn folded flexure pair could obtain ideal tangential stiffness with minimized normal deviation, it is not suitable for the sensing bush of the rotary testing device due to the space limitation. To ensure a stable pressure at the contact region during the interaction of the sensing bush and the driving ring, an
S-shape folded flexure was developed in this project.

As shown in Figure 3.13, the S-shape folded flexure consists of folded beam segments at both sides of the central beam AB. One end of the flexure, node A, is fixed while loads acting on the other end, node L. Using this S-shape design, the lateral deviation caused by the normal load can be reduced to a negligible range according to the analysis below.

Considering that one S-shape flexure compromises eleven beam segments which have different lengths, numerical derivation would be too complicated to be performed. In this project, the S-shape flexure was only analyzed by using ANSYS. Noting that the stiffnesses obtained by using beam element with ANSYS is very close to the values obtained by using numerical analysis in the previous research [103, 104], finite element analysis is a reliable method to achieve mechanical characteristics in MEMS devices.

A finite element model of a S-shape flexure by using ANSYS is shown in Figure 3.14(a). Corresponding parameters of this flexure model were listed in Table 3.5. The model was analyzed under two different conditions, with a tangential load only, and
Table 3.5 Characteristics of the S-shape flexure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the bending beam, ( L_b )</td>
<td>250</td>
</tr>
<tr>
<td>Increment of the bending beam, ( L_s )</td>
<td>25</td>
</tr>
<tr>
<td>Length of the connecting beam, ( L_c )</td>
<td>16</td>
</tr>
<tr>
<td>Length of the starting segment, ( L_a )</td>
<td>32.5</td>
</tr>
<tr>
<td>Length of the ending segment, ( L_d )</td>
<td>35</td>
</tr>
<tr>
<td>Width of beam segment, ( w_b )</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.6 Characteristics of the S-shape flexure

<table>
<thead>
<tr>
<th></th>
<th>S-Shape</th>
<th>Multi-Turn</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_t ) (N/m)</td>
<td>7.91</td>
<td>79.8</td>
<td>101</td>
</tr>
<tr>
<td>( k_n ) (N/m)</td>
<td>716</td>
<td>219</td>
<td>3.97 \times 10^5</td>
</tr>
<tr>
<td>( k_n/k_t )</td>
<td>90.5</td>
<td>2.75</td>
<td>3914</td>
</tr>
</tbody>
</table>

with a moment only. To achieve a moment in ANSYS, an equivalent moment was generated by using a rigid segment. As shown in Figure 3.14(b), a rigid segment with a length of \( H_r \) was added to the free end of the flexure. Two opposite tangential forces, \( F \), were acting on the upper and lower ends of the rigid segment, respectively. Hence, an equivalent moment \( M_e = FH_r \) was obtained. Combining the analysis results obtained under these two conditions, an expression of the S-shape flexure with given dimensions was found to be

\[
\begin{cases}
    u = 0.126F_x + 537M \\
    \theta = 537F_x + 2.73 \times 10^6M
\end{cases}
\]  

(3.59)

where the coefficients for the applied loads would be varying with the change of the parameters of the S-shape flexure.

As a comparison of the performance of different flexure designs, the normal stiffness, \( k_n \), the tangential stiffness, \( k_t \), and the ratio of \( k_n/k_t \) for an S-shape flexure, a multi-turn folded flexure with the same number of bending beams as that of the S-shape...
design of MEMS friction testing devices

flexure, i.e., a two-turn folded flexure, and a cantilever beam with the same length of the bending beam as that of the two-turn folded flexure, were listed in Table 3.6. Although the cantilever beam has the maximum value of \( k_n/k_t \), it is much stiffer comparing to the S-shape folded flexure in the tangential direction. Thus, the S-shape folded flexure was chosen as the supporting flexure for the sensing bush as a comprehensive consideration of stiffnesses in both tangential and normal directions.

According to the results of the finite element analysis, the length of the bending beam, \( L_b \), and that of the connecting beam, \( L_c \), influenced on the stiffness of the S-
shape folded flexure. In this project, the applied tangential and normal loads were $F_x = 50 \, \mu N$ and $F_y = 50 \, \mu N$, respectively.

As shown in Figure 3.15(a), the length of the connecting beam has a small influence
on the tangential stiffness $k_t$ while having a great effect on the ratio of $k_n/k_t$. Noting
that the connecting beam was treating as rigid in the S-shape flexure, increasing the
value of $L_c$ would more rigid body tilting along the normal direction, and, hence, de­
crease the value of $k_n/k_t$. However, the symmetrically placed bending beams in the
S-shape flexure would cancel the most of the rigid body displacement along the tan­
gential direction. Thus, the value of $k_t$ had a small variation with the change of $L_c$.
As shown in Figure 3.15(b), increasing the length of the bending beam, $L_b$ decreased
the tangential stiffness $k_t$ while increased the ratio of $k_n/k_t$. Thus, the S-shape folded
flexure has a combination of tangential flexibility and normal stiffness and is suitable to
be adopted as the supporting flexure to supply stable motion.

3.4 Flexure Design for Rotary Oscillation

In a rotary friction testing device, the driving ring was designed to oscillate about its
center. Thus, supporting flexures are required to be flexible along the circular direction
while be stiff along the transverse direction. As introduced above, four symmetrically
placed beams were introduced as the supporting flexure combination (flexure combina­
tion $C$) for rotary motion in the circular testing device (Figure 3.4).

Noting that these four identical beams are equivalent to a parallel connected flexure
combination, the equivalent stiffness of the flexure combination is four times as that
of a single beam. The bending effect of one beam connecting with the driving ring is
shown in Figure 3.16. The driving ring is driven by a torque $T$ (given by Eq. (3.13)),
$F_t$, $F_I$ and $M_B$ are the loads at the point $B$ which is the joint point of the beam and the
driving ring. The length of the cantilever beam is denoted as $L_r$, and the outer radius
of the driving ring is $r_d$. During the operation, tilting angle of point $B$, $\beta$, is always
the same as the rotation angle of the ring. Treating the beam as a constrained cantilever
beam, the displacement along the $y$-direction and the tilting angle of the beam based on
beam theory are given by

\[ y(L_R) = \frac{2F_t L_r^3 - 3M_B L_r^2}{6EI} \]  

(3.60)

and

\[ \beta = \frac{F_t L_r^2 - 2M_B L_r}{2EI} \]  

(3.61)

respectively, where \( E \) is Young’s modulus, and \( I \) is the area moment of inertia. Considering that only a small amount of rotation would be involved, the displacement along the \( y \)-direction was approximately equal the arc length corresponding to the tilting angle. Thus, the tilting angle can be expressed as

\[ \beta = -\frac{y(L_r)}{r_d} \]  

(3.62)

where the negative sign represents the rotation direction. Combining Eqs. (3.60) and
Chapter 3

Design of MEMS Friction Testing Devices

(3.62), the moment at point $B$ is found to be

$$M_B = \frac{L_T(2L_T + 3r_d)}{3(L_T + 2r_d)} F_t$$

(3.63)

Substitution of Eq. (3.63) into (3.60) gives the displacement of the beam

$$y(L_T) = \frac{r_d L_T^3}{6EI(L_T + 2r_d)} F_t$$

(3.64)

According to the free body diagram of the driving ring (Figure 3.16), the equilibrium of the driving torque and the loads at the joint point $B$ gives

$$T = M_B + F_t r_d$$

(3.65)

Combining Eqs. (3.63) and (3.65), a relationship between $T$ and $F_t$ is found to be

$$T = F_t \left( \frac{2L_T^2 + 6L_T r_d + 6r_d^2}{3(L_T + 2r_d)} \right)$$

(3.66)

Combining Eqs. (3.62) and (3.64), the tilting angle of the beam, i.e., the rotating angle of the ring is found to be

$$\beta = -F_t \frac{L_T^3}{6EI(L_T + 2r_d)}$$

(3.67)

Combining Eqs. (3.66) and (3.67) with noting that the angular stiffness of a flexure is defined as $k_\beta = T/\beta$, the angular spring constant of one beam, $k_{\beta 0}$ is given by

$$k_{\beta 0} = 4EI \frac{L_T^2 + 3L_T r_d + 3r_d^2}{L_T^3}$$

(3.68)

Thus, the equivalent angular stiffness of the flexure combination $C$ is given by

$$k_\beta = 4k_{\beta 0} = 16EI \frac{L_T^2 + 3L_T r_d + 3r_d^2}{L_T^3}$$

(3.69)
To verify the obtained angular stiffness, a finite element model was developed by using ANSYS in this project, as shown in Figure 3.17. Following the designed dimensions of the MEMS device, the widths of the central ring and the driving arms were about four times wider than that of the supporting beam. Based on the results in the finite element analysis, these widths were large enough to make the ring and the arms rigid.

Comparison of the angular stiffnesses obtained by using the numerical analysis and the finite element analysis were given in Figure 3.18. It was found that the length of the supporting beam significantly affects the angular stiffness of the device. The accuracy of the derived numerical expression was proved by its high integrity of the results from the finite element analysis. Thus, according to Eq. (3.69), the influence of the radius on the angular stiffness can be neglected when the radius of the central ring is much
smaller than the length of the beams.

3.5 Summary

In this chapter, working principles of both the linear friction testing device and the rotary friction testing device were first introduced. By using electrostatic comb actuators, both linear and rotary actuators have been developed. For the linear comb drive, the electrostatic force is independent of lateral displacement, and a good linearity of the electrostatic force for actuating application is achieved. For the rotary comb actuators, the comb fingers lie on arcs of concentric circles and actuate rotary motion about the center of the circles.

To achieve stable actuation and sensing applications, this project has exploited different profiles of supporting flexures. A general method of combination of identical flexure units to a multi-turn flexure was given. Moreover, a novel design of S-shape folded flexure was developed in this project. By using both numerical analysis and finite element analysis, it was proved to be an ideal flexure to achieve large tangential displacement with minimized normal deviation. A general expression for the angular...
Chapter 3  

Design of MEMS Friction Testing Devices

stiffness of the rotary friction testing device was also derived in this chapter. With a high integrity of the results obtained via finite element analysis, this simple expression has wide application in MEMS devices.
Chapter 3

Design of MEMS Friction Testing Devices

- 68 -

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library
Chapter 4

Fabrication of MEMS Testing Devices

This chapter first describes the fabrication methods adoptable for the designed MEMS testing devices. Based on the chosen fabrication method, the process of the fabrication procedure, the key fabrication processes, and some techniques for optimizing the fabrication results are highlighted. Measurement methods of the prototypes of the developed devices are also introduced in this chapter. Image processing techniques are adopted to perform the measurements of the prototypes. Characterizations of the MEMS devices are conducted and the effects of the dimensional tolerance are also studied.

4.1 Fabrication Process

As reviewed in Chapter 2, both silicon-on-insulator (SOI) wafers and silicon-on-glass (SOG) wafers could be adopted in the fabrication process of the friction testing devices. To achieve a high cost performance ratio, process with SOG wafers were chosen as the fabrication method in this project.

The key steps in the fabrication process based on SOG wafers are shown in Figure 4.1. First, photolithography was performed to transfer patterns of one mask to the bottom side of a silicon wafer, as shown in Figure 4.1(a). Using the patterned photoresist as an etching mask, a deep reactive ion etching (DRIE) process was subsequently per-
Chapter 4

Fabrication of MEMS Testing Devices

Figure 4.1 Schematic diagram of the fabrication process flow of the MEMS devices based on anodic bonding.

formed to etch cavities at the bottom side of the silicon wafer. The depth of the etched cavities is about a half of the wafer thickness, i.e., 75-μm-deep cavities for 150-μm-thick wafers, and 150-μm-deep cavities for 300-μm-thick wafers. The etching depths were controlled by the etching rate and the etching time together. Figure 4.1(b) shows schematically the etched silicon wafer with patterned photoresist. A photolithography
Chapter 4  

Fabrication of MEMS Testing Devices

process was also conducted on a glass wafer (Pyrex 7740), as shown in Figure 4.1(c). After the sputtering of Cr/Au (10 nm/100 nm) on the patterned glass wafer and a lift-off process, contact pads for applying driving voltages to the comb-drive actuators of the testing device were formed, as shown in Figure 4.1(d). By using anodic bonding, the silicon wafer and the glass substrate were bonded, as shown in Figure 4.1(e). Finally, another photolithography process was performed on the top side of the bonded silicon wafer, and the DRIE process was performed to etch through the silicon wafer with cavities to form the floating comb-drive and flexure micro-structures (Figure 4.1(f)).

After the final DRIE process, the remaining photoresist on the top of silicon was removed using oxygen plasma treatment (reactive ion etching or RIE). A Technics 800-II Series RIE System equipped with a 300-W solid state radio-frequency generator operating at 13.56 MHz was adopted for the photoresist striping operation. Noting that there was no wet process involved during the etching and cleaning, clean surfaces could be achieved after the DRIE and RIE processes, and no more post-fabrication cleaning treatments were needed.

4.2 Critical Processes in Fabrication

In this project, a cost-effective fabrication process based on silicon on glass (SOG) was chosen as the alternative to the process using SOI wafers on consideration of the robustness requirement of the MEMS testing devices [105]. Moreover, considering the similar coefficient of thermal expansion to single crystalline silicon, Pyrex® 7740 glass was chosen as the glass substrate to avoid residual stress caused during the bonding process [106]. The silicon wafers used were N-type (100) single-crystal wafers with a resistivity of 0.1-1 Ω-cm and a [110] surface was used as the contact sidewall interfaces. In the fabrication of the friction testing devices, patterning of electrode pads, bonding of silicon and glass wafers, and etching of device silicon wafers were the most critical processes. Details of these fabrication processes, i.e., deposition of metal, anodic
bonding and DRIE, are given below.

4.2.1 Deposition of Metal Layer

In this project, patterned metal layer was adopted as electrode pads on the topside of glass wafers. Comparing to the evaporation process, sputtering has a faster deposition rate, better uniformity, and better adhesion to substrate. Thus, sputtering was adopted in thin-film deposition processes in this project.

During the deposition of a metal layer, a glass substrate was first patterned with a layer of AZ9260 photoresist (about 5-μm-thick). Then, a thin-film of Cr/Au (10 nm/100 nm) was deposited on the topside of the glass substrate by using a DC magnetron sputtering machine. The deposited chrome layer was adopted as an adhesive layer before further depositing a gold layer. The deposited gold layer would be the actual layer working as wiring pads. After soaking the sputtered wafer into acetone solution, i.e., the lift-off process, patterned metal layer was formed. Figure 4.2 shows a metal pad formed by deposition of a layer of Cr/Au.

![Figure 4.2 Micrograph of a deposited Cr/Au pattern.](image)

4.2.2 Anodic Bonding

As reviewed in Chapter 2, anodic bonding is a hermetically bonding method to achieve permanent joins between glass to silicon without using adhesives. In this project, the anodic bonding was carried out with the SB6 wafer bonder (Karl Suss). To avoid bending or buckling of the bonded wafer, a 400-°C-temperature was adopted in the bonding process [68]. The central bonding was processed at a 2000-V-voltage for two minutes, and the area bonding was processed at a 1000-V-voltage for about fifteen minutes.
Chapter 4  

Fabrication of MEMS Testing Devices

Figure 4.3 Micrograph of the result of the anodic/eutectic bonding between silicon to glass with electrodes.

Noting that the roughness tolerance is about 0.1 μm in the anodic bonding, the patterned contact pad (about 100-nm-thick) on the glass substrate would not affect the final bonding strength. The large portion of each anchor formed in the previous process was bonded to the glass substrate by the anodic bonding between the silicon and Pyrex glass, while a small portion of the anchor was bonded by the eutectic bonding between the silicon and the thin Au layer (on the top of a Cr layer) sputtered on the glass substrate, as shown in Figure 4.1(e). This eutectic bonding formed the electric connection from the contact pads to the silicon micro-drive actuators [107]. Before the final DRIE of the SOG wafers, the fabrication yield achieved was about 95%. The anodic bonding results are shown in Figure 4.3.

4.2.3 Deep Reactive Ion Etching (DRIE)

In this project, the DRIE process was carried out on a Surface Technology Systems Multiplex inductively coupled plasma system (ICP). Two independent 13.56-MHz radio frequency (RF) power sources, a 1000-W coil power to create plasma for etching and a 300-W electrode power to vary the RF bias potential of the loaded wafer with respect
to the plasma, are included in the equipment. As shown in Figure 4.4, etching and passivating gases enter from the top of the etching chamber through a single gas feed. Before the gases access the chamber, they are distributed evenly across the loaded wafer. Back-side helium pressurization is used to provide heat transfer between the wafer and the electrode to maintain a constant wafer temperature [108].

In the fabrication of the MEMS friction testing devices, DRIE was adopted as the etching method in the etching of both the backside and the topside of the device silicon wafer, as shown in Figure 4.1(b) and (f). For a 300-μm-thick silicon wafer used in this project, the depth of the etched cavities was about 150 μm, i.e., a half of the wafer thickness. Noting the smallest gap was designed to be 8 μm, the aspect-ratio of trenches was about 20:1. To achieve this high aspect-ratio, optimizations of fabrication were performed and were discussed in the following sections. Detailed parameters of the etching recipes are given in Appendix C.

Figure 4.5 shows the SEM image of a prototype of the linear friction testing device, and Figure 4.6 shows that of a rotary friction testing device. The fabricated linear comb actuators and rotary comb actuators are shown in Figures 4.7 and 4.8, respectively.
Figure 4.5 SEM image of a linear friction testing device.

Figure 4.6 SEM image of a rotary friction testing device.
Moreover, the SEM image of the sensing bush of the rotary friction testing devices is shown in Figure 4.9. In these images, deep trenches with high anisotropy and good uniformity of structure after the DRIE process were achieved.

The anodic bonding and DRIE based SOG wafers has a good yield and feasibility. The fabrication yield before the final DRIE process on the SOG wafer was about 95% for one wafer scale. However, this rate dropped significantly after the final DRIE process due to technical limitations of the fabrication systems being used. To improve the yield, optimizations to overcome the processing challenges in the final DRIE process were carried out in this project.

4.3 Challenges in Deep Reactive Ion Etching

In this project, the complicated design (over thousands of comb fingers) and relative large cell size (about 5.6 mm by 7.1 mm) make some inherent limitations of the DRIE
Chapter 4  
Fabrication of MEMS Testing Devices

**Figure 4.8** SEM image of a partial view of the comb actuator of the rotary friction testing device.

**Figure 4.9** SEM image of the sensing bush of the rotary friction testing device.
process even more severe. Aiming at these challenges, optimizations of operation parameters and layout designs were carried out to improve the final fabrication yield.

4.3.1 Heat Accumulation

During the reactive ion etching of the loaded substrate, heat would accumulate on the surface of the substrate and the regional temperature would be increasing as etching lasting. Noting that high temperature accelerates the etching rate of the DRIE process, this heat accumulation brings a non-uniform etching rate, i.e., some parts of the structure would be over etched while other parts are not done.

As an inherent phenomenon of the ICP process, the accumulated heat is very hard to be eliminated. Some etching machines, such as Alcatel 601E ICP etching machine, use cooling mechanism to adjust the temperature of the substrate to solve this problem. However, for the STS system adopted in this project, the temperature of the substrate was not adjustable. Moreover, the SOG-wafer-based design would even slow down the heat dissipation during the DRIE process since glass is a heat insulating material.

![Figure 4.10 SEM picture of the damage caused by accumulated heat during the DRIE process.](image-url)
Besides the inherent factor, the structure of flexures adopted in this project was the external factor for the heat accumulation problem. For each folded flexure, it is a combination of several beam segments and working as a flexible connection of two rigid bodies. Before releasing of the supporting flexure, accumulated heat could be dissipated via the whole wafer. After the structure of the flexure being fully released, accumulated heat on the flexure could only be transferred via the narrow joints between the flexure and the rigid bodies. Thus, the speed of heat dissipation would be slowed down a lot, and the temperature at the flexure part would then increase. Once the temperature increased to a certain value, burning of the beam would occur and the structure was damaged, as shown in Figure 4.10.

To overcome the effect of heat accumulation, two methods were utilized in this project. For optimization of the design of the mask layout, dimensions of the rigid bodies being connected by the supporting flexures were increased. Thus, these rigid parts could be treated as heat sinks and could transfer more heat during the DRIE process. For optimization of the fabrication process, shorter etching intervals were applied to break the heat accumulating progress. During the final DRIE process, the etching process of the loaded wafer was monitored. As soon as the most of the exposed areas of the wafer being etched through, the etching process was interrupted to cool the wafer down. After that, the interval of the etching process was set as one minute to prevent the damage caused by accumulated heat. Based on the fabrication results, the damage caused by accumulated heat was significantly reduced and good etching results of the final DRIE process were obtained, as shown in Figure 4.12.

4.3.2 RIE Lag Effect

As reviewed in Chapter 2, low chamber pressure, low platen power and short etching cycle duration are applicable to reduce the RIE lag effect during the DRIE process. However, these relative low parameters decrease the etch rate and end up with a more
positive trench profile [109], i.e., the bottom width of a trench is much narrower than
its top width. Moreover, longer etching time due to a lower etch rate would bring
more severe heat accumulation problem which could burn slender features, as shown
in Figure 4.11. To achieve good trench profiles of the MEMS testing devices with
minimum RIE lag effect, optimized fabrication steps were carried out in this project.

![Smaller Gap](image)

**Figure 4.11** Micrograph of the RIE lag effect.

The optimized DIRE process included two critical steps, faster etching for good
trench profiles and slower etching for less RIE lag. Etching parameter for both fast and
slow etching rates were listed in Table 4.1. For a SOG wafer prepared for the final DRIE
process, a combination of photoresist and 1-μm-thick silicon oxide was adopted as the
etching mask. The faster etching step was first conducted till most of large patterns such
as anchors and frames were released. At this time, the areas with small features such
as comb fingers and flexures were etched through due to the RIE lag effect. To release
these small features, the slower etching step was performed. To avoid the burning of
slender features due to the heat accumulation, the duration of the slower etching step
was 5 minutes. An one-minute faster etching step was adopted to clear the Teflon film
accumulated on the bottom of the trenches immediately after the slower etching step.
This two steps were carried out alternately till all the features were finally released. The
etching results obtained by using the two-step etching technique is illustrated in Figure
4.12.
Figure 4.12 SEM picture of etching result after the final DRIE process by using optimized etching steps.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Faster Etching</th>
<th>Slower Etching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etching duration (s)</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Passivation duration (s)</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

4.3.3 Microloading Effect

Microloading is another inherent limitation of the DRIE process that affects the etching rate of the DRIE process.

In this project, large open areas were intended designed in the mask layouts to ensure enough space for flowing of etching radicals and for further probe manipulating. Moreover, larger distance between each group of comb actuators were also necessary to eliminate unwanted attraction force. Thus, open areas with nonuniform dimension were unavoidable, i.e., the feature density at the comb drive areas is much higher than
Chapter 4  

Fabrication of MEMS Testing Devices

that at the anchor areas, as shown in Figure 4.13. As reviewed in Chapter 2, the etching rate of DRIE process decreases with the feature density of the areas to be etched. As observed during the fabrication process, the etch rate at the areas of flexures was higher than that of the comb finger. Noting that the width of the flexure beams are similar with that of the comb fingers, flexures would be over etched a lot or even etched away before releasing of all the comb fingers, as shown in Figure 4.14.

![Figure 4.13 Schematic diagram of the nonuniform dimensions of open areas due to SEM image of the damage of the supporting flexure caused by over etching during the DRIE process.](image)

To minimize the microloading effect (Figure 4.15) during the DRIE process, an optimal geometry in the layout design, i.e., beam protections close the supporting beams, was adopted in this project. The gap between the beam and the protections is 20 \( \mu \text{m} \). A 25 \( \mu \text{m} \)-distance between each protector was given to allow the flow of the chemical radicals. The etching result with etching protection is shown in Figure 4.16. It can be found out that the beam and the comb fingers were both released without much damage.

4.4 Setup of the Testing Platform

In this project, all the experiments were conducted in a customized testing platform mounted on a vibration isolation table (Newport Electronics, Inc.) in a clean room (Class 1000, 22°C and 40% relative humidity). Voltages were applied to the MEMS
Figure 4.14 SEM image of the damage of the supporting flexure caused by over etching during the DRIE process.

Figure 4.15 Micrograph of the microloading effect during the DRIE process.
device under test through tungsten probe tips. A schematic diagram of the testing apparatus is shown in Figure 4.17.

Up to six individual tungsten probes were used to provide electrical connections to each device under test. The electrostatic forces for actuating the friction testing devices in different directions were obtained by applying bias voltages generated from a DC power supply (FLUKE PM2811) and a waveform generator (HP 33120A), respectively. To isolate the electrical signals from the environmental noise, external instruments were connected to the probes by coaxial cables using BNC connectors. The displacements of the motion of the sensing components of the friction testing devices were monitored with a microscope (Mitutoyo, FS series). Digital images and motion videos were obtained with a charge-coupled device camera (Sony, SSC-DC54A).

4.5 Static Measurements

Static measurements of the prototypes of the MEMS devices were carried out to find out the dimensions of the testing devices, and the moving displacements of the sensing
4.5.1 Measurements of the Feature Dimensions

After setting up the testing platform, a device under test was put on the probe station. After manipulating probes in contact with the electrode pads of the test specimen, both DC voltage and waveform were then applied to the probes. Video of the motion of the device under test was then captured.

The calibration for the displacement of the driving plate in the tangential direction with a given driving voltage, $V_t$, and the calibration for the displacement for the sensing plate in the normal direction with a given voltage, $V_n$, were performed under the conditions of no contact between the bumps and the sensing plate surface. After fabrication, the dimensions of the devices, such as the finger gaps, the beam widths of the flexures, etc., were verified based on the optical micrographs and SEM images. Noting that the pitch values remained unchanged during the fabrication, a ratio of the measured pitch value to the design pitch value, $r_f$, was calculated and the unit is pixel/μm. After ob-
Chapter 4  

Fabrication of MEMS Testing Devices

Figure 4.18 Schematic diagram of the principles of the measurement pitches and dimensions of (a) a folded flexure; (b) a rotary comb drive.

tained an average beam width or finger gap (pixels) in the measurement, the physical distance can then be calculated from the pixels value by dividing $r_f$.

4.5.2 Measurements of Displacement

In the first step of displacement measurement, the captured video files were converted to multi-frame still images, and only the intensity information was kept. The intensity of the points in a captured image frame varies in the range from 0 to 255 digital levels, where 0 represents the darkest area and 255 represents the brightest area. In the captured images, an empty area is always darker than the feature area with the material. Since the camera position was fixed during the capturing process, batch measurements with large number of image frames are doable, and functions based on the MATLAB software
have been developed to process the acquired image frames.

After analyzing the intensity distribution of a still video frame, the mid-value of the intensity was chosen as a threshold for judging bright area (silicon material) from dark area (empty space). To measure the displacements of the driving and sensing plates, reference and measuring points are necessary. These points for the anchor and the sensing plate are shown in a still partial frame of the digital video image (Figure 4.19). The reference point is near the anchor area of the testing device. The measuring points are close to the sensing plate and driving plate, respectively. Both vertical and horizontal scans were performed to detect the edge of each plate by noting a jump in the intensity value and to count the dark dots (pixels) in the image between the point of interest (either the reference point or the measuring points) and the edge of the feature (silicon material) which appears as a bright area. Thus, the relative distance between the respective points to the anchor, sensing plate and driving plate can be obtained. For the multi-image frames, the variation of the relative distance to the anchor reflects the deviation coming from environmental vibration. After deducting this deviation from the results of the relative distances obtained based on the measuring points, the actual displacements of the sensing and driving plate versus time were determined. The vernier teeth shown in Figure 4.19 were designed just for use in confirming the computed displacement values.

During the acquisition process, a numerical operation was involved to treat those image frames which had visible artifacts caused by a standard video process called interlacing [110]. Since the PAL (Phase Alternating Line) analogue TV broadcast system was used for the video camera, all the captured video images were interlaced, i.e., each frame actually consists of two half-resolution frames, with odd and even scan lines, respectively. For an object moving in the scanning direction, an edge not parallel to the scan lines would appeal as a "double image" in the capture digital video image file. This
gives difficulty in determine the edges of objects. To solve this problem, a line doubling operation was adopted by replacing the intensities of the pixels on every other scan line with the averaged intensities of the two adjacent scan lines. Thus, smooth edges of the objects under consideration appeared, making it possible to determine their displacements in the video scanning direction. For a fixed number of pixels per video image frame, the resolution of the measured displacement is about 0.32 μm for the present research, which gives enough accuracy for the subsequent data analysis.

4.6 Correction of Measurement Results

Due to the fabrication limitation, the actual dimensions of the devices might differ from the designed values, such as profiles of the sidewall surfaces, beam widths of the supporting flexures, and air gaps of the comb actuators. As discussed in previous chapters, those values are critical to the mechanical performance of the MEMS testing devices. Thus, corrections based on the measured dimension were given to compensate the theoretical results.

As the only silicon etching method adopted in this project, the DRIE process is the source for the variation of dimensions, or fabrication tolerance. For the DRIE process, the etched profile and anisotropy are greatly dependent on the operation pa-
Chapter 4  Fabrication of MEMS Testing Devices

Figure 4.20 schematic diagram of a positive and a negative profiles after the DRIE process.

rameters, such as gas flow rate, electrode power, pressure, temperature, cycling time [76, 111, 112]. Although ultra vertical profiles of the etched sidewall surfaces can be achieved with optimized process parameters, the fabrication tolerance is inevitable in most conditions, especially for long time etching processes.

4.6.1 Effect of the Profile Tolerance

Generally, two kinds of profiles are encountered in the DRIE of silicon, positive profiles and negative profiles. The dimensions of the profiles of etched trenches are shown in Figure 4.20, where \( g \) is the top gap spacing, \( H \) is the height of the trenches, and \( \varphi \) is the angle between the sidewall of the trench and the vertical reference line.

Based on Eqs. (3.5), (3.7) and (3.53), the values of the electrostatic force of the comb drives and the stiffnesses of the supporting flexures are relative with the fabrication tolerance. Analysis of the effect of the fabrication tolerance was conducted based on the positive profile, and a similar derivation is applicable to the negative profile by using a negative value of \( \varphi \).

4.6.1.1 Effect on Electrostatic Force

A positive profile of a pair of comb fingers is shown in Figure 4.21. Similar as Figure 3.3, the overlapping length of the fixed and movable finger is \( x_0 \), the width of each
finger is \( w \), and the tapered angle of the profile is \( \varphi \). Based on Eq. (3.6), the changed capacitance of the dark zone of a finger pair (Figure 4.21) as the overlapping length increasing \( \Delta x \) is given by

\[
\Delta C = \frac{\varepsilon h \Delta x}{g - 2h \tan \varphi}
\]  

(4.1)

Combining Eq. (3.7) and Eq. (4.1), the electrostatic force generated by one pair of fingers with a positive tapered angle is found to be

\[
F(\varphi) = 2\int_0^H \frac{\varepsilon V^2}{g - 2h \tan \varphi} dh = -\frac{\varepsilon V^2}{\tan \varphi} \ln \left(1 - 2\frac{H}{g} \tan \varphi\right)
\]  

(4.2)

which is the same as Eq. (3.8) which is obtained under an idealistic condition, i.e., \( \varphi \to 0 \). Thus, the variation ratio of the electrostatic force after the DRIE process is given by

\[
\xi_F = \frac{F(\varphi)}{F(0)} = -\frac{g}{2H \tan \varphi} \ln \left(1 - 2\frac{H}{g} \tan \varphi\right)
\]  

(4.3)

Noting that \( \tan \varphi = \Delta w / 2H \), where \( \Delta w \) is the width difference between the top and the bottom of one comb finger. Combining the foregoing relation with Eq. (4.3), the variation ratio is found to be

\[
\xi_F = -\frac{g}{\Delta w} \ln \left(1 - \frac{\Delta w}{g}\right)
\]  

(4.4)

For a negative profile, Eqs. (4.2) to (4.3) are still applicable by substituting \( -\varphi \) as the tapered angle, i.e., \( \Delta w \) is a negative value.

### 4.6.1.2 Effect on Flexure Stiffness

The stiffnesses of the supporting flexures adopted in the testing devices were also affected after the DRIE process due to the imperfect profile of the flexure beams. The schematic diagram of the cross section of a flexure beam is shown in Figure 4.22. The
top width and height of the trapezoidal cross-section of the beam are \( w \) and \( H \), respectively, and the cross profile has an angle of \( \phi \). The width of the beam of the flexures is a function of its depth given by

\[
w(h) = w + 2h \tan \phi
\]  

(4.5)

Thus, the area moment of inertia \( I_z \) of the trapezoidal beam is found to be

\[
I_z = \frac{1}{12} \int_0^H (w + 2h \tan \phi)^3 \, dh = \frac{H}{12} \left( w^3 + 3H w^2 \tan \phi + 4H^2 w \tan^2 \phi + 2H^3 \tan^3 \phi \right)
\]

(4.6)

The area moment of inertia of an idealistic beam section can be obtained from the above relation when \( \phi = 0 \).

Based on Eqs. (3.58) and (3.69), it is noticed that the stiffnesses of the supporting flexures have a linear relationship with the value of \( I \). Thus, the variation ratio of the
stiffnesses after the DRIE process is given by

\[ \xi_k = \frac{I_z}{I} = \frac{w^3 + 3H w^2 \tan \phi + 4H^2 w \tan^2 \phi + 2H^3 \tan^3 \phi}{w^3} \]  

(4.7)

For a negative profile, Eq. (4.7) is still applicable by substituting \(-\phi\) as the tapered angle.

4.6.2 Effect of the Undercut Tolerance

Besides the profile tolerance, undercut is another common phenomenon introduced by DRIE process. Generally, dry etching of silicon is an isotropic process where the etching rates in vertical and transverse directions are almost the same. Figure 4.23(a) shows the profile of an isotropic etching result. The undercut length \(L_u\) is almost the same as the etching depth \(H_u\). Anisotropic DRIE was developed to obtain profiles with high-aspect-ratio and vertical side walls, as shown in 4.23(b).

Supposing the undercut of comb fingers and flexure beams being \(\Delta d\) and \(\Delta w\), respectively, the actual finger gap and flexure width turns to be \(w_f + 2\Delta d\) and \(w_b - 2\Delta w\), respectively. According to Eqs. (3.23) and (5.19), the electrostatic force and the stiffness of supporting flexures would be decreased at the same time.

In conclusion, the influence of the fabrication tolerance on the performance of the friction testing device has been analytically studied. By using those obtained numerical equations, prediction and evaluation of the actual performance of the testing device can
4.7 Dimension Characterization

The critical dimensions of the fabricated microactuators are measured with an optical microscope, and the tolerance of the dimensions has been observed. Figures 4.24 shows the top view and the back view etched comb fingers. It can be found that difference of the gap at two sides was formed after the DRIE process.

By measuring the fabricated comb fingers and flexure with optical method, dimensions of the fabrication results were listed in Table 4.2. Based on the measured dimensions, the slope angles of comb electrodes and flexures are given by

\[ \theta = \tan^{-1} \left( \frac{(5.3 - 4.8)}{(2 \times 100)} \right) = \tan^{-1} (0.0025) \cong 0.15^\circ \]

and

\[ \phi = \tan^{-1} \left( \frac{(4.3 - 4.8)}{(2 \times 100)} \right) = \tan^{-1} (-0.0025) \cong -0.15^\circ \]
respectively.

4.8 Summary

Fabrication processes of the linear and rotary MEMS friction testing devices were first presented in this chapter. Based on the working principles of silicon-on-glass (SOG), anodic bonding was performed to bond the glass and silicon wafers with intermediate electrode layer. As the most important process, the operation of high-aspect-ratio DRIE on the SOG wafer was discussed in detail. To achieve prototypes of the MEMS devices, optimizations of the design of the mask layouts, the etching parameters and the operation techniques were carried out. As the measurement method, the idea of using image processing techniques has also been introduced. By using a customized testing platform, both stationary images and motion videos have been captured. Furthermore, effects of different profiles after fabrication on the testing results have been analyzed. Calculation based on the experimental results of the testing devices was also given. As a result, the feasibility of those optimizations have been proved by the fabrication and measurement results.
Chapter 5

Measurement and Analysis of the Linear Friction Testing Device

In this chapter, analysis of the contact interface of the linear friction testing device based on contact mechanics is first carried out. For understanding the friction behaviors of the linear devices, a quasi-static stick-slip model is then developed to analyze the characteristics of the friction at the contacting sidewall surfaces and to predict the transitions between static and kinetic friction. The measured displacements of the driving and sensing plates versus time are in good agreement with the trend predicted by the quasi-static stick-slip model. Based on the quasi-static stick-slip model, a saturation phenomenon of the kinetic friction at the silicon sidewall surfaces with an increasing normal load is observed.

5.1 Contact Mechanics Analysis

It is desirable to know whether the friction force measured is due to the macroscopic plastic deformation of the bumps (at the scale of the bump geometry) or finer structure of the surface roughness. To understand the state of the deformation, the contact between the bumps and the sensing plate was analyzed based on the Hertz theory for
elliptical contacts.

**Figure 5.1** Scanning electron microscope (SEM) image of a sidewall surface with waviness.

The radius of each cylindrical bump in the testing device is denoted by $R_x$. Due to the alternating operations of etching and passivation in the DRIE (Bosch) process, nearly sinusoidal ridges (scallops) were formed on the sidewall surfaces along a direction perpendicular to the depth direction of the silicon wafer, as shown in Figure 5.1. Considering that the two opposing side walls were fabricated in the same process with synchronized passivation and etching cycles, peak-to-peak contact between the two surfaces would occur, as shown in Figure 5.2(a).

Treating a half cycle of this waviness as a circular arc (Figure 5.2(a)), the amplitude can be described as

$$A = R_y - \sqrt{R_y^2 - \left(\frac{\lambda}{4}\right)^2}$$

(5.1)
Chapter 5  
*Measurement and Analysis of the Linear Friction Testing Device*

![Diagram](image)

**Figure 5.2** Schematic diagram of the contact region: (a) profile of the peak-to-peak contact at the sidewall surfaces and one sinusoidal ridge as well as the simplified arc of the profile; (b) contact of a smooth bump with the ridges as equivalent radius $R_*$ applied.

![Diagram](image)

**Figure 5.3** Schematic diagram of the elliptical contact areas along the contact sidewall.
where $R_y$ is the radius of the arc, $A$ is the amplitude of the ridge wave, and $A$ is the wavelength of the sidewall surfaces. Therefore, for both side walls, the sinusoidal ridges can be simplified as a combination of jointed arcs, with the radii of the two sides being the same. To analyze the contact areas at the side walls, an equivalent radius, $R^*$, and an equivalent elastic modulus, $E^*$, are defined, respectively, by $R^* = R_y/2$ and $E^* = 2(1 - \nu^2)/E$, where $E$ is Young’s modulus and $\nu$ is the Poisson’s ratio of the material of the contact surfaces. For a bulk-machined device based on single crystal silicon, $E = 165$ GPa, and $\nu = 0.22$ [113]. Therefore, the equivalent elastic modulus is $E^* = 86.7$ GPa.

Considering that the etched side walls are not exactly vertical after the DRIE process [114], i.e., a tapered sidewall profile, only partial contact would occur between the two sidewall surfaces. Thus, the contact length, $L$, is approximately given by $L = \eta h$, where $\eta$ is a contact length ratio and $h$ is the thickness of the sidewall surfaces. The number of peaks being in contact along the side walls is given by $n^* = L/\lambda$.

A schematic diagram of the contact of an equivalent bump and the side wall is shown in Figure 5.2(b) and the contact areas along the depth direction based on Hertz contact theory for an elliptical contact are shown in Figure 5.3, where $a$ and $b$ are the minor and major axes of the elliptical areas, respectively. For the elliptical contact, the equivalent relative curvature is defined by $R_e = (R'R'')^{1/2}$, where $R' = R_x$ and $R'' = R_y^*$. According to the Hertz contact theory, the equivalent contact radius of the contact region, $c$, is given by [115]

$$c = \left(\frac{3PR_e}{4E^*}\right)^{1/3} F_1$$

(5.2)

where $F_1$ is a modifying factor, which depends only on the ratio, $R'/R''$, and $P$ is the
Chapter 5  
Measurement and Analysis of the Linear Friction Testing Device

Table 5.1 Technical data of the etched sidewall surfaces

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of the bump, ( m (\mu m) )</td>
<td>2</td>
</tr>
<tr>
<td>Radius of the bump, ( R_e (\mu m) )</td>
<td>2376</td>
</tr>
<tr>
<td>Amplitude of the ridges, ( A (nm) )</td>
<td>99.3</td>
</tr>
<tr>
<td>Wavelength of the ridges, ( \lambda (nm) )</td>
<td>445.8</td>
</tr>
<tr>
<td>Equivalent radius of the ridge, ( R_e^* (nm) )</td>
<td>56.1</td>
</tr>
<tr>
<td>Equivalent relative curvature, ( R_e (\mu m) )</td>
<td>11.5</td>
</tr>
<tr>
<td>Modifying factor, ( F_1 )</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Local load per contact peak given by

\[
P = \frac{N}{2n^*}
\]  \quad (5.3)

The maximum contact pressure at the contact area is found to be [115]

\[
p_0 = \frac{3P}{2\pi c^2}
\]  \quad (5.4)

With an estimation of \( A \) and \( \lambda \) from a side-view SEM image of a side wall created with the DRIE process [3], \( R_e \) was calculated by solving Eq.(5.1). All relevant parameters of the profile of the etched sidewall surfaces are listed in Table 5.1. For devices fabricated from 150-\( \mu m \)-and 300-\( \mu m \)-thick wafers, the thicknesses of the sidewall surfaces \( L \) are about 75 \( \mu m \) and 150 \( \mu m \), respectively, i.e., a half of the wafer thickness. The value of the local load \( P \) obtained for different wafer thicknesses corresponding to the conditions that the normal actuator was driven at \( V_n = 23.5 \) V, and 33 V, respectively. Based on the parameters listed in Table 5.1, the calculated values of \( P \) and the corresponding maximum contact pressure \( p_0 \) for different values of the contact length ratio \( \eta \) are listed in Table 5.2.

Noting that the yield strength of single crystal silicon \( \sigma_y = 7 \) GPa [116], the value
Table 5.2 Local load per contact peak and maximum contact pressure for different conditions

<table>
<thead>
<tr>
<th>( n )</th>
<th>( P (\mu N) )</th>
<th>( P_0 ) (GPa)</th>
<th>( P (\mu N) )</th>
<th>( P_0 ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04</td>
<td>0.28</td>
<td>0.06</td>
<td>0.33</td>
</tr>
<tr>
<td>0.75</td>
<td>0.05</td>
<td>0.31</td>
<td>0.08</td>
<td>0.36</td>
</tr>
<tr>
<td>0.5</td>
<td>0.08</td>
<td>0.36</td>
<td>0.13</td>
<td>0.42</td>
</tr>
<tr>
<td>0.25</td>
<td>0.16</td>
<td>0.45</td>
<td>0.25</td>
<td>0.53</td>
</tr>
<tr>
<td>0.125</td>
<td>0.32</td>
<td>0.57</td>
<td>0.51</td>
<td>0.66</td>
</tr>
</tbody>
</table>

The contact length ratio, \( \eta \), was estimated to be 0.5 based on the observation of the sidewalls. From Table 5.2, the calculated values of the maximum contact pressure, \( P_0 \), are 0.36 and 0.42 GPa, for the 150-\( \mu \)m- and 300-\( \mu \)m-thick wafers, respectively. Because \( P_0 \) is much lower than \( (p_0)_y \) for both wafer thicknesses involved, it can be concluded that no plastic deformation occurs at the contact interface on the scale of the designed bumps and the scallops (waviness) caused by the DRIE process. It can also be seen from Table 5.2 that a slightly different value of \( \eta \) will not change the conclusion from this analysis.

For more advanced analysis of the surface interaction at finer structures including those between asperities, an expansion of the apparent area of contact should be considered. Relevant theories on the expansion of the contact semi-width and the effects of multi-scale fractal-regular surfaces can be found in [117] and [118].
Chapter 5  
Measurement and Analysis of the Linear Friction Testing Device

5.2 Development of a Quasi-Static Stick-Slip Model

By considering the transitions between the static and kinetic friction, a friction model was developed to describe the sliding process of the contact sidewall surfaces and to predict the waveforms of the displacement of the driving plate and that of the sensing plate subjected to a given oscillating electrostatic driving force in the tangential direction, $F_t$.

Considering the small feature size (on a micro-scale) and the low working frequency (0.1 to 0.5 Hz) of the testing device, no mass or delay in response were considered during the analysis of the motion of the testing device. Thus, the performance of the device was represented by a simplified quasi-static model. The relative motion of the contact sidewall surfaces was treated as that in either a sticking or a slipping states indicating transitions of the friction force at the side walls.

To find out the critical instants of the transitions, a trapezoidal waveform $V_t(t)$ was adopted to drive the tangential comb actuators in the present study. In the ascending and descending periods of each cycle of the waveform, the voltage has a linear relationship with the elapsed time, which helps avoid the measurement error that could be brought in by a square wave. For simplicity in the analysis, the tangential driving force, $F_t$, has been assumed to be linear with time. In the rising and falling stages, this linear relationship provides a simple way to find out the exact value of the driving voltage when kinetic friction starts to occur, and the critical driving force to generate relative sliding at the contacted surfaces can be found out. Actually, $F_t$ is a quadratic function of time for a trapezoidal voltage waveform. Nevertheless, the displacement values at the critical instants predicted by the following analysis remain unchanged. A complete cycle of the tangential driving force with the critical instants and the corresponding displacements of the driving plate, $x$, and the sensing plate, $s$, are shown in Figure 5.4.

As described earlier, the waveform was applied after the sensing plate had been
brought into contact with the driving plate. After the first cycle of the waveform, due to residual stresses of the supporting flexures, the equilibrium positions of the driving plate and the sensing plate would deviate slightly from their initial positions. The offset is given by

$$x_A - s_A = \Delta$$  \hspace{1cm} (5.5)

Moreover, $x$ and $s$ at instant $A$ satisfy the condition, $x_A k + s_A k_s = 0$, where $k$ and $k_s$ are the stiffnesses of flexure combinations $T$ and $S$, respectively. From instant $A$ to instant $B$, the tangential driving force $F_t$ keeps increasing, and the sensing plate moves together with the driving plate due to the static friction force, $f_s$. At instant $B$, slip at the contact area occurs and the static friction force reaches its maximum value, $f_{sm}$. Thus, $x_B - x_A = s_B - s_A$. Combining the foregoing relation with Eq. (5.5) and noting that $F_t - f_s = k x$ and $f_s = k_s s$, the displacements of the driving plate and sensing
Chapter 5  

Measurement and Analysis of the Linear Friction Testing Device

plate, $x_B$ and $s_B$, can be expressed, respectively, as

$$x_B = s_B + \Delta = (F_{tB} - f_{sm}) / k$$  \hspace{1cm} (5.6)

and

$$s_B = f_{sm} / k_s$$  \hspace{1cm} (5.7)

where $f_{sm} = \mu_s N$, and $\mu_s$ is the coefficient of static friction. Immediately after instant $B$, a transition from static friction to kinetic friction occurs so that a jump happens in both $x$ and $s$, assuming that the kinetic friction force, $f_k$, is smaller than $f_{sm}$. This assumption will be shown to be valid by observing the similar trends of the $x$ and $s$ curves, respectively, obtained from experiments as compared to the predicted curves.

Similarly, the displacement of the sensing plate at instant $C$ is given by

$$s_C = f_k / k_s$$  \hspace{1cm} (5.8)

Since the tangential driving force $F_t$ at instant $C$ is the same as that of at instant $B$, the displacement of the driving plate changes to

$$x_C = (F_{tC} - f_k) / k$$  \hspace{1cm} (5.9)

Comparing Eq. (5.9) to (5.6), the jumped distance of the driving plate is found to be

$$x_C - x_B = (f_{sm} - f_k) / k$$  \hspace{1cm} (5.10)

From instant $C$ to $D$, the sensing plate maintains at its position while the displacement of the driving plate keeps increasing to

$$x_D = (F_{tD} - f_k) / k = (F_{tD} - \mu_k N) / k$$  \hspace{1cm} (5.11)
where \( F_{t_D} \) can be calculated by using Eq. (3.11) with the known voltage at instant \( D \), and \( \mu_k \) is the coefficient of kinetic friction. Note that \( f_k = \mu_k N \). From instant \( D \) to \( E \), both plates remain stationary since \( F_t \) is constant in this time interval.

From instant \( E \) to \( F \), the static friction force makes the two plates stick together in a return trip, so that \( x_F - x_E = s_F - s_E < 0 \). In the time range of \( C \rightarrow D \rightarrow E \rightarrow F \), contact interface is in a static friction state. At instant \( F \), the static friction force reaches its maximum value again, but in an opposite direction. The displacements of the two plates at instant \( F \) can, then, be expressed, respectively, as

\[
x_F = \left( F_{t_F} + f_{sm} \right)/k \tag{5.12}
\]

and

\[
s_F = -f_{sm}/k_s \tag{5.13}
\]

Immediately after instant \( F \), the second slip occurs. Similar to the jump from instant \( B \) to \( C \), the displacement of the jump from instant \( F \) to \( G \) for the driving plate is given by

\[
x_F - x_G = (f_{sm} - f_k)/k \tag{5.14}
\]

and the displacement of the sensing plate at instant \( G \) is

\[
s_G = -s_C = -f_k/k_s \tag{5.15}
\]

From instant \( G \) to \( H \), the sensing plate remains at its position corresponding to the friction force, \( f_k \), i.e., \( s_H = s_G \), while the displacement of the driving plate keeps decreasing until it stops at its equilibrium position \( x_H = f_k/k \). From this relationship and Eq. (5.15), it can be shown that \( x_H \) and \( s_H \) satisfy the condition, \( x_H k + s_H k_s = 0 \), just as that for \( x_A \) and \( s_A \). In the time range of \( F \rightarrow G \rightarrow H \), contact interface is in a
Chapter 5  Measurement and Analysis of the Linear Friction Testing Device

kinetic friction state. Starting at instant \( H \) until instant \( B \) of the next cycle, the interface is in a static friction state again.

According to this stick-slip model, the jumped distances of the driving and sensing plates are determined by the values of coefficient of friction when the normal load \( N \) is given. Equations (5.10) and (5.14) can be rewritten as

\[
x_C - x_B = x_F - x_G = (\mu_s - \mu_k) \frac{N}{k}
\]

(5.16)

Combining Eq. (5.9) with Eq. (5.15) yields

\[
s_C - s_G = 2\mu_k \frac{N}{k_s}
\]

(5.17)

Since the foregoing jumped distance in Eq. (5.16) and (5.17) can easily be found from experiment data of \( x \) and \( s \) versus time, Eq. (5.16) and (5.17) can readily be used to determine the coefficient of static friction, \( \mu_s \) and the coefficient of kinetic friction, \( \mu_k \).

The following derivation shows that the stiffness ratio, \( r \), defined as \( r = k/k_s \), can be obtained from the measured curves of \( x \) and \( s \) together with a reference displacement of the driving plate, \( x_{D0} \). This reference quantity is defined as the \( x_D \) value for \( f_k = 0 \) (no contact). By Eq. (5.11), \( x_{D0} = F_{tD}/k \), and \( f_k = k(x_{D0} - x_D) \). Combining the last relationship with Eq. (5.17) yields

\[
r = \frac{k}{k_s} = \frac{s_C - s_G}{2(x_{D0} - x_D)}
\]

(5.18)

The contact interface will enter a no-slip regime when the normal load is large enough to ensure that only static friction could occur. In this regime, the displacements of the driving and sensing plates will be the same at any instant, with a largest displace-
Chapter 5  Measurement and Analysis of the Linear Friction Testing Device

ment, \( x_{Dn} \), which represents the counterpart part of \( x_D \). Due to the no-slip condition, the sum of the flexure stiffnesses is given by

\[
k + k_s = \frac{F_{tD}}{x_{Dn}} \tag{5.19}
\]

When the tangential force \( F_{tD} \) is calculated reasonably, \( k \) and \( k_s \) can be readily obtained from the measured displacement values by using Eq. (5.18) and (5.19).

Combining Eqs. (3.11), (3.12), (5.16), (5.18), and (5.19) with noting \( f_k = k(x_{po} - x_D) \), the coefficient of kinetic friction is found to be

\[
\mu_k = q \frac{x_{Dn} - x_D}{x_{Dn}} \tag{5.20}
\]

and the difference between the coefficients of static and kinetic friction can be expressed in term of measurable quantities as follows

\[
\mu_s - \mu_k = q \frac{x_C - x_B}{x_{Dn}} \tag{5.21}
\]

where \( q \) is a displacement-friction factor, defined as

\[
q = \frac{c_t}{c_n} \frac{r}{1 + r} \frac{V_{1D}^2}{V_{tD}^2 - V_{nc}^2} \tag{5.22}
\]

Note that the jumped distance, \( x_C - x_B \), in Eq. (5.21) can also be replaced with \( x_F - x_G \) according to Eq. (5.16).

Coefficients of static and kinetic friction, \( \mu_s \) and \( \mu_k \), respectively, can be obtained from Eq. (5.20) and (5.21) with the aid of Eq. (5.18) and (5.22). Note that all required values in these equations are measurable quantities (displacements and voltages) except for \( c_t \) and \( c_n \), which are calculated based on their definitions given just below Eq. (3.11). Because any errors in the calculation due to inaccuracy in the gap, \( g_n \) or \( g_t \), and the...
Chapter 5  
*Measurement and Analysis of the Linear Friction Testing Device*

### Table 5.3  
Stiffnesses of the supporting flexures adopted in the testing devices.

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Device wafer thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>150 μm</td>
</tr>
<tr>
<td>Flexure combination $N, k_n$ (N/m)</td>
<td>8.25</td>
</tr>
<tr>
<td>Flexure combination $S, k_s$ (N/m)</td>
<td>16.38</td>
</tr>
<tr>
<td>Flexure combination $T, k$ (N/m)</td>
<td>1.66</td>
</tr>
</tbody>
</table>

thickness, $H$, or due to neglecting the fringe field effects would drive the values of $c_t$ and $c_n$ in the same direction, the calculated value of $c_t/c_n$ should be very close to the realistic value. Thus, the coefficients of static and kinetic friction determined based on the present scheme are insensitive to the errors in the calculation of the electrostatic forces.

#### 5.3 Friction Behavior of the Linear Device

To characterize the motion of the driving plate, its displacement under the condition without a normal contact ($V_n = 0$) was first measured. A 0.5-Hz trapezoidal waveform with a voltage range set as 0–20 V, with a maximum measured voltage as $V_{t,\text{max}} = V_D = 19.4$ V, was applied to the tangential actuator for the device made from a 150-μm-thick wafer (for the 300-μm case, parameters are given under Figure 5.7). The displacement of the driving plate was obtained by processing the digital video images, with $x_{DSP}$ readily identified (Figure 5.5). As the friction force affects the displacement of the driving plate, the maximum displacement under a given $V_t$ was used as a reference to judge whether friction occurred between the driving plate and the sensing plate.

By using Eqs. (5.18) and (5.19) with measured displacements of the driving and sensing plates as well as the corresponding driving voltages, the stiffness values of all the flexures for different wafer thicknesses were obtained, as listed in Table 5.3.

The friction behavior of the sidewall surfaces was first studied based on the testing
device fabricated from a 150-μm-thick silicon wafer. The experimental curves of the
displacement of the driving plate, $x$, and the displacement of the sensing plate, $s$, versus
time, as shown in Figure 5.6(a) for $V_n = 23.5$ V and $V_{t,\text{max}} = 19.4$ V, follow closely the
theoretically predicted curves of $x$ and $s$, respectively, shown in Figure 5.4. The two
distances of interest were found to be $x_c - x_B = 5.0$ μm and $s_c - s_G = 0.71$ μm. The
voltage corresponding to the first occurrence of contact, $V_{nc}$, the reference displacement
with no contact, $x_{DO}$, and the maximum displacement for the no-slip condition, $x_{Dn}$,
were found to be 21 V, 8.51 μm and 2.02 μm, respectively. By using Eqs. (5.20)
and (5.21), the coefficients of static and kinetic friction at the sidewall surfaces were
found to be $\mu_s = 0.75$ and $\mu_k = 0.40$, respectively. Combining the results for different
jumps in all observed cycles, the coefficients of static and kinetic friction at the sidewall
surfaces of the 150-μm-thick device were found to be $\mu_s = 0.73 \pm 0.03$ and $\mu_k =
0.39 \pm 0.01$ (mean ± standard deviation), respectively.

Similar measurements were performed on a testing device fabricated from a 300-
μm-thick silicon wafer. Note that, theoretically, the displacement due to the electrostatic
force does not depend on the thickness of the silicon device because both this force and
the flexure stiffness are directly proportional to the thickness. The measured displace-
Figure 5.6 Measured displacements of the driving plate and the sensing plate of a device made from a 150-μm-thick silicon wafer: (a) \( V_n = 23.5 \text{ V} \) and \( V_{t,\text{max}} = 19.4 \text{ V} \); (b) \( V_n = 22.0 \text{ V} \) and \( V_{t,\text{max}} = 19.4 \text{ V} \).
Figure 5.7 Measured displacements of the driving plate with bumps and sensing plate of a device made from a 300-μm-thick silicon wafer at 0.1 Hz, $V_n = 33$ V and $V_{t,\text{max}} = 20$ V.

The variation of the kinetic friction force with a ramping normal force was also studied with the sidewall friction testing device fabricated from a 150-μm-thick silicon wafer. During the experiment, the normal driving voltage, $V_n$, varied from 19.0 to 23.5 V with an increment of 0.5 V while the tangential waveform was maintained at $V_{t,\text{max}} = 19.4$ V. In Figure 5.8(a), the measured driving plate displacement at instant $D$, $x_D$, (as
Chapter 5  
Measurement and Analysis of the Linear Friction Testing Device

defined in Figure 5.4) is plotted against $V_n^2$. The normal force values calculated from Eq. (3.12) are given on the top horizontal axis in Figure 5.8(b). When $V_n^2$ varied from 361 to 441 V$^2$, $x_D$ is maintained at a maximum value, and the measured displacement waveform of the driving plate at this stage (similar to that in Figure 5.5) indicated no contact between the opposing sidewall surfaces. As $V_n^2$ increased from 441 to 506 V$^2$, contact occurred as seen from the waveforms of $x$ and $s$ shown in Figure 5.6(b). In this range of $V_n^2$, an approximately linear relationship between $x_D$ and $V_n^2$ was observed, as shown in Figure 5.8(a). However, while $V_n^2$ varied from 506 to 552 V$^2$, the slope of the fitting line for $x_D$ versus $V_n^2$ reduced to almost zero while sliding between the driving and sensing plates was still observed by examining $x$ and $s$. According to the developed quasi-static model, the displacement of the driving plate at instant $D$, $x_D$, is given by Eq. (5.11). Considering that both $k$ and $F_{t_D}$ are constant, the slope for $x_D$ against $V_n^2$ (which varies linearly with $N$ according to Eq. (3.12)) is determined by the coefficient of kinetic friction $\mu_k$. The variation of the slope of $x_D$ is an indicator of a change of the kinetic friction force as well as the coefficient of kinetic friction during the sliding process.

The calculated kinetic friction force and coefficient versus the square of the DC voltage for the normal actuator, $V_n^2$, is shown in Figure 5.8(b). From 462 to 506 V$^2$, where $x_n$ has a descending slope in Figure 5.8(a), the kinetic friction force $f_k$ has an ascending slope while the kinetic friction coefficient $\mu_k$ is maintained at a relatively high level (Figure 5.8(b)). From 506 to 552 V$^2$, where $x_D$ is almost leveled in Figure 5.8(a), the kinetic friction force $f_k$ is almost at a constant level of about 5.3 μN while the coefficient of kinetic friction $\mu_k$ has a descending slope (Figure 5.8(b)) due to the continued increase of the normal force.

Based on the contact analysis in this study, as discussed earlier, the friction behavior of the sidewall surfaces cannot be explained by assuming shearing of macroscopic
Figure 5.8 Effects of a ramping normal load: (a) variation of the displacement of the driving plate at instant $D_i$, (b) variations of the kinetic friction force and coefficient with $V_{nc}^2$ (bottom axis) and the normal load $N$ (top axis).
plastically deformed junctions. The saturation phenomenon of the kinetic friction force observed from Figure 5.8(b) can be explained based on the friction mechanisms at molecular level with adhesion considered [119]. Thus, the kinetic friction force can be expressed as

\[ f_k = \mu_k N + \mu A_e \]  

(5.23)

where \( \mu \) is a coefficient determined by the surface contact characteristics, and \( A_e \) is the effective area. While the normal load \( N \) is low, contact occurs only on higher asperities. The adhesion effect is not significant due to the large average gap between the two contacting surfaces. Thus, the friction force is linear with the normal force \( N \) according to the Amonton's law of friction. As \( N \) increases, the average gap between the contacting surfaces becomes smaller due to elastic deformation, and the contact enters an adhesion-dominant regime. Thus, the friction force becomes strongly dependent on \( A_e \), which is closely related to the apparent contact area. The fact that the kinetic friction force becomes almost a constant suggests that the effective constant area \( A_e \) is nearly a constant in the adhesion-dominant regime for the conditions of the present experiment. This observed saturation phenomenon for the kinetic friction force is in agreement with the experimented results obtained by Guo et al. [54] under similar testing conditions. Some possibly deep physical roots of this friction saturation behavior may constitute an interesting topic for future exploration.

5.4 Summary

Friction behavior at the silicon sidewall surfaces etched by DRIE MEMS was studied in this chapter. The dual-bump design of the friction testing device ensures the repeatability of the contact location. A quasi-static stick-slip model was developed to analyze the characteristics of the friction at the contacting sidewall surfaces and to predict the transitions between static and kinetic friction. Due to the generality in the basic as-
Chapter 5  
*Measurement and Analysis of the Linear Friction Testing Device*

...sumptions, this model is applicable to any friction-related dynamic system that can be described by a similar free-body diagram (Figure 5.4).
Chapter 6

Measurement and Analysis of the Rotary Friction Testing Device

In this chapter, the stick-slip phenomenon of bush-shaft like structures based on the developed quasi-static stick-slip model is further expanded. A general formulation is developed to determine the equivalent tangential stiffness of the contact point. The prediction of the regions with sliding wear is then introduced based on the developed formulation. At last, the coefficient of static friction is found from the experimental results by using the quasi-static stick-slip model.

6.1 Investigation of the Stick-Slip Phenomenon

6.1.1 Quasi-Static Stick-Slip Model

Considering that only a small amount of rotation would be involved, the tangential motions of the driving ring and the sensing bush at the contact region can approximately be treated as one-dimensional, with the driving torque due to the circular comb drive being converted to an equivalent tangential force acting at the radical position of the contact interface. An equivalent system in one dimension with static and kinetic friction
Chapter 6  \textit{Measurement and Analysis of the Rotary Friction Testing Device}

behaviors can, then, be described by the quasi-static stick-slip model developed by Wu et al. [120][121]. According to this model, the tangential driving force, $F_{ce}$, tangential displacement of the driving ring, $x$, and tangential displacement of the sensing bush, $s$, are shown schematically in Figure 6.1. For clarity, all the curves are represented by straight-line segments. It should be understood that the actual curve for $F_{ce}$ should have parabolic side segments because the electrostatic force is proportional to the square of the driving voltage, thus requiring all inclined lines to be curved in reality. Nevertheless, the levels of the vertical coordinates remain unchanged in this simplified representation.

The jumped distance of the driving ring is given by

$$\Delta x = x_c - x_B = x_f - x_C = \left( f_{sm} - f_k \right) / k_c$$  \hspace{1cm} (6.1)

where $k_c$ is the effective tangential stiffness of flexure combination $C$, and that of the sensing bush is given by

$$\Delta s = s_C - s_B = 2\mu k_s N$$  \hspace{1cm} (6.2)

where $k_s$ is the equivalent tangential stiffness of flexure $S$. In the present study, the ratio of the stiffnesses of flexure combination $C$ and flexure $S$, was expected to be much larger than that of in [120] due to the different design of supporting flexures being adopted. Thus, the predicted waveforms of the tangential displacements of the driving ring, $x$, and the sensing bush, $s$, are shown in Figure 6.1.

From instant $A$ to $B$ in Figure 6.1, only static friction force $f_s$, acts on the sensing bush. The maximum static friction force, $f_{sm}$, can then be obtained during the transition from static friction to kinetic friction, i.e., $f_{sm} = s_B k_s$, where $s_B$ is the displacement of the sensing bush at the transition point, $B$. Due to the similarity of the motion model, the relation for the linear friction testing device in Eq. 3.14 is still applicable for the rotary friction testing device. Thus, combination of Eq. 3.14 with the foregoing relation,
the coefficient of static friction, $\mu_s$, is found to be

$$\mu_s = \frac{s_{bl} k_s}{c_n (V_n^2 - V_{nc}^2)} \quad (6.3)$$

6.1.2 Displacement Measurements

To characterize the motion of the driving ring, a 0.5-Hz trapezoidal waveform with a voltage range set as 0–50 V, and a maximum measured voltage as $V_{n,\text{max}} = V_B = 48.6$ V, was applied to the circular actuator of the testing device. The tangential displacements of the driving ring and sensing bush were obtained by processing the captured digital video images.

In the present study, six rounds of measurements have been carried out, and a ramping normal driving voltage, $V_n$, was applied to the sensing bush, i.e., a ramping normal force between the contacting sidewall surfaces was obtained. During each round of the measurement, $V_n$ was first increased from the minimum value, 26.0 V, to the maximum
value, 27.5 V using 0.1 V step. After that, $V_n$ was decreased in the same step size to finish one round of the measurements. The experimental curves of the tangential displacement of the driving ring, $x$, and that of the sensing bush, $s$, versus time in the increasing position of the first round are shown in Figure 6.2 for $V_n = 27.1$ V and $V_{c,max} = 48.6$ V. The measured displacements follow closely the theoretically predicted curves of $x$ and $s$ (Figure 6.1).

### 6.2 Analysis of Equivalent Stiffness and Contact Point

#### 6.2.1 Observation of the Sensing Bush

During the motion of the rotary friction testing device, variation of the gap between the sensing bush and the driving ring has been observed. As shown in Figure 6.3(a), initially the contact point was located on the central line of the sensing bush ($V_c = 0$). Due to the tapered profile of the sidewall surfaces after fabrication, a small open area
at the top surface (the dark line between the bush and the ring) was observed when the contact of the bush and ring occurred. When $V_c$ varied from 0 to $V_{c,\text{max}}$, the driving ring rotated in a counter-clockwise sense. Expansion of the gap between the bush and ring and the deformation of the supporting flexure were observed, as shown in Figure 6.3(b).

The variation of the gap suggests that the contact point is not a stationary point. It is expected that the equivalent tangential stiffness of the flexure-bush assembly, $k_s$, plays a key role in the motion status of this bush-shaft structure. To obtain a general expression of $k_s$ and to explain the motion of the contact point, two mechanics-based models, namely, a bush-flat model and a bush-shaft model, were developed with some generalization in geometry for a range of device applications. The simpler bush-flat model will be presented first to explain the basic concepts and strategy in treating the motion of the rotary testing device. As an expansion of the bush-flat model, the bush-shaft model reduces the two-dimensional motion into a one-dimensional equivalent with considerations of the rotations of both the driving ring and the sensing bush. In the analysis, a counter-clockwise sense is defined as positive.

6.2.2 Bush-Flat Model

In the bush-flat model, the sensing bush is extended to form a sensing ring with a radius $r_{eq}$, and the driving ring is simplified into a driving flat. This geometric configuration
is expected in some devices with a contact interface. The thickness of the sensing ring is denoted by \( H_b \). The sensing ring with a thickness, \( H_b \), is supported by Flexure \( S \), as shown in Figure 6.4(a). The driving flat is connected to flexure combination \( C \) and is pulled by the effective tangential driving force \( F_{ce} \). Point \( C \) is the center of the sensing ring, \( Q \) is the end point of Flexure \( S \), \( P_s \) is a material point on the sensing ring, and \( P_c \) is the contact point. Equilibrium of the sensing ring is achieved by the combination of the normal force, \( N \), the friction force at the contacting surfaces, \( F_s \), and the elastic restoring force and moment supplied by Flexure \( S \). Note that this flexure is more than a linear spring despite the simple spring symbol used.

Based on the radii of the sensing bush and the driving ring are \( r_s \) and \( r_d \), respectively, the equivalent radius of the sensing ring, \( r_{eq} \), is defined by

\[
\frac{1}{r_{eq}} = \frac{1}{r_s} \pm \frac{1}{r_d}
\]  

(6.4)

where the positive and negative signs should be taken for convex and concave sensing ring surfaces, respectively.

Before the motion occurs, points \( P_s \) and \( P_c \) are overlapped, points \( P_s' \) and \( P_c' \) are the material point and contact point after the motion of the sensing ring, respectively, as shown in Figure 6.4(b). The tangential displacement of the material point, \( s \), is a combination of the displacements acquired from translation of the center point \( C \), \( s_c \), and the distance between \( P_s' \) and \( P_c' \), \( \Delta \), where \( \Delta = r_{eq}\theta \).

Considering that the thickness of the sensing ring is \( H_b \), and segment \( P_sQ \) is treated as a rigid body, as shown in Figure 6.5. Thus, a general matrix equation for the tangential displacement, \( s_g \), and the rotation angle, \( \theta_q \), at point \( Q \) is given by

\[
\begin{bmatrix}
    s_g \\
    \theta_q
\end{bmatrix} =
\begin{bmatrix}
    \lambda_{11} & \lambda_{12} \\
    \lambda_{21} & \lambda_{22}
\end{bmatrix}
\begin{bmatrix}
    F_q \\
    M_q
\end{bmatrix}
\]  

(6.5)
where $F_q$ and $M_q$ are the tangential force and moment at point $Q$, respectively. Based on the geometric relation between points $P_s$ and $Q$ as well as the rigid body translation of segment $P_sQ$, the tangential displacement, $s$, of the material point $P$ and the tilting angle of the bush ring, $\theta$, are found to be

\[
\begin{aligned}
s &= s_q + H_b \theta \\
\theta &= \theta_q
\end{aligned}
\]  

(6.6)
Moreover, the relationships between the loads on points $P_s$ and $Q$ according to Figure 6.5 are given by

\[
\begin{align*}
F_s &= F_q \\
M_s + F_s H_b &= M_q
\end{align*}
\] (6.7)

Substituting Eqs. (6.6) and (6.7) into (6.5) gives the displacements of the material point $P_s(\mathbb{P}_s')$ as

\[
\begin{cases}
\begin{align*}
s &= \left[\lambda_{11} + (\lambda_{12} + \lambda_{21}) H_b + \lambda_{22} H_b^2\right] F_s + (\lambda_{12} + \lambda_{22} H_b) M_s \\
\theta &= (\lambda_{21} + \lambda_{22} H_b) F_s + \lambda_{22} M_s
\end{align*}
\end{cases}
\] (6.8)

Since $N$ and $F_s$ are both forces acting on the contact point (Figure 6.4(b)), the driving moment, $M_s$, is found to be

\[
M_s = -N\Delta = -Nr_{eq}\theta
\] (6.9)

Combining Eq. (6.8) and (6.9), the equivalent stiffness at the material point $P_s$ in the
tangential direction and the rotation angle are, respectively, given by

\[ k_s = \frac{F_s}{s} = \frac{1 + \lambda_{22} N r_{eq}}{(\lambda_{12} + \lambda_{21} + \lambda_{22}) H_b + (\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}) N r_{eq} + \lambda_{11}} \]  

(6.10)

and

\[ \theta = \frac{\lambda_{21} + \lambda_{22} H_b F_s}{1 + \lambda_{22} N r_{eq}} \]  

(6.11)

Noting that \( \Delta = r_{eq} \), the distance between \( P'_s \) and \( P'_c \) based on Eq. 6.11 is found to be

\[ \Delta = \frac{r_{eq} (\lambda_{21} + \lambda_{22} H_b)}{1 + \lambda_{22} N r_{eq}} F_s \]  

(6.12)

6.2.3 Bush-Shaft Model

As an expansion of the bush-flat model, a bush-shaft model has been developed to simulate the motions of structures with a conformal contact interface. In this model, the sensing bush is still extended to form a sensing ring with a radius \( r_s \), but it has a concave sidewall surface as viewed from the opposing surface. The driving ring, such as that of the present device, is simplified into a hollow driving shaft, with a radius \( r_d \), which can only rotate about its fixed center, Point \( O \), as shown in Figure 6.6(a). Similar to the bush-flat model, the sensing ring is supported by Flexure \( S \). The friction force, \( F_s \), and the normal force, \( N \), are acting on the contact point \( P_c \). Point \( Q \) is the end point of Flexure \( S \), and \( P_s \) is a material point on the sensing ring. Before motion occurs, points \( P_s \) and \( P_c \) are overlapped (Figure 6.6(a)). If the hollow driving shaft rotates counterclockwise, points \( P_s \) and \( P_c \) will move to \( P'_s \) and \( P'_c \), respectively. The angle of rotation of the sensing ring about its own center is \( \theta \), and that of the contact point about the center of the driving ring is \( \phi \), as shown in Figure 6.6(b).

Considering small rotation only, the displacements of the contact point \( P_c \) and the
material point $P_s$ are given as follows

$$s_c = \overrightarrow{P_cP_c'} = -r_d \phi$$  \hspace{1cm} (6.13)

and

$$s = \overrightarrow{P_sP_s'} = -r_s \theta$$  \hspace{1cm} (6.14)

Note that $\theta > 0$ and $\phi < 0$ for the situation shown in Figure 6.6(b). Since the two radii are almost equal, i.e., $r_s \approx r_d$, the arc length between the material point, $P_s'$, on the sensing ring and the contact point, $P_c'$, after motion is given by

$$\Delta = \overrightarrow{P_sP_c'} = s_c - s = r_s (\theta - \phi)$$  \hspace{1cm} (6.15)

Noting the conditions of Flexure $S$ (Figure 6.5), the thickness of the sensing ring and the forces on the contact point $P_c'$ are the same as those of the bush-flat model. Hence, the relations in Eq. (6.8) are still applicable to the bush-shaft model. Combination of
the two relations in Eq. (6.8) gives

\[ s = (\lambda_{11} + \lambda_{12} H_b) F_s + \lambda_{12} M_s + H_b \theta \]  

(6.16)

Combination of Eq. (6.8) with the foregoing relation gives

\[ \lambda_{22} s - \lambda_{12} \theta = (\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}) F_s + \lambda_{22} H_b \theta \]  

(6.17)

Thus, combination of the foregoing relation with Eq. (6.14) gives the tangential displace­ment and the rotation angle of the material point \( P_s \) as

\[ s = \frac{\tau_s (\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21})}{\lambda_{12} + \lambda_{22} (\tau_s + H_b)} F_s \]  

(6.18)

and

\[ \theta = \frac{\lambda_{12} \lambda_{21} - \lambda_{11} \lambda_{22}}{\lambda_{12} + \lambda_{22} (\tau_s + H_b)} F_s \]  

(6.19)

respectively. The equivalent tangential stiffness is then found to be

\[ k_s = \frac{F_s}{s} = \frac{\lambda_{12} + \lambda_{22} (\tau_s + H_b)}{\tau_s (\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21})} \]  

(6.20)

Figure 6.7 Schematic diagram of a part of the sensing ring and Flexure \( S \): (a) forces acting at the contact region; (b) equivalent forces and moment at the material point close to the end of Flexure \( S \).

At the material point, \( P_s' \), close to the end of Flexure \( S \), the equivalent moment, \( M_s \),
Chapter 6  

Measurement and Analysis of the Rotary Friction Testing Device

is assumed to be caused by the normal load, \( N \), only (Figure 6.7), while the contribution of the tangential force, \( F_s \), to this moment is of a second order and, hence, negligible. For consistency with Eq. (6.20), \( F_s \) is drawn in Figure 6.7 in its positive direction (to the right), which is just opposite to the actual direction. Since \( \theta \) and \( \phi \) are small angels, \( \cos \theta \approx \cos \phi \approx 1 \), and \( M_s \) is approximately given by

\[
M_s = N \Delta \quad (6.21)
\]

Similar to the derivation for the equivalent tangential stiffness, \( k_s \), the moment, \( M_s \), can be related to the tangential displacement by

\[
M_s = -k_M s \quad (6.22)
\]

where \( k_M (>0) \) is an interaction stiffness of the flexure and has the dimension of force because it is the ratio of a moment to a linear displacement. Note that \( M_s \) is positive if counter-clockwise. Combining Eqs. (6.15), (6.21) and (6.22) gives

\[
s_c = \left(1 - \frac{k_M}{N}\right) s \quad (6.23)
\]

Noting that \( s_c = -r_s \), the above relation can then be rewritten as

\[
\phi = \left(\frac{k_M}{N} - 1\right) \frac{s}{r_s} \quad (6.24)
\]

Substituting Eq. (6.22) into Eq. (6.8) and noting that \( s = -r_s \), the interaction stiffness of the flexure, \( k_M \), is found to be

\[
k_M = \frac{\lambda_{11} + \lambda_{21} r_s + \left(\lambda_{12} + \lambda_{22} + \lambda_{22} r_s\right) H_b + \lambda_{22} H_b^2}{\lambda_{12} + \lambda_{22} (r_s + H_b)} k_s \quad (6.25)
\]
Furthermore, combining Eq. (6.8) and (6.21), the tangential displacement of the contact point, $A$, is found to be

$$
\Delta = \frac{\lambda_{11} + \lambda_{21} r_s + (\lambda_{12} + \lambda_{21} + \lambda_{22} r_s) H_b + \lambda_{22} H_b^2 F_s}{\lambda_{12} + \lambda_{22} (r_s + H_b)} N
$$

(6.26)

As general expressions, Eqs. (6.20) and (6.25) are applicable to all flexure types as Flexure $S$. An example is given by using a cantilever beam as Flexure $S$ for a better understanding of the bush-shaft model, shown in Figure 6.6(a). A cantilever beam with a length $L$ is adopted as Flexure $S$ for this specific device designed. Thus, displacements of point $Q$ based on beam bending theories are given by

$$
\begin{bmatrix}
\theta_q \\
\phi_q
\end{bmatrix} =
\begin{bmatrix}
L^3/3EI & L^2/2EI \\
L^2/2EI & L/EI
\end{bmatrix}
\begin{bmatrix}
F_q \\
M_q
\end{bmatrix}
$$

(6.27)

Substitution of Eq. (6.27) into Eq. (6.20) reduces this matrix equation to a one-dimensional formulation in which the equivalent stiffness is given by

$$
k_s = \frac{6 (L + 2r_s + 2H_b) EI}{r_s L^3}
$$

(6.28)

Similarly, substitution of Eq. (6.27) into Eq. (6.26) gives the displacement of the contact point as

$$
\Delta = \frac{-2L^2 + 3Lr_s + 6 (L + r_s) H_b + 6H_b^2 F_s}{3L + 6 (r_s + H_b)} N
$$

(6.29)

6.3 Prediction of the Regions with Sliding Wear

During the motion of the friction testing device, the values of $V_c, \max$ and $V_n$ not only determine the tangential displacement of the sensing bush, but also determine whether the sensing bush is in pure rolling or rolling with slipping. The method of predicting its motion status is given below based on the equivalent stiffness.

- 127 -
6.3.1 Condition for Pure Rolling

As shown in the stick-slip model (Figure 6.1), the motions of the driving ring and the sensing bush are dependent on an effective driving force, $F_{ce}$, which is determined by the rotary actuating force, $F_{cm}$ (see Figure 6.8). According to the working principles of rotary comb actuators, $F_{cm}$ is given by

$$F_{cm} = \frac{n_c \varepsilon h V_{c,\text{max}}^2}{g}$$  \hspace{1cm} (6.30)

where $n_c$ is the number of the rotary comb finger pairs, and $g_c$ is the gap between the comb fingers. Considering the diagonally placed rotary actuators (Figure 3.4), the driving torque is given by $T_c = 2r_c F_{cm}$, where $r_c$ is the effective moment arm (Figure 6.8). Thus, the maximum of the effective tangential driving force at the contact interface of the bush and the ring, $F_{cem}$, is given by $F_{cem} = T_c / r_r = 2r_c F_{cm} / r_r$, where $r_r$ is the outside radius of the driving ring. Combining Eq. (6.30) with the foregoing relation, the maximum effective driving force, $F_{cem}$, is given by

$$F_{cem} = 2n_c \varepsilon h V_{c,\text{max}}^2 / g r_r c V_{c,\text{max}}^2$$  \hspace{1cm} (6.31)

where $c_e$ is the electrostatic force coefficient for the rotary actuators, defined as $c_e = 2n_c \varepsilon (h/g_c)(r_c/r_r)$.

Referring to Figure 6.8, the tangential forces at the contacting surfaces are equilibrated to give

$$F_{ce} = F_s + R_c$$  \hspace{1cm} (6.32)

where $F_s$ is the friction force at the contacting surfaces, $R_c$ is the restoring force of flexure combination $C$, given by $R_c = k_c x$ and $k_c$ is the equivalent tangential stiffness of flexure combination $C$ for the driving ring. An equivalent tangential stiffness, $k_s$,
Chapter 6  
Measurement and Analysis of the Rotary Friction Testing Device

Figure 6.8 Free-body diagram for analyzing the working principle of the friction testing device: (a) the left part of the testing device; (b) the driving ring.

is adopted to express the relation between $F_s$, and the tangential displacement of the sensing bush, $s$, by

$$F_s = k_s s \tag{6.33}$$

The tangential displacements of the driving ring and the sensing bush are the same during pure rolling (with sticking), i.e., $x = s$. Combining Eqs. (6.32), (6.33), the foregoing relation and $R_c = k_c x$, the effective driving force is found to be

$$F_{ce} = (k_c + k_s) s \tag{6.34}$$

Combining Eqs. (6.32) and (6.33) gives $F_s = F_{ce} k_s / (k_c + k_s)$. Thus,

$$f_{sm} = (F_s)_{max} = F_{cem} k_s / (k_c + k_s) \tag{6.35}$$

Based on Eqs. (6.31) and (6.35), the condition for pure rolling, $F_s / N \leq \mu_s$ for all time,
Table 6.1 Characteristics of the Fabricated Testing Device

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness of flexure $S$, $k_s$ (N/m)</td>
<td>2.04</td>
</tr>
<tr>
<td>Stiffness of flexure combination $N$, $k_n$ (N/m)</td>
<td>14.14</td>
</tr>
<tr>
<td>Stiffness of flexure combination $C$, $k_c$ (N/m)</td>
<td>46.5</td>
</tr>
<tr>
<td>Coefficient of normal driving force, $c_n$ (N/V$^2$)</td>
<td>$8.66 \times 10^{-8}$</td>
</tr>
<tr>
<td>Coefficient of circumferential driving force, $c_c$ (N/V$^2$)</td>
<td>$6.45 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

can be rewritten as

$$\frac{c_c}{c_n} \frac{V_{c,\text{max}}^2}{V_n^2 - V_{nc}^2} \leq \mu_s \left( 1 + \frac{k_c}{k_s} \right) \quad (6.36)$$

Characteristics based on the measured dimensions of the fabricated device are listed in Table 6.1. The stiffnesses of flexure $S$ and flexure combinations $N$, and $C$ were obtained by numerical calculation with considerations of the effects of tapered cross-sectional profiles due to DRIE as discussed in Chapter 5 and were verified by finite element analysis (with the ANSYS software). The values obtained with these two methods are in good consistency.

Based on Eq. (6.36), the relationship between a dimensionless voltage parameter, $c_c/V_{c,\text{max}}^2/c_n(V_n^2 - V_{nc}^2)$, and $k_c/k_s$, are plotted in Figure 6.9. The position of the curve varies with the change of the coefficient of static friction for different contact interfaces. For designs with similar bush-shaft structures and actuating mechanisms, the values of $c_c$, $c_n$, $k_c$, $k_s$ and will be determined for fabricated devices. After the values of $V_{c,\text{max}}^2$ and $V_n^2$ being chosen for actuation, an operating point reflecting the status of the experimental parameters can be plotted in the place of coordination in Figure 6.9. If the operating point is below the curve of a particular value of the coefficient of static friction, $\mu_s$, only pure rolling will occur. Conversely, stick-slip motion can be achieved if the point is above the corresponding curve. Thus, as the dimensionless voltage increase, the motion of the interface can experience a transition from pure rolling to rolling with...
some slipping.

![Figure 6.9 Diagram of the boundaries for pure rolling (occurring above a particular line for a given $\mu_s$).](image)

### 6.3.2 Tilting of the Sensing Bush

Since the sensing bush has a profile of a partial circular arc, tilting would occur when the contact point $P_c$ moves to the corner of the sensing bush. In such a case, the device would not be able to maintain the desired motion. Therefore, prevention of tilting of the sensing bush is necessary in MEMS devices with bush-shaft configurations.

![Figure 6.10 Schematic diagram of the dimension of the sensing bush.](image)

As shown in Figure 6.10, the distance between the contact point and the material point during motion, $\Delta$, is limited by the semi-width of the sensing bush, $D$. Hence, to avoid the tilting of the sensing bush, the following inequality must be satisfied.

$$|\Delta| \leq D$$  \hspace{1cm} (6.37)
Chapter 6  
Measurement and Analysis of the Rotary Friction Testing Device

For a sensing bush using a cantilever beam as the supporting flexure, combining Eqs. (6.37) and (6.26) gives the non-tilting oscillation as

\[
\frac{D}{L} \geq \frac{2 + 3 \frac{r_s}{L} + 6 \left(1 + \frac{r_s}{L}\right) \frac{H_b}{L} + 6 \left(\frac{H_b}{L}\right)^2 \frac{F_s}{N}}{3 + 6 \left(\frac{r_s}{L} + \frac{H_b}{L}\right)}
\]  

(6.38)

6.4 Friction Behaviors of the Rotary Device

The coefficient of static friction, \(\mu_s\), defined as \(\mu_s = \frac{f_{sm}}{N}\) is actually the \textit{engineering} coefficient of static friction because the contribution of the interfacial adhesion force is not included in the normal load \(N\) [55]. Under experimental conditions simpler to those in the present study, the adhesion force affecting the normal load is usually due to the van der Waals forces (all testing environments) and the capillary force (mainly in humid atmosphere) [122, 123].

\[\text{Figure 6.11 Averaged forward and backward displacements of the sensing bush at instant } B \text{ for } V_{c,\text{max}} = 48.6 \text{ V.}\]

The averaged values of the displacement of the sensing bush at instant \(B, s_B\), in-
including both the increasing and the decreasing routines, are shown in Figure 6.11. Note that the maximum static friction force is given by \( f_{sm} = k_s s_B \), the forward and backward values are very close, indicating no hysteresis. From the measurement results for eight continuous cycles (only five cycles shown in Figure 6.11, the displacement of interest is found to be \( s_B = 2.00 \pm 0.11 \) \( \mu m \) (mean ± standard deviation). Based on the flexure stiffnesses (in Table 6.1) and electrostatic driving forces with a correction for the effects of the tapered cross sections as discussed in Chapter 5, the variations of the coefficient of static friction at the sidewall surfaces, \( \mu_s \), under both increasing and decreasing normal loads (forward and backward friction traces) were obtained (Figure 6.12). The error bar of each point indicates one standard deviation above and below the corresponding average value.

**Figure 6.12** Averaged coefficients of static friction for \( V_{c_{\text{max}}} = 48.6 \) V (solid line for the fitting curve under increasing normal load and dashed line for the fitting curve under decreasing normal load).

The nonlinear variation of the coefficient of static friction, \( \mu_s \), as observed from Figure 6.12, may be explained by the effects of adhesion between the contacting surfaces. The static friction is generated by the tangential action of the interface under both the
normal load and the adhesion force. Thus

$$\mu_s = \mu_t + \mu_t N_{ad}/N$$  \hfill (6.39)$$

where \(\mu_t\) is the true coefficient of static friction, and \(N_{ad}\) is the adhesion force. For a given (external) normal load, \(N\), the adhesion due to the inter-molecular forces is equivalent to an additional load. Thus, a high coefficient of static friction arises for a very low normal load, for which the inter-molecular forces become dominant. The value of \(\mu_s\) would become infinite if \(N\) approached zero. However, with increasing normal load, the inter-molecular forces gradually become secondary, and \(\mu_s\) asymptotically approaches a constant value, as shown in Figure 6.13.

![Figure 6.13 Averaged adhesion force for \(V_{c,\text{max}} = 48.6\, \text{V}\) (solid line for fitting curve under increasing normal loads and dashed line for fitting curve under decreasing normal loads).](image_url)

The measured coefficient of static friction exhibits a nonlinear dependence on the normal load that can be described by

$$\mu_s = A_s N^{-B_s} + \mu_t$$  \hfill (6.40)$$

where \(A_s\) and \(B_s\) are constants [55]. Fitting the curves in Figure 6.12 yields \(\mu_t = 0.64\), \(\mu_s = 0.64\).
Chapter 6  

**Measurement and Analysis of the Rotary Friction Testing Device**

$A_s = 2.92$, and $B_s = 0.92$. Considering Eq. (6.39), a value of $B_s$ slightly below unity indicates that the adhesion force, $N_{ad}$, increases slightly with the normal load $N$, in consistency with the previous results for nominally flat surfaces [55].

### 6.5 Summary

In this chapter, friction behavior at the curved silicon sidewall surfaces was studied by using the developed rotary MEMS devices. The characteristics of the friction at the contacting sidewall surfaces were analyzed by applying a previously developed quasi-static stick-slip model to an equivalent configuration and the coefficient of static friction was found under different operating conditions. General formulations fit two different analysis models were developed to determine the equivalent tangential stiffness of the bush-flexure assembly. Based on the equivalent tangential stiffness, the contact-affected region of the bush surface was predicted, and the condition for maintaining a pure rolling of a bush-shaft interface was determined.
Chapter 6  
*Measurement and Analysis of the Rotary Friction Testing Device*
Chapter 7

Conclusions and Future Work

This final chapter summarizes the contributions in this project. To further improve and utilize the developed testing devices, discussions of the possible future work can be done are also given.

7.1 Conclusions

Aimed at studying of friction behaviors at micro-machined sidewall surfaces of MEMS devices, two types of MEMS devices, linear and rotary friction testing devices, have been developed in this project. Using silicon-on-glass (SOG) wafers, prototypes of those MEMS testing devices were fabricated via bulk micro-machining techniques.

For the linear friction testing devices, a dual-bump design was adopted to ensure that friction and wear take place at the same regions on the bumps in every cycle because the orientation of the flat sensing plate surface is uniquely determined by its engagement with the two bumps measurement of the friction and wear. The coefficients of static and kinetic friction determined with experimentally obtained flexure stiffnesses are insensitive to the errors in the calculation of the electrostatic forces of the comb-drive actuators. Based on the testing of the fabricated linear friction testing device, the following conclusions can be drawn:
Chapter 7

Conclusions and Future Work

(1) A quasi-static stick-slip model was developed to predict the motion of the sensing part of the friction testing devices.

(2) The measured displacements of the driving and sensing plates versus time are in good agreement with the trend predicted by the quasi-static stick-slip model.

(3) Both the coefficient of static friction and the coefficient of kinetic friction can be determined from the measured displacement data of the driving and sensing plates by using the quasi-static stick-slip model.

(4) Based on the present scheme, the coefficients of static and kinetic friction determined with experimentally obtained flexure stiffnesses are insensitive to the errors in the calculation of the electrostatic forces of the comb-drive actuators.

(5) The observed average coefficients of static friction for the devices fabricated from the 150-μm- and 300-μm-thick single-crystal silicon wafers are in the range of 0.73-0.75.

(6) A contact analysis indicates that the silicon sidewall contact interface is in an elastic regime at the scale of the designed silicon bumps and the surface scallops (waviness) caused by the DRIE process. Thus, the observed friction phenomena are not due to the shear of a plastic junction at this scale, but should be contributed to the finer structure interactions, including those at the asperity level.

(7) Based on the quasi-static stick-slip model, a saturation phenomenon of the kinetic friction at the silicon sidewall surfaces was observed while the normal load was increased.

For the rotary friction testing devices, it is a novel design to study friction behavior occurred at curved sidewall surfaces. Due to the generality in the basic assumptions, the quasi-static slip-slip model is also applicable to describe the motion of the rotary
friction testing device. With obtained general analysis for bush-shaft like structures, equivalent stiffness for the sensing bush of this type of device has been derived and it can easily be apply to different of flexure. Based on the testing results of the fabricated prototypes, conclusions for the rotary friction testing devices can be drawn as:

(1) The measured tangential displacements of the driving ring and the sensing bush versus time are in good agreement with the trends predicted by the quasi-static stick-slip model.

(2) A simple bush-flat model has been developed to explain the basic concepts and strategy in treating the motion of the rotary testing device. As an expansion of the bush-flat model, the bush-shaft model reduces the two-dimensional motion into a one-dimensional equivalent with considerations of the rotations of both the driving ring and the sensing bush.

(3) The motion occurring at the bush-shaft interface can be easily determined by the stiffnesses of the flexures of the driving and sensing parts, a dimensionless voltage parameter, and the coefficient of static friction of the contact interface.

(4) The measured coefficient of static friction exhibits a non-linear dependence on the normal load. This non-linear variation can be attributed to the presence of adhesion at the contact interface.

(5) The true coefficient of static friction, $\mu_t$, represented by an assumption value of the average coefficient of static friction, was found to be 0.64 for the tested silicon sidewall surfaces.

### 7.2 Future Work

The developed friction testing devices in the present research provide substantial means to understand the characteristics of friction as well as wear at sidewall surfaces of...
MEMS devices. However, there are yet many technical challenges to be solved before these friction testing devices can be introduced in industrial applications. Based on the contributions of this project, more research can be carried out in the future work as further improvement of the present research.

7.2.1 Improvement of Calibration

In the present research, the value of the stiffness of the supporting flexures adopted in both linear and rotary testing devices were obtained via finite element analysis. Those values were calibrated were performed under the condition of no contact between the driving and the sensing components of those testing devices. In the future, a direction measuring of the stiffness of the flexures can be carried out for a more accurate calibration.

The calibration setup is shown in Figure 7.1(a). In this setup, a probe is fixed on a screw micrometer stage which provides the probe a linear displacement. The connection between the the probe and the stage is considered to be rigid. The stage is firstly manipulated to ensure a initial contact between the tip of the probe and the end of the sensing part, as shown in Figure 7.1(b). Then, a distance $x_1$ along the tangential direction is applied to the probe. In the mean time, the corresponding tangential displacement of the sensing part, $x_2$, is measured via the microscope. The corresponding deformation of the tip of the probe is then found to be $x_1 - x_2$. According to Newton’s third law, the restoring force of the probe is equal to that of the sensing part to be calibrated, i.e.,

$$k_p (x_1 - x_2) = k_s x_2$$  \hspace{1cm} (7.1)

where $k_p$ and $k_s$ are the stiffnesses of the probe and the sensing part in the tangential direction, respectively. Noting that the value of $k_p$ can be easily found out, the value of $k_s$ is then obtained.
Chapter 7

Conclusions and Future Work

Figure 7.1 The principle of the calibration of the flexure stiffness: (a) setup of the calibration, (b) schematic diagram of the probe and the flexure (zoomed view of the sensing bush of the rotary testing device.)
7.2.2 Improvement of Measurement

In the present research, all the measurements were carried out based on off-line image processing of captured static images and motion videos. The measurement resolution is limited by the value of the workable magnification of the optical microscope connected to the video camera. This workable magnification must ensure that both a moving part and an anchored part (for reference) could be seen in the same image. Thus, it is possible to improve the displacement resolution and, hence, the force resolution by designing an additional anchored part very close to the moving part such that both parts can be captured by the video camera using a higher magnification of the microscope.

Besides the limitation of the measurement resolution, measuring errors might be also brought in as the threshold of gray scale was chosen based on human experience. Thus, an embedded sensing mechanism would improve the accuracy of the measurement result. As shown in Figure 7.2(a), doped piezoresistance can be adopted as the embedded sensors in the future work.

Piezoresistance is defined as the fractional change in bulk resistivity induced by small mechanical stresses applied to a material. The resistance change can be calculated as a function of the stress of a membrane or a cantilever beam. The contribution to resistance from stresses, longitudinal, $\sigma_l$ or transverse, $\sigma_t$, with respect to the current flow is given by [58]

$$\frac{\Delta R}{R} = \sigma_l \pi_l + \sigma_t \pi_t$$  \hspace{1cm} (7.2)

where $\pi_l$ is the longitudinal piezoresistance coefficient, and $\pi_t$ is the transversal piezoresistance coefficient. To convert the piezoresistive effect into a measurable signal, a Wheatstone bridge will be needed in the future work, as shown in Figure 7.2(b). In this bridge, $R_1$ and $R_3$ are the two piezoresistive sensors while $R_2$ and $R_4$ are two resistances, or compensators, for temperature compensation. After setting $R_1 = R_2 =$
Figure 7.2 Schematic diagram of a structure with embedded piezoresistive sensors: (a) zoomed piezoresistive sensors for the linear testing device; (b) zoomed piezoresistive sensor for the rotary testing devices; (c) the working principle of a Wheatstone bridge.
Chapter 7  

Conclusions and Future Work

\[ R_3 = R_4 = R, \] the output voltage is found out to be

\[ V_{out} = \frac{\Delta R}{2R} V_{in} \]  \hspace{1cm} (7.3)

Therefore, with recording the variation of the value of \( V_{out} \) during the sliding of the sensing and driving parts of the testing devices, an on-line measurement system of the friction can then be built.

**Figure 7.3** Schematic diagram of the concept of time window for on-line measurement.

To handle the massive data during this kind of on-line measurement, an idea using "time windows" to record the specified data can be adopted. As shown in Figure 7.3, a "time windows" is a recording period during the measurement. In each time window, only the maximum and average data will be recorded. After continuous running the testing devices for a lone period, the trend of the friction behavior at the contacted sidewall surfaces can then be found out. With this kind of long period running results, the research areas of the linear and rotary friction testing devices can be extended to reliability test, failure analysis and control, etc.

### 7.2.3 Expansion of Research Objectives

In this project, study of friction and wear at sidewall surfaces were focused on single crystal silicon by using the developed MEMS devices. In the future, research work for
different aspects can be carried out based on the same friction testing devices.

For MEMS devices with sliding interfaces, lifespan of the contact interface of a MEMS friction testing device under different working environments are valuable data to improve the performance of those devices. For the linear and rotary friction testing devices developed in this project, life spans can be first obtained the by making the driving parts continuously sliding against the sensing parts. Moreover, the failed surfaces of the contact components can be studied by atomic force microscopy (AFM) or other microscopies to verify the range of wear areas predicted by the models.

As an expansion of the working scope of the friction testing devices developed in this project, tribological characteristic of other of materials can be studied by deposition corresponding materials on the sidewall surfaces in the future work. Moreover, by using other instruments, such as AFM, Raman spectroscopy, etc., specifics of wear and material conversion can also be studied based on the developed MEMS devices in this project.

Furthermore, adding one more sensing plate for the normal displacement to the linear friction testing device, adhesion force at the contact interface can then be mea-
surable. The basic idea of the structure for measuring adhesion is shown in Figure 7.4. During the operation of the testing device, the adhesion force may work as a negative normal force which can be reflected by the displacement of the added sensing plate.

7.3 Summary

This project focuses on friction of micro-machined sidewall surfaces, both linear and curved, for MEMS devices. Tribological behaviors relating to both the fundamental origins of friction, and the application of MEMS have been described. This project has achieved the objectives listed in Chapter 1. With the expansions listed in the section of Future Work, this research can be further extended to meet the needs of understanding the characteristics of both fiction and wear at sidewall surfaces of MEMS devices.
Appendix A

Analysis of Basic Folded Flexure

For a basic folded flexure, it can be treated as a multi-segment structure and each beam segment can be treated as an one-end fixed and one-end free cantilever beam, as shown in Figure 3.7.

For segment 12, the bending effect under the exerted loads is shown in Figure 3.8. Based on the force and moment equilibrium conditions, relations are found to be

\[ \sum F(x) = 0 : F_{x2} = F_x \] \hspace{0.5cm} (A.1)

\[ \sum F(y) = 0 : F_{y2} = F_y \] \hspace{0.5cm} (A.2)

\[ \sum M(2) = 0 : (M - bF_x + 2aF_y - M_2) = 0 \] \hspace{0.5cm} (A.3)

As derived in Chapter 3, displacements and tilting angle of segment 12 are given by Eqs. (3.44), (3.45), and (3.46).

Similarly, the bending effect of segment 23 is shown in Figure A.1. Based on the force and moment equilibrium conditions, relations are found to be

\[ \sum F(x) = 0 : F_{x3} = F_x \] \hspace{0.5cm} (A.4)
Appendix A

Analysis of Basic Folded Flexure

Figure A.1 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment 23 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment 23 with deflection.

\[ \sum F(y) = 0 : F_{y3} = F_y \]  \hspace{1cm} (A.5)

\[ \sum M(3) = 0 : (M - bF_x + aF_y - M_3) = 0 \]  \hspace{1cm} (A.6)

Combining the foregoing relations, the moment on node 3 is found to be

\[ M_3 = M - bF_x + aF_y \]  \hspace{1cm} (A.7)

As mentioned before, segment 23 is treated as rigid. Thus, the relative displacements of segment 23 based on beam bending theory are given by

\[ u_{3r} = 0 \]  \hspace{1cm} (A.8)

\[ v_{3r} = 0 \]  \hspace{1cm} (A.9)

\[ \theta_{3r} = 0 \]  \hspace{1cm} (A.10)
Appendix A

Analysis of Basic Folded Flexure

The rigid-body displacement of segment 23 with noting beam tilting is given by

\[ u_{3b} = u_2 \quad (A.11) \]

\[ v_{3b} = v_2 + a\theta_2 \quad (A.12) \]

Combining Eqs. (3.44), (3.45), (A.11) and (A.12), the rigid body displacements of node 3 are found to be

\[ u_{3b} = \frac{3b^2M - b^3F_x + 6ab^2F_y}{6EI} \quad (A.13) \]

\[ v_{3b} = \frac{6abM - 3ab^2F_x + 12a^2bF_y}{6EI} \quad (A.14) \]

Combining Eqs. (A.8) and (A.13) with noting \( u_3 = u_{3b} + u_{3r} \), the lateral displacement of segment 23 is found to be

\[ u_3 = \frac{3b^2M - b^3F_x + 6ab^2F_y}{6EI} \quad (A.15) \]

Combining Eqs. (A.9) and (A.14) with noting \( v_3 = v_{3b} + v_{3r} \), the transverse displacement of segment 23 is

\[ v_3 = \frac{6abM - 3ab^2F_x + 12a^2bF_y}{6EI} \quad (A.16) \]

Based on Eq. (A.10) with noting \( \theta_3 = \theta_2 + \theta_{3r} \), the tilting angle of segment 23 is found to be

\[ \theta_3 = \frac{2bM - b^2F_x + 4abF_y}{2EI} \quad (A.17) \]

For segment 34, free-body diagrams of beam bending effect are shown in Figure A.2. Based on the force and moment equilibrium conditions, relations are found to be

\[ \sum F(x) = 0 : F_{z4} = F_x \quad (A.18) \]
Figure A.2 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment 34 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment 34 with deflection.

\[ \sum F(y) = 0 : F_{y4} = F_y \]  
\[ \sum M(4) = 0 : (M - bF_x + aF_y - M_4) = 0 \]

Combining the foregoing relations, the moment on node 4 is found to be

\[ M_4 = M + bF_x + aF_y \]

The relative displacements of segment 34 based on beam bending theory are given by

\[ u_{4r} = -\frac{4b^2M_4}{2EI} + \frac{8b^3F_{x4}}{3EI} \]  
\[ v_{4r} = 0 \]  
\[ \theta_{4r} = \frac{2bM_4}{EI} - \frac{4b^2F_{x4}}{2EI} \]

Combining Eqs. (A.18), (A.21), (A.22), (A.23) and (A.24), the relative displacements
Appendix A

Analysis of Basic Folded Flexure

of node 4 are found to be

\[
u_{4r} = \frac{-12b^3M + 4b^3F_x - 12ab^2F_y}{6EI}\]  (A.25)

\[
\theta_{4r} = \frac{4bM + 4abF_y}{2EI}\]  (A.26)

The rigid-body displacement of segment 34 with noting beam tilting is given by

\[
u_{4b} = u_3 - 2b\theta_3\]  (A.27)

\[
v_{4b} = v_3\]  (A.28)

Combining Eqs. (A.15), (A.16), (A.27) and (A.28), the rigid body displacements of node 4 are found to be

\[
u_{4b} = \frac{-9b^2M + 5b^3F_x - 18ab^2F_y}{6EI}\]  (A.29)

\[
v_{4b} = \frac{6abM - 3ab^2F_x + 12a^2bF_y}{6EI}\]  (A.30)

Combining Eqs. (A.25) and (A.29) with noting \(u_4 = u_{4b} + u_{4r}\), the lateral displacement of segment 34 is found to be

\[
u_4 = \frac{-21b^2M + 9b^3F_x - 30ab^2F_y}{6EI}\]  (A.31)

Combining Eqs. (A.23) and (A.30) with noting \(v_4 = v_{4b} + v_{4r}\), the transverse displacement of segment 34 is

\[
v_4 = \frac{6abM - 3ab^2F_x + 12a^2bF_y}{6EI}\]  (A.32)

Based on Eq. (A.26) with noting \(\theta_4 = \theta_3 + \theta_{4r}\), the tilting angle of segment 34 is found
Appendix A

Analysis of Basic Folded Flexure

**Figure A.3** Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment 45 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment 45 with deflection.

\[ \theta_4 = \frac{6bM - b^2 F_x + 8abF_y}{2EI} \]  \hspace{1cm} (A.33)

For segment 45, free-body diagrams of beam bending effect are shown in Figure A.3. Based on the force and moment equilibrium conditions, relations are found to be

\[ \sum F(x) = 0 : F_{x5} = F_x \]  \hspace{1cm} (A.34)

\[ \sum F(y) = 0 : F_{y5} = F_y \]  \hspace{1cm} (A.35)

\[ \sum M(5) = 0 : (M - bF_x) - M_5 = 0 \]  \hspace{1cm} (A.36)

Combining the foregoing relations, the moment on node 4 is found to be

\[ M_5 = M - bF_x \]  \hspace{1cm} (A.37)

Noting that segment 45 is a rigid beam, relative displacements of segment 45 are given by

\[ u_{5r} = 0 \]  \hspace{1cm} (A.38)
Appendix A

Analysis of Basic Folded Flexure

\[ v_{5r} = 0 \]  \hspace{1cm} (A.39)

\[ \theta_{5r} = 0 \]  \hspace{1cm} (A.40)

and rigid-body displacements of segment 45 with noting beam tilting are given by

\[ u_{5b} = u_4 \]  \hspace{1cm} (A.41)

\[ v_{5b} = v_4 + a\theta_4 \]  \hspace{1cm} (A.42)

Combining Eqs. (A.31), (A.32), (A.41) and (A.42), the rigid body displacements of node 5 are found to be

\[ u_{5b} = \frac{-21b^2M + 9b^3F_x - 30ab^2F_y}{6EI} \]  \hspace{1cm} (A.43)

\[ v_{5b} = \frac{24abM - 6ab^2F_x + 36a^2bF_y}{6EI} \]  \hspace{1cm} (A.44)

Combining Eqs. (A.38) and (A.43) with noting \( u_5 = u_{5b} + u_{5r} \), the lateral displacement of segment 45 is found to be

\[ u_5 = \frac{-21b^2M + 9b^3F_x - 30ab^2F_y}{6EI} \]  \hspace{1cm} (A.45)

Combining Eqs. (A.39) and (A.44) with noting \( v_5 = v_{5b} + v_{5r} \), the transverse displacement of segment 45 is

\[ v_5 = \frac{24abM - 6ab^2F_x + 36a^2bF_y}{6EI} \]  \hspace{1cm} (A.46)

Based on Eq. (A.40) with noting \( \theta_5 = \theta_4 + \theta_{5r} \), the tilting angle of segment 45 is found to be

\[ \theta_5 = \frac{6bM - b^2F_x + 8abF_y}{2EI} \]  \hspace{1cm} (A.47)
Appendix A

Analysis of Basic Folded Flexure

Figure A.4 Schematic of the bending effect of the basic folded flexure: (a) free-body diagram of segment 56 and the rest of beam segments of the basic folded flexure; (b) diagram of displacement of segment 56 with deflection.

For segment 56, free-body diagrams of beam bending effect are shown in Figure A.4. Thus, relative displacements of segment 56 based on beam bending theories are given by

\[ u_{6r} = \frac{3b^2 M + 2b^3 F_x}{6EI} \]  
\[ v_{6r} = 0 \]  
\[ \theta_{6r} = \frac{2bM + bF_x}{2EI} \]  

The rigid-body displacements of segment 45 with noting beam tilting are given by

\[ u_{6b} = u_5 + b\theta_5 \]  
\[ v_{6b} = v_5 \]  

Combining Eqs. (A.45), (A.46), (A.41) and (A.42), the rigid body displacements of node 6 are found to be

\[ u_{6b} = \frac{-3b^2 M + 6 b^3 F_x - 6ab^2 F_y}{6EI} \]  
\[ v_{6b} = \frac{24abM - 6ab^2 F_x + 36a^2 b F_y}{6EI} \]
Appendix A

Analysis of Basic Folded Flexure

Combining Eqs. (A.48) and (A.53) with noting \( u_6 = u_{6b} + u_{6r} \), the lateral displacement of segment56 is found to be

\[
  u_6 = \frac{8b^3F_x - 6ab^2F_y}{6EI} \quad (A.55)
\]

Combining Eqs. (A.49) and (A.54) with noting \( v_6 = v_{6b} + v_{6r} \), the transverse displacement of segment56 is

\[
  v_6 = \frac{24abM - 6ab^2F_x + 36a^2bF_y}{6EI} \quad (A.56)
\]

Based on Eq. (A.50) with noting \( \theta_6 = \theta_5 + \theta_{6r} \), the tilting angle of segment56 is found to be

\[
  \theta_6 = \frac{8bM + 8abF_y}{2EI} \quad (A.57)
\]
Appendix B

Analysis of Three-Turn Folded Flexure

For a three-turn folded flexure, as shown in Figure 3.11, three basic (complete-cycle) flexure units and a half-cycle flexure unit are included. As the same as basic flexure units, the vertical segments with a length $b$ were treated as bending beams, and the short horizontal segments with a length, $a$, were treated as rigid connecting beams in a half-cycle unit.

Free-body diagrams of beam bending effect the half-cycle unit is shown in Figure B.1. Noting the similarity of the structure of a half-cycle flexure unit with that of a basic flexure unit, similar analysis based on beam bending theories can be carried out on the half-cycle flexure unit. Substitution of beam lengths of the half-cycle unit in Eqs. (A.31), (A.32) and (A.33), the displacements of a half-cycle unit are given by

$$u_h = \frac{-6b^2M + 4b^3F_x - 3ab^2F_y}{6EI}$$  \hspace{1cm} (B.1)

$$v_h = \frac{6abM - 3ab^2F_x + 6a^2bF_y}{6EI}$$  \hspace{1cm} (B.2)

and

$$\theta_h = \frac{4bM - 2b^2F_x + 2abF_y}{2EI}$$  \hspace{1cm} (B.3)
Appendix B

Analysis of Three-Turn Folded Flexure

Figure B.1 Schematic of a half-cycle flexure unit: (a) the half-cycle flexure unit before bending; (b) deformation of a half-cycle flexure unit (dashed line is the beam with deflection).

Figure B.2 Free body diagram of unit 1 of a three-turn folded flexure.

respectively.

In a three-turn folded flexure, each flexure unit can be treated as a rigid segment in a flexure. Thus, add-up of rigid body displacement, the same as the beam analysis for a folded flexure, of each unit can then be carried out. For unit 1, the free body diagram under exerted loads is shown in Figure B.2. Based on the force and moment equilibrium conditions, relations are found to be

\[ \sum F(x) = 0 : F_{xa} = F_x \]  

\[ \sum F(y) = 0 : F_{ya} = F_y \]  

\[ \sum M(6\alpha) = 0 : (M + 5\alpha F_y - M_a) = 0 \]
Thus, $F_{xa} = F_x$, $F_{ya} = F_y$ and $M_a = M + 5aF_y$. Substitution of the foregoing relations into the displacements of a basic flexure unit given by Eqs. (3.47), (3.48) and (3.49) gives the relative displacements of unit 1 as

$$
\begin{align*}
\nu_{ua} &= \frac{8b^3F_x - 6ab^2F_y}{6EI} \\
\theta_{ua} &= \frac{4abM - ab^2F_x + 26a^2bF_y}{EI} \\
\end{align*}
$$

(B.7)

For unit 2, the free body diagram under exerted loads is shown in Figure B.3. Based on the force and moment equilibrium conditions, relations are found to be

$$
\begin{align*}
\sum F(x) &= 0 : F_{xa} = F_x \\
\sum F(y) &= 0 : F_{ya} = F_y \\
\sum M(6b) &= 0 : (M + 3aF_y - M_b) = 0
\end{align*}
$$

(B.8) (B.9) (B.10)

Thus, $F_{xb} = F_x$, $F_{yb} = F_y$ and $M_b = M + 3aF_y$. Substitution of the foregoing relations into the displacements of a basic flexure unit given by Eqs. (3.47), (3.48) and (3.49)
Appendix B

Analysis of Three-Turn Folded Flexure

Figure B.4 Free body diagram of unit 3 of a three-turn folded flexure.

gives the relative displacements of unit 2 as

\[
\begin{align*}
    u_{ub} &= \frac{8b^3 F_x - 6ab^2 F_y}{6EI} \\
    v_{ub} &= \frac{4abM - ab^2 F_x + 18a^2 b F_y}{EI} \\
    \theta_{ub} &= \frac{4bM + 16abF_y}{EI}
\end{align*}
\]  

(B.11)

For unit 3, the free body diagram under exerted loads is shown in Figure B.4. Based on the force and moment equilibrium conditions, relations are found to be

\[
\sum F(x) = 0 : F_{xc} = F_x \quad \text{(B.12)}
\]

\[
\sum F(y) = 0 : F_{yc} = F_y \quad \text{(B.13)}
\]

\[
\sum M(6c) = 0 : (M + aF_y - M_b) = 0 \quad \text{(B.14)}
\]

Thus, \( F_{xc} = F_x, F_{yc} = F_y \) and \( M_c = M + aF_y \). Substitution of the foregoing relations into the displacements of a basic flexure unit given by Eqs. (3.47), (3.48) and (3.49)
Appendix B

Analysis of Three-Turn Folded Flexure

gives the relative displacements of unit 3 as

\[
\begin{align*}
    u_{uc} &= \frac{8b^3 F_x - 6ab^2 F_y}{6EI} \\
    v_{uc} &= \frac{4ab M - ab^2 F_x + 10a^2 b F_y}{EI} \\
    \theta_{uc} &= \frac{4b M + 8ab F_y}{EI}
\end{align*}
\]  

(B.15)

For the half-cycle unit, the force and moment equilibrium conditions are the same as that shown in Figure B.1. Thus, the relative displacements of the half-cycle unit are given by Eqs. (B.1), (B.2) and (B.3), respectively. Accumulation of these displacement based on rigid body translation gives the displacement of a three-turn folded flexure

\[
\begin{align*}
    u_{eq} &= u_{ua} + u_{ub} + u_{uc} + u_h \\
    v_{eq} &= 5\alpha \theta_{ua} + 3a \theta_{ub} + a \theta_{uc} + v_{ua} + v_{ub} + v_{uc} + v_h \\
    \theta_{eq} &= \theta_{ua} + \theta_{ub} + \theta_{uc} + \theta_h
\end{align*}
\]  

(B.16)

Substituting Eqs. (B.7) to (B.15) and (B.1) to (B.3) into the above relations with noting \( \theta_{eq} = 0 \) due to the constrains of the end of the flexure, the lateral displacement of the three-turn folded flexure is found to be

\[
u_t = \frac{193b^3}{42EI} F_x
\]  

(B.17)

Thus, the equivalent stiffness of this flexure is found to be

\[
k_t = \frac{42EI}{193b^3}
\]  

(B.18)
Appendix C

Details of Micro-Fabrication Processes

C.1 Photolithography

C.1.1 Photoresist AZ7220 (Clariant)

- Spinning: 500 rpm for 10 seconds (uniforming), 3000 rpm for 45 seconds (thinning)

- Photoresist thickness: 2.2 μm

- Baking: 100 °C 90 seconds for pre-exposure baking, 110 °C 1 minute for post-baking, 120 °C 30 minute for hard-baking (optional)

- Aligner machine: Karl Suss MA6

- Contact type: soft contact

- Exposure time: 3.4 seconds

- Development: 50 sec in MIF 300 developer, then rinse in DI water and spin-dry
Appendix C  

Details of Micro-Fabrication Processes

C.1.2  Photoresist type: AZ9260 (Clariant)

- Spinning: 500 rpm for 10 seconds (uniforming), 3000 rpm for 45 seconds (thinning)

- Photoresist thickness: 8 μm

- Baking: 4 minutes for pre-exposure baking at 110 °C, no post-baking, 30 minutes hard baking (optional) at 120 °C

- Aligner machine: Karl Suss MA6

- Contact type: soft contact

- Exposure time: 20 seconds

- Development: 60–70 sec in AZ 400K developer, then rinse in DI water and spin-dry

C.2  Deposition and patterning the electrodes

- Machine: DC/RF sputtering system

- Deposition pressure: 2 mtorr

- Gas: Ar 20 sccm

- Deposition temperature: room temperature (25 °C)

- Deposition power: DC 200 W

- Deposition rate: 10 nm/min for Cr, 25 nm/min for Au, 10 nm/min for Ti, 15 nm/min for Pt
Appendix C

Details of Micro-Fabrication Processes

C.3 Eutectic bonding of silicon to glass wafers

- Machine: Karl Suss WB6 (and BA6 for pre-alignment)
- Temperature: 380 °C
- Pressure: 2000 mbar
- Bonding time: 15 minutes
- Bonding yield: 85% bonded, but with a few distinct bubbles

C.4 Anodic bonding of silicon to glass wafers

- Machine: Karl Suss WB6 (and BA6 for pre-alignment)
- Temperature: 360–400 °C
- Pressure: 2000 mbar
- Voltage: 2000 V for center bond 2 minutes, 1000 V for area bond 10 minutes
- Bonding time: 12 min
- Bonding yield: 99% bonded, no distinct bubbles observed

C.5 DRIE of Silicon

- Machine: STS ICP Multiplex system
- Gas: SF₆ 100 sccm, O₂ 13 sccm, C₄F₈ 130 sccm
- Cycle time: etching 14 sec and passivation 8 sec (starting with passivation) or etching 8 seconds and passivation 5 seconds (starting with passivation)
- Pressure: 5 mtorr
Appendix C

Details of Micro-Fabrication Processes

- Power: Coil power 800 W, platen power 10–12 W, multiple 10

- Etch rate: 2–3 μm for 14s/8s (etching/passivation time), 1.4–1.8 μm for 8s/5s (etching/passivation time), which is dependent on the exposed area and feature size
Appendix D

List of Publications


References


References


References


References


References


References


References


References
