Flexible Control of Transmission and Distribution Networks through Applications of Power Electronics Technology

Wang Qi

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University in FULFILMENT of the requirement for the degree of Doctor of Philosophy

2007
I am greatly indebted to my supervisor, Prof. Choi San Shing, for his knowledge, instruction, dedication and exemplary academic standards. He always provided thoughtful guidance that contributed a lot to this thesis. I am deeply impressed by his rigorous attitude to do research that will have effect on my future research and work.

The research would not be possible without the financial support provided by Nanyang Technological University in the form of a research scholarship. For this I am most grateful.

My special thanks go to Associate Professor M. H. Haque, Assistant Professor D. M. Zhang, and Assistant Professor S. Rajakaruna, for their invaluable advice and help throughout my research.

Part of my capabilities and thinking process are due to my excellent graduate education at Nanyang Technological University. I would like to thank all my teachers, and especially Associate Professor Y. Y. Wang, Associate Professor H. B. Gooi, Associate Professor P. L. So, Associate Professor P. Wang, Assistant Professor L. P. Chiang, and Assistant Professor E. K. K. Sng.

I greatly appreciate Professor, Lalit Kumar Goel, for his concerns, particular suggestion and prompt help.

I am also deeply thankful to Ms Chew-Sim Annie, Mr. Yeoh Tiow Koon, and Mr. Lee Ting Yeng for the technical support they have given me in the course of my research.

Last but not at least, to my husband, Long Fei, for his unfailing support, patience and cooperation that have made the completion of the research work possible. And also my parents, brother and sister for their invaluable support and understanding over these three years of my graduate program.
Summary

The fast-changing re-structured electricity industry brings with it fresh opportunities as well as challenges, of which system stability and supply quality have received increasing attention. In an attempt to address these two issues, this thesis describes the results of an investigation into the flexible control of transmission and distribution networks through applications of power electronics technology. By flexible control, the structures of networks, the functions and the relationships between the various controllers are allowed to change depending on prevailing system conditions.

This thesis begins by considering a coordinated hierarchical control scheme for a radial transmission system. The proposed control scheme is designed to improve system voltage stability as well as to manage the reactive power resources in an equitable way. A method to determine the control parameters of the primary and the secondary controllers is developed, using the well-known frequency response technique. Practical realization of the scheme is through power electronics-based static reactive power compensators.

The beneficial effect of shunt reactive power compensation for long-distance radial transmission is also re-examined from the theoretical basis. The analysis, based on the \( \pi \)-circuit transmission line model which takes into account line losses and capacitance, includes source impedances in the network. Analytical results governing the real power flows and voltage angle between the two end-sources are derived. A computational procedure is then proposed to determine the amount of the reactive power needed at an intermediate in the line, for any given power transfer level. Steady-state stability limit improvement of the transmission system through mid-point shunt compensation is then evaluated using the analytical expressions obtained. The analysis demonstrates that due to the presence of line resistances and the distributed nature of the line shunt capacitances, the maximum steady-state power transfer level is typically 1.6 times that of the uncompensated line. This transfer improvement factor is significantly less than that commonly derived using simplified line model, in which the factor is predicted to be 2. Digital simulation studies verify the accuracy of the analysis.
Summary

The derived method also allows the optimal location of shunt compensator to be determined for a given radial transmission system, in which the aim is to achieve the highest possible power transfer and at reduced transmission angle. The proposed computational procedure extends the results to include the determination of the optimal multiple compensator locations for a general symmetric transmission system. Additional insight into the steady-state transfer limit of the compensated system is gained.

In extending the technique of shunt compensation, the application of both series and shunt compensation devices into a form similar to the Unified Power Flow Controller (UPFC) is also considered. Significantly, the unified controller described in the thesis incorporates an active power (AP) source. The AP source is in the form of a battery energy storage system (BESS) which is assumed to obtain its input energy from a renewable source. The design of the UPFC-AP is formulated as an optimization problem where the objective includes the minimization of the AP capacity or power transfer angle, or the maximization of the power transfer level. The design procedure also takes into account practical operating constraints of the network and that of the UPFC-AP. The concept of feasible operating regime (FOR) is then introduced. FOR prescribes the range of the AP output power that can be dispatched for given load demand, subject to the network operating within the specified constraints. With FOR and the application of a proposed economic dispatch strategy, the relationship between the minimum total generation cost and the load demand is established. The analysis forms the basis of an optimal load tracking scheme which leads to the economic dispatch of generation and minimum power angle across the transmission link.

Lead-acid battery has been considered as the suitable AP source in the UPFC-AP. A possible battery energy storage system (BESS) change-over strategy is shown in the thesis. The strategy determines when the change-over from the in-service BESS to the stand-by BESS should occur. Furthermore, a method has been derived to predict the scheduling of the battery bank change-over for a given load demand. In this way, it is possible to arrive at the suitable BESS capacity so that the overall power system can be operated under the economic dispatch condition. By varying the capacity of the BESS to suit the available
Summary

energy profile of the renewable source, one ensures that the UPFC-AP system will guarantee the transfer capability of the compensated system is maximized.

Based on the series-shunt compensation technique, the thesis also explores the design of flexible electric power distribution system through the concept of Power Quality Control Center (PQCC). A new series compensation technique is proposed as part of the PQCC scheme. The series compensator (SC) is fed from an AP source but with its instantaneous output power change constrained. The technique also takes into account the apparent power rating of the SC. Under a voltage sag/swell and through the proposed voltage injection strategy, the technique allows the compensation to be carried out at minimum energy level and thus offers an increased ride-through range for the compensated load under voltage sag/swell. Furthermore, the load ride-through ability is shown to be insensitive to voltage phase jump introduced during the disturbance. Analytical expressions pertaining to the injected voltage magnitude and phase angle are derived and from which the extent of the sag/swell that can be ridden through is readily determined. The PQCC scheme is seen to enhance the supply quality significantly, and requires only the addition of a SC of modest capacity.
# Table of Contents

Acknowledgements i  
Summary ii  
Table of Contents v  
List of Figures x  
List of Tables xv  

## Chapter 1  Introduction

1.1 Motivation 1  
1.2 Major Contributions of the Thesis 6  
1.3 Organization of the Thesis 10  

## Chapter 2  A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

2.1 Power System Stability 12  
2.2 Power Quality 14  
2.2.1 Some common disturbances 15  
2.2.2 CBEMA and ITIC curves 17  
2.3 Flexible AC Transmission Systems 19  
2.3.1 Basic operational principle of FACTS 19  
2.3.2 Main FACTS devices 20  
2.3.2.1 Shunt connected controllers 21  
2.3.2.2 Series connected controllers 23  
2.3.2.3 Combined shunt-series controllers 25  
2.3.2.4 Distributed FACTS 30  
2.4 Power Quality Enhancement Technology 30  
2.4.1 Distributed generation 30  
2.4.1.1 Types of distributed generators 31  
2.4.1.2 Energy storage 34
Table of Contents

2.4.2 Impacts of DGs on power system stability and power quality 38
2.5 Custom Power 39
  2.5.1 Compensating type 40
  2.5.2 Reconfiguration type 44
  2.5.3 Custom Power Park 44
  2.5.4 Flexible, Reliable and Intelligent Electric Energy Delivery System (FRIENDS) 45
  2.5.5 Power Quality Control Center 46
    2.5.5.1 UPS-type PQCC structure 47
2.6 Hierarchical Network Control 48
  2.6.1 A short review of hierarchical voltage control principles 50
    2.6.1.1 Pilot point selection 51
    2.6.1.2 Secondary voltage control law 52
    2.6.1.3 Improved secondary voltage control law 53
  2.6.2 A historical perspective of the hierarchical voltage control 54
2.7 Conclusions 58

Chapter 3 A Coordinated Hierarchical Voltage Control Scheme

3.1 Introduction 59
3.2 System Description 60
  3.2.1 Generator model 61
  3.2.2 Load model 62
  3.2.3 Static Var Compensator (SVC) 63
3.3 Frequency Response Technique 64
3.4 Coordinated Hierarchical Voltage Control Scheme 67
3.5 Analysis 71
  3.5.1 Network model derivation 71
  3.5.2 Design of Primary SVC Controller 73
  3.5.3 Design of Secondary Generator Controller (SGC) 75
3.6 Numerical Examples 77
  3.6.1 Design considerations 78
  3.6.2 Final controller design 84
### Table of Contents

3.6.3 Time response study | 85

3.7 Conclusions | 90

**Chapter 4  Shunt Reactive Power Compensation of Long Transmission Lines**

4.1 Introduction | 91

4.2 Compensation at Line Terminals | 93

4.2.1 Numerical examples | 97

4.3 Transmission Line $\pi$-Circuit Model | 99

4.4 Power-Angle Relationship | 101

4.5 Shunt Compensation | 103

4.6 Maximum Power Transfer | 110

4.6.1 General expressions of midpoint compensation | 111

4.7 Numerical Examples | 112

4.7.1 Effect of terminal impedances on power transfer | 113

4.7.2 Effect of line losses on steady-state stability limit | 113

4.7.3 General observation on the benefit of midpoint compensation | 114

4.7.4 Time-response simulation | 116

4.7.4.1 Dynamic working of the midpoint compensator | 116

4.7.4.2 Effect of line resistance on transmission system | 117

4.8 Optimal Location: Single Shunt Compensator | 118

4.9 Steady-State Stability | 123

4.10 Optimal Location: Multiple Shunt Compensators | 128

4.11 Conclusions | 133

**Chapter 5  Unified Power-Flow Controller Incorporated with an Active Power Source**

5.1 System Description | 137

5.2 Problem Formulation | 139

5.2.1 Mathematical model and preliminary analysis | 139

5.2.2 UPFC-AP design subject to system constraints | 143

5.3 UPFC-AP Optimal Steady-state Operation | 147
### Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3.1 Feasible Operating Regime</td>
<td>147</td>
</tr>
<tr>
<td>5.3.2 Optimal economic dispatch</td>
<td>150</td>
</tr>
<tr>
<td>5.3.3 An optimal load tracking scheme</td>
<td>153</td>
</tr>
<tr>
<td>5.4 Illustrative Examples</td>
<td>155</td>
</tr>
<tr>
<td>5.4.1 Example 1. Optimal load tracking</td>
<td>155</td>
</tr>
<tr>
<td>5.4.2 Example 2. Determination of the optimal capacity of the UPFC-AP</td>
<td>157</td>
</tr>
<tr>
<td>5.4.3 Example 3. Optimal load tracking</td>
<td>158</td>
</tr>
<tr>
<td>5.5 Conclusions</td>
<td>160</td>
</tr>
</tbody>
</table>

#### Chapter 6 Battery Energy Storage System in the UPFC-AP

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 Introduction</td>
<td>161</td>
</tr>
<tr>
<td>6.2 Electrochemical Reactions</td>
<td>162</td>
</tr>
<tr>
<td>6.3 Equivalent Circuit of Battery Bank</td>
<td>164</td>
</tr>
<tr>
<td>6.4 A Possible BESS Change-over Strategy</td>
<td>166</td>
</tr>
<tr>
<td>6.5 Determination of BESS Capacity</td>
<td>168</td>
</tr>
<tr>
<td>6.5.1 Load demand profile</td>
<td>169</td>
</tr>
<tr>
<td>6.5.2 Estimation of terminal voltage per cell</td>
<td>170</td>
</tr>
<tr>
<td>6.5.3 A computational procedure</td>
<td>171</td>
</tr>
<tr>
<td>6.6 Illustrative Examples</td>
<td>173</td>
</tr>
<tr>
<td>6.6.1 Example 1. BESS change-over strategy</td>
<td>173</td>
</tr>
<tr>
<td>6.6.2 Example 2. BESS scheduling strategy</td>
<td>176</td>
</tr>
<tr>
<td>6.7 Conclusions</td>
<td>179</td>
</tr>
</tbody>
</table>

#### Chapter 7 Series-Shunt Compensation Technique for Power Quality Enhancement

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 Introduction</td>
<td>181</td>
</tr>
<tr>
<td>7.2 Fuel Cell Distributed Generator Model</td>
<td>183</td>
</tr>
<tr>
<td>7.3 Some Basic Considerations</td>
<td>189</td>
</tr>
<tr>
<td>7.3.1 General description</td>
<td>189</td>
</tr>
<tr>
<td>7.3.2 Proposed operational scheme</td>
<td>190</td>
</tr>
<tr>
<td>7.3.2.1 Pre-disturbance condition</td>
<td>190</td>
</tr>
<tr>
<td>Table of Contents</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td></td>
</tr>
<tr>
<td>7.3.2.2 Under-disturbance condition</td>
<td>190</td>
</tr>
<tr>
<td>7.4 Compensation Strategy</td>
<td>192</td>
</tr>
<tr>
<td>7.4.1 Preliminary analysis</td>
<td>192</td>
</tr>
<tr>
<td>7.4.2 Maximum sag/swell ride-through</td>
<td>194</td>
</tr>
<tr>
<td>7.4.3 Energy-saving compensation strategy</td>
<td>197</td>
</tr>
<tr>
<td>7.4.4 An implementation scheme</td>
<td>201</td>
</tr>
<tr>
<td>7.5 Numerical Illustration</td>
<td>203</td>
</tr>
<tr>
<td>7.5.1 Load ride-through range</td>
<td>203</td>
</tr>
<tr>
<td>7.5.2 Time-domain simulation</td>
<td>207</td>
</tr>
<tr>
<td>7.6 Conclusions</td>
<td>210</td>
</tr>
</tbody>
</table>

Chapter 8 Conclusions and Recommendations

| 8.1 Conclusions | 212 |
| 8.2 Recommendations | 215 |

References

Appendices

| Appendix A | List of Symbols | 240 |
| Appendix B | Transfer Functions and System Coefficients in Chapter 3 | 244 |
| Appendix C | Derivation of Equation (3.5.5) | 253 |
| Appendix D | Detailed Data for the Studied Power System Used in Section 3.6 | 257 |
| Appendix E | Detailed Data for the Studied Power System Used in Section 4.7 | 259 |
| Appendix F | Derivation of Equations (5.2.6) - (5.2.14) | 262 |
| Appendix G | Derivation of $f(\delta_{sw})$ | 265 |
| Appendix H | Typical 100 kW SOFC Power Plant Experimental Data | 267 |
| Appendix I | Phasor Diagram Corresponding to the Conditions Described in Table 7-1 | 268 |

Vita

ix
### List of Figures

| Figure 2-1 | CBEMA curve [103] | 18 |
| Figure 2-2 | ITIC curve [104] | 19 |
| Figure 2-3 | A schematic diagram describing FACTS control actions | 20 |
| Figure 2-4 | Single-line diagram of a typical SVC | 22 |
| Figure 2-5 | V/I characteristic of SVC | 22 |
| Figure 2-6 | Single-line diagram of a typical STATCOM [55] | 23 |
| Figure 2-7 | Single-line diagram of a typical TCSC | 23 |
| Figure 2-8 | Single-line diagram of a typical SSSC | 24 |
| Figure 2-9 | Single-line diagram of a typical TCPS | 25 |
| Figure 2-10 | Single-line diagram of a typical UPFC [55] | 26 |
| Figure 2-11 | Basic SOFC electrode reaction [110] | 34 |
| Figure 2-12 | A possible structure of DVR: showing the injection voltage in phase b | 41 |
| Figure 2-13 | General concept of FRIENDS [87] | 46 |
| Figure 2-14 | Configuration of UPS-type PQCC [89] | 48 |
| Figure 2-15 | Power system area and cluster controllers in a multi level hierarchical arrangement | 49 |
| Figure 2-16 | General principle of secondary voltage control loops [9] | 53 |
| Figure 3-1 | Single-line diagram of the radial transmission system | 60 |
| Figure 3-2 | Voltage-current characteristic of SVC | 63 |
| Figure 3-3 | Voltage-current relation of SVC | 64 |
| Figure 3-4 | Bode plot: phase and gain margins | 66 |
| Figure 3-5 | Structure of the two-level hierarchical voltage control loop | 69 |
| Figure 3-6 | Block diagram of the i-th primary SVC control loop | 70 |
| Figure 3-7 | Block diagram of the i-th secondary generator control loop | 71 |
| Figure 3-8 | Equivalent circuit diagrams of Figure 3-1 | 72 |
| Figure 3-9 | Detailed block diagram of the i-th secondary generator control loop | 75 |
| Figure 3-10 | AVR model - conventional excitation regulator type | 78 |
**List of Figures**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-11</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SVC1}$ at different power transfer levels</td>
</tr>
<tr>
<td>3-12</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SVC2}$ at different power transfer levels</td>
</tr>
<tr>
<td>3-13</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SGC1}$ at different power transfer levels</td>
</tr>
<tr>
<td>3-14</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SGC2}$ at different power transfer levels</td>
</tr>
<tr>
<td>3-15</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SVC1}$</td>
</tr>
<tr>
<td>3-16</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SVC2}$</td>
</tr>
<tr>
<td>3-17</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SGC1}$</td>
</tr>
<tr>
<td>3-18</td>
<td>Open-loop Bode plot for $\Delta V_p/\Delta V_{SGC2}$</td>
</tr>
<tr>
<td>3-19</td>
<td>Response following a 40MVAr reactor switching on</td>
</tr>
<tr>
<td>3-20</td>
<td>Response following a 40MVAr reactor switching on without SVC1</td>
</tr>
<tr>
<td>4-1</td>
<td>Transmission line model</td>
</tr>
<tr>
<td>4-2</td>
<td>Reactive compensation requirements of a 350-km, 500-kV line at different load level: comparison of results with and without line losses included in the model</td>
</tr>
<tr>
<td>4-3</td>
<td>Shunt reactive power compensation required at the terminals of a 500 kV line: as function of transferred power and line length</td>
</tr>
<tr>
<td>4-4</td>
<td>Shunt reactive compensation required for a 700 km, 500 kV 2-section double-circuit line: mid-point compensation at node 2</td>
</tr>
<tr>
<td>4-5</td>
<td>Equivalent π circuit model of a transmission line</td>
</tr>
<tr>
<td>4-6</td>
<td>$P-\delta$ characteristics of a transmission line</td>
</tr>
<tr>
<td>4-7</td>
<td>A schematic diagram showing a transmission system with shunt reactive power compensation</td>
</tr>
<tr>
<td>4-8</td>
<td>Equivalent circuit of a radial line with shunt compensation and source impedances</td>
</tr>
<tr>
<td>4-9</td>
<td>$P_r-\delta$ characteristics of a shunt-compensated line with $V_s = V_r = V_0$</td>
</tr>
<tr>
<td>4-10</td>
<td>Effect of line terminal impedances on (a) the $P_r-\delta$ characteristic and (b) the steady-state stability limit of a mid-point compensated line</td>
</tr>
</tbody>
</table>
List of Figures

Figure 4-11 Effect of line resistances on (a) the $\delta$-$P_r$ characteristic and (b) reducing the benefit under dynamic midpoint compensation 114

Figure 4-12 Example of a mid-point compensated transmission network 116

Figure 4-13 Rotor angle difference between $G_1$ and $G_2$ following a -1% step change in $G_1$ AVR input 117

Figure 4-14 Rotor angle difference between $G_1$ and $G_2$ following reactor energization and tripping 118

Figure 4-15 (a) Power flows of the compensated line, (b) Maximum RE power of section 2 ($P_{r2}$) vs. shunt compensator location $k$, (c) Maximum $P_r$, $P_{s2}$ and $P_{r2}$ as function of $k$ 119

Figure 4-16 (a) Power-angle characteristics of the compensated transmission line using the proposed method, (b) an expanded view when $k = 0.46$ 120

Figure 4-17 Power-angle characteristics for (a) using the method proposed in [32], i.e. at $k = 0.447$, (b) $k = 0.452$ 121

Figure 4-18 Power-angle characteristics for (a) $k$ decreases, (b) $k$ increases from $k=0.452$ 122

Figure 4-19 $P$-$\delta$ characteristics of transmission lines with different shunt compensator location 124

Figure 4-20 Power-angle characteristics of line sections 1 and 2: (a) $k = 0.4$, (b) $k = 0.5$ 125

Figure 4-21 Power-angle characteristics of line sections 1 and 2: (a) $k < 0.447$, (b) $k > 0.447$ ($k = 0.5$) 127

Figure 4-22 A transmission system with two shunt reactive power compensators 129

Figure 5-1 Schematic diagram of a transmission system incorporated with the UPFC-AP 137

Figure 5-2 Schematic of a wind turbine-battery energy storage system (BESS): single-line representation 138

Figure 5-3 Circuit model of a loop transmission system with the UPFC-AP 140

Figure 5-4 Equivalent circuit of battery 141

Figure 5-5 Feasible operating area of the UPFC-AP where $S_{m,1} > S_{m,2}$ 148

Figure 5-6 Optimal economic dispatch of the UPFC-AP system within the FOR 148
<table>
<thead>
<tr>
<th>List of Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Figure 5-7</strong> Proposed UPFC-AP load-tracking optimal power dispatch scheme 153</td>
</tr>
<tr>
<td><strong>Figure 5-8</strong> Optimal capacity of the UPFC and the battery 157</td>
</tr>
<tr>
<td><strong>Figure 5-9</strong> Load tracking by the UPFC-AP following a 0.3 p.u. ramp load increase in 3 s 159</td>
</tr>
<tr>
<td><strong>Figure 5-10</strong> Load tracking (a) with UPFC-AP and following a 0.1 p.u. step power increase in ( P_r ) and (b) with UPFC and 0.01 p.u. step power increase in ( P_r ) 160</td>
</tr>
<tr>
<td><strong>Figure 6-1</strong> (a) Schematic of a battery bank composed of ( n ) parallel branches, with ( m ) battery cells in series in each branch; (b) Equivalent circuit of each battery cell 164</td>
</tr>
<tr>
<td><strong>Figure 6-2</strong> (a) Equivalent circuit of Figure 6-1; (b) Lumped-parameter Model of the Battery Bank 166</td>
</tr>
<tr>
<td><strong>Figure 6-3</strong> Typical daily load curve ( (P_r) ) over a work day 169</td>
</tr>
<tr>
<td><strong>Figure 6-4</strong> Typical discharge characteristics of battery, per cell 170</td>
</tr>
<tr>
<td><strong>Figure 6-5</strong> ( V_{dc,\text{max}} ) over the range ( P_{r,l} &lt; P_r &lt; P_{r,i} ) under varied BESS state of discharge condition 174</td>
</tr>
<tr>
<td><strong>Figure 6-6</strong> (a) ( V_{dc,\text{max}} ) as a function of BESS state of discharge over the range ( P_r &gt; P_{r,l} ); (b) ( V_{dc,\text{min}} ) over the range ( P_r &gt; P_{r,l} ) and ( V_{dc,\text{lim}} = 1.05 V_{dc,\text{min}} ) 175</td>
</tr>
<tr>
<td><strong>Figure 6-7</strong> ( V_{dc,\text{max}} \cdot f' ) and ( V_{dc,\text{lim}} \cdot P_r ) for ( P_r &gt; P_{r,l} ) 175</td>
</tr>
<tr>
<td><strong>Figure 6-8</strong> Variation of ( f_m ) with ( P_r ) 176</td>
</tr>
<tr>
<td><strong>Figure 6-9</strong> Simulation results for ( n = 8 ) 178</td>
</tr>
<tr>
<td><strong>Figure 6-10</strong> Simulation results for ( n = 12 ) 180</td>
</tr>
<tr>
<td><strong>Figure 7-1</strong> Ideal and actual fuel cell current and voltage characteristics [71] 184</td>
</tr>
<tr>
<td><strong>Figure 7-2</strong> Schematic diagram of a SOFC [157, 158] 185</td>
</tr>
<tr>
<td><strong>Figure 7-3</strong> Feasible steady-state operating area of a SOFC [158] 186</td>
</tr>
<tr>
<td><strong>Figure 7-4</strong> PQCC-SC scheme: single-phase representation 189</td>
</tr>
<tr>
<td><strong>Figure 7-5</strong> (a) Schematic and (b) phasor diagrams describing the proposed scheme during voltage restoration stage 192</td>
</tr>
<tr>
<td><strong>Figure 7-6</strong> Phasor diagram illustrating PQCC series compensation: for condition when ( V_{\text{thr}} \cdot \sin \theta &lt; V_{i,n} \leq V_{i,p}/\cos \theta ) 195</td>
</tr>
</tbody>
</table>
List of Figures

Figure 7-7  Phasor diagram illustrating energy-saving compensation method for operating condition $V_{th} \sin \theta < V_{i_m} \leq V_{ip1} / \cos \theta$ and $\bar{V}_i$ originating from the AB sector 199

Figure 7-8  Phasor diagram illustrating energy-saving compensation method for operating condition $V_{th} \sin \theta < V_{i_m} \leq V_{ip1} / \cos \theta$ and $\bar{V}_i$ originating from the: (a) BC sector; (b) CD sector; (c) DE sector and (d) EF sector 200

Figure 7-9  HQ load voltage sag/swell ride-through range vs SC capacity at (a) varied load power factor: $S_{HQ} = 0.8$ p.u.; (b) varied $S_{HQ}$: constant load power factor of 0.95 lag 205

Figure 7-10 Maximum voltage sag ride-through capability of HQ load for $S_{max} = 0.31$ pu, load power factor of 0.95 (lag), $S_{HQ} = 0.8$ pu 206

Figure 7-11 Simulation results for an upstream voltage sag of $V_p = 0.61$ pu 208

Figure 7-12 Simulation results for $V_p = 0.97$ pu 209

Figure 7-13 Simulation results for $V_p = 1.1$ pu 210

Figure 8-1  (a) Idealized multi-machine-infinite bus system, (b) the i-th single-machine-equivalent generator system. 217

Figure E-1  AVR model --- IEEE type 260

Figure 1-1  Phasor diagram corresponding to Case 1 of Tables 7-1 and 1-1 268

Figure 1-2  Phasor diagram corresponding to Case 2 of Tables 7-1 and 1-1 268

Figure 1-3  Phasor diagram corresponding to Case 4 of Tables 7-1 and 1-1 269

Figure 1-4  Phasor diagram corresponding to Case 5 of Tables 7-1 and 1-1 269

Figure 1-5  Phasor diagram corresponding to Case 6 of Tables 7-1 and 1-1 270

Figure 1-6  Phasor diagram corresponding to Case 7 of Tables 7-1 and 1-1 270

Figure 1-7  Phasor diagram corresponding to Case 8 of Tables 7-1 and 1-1 271

Figure 1-8  Phasor diagram corresponding to Case 9 of Tables 7-1 and 1-1 271

Figure 1-9  Phasor diagram corresponding to Case 10 of Tables 7-1 and 1-1 272
List of Tables

Table 3-1  Phase margins of different power transfer levels 79
Table 4-1  Effect of line model representation on steady-state stability limits 115
Table 4-2  Shunt compensated 345kV transmission line example of [32] for up to 4 compensators 132
Table 5-1  Optimal load-tracking with UPFC-AP operating trajectory OHIJB 156
Table 5-2  Load tracking of UPFC operating trajectory OC 156
Table 5-3  Optimal results for maximum power transfer for different series and shunt converter transformer ratings 158
Table 7-1  Maximum sag/swell load ride-through capability for given $V_{min}$, $\theta$ and $V_{lm}$ 197
Table 7-2  Parametric values of the PQCC and power system 203
Table D-1  Generator parameters in studied power system 257
Table D-2  Parameters of loads in studied power system 257
Table D-3  Parameters of transmission lines in studied power system 258
Table D-4  AVR parameters for generators in studied power system 258
Table E-1  Generator parameters in studied power system 259
Table E-2  Parameters of transmission lines in studied power system 259
Table E-3  Parameters of transformer 260
Table E-4  AVR parameters for generators in studied power system 261
Table E-5  SVC parameters for studied power system 261
Table H-1  100 kW SOFC power plant data [157] 267
Table I-1  Expressions for the computation of injection voltage under the minimum energy injection/absorption strategy 273
Chapter 1

Introduction

In this chapter, a brief overview of the motivations of flexible control of transmission and distribution networks is provided, contributions made are described and the organization of the thesis is also included.

1.1 Motivation

Electric power systems are experiencing dramatic changes in operational requirements as a result of deregulation. Continuing electric load growth and ever higher regional power transfers in a large interconnected network would lead to increasingly complex and possibly less secure power system operation. Unfortunately, often power generation and transmission facilities have not been able to grow to meet these new demands as a result of economic, environmental, technical, and governmental regulatory constraints [1]. Power system engineers would need to find solutions which would allow the electricity systems to be operated satisfactorily under such challenging conditions.

In recent years, an approach to achieve these desirable outcomes is through the application of power electronic technology. The research work to be described in this thesis is an attempt to make a contribution in the area.

One important issue pertaining to power system operation is the need to maintain steady and acceptable voltage level under normal as well as disturbed operating conditions. This is often referred to as voltage regulation consideration. The significance of voltage-reactive power control relationship increases considerably after many incidents of voltage-related operational problems which have been reported worldwide [2-5]. Voltage stability is related intricately to the generation, transmission and load systems. During the process of power transfer from power generators into customers' facilities, electrical energy must transit transmission system. The characteristic of the transmission system is
Chapter 1: Introduction

one important and determining factor in voltage stability. Due to the characteristic of reactive power, it is very difficult to transit reactive power over long distance [2-5]. Thus how to control voltage along a long-distance transmission line system is of great challenge. While conventional generator excitation controllers and mechanical switched capacitors are helpful in enhancing voltage regulation, these conventional methods of reactive power control may not be sufficient under fault conditions. Indeed, it has been established that power electronics-based shunt compensator, such as a SVC, can achieve fast and precise regulation of terminal voltage. So combinations of the conventional controllers and the SVC seem a natural development and have good prospect. However, once a large number of these controllers are deployed in the system, the coordination and overall control to provide maximum system benefits and prevent undesirable interactions under the myriad of system configurations, often conflicting objectives and system operating conditions, will present a difficult technological challenge. In this regard, Chapter 3 examines the design of a typical radial transmission system, and proposes a coordinated hierarchical control scheme to achieve voltage control. Coordinated voltage control problem had been well researched and documented [3-23]. Often the design technique entails a trial-and-error process and can be time-consuming. Furthermore, there is no guarantee that the controller so obtained is robust as the power system operating conditions can vary. Thus an important motivation of the project is to devise a systematic procedure and tuning method, as described in Chapter 3, by which closed-loop stability can be assured.

At the transmission-level, power system transfer capability indicates how much inter-area power transfers can be increased without compromising system security. Accurate identification of this capability provides vital information for both the planning and operation of the bulk power network. Planners need to know the system bottlenecks and system operators must not implement transfers which exceed the transfer capability. Estimates of transfer capabilities are needed to ensure that the power transfers do not cause an undue risk of system overloads, equipment damage, or blackouts. However, an overly conservative estimate of transfer capability unnecessarily limits the power transfers and results in costly and inefficient use of the network. One must note that power transfers are increasing both in amount and in variety as deregulation proceeds. Indeed, such power transfers are necessary for utility to operate in a competitive electricity market.
Chapter 1: Introduction

Hence, there is a very strong economic incentive to improve the accuracy and effectiveness of transfer capability computations for use by system operators, planners and power marketers. Indeed, one of the most important considerations in AC transmission is centered on how to increase power transfer limit in the most economical manner. It is a well-known fact that transmission system power capabilities can be directly influenced by shunt and series compensation [24]. The appearance of Flexible AC Transmission System (FACTS) controllers, which are power electronics-based devices designed for the direct control of AC transmission lines, changes significantly the way transmission systems can be controlled and operated [25, 26]. Most FACTS controllers are essentially based on variable shunt and/or series compensation of transmission systems; hence, it is important to study the effect of these controllers on power transfer capability, so that design techniques can be developed to maximize power transfer capability.

The use of shunt compensation to enhance the power transfer capacity is well-established and the technique is known to be economically attractive. As pointed out and theoretically proven in, for example [27-29], a mid-point compensated line can transmit up to 2 times the power of the uncompensated line while maintaining steady-state stability. However, this observation is only valid if simplified lossless line model is assumed. In practice, the benefit of the shunt compensation is significantly less than that envisaged in [27-30]. Indeed, reference [31] has indicated that the factor is closer to 1.6 for a typical shunt-compensated line. The authors in [31] attributed line losses and capacitance to have contributed to some extents toward the reduction. Unfortunately, they have not provided quantitative evidence to support the claim. Thus a re-evaluation of the contribution of the midpoint shunt compensator to enhance stability is needed, as is shown in Chapter 4.

Moreover, using the simplified line model, previous studies [2, 26-30] have drawn the conclusion that the optimal compensation location is the mid-point of the line to achieve the maximum receiving-end (RE) power. As the transmission systems do contain losses, the optimal location of shunt compensation will be affected [32]. Thus another aspect of AC transmission is to re-examine the optimal location of the shunt compensator. Furthermore, in many long-distance transmission systems, two or more shunt compensators need to be installed to achieve satisfactory voltage control. Thus the determination of optimal locations for multiple compensators is also investigated.
Chapter 1: Introduction

At the generation level, the utilization of renewable energy resources for electricity production has received considerable attention in recent years. The motivation for this is due to the adverse environmental impacts and fuel cost escalation associated with conventional modes of carbon-based generation [33-40]. At present, wind and/or solar energy sources are utilized to generate electric power in many applications [33-40]. These sources have excellent potential to become important contributors for electricity production in many parts of the world because of their attractive environmental, social and economic benefits.

Unfortunately, often the availability of wind and sunlight is a much more variable, and they behave far differently than conventional sources. Energy storage systems are therefore required to smooth the fluctuating nature of the energy conversion process. Fortunately, recent developments and advances in energy storage and power electronics technologies make the application of energy storage technologies a viable solution for modern power system applications [41-54]. As mentioned earlier, in an open access environment of many power systems, transmission systems are required to provide increased bulk power transfer capability and to accommodate a much wider range of possible generation scenario. This has led to an increased focus on transmission system constraints and on the means by which such constraints can be alleviated. The Unified Power Flow Controller (UPFC) offers a versatile and viable alternative to conventional reinforcement methods [55-65]. However, hitherto economic factor limits wider UPFC applications in power systems. This may be reversed if additional services and enhanced performance can be achieved at an incremental cost which is modest compared to that of a stand-alone UPFC. Chapter 5 therefore proposes a UPFC-AP scheme which includes an active power (AP) energy storage system. The chapter is to report an analysis into the compensated system and to show the extent by which the proposed scheme can enhance transmission system performance.

By harnessing energy from a renewable source, a battery bank is considered as the medium for energy-storage. The battery system forms an AP source in the new UPFC scheme. When the UPFC-AP is used to support power transfer, the battery bank in service is operating under the discharge mode and the energy in the battery is drained continuously. Thus it is necessary to derive a method to assess the expected frequency of
Chapter 1: Introduction

the battery change-over, as this information will be needed in matching the renewable energy source with the power demand. This work will be described in Chapter 6.

On the distribution-level, power electronics technology has been used in a much more extensive way. Due to the wide-spread use of computerized and automated equipment [66], these loads are sources of voltage/current harmonics. Their operations can also be sensitive to any degradation of the voltage waveforms, and hence, they demand high quality of supply. On the other hand, there are still significant proportions of present-day loads which can accept lower-quality supply but demand it to be met at lower prices. Therefore it would be highly desirable if utility industry can provide an unbundled power quality service [67] and enables the delivery of different power quality levels to match customer requirements and expectation. Parallel with the above developments in recent years is the increasing amount of small-scale distributed generation (DG) that has been introduced on the customer-side of distribution systems. Often, this is due to environmental and/or economic considerations [68] if the DG takes the form of a renewable energy source. However, introducing DG into an existing power system would pose new technical challenges [68-77]. For example, DG can cause power flow along unintended paths under certain operational conditions. This could result in undesirable consequences.

With these backgrounds, it is therefore not surprising to find that much of recent research attention has been directed towards exploring the design of flexible electric power distribution system for the future. The concept of “Custom Power” has been proposed in [78-81], in which the application of power electronics to control power flows in distribution systems has been considered. Studies for bringing “unbundled power quality services” to realization have been promoted by many researchers, see e.g. [82]. Flexible, Reliable and Intelligent Energy Delivery System (FRIENDS) is yet another attempt in this development [83-86]. Using DG, power electronics technologies, demand side control, high-level communication technologies and dispersed intelligent facilities, FRIENDS is intended to have many desirable features. The most significant one is the new conceptual facility named Power Quality Control Center (PQCC) [87-93]. A PQCC plays a vital role in FRIENDS operation, such as providing unbundled power quality services. The Center is located between the high voltage distribution lines and the customers. It consists of
power electronic devices for controlling power flows, static transfer switches for flexible network configuration and computers for information processing, among other functions. Distributed generators are also installed in the PQCC for efficient use of electrical energy production.

At present, much progress has been reported in [87-97], where the intention is to design systems which can realize unbundled power quality supply and allow flexible changes in the configurations of distribution networks without degrading system reliability and security. Although there are several possible strategies in achieving voltage restoration through series compensation (SC) [98, 99], it is worthwhile considering the energy-saving possibility because of the presence of constrained instantaneous output power change. If it is possible to reduce the SC injection/absorption power, it will mean increased ride-through range for the load. Furthermore, reduced variations on the SC power would lead to lesser changes on the DC-link voltage. Thus, unlike the SC design described in previous works, a new SC compensation technique is proposed in Chapter 7 as part of a PQCC scheme in which the SC is fed from an active source which has its instantaneous output power change constrained. Moreover, the method has the very desirable feature in that it allows the compensation to be carried out at minimum energy level. Thus it maximizes the ride-through range offered to the compensated load. Furthermore, the new technique takes into account the apparent power rating of the SC. The resulting PQCC-SC control strategy is shown to be superior in terms of sag ride-through ability compared to that given in previous works.

1.2 Major Contributions of the Thesis

As described briefly earlier, the research work has resulted in original contributions being made in the flexible control of both transmission and distribution networks through applications of power electronics technology.

A. A hierarchical voltage-reactive power control scheme for a radial transmission system. At present, voltage-reactive power control is one of the most important considerations in the operation of modern power systems. As a result, many control devices have been installed to provide voltage-reactive power control.
Chapter 1: Introduction

Thus it is of great importance to coordinate the control of these devices. Unfortunately, there has been limited published theoretical analysis on how to design the coordinated control system. Chapter 3 describes an attempt to close the gap. A coordinated hierarchical control scheme for a long distance radial transmission system is proposed. A model of this system suitable for detailed analysis is developed and the state space model is obtained using small-signal analysis technique. Using the well-known frequency response technique, a tuning method of the control parameters of the primary and the secondary controllers is developed. A significant finding of the subsequent analysis of the power system shows that the controller should be designed based on the most onerous condition, which is under the maximum power transfer level. A practical power system is used as an illustrative example of the proposed method. The simulation results show clearly the validity of the design technique. They also show that with the proposed coordinated hierarchical control scheme, the voltages of the transmission system can be quickly restored to the nominal values following contingencies, and reactive power resources are continuously well managed to improve network security.

B. Re-evaluation of the mid-point shunt compensation to enhance stability. In many modern power systems, power electronics-based shunt compensators, such as SVC or STATCOM, are used to provide very fast and effective reactive power support and voltage control. However, some aspects of the compensation technique need to be re-examined. Shunt compensation on long-line has been considered in Chapter 4. Generalized expressions with exact representation of the transmission line have been obtained to predict the maximum power transfer ($P_{\text{max}}$) level. A computational procedure is proposed to determine the reactive power needed at the line midpoint at any given reasonable power transfer level. The amount of the reactive compensation needed for a given power transfer can be determined much more accurately using the exact long-line model which includes line losses, compared to that based on simplified line model. Furthermore, the contribution of the midpoint shunt compensator to enhance stability is re-evaluated and the analysis indicates that the improvement is (typically) only by a factor of 1.67 instead of the well known factor of 2. The decrement is accounted
Chapter 1: Introduction

for due to the presence of line resistances and the distributed nature of the line shunt capacitances. If the terminal impedances at the end(s) of the line are also included, there is a further reduction in the factor to typically around 1.6. The analysis provides an improved evaluation of the contribution made by midpoint shunt compensator in enhancing network stability. It quantifies the extent of the improvement in stability through mid-point compensation, as it is affected by such factors as line losses, line length, and SE and RE terminal impedances.

C. **Power transfer capability of long transmission lines as affected by shunt compensation location.** The power transfer limit varies when the location of shunt compensator changes. The conclusion that the mid-point of the line is the optimal compensation location to achieve the RE power has been shown in previous studies [2, 26-30]. However, this result is based on the simplified line model. Whereas by including the line losses, the analysis in [32] shows that the device needs to be placed slightly off-centre to get the maximum power transfer level at the intermediate node where the compensator is connected. As the receiving end (RE) power is of more practical interest, it is necessary to investigate the optimal location of the shunt compensator to achieve the maximum RE power. Expressions that can be used to achieve the above aim are derived systematically in Chapter 4. In addition, extension of the one shunt-compensator line to include multiple compensators is also considered. A computational procedure which can be used to determine the optimal compensator locations is proposed. By adopting the optimal shunt compensator locations, a significant increase in power transfer level of the transmission system is confirmed by simulation results. Through the graphical examination of the $P$-$\delta$ curves, stability limit of the compensated system is also elaborated on.

D. **Optimal design and dispatch strategy for a UPFC incorporated with an AP Source.** UPFC has been the most versatile FACTS device due to its ability to control real and reactive power flows on transmission lines while controlling the voltage of the bus to which it is connected to. As a means to improve and expand the applications of UPFC device, a new scheme of a power compensation system which incorporates an active power (AP) source into a UPFC is proposed in
Chapter 1: Introduction

Chapter 5. The AP source is composed of battery banks which harness energy from renewable sources, such as that from wind turbines. An optimization problem is then formulated to determine the capacity of the UPFC-AP. The design objective could include the minimization of the AP capacity or the power transfer angle, or the maximization of the power transfer level. Operating constraints of the network and that of the UPFC-AP have been considered in the numerical design procedure, and the concept of feasible operating regime (FOR) has also been introduced in the design process. Without violating any of the operating constraints, the range of the AP output power \( P_d \) that can be dispatched for given load demand \( P_r \) is shown in a graphical way. The optimal capacity of the UPFC-AP then can be determined using the FOR. Furthermore, a simple relationship between the output power of the AP and the load demand has been established using the FOR and the result of an economic dispatch strategy applied to the total generation cost of the generators. An optimal load tracking strategy has been proposed to realize economic dispatch of power from the upstream source and the AP while keep the power angle across the transmission system to the minimum. Numerical examples have demonstrated the effectiveness of the proposed strategy and the theoretical analysis.

E. Battery Energy Storage System in the UPFC-AP. Lead-acid battery [47, 48] is considered as a suitable form for the AP in the proposed UPFC-AP scheme. A possible BESS change-over strategy has been proposed to determine when the change-over from the in-service BESS to the stand-by BESS should occur. Furthermore, a method to determine the scheduling of the battery bank change-over for a given load demand has been derived. In this way, suitable BESS capacity is determined such that the overall power system can be operated under the economic dispatch condition. By varying the capacity of the BESS to suit the available energy profile of the renewable source, one also ensures that the UPFC-AP system will guarantee the transfer capability of the compensated system is maximized.

F. An energy-saving series compensation strategy subject to injected voltage and input power limits. A new series compensation (SC) method as part of a Power
Chapter 1: Introduction

Quality Control Center scheme is considered. The SC is fed from an active source whose allowable instantaneous output power change is constrained. By introducing the injection voltage to effect a phase angle adjustment on the load-side voltage, maximum load ride-through capability offered by the SC is obtained for given SC rating. The proposed energy-saving voltage injection strategy requires minimum injection/absorption energy to achieve the voltage restoration and the variation on the DC-link voltage in the PQCC is reduced during the restoration process. Analytical expressions pertaining to the injected voltage magnitude and phase angle, in terms of given voltage sags/swells and load power factor and SC capacity, are derived and from which the extent of the sag/swell that can be ridden through is readily determined. Furthermore, through analysis and simulation, it is shown that the proposed scheme is superior compare to that of existing methods, in terms of its higher load ride-through capability and lower energy requirements. The simulation results also indicate that a reasonable range of load voltage sag/swell ride-through can be achieved for a modestly-sized SC, suggesting the advantage of the proposed technique for power quality enhancement.

1.3 Organization of the Thesis

The focus of the thesis is on the applications of advanced power electronics technologies for power systems. In this aspect, Chapter 1 provides a brief description of the motivation and lists the major contributions made in this project. A review of the basic power electronics-based power flow controllers, distributed generation, the issues on power quality and stability will be given in Chapter 2. A detailed literature review pertaining to the research area is included.

The voltage-reactive power control problem of the long-line radial transmission system is examined in Chapter 3. A coordinated hierarchical control scheme is proposed and a tuning method of the control parameters of the primary and the secondary controllers is developed using the well-known frequency response technique.

Shunt compensation of a long-line is considered in Chapter 4. A computational procedure
Chapter 1: Introduction

to determine the reactive power needed at the line midpoint for given power transfer level is proposed. The effectiveness of shunt reactive power compensation for long-distance radial transmission is re-examined and generalized expressions on the power transfer limit are obtained. Steady-state stability limit of the transmission system through mid-point compensation can be readily evaluated using the analytical expressions obtained. A computational procedure is proposed in which the optimal locations of the shunt compensators can be determined. This is for the purpose to maximize the power transfer ability of the compensated line.

A new scheme of a power compensation system which incorporates an active power (AP) source into a UPFC is investigated in Chapter 5. The design of the UPFC-AP is formulated as an optimization problem. The concept of feasible operating regime (FOR) is then introduced. With FOR and the application of a proposed economic dispatch strategy, the relationship between the minimum total generation cost and the load demand is established. The analysis forms the basis of an optimal load tracking scheme, which leads to the economic dispatch of generation and minimum power angle across the transmission link.

The AP described in Chapter 5 is in the form of a battery energy storage system (BESS). In Chapter 6, a possible BESS change-over strategy is proposed to determine when the change-over from the in-service BESS to the stand-by BESS should occur. A method to determine the scheduling of the battery bank change-over for a given load demand is also derived. In this way, the capacity of the BESS needed to support the power transfer can be determined.

In Chapter 7, a new series compensation method as part of a Power Quality Control Center scheme is described. An energy-saving voltage injection strategy is proposed. Analytical expressions for the injected voltage phasor, in terms of given voltage sags/swells, load power factor and SC capacity, are derived. The superiority of the proposed method over existing techniques is shown through analysis and simulation.

The main findings of the research and recommendations for future works are given in Chapter 8.
Chapter 2

A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

As pointed out in Chapter 1, the present day evolving power systems create a growing need for flexible, reliable, fast and accurate control actions on the generation, transmission, distribution, and utilization of electricity. In this Chapter, a review of the various aspects and issues pertaining to the control of power systems will be given. A detailed literature review pertaining to the research area is included.

This Chapter is organized as follows. Section 2.1 and 2.2 will provide a brief introduction on power system stability and power quality issues respectively. Some widely used Flexible AC Transmission Systems (FACTS) devices and power quality improvement methods will then be shown in Section 2.3 and 2.4 respectively. Various custom power technologies and their possible effects on power quality will be reviewed in Sections 2.5. Finally, some basic knowledge on hierarchical network control will be described in Section 2.6.

2.1 Power System Stability

Power system stability was first recognized as an important consideration in 1920 [1], and since then, it has evolved to become an extremely complex engineering problem due to the immense size of typical modern networks. A widely-used definition of power system stability is:

- Power system stability may be broadly defined as that property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance [3].
Traditionally, power system stability is classified in two main categories: angle stability and voltage stability.

- **Power System Angle Stability**

  **Angle stability** refers to the capability of the synchronous machines interconnected in a power system to maintain synchronism. It can be further categorized as two sub-classes: small signal stability and transient stability.

  **Small-signal stability** is defined as the ability of a power system to remain in synchronism when subjected to small disturbances such as the small variations in loads. Small-signal stability problem is largely a problem of insufficient damping of oscillation. Small-signal stability usually could be improved using either power system control equipment such as power system stabilizers or by appropriate strengthening of the transmission system.

  **Transient stability** is the ability of a power system to maintain synchronism under serious transient disturbances, such as under three-phase-to-ground short circuit contingencies. The faults can usually be cleared by isolating the faulted part from the power system through the opening of appropriate circuit breakers.

- **Voltage Stability**

  A power system at a given operating state is small-disturbance voltage stable if, following any small disturbance, voltages near loads are identical or close to the pre-disturbance values [4].

  A power system at a given operating state and subject to a given disturbance is voltage stable if voltages near loads approach post-disturbance equilibrium values. The disturbed state is within the region of attraction of the stable post-disturbance equilibrium [4].

  A power system at a given operating state and subject to a given disturbance undergoes voltage collapse if post-disturbance equilibrium voltages are below acceptable limits. Voltage collapse may be total (blackout) or partial [4].
Voltage stability and voltage collapse problems are largely due to the fact that power systems cannot supply enough reactive power to meet the reactive load demand and maintain acceptable bus voltage levels. Many reactive control techniques such as that from generation excitation systems, shunt capacitor banks, load tap changers, and Flexible AC Transmission System equipment (FACTS), can play important roles in maintaining desired voltage profiles.

In general, a typical power system must maintain as large as possible steady-state and dynamic stability margins under its normal and fault conditions. This may be achieved by properly configuring generation pattern, re-arrangement of transmission network, switching in/out of reactive control equipment in the system.

Power systems are often forced to operate ever closer to their critical power stability limits and with lesser stability reserve margins as load demand increases. To counter-act this trend, techniques must be found to enhance the stability. The proposed solution methods must be practical and economical. In many instances, the control devices on today’s power systems are electro-mechanical types. The control actions are often too slow as to be effective. Also, switching of such devices cannot be done frequently due to the fear of wearing out of the mechanical devices. This translates into somewhat inefficient operation since it means that network security is met mainly through generation scheduling. In some cases, it may even lead to the over-design of power systems. Fortunately, technological developments in power electronics-based devices have provided alternative solutions to the above mentioned problems. The details of the modern techniques will be described in latter sections.

2.2 Power Quality

Power quality is another important attribute of modern power systems. Power quality is a broad concept related to voltage and current quality. It is concerned with deviations of voltage and/or current from the ideal single-frequency sinusoidal wave of constant frequency and constant magnitude in an AC system. A widely used definition of power quality is [100, 101]:

14
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

- The ability of a power system to operate loads without disturbing or damaging them, a property mainly concerned with voltage quality at points of common coupling.
- The ability of loads to operate without disturbing or reducing the efficiency of the power system, a property mainly, but not exclusively, concerned with the quality of the current waveform.

With the present the power supply system, it can only control the quality of the voltage and has little control over the current particular loads might draw, hence in most cases, power quality actually means the quality of the voltage.

Both electric utilities and industry end-users of electricity are increasingly aware of the need for high quality of supply since the late 1980s.

2.2.1 Some Common Disturbances

Some of the most common disturbance phenomena seen in power systems are voltage sags/swells, undervoltages, overvoltages, voltage interruptions, transients, flicker, harmonics, and frequency variations. The phenomena are defined as follows [66, 100-102]:-

Voltage sags (Dips): Voltage sag is considered one of the most common power quality phenomena in practice. A voltage sag is defined as a decrease in voltage to between 0.1 and 0.9 p.u. in rms value at power frequency for a duration of 0.5 cycles to 1 minute. Normally, voltage sags are caused by faults or the switching of heavy loads. Since many types of modern load equipment are quite sensitive to voltage sags, the loads are often automatically disconnected from the supply system when the rms voltage drops below 90% for longer than one or two cycles. This would lead to disruption to processes/operations and incur economic loss to the customers.

Voltage swells: As the opposite of voltage sag, a voltage swell is defined as an increase to between 1.1 and 1.8 p.u. in rms value at the power frequency for durations from 0.5 cycle to 1 min. Similar to sags, swells are usually associated with unbalanced system fault conditions. For example, a swell can occur on the unfaulted phases during a single-line-
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

grounded (SLG) fault. On an ungrounded system, with infinite zero-sequence impedance, the line-to-ground voltages on the ungrounded phases will be 1.73 p.u. under a SLG condition. Swells can also be caused by switching off a large load or energizing a large capacitor bank. Excursive voltage swell can lead to accelerated degradation of insulation material, and flashover.

**Voltage interruptions:** Voltage interruptions mean the complete loss of voltage of below 0.1 p.u. on one or more phase. Momentary interruptions are defined as those incidents lasting between 0.5 cycles and 3s, temporary interruptions have a time span between 3s and 60s, and sustained interruptions are those that last for a period longer than 60s.

**Transients:** Transients is a phenomenon of supply voltage which is varying between two consecutive steady states. It can be a unidirectional impulse of either polarity, or damped oscillatory wave with the first peak occurring in either polarity. Although most transients are usually generated near the user due to the operation of other equipment, switching operations on the utility network can pose a more serious problem. For example, capacitor switching can lead to transients with a magnitude of 2~3 p.u. and the high energy levels can considerably shorten the life of surge protection devices.

**Overvoltage:** Similar with voltage swell, overvoltage is also defined as an increase in rms voltage at the power frequency, but the main difference between these two terms is that the later is usually used to describe a specific type of long-duration variation. Normally this duration could last more than 1 minute. The other difference is the range of voltage swell is about 1.1 to 1.9 pu while the typical values of overvoltage are 1.1 to 1.2 pu.

**Undervoltage:** Comparing to voltage sag, undervoltage also means a value of less than the nominal voltage despite its lasting time is generally greater than one minute. Its typical values are 0.8 to 0.9 pu.

**Harmonics:** Harmonics is another common power quality issue in the utility industry. Harmonics are defined as sinusoidal voltages or currents having frequencies that are multiples of the fundamental power frequency. Generally harmonics are caused by nonlinear characteristics of power system devices and loads. For example, the switches of
power electronics devices in the adjustable speed drives could inject higher harmonics
components into the power system and leads to the distorted waveforms in the current. In
recent years, in particular with the growth of non-linear loads such as personal computers
and power electronic devices, the combined effect of many such devices can result in
harmonic distortion of voltage/current greater than 5~10% levels and exceed the normal
industry regulation standards.

Notches: Periodic voltage disturbances lasting less than 0.5 cycles. Mainly power
electronics devices can cause notching when the current is commutated from one phase to
another during the momentary short circuit between the two participating phases.

Voltage fluctuations/flickers: Voltage fluctuations are systematic variations in the
envelope or a series of random voltage changes with magnitudes that do not normally
exceed the voltage range of 0.9 to 1.1 p.u. Typical cause of flickers includes the
operations of power converters on electrical devices.

2.2.2 CBEMA and ITIC Curves

The voltage quality of a power system with respect to voltage interruptions, sags and
swells can be evaluated by the well-known Computer Business Equipment Manufacturer
Association (CBEMA) curve [103], shown in Figure 2-1. This figure shows that the curve
describing the magnitude of voltage sags/swells against the duration of voltage variations
which can be withstood by equipment. The smaller (larger) the magnitude of the sag
(swell), the lesser is the equipment withstand time. The region between the two
boundaries is the tolerance envelope, within which electronic equipment is expected to
operate reliably. The region below the envelope is presumed to cause the load to drop out
due to lack of energy. Conversely, the region above the envelope can cause other
malfunctions such as insulation failure, over-voltage trip and over-excitation.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

Short duration impulses are most frequently measured as maximum volts deviation from the power frequency sine wave, or as peak-to-peak volts measured through a high-pass filter or a coupling capacitor. It is sometimes expressed in percent of nominal peak or peak-to-peak voltage.

RMS voltage measurements of voltage envelope are appropriate for 1/4 cycle or longer.

Voltage breakdown concern area

Lack of stored energy in some manufacturer's equipment

Nois and voltage breakdown problems

Energy flow related problems

0% 100%
0.001 1 10 100 1000

100us 1ms 8.33 0.1 0.5 2

Seconds

Duration in cycles at 60Hz and in seconds

The CBEMA Curve from FIPS No. 94.

Figure 2-1 CBEMA curve [103]

Due to the prominence of the CBEMA curve among the computer and electronic industries, the Information Technology Industry Council (ITIC) has developed a revised version of CBEMA curve [104] as shown in Figure 2-2. The withstand limits at different durations of the ITIC curve are very similar to that of the original CBEMA curve. Under a voltage sag (swell) condition, the withstand time decreases when the voltage magnitude reduces (increases). The main difference is that the ITIC version is piecewise and hence is easier to digitize than the continuous CBEMA curve. The ITI curve would be used in conjunction with the load voltage-sag tolerance characteristic in Chapter 6.


Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

2.3 Flexible AC Transmission Systems

Having described the two technical issues of stability and power quality, the remainder part of this Chapter is to review selected mitigation methods/techniques which can be used to deal with the problems.

2.3.1 Basic Operational Principle of FACTS

As mentioned in Section 2.1, power electronics-based FACTS devices are effective means of achieving improved performance of power transmission system. The philosophy of FACTS is to use power electronic controlled devices to control the basic electrical parameters including voltage magnitude, phase angle, impedance and network topology in a transmission network, thereby allowing the power system to be operated in a more flexible, secure and economic way.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

The operational principle of FACTS can be illustrated by the following simple power transfer equation pertaining to a transmission line:

\[ P = \frac{V_1 V_2 \sin \delta_{12}}{X} \]  

(2.3.1)

where \( P \) is the active power transferred through the transmission line; \( V_1 \) and \( V_2 \) are the voltage magnitudes at the line terminals; \( \delta_{12} \) is the phase difference between the two terminal voltages; \( X \) is the total reactance of the transmission line.

(2.3.1) shows that the power transfer can be influenced by the three parameters: voltage magnitudes, reactance and voltage angle difference. There are a variety of FACTS devices for controlling one or more of these three parameters. For example, Thyristor Controlled Series Compensator (TCSC) directly modulates the reactance of transmission line. Static Var Compensation (SVC) mainly serves to influence the magnitude of the voltage(s) by producing or consuming reactive power, while Thyristor Controlled Phase Angle Regulator (TCPAR) only changes the phase angle difference \( \delta_{12} \). Unified Power Flow Controller (UPFC) is designed to control all the three parameters at the same time. In fact, it consolidates the effects of the SVC, TCPAR, and TCSC.

### 2.3.2 Main FACTS Devices

As mentioned, different FACTS devices target different parameters in the transmission system. A simple transmission line with a FACTS controller, shown in Figure 2-3, may now be used to describe the control actions of each of the devices.

![Figure 2-3 A schematic diagram describing FACTS control actions](image-url)
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

Based on the connection of the FACTS device with the transmission line, there are three types of control actions: shunt compensation, series compensation and combined shunt-series compensation.

**Shunt Compensation:** In shunt compensation, the FACTS are connected in shunt with the power system. It works as a controllable current source.

**Series Compensation:** In series compensation, FACTS is connected in series with the power system. It functions as a controllable voltage source.

**Combined Shunt-Series Compensation:** In shunt-series compensation, the controllers are connected both in shunt and in series with the power system.

In the following sub-sections, a brief description of several well-known FACTS devices will be given.

2.3.2.1 **Shunt Connected Controllers**

- **Static VAr Compensator (SVC) [55]**

SVC, of first generation of FACTS, is shunt-connected var generator and/or absorber whose output is varied so as to control specific parameters of the power system. The term “static” is used to indicate the SVC, unlike synchronous condensers, have no moving or rotating components. Generally, SVC consist of the following elements: (i) Fixed Capacitor (FC) that provides a permanently connected source of reactive power, often designed also to act as a harmonic filter; (ii) Thyristor-Controlled Reactors (TCR) that consists of bi-directional thyristor valves in series with shunt reactors, usually connected in delta configuration and (iii) Thyristor-Switched Capacitors (TSC), which consists of a capacitor, a bi-directional thyristor valve, and a relatively small surge current limiting reactor. These elements are connected in parallel to the electrical system. They are capable of controlling the voltages of the buses to which they are connected. Figure 2-4 gives a typical structure of the SVC which consists of a TCR, two TSCs, a FC and a harmonic filter (for filtering harmonics generated by the TCR). The TCR and TSC are controlled in such a manner that the MV bus voltage is kept at or close to a constant level depending on the control scheme used. The reactive elements of compensator are
connected to the transmission line through a transformer so that the elements do not have to withstand full system voltage.

From the viewpoint of power system operation, the SVC is equivalent to a shunt capacitor and a shunt inductor, both of which can be adjusted to control voltage and reactive power at its terminals in a prescribed manner. Ideally, the SVC should hold the bus voltage at pre-specified value, possess unlimited var generation/absorption capability with no active and reactive power losses. In this ideal situation, the performance of the SVC can be visualized on a graph of controlled AC bus voltage ($V_{svc}$) plotted against the SVC reactive current ($I_{svc}$). This is the so-called V/I characteristic of SVC, as shown in Figure 2-5. From Figure 2-5, the V/I characteristic of the SVC is:

$$V_{svc} = V_{ref} + X_{SL}I_{svc}$$

(2.3.2)

where $X_{SL}$ is the slope reactance. This is applicable over the linear operating range $I_{cr} < I_{svc} < I_{Lr}$.

• Static Synchronous Compensator (STATCOM) [55]

STATCOM is of second generation of FACTS device. The basic principle of a STATCOM is the generation of a controllable AC voltage by a voltage-source converter connected to an energy storage unit (DC capacitor). The development of STATCOM is based on the use of Gate-Turn-off (GTO) thyristors which can emulate reactors and capacitors electronically. Its basic building block is illustrated in Figure 2-6. The advantage over the SVC is the improved operating characteristic. The compensation is
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

independent of the actual voltage level at the bus and the time delay between changes in the power system and device response is decreased.

2.3.2.2 Series Connected Controllers

- Thyristor Controlled Series Compensation (TCSC) [55]

TCSC is also of a first generation FACTS device. The TCSC arrangement resembles the SVC topology but is used as a series device between two power system buses. It can control the line impedance through the introduction of a thyristor-controlled capacitor in series with the transmission line. The major benefit of TCSC is its ability to rapidly modulate the effective impedance in a continuous manner. A single module of the TCSC is shown in Figure 2-7. It consists of a fixed series capacitor \( C \) connected in parallel with a thyristor-connected reactor \( L \). By varying the firing angle of the thyristors, the TCSC's impedance can be smoothly controlled over a wide range.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

- Static Synchronous Series Compensator (SSSC) [55]

SSSC, a second generation FACTS device, is like a STATCOM except its output transformer is connected in series with the transmission line, as shown in Figure 2-8. The advantages over the TCSC are an improved overall performance widely independent of the actual operating conditions and the nonexistence of the subsynchronous resonance problem.

\[ \vec{V} \]

Transmission Line

Series Transformer

\[ \vec{I} \]

Switching Power Converter

\[ V_{dc} \]

Storage (dc) Capacitor

Figure 2-8 Single-line diagram of a typical SSSC

- Thyristor Controlled Phase Shifter (TCPS) [55]

TCPS, a first generation FACTS device, functions just like the conventional phase shifters with mechanical tap-changers. It operates by injecting a voltage \( \vec{V}_q \) in quadrature with the supply voltage \( \vec{V} \) to introduce phase shifting \( \alpha \) between the supply voltage \( \vec{V} \) and terminal voltage \( \vec{V}' \), as shown in Figure 2-9. The injected voltage \( \vec{V}_q \) is made variable with a variety of power electronics topologies. This controller offers control of the transmission angle, making it possible to increase power transfer capability and enhance system stability.
2.3.2.3 Combined Shunt-Series Controllers

- Unified Power Flow Controller (UPFC)

UPFC is of the third generation of FACTS and is the most versatile FACTS controller. It consists of the shunt and series branches. As shown in Figure 2-10, the shunt converter 1 is connected to the line through the supply (shunt) transformer. The series converter 2 is connected to the transmission line through the series transformer. The two converters are coupled by a common dc link capacitor, as shown. This arrangement functions as an ideal ac-to-ac power converter in which active power can freely flow in either direction between the two converters. The series branch injects a voltage $\bar{V}_p$, in series with the line, whose phase angle can vary between 0 to $2\pi$ with respect to the terminal voltage $\bar{V}$ and whose magnitude can vary from 0 to a maximum value determined by the device rating. Thus it provides both real and reactive power regulations over the transmission line. Operation of the shunt converter involves drawing a control current from transmission line. One component of the current ($I_r$) is automatically determined by the requirement to balance the real power of the series converter. The remaining component of the current...
(\tilde{I}_q) is reactive and can be set to any desired level (capacitive or inductive) within the operational range of the converter. The shunt branch primarily provides the real power required by the series converter by regulating the dc bus voltage at a desired value. The dc link power is converted back to ac and coupled to the transmission line via the series transformer. The shunt branch can act as a STATCOM, i.e. it provides independently controllable shunt reactive compensation. The series branch can act as an SSSC. Thus the UPFC can simultaneously control all three parameters of power flow, i.e. voltage magnitude, line impedance and phase angle. The common dc capacitor provides a direct voltage support for the converter operation and also functions as an energy storage device.

![Figure 2-10 Single-line diagram of a typical UPFC](image)

From the above description, it can be seen that the UPFC is a multifunction power flow controller because of its unique feature to simultaneously or selectively control voltage magnitude, phase angle and line impedance. So it has the potential benefits for power flow control, loop-flow control, load sharing among parallel corridors, enhancement of transient stability, mitigation of system oscillations and voltage regulation. Since UPFC was brought to attention in 1991, it has been widely recognized as the most promising FACTS device. In the remaining part of this section, a literature review helps identify various aspects of research and development activities on UPFC since its advocacy. The reason for including the review here is because in Chapter 5, a variation form of the UPFC would be considered in details.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

Since UPFC has been incorporated as a member of the FACTS family [56], EPRI and Westinghouse took the first initiative to develop UPFC. The first UPFC in the world was demonstrated at Inez area of AEP [57], and a set of normal and contingency power system operating conditions were studied in the AEP system [58]. The authors showed that the UPFC could provide functional flexibility to address many issues, such as voltage depression and/or thermal overload in several single contingency conditions facing the Inez area transmission system. In a planning stage, major equipment ratings of UPFC including maximum shunt current, maximum shunt voltage, maximum series current, maximum series inserted voltage and maximum DC power transfer have to be considered. AEP developed a load flow model to determine these parameters but the model was not convenient for operations and routine planning purposes.

Basic control, sequencing and protection philosophies that govern the operation of the UPFC were described in [59]. The fundamental operating constraints of UPFC were discussed there. A concept of a high-level Line Optimization Control (LOC) was suggested as a supervisory control, by which it computes the optimum reference values for the UPFC from observed conditions in the network while maximizes its benefit for the power network. Specifically, an example was given when the requested values of active power P and reactive power Q could not be achieved within the allowable UPFC series injected voltage range. Then the LOC gave top priority to maximizing the real power flow P while allowing the reactive power Q to deviate from the desired value, i.e., the reactive power set-point Q is reduced until the active power set-point P can be satisfied while keeping the UPFC series injected voltage magnitude at its limit. This dispatch, however, may not result in a satisfactory to-bus voltage. Furthermore, the LOC does not address the situation when the set-point P is infeasible.

In order to analyze UPFC’s effects on power systems, power flow representation of UPFC using auxiliary capacitors was introduced in [60], and a strategy for handling the UPFC limits was proposed. In the method, for the shunt Voltage Source Converter (VSC), the active power injection by the shunt converter $P_{sh}$ is left unconstrained, while the reactive power injection by the shunt converter $Q_{sh}$ is set at $Q_{sh}^2 = I_{sh max}^2 V_{sh}^2 - P_{sh}^2 \geq 0$. By so doing, it imposes the limit on the shunt current magnitude to $I_{sh max}$. When $I_{sh max}^2 V_{sh}^2 - P_{sh}^2$ becomes
negative, $Q_s$ is set to be zero and the shunt current will no longer be limited, $P_{sh}$ is treated with a higher priority to satisfy the demand by the series VSC. For the series VSC, in order to address the limit violation of the to-bus voltage, the series injection voltage in all four quadrants with respect to the from-bus voltage was also examined in [60]. For each of the four quadrants, the magnitude and the angle of the series injection voltage were estimated when the to-bus voltage was set at its lower limit or upper limit, depending on the situation. This strategy is difficult to incorporate in a load flow algorithm or in a real-time operation control environment. In addition, the heuristic strategies for this to-bus voltage constraint cannot be extended to handle other constraints, and could be contradictory when multiple limit violations take place.

A more detailed study for the UPFC equipment size and operating constraints was reported in [61]. The authors demonstrate a methodology for defining a control system for a UPFC to increase power transfer between two large power systems. This strategy is mainly concerned with the series converter control, with the assumption that the from-bus voltage set-point is properly set, and can handle all of the limits for the series converter except for the MVA limits. The shunt VSC is modeled as a current source. Assuming that the shunt reactive power compensation is small enough, the shunt current magnitude limit can always be satisfied by only adjusting the power circulation demanded by the series injection voltage.

For a given pair of real and reactive power set-points $P_d$ and $Q_d$ of to-bus, this method calculates the magnitude ($V_{se}$) and the angle ($\delta_{se}$) of the series injection voltage and checks the limits for the series converter and the shunt current magnitude limit for the shunt converter. If any of the constraints are violated, the magnitude of the series injection voltage is fixed at the present value, and the angle of the series injection voltage is searched to check for a feasible range so as to avoid the violations. If no feasible range for $\delta_{se}$ could be found, $V_{se}$ is reduced and $\delta_{se}$ is searched again until a feasible range for $\delta_{se}$ is found, from which the values of $\delta_{se}$ to achieve the maximum line flow is computed. In this approach, the search process for the optimal $V_{se}$ and $\delta_{se}$ is based on the assumption that all the bus voltages do not change. This is, however, an approximation, and a load flow should be solved at each step, requiring substantial computational effort.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

Furthermore, if the shunt control is also considered in the process, an additional outer loop has to be introduced.

In an attempt to overcome the above shortcomings, Cai et al in [62] investigated the power transfer capacity for systems with a UPFC. The combined effects of equipment constraints, system topology and installation locations on the UPFC real power transfer are considered. The P-Q characteristics of UPFC operation under different sets of constraints are determined. Based on the assumption that all the bus voltages are fixed, and by not considering the shunt VSC control, the series control variables $V_{se}$ and $\delta_{se}$ are adjusted to maximize the active power flow on the UPFC line.

Yet another dispatch strategy for UPFC operating at rated capacity has been proposed by Wei et al in [63], in which a single quantity, namely, the circulation power between the coupled VSCs, is used as the parameter to optimize the power transfer. In this way, the circulation power $P_c$ is specified when capacity saturation takes place. When the shunt VSC saturates, the circulation power $P_c$ and one of the limits, MVA rating ($S_{shmax}$), the maximum current magnitude ($I_{shmax}$), or injected voltage magnitude ($V_{shmax}$) are enforced as appropriate. Similarly, when the series VSC saturates, the circulation power $P_c$ and one of the limits, MVA rating ($S_{semax}$), the maximum current magnitude ($I_{semax}$), or injected voltage magnitude ($V_{semax}$) are enforced. Similarly, when the UPFC reaches other operating limits, the method in [63] enforces the active power limit, and specify power circulation. For the VSC injected voltage magnitude limits, the limit equations are to enforce the shunt and series injected voltage magnitudes at their maximum limits. For the current magnitude limits, the limit equations are to enforce the shunt and series currents at their maximum limits. By specifying the power circulation $P_c$, it evaluates the amount of coupling to achieve optimal results. If $P_c = 0$, then the two VSCs are operated separately as a STATCOM for the shunt VSC and an SSSC for the series VSC. It is expected that by increasing the coupling, that is, increasing or decreasing $P_c$ from zero, the power transfer would be improved, until the power circulation provides no further benefit or the load flow ceases to have a solution. To implement this strategy in the Newton-Rapson (NR) load flow algorithm, the ratings of the UPFC need to be monitored at the end of each iteration. Once it is determined that either the shunt VSC or the series VSC or both VSCs will be operated at rated capacities, the proper limit equations with the power circulation
set-point will be utilized as the VSC equations, and the corresponding modification in the Jacobian matrix with respect to the VSC equations will be carried out. Voltage stability curves are used to illustrate the effectiveness of the strategy to maximize voltage stability limited power transfer.

2.3.2.4 Distributed FACTS

Another family of FACTS devices has been described in [147]. It is termed Distributed FACTS and is used to illustrate the concept of a distributed approach for realizing the functionality of FACTS devices. The implementation of distributed FACTS is similar to that of high power FACTS devices. However, distributed FACTS can be clipped on to a power line and can, dynamically and statically, change the impedance of the line so as to control power flow. In this way, it has been reported that distributed FACTS is a higher-performance and lower-cost method for enhancing transmission and distribution system reliability and controllability. It can improve asset utilization and end-user power quality, while minimizing system cost and environmental impact.

2.4 Power Quality Enhancement Technology

Having described the various power electronics-based technology for power system stability improvement, attention is now directed to that for power quality enhancement. Two kinds of power devices have been proposed to enhance power quality. The techniques can be categorized into two types:- that based on Distributed Generation (DG) [68-77] and that on Custom Power [78-81].

2.4.1 Distributed Generation

The growing demand for electrical energy and increasing fuel costs throughout the world have created an urgent need to explore new energy sources. Often, these sources are derived from a broad spectrum of resources, all of which are based on self-renewing energy sources such as sunlight, wind, flowing water and biomass.
Compared to competing conventional technologies, renewable energy systems are relatively capital intensive but they may have lower operating and maintenance costs. For example, the fuel costs of both solar and wind systems are zero. Thus as a whole the economics of renewable energy technologies improve in comparison with conventional technologies, especially in the regions of the world where world fuel prices are relatively high. This benefit will be even more significant in the future as fuel prices increase. As these sources can be placed close to loads, the cost of transmission/distribution becomes less. Another advantage of renewable energy systems is that they generate little, if any, waste or pollutants that contribute to acid rain, urban smog, and health problems. They do not require environmental cleanup costs or waste disposal fees.

During the past two decades, much research works have been carried out and dramatic improvements have been achieved in terms of the cost, performance, and reliability of renewable energy systems. Indeed, many of the renewable sources are incorporated in power system to form the Distributed Generation in the networks.

DG increases the generating capacity and with significant DG sources installed, the nearby load variation can be readily tracked, thus the supply frequency variations can be reduced. Some main types of DG and their effects on power system stability and power quality will be discussed next.

2.4.1.1 Types of Distributed Generators

- Wind turbines

Wind power installations convert the kinetic energy in the wind into mechanical energy, and then into electricity. Wind turbine (WT) is the most widely used DG which is based on renewable energy resource. Typical sizes for a wind turbine range from 200–750 kW, with electricity produced within a specific range of wind speeds. By 2020, WTs are expected to provide up to 10% of the residential power in the world [71].

Wind power generation harnesses the kinetic energy in the wind to drive the WT. The WT has a same rotor with an induction motor (IM), which is operated in the generation mode and converts the mechanical power to electrical power. As the output AC power of the
WT depends on the wind speed, AC/DC and DC/AC converters would be needed to interconnect to the grid.

Attractive features of wind generation include no fuel charge and non-polluting. However, wind power generation is strongly dependent of the weather condition. Wind fluctuation may cause power quality issue such as voltage flicker. In order to overcome this problem, WT are always operating in parallel with other DG or with energy storage systems.

- Photovoltaics

Photovoltaics (PV) systems convert sunlight directly into electrical energy through the complex interaction of light photons with semiconductor materials such as silicon. Modules can be interconnected in larger groups to form arrays to produce sufficient power. Using a DC/AC converter, the DC output power of PV systems can be converted into AC which is then used to supply local loads or fed back to the grid.

The advantages of PV systems are: emission-free operation, no fossil fuel consumption, excellent modularity, negligible maintenance, and excellent part-load efficiency. However, PV is strongly dependent of the weather condition and it cannot provide power at night, batteries or energy storage systems are often needed [71].

- Internal Combustion Engines

An Internal Combustion Engine (ICE) is a traditional technology for emergency power generation used all over the world. Its fuel is burnt inside the engine and produces expanding gases, which are used directly to provide mechanical power and drive a generator [105]. The cost of ICE unit is also the least of any DG technology. However, ICE also shows some undesirable features, such as the high NO\textsubscript{x} emissions and the high noise level [71]. The low frequency noise can be attenuated at low costs with noise abatement material and enclosure. Natural gas ICE generators can offer a solution to the emissions problem but do not solve it entirely.

- Micro-turbines

Micro-turbine (MT) is a Brayton cycle gas combustion turbine, which produces shaft power using atmospheric air and natural gas fuel [106, 107]. Unlike the traditional backup
generators, a MT is designed to operate for extended periods of time. MT can be used for standby and is feasible for peak-load shaving and providing high quality power to the utilities. MT can also be operated together with the fuel cell power plant to build a cogeneration system, which can have a very high efficiency. However, there are still some barriers to MT usage, such as MT is sensitive to ambient air temperature, the maintenance cost is high, and it emits high frequency noise [71].

Fuel cells
A fuel cell (FC) is a device in which hydrogen and oxygen combine without combustion to produce electricity [108-110]. It consists of a pair of electrodes (anode and cathode) and an electrolyte. Typical capacity of a FC at present technological development ranges from 50 kW to 2 MW [71]. The FC generates the truly clean power as its only byproduct is water. Generally, FC is characterized by the type of electrolyte used [108-110]. There are four major types of FC: phosphoric acid, proton exchange membrane, molten carbonate and solid oxide. They have similar structures and chemical reactions. As the Solid Oxide Fuel Cell (SOFC) is the device which would be considered in Chapter 7, its operations will be explained in slightly greater detail as follows.

SOFC is primarily designed for medium or large-scale stationary power generation applications. A solid ceramic material is used as the electrolyte at operating temperature of 800 – 1000 °C. Figure 2-11 illustrates the basic components of a SOFC fuel stack [110]. Hydrogen fuel is supplied to the anode, where the fuel is oxidized, generating electrons ($e^-$), which travel through the external circuit. At the cathode, the oxygen, which is normally air, is reduced, consuming the electrons from the external circuit. Oxygen ions ($O^{2-}$) travel through the electrolyte to balance the flow of electrons. DC current ($I_{FC}$) in the external circuit connected across the two electrodes is generated due to the releasing of the electrons ($e^-$) at the anode. The fundamental electrode reactions are [108-110],

\begin{align*}
\text{Anode:} & \quad H_2 + O^{2-} \rightarrow H_2O + 2e^- \\
\text{Cathode:} & \quad \frac{1}{2} O_2 + 2e^- \rightarrow O^{2-}
\end{align*}

(2.4.1)

It can be seen from the external circuit shown in Figure 2-11 that the FC is a power source supplying the load with an internal EMF. This EMF can be derived from the thermodynamic principles and is defined by the well-known Nernst equation [108, 109],

\begin{align*}
\text{FC EMF} = nF \times \frac{RT}{Z} \times \ln \left( \frac{P_{H_2O} \times P_{O_2} \times P_{H_2} \times P_{O_2}}{P_{H_2} \times P_{O_2} \times P_{H_2O}} \right)
\end{align*}

where $n$ is the number of electrons, $F$ is the Faraday constant, $R$ is the gas constant, $T$ is the temperature, and $Z$ is the stoichiometric coefficient.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

\[ E = N_0 E_0 + \frac{N_0 RT}{2F} \ln \left( \frac{P_{H_2}P_{O_2}^{0.5}}{P_{H_2O}} \right) \]  \hspace{1cm} (2.4.2)

where \( E_0 \) is the EMF associated with the reaction-free energy of a cell at standard pressure, \( N_0 \) is the number of stack cells in series, \( R \) is the gas constant (8.31 J/mol*K), \( T \) is the operating temperature, \( F \) is the Faraday constant (96487 C/mol), \( p_{H_2}, p_{O_2} \) and \( p_{H_2O} \) are the reactant partial pressures of hydrogen, oxygen and water respectively.

![Figure 2-11 Basic SOFC electrode reaction [110]](image)

2.4.1.2 Energy Storage

As is evident from the preceding discussion in Section 2.4, wind energy has become one of the most important of renewable energy and has experienced increased growth internationally. There do exist, however, various drawbacks such as the output power is variable as a result of the dynamics of the wind, requiring a stand-by power source to meet the load demand. The integration of wind generation and an energy storage system (ESS) proposes a solution to these challenges. Energy storage systems are, therefore, often required to smooth the fluctuating nature of the energy conversion systems and to match the customer energy demands.

Recent developments and advances in energy storage and power electronics technologies are making the application of energy storage technologies a viable solution for modern...
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

power applications. Viable storage technologies include batteries, flywheels, supercapacitors, and superconducting energy storage systems. Although several of these technologies were initially envisioned for large-scale load-leveling applications, energy storage is now seen more as a tool to enhance system stability, aid power transfer, and improve power quality in power systems. In this sub-section, a brief description of supercapacitor and battery energy storage systems will now be given, as they will be used in subsequent chapters.

* Supercapacitors

Supercapacitors are simply capacitors using plates with extremely high surface areas, thus providing a high storage capacity. Capacitors store electric energy by accumulating positive and negative charges (often on parallel plates) separated by an insulating dielectric. The capacitance depends on the permittivity of the dielectric, the area of the plates, and the distance between the plates. The amount of energy a capacitor is capable of storing can be increased by either increasing the capacitance or the voltage stored on the capacitor.

Capacitors are used in many ac and dc applications in power systems. DC storage capacitors can be used for energy storage for power applications. They have long been used in pulsed power applications for high-energy physics applications. Capacitors are often used for very short-term storage in power converters. Additional capacitance can be added to the dc bus of motor drives and consumer electronics to provide added ability to ride through voltage sags and momentary interruptions. The main transmission or distribution system application where conventional dc capacitors are used as large-scale energy storage is in the distribution dynamic voltage restorer (DVR), a custom power device that compensates for temporary voltage sags on distribution systems [111]. The DVR uses energy stored in dc capacitors to supply a component of the real power needed by the load. Several varieties of advanced capacitors are in development, with several available commercially for low power applications. These capacitors have significant improvements in one or more of the following characteristics: higher permittivities, higher surface areas, or higher voltage-withstand capabilities.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

- Battery Energy Storage System (BESS)

Batteries are one of the most cost-effective energy storage technologies available, with energy stored electrochemically. A battery system is made up of a set of low-voltage/power battery modules connected in parallel and series to achieve a desired electrical characteristic. Batteries are "charged" when they undergo an internal chemical reaction under a potential applied to the terminals. They deliver the absorbed energy, or "discharge," when they reverse the chemical reaction. Key factors of batteries for storage applications include: high energy density, high energy capability, round trip efficiency, cycling capability, life span, and initial cost [43].

Batteries store dc charge, so power conversion is required to interface a battery with an ac system. Small, modular batteries with power electronic converters can provide four-quadrant operation (bidirectional current flow and bidirectional voltage polarity) with rapid response. Advances in battery technologies offer increased energy storage densities, greater cycling capabilities, higher reliability, and lower cost [44]. Battery energy storage systems (BESS) have recently emerged as one of the more promising near-term storage technologies for power applications, offering a wide range of power system applications such as area regulation, area protection, spinning reserve, and power factor correction [45]. Several BESS units have been designed and installed in existing systems for the purposes of load leveling, stabilizing, and load frequency control [46]. Optimal installation site and capacity of BESS can be determined depending upon its application. This has been done for load leveling applications. Also, the integration of battery energy storage with a FACTS power flow controller can improve the power system operation and control. This thesis will address this aspect of the BESS application in a latter chapter.

The state of charge (SOC) of a battery is its available capacity expressed as a percentage of its rated capacity. Knowing the amount of energy left in a battery compared with the energy it had when it was new gives the user an indication of how much longer a battery will continue to perform before it needs recharging. The SOC of the battery is a complex function of these parameters, as well as the service history. Several practical techniques are available to estimate SOC of lead–acid batteries [49-54].
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

❖ Direct Measurement

If the battery could be discharged at a constant rate, the charge in a battery is then equal to the current multiplied by the time for which it flowed. However, in all practical batteries, the discharge current is not constant but diminishes as the battery becomes discharged, usually in a non-linear way. Any measurement device must therefore be able to integrate current over time.

The coulometric measurement is a measure of how much usable energy is available during discharging compared with the energy used to charge the cell. In coulometric measurement, the amount of capacity taken out or put into a battery is measured in terms of ampere-hours. However, this method is impractical in the long term due to accumulation of error. The use of correction factors is required when different discharge rates and ambient temperatures are considered. A monitoring technique combining the open-circuit voltage under no-load condition, and coulometric measurements under constant load has been implemented in [49] on a microcomputer-based circuit.

❖ SOC from Specific Gravity (SG) Measurements

The term specific gravity describes the ratio of the density of electrolyte to the density of water. As the battery discharges the active electrolyte, sulphuric acid is consumed and the concentration of the sulphuric acid in water is reduced. This in turn reduces the SG of the battery acid in direct proportion to the SOC. The SG of the battery acid or electrolyte therefore can be used to determine the battery’s SOC. Hydrometer can be used to measure the density of the electrolyte to give an indication of the SOC. Furthermore, with the development of the electronic technology, electronic sensors which provide a digital measurement of the SG of the electrolyte have been developed to give a continuous reading of the battery condition by incorporating directly into the cells.

❖ Voltage Based SOC Estimation

This method uses the voltage of the battery cell as the basis for SOC calculation. Estimates of SOC have been made by redrawing manufacturer’s discharge voltage versus time characteristics based on changes in internal battery parameters [50] to be measured. Some curve-fitting models have been employed to match the discharge voltage versus
time characteristics to exponential, parabolic, and hyperbolic curves [51]–[53]. A parameter set is then required for each discharge condition, besides the necessity for accurate voltage measurements over a narrow range.

**Current Based SOC Estimation**

This method measures the current entering and leaving the cells as a basis for calculating SOC. The charge transferred into or out of the cell is obtained by accumulating the current drain over time. One of the simplest methods of determining the current is by measuring the voltage drop across a low ohmic value series resistor. Compared with the voltage based method, this method provides higher accuracy but it needs compensation for the operating conditions, which will cause a slight power loss in the current path and also heats up the battery. Hall effect and magneto-resistive transducers avoid this problem but they are more expensive.

**Other State of Charge Measures**

The composition of the active chemicals in the cell changes during the cell charge-discharge cycles, as the chemicals are converted between the charged and discharged states. This will be reflected in changes to the cell impedance. The measurements of cell internal impedance therefore can also be used to calculate SOC. However, this method is not widely used due to difficulties in measuring the impedance while the cell is active.

Recently, an approach has been presented for online SOC estimation in [54]. The technique is based on discharge time versus discharge rate data given in manufacturer's data sheets, combined with coulometric measurements.

### 2.4.2 Impacts of DGs on Power System Stability and Power Quality

With the above introduction to DG and energy storage systems, the impacts of them on power system will now be described. Although DG incorporated into distribution system can be seen to result in much benefits, however, it also increases the complexity of the system operation [68]. Depending on both the DG and distribution system operating characteristics and conditions, not all positive impacts will be fulfilled at the same time.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

Since power system stability and power quality are the main topics of this thesis, the technical impacts of DGs on power system stability and power quality will be discussed [68-77].

- Stability
Generally, a small number of quite small-size DG units, compared to the large centralized power stations, will not tend to influence the power system stability significantly. However, when large numbers of DG units with higher capacities are introduced into the network, the overall dynamics of power systems will be greatly impacted by the DG. For example, if the DG can supply relatively large amount of power in comparison with the upstream central generator, then the tripping of the DG may cause significant system frequency deviation, quite apart from the significant impact this will have on the voltage level.

- Power quality
DG may cause power quality problems such as voltage flicker and harmonic voltage distortion. Visible voltage flicker may be caused by DG starting (such as in induction generator) and its output step changes (e.g. wind turbine and photovoltaic). Similarly, harmonics which can lead to unacceptable network voltage distortion may be injected by incorrectly designed or power electronics-interfaced DG. Furthermore, voltage unbalance may become significant if large numbers of single-phase generators are connected in the network. In general, DGs may improve network power quality by effectively increasing the short-circuit level of the distribution network or deteriorate the power quality by introducing distorted current.

2.5 Custom Power

The concept of Custom Power was first introduced by N. G. Hingorani in [78-81]. Just like Flexible AC Transmission Systems (FACTS) described in Section 2.3 for bulk power transfer, the term Custom Power concerns the use of power electronic controllers for distribution systems. Just as FACTS improves the reliability and quality of power transmission by simultaneously enhancing both the level of power transfer and stability,
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

the Custom Power enhances the quality and reliability of power that is delivered to customers.

Custom Power devices include static switches, inverters, converters, injection transformers, master control modules, and/or energy storage modules. These devices are capable of performing current interruption and voltage regulation functions in a distribution system, for the purpose to improve system reliability and/or power quality. The Custom Power devices can again be sub-divided into two types: compensating type and network reconfiguring type [112, 113].

2.5.1 Compensating Type

The compensating type of devices is used for active filtering, load balancing, power factor correction and voltage regulation. Some of these devices are used in load compensation while others are operated to provide balanced, harmonic-free voltage to the customers. These devices are either connected in shunt or in series or a combination of both [100]. Family members of compensating type devices are:-

Uninterruptible Power Supply (UPS): This is the most widely used shunt compensation device that can help sensitive loads ride through disturbances. UPS can be operated on-line or on stand by mode [100].

Distribution STATCOM (DSTATCOM): This is a shunt-connected device that when connected to the load terminals, it can perform load compensation, such as power factor correction, harmonic filtering and load balancing. It can also perform voltage regulation, when connected to a distribution bus, by injecting an unbalanced and harmonically distorted current to the poor power quality feeder. The protected load will receive a distortion-free voltage [114].

Dynamic Voltage Restorer (DVR) [98, 112, 115, 116]: This is one of the most promising series-connected devices to mitigate voltage sag/swell. A DVR is a forced commutated voltage source inverter that injects a dynamically controlled voltage in series with the supply voltage through a booster transformer to correct the load voltage. By
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

injecting a voltage of required magnitude and frequency, DVR can rapidly compensate for balanced or unbalanced sag/swell in the upstream supply voltage [98, 115, 116]. As it is the device which would be considered in Chapter 6, its operations will be explained in greater detail as follows.

Figure 2-12 shows a possible configuration of a three-phase DVR. A DVR consists of the following main functional components [112]:

- Voltage Source Converter (VSC)

A three single-phase full-leg PWM controlled VSC is used in a DVR. The VSC is composed of four pairs of IGBT and diode as shown in Figure 2-12 and is capable of injecting a set of three-phase AC output voltages \((V_{inj_a}, V_{inj_b}, V_{inj_c})\) which is in series and synchronized with the upstream distribution feeder voltages \(V_s\). Thus the real and reactive power exchange between the DVR and the distribution system can be controlled, as the amplitude and phase angle of the injected voltage are variable.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

- Energy Storage Device
  The energy storage device would act as a buffer by exchanging energy with the external system. The load ride-through capability afforded by the DVR is to a large extent determined by the energy storage capacity. The energy storage device can be an electrolytic capacitor bank, a supercapacitor or a battery bank for most DVR applications.

- Injection Transformer
  The injection transformer is used to boost and interconnect the injection voltage generated by the VSC to the incoming supply voltage. In order to enable the VSC to compensate for the deepest voltage dips at the minimum DC-link voltage, the winding ratio of the transformer has to be cautiously selected.

- Harmonic Filter
  A filtering system is necessary to offer a clean injected voltage on the primary-side of the injection transformer, as the switching of the VSC will generate a large amount of high-order harmonics. The filtering system can be placed either on the primary side (line-side filter system) or on the secondary side (inverter-side filtering system). Generally, the filter system will include a LC section. The values of the filter components will be related with the switching frequency of the VSC.

Although not shown in Figure 2-12, for the proper operation of a DVR, a control and protection system is needed. The control system of the DVR will determine the magnitude and phase of the injection voltage once a voltage disturbance is detected. The DVR is required to possess quick response characteristic and guarantee a high quality waveform.

At present, three voltage restoration strategies: pre-sag compensation, in-phase injection and energy-saving injection, are used to compensate for voltage disturbance [99, 117, 118].

The pre-sag compensation strategy [117, 118] is to inject voltage in phase with the incoming three-phase network voltages to compensate for the difference between the sag/swell and pre-sag/swell voltages. This strategy applies to both balanced and
unbalanced voltage disturbances. From the perspective of voltage restoration, this method is superior to others as the restored voltage will be (theoretically) the same as the pre-disturbance voltage. However, this strategy is based on the assumption that the capacity of energy storage device and the voltage injection capability of a DVR are sufficiently large.

The in-phase injection strategy [117] is the most simple restoration scheme among the three. For this strategy, the injected voltage generated by the DVR is always in phase with the measured supply voltage, regardless of the load current and the pre-disturbance voltage. It is especially suitable for single-phase DVR, since it would ignore the phase shift of the supply voltage. For three-phase system, the situation is quite different because three-phase disturbances could be unbalanced, particularly when the disturbances have different phase shift on each phase. If the in-phase compensation is adopted, the three-phase compensated voltages are likely to remain unbalanced unless the phase shift has been taken into account.

Notice that if the power injection by the DVR is minimized, the same energy storage can be used for a longer period. While the previously discussed two restoration schemes may require a considerable amount of active power injection into the system, they are only suitable if the capacity of the energy storage device of the DVR is sufficiently large and no limit is placed on the injection voltage magnitude. The third strategy, the energy-saving compensation strategy [99], therefore tries to make the same correction with a lower value of active power injection. It uses information about the load current to minimize the depletion of the stored energy while maintaining the load voltage magnitude constant. The method is very attractive as the ride-through ability of the DVR can be increased. However a voltage phase shift at the load terminals will be caused.

**Unified Power Quality Conditioner** (UPQC): This is a versatile device that can inject current in shunt and voltage in series simultaneously in a dual control mode. Therefore, it can perform both the functions of load compensation and voltage control at the same time by injecting unbalanced and distorted voltage and current to the feeder [119].
2.5.2 Reconfiguration Type

The network reconfiguration devices which can be thyristor-based or GTO-based are essentially switchgear, including current limiting, current breaking and current transferring devices. The devices include the following:

**Solid State Current Limiter (SSCL):** This is a GTO-based device that inserts a fault current limiting inductor in series with the faulted circuit as soon as the fault is detected. The inductor is removed from the circuit once the fault is cleared.

**Solid State Circuit Breaker (SSCB):** This is a device that can rapidly interrupt a fault current and also performs auto-reclosing function. Based on a combination of GTO and thyristor switches, this device is much faster than the mechanical circuit breaker and is an ideal device for Custom Power applications.

**Solid State Transfer Switch (SSTS):** This thyristor-based device is used to protect sensitive loads from sag/swell. It can transfer the load undergoing a voltage sag/swell to an alternative feeder within sub-cycle. A SSTS can also be used as a bus coupler between two incoming feeders.

2.5.3 Custom Power Park [79, 112, 120]

By the application of Custom Power devices described earlier power quality can be improved according to customer needs. The concept of “Custom Power Park” integrates multiple power quality technologies and utilizes a communications network between the devices, thus distinguishes it from previous technologies which rely only on stand-alone piece of equipment.

Custom Power Park is installed on the demand side of the network. Custom Power technologies have been rapidly developed to meet the increased demand for high quality electrical power. Various Custom Power devices have been proposed and installed to provide protection against power disturbances. For example, the world's first distributed Power Quality Park has been developed in Ohio by AEP, S&C and in conjunction with
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

EPRI. Some of the results pertaining to the proposed Power Park have been reported in [120]. Implementation of the scheme and monitoring its performance have been reported there as parts of the future work.

2.5.4 Flexible, Reliable and Intelligent Electric Energy Delivery System (FRIENDS)

FRIENDS is another possible form of electric power delivery system for the future [83-92]. Nara and Hasegawa proposed the concept of “FRIENDS” in 1994. FRIENDS is intended to achieve the following functions [85]: (a) Flexibility in reconfiguration of the system in normal and fault states; (b) High Reliability in power supply; (c) Multi-menu services or customized power quality services to allow consumers to select the quality of electric power and supplier; (d) Load leveling and energy conservation; (e) Enhancement of information services to customers; and (f) Efficient demand side management.

Figure 2-13 shows the general concept of FRIENDS [87]. FRIENDS is of a more global concept compared with the Custom Power Park concept. It is seen from Figure 2-13 that FRIENDS is developed to integrate the whole network operation, whereas Custom Power Park is designed toward a local system, such as industrial and commercial application. In FRIENDS, each consumer can select the quality of electrical power independently through the so-called “Power Quality Control Centers” (PQCCs). Similar as the Custom Power Park concept, PQCC is essentially a power quality enhancement facility installed between the high voltage distribution lines and the customers.

As a potential candidate of the fundamental system technologies needed for the future power distribution system, much research work has to be carried out to realize FRIENDS. The overall advantages of FRIENDS over the existing system, including environmental and economic factors, must be quantified. The major apparatus to be installed in PQCC are being developed due to the development of Custom Power and FACTS. Therefore in some regions where the basic of power supply is insufficient, PQCC may be installed for the gradual development towards the form of FRIENDS. The early stage of PQCC would be similar to the idea of Custom Power Park.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

2.5.5 Power Quality Control Center

PQCC is the most significant feature of FRIENDS. PQCC is installed between loads and distribution substations. PQCC permits switching between the multiple distribution network lines fed to the PQCC so as to reduce power losses in the distribution network and ensures continuity of supply during network faulted state. PQCC is also to provide unbundled power quality services for customers connected to it. The main components of PQCC include [92]:

- Thyristor switch: it is used for achieving optimal network operation by reducing losses in distribution lines. And it can avoid interruptions of power supply during upstream disturbances by using seamless switching control method for the switch.
- Voltage regulator (DVR, D-STATCOM): DVR maintain the load voltage over a short-term during upstream disturbances. D-STATCOM compensates the line voltage fluctuations or flickers.

![Diagram of Power Quality Control Center](image)

Figure 2-13 General concept of FRIENDS [87]
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

- **Uninterrupted Power Supply (UPS):** UPS is for short term outages, typically for duration of less than 5 minutes.
- **Distributed Generator (DG):** DG can be used as a co-generator or for emergency situation. As a co-generator, it is online at all time. For emergency generation, it starts to operate only after the UPS is no longer able to support the loads.
- **Active Filter (AF)** [121]: it compensates for unbalance loads and harmonics produced by loads.

There are many ways in combining the above devices to form a PQCC. The optimal combination should be selected by considering its location, costs and functions. Hitherto, several possible structures of PQCC have been proposed [89-95]. In conjunction with the PQCC structures, the authors of [89-95] have also used the following terms to describe power quality levels: normal or ordinary quality, high quality power and super premium power. These are defined as follows:

- **Ordinary Quality (OQ):** the PQCC does not improve quality. The quality supplied to loads is the same as that in the upstream system. The advantage of this is lower tariff for customers.
- **High Quality (HQ):** Voltage disturbances are corrected in the PQCC using some voltage regulating devices. For example, voltage sags of larger than 70% nominal magnitude, and for up to one second are compensated, whereas sags of higher severity are not compensated.
- **Super Premium Quality (SP):** Not only all sags but also outages are compensated for by the PQCC using energy storage devices and local generators.

### 2.5.5.1 UPS-Type PQCC Structure

The configuration of the UPS-type PQCC is shown in Figure 2-14 [89, 90]. It consists of two PWM-controlled inverters (Inv.1 and Inv.2) and a DC-bus, with a DG on the DC-bus. A step-down transformer is envisaged between the upstream power system and the PQCC. Under such an arrangement, three different quality levels for AC loads and one quality level for DC loads have been realized. The operation of the UPS-type PQCC is given as follows [89, 90]:

---

47
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

Figure 2-14 Configuration of UPS-type PQCC [89]

- Normal state operation: the OQ and HQ loads have the same waveform quality levels as they are supplied by the upstream network directly through a transformer. The SP load has very high level of waveform quality which is improved by the two PWM-controlled converters. Under normal state, although the SP and DC load are supplied by the upstream system, it is also expected that the DG meets part of these load demands. Therefore, Inv.1 is operated in rectifier mode.

- Faulted state operation: the OQ load will be disconnected from the network by the solid-state switch if the PQCC cannot be supplied by the upstream network due to an upstream fault. The remaining loads are supplied by the DG. With this operation, the supply reliability of HQ, DC and SP loads can be improved. In the process, Circuit Breaker (CB) is tripped. Inv.1 enters the inversion mode and transfers power from the DG to meet the HQ load requirement. Inv.2 is still operated in inversion mode to supply the SP load.

2.6 Hierarchical Network Control

Having described the various techniques to improve power system stability and supply quality, it could be useful to review how voltage stability control is achieved in typical electrical systems. In particular, the concept of hierarchical network control is examined next. This is because power systems are naturally decentralized in structure and, therefore, in need of multilevel hierarchical control schemes to attain wider-area objectives for optimum operation.
In general, a multilevel hierarchy can be defined as a vertical arrangement of a number of subsystems. The subsystems are arranged with defined priority of action and rights of intervention. The objective is to help achieve system-wide goals. Depending on the system-wide objectives and the structure of the system, the subsystems may have cooperative or competitive objectives. Higher-level controllers can therefore serve in a conflict resolution role. Figure 2-15 illustrates a multilevel hierarchical control structure. The system-wide controller/coordinator are concerned with a larger portion or broader aspects of the system behavior, dealing with slower phenomena and in this way, allows more time for decision-making and control computation.

Figure 2-15 Power system area and cluster controllers in a multi level hierarchical arrangement

In the context of a power system, it is divided into a number of zones. The zone central controllers exchange information with the higher level (and possibly with each other) to jointly compute the control action required to achieve zone-wide and system-wide performance objectives. Furthermore, each zone is subdivided into a number of clusters, each containing one or more controllers. Each cluster controller will then only report to and receive coordination/control signals from the higher level, i.e., the area central controller, to help attain system-wide performance goals. A natural time decomposition accompanies this control functionality decomposition. The resulting scheme depends on
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

the geographic at location of the subsystems, the communications links available, and the time delays involved.

In power systems, inherent multiple time-scales and weak coupling between real and reactive power dynamics have allowed a simplified form of this hierarchical centralized coordination. Nowadays there has been no system-wide coordinated closed-loop voltage control system implemented in the United States. In Europe, this work has been ongoing over the past decade, and recent work has also been reported in Brazil. To the author’s knowledge, there are no automatic system-wide MW injection/flow coordination and control schemes currently in existence. Instead, this chapter will focus on the coordinated hierarchical voltage control system which has been examined by several researches.

2.6.1 A Short Review of Hierarchical Voltage Control Principles

In order to achieve the regulation of the voltage within allowable range, a continuous control of the generators voltage set-points and of the compensation means has been implemented on practical networks, such as that on the French EHV power system [6]. The possible actions have been organized in three hierarchical levels, which concern distinct geographical sizes and time constants:

- **Primary Level**: at this level, it consists basically generator unit and plant control. The first action involves the adjustments of automatic voltage regulators installed in generators. The second one maintains the high-side voltage of step-up transformers to specific values, so as to avoid reactive power interchange among generating units. Primary actions are very fast, in a time frame of up to a few seconds. It is considered a local control;

- **Secondary Level**: the main objective of this level is to adjust and to maintain the voltage profile inside a network area. Control actions in this level are carried out by reactive power compensation devices such as capacitors, inductors, synchronous or static voltage compensators and by under-load tap changers (ULTC). The principle of the secondary voltage control is to divide the power network into distinct geographical parts, called “zones”, and to control the voltage profile separately in each zone by the automatic adjustments of, for example, the
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

AVR of some units (called “controlling generators”) located in the zone. These adjustments lead to variations of the reactive power supplied by the controlling units. The size of the adjustment is determined by the difference between a set-point value and the voltage value of a special node in the zone, sometimes called “pilot point”. Definition and implementation of the secondary voltage control level are quite dependent on the philosophy of each utility. The time scale for secondary level is from several seconds to a minute. It is considered a regional control;

• Tertiary Level: the purpose of this level is to determine an optimal voltage profile of the network and to coordinate the secondary controllers, according to safety and economic criteria. The Tertiary Voltage Controller (TVC) operates at the highest hierarchical level, and coordinates in a centralized way the actions of the Regional Voltage Controllers (RVCs) by determining the optimal voltage values of the pilot buses. The most suitable values of the RVCs set points are determined and updated, instant by instant, by the TVC on the basis of the optimal forecast voltages and of the real time measurements of the system stage. The time frame of this control is several minutes or on demand.

The success of the above described control scheme mainly depends upon the way of selecting the pilot buses and the control generators, and upon the coordination of the regional controllers. These will be described in greater detail, as follows.

2.6.1.1 Pilot Point Selection

The selection of load buses as pilot point candidates is made so that, although there are few of them, the information from them is sufficient to control the voltage profile of the system, i.e., the pilot points must be chosen so as to have voltage variations representative of the voltage evolutions throughout the zone. This condition is fulfilled if the electrical distance between the pilot point and the nodes is short. In [9], the pilot points for the French system are selected in the following way: first the system is divided into regions typically associated with the particular power pools. One pilot point is selected for each region which is to be the load bus with the largest short circuit level.
2.6.1.2 Secondary Voltage Control Law

Once the pilot points are found, a control scheme for resetting reference (set) points for voltage controlling devices (on the French system, only automatic voltage regulators of generators) is implemented such that all of the generators in a certain region are dedicated to controlling the voltage at the pilot point, and the information available to all generators is only the voltage at the pilot point in the specific area. Furthermore, all generators in a given region are operated at the same relative portion of reactive power described below. Thus control of the pilot point voltage in a given region involves only one measurement and one control decision making. Once the control is obtained for one generator, the others in the same region operate with the same portion of the maximum reactive power that they can produce.

Figure 2-16 shows how the control is performed. The reactive power of the controlling units is adjusted by two control loops, which are superimposed on the AVRs of these generators. A control signal \( N (N = \alpha \int_{0}^{t} \frac{V_c - V_p}{V_n} dt + \beta \frac{V_c - V_p}{V_n}) \) is obtained from the difference between the pilot node’s measured voltage \( V_p \) and the set-point \( V_c \) (which can be determined by the tertiary control system), using a proportional integral law. The control signal is generated in a dedicated microcomputer located in the regional control center; then it is transmitted to each controlling unit, to be used as the input of a second control loop (the reactive power loop) which modifies the AVR’s set-point value, taking the possible participation factor \( Q_r \) of the generator into account, in order to make its reactive power output equal to \((N \cdot Q_r)\).
This intermediate control, performed through a reactive power loop, provides a simple coordination of the reactive outputs of the different controlling generators.

Note that the success of a coordinated secondary control for each region briefly described strongly depends on a good choice of a voltage control region. A region is well defined if the following three assumptions hold [8]:

- **Assumption 1**: When the voltage of pilot node is maintained at a steady level, the variations of the other load voltages in the region remain small even with the load variations.
- **Assumption 2**: The control actions in a given zone do not cause significant voltage variations in the other zones.
- **Assumption 3**: The zone has sufficient voltage control capacity to keep the pilot point voltages steady in each region, under both normal and emergency conditions.

### 2.6.1.3 Improved Secondary Voltage Control Law

In the above control scheme, the voltage control areas (the "zones") are assumed to be decoupled. If this condition is fulfilled, the multivariable system (i.e. the power system) can be dealt with as an aggregation of separate and independent mono-variable subsystems (i.e. the zones). However, as the mesh of the power system is getting increasingly...
dense, it would become difficult to define appropriate zones in some regions, with sufficient homogeneity and independence with regard to voltage control. In order to cancel the effects of the neighboring regions on a regional performance criterion, an improved secondary voltage control scheme was proposed by Ilic [10]. Unlike the EDF secondary voltage regulator which is based only on the regional measurements, i.e. regional pilot point voltages, this improved secondary voltage control takes into consideration the effect of interconnections, while preserving its decentralized nature. The proposed control law is that it cancels out the effect of interactions based on additional feedback signals which use the reactive power tie-line flow measurements. It must be stressed that the other two conditions must be fulfilled to ensure a good control: (i) Sufficient reactive power must be available in the zone; (ii) The electrical distance between the pilot point and the nearest adjacent zones must be large enough to prevent undesirable influence between different zones. This latter condition is generally easily satisfied, because voltage problem tends to be localized.

2.6.2 A Historical Perspective of the Hierarchical Voltage Control

In the early eighties, while the voltage magnitude can be changed through state estimator, the data were typically not utilized for any direct voltage control. This is mainly because it is much easier to monitor and understand smaller data sets (such as the system frequency and tie line flows), which clearly reflect the system-wide active power imbalances, than the large amount of the voltage magnitudes associated with the load buses throughout the system. Moreover, voltage-related problems tend to be more local in a geographical sense. Therefore, voltage security needs to be monitored and controlled locally across the entire power grid.

After many incidents of voltage-related operational problems had been reported worldwide [2], the significance of reactive power control has increased considerably. In order to work with fewer voltage data and make the real-time monitoring and control more manageable, many efforts were made. The hierarchical information and control structure based on pilot point was introduced in France for the first time, and it has been implemented by EDF since 1979 [8]. As was explained earlier, a pilot point is a load bus at which the voltage is measured in real-time and utilized for control actions. In this
infrastructure, voltage-control devices (automatic voltage regulator of generators, load tap changing transformers, capacitor banks, etc) attempt to maintain the voltages within a desired threshold of the reference voltages. At this primary control level, only local information is utilized. Then the system is divided into several regions, a secondary control scheme using information from the pilot points in each region will update the reference voltages of the primary control devices.

The pilot points are selected to obtain enough information from them to control the voltage profile of the region, although the number of them is few. At the beginning of the implementation in the EDF system, the load bus with the largest short circuit current is selected as the pilot point for each region [9]. Once the pilot points are found, a control scheme is implemented with all generators in a given region operating at the same proportion of relative reactive power ("aligned" operation). Thus control of the pilot point voltage in a given region involves only one measurement and one control decision. Several other algorithms for pilot points selections are explained in [10, 11].

The above concept has two assumptions. Firstly the voltage control is regional, secondly it is necessary to have a system-wide coordination of the reactive power flow between different regions. This control level is referred to as the tertiary control. The French control concept is only one version of the general reduced voltage information structure for monitoring and control.

The secondary voltage control assumes that interaction within the neighboring regions is negligible. As the power system has become increasingly meshed and is operated closer to its transmission limits, an improved control design at the secondary level was proposed, which uses additional measurements to counteract the effects of the neighboring regions on a regional performance criterion [12]. However, it is efficient only when there are sufficient reactive power reserves. EDF and some other utilities also considered a coordinated secondary voltage control scheme, which takes into account operating constraints in order to manage interaction between coupled areas [11, 13, 14].

The tertiary voltage control operates at the highest hierarchical level. It coordinates the actions of the regional voltage control in a centralized way, which defines and actuates the
optimal voltage pattern of the pilot nodes in real-time [13, 14]. In Italy ENEL, the tertiary voltage regulator has strong interaction with the reactive power scheduling functions [13]. In Belgium [14], coordinated voltage control is in operation since 1998. Every 15 minutes or on request (e.g. following important disturbances), a tertiary voltage control scheme is computed using an optimal power flow with dedicated objective function. It optimizes system-wide generator reactive reserves and shunt capacitor bank switchings under constraints of voltage limits and reactive power area balance. The actual optimization is completed by linear programming, with the quadratic objective function linearized into segments.

References [15, 16] introduced an on-line voltage control using full SCADA information implemented in the New England system. This control is also generation-based. It maintains least square minimization of voltage deviations from their desired, “optimal” voltage profile as system condition changes. The localized echelon-based approach was also proposed to achieve the solution for a small portion of the system.

On the on-line control, most of today’s power systems are mechanically controlled with mechanical devices, such as circuit breakers. Control cannot be initiated frequently because mechanical devices tend to wear out quickly compared to static devices. In [16], it was indicated that the software developed based on [15] is not being used by the utility, partly because the operators hesitate to switch capacitors frequently.

Technology developments in power electronics combined with sophisticated electronic control methods have made the implementation of fast Static Var Compensators (SVC) possible in the later 1970s [17]. Today, there are hundreds of SVC installations, worldwide [18]. SVC is considered an early version of a FACTS device.

The SVC is a common shunt compensation device. It was first used in Basin Electric Power Cooperative in 1977 in order to provide automatic and continuous voltage control on an 115kV network in Western Nebraska [17]. Engineering studies pertaining to the application of a TCR type SVC are described in [17] and the results were validated by some staged tests in [18]. Excellent overviews on SVC applications are contained in [19].
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

These are useful reference guides for prospective system planners contemplating using an SVC as a means of stabilizing network voltages.

Since the invention of the SVC, much research attentions have been directed towards the applications of FACTS. For example, Mahran et al [20] investigated a system in which SVC was located at the terminal of the generator terminal. It means control signals are locally available, but this is not practical because most SVC are geographically distributed within networks. In 2001, Wang [21] proposed a coordination control strategy based on multi-agent request-and-answer type of protocol between any two agents. However, this secondary voltage control can only cover locations where voltage controllers are installed. Paserba [22] discussed the concept of coordinating a STATCOM with local voltage-var control devices such as load-tap changers (LTCs) and capacitor banks, for long term voltage-var management. Long term voltage-var management was considered for any one of the following three objectives: (1) Resetting a STATCOM by a simple reactive power runback function so that it would be available for the “next” dynamic event on the system; (2) Improving the overall system voltage profile by coordinating the STATCOM with local LTCs and/or capacitor banks; (3) Reducing LTC tap movements by coordinating the STATCOM with local LTCs and/or capacitor banks. The advantages and disadvantages of applying secondary controls to STATCOMS for each of the above-listed objectives are also discussed. In a later work, Paserba [23] presented actual cases of secondary voltage-var controls applied to Static Compensators (STATCOM) for fast voltage control and long term Var management. The secondary control is used to ensure that an adequate range of the STATCOM dynamic capability is available for major system disturbances. The output of the secondary controls calls for the switching of capacitor banks to “reset” the reactive power output of the STATCOM to a pre-specified level after a system event (long term), or during the course of a daily load cycle (long term), or during an event for voltage control (fast). The voltage control structure is determined by experimental method.
Chapter 2: A Review of Selected Power Electronics-Based Power Flow Controllers and Custom Power Control Techniques

2.7 Conclusions

The Chapter contains a brief description of the power stability problems encountered in electrical networks. Various types of FACTS devices which can be used to improve power system stability are also described. The basic principle of the stabilization through FACTS has been included.

On power quality, the various types of disturbances and the techniques to enhance supply quality are also included. The provision of Unbundled Power Quality service is also introduced, together with a review of earlier works pertaining to the realization of such service in the future power distribution systems. The systems described include the Custom Power Park and FRIENDS. A description of Power Quality Control Center (PQCC) structure and functions is also given. Within the PQCC, a number of technical details on its design remain to be investigated. One of the following Chapters will attempt to address the issues of the compensation strategy.

Finally on network voltage control, a literature review of the research area shows a strong interest in the coordinated voltage control of power systems. In the next chapter, the results of an attempt to place the control scheme on a more theoretical basis will be presented.
Chapter 3

A Coordinated Hierarchical Voltage Control Scheme

This chapter is organized as follows. In Section 3.1, some background on hierarchical voltage-reactive power control is provided. The system description is presented in Section 3.2. In Section 3.3, the frequency response technique is introduced. The proposed coordinated hierarchical voltage control scheme is elaborated in Section 3.4. Mathematical analysis of proposed control scheme and the simulation results are presented in Section 3.5 and 3.6 respectively. Finally Section 3.7 concludes this chapter by highlighting contributions made.

3.1 Introduction

As was explained in Chapter 2, voltage-reactive power control is one of the most important considerations in the operation of modern power systems. Voltage stability is related to power generating, transmission and load systems. During the process of power flow from power plants into customers' facilities, electric energy must transit transmission system, which is constituted by transmitting line, transformer, etc. The characteristic of the transmission system is a determining factor for voltage stability. Due to the characteristic of reactive power, it is very difficult to transit reactive power over long distance [4]. Thus how to control the voltage over a long distance radial transmission line system is of great challenge.

This chapter examines a typical radial transmission system and proposes a coordinated hierarchical control scheme to control the voltage of this system. It is the most convenient way to demonstrate the principle of the control scheme by making use of an actual operating power system. Some of the materials contained in this chapter have been reported in [122].
3.2 System Description

The particular network considered in this chapter is a radial transmission system shown in Figure 3-1. It interconnects the main generating station at A with a load center at G. Node A at the sending end is connected to the equivalent generation ($G_1$) in the main grid, and a much smaller generation unit ($G_2$) is connected to node G. The main distribution system is connected to node G, however, there are also small loads supplied at the intermediate points C, D and E along the route of the interconnection. At D which is about the midpoint of the interconnection, there is one Static VAr Compensator (SVC). At F, the receiving end of the interconnection, there are also two SVCs.

In this study, each of the SVC is represented by a thyristor controlled reactor (TCR). Associated with each TCR compensator is a set of harmonic filters comprising the second-harmonic filter, the third-harmonic filter and the high-pass filter. Each set of harmonic filters is denoted by $H$ in Figure 3-1. For each TCR compensator, there is a switched capacitor bank (FC) and it is under independent circuit-breaker switching control.
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

In this power system, there are two equivalent generating units at the two ends of the long distance transmission system, and three SVCs (one at the mid-point of the transmission system, another two at the end of the transmission system). It is a suitable model for the application of the proposed coordinated hierarchical control scheme for long distance radial transmission system.

It is also important to examine the role play by node D. Node D is the midpoint of the transmission system. As will be described in Chapter 4, if the voltage of this node can be maintained constant and if losses in the system can be ignored, then it is equal to a electrical source node. Thus the electrical distance between electrical source ($G_i$) and load ($F$) is shortened to half, and the transmitting capacity of the power systems is doubled. Dynamic voltage stability will also be improved. Hence, Node D in this study is chosen as the pilot bus.

3.2.1 Generator Model

Each of the generator $G_i$, $i = 1, 2$, is represented by a third-order model by assuming negligible amortisseur effect, armature resistance and by ignoring the armature $d\psi/dt$ terms and saturation effect. Such representation is considered sufficient for the purpose of this voltage control study.

Hence, for the $i$-th generator, the following equivalent generator equations can be obtained [123]:

\[
\begin{align*}
\frac{d\omega_i}{dt} &= \left(\frac{1}{T_{qi}}\right) \left[ P_{gi} - P_{li} \right] \\
\frac{d\delta_i}{dt} &= 2\pi f_0 \left(\frac{\omega_i}{\omega_j} - 1\right) \\
\frac{dE'_{qi}}{dt} &= \left(\frac{1}{T_{d(qi)}}\right) \left[ E_{ghi} - E_{gqi} - (X_{qi} - X_{di})i_{di} \right] \\
E'_{qi} &= V_{qi} + i_{di}X_{qi} \\
0 &= V_{di} - i_{qi}X_{qi}
\end{align*}
\]

(3.2.1)

The meaning of each of the symbols is included in Appendix A. Since the main concern is on the steady-state stability behavior of the power system, the generator equations under small perturbation conditions will be of interest and thus (3.2.1) becomes:
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

\[
\begin{align*}
\frac{d(\Delta \omega_j)}{dt} &= (1/T_{v_j})[\Delta P_{v_j} - V_{q0}\Delta i_{qj} - V_{d0}\Delta i_{dj} - i_{q0}\Delta V_{qj} - i_{d0}\Delta V_{dj}] \\
\frac{d(\Delta \delta_j)}{dt} &= 2\pi f_c \Delta \omega_j \\
\frac{d(\Delta E_{qj})}{dt} &= (1/T_{m_j})[\Delta E_{qj} - \Delta E_{qj} - (X_{qj} - X_{dj})\Delta i_{dj}] \\
\Delta E_{qj} &= \Delta V_{qj} + X_{qj} \Delta i_{qj} \\
0 &= \Delta V_{ai} - X_{ai} \Delta i_{qj}
\end{align*}
\]

(3.2.2)

### 3.2.2 Load Model

The choice of load model is a compromise between the model accuracy and the convenience for controller designs. At this stage, the load model is to be of constant impedance.

In general, loads may be of constant-impedance, constant-power, constant-current, or any combination of these three. The so-called ZIP [124] model can be used to represent each of the 3 types or combination of load types, as follows:

\[
P = P_0 \left[ Z_p \left( \frac{V}{V_0} \right)^2 + I_p \left( \frac{V}{V_0} \right) + P_p \right] \\
Q = Q_0 \left[ Z_q \left( \frac{V}{V_0} \right)^2 + I_q \left( \frac{V}{V_0} \right) + P_q \right]
\]

(3.2.3)

where \( Z_p, Z_q, I_p, I_q, P_p \) and \( P_q \) are constant impedance, constant current and constant power fractions, respectively. \( P_0, Q_0 \) are the steady-state load real and reactive powers at the nominal voltage level \( V_0 \). This model assumes a second order polynomial relationship between power and voltage, which can represent some simple nonlinear loads.

The ZIP model is in a form inconvenient for controller design and further simplification is necessary. Suppose the composition of the load is known, i.e. the constant impedance fraction \( Z_p \) and \( Z_q \), constant current fraction \( I_p \) and \( I_q \), and constant power fraction \( P_p \) and \( P_q \) are also known. Since the main concern at this stage is on the steady-state stability behavior of the power system, the proposed scheme aims to keep voltage constant to the pre-disturbance level, or as close to it as possible. A successful coordinated hierarchical control scheme would ensure the voltages of the transmission system to be quickly restored to the nominal values following contingencies and the load bus voltage variation.
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

$\Delta V$ would be expected to be small under small perturbation conditions. Linearize the system at the operating $P_0, Q_0, V_0$, (3.2.3) can be re-written as:

$$
\begin{align*}
\Delta P &= \frac{P_0}{V_0} \left( 2Z_p + I_p \right) \Delta V \\
\Delta Q &= \frac{Q_0}{V_0} \left( 2Z_q + I_q \right) \Delta V
\end{align*}
$$

(3.2.4) shows that the load demand variation $\Delta P$ and $\Delta Q$ with respect to the load bus voltage variation $\Delta V$. Since $\Delta V$ is small, $\Delta P/P_0$ and $\Delta Q/Q_0$ will also be small. Hence the load demand would remain close to the pre-fault level. Thus the load can be represented by an equivalent constant impedance

$$
\bar{Z} = \frac{V^2}{P - jQ} = \frac{1}{G - jB} = \frac{G}{G^2 + B^2} + j \frac{B}{G^2 + B^2}
$$

(3.2.5)

where $G = \frac{P}{V^2} = \frac{P_0}{V_0^2} \left( Z_p + I_p + P_p \right)$ and $B = \frac{Q}{V^2} = \frac{Q_0}{V_0^2} \left( Z_q + I_q + P_q \right)$. It is therefore seen that $G$ and $B$ can be expressed in terms of $Z_p$, $Z_q$, $I_p$, $I_q$, $P_p$ and $P_q$. Therefore there is no loss of generality when the constant impedance load model is used in subsequent sections when analyzing the proposed scheme.

3.2.3 Static Var Compensator (SVC)

As described in Chapter 2, Static Var Compensators (SVCs) are shunt-connected var generators and/or absorbers whose outputs are varied so as to control specific parameters of the power system. The voltage-current characteristic and relation of SVC are shown in Figure 3-2 and Figure 3-3, respectively.

![Figure 3-2 Voltage-current characteristic of SVC](image)
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

\[ V_{SVCi} \]

\[ I_{SVCi} \]

\[ B_{SVCi} \]

Figure 3-3 Voltage-current relation of SVC

From Figure 3-2, the characteristic of the i-th SVC is:

\[ V_{SVCi} = V_{ref} + X_{SLi} I_{SVCi} \]  

(3.2.6)

Where, \( X_{SLi} \) is the slope reactance. This is applicable over the linear operating range \( I_{or} < I_{SVC} < I_{Lr} \).

After linearization, (3.2.6) can be re-written as:

\[ \Delta V_{SVCi} = X_{SLi} \Delta I_{SVCi} \]  

(3.2.7)

From Figure 3-3, \( V_{SVCi} \) is also related to \( I_{SVCi} \) through the SVC reactance \( B_{SVCi} \), as follows:

\[ I_{SVCi} = B_{SVCi} V_{SVCi} \]  

(3.2.8)

After linearization, (3.2.8) can be re-written as:

\[ \Delta I_{SVCi} = B_{SVCi0} \Delta V_{SVCi} + V_{SVCi0} \Delta B_{SVCi} \]  

(3.2.9)

Substituting \( \Delta I_{SVCi} \) from (3.2.9) into (3.2.10), \( \Delta V_{SVCi}/\Delta B_{SVCi} \) can be solved,

\[ \frac{\Delta V_{SVCi}}{\Delta B_{SVCi}} = \frac{X_{SLi} V_{SVCi0}}{1 - X_{SLi} B_{SVCi0}} \]  

(3.2.10)

3.3 Frequency Response Technique

As the well-known frequency response technique will be adopted as the main analytical tool in this chapter, it is therefore prudent to provide a brief revision of some basic concepts here.

The frequency response of a system can be viewed in two different ways: via the Bode plot or via the Nyquist diagram. Both methods display the same information. The difference lies in the way the information is presented. Bode plot method is introduced in
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

this section since it is widely used in this research work. As stability analysis using frequency response method has long been well established [125], only a brief introduction will be given here.

- **Bode Plot**

A sinusoidal transfer function may be represented by two separate plots, one giving the magnitude versus frequency and the other the phase angle (in degrees) versus frequency. A Bode plot consists of two graphs: one is a plot of the logarithm of the magnitude of a sinusoidal transfer function; the other is a plot of the phase angle; both are plotted against the frequency in logarithmic scale. If \( G(s) \) is the open loop transfer function of a system and \( \omega \) is the frequency vector, Bode plot is used to plot both the magnitude and phase of \( G(j\omega) \) vs. \( \omega \). The standard representation of the logarithmic magnitude of \( G(j\omega) \) is \( 20\log|G(j\omega)| \), where the base of the logarithm is 10. The unit used in this representation of the magnitude is the decibel. In the logarithmic representation, the curves are drawn using the log scale for frequency and the linear scale for either magnitude or phase angle.

The main advantage of using Bode plot is that multiplication of magnitudes can be converted into addition. Furthermore, a simple method for sketching an approximate log-magnitude curve is available. It is based on asymptotic approximations.

- **Gain and Phase Cross-Over Frequencies**

The gain cross-over frequency is the frequency at which \( |G(j\omega)| \), the magnitude of the open-loop transfer function is unity. The phase cross-over frequency is the frequency at which the phase angle of the open-loop transfer function is equal to \(-180^\circ\). The gain and phase cross-over frequencies are illustrated in Figure 3-4 as \( \omega_{gc} \) and \( \omega_{pc} \) respectively.

- **Phase and Gain Margins**

The phase margin is the difference in phase between the phase curve and \(-180^\circ\) at the point corresponding to the gain cross-over frequency \( \omega_{gc} \). The phase margin is that amount of additional phase lag at \( \omega_{gc} \) required to bring the system to the verge of instability. Thus, the phase margin is equaled to the change in the open loop phase shift which will make the closed-loop system unstable. Likewise, the gain margin is the
difference between the magnitude curve and 0dB at the point corresponding to $\omega_{pc}$. The gain margin is defined as the change in open loop gain required to making the system unstable. Systems with greater gain margin can withstand greater change in system parameters before becoming unstable in closed loop.

For a minimum phase system, both the phase and the gain margins must be positive for the system to be stable. Negative margins indicate instability. In other words, if the gain cross-over frequency is less than the phase cross-over frequency (i.e. $\omega_{gc} < \omega_{pc}$), then the closed-loop system will be stable [15]. This conclusion is clearly shown in Figure 3-4.

Indeed, in designing a control system, not only the system must be stable but it is also necessary that the stable system have adequate degree of stability, i.e., adequate relative stability. Control systems with adequate phase and gain margins ensure stability against variations in the system components. The two stability margins bound the behavior of the closed-loop system near the resonant frequency. For satisfactory performance, the phase margin should be typically between $30^\circ$ and $60^\circ$ and the gain margin should be greater then 6db. With these values, a minimum phase system has guaranteed stability, even if the open-loop gain and the time constants of the components vary to a certain extent.

![Bode Diagram](image)

**Figure 3-4 Bode plot: phase and gain margins**
3.4 Coordinated Hierarchical Voltage Control Scheme

In line with the methods of secondary voltage control of power systems described in [6-14], the proposed control scheme is organized in three hierarchical levels, which concern distinct geographical sizes and time constants. The description of the scheme is as follows.

At the highest level or the tertiary level, a scheduling function ensures that loads are supplied, adequate reserves for ancillary services are in place, network operating constraints and transmission contracts are respected, and maximum possible efficiency prevails. Through optimal power flow (OPF) calculation, initial set points of all controllers are obtained. The timeframe for the tertiary level is from one or several hours to the day ahead. This function is executed at a coordinating level such as a power pool or an Independent System Operator (ISO).

At the secondary level, control actions are carried out by adjusting set points for generator automatic voltage regulators (AVRs) and slowly reactive power compensation devices, such as mechanically switched capacitors. The time frame for secondary level is from several seconds to a few minutes.

At the primary level, control actions are carried out by the adjustment of AVRs installed in generators, and by fast reactive power compensation devices which are, at this case, the SVCs. Primary actions are very fast, the time frame for primary level is from several milliseconds to a few seconds.

At this stage, the project is focused on the later two levels, i.e., the secondary and the primary levels.

Control functions of this scheme include both voltage control and reserve capacity control. Both of these two functions concern both primary and secondary control levels. The realization of those two control functions is as follows.
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

❖ Voltage Control
At the primary level, the AVR monitors the generator terminal voltage. If the generator terminal voltage is below (or over) the initial value, the AVR regulates generator to output (or absorb) reactive power to recover the generator terminal voltage quickly. The response time of AVR is typically within 500 milliseconds. The primary SVC controller monitors the pilot bus voltage. If the pilot bus voltage is below (or over) the initial value, the primary SVC controller regulates SVC to support the pilot bus voltage by injecting or absorbing reactive current quickly. The response time of SVC is typically around 20 to 100 milliseconds.

The secondary voltage control also monitors the voltage of the pilot bus. If the voltage of the pilot bus is below (or over) the initial value, the secondary generator controller regulates generator to output (or absorb) reactive power by adjusting set point for the generator AVR. If the voltage error (typically +/−2%) of the pilot bus exceeds a threshold for a specified time (typically seconds), then a connect (for low voltage conditions) or disconnect (for high voltage conditions) signal is given to the mechanically switched capacitors, these capacitors will be switched on or off. The time interval before a subsequent switch signals is typically tens of seconds or a few minutes.

❖ Reserve Capacity Control
The reserve capacity control is designed to enable the operating point of the SVC to be offset into the inductive region so that a desired "net capacitive range" or "reserve capacity" can be achieved. Hence, the SVC will retain a better capacity to respond quickly to disturbances. Reserve capacity is defined as the available net change in SVC output towards the capacitive region from a given operating point. For example, if the SVCs are operating with zero net output, the reserve capacity will be equal to 0 MVAR. If the operating point is biased into the inductive region, for example to 17 MVAR or 21 MVAR inductive, then the reserve capacity will be 17 MVAR or 21 MVAR, respectively.

The desired reserve capacity is a function of the system loading conditions with generally higher reserve capacity (i.e., more biasing into the inductive region) required under heavy load conditions. Under light load conditions the system requirements for reserve capacity
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

are lower and it is advantageous to operate the SVC at the low or medium reserve capacity settings.

The reserve capacity requirement is achieved by the secondary generator control and automatically connecting or disconnecting shunt capacitors, i.e., after disturbance, if the pilot point voltage can be brought back to its initial value and generators still have reactive power margin, secondary generator controller drives the generators to continue to increase their reactive power output by adjusting the set points for generator AVRs. If the reactive power output of SVC is outside a settable threshold for a specified time, i.e., the reactive power output of SVC is bigger than the capability of one mechanically switched capacitor, then a connect signal is given to this capacitor. This capacitor will be switched on. After some delay equivalent to the charge time of this capacitor, this capacitor will be on service. Then the output of SVC can be biased into the inductive region so that a desired “net capacitive range” or “reserve capacity” can be achieved.

Figure 3-5 depicts the two-level hierarchical control structure as just described. The detail of Primary SVC Controller and Secondary Generator Controller are shown in Figure 3-6 and Figure 3-7 respectively.

![Figure 3-5 Structure of the two-level hierarchical voltage control loop](image)

- Primary SVC Control

Figure 3-6 shows the block diagram for the Primary SVC controller. The set-point is subtracted from the pilot bus voltage to form the error signal. This signal is fed to the
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

voltage proportional-integral (PI) regulator, which is modeled as \(k_p + \frac{k_i}{S}\). The output of the regulator drives the thyristor-reactor unit, which is represented by the ratio \(L_{\text{base}} / S_{\text{base}}\) and a small time constant \(T_{2i}\). \(L_{\text{base}}\) is equal to the MVA range that is under the control of thyristor-controlled reactors and \(S_{\text{base}}\) is equal to system MVA base. The thyristor-unit output is then subtracted from that of a fixed capacitor output, to form the SVC network admittance. \(T_{2i}\) represents the time lags inherent in the thyristor-reactor unit. A typical assumption is 0.025 seconds. The network in turn responds to changes in SVC output by changing its voltage, therefore closing the voltage-control loop. The pilot bus voltage response to changes in SVC admittance is represented by \(\Delta V_p / \Delta B_{\text{SVCI}}\). Parameters \(k_p\) and \(k_i\) are application-dependent and must be tuned accordingly. More will be said in Section 3.5 on the tuning of these parameters.

![Figure 3-6 Block diagram of the i-th primary SVC control loop](image)

- Secondary Generator Control

Figure 3-7 shows the configuration of the secondary generator control (SGC) loop. The set-point is subtracted from the pilot bus voltage to form the error signal. This signal is fed to the voltage PI regulator, which is modeled as \(K_{SGC} \frac{T_p S + 1}{T_n S}\). The signal output of the regulator is sent to an inertia loop \(\frac{1}{T_n S + 1}\), \(T_n\) is time constant which is about 5 seconds. It can be used to realize the decoupling of secondary voltage controller and primary voltage controller. The signal output of the inertia loop is sent to the generator AVR. It will adjust the set-point for the generator AVR, and regulate the generators’ reactive power output. The network in turn responds to changes in generator reactive power output by changing its voltage, therefore closing the voltage-control loop. The pilot bus voltage response to changes in generator terminal voltage is represented by the
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

transfer function block \( \Delta V_p / \Delta V_i \cdot K_{SGC} \cdot T_{si} \) and \( T_{pi} \) are application-dependent and must be tuned accordingly. This will be discussed in Section 3.5.3.

A supplementary input signal \( \Delta V_{ri} \) is injected into the error-summing junction point of SGC loop as shown in Figure 3-7. This signal is used for reserve capacity control. At the normal operation condition, \( \Delta V_{ri} \) is set to be zero. When disturbance occurs, \( \Delta V_{ri} \) is set to be a small value (say, 0.1%), thus the secondary generator controller will drive the generator to continue increasing its reactive power until the output of the SVC is within a specified deadband (say, 1.2MVar). Hence, the output of SVC can then be biased into the inductive region so that a desired “reserve capacity” can be achieved.

![Figure 3-7 Block diagram of the i-th secondary generator control loop](image)

Capacitor Bank Selection

The secondary control (voltage control or the reserve capacity control) sends a signal when a capacitor bank switching event (connect or disconnect) is requested. The algorithm adopted for the capacitor bank selection is that: First they are switched on or off based on their bus voltages (i.e., lowest voltage on first, highest voltage off first). If a selected capacitor bank is already on-line or is disabled, the selection controller searches for the next one in the hierarchy.

3.5 Analysis

3.5.1 Network Model Derivation

In this analysis, it is necessary to obtain a model to describe the transmission network. The detail derivation of the following results is tedious and would only be included in
Appendix B. Briefly, by referring to Figure 3-1 and through a series of $\Delta$-Y transformation, one can derive the equivalent circuit shown in Figure 3-8 (a). Further analysis yields the equivalent circuits 3-8 (b)-(d).

Figure 3-8 Equivalent circuit diagrams of Figure 3-1

where,

\[
\bar{Z}_{\text{equ1}} = \bar{Z}_{L1}^* \parallel \bar{Z}_{12}^* = \frac{R_{12}^* + jX_{12}^*}{(1 + R_{12}^*G_{12}^* - X_{12}^*B_{12}^* - X_{12}^*B_{\text{SVC2}}^*)} \left(1 + j(B_{L1}^* + B_{\text{SVC1}}^*)\right) + j(X_{12}^*G_{12}^* + R_{12}^*B_{L1}^* + R_{12}^*B_{\text{SVC1}}^*)
\]  
(3.5.1)

\[
\bar{Z}_{\text{equ2}} = \bar{Z}_{L2}^* \parallel jX_2 = \frac{1}{G_{L2}^* + j(B_{L2}^* + B_{\text{SVC2}}^*)} + \frac{1}{jX_2^*} = \frac{1}{\left[1 - X_2^*(B_{L2}^* + B_{\text{SVC2}}^*)\right]} + jX_2^*G_{L2}^*
\]  
(3.5.2)
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

\[ V_{e1} \Delta \delta_{e1} = \frac{\bar{Z}_{L1}^2}{\bar{Z}_{L1}^2 + \bar{Z}_{L2}^2} E_{q1} \Delta \delta_{1} = \frac{1}{(1 + R \bar{G}_{L1} - X \bar{B}_{L1} - X \bar{B}_{SVC1}) + j(X \bar{G}_{L1} + R \bar{B}_{L1} + R \bar{B}_{SVC1})} E_{q1} \Delta \delta_{1} \]  
(3.5.3)

\[ V_{e2} \Delta \delta_{e2} = \frac{\bar{Z}_{L2}^2}{\bar{Z}_{L2}^2 + jX_2} E_{q2} \Delta \delta_{2} = \frac{1}{[1 - X_2^2(B \bar{L}_{2}^2 + B \bar{SVC2})] + jX_2^2 \bar{G}_{L2}} E_{q2} \Delta \delta_{2} \]  
(3.5.4)

Since we still study steady-state stability, we can use small-signal analysis. From this equivalent model, one can derive a small-signal model of this system. The detailed analysis is shown in Appendix C and the state space model of the system can be obtained as:

\[ \Delta \delta = 2\pi f_0 \Delta \omega \]

\[ \begin{bmatrix} \Delta \omega_i \\ \Delta E_{qi} \end{bmatrix} = \begin{bmatrix} -K_{1i}/T_{ji} & -K_{2i}/T_{ji} \\ -K_{4i}/T_{d0i} & 1/(K_{3i} T_{d0i}) \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta E_{qi} \end{bmatrix} + \begin{bmatrix} -K_{14i}/T_{d0i} & -K_{8i}/T_{d0i} \\ K_{12i}/T_{d0i} & K_{9i}/T_{d0i} \end{bmatrix} \begin{bmatrix} \Delta E_{q1} \\ \Delta E_{q2} \end{bmatrix} \]

\[ + \begin{bmatrix} -K_{b1i}/T_{d0i} & -K_{b2i}/T_{d0i} \\ K_{b5i}/T_{d0i} & K_{b6i}/T_{d0i} \end{bmatrix} \begin{bmatrix} \Delta B_{svc1} \\ \Delta B_{svc2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/T_{d0i} \end{bmatrix} \begin{bmatrix} \Delta E_{5i} \\ \Delta E_{6i} \end{bmatrix} \]  
(3.5.5)

\[ \begin{bmatrix} \Delta V_{i1} \\ \Delta V_p \\ \Delta Q_{SVC1} \end{bmatrix} = \begin{bmatrix} K_{5i} & -K_{5i} & K_{6i} \\ K_{p1i} & K_{p2i} & 0 \\ K_{q1i} & K_{q2i} & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \delta_2 \\ \Delta E_{qi} \end{bmatrix} + \begin{bmatrix} K_{13j} & K_{7j} & K_{b5j} & K_{b6j} \\ K_{p3j} & K_{p4j} & K_{p5j} & K_{p6j} \\ K_{q3j} & K_{q4j} & K_{q5j} & K_{q6j} \end{bmatrix} \begin{bmatrix} \Delta E_{q1} \\ \Delta E_{q2} \\ \Delta B_{svc1} \\ \Delta B_{svc2} \end{bmatrix} \]

In these equations, \( V_i \) is the i-th generator terminal voltage, \( V_p \) is the input to the primary SVC controller and the secondary generator controller. \( Q_{SVC1} \) is the reactive power output of the i-th SVC. Once the state space model is known, the transfer functions between the state variables \( \Delta V_p \) and the control variables \( \Delta Q_{SVC1}, \Delta V_{ref} \) can be obtained. Again it is to be reminded that the system coefficients and some transfer functions which would be used in the chapter are given in Appendix B.

3.5.2 Design of Primary SVC Controller

Primary SVC control actions need to be tuned to respond quickly to the change, which will have time constant much less than the secondary generator control (SGC). In this
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

case, when considering primary SVC controller design, the SGC can be neglected. As will be illustrated by the numerical example in the next section, this simplification is seen to be accurate in so far as for the design of the Primary SVC Controller.

Figure 3-6 shows the block diagram for the primary SVC controller. The aim is to determine the values of \( k_p \) and \( k_h \). At this stage of the investigation, it is proposed that the frequency response method is used. The method is as follows.

The new cross-over frequency \( \omega_c \) should be chosen close to the original cross-over frequency when the primary SVC controller is taken out. Although varied operating conditions will result in different Bode plot, the shape of the plots is similar. In order to ensure closed-loop stability in all cases, \( \omega_c \) should be chosen under the most onerous system condition. \( \omega_c \) should be a little smaller than the original cross-over frequency.

Before the detailed design of the primary control system for the SVC is described, it will be useful to examine the functional relationship between the state and control variables. Within the primary SVC control loop, it is then necessary to examine the changes in the pilot bus voltage \( V_p \) with the control variable \( B_{SVC} \). The transfer functions of \( \Delta V_p/\Delta B_{SVC} \), as given in Appendix B are of the form:

\[
\frac{\Delta V_p}{\Delta B_{SVC}} = \frac{M'_{p1}S^5 + M'_{p2}S^4 + M'_{p3}S^3 + M'_{p4}S^2 + M'_{p5}S + M'_{p6}}{M'_{q1}S^5 + M'_{q2}S^4 + M'_{q3}S^3 + M'_{q4}S^2 + M'_{q5}S + M'_{q6}}.
\]

For the \( i \)-th SVC, the primary SVC controller can be designed according to the Bode plots of the open-loop transfer function \( \Delta V_p/\Delta V_{SVC} \). Assuming that \( G_i(S) \) represents the transfer function of \( \Delta V_p/\Delta V_{SVC} \). According to the basic frequency response technique introduced in Section 3.3, after adding the primary SVC controller, at the cross-over point \( S = j\omega_c \), the desired system open-loop gain should be \( G_{SVC}(S)G_i(S) = 1 \) and the phase angle should be \(-180^\circ + PM\), where PM is the desired phase margin at the cross-over point.

Thus,
(3.5.7)

\[ M_{p_i} \text{ and } \theta_{p_i} \text{ are the gain and phase angle of } G_i(j\omega) \text{ at the point } \omega = \omega_c. \]
Equation (3.5.7) can be written as

\[ \left( k_{p_i} + \frac{k_{n}}{j\omega_c} \right) M_{p_i} \cos \theta_{p_i} + j \sin \theta_{p_i} = \cos(-180 + PM) + j \sin(-180 + PM) \]

Separate the above equation into its real and imaginary parts, \( T_p \) and \( T_n \) can be derived

\[ k_{p_i} = \frac{-\cos(PM - \theta_{p_i})}{M_{p_i}} \]
\[ k_{n} = \frac{\omega_c \sin(PM - \theta_{p_i})}{M_{p_i}} \]  

(3.5.8)

A good damping factor \( \xi \) of closed-loop system is 0.707, the necessary phase margin PM should approximately be 70°. To obtain the desired phase margin, it is usual to make the targeted phase margin a few degrees higher (say by 5°). This is because the primary SVC control model introduces an additional zero to the system. The zero will make the final cross-over frequency \( \omega_c \) slightly higher. The recommended PM is therefore 75°.

Once knowing \( \omega_c, M_{p_i} \text{ and } \theta_{p_i}, \) equation (3.5.8) permits \( k_{p_i} \text{ and } k_{n} \) to be determined readily.

3.5.3 Design of Secondary Generator Controller (SGC)

Figure 3-9 shows the detailed block diagram for the i-th secondary generator controller. Before the detailed design of the secondary generator controller is considered, it will be useful to examine the functional relationship between the state and control variables.
Within the secondary generator control loop, it is then necessary to examine the changes in the pilot bus voltage $V_P$ with the control variable $V_{ti}$. The transfer functions of $\Delta V_P/\Delta V_a$ and $\Delta V_a/\Delta E_{fdj}$, as shown in Appendix B are of the form:

$$\frac{\Delta V_P}{\Delta V_a} = \frac{K_p K_p' T_p S^2 + 2\pi f_s K_p' [K_p' (D_i + D_j H_{ij} + D_k H_{kj})] - K_p' (D_i + D_j H_{ij} + D_k H_{kj})}{K_p K_p' T_p S^2 + 2\pi f_s K_p' [K_p' (D_i + D_j H_{ij} + D_k H_{kj})] - K_p' (D_i + D_j H_{ij} + D_k H_{kj})}$$

(3.5.9)

$$\frac{\Delta V_a}{\Delta E_{fdj}} = \frac{K_p' T_p S^2 + 2\pi f_s K_p' [K_p' (D_i + D_j H_{ij} + D_k H_{kj})] - K_p' (D_i + D_j H_{ij} + D_k H_{kj})}{T_p S^2 + 2\pi f_s K_p' [K_p' (D_i + D_j H_{ij} + D_k H_{kj})] - K_p' (D_i + D_j H_{ij} + D_k H_{kj})}$$

(3.5.10)

For the i-th generator, the SGC can be designed according to the Bode plots of the open-loop transfer function $\Delta V_P/\Delta V_{SGC}$. Suppose the form of the SGC transfer function is $K_{SGC} T_{pi} S^2 + 1$, the aim is to determine the values of $K_{SGC}, T_{pi}$ and $T_{ni}$. The technique is identical to that of the design of the primary SVC controller, i.e. the new cross-over frequency $\omega_{ci}$ should be a little smaller than the original cross-over frequency under the most onerous system condition.

Assuming that $G_i(S)$ represents the transfer function of $\Delta V_P/\Delta V_{SGC}$. According to the basic frequency response technique, after adding the SGC, at the cross-over point $S = j \omega_{ci}$, the desired system open-loop gain should be $G_{SCG}(S)G_i(S) = 1$ and the phase angle should be $-180^\circ + \text{PM}$, where PM is the desired phase margin at the cross-over point.

Thus,

$$G_{SCG}(j \omega_{ci})G_i(j \omega_{ci}) = K_{SGC} \frac{j \omega_{ci} T_{pi} + 1}{j \omega_{ci} T_{ii}} \frac{M_{ci} e^{j\theta_{ci}}}{M_{ci} e^{j\theta_{ci}}} = e^{j(180^\circ + \text{PM})}$$

(3.5.11)

where $M_{ci}$ and $\theta_{ci}$ are the gain and phase angle of $G_i(j \omega)$ at the point $\omega = \omega_{ci}$. Equation (3.5.11) can be written as

$$K_{SGC} \frac{j \omega_{ci} T_{pi} + 1}{j \omega_{ci} T_{ii}} M_{ci} (\cos \theta_{ci} + j \sin \theta_{ci}) = \cos(-180^\circ + \text{PM}) + j \sin(-180^\circ + \text{PM})$$

76
Separate the above equation into its real and imaginary parts, \( T_{pi} \) and \( T_{ui} \) can be derived

\[
T_{pi} = \frac{\cos(PM - \theta_{Gi})}{-\omega_{ci}\sin(\omega_{c1}M_{Gi}\theta_{Gi})}
\]

\[
T_{ui} = \frac{K_{SGCi}M_{Gi}}{\omega_{ci}\sin(PM - \theta_{Gi})}
\]

(3.5.12)

To ensure a small static error, \( K_{SGCi} \) should not be chosen too small. Typical value is \( K_{SGC} = 5 - 10 \). The selection of damping factor and necessary phase margin (PM) follows the similar method as for the primary SVC control.

Once \( K_{SGCi} \) and PM are fixed and knowing \( \omega_{ci} \), \( M_{Gi} \) and \( \theta_{Gi} \), equation (3.5.12) permits \( T_{pi} \) and \( T_{ui} \) to be determined readily.

### 3.6 Numerical Examples

The procedure described earlier may now be illustrated by using an actual 220 kV network, as shown in Figure 3-1. It is operated by Western Power, Australia and interconnects the main generating station at Muja (A) with the mining district of Eastern Goldfields (G). Node A at the sending end is connected to the generation (\( G_1 \) of about 800MW) in the main grid, and a much smaller (about 30MW) generation unit (\( G_2 \)) is connected to node G. The main distribution system is connected to node G, however, there are also small loads supplied at the intermediate points C, D and E along the route of the interconnection. The total length from B to F is about 650 km. At D which is about the mid-point of the interconnection, there is one Static VAr Compensator (SVC). At F, the receiving end of the interconnection, there are also two SVCs.

In this study, Each TCR has a rating of 55 MVAr at supply frequency. On a base of 100 MVA, the inherent slope impedance is 2% when referred to the 132 kV busbar. Tap changing on the 220 kV windings of the compensator transformers allows 220 kV voltage profiles to be established almost independently of operating conditions in the 132 kV system. Compensators are connected to the primary transmission system through the 220/132/29.5 kV 3-winding autotransformers. Associated with each TCR compensator is a set of harmonic filters and the total reactive-power generation from each set at nominal
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

voltage is 22.6 MVAr. For each TCR compensator, there are two switched capacitor banks rated at 21 MVAr and 17 MVAr respectively and they are under independent circuit-breaker switching control.

AVR model is illustrated in Figure 3-10. The parameters of AVR are also given in Appendix D.

![AVR model - conventional excitation regulator type](image)

As there are many possible system conditions to be investigated, it is practical that only the most representative and interesting cases be included in this section.

### 3.6.1 Design Considerations

Figures 3-11 - 3-14 shows the open-loop plot of $\Delta V_p/\Delta V_{SVC_1}$, $\Delta V_p/\Delta V_{SVC_2}$, $\Delta V_P/\Delta V_{SGCl}$ and $\Delta V_P/\Delta V_{SGC_2}$ under various power transfer level conditions (by varying load power, i.e. $P_{L2}$, $Q_{L2}$). As the power transfer level increases (i.e., $P_{L2}$, $Q_{L2}$ become larger), the magnitudes of $\Delta V_p/\Delta V_{SVC_1}$ and $\Delta V_P/\Delta V_{SVC_2}$ increase. Moreover, as the power transfer level increases, the magnitudes of $\Delta V_p/\Delta V_{SGCl}$ and $\Delta V_P/\Delta V_{SGC_2}$ increase over the frequency range below the cross-over frequency $\omega_c$. There is however little change in the magnitude of $\Delta V_p/\Delta V_{SGCl}$ and $\Delta V_P/\Delta V_{SGC_2}$ for frequency above $\omega_c$.

Furthermore, as the power transfer level increases, the phase margin decreases. In this example, the phase margins of $\Delta V_p/\Delta V_{SVC_1}$, $\Delta V_P/\Delta V_{SVC_2}$, $\Delta V_p/\Delta V_{SGCl}$ and $\Delta V_P/\Delta V_{SGC_2}$ at $P_{L2} = 150$MW and 200MW are listed in Table 3-1.
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

Table 3-1: Phase margins of different power transfer levels

<table>
<thead>
<tr>
<th>$P_{L2} (MW)$</th>
<th>Phase Margin of $\Delta V_P / \Delta V_{SVC1}$</th>
<th>Phase Margin of $\Delta V_P / \Delta V_{SVC2}$</th>
<th>Phase Margin of $\Delta V_P / \Delta V_{SGC1}$</th>
<th>Phase Margin of $\Delta V_P / \Delta V_{SGC2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>96.6°</td>
<td>111.2°</td>
<td>128.8°</td>
<td>124.5°</td>
</tr>
<tr>
<td>200</td>
<td>93.5°</td>
<td>106.9°</td>
<td>113.9°</td>
<td>109.8°</td>
</tr>
</tbody>
</table>

This result shows that as the power transfer level increases, the damping level of the closed-loop system becomes less. Hence for practical design of the Primary SVC Controller and the Secondary Generator Controller, the controllers should be designed under the maximum power transfer level conditions (as this is the worst-case scenario). In this way, it will guarantee that the closed-loop system will still be stable under all conceivable power transfer condition. For the case of the 220 kV power system, for instance, commissioning of the Primary SVC Controller and the Secondary Generator Controller should be carried out when the load at G is 200MW.

From Figures 3-11 – 3-14, it can also be found that $\Delta V_P / \Delta V_{SVC1}$ and $\Delta V_P / \Delta V_{SVC2}$ have greater change than $\Delta V_P / \Delta V_{SGC1}$ and $\Delta V_P / \Delta V_{SGC2}$ as the power transfer level changes. This result illustrates that primary SVC control is more sensitive to the change of pilot bus voltage than secondary generator control.

Figure 3-11 Open-loop Bode plot for $\Delta V_P / \Delta V_{SVC1}$ at different power transfer levels
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

Figure 3-12 Open-loop Bode plot for $\Delta V_P/\Delta V_{SVC2}$ at different power transfer levels

Figure 3-13 Open-loop Bode plot for $\Delta V_P/\Delta V_{SGC1}$ at different power transfer levels
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

Figure 3-14 Open-loop Bode plot for $\Delta V_P/\Delta V_{SGC2}$ at different power transfer levels

As the purpose of the investigation is the coordination in the design of the primary SVC controller and Secondary Generator controller, it will therefore be instructive to examine the open-loop transfer functions pertaining to the two control devices. From Figures 3-15–3-18, it can be found that the cross-over frequencies of $\Delta V_P/\Delta V_{SVC1}$ and $\Delta V_P/\Delta V_{SVC2}$ are about 667 rad/sec and 128 rad/sec respectively, while the cross-over frequencies of $\Delta V_P/\Delta V_{SGC1}$ and $\Delta V_P/\Delta V_{SGC2}$ are about 0.362 rad/sec and 0.147 rad/sec respectively. As the cross-over frequency of the primary SVC controller is almost three orders away from the cross-over frequency of the Secondary Generator controller, it can be concluded that the proposed primary SVC controller tuning method will result in its control action having little effect on the Secondary Generator controller. Hence the design of the primary SVC controller and Secondary Generator controller can be carried out quite independently of each other.
Figure 3-15 Open-loop Bode plot for $\Delta V_p/\Delta V_{SVC1}$

Figure 3-16 Open-loop Bode plot for $\Delta V_p/\Delta V_{SVC2}$
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

Figure 3-17 Open-loop Bode plot for $\Delta V_p/\Delta V_{SGC1}$

Figure 3-18 Open-loop Bode plot for $\Delta V_p/\Delta V_{SGC2}$
3.6.2 Final Controller Design

a) Primary SVC1 Controller design
Suppose SVC1 is of the form \( k_{p1} + \frac{k_{i1}}{S} \). And the desired phase-margin is specified as 75° at the cross-over frequency of 667 rad/sec. From Figure 3-15, this gives the gain and phase angle of 0.0355 dB and -86.5°. Substitute these values into (3.5.8), the values of \( k_{pl} = 0.95 \) and \( k_{i1} = 211.86 \) are obtained. Hence the primary SVC1 controller is:

\[
G_{SVC1}(S) = 0.95 + \frac{211.86}{S}.
\]

b) Primary SVC2 Controller design
Similarly, suppose SVC2 is of the form \( k_{p2} + \frac{k_{i2}}{S} \). And the desired phase-margin is specified as 75° at the cross-over frequency of 128 rad/sec. From Figure 3-16, this gives the gain and phase angle of 0.0262 dB and -73.1° respectively. Substitute these values into (3.5.8), the values of \( k_{p2} = 0.85 \) and \( k_{i2} = 67.48 \) are obtained. Hence the SVC2 controller is:

\[
G_{SVC2}(S) = 0.85 + \frac{67.48}{S}.
\]

c) SGC1 design
Suppose SGC1 is of the form \( K_{SGC1} \frac{T_{p1}S + 1}{T_{i1}S} \). Choose \( K_{SGC1} = 5 \) and the desired phase-margin is specified as 75° at the cross-over frequency of 0.362 rad/sec. From Figure 3-17, this gives the gain and phase angle of 0.0161 dB and -66.1°. Substitute these values into (3.5.12), the values of \( T_{p1} = 3.4235 \) and \( T_{i1} = 22.036 \) are obtained. Hence the SGC1 controller is:

\[
G_{SGC1}(S) = 5 \frac{3.4235S + 1}{22.036S}.
\]
d) SGC2 design

Similarly, suppose SGC2 is of the form \( K_{SGC2} \frac{T_{p2}S + 1}{T_{n2}S} \). Choose \( K_{SGC2} = 5 \) and the desired phase-margin is specified as \( 75^\circ \) at the cross-over frequency of \( 0.147 \text{rad/sec} \). From Figure 3-18, this gives the gain and phase angle of 0.0387dB and \(-70.2^\circ\). Substitute these values into (3.5.12), the values of \( T_{p2} = 9.7878 \) and \( T_{n2} = 59.8645 \) are obtained. Hence the SGC2 controller is:

\[
G_{SGC2}(S) = 5 \frac{9.7878S + 1}{59.8645S}.
\]

### 3.6.3 Time Response Study

In order to assess the effectiveness of the proposed voltage control scheme, time-response simulation studies of the power system have been carried out.

First consider the test case when one 40MVAr reactor at pilot bus is switched on. The system responses are as shown in Figure 3-19 (a1)-(f1) are the responses with only primary control and mechanically-switched capacitor. (a2)-(f2) are the responses with both primary control and secondary control. (a1)-(c1) and (a2)-(c2) are the first ten seconds of simulation. (d1)-(f1) and (d2)-(f2) are long-time simulation of 100 seconds. It is seen that when reactor is switched on, control actions are carried out by the adjustments of automatic voltage regulators and SVC quickly. Reactive power outputs of Generator 1 and Generator 2 are quickly increased to maintain their terminal voltages, and susceptances of SVC1 and SVC2 are increased quickly to maintain the voltage of pilot bus. The pilot bus is restored most rapidly to its initial value. Some seconds later, the 21 MVAr mechanically switched capacitor is switched on service to push the outputs of SVC1 and SVC2 closer into the maximum inductive region.
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

(a1) Voltage profiles

(b1) Susceptances of SVCs

(c1) Reactive power outputs of generators

(a2) Voltage profiles

(b2) Susceptances of SVCs

(c2) Reactive power outputs of generators
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

Figure 3-19 Response following a 40MVAR reactor switching on
It can also be found that with the secondary control, after the voltage of pilot bus is brought back to its initial value, generator 1 and generator 2 still continue to increase reactive power output to push the outputs of SVC1 and SVC2 into the maximum inductive region. Susceptances of SVC1 and SVC2 will be closer to the minimum susceptance region (i.e., the maximum inductive region) with secondary voltage control. In this way the SVC will retain a better capacity to respond quickly to subsequent disturbances.

Next, we will consider the case when SVC1 is out of service, and one 40MVAr reactor at pilot bus is switched on. The system responses are shown in Figure 3-20. It can be found that when disturbance occurs, the susceptance of SVC2 increases quickly to maintain the voltage of the pilot bus. Although this response increases the voltage of pilot bus, the voltages of SVC2 and generator 2 are also increased, causing the reactive power output of generator 2 to decrease. The voltage of SVC2 decreases when the reactive power output of generator 2 decreases. Hence, the reactive output of SVC2 decreases, the susceptance of SVC2 has to continue to increase to maintain the voltage of the pilot bus. This in turn causes the reactive power output of generator 2 to reduce further. Finally the under excitation limit of generator 2 is breached, the generator angle continues to increase, the system will lose synchronism and out of stability.

This later case shows the important role played by SVC1 at the intermediated point of the transmission system. It also points to the need to coordinate the under-excitation operation of $G_2$ with that of the SVC.
Chapter 3: A Coordinated Hierarchical Voltage Control Scheme

(a) Voltage profiles
(b) Susceptance of SVC2
(c) Reactive power outputs of generators
(d) Real power outputs of generators
(e) Rotor angle difference between $G_1$ and $G_2$

Figure 3-20 Response following a 40MVAR reactor switching on without SVC1
3.7 Conclusions

In this chapter, a coordinated hierarchical method for a long-distance radial transmission system is studied. A model of this system suitable for detailed analysis is developed and the state space model is obtained. The functional relationship between the primary voltage control and secondary voltage control loops has been examined by using the well-known frequency response technique. It has been shown that the tuning of the primary voltage controller and the secondary voltage controller can be carried out separately and without having to consider the interaction between the two control loops. The simulation results show that the proposed coordinated hierarchical control scheme will guarantee system stability while performing the intended functions of the reactive power devices.

Using the proposed method, a systematic coordination of controllers study procedure can be carried out during the system planning stage such that a more cost effective and economical transmission network reactive power compensation scheme can be obtained.

The result also shows that when SVC1 is out of service, large disturbance may cause the under excitation limit of generator being breached. The system will lose stability as a result. Therefore, it is possible to conclude that shunt compensation has limitation in terms of transfer ability, thus it is needed to extend compensation to include series compensation, say, the more general UPFC. This will be studied in Chapter 5.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Chapter 4

Shunt Reactive Power Compensation of Long Transmission Lines

This chapter is organized as follows. In Section 4.1, some background of shunt reactive power compensation is provided. The issue of compensation at line terminals is brought out in Section 4.2. In Section 4.3, transmission line π-circuit model is described, followed by power-angle relationship of transmission line in Section 4.4. The shunt compensation problem is elaborated in Section 4.5. A method of determining the maximum power transfer of a shunt compensated line is proposed in Section 4.6. In Section 4.7, numerical results to demonstrate the accuracy of the predicted small-signal stability limit are presented. The optimization problem of single shunt compensator is described in Section 4.8. In Section 4.9, steady-state stability of a shunt compensated line is studied. The optimization problem of multiple shunt compensators is then brought out in Section 4.10. Finally Section 4.11 concludes this chapter.

4.1 Introduction

As pointed out in Chapters 2 and 3, a fundamental requirement in AC power transmission is the maintenance of appropriate network voltage levels. Modern power systems are not very tolerant of abnormal voltages, even for short periods. Whereas active power is generated and absorbed at specific points in the system, reactive power is generated and absorbed in significant quantities throughout the whole system. It also tends to vary with system conditions [131]. In situations where it is difficult to install new transmission circuits, existing facilities are relied on to transfer ever more power. The voltage control problem is exacerbated and the electrical networks are even more stressed. Thus research in AC transmission is centered on how to increase power transfer limit in the most economical manner.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

In considering a transmission line, a lossless line operating at its surge impedance loading (SIL) has a flat voltage profile; that is, the voltage magnitude is the same everywhere along the line. No reactive power flow will be observed on the line and the power delivery system is most efficient in terms of the delivery of real power. However, as it is not practicable to force the load on a transmission line to coincide with the line SIL at all time, the alternative is to control the SIL so that it matches exactly with the load. Reference [30] describes very well this concept of modifying the surge impedance of a transmission line through rapid shunt compensation control so that the corresponding line SIL equals to the actual loading. Practical shunt compensation scheme would consist of reactive compensators being connected at line ends and at strategic intermediate nodes along the line. Power electronics-based fast-acting shunt compensators such as Static VAr Compensators (SVC) are used to realize the scheme. Voltage support capability can be exploited and the transmission distance is reduced artificially. As pointed out and theoretically proven in [27, 28], it is well known a mid-point compensated line can transmit up to 2 times the power of the uncompensated line while maintaining steady-state stability. However in a recent article [31], Huang and Ooi presented results of digital simulation studies of such a mid-point compensated transmission system and showed that the ratio is actually close to 1.59. The authors attributed the reduction in terms of the distributed capacitance of the line, the finite source-end and receiving-end reactances used in their studies and the presence of line resistances in contributing toward the reduction in the steady-state stability limit. Unfortunately, the authors have not provided any quantitative evidence to support the claim. In the present work, the exact long-line model would be used in the analysis. The purpose is to determine the required shunt compensation to achieve a given power transfer. It is shown that line resistance can affect the required amount of the line reactive power compensation significantly, especially when the transmitted power is above the line natural load and as the line length increases. On the prediction of maximum power transfer \( P_{\text{max}} \), although \( P_{\text{max}} \) may be obtained using the approach based on simplified line model [3, 4], generalized expressions with exact representation of the transmission line have yet to be obtained. Such expressions will be included in this chapter. The results are obtained by including the sending-end (SE), receiving-end (RE) impedances and the application of mid-point compensation. The accuracy of the analysis is supported through digital simulation studies which show that steady-state stability limit can indeed be predicted more accurately compared to previous
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

works. The analysis provides an improved evaluation of the contribution made by midpoint shunt compensator in enhancing network stability. It quantifies the extent of the improvement in stability through mid-point compensation, as it is affected by such factors as line losses, line length, and SE and RE terminal impedances.

The power transfer limit is known to vary when the location of shunt compensator changes. Whereas in the previous studies [3, 4, 27-30], the optimal compensation location is considered as the mid-point of the line based on the simplified line model. The analysis in [32] shows that the device needs to be placed slightly off-centre in order to increase the power transfer capability and stability of the system. This finding contradicts the results found in [3, 4, 27-30] essentially because line losses have been included in [32]. On the other hand, the approach used in [32] is only concerned with maximizing the power transfer level to the intermediate node where the compensator is connected. It does not result in the maximum power deliverable at the RE of the line. As it is the RE power which is of practical interest, the optimal location of the shunt compensator to achieve the maximum RE power need to be investigated. The analytical technique is then extended to deal with the case of multiple shunt compensated system. A method to determine the optimal compensator locations is then proposed.

Nowadays, the shunt compensation technique is based on using fast and accurate power electronics–based converters, to realize the fast control aim. This chapter is intended to establish the theoretical basis on the power transfer capability. Some of the materials contained in this chapter have been reported in [126, 127].

4.2 Compensation at Line Terminals

As explained earlier, practical compensation scheme would include shunt compensators connected at the terminals of long-lines. The problem thus becomes one of predicting the amount of reactive power needed at each line terminal so as to maintain the terminal voltage at its nominal value. In previous works [3, 4, 27-30], the authors used various types of approximated line model in their analysis. Of course, physical transmission lines do contain resistances and shunt capacitances. Hence, the so-called exact long-line model which includes line loss would be the starting point of the investigation.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

The long-line model is as shown in Figure 4-1, where \( \tilde{z} = R + j\omega L \) is the series impedance per unit length/phase, and \( \tilde{y} = G + j\omega C \) is the shunt admittance per unit length/phase of the line. The physical meanings of the various parameters are described in [128]. Line length is \( l \). The focus of the present work is on the behavior of the network at fundamental frequency. Mutual coupling between phases has been ignored. Assume that voltage \( \tilde{V}_r \) and current \( \tilde{I}_r \) are known at the RE \( (x = 0) \), thus the general expressions for voltage and current at a distance \( x \) from the RE are

\[
\tilde{V}_x = \left[ (\tilde{V}_r + \tilde{Z}_c \tilde{I}_r) e^{jx} + (\tilde{V}_r - \tilde{Z}_c \tilde{I}_r) e^{-jx} \right] / 2
\]

\[
\tilde{I}_x = \left[ (\tilde{V}_r / \tilde{Z}_c + \tilde{I}_r) e^{jx} - (\tilde{V}_r / \tilde{Z}_c - \tilde{I}_r) e^{-jx} \right] / 2
\]

where,

\( \tilde{Z}_c = \sqrt{\tilde{z}/\tilde{y}} \) and \( \tilde{y} = \sqrt{\tilde{z}} = \alpha + j\beta \). \( \tilde{Z}_c \) is called the characteristic impedance of the line.

\( \gamma \) is the line propagation constant: \( \alpha \) being the line attenuation constant and \( \beta \) the line phase constant. These equations constitute the distributed parameter model of the transmission line. Note that line losses and shunt capacitance have been included. For a lossless line, \( \tilde{Z}_c \) is commonly referred to as the line surge impedance \( Z_s = \sqrt{L/C} \) and has the dimension of a pure resistance. The power delivered by a transmission line when it is terminated by its surge impedance is known as the natural load or SIL denoted as \( P_0 \),

\[
P_0 = V_o^2 / Z_0
\]

where \( V_o \) is the rated voltage of the line.

From (4.2.1), with \( x = l \), (4.2.4) can be obtained.

\[
\tilde{V}_s = \left[ (\tilde{V}_r + \tilde{Z}_c \tilde{I}_r) e^{jn} + (\tilde{V}_r - \tilde{Z}_c \tilde{I}_r) e^{-jn} \right] / 2
\]
Hence, the current at the RE of a transmission line can be written as:

\[
I_r = \frac{[2V_s - \bar{V}_r(e^{\imath \theta} + e^{-\imath \theta})]/[\bar{Z}_c(e^{\imath \theta} - e^{-\imath \theta})] \quad (4.2.5)
\]

From Equations (4.2.2) and (4.2.5), with \( x = l \), (4.2.6) can be obtained.

\[
I_s = \frac{\bar{V}_s[e^{\imath \theta} (\cos \theta + j \sin \theta) + e^{-\imath \theta} (\cos \theta - j \sin \theta)] - 2\bar{V}_r}{(R_c + jX_c)[e^{\imath \theta} (\cos \theta + j \sin \theta) - e^{-\imath \theta} (\cos \theta - j \sin \theta)]} \quad (4.2.6)
\]

Let \( \bar{Z}_c = R_c + jX_c \) and the line angle \( \theta = \beta l \). Select \( \bar{V}_r \) as the reference voltage and denote \( \tilde{S} \) as the phase angle difference by which \( V_s \) leads \( V_r \). That is \( \bar{V}_s = V_r \angle 0 \), \( \bar{V}_s = V_r \angle \delta \). (4.2.5) and (4.2.6) can be rewritten as (4.2.7) and (4.2.8), respectively:

\[
\tilde{S}_s = P_s + jQ_s = \bar{V}_r \bar{I}_r = \frac{2\bar{V}_r V_s (\cos \delta - j \sin \delta) - V_s^2 - 2V_s V_r (\cos \theta - j \sin \theta) + e^{\imath \theta} (\cos \theta + j \sin \theta)]}{(R_c + jX_c)[e^{\imath \theta} (\cos \theta - j \sin \theta) - e^{-\imath \theta} (\cos \theta + j \sin \theta)]} \quad (4.2.7)
\]

\[
\tilde{S}_r = P_r + jQ_r = \bar{V}_s \bar{I}_s = \frac{V_r^2 e^{\imath \theta} (\cos \delta + j \sin \delta) - V_r^2 (\cos \theta + j \sin \theta) + e^{\imath \theta} (\cos \theta - j \sin \theta)]}{(R_c + jX_c)[e^{\imath \theta} (\cos \theta + j \sin \theta) - e^{-\imath \theta} (\cos \theta - j \sin \theta)]} \quad (4.2.8)
\]

It can be derived that the apparent powers at the RE and SE of a transmission line are given by Equations (4.2.9) and (4.2.10), respectively.

\[
\tilde{S}_s = P_s + jQ_s = \bar{V}_r \bar{I}_r = \frac{2\bar{V}_r V_s (\cos \delta - j \sin \delta) - V_s^2 + 2V_s V_r (\cos \theta - j \sin \theta) + e^{\imath \theta} (\cos \theta + j \sin \theta)]}{(R_c + jX_c)[e^{\imath \theta} (\cos \theta - j \sin \theta) - e^{-\imath \theta} (\cos \theta + j \sin \theta)]} \quad (4.2.9)
\]

\[
\tilde{S}_r = P_r + jQ_r = \bar{V}_s \bar{I}_s = \frac{V_r^2 + 2V_r V_s (\cos \delta + j \sin \delta) - 2V_r V_s (\cos \theta + j \sin \theta) - 2V_r^2 (\cos \theta - j \sin \theta)]}{(R_c + jX_c)[e^{\imath \theta} (\cos \theta + j \sin \theta) - e^{-\imath \theta} (\cos \theta - j \sin \theta)]} \quad (4.2.10)
\]

Suppose the intention is to make the SE and RE voltage magnitudes equal to \( V_0 \). The active and reactive powers at the receiving and sending ends of the line can be derived and expressed in term of \( P_0 \). The normalized values with respect to \( P_0 \) and \( Q_0 \) for \( P_s \) and \( Q_s \) can be obtained as shown in Equations (4.2.11) –(4.2.14).
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

\[ P_r/P_0 = \text{Re} \left( V_r I_r^* \right)/P_0 \]
\[ = Z_0 R_c \left[ 2V_r V_s e^{j\theta} \cos(\theta - \delta) - 2V_r V_s e^{j\theta} \cos(\theta + \delta) + V_s^2 (e^{-2j\theta} - e^{2j\theta}) \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]
\[ - Z_0 X_c \left[ 2V_r V_s e^{j\theta} \sin(\theta - \delta) + 2V_r V_s e^{j\theta} \sin(\theta + \delta) - 2V_r^2 \sin 2\theta \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]

\[ (4.2.11) \]

\[ Q_r/P_0 = \text{Im} \left( V_r I_r^* \right)/P_0 \]
\[ = Z_0 X_c \left[ 2V_r V_s e^{j\theta} \cos(\theta - \delta) - 2V_r V_s e^{j\theta} \cos(\theta + \delta) + V_s^2 (e^{-2j\theta} - e^{2j\theta}) \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]
\[ + Z_0 R_c \left[ 2V_r V_s e^{j\theta} \sin(\theta - \delta) + 2V_r V_s e^{j\theta} \sin(\theta + \delta) - 2V_r^2 \sin 2\theta \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]

\[ (4.2.12) \]

\[ P_s/P_0 = \text{Re} \left( V_s I_s^* \right)/P_0 \]
\[ = Z_0 X_c \left[ (e^{j\theta} - e^{-j\theta}) V_s^2 - 2V_r V_s e^{j\theta} \cos(\theta + \delta) + 2V_r V_s e^{j\theta} \cos(\theta - \delta) \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]
\[ - Z_0 R_c \left[ 2V_r^2 \sin 2\theta - 2V_r V_s e^{j\theta} \sin(\theta + \delta) - 2V_r V_s e^{j\theta} \sin(\theta - \delta) \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]

\[ (4.2.13) \]

\[ Q_s/P_0 = \text{Im} \left( V_s I_s^* \right)/P_0 \]
\[ = Z_0 X_c \left[ (e^{j\theta} - e^{-j\theta}) V_s^2 - 2V_r V_s e^{j\theta} \cos(\theta + \delta) + 2V_r V_s e^{j\theta} \cos(\theta - \delta) \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]
\[ + Z_0 R_c \left[ 2V_r^2 \sin 2\theta - 2V_r V_s e^{j\theta} \sin(\theta + \delta) - 2V_r V_s e^{j\theta} \sin(\theta - \delta) \right]/\left[ V_0^2 (R_c^2 + X_c^2)(e^{2j\theta} + e^{-2j\theta} - 2 \cos 2\theta) \right] \]

\[ (4.2.14) \]

There are now sufficient analytical expressions obtained from which the reactive power requirements at line terminals can be determined. This is because once \( V_r, V_s, P_r \) (or \( P_s \)) are specified, the power angle \( \delta \) can be obtained using (4.2.11) (or (4.2.13)) as line parameters \( \theta \) and \( \alpha \) are known. The required \( Q_r \) and \( Q_s \) can then be determined using (4.2.12) and (4.2.14) respectively.

The case of lossless lines becomes a special case of the above when \( R = G = 0 \). Equations (4.2.11) - (4.2.14) then degenerate into (4.2.15)-(4.2.16). These are the familiar results shown in previous works, e.g. in [3, 4, 27-30].

\[ P_r/P_0 = P_s/P_0 = \sin \delta / \sin \theta \]

\[ Q_r/P_0 = Q_s/P_0 = (\cos \delta - \cos \theta) / \sin \theta \]

The analytical expressions obtained earlier can also be used to determine the required reactive power for multi-circuit lines. In approaching the ideal practice of having close to
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

a flat voltage profile across the line, the amount of the shunt reactive compensation at each bus is the sum of all the shunt compensation required of each line end connected to the bus. The required shunt compensation at the terminals of each line is of course governed by (4.2.11)-(4.2.14).

It is now appropriate to provide some numerical examples to illustrate the results obtained thus far.

4.2.1 Numerical Examples

Using the expressions (4.2.11)- (4.2.14), Figure 4-2 shows the relationship between $P_r$ and $Q_r$, $P_s$ and $Q_s$ for a 350-km, 500-kV ACSR overhead transmission line which have the following parameters: $R = 0.01755 \ \Omega/km$, $L = 0.0008737 \ \text{H/km}$, $C = 0.01333 \ \text{uF/km}$ and $G = 0$. The data are taken from [129]. In Figure 4-2, $P_r$, $Q_r$, $P_s$ and $Q_s$ are scaled as a ratio of the line natural load ($P_0$) of 976.5 MW. The corresponding results without the line series resistance ($R = 0$) have also been included in the figure for comparison purpose. It can be seen that generally the effect of line resistance is to increase the compensation reactive power needed to support the real power transfer. As $P_r$ increases beyond the line natural load $P_0$, the increase is at a much more rapid rate than that under the lossless line assumption.

Figure 4-2 Reactive compensation requirements of a 350-km, 500-kV line at different load level: comparison of results with and without line losses included in the model
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

The effect of line length can also be studied by varying the line angle \( \theta \). The results are as shown in Figure 4-3. It shows that when the power transfer level is below the SIL (approximately), inductive shunt compensation would be necessary at the line ends to compensate for the inherent capacitance of the long line. In contrast, when the power transfer level is beyond \( P_0 \), capacitive shunt compensation would be needed. Not surprisingly, Figure 4-3 also indicates that the required shunt compensation increases with the line length (\( \theta \)). The increases become increasingly significant as the transfer level exceeds \( P_0 \).

(a) \( P_r - Q_r \) characteristics
(b) \( P_r - Q_s \) characteristic
(c) \( P_s - Q_r \) characteristic
(d) \( P_s - Q_s \) characteristic

Figure 4-3 Shunt reactive power compensation required at the terminals of a 500 kV line: as function of transferred power and line length

The analytical expressions obtained earlier can also be used to determine the required reactive power for multi-circuit lines. In approaching the ideal practice of having close to
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

a flat voltage profile across the line, the amount of the shunt reactive compensation of each bus is the sum of all the shunt compensation required of each line end which is connected to the bus. The required shunt compensation at the terminals of each line is of course governed by (4.2.11)-(4.2.14). Again using the 500 kV line example, Figure 4-4 shows a double-circuit double-section line delivering 3 times the SIL of one circuit, that is, \(1.5P_n\) per circuit. Suppose the total transmission distance is 700km, and the lines have a mid-point compensation at node 2. The corresponding length of each section of the line is 350km, which is identical to that used to produce the results of Figure 4-2. Hence, \(\theta = 25.81^\circ\). In order to maintain \(V_1 = V_2 = V_3 = V_0 = 500\) kV, from Figure 4-3, the required shunt compensator at Bus2 is \(0.2782P_0 \times 2 + 0.4984P_0 \times 2 = 1.5532P_0\). \(Q_I\) and \(Q_3\) at the line terminals (nodes 1 and 3) are the total amount of reactive power needed and these can be supplied by the end sources and/or shunt reactive power compensators.

Clearly the concept of multi-circuit lines can be extended to networks.

\[
\begin{align*}
\bar{V}_1 & = 1.6276P_0 \\
& = P_s = 1.6276P_0 \\
& \Downarrow Q_1 = 0.6688P_0 \\
\bar{V}_2 & = P_s = 1.6276P_0 \\
& \Downarrow Q_2 = 1.5532P_0 \\
\bar{V}_3 & = P_I = 1.5P_0 \\
& \Downarrow Q_3 = 0.8714P_0
\end{align*}
\]

Figure 4-4 Shunt reactive compensation required for a 700 km, 500 kV 2-section double-circuit line: mid-point compensation at node 2.

4.3 Transmission Line \(\pi\)-Circuit Model

The long-line model in which the transmission line parameters are uniformly distributed provides a complete description of the voltage/current relationship of the line. However, for purposes of analysis involving interconnection with other network elements, it is more convenient to use equivalent circuits which only describe the characteristics of the lines at their terminals. As far as the end conditions are concerned, the exact equivalent circuit of a transmission line in the form of a \(\pi\)-circuit, as shown in Figure 4-5, can be used for this
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

purpose. It is also known as the Telegraph model [128]. The line parameters of Figure 4-5 are given by \( Z_e = Z_c \sinh(\gamma l) \) and \( \frac{\gamma l}{2} = \tanh(\gamma l/2)/Z_c \), where \( l, \ Z_c, \ \gamma \) are the line length, characteristic impedance and line propagation constant respectively. Let \( \bar{z} = R + j\omega L \) be the series impedance per unit length/phase, and \( \bar{y} = G + j\omega C \) be the shunt admittance per unit length/phase of the line. From [128], it is known that \( Z_c = \sqrt{\bar{z}/\bar{y}} \) and \( \gamma = \sqrt{\bar{y}^2} \).

\[
\begin{align*}
\bar{Z}_e &= \frac{1}{Z_c} + \frac{1}{2} \bar{Y}_e \\

\frac{\gamma l}{2} &= \tanh(\gamma l/2)/Z_c.
\end{align*}
\]

Figure 4-5 Equivalent \( \pi \) circuit model of a transmission line

Suppose one assigns the receiving-end voltage as the reference phasor \( (\bar{V}_r = V_r e^{0^\circ}) \) such that the sending-end voltage lead it by an angle \( \delta \), i.e. \( \bar{V}_s = V_s e^{\delta} \). In terms of \( \bar{V}_s \) and \( \bar{V}_r \), the sending-end (SE) and receiving-end (RE) currents of the line can be expressed as

\[
\begin{align*}
\bar{I}_s &= \left(1/\bar{Z}_e + \bar{Y}_e / 2\right) \bar{V}_s - \bar{V}_r / \bar{Z}_e \\
\bar{I}_r &= \bar{V}_s / \bar{Z}_e - \left(1/\bar{Z}_e + \bar{Y}_e / 2\right) \bar{V}_r.
\end{align*}
\]

The active power flows at the SE and RE of the line can then be written as

\[
\begin{align*}
P_s &= \text{Re}(\bar{V}_s \bar{I}_s^*) = -V_s V_r (G_t \cos \delta + B_t \sin \delta) + V_s^2 (G_t + G_c / 2) \\
P_r &= \text{Re}(\bar{V}_r \bar{I}_r^*) = V_s V_r (G_t \cos \delta - B_t \sin \delta) - V_r^2 (G_t + G_c / 2)
\end{align*}
\]

where,
\[
G_t = \text{Re}(1/\bar{Z}_e); \quad B_t = \text{Im}(1/\bar{Z}_e); \quad G_c = \text{Re}(\bar{Y}_e).
\]

\( \text{Re(\cdot)} \) and \( \text{Im(\cdot)} \) mean the real and imaginary part of \( \cdot \), respectively.

Note that the values of \( G_t, B_t \) and \( G_c \) depend on the line parameters \( (R, L \text{ and } C) \) and the line length \( l \).
4.4 Power-Angle Relationship

Since it is highly desirable to exercise voltage control on the line, the practical assumption that \( V_s = V_r = V_0 \) can be made. \( P_s \) and \( P_r \) in (4.3.3) and (4.3.4) can be normalized in term of \( P_0 \) to yield

\[
\frac{P_s}{P_0} = P_{\text{max}} \sin (\delta - \varphi) + \xi \tag{4.4.1}
\]
\[
\frac{P_r}{P_0} = P_{\text{max}} \sin (\delta + \varphi) - \xi \tag{4.4.2}
\]

where

\[
P_{\text{max}} = Z_0 \sqrt{G_l^2 + B_l^2}
\]
\[\varphi = \tan^{-1} \left( -\frac{G_l}{B_l} \right) \]
\[\xi = Z_0 \left( \frac{G_l + G_c}{2} \right) \tag{4.4.3}\]

Note that \( \varphi \) depends on \( G_l / B_l \) ratio and hence its value mainly depends on the \( R/\omega L \) ratio of the line and is very insensitive to the line length \( l \), while \( \xi \) depends on both \( R \) and \( l \).

Equations (4.4.1) and (4.4.2) represent the \( P_s - \delta \) and \( P_r - \delta \) characteristics, respectively, of the line. Upon closer examination of (4.4.1) when \( P_s = 0 \), it can be readily shown that the angle \( \delta \) across the transmission line will be either

\[
\delta_{01,s} = -\sin^{-1} \left( \frac{\xi}{P_{\text{max}}} \right) + \varphi \tag{4.4.4}
\]

or

\[
\delta_{02,s} = \pi + \sin^{-1} \left( \frac{\xi}{P_{\text{max}}} \right) + \varphi \tag{4.4.5}
\]

The subscript “0” is used to denote zero power flow. Similar analysis on (4.4.2) also concludes that \( P_r = 0 \) means that either

\[
\delta_{01,r} = \sin^{-1} \left( \frac{\xi}{P_{\text{max}}} \right) - \varphi \tag{4.4.6}
\]

or

\[
\delta_{02,r} = \pi - \sin^{-1} \left( \frac{\xi}{P_{\text{max}}} \right) - \varphi \tag{4.4.7}
\]

Since \( \delta_{01,s} \) and \( \delta_{02,r} \) are not equal to \( \delta_{01,s} \) and \( \delta_{02,s} \), this means that even when the receiving-end (RE) power \( (P_r) \) is zero, the sending-end (SE) power \( (P_s) \) is not exactly zero because of the line losses.
In practice, the leakage conductance of a transmission line is usually negligible i.e. $G_c \approx 0$. Thus $\delta_{01,s} = 0$, $\delta_{02,s} = \pi + 2\varphi$, $\delta_{01,r} = 0$ and $\delta_{02,r} = \pi - 2\varphi$. It is clear from (4.4.1) that $P_s$ reaches the maximum value when

$$\delta = \delta_{cr,s} = \pi/2 + \varphi$$

(4.4.8)

Similarly, analysis on (4.4.2) shows that maximum $P_r$ occurs when

$$\delta = \delta_{cr,r} = \pi/2 - \varphi$$

(4.4.9)

Using (4.4.8) and (4.4.9), it is interesting to note that $\delta_{cr,s} > \delta_{cr,r}$ because $\varphi > 0^\circ$. It can be easily determined that the maximum value of $P_s$ is larger than that of $P_r$ by an amount $2\zeta$.

Hence in general, a given line will have its $P_s-\delta$ and $P_r-\delta$ curves having the forms shown in Figure 4-6, with the respective values of $\delta_{02,s}$ and $\delta_{02,r}$ as indicated. $\delta_{cr,s}$ is exactly equal to half of the angle $\delta_{02,s} = \pi + 2\varphi$ where $P_s = 0$. The $P_s-\delta$ curve is symmetric about $\delta_{cr,s}$ and assumes the $\delta$ range $(0, \pi + 2\varphi)$ for $P_s \geq 0$. Similarly for the $P_r-\delta$ curve, $P_r$ reaches the maximum value at exactly half of $\delta_{02,r} = \pi - 2\varphi$. The curve is symmetric about $\delta_{cr,r}$ and spans over the range $(0, \pi - 2\varphi)$ for $P_r \geq 0$.

![Diagram showing P-\delta characteristics of a transmission line](image)

Figure 4-6 $P-\delta$ characteristics of a transmission line: (a) $P_s$ vs $\delta$, (b) $P_r$ vs $\delta$ and (c) $P$ vs $\delta$ for lossless line.

If line losses and capacitances have been excluded, i.e., assuming the simplified model of the line, (4.4.1) and (4.4.2) reduce to the familiar equation

$$P_s / P_0 = P_r / P_0 = P_{\text{max}}' \sin \delta$$

(4.4.10)

where $P_{\text{max}}' = Z_0 B_l$. This is shown as curve (c) in Figure 4-6 and has been included in many references [3, 4, 27-30].
Since $\varphi$ mainly depends on the $R/\omega L$ ratio of the line and is very insensitive to the line length $l$, it can therefore be concluded that in practice, transmission lines of practical lengths but of the same construction and type will have their $P_r-\delta$ curves reaching their respective maximum $P_r$ values at almost the same angle $\delta_{cr,cr}$. As the line length decreases, $P_{max-\delta}$ increases. The $P_r-\delta$ curve rises in magnitude but spans over the same range $(0, \pi - 2\varphi)$. Similar statement can also be made concerning the $P_s-\delta$ curves.

Having derived and explained some of the important characteristics governing the real power and angle relationships of transmission lines, the next step is to examine how shunt reactive power compensation can modify the power transfer capability of the lines.

### 4.5 Shunt Compensation

As was described in [3, 4, 27-30], one practical way to increase the power transfer capability of a long line is to apply the technique of sectioning. This technique can be readily studied by considering the radial power system shown in Figure 4-7 where the transmission line between nodes C and D interconnects a generating source at A to another source at B. The total length of the line is $l$ and a reactive power shunt compensation device is installed at the node "m" on the line, as shown. As is well-known, the compensator could be in the form of a SVC or STATCOM. A parameter $k$ may then be used to indicate that $kl$ equals to the line length between C and m. The transmission line is therefore divided into sections 1 and 2. Each section can be represented by its $\pi$-equivalent circuit discussed in Section 4.3, with its respective series impedance and shunt susceptance values. The resulting equivalent circuit of the power system is as shown in Figure 4-8 (a). The terminal synchronous generators $G_1$ and $G_2$ are assumed to be ideal voltage sources and the voltages are represented by the voltage phasor quantities $\vec{V}_s$ and $\vec{V}_r$. In practical networks, of course, the synchronous sources and their associated transformers connected to the line ends would have their respective impedances. Therefore these impedances should be included in the analysis [31, 126, 127]. In Figure 4-8 (a), the values of these impedances are denoted as $R_i+jX_i$ at the respective line terminal, $i = 1, 2$. In the present investigation, power flow is assumed from $G_1$ to $G_2$, i.e. 1 denotes the SE whereas 2 denotes the RE of the line.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Figure 4-7 A schematic diagram showing a transmission system with shunt reactive power compensation

![Diagram showing a transmission system with shunt compensation](image)

Figure 4-8 Equivalent circuit of a radial line with shunt compensation and source impedances

![Equivalent circuit diagram](image)

Suppose the compensator is lossless and its susceptance is $B_r$. In the case of a SVC or STATCOM, $B_r$ would be variable and when $B_r$ is combined with the line section susceptances ($B_{c1}/2$ and $B_{c2}/2$) at $m$, the resulting susceptance is shown in Figure 4-8 (a) in the form of a variable shunt. In this representation, the degree of shunt compensation [29] of the central half of the line is defined as
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

\[ k_m = \frac{B_r}{[0.5(B_{cl} + B_{c2})]} \]  
(4.5.1)

\( k_m \) is arbitrarily taken as positive if \( B_r \) is inductive and negative if \( B_r \) is capacitive.

At fundamental frequency and through a series of \( \Delta-Y \) transformations, the equivalent circuit shown in Figure 4-8 (a) reduces to Figure 4-8 (b) where it can be shown that

\[ \begin{align*}
\bar{Z}_{eq,i} &= R_i + jX_i + R_i + mX_i + jB_{ci} (R_i + jX_i) (R_i + jX_i) / 2 \\
\bar{Z}_{eq,i} &= R_i + jX_i - j/2 (R_i + jX_i) / \left[ B_{ci} (R_i + jX_i) \right] \\
\bar{Y}_{eq,i} &= 1/\bar{Z}_{eq,i} + 1/\bar{Z}_{eq,i} + j (B_{ci} + B_{c2}) 1/2 \\
\bar{Z}_{eq,i} &= R_i + jX_i - j/2 (R_i + jX_i) / \left[ B_{ci} (R_i + jX_i) \right]
\end{align*} \]  
(4.5.2)

The above expressions show that once the line parameters and source equivalent impedances are known and the degree of compensation \( (k_m) \) is given at the compensation location \( k \), the equivalent network impedances and admittance of Figure 4-8 (b) can be readily determined.

Suppose the voltage at \( m \) is treated as the reference phasor, i.e. \( \bar{V}_m = V_m \angle 0^\circ \). Denote \( \bar{V}_s = V_s \angle \delta_s \) and \( \bar{V}_r = V_r \angle \delta_r \). Note that in this case, the power angle \( \delta \) between the two voltage sources is given as \( \delta = \delta_s - \delta_r \). From Figure 4-8 (b), the following circuit equations are obtained:

\[ \begin{align*}
(\bar{V}_s - \bar{V}_m) / \bar{Z}_{eq,1} - (\bar{V}_m - \bar{V}_r) / \bar{Z}_{eq,2} &= \bar{Y}_{eq,i} \bar{V}_m \\
\bar{I}_r &= (\bar{V}_m - \bar{V}_r) / \bar{Z}_{eq,2} - \bar{V}_r / \bar{Z}_{eq,2}
\end{align*} \]  
(4.5.3)

(4.5.3) can be rewritten as,

\[ \begin{align*}
\bar{V}_m &= V_s (\alpha_{11} \cos \delta_s - \alpha_{12} \sin \delta_s) + V_r (\alpha_{21} \cos \delta_r - \alpha_{22} \sin \delta_r) \\
&+ j \left[ V_s (\alpha_{11} \sin \delta_s + \alpha_{12} \cos \delta_s) + V_r (\alpha_{21} \sin \delta_r + \alpha_{22} \cos \delta_r) \right] \\
\end{align*} \]  
(4.5.4)

where,

\[ \begin{align*}
\alpha_{11} &= \text{Re} \left[ \bar{Z}_{eq,1} / \left( \bar{Y}_{eq,i} \bar{Z}_{eq,1} + \bar{Z}_{eq,2} \right) \right] ; \alpha_{12} &= \text{Im} \left[ \bar{Z}_{eq,2} / \left( \bar{Y}_{eq,i} \bar{Z}_{eq,1} + \bar{Z}_{eq,2} \right) \right] \\
\alpha_{21} &= \text{Re} \left[ \bar{Z}_{eq,1} / \left( \bar{Y}_{eq,i} \bar{Z}_{eq,1} + \bar{Z}_{eq,2} \right) \right] ; \alpha_{22} &= \text{Im} \left[ \bar{Z}_{eq,2} / \left( \bar{Y}_{eq,i} \bar{Z}_{eq,1} + \bar{Z}_{eq,2} \right) \right]
\end{align*} \]

Equating the real and imaginary parts of (4.5.5),

\[ \begin{align*}
V_m &= V_s (\alpha_{11} \cos \delta_s - \alpha_{12} \sin \delta_s) + V_r (\alpha_{21} \cos \delta_r - \alpha_{22} \sin \delta_r) \\
V_s (\alpha_{11} \sin \delta_s + \alpha_{12} \cos \delta_s) + V_r (\alpha_{21} \sin \delta_r + \alpha_{22} \cos \delta_r) &= 0
\end{align*} \]  
(4.5.6)

(4.5.7)
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Since $\delta = \delta_r - \delta_n$, multiply (4.5.6) by $\cos \delta$, and multiply (4.5.7) by $\sin \delta$, and add the resulting expressions yield

$$V_m \cos \delta_r = a_{11} V_s \cos \delta - a_{12} V_s \sin \delta + a_{21} V_r$$

(4.5.8)

Also multiply (4.5.6) by $\sin \delta_r$ and multiply (4.5.7) by $\cos \delta_r$ and subtract the resulting expressions yield

$$V_m \sin \delta_r = -a_{11} V_s \sin \delta - a_{12} V_s \cos \delta - a_{22} V_r$$

(4.5.9)

From (4.5.4), $P_r$ can be derived:

$$P_r = \text{Re} \left( \vec{V}_r \vec{V}_r^* \right) = V_r V_m \left( G_{eq1,2} \cos \delta_r + B_{eq1,2} \sin \delta_r \right) - V_r^2 \left( G_{eq1,2}^2 + G_{eq2,2} \right)$$

(4.5.10)

where,

$$G_{eq,k} = \text{Re} \left( 1/Z_{eq,k} \right); \quad B_{eq,k} = \text{Im} \left( 1/Z_{eq,k} \right); \quad (k = 1, 2, m; \ i = 1, 2)$$

The last expression (4.5.10) is rather general. However, practical consideration indicates that the source voltages should be controlled to close to their nominal values ($V_0$). Hence one may assume that $V_s = V_r = V_0$. Substitute (4.5.8) and (4.5.9) into (4.5.10), and if $V_s = V_r = V_0$, the value of $P_r$, normalized to $P_0$ is then given by

$$P_r/P_0 = Z_0 \left[ (\alpha_{11} G_{eq1,2} - \alpha_{12} B_{eq1,2}) \cos \delta - Z_0 (\alpha_{11} B_{eq1,2} + \alpha_{12} G_{eq1,2}) \sin \delta + Z_0 (\alpha_{21} G_{eq1,2} - \alpha_{22} B_{eq1,2} - G_{eq2,2}) \right]$$

(4.5.11)

Further simplification of (4.5.11) shows that

$$P_r/P_0 = -Z_0 \sqrt{\left( \alpha_{11}^2 + \alpha_{12}^2 \right) G_{eq1,2}^2 + B_{eq1,2}^2} \sin (\delta - \psi) + \zeta$$

(4.5.12)

where

$$\tan \psi = \left( \alpha_{11} G_{eq1,2} - \alpha_{12} B_{eq1,2} \right) \left( \alpha_{11} B_{eq1,2} + \alpha_{12} G_{eq1,2} \right)$$

$$\zeta = Z_0 \left[ (\alpha_{21} G_{eq1,2} - \alpha_{22} B_{eq1,2} - G_{eq2,2}) \right]$$

(4.5.13)

The parameters $\psi$ and $\zeta$ are only dependent of the line constants, the sources impedances and the amount of shunt compensation $B_y$. Very much similar to (4.4.2) in relating $P_r$ to $\delta$ for a transmission line, (4.5.12) shows that $P_r$ vs $\delta$ in this more general case would again be a sine function. As an illustration, Figure 4-9 shows typical $P_r$ - $\delta$ curves obtained using (4.5.12) for the 700-km, 500-kV line described in [129]. Each of the curves (a)-(c) has been determined for a fixed value of $B_y$. Note that the amplitudes of these constant- $B_y$ curves increase with $B_y$. 

106
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Figure 4-9 $P_r-\delta$ characteristics of a shunt-compensated line with $V_s = V_r = V_0$: (a) $B_\gamma = 0$; (b) $B_\gamma = 0.5/Z_0$; (c) $B_\gamma = B_{\gamma\text{max}}$; (d) exact line model with variable $B_\gamma$ and $V_m = V_0$; (e) lossless line model with variable $B_\gamma$, $V_m = V_0$

Instead of making $B_\gamma$ a fixed shunt susceptance at $m$, suppose $B_\gamma$ is allowed to be regulated in such a way as to keep $V_m$ constant also at the level $V_0$. Substitute $V_m = V_0$ into (4.5.3), (4.5.3) can be rewritten as,

$$\left[(G_{eq,1} + G_{eq,2} + G_{eqsh}) + j\left(B_{eq,1} + B_{eq,2} + B_{eqsh}\right)\right]V_m = \left(G_{eq,1} + jB_{eq,1}\right)V_s + \left(G_{eq,2} + jB_{eq,2}\right)V_r$$

(4.5.14)

where,

$$G_{eqsh} = \text{Re}\left(\gamma_{eqsh}\right) = G_{eq,1} + G_{eq,2}$$

(4.5.15)

$$B_{eqsh} = \text{Im}\left(\gamma_{eqsh}\right) = B_{eq,1} + B_{eq,2} + (B_{c1} + B_{c2})(1 - k_m)/2$$

(4.5.16)

Separating the real and imaginary parts of (4.5.14) leads to the following identities:

$$(G_{eq,1} + G_{eq,2} + G_{eqsh}) V_m = (G_{eq,1} \cos \delta - B_{eq,1} \sin \delta) V_s + (G_{eq,2} \cos \delta - B_{eq,2} \sin \delta) V_r$$

(4.5.17)

$$(B_{eq,1} + B_{eq,2} + B_{eqsh}) V_m = (G_{eq,1} \sin \delta + B_{eq,1} \cos \delta) V_s + (G_{eq,2} \sin \delta + B_{eq,2} \cos \delta) V_r$$

(4.5.18)

Equation (4.5.17) can be rewritten as:

$$G_{eq,1} + G_{eq,2} + G_{eqsh} = (G_{eq,1} \cos \delta - B_{eq,1} \sin \delta + G_{eq,2}) \cos \delta - (G_{eq,1} \sin \delta + B_{eq,1} \cos \delta + B_{eq,2}) \sin \delta$$

(4.5.19)
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

\[
\delta_1 = -\cos^{-1}\left[\frac{\left(G_{eq,l1} \cos \delta - B_{eq,l1} \sin \delta + G_{eq,l2}\right)}{\sqrt{\left(G_{eq,l1} \cos \delta - B_{eq,l1} \sin \delta + G_{eq,l2}\right)^2 + \left(G_{eq,l1} \sin \delta + B_{eq,l1} \cos \delta + B_{eq,l2}\right)^2}}\right] \tag{4.5.20}
\]

\[
\delta_2 = -\cos^{-1}\left[\frac{\left(G_{eq,l1} + G_{eq,l2} + G_{eq,h}\right)}{\sqrt{\left(G_{eq,l1} \cos \delta - B_{eq,l1} \sin \delta + G_{eq,l2}\right)^2 + \left(G_{eq,l1} \sin \delta + B_{eq,l1} \cos \delta + B_{eq,l2}\right)^2}}\right] \tag{4.5.21}
\]

Substitute (4.5.20) and (4.5.21) into (4.5.19) and express the resulting equation in term of \(\delta_1, \delta_2\) and \(\delta_r\). Then the following equation can be obtained:

\[
\cos \delta_2 = \cos \delta_1 \cos \delta_r - \sin \delta_1 \sin \delta_r \cos \delta_1 \sin \delta_1 \tag{4.5.22}
\]

(4.5.22) allows \(\delta_r\) to be solved,

\[
\delta_r = \delta_2 - \delta_1 \tag{4.5.23}
\]

Substitute Equations (4.5.20), (4.5.21) and (4.5.23) into (4.5.10), (4.5.24) can be obtained,

\[
\frac{P}{P_0} = -Z_0 \left\{G_{eq,l2} + G_{eq,h}\right\} + \frac{Z_0 \left\{\beta_4 G_{eq,l1} - B_{eq,h} \sqrt{\beta_3 + \beta_3 \cos \delta - \beta_3 \sin \delta} \right\}\left(G_{eq,l1} \cos \delta - B_{eq,l1} \sin \delta + G_{eq,l2}\right)}{\beta_1 + \beta_1 \cos \delta - \beta_1 \sin \delta} \tag{4.5.24}
\]

where,

\[
\beta_4 = G_{eq,l1}^2 + G_{eq,l2}^2 + B_{eq,l1}^2 + B_{eq,l2}^2; \quad \beta_3 = 2\left(G_{eq,l1} G_{eq,l2} + B_{eq,l1} B_{eq,l2}\right); \quad \beta_1 = 2\left(B_{eq,l1} G_{eq,l2} - G_{eq,l1} B_{eq,l2}\right); \quad \beta_2 = G_{eq,l1}^2 + G_{eq,l2}^2 + G_{eq,h}; \quad \beta_5 = B_{eq,l1}^2 + B_{eq,l2}^2 - G_{eq,h} - 2G_{eq,l1} G_{eq,l2} - 2G_{eq,l1} G_{eq,h} - 2G_{eq,l2} G_{eq,h};
\]

From (4.5.18), one can readily show that

\[
B_{eq,h} = G_{eq,l1} \sin \delta + B_{eq,l1} \cos \delta + G_{eq,l2} \sin \delta + B_{eq,l2} \cos \delta - B_{eq,l1} - B_{eq,l2} \tag{4.5.25}
\]

Also, from (4.5.16), one can show that the degree of compensation \(k_m\) is given by

\[
k_m = 1 - 2\left(B_{eq,h} - B_{eq,m} - B_{eq,m} - B_{eq,m}\right)/(B_{q1} + B_{q1}) \tag{4.5.26}
\]

Hence, for a given receiving-end power \(P_r\), the following steps can be used to evaluate the shunt reactive compensation required at the midpoint of a transmission line in order to keep \(V_m = V_s = V_r = V_0\):

Step 1: For a given \(P_r\), use equation (4.5.24) to determine \(\delta\) through numerical means as all the other parameters on the RHS of (4.5.24) are known.

Step 2: Determine \(\delta_1\) and \(\delta_2\) using the expressions (4.5.20) and (4.5.21).

Step 3: Use (4.5.23) to determine \(\delta_r\). \(\delta_r\) can then be obtained since \(\delta_r = \delta_2 - \delta_1\).
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Step 4: Equation (4.5.25) is used to solve for $B_{eqh}$ and use (4.5.26) to obtain $k_m$.

Step 5: From the definition of $k_m$ in (4.5.1), reactive compensation $B_r$ needed at the mid-point is determined as $B_r = 0.5 k_m (B_{c1} + B_{c2})$.

(4.5.24) describes the $P_r - \delta$ relationship of the shunt-compensated system shown in Figure 4-7. If the shunt compensator can control $V_m$ through the adjustment in $B_r$ such that $V_m$ is maintained at the level $V_0$, then the voltage angle difference ($\delta$) between $G_1$ and $G_2$ would automatically satisfy (4.5.24) for a given RE power $P_r$. The requirement of $V_m = V_0$ can be met using, for example, a shunt FACTS device such as a SVC.

The controlling effect of the compensator at $m$ constrains $P_r$ and $\delta$ to move along the locus governed by (4.5.24). Using the same 500 kV line example as before, and assuming the compensator is located at the mid-point of the transmission line, the curve (d) in Figure 4-9 describes such a locus. The formation of this locus can be visualized as if the compensator is regulating its susceptance ($B_r$) continuously, such that $P_r$ and $\delta$ of the power system transit smoothly from one constant-$B_r$ curve to another as $P_r$ varies.

Maximum receiving end power $P_r$ is seen to occur at an angle $\delta$ close to 142.5° in this example. The locus (d) was obtained by assuming that there is no limit placed on $B_r$. With a practical compensator, of course, the limit on $B_r$ must be considered. When the limit on $B_r$ is reached, the compensator ceases its ability to maintain $V_m = V_0$ and instead, it behaves as a fixed susceptance. The operating point then leaves the characteristic curve (d) and moves onto the constant $B_r$ curve when $B_r = B_{rmax}$, i.e. curve (c). The corresponding maximum power that can be transferred corresponds to point A in Figure 4-9.

If source resistances, line losses and capacitances have been excluded from (4.5.24), one can re-evaluate the $P_r - \delta$ curve. This is shown as curve (e) in Figure 4-9. From the figure, it can also be seen that with the same maximum level of compensation $B_{rmax}$, the $P_r - \delta$ curve derived using the simplified line model (curve (e)) is above that for the exact line model (curve (d)). Hence the predicted maximum power transfer level based on the lossless line assumption, such as that described in [3, 4, 27-30], would be over-optimistic.
4.6 Maximum Power Transfer

There are sufficient results contained in Section 4.5 for one to determine the condition of maximum power transfer in a shunt compensated line. One could make use of (4.5.24) by differentiating $P_r$ with respect to $\delta$ and set the resulting equation $f(\delta)$ to zero. The expression so obtained, $f(\delta)$, is shown in (4.6.1).

$$f(\delta) = \frac{dP_r}{d\delta} = V_0^2 \left( \frac{\left( \beta_1 \sin \delta + \beta_2 \cos \delta \right) \left[ \beta_1 \left( G_{eq,1,2}^2 - B_{eq,1,2}^2 \right) - 2G_{eq,1,2}B_{eq,1,2} \sqrt{\beta_3 + \beta_4 \cos \delta - \beta_5 \sin \delta} \right]}{\left( \beta_1 + \beta_2 \cos \delta - \beta_5 \sin \delta \right)^2} \right)$$

$$\left( G_{eq,1,2}^2 - B_{eq,1,2}^2 \right) \left( \beta_3 \beta_4 - \beta_3 \beta_5 \cos \delta - \beta_5 \sin \delta - \beta_4 \sqrt{\beta_3 + \beta_4 \cos \delta - \beta_5 \sin \delta \cos \delta - \beta_5 \sin \delta} \right)$$

$$= \left( G_{eq,1,2}^2 + G_{eq,1,2}^2 \right) \left( \beta_3 \beta_4 + \beta_3 \beta_5 \cos \delta - \beta_5 \sin \delta + \beta_4 \beta_5 \cos \delta - \beta_5 \sqrt{\beta_3 + \beta_4 \cos \delta - \beta_5 \sin \delta \cos \delta - \beta_5 \sin \delta} \right)$$

$$\left( \beta_1 + \beta_2 \cos \delta - \beta_5 \sin \delta \right)^2$$

$$+ \left( \beta_3 \sin \delta + \beta_4 \cos \delta \right) \left[ \left( G_{eq,1,2}^2 - B_{eq,1,2}^2 \right) \sin \delta + \left( G_{eq,1,2}^2 B_{eq,1,2} + G_{eq,1,2}^2 B_{eq,1,2} \right) \cos \delta + 2G_{eq,1,2}B_{eq,1,2} \right]$$

$$2 \left( \beta_1 + \beta_2 \cos \delta - \beta_5 \sin \delta \right) \sqrt{\beta_3 + \beta_4 \cos \delta - \beta_5 \sin \delta}$$

$$= \left( 4.6.1 \right)$$

No analytical solution to $f(\delta)$ can be readily found unless some rather restrictive assumptions are made. Instead, it is proposed that through numerical means (e.g. that based on the Bisection method [130]), one solves for the value of $\delta$ by which maximum $P_r$ occurs. This $\delta$ value is denoted as the critical angle, $\delta_{crw}$. Essentially the numerical iterative procedure used to find the roots of $f(\delta) = 0$ can be summarized as follows:

Step 1: Set initialization values of $a(\cdot)$ and $b(\cdot)$, where $a(1) = 1.57$ rad and $b(1) = 3.14$ rad, and the initial iteration number $i = 0$;

Step 2: Set iteration number $i = i + 1$;

Step 3: Find $\delta_{mp} = \frac{a(i) + b(i)}{2}$;

Step 4: If $\delta_{mp}$ satisfies the convergence criterion

$$\left| f(\delta_{mp}) \right| \leq \varepsilon,$$

take the desired root as $\delta_{crw} = \delta_{mp}$ and stop the procedure. Otherwise, go to step 5.

110
Step 5: If \( f(\delta_{\text{imp}}) \cdot f(a(i)) > 0 \), both \( f(\delta_{\text{imp}}) \) and \( f(a(i)) \) will have the same sign, hence, set \( a(i+1) = \delta_{\text{imp}} \), \( b(i+1) = b(i) \) and go to step 2.

Step 6: If \( f(\delta_{\text{imp}}) \cdot f(a(i)) < 0 \), \( f(\delta_{\text{imp}}) \) and \( f(a(i)) \) will have opposite signs, hence, set \( b(i+1) = \delta_{\text{imp}} \), \( a(i+1) = a(i) \) and go to step 2.

\( \varepsilon \) is a specified small number, say, \( \varepsilon = 0.000001 \).

The maximum power, denoted herewith as \( P_{\text{max}} \), can be obtained by substituting \( \delta \) with \( \delta_{\text{crw}} \) into (4.5.24), having determined the value of \( \delta_{\text{crw}} \) from numerical means. \( P_{\text{max}} \), expressed in terms of \( P_0 \), is given by

\[
P_{\text{max}} = \frac{P_0}{Z_0} \left( \beta_1 G_{eq,1,2} - B_{eq,1,2} \sqrt{\beta_3 + \beta_4 \cos \delta_{\text{crw}} - \beta_3 \sin \delta_{\text{crw}}} \right) \left( G_{eq,1,1} \cos \delta_{\text{crw}} - B_{eq,1,1} \sin \delta_{\text{crw}} + G_{eq,1,2} \right) \frac{\beta_1 + \beta_3 \cos \delta_{\text{crw}} - \beta_1 \sin \delta_{\text{crw}}}{\beta_1 + \beta_3 \cos \delta_{\text{crw}} - \beta_3 \sin \delta_{\text{crw}}} \]

\[
- Z_0 \left( G_{eq,1,2} + G_{eq,2,2} \right) \] (4.6.2)

4.6.1 General Expressions of Midpoint Compensation

The previous results are applicable for a compensator located at any arbitrary intermediate node. For the particular case when the compensator is located at the middle of the line and if all losses are ignored, then Equation (4.6.2) reduces to Equation (4.6.3).

\[
P_{\text{max}} = \frac{Z_0 B_{eq,1,1} B_{eq,1,2} \sin \delta_{\text{crw}}}{P_0 \sqrt{B_{eq,1,1}^2 + B_{eq,1,2}^2 + 2B_{eq,1,1} B_{eq,1,2} \cos \delta_{\text{crw}}}} \] (4.6.3)

and

\[
\delta_{\text{crw}} = \begin{cases} 
\cos^{-1} \left( - \frac{B_{eq,1,2}}{B_{eq,1,1}} \right) & B_{eq,1,1} \leq B_{eq,1,2} \\
\cos^{-1} \left( - \frac{B_{eq,1,1}}{B_{eq,1,2}} \right) & B_{eq,1,1} > B_{eq,1,2} 
\end{cases} \] (4.6.4)

Furthermore, if it is assumed that the two terminal reactances are identical, i.e., \( X_1 = X_2 \), then Equation (4.6.3) becomes even simpler and the maximum power transfer is:

\[
P_{\text{max}} = - \frac{V_0^2}{2} B_{eq,1,1} \sin \delta_{\text{crw}} = \frac{V_0^2}{2Z_0 \sin (\beta l/2) + X_0 \cos (\beta l/2) \sin \delta_{\text{crw}}} \] (4.6.5)
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

This is the result shown in [31]. If the terminal reactances are ignored, i.e., \( X_t = 0 \), the last expression degenerates to the familiar result described in [3, 4, 27-30],

\[
P_{r_{\text{max}}} = \frac{2V_0^2}{X_t} \sin \left( \frac{\delta_{cr}}{2} \right)
\]  

(4.6.6)

In the last case, the maximum steady-state limit occurs when the sine function in (4.6.6) reaches its maximum, i.e. \( \delta_{cr} = 180° \). Notice that the term \( V_0^2/X_t \) corresponds to the maximum power transferable when the terminal voltages are at nominal value and without shunt compensation. Hence under the lossless condition, the power limit is twice that without compensation. Again this observation is well-known [3, 4, 27-30].

Further simplification of the line model is possible. For example, a simplified model could consist of the series impedance \( \bar{Z}_s \) while \( \bar{Y}_s = 0 \). With midpoint shunt compensator, the same analysis as shown earlier can be used. In this case \( \bar{Z}_{eq,i} = R_i/2 + jX_i/2 + R_n + jX_n \), \( G_{eq,i} = 0 \) (i=1, 2), \( \bar{Y}_{eqsh} = jB_y \).

As described in Section 4.5, in practice there is an economic limit placed on the reactive power rating of the compensator. When the limit is reached the compensator ceases to maintain constant voltage at its terminals and behaves like a fixed susceptance. The operating point then moves on to the constant \( B_y \) curve corresponding to the maximum value of \( B_y \), i.e., \( B_{y_{\text{max}}} \). The corresponding maximum power can be evaluated by reversing the numerical procedure described in Section 4.5. As shown in Figure 4-9, note that even without a limit on \( B_y \), maximum power does not occur at \( \delta = 180° \) when line losses are included, unlike the case under the lossless line assumption described in [3, 4, 27-30].

4.7 Numerical Examples

The following examples are based on the same 500-kV transmission line used in Section 4.5. For convenience, it is also assumed that the terminal impedances are identical, i.e. \( \bar{Z}_1 = \bar{Z}_2 = Z, \angle \theta \). Also, it is assumed that the rating of the shunt compensator is large enough to supply the reactive power required to maintain \( V_m = V_0 \).
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

4.7.1 Effect of Terminal Impedances on Power Transfer

The total impedances appearing at the end(s) of the line add to the series transfer impedances and alter the phase angle difference between the SE and RE voltages. The effect of the impedances is therefore to increase $\delta$ for a given $P_r$, as illustrated in Figure 4-10 (a). Figure 4-10 (b) shows the ratio of the maximum power transfer with mid-point compensation ($P_{r_{\text{max}}}$) to that without midpoint compensation ($P_{r_{\text{max}0}}$). This ratio is plotted against $Z_t$ under varied line length. It can be seen that $Z_t$ will reduce the benefit obtainable from the midpoint compensator. The reduction increases more significantly with the increase in line length. For example, Figure 4-10 (b) shows that with the common practice of mid-point shunt compensation at a regular 350-km (200-miles) interval, a 700-km line would see its maximum power level at about 1.6 times that without compensator and if $Z_t = 0.11 Z_0$.

![Figure 4-10](image.png)

Figure 4-10 Effect of line terminal impedances on (a) the $P_r-\delta$ characteristic and (b) the steady-state stability limit of a mid-point compensated line

4.7.2 Effect of Line Losses on Steady-State Stability Limit

Figure 4-11 shows the $P_r-\delta$ characteristic of the 700-km line when it is assumed that $Z_t = 0.11 Z_0$ and has an impedance angle of 89°. From Figure 4-11 (a), it can be seen that as the line becomes more resistive, $P_{r_{\text{max}1}}$ reduces. For the same level of power transmission, $\delta$ tends to increase as $R_t/\chi_t$ increases. Furthermore, the line resistance will reduce the
benefit that can be derived from the midpoint compensator. The larger the \( R_i/X_i \) ratio, the larger reduction in the benefit of the mid-point compensation is observed. This is indicated clearly in Figure 4-11 (b).

![Figure 4-11](image)

Figure 4-11 Effect of line resistances on (a) the \( \delta-P_r \) characteristic and (b) reducing the benefit under dynamic midpoint compensation

### 4.7.3 General Observation on the Benefit of Midpoint Compensation

Steady-state transfer capability of long transmission lines and the extent such capability can be enhanced through midpoint shunt compensation can now be elaborated on. Table 4-1 summarizes the maximum power transfer calculated using the expressions contained in Section 4.6 for the same 700-km, 500-kV line and terminal conditions as given in Section 4.5. Assume that the compensator is able to maintain the midpoint voltage at \( V_0 \) with ideal instantaneous response. Four line models have been considered. Using the simplified line model in which only the line series reactance has been included, it has been shown earlier that with mid-point compensation, the maximum power that can be transmitted with respect to the steady-state stability limit is increased by a factor of 2. This can be seen by examining the results of Case 1 in Table 4-1. However this factor is an overly optimistic assessment in predicting the beneficial effect of the shunt compensator. When line losses are included (Case 2), this factor reduces to 1.87. In Case 3 in which the equivalent \( \pi \)-circuit model with no losses shows that the steady-state stability limit is 2.2976\( P_0 \) as against 2.2208\( P_0 \) shown in Case 1. The difference is due to the presence of the line capacitance: the capacitance provides additional support on the
power transfer, an effect the simplified line model does not take into consideration. Hence the simplified line model over-estimates the beneficial effect of mid-point compensator on improving steady-state stability limit. With the line losses included in the model, it predicts a further decrease of the limit by some 13%. This can be seen by comparing the results of Cases 3 and 4.

The most accurate line model amongst the four would be the one based on the Telegraph equations. When the terminal impedance is included in the example, a further reduction of the limit is obtained, as shown in Cases 5-8. Case 8 shows that the beneficial effect of the mid-point compensation is about 1.59 times that without the compensation. This agrees fairly closely with that obtained in [31]. Unlike the observations shown in [31] which were based on computer simulation, the predicted values shown in Table 4-1 were obtained using the analytical expressions derived in Section 4.6.1. Hence the analytical expressions shown are most useful: they provide the theoretical basis in quantifying the various factors that could affect the steady-state stability limit of a compensated line.

Table 4-1: Effect of line model representation on steady-state stability limits

<table>
<thead>
<tr>
<th>Case</th>
<th>Without terminal impedances</th>
<th>With terminal impedances</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{r_{\text{max}0}}/P_0$</td>
<td>$P_{r_{\text{max}1}}/P_0$</td>
</tr>
<tr>
<td>1: Simplified lossless line model</td>
<td>1.1104</td>
<td>2.2208</td>
</tr>
<tr>
<td>2: Simplified line model with losses</td>
<td>1.0498</td>
<td>1.9582</td>
</tr>
<tr>
<td>3: Telegraph lossless line model</td>
<td>1.2760</td>
<td>2.2976</td>
</tr>
<tr>
<td>4: Telegraph line model with losses</td>
<td>1.2161</td>
<td>2.0338</td>
</tr>
<tr>
<td>5: Simplified lossless line model</td>
<td>0.8884</td>
<td>1.7768</td>
</tr>
<tr>
<td>6: Simplified line model with losses</td>
<td>0.8465</td>
<td>1.5911</td>
</tr>
<tr>
<td>7: Telegraph lossless line model</td>
<td>1.0947</td>
<td>1.8637</td>
</tr>
<tr>
<td>8: Telegraph line model with losses</td>
<td>1.0522</td>
<td>1.6774</td>
</tr>
</tbody>
</table>
4.7.4 Time-Response Simulation

In order to verify the accuracy of the earlier analysis, time-response simulation studies of an example power system have been carried out.

![Diagram of a mid-point compensated transmission network](Figure 4-12 Example of a mid-point compensated transmission network)

The example power system is shown in Figure 4-12. A 2000-MW hydro-generation plant \( G_1 \) is connected to a load center through the same 700-km, 500-kV transmission line and terminal impedances considered earlier. The load center is modeled by a 5000 MW resistive load. The load is fed by \( G_1 \) and a local generation \( (G_2) \) of 3500 MW capacity. As \( G_2 \) capacity is larger than \( G_1 \), \( G_2 \) is treated as the reference bus. The system surge impedance loading (SIL) is about 976.5 MW. The two machines are assumed to be equipped with excitation system. Pertinent parameters are given in Appendix E [129]. In order to maintain system stability, the transmission line is shunt compensated at its center by a SVC of 1050MVAr capacitive rating. The line is represented by the equivalent \( \pi \) circuit model. As there are many possible system conditions to be investigated; only the most representative and interesting cases will be included in this section. The base values used are 500 kV, 976.5 MVA.

4.7.4.1 Dynamic Working of the Midpoint Compensator

Assessment on the accuracy of the predicted small-signal stability limit is examined first. Consider the case in which the SVC is at its rated output, i.e. \( B_y = B_{\text{max}} = 1.075/Z_o \). The receiving-end power is calculated using the method proposed in Section 4.5 and is seen to be \( 1.4671P_0 \). Simulation study shows that the transmission system can indeed deliver \( 1.4265P_0 \). The predicted result therefore agrees well with the simulation.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Figure 4-13 (a) shows the response of the system under this maximum transfer condition of $B_{r_{\text{max}}} = 1.075/\Z_0$, $P_r = 1.4265 P_0$, when a -1% step change is introduced into the AVR summing junction of $G_1$. The simulation results show that the system is stable. On the other hand, Figure 4-13 (b) shows that when $P_r$ is increased slightly to 1.430 $P_0$ and the same voltage disturbance is introduced, it has resulted in instability. Hence, it can be concluded that the small-signal stability limit of the studied system has been predicted with great degree of accuracy.

![Figure 4-13 Rotor angle difference between $G_1$ and $G_2$ following a -1% step change in $G_1$ AVR input: (a) $P_r = 1.4265 P_0$, (b) $P_r = 1.430 P_0$.](image)

### 4.7.4.2 Effect of Line Resistance on Transmission System

Another test case in which the effect of line resistance on steady-state stability is assessed is when a 0.1 $\Z_0$ reactor at the line midpoint bus C is switched on and then off some 600 ms later. The system responses are as shown in Figure 4-14. Figure 4-14(a) shows the rotor angle of $G_1$ when the line losses have been ignored and $P_r = 1.4849 P_0$. It shows that stability is maintained. Figure 4-14 (b) and (c) show the results for the same disturbance but with the line losses included but at two slightly different transfer levels. When the line resistance has been included, even when $P_r = 1.430 P_0$ the system is unstable. Only when $P_r$ is reduced to 1.4265 $P_0$ will stability be regained, as shown in Figure 4-14(c).
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Figure 4-14 Rotor angle difference between G1 and G2 following reactor energization and tripping: (a) No line resistance, $P_r = 1.4849 \ P_0$ (b) With line resistance, $P_r = 1.430 \ P_0$, (c) With line resistance, $P_r = 1.4265 \ P_0$

4.8 Optimal Location: Single Shunt Compensator

From the observations contained in Section 4.6, it is clear that when a shunt reactive power compensator is connected at a distance $kl$ from the SE, the value of the maximum power transfer depends on the location factor $k$. The main purpose of this section is to extent the above results to determine the optimal location of the shunt compensator such that maximum power transfer can be achieved at the RE of the power system. It will be shown that in comparison with the method described in [32], the proposed method will result in a higher transfer level. For ease of comparison with the results obtained in [32], it is also assumed that the terminal impedances are negligible. Furthermore, it would be convenient if one is to use an example to illustrate the concept involved. The same 450-
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

km, 345 kV transmission line considered in [32] is used again so that easy comparison can be made with the results obtained there. As described in [32], each phase of the line has a bundle of two conductors of size one million c-mils each and the conductors are fully transposed. The series impedance and shunt admittance of the line are found to be $z = (0.02986 + j0.2849) \Omega/km$ and $y = j3.989 \times 10^{-6} \text{S/km}$, respectively, at 50 Hz. The results are presented in p.u. on a 100 MVA, 345 kV base. Similar to Figure 4-7, the two line sections are denoted as sections 1 and 2. The corresponding powers at the sending and receiving ends of the sections are $P_{s1}$, $P_{r1}$, $P_{s2}$ and $P_{r2}$ respectively, as show in Figure 4-15 (a).

Figure 4-15 (a) Power flows of the compensated line, (b) Maximum RE power of section 2 ($P_{r2}^m$) vs. shunt compensator location $k$, (c) Maximum $P_{r1}$, $P_{s2}$ and $P_{r2}$ as function of $k$

Figure 4-15 (b) shows the variation of maximum RE power ($P_{r2}^m$) with respect to the compensator location factor $k$ obtained via the numerical solution of (4.5.24). It indicates that maximum $P_{r2}$ increases from 8.64 p.u. for $k = 0$ to 15.38 p.u. for $k = 0.46$. Therefore, in terms of realizing the maximum power deliverable at the RE of section 2, the optimal location of the compensator should be placed at $k = 0.46$ in this compensated line example.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Figure 4-15 (c) depicts the variation of the maximum RE power of section 1 \( P_{r1}^m \) and the maximum SE power of section 2 \( P_{r2}^m \) against the value of \( k \). It can be seen from Figure 4-15 (c) that the \( P_{r1}^m \) and \( P_{r2}^m \) curves intercept at \( k = 0.447 \). The design approach described in [32] is based on locating the compensator at this value of \( k \) such that \( P_{r1}^m \) is equal to \( P_{r2}^m \). However, this approach does not guarantee that the \( k \) value so obtained would result in the maximum power being obtained at the RE of line section 2. Indeed, in this example, when \( k = 0.447 \), the maximum \( P_{r2} \) is 14.81 p.u., which is less than the value of 15.38 p.u. determined using (4.5.24). Thus to secure the highest benefit in terms of the power transfer capability, the optimal location of the shunt compensator should be determined using (4.5.24).

Figure 4-16 illustrates the variations of \( P_{z2}, P_{r1} \) and \( P_{r2} \) against the angle (\( \delta_z \) or \( \delta_r \)) of the respective line section, with the shunt compensator installed at the optimal location determined using the proposed method, i.e. \( k = 0.46 \). Figure 4-16 (b) is an expanded view of the rectangular area of Figure 4-16 (a). Since the compensator does not absorb or deliver any active power, thus \( P_{r1} = P_{z2} \). With this compensator location, section 1 of the line can deliver up to the maximum power of 18.14 p.u. at its receiving end (bus \( m \)). This situation is represented by the point \( a \), corresponding to the angle \( \delta_z = 84.07^\circ \) across the two ends of section 1 of the line. The corresponding power flow status of section 2 will be represented by the points \( b \) i.e. \( P_{z2} = P_{r1} = 18.14 \) p.u., and \( c \) where \( P_{r2} = 15.38 \) p.u.. The angle across the line section 2 is \( 77.04^\circ \), i.e. \( \delta_r = -77.04^\circ \). Thus the total transmission angle at this maximum power transfer condition is \( \delta_{max} = \delta_z - \delta_r = 161.11^\circ \).

Figure 4-16 (a) Power–angle characteristics of the compensated transmission line using the proposed method, (b) an expanded view when \( k = 0.46 \)
Figure 4-17 (a) shows the power–angle curves for the same transmission line with the shunt compensator installed at the location determined using the method described in [32], i.e. at \( k = 0.447 \). By the same reasoning as before, section 1 delivers the maximum power \( P_{r1} = 18.65 \) p.u. at its receiving end (bus \( m \)). This situation is represented by the point \( a \) where the angle \( \delta_r = 84.06^\circ \). The corresponding operating points of Section 2 are represented by \( b \) where \( P_{r2} = P_{r1} \), and \( c \) on the \( P_r2 \) curve. The RE power of section 2 is \( P_{r2} = 14.81 \) p.u. with \( \delta_r = -95.91^\circ \). Thus the total transmission angle at the maximum power transfer is \( \delta_{\text{max}} = \delta_r - \delta_r = 179.97^\circ \), which is some 18.8° higher than \( \delta_{\text{max}1} \), the angle resulting from the design method proposed in this paper.

It is interesting to note that in this example, should the compensator be located at the midpoint of the line (i.e. \( k = 0.5 \)), the maximum \( P_{r2} = 14.89 \) p.u. and occurs at \( \delta = 142.52^\circ \).

Using this graphical way, it demonstrates that the proposed method to determine the optimal compensator location allows a higher power transfer level \( P_{r2} \) to be achieved and at a smaller \( \delta \). From the observation made on Figure 4-16 and Figure 4-17 (a), it is obvious that as the shunt compensator location \( k \) changes, the \( P - \delta \) characteristics for the transmission line also varies. Figure 4-17 (b) shows a particularly interesting case. It shows that when the compensator is connected at \( k = 0.452 \), the curves of \( P_{r1} \) and \( P_{r2} \) intercept at the point \( (a_1 \text{ or } b_1) \) where \( P_{r1} \) is at its maximum value. From the results of Section 4.4, if one assumes the two line sections are of the same construction and type,
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

$P_{r1}$ and $P_{r2}$ curves would both reach their peak values at almost the same $\delta$ value. Therefore, it also means that $P_{r2}$ will be at the maximum value $c_1$. Under this operating condition, the power angles of section 1 ($\delta_1$) and section 2 ($\delta_2$) are of the same magnitude. With $k = 0.452$, the maximum power transfer capability of the system is limited by $P_{r1}$, the maximum RE power of section 1.

As $k$ reduces from 0.452, Figure 4-18 (a) illustrates that the curve of $P_{r1}$ rises to $P'_{r1}$ because the length of section 1 reduces and hence, an increased amount of real power can be transmitted in this section. At the same time the curves $P_{s2}$ and $P_{r2}$ are lowered to $P'_{s2}$ and $P'_{r2}$, as section 2 becomes longer and less power can be transmitted. By observing the manner of the movement of these curves, one concludes that the maximum power transfer capability of the system is then determined by the maximum of the RE power of section 2 shown as $c_2$ in Figure 4-18 (a). It corresponds to the operating level $b_2$ for $P_{s2}$ and $a_2$ for $P_{r1}$. The point $c_2$ is seen to be at a level lower than that obtained when $k = 0.452$, i.e. $c_1$. On the other hand, as $k$ increases beyond 0.452, the curve $P_{r1}$ is lowered to $P''_{r1}$, while that of $P_{s2}$ and $P_{r2}$ rise to $P'_{s2}$ and $P'_{r2}$ respectively. The maximum power transfer capability is limited by maximum RE power of section 1, as shown in Figure 4-18 (b). Hence, it indicates that $k$ should be increased further until the decrease in the level in $P_{r1}$ more than offset the rise in $P_{s2}$, and results in the beginning of a decrease in $c_2$ level. Thus in general, the optimal location is determined by the balancing of these two conflicting movements until $c_2$ reaches its maximum value. In the present example, Figure 4-16 shows the condition when the maximum $P_{r2}$ is obtained at $k = 0.46$.

![Figure 4-18 Power-angle characteristics for (a) $k$ decreases, (b) $k$ increases from $k=0.452$](image)
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Having shown how the location of the shunt compensator affects the maximum power transfer under steady-state, it is also necessary to consider the steady-state stability limit of the compensated system.

4.9 Steady-State Stability

Return to the power-angle curve (d) shown in Figure 4-9 obtained by solving (4.5.24) numerically. As the compensator location is varied, i.e. by varying \( k \), a family of curves such as that shown in Figure 4-19 can be obtained. Figure 4-19 (a) was obtained when the line losses and line capacitance are ignored. It can be noticed from Figure 4-19 (a) that for \( k < 0.5 \), all \( P_r - \delta \) curves approach the point \( a (180^\circ) \) at zero real power. However for \( k = 0.5 \), \( P_r \) actually reaches its maximum value at \( \delta = 180^\circ \). The reason is that for \( k \neq 0.5 \), the reactances of the two line sections (\( X_{11} \) and \( X_{12} \)) are different. Hence when the real power flow is zero, \( \delta \) across the whole transmission line must be equal to either 0° or 180°. Also, the power angle across each of the two line sections (\( \delta_i \) and \( \delta_r \)) have to be either 0° or 180° since there is no real power flow in either section. When \( k = 0.5 \), \( X_{11} = X_{12} \). Zero real power flow occurs when the power angle of the whole line is 0°. As \( \delta \) increases, the real power transfer level also increases. When \( \delta = 180^\circ \) and since the section line reactances are the same, it means that the angles across the two line section must be equal. Hence \( \delta_r = -\delta_r = 90^\circ \). As the line section reactance is now halved of that of the whole line, therefore, one notes that the maximum \( P_r \) will be doubled that when the compensator is placed at \( k = 0 \) where its reactive compensation does not play an active role in affecting the transfer limit. In this way maximum \( P_r \) is obtained when \( \delta = 180^\circ \). As stated in Section 4.6.1, this phenomena of doubling the transfer limit is well-known and can be found in, for example, [3, 4, 27-30].

If the exact line model shown Figure 4-5 is used instead, it can be observed from Figure 4-19 (b) that all \( P_r - \delta \) curves assume different \( \delta \) values at zero real power for \( k < 0.5 \). Indeed, it is seen in Figure 4-19 (c) and (d) that for \( k > 0.5 \), all \( P_{r2} - \delta \) curves reach the point \( a (\delta = 168^\circ) \) when \( P_{r2} = 0 \), and for \( k < 0.5 \), all \( P_{s1} - \delta \) curves approach the point \( b (\delta = 192^\circ) \) when \( P_{s1} = 0 \). The reason for the shifting of the \( P-\delta \) curves is further investigated through stability consideration of the power system, as follows.
Figure 4-19 $P-\delta$ characteristics of transmission lines with different shunt compensator location: (a) $P_{r2}$ vs $\delta$ for simplified transmission line model (Note: $P_{s1} = P_{r2}$); (b) $P_{r2}$ vs $\delta$ for $k \leq 0.5$ (c) $P_{r2}$ vs $\delta$ for $k \geq 0.5$ (d) $P_{s2}$ vs $\delta$ for $k \leq 0.5$, for $\pi$ circuit model of transmission lines.

From the analysis of Section 4.4, it is noted that $P_{r1}$ and $P_{r2}$ of the respective line sections reach their maximum values at about the same angle $\delta_{cr}$, if the line sections are of the same type. Similar statement can also be made regarding the $P_{s1}$ and $P_{s2}$, and its critical angle $\delta_{cr}$. Figure 4-20 (a) and (b) show $P_{r1}$ and $P_{s2}$ plotted against the angle $\delta$, or $\delta$, of the respective line section, with $k=0.4$ and $0.5$, respectively. The choice of these cases is intended to study the conditions when $k$ is below or above the value of 0.447 used in Figure 4-17 (a).

From Figure 4-17 (a) it is seen that with $k=0.447$, $P_{r1}^m$ and $P_{s2}^m$ are the same. If $k$ were to decrease (i.e., $k<0.447$), $P_{r1}^m$ would increase while $P_{s2}^m$ would decrease. Thus point a in
Figure 4-17 (a) moves upwards and point b goes downwards. Hence, a would be higher than b (as shown in Figure 4-20 (a)). Suppose the initial operating situation of sections 1 and 2 correspond to the points “a₁” and “b₁” as shown in Figure 4-20 (a). Of course, a₁ and b₁ are at the same power level. If for whatever reason the transmission angle were to increase, the operating points of $P_{r1}$ and $P_{s2}$ will increase, say, to “a₂” and “b₂”, respectively. As indicated by the arrows, such increase in $\delta$ $(=\delta_{r}-\delta_{r})$ would be met by a corresponding increase in $P_{r1}$ and $P_{s2}$ until $P_{s2}$ reaches its maximum value, point b. Any further attempt to increase $\delta$ would cause $P_{s2}$ to decrease such as indicated by “c₂” in the figure. The corresponding operating point of section 1 at this stage is represented by the point “a₃”. Thus section 2 would operate in the unstable region. Continuing this way, zero power at bus m (i.e. $P_{r1} = P_{s2} = 0$) will eventually occur at the angles shown in Figure 4-20 (a) when $\delta_s \approx 0°$ and $\delta_s = 2(\pi-\delta_{cr,r})$. Thus the overall $P - \delta$ curve approaches the angle $\delta =\delta_{r}-\delta_{r}=2(\pi-\delta_{cr,r})$. Furthermore, from the analysis of Section 4.4, it has been established that all the $P_{r} - \delta$ curves of the same line type and practical length will reach the maximum power point at almost the same angle of $\pi-\delta_{cr,r}$. Hence, for $k < 0.447$, all the $P - \delta$ curves will approach the angle $2(\pi-\delta_{cr,r})$ or 192° for this line example, as shown in Figure 4-19 (d). For this range of $k$ value, the maximum power transfer is governed by the level of “b”.

![Figure 4-20 Power-angle characteristics of line sections 1 and 2: (a) k = 0.4, (b) k = 0.5](image)

On the other hand, for higher values of $k (>0.447)$, the maximum $P_{r1}$ would decrease while the maximum $P_{s2}$ increases when compared to that depicted in Figure 4-17 (a). Thus the point “b” in Figure 4-17 (a) moves upwards and point a goes downwards. Hence, a is lower than b (as shown in Figure 4-20 (b)). Now section 1 operates in the unstable region while section 2 operates in the stable region if, during any transient process, the
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

power flow transits beyond the maximum power point “a”. Using the same reasoning as before, it means that power at bus \( m \) reduces to zero when \( \delta_s = 2\delta_{cr,r} \) and \( \delta_r \approx 0^\circ \). Thus the overall \( P-\delta \) curve approaches the angle \( \delta = \delta_s - \delta_r = 2\delta_{cr,r} \) or \( 168^\circ \) as illustrated as Figure 4-19 (c) for this compensated line. The transition of the shifting of the \( P-\delta \) curve takes place at the particular location factor \( k \) when \( P_{rl}^n = P_{r2}^n \). This conclusion is the same as that obtained in [32].

Figure 4-21 (a) and (b) show \( P_{rl}, P_{r2} \) and \( P_{s2} \) plotted against the angle \( \delta_s \) or \( \delta_r \) of the respective line section, with \( k < 0.447 \) and \( k > 0.447 \), respectively. The aim is intended to study the reason why all \( P_{rl} - \delta \) curves approach different \( \delta \) values at zero real power for \( k < 0.447 \) as shown in Figure 4-19 (b) while all \( P_{s1} - \delta \) curves approach different \( \delta \) values at zero real power for \( k > 0.447 \) as shown in Figure 4-19 (d). The reason for the shifting of the \( P-\delta \) curves is further investigated through stability consideration of the power system, as follows.

For \( k < 0.447 \), as \( k \) reduces from \( k_l \) to \( k_2 \) (suppose initially \( k_l > k_2 \)), Figure 4-21 (a) illustrates that the curve of \( P_{rl} \) rises to \( P'_{rl} \) because the length of section 1 reduces and hence, an increased amount of real power can be transmitted in this section. At the same time the curves \( P_{s2} \) and \( P_{r2} \) are lowered to \( P'_{s2} \) and \( P'_{r2} \), as section 2 becomes longer and less power can be transmitted. Furthermore, from the analysis of Section 4.4, it has been established that all the \( P_{rl} - \delta \) curves of the same line type and practical length will reach the zero power point at almost the same angle of \( 2\delta_{cr,r} \). Thus zero power at RE of section 2 for \( k = k_l \) or \( k = k_2 \) (i.e. \( P_{r2} = 0 \) or \( P'_{r2} = 0 \)) will both occur at the angle of \( 2\delta_{cr,r} \) or \( 168^\circ \) for this line example, as shown in Figure 4-21 (a). They correspond to the operating level \( a_1 \) for \( P_{s2}, b_1 \) for \( P_{rl}, a_2 \) for \( P'_{s2} \) and \( b_2 \) for \( P'_{rl} \), respectively. Since \( P'_{s2} \) is lower than \( P_{s2} \) for the same angle, \( P_{r2} \) at operating point \( a_1 \) is larger than \( P'_{r2} \) at operating point \( a_2 \). As the compensator does not absorb or deliver any active power, thus \( P_{rl} (= P_{s2}) \) is larger than \( P'_{rl} (= P'_{s2}) \). Since \( P'_{rl} \) is higher than \( P_{rl} \) for the same angle, which means the angle of section 1 for \( k = k_l \) should be larger than that for \( k = k_2 \) for the same RE power of section 1 (i.e. \( P_{rl} = P'_{r1} \)), therefore the angle of section 1 for \( k = k_l \) is larger than that for \( k = k_2 \), i.e., \( \delta_{b1} > \delta_{b2} \), as shown in Figure 4-21 (a). Thus zero power at RE of section 2 for \( k = k_l \)
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

(i.e. $P_{r2} = 0$) will occur at the angles shown in Figure 4-21 (a) when $\delta = \delta_{b1}$ and $-\delta = 2\delta_{cr,r}$. Thus the $P - \delta$ curve approaches the angle $\delta = \delta_{b1} - \delta_{r} = 2\delta_{cr,r}$ for $P_{r2} = 0$. At the same time, zero power at RE of section 2 for $k = k_2$ (i.e. $P'_{r2} = 0$) will occur at the angle $\delta = \delta_{b2} + 2\delta_{cr,r}$. Since $\delta_{b1} > \delta_{b2}$, thus the $P_{r2} - \delta$ curve approach larger $\delta$ values at zero real power for $k = k_2$ than $k = k_1$, as shown in Figure 4-19 (b). It is therefore seen that all $P_{r2} - \delta$ curves approach different $\delta$ values at zero real power for $k < 0.447$, as shown in Figure 4-19 (b).

![Figure 4-21](image_url)

(a)

(b)

Figure 4-21 Power–angle characteristics of line sections 1 and 2: (a) $k < 0.447$, (b) $k > 0.447 (k = 0.5)$
For $k > 0.447$, from the analysis of Section 4.4, it has been established that all the $P_s - \delta$ curves of the same line type and practical length will reach the zero power point at almost the same angle of $2(\pi-\delta_{cr,r})$. Thus zero power at SE of section 1 (i.e. $P_{s1} = 0$) will occur at the angle of $2(\pi-\delta_{cr,r})$ or 192° for this line example, as shown in Figure 4-21 (b). It corresponds to the operating level $a$ for $P_{r1}$ and $b$ for $P_{r2}$ respectively. Thus zero power at SE of section 1 will occur at the angles shown in Figure 4-21 (b) when $\delta = 2(\pi-\delta_{cr,r}) + \delta_b$. Thus the $P - \delta$ curve approaches the angle $\delta = \delta_r - \delta_r = 2(\pi-\delta_{cr,r}) + \delta_b$ for $P_{s1} = 0$. It is therefore seen that $P_{s1} - \delta$ curves approach different $\delta$ values at zero real power for $k > 0.447$ as shown in Figure 4-19 (d).

Stability studies involve many issues and EAC is only one approximate method which can be used for a quick prediction of stability of a single-machine-infinite-bus system. Further work will be needed in extending the above results for more complex networks.

### 4.10 Optimal Location: Multiple Shunt Compensators

The above results may now be extended to include lines with multiple shunt compensators. With sources at nodes A and B, consider firstly a long transmission line compensated by two shunt FACTS devices at nodes “$N_1$” and “$N_2$”, as shown in Figure 4-22 (a). The total line length is $l$ and $k_1$ and $k_2$ are used to denote the fractions of line sections so that $k_1 l$, $(k_2-k_1) l$ and $(1-k_2) l$ equal to the line lengths between A and $N_1$, $N_1$ and $N_2$, $N_2$ and B, respectively. The transmission line is therefore divided into three sections (1, 2 and 3), and each section can be represented by its $\pi$-equivalent circuit described in Section 2. The corresponding powers at the sending and receiving ends of the sections are as shown in Figure 4-22 (a).

Denote line section 12 as that part of the line between nodes A and $N_2$. Its length is therefore $k_2 l$. For a fixed value of $k_2$, in order to attain the maximum deliverable power $P_{r12}$ at the RE of section 12 (i.e. at node $N_2$), the optimal location ($k_1$) of Compensator1 can be determined using the method described in the Section 4.8.
Figure 4-22 A transmission system with two shunt reactive power compensators: (a) schematic diagram, (b) $P-\delta$ characteristics of line sections 12 and 3 when $k_1=0.46$, $k_2=0.602$; (c) when $k_1=0.46$, $k_2 < 0.602$, (d) when $k_1=0.46$, $k_2 > 0.602$

For the convenience of discussion, the same transmission line example described earlier is re-used herewith except that the line length is increased to 600-km. It is noted from Section 4.9 that as the shunt compensator locations change, the $P-\delta$ characteristics for the transmission line also vary. Figure 4-22(b) shows the interesting case when Compensator 2 is connected at $k_2 = 0.602$, the operating point $b$ for $P_{r3}$ corresponding to the operating level where $P_{r12}$ and $P_{r3}$ are both at their maximum values ($a$ and $c$, respectively).
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

respectively). Once this has been achieved, from the analysis shown in Section 3, the optimal location of Compensator1 is when $k_1 = 0.46k_2$ or 0.2769. Thus, with $k_2 = 0.602$, $k_1 = 0.46k_2$, the maximum power transfer capability of the system is limited by $P_{r12}$ or $P_{r3}$.

As $k_2$ reduces from 0.602, the curve $P_{r12}$ rises to $P'_{r12}$ because section 12 becomes shorter and hence, an increased amount of real power can be transmitted over this line section. This is shown in Figure 4-22 (c). At the same time $P_{s3}$ and $P_{r3}$ are lowered to $P'_{s3}$ and $P'_{r3}$, as section 3 becomes longer and less power can be transmitted. Hence even though section 12 is capable of delivering more power to $N_2$, section 3 is unable to transfer that power to $B$. Thus, one concludes that the maximum power transfer capability of the power system is then limited by the maximum of the RE power of Line section 3 shown as $c1$ in Figure 4-22 (c). It corresponds to the operating level $b1$ for $P'_{s3}$ and $a1$ for $P'_{r12}$. The point $c1$ is seen to be at a level lower than that obtained when $k_2 = 0.602$, i.e. $c$ in Figure 4-22 (b).

On the other hand, as $k_2$ increases beyond 0.602, the curve $P_{r12}$ is lowered to $P'_{r12}$, while that of $P_{s3}$ and $P_{r3}$ rise to $P'_{s3}$ and $P'_{r3}$ respectively. By the same reasoning as before, it leads one to the conclusion that the maximum power transfer capability of the system is limited by the maximum RE power of section 12, as shown in Figure 4-22 (d). Hence, $k_2$ should be increased further until the decrease in the level in $P_{r12}$ more than offset the rise in $P_{s3}$, and results in the beginning of a decrease in $c1$. Thus similar to the one-compensator case, the optimal location for Compensator1 is determined by the balancing of these two conflicting movements until $c1$ reaches its maximum value.

Based on the above reasoning, it is now proposed that the following procedure be used to determine the optimal compensator locations so that maximum power transfer level $P_{rw}^m$ can be achieved for a general radial and symmetric network. Starting from the 2-compensator system shown in Figure 4-22 (a), the optimal locations are denoted as $k_{1,2}$ and $k_{2,2}$ for Compensator1 and Compensator2 respectively. The new index $k_{m,n}$ is introduced to denote the optimal location of the $m^{th}$ compensator on a transmission line which has $n$ compensators connected to it. The design procedure for $n = 2$ is:
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

Step 1: Determine the optimal location \( k_{1,1} \) in which only one shunt compensator is considered using the method proposed in Section 4.8. Set another index \( j = 1 \). The remaining fraction of the line section toward the RE of the line is 1 - \( k_{1,1} \). Divide this line section into \( h \) equal sections, assume that the intention is to determine the optimal compensator location to within a location accuracy of \((1 - k_{1,1})l/h\).

Step 2: Set Compensator2 location to \( k_2^j = k_{1,1} + (1 - k_{1,1}) * j / h \).

Step 3: For this value of \( k_2^j \), find the maximum deliverable power of \( P_{r12}^j \) using the method proposed in Section 4.8.

Step 4: Determine the critical angle \( \delta_{cr,r3} \) for the line section 3 using (4.4.9).

Step 5: Obtain the maximum RE power (\( P_{r3}^j \)) and the corresponding SE power (\( P_{s3,c} \)) for the line section 3 by replacing \( \delta \) in (4.4.2) and (4.4.1) with \( \delta_{cr,r3} \), respectively.

Step 6: If \( P_{r12}^j > P_{s3,c} \), this corresponds to the situation shown in Figure 4-22 (c), the maximum \( P_f \) for the whole line for this given value of \( k_2^j \) is \( P_{r12}^j \). Otherwise, set \( P_{s3} = P_{r12}^j \) and find the corresponding angle \( \delta_3 \) using (4.4.1), i.e.

\[
\delta_3 = \sin^{-1} \left( \frac{P_{s3}}{P_0 - \xi_3} \frac{P_{\text{max},3}}{P_{r3}} \right) + \varphi_3
\]

Then obtain \( P_f^j \) by replacing \( \delta \) in (4.4.2) with \( \delta_3 \).

Step 7: Repeat Steps 2-7 for \( j = 2, \ldots, h-1 \) and obtain the corresponding maximum \( P_f^j \) for each \( j \).

Step 8: Determine the maximum RE power for the whole line: \( P_{rw}^m = \max_j \{ P_f^j \}, j = 1, \ldots, h-1 \).

Suppose \( f^m \) is the index corresponding to the maximum power transfer situation \( P_{rw}^m \).

Hence the optimal location for Compensator2 is \( k_{2,2} = k_{1,1} + (1 - k_{1,1}) * f^m / h \), and the optimal location for Compensator1 is \( k_{1,2} = k_{1,1} k_{2,2} \).

Using \( k_{1,2} \) and \( k_{2,2} \) for a line with 2 shunt compensators, the optimal locations \( k_{1,3}, k_{2,3} \) and \( k_{3,3} \) for a line with 3 shunt devices can be deduced in a similar procedure as described. Thus, the optimal location \( k_{3,3} \) for the third shunt compensator is determined using \( k_{3,3} = k_{2,2} + (1 - k_{2,2}) * f^m / h \), where \( f^m \) is determined through steps 1-8 above and from which the other compensator optimal locations are given as \( k_{1,3} = k_{1,2} k_{3,3}, k_{2,3} = k_{2,2} k_{3,3} \).
In general, suppose the optimal locations \{k_{i,M-2}, i = 1, \ldots, M-2\} for a long radial transmission line with \(M-2\) \((M > 4)\) shunt compensating devices at nodes \(N_1, N_2, \ldots, N_{M-2}\) along the line are already obtained. The optimal locations \{k_{i,M-1}, i = 1, \ldots, M-1\} for a line with \(M-1\) shunt reactive power compensators can then be determined by repeating the above iterative procedure. The optimal location for the \((M-1)^{th}\) shunt compensator is determined using the equation

\[ k_{M-1,M-1} = k_{M-2,M-2} + (1 - k_{M-2,M-2}) \cdot \frac{f''}{h} \]  

The remaining compensator locations are given by

\[ k_{i,M-1} = k_{i,M-2} \cdot k_{M-1,M-1}, \]  

for \(i = 1, \ldots, M-2\).

As an illustration, when applied to the 600-km, 345-kV line example for up to 4 compensators, the above procedure results in the optimal locations shown in Table 4-2. Also shown in the last column of Table 4-2 is the corresponding maximum power transfer level \((P''_{m})\) when the compensators are placed at equal intervals along the line. Clearly the optimally located compensators have increased considerably the maximum power transmittable.

Table 4-2: Shunt compensated 345kV transmission line example of [32] for up to 4 compensators

<table>
<thead>
<tr>
<th>n-1</th>
<th>(k_1)</th>
<th>(k_2)</th>
<th>(k_3)</th>
<th>(k_4)</th>
<th>(P''_{m}) (p.u.)</th>
<th>(P'^{m'}_{m}) (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.46</td>
<td></td>
<td></td>
<td></td>
<td>11.64</td>
<td>11.26</td>
</tr>
<tr>
<td>2</td>
<td>0.46*0.62 (\approx 0.285)</td>
<td>0.62</td>
<td></td>
<td></td>
<td>16.00</td>
<td>15.16</td>
</tr>
<tr>
<td>3</td>
<td>0.46<em>0.62</em>0.7 (\approx 0.2)</td>
<td>0.62*0.7 (\approx 0.434)</td>
<td>0.7</td>
<td></td>
<td>19.80</td>
<td>18.49</td>
</tr>
<tr>
<td>4</td>
<td>0.46<em>0.62</em>0.7*0.75 (\approx 0.15)</td>
<td>0.62<em>0.7</em>0.75 (\approx 0.326)</td>
<td>0.7*0.75 (= 0.525)</td>
<td>0.75</td>
<td>23.15</td>
<td>21.37</td>
</tr>
</tbody>
</table>

Note: \(P''_{m}\) = Maximum Power Transfer Capacity of Compensated Line with Optimal Compensator Placement. \(P'^{m'}_{m}\) = Maximum Power Transfer Capacity of Compensated Line with the Compensators Equally Spaced.
Chapter 4: Shunt Reactive Power Compensation of Long Transmission Lines

4.11 Conclusions

Shunt compensation of a long-line at terminals and at intermediate point(s) has been considered. The results of the analysis allow the effect of line resistance and capacitance on the amount of the reactive compensation needed for power transfer to be determined much more accurately by utilizing the exact long-line model which includes line losses.

In considering compensation at line terminals and based on the exact π-circuit model of a transmission line, some important characteristics governing the real power flow and voltage angle across the line have been derived and elaborated on. The results are then applied to the case of shunt reactive power compensation at intermediate node of long-lines. Impedances of the line terminal synchronous sources are also taken into account. The analysis provides a much more accurate assessment of the maximum power transfer capability of the shunt compensated system.

An important application of the results of the analysis is that the reactive power needed at the line intermediate point at any given power transfer level can be determined using the computational procedure described. Re-evaluation of the contribution of the midpoint shunt compensator to enhance stability shows that whereas in previous works based on simplified line model, the improvement in power transmissibility with respect to the steady-state stability limit is predicted to increase by a factor of 2, the present analysis indicates that the improvement is only by a factor of 1.67. The decrement is accounted for due to the presence of line resistances and the distributed nature of the line shunt capacitances. If the terminal impedances at the end(s) of the line are also included, there is a further reduction in the factor to around 1.59. A more accurate assessment of the beneficial contribution of shunt reactive power compensation has been provided by the analysis and additional insights into the steady-state transfer capability of long transmission lines have therefore been gained.

Another important application of the results of the analysis is to determine the optimal location of the shunt reactive power compensator to achieve maximum power transfer.
Expressions which can be used in a computational procedure to achieve this aim have been derived. The effects of the shunt compensator location on the power transfer level have been examined. Whereas in previous works which are based on simplified line model, the midpoint of a transmission line is considered the optimal location for shunt reactive power support. The present analysis indicates that the shunt compensator needs to be placed slightly off-centre to get the highest possible benefit. Both the power transfer capability and stability of the system can be improved. Stability limit of the compensated system has been elaborated on through the graphical examination of the $P-\delta$ curves.

Additional investigation has been carried out on optimal locations of multiple shunt compensators. A computational procedure in which the optimal compensator locations can be determined is included. Simulation results show that increased power transfer of the transmission system can be achieved by adopting the optimal shunt compensator locations.
Chapter 5

Unified Power Flow Controller Incorporated with an Active Power Source

This chapter is organized as follows. In Section 5.1, the system description is presented. A mathematical network model of the transmission system with the Unified Power Flow Controller –Active Power (UPFC-AP) is analyzed in Section 5.2. Next UPFC-AP subject to system constraints is designed. In Section 5.3, the feasible system operating regime is derived and an optimal economic dispatch and an optimal load tracking scheme are developed. The simulation results are demonstrated in Section 5.4. Finally Section 5.5 concludes this chapter by highlighting the contributions.

As introduced in Chapter 2, in an open access environment, transmission systems are often required to provide higher bulk power transfer capability and to accommodate a much wider range of possible generation scenario. This has led to an increased focus on locating transmission system bottle-necks and on the means by which such constraints can be alleviated. With this in mind, the SVC has been chosen in Chapters 3 and 4 for consideration as a fast reactive power compensation device. However, the results have shown that large disturbance may cause the system loses stability when only shunt compensator is in service. Therefore, it is needed to extend compensation to include series compensation. In this regard, Chapter 2 explains how the Unified Power-Flow Controller (UPFC) offers a versatile and viable alternative to conventional reinforcement methods in solving the transmission problem. The UPFC is able to control simultaneously or selectively system parameters, such as voltage magnitudes and phases, which determine power flow patterns in a transmission network [56]. These basic capabilities have resulted in the UPFC being recognized as a most powerful device available in the control of large transmission systems. Consequently, extensive research attention has been directed toward studying the behavior and control of UPFC under the steady-state and dynamic conditions [132]–[142]. UPFC has been successfully installed in several power systems.
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

[137] and has been proven to be particularly useful in reactive power compensation [138], in power quality improvement [139], and in damping inter-area oscillations [140], or combination of these applications. For example, [141] has proposed a new controller for UPFC designed to eliminate voltage flicker and current harmonics while also achieves power flow control for load-tracking.

Just like any other physical equipment, however, the UPFC has its own internal limits which have to be considered in practical network design. The works described in [132]-[141] do not take these limits into consideration. In an attempt to address these issues, a strategy has been proposed in [61] and uses the phase angle of the series-injected voltage to alleviate some of the UPFC voltage and power ratings constraints. The design objective is to achieve maximum power transfer. Unfortunately, the authors have not considered the effects the magnitudes of the series-injected voltage and shunt voltage on the power transfer level. In a more recent work, the authors of [63] have presented a dispatch strategy for a UPFC to maximize power transfer under which voltage stability limit has been included. The proposed strategy requires the UPFC to change its power circulation set-point control, so as not to exceed one or both of the UPFC Voltage Source Converter (VSC) capacities.

While significant developments in the power-flow controller have been made, factors inhibiting wider UPFC applications must surely include those pertaining to equipment reliability and cost. With the continued advancements made in power electronics devices and associated technology, reliability of UPFC can be expected to improve over time. It may also be argued that wider acceptance of the UPFC could materialize if additional functions/benefits derived from its application on network control can be realized. If the extra functions/benefits can be achieved with only a modest incremental investment, clearly the economic competitiveness of UPFC can be enhanced. The intent of the present work is to explore this possibility through the incorporation of an active power (AP) source into the conventional UPFC.

Hence unlike previous works on UPFC, this chapter proposes a UPFC-AP scheme which includes an energy storage system. The aim is to report an analysis into the compensated
system and to show the extent by which the proposed scheme can further enhance transmission system performance.

5.1 System Description

With reference to Figure 5-1, one could visualize the situation in which along a long-distance transmission route interconnecting a bulk generation source S and a load center R, a UPFC installed at an intermediate node T on one of the transmission links can significantly enhance the transmission system performance. Suppose a relatively smaller capacity RE resource also exists close to T. If the energy from the RE resource can be harnessed and the resulting AP source incorporated into the UPFC, the objective is then to design the combined UPFC-AP system to support a larger range of transmission services and network control functions. And if these functions can be achieved at a modest incremental cost compared to that of a stand-alone UPFC, the additional services and enhanced performance will certainly increase the appeal of the UPFC to the utility industry, the transmission service provider and the RE operator.

![Figure 5-1 Schematic diagram of a transmission system incorporated with the UPFC-AP](image)

Figure 5-1 Schematic diagram of a transmission system incorporated with the UPFC-AP
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

It must be noted that another main reason for introducing the UPFC is to ensure a more equitable sharing of loads on parallel links [56]. This can easily be achieved by the UPFC-AP, as will be described shortly.

On closer examination of the above proposal, one of the foremost technical concerns is the impacts of the RE generation on network frequency and voltages. For example, if the source obtains its energy from a wind-farm, the RE output power depends very much on the wind speed which tends to fluctuate. For this reason, it will not be satisfactory to directly introduce the AP source into the network without some special measures being taken. In the context of the present design, one possible solution to the problem is to include a battery energy storage system (BESS), in an arrangement shown in Figure 5-2. The schematic diagram shows that the RE source is used to charge a stand-by battery bank (BESS1) while a second battery bank (BESS2) becomes the in-service AP source, connected across the DC bus of the UPFC. In this way, fluctuations in the RE source input would be decoupled from the network. BESS is a proven technology and the relatively lower costs of the battery are sound reasons for considering its inclusion in the scheme. Indeed, the uses of BESS in conjunction with wind RE resources have been considered by others, see e.g. [143].

Figure 5-2 Schematic of a wind turbine-battery energy storage system (BESS): single-line representation
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

External to the UPFC-AP would be the transmission system and in this investigation, it is shown in a loop configuration in Figure 5-1. Under normal mode of operation, power transfer is from S to R. As in conventional UPFC, the UPFC-AP consists of a parallel branch and a series branch. Each branch will contain a transformer, a voltage-source converter (VSC) based on turn-off-capable power semiconductor devices. The dc circuit interconnects the two converters. The basic function of Converter 1, connected to UPFC-AP terminal node T, is to supply the real power demanded by Converter 2 through the common dc link. Converter 1 can also generate or absorb controllable reactive power. On the other hand, Converter 2 is connected in series with the transmission line through the series transformer. The real and reactive power flows in the transmission line can be quickly regulated by changing the magnitude ($V_{se}$) and phase angle ($\delta_{se}$) of the series-injected voltage produced by Converter 2 [56].

In conventional UPFC system, the DC link is supported by a dc storage capacitor. The real power demanded by Converter 2 can only be supplied by Converter 1 drawn from the AC system through the common dc link. In order to keep the voltage $V_{dc}$ across the storage capacitor constant, the active power supplied by the series converter should be equal to the active power drawn from the AC system through the shunt converter, i.e. $P_{se} = P_{sh}$. However, when the UPFC is equipped with an AP source, i.e. the BESS described earlier, the real power demanded by Converter 2 is supplied by both the battery bank and that from Converter 1 drawn from the AC system. Thus the active power supplied by the series converter is the sum of the active power ($P_d$) provided by the AP source and that drawn from the AC system through the shunt converter, i.e.

$$P_{se} = P_d + P_{sh}. \quad (5.1.1)$$

Since Battery Energy Storage System (BESS) forms the AP source, it is necessary to review the characteristics of the BESS.

5.2 Problem Formulation

5.2.1 Mathematical Model and Preliminary Analysis

A mathematical network model will now be used to describe the relationship between the transmission system and UPFC-AP under steady-state and quasi-steady-state conditions.
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

It is assumed that the transmission system has negligible losses and appropriate filters would have been installed in conjunction with the UPFC-AP such that harmonic levels will be minimized and the dc-bus voltage has a negligible level of ripples. Hence, the analysis to follow would only consider the power frequency component of the voltages/currents on the AC circuits. The single-line diagram of this model is shown in Figure 5-3 where the UPFC-AP has sectionised the line into two sections with reactances $X$ and $X_i$, respectively. The alternate parallel link between S and R is shown to have the effective transfer reactance $X_2$. The line reactances are assumed known. The power-flow control over the lines can be evaluated through power balance analysis.

\[ P_s + jQ_s = V_s I_s \]
\[ V_{se} = jX_{se} I_{se} \]
\[ V_{sh} = jX_{sh} I_{sh} \]

Figure 5-3 Circuit model of a loop transmission system with the UPFC-AP

In the network model, $V_{se}$ is shown as the magnitude of the series injected voltage whereas the other voltage/current phasor quantities $\bar{V}_s$, $\bar{V}_r$, $\bar{I}_s$, $\bar{I}_r$, $\bar{V}_{sh}$, $\bar{V}_{set}$ are as defined in Figure 5-3. The corresponding voltage/current magnitudes are shown without the "→" sign above the respective symbols. Furthermore, let $S_{se}$ and $S_{sh}$ denote the series and shunt converter transformer ratings, respectively. The UPFC controls power flow of the line through continuous control of $V_{se}$ and $\delta_{se}$, achieved by adjusting the amplitude modulation index $m_B$ and phase-angle $\delta_B$ of Converter 2 control signals, as mentioned earlier. Depending upon the system operating condition, voltage source $V_{se}\angle \delta_{se}$ exchanges real and reactive powers with the external system. Since a UPFC can neither absorb nor deliver real power (losses are neglected), phase angle $\delta_E$ on Converter 1 is adjusted to regulate the real power exchange between the series converter and the system. The
amplitude modulation index $m_c$ can be used to control $V_{sh}$ and consequently, the reactive power exchange between $\delta_{sh}$ and the external system can be regulated [64].

In the proposed UPFC-AP scheme, however, the BESS is connected across the dc-bus of the UPFC as the AP source. While the electrochemical dynamics of BESS are complex, a simplified battery model adopted from [144] is used in the present analysis. In this model shown in Figure 5-4, an internal EMF $E$ is connected in series with a resistor $r$. $E$ and $r$ are functions of the battery state of discharge $f$. Mathematically,

$$E = E_0 - k f$$
$$r = r_0 - k_r f$$

(5.2.1)

where $E_0$ is the no load voltage when the battery is fully charged, $f$ is the state of discharge; $r_0$ is the internal resistance when the battery is fully charged, $k$ and $k_r$ are constants obtained from experiments. Currently there are many methods to measure $f$ on a continuous basis and some of those methods will be briefly described in Chapter 6.

![Figure 5-4 Equivalent circuit of battery](image)

If the battery state of discharge is known, the values of $E$ and $r$ can be determined from (5.2.1), respectively. Therefore, from Figure 5-4, the BESS steady-state output voltage $V_{dc}$ is

$$V_{dc} = E - r I_b$$

(5.2.2)

and the output power $P_d$ of the battery is then given by

$$P_d = V_{dc} I_b$$

(5.2.3)

Hence voltage $V_{dc}$ can be determined in terms of the known or specified system parameters $P_d, E$ and $r$:

$$V_{dc} = \left( E + \sqrt{E^2 - 4r P_d} \right) / 2$$

(5.2.4)
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

One can therefore obtain given active power output \( P_d \) by appropriately regulating \( V_{dc} \).

Select the voltage \( V_r \) as the reference voltage phasor and denote \( \delta \) as the phase angle difference between \( V_r \) and the voltage \( V_s \), i.e. \( V_r = V_r \angle 0 \), \( V_s = V_s \angle \delta \). In order to study how the scheme will perform under combinations of line reactance/transfer reactance values, \( X_1 \) and \( X_2 \) are expressed in terms of \( X \) such that

\[
\begin{align*}
X_1 &= k_1 X \\
X_2 &= k_2 X
\end{align*}
\] (5.2.5)

From Figure 5-3, the steady-state active and reactive powers at the sending and receiving ends of the transmission system can be derived as (5.2.6) - (5.2.9). The detailed derivation is presented in Appendix F.

\[
P_s = V_s V_r \sin \delta /[k_2 X] + [(X_{se} + k_1 X) V_s V_r \sin (\delta - \delta_{se}) + X_{sh} V_r \sin \delta - X_{sh} V_s \sin (\delta - \delta_{se})]/X_{eq}^2
\] (5.2.6)

\[
Q_s = (V_s^2 - V_r^2 \cos \delta)/[k_2 X] + [(X_{se} + k_1 X + X_{sh}) V_s^2 - X_{sh} V_s V_r \cos \delta]/X_{eq}^2 + [X_{sh} V_s \cos (\delta - \delta_{se}) - (X_{se} + k_1 X) V_s \cos (\delta - \delta_{se})]/X_{eq}^2
\] (5.2.7)

\[
P_r = V_r V_r \sin \delta /[k_2 X] + (X_{se} V_r \sin \delta + X V_r \sin \delta_{sh})/X_{eq}^2 + [(X + X_{sh}) V_{se} \sin \delta_{se}]/X_{eq}^2
\] (5.2.8)

\[
Q_r = (V_r^2 \cos \delta - V_r^2)/[k_2 X] + (X_{se} V_r \cos \delta + X V_r \cos \delta_{sh})/X_{eq}^2 + [X + X_{sh}) V_{se} \cos \delta_{se} - (X + X_{sh}) V_r^2]/X_{eq}^2
\] (5.2.9)

where \( X_{eq}^2 = (X_{se} + k_1 X) X + XX_{sh} + (X_{se} + k_1 X) X_{sh} \). Note that the powers are expressed in terms of network parameters and the UPFC control parameters. Similarly it can be shown that the real power exchange \( P_{se} \) of the series voltage source \( V_{se} \angle \delta_{se} \) with the external system is:

\[
P_{se} = [(X + X_{sh}) V_{se} V_r \sin \delta_{se} + X_{sh} V_s V_{se} \sin (\delta - \delta_{se})]/X_{eq}^2 + X_{sh} V_{se} \sin (\delta - \delta_{se})]/X_{eq}^2
\] (5.2.10)

And the real power exchange \( P_{sh} \) of the shunt voltage source \( V_{sh} \angle \delta_{sh} \) with the external system is:

\[
P_{sh} = [X V_{sh} V_{se} \sin (\delta_{sh} - \delta_{se}) - X V_{sh} V_r \sin \delta_{sh}]/X_{eq}^2 + (X_{se} + k_1 X) V_{sh} V_{se} \sin (\delta - \delta_{se})]/X_{eq}^2
\] (5.2.11)

Correspondingly, the real and reactive powers at the downstream output terminal of UPFC-AP are:
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

\[ P_{set} = V_r \left[ X_{sh} V_s \sin \delta + X V_{sh} \sin X_{sh} \right] \left( X + X_{sh} \right) \sin X_{se} \left( \delta - X_{se} \right) \right] / X_{eq}^2 \] (5.2.12)

\[ Q_{set} = V_r \left[ X_{sh} V_s \cos \delta + X V_{sh} \cos X_{sh} \right] \left( X + X_{sh} \right) \cos X_{se} \left( \delta - X_{se} \right) \right] / X_{eq}^2 \]

\[ + 2k_i \left[ X_{sh} V_{sh} \left( V_{se} \cos \delta \right) - V_r \cos \delta \right] \left( X + X_{sh} \right) V_{se} \cos \left( \delta - X_{se} \right) \right] / X_{eq}^2 \]

\[ + 2k_i \left[ X_{sh} V_{sh} \left( V_{se} \cos \delta \right) - V_r \cos \delta \right] \left( X + X_{sh} \right) V_{se} \cos \left( \delta - X_{se} \right) \right] / X_{eq}^2 \]

\[ + k_i \left[ X_{sh} V_{sh} \left( V_{se} \cos \delta \right) - V_r \cos \delta \right] \left( X + X_{sh} \right) V_{se} \cos \left( \delta - X_{se} \right) \right] / X_{eq}^2 \] (5.2.13)

Finally, the magnitude of the voltage \( V_{sh} \), at \( T \) is:

\[ |V_{sh}| = \sqrt{V_{sh}^2 + V_{sh}^2} / X_{eq} \] (5.2.14)

where

\[ V_{sh} = \left( X_{se} + k_i X \right) X_{sh} V_r \sin \delta + X \left( X_{se} + k_i X \right) V_{sh} \sin \delta \right] \left( X + X_{sh} \right) V_{se} \sin \left( \delta - X_{se} \right) \]

\[ V_{se} = \left( X_{se} + k_i X \right) X_{sh} V_r \sin \delta + X \left( X_{se} + k_i X \right) V_{sh} \sin \delta \right] \left( X + X_{sh} \right) V_{se} \sin \left( \delta - X_{se} \right) \]

The above expressions are derived, as shown in Appendix F, in order to relate \( P_{set}, Q_{set} \) and \(|V_{sh}|\) to the UPFC-AP operating states. They are useful expressions which could be used to advantage, in terms of the design of control strategy of the network under varied conditions. For example, it is interesting to note that maximum \( P_r \) occurs at \( \delta_{se} = 90^\circ \). The proof is as shown in Appendix G. Hence one could envisage Converter 2 injected voltage \( V_{se} \) to be perpendicular to \( V_r \), in order to achieve maximum power transfer capacity through the network.

5.2.2 UPFC-AP Design Subject to System Constraints

While the above analysis is useful, unfortunately it does not consider the physical constraints that would be imposed in practical UPFC-AP installation and also on the network. This aspect of the network constraints will now be described and incorporated in the design.

In general, the capacity of the UPFC-AP real and reactive powers exchanges with the power system depends on the VA ratings of the UPFC-AP. The magnitude of the series injected voltage, \( V_{se} \), is limited by the maximum voltage ratings of the series converter and its associated transformer. The maximum current through Converter 2 primary circuit
can be taken to be the thermal line current limit, which essentially implies that the converter GTO switches have to be rated to the same level of current, taking due consideration of the injection transformer turns-ratio.

GTO thyristors are limited in their continuous current loading due to thermal considerations. Each converter therefore has a nominal current rating, at which level it can operate continuously. The UPFC ability to raise or lower $V_{set}$ (the UPFC-AP downstream voltage) depends on $V_{set}$. In most applications, the UPFC will be located in a substation where $V_{shl}$ will be the voltage at the regulated substation bus, and $V_{set}$ will be the voltage on the outgoing line side of the equipment. As the UPFC has the intrinsic ability to raise or lower the magnitudes of $\bar{V}_{shl}$ and $\bar{V}_{set}$, this can present a problem from the point of view of voltage stresses on the line, or if the line would be tapped for any purpose [61]. Therefore, it will be necessary to impose limits on the allowable magnitudes of $\bar{V}_{shl}$ and $\bar{V}_{set}$. Clearly, if both $\bar{V}_{shl}$ and $\bar{V}_{set}$ were to be tightly regulated in magnitude, then the only tolerable values of $\bar{V}_{set}$ and $\bar{V}_{shl}$ would be those that produce phase shifts on these two voltage phasors. In practice it is acceptable to permit some variations in the magnitudes of $\bar{V}_{shl}$ and $\bar{V}_{set}$, and this restriction is enforced electronically by the UPFC control system.

Finally on the line-side, the potential difference across the series transformer reactance $X_{se}$ has to be taken into account. This voltage component depends on the line current and, therefore, can be significant for a heavily loaded transmission line.

On the AP source, in general the battery terminal voltage will fall progressively throughout a battery discharge cycle [48]. The amount of the voltage drop increases with the magnitude of the discharge current. Hence for sustained UPFC-AP operations, this could give rise to problems for high power applications towards the end of the cycle. Since the discharge current depends on the battery discharge capacity, another important restriction on the operation of the UPFC-AP is the battery discharge capacity: the active power output from the AP source should be kept within limit.

Based on the practical limits imposed on the UPFC-AP described earlier and together with the prevailing power system conditions, they essentially define a feasible operating regime for the network. This has given rise to the idea of the development of an Optimal
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

Control Scheme (OCS). The optimum reference states of the UPFC-AP would be determined from the prevailing and observed network conditions in order to ensure the network is operating within the practical limits stated earlier, while maximizing (or minimizing) certain pre-specified benefit or objective function. Based on the computed optimal reference states, the UPFC-AP control system would adjust the gating patterns of the shunt and series converters to realize the optimal operating condition.

With reference to Figure 5-3 and the network constraints described earlier, the OCS can be formulated based on satisfying the following constraints:

\[
\begin{align*}
\text{magnitude of the series injected voltage } & V_{se}: 0 \leq V_{se} \leq V_{se,\text{max}} \\
\text{magnitude of the shunt VSC voltage } & V_{sh}: 0 \leq V_{sh} \leq V_{sh,\text{max}} \\
\text{line current through the series inverter } & I_s: I_s \leq I_{th} \\
\text{line current through transmission-line "ST" section } & \overline{I}: I \leq I_{th} \\
\text{magnitude of the shunt VSC current } & I_{sh} \leq I_{sh,\text{max}} \\
\text{UPFC-AP from-bus voltage } & V_{sht} \leq V_{sht,\text{max}} \\
\text{UPFC-AP to-bus voltage } & V_{set} \leq V_{set,\text{max}} \\
\text{series converter transformer rating } & S_s \leq S_{se,\text{max}} \\
\text{shunt converter transformer rating } & S_{sh} \leq S_{sh,\text{max}}
\end{align*}
\]

where \( V_{se,\text{max}}, V_{sh,\text{max}}, I_{th}, I_{sh,\text{max}}, V_{sht,\text{max}}, V_{set,\text{max}}, S_{se,\text{max}}, S_{sh,\text{max}} \) are the respective specified limits.

The constrained operating conditions are subject to achieving the maximization (or minimization) of pre-specified objective function. In this thesis, three objective functions are considered. These are:

- **Design Objective no. 1: Maximization of Power Transfer Capacity**
  
  Real power flow control is very important during both steady-state and transient operations. Therefore, the first objective function considered is to maximize the real power transfer capability of the transmission system while satisfying constraints (5.2.15). In this way, an appropriate objective function can be described as:

\[
\begin{align*}
\text{max } P_r &= g (\delta, V_{se}, \delta_{se}, V_{sh}, \delta_{sh}, P_d) \\
\text{subject to power balance equation } & (5.1.1), \text{ i.e. } P_{se} = P_d + P_{sh} \text{ and } g (\bullet) \text{ is described by } (5.2.8). \text{ Note that } V_{se}, \delta_{se}, V_{sh}, \delta_{sh} \text{ are the control variables in the design problem.}
\end{align*}
\]
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

- Design Objective no. 2: Minimization of battery output $P_d$ for a given $P_r$

As mentioned previously, the energy storage capacity of a BESS is limited. If the discharged power output of the battery bank can be minimized, then for a given BESS energy storage capacity, the battery can be expected to be effective over a longer period and this could be desirable. Hence another operating objective can be described as the determination of the value of the minimum power output of the battery:

$$\text{min } P_d$$

subject to satisfying constraints (5.2.15) to deliver $P_{r_0}$ at R, and also meeting the power balance $P_{se} = P_d + P_{sh}$, i.e. (5.1.1).

- Design Objective no. 3: Minimization of $\delta$ for a given $P_r$

The aim of Design Objective no. 1 is to achieve maximum power transfer capacity. However, power systems do not operate at the highest possible transfer level in all instances. Thus, it is important to consider the stability aspect of the network operation. In other words, in order to delivery a given level of power, $P_{r_0}$, power angle difference $\delta$ between S and R should be controlled to be as small as possible, so that the power system can have a higher stability margin. Therefore, the third design objective considered in this investigation is to minimize $\delta$ for a given $P_r$. Thus the optimal design problem can be described as:

$$\text{min } \delta$$

subject to $P_r = P_{r_0}$ and $P_{se} = P_d + P_{sh}$.

With the UPFC-AP design problem so formulated, a trust region approach for nonlinear minimization subject to bounds can be used to determine the solutions. Trust region idea is a powerful concept in optimization. Consider the general minimization problem to minimize $f(x)$, where the function takes vector arguments and returns scalars. Suppose at a point $x$ in n-space, the intention is to determine a new $x$ which will result in a lower function value. The basic idea is to approximate $f$ with a simpler function $g$, which reasonably reflects the behavior of function $f$ in a neighborhood $N$ around $x$. This neighborhood is the trust region. A trial step $s$ is computed by minimizing (or approximately minimizing) over $N$. This is the trust region subproblem, min $\{g(s), s \in N\}$. 

146
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

The current point is updated to be \( x + s \) if \( f(x + s) < f(x) \); otherwise, the current point remains unchanged and \( N \), the region of trust, is shrunk and the trial step computation is repeated. As detail of the method is rather involved, it will not be described here but interested readers may refer to [145]. Furthermore, there are likely to be other design objectives but the above three are the most obvious ones to consider at this stage. The next step is therefore to explore how each of the objectives can be applied and thus lead to optimal UPFC-AP steady-state operation.

5.3 UPFC-AP Optimal Steady-State Operation

5.3.1 Feasible Operating Regime

As it would be highly desirable to keep to the minimum the additional cost incurred due to the inclusion of the AP source into the UPFC, the first step of the investigation would be to determine the minimum AP capacity that would be needed to achieve a given transfer level \( P_r \). The approach is based on achieving Design Objective no. 2 shown earlier. The solution obtained from the trust region optimization technique in solving (5.2.17) would be the value of the minimum \( P_d \) which would contribute toward meeting the demand \( P_{r,0} \) while ensuring that all the constraints (5.2.15) are satisfied. In this manner, a steady-state feasible operating regime (FOR) of the UPFC-AP can be obtained. The FOR concept is best illustrated using an example, such as that based on typical UPFC data given in [64]. Figure 5-5 shows a family of curves (a)-(c) describing the relationship between \( P_r \) and \( P_d \) obtained through the application of (5.2.15) and (5.2.17) for a range of UPFC capacity, up to the level when \( S_{sh,max} = S_{se,max} = S_{m,l} \). By applying the optimization design procedure, the curve describing the minimum capacity of the battery bank \( P_d \) that can be used to support a range of transfer level \( P_r \) can be obtained. In Figure 5-5, BCF was obtained when solving the optimal design problem (5.2.17), given \( S_{sh,max} = S_{se,max} = S_{m,l} \), and the network operation satisfies (5.2.15) while minimizing \( P_d \).
Figure 5-5 Feasible operating area of the UPFC-AP where $S_{m,1} > S_{m,2}$: (a) $S_{sh, max} = S_{se, max}$

= $S_{m,1}$; (b) $S_{sh, max} = S_{se, max} = S_{m,2}$; (c) $S_{sh, max} = S_{m,1}, S_{se, max} = 0$

One can also carry out similar study to obtain the curves corresponding to maximizing $P_d$ for a given $P_r$ and BD is one such curves obtained when $S_{sh, max} = S_{se, max} = S_{m,1}$. Figure 5-5 shows that point “B” defines the maximum $P_r$, denoted as $P_{r,B}$ in Figure 5-5, that can be transferred subject to the practical network constraint (5.2.15) being satisfied for the given UPFC capacity. It shows that even if one were to attempt to raise the power output of the battery $P_d$, $P_r$ cannot be increased further. The reason is that at “B”, the UPFC-AP voltage $\hat{v}_{sh}$ reaches $V_{sh, min}$ and $\hat{v}_{se}$ reaches $V_{se, max}$. Both the loadings on the series and shunt converter transformers have reached their power ratings. It can be observed from numerical solution that if one were to increase $P_d$ beyond $P_{d,B}$, $\hat{v}_{sh}$ will increase, which in turn causes $P_s$ to reduce. The total power transfer level decreases. Thus, it can be concluded that there is no advantage in sizing the battery power capacity larger than $P_{d,B}$ since $P_r$ cannot be increased beyond $P_{r,B}$ unless the UPFC capacity is increased. In conclusion, the power rating of the battery may be chosen to be $P_{d,B}$ for the given UPFC capacity.

The above is pertaining to the battery under the discharge mode. Although the intent of this investigation is to explore the harnessing of RE resource to charge the BESS, there is of course the possibility of using any surplus power from the source S to charge the BESS,
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

if it is shown to be desirable. In which case and if the battery power limit $P_{d,B}$ is to be used for both the battery charging as well as discharging modes, these would appear as constraints placed on $P_d$ and could be represented by the boundary lines AB and EF in Figure 5-5.

Yet another steady-state consideration is, in the event $P_r$ is less than $P_d$, the surplus power would flow reversibly back to S. Perhaps due to contractual reasons, assume such reverse power flow is prohibited. Therefore, it is proposed that the active power output of the battery cannot exceed $P_r$. This constraint condition is represented by the curve OA.

Above the dashed line OC, the area OABC corresponds to the condition of the battery discharging power to the external source. Conversely, OCFE corresponds to the battery being charged by the source S. BC describes the minimum $P_d$ for given $P_r$ while satisfying (5.2.15) during battery discharge stage. The curve prescribes the minimum amount of discharged battery output $P_d$ needed when $P_r$ is between $P_{r,c}$ and $P_{r,B}$. Of course when $P_r$ is within this range, the power output from the battery can be above this minimal level if required, but this is at the expense of the battery only being able to support a shorter discharge duration. Clearly, this reasoning indicates that the power dispatch from the BESS would have to be based on other consideration. This point will be addressed in the next sub-section.

For $P_r < P_{r,c}$, the minimum $P_d$ level shown by the curve CF corresponds to the maximum battery charging level: this is the maximum $P_d$ that can be absorbed by the battery. Battery power absorption larger than this level means that some of the network constraints (5.2.15) cannot be satisfied. Of course, when $P_r < P_{r,c}$, the battery can still operate in the discharging mode, i.e. operates in the FOR area above OC, while satisfying all the constraints in (5.2.15).

Taking all of these into consideration, the FOR of the UPFC-AP must therefore be within the area OABCFOE of Figure 5-5 for the case $S_{sh,max} = S_{se,max} = S_{m,l}$ if the battery charging mode is allowed, or OABCO if the battery charging mode is disallowed. The $P_d$ vs $P_r$ operating state must remain within these boundaries in order to satisfy the network constraints (5.2.15) as well as the battery charging/discharging limits.
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

It is also interesting to note from Figure 5-5 that the operating range of the UPFC without the AP source is the line OC, while the operating range of the UPFC with the battery is the area within OABCPEO. From Figure 5-5, it can be seen that the larger the UPFC-AP power rating, the larger will be the FOR. For the case $S_{sh,max} = S_{se,max} = S_{m,l}$, the maximum power transfer level for the compensated system is $P_{r,B}$, which is significantly higher than that of the uncompensated system (i.e. without the UPFC-AP) where the maximum power transfer level is actually Point "G" in Figure 5-5 for this example. $P_{r,B}$ is also higher than the maximum transferable power level ($P_{r,c}$) obtained in the case when the UPFC is without the AP. In this way, it can be readily concluded that the UPFC-AP has resulted in higher power transfer capability as compared to the UPFC without the AP. Finally, Figure 5-5 also shows that greater flexibility on power transfer can be achieved with both the shunt and series compensators in the UPFC-AP operating (curve (a)), as compared to the case when only the shunt compensator is used (curve (c)).

If the maximum power transfer level of $P_r$ intended for the power system is known, one can determine the UPFC rating and the rating of the battery using the FOR constructed through the optimization study. In Figure 5-5 for example, if the maximum receiving end power is $P_{r,B}$, then the shunt and series converter transformer ratings of the UPFC-AP can be $S_{sh,max} = S_{se,max} = S_{m,l}$, the rating of the battery can be $P_{d,B}$.

### 5.3.2 Optimal Economic Dispatch

Within the FOR and as described earlier, one notes that there is considerable flexibility in terms of the scheduling of $P_d$ for a given $P_r$. Hence the next step would be to take advantage of the flexibility to design a dispatch strategy for the AP source. A most obvious approach is based on achieving the most economical operating condition through minimizing the total generation cost in the transmission system. While several authors have examined the use of UPFC to achieve certain economic dispatch objective, see e.g. [65], the present work is more general in that it examines the scheduling of upstream bulk generation $S$ as well as that of the AP to achieve minimum total generation cost. In general, generation cost includes capital cost and the cost to actually produce electricity. The cost components can vary widely, with different generation types complement each
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

other in terms of cost effectiveness. Generator cost curves are usually not smooth functions. However the curves can usually be adequately approximated using a quadratic function [146]. For the problem in hand, suppose the generation cost of generator $S$ and the AP can be represented by (5.3.1) and (5.3.2), respectively.

\[
C_s(P_s) = \alpha_s + \beta_s P_s + \gamma_s P_s^2
\]  
(5.3.1)

\[
C_d(P_d) = \alpha_d + \beta_d P_d + \gamma_d P_d^2
\]  
(5.3.2)

where $\alpha_s$, $\beta_s$ and $\gamma_s$ are coefficients of the quadratic cost function in terms of the generator $S$ output $P_s$, while $\alpha_d$, $\beta_d$ and $\gamma_d$ are that for the AP. In this formulation, assume that $\alpha_s > 0$, $\beta_s > 0$, $\gamma_s > 0$, $\alpha_d > 0$ and $\beta_d > 0$ as they reflect the respective capital costs. The marginal running costs, $\beta_s + 2 \gamma_s P_s$ and $\beta_d + 2 \gamma_d P_d$, are positive and increase (linearly) with $P_s$ and $P_d$, respectively. The following analysis only consider the case for $P_s > 0$ and $P_d > 0$ since (5.3.1) and (5.3.2) are only applicable when $P_s > 0$ and $P_d > 0$.

Economic dispatch involves the allocation of power output between the available generation units to meet the demand such that the total cost of operation, $C(P_s, P_d)$, is minimized. Thus the economic dispatch problem can be defined by the optimization problem:

\[
\text{Min } C(P_s, P_d) = C_s(P_s) + C_d(P_d)
\]  
(5.3.3)

This is subjected to

\[
P_s = P_r - P_d
\]  
(5.3.4)

for a given $P_r$. Substitute (5.3.1), (5.3.2) and (5.3.4) into (5.3.3), the total cost of generation $C$ is

\[
C = (\gamma_s + \gamma_d) P_d^2 + (\beta_d - 2 \gamma_s P_r - \beta_s) P_d + \alpha_d + \gamma_s P_r^2 + \beta_s P_r + \alpha_s
\]  
(5.3.5)

In order to achieve minimum total generation cost, one can differentiate $C$ with respect to $P_d$ and set the resulting equation to zero. One can then show that the economic dispatch of the AP source output $P_d$ in meeting the load demand $P_r$ is when

\[
P_d = \frac{\gamma_s}{(\gamma_s + \gamma_d)} P_r + \frac{(\beta_s - \beta_d)}{2(\gamma_s + \gamma_d)}
\]  
(5.3.6)

The last equation indicates that the economic dispatch relationship between the AP output $P_d$ and the required load demand $P_r$ is linear. In addition, from (5.3.6) we can obtain the
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

corresponding values of $P_r$ when $P_d = 0$ and $P_d = P_{d,B}$, and define these values as $P_{r,H}$ and $P_{r,I}$ respectively, which are shown as follows.

$$P_{r,H} = (\beta_d - \beta_s)/2\gamma_s,$$
$$P_{r,I} = [2P_{d,B}(\gamma_s + \gamma_d) + \beta_d - \beta_s]/2\gamma_s.$$

Since the intent of this investigation is to explore the harnessing of RE resource to charge the BESS, thus only the battery operating at discharge mode is considered in this work, therefore the AP output $P_d$ should be no smaller than 0, i.e. $P_d \geq 0$. Thus, for $0 < P_r \leq P_{r,H}$, only generator S should be scheduled to meet $P_r$, while the AP output $P_d = 0$. Conversely, when $P_r > P_{r,H}$, it would be more economical if both generator S and the AP are operating, with $P_d$ scheduled in the manner described by (5.3.6). Moreover, when $P_r \geq P_{r,I}$, the BESS output power $P_d$ will maintain constant as $P_{d,B}$ because the battery has reached the power rating of the battery. In summary, the optimal economic dispatch strategy would be

$$P_d = \left\{\begin{array}{ll}
0 & \text{for } 0 < P_r \leq P_{r,H} \\
\frac{\gamma_s}{(\gamma_s + \gamma_d)}P_r + \frac{(\beta_s - \beta_d)}{2(\gamma_s + \gamma_d)} & \text{for } P_{r,H} \leq P_r < P_{r,I} \\
P_{d,B} & \text{for } P_r \geq P_{r,I}
\end{array}\right.$$  (5.3.7)

where $P_{r,H} = (\beta_d - \beta_s)/2\gamma_s$, $P_{r,I} = [2P_{d,B}(\gamma_s + \gamma_d) + \beta_d - \beta_s]/2\gamma_s$.

The optimal economic dispatch strategy (5.3.7) can be superimposed onto the FOR to produce the curve OHIB in Figure 5-6. When the demand $P_r$ is less than $P_{r,H}$, power is supplied totally by generator S since it is most economical to do so under this light load level. The dispatch characteristic will be the line OH, similar to the operation of traditional UPFC without the AP. However, when $P_{r,H} \leq P_r \leq P_{r,I}$, the BESS will operate in the discharge mode and the $P_d$ dispatch will follow the line HI, up to $P_r = P_{r,I}$. For $P_r \geq P_{r,I}$, the BESS output power is at the battery rated value of $P_{d,B}$. Beyond $P_{r,I}$, the UPFC-AP will still be able to support $P_r$ up to $P_{r,B}$ since the marginal cost of the AP at $P_r = P_{r,B}$ is lower than that of the source S for $P_{r,I} < P_r < P_{r,B}$. If the load is higher than $P_{r,B}$, the transmission system is unable to support the demand. A larger capacity UPFC-AP is needed (e.g. corresponding to curve MN in Figure 5-6) or the transmission system has to be reinforced.
In fact, one can proceed to determine the optimal capacity of the UPFC-AP once the economic dispatch results are known. By not imposing constraint on the UPFC-AP capacity, the economic dispatch strategy is represented by the line OHIK. A series of curves similar to CBD and corresponding to different UPFC ratings can be constructed using (5.2.15) and (5.2.17). The particular curve MN which intercepts OHIK precisely at its maximum power transfer point would correspond to the optimal UPFC rating. The corresponding optimal battery capacity $P_{d,M}$ is also determined. This interesting development will be illustrated later through a numerical example.

![Figure 5-6 Optimal economic dispatch of the UPFC-AP system within the FOR](image)

### 5.3.3 An Optimal Load Tracking Scheme

Based on the above considerations, one can devise a strategy on the dispatch of power $P_s$ from the upstream source S as well as from the UPFC-AP. The dispatch can be realized through the control mechanism of the UPFC. With reference to Figures 5-1 and 5-3, a common method is to specify the four desired set-points for the UPFC. These set-points are pertaining to that of the DC voltage $V_{dc}$, the from-bus voltage $V_{sh}$, and the line active and reactive power flows $P_{set}$ and $Q_{set}$, respectively at the downstream output terminal of the UPFC.

Furthermore, if the optimal dispatch strategy described earlier is to be adopted, there is still enough degree of design freedom offered by the UPFC for one to achieve other
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

desirable objective(s). In the present investigation, it is proposed that once the optimal dispatch policy is applied, Design Objective no. 3 described in Section 5.2.2 is incorporated into the UPFC-AP control strategy. The final design would be a load-tracking mechanism in which not only the power system would operate under the minimum generation cost but also has the desirable feature of having the minimum power transfer angle across the transmission link.

In this way, the final optimal load tracking scheme for the UPFC-AP can be constructed as follows: firstly, \( P_r \), at the receiving end R is given, thus the corresponding reference for \( P_d \), denoted as \( P_d^* \), can be determined from (5.3.7). The reference value \( (V_{dc}^*) \) for \( V_{dc} \) can then be determined using (5.2.4), assuming the battery state of discharge \( f \) is known. The control variables, \( V_{se}, \delta_{se}, V_{sh} \), and \( \delta_{sh} \) are determined through minimizing \( \delta \) described by (5.2.18) while satisfying constraints (5.2.15) for the known \( P_r \) and \( P_d \). The optimization procedure ensures that the control action determined for the UPFC-AP would result in the minimum angle \( \delta \) across the transmission system. The respective reference values, \( P_{set}, Q_{set} \), and \( V_{shl} \), for \( P_{set}, Q_{set} \), and \( V_{shl} \) can be calculated by substituting \( V_{se}, \delta_{se}, V_{sh}, \delta_{sh} \), and \( \delta \) into (5.2.12), (5.2.13), and (5.2.14), respectively. Once the references are determined, the switching signals for the UPFC converters 1 and 2 can be generated using the control mechanism similar to that described in [61]. The schematic diagram showing the optimal load tracking scheme is as shown in Figure 5-7.

Figure 5-7 Proposed UPFC-AP load-tracking optimal power dispatch scheme
5.4 Illustrative Examples

The following examples are based on the 345-kV transmission system shown in Figure 5-3, with the line parameters taken from [147]. The line length is assumed to be 500 miles between busbars S and R. The line thermal capacity is 1195 MVA and has the reactance value of 0.6 ohms/mile. With \(V_s = V_r = 345\) kV and the stated line thermal capacity, the current carrying capacity \(I_{th} = 2000\) A. Accordingly the power and voltage base values are chosen to be \(S_{base} = 1000\) MVA and \(V_{base} = 345\) kV. Thus the above values translated into: \(V_s = V_r = 1\) p.u., \(I_{th} = 1.195\) p.u. and \(S_{th} = 1.195\) p.u.

The UPFC parameters and the realistic network-UPFC limits are taken from [64]: shunt converter transformer is rated 400 MVA, \(X_{sh} = 0.2\) p.u. and the voltage rating is 345 /100 kV, series converter transformer is also rated 400 MVA, \(X_{se} = 0.04\) p.u. and the voltage rating is 172.5/100 kV, \(0 \leq V_{se} \leq 0.4\) p.u., i.e. \(V_{se,max} \leq 0.4\) p.u., \(0 \leq \delta_{se} \leq 2\pi\), \(V_{sh,max} = 1.1\) p.u., \(V_{sh,min} = 0.9\) p.u., \(V_{set,max} = 1.1\) p.u., \(V_{set,min} = 0.9\) p.u., \(V_{set,max} = 1.1\) p.u., \(S_{se,max} = 0.4\) p.u., \(S_{sh,max} = 0.4\) p.u.. Suppose the generator busbar S is a thermal unit and the AP source obtains its energy from a wind-farm. The capacity of AP source is 268MW. The quadratic cost function coefficients for the generator S and AP are [146, 148]: \(\alpha_s = 1021\) $/h, \(\beta_s = 14.1\) $/MWh and \(\gamma_s = 0.0235\) $/ MW^2h; \(\alpha_d = 919\) $/h, \(\beta_d = 17.16\) $/MWh and \(\gamma_d = 0.0167\) $/ MW^2h.

Consider a radial single-circuit transmission link between busbars S and R, i.e. \(X_2 \) or \(k_2 \) in (5.2.5) is infinite and the UPFC-AP is located at the mid-point of the transmission line, i.e. \(k_1 = 1\). Thus, \(X = X_r = 250\) miles x 0.6 ohms/mile = 1.26 p.u..

5.4.1 Example 1. Optimal Load Tracking

The examples in this section will be used to illustrate the effectiveness of the UPFC-AP in tracking the load demand \(P_r\) using the derivation given in Section 5.3.2 and the cost functions provided. It was determined that \(P_{r,H} = 0.065\) p.u. and \(P_{r,I} = 0.523\) p.u. Suppose the optimal load tracking scheme is implemented, Table 5-1 shows the various states of the power system and the total generation cost \(C\) for a range of transfer level \(P_r, P_d\) is also seen to have tracked the optimal trajectory OHIB shown in Figure 5-6.
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

Table 5-1: Optimal load-tracking with UPFC-AP operating trajectory OHIB

<table>
<thead>
<tr>
<th>( P_r )</th>
<th>( P_d )</th>
<th>( \delta^\circ )</th>
<th>( V_{se} )</th>
<th>( \delta_{se}^\circ )</th>
<th>( V_{sh} )</th>
<th>( \delta_{sh}^\circ )</th>
<th>( C($/h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.1</td>
<td>0</td>
<td>1940</td>
<td></td>
</tr>
<tr>
<td>0.065</td>
<td>0.0</td>
<td>4.2</td>
<td>0.22</td>
<td>157.5</td>
<td>1.1</td>
<td>0</td>
<td>2956</td>
</tr>
<tr>
<td>0.2</td>
<td>0.079</td>
<td>7.0</td>
<td>0.35</td>
<td>127.1</td>
<td>1.1</td>
<td>0</td>
<td>5450</td>
</tr>
<tr>
<td>0.323</td>
<td>0.151</td>
<td>12.9</td>
<td>0.4</td>
<td>98.0</td>
<td>1.1</td>
<td>3.0</td>
<td>8031</td>
</tr>
<tr>
<td>0.40</td>
<td>0.196</td>
<td>20.8</td>
<td>0.4</td>
<td>103.2</td>
<td>1.1</td>
<td>9.2</td>
<td>9800</td>
</tr>
<tr>
<td>0.45</td>
<td>0.225</td>
<td>26.1</td>
<td>0.4</td>
<td>106.5</td>
<td>1.1</td>
<td>13.3</td>
<td>11009</td>
</tr>
<tr>
<td>0.50</td>
<td>0.255</td>
<td>31.6</td>
<td>0.4</td>
<td>109.9</td>
<td>1.1</td>
<td>17.7</td>
<td>12267</td>
</tr>
<tr>
<td>0.523</td>
<td>0.268</td>
<td>34.2</td>
<td>0.4</td>
<td>111.4</td>
<td>1.1</td>
<td>19.8</td>
<td>12856</td>
</tr>
<tr>
<td>0.6</td>
<td>0.268</td>
<td>46.8</td>
<td>0.4</td>
<td>116.3</td>
<td>1.1</td>
<td>26.6</td>
<td>15012</td>
</tr>
<tr>
<td>0.63</td>
<td>0.268</td>
<td>52.1</td>
<td>0.4</td>
<td>118.1</td>
<td>1.09</td>
<td>29.5</td>
<td>15925</td>
</tr>
<tr>
<td>0.72</td>
<td>0.268</td>
<td>75.6</td>
<td>0.4</td>
<td>107.5</td>
<td>0.98</td>
<td>40.3</td>
<td>18919</td>
</tr>
<tr>
<td>0.73</td>
<td>0.268</td>
<td>78.2</td>
<td>0.4</td>
<td>106.7</td>
<td>0.96</td>
<td>41.5</td>
<td>19096</td>
</tr>
</tbody>
</table>

For comparison, the conventional UPFC of the same rating is also used for the same load tracking exercise. Thus the operating trajectory is the line OC shown in Figure 5-5. The corresponding results are summaries in Table 5-2.

Table 5-2: Load tracking of UPFC operating trajectory OC

<table>
<thead>
<tr>
<th>( P_r )</th>
<th>( \delta^\circ )</th>
<th>( V_{se} )</th>
<th>( \delta_{se}^\circ )</th>
<th>( V_{sh} )</th>
<th>( \delta_{sh}^\circ )</th>
<th>( C ($/h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
<td>0.0</td>
<td>1.1</td>
<td>0</td>
<td>1940</td>
<td></td>
</tr>
<tr>
<td>0.065</td>
<td>4.2</td>
<td>0.22</td>
<td>157.5</td>
<td>1.1</td>
<td>0</td>
<td>2956</td>
</tr>
<tr>
<td>0.2</td>
<td>13.1</td>
<td>0.35</td>
<td>131.2</td>
<td>1.1</td>
<td>0</td>
<td>5702</td>
</tr>
<tr>
<td>0.323</td>
<td>23.5</td>
<td>0.4</td>
<td>97.0</td>
<td>1.1</td>
<td>1.5</td>
<td>8950</td>
</tr>
<tr>
<td>0.4</td>
<td>35.2</td>
<td>0.4</td>
<td>101.5</td>
<td>1.1</td>
<td>7.2</td>
<td>11346</td>
</tr>
<tr>
<td>0.45</td>
<td>43.2</td>
<td>0.4</td>
<td>104.3</td>
<td>1.1</td>
<td>11.0</td>
<td>13052</td>
</tr>
<tr>
<td>0.5</td>
<td>51.6</td>
<td>0.4</td>
<td>107.0</td>
<td>1.1</td>
<td>15.0</td>
<td>14875</td>
</tr>
<tr>
<td>0.523</td>
<td>55.8</td>
<td>0.4</td>
<td>108.1</td>
<td>1.1</td>
<td>17.0</td>
<td>15753</td>
</tr>
<tr>
<td>0.6</td>
<td>71.0</td>
<td>0.4</td>
<td>111.8</td>
<td>1.1</td>
<td>23.8</td>
<td>18874</td>
</tr>
<tr>
<td>0.63</td>
<td>80.0</td>
<td>0.4</td>
<td>123.4</td>
<td>1.09</td>
<td>29.1</td>
<td>20166</td>
</tr>
</tbody>
</table>
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

From Tables 5-1 and 5-2, it can be seen that compared with the conventional UPFC compensation, the UPFC-AP has resulted in smaller power angle $\delta$ being achieved for given $P_r > P_{r,H} = 0.065$ p.u. Furthermore, the maximum power transfer is also increased by some 100 MW through the actions of the UPFC-AP. In terms of the total generation cost $C$, over the range $P_{r,H} < P_r < P_{r,C}$ i.e. between $0.065 < P_r < 0.63$ p.u., the UPFC-AP optimal power dispatch scheme has resulted in significant generation cost saving. Hence it shows that the addition of the AP into the UPFC has distinct advantages over the original UPFC.

5.4.2 Example 2. Determination of the Optimal Capacity of the UPFC-AP

As described in Section 5.3.2, Figure 5-8 can be used to determine the optimal capacity of the UPFC and the BESS. In this example, the economic dispatch results are shown as the trajectory OHIB on Figure 5-8. Through the optimization design procedure based on achieving Design Objective 2, i.e. (5.2.17), a series of curves corresponding to different UPFC ratings have been determined and these are shown as curves (a) - (d) in the figure.

From Figure 5-8, it can be seen that curve (d), corresponding to 0.7 p.u. UPFC rating, intercepts precisely the economic dispatch line IB at its maximum power transfer point “B”. Thus the optimal capacity of the UPFC is 0.7pu in this example and from the figure, the optimal capacity of the battery is $P_{d,B} = 0.47$ pu.

![Figure 5-8 Optimal capacity of the UPFC and the battery: (a) $S_{sh,\ max} = S_{se,\ max} = 0.16$ pu; (b) $S_{sh,\ max} = S_{se,\ max} = 0.3$ pu; (c) $S_{sh,\ max} = S_{se,\ max} = 0.4$ pu; (d) $S_{sh,\ max} = S_{se,\ max} = 0.7$ pu](image-url)
Figure 5-8 also shows that the larger the series and shunt converters ratings, the smaller the active power output $P_d$ is needed for the delivery of a given $P_r$. Indeed, Table 5-3 shows the optimal results for maximum power transfer ($P_{r_{max}}$) for combinations of series and shunt converter ratings. From the Table, it is not surprising to note that the larger the converter rating, the higher is $P_{r_{max}}$.

Table 5-3: Optimal results for maximum power transfer for different series and shunt converter transformer ratings

<table>
<thead>
<tr>
<th>$S_{sh}$ (pu)</th>
<th>$S_{se}$ (pu)</th>
<th>$V_{se}$ (pu)</th>
<th>$\delta_{se}$ (°)</th>
<th>$V_{sh}$ (pu)</th>
<th>$\delta_{sh}$ (°)</th>
<th>$\delta$ (°)</th>
<th>$P_d$ (pu)</th>
<th>$P_{r_{max}}$ (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.086</td>
<td>0.15</td>
<td>122.1</td>
<td>0.93</td>
<td>37.1</td>
<td>71.4</td>
<td>0.0872</td>
<td>0.5026</td>
</tr>
<tr>
<td>0.16</td>
<td>0.16</td>
<td>0.27</td>
<td>114.9</td>
<td>0.93</td>
<td>32.1</td>
<td>68.7</td>
<td>0.1007</td>
<td>0.5398</td>
</tr>
<tr>
<td>0.2</td>
<td>0.09</td>
<td>0.15</td>
<td>116.9</td>
<td>0.94</td>
<td>40.0</td>
<td>73.9</td>
<td>0.1174</td>
<td>0.5326</td>
</tr>
<tr>
<td>0.25</td>
<td>0.096</td>
<td>0.15</td>
<td>110.4</td>
<td>0.95</td>
<td>43.7</td>
<td>77.1</td>
<td>0.1556</td>
<td>0.5687</td>
</tr>
<tr>
<td>0.3</td>
<td>0.102</td>
<td>0.15</td>
<td>104.4</td>
<td>0.96</td>
<td>47.5</td>
<td>80.2</td>
<td>0.1944</td>
<td>0.6032</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>104.8</td>
<td>0.95</td>
<td>36.1</td>
<td>73.3</td>
<td>0.2279</td>
<td>0.685</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>106.7</td>
<td>0.96</td>
<td>41.5</td>
<td>78.2</td>
<td>0.268</td>
<td>0.725</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.987</td>
<td>62.5</td>
<td>92.4</td>
<td>0.2157</td>
<td>0.6059</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>0.5</td>
<td>137.8</td>
<td>1.00</td>
<td>69.1</td>
<td>96.3</td>
<td>0.4698</td>
<td>0.8706</td>
</tr>
</tbody>
</table>

5.4.3 Example 3. Optimal Load Tracking

The examples in this section will be used to illustrate the effectiveness of the UPFC-AP in tracking slow variations of the load. In the following illustrative condition, it is assumed that $P_r$ ramps up from its initial value of 0.4 pu to 0.7 p.u. in 3s. The load increase is to be met through the optimal dispatch strategy OHIB described in Section 5.3.2. Figure 5-9 shows the response of the power system following the load increase. From the simulation results, it can be seen that the load is indeed properly tracked along the optimal trajectory OHIB shown in Figure 5-6 over the complete interval of the load variations.
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

For comparison, two more cases are shown in Figure 5-10. Figure 5-10 (a) corresponds to the case when the UPFC-AP operating trajectory is OHIB while Figure 5-10 (b) is that when the operating trajectory is OC, i.e. without the AP. It is assumed that the load demand $P_r$ is initially 0.63 p.u. before a load step increase is applied. Figure 5-10 (a) shows that the system remains stable after a 0.1 p.u. step load increase when the UPFC-AP is in service. On the other hand, Figure 5-10 (b) shows that even with a 0.01pu load increase, the system with the conventional UPFC (i.e. without the AP) is unable to maintain stable operation. Thus it can be concluded that compared with the conventional UPFC compensation, the maximum power transfer level or dynamic stability level has been increased by some 0.1 p.u. or 100 MW, through the actions of the UPFC-AP.
Chapter 5: Unified Power-Flow Controller Incorporated with an Active Power Source

![Graph showing load tracking](image)

Figure 5-10 Load tracking (a) with UPFC-AP and following a 0.1 p.u. step power increase in $P_r$, and (b) with UPFC and 0.01 p.u. step power increase in $P_r$.

5.5 Conclusions

A new transmission enhancement scheme which incorporates an active power (AP) source into a UPFC has been investigated. The AP source is based on harnessing the energy from a renewable resource and a battery bank is used as the medium for energy storage. The determination of the UPFC-AP capacity is formulated as an optimization problem in which the design objectives include the minimization of the AP capacity or the power transfer angle, and/or the maximization of the power transfer level. The design procedure also takes into account operating constraints of the network and that of the UPFC-AP. The feasible operating regime (FOR) of the UPFC-AP shows in a graphical way the range of the AP output power $P_d$ that can be dispatched for a given load demand $P_r$, while ensuring none of the operating constraints have been violated. Furthermore, using the FOR and the result of an economic dispatch strategy applied to the total generation cost of the power system, the relationship between the output power of AP and the load demand has been established. The analysis forms the basis of a proposed optimal load tracking strategy, such that the dispatch of power from the upstream source and the AP can be realized in the most economic manner while keeping the power angle across the transmission system to the minimum. Knowing the optimal economic dispatch strategy and the FOR also allows the optimal capacity of the UPFC-AP to be determined. The effectiveness of the proposed strategy and the theoretical analysis has been demonstrated through numerical examples.
Chapter 6

Battery Energy Storage System in the UPFC-AP

This chapter is organized as follows. In Section 6.1, some background of UPFC-AP and BESS is provided. Next, the electrochemical reactions in rechargeable battery are presented in Section 6.2. The equivalent circuit of battery bank is introduced in Section 6.3. In Section 6.4, an operational strategy pertaining to the change-over from the in-service BESS to the stand-by BESS is proposed. Furthermore, a method to determine the battery bank change-over schedule for a given load demand is derived in Section 6.5. The numerical results are demonstrated in Section 6.6. Section 6.7 concludes with main findings of this chapter.

6.1 Introduction

As described in Section 5.3.2, when the UPFC-AP is used to support power transfer $P_r > P_{r,H}$, the BESS in service is operating under the discharge mode and the energy in the battery is drained continuously. Thus in order to satisfy the load demand according to the economic dispatch strategy proposed in Chapter 5, it will be useful to derive a method to assess the expected frequency of the battery change-over, as this information will be needed in matching the renewable energy source with the power demand $P_r$. Hence the objective of this chapter is to derive such a method.

As mentioned in Chapter 2, BESS has emerged in recent years as one of the more promising near-term storage technologies for power applications. Among a number of battery types under consideration for large-scale energy storage, lead-acid batteries represent an established, mature technology. Lead-acid batteries can be designed for bulk energy storage or for rapid charge/discharge. Improvements in energy density and charging characteristics are still on-going. At present, lead-acid batteries still represent a
low-cost option for most applications which require large energy storage capabilities. Thus in this chapter, lead-acid battery would be considered as a suitable form to be included in the UPFC-AP.

### 6.2 Electrochemical Reactions

A battery "cycle" is defined as one complete discharge and recharge cycle. A cycle is usually considered to be discharging from 100% to some point not lower than 20%, and then re-charged back up to 100%. The fundamental discharge and charge reactions of the lead-acid cell involve dissolution/precipitation mechanisms, which are described by the "double-sulphate theory" [149].

**Discharge reaction**

The main reaction on the positive electrodes is the conversion of active mass PbO₂ to lead sulphate PbSO₄:

\[
PbO₂ + 4H^+ + SO₄^{2-} + 2e^- \rightarrow PbSO₄ + 2H₂O \quad (6.2.1)
\]

and on the negative electrodes, of lead metal Pb to lead sulphate PbSO₄:

\[
Pb + SO₄^{2-} \rightarrow PbSO₄ + 2e^- \quad (6.2.2)
\]

The dependence of the cell electromotive force \(E\) on concentration can be determined by the Nernst equation [150]:

\[
E = E^* + RT \frac{\ln \left( \frac{[H^+]^2 [HSO₄]^2}{[H₂O]^2} \right)}{nF} \quad (6.2.3)
\]

where:

- \(E^*\) is the standard potential of the cell;
- \(R = 8.314510 \text{ J K}^{-1} \text{ mol}^{-1}\), the gas constant;
- \(T\) is the absolute temperature in Kelvins (Kelvins = 273.15 + °C);
- \(F\) is the Faraday constant, equals to 9.6485309*10⁴ C/mol;
- \(n = \) number of exchange electronic equivalents, in this case \(n = 2\);
- \(nF\) means the amount of electrical charge connected with the reaction;
Chapter 6: Battery Energy Storage System in the UPFC-AP

\([H^+]\) is the activity of the hydrogen ion reactant, which is approximately equal to its concentration (mole/liter). It is similar for the other reactances.

A battery's state of discharge is a percentage figure giving the amount of energy moving out of the battery. A 400 ampere-hour battery at a 20% state of discharge will contain 320 ampere-hours of energy. At a 50% state of charge the same battery will contain 200 ampere-hours. A battery which is discharged to a 80% or more state of discharge is said to be "deep cycled". Shallow cycle service withdraws less than 10% of the battery's energy per cycle. This term is important. It is critical for users to know when the battery is nearly empty and should be charged. In order to properly cycle the battery, one must know the battery's state of discharge.

(6.2.1) and (6.2.2) show that the discharge reaction reduces \(H_2SO_4\) and increases \(H_2O\). Thus the density of the electrolyte decreases, as the state of discharge increases. (6.2.3) indicates the electromotive force \((E)\) decreases, as the state of discharge increases. As the amount of discharge increases, the active material in the battery decreases. As a result, the internal resistance increases. For the specified battery duty cycle and the cell size selected, the average cell voltage will not drop below the specified minimum value, such as 1.75 V/Cell at any point in the duty cycle.

- Charge reaction

In the charging process, the oxidation and reduction reactions can be described using (6.2.1) and (6.2.2) in the reverse direction. An additional reaction is water electrolysis, producing oxygen at the positive electrode:

\[H_2O \rightarrow \frac{1}{2} O_2 + 2H^+ + 2e^-\]  (6.2.4)

and hydrogen at the negative electrode:

\[2H^+ + 2e^- \rightarrow H_2\]  (6.2.5)

A battery's state of charge is a percentage figure giving the amount of energy remaining in the battery. A battery at a 80% state of charge is also at a 20% state of discharge. (6.2.4) and (6.2.5) show that when most of the active material is converted, oxygen gas is evolved at the positive plate, and hydrogen gas is evolved at the negative plate.
Chapter 6: Battery Energy Storage System in the UPFC-AP

6.3 Equivalent Circuit of Battery Bank

Having explained the electrochemical reactions of the lead-acid battery and relates them to $E$ and battery internal resistance, one can return to the battery equivalent circuit described in Section 5.2.1. In the model, an internal EMF $E$ is connected in series with a resistor $r$. From the above description, it is obvious that $E$ and $r$ are functions of the battery state of discharge. Since a battery is usually composed of a number of identical cells connected in series, the voltage of the battery is the voltage of a cell multiplied by the number of cells in series. If cells of sufficiently large capacity are not available, then two or more strings (with equal numbers of series-connected cells per string) may be connected in parallel to obtain the necessary capacity. The capacity of such a battery is the sum of the capacities of the strings. Figure 6-1 (a) shows a battery bank which includes $n$ parallel branches with $m$ identical cells connected in series in each branch. Figure 6-1 (b) is the equivalent circuit of the battery cell.

![Figure 6-1 (a) Schematic of a battery bank composed of n parallel branches, with m battery cells in series in each branch; (b) Equivalent circuit of each battery cell](image)

The equivalent circuit of the battery bank is as shown in Figure 6-2 (a), and the lumped-parameter model of the battery bank is as shown in Figure 6-2 (b). In the equivalent circuit, $E_{bc(i,j)}$ and $r_{bc(i,j)} (i = 1, 2, ..., m; j = 1, 2, ..., n)$ are the internal EMF and resistance of the jth series battery cell in the ith parallel branch, respectively. $E_{bc(i,j)}$ and $r_{bc(i,j)} (i = 1, 2, ...,
Chapter 6: Battery Energy Storage System in the UPFC-AP

\( n; j = 1, 2, \ldots, m \) are functions of the battery state of discharge. Mathematically, the relationship can be expressed as

\[
\begin{align*}
E_{bci,j} &= E_{obci,j} - k_{bei,j}f \\
r_{bei,j} &= r_{0bei,j} - k_{rbei,j}f
\end{align*}
\]  

(6.3.1)

where \( E_{obci,j} \) is the no load voltage when the battery cell is fully charged, \( f \) is the state of discharge; \( r_{0bei,j} \) is the internal resistance when the battery cell is fully charged, \( k_{bei,j} \) and \( k_{rbei,j} \) are constants obtained from experiments [144]. For ease of analysis, suppose all the battery cells are identical and the cells are at the same state of discharge. Thus for the battery bank

\[
\begin{align*}
E &= E_{bci,j} \\
r &= \frac{mr_{bei,j}}{n}
\end{align*}
\]  

(6.3.2)

Also

\[
\begin{align*}
E_0 &= mE_{obci,j} \\
k &= mk_{bei,j} \\
r_0 &= mr_{0bei,j}/n \\
k_r &= mk_{rbei,j}/n
\end{align*}
\]  

(6.3.3)

For the equivalent bank, therefore

\[
\begin{align*}
E &= E_0 - kf \\
r &= r_0 - k_r f
\end{align*}
\]  

(6.3.4)

\( E_0 \) will have to be larger than the nominal DC-link voltage \( V_{dc} \) value, thus \( m \) would have to be determined based on the nominal value of \( V_{dc} \) to satisfy the voltage rating. It may be necessary to use a DC-DC converter, without which the number of series battery cells needed is too large as to make the scheme impractical. At this stage, such a converter has not been included for ease of analysis. The number of parallel strings \( n \) has to be decided on the energy stored and needed for the active power output \( P_d \). A method in Section 6.5 can be used to determine \( n \). Note that in terms of BESS capacity, at full charge, the capacity is proportional to \( n \).
Chapter 6: Battery Energy Storage System in the UPFC-AP

Figure 6-2 (a) Equivalent circuit of Figure 6-1; (b) Lumped-parameter Model of the Battery Bank

From Figure 6-1 (b), the following relationship of the internal EMF, the resistance of the battery cell and battery cell terminal voltage $V_{bc}$ can be obtained,

$$V_{bc} = E_{bc} - r_{bc}I_{bc}$$

(6.3.5)

If the battery cell terminal voltage $V_{bc}$ is known, the battery cell current $I_{bc}$ can be determined in terms of known or specified system parameters

$$I_{bc} = P_{bc}/V_{bc}$$

(6.3.6)

where $P_{bc}$ is the battery cell output power and $I_{bc}$ is the battery cell current. Furthermore, from (6.3.1), (6.3.5) and (6.3.6), the cell state of discharge can be determined,

$$f = \left( E_{0bc}V_{bc} - P_{bc}r_{0bc} - V_{bc}^2 \right) / (k_{bc}V_{bc} - k_{rbc}P_{bc})$$

(6.3.7)

The last equation is useful only if the battery parameters $E_{0bc}$, $r_{0bc}$, $k_{bc}$ and $k_{rbc}$ are known.

6.4 A Possible BESS Change-over Strategy

The next task is to examine how the battery discharge will impact on the operation of the UPFC-AP, and from such knowledge, the change-over of the in-service BESS to the stand-by BESS can be effected.

Since the UPFC-AP is used to support power transfer when $P_r > P_{r,th}$, the in-service BESS
Chapter 6: Battery Energy Storage System in the UPFC-AP

state of discharge \( f \) will increase with time. Hence one would need to determine when the change-over from the in-service BESS (denoted as BESS2 in Section 5.1) to the stand-by BESS (called BESS1) should occur, given the present status of the BESS2 and \( P_r \). From (6.3.4), it is seen that the battery parameters \( E \) and \( r \) are functions of BESS2 discharge state \( f \). Since \( f \) can be measured on line as described in Section 2.4.1.2, the values of \( E \) and \( r \) obtained from (6.3.4) can be determined in real-time and these are denoted as \( E_i \) and \( r_i \) respectively.

On the other hand, the optimal economic dispatch strategy shown as (5.3.7) also allows one to determine the corresponding \( P_a^* \) for the given \( P_r \). Thus the corresponding value of \( V_{dc} \), denoted as \( V_{dc,\text{max}} \), that would allow BESS2 at its present status \( f \) (i.e. \( E_i \) and \( r_i \)) to support \( P_r \) can be determined from (5.2.4), i.e.

\[
V_{dc,\text{max}} = \left( E_i + \sqrt{E_i^2 - 4r_i P_r^*} \right)/2
\]  

Also, the optimization procedure described earlier and shown in Figure 5-7 allows one to determine the required converter output voltages, \( V_{se} \) and \( V_{sh} \), for the given \( P_r \). In general, one notes that the VSC AC voltage (\( V_{con} \)) is related to \( V_{dc} \) through the expression

\[
V_{con} = \sqrt{3} m V_{dc}/2\sqrt{2} = 0.6124m V_{dc}
\]

where \( m \) is the converter modulation index. Thus the series and shunt converter output voltages (\( V_{se} \) and \( V_{sh} \)) needed to support \( P_r \) would have the corresponding minimum value of \( V_{dc} \) denoted as \( V_{dc,\text{min}} \), when \( m = 1 \). Hence \( V_{dc,\text{min}} \) is given by

\[
V_{dc,\text{min}} = \max\{V_{se}/0.6124, V_{sh}/0.6124\}
\]  

where \( \max\{a, b\} \) returns the bigger value of \( a \) and \( b \). Thus if \( V_{dc,\text{max}} > V_{dc,\text{min}} \), it means that BESS2 at its present state of discharge \( f \) can still meet the demand \( P_r \) in the economic dispatch manner, at least for a duration until \( f \) decreases so significantly that \( V_{dc,\text{max}} < V_{dc,\text{min}} \). When that happens, the stand-by BESS1 will need to be switched in and connect to the UPFC, assuming (of course) BESS1 state of discharge \( f \) is lower than that of BESS2.

Indeed, one can determine the state of discharge of BESS1 by which the change-over will be acceptable, in the following way. Include a small (say 5%) design safety margin in \( V_{dc} \)
and denote this value as $V_{dc,lim}$, i.e. $V_{dc,lim} = 1.05V_{dc,min}$. From (5.2.4), (6.3.4) and (6.4.2), the BESS change-over will be beneficial if it can be shown that BESS1 state of discharge $f$ should be less than a value $f_m$ where

$$f_m = \left(1.05E_0V_{dc,min} - 1.1025V_{dc,min}^2 - r_0 P_d^*\right)/\left(1.05kV_{dc,min} - k_r P_d^*\right)$$  \hspace{1cm} (6.4.3)

One may also wish to take into consideration that battery life is limited by the charge/recharge cycles and depth of discharge/charge. In order to prevent battery from going into deep discharge, the allowable maximum battery SOD $f_m$ could be constrained to be less than (say) $f_{m,\text{max}}$. Thus if $f_m$ obtained from (6.4.3) is larger than $f_{m,\text{max}}$, $f_m$ should be set equal to $f_{m,\text{max}}$.

A numerical example will be used in a latter section to illustrate the proposed BESS change-over strategy.

### 6.5 Determination of BESS Capacity

The change-over strategy described in the earlier section is designed with the practical implementation of the UPFC-AP in mind. However, there remains the question of whether such change-over scheme would be sustainable, as the switching-out of BESS2 assumed that BESS1 is sufficiently charged to go into service. Clearly whether BESS1 would be ready depends on very much the availability of the renewable energy. For example, if the source of the energy is from the wind, the availability of which is highly time-varying. With the intense interest and developments in many parts of the world in the harnessing of wind energy, it is reasonable to assume that techniques to predict the amount of wind energy available at a given time and location will continue to improve. One can therefore expect the prediction on the availability of the wind can be achieved with a high level of confidence. If the premise is made herewith that the design of the UPFC-AP can assume there is sufficient energy from the renewable source, then one can remove one of the major uncertainties concerning the availability of the input energy to the AP, and focuses instead on the determination of the BESS capacity so that the change-over strategy described in the earlier section can be applied. In what follows, therefore, the objective is to obtain the battery change-over schedule for given load demand profile.
Chapter 6: Battery Energy Storage System in the UPFC-AP

and the information is then used to size the capacity of the BESS. It involves the use of the battery typical discharge characteristics so that the overall system will operate in the most economic power dispatch manner for the given $P_r(t)$. Hence, unlike Section 6.4 where the change-over strategy described there is focused on the operational aspect of the scheme, the intent of this Section is to derive a design method which would be useful for the planning of the UPFC-AP system.

6.5.1 Load Demand Profile

Figure 6-3 shows a typical load demand profile over a day for a sub-station supplying for example predominantly domestic-commercial customers. It shows that over the mid-night and the early morning period, the demand would be light. The demand would then increase rapidly in the morning when commercial and household activities commence, usually reaching a peak load condition close to noon. The load stays high over the early afternoon period, except for a short period during the lunch hours. From the late afternoon to the early evening period, the demand decreases progressively and the load cycle is repeated. Such load profile is usually available for the system planner, and hence, it will be used as the starting point of the design of the AP system.

In this Section, the load demand profile is considered over a 24-hr period because such a profile can be readily related to customers' activities. Hence it is attractive in using such a daily load cycle to illustrate the method to be described, as follows.

![Figure 6-3 Typical daily load curve ($P_r$) over a work day](image)
Chapter 6: Battery Energy Storage System in the UPFC-AP

The 24-hr period can be divided into $N$ equal segments, with each segment corresponding to $24/N$ hours. Over each segment, one can approximate the load demand as constant and therefore, the cumulative ampere-hours drawn from the in-service BESS will be a constant value, if $V_{dc}$ varies little over the interval. Furthermore, at the initial starting time ($t = 0$) of the study period, assume that the in service-battery is fully charged. Thus the initial ampere-hours "removed" from the cell is 0.

6.5.2 Estimation of Terminal Voltage per Cell

In order to arrive at the suitable BESS capacity so that the overall power system can be operated under the economic dispatch condition, the scheduling of the battery bank change-over for a given load profile has to be determined. However, at the planning stage, $f$ is not known or measurable, thus one has to use the data supplied by battery manufacturers to predict how the cell state of discharge $f$ and terminal voltage $V_{bc}$ would behave for a given $P_r$.

Figure 6-4 illustrates the typical discharge characteristic curve obtained from battery manufacturers. It is the so-called "fan" curve [151]. The figure shows a family of lines $V_{bc}^1, V_{bc}^2, ..., V_{bc}^n$ which are the cell terminal voltage where $V_{bc}^1 > V_{bc}^2 > ... > V_{bc}^n$. If one starts from an initial terminal voltage of $V_{bc}^1$, and after a certain amount of amp-hours discharge at a given current, one can use the curve to estimate what is the cell terminal voltage at the end of the period.

![Figure 6-4 Typical discharge characteristics of battery, per cell](image-url)
Suppose the given load for the operating period $M$ is $P_r^M$. Using the optimal economic dispatch equation (5.3.7), the corresponding battery output power $P_d^M$ can be determined. Thus the required output power per cell is $P_{bc}^M = P_d^M / mn$. The proposed method of calculating the battery cell terminal voltage for this operating period $M$ is an iterative process. Suppose $V_{bc}^1$ is the cell terminal voltage at the beginning of the $M^{th}$ operating period. Assume initially the cell terminal voltage remains constant at $V_{bc}^1$ over the next time interval $\Delta T = 24/N$ Hrs. Thus the cell current over this operating point can be determined by substituting $V_{bc}^1$ into (6.3.6), i.e., $I_{bc}^M = P_{bc}^M / V_{bc}^1$. The cumulative total of the ampere-hours “removed” from the cell can be calculated:

$$IT^M = IT^{M-1} + I_{bc}^M \Delta T \quad (6.5.1)$$

where $IT^{M-1}$ is the total ampere-hours removed from the cell from $t = 0$ to the period $M$. Once $IT^M$ is known, the operating point “$M$” in the fan curve whose x-axis value is $I_{bc}^M$ and y-axis value is $IT^M$ is determined, as shown in Figure 6-4. Thus from Figure 6-4, the cell terminal voltage of the operating point $M$ can be estimated as,

$$V_{bc}^M = V_{bc}^1 \cdot \frac{(I_{bc}^{i+1} - V_{bc}^{i+1}) (I_{bc}^{i+1} - I_{bc}^M)}{(I_{bc}^{i+1} - I_{bc}^i)} \quad (6.5.2)$$

where $I_{bc}^i$ is the x-axis value of the intersection of the $V_{bc}^i$ curve and the horizontal line whose y-axis value equals to $IT^M$, and $I_{bc}^{i+1}$ is that corresponding to the intersection with the next $V_{bc}$ curve closest to $V_{bc}^i$.

If the magnitude of the error between $V_{bc}^M$ and $V_{bc}^i$ is smaller than a pre-specified tolerance $\varepsilon$, then the cell terminal voltage is $V_{bc}^M$. Otherwise substitute the new value ($V_{bc}^M$) for $V_{bc}$ using (6.3.6) i.e. $I_{bc}^M = P_{bc}^M / V_{bc}^M$, and repeat the procedure described above until the magnitude of the error between $V_{bc}^M$ and $V_{bc}$ is smaller than $\varepsilon$.

### 6.5.3 A Computational Procedure

Since the battery bank has been assumed to compose of identical cells, therefore the state of discharge of the battery bank equals that of each cell. In Section 6.5.2, a method has
been proposed to predict the battery cell terminal voltage $V_{bc}$ and the battery cell state of discharge $f$ for a given load $P_r$. In this Section we will use the results of Section 6.3, Section 6.4 and Section 6.5.2 to obtain the total bank terminal voltage $V_{dc}$, and $r$ for given load profile $P_r$. Furthermore, combined with the result of Section 6.4, where the allowable maximum $f$ value $f_m$ for a given load demand has been determined, a strategy of scheduling of the battery bank change-over for a given load demand will be derived, such that the overall power system can be operated under the economic dispatch condition.

The computational procedure for the $M$th time segment is described as follows. In the procedure, one notes that Step 1 assumes the in-service battery bank is fully charged. Steps 2-3 calculate $P_{bc}^M$ for the $P_r^M$. Steps 4-5 are needed to calculate the cell $E_{bc}$, $r_{bc}$ for the given $f^M$. Steps 6-9 are used to carry out the iterative calculation of the battery cell terminal voltage $V_{bc}$ described earlier. Steps 10-11 calculate the battery state of discharge $f$ using (6.3.7) and compare the $f$ so obtained with the allowable maximum battery state of discharge i.e. $f_m$ obtained from (6.4.3), subject to $f_m \leq f_{m,max}$. If $f$ is larger than $f_m$, or $V_{bc}$ is smaller than the allowable minimum battery cell terminal voltage $V_{bc,min}$, the change-over of the in-service battery bank to the fully charged stand-by BESS is initiated. Steps 12-13 calculate the total battery bank internal EMF, resistance and terminal voltage. Steps 14-15 are to update the time segment to the next $(M+1)$th time segment and $V_{bc}$ and $f$ are updated before step 16 where return to step 1 to begin a new daily cycle.

Step 1: Let $r^M = 0$, and switch the fully charged battery bank into service, thus $f^M = 0$.

Step 2: For a given $P_r^M$, corresponding to time $r^M$, use equation (5.3.7) to determine the needed corresponding battery output power $P_{d}^M$ and calculate the maximum $f_m^M$ by using the analytical expression (6.4.3). If $f_m^M > f_{m,max}$, let $f_m^M = f_{m,max}$.

Step 3: Determine the required output power from each battery cell by using expression $P_{bc}^M = P_{d}^M / mn$.

Step 4: Identify the battery cell internal EMF $E_{bc}^M$ and resistor $r_{bc}^M$ by substituting $f^M$ into expression (6.3.1).

Step 5: Determine the battery cell terminal voltage $V_{bc}^{M(0)}$ using expression (5.2.4), i.e.

$$V_{bc}^{M(0)} = \left( E_{bc}^M + \sqrt{(E_{bc}^M)^2 - 4r_{bc}^M P_{bc}^M} \right)/2.$$
Step 6: Obtain the discharge cell current $I_{bc}^M$ using $I_{bc}^M = P_{bc}^M / V_{bc}^{M(0)}$.

Step 7: Calculate the cumulative total of the ampere-hours “removed” from the cell $IT_{bc}^M = IT_{bc}^{M-1} + I_{bc}^M * 24/N$.

Step 8: Using the “fan” discharge characteristic curve, determine the new value of battery terminal voltage $V_{bc}^{M(j)}$ using (6.5.2).

Step 9: If $\|V_{bc}^{M(j)} - V_{bc}^{M(0)}\| \leq \epsilon$, go to step 10; else $V_{bc}^{M(0)} = V_{bc}^{M(j)}$, go back to step 6.

Step 10: Determine the battery cell state of discharge using expression (6.3.7), i.e. 

$$f^M = \left( E_{0bc} V_{bc}^{M(0)} - r_{0bc} P_{bc}^M - \left[ V_{bc}^{M(0)} \right]^2 \right) \left( k_{bc} V_{bc}^{M(0)} - k_{bc} P_{bc}^M \right) .$$

Step 11: If $f^M \leq f_{m}$ or $V_{bc}^{M(0)} \leq V_{bc, min}$, go to step 12. Otherwise a battery bank change-over is needed, Let $f^M = 0$, $IT^M = 0$, go back to step 4.

Step 12: Calculate the battery cell internal EMF $E_{bc}^M$ and resistor $r_{bc}^M$ by substituting $f = f^M$ into expression (6.3.1).

Step 13: Calculate the BESS internal EMF, resistance and terminal voltage using $E^M = m E_{bc}^M$, $r^M = m r_{bc}^M / n$, and $V_{dc}^M = \left( E^M + \sqrt{\left( E^M \right)^2 - 4 r^M P_{dc}^M} \right) / 2$.

Step 14: Update time step $t_{M+1} = t_{M} + 24/N$.

Step 15: If $t_{M+1} \geq 24$, go to step 16, else let $V_{bc}^{M+1(0)} = V_{bc}^{M(0)}$, $f_{M+1} = f^M$, $M = M + 1$, go back to step 2.

Step 16: let $t_{M+1} = 0$, return to step 1 to start a new daily cycle.

6.6 Illustrative Examples

The following examples are based on the 345-kV transmission system described in Section 5.4. The UPFC parameters and the realistic network-UPFC limits are the same as that shown in Section 5.4.

6.6.1 Example 1. BESS Change-over Strategy

In this example, assume the BESS parametric values are:

$$E_{0bc} = 2.115V, \quad k_{bc} = 0.249, \quad r_{0bc} = 1.667 \text{ m}\Omega, \quad k_{rbc} = -0.0003934 \quad [152], \quad f_{m, max} = 80\% \quad [47].$$
Assume a 5% \( V_{dc} \) voltage safety margin. On the DC-bus, the voltage base is 220 kV.

From Table 5-1, it is seen that for \( P_{r,h} < P_r \leq P_{r,t} \), the optimal dispatch strategy requires that \( P_d \) increases linearly as \( P_r \) while \( V_{se} \leq 0.4 \) and \( V_{sh} \) maintains constant at 1.1 p.u.. Hence for \( P_{r,h} < P_r \leq P_{r,t} \), it is determined that

\[
V_{dc,min} = \max \{ V_{se}/0.6124, V_{sh}/0.6124 \}
\]

\[
= \max \{ 0.4*345*(100/172.5)/0.6124/220, 1.1*345*(100/345)/0.6124/220 \}
\]

\[
= 0.82 \text{ p.u.}
\]

Thus if \( V_{dc,max} \) calculated from (6.4.1) for the given \( P_d \) is such that \( V_{dc,max} \) is above \( V_{dc,lim} = 1.05 \ V_{dc,min} \) or 0.86 p.u., the in-service BESS will still be able to meet the power demand \( P_d \). Figure 6-5 shows the family of curves describing the relationship between \( P_d \) and \( V_{dc,max} \) obtained through the application of (6.4.1) for a range of \( f \). The area above the dashed line \( V_{dc,lim} \) corresponds to the battery discharge state when the in service BESS can meet the demand \( P_d \).

![Figure 6-5 V_{dc,max} over the range P_{r,h} < P_r < P_{r,t} under varied BESS SOD condition](image)

Table 5-1 also shows that for \( P_r > P_{r,t} \), \( P_d \) is constant at the maximum value of 0.268 p.u. while \( V_{se} \) and \( V_{sh} \) decrease as \( P_r \) increases. Again from (6.4.1), one can obtain the characteristic curve showing how \( V_{dc,max} \) varies with \( f \). This is given in Figure 6-6 (a). Figure 6-6 (b) shows how \( P_r \) varies with \( V_{dc,min} \). It shows that beyond \( P_{r,L} \approx 0.63 \) p.u., \( V_{dc,min} \) begins to decrease as \( P_r \) increases because either \( V_{se} \) and/or \( V_{sh} \) are decreasing from
their rated values. The area above $V_{dc,lim}$ corresponds to the condition the in-service BESS is able to meet the required $P_r$. With the 5% design margin, $V_{dc,lim} = 1.05 \ V_{dc,min}$ and $V_{dc,lim}$ is also shown in Figure 6-6 (b). Figure 6-7 is the combination of Figure 6-6 (a) and Figure 6-6 (b) and it can be used in the following way. For example, suppose $P_r$ is 0.7 p.u. From the $V_{dc,lim} : P_r$ curve, one notes that $V_{dc,lim} = 0.81$ p.u. At this $V_{dc}$ level and from the $V_{dc,max} : f$ curve, the corresponding $f$ is 0.64. It indicates that when the in-service BESS $f \leq 0.64$, the voltage of the DC-link $V_{dc,max}$ predicted will be higher than $V_{dc,lim}$, and the BESS can meet the required $P_r$.

![Figure 6-6](image1.png)

Figure 6-6 (a) $V_{dc,max}$ as a function of BESS state of discharge over the range $P_r > P_{r,1}$; (b) $V_{dc,min}$ over the range $P_r > P_{r,1}$ and $V_{dc,lim} = 1.05 \ V_{dc,min}$

![Figure 6-7](image2.png)

Figure 6-7 $V_{dc,max} : f$ and $V_{dc,lim} : P_r$ for $P_r > P_{r,1}$
While the above example shows how the maximum BESS $f$ can be determined in a graphical manner for a given $P_r$, one may also use the analytical expression (6.4.3) to obtain the maximum $f$ value i.e. $f_m$. Hence by tracking $P_r$ and the in-service BESS $f$ online, one must ensure that $f$ must not be higher than $f_m$. Figure 6-8 shows how $f_m$ varies with $P_r$. Thus, in the implementation of the scheme, $f_m$ can be compared with the on-line measured $f$. If the measured $f$ approaches close to the value $f_m$, a change-over of the BESS is required. The stand-by BESS is then connected to the UPFC dc-bus, while the in-service BESS will be disconnected from the bus. This is assuming that the stand-by BESS $f$ is below $f_m$. Note that below $P_{r,H} = 0.264$ p.u., $f_m$ is a constant value (0.8). The reason is for $P_r < P_{r,f}$, $f_m$ obtained from (6.4.3) is larger than $f_{m,max}$, in order to prevent battery from going into deep discharge, thus $f_m$ is set equal to $f_{m,max}$.

![Figure 6-8 Variation of $f_m$ with $P_r$](image)

**6.6.2 Example 2. BESS Scheduling Strategy**

In this example, assume the battery cell parametric values are: $E_{0bc}=2.115\text{V}$, $k_{bc} = 0.249$, $r_{0bc} = 1.667 \text{m}\Omega$, $k_{rbc} = -0.0003934$ [152], $V_{bc,min} = 1.75\text{V}$[151], $f_{m,max} = 80\%$ [47]. Suppose the no load voltage of the DC-link is 220 kV, thus the number of series cells required $m = 220*10^3 / 2.115 \approx 104020$. This appears to be a particularly large number of cells connected in series. Although not considered in this study, in practice, one may need to include a DC-DC boost converter. In which case, $m$ can be reduced considerably.
Chapter 6: Battery Energy Storage System in the UPFC-AP

Figure 6-9 shows the power system operating profile when there are 8 parallel strings of the cells, i.e. \( n = 8 \). Figure 6-9 (a) shows the load/time profile and the corresponding real power output of the AP according to the optimal economic dispatch strategy shown as (5.3.7). Figure 6-9 (b) illustrates the variation of \( f_m \) with \( P_r \). Figures 6-9 (c)-(g) show the corresponding battery cell terminal voltage \( V_{bc} \), DC-link voltage \( V_{dc} \), battery internal EMF \( E \), battery state of discharge \( f \) and battery internal resistance \( r \) profile over the 24-hr period, respectively. It is seen that at 0000 hr when \( P_r \) is 0.15 p.u., BESS2 begins to output power. From then on, the battery SOD \( f \) increases, while the battery internal EMF \( E \) reduces and \( r \) increases. At about 0836 hr, \( P_r \) increases to 0.39 p.u. and the corresponding \( f_m \) is 0.47, but the battery state of discharge \( f \) has reached 0.48. Hence BESS2 cannot meet the required battery power output. The stand-by BESS1 will be connected to the UPFC dc-bus, while BESS2 will be disconnected from the bus. The in-service BESS internal EMF \( E \), state of discharge \( f \) and internal resistance \( r \) are reset to their initial values. In the study, it is assumed that the stand-by BESS1 is full charged. In this way, the battery discharge and switch-in and switch-out process continue in the manner as shown in the figure.

Figure 6-9 indicates that the BESS banks need to be switched 10 times over the period of a day. The average time between the switching in and the switching out of one BESS is approximately 2.4 hours, in the interval 00:00-24:00 hrs. One notes that the period between the switching in and switching out of the BESS is not uniform over the 24-hr period. Indeed, when \( P_r \) is at peak level, say over the 08:00-20:00 hrs, the switch-in and switch-out period is the shortest (typically 1.24 hours) amongst the 10 switching events. This is the result of the combination of \( f_m \) being of the lowest value and \( P_d \) is at its peak. Hence, a crucial design consideration would be that over this period, the stand-by BESS has to be sufficiently charged to takeover from the in service BESS. From Figure 6-9 (c) and (f), it can be found that the battery cell terminal voltage excursive is above the minimum allowable voltage 1.75 V, and the SOC never drops to below 80%.
Figure 6-9 Simulation results for $n = 8$: (a) $P_r$ and $P_d$; (b) Variations of $f_m$ with $P_r$; (c) battery cell terminal voltage $V_{bc}$; (d) DC-link voltage $V_{dc}$; (e) Battery internal EMF $E$; (f) Battery State of discharge $f$; (g) Battery internal resistance $r$
Chapter 6: Battery Energy Storage System in the UPFC-AP

Suppose the previous results have indicated that the period between switching in and switching out of the BESS is too short for BESS1 to be fully charged. One solution would be to increase the number of parallel cells \( n \) in the BESS, so that the period can be increased. Of course, this is assuming that there is enough energy in the renewable source. By increasing the capacity of the BESS, one is to ensure that more of the renewable energy is harnessed and stored in the BESS.

Figure 6-10 shows the results when the BESS capacity is increased by making \( n = 12 \). Compare Figure 6-10 with Figure 6-9, it can be seen that when \( n \) increases, the value of \( f_m \) corresponding to the same \( P_r \) will increase. The reason is that when \( n \) increases, from (6.3.3), \( E_0 \) and \( k \) will not change while \( r_0 \) and \( k_r \) will decrease. Thus from (6.4.3), \( f_m \) will increase. Figure 6-10 also shows that when \( n \) increases to 12, the battery banks only need to be switched 7 times a day, far less than the 10 times a day predicted when \( n = 8 \). Thus it shows that the battery bank will need to be changed over less frequently when the capacity of the bank increases.

6.7 Conclusions

In this chapter, lead-acid battery has been considered as a suitable form to be included in the UPFC-AP. An operational strategy has been proposed to pertaining to the change-over from the in-service BESS to the stand-by BESS. Furthermore, a method to determine the battery bank change-over schedule for a given load demand has been derived. From the schedule so obtained, one can arrive at the suitable BESS capacity so that the overall power system can be operated under the economic dispatch condition. By varying the capacity of the BESS to suit the available energy profile of the renewable, one also ensures that the UPFC-AP system will guarantee the transfer capability of the compensated system is maximized.
Figure 6-10 Simulation results for \( n = 12 \): (a) \( P_r \) and \( P_d \); (b) Variations of \( f_m \) with \( P_r \); (c) battery cell terminal voltage \( V_{bc} \); (d) DC-link voltage \( V_{dc} \); (e) Battery internal EMF \( E \); (f) Battery State of discharge \( f \); (g) Battery internal resistance \( r \)
Chapter 7

Series-Shunt Compensation Technique for Power Quality Enhancement

This chapter is organized as follows. In Section 7.1, some background information on Power Quality Control Center-Series Compensator (PQCC-SC) is provided. In Section 7.2, the fuel cell distributed generator model is presented. Next, the PQCC-SC system model is introduced and an operational scheme is proposed in Section 7.3. Furthermore, a compensation scheme is developed in Section 7.4. Simulation results of the proposed control scheme are presented in Section 7.5. Finally Section 7.6 concludes this chapter by highlighting the contributions.

7.1 Introduction

In Chapter 5, a new scheme which incorporates an active power (AP) source into a UPFC for transmission enhancement has been investigated. The design objective includes the increase in power transfer capacity. On the other hand, at the distribution level, the quality of supply has become an increasingly important consideration. This is due to the widespread use of computerized and power electronic equipment [66] in the load area. Thus there have been much research works on the use of power electronics-based compensators to improve supply quality for sensitive loads. These works have been described in Sections 2.4 and 2.5.

Indeed, in Section 2.5, it has been described that while many loads are in great need of high power quality, there are still significant proportions of present-day loads which can accept lower-quality supply but demand it to be met at lower prices. Hence it would be highly desirable if utility industry can provide an unbundled power quality service [67]
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

and enables the delivery of different power quality levels to match customer requirements and expectation. Parallel with the above developments in recent years is the increasing amount of small-scale distributed generation (DG) that has been introduced on the customer-side of distribution systems. This is due to environmental and/or economic considerations [68]. For example, reference [153] proposes an inverter-interfaced DG scheme intending to provide improved power quality to loads. Another development is based on the concept of Power Quality Control Center (PQCC) [89, 92], also described in Section 2.5, in which the attempt is to design a power system which has the feature of unbundled power quality supply. The Center also requires the incorporation of DG, although the impacts of the DG on system performance have not been clearly explained. In an attempt to fill this gap and by assuming the DG in the form of a Solid-Oxide Fuel Cell (SOFC), a more practical PQCC operational scheme has been proposed in [154]. Unfortunately as the DG is shunt-connected to the load, it is shown that the voltage sag ride-through capability provided by the PQCC system appears to be restricted. One main reason is because like any practical generators, the instantaneous active power change the SOFC can accommodate is constrained. An attempt to overcome such shortcoming is described in [155] in which it is shown that the voltage sag ride-through capability of the scheme can be improved by introducing a Series Compensator (SC) into the PQCC structure. The SC operates very much like the DVR described in Section 2.5.1.

The work to be described in this chapter differs from previous works [98, 99, 154, 155, 156] in several aspects. The role of the SC is to inject a voltage component in series with the protected load. In the process of the injection, power exchange between the SC and the load shall occur. Unlike the SC design described in previous works, a new SC compensation technique is proposed in which the SC is fed from an active source which has its instantaneous output power change constrained. Moreover, the method has the very desirable feature in that it allows the compensation to be carried out at minimum energy level. Thus it maximizes the ride-through range offered to the compensated load. Furthermore, the new technique takes into account the apparent power rating of the SC. The resulting PQCC-SC control strategy is shown to be superior in terms of sag ride-through ability compared to that given in previous works. There is also an added advantage of the technique: the sag ride-through ability is shown to be insensitive to voltage phase jump introduced during a disturbance. The method is therefore seen to
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

contain some degree of robustness in terms of its effectiveness to overcome voltage sag. Analytical expressions for the injected voltage magnitude and angle, in terms of voltage sag/swell severity and load power factor, are also derived. Simulation results are given to show the efficacy of the proposed method, and the voltage sag/swell ride-through which can be achieved for a modestly-sized SC.

7.2 Fuel Cell Distributed Generator Model

As introduced in Section 2.4, fuel cells have emerged as attractive power sources in recent years as they are energy-efficient, friendly to the environment and can provide reliable service [108-110]. Due to the impressive service record of a number of demonstration units of the high temperature solid-oxide fuel cell (SOFC) currently in operation [108], SOFC power plant has been chosen as the form of DG in the present work.

The SOFC internal electrochemical process has been described in Chapter 2. According to the SOFC Nernst Equation given in (2.4.2), it is seen that the ideal SOFC open-circuit EMF $E$ should be constant [71, 108, 109]. However, there are several physical and chemical processes which introduce losses to the performance of fuel cells, such as transport of reactants to the electrolyte interface and absorption of electro-active species onto the electrode. Therefore the operation voltage is less than the theoretical value, as shown in Figure 7-1. It illustrates a comparison of the ideal and actual voltage versus fuel-cell stack current ($I_{FC}$) characteristics of a typical fuel cell operating state [71]. It indicates that the differences can be divided into three regions. The region of activation polarization is generally a result of losses associated with slow chemical reactions. It is prominent when current densities are low. The region of Ohmic polarization is the linear part of the voltage and varies directly with current. Ohmic polarization is loss due to the flow of electricity through the fuel cell which resists the flow. The region of concentration polarization is dominant at large current densities and is caused by transport phenomena which lead to lower concentrations of reactants at the electrochemical surface.

For simplify of analysis, SOFC would be assumed herewith to operate in the linear Ohmic polarization region and the Ohmic loss is the main voltage loss under steady-state. Therefore, under steady-state the electrical characteristics of the SOFC can be expressed
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

as,

\[ V_{dc} = E - rI_{FC} \]  \hspace{1cm} (7.2.1)

where \( V_{dc} \) is the SOFC terminal voltage, \( E \) is the SOFC open-circuit EMF assumed to be essentially constant under steady-state and short-duration disturbance condition, \( r \) accounts for the Ohmic loss. Therefore, the SOFC output power \( (P_{dg}) \) is

\[ P_{dg} = V_{dc}I_{FC} \]  \hspace{1cm} (7.2.2)

Figure 7-1 Ideal and actual fuel cell current and voltage characteristics [71]

Fuel cell converts chemical energy in hydrogen \( (H_2) \) and oxygen \( (O_2) \) directly into electrical energy. The detailed energy conversion process is highly complex and again, for the purpose of the present investigation, it would be sufficient to consider the schematic diagram of the SOFC shown in Figure 7-2 [157, 158]. Two main parts of the DG can be readily identified. The part labeled as the balance of plant (BOP) consists of the natural gas fuel storage, fuel valve and the fuel processor. The fuel processor reforms the natural gas, of input flow rate \( N_f \), to the hydrogen-rich fuel with flow rate of \( N_{H_2}^{in} \). A first-order lag model of time constant \( \tau \) can be used to approximate the processor dynamic [157]. The part called the cell stack is where the complex thermodynamic and electrochemical processes take place.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

Figure 7-2 Schematic diagram of a SOFC [157, 158]

According to [108, 109], two practical constraints of the operation of SOFC have to be taken into account. The first concerns the fuel utilization factor $u$ which is the most important operating variable that affects the performance of SOFC and is defined as

$$u = \frac{N_{i}^{\text{in}} - N_{i}^{\text{out}}}{N_{i}^{\text{in}}}$$

where $N_{i}^{\text{in}}$ and $N_{i}^{\text{out}}$ are the input and output flow rates of hydrogen in the fuel cell stack. It is seen in [157] that $u$ can also be expressed in term of $I_{FC}$ as

$$u = \frac{2K_{r}I_{FC}}{N_{i}^{\text{in}}}$$

(7.2.3)

where $K_{r}$ is a modeling parameter. Typical desirable range of $u$, expressed as $u_{\min} \leq u \leq u_{\max}$, is from 0.7 to 0.9. When operating under underused-fuel condition ($u < 0.7$), it could lead to undesirably high cell voltages. On the other hand, overused-fuel situation ($u > 0.9$) results in permanent damage to the cells due to fuel starvation.

The second constraint is pertaining to the fuel cell output power $P_{dg}$ which should be maintained within the range

$$P_{\min} \leq P_{dg} \leq P_{\max}$$

(7.2.4)

The minimum power constraint $P_{\min}$ is necessary for maintaining stable stack operating temperature and acceptable power plant efficiency [159]. The maximum power constraint $P_{\max}$ indicates the design power rating of the SOFC.

Due to the above constraints, a steady-state feasible operating area (FOA) of the SOFC, using the typical data given in Appendix H, can be derived. This is shown in Figure 7-3. The authors of [158, 160] have described the detailed construction of the FOA. In deriving the FOA, the operating temperature of the fuel cell stack has been assumed
constant and the fuel cell rated power and output voltage are used as the base quantities. The horizontal lines AD and BC denote the constraints placed on \( u \). The curves AB and CD represent the limits on \( P_d \). The FOA of the SOFC would correspond to the area ‘ABCDA’. The SOFC must operate within the FOA as operation outside of FOA will reduce cell life and is unacceptable. The operational characteristic of the SOFC under a constant \( V_{dc} \) condition is shown, for example by the curve XY, which indicates \( V_{dc} \) operating within the voltage limits \( V_{dc,\text{max}} \) and \( V_{dc,\text{min}} \).

![Figure 7-3 Feasible steady-state operating area of a SOFC [158]](image)

Equation (7.2.3) shows that the operation of the fuel cell stack with a fuel input \( N_{\text{H}_2}^{\text{in}} \) proportional to the stack current \( I_{FC} \) results in a constant utilization factor under steady-state. Such a constant utilization factor operation in turn brings on very small deviations in steady-state \( V_{dc} \) due to changes in stack current, as can be seen in Figure 7-3. However, such small voltage changes can be easily handled through a DC/DC converter at the terminals of the SOFC. Thus, the fuel cell stack can be controlled to operate with a constant steady-state utilization factor by controlling the natural gas input to the stack in the following manner:

\[
N_f = \frac{2K_rI_{FC}}{u_s} \quad (7.2.5)
\]

where \( u_s \) is the desired utilization factor under steady-state. Such an input fuel control scheme produces a characteristic which can also be seen in Figure 7-2 and Figure 7-3. By adopting a constant \( u \) operational scheme such as described by the line EF in Figure 7-3, where \( u_s \) is shown as 0.8, \( V_{dc} \) would vary as \( I_{FC} \) changes.
Furthermore, with the adoption of this fuel input control strategy and in conjunction with the fuel processor dynamics described in Figure 7-2, the relationship between a small change of stack current $\Delta I_{FC}$ and a small change of hydrogen input $\Delta N_{H_2}^{in}$ fed to the fuel cell stack can be derived as

$$\Delta N_{H_2}^{in} = \frac{2K_r}{u_s(1+\tau_f s)} \Delta I_{FC}$$  \hspace{1cm} (7.2.6)

From (7.2.3), the small-signal equation relating $\Delta u_s$ $\Delta I_{FC}$ and $\Delta N_{H_2}^{in}$ about the initial steady-state $N_{H_2,0}$ and $u_s$ can be written as

$$\Delta I_{FC}(s) = \frac{N_{H_2,0}}{2K_r} \Delta u_s(s) + \frac{u_s}{2K_r} \Delta N_{H_2}^{in}(s)$$  \hspace{1cm} (7.2.7)

The subscript '0' is used herewith to denote the respective steady-state quantity. Substituting (7.2.3) and (7.2.6) into (7.2.7), one obtains

$$\Delta u_s(s) = \frac{u_s \tau_f s}{I_{FC,0}(1+\tau_f s)} \Delta I_{FC}(s)$$  \hspace{1cm} (7.2.8)

where $I_{FC,0}$ is the initial stack current and is governed by the initial value of $N_f$ (i.e., $N_{H_2,0}$) through (7.2.5). Suppose the disturbance is due to an external power variation and has resulted in a step change in $I_{FC}$ of magnitude $\Delta I_{FC}$ pu. Expressed in the s-domain, it yields $\Delta I_{FC}(s) = \Delta I_{FC}/s$. Substituting it into (7.2.8), the SOFC step response in $u$ can be obtained as

$$\Delta u(t) = \frac{u_s \Delta I_{FC}}{I_{FC,0}} \exp(-t/\tau_f)$$  \hspace{1cm} (7.2.9)

It is seen from (7.2.9) that $\Delta u(t)$ will experience a maximum change of magnitude $(u_s \Delta I_{FC}/I_{FC,0})$ initially, and then $\Delta u(t)$ will decay in an exponential manner. The rate of decay corresponds to the time constant of the fuel processor. A new variable $\varepsilon$ in term of the upper and lower limits on $u$ may then be defined as

$$\varepsilon = \begin{cases} (u_{max} - u_s)/u_s & \text{for positive } I_{FC} \text{ step} \\ (u_{min} - u_s)/u_s & \text{for negative } I_{FC} \text{ step} \end{cases}$$  \hspace{1cm} (7.2.10)

From the definition of $\varepsilon$ shown in (7.2.10), clearly the maximum change in $u$ should be constrained such as

$$u_s \Delta I_{FC}/I_{FC,0} \leq u_s |\varepsilon|$$
i.e.,
\[ \Delta I_{FC} \leq |e| I_{FC,0} \]  \hspace{1cm} (7.2.11)

Thus, (7.2.11) describes the maximum step change in \( I_{FC} \) which can be accommodated by the SOFC. This is to ensure that \( u_{min} \leq u \leq u_{max} \).

Conversely, based on the above discussion the maximum allowable instantaneous real power change imposed on the cell can also be calculated approximately. Over the initial period in which \( E \) is constant in (7.2.1) and applying the small-signal analysis to (7.2.2), the real power change \( \Delta P_{dg} \) can be expressed as,
\[ \Delta P_{dg} = V_{dc,0} \Delta I_{FC,0} + I_{FC,0} \Delta V_{dc} = (V_{dc,0} - rI_{FC,0}) \Delta I_{FC} \]

Substituting (7.2.11) into the above equation, therefore one obtains the constraint equation
\[ \Delta P_{dg} \leq |e|(V_{dc,0} - rI_{FC,0})I_{FC,0} = |e| P_{dg,0} \]

or
\[ \Delta P_{dg} \leq |e| P_{dg,0} \] \hspace{1cm} (7.2.12)

where \( P_{dg,0} = V_{dc,0}I_{FC,0} \) is the initial operating power level. One makes use of the observation that the internal voltage drop term \( rI_{FC,0} \) is much smaller than \( V_{dc,0} \) in arriving at the last approximation. Hence, (7.2.12) indicates that in order to maintain operation within the FOA [160], the instantaneous power change (\( \Delta P_{dg} \)) imposed on the SOFC should be less than \( |e| P_{dg,0} \).

Consider the typical values of \( u_s = 0.8, u_{min} = 0.7 \) and \( u_{max} = 0.9 \). It yields \( \varepsilon = 0.125 \) from (7.2.10). Therefore, the maximum instantaneous power change that can be accommodated by the SOFC is 12.5% of its initial loading, i.e., \( 0.125 P_{dg,0} \). Demand for an instantaneous power change of less than 12.5% of \( P_{dg,0} \) can be met by the SOFC instantaneously in a single step change in the output power. For a larger power change demand, however, the SOFC would require several tens of seconds for the fuel processor to adjust the fuel input, and that through the internal electro-chemical process, before the targeted power level can be reached. In a conclusion, there is a practical limit in terms of the magnitude and the speed by which the SOFC can respond to a power change demand. The above results have been reported in [154].
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

7.3 Some Basic Considerations

The particular PQCC scheme and the basic design considerations in this chapter will first be described. In what follows, a voltage/current quantity with a "~" overhead denotes a phasor, its magnitude is without the "~" sign but is shown in capital letter.

7.3.1 General Description

The proposed scheme is adopted from the UPS-type PQCC described in [89, 92] but with the addition of a SC. The schematic diagram of the scheme is as shown in Figure 7-4. It uses the basic idea of Uninterruptible Power Supply (UPS) system and consists of two PWM-controlled inverters (Inv.1 and Inv.2) and a DC bus. Unlike conventional UPS system, however, the PQCC is equipped with a SOFC DG in the DC side. The SC is connected to the DC bus and coupled to the AC system through a DC/AC converter and an injection transformer. Details of the SC can be found in [154]. Also from [154], it is shown that unbundled power quality service could be realized with such an arrangement: there are three levels of power quality in the AC supply, named Ordinary Quality (OQ), High Quality (HQ), and Super Premium Quality (SP), plus one quality level in DC. As the names imply, the SP load enjoys the highest quality level amongst the AC loads, followed by the HQ load. The OQ load would have the lowest level of quality and reliability, as will be elaborated on later.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

7.3.2 Proposed Operational Scheme

7.3.2.1 Pre-Disturbance Condition

It is seen from Figure 7-4 that the complex upstream system is represented simply by the Thevenin equivalent voltage source $\bar{V}_s$ and a source inductor $L$. In this way, an upstream voltage disturbance due to a fault or load switching event could be approximated as the corresponding change in the magnitude and phase angle of $\bar{V}_s$. The survey shown in [161] indicates that most distribution system voltage disturbances have resulted in $V_s$ to be within the range 0.7 - 0.9 p.u. and its phase angle changes, often called phase angle jump, of within ±20°.

Under normal operating conditions of the PQCC-SC scheme, the SC will play no active role and its converter will be in the off state, i.e., $P_l + j Q_l = 0$. The DG supplies all the DC and SP loads, and also injects a surplus power $P_{d.o}$ to the PCC. The power factor at point ‘A’ is maintained at unity. This is considered an efficient way for power transfer from the upstream system to the PQCC.

7.3.2.2 Under-Disturbance Condition

When an upstream voltage sag/swell occurs, $V_s$ reduces/increases while its phase angle may also undergo a sudden change. The OQ load would be disconnected if $V_p$ (i.e. the standing voltage $V_L$) consequently decreases/increases below/over a pre-specified threshold value $V_{thv}$. Typically $V_{thv}=0.9$ p.u. for an undervoltage (sag) event and $V_{thv}=1.1$ p.u. for an overvoltage (swell) condition [104]. If $V_p$ remains below (more likely) or above $V_{thv}$ after the shedding of the OQ load, the SC will attempt to maintain supply to the HQ load by helping to restore $V_L$ to $V_{thv}$. The DC and SP loads would still be supplied by the DG and would not be affected by the sag/swell as long as $V_{dc}$ is in a reasonable range and Inv.2 can maintain the SP load terminal voltage. During the sag/swell, it is proposed that Inv.1 is also controlled to maintain unity power factor at ‘A’. 
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

In attempting to maintain the HQ load terminal voltage, the SC has to inject/absorb a certain amount of active power ($P_i$) through the injection transformer during the above voltage restoration interval. From Figure 7-4, the active power flow within the PQCC-SC is given by

$$P_i = P_d + P_r$$

(7.3.1)

If following a sag/swell, $P_i > P_d$. This means that active power will be absorbed from the PCC bus through Inv.1. Conversely a disturbance which results in $P_i < P_d$ means that surplus active power will be supplied to the PCC through Inv.1. The changes to the power flow through Inv.1 can be easily achieved by adjusting its switch control signals.

As stated in Section 7.2, the SOFC DG has only a limited instantaneous power change capacity. This is denoted by $\Delta P_{dg,max}$. As a consequence of this consideration, there is an additional constraint under the steady-state pre-disturbance system operating conditions: in the event of the sudden loss of the AC link between the PCC and Inv.1, the change in the DG output power must not exceed $\Delta P_{dg,max}$ in order to satisfy the DG instantaneous power change limit. Hence it is necessary to ensure that $P_r \leq \Delta P_{dg,max}$. Since $P_{d,0} = P_r$, therefore,

$$0 \leq P_{d,o} \leq \Delta P_{dg,max}$$

(7.3.2a)

In response to the disturbance, the DG can be allowed to instantaneously vary its active output power by up to the amount $\Delta P_{dg,max}$. Thus instantaneously after the disturbance, the maximum power flow from the DG to Inv.1 is $P_{d,max} = P_{d,0} + \Delta P_{dg,max}$ and the minimum power flow from the DG to Inv.1 is $P_{d,min} = P_{d,0} - \Delta P_{dg,max}$. Taking these factors into consideration, thus, the maximum amount of active power that the SC can inject into the external interconnected system through the injection transformer is

$$P_{inj,max} = P_{d,max} + \Delta P_{dg,max} = P_{d,0} + 2\Delta P_{dg,max}$$

(7.3.2b)

while the maximum amount of active power that the SC can absorb from the external system is

$$P_{abs,max} = \Delta P_{dg,max} - P_{d,min} = 2\Delta P_{dg,max} - P_{d,0}$$

(7.3.2c)

Hence $P_i$ must be limited to the range

$$-(2\Delta P_{dg,max} - P_{d,0}) \leq P_i \leq P_{d,0} + 2\Delta P_{dg,max}$$

(7.3.2d)

where the negative sign indicates an absorption of the real power.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

7.4 Compensation Strategy

While the results described so far have been reported in [154], the potential of the compensation strategy of the PQCC-SC scheme has not been fully explored there. Hence, attention will be directed toward the design of the injection strategy for the PQCC-SC.

7.4.1 Preliminary Analysis

The relationship between the voltage quantities during the steady-state voltage restoration interval can be described by the phasor diagram shown in Figure 7-5. As shown, the PCC voltage phasor before the voltage sag/swell is chosen as the reference quantity and is denoted as $V_{p0}$. Under the pre-disturbance normal operating conditions, $V_{p0}$ would be equal to the HQ load voltage. The pre-disturbance source voltage $V_s$ is denoted as $V_{s0}$. The phase angle of $V_{s0}$ is $\varphi_0$.

![Diagram (a) Schematic and (b) phasor diagram describing the proposed scheme during voltage restoration stage](image)

Figure 7-5 (a) Schematic and (b) phasor diagram describing the proposed scheme during voltage restoration stage
Due to an upstream fault for example, the source voltage becomes $V_s$ which results in not only a sudden decrease in its magnitude but also causes a phase jump of $\alpha$. In this way, $V_s$ can therefore be expressed as

$$V_s = V_s \cos \varphi + j V_s \sin \varphi$$  \hspace{1cm} (7.4.1)

where $\varphi = \alpha + \phi_0$. As described earlier, the OQ load is disconnected and the HQ load voltage $V_L$ is restored to the minimum value of $V_{thv}$ such that

$$V_L = V_{thv} \cos \beta + j V_{thv} \sin \beta$$  \hspace{1cm} (7.4.2)

Note that in the present formulation and unlike previous work [154] which assume that the HQ load voltage after restoration equals to that before the sag, a phase jump $\beta$ between the pre-sag and post restoration HQ load voltage is allowed. Just as in previous works, the HQ load power factor $\cos \theta$ is assumed unchanged and is lagging. Hence $I_L$ is

$$I_L = I_L \cos (\theta - \beta) - j I_L \sin (\theta - \beta)$$  \hspace{1cm} (7.4.3)

The upstream line current $I_s$ during the sag would be of the form

$$I_s = I_s \cos \delta + j I_s \sin \delta$$  \hspace{1cm} (7.4.4)

Following the proposed scheme, the power factor at point 'A' is maintained at unity. Therefore the PCC voltage $V_p$ is in phase with $I_s$. From Figure 7-5, $V_p$ can be represented as

$$V_p = V_s \cos (\varphi - \delta) \cos \delta + j V_s \cos (\varphi - \delta) \sin \delta$$  \hspace{1cm} (7.4.5)

and at the power frequency $\omega$,

$$I_s = \frac{V_s \sin (\varphi - \delta) - j V_s \cos (\varphi - \delta) \sin \delta}{\omega L}$$  \hspace{1cm} (7.4.6)

From (7.4.2) and (7.4.5), it then follows that the SC injected voltage is

$$\vec{V}_i = \vec{V}_L - \vec{V}_p = V_{thv} \cos \beta - V_s \cos (\varphi - \delta) \cos \delta + j [V_{thv} \sin \beta - V_s \cos (\varphi - \delta) \sin \delta]$$  \hspace{1cm} (7.4.7)

from which it can be shown that the magnitude of $\vec{V}_i$ is

$$|\vec{V}_i| = \sqrt{V_{thv}^2 + [V_s \cos (\varphi - \delta)]^2 - 2V_{thv}V_s \cos (\varphi - \delta) \cos (\beta - \delta)}$$  \hspace{1cm} (7.4.8)

In the last equation, $|\vec{V}_i|$ is expressed in terms of the independent system or pre-specified design parameters.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

Under this scheme, the current in the primary winding of the injection transformer of the SC is $\tilde{I}_L$. Therefore from (7.4.3) and (7.4.7), the apparent power loading of the SC under this scheme is

$$S = V_t I_L$$  \hspace{1cm} (7.4.9)

Let $S_{HQ}$ denote the apparent power drawn by the HQ load at $V_{thv}$. Substituting $I_L = S_{HQ} / V_{thv}$ into (7.4.9), it yields

$$S = S_{HQ} V_t / V_{thv}$$  \hspace{1cm} (7.4.10)

Suppose the power rating of the SC is $S_{max}$. Thus from (7.4.10), the maximum injection voltage which can be accommodated is

$$V_{i,m} = S_{max} V_{thv} / S_{HQ}$$  \hspace{1cm} (7.4.11)

Returning to Figure 7-5(b), the real power loading on the SC is

$$P_i = V_i I_L \cos \gamma = S_{HQ} \cos \theta - S_{HQ} V_P \cos (\delta - \beta + \theta) / V_{thv}$$  \hspace{1cm} (7.4.12)

where $\gamma$ is the phase difference between $\tilde{I}_L$ and $\tilde{V}_i$. It is defined such that if $\tilde{I}_L$ lags $\tilde{V}_i$, $\gamma$ is positive. Equation (7.4.12) shows that $P_i$ is a function of the system parameters $S_{HQ}$, $V_{thv}$, $\theta$, $\phi$ (or phase jump $\alpha$) and the controllable phase shift $\beta$.

The above analysis is for voltage sag scenario, the case for voltage swell can be studied in a similar manner.

### 7.4.2 Maximum Sag/Swell Ride-through

The purpose of the following analysis is to determine the maximum HQ load voltage sag/swell ride-through this scheme can provide when the apparent power rating ($S_{max}$) of the SC and the limit on the instantaneous power change capability of DG are both taken into consideration.

Suppose one uses $\tilde{I}_L$ as the reference phasor, the restored HQ load terminal voltage $\tilde{V}_L$ has attained the magnitude $V_{thv}$ as shown by the line OD in Figure 7-6. An upstream voltage disturbance has resulted in the PCC voltage $\tilde{V}_P$, where $\zeta$ is the phase angle difference between $\tilde{V}_P$ and $\tilde{I}_L$. One could inject a voltage $\tilde{V}_i$, as shown, to restore the load terminal voltage to $\tilde{V}_L$. Since (7.4.11) governs the maximum injection voltage $V_{i,m}$ for the given $S_{max}$, $S_{HQ}$ and $V_{thv}$, one can denote the presence of this voltage limit by super-
imposing onto the phasor diagram a circle with radius $V_{i,m}$ centered at D. Therefore $V_i$ must not be greater than $V_{i,m}$.

![Figure 7-6 Phasor diagram illustrating PQCC series compensation: for condition when $V_{thw} \sin \theta < V_{i,m} \leq V_{ip1}/\cos \theta$.](image)

Figure 7-6 Phasor diagram illustrating PQCC series compensation: for condition when $V_{thw} \sin \theta < V_{i,m} \leq V_{ip1}/\cos \theta$.

Next define two new quantities $V_{ip1}$ and $V_{ip2}$ where

$$V_{ip1} = \frac{P_{\text{inj, max}}}{I_L}$$

$$V_{ip2} = \frac{P_{\text{abs, max}}}{I_L}$$

(7.4.13) (7.4.14)

$P_{\text{inj, max}}$ and $P_{\text{abs, max}}$ are as given by (7.3.2b) and (7.3.2c), which limit $P_i$ injected and absorbed active power due to the instantaneous power change constraint imposed on the DG. Construct a line $L1$ perpendicular to $I_L$ and which passes through D, and two lines $L2$ and $L3$ parallel to but at a distance $V_{ip1}$ and $V_{ip2}$ respectively from $L1$. From Figure 7-6, it can be readily deduced that any voltage sag/swell which results in $\bar{V}_p$ appearing to the left of $L1$ will cause the SC to inject active power into the system. This is because $P_i = V_i I_L \cos \gamma > 0$ since $|\gamma| < 90^\circ$. However, as the maximum amount of active power that the SC can inject into the external interconnected system is $P_{\text{inj, max}}$, therefore voltage restoration is possible only when $\bar{V}_p$ does not lie to the left of $L2$. Conversely, if the voltage disturbance has resulted in $\bar{V}_p$ appearing to the right of $L1$, the SC will absorb active power from the external system since $|\gamma| > 90^\circ$. Again as the maximum amount of active
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

power that the SC can absorb from the external system is \( P_{\text{abs,max}} \), therefore voltage restoration is possible if \( \bar{V}_p \) does not appear to the right of \( L3 \).

As depicted in Figure 7-6, the arc \( C_1C_2 \) represents the locus of the compensated HQ load voltage \( \bar{V}_L \) such that its magnitude has been restored to \( V_{thv} \). If the magnitude of the PCC voltage \( \bar{V}_p \) is smaller than \( V_{thv} \), the power system is said to be under a voltage sag condition. Otherwise, the PQCC system is experiencing a voltage swell. Combining the limits on \( V_i \) and that on \( P_h \), it can be readily seen that the conceivable range of \( \bar{V}_p \) for which restoration is achievable must fall within the shaded area in the figure.

Also from the figure, it should be noted that the phasor diagram depicts the particular system condition when \( V_L = V_{thv} \) and \( V_{thv} \sin \theta < V_{i,m} \leq V_{ip1}/\cos \theta \). These conditions can be seen from the geometry of the phasor diagram. From the above analysis and based on the phasor equation \( \bar{V}_L = \bar{V}_p + \bar{V}_i \), it can be concluded from Figure 7-6 that the most severe voltage sag the scheme can restore \( \bar{V}_L \) is when \( |\bar{V}_p| = |OA| = V_{thv} - V_{i,m} \). When this sag occurs, it calls for the application of the in-phase injection method described in [98] when \( \bar{V}_i \) is injected in phase with the load side voltage \( \bar{V}_L \), i.e., depicting the condition of \( \beta = \delta \) in Figure 7-5 (a). By similar observation, the most severe PCC voltage swell that the SC can compensate for corresponds to the voltage magnitude \( V_p = |\bar{O}F| \), when the \( V_i \) limit circle intercepts \( L3 \). The magnitude of \( \bar{V}_p \) corresponding to this most severe swell is

\[
|\bar{O}F| = \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m} \cos \eta}, \quad \text{where} \quad \eta = \arccos \left( \frac{V_{ip2}}{V_{im}} \right) - \theta.
\]

Again as \( V_{thv}, V_{i,m}, V_{lp1}, V_{ip2} \) and \( \theta \) are pre-specified parameters, the most severe sag/swell conditions can be readily determined.

Without doubt, there would be other system conditions over which sets of the most severe sag/swell conditions can also be similarly derived. The analysis has been carried out and the results summarized in Table 7-1. The corresponding phasor diagrams are also as indicated in Appendix I.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

Table 7-1: Maximum sag/swell load ride-through capability for given \( V_{thv}, \theta \) and \( V_{Lm} \)

<table>
<thead>
<tr>
<th>Case</th>
<th>( V_{thv} \sin \theta )</th>
<th>( V_{Lm} )</th>
<th>Max Sag Restorable</th>
<th>Max Swell Restorable</th>
<th>Phasor Diag.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( V_{thv} \sin \theta \leq \frac{V_{ip2}}{\cos \theta} )</td>
<td>( V_{thv} - V_{Lm} )</td>
<td>( V_{thv} - V_{Lm} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-1</td>
</tr>
<tr>
<td>2</td>
<td>( \leq \frac{V_{thv}}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} - V_{Lm} )</td>
<td>( \sqrt{V_{thv}^2 + V_{Lm}^2 + 2V_{thv}V_{Lm}\cos \eta} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-2</td>
</tr>
<tr>
<td>3</td>
<td>( &lt; \frac{V_{thv} \sin \theta}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} - V_{Lm} )</td>
<td>( \sqrt{V_{thv}^2 + V_{Lm}^2 + 2V_{thv}V_{Lm}\cos \eta} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-3</td>
</tr>
<tr>
<td>4</td>
<td>( \geq \frac{V_{thv} \sin \theta}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-4</td>
</tr>
<tr>
<td>5</td>
<td>( \geq \frac{V_{thv}^2 \sin ^2 \theta + V_{ip1}^2}{V_{thv}^2 \sin ^2 \theta + V_{ip1}^2} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-5</td>
</tr>
<tr>
<td>6</td>
<td>( \geq \frac{V_{ip2} / \cos \theta}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} - V_{Lm} )</td>
<td>( V_{L} + V_{Lm} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-6</td>
</tr>
<tr>
<td>7</td>
<td>( \geq \frac{V_{ip2} / \cos \theta}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} - V_{Lm} )</td>
<td>( V_{L} + V_{Lm} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-7</td>
</tr>
<tr>
<td>8</td>
<td>( \geq \frac{V_{ip2} / \cos \theta}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-8</td>
</tr>
<tr>
<td>9</td>
<td>( \geq \frac{V_{ip2} / \cos \theta}{V_{ip2} / \cos \theta} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} \cos \theta - V_{ip1} )</td>
<td>( V_{thv} + V_{Lm} )</td>
<td>Fig. I-9</td>
</tr>
</tbody>
</table>

Note: \( e = \arccos(V_{ip1}/V_{Lm}) - \theta \) and \( \eta = \arccos(V_{ip2}/V_{Lm}) - \theta \).

7.4.3 Energy-Saving Compensation Strategy

Having determined the maximum voltage sag/swell this PQCC-SC scheme can provide the HQ load ride-through, attention is now directed toward achieving the so-called the energy-saving compensation strategy for this scheme. Although there are several possible strategies in achieving voltage restoration through series compensation [98, 99], it is worthwhile considering the energy-saving possibility because of the presence of \( P_{inj,max} \) and \( P_{abs,max} \) described previously. If it is possible to reduce the SC injection/absorption power, it will mean increased ride-through range for the HQ load. Furthermore, reduced variations on the SC power leads to lesser changes on the DC-link voltage \( V_{dc} \) and therefore minimizes the impact on the DG.

The energy-saving voltage injection strategy considered here is modified from that described in [155]. Unlike [155], the technique calls for the introduction of \( V_i \) to effect a
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

phase angle adjustment in $\bar{V}_L$ so as to maximize the load ride-through capability offered by the SC. The phasor diagram describing the proposed scheme is that shown in Figure 7-5 (b) which shows the load-side compensated voltage $\bar{V}_L$ having a phase shift $\beta$ with respect to its pre-fault phasor $\bar{V}_{p0}$. By adjusting $\beta$ through the control of the DC/AC converter, the injected active power through the SC could be controlled as described by (7.4.12). In this way, maximum ride-through potential for the PQCC-SC scheme can be realized.

In general, the combination of the system parameters $S_{H0}$, $V_{thv}$, $\theta$, $V_{Im}$, $P_{inj,max}$, $P_{abs,max}$ and $\bar{V}_p$ determines the minimum energy injection operating point for the SC. Referring to the system condition depicted in Figure 7-7, suppose an upstream disturbance has resulted in a voltage sag such that the magnitude of $\bar{V}_p$ is $|OA_2|$. The injection voltage must be such that $\bar{V}_L = \bar{V}_p + \bar{V}_i$. Assuming the magnitude of $\bar{V}_p$ is such that it remains constant during the voltage restoration stage, it is now proposed that $\bar{V}_i$ is adjusted through the SC such that the $\bar{V}_i$ phasor would start from a point on the arc $A_2A_1$ and terminates at D. In adjusting $\bar{V}_i$ and as $\gamma$ increases, the phase of $\bar{V}_p$ (i.e. $\zeta$) decreases and the corresponding SC injected active power $P_i$ also decreases because the component of $\bar{V}_i$ in phase with $I_L$ reduces. When $\bar{V}_i$ has been shifted to the precise position $A_4D$, this corresponds to the condition of minimum energy injection and the SC voltage injection limit $V_{Im}$ is also reached. Indeed, on closer examination of Figure 7-7, one notes that for any PCC voltage $\bar{V}_p$ such that $OA < |\bar{V}_p| < OB$, or from the geometry of the phasor diagram, it means that

$$V_{thv} - V_{Im} \leq |\bar{V}_p| \leq V_{thv} \cos \theta - \sqrt{V_{Im}^2 - V_{thv}^2 \sin^2 \theta}$$

$\bar{V}_i$ should be adjusted to start from the point on which the arc AB and the circle of radius $V_p$ intercepts, and terminates at D. This will ensure minimum power injection. From Figure 7-7, the minimum power injection voltage phasor is readily determined where

$$|\bar{V}_i| = V_{Im} \quad (7.4.15a)$$
and its phase angle is
\[
\gamma = \theta + \arccos\left[\frac{V_{ihv}^2 + V_{i,m}^2 - V_p^2}{2V_{ihv}V_{i,m}}\right] \tag{7.4.15b}
\]

The injection voltage can be realized through control of the SC converter duty ratio and phase angle, as the quantities on the RHS of (7.4.15a) and (7.4.15b) are known or can be measured online.

Figure 7-7 Phasor diagram illustrating energy-saving compensation method for operating condition \( V_{ihv} \sin \theta < V_{i,m} \leq V_{ip1}/\cos \theta \) and \( \vec{V}_i \) originating from the AB sector

Furthermore and through similar reasoning as before, if a voltage sag event has resulted in
\[
|\overrightarrow{OB}| \leq |\overrightarrow{V}_p| \leq |\overrightarrow{OC}|
\]
i.e. from Figure 7-8 (a), when \( V_p \) is such that
\[
V_{ihv} \cos \theta - \sqrt{V_{i,m}^2 - V_{ihv}^2 \sin^2 \theta} \leq V_p \leq V_{ihv} \cos \theta
\]
\( \vec{V}_i \) phasor should be adjusted to start from the line segment BC and the phasor terminates at D to realize the minimum power injection, as shown in Figure 7-8 (a). The magnitude of the injection voltage is
\[
|\overrightarrow{V}_i| = \sqrt{V_{ihv}^2 + V_p^2 - 2V_{ihv}V_p \cos \theta}
\]
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

and

\[ \gamma = \arcsin \left( \frac{V_{thv} \sin \theta}{\sqrt{V_{thv}^2 + V_p^2 - 2V_{thv}V_p \cos \theta}} \right). \]

Figure 7-8 Phasor diagram illustrating energy-saving compensation method for operating condition \( V_{ihv} \sin \theta < V_{i,m} < V_p \cos \theta \) and \( \vec{V}_i \) originating from the: (a) BC sector; (b) CD sector; (c) DE sector and (d) EF sector

One can continue through similar analysis and arrives at the following sag/swell cases:

- When \( |OC| \leq |OP| \leq |OE| \), i.e., \( V_{ihv} \cos \theta \leq V_p \leq \sqrt{V_{ihv}^2 + V_{i,m}^2 + 2V_{ihv}V_{i,m} \sin \theta} \), zero power injection can be realized. The corresponding phasor diagram for this injection
strategy is as shown in Figure 7-8 (b) where $|\vec{V}_i| = V_{thv} \sin \theta - \sqrt{V_p^2 - V_{thv}^2 \cos^2 \theta}$ and $\gamma = 90^\circ$ if $|\vec{V}_p| \leq |\vec{V}_L|$. Conversely, Figure 7-8 (c) illustrates the case of zero power injection where $|\vec{V}_i| = -\sqrt{V_p^2 - V_{thv}^2 \cos^2 \theta} - V_{thv} \sin \theta$, $\gamma = -90^\circ$ when $|\vec{V}_p| > |\vec{V}_L|$.

- When $OE \leq |\vec{V}_p| \leq OF$, $\vec{V}_i$ is as shown in Figure 7-8 (d) where it begins from the arc EF to end at D, in order to achieve the minimum power absorption. In this instance, $|\vec{V}_i| = V_{i,m}$ and $\gamma = -\arccos \left( \frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}} \right) + \theta$.

The above description provides the theoretical basis for the determination of the injection strategy under the minimum energy compensation scenario for the specific case of $V_{thv} \sin \theta < V_{i,m} \leq V_{ip1} / \cos \theta$. Again one can consider in similar manner other system conditions described in Table 7-1 and the results are summarized in Table I-1 in Appendix I. The corresponding phasor diagrams are also as indicated in Appendix I.

7.4.4 An Implementation Scheme

The previous section has described the derivation for the injection voltages to achieve minimum active power injection/absorption under various voltage sag/swell conditions, subject to the injection voltage limit and instantaneous power change constraint impose by the DG. A possible implementation scheme will now be described. Since $\vec{V}_p$, $S_{HQ}$, $\theta$ and $P_{dg}$ can be readily measured and tracked on-line, it is a relatively simple matter to determine $\vec{V}_i$ using the corresponding expressions of $V_i$ and $\gamma$ shown on Table I-1 for the respective system conditions, and introduce the injection voltage dynamically. The procedure can be described as follows:

Step 1: Through on-line tracking of the power system operating conditions, $\Delta P_{dg,max}$ can be determined based on the DG capability.

Step 2: From the specified $V_{thv}$, $S_{HQ}$ and the measured $\theta$, $P_{dg}$ and $P_{d}$, calculate $I_L = S_{HQ} / |V_{thv}|$. Also evaluate $P_{inj,max}$, $P_{abs,max}$, $V_{ip1}$ and $V_{ip2}$ using (7.3.2b), (7.3.2c), (7.4.13) and (7.4.14). Determine $V_{i,m}$ using (7.4.11) for a given $S_{max}$, the apparent power rating of the SC.
Step 3: Once an upstream voltage disturbance is detected, compute the voltage injection $\bar{V}_i$ using the appropriate expressions given on Table 1-1.

Step 4: SC injects voltage $\bar{V}_i$.

Step 5: Upon the recovery of $\bar{V}_p$, the SC goes into standby operation mode.

The above procedure outlines the steps needed to implement the SC mitigation of voltage sag/swell. In terms of the practical implementation of the scheme, a digital microcontroller can be used, through which $\bar{V}_p$ estimation and the proposed strategy can be implemented. A software phase locked loop (PLL) can be applied to generate the pre-sag load voltage. Once a disturbance is detected, the PLL records the pre-disturbance voltage phasor while the controller detects the sag/swell voltage and phase shift. From the results of previous section, the required $\bar{V}_i$ is evaluated. $\bar{V}_i$ forms the reference voltage signal which is sent to the pulse width modulator of the microcontroller. The modulator will then direct the gate signals to the switching devices in the converter, according to the modulated signal to synthesize $\bar{V}_i$ [162].

It is worth to note that for a typical phase-locked loop (PLL), the jitter performance and the transient behavior mainly depend on the dynamic parameters, i.e., damping factor and natural frequency [163]. Nowadays, with technological advancement, faster acquisition by a PLL is achieved. In reference [163], using a time-constant calibration circuit and the variable capacitance multiplier (VCM), the measured locked time within 2% frequency error is 2.5 $\mu$s. While typical IGBT switch frequencies are around 3 kHz to 4 kHz [164], these correspond to a much smaller frequency range than the PLL acquisition frequency. Thus in the implementation of the above scheme, it is deemed that by ignoring the internal dynamics of PLLs, it will not introduce significantly large error.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

7.5 Numerical Illustration

Based on the power system example shown on Figure 7-4, the results of the above analysis can be demonstrated using the parameters given in Table 7-2. In the example, the upstream 50-Hz source inductance $X_L$ is obtained by assuming the upstream fault level is 20 times that of the total PQCC loads. This short-circuit ratio is typical for distribution systems. The SOFC power plant has been chosen as the form of the DG in the example. The SOFC parameters are obtained from [157]. The SOFC is rated 100 kW and has a rated DC link voltage of 330 V. Accordingly the power and voltage base values are chosen to be $S_b=100$ kVA and on the DC-bus, the voltage base is 330 V. From [165], it is readily obtained that the corresponding AC voltage base at the PCC bus is 150 V, when Inv.1 is assumed to be operating under the sinusoidal PWM scheme with the nominal modulation index of 0.74. The pre-sag SOFC output is assumed to be 0.9 p.u., of which 0.1 p.u. is diverted to the PCC. $V_{thv}$ is assumed to be 1.0 p.u..

Table 7-2: Parametric values of the PQCC and power system

(Values are shown in p.u., on 100 kVA, 150V base AC)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_L$</td>
<td>0.0156</td>
</tr>
<tr>
<td>$P_{dg0}$</td>
<td>0.9</td>
</tr>
<tr>
<td>$P_{DC}$</td>
<td>0.3</td>
</tr>
<tr>
<td>$P_{d,0}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$V_{thv}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$P_{OQ}$</td>
<td>1.6</td>
</tr>
<tr>
<td>$P_{SP}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Delta P_{dg,max}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

7.5.1 Load Ride-through Range

In illustrating how the HQ load ride-through capability will be affected by the capacity of the SC, a range of SC VA rating from 0 to 0.8 p.u. (i.e. up to 80 kVA) has been considered. Using (7.4.11) and from Table 7-1, Figure 7-9(a) shows the range of the sag/swell ride-through capability under this PQCC scheme for varied SC VA rating ($S_{max}$) and HQ load power factor ($cos\theta$). $S_{HQ}$ is assumed constant at 0.8 p.u. in all cases. The lower half of the curve shown in Figure 7-9(a) indicate that the HQ load voltage sag ride-through capability of this scheme would increase almost linearly with the VA rating of the
SC ($S_{\text{max}}$) until a certain minimum value is reached, beyond which the load ride-through capability will remain constant no matter how large $S_{\text{max}}$ is. The reason for this can be seen using (7.4.11) and the results in Table 7-1. When $S_{\text{max}}$ increases, (7.4.11) shows that $V_{i,m}$ will increase. For this particular example, $V_{ihv} \sin \theta < V_{ip1}/\cos \theta$ which corresponds to Cases 1-5 of Table 7-1. When $V_{i,m} \leq V_{ip1}/\cos \theta$, i.e. from Case 1 of the Table, the most severe sag restorable is when $V_p = V_{ihv} - V_{i,m}$. However, when $V_{i,m}$ is increased to within the range $V_{ip1}/\cos \theta \leq V_{i,m} < \sqrt{V_{ihv}^2 \sin^2 \theta + V_{ip1}^2}$, i.e. corresponding to Case 4 in Table 7-1, the most severe restorable sag is $V_p = V_{ihv} \cos \theta - V_{ip1}$. It shows that the most severe restorable sag is independent of $V_{i,m}$, i.e., it is independent of $S_{\text{max}}$. Similar analysis can also be made for those cases when $V_{ihv} \sin \theta \geq V_{ip1}/\cos \theta$ and for voltage-swell situation. The conclusion would be in accordance to the results shown on Figure 7-9(a).

The figure also indicates that when $S_{\text{max}}$ is relatively small compared to the SOFC capacity, the ride-through capability is not affected by the HQ load power factor. The phenomenon can again be inferred from (7.4.11) and Table 7-1. When $S_{\text{max}}$ is small, and when $V_{i,m}$ is lower than $V_{ip2}/\cos \theta$, from Case 1 of Table 7-1, the most severe restorable swell is $V_p = V_{ihv} + V_{i,m}$; it is independent of HQ load power factor ($\cos \theta$). Case 3 of Table 7-1 also shows that when $V_{i,m} < V_{ip1}/\cos \theta$, the most severe restorable sag is $V_p = V_{ihv} - V_{i,m}$, again it is independent of the HQ load power factor.

Furthermore, Figure 7-9(a) shows that as the load power factor $\cos \theta$ becomes closer to unity, the ride-through capability afforded by the scheme reduces. The reason is that when $V_{i,m}$ increases to be greater than $V_{ip1}/\cos \theta$ i.e. Cases 4 and 5 of Table 7-1, the most severe restorable sag is $V_p = \sqrt{V_{ihv}^2 + V_{i,m}^2 - 2V_{ihv}V_{i,m} \cos \epsilon}$ when $V_{i,m} < \sqrt{V_{ihv}^2 \sin^2 \theta + V_{ip1}^2}$ and $V_p = V_{ihv} \cos \theta - V_{ip1}$ when $V_{i,m} \geq \sqrt{V_{ihv}^2 \sin^2 \theta + V_{ip1}^2}$. Both of these two cases indicate that the
larger the value of $\cos \theta$, the larger the value of $V_p$ will be, i.e., the HQ load ride-through capability will decrease as $\cos \theta$ increases. Similar analysis can be made for voltage-swell situation and for those cases when $V_{thv} \sin \theta \geq V_{ip1}/\cos \theta$.

The corresponding HQ load ride-through range for different SC capacity but at varied HQ load power is as shown in Figure 7-9 (b). Again similar observation as that shown in Figure 7-9 (a) can be made. Figure 7-9(b) shows that as $S_{HQ}$ increases, the ride-through capability reduces. The reason for this trend can again be derived using (7.4.11) and the results of Table 7-1 and would not be elaborated further.

Based on the above results, it would appear that a reasonable range for HQ load voltage sag/swell ride-through can be achieved using a modestly sized SC. Specifically, Figure 7-9 appears to suggest that $S_{max}$ of approximately 0.3 p.u. would suffice for most of the voltage sags/swells encountered in practice and for the HQ load power factor $\cos \theta > 0.9$ lagging.
An additional factor which has not been considered in the above example is with regard to the appearance of the phase shift $\alpha$ in $V_s$ following an upstream disturbance. $\alpha$ has been indicated in Figure 7-5 but it is an independent and uncontrollable parameter. Hence its effect on the design of the PQCC has to be assessed. Based on the results of Figure 7-9 (a) and (b), suppose one selects $S_{\text{max}}=0.31\text{pu}$ and for the HQ load power factor $\cos \theta =0.95$ and $S_{\text{HQ}} = 0.8 \text{ pu}$. Curve (a) in Figure 7-10 shows the most severe voltage sag ride-through for varied $\alpha$ when the proposed minimum energy strategy has been applied. Curve (b) was obtained when the injection strategy without the phase shift (i.e., $\beta =0$), such as that described in [154], had been adopted. In the latter case, the HQ load ride-through capability would deteriorate as $|\alpha|$ increases. Unlike the strategy proposed in [154] however, curve (a) shows that the HQ load ride-through is un-affected by $\alpha$. This phenomenon can again be inferred from Cases 1-3, Table 7-1, which indicate that the voltage ride-through capability of the HQ load is independent of the phase shift $\alpha$. Hence, the proposed method enjoys a certain degree of robustness in terms of its effectiveness in overcoming voltage sags/swells for it is not affected by the change in $\alpha$. From the figure, it is also clear that the energy-saving injecting strategy allows the HQ load to ride through more severe voltage sag for given $S_{\text{max}}$ as compared to that afforded by the technique described in [155].

Figure 7-10  Maximum voltage sag ride-through capability of HQ load for $S_{\text{max}}=0.31\text{pu}$, load power factor of 0.95 (lag), $S_{\text{HQ}} = 0.8 \text{ pu}$: (a) energy-saving voltage restoration strategy, (b) voltage restoration strategy of [155]
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

7.5.2 Time-Domain Simulation

Using Matlab/Simulink, studies have also been carried out to verify the performance of the proposed compensation strategy. In simulating the restoration process of the SC, the injection strategy based on that described in Section 7.4 was used. The SC and Inv. 1 models described in [154] are adopted. The SC consists of a DC/AC converter, an injection transformer and a DC-link. Inv. 1 and Inv. 2 are three-phase SPWM converters. Each converter is represented by six on-off switches, with each switch having a finite but small resistance. The switches are composed of an IGBT with a freewheeling diode. The SP, DC, HQ and OQ loads are also assumed to be linear and will not introduce harmonics into the power system. The system parameters used are identical to those shown in Table 7-2. It is assumed that $S_{HQ} = 0.8$ pu, load power factor $\cos \theta = 0.95$ lagging, $S_{max} = 0.31$ pu.

From Figure 7-6 and previous analysis, it is then established that when $S_f = 0.31$ pu, $|OA| = 0.61$ pu, $|OB| = 0.72$ pu, $|OC| = 0.95$ pu, $|OD| = 1.0$ pu, $|OE| = 1.18$ pu and $|OF| = 1.27$ pu. Therefore, the most severe sag this scheme can compensate for is 0.61 pu and at this condition, the corresponding active and reactive power from the SC are $P_f = 0.295$ pu, $Q_f = 0.097$ pu, respectively. Simulation results under the sag condition of $V_p = 0.61$ p.u. are shown in Figure 7-11. Figure 7-11 (b) shows that indeed the HQ terminal voltage has been restored essentially to the required level. Inv. 1 exports $P_r$ of 0.1 p.u. from the SOFC to the PCC prior to the disturbance. However, during the sag, Inv. 1 attempts to import active power from the upstream system. Indeed the amount imported is 0.095 p.u. during the sag, as is shown in Figure 7-11 (c). Therefore the active power contribution from the SOFC is increased to $P_d = 0.2$ p.u. through reducing the DC link voltage $V_{dc}$ to 0.988 p.u. (i.e. 326V), as can be seen from Figure 7-11 (f). The active power absorbed by Inv. 1 and the surplus active power from the SOFC are then transferred to the SC, as intended by the design. It has resulted in the injected active power from the SC as $P_i = P_r + P_d = 0.295$ p.u, as predicted. The power factor at A of the upstream line has also been maintained at unity through Inv. 1 actions, as indicated by $Q_i = 0$ in Figure 7-11 (d). Also in Figure 7-11 (f), the VA loading on the SC is seen to be at its maximum value of 0.31 p.u. during the sag, again as intended by the design.
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

Figure 7-11. Simulation results for an upstream voltage sag of $V_p = 0.61\text{pu}$: (a) PCC voltage (phase a); (b) HQ load voltage; (c) Active and reactive power supplied by Inv.1; (d) Active and reactive power supplied by the upstream system; (e) Active and reactive power injected by the series compensator; (f) Apparent power of SC and DC link voltage
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

The next case to be shown illustrates the case of zero injection power during a voltage sag. From the analysis in Section 7.4.3, it is predicted that when \( V_p = 0.97 \) pu, \( \bar{V}_i \) should lie on the sector CD if one uses the proposed energy-saving strategy. Using the results of the analysis, it can be evaluated that the corresponding active and reactive power from the SC are \( P_i = 0.0 \) pu, \( Q_i = 0.093 \) pu, with the SC VA loading of 0.093 pu. Simulation results under this sag condition are shown in Figure 7-12. The results are consistent with that obtained from the analysis, i.e. Figure 7-12(a) shows \( P_i \approx 0 \) p.u., \( Q_i = 0.093 \) p.u.. As intended, the DC link voltage is kept constant, as can be seen from Figure 7-12 (b). This shows that the zero-power injection is obtained using the proposed energy-saving compensation strategy.

Previous analysis also indicates that zero injection power is also possible when a voltage swell occurs, i.e. \( 1 < V_p < 1.18 \) p.u. for the system condition considered. Suppose \( V_p = 1.1 \) p.u. and according to the proposed strategy, \( \bar{V}_i \) should lie along the sector DE, i.e. Figure 7-8 (c). Whence it can be determined that the corresponding active and reactive power from SC are \( P_i = 0.0 \) pu and \( Q_i = -0.19 \) pu. Simulation results under this swell condition are shown in Figure 7-13 which shows that the results are again in line with the analysis: \( P_i = 0 \) p.u. throughout the study interval and during the swell, the SC attempts to absorb reactive power from the upstream system. Therefore \( Q_i \) becomes negative as shown in Figure 7-13 (a). The reactive power from series compensator \( Q_i \) is met through control of the series compensator converter. Also there is no exchange of real power through the SC. Hence, the SOFC does not change its output power, and the DC link voltage is kept
constant (Figure 7-13 (b)). This is desirable because it will not disturb the supply to the SP and DC load. Inv.1 and the SC control systems have therefore ensured the operations of the SOFC have not been disturbed. This shows that the zero-power injection has been realized through the proposed energy-saving compensation strategy.

![Figure 7-13 Simulation results for $V_p = 1.1$ pu: (a) Active and reactive power injected by the series compensator; (b) DC link voltage](image)

### 7.6 Conclusions

A new series compensation method has been proposed as part of a Power Quality Control Center (PQCC) scheme. In stead of at the transmission level, the scheme involves the application of both series-shunt compensation and power converters at distribution level so that supply quality can be enhanced within the PQCC, the SC is fed from an active source which has an output power change constraint. It is seen that for given apparent power rating of the SC the maximum load ride-through capability offered by the SC is improved by introducing the injection voltage to effect a phase angle adjustment on the load-side voltage. Furthermore, the proposed energy-saving voltage injection strategy requires minimum injection/absorption energy to implement the voltage restoration. The variation on the DC-link voltage in the PQCC is also reduced during the restoration process. Analytical expressions for the injected voltage phasor, in terms of given voltage sags/swells and load power factor and SC capacity, have also been derived. The validity of the analysis has been demonstrated by simulation results. Through analysis and simulation, the proposed scheme is shown to be superior compare to that of existing
Chapter 7: Series-Shunt Compensation Technique for Power Quality Enhancement

methods, in terms of its higher load ride-through capability and lower energy requirements. From the simulation results, it is seen that a reasonable range of HQ load voltage sag/swell ride-through can be achieved for a modestly-sized SC. This indicates that the proposed technique can be used to advantage for power quality enhancement. It therefore expands considerably the usefulness of power electronics-based compensators for power system applications.
Chapter 8: Conclusions and Recommendations

Chapter 8

Conclusions and Recommendations

This chapter summarizes the main findings of the research work elaborated in the thesis. In addition, some possible future research directions are also presented.

8.1 Conclusions

This thesis focuses on the application of power electronics technology and control techniques for the purpose of providing flexible control of transmission and distribution networks. The power electronics-based FACTS Controllers offer potentials to improve power transfer capability of transmission systems. Depending on the FACTS Controller configurations, the shunt compensator and the series compensator can provide bus voltage support, or power flow regulation, or both on the transmission system. Likewise, the power electronics-based Power Quality Control Center (PQCC) offers potentials to mitigate the undesirable impacts of disturbances on sensitive loads and provide unbundled power quality services to customers. Equipped with various Custom Power devices, distributed generators (DG), computers and communication channels, the PQCC is designed to meet the demand of loads for varied quality levels, through flexible changes of network configuration.

The thesis begins with the description of an investigation into the use of a power electronics-based shunt compensator, the SVC, together with other conventional voltage controllers in regulating voltages over long-distance radial transmission line systems. A coordinated hierarchical control scheme for the transmission system has been proposed in Chapter 3. Using the well-known frequency response technique, a tuning method of the control parameters of the primary and the secondary controllers has been developed. A significant finding of the subsequent analysis of the power system shows that the controllers should be designed based on the most onerous condition, which is the
maximum power transfer level. A practical power system has been used as an illustrative example of the proposed method. The simulation results show clearly the validity of the design technique. Using the proposed coordinated hierarchical control scheme, the voltages of the transmission system can be quickly restored to their nominal values following contingencies. The reactive power resources are continuously being well-managed and equitably shared among the reactive power sources to achieve improved network security.

Extension of the research work of Chapter 3 is to re-examine the potential of the shunt compensation to increase power transfer level and steady-state stability. In Chapter 4, shunt compensation of a long-line at terminals and at mid-point has been considered. Generalized expressions based on exact representation of the transmission line have been obtained to predict the maximum power transfer ($P_{\text{max}}$). A computational procedure has been proposed to determine the reactive power needed at the line intermediate point at any given reasonable power transfer level. The amount of the reactive compensation needed for a given power transfer can be determined much more accurately using the exact long-line model, compared to that based on simplified line model. Furthermore, the contribution of the midpoint shunt compensator to enhance stability has been re-evaluated and the analysis indicates that typically, the improvement is only by a factor of 1.67 instead of the well-known value of 2. The decrement is accounted for due to the presence of line resistances and the distributed nature of the line shunt capacitances. If the terminal impedances at the end(s) of the line are also included, there is a further reduction in the factor to (typically) around 1.59.

The power transfer limit varies when the location of shunt compensator changes. The observation that the mid-point of the line is the optimal compensation location to achieve maximum receiving-end (RE) power has been shown in previous studies but based on the simplified line model. Whereas by including the line losses, the analysis in Chapter 4 has shown that the shunt compensator needs to be placed slightly off-centre to get the maximum power transfer level at the RE. Expressions used to determine the optimal location of the shunt compensator to achieve the maximum RE power have been derived systematically in the chapter. In addition, extension of the one shunt-compensator line to include the case of multiple-compensators line has also been considered. A computational
procedure to determine the optimal compensator locations has been included. Simulation results show that by adopting the optimal shunt compensator locations, increased power transfer of the transmission system can be realized.

From the results obtained in Chapters 3 and 4, it is found that the transfer ability of the shunt-compensated system would be limited. For example, for a single compensator case, the limit is 1.6 times of the power transfer level of uncompensated case. Such a limit could place a constraint on the economic operation of transmission systems. In order to alleviate this constraint, Chapter 5 considers the inclusion of a Series Compensator into the compensation structure. It leads to the more general Unified Power-Flow Controller (UPFC). In order to further improve and expand the applications of the UPFC, an energy storage system has been incorporated into the DC-link. By harnessing energy from a renewable source, such as a wind turbine, a battery bank is used as the medium for energy-storage. The battery system forms an active power (AP) source in this new UPFC scheme. An optimization problem has been formulated to determine the capacity of the UPFC-AP. The design objective could include the minimization of the AP capacity or the power transfer angle, or the maximization of the power transfer level. The design procedure also takes into account practical operating constraints of the network and that of the UPFC-AP. The concept of feasible operating regime (FOR) has also been introduced in the design process. FOR shows in a graphical way the range of the AP output power \( P_d \) that can be dispatched for given load demand \( P_r \), without violating any of the operating constraints. Furthermore, by using the FOR and the result of an economic dispatch strategy applied to the total generation cost of the generators, a simple relationship between the output power of the AP and the load demand has been established. An optimal load tracking strategy has been proposed to realize economic dispatch of power from the upstream source and the AP while keeping the power angle across the transmission system to the minimum.

In Chapter 6, lead-acid battery has been considered as a suitable form to be included in the UPFC-AP. An operational strategy has been proposed pertaining to the change-over from the in-service BESS to the stand-by BESS. Furthermore, a method to determine the battery bank change-over schedule for a given load demand has been derived. In this way, one can arrive at the suitable BESS capacity so that the overall power system can be
operated under the economic dispatch condition. By varying the capacity of the BESS to suit the available energy profile of the renewable, one also ensures that the UPFC-AP system will guarantee the transfer capability of the compensated system is maximized.

While previous chapters have explored techniques to enhance transmission system power transfer capability, it is important to ensure that the delivered power is of acceptable quality. Power quality has become an increasingly important consideration due to the wide-spread use of computerized and power electronic equipment. Thus in Chapter 7, the use of Power Quality Control Center (PQCC) to improve system voltage sag/swell ride-through for sensitive loads has been examined. A new series compensation method has been proposed as part of a Power Quality Control Center scheme. The series compensator (SC) is fed from an active power source whose allowable instantaneous output power change is constrained. For given SC rating, the maximum load ride-through capability offered by the SC has been derived through a detailed analysis. The technique used is through the injection of a series voltage to effect a phase angle adjustment on the load-side voltage. Minimum injection/absorption energy is required using the proposed energy-saving voltage injection strategy, and the variation on the DC-link voltage in the PQCC is reduced during the restoration process. Analytical expressions for the injected voltage phasor, in terms of given voltage sags/swells and load power factor and SC capacity, have also been derived. Furthermore through analysis and simulation, the proposed scheme is shown to be superior, compared to that of existing methods, in terms of its higher load ride-through capability and lower energy requirements. The simulation results also show that a reasonable range of HQ load voltage sag/swell ride-through can be achieved for a modestly-sized SC, which suggests that the proposed technique can be used to advantage for power quality enhancement.

8.2 Recommendations

In view of the progress described in the previous Chapters, the following directions are suggested as possible areas for further investigation.
Chapter 8: Conclusions and Recommendations

In the work of Chapter 3, as small-signal analysis method has been used to linearize the system, the controller would only be effective in the vicinity of the steady-state operating point. For example, in the event of a more severe fault than that considered, the performance of the control system could deteriorate. It is necessary to consider the nonlinearity of power systems in the controller design. In recent years, increasing interest has been shown in using nonlinear control theory in power systems [166-171]. Instead of using approximate linear model, nonlinear models are used and nonlinear feedback linearization (NFL) techniques [166] applied on the power system models, thereby alleviating the operating point dependent nature of the linear designs. Among the techniques, direct feedback linearization is quite an appealing design method for nonlinear systems. Essentially, the feedback linearization uses state feedback to cancel nonlinearities in the model, thus allowing the application of a linear control design to a nonlinear model, without coordinate transformation [168]. Using the DFL technique and by incorporating the robust control theory, references [169, 170, 171] have developed integrated nonlinear voltage controllers to achieve simultaneous voltage regulation and system stability enhancement. The application of nonlinear control techniques could be a fruitful area for further work under the hierarchical voltage control scheme.

Furthermore, the proposed control scheme is built on the basis of a radial transmission system model. Unfortunately, most power systems do contain meshed networks. In order to develop a theoretically sound technique for power system planning and operations, it is necessary to study coordinated hierarchical control for more general meshed electrical networks.

A general multi-machine environment is shown in Figure 8-1 (a). Generators 1 to n in Figure 8-1 (a) are not necessarily at the same power station. However, using a series of star-delta transformation, it should be possible to transform the mesh or loop circuit into its "star" equivalent and finally into the form shown in Figure 8-1(a). Hence Figure 8-1 (a) is completely general and will be in a form suitable for the design of controllers.
Figure 8-1 (a) Idealized multi-machine-infinite bus system, (b) the $i$-th single-machine-equivalent generator system.

Using Thevenin's theorem, the $i$-th generator in Figure 8-1 (a) is transformed to that of Figure 8-1 (b), the $i$-th single-machine-equivalent generator system. Evaluation of the parameters of the $i$-th equivalent machine $G_{ei}$ yields the following:

$$V_{ei} < \delta_{ei} = \bar{Z}_{ei} \left( \sum_{j=1}^{n} \frac{E_{ej}/\delta_{ej}}{jX_{dj} + jX_{ej} + \bar{Z}_{ej}} \right) + \frac{\bar{Z}_{ei}}{Z} V < 0$$

(8.2.1)

where $\bar{Z}_{ei} = \left( \sqrt{1 + \sum_{j \neq i}^{n} \left[ \frac{1}{\bar{Z} + jX_{ij} + jX_{ej} + \bar{Z}_{ej}} \right]^{2}} \right)$. 

In this way, thus the method proposed in Chapter 3 can be applied for this multi-machine power system controller design.
For the scheduling of the battery problem described in Chapter 6, assumption has been made that the standby battery energy storage system will be fully charged before the battery bank change-over. Clearly whether the standby BESS would be ready depends on the availability of the renewable energy. For example, if the source of the energy is from the wind, the availability of the wind energy is highly time-variable. Possible work may be related to take into consideration the availability of the renewable energy. Even assuming the renewable energy is sufficient, there is also the need to determine the capacity and the number of the stand-by battery needed to support $P_r$. If one standby battery is not enough, one may choose two or more standby battery.

Also in Chapter 6, battery banks are used as active power source for the proposed UPFC-AP scheme. This arrangement may require a particularly large number of cells connected in series. One solution could be to use a DC-DC converter system so that a much lower voltage rating battery bank can be used to meet the DC-link voltage requirement by boosting dc voltage using the converter. In this way, the number of series cell can be reduced considerably.

It is noted that the research work described in Chapter 7 is mainly focused on examining the PQCC operation under upstream system voltage sag/swell condition. Possible strategies have been proposed to improve the sensitive load voltage sag/swell ride-through capability. However, for the purpose of simplification, the analysis given in the Chapter 7 is based on the assumption that the upstream voltage sag/swell is balanced. In practice, most faults in power systems involve only single phase or two phases [66]. The unbalance upstream voltage results in the appearance of even harmonics at the DC link voltage and odd harmonics in the input current, which could interfere with Inv.1 control and the DG operation. It is therefore necessary to re-examine the PQCC operations under such unbalanced sag/swell conditions. It is envisaged that symmetrical components analysis can be used to design Inv.1 control scheme under the unbalanced sag/swell conditions.

Another possible area for future work is pertaining to the internal dynamics of the SOFC. They include the dynamics of the fuel processor and the electrochemical process. Typically, the processes require several tens of seconds for the SOFC to achieve a significant output power change. Such a slow response of SOFC may affect the operation
of the PQCC system if the sag/swell persists. Inclusion of such dynamics is necessary to provide a better estimate of the compensation ability of the PQCC.

Under such severe sag/swell condition, it may be necessary to include a buffer energy storage device in the PQCC. The storage device could be a battery or a supercapacitor, intended to complement for the slow dynamics of the SOFC. If the output power change cannot be satisfied by the SOFC instantaneously, the energy storage device will supply the balanced active power until the internal dynamics of SOFC has adjusted sufficiently to meet the new power demand. The combination of the SOFC and energy storage device to realize unbundled power quality service needs further investigation.