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## Appendices

### Appendix A

### List of Symbols

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\bar{z}$</td>
<td>Series impedance per unit length/phase of the line</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>Shunt admittance per unit length/phase of the line</td>
</tr>
<tr>
<td>$\bar{Z_c}$</td>
<td>Characteristic impedance of the line</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>Line propagation constant</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Line surge impedance</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Natural load or SIL</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Rated voltage of the line</td>
</tr>
<tr>
<td>$\bar{V}_s$</td>
<td>Sending-end voltage</td>
</tr>
<tr>
<td>$\bar{V}_r$</td>
<td>Receiving-end voltage</td>
</tr>
<tr>
<td>$\bar{I}_s$</td>
<td>Current at the sending-end</td>
</tr>
<tr>
<td>$\bar{I}_r$</td>
<td>Current at the receiving-end</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Phase angle difference between RE and SE generator rotor angle</td>
</tr>
<tr>
<td>$\bar{S}_s$</td>
<td>Apparent powers at the SE</td>
</tr>
<tr>
<td>$\bar{S}_r$</td>
<td>Apparent powers at the RE</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Active powers at the SE</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Active powers at the RE</td>
</tr>
<tr>
<td>$Q_s$</td>
<td>Reactive powers at the SE</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>Reactive powers at the RE</td>
</tr>
<tr>
<td>$R$</td>
<td>Series resistance per unit length/phase of the line</td>
</tr>
<tr>
<td>$L$</td>
<td>Series inductance per unit length/phase of the line</td>
</tr>
<tr>
<td>$G$</td>
<td>Shunt conductance per unit length/phase of the line</td>
</tr>
<tr>
<td>$C$</td>
<td>Shunt capacitance per unit length/phase of the line</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Nominal system frequency</td>
</tr>
<tr>
<td>$E_q$</td>
<td>Generator internal voltage</td>
</tr>
<tr>
<td>$E'_q$</td>
<td>Generator transient voltage</td>
</tr>
<tr>
<td>$E_{fd}$</td>
<td>Generator field voltage</td>
</tr>
<tr>
<td>$X_d$</td>
<td>Generator reactance</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Inertia constant</td>
</tr>
<tr>
<td>$T_{d0}$</td>
<td>Field open circuit time constant</td>
</tr>
<tr>
<td>$X'_d$</td>
<td>Generator transient direct axis reactance</td>
</tr>
<tr>
<td>$X'_q$</td>
<td>Generator transient quadrature axis reactance</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Armature voltage, direct axis component</td>
</tr>
<tr>
<td>$V_q$</td>
<td>Armature voltage, quadrature axis component</td>
</tr>
<tr>
<td>$I_d$</td>
<td>Armature current, direct axis component</td>
</tr>
<tr>
<td>$I_q$</td>
<td>Armature current, quadrature axis component</td>
</tr>
<tr>
<td>$X_t$</td>
<td>Transformer equivalent reactance</td>
</tr>
<tr>
<td>$X$</td>
<td>Transmission line reactance</td>
</tr>
<tr>
<td>$B$</td>
<td>Transmission line susceptance</td>
</tr>
<tr>
<td>$B_{SVC}$</td>
<td>Susceptance of SVC</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Load resistance</td>
</tr>
<tr>
<td>$X_L$</td>
<td>Load reactance</td>
</tr>
<tr>
<td>$X_{se}$</td>
<td>Series transformer reactance</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Total internal resistance of a fully charged battery</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Open circuit voltage of a battery cell when fully charged</td>
</tr>
<tr>
<td>$k, k_r$</td>
<td>Experimental constant</td>
</tr>
<tr>
<td>$f$</td>
<td>State of discharge of the battery</td>
</tr>
<tr>
<td>$r$</td>
<td>Equivalent internal resistor of the battery/SOFC</td>
</tr>
<tr>
<td>$E$</td>
<td>Equivalent open-circuit voltage of the battery/SOFC</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>DC link voltage of the PQCC/UPFC system</td>
</tr>
<tr>
<td>$I_b$</td>
<td>Instantaneous current of the battery</td>
</tr>
<tr>
<td>$P_d$</td>
<td>Output power of the battery</td>
</tr>
</tbody>
</table>
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\[ m_{E_1}, m_{B_2} \]
Modulation indexes of Converter 1 and Converter 2 respectively

\[ \delta_{E_1}, \delta_{B_2} \]
Phase shift of Converter 1 and Converter 2 respectively

\[ \bar{v}_{se} \]
Series injected voltage of the UPFC system

\[ V_{se} \]
rms value of series injected voltage

\[ \delta_{se} \]
Phase angle of series injected voltage

\[ \bar{V}_{sh} \]
Shunt injected voltage of the UPFC system

\[ V_{sh} \]
rms value of shunt injected voltage

\[ \delta_{sh} \]
Phase angle of shunt injected voltage

\[ \bar{I} \]
Line current through transmission line "AC" section

\[ \bar{I}_1 \]
Line current through the series inverter

\[ \bar{I}_{sh} \]
Shunt VSC current

\[ \bar{v}_{sh} \]
UPFC-AP from-bus voltage

\[ \bar{v}_{set} \]
UPFC-AP to-bus voltage

\[ S_{se} \]
Series converter transformer rating

\[ S_{sh} \]
Shunt converter transformer rating

\[ P_{sh} \]
Active power exchange of the shunt VSC and the external system

\[ P_{se} \]
Active power exchange of the series VSC and the external system

\[ P_{set} \]
Active power at the downstream output terminal of UPFC-AP

\[ Q_{set} \]
Reactive power at the downstream output terminal of UPFC-AP

\[ \alpha \]
Phase angle jump of the voltage sag

\[ \varphi \]
Phase angle of the upstream source voltage

\[ \beta \]
Phase shift between \( \bar{v}_{p0} \) and \( \bar{v}_{L} \)

\[ \gamma \]
Phase difference between \( \bar{I}_L \) and \( \bar{v}_{L} \)

\[ \delta \]
Phase angle of \( \bar{I}_S \)

\[ \cos \theta \]
Power factor of the HQ load

\[ I_{FC} \]
SOFC stack current

\[ L_f \]
Link inductor between the Inv.1 and the PCC

\[ \bar{I}_L \]
HQ load current

\[ \bar{I}_S \]
Upstream source current

\[ \bar{V}_L \]
HQ load terminal voltage

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Appendices

\[ \bar{V}_s \]
Upstream source voltage

\[ \omega \]
Nominal system frequency

\[ V_s \]
rms value of the upstream source voltage

\[ I_s \]
rms value of the upstream source current

\[ V_{sag} \]
Upstream source voltage during voltage sag

\[ V_{sag,max} \]
The amplitude of the most severe sag the system can ride-through

\[ \bar{V}_p \]
Phase vector of the PCC voltage

\[ V_i \]
Inv.1 front-end voltage

\[ V_{thv} \]
Minimum voltage level for prolonged operation of the loads

\[ \bar{V}_i \]
Injected phase voltage of the series compensator

\[ V_{lm} \]
Maximum injected voltage of the series compensator

\[ P_i, Q_i \]
Injected active and reactive power of the series compensator

\[ P_{inj,max}, P_{abs,max} \]
Maximum injected and absorbed active power of the SC

\[ S_{max} \]
Power rating of the series compensator

\[ P_{DC}, P_{SP} \]
Active power demand of the DC, SP respectively

\[ P_{HQ}, P_{OQ} \]
Active power demand of the HQ, OQ loads respectively

\[ P_{dg} \]
Output power of the DG

\[ \Delta P_{dg,max} \]
Maximum instantaneous power change of the SOFC

\[ P_{d,max}, P_{d,min} \]
Maximum and minimum power flow from the DG to Inv.1

\[ P_r, Q_r \]
Active and reactive power transferred from the Inv.1 to the PCC

\[ u \]
Fuel utilization factor of the SOFC

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Appendix B

Transfer Functions and System Coefficients in Chapter 3

The transfer functions, which are used in Chapter 3, are given below.

\[
\Delta V_p = \frac{M'_{p1,2}S^5 + M'_{p2,2}S^4 + M'_{p3,2}S^3 + M'_{p4,2}S^2 + M'_{p5,2}S + M'_{p6,2}}{M'_{q1,2}S^5 + M'_{q2,2}S^4 + M'_{q3,2}S^3 + M'_{q4,2}S^2 + M'_{q5,2}S + M'_{q6,2}}
\]

(B.1)

\[
\Delta V_{\phi} = \frac{K'_{p1,2}T_{1,2}S^5 + 2\beta'_{p1,2}K'_{p1,2}(D_{1,2} + D_{3,2}H_{1,2} + D_{4,2}H_{3,2}) - K'_{p1,2}(D_{2,2} + D_{3,2}H_{2,2} + D_{4,2}H_{4,2})}{K'_{q1,2}T_{1,2}S^5 + 2\beta'_{q1,2}K'_{q1,2}(D_{1,2} + D_{3,2}H_{1,2} + D_{4,2}H_{3,2}) - K'_{q1,2}(D_{2,2} + D_{3,2}H_{2,2} + D_{4,2}H_{4,2})}
\]

(B.2)

\[
\Delta V_{\psi} = \frac{K'_{p1,2}T_{1,2}S^5 + 2\beta'_{p1,2}K'_{p1,2}(D_{1,2} + D_{3,2}H_{1,2} + D_{4,2}H_{3,2}) - K'_{p1,2}(D_{2,2} + D_{3,2}H_{2,2} + D_{4,2}H_{4,2})}{K'_{q1,2}T_{1,2}S^5 + 2\beta'_{q1,2}K'_{q1,2}(D_{1,2} + D_{3,2}H_{1,2} + D_{4,2}H_{3,2}) - K'_{q1,2}(D_{2,2} + D_{3,2}H_{2,2} + D_{4,2}H_{4,2})}
\]

(B.3)

where,

\[
M'_{p1,1} = -K_{p1,1}M_{p1,1}T_{1,2}T_{2,2}; \quad M'_{p1,2} = -K_{p1,2}(M_{p1,2} + M_{p6,2})T_{1,2};
\]

\[
M'_{p2,1} = -K_{p2,1}M_{p2,1}T_{1,2}T_{2,2}; \quad M'_{p2,2} = -K_{p2,2}(M_{p2,2} + M_{p6,2})T_{1,2};
\]

\[
M'_{p3,1} = K_{p3,1}(M_{p2,1} + M_{p3,1}T_{1,2})T_{2,2}; \quad M'_{p3,2} = K_{p3,2}(M_{p3,2}T_{1,2} - M_{p2,1} - M_{p3,1}T_{2,2});
\]

\[
M'_{p4,1} = K_{p4,1}(M_{p1,1} + M_{p3,1} + M_{p5,1} + M_{p7,1}); \quad M'_{p4,2} = K_{p4,2}(M_{p4,2} + M_{p6,2})T_{1,2};
\]

\[
M'_{p5,1} = K_{p5,1}(M_{p1,1} + M_{p7,1})T_{2,2}; \quad M'_{p5,2} = K_{p5,2}(M_{p1,2} + M_{p7,2}T_{1,2});
\]

\[
M'_{p6,1} = K_{p6,1}(M_{p1,1} + M_{p3,1} + M_{p5,1} + M_{p7,1}); \quad M'_{p6,2} = K_{p6,2}(M_{p4,2} + M_{p6,2})T_{1,2};
\]

\[
M'_{p7,1} = M'_{p7,2} = K_{p7,1}K_{p7,2}T_{1,2}T_{2,2}; \quad M'_{p7,2} = K_{p7,2}(M_{p1,2} + M_{p3,2} + M_{p5,2} + M_{p7,2});
\]

\[
M'_{q1,1} = -M_{q6,1}T_{1,2}T_{2,2}; \quad M'_{q1,2} = -M_{q6,1}T_{1,2}T_{2,2};
\]

\[
M'_{q2,1} = -(M_{q4,1} + M_{q6,1})T_{1,2}; \quad M'_{q2,2} = -(M_{q4,2} + M_{q6,2})T_{1,2};
\]

\[
M'_{q3,1} = -(M_{q4,1} + M_{q6,1} - M_{q4,1}T_{2,2}); \quad M'_{q3,2} = -(M_{q4,2} + M_{q6,2} - M_{q4,2}T_{2,2});
\]

\[
M'_{q4,1} = M_{q4,1}T_{1,2} - M_{q2,1} - M_{q4,1} - M_{q6,1} + M_{q4,1}T_{2,2}; \quad M'_{q4,2} = M_{q4,2} - M_{q4,2}T_{1,2};
\]

\[
M'_{q5,1} = (M_{q4,1} + M_{q7,1})T_{2,2}; \quad M'_{q5,2} = K_{q5,2}(M_{q1,2} + M_{q7,2})T_{2,2};
\]

\[
M'_{q6,1} = M_{q4,1} + M_{q3,1} + M_{q4,1} + M_{q7,1}; \quad M'_{q6,2} = M_{q4,2} + M_{q6,2} + M_{q7,1}T_{1,2};
\]

\[
M'_{q7,1} = M_{q4,1} + M_{q3,1} + M_{q4,1} + M_{q7,1}; \quad M'_{q7,2} = M_{q4,2} + M_{q6,2} + M_{q7,2}T_{1,2};
\]
Appendices

\[ K_{p01} = -\frac{V^2}{V_{SCV01}} \left[ 1 + \frac{2B_{SCV01}X_{SL1}}{1 - X_{SL1}B_{SCV01}} \right]; \]

\[ K_{p02} = -\frac{V^2}{V_{SCV02}} \left[ 1 + \frac{2B_{SCV02}X_{SL2}}{1 - X_{SL2}B_{SCV02}} \right]; \]

\[ M_{p1,1} = 2\pi\theta_0K_{p1,1}N_{5,1r} - 2\pi\theta_0K_{p2,1}N_{3,1r}; \]

\[ M_{p3,1} = 2\pi\theta_0K_{p1,1}N_{6,1r} - 2\pi\theta_0K_{p2,1}N_{2,1r}; \]

\[ M_{p5,1} = -2\pi\theta_0K_{p2,1}N_{1,1r}; \]

\[ M_{p7,1} = 2\pi\theta_0K_{p1,1}N_{7,1r} - 2\pi\theta_0K_{p2,1}N_{4,1r}; \]

\[ M_{q2,1} = T_{j1}K_{p1,1}N_{8,1r}; \]

\[ M_{p4,1} = T_{j1}K_{p2,1}N_{9,1r}; \]

\[ M_{p6,1} = T_{j1}K_{p2,1}N_{10,1r}; \]

\[ M_{q3,1} = 2\pi\theta_0K_{Q1,1}N_{6,1r} - 2\pi\theta_0K_{Q1,1}N_{2,1r}; \]

\[ M_{q5,1} = -2\pi\theta_0K_{Q1,1}N_{1,1r}; \]

\[ M_{q7,1} = 2\pi\theta_0K_{Q1,1}N_{7,1r} - 2\pi\theta_0K_{Q1,1}N_{4,1r}; \]

\[ N_{1,1r} = (K_{1,1r}K_{4,2r} - K_{2,2r}K_{3,2r})(D_{3,1}R_{3,1}R_{3,1} - D_{4,1}R_{4,1}R_{4,1}); \]

\[ N_{2,1r} = K_{3,2r}D_{4,1}R_{1,2} + K_{4,2r}(D_{3,1}R_{3,1}R_{1,1} + D_{4,1}R_{4,1}R_{1,1}); \]

\[ N_{3,1r} = K_{1,2r}(D_{3,1} - D_{3,1}R_{3,1}R_{3,2} + D_{4,1}R_{4,1}R_{3,2} - D_{4,1}R_{4,2}); \]

\[ M_{p2,2} = T_{j2}K_{p2,2}N_{8,2r}; \]

\[ M_{p4,2} = T_{j2}K_{p2,2}N_{9,2r}; \]

\[ M_{p6,2} = T_{j2}K_{p2,2}N_{10,2r}; \]

\[ M_{q2,2} = T_{j2}K_{Q2,2}N_{8,2r}; \]

\[ M_{q4,2} = T_{j2}K_{Q2,2}N_{9,2r}; \]

\[ M_{q6,2} = T_{j2}K_{Q2,2}N_{10,2r}; \]

\[ M_{q7,2} = 2\pi\theta_0K_{Q2,1}N_{7,2r} - 2\pi\theta_0K_{Q2,2}N_{4,2r}; \]

\[ N_{1,2r} = (K_{1,1r}K_{4,2r} - K_{2,2r}K_{3,2r})(D_{3,2}R_{3,1}R_{3,2} + D_{4,2}R_{4,1}R_{4,2}); \]

\[ N_{2,2r} = K_{3,2r}D_{4,2}R_{1,2} + K_{4,2r}(D_{3,2}R_{3,1}R_{1,1} - D_{4,2}R_{4,1}R_{1,1}); \]
\[ N_{3,2t} = K_{1,1t}(D_{3,2} - D_{3,2}'R_3\dot{R}_{3,2} + D_{4,2}'R_{2,1}' - D_{4,2}'R_3\dot{R}_{3,2}); \]
\[ + K_{2,1t}(D_{1,2}'R_3\dot{R}_{3,1} - D_{1,2}'R_{2,1}' + D_{3,2}'R_4\dot{R}_{4,2} - D_{3,2}'R_4\dot{R}_{4,2}); \]
\[ N_{4,2t} = -D_{1,2}'R_4\dot{R}_{3,1} + D_{1,2} + D_4'\dot{R}_3\dot{R}_{3,2} - D_{4,2}'R_{4,1}; \]
\[ N_{5,2t} = K_{2,1t}(-D_{2,2}'R_{1,2}' + D_{3,2}'R_3\dot{R}_{3,1} + D_{4,2}'R_{1,2}'R_{2,1}); \]
\[ N_{6,2t} = K_{4,1t}(D_{2,2}'R_3\dot{R}_{3,1} + D_{4,2}'R_{1,2}'\dot{R}_{1,1}); \]
\[ N_{7,2t} = D_{2,2} - D_{2,2}'R_3\dot{R}_{3,1} - D_{4,2}'R_3\dot{R}_{3,2}; \]
\[ N_{8,2t} = K_{4,1t}(R_3\dot{R}_{2,1}' - R_{2,2}'); \]
\[ N_{9,2t} = K_{4,1t}(R_3\dot{R}_{2,2}' - R_{2,2}'); \]
\[ A_{1,1} = G_{eq20}\sin\delta_{10} - B_{eq20}\cos\delta_{10}; \]
\[ A_{1,2} = G_{eq20}\sin\delta_{10} - \beta_1E_{q10} - \beta_2E_{q20}\cos\delta_{10}; \]
\[ A_{1,3} = B_{eq20}\sin\delta_{10} + G_{eq20}\cos\delta_{10}; \]
\[ A_{1,4} = \alpha_1E_{q10} + \alpha_2E_{q20}\cos\delta_{10} + \beta_1E_{q10}\sin\delta_{10}; \]
\[ A_{1,5} = -B_{eq10} - A_{1,10} = G_{eq10}; \]
\[ A_{1,6} = -B_{eq10} - b_{2,2}E_{q20} - b_{2,1}E_{q10}\cos\delta_{10}; \]
\[ A_{1,7} = a_{2,1}E_{q10}\sin\delta_{10} - b_{2,1}E_{q20} - b_{2,1}E_{q10}\cos\delta_{10}; \]
\[ A_{1,8} = -d_{2,10}\sin\delta_{10} - B_{eq10}\cos\delta_{10}; \]
\[ A_{1,9} = -G_{eq10}M_{1,1} - B_{eq10}M_{1,1} - B_{eq10}M_{1,12}; \]
\[ a_{1,11} = G_{eq110}M_{1,13} + G_{eq120}M_{1,11} - B_{eq110}M_{1,14} - B_{eq120}M_{1,12}; \]
\[ a_{1,12} = G_{eq110}N_{1,13} + G_{eq120}N_{1,11} - B_{eq110}N_{1,14} - B_{eq120}N_{1,12}; \]
\[ b_{1,11} = G_{eq110}M_{1,14} + G_{eq120}M_{1,12} + B_{eq110}M_{1,13} + B_{eq120}M_{1,11}; \]
\[ b_{1,12} = G_{eq110}N_{1,14} + G_{eq120}N_{1,12} + B_{eq110}N_{1,13} + B_{eq120}N_{1,11}; \]
\[ a_{1,21} = G_{eq120}M_{1,23} + G_{eq220}M_{1,21} - B_{eq120}M_{1,24} - B_{eq220}M_{1,22}; \]
\[ a_{1,22} = G_{eq120}N_{1,23} + G_{eq220}N_{1,21} - B_{eq120}N_{1,24} - B_{eq220}N_{1,22}; \]
\[ b_{1,21} = G_{eq120}M_{1,24} + G_{eq220}M_{1,22} + B_{eq120}M_{1,23} + B_{eq220}M_{1,21}; \]
\[ b_{1,22} = G_{eq120}N_{1,24} + G_{eq220}N_{1,22} + B_{eq120}N_{1,23} + B_{eq220}N_{1,21}; \]
\[ a_{2,11} = G_{eq2110}M_{1,13} + G_{eq120}M_{2,11} - B_{eq2110}M_{1,14} - B_{eq120}M_{2,12}; \]
\[ a_{2,12} = G_{eq2110}N_{1,13} + G_{eq120}N_{2,11} - B_{eq2110}N_{1,14} - B_{eq120}N_{2,12}; \]
\[ \text{(continued)} \]
\[ b_{2,11} = G_{eq2,110}M_{14} + G_{eq120}M_{2,12} + B_{eq2,110}M_{13} + B_{eq120}M_{2,11}; \]
\[ b_{2,12} = G_{eq2,110}N_{14} + G_{eq120}N_{2,12} + B_{eq2,110}N_{13} + B_{eq120}N_{2,11}; \]
\[ a_{2,11} = G_{eq2,210}M_{23} + G_{eq220}M_{2,21} - B_{eq2,210}M_{24} - B_{eq220}M_{2,22}; \]
\[ a_{2,22} = G_{eq2,210}N_{23} + G_{eq220}N_{2,21} - B_{eq2,210}N_{24} - B_{eq220}N_{2,22}; \]
\[ b_{2,21} = G_{eq2,210}M_{24} + G_{eq220}M_{2,22} + B_{eq2,210}M_{23} + B_{eq220}M_{2,21}; \]
\[ b_{2,22} = G_{eq2,210}N_{24} + G_{eq220}N_{2,22} + B_{eq2,210}N_{23} + B_{eq220}N_{2,21}; \]
\[ \alpha_1 = R_{eq}a_{1,11} - X_{eq}b_{1,11}; \quad \alpha_{12} = R_{eq}a_{1,12} - X_{eq}b_{1,12}; \quad \beta_1 = X_{eq}a_{1,11} + R_{eq}b_{1,11}; \]
\[ \alpha_{12} = R_{eq}a_{1,12} + X_{eq}b_{1,12}; \quad \alpha_{21} = R_{eq}a_{1,21} - X_{eq}b_{1,21}; \quad \alpha_{22} = R_{eq}a_{1,22} - X_{eq}b_{1,22}; \]
\[ \beta_{21} = X_{eq}a_{1,21} + R_{eq}b_{1,21}; \quad \beta_{22} = X_{eq}a_{1,22} + R_{eq}b_{1,22}; \]
\[ K_{ld1,1} = \frac{X_{el} + X_{ql}}{R_{el}^2 + (X_{el} + X_{ql})(X_{el} + X_{ql})}; \]
\[ K_{ld1,2} = -\frac{A_{0,1}(X_{el} + X_{ql}) - R_{el}A_{4,1}}{R_{el}^2 + (X_{el} + X_{ql})(X_{el} + X_{ql})}; \]
\[ K_{ld1,3} = -\frac{A_{0,1}(X_{el} + X_{ql}) - R_{el}A_{4,1}}{R_{el}^2 + (X_{el} + X_{ql})(X_{el} + X_{ql})}; \]
\[ K_{ld1,4} = -\frac{A_{0,1}(X_{el} + X_{ql}) - R_{el}A_{4,1}}{R_{el}^2 + (X_{el} + X_{ql})(X_{el} + X_{ql})}; \]
\[ K_{ld1,5} = -\frac{A_{0,1}(X_{el} + X_{ql}) - R_{el}A_{4,1}}{R_{el}^2 + (X_{el} + X_{ql})(X_{el} + X_{ql})}; \]
\[ K_{ld1,6} = -\frac{A_{0,1}(X_{el} + X_{ql}) - R_{el}A_{4,1}}{R_{el}^2 + (X_{el} + X_{ql})(X_{el} + X_{ql})}; \]
\[ K_{ld1,7} = \frac{1}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,1} = -\frac{A_{0,2}}{X_{el} + X_{d2}']; \]
\[ K_{ld2,2} = -\frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,3} = -\frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,4} = -\frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,5} = -\frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,6} = -\frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,7} = 0; \]
\[ K_{ld2,8} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,9} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,10} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,11} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,12} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,13} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,14} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,15} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,16} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,17} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,18} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,19} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,20} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,21} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,22} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,23} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,24} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,25} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]
\[ K_{ld2,26} = \frac{A_{0,2}}{X_{el} + X_{d2}'}; \]

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\[ K_{51} = \frac{V_{d10}(A_{i2} + R_e K_{ld1.4} - X_e K_{ld1.4})}{V_{10}} + \frac{V_{q10}(A_{i6} + R_e K_{ld1.4} + X_e K_{ld1.4})}{V_{10}}; \]

\[ K_{61} = \frac{V_{d10}(R_e K_{ld1.4} - X_e K_{ld1.4})}{V_{10}} + \frac{V_{q10}(R_e K_{ld1.4} + X_e K_{ld1.4})}{V_{10}}; \]

\[ K_{71} = \frac{V_{d10}(A_{i1} + R_e K_{ld1.3} - X_e K_{ld1.3})}{V_{10}} + \frac{V_{q10}(A_{i5} + R_e K_{ld1.3} + X_e K_{ld1.3})}{V_{10}}; \]

\[ K_{131} = \frac{V_{d10}(A_{i1} + R_e K_{ld1.2} - X_e K_{ld1.2})}{V_{10}} + \frac{V_{q10}(A_{i4} + R_e K_{ld1.2} + X_e K_{ld1.2})}{V_{10}}; \]

\[ K_{b51} = \frac{V_{d10}(A_{i3} + R_e K_{ld1.5} - X_e K_{ld1.5})}{V_{10}} + \frac{V_{q10}(A_{i7} + R_e K_{ld1.5} + X_e K_{ld1.5})}{V_{10}}; \]

\[ K_{b61} = \frac{V_{d10}(A_{i4} + R_e K_{ld1.6} - X_e K_{ld1.6})}{V_{10}} + \frac{V_{q10}(A_{i8} + R_e K_{ld1.6} + X_e K_{ld1.6})}{V_{10}}; \]

\[ K_{52} = \frac{V_{d10}(A_{i2} - X_e K_{ld1.4})}{V_{20}} + \frac{V_{q10}(A_{i6} + X_e K_{ld1.4})}{V_{20}}, \quad K_{62} = \frac{-V_{d10}X_e K_{ld1.4}}{V_{20}} + \frac{V_{q10}X_e K_{ld1.4}}{V_{20}}; \]

\[ K_{72} = \frac{V_{d10}(A_{i1} - X_e K_{ld1.3})}{V_{20}} + \frac{V_{q10}(A_{i5} + X_e K_{ld1.3})}{V_{20}}, \quad K_{32} = \frac{-V_{d10}(A_{i9} - X_e K_{ld1.2})}{V_{20}} + \frac{V_{q10}(A_{i4} + X_e K_{ld1.2})}{V_{20}}; \]

\[ K_{62} = \frac{V_{d10}(A_{i1} - X_e K_{ld1.6})}{V_{20}} + \frac{V_{q10}(A_{i6} + X_e K_{ld1.6})}{V_{20}}, \quad K_{52} = \frac{-V_{d10}X_e K_{ld1.6}}{V_{20}} + \frac{V_{q10}X_e K_{ld1.6}}{V_{20}}; \]

\[ B_{l11} = -G_e e_{110} E_{q10} \sin \delta_{10} - B_{e110} E_{q10} \cos \delta_{10}; \quad B_{l12} = G_e e_{110} \cos \delta_{10} - B_{e110} \sin \delta_{10}; \]

\[ B_{l13} = -G_e e_{130} E_{q20} \sin \delta_{20} - B_{e130} E_{q20} \cos \delta_{20}; \quad B_{l14} = G_e e_{130} \cos \delta_{20} - B_{e130} \sin \delta_{20}; \]

\[ B_{l15} = a_{11} e_{110} \cos \delta_{10} - b_{11} E_{q10} \sin \delta_{10} - a_{i1} E_{q20} \cos \delta_{20} + b_{i1} E_{q20} \sin \delta_{20}; \]

\[ B_{l16} = a_{12} E_{q10} \cos \delta_{10} - b_{12} E_{q10} \sin \delta_{10} - a_{i2} E_{q20} \cos \delta_{20} + b_{i2} E_{q20} \sin \delta_{20}; \]

\[ B_{l17} = G_e e_{170} E_{q10} \cos \delta_{10} - B_{e170} E_{q10} \sin \delta_{10}; \quad B_{l18} = G_e e_{170} \sin \delta_{10} + B_{e170} \cos \delta_{10}; \]

\[ B_{l19} = G_e e_{190} E_{q20} \cos \delta_{20} - B_{e190} E_{q20} \sin \delta_{20}; \quad B_{l110} = G_e e_{190} \sin \delta_{20} + B_{e190} \cos \delta_{20}; \]

\[ B_{l111} = a_{11} E_{q10} \sin \delta_{10} + b_{11} E_{q10} \cos \delta_{10} + a_{i1} E_{q20} \sin \delta_{20} + b_{i1} E_{q20} \cos \delta_{20}; \]

\[ B_{l112} = a_{12} E_{q10} \sin \delta_{10} + b_{12} E_{q10} \cos \delta_{10} + a_{i2} E_{q20} \sin \delta_{20} + b_{i2} E_{q20} \cos \delta_{20}; \]

\[ K_{SYC1j} = \frac{V_{SYC0j} B_{l1j} + V_{SYC0j} B_{l1,j}}{V_{SYC0j}}, \quad K_{SYC2j} = \frac{V_{SYC0j} B_{l1,2} + V_{SYC0j} B_{l1,9}}{V_{SYC0j}}; \]

\[ K_{SYC3j} = \frac{V_{SYC0j} B_{l1,2} + V_{SYC0j} B_{l1,8}}{V_{SYC0j}}, \quad K_{SYC4j} = \frac{V_{SYC0j} B_{l1,4} + V_{SYC0j} B_{l1,10}}{V_{SYC0j}}; \]

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Appendices

\[ K_{SCP,ij} = \frac{V_{SCP0,i}B_{Li,5} + V_{SCP0,i}B_{Li,1}}{V_{SCP0,j}}; \quad K_{SCP,6,j} = \frac{V_{SCP0,i}B_{Li,6} + V_{SCP0,j}B_{Li,3}}{V_{SCP0,j}}; \]

\[ K_{i,j} = \frac{1}{1 + K_{ji,i}(X_{di} - X_{di}')}; \quad K_{4,i} = K_{li,i}(X_{di} - X_{di}'); \quad K_{9,i} = -K_{li,3}(X_{di} - X_{di}'); \]

\[ K_{12,j} = -K_{li,2}(X_{di} - X_{di}'); \quad K_{k3,1} = -K_{li,5}(X_{m} - X_{m}'); \quad K_{k4,1} = -K_{li,6}(X_{di} - X_{di}'); \]

\[ K_{1,j} = A_{6,i}q_{i0} + A_{2,i}d_{i0} + K_{li,i}(R_{ei}q_{i0} - X_{ei}d_{i0} + V_{q0}) + K_{li,i}(X_{ei}q_{i0} + R_{ei}d_{i0} + V_{d0}); \]

\[ K_{2,i} = K_{qi,i}(R_{ei}q_{i0} - X_{ei}d_{i0} + V_{q0}) + K_{li,i}(X_{ei}q_{i0} + R_{ei}d_{i0} + V_{d0}); \]

\[ K_{s,j} = A_{2,i}q_{i0} + A_{4,i}d_{i0} + K_{li,i}(R_{ei}q_{i0} - X_{ei}d_{i0} + V_{q0}) + K_{li,i}(X_{ei}q_{i0} + R_{ei}d_{i0} + V_{d0}); \]

\[ K_{14,j} = A_{4,i}q_{i0} + A_{2,i}d_{i0} + K_{li,i}(R_{ei}q_{i0} - X_{ei}d_{i0} + V_{q0}) + K_{li,i}(X_{ei}q_{i0} + R_{ei}d_{i0} + V_{d0}); \]

\[ K_{i,j} = A_{2,i}q_{i0} + A_{4,i}d_{i0} + K_{li,5}(R_{ei}q_{i0} - X_{ei}d_{i0} + V_{q0}) + K_{li,5}(X_{ei}q_{i0} + R_{ei}d_{i0} + V_{d0}); \]

\[ K_{Q1,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP1,i}; \quad K_{Q2,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP2,i}; \]

\[ K_{Q3,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP3,i}; \quad K_{Q4,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP4,i}; \]

\[ K_{Q5,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP5,i} - V_{SCP0,j}^2; \quad K_{Q5,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP5,i} - V_{SCP0,j}^2; \]

\[ K_{Q6,j} = -2V_{SCP0,j}B_{SCP0,j}K_{SCP6,i}; \]

\[ R_{i,1} = \frac{1 + K_{li,1}(X_{q1} - X_{d1})}{1 - K_{li,1}(X_{q1} - X_{d1})}; \quad R_{2,i} = \frac{K_{li,4}(X_{q1} - X_{q1}')}{1 - K_{li,2}(X_{q1} - X_{d1})}; \quad R_{3,i} = \frac{-K_{li,3}(X_{q1} - X_{d1})}{1 - K_{li,2}(X_{q1} - X_{d1})}; \]

\[ R_{3,1} = \frac{K_{li,5}(X_{q1} - X_{q1}')}{1 - K_{li,2}(X_{q1} - X_{d1})}; \quad R_{3,2} = \frac{-K_{li,6}(X_{q1} - X_{q1}')}{1 - K_{li,2}(X_{q1} - X_{d1})}; \]

\[ R_{2,2} = \frac{K_{li,4}(X_{q2} - X_{d2})}{1 - K_{li,2}(X_{q2} - X_{d2})}; \quad R_{3,2} = \frac{-K_{li,5}(X_{q2} - X_{q2}')}{1 - K_{li,2}(X_{q2} - X_{d2})}; \]

\[ R_{h1,2} = -K_{li,2}(X_{q2} - X_{d2}) \]

\[ K_{P1} = K_{SCP1,i}; \quad K_{P2} = K_{SCP2,i}; \quad K_{P3} = K_{SCP3,i}; \quad K_{P4} = K_{SCP4,i}; \quad K_{P5} = K_{SCP5,i}; \quad K_{P6} = K_{SCP6,i}; \]

\[ K_{P7,1} = -K_{P5,1}G_{SCP1,j} - K_{P6,1}G_{SCP2,j}; \]

\[ K_{P1,1} = \frac{1}{K_{P7,1}}[K_{P1,1} + K_{P3,1}R_{2,1} + H_{1,1}(K_{P2,1} - K_{P3,1}R_{4,1}) + H_{3,1}(K_{P4,1} - K_{P3,1}R_{6,1})]. \]


\( K_{p2,1} = \frac{1}{K_{p1,1}} [K_{p3,1} R_{i,1} + H_{2,1}(K_{p2,1} - K_{p3,1} R_{i,1}) + H_{4,1}(K_{p4,1} - K_{p3,1} R_{i,1})]; \\
K_{Q1,1}' = K_{Q1,1} + K_{Q1,1'} R_{2,1} + H_{1,1}'(K_{Q1,1} - K_{Q1,1'} R_{2,1}) + H_{3,1}'(K_{Q1,1} + K_{Q1,1'} R_{2,1}); \\
K_{Q2,1}' = K_{Q1,1}' R_{1,1} + H_{2,1}'(K_{Q1,1}' - K_{Q1,1'} R_{2,1}) + H_{4,1}'(K_{Q1,1}' - K_{Q1,1'} R_{2,1}); \\
H_{1,1}' = \frac{H_{21} H_{31} + H_{41}(1 - H_{41})}{(H_{71} - 1)(H_{41} - 1) - H_{31} H_{81}}; \\
H_{2,1}' = \frac{H_{21} H_{41} + H_{61}(1 - H_{61})}{(H_{71} - 1)(H_{41} - 1) - H_{31} H_{81}}; \\
H_{3,1}' = \frac{H_{21} H_{61} + H_{21}(1 - H_{21})}{(H_{71} - 1)(H_{41} - 1) - H_{31} H_{81}}; \\
H_{4,1}' = \frac{H_{21} H_{81} + H_{21}(1 - H_{21})}{(H_{71} - 1)(H_{41} - 1) - H_{31} H_{81}}; \\
H_{1,2} = R_{i,1} G_{i,4,1} + R_{i,2} G_{i,4,2} - R_{i,2} R_{i,2} G_{i,2,1} + R_{i,1} (R_{i,1} G_{i,4,2} - R_{i,2} G_{i,4,1} - R_{i,2} R_{i,2}); \\
H_{2,1} = R_{i,1}'(R_{i,2}' G_{i,4,1} - R_{i,2}' G_{i,4,2} - R_{i,2}' R_{i,2}) + R_{i,2}' (R_{i,2}' G_{i,4,2} - R_{i,2}' G_{i,4,1} - R_{i,2}' R_{i,2}); \\
H_{3,2} = R_{i,2}' (R_{i,1}' G_{i,1,1} - R_{i,1}' G_{i,2,1} - R_{i,1}' R_{i,2}'), \\
H_{4,2} = R_{i,2}' (R_{i,1}' G_{i,1,1} + R_{i,2}' G_{i,2,1} - R_{i,1}' R_{i,2}'), \\
H_{5,2} = G_{i,2}' R_{i,1} + G_{i,2}' R_{i,2}; \\
H_{6,2} = G_{i,2}' R_{i,1} + G_{i,2}' R_{i,2}; \\
H_{7,2} = G_{i,2}' R_{i,1} + G_{i,2}' R_{i,2}; \\
H_{8,2} = G_{i,2}' R_{i,1} + G_{i,2}' R_{i,2}; \\
D_{3,1} = K_{s,1} + K_{s,1}' R_{i,1}; \\
D_{3,2} = K_{s,2} + K_{s,1}' R_{i,1}; \\
D_{4,1} = K_{s,1} - K_{s,1}' R_{i,1}; \\
D_{4,2} = K_{s,2} - K_{s,2}' R_{i,1}; \\
T_{1,1} = \frac{T_{1,1}}{2 \pi D_{1,1}}; \\
K_{1,1} = \frac{D_{1,1}}{D_{1,1}}; \\
K_{2,1} = \frac{D_{1,1}}{D_{1,1}}; \\
T_{1,2} = \frac{T_{1,2}}{2 \pi D_{1,2}}; \\
K_{1,2} = \frac{D_{1,2}}{D_{1,2}}; \\
K_{2,2} = \frac{D_{1,2}}{D_{1,2}}; \\
K_{3,1} = \frac{K_{s,1} G_{i,3,1}}{1 + G_{i,2}' K_{s,1} - \frac{D_{2,1}' K_{s,1} D_{4,1}}{D_{1,1}}}; \\
K_{3,2} = \frac{K_{s,1} G_{i,3,2}}{1 + G_{i,2}' K_{s,1} - \frac{D_{2,2}' K_{s,1} D_{4,2}}{D_{1,2}}}; \\
K_{4,1} = \frac{K_{s,1} G_{i,4,1}}{1 + G_{i,2}' K_{s,1} - \frac{D_{2,1}' K_{s,1} D_{4,1}}{D_{1,1}}}; \\
K_{4,2} = \frac{K_{s,1} G_{i,4,2}}{1 + G_{i,2}' K_{s,1} - \frac{D_{2,2}' K_{s,1} D_{4,2}}{D_{1,2}}};
\[
\begin{align*}
K_{3,2r} &= \frac{K_{3,2}G_{3,2}'}{1 + G_{3,2}K_{3,2} - \frac{D_{3,2}K_{3,2}D_{4,2}}{D_{1,2}}}; \\
K_{4,2r} &= \frac{K_{3,2}G_{4,2}'}{1 + G_{3,2}K_{3,2} - \frac{D_{3,2}K_{3,2}D_{4,2}}{D_{1,2}}}.
\end{align*}
\]

\[
\begin{align*}
\frac{1}{K_{3,1}'} &= \frac{1}{K_{3,1}} - K_{12,1}R_{1,1}' - H_{1,1}'(K_{16,1} - K_{12,1}R_{4,1}') - H_{4,1}'(K_{9,1} - K_{12,1}R_{3,1}'); \\
K_{4,1} &= K_{4,1}' - K_{12,1}R_{2,1}' - H_{1,1}'(K_{16,1} - K_{12,1}R_{4,1}') - H_{3,1}'(K_{9,1} - K_{12,1}R_{3,1}'); \\
\frac{1}{K_{3,2}'} &= \frac{1}{K_{3,2}} - K_{9,2}R_{1,1}' + H_{1,1}'(K_{4,2}' - K_{9,2}R_{2,1}') - H_{4,2}'(K_{12,2} - K_{9,2}R_{3,1}'); \\
K_{4,2} &= -K_{16,2} + K_{9,2}R_{4,2}' + H_{1,2}'(K_{4,2}' - K_{9,2}R_{2,2}') - H_{3,2}'(K_{12,2} - K_{9,2}R_{3,2}'); \\
K_{5,1} &= K_{5,1}' + K_{13,1}R_{1,1}' - H_{1,1}'(K_{17,1} + K_{13,1}R_{4,1}') + H_{3,1}'(K_{7,1} - K_{13,1}R_{3,1}'); \\
K_{6,1} &= K_{6,1}' + K_{13,1}R_{1,1}' - H_{1,1}'(K_{17,1} + K_{13,1}R_{4,1}') + H_{4,1}'(K_{7,1} - K_{13,1}R_{3,1}'); \\
K_{5,2} &= -K_{17,2} - K_{7,2}R_{4,2}' + H_{1,2}'(K_{5,2}' + K_{7,2}R_{2,2}') + H_{3,2}'(K_{13,2} - K_{7,2}R_{3,2}'); \\
K_{6,2} &= K_{6,2}' + K_{7,2}R_{1,1}' + H_{1,2}'(K_{5,2}' + K_{7,2}R_{2,2}') + H_{4,2}'(K_{13,2} - K_{7,2}R_{3,2}'); \\
K_{p,1} &= \frac{1}{K_{p,1}}[K_{p,1}' + K_{p,3,1}R_{1,1}' + H_{1,1}'(K_{p,2,1} - K_{p,3,1}R_{4,1}') + H_{3,1}'(K_{p,4,1} - K_{p,3,1}R_{3,1}')]; \\
K_{p,2} &= \frac{1}{K_{p,2}}[K_{p,3,1}R_{1,1}' + H_{1,1}'(K_{p,2,1} - K_{p,3,1}R_{4,1}') + H_{3,1}'(K_{p,4,1} - K_{p,3,1}R_{3,1}')]; \\
K_{p,3,2} &= \frac{1}{K_{p,3,2}}[K_{p,2,1} - K_{p,3,1}R_{4,2}' + H_{1,2}'(K_{p,1,2} + K_{p,3,1}R_{2,2}') + H_{3,2}'(K_{p,3,1} - K_{p,4,1}R_{3,1}')]; \\
K_{p,4,2} &= \frac{1}{K_{p,4,2}}[K_{p,4,1}R_{1,2}' + H_{1,2}'(K_{p,1,4} + K_{p,4,1}R_{2,2}') + H_{3,2}'(K_{p,3,1} - K_{p,4,1}R_{3,1}')]; \\
K_{p,5} &= K_{p,5}' + K_{p,3,1}R_{1,1}' + H_{1,1}'(K_{p,4,2} - K_{p,3,1}R_{4,1}') + H_{3,1}'(K_{p,4,1} - K_{p,5,1}R_{3,1}'); \\
K_{p,6} &= K_{p,6}' + K_{p,3,1}R_{1,1}' + H_{1,1}'(K_{p,4,2} - K_{p,3,1}R_{4,1}') + H_{3,1}'(K_{p,4,1} - K_{p,6,1}R_{3,1}'); \\
K_{p,7} &= K_{p,7}' - K_{p,4,2}R_{1,2}' + H_{1,2}'(K_{p,5} - K_{p,4,2}R_{2,2}') + H_{3,2}'(K_{p,5,2} - K_{p,4,2}R_{3,2}'); \\
K_{p,8} &= K_{p,8} + \frac{K_{p,1,1}}{K_{p,7,1}}(K_{p,1,1}G_{SVC1} + K_{p,2,1}G_{SVC2}); \\
K_{p,9} &= K_{p,9} + \frac{K_{p,1,1}}{K_{p,7,1}}(K_{b,1,1}G_{SVC1} + K_{b,2,1}G_{SVC2}); \\
K_{p,14} &= K_{p,14} + \frac{K_{p,3,1}}{K_{p,7,1}}(K_{b,1,1}G_{SVC1} + K_{b,2,1}G_{SVC2}); \\
K_{p,15} &= K_{p,15} + \frac{K_{p,1,1}}{K_{p,7,1}}(K_{b,1,1}G_{SVC1} + K_{b,2,1}G_{SVC2});
\end{align*}
\]
\[ K_{12} = K_{12} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{142} = K_{142} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{152} = K_{152} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{41} = K_{41} - \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{121} = K_{121} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{42} = K_{42} - \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{122} = K_{122} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{51} = K_{51} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{131} = K_{131} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{52} = K_{52} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ K_{132} = K_{132} + \frac{K_{p1}}{K_{p1}} (K_{p1} G_{SVC1} + K_{b1} G_{SVC2}) \]

\[ R_{11} = \frac{K_{p1} R_{11}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{21} = \frac{K_{p1} R_{21}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{31} = \frac{K_{p1} R_{31}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{41} = \frac{K_{p1} R_{41}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{12} = \frac{K_{p1} R_{12}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{22} = \frac{K_{p1} R_{22}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{32} = \frac{K_{p1} R_{32}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ R_{42} = \frac{K_{p1} R_{42}}{K_{p1} + K_{p1} (R_{p1} G_{SVC1} + R_{b1} G_{SVC2})} \]

\[ K_{Q1} = K_{Q1} + \frac{K_{p1}}{K_{p1}} (K_{Q1} G_{SVC1} + K_{Q1} G_{SVC2}) \]

\[ K_{Q2} = K_{Q2} + \frac{K_{p1}}{K_{p1}} (K_{Q1} G_{SVC1} + K_{Q1} G_{SVC2}) \]

\[ K_{Q3} = K_{Q3} + \frac{K_{p1}}{K_{p1}} (K_{Q1} G_{SVC1} + K_{Q1} G_{SVC2}) \]

\[ K_{Q4} = K_{Q4} + \frac{K_{p1}}{K_{p1}} (K_{Q1} G_{SVC1} + K_{Q1} G_{SVC2}) \]
Appendix C

Derivation of Equation (3.5.5)

From Figure 3-8(d), the voltage of SVC1 and SVC2, i.e., $\vec{V}_{SVC1}$ and $\vec{V}_{SVC2}$ can be obtained:

$$\vec{V}_{SVC1} = \left[ (\tilde{Z}_{eq2} + \tilde{Z}_{45}) V_{el} - \delta_{el} + \tilde{Z}_{equ} V_{el} - \delta_{el} \right] / (\tilde{Z}_{equ} + \tilde{Z}_{45})$$  \hspace{1cm} (C.1)

$$\vec{V}_{SVC2} = \left[ \tilde{Z}_{equ} V_{el} - \delta_{el} + (\tilde{Z}_{equ} + \tilde{Z}_{45}) V_{el} - \delta_{el} \right] / (\tilde{Z}_{equ} + \tilde{Z}_{45})$$  \hspace{1cm} (C.2)

From Figure 3-8(b), the terminal voltage of $G_1$, i.e., $\vec{V}_1$ can be derived:

$$\vec{V}_1 = \left( \tilde{Z}_{LS} \vec{V}_{SVC1} + \tilde{Z}_{LS} \vec{I}_1 \right) / (\tilde{Z}_{LS} + \tilde{Z}_{12})$$  \hspace{1cm} (C.3)

From Figure 3-8(c), the terminal voltage of $G_2$, i.e., $\vec{V}_2$ can also be obtained:

$$\vec{V}_2 = \vec{V}_{SVC2} + j X_{12} \vec{I}_2$$  \hspace{1cm} (C.4)

Let,

$$G_{el1} + j B_{el1} = \frac{(\tilde{Z}_{equ} + \tilde{Z}_{45})}{(\tilde{Z}_{equ} + \tilde{Z}_{equ} + \tilde{Z}_{45}) \left[ (1 + R_{12} G_{el1} - X_{12} B_{el1} - X_{12} B_{SVC1} + j (X_{12} G_{el1} + R_{12} B_{el1} + R_{12} B_{SVC1}) \right]}$$;

$$G_{el2} + j B_{el2} = \frac{\tilde{Z}_{equ}}{(\tilde{Z}_{equ} + \tilde{Z}_{equ} + \tilde{Z}_{45}) \left[ (1 - X_{12}^2 (B_{el2}^* + B_{SVC2}) + j X_{12}^2 G_{el2}^* \right]}$$;

$$G_{el2} + j B_{el2} = \frac{\tilde{Z}_{equ}}{(\tilde{Z}_{equ} + \tilde{Z}_{equ} + \tilde{Z}_{45}) \left[ (1 + R_{12} G_{el2} - X_{12} B_{el2} - X_{12} B_{SVC1} + j (X_{12} G_{el2} + R_{12} B_{el2} + R_{12} B_{SVC1}) \right]}$$;

$$G_{el2} + j B_{el2} = \frac{\tilde{Z}_{equ}}{(\tilde{Z}_{equ} + \tilde{Z}_{equ} + \tilde{Z}_{45}) \left[ (1 - X_{12}^2 (B_{el2}^* + B_{SVC2}) + j X_{12}^2 G_{el2}^* \right]}$$;

Thus,

$$\vec{V}_{SVC1} = (G_{el1} + j B_{el1}) E_{q1} \angle \delta_{1} + (G_{el1} + j B_{el1}) E_{q2} \angle \delta_{2}$$  \hspace{1cm} (C.5)

$$\vec{V}_{SVC2} = (G_{el2} + j B_{el2}) E_{q1} \angle \delta_{1} + (G_{el2} + j B_{el2}) E_{q2} \angle \delta_{2}$$  \hspace{1cm} (C.6)

Substitute (C.5) into (C.3),

$$\vec{V}_1 = \frac{\tilde{Z}_{LS}}{(\tilde{Z}_{LS} + \tilde{Z}_{12})} \left[ (G_{el1} + j B_{el1}) E_{q1} \angle \delta_{1} + (G_{el1} + j B_{el1}) E_{q2} \angle \delta_{2} \right] + \frac{\tilde{Z}_{LS} \tilde{Z}_{12} \tilde{I}_1}{(\tilde{Z}_{LS} + \tilde{Z}_{12})}$$  \hspace{1cm} (C.7)

Let,
Appendices

\[ G_{eq1} + jB_{eq1} = \frac{\tilde{Z}_{15}}{\tilde{Z}_{15} + \tilde{Z}_{12}} (G_{el1} + jB_{el1}) ; \quad G_{eq2} + jB_{eq2} = \frac{\tilde{Z}_{15}'}{\tilde{Z}_{15} + \tilde{Z}_{12}'} (G_{el1} + jB_{el1}) ; \]

\[ R_{el} + jX_{el} = \frac{\tilde{Z}_L'}{\tilde{Z}_L + \tilde{Z}_{12}'} \]

Thus,

\[ \tilde{V}_1 = (G_{eq1} + jB_{eq1})E_{q1} \angle \delta_1 + (G_{eq2} + jB_{eq2})E_{q2} \angle \delta_2 + (R_{el} + jX_{el})I_1 \]

(C.8)

Substitute (C.6) into (C.4)

\[ \tilde{V}_2 = (G_{el1} + jB_{el1})E_{q1} \angle \delta_1 + (G_{el2} + jB_{el2})E_{q2} \angle \delta_2 + jX_{el}I_2 \]

(C.9)

Transform equations (C.8) and (C.9) to d-q systems and obtain the d- and q- components:

\[
\begin{align*}
V_d &= -B_{eq1}E_{q1} + G_{eq2}E_{q2} \sin \delta - B_{eq2}E_{q2} \cos \delta + R_{el}I_d - X_{el}I_q \\
V_q &= G_{eq1}E_{q1} + G_{eq2}E_{q2} \cos \delta + B_{eq2}E_{q2} \sin \delta + R_{el}I_q + X_{el}I_d \\
V_{d2} &= -G_{el1}E_{q1} \sin \delta - B_{el2}E_{q1} \cos \delta - B_{el2}E_{q2} - X_{el}I_q \\
V_{q2} &= G_{el1}E_{q1} \cos \delta - B_{el2}E_{q1} \sin \delta + G_{el2}E_{q2} + X_{el}I_d
\end{align*}
\]

(C.10)

where,

\[ \delta' = \delta_1 - \delta_2 \]

Linearization is carried out at the following operating point: generator \( G_1 \) internal voltage is \( E_{q10} \), its rotor angle is \( \delta_{10} \); generator \( G_2 \) internal voltage is \( E_{q20} \), its real power output is \( P_{20} \), the susceptance of SVC1 is \( B_{SVC1} \), the susceptance of SVC2 is \( B_{SVC2} \). The main load connected to node G is \( R_{L2} + jX_{L2} \), while the loads at the sending end A and the intermediate points C, D and E are \( R_{L3} + jX_{L3}, R_{L4} + jX_{L4} \), respectively.

After linearization, equation (C.10) can be re-written as:

\[
\begin{align*}
\Delta V_{d1} &= A_{d1} \Delta E_{q1} + A_{d2} \Delta E_{q2} + A_{d3} \Delta \delta_1 + A_{d4} \Delta B_{SVC1} + A_{d5} \Delta B_{SVC2} + R_{el} \Delta I_d - X_{el} \Delta I_q \\
\Delta V_{q1} &= A_{q1} \Delta E_{q1} + A_{q2} \Delta E_{q2} + A_{q3} \Delta \delta_1 + A_{q4} \Delta B_{SVC1} + A_{q5} \Delta B_{SVC2} + R_{el} \Delta I_q + X_{el} \Delta I_d \\
\Delta V_{d2} &= A_{d2} \Delta E_{q1} + A_{d3} \Delta E_{q2} + A_{d4} \Delta \delta_1 + A_{d5} \Delta B_{SVC1} + A_{d6} \Delta B_{SVC2} + X_{el} \Delta I_d \\
\Delta V_{q2} &= A_{q2} \Delta E_{q1} + A_{q3} \Delta E_{q2} + A_{q4} \Delta \delta_1 + A_{q5} \Delta B_{SVC1} + A_{q6} \Delta B_{SVC2} + X_{el} \Delta I_d
\end{align*}
\]

(C.11)

Substitute (3.2.2) into (C.11), \( \Delta I_d \) and \( \Delta I_q \) can be obtained.
After linearization, equation (C.17) can be re-written as:

$$\Delta i_{q1} = K_{i_{q1},1} \Delta E_{q1} + K_{i_{q1},2} \Delta E_{q1} + K_{h_{q1},4} \Delta E_{q2} + K_{h_{q1},5} \Delta B_{svc1} + K_{h_{q1},6} \Delta B_{svc1} \tag{C.12}$$

$$\Delta i_{q2} = K_{i_{q2},1} \Delta E_{q1} + K_{i_{q2},2} \Delta E_{q2} + K_{h_{q2},3} \Delta E_{q1} + K_{h_{q2},4} \Delta E_{q2} + K_{h_{q2},5} \Delta B_{svc1} + K_{h_{q2},6} \Delta B_{svc2} \tag{C.13}$$

$$\Delta i_{q2} = K_{i_{q2},1} \Delta E_{q1} + K_{i_{q2},2} \Delta E_{q2} + K_{h_{q2},3} \Delta E_{q1} + K_{h_{q2},4} \Delta E_{q2} + K_{h_{q2},5} \Delta B_{svc1} + K_{h_{q2},6} \Delta B_{svc2} \tag{C.14}$$

Use (3.2.2) and (C.12), $\Delta E_{q1}$ can also be solved:

$$\Delta E_{q1} = \frac{K_{i_{q1}}}{1 + K_{i_{q1}}/\omega_0} \left( -K_{i_{q1},4} \Delta i_{q1} + \Delta E_{q1} + K_{h_{q1},5} \Delta E_{q2} + K_{h_{q1},6} \Delta B_{svc1} + K_{h_{q1},6} \Delta B_{svc2} \right) \tag{C.15}$$

In (3.2.2), $d(\Delta \omega_1)/dt$ can be obtained using (C.11) and (C.12),

$$S \Delta \omega_1 = \frac{1}{T_{ji}} \left( K_{i_{q1},4} \Delta i_{q1} + K_{h_{q1},5} \Delta E_{q2} + K_{h_{q1},6} \Delta B_{svc1} + K_{h_{q1},6} \Delta B_{svc2} \right) \tag{C.16}$$

The input to the AVR is:

$$\Delta V_{ii} = (V_{d0}/V_{q0}) \Delta V_{di} + (V_{q0}/V_{q0}) \Delta V_{qi} \tag{C.17}$$

Substitute (C.11) and (C.12) into (C.15), $\Delta V_{ii}$ can be obtained:

$$\Delta V_{ii} = K_{i_{q1}} \Delta i_{q1} + K_{h_{q1},5} \Delta E_{q2} + K_{h_{q1},6} \Delta B_{svc1} + K_{h_{q1},6} \Delta B_{svc2} \tag{C.18}$$

Separating (C.5) and (C.6) into the real and imaginary parts:

$$V_{svc,r} = G_{e_{rl}} E_{q1} \cos \delta - B_{el,1} E_{q1} \sin \delta + G_{e_{rl}} E_{q2} \sin \delta - B_{el,2} E_{q2} \sin \delta \tag{C.19}$$

$$V_{svc,l} = G_{e_{rl}} E_{q1} \sin \delta + B_{el,1} E_{q1} \cos \delta + G_{e_{rl}} E_{q2} \sin \delta + B_{el,2} E_{q2} \cos \delta \tag{C.20}$$

After linearization, equation (C.17) can be re-written as:

$$\Delta V_{svc,r} = B_{l_{rl}} \Delta \delta + B_{l_{rl}} \Delta E_{q1} + B_{l_{rl}} \Delta E_{q2} + B_{l_{rl}} \Delta B_{svc1} + B_{l_{rl}} \Delta B_{svc2} \tag{C.21}$$

$$\Delta V_{svc,l} = B_{l_{rl}} \Delta \delta + B_{l_{rl}} \Delta E_{q1} + B_{l_{rl}} \Delta E_{q2} + B_{l_{rl}} \Delta B_{svc1} + B_{l_{rl}} \Delta B_{svc2} \tag{C.22}$$

The reactive power output of the $i$-th SVC is,

$$Q_{svc,i} = -V_{svc,i}^2 B_{svc,i} \tag{C.23}$$

After linearization, the above equation can be written as:

$$\Delta Q_{svc,i} = -2 V_{svc,i} B_{svc,i} \Delta V_{svc,i} - V_{svc,i}^2 \Delta B_{svc,i} \tag{C.24}$$

Substitute (C.20) into (C.21):

$$\Delta Q_{svc,i} = K_{q_{rl}} \Delta \delta + K_{q_{rl}} \Delta E_{q1} + K_{q_{rl}} \Delta E_{q2} + K_{q_{rl}} \Delta B_{svc1} + K_{q_{rl}} \Delta B_{svc2} \tag{C.25}$$

$$255$$
From the definition of generator internal voltage,
\[
\Delta E_{qi} = \Delta E'_{qi} + (X'_{qi} - X'_d) \Delta i_{di}
\]
Substitute (C.12) into \(\Delta E_{qi}\):
\[
\begin{align*}
\Delta E_{q1} &= R_{i,1} \Delta E'_{q1} + R_{2,1} \Delta \delta_1 - R_{3,1} \Delta E_{q2} + R_{b1,1} \Delta B_{svc1} + R_{b2,1} \Delta B_{svc2} \\
\Delta E_{q2} &= R_{i,2} \Delta E'_{q2} + R_{2,2} \Delta \delta_2 - R_{3,2} \Delta E_{q1} + R_{b1,2} \Delta B_{svc1} + R_{b2,2} \Delta B_{svc2}
\end{align*}
\]
(C.23)

The input to the SVC and SGC is:
\[
\Delta V_P = \Delta V_{SVC,1} = K_{pc1} \Delta \delta_1 + K_{pc2} \Delta \delta_2 + K_{pc3} \Delta E_{q1} + K_{pc4} \Delta E_{q2} + K_{pc5} \Delta B_{svc1} + K_{pc6} \Delta B_{svc2}
\]
(C.24)

Therefore, combining (C.13), (C.14), (C.16), (C.22), and (C.24), the state space model of
the system can be obtained as (3.5.5).
Appendix D

Detailed Data for the Studied Power System Used in Section 3.6

Nominal frequency: 50 Hz.

The generator parameters used in the project is given in Table D-1. All per unit values are based on its own generator unit rating.

Table D-1 Generator parameters in studied power system

<table>
<thead>
<tr>
<th>Generator Identifier</th>
<th>( G_1 )</th>
<th>( G_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Rating (MVA)</td>
<td>1000</td>
<td>50</td>
</tr>
<tr>
<td>Unit Power Rating (MW)</td>
<td>800</td>
<td>30</td>
</tr>
<tr>
<td>Direct axis synchronous reactance ( X_{d^*} ), (p.u.)</td>
<td>2.127</td>
<td>0.277</td>
</tr>
<tr>
<td>Direct axis transient reactance ( X_{d^t} ), (p.u.)</td>
<td>0.277</td>
<td>0.2</td>
</tr>
<tr>
<td>Direct axis sub-transient reactance ( X_{d^s} ), (p.u.)</td>
<td>0.176</td>
<td>0.13</td>
</tr>
<tr>
<td>Quadrature axis synchronous reactance ( X_{q^*} ), (p.u.)</td>
<td>2.127</td>
<td>0.49</td>
</tr>
<tr>
<td>Quadrature axis transient reactance ( X_{q^t} ), (p.u.)</td>
<td>0.49</td>
<td>0.42</td>
</tr>
<tr>
<td>Quadrature axis sub-transient reactance ( X_{q^s} ), (p.u.)</td>
<td>0.191</td>
<td>0.18</td>
</tr>
<tr>
<td>Negative sequence reactance ( X_{2^*} ), (p.u.)</td>
<td>0.14</td>
<td>0.134</td>
</tr>
<tr>
<td>Direct axis open circuit transient time constant ( T_{d^t} ) (s)</td>
<td>5.86</td>
<td>4.2</td>
</tr>
<tr>
<td>Direct axis open circuit sub-transient time constant ( T_{d^s} ) (s)</td>
<td>0.05</td>
<td>0.032</td>
</tr>
<tr>
<td>Quadrature axis open circuit transient time constant ( T_{q^t} ) (s)</td>
<td>1.5</td>
<td>0.57</td>
</tr>
<tr>
<td>Quadrature axis open circuit sub-transient time constant ( T_{q^s} ) (s)</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Inertia constant ( T_j )</td>
<td>5.6</td>
<td>6.14</td>
</tr>
</tbody>
</table>

The parameters of loads are listed in Table D-2.

Table D-2 Parameters of loads in studied power system

<table>
<thead>
<tr>
<th>Load Name</th>
<th>( P_L ) (p.u.)</th>
<th>( Q_L ) (p.u.)</th>
<th>Power Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.82</td>
<td>3.632</td>
<td>0.85</td>
</tr>
<tr>
<td>C</td>
<td>0.10</td>
<td>0.088</td>
<td>0.75</td>
</tr>
<tr>
<td>D</td>
<td>0.20</td>
<td>0.176</td>
<td>0.75</td>
</tr>
<tr>
<td>E</td>
<td>0.10</td>
<td>0.088</td>
<td>0.75</td>
</tr>
<tr>
<td>G</td>
<td>2.00</td>
<td>1.76</td>
<td>0.75</td>
</tr>
</tbody>
</table>
The parameters of transmission lines are listed in Table D-3.

Table D-3 Parameters of transmission lines in studied power system

<table>
<thead>
<tr>
<th>Line Name</th>
<th>R (p.u.)</th>
<th>X (p.u.)</th>
<th>B/2 (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>0.038</td>
<td>0.066</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>0.0165</td>
<td>0.0287</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>0.0198</td>
<td>0.0345</td>
</tr>
<tr>
<td>E</td>
<td>F</td>
<td>0.0331</td>
<td>0.0574</td>
</tr>
</tbody>
</table>

The four 3-winding autotransformers in Figure 3-1 are identical. Each is of YYD11 connection and has winding voltages of 220 kV, 132 kV and 29.5 kV. Star-circuit equivalent reactances in per unit on a base of 100 MVA are $X_H = 0.0529; X_L = -0.0004; X_T = 0.0921$; H denotes the 220 kV windings, L denotes the 132 kV windings, and T denotes the 29.5 kV windings. Corresponding value on a base of 100 MVA for the 2-winding transformer in Figure 3-1 is $X_r = 0.053$.

The parameters of AVR are listed in Table D-4.

Table D-4 AVR parameters for generators in studied power system

<table>
<thead>
<tr>
<th>AVR Location</th>
<th>G1</th>
<th>G2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_r$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_r$ (s)</td>
<td>0.012</td>
<td>0.04</td>
</tr>
<tr>
<td>$K_A$</td>
<td>100</td>
<td>84</td>
</tr>
<tr>
<td>$T_a$ (s)</td>
<td>0.08</td>
<td>0.014</td>
</tr>
<tr>
<td>$T_e$ (s)</td>
<td>0.56</td>
<td>0.49</td>
</tr>
<tr>
<td>$K_f$</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>$T_f$ (s)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$T_{dc}$ (s)</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>$T_{sa}$ (s)</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$V_{RMAX}$</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>$V_{RMIN}$</td>
<td>-4.5</td>
<td>-4.5</td>
</tr>
<tr>
<td>$E_{fRMAX}$</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>$E_{fRMIN}$</td>
<td>-4.5</td>
<td>-4.5</td>
</tr>
</tbody>
</table>
Appendix E

Detailed Data for the Studied Power System Used in Section 4.7

Nominal frequency: 60 Hz.

The generator parameters used in the project is given in Table E-1. All per unit values are based on its own generator unit rating.

Table E-1 Generator parameters in studied power system

<table>
<thead>
<tr>
<th>Generator Identifier</th>
<th>G1</th>
<th>G2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Rating (MVA)</td>
<td>2000</td>
<td>5000</td>
</tr>
<tr>
<td>Unit Power Rating (MW)</td>
<td>2000</td>
<td>5000</td>
</tr>
<tr>
<td>Direct axis synchronous reactance $X_d$, (p.u.)</td>
<td>1.305</td>
<td>1.305</td>
</tr>
<tr>
<td>Direct axis transient reactance $X_d^t$, (p.u.)</td>
<td>0.296</td>
<td>0.296</td>
</tr>
<tr>
<td>Direct axis sub-transient reactance $X_d^{*}$, (p.u.)</td>
<td>0.252</td>
<td>0.252</td>
</tr>
<tr>
<td>Quadrature axis synchronous reactance $X_q$, (p.u.)</td>
<td>0.474</td>
<td>0.474</td>
</tr>
<tr>
<td>Quadrature axis sub-transient reactance $X_q^{*}$, (p.u.)</td>
<td>0.243</td>
<td>0.243</td>
</tr>
<tr>
<td>Negative sequence reactance $X_0$, (p.u.)</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>Direct axis open circuit transient time constant $T_{d0}^t$ (s)</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Direct axis open circuit sub-transient time constant $T_{d0}^{*}$ (s)</td>
<td>0.053</td>
<td>0.053</td>
</tr>
<tr>
<td>Quadrature axis open circuit sub-transient time constant $T_{q0}^{*}$ (s)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Inertia constant $T_j$</td>
<td>3.7 0</td>
<td>3.7 0</td>
</tr>
</tbody>
</table>

The parameters of transmission lines are listed in Table E-2.

Table E-2 Parameters of transmission lines in studied power system

<table>
<thead>
<tr>
<th>Line Name</th>
<th>Resistance (Ohms/km)</th>
<th>Inductance (H/km)</th>
<th>Capacitance (F/km)</th>
<th>Line length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B C</td>
<td>0.01755</td>
<td>0.8737e-3</td>
<td>13.33e-9</td>
<td>350</td>
</tr>
<tr>
<td>C D</td>
<td>0.01755</td>
<td>0.8737e-3</td>
<td>13.33e-9</td>
<td>350</td>
</tr>
</tbody>
</table>
The parameters of transformer are listed in Table E-3.

<table>
<thead>
<tr>
<th>Transformer Name</th>
<th>T1</th>
<th>T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal power (MVA)</td>
<td>2000</td>
<td>5000</td>
</tr>
<tr>
<td>Nominal frequency (Hz)</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Winding 1 connection</td>
<td>Delta (D1)</td>
<td>Delta (D1)</td>
</tr>
<tr>
<td>V1 Ph-Ph (Vrms)</td>
<td>13.8e3</td>
<td>13.8e3</td>
</tr>
<tr>
<td>R1 (pu)</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>L1 (pu)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Winding 2 connection</td>
<td>Yg</td>
<td>Yg</td>
</tr>
<tr>
<td>V2 Ph-Ph (Vrms)</td>
<td>500e3</td>
<td>500e3</td>
</tr>
<tr>
<td>R2 (pu)</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>L2 (pu)</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Magnetization resistance Rm (pu)</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Magnetization reactance Lm (pu)</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

AVR model is illustrated in Figure E-1 and the parameters of AVR are listed in Table E-4.

Figure E-1. AVR model ---IEEE type
Table E-4: AVR parameters for generators in studied power system

<table>
<thead>
<tr>
<th>AVR Location</th>
<th>$G_1$</th>
<th>$G_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$ (s)</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$K_d$</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$T_a$ (s)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$K_e$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_c$ (s)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$T_b$ (s)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$T_c$ (s)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_f$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$T_f$ (s)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$E_{RMIN}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E_{RMAX}$</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$K_p$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_{RMIN}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_{RMAX}$</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

The parameters of SVC are listed in Table E-5.

Table E-5 SVC parameters for studied power system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal voltage (V&lt;sub&gt;rms&lt;/sub&gt; Ph-Ph)</td>
<td>500e3</td>
</tr>
<tr>
<td>Nominal reactive power (MVAr)</td>
<td>1050</td>
</tr>
<tr>
<td>Reference voltage $V_{ref}$ (pu)</td>
<td>1.009</td>
</tr>
<tr>
<td>Droop $X_s$ (pu)</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Appendices

Appendix F

Derivation of Equations (5.2.6)- (5.2.14)

From Figure 5-3, the following basic circuit equations are obtained:

\[ I = \left( V_s - V_{sh} \right) / (jX) \]  \hfill (F.1)

\[ I_1 = \left( V_{sh} + V_s - V_r \right) / j \left( X_{se} + X_1 \right) \]  \hfill (F.2)

\[ I_2 = \left( V_r - V_r \right) / (jX_2) \]  \hfill (F.3)

\[ I_s = I_1 + I_2 \]  \hfill (F.4)

\[ I_r = I_1 + I_2 \]  \hfill (F.5)

\[ I_{sh} = \left( V_{sh} - V_r \right) / (jX_{sh}) \]  \hfill (F.6)

\[ I = I_1 + I_{sh} \]  \hfill (F.7)

\[ V_{set} = V_{sh} + V_{se} \]  \hfill (F.8)

From (F.1), (F.2), (F.6) and (F.7), \( V_{sh} \) can be derived:

\[ V_{sh} = \left[ X_{sh} \left( X_{se} + X_1 \right) \bar{V}_z + X \left( X_{se} + X_1 \right) \bar{V}_{sh} + XX_{sh} \bar{V}_r - XX_{sh} \bar{V}_{se} \right] / X_{eq}^2 \]  \hfill (F.9)

where \( X_{eq}^2 = \left( X_{se} + k_1 X \right) X + XX_{sh} + \left( X_{se} + k_1 X \right) X_{sh} \).

From (F.8) and (F.9), \( V_{set} \) can be derived:

\[ V_{set} = \left[ X_{sh} \left( X_{se} + X_1 \right) \bar{V}_z + X \left( X_{se} + X_1 \right) \bar{V}_{sh} + XX_{sh} \bar{V}_r + \left( X_{se} + X_1 \right) \left( X + X_{sh} \right) \bar{V}_{se} \right] / X_{eq}^2 \]  \hfill (F.10)

From (F.1), (F.3), (F.4) and (F.9), \( I_s \) can be derived:

\[ I_s = \left( \bar{V}_s - \bar{V}_r \right) / (jX_2) + \left[ \left( X_1 + X_{se} + X_{sh} \right) \bar{V}_z - \left( X_1 + X_{se} + X_{sh} \right) \bar{V}_{sh} - X_{sh} \bar{V}_r + X_{sh} \bar{V}_{se} \right] / (jX_{eq}^2) \]  \hfill (F.11)

From (F.1), (F.3), (F.5) and (F.9), \( I_r \) can be derived:

\[ I_r = \left( \bar{V}_s - \bar{V}_r \right) / (jX_2) + \left[ X_{se} \bar{V}_z + X \bar{V}_{sh} + \left( X + X_{se} \right) \left( \bar{V}_r - \bar{V}_r \right) \right] / (jX_{eq}^2) \]  \hfill (F.12)

From (F.6) and (F.9), \( I_{sh} \) can be derived:

\[ I_{sh} = \left[ \left( X_1 + X_{se} \right) \bar{V}_z + X \left( \bar{V}_r - \bar{V}_{se} \right) - \left( X + X_1 + X_{se} \right) \bar{V}_{ra} \right] / (jX_{eq}^2) \]  \hfill (F.13)

From (F.1) and (F.9), \( I \) can be derived:
From (F.2) and (F.9), \( I_1 \) can be derived:
\[
I_1 = \left[ X_{sh} \bar{V}_s + X \bar{V}_{sh} + (X + X_{sh}) \left( \bar{V}_{sh} - \bar{V}_r \right) \right] / (jX_{eq})
\] (F.15)

The sending-end power and the receiving-end power are expressed as:
\[
\bar{S}_s = \bar{V}_s \bar{I}_s^* = \frac{V_s V_r \sin \delta}{X_2} + \frac{(X_1 + X_{se}) V_{se} V_r \sin (\delta - \delta_{se}) + X_{sh} V_{sh} \sin \delta - X_{sh} V_{sh} \sin (\delta - \delta_{se})}{X_{eq}^2}
\]
\[+ j \left[ \frac{X_{sh} V_{sh} \sin \delta + X V_{sh} \sin \delta + X V_{sh} \sin \delta_{sh} + (X + X_{sh}) V_{sh} \sin \delta_{sh}}{X_{eq}^2} \right] \]
(F.16)

\[
\bar{S}_r = \bar{V}_r \bar{I}_r^* = \frac{V_s V_r \sin \delta}{X_2} + \frac{(X_1 + X_{se}) V_{se} V_r \sin (\delta - \delta_{se}) + X_{sh} V_{sh} \sin \delta - X_{sh} V_{sh} \sin (\delta - \delta_{se})}{X_{eq}^2}
\]
\[+ j \left[ \frac{X_{sh} V_{sh} \cos (\delta - \delta_{sh}) + X V_{sh} \cos (\delta - \delta_{sh}) + (X + X_{sh}) (V_{se}^2 - V_{sh} V_r \cos \delta_{se})}{X_{eq}^2} \right] \]
(F.17)

From (F.16) and (F.17), \( P_s, Q_s, P_r \) and \( Q_r \) can be derived.

The power exchange of the voltage sources \( V_{se} \angle \delta_{se} \) and \( V_{sh} \angle \delta_{sh} \) with the system are:
\[
\bar{S}_{se} = \bar{V}_{se} \bar{I}_1^* = \frac{X_{sh} V_{sh} \sin (\delta - \delta_{se}) + X V_{sh} \sin (\delta_{sh} - \delta_{se}) + (X + X_{sh}) V_{sh} \sin \delta_{se}}{X_{eq}^2}
\]
\[+ j \left[ \frac{X_{sh} V_{sh} \cos (\delta - \delta_{se}) + X V_{sh} \cos (\delta_{sh} - \delta_{se}) + (X + X_{sh}) (V_{se}^2 - V_{sh} V_r \cos \delta_{se})}{X_{eq}^2} \right] \]
(F.18)

\[
\bar{S}_{sh} = \bar{V}_{sh} \bar{I}_{sh}^* = \frac{(X_1 + X_{se}) V_{se} V_{sh} \sin (\delta - \delta_{sh}) - X V_{sh} \sin \delta_{sh} + X V_{sh} \sin (\delta_{sh} - \delta_{se})}{X_{eq}^2}
\]
\[+ j \left[ \frac{(X_1 + X_{se}) V_{se} \cos (\delta_{sh} - \delta_{se}) + X V_{sh} \cos (\delta_{sh} - \delta_{se}) - (X + X_{se}) V_{sh}^2}{X_{eq}^2} \right] \]
(F.19)

From (F.18) and (F.19), \( P_{se} \) and \( P_{sh} \) can be derived.

The power at the terminal of UPFC is:
\[\bar{S}_{set} = \bar{V}_{set}^* = \frac{V_r \left[ X_{sh} V_s \sin \delta + X V_{sh} \sin \delta_{sh} + (X + X_{sh}) V_{se} \sin \delta_{se} \right]}{X^2_{eq}} \]

\[+ j \left\{ \frac{V_r \left[ X_{sh} V_s \cos \delta + X V_{sh} \cos \delta_{sh} + (X + X_{sh}) (V_w \cos \delta_{se} - V_{se}) \right]}{X^2_{eq}} \right\} \]

\[+ \frac{2X_t X_{sh} V_s \left\{ (X + X_{sh}) \left[ V_{se} \cos (\delta - \delta_{se}) - V_{r} \cos \delta \right] + X V_{sh} \cos (\delta - \delta_{sh}) \right\}}{X^4_{eq}} \]

\[+ j \frac{2X_t (X + X_{sh}) \left\{ X_{sh} \left[ V_{se} \cos (\delta_{sh} - \delta_{se}) - V_{r} \cos \delta_{sh} \right] - (X + X_{sh}) V_{se} V_{r} \cos \delta_{se} \right\}}{X^2_{eq}} \]  \hspace{1cm} (F.20)

From (F.20), \( P_{set} \) and \( Q_{set} \) can be derived.

From (F.9), the magnitude of voltage \( V_{sh} \) can be derived as (5.2.14).
Appendix G

Derivation of $f(\delta_{se})$

From Figure 5-3,

$$P_s = \frac{V_sV_r\sin\delta}{k_2X} + \frac{V_sV_{sh}\sin(\delta - \delta_{sh})}{X}$$  \hspace{1cm} (G.1)

$$P_r = \frac{V_sV_r\sin\delta}{k_2X} + \frac{(V_sV_{sh}\sin\delta_{sh} + V_sV_{se}\sin\delta_{se})}{(X_{se} + k_1X)}$$  \hspace{1cm} (G.2)

Combine (G.1), (G.2) and $P_r = P_s + P_d$, the following equation can be obtained,

$$(X_{se} + k_1X)V_sV_{sh}\sin(\delta - \delta_{sh}) - V_rV_{sh}X\sin\delta_{sh} - V_rV_{se}X\sin\delta_{se} + (X_{se} + k_1X)XP_d = 0$$  \hspace{1cm} (G.3)

(G.3) can be rewritten as,

$$V_{sh}\left[V_r(X_{se} + k_1X)\sin\delta\left(1 - \sin^2\delta_{sh} - \cos\sin\delta_{sh}\right) - V_rX\sin\delta_{sh}\right] = V_sV_{se}X\sin\delta_{se} - (X_{se} + k_1X)XP_d$$  \hspace{1cm} (G.4)

From (G.4), $V_{sh}\sin\delta_{sh}$ can be expressed as,

$$V_{sh}\sin\delta_{sh} = \frac{[(X_{se} + k_1X)V_s\cos\delta + V_rX][X_{se} + k_1X]XP_d - V_rV_{se}X\sin\delta_{se}]}{(X_{se} + k_1X)^2V_s^2 + V_r^2X^2 + 2(X_{se} + k_1X)XV_r\cos\delta}$$

$$+ \frac{(X_{se} + k_1X)V_s\sin\delta\left((X_{se} + k_1X)^2V_s^2 + X^2V_r^2 + 2(X_{se} + k_1X)XV_r\cos\delta\right) - V_rV_{se}X\sin\delta_{se} - (X_{se} + k_1X)XP_d}{(X_{se} + k_1X)^2V_s^2 + V_r^2X^2 + 2(X_{se} + k_1X)XV_r\cos\delta}$$  \hspace{1cm} (G.5)

Substitute (G.5) into (G.2)

$$P_r = \frac{V_sV_r\sin\delta}{k_2X} + \frac{V_sV_{sh}\sin\delta_{sh}}{(X_{se} + k_1X)} + \frac{V_r\left[(X_{se} + k_1X)V_s\cos\delta + V_rX\right][X_{se} + k_1X]XP_d - V_rV_{se}X\sin\delta_{se}}{(X_{se} + k_1X)[(X_{se} + k_1X)^2V_s^2 + V_r^2X^2 + 2(X_{se} + k_1X)XV_r\cos\delta]}$$

$$+ \frac{V_sV_r\sin\delta\left((X_{se} + k_1X)^2V_s^2 + X^2V_r^2 + 2(X_{se} + k_1X)XV_r\cos\delta\right) - V_rV_{se}X\sin\delta_{se} - (X_{se} + k_1X)XP_d}{(X_{se} + k_1X)^2V_s^2 + V_r^2X^2 + 2(X_{se} + k_1X)XV_r\cos\delta}$$  \hspace{1cm} (G.6)
There are sufficient results now for one to determine the condition of maximum power transfer in a UPFC-DG compensated line. One could make use of (G.6) by differentiating $P_r$ with respect to $\delta_{se}$ and set the resulting equation $f(\delta_{se})$ to zero. The expression so obtained, $f(\delta_{se})$, is as shown in (G.7).

$$f(\delta_{se}) = \frac{d P_r}{d \delta_{se}} = V_r V_{se} \cos \delta_{se} \left\{ \frac{1}{(X_{se} + k_i X)} - \frac{(X_{se} + k_i X)V \cos \delta - X^2 V_r^2}{(X_{se} + k_i X)^2 V_r^2 + X^2 V_r^2 + 2(X_{se} + k_i X)V \cos \delta} \right\} \frac{V_r \sin \delta[V_r V_{se} \sin \delta_{se} - (X_{se} + k_i X) P_d]}{\sqrt{[(X_{se} + k_i X)^2 V_r^2 + X^2 V_r^2 + 2(X_{se} + k_i X)V \cos \delta] \left[V_r V_{se} X \sin \delta_{se} - (X_{se} + k_i X) X P_d\right]^2}}$$

(G.7)

From (G.7), it can be readily found that maximum $P_r$ occurs when $\delta_{se} = 90^\circ$. This $\delta_{se}$ value is denoted as the critical angle, $\delta_{se, cr}$. The maximum power that can be transferred is denoted herewith as $P_{r \text{max}}$. It can be obtained by substituting $\delta_{se}$ with $\delta_{se, cr}$ into (G.6).
Appendices

Appendix H

Typical 100 kW SOFC Power Plant Experimental Data Used for Simulation Studies

Table H-1: 100 kW SOFC power plant data [157]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Representation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rate}$</td>
<td>Rated output power</td>
<td>100 kW</td>
</tr>
<tr>
<td>$V_{dc,rate}$</td>
<td>Rated FC terminal voltage</td>
<td>330 V</td>
</tr>
<tr>
<td>$T$</td>
<td>Operation temperature</td>
<td>1273 °K</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Ideal standard potential</td>
<td>1.18 V</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Number of series cells in the stack</td>
<td>384</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Modeling constant</td>
<td>$0.993 \times 10^{-3}$ mol/(s.atm)</td>
</tr>
<tr>
<td>$u_s$</td>
<td>Fuel utilization factor setting</td>
<td>0.8</td>
</tr>
<tr>
<td>$K_{H2}$</td>
<td>Hydrogen valve molar constant</td>
<td>0.843 mol/(s.atm)</td>
</tr>
<tr>
<td>$K_{H2O}$</td>
<td>Water valve molar constant</td>
<td>0.281 mol/(s.atm)</td>
</tr>
<tr>
<td>$K_{O2}$</td>
<td>Oxygen valve molar constant</td>
<td>2.52 mol/(s.atm)</td>
</tr>
<tr>
<td>$\tau_{H2}$</td>
<td>Hydrogen flow response time</td>
<td>26.1 s</td>
</tr>
<tr>
<td>$\tau_{H2O}$</td>
<td>Water flow response time</td>
<td>78.3 s</td>
</tr>
<tr>
<td>$\tau_{O2}$</td>
<td>Oxygen flow response time</td>
<td>2.91 s</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>Fuel processor response time</td>
<td>5 s</td>
</tr>
<tr>
<td>$r$</td>
<td>Ohmic loss</td>
<td>0.126 Ω</td>
</tr>
<tr>
<td>$r_{H,O}$</td>
<td>Ratio of hydrogen and oxygen</td>
<td>1.145</td>
</tr>
<tr>
<td>$b_{act}$</td>
<td>Tafel slope</td>
<td>0.11</td>
</tr>
<tr>
<td>$a_{act}$</td>
<td>Tafel constant</td>
<td>0.05</td>
</tr>
<tr>
<td>$a_{con}$</td>
<td>Concentration loss constant</td>
<td>$10^{-4}$ V</td>
</tr>
<tr>
<td>$b_{con}$</td>
<td>Concentration loss constant</td>
<td>$8 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Appendices

Appendix I

Phasor Diagram Corresponding to the Conditions Described in Table 7-1

Figure I-1 Phasor diagram corresponding to Case 1 of Tables 7-1 and I-1

Figure I-2 Phasor diagram corresponding to Case 2 of Tables 7-1 and I-1
Figure I-3 Phasor diagram corresponding to Case 4 of Tables 7-1 and I-1

Figure I-4 Phasor diagram corresponding to Case 5 of Tables 7-1 and I-1
Figure I-5 Phasor diagram corresponding to Case 6 of Tables 7-1 and I-1

Figure I-6 Phasor diagram corresponding to Case 7 of Tables 7-1 and I-1
Figure I-7 Phasor diagram corresponding to Case 8 of Tables 7-1 and I-1

Figure I-8 Phasor diagram corresponding to Case 9 of Tables 7-1 and I-1
Figure I-9 Phasor diagram corresponding to Case 10 of Tables 7-1 and I-1
Table 1-1: Expressions for the computation of injection voltage under the minimum energy injection/absorption strategy

| Case | $|V_p|$ | $|V_i|$ | $\gamma$ | Sector |
|------|--------|--------|----------|--------|
| 1(a) | $\leq \sqrt{V_{thv}^2 + V_{i,m}^2 - 2V_{thv}V_{i,m}\sin \theta}$ | $\geq V_{thv} - V_{i,m}$ | $\theta + \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | AB in Fig. 1-1 |
| 1(b) | $\geq \sqrt{V_{thv}^2 + V_{i,m}^2 - 2V_{thv}V_{i,m}\sin \theta}$ | $V_{i,m}$ | $90^\circ$ | BD in Fig. 1-1 |
| 1(c) | $\leq \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m}\sin \theta}$ | $\geq V_{thv}$ | $-90^\circ$ | DE in Fig. 1-1 |
| 1(d) | $\geq \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m}\sin \theta}$ | $V_{i,m}$ | $\theta - \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | EF in Fig. 1-1 |
| 2(a) | $\leq \sqrt{V_{thv}^2 + V_{i,m}^2 - 2V_{thv}V_{i,m}\sin \theta}$ | $\geq V_{thv} - V_{i,m}$ | $\theta + \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | AB in Fig. 1-2 |
| 2(b) | $\geq \sqrt{V_{thv}^2 + V_{i,m}^2 - 2V_{thv}V_{i,m}\sin \theta}$ | $V_{i,m}$ | $90^\circ$ | BD in Fig. 1-2 |
| 2(c) | $\leq \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m}\sin \theta}$ | $\geq V_{thv}$ | $-90^\circ$ | DE in Fig. 1-2 |
| 2(d) | $\geq \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m}\sin \theta}$ | $V_{i,m}$ | $\theta - \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | EF in Fig. 1-2 |
| 3(a) | $\leq V_{thv}\cos \theta - \sqrt{V_{thv}^2 - V_{i,m}^2 \sin^2 \theta}$ | $\geq V_{thv} - V_{i,m}$ | $\theta + \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | AB in Fig. 7-6 |
| 3(b) | $\geq V_{thv}\cos \theta - \sqrt{V_{thv}^2 - V_{i,m}^2 \sin^2 \theta}$ | $V_{i,m}$ | $\arcsin \left(\frac{V_{thv}\sin \theta}{\sqrt{V_{thv}^2 + V_p^2 - 2V_{thv}V_p \cos \theta}}\right)$ | BC in Fig. 7-6 |
| 3(c) | $\leq V_{thv}\cos \theta$ | $\geq V_{thv}$ | $90^\circ$ | CD in Fig. 7-6 |
| 3(d) | $\geq V_{thv}\cos \theta$ | $V_{i,m}$ | $\theta - \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | DE in Fig. 7-6 |
| 3(e) | $\geq \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m}\sin \theta}$ | $\geq V_{thv}$ | $\theta + \arccos \left(\frac{V_{thv}^2 + V_{i,m}^2 - V_p^2}{2V_{thv}V_{i,m}}\right)$ | EF in Fig. 7-6 |
| 4(a) | $\leq V_{thv}\cos \theta - \sqrt{V_{thv}^2 - V_{i,m}^2 \sin^2 \theta}$ | $\geq V_{thv} + V_{i,m}$ | $90^\circ$ | AB in Fig. 1-3 |

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### Appendices

<table>
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<th>Value of ( \theta )</th>
<th>Region</th>
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<td>4(b)</td>
<td>( V_{thv} \cos \theta - \sqrt{V_{i,m}^2 - V_{thv}^2 \sin^2 \theta} \leq V_{thv} )</td>
<td>( \theta + \arccos \left( \frac{V_{thv}^2 + V_{i,m}^2 - V_{p}^2}{2V_{thv}V_{i,m}} \right) )</td>
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<td>4(c)</td>
<td>( V_{thv} \sin \theta - \sqrt{V_{p}^2 - V_{thv}^2 \cos^2 \theta} \leq V_{thv} )</td>
<td>( 90^\circ )</td>
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<td>4(d)</td>
<td>( \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m} \sin \theta} \leq V_{thv} )</td>
<td>( -90^\circ )</td>
<td>DE in Fig. 1-3</td>
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<td>4(e)</td>
<td>( \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m} \cos \eta} \leq V_{thv} )</td>
<td>( V_{i,m} )</td>
<td>EF in Fig. 1-3</td>
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<td>5(a)</td>
<td>( V_{thv} \sin \theta - \sqrt{V_{p}^2 - V_{thv}^2 \cos^2 \theta} \leq V_{thv} )</td>
<td>( \theta - \arccos \left( \frac{V_{thv}^2 + V_{i,m}^2 - V_{p}^2}{2V_{thv}V_{i,m}} \right) )</td>
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<td>5(c)</td>
<td>( \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m} \sin \theta} \leq V_{thv} )</td>
<td>( -90^\circ )</td>
<td>DE in Fig. 1-4</td>
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<td>5(d)</td>
<td>( \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m} \cos \eta} \leq V_{thv} )</td>
<td>( V_{i,m} )</td>
<td>EF in Fig. 1-4</td>
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<td>6(a)</td>
<td>( \sqrt{V_{thv}^2 + V_{i,m}^2 - 2V_{thv}V_{i,m} \sin \theta} \leq V_{thv} )</td>
<td>( V_{i,m} )</td>
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<td>( V_{thv} - V_{i,m} \leq \sqrt{V_{thv}^2 + V_{i,m}^2 - 2V_{thv}V_{i,m} \sin \theta} )</td>
<td>( 90^\circ )</td>
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<tr>
<td>7(b)</td>
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<td>( 90^\circ )</td>
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<td>DE in Fig. 1-6</td>
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<tr>
<td>7(d)</td>
<td>( \sqrt{V_{thv}^2 + V_{i,m}^2 + 2V_{thv}V_{i,m} \cos \eta} \leq V_{thv} )</td>
<td>( V_{i,m} )</td>
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Fig. 1-3: ABCD

Fig. 1-4: DEFG

Fig. 1-5: ABD

Fig. 1-6: ABD
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<th>Solution</th>
<th>Area</th>
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<td>8(a)</td>
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<td>8(d)</td>
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<td>$\leq \sqrt{V_{th}^2 + V_{i,m}^2 + 2V_{th}V_{i,m} \cos \eta}$</td>
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<td>9(e)</td>
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<td>$V_{th} \sin \theta$</td>
<td>EF in Fig. 1-9</td>
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Vita

Wang Qi was born in P. R. China. She received her B.E. and M.S. degrees from Tsinghua University, China, in 1999 and 2002 respectively. She joined the Nanyang Technological University in March 2003 and is now pursuing her Ph.D. degree. Her research areas include power system analysis and control, power quality and the use of Power Electronics devices for power system applications.

Research related to this dissertation has resulted in the following publications:


