ULTRASONIC TRAPPING OF SMALL PARTICLES

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Particle trapping by acoustic method has been developed for decades. For conventional trapping of particles, there are two methods: magnetic trapping and optical trapping. Magnetic trapping needs the magnetization of the treated particles. Optical trapping typically uses controlled laser beams. Its weakness is that the reflection and absorption of particles have great influence on the trapping capability of particles; moreover it is not suitable to trap large number of particles. In conventional acoustic methods of trapping small particles, standing ultrasonic wave and focused ultrasonic beam are used. Standing ultrasonic wave uses ultrasonic far-field to trap particles, which may result in low utilization energy efficiency; particles can be trapped only at nodal points, so the capability of manipulating a particular particle is weak. Ultrasonic focused beams use ultrasonic far-field to trap particles as well; the capability of manipulating a particular particle by focused beams is relatively strong, but it can’t trap large number of particles. Also devices in these methods are not compact.

To response to the above stated challenges, in the work reported by the thesis, a new operating principle of using ultrasonic field near the radiation surface to trap small particles has been proposed. Based on this principle, three novel compact ultrasonic transducers have been designed to trap different targeted particles, respectively. They are W-shaped transducer, acoustic needle and vibrating rod. The trapping mechanism of the transducers has been quantitatively analyzed, and performance of the transducers has been measured.
and analyzed. Based on the experimental and theoretical analysis, guidelines to optimize the trapping capability for each transducer have been proposed.

The W-shaped transducer has merits of trapping large and heavy particles. It uses the whole sharp edges of the two metal strips with large vibration to trap particles. Heavy particles up to a weight of 256 milligrams per particle can be trapped onto the sharp edge at a driving voltage of 65 $V_{\text{rms}}$ at resonance.

Acoustic needle which has the most compact structure can be used to trap small and light particles at a particular location as it uses the tip of needle to trap particle. It has advantages in transportation and separation of small particles. Small particles with a minimum radius of 5 $\mu$m can be trapped in water.

The ultrasonically vibrating rod has potential capability to trap more particles. The vibrating surface along the whole length of the rod is used to trap particles onto the surface. More than 50 flying color seeds can be trapped one time onto the surface of the vibrating rod at a driving voltage of 50 $V_{\text{rms}}$ at resonance.

Furthermore, a method to calculate the acoustic radiation force on a particle with arbitrary shape in an arbitrary sound field has been proposed and developed in this thesis. It is based on the combination of the analysis of Finite Element Method (FEM) and the energy density difference theory of acoustic radiation force. The conventional method is not suitable for calculating the acoustic radiation force acting on particle with other shape than sphere in
complicated sound fields, such as the one near radiation surface. These
limitations can by improved by the calculation method developed in this thesis.

In addition, the sound speed for some materials which are commonly
seen in daily life but whose sound speeds are not reported yet, has been
measured by a sound speed meter. It is very useful in the calculation of the
acoustic radiation force acting on particles.

This research work has potential applications in many areas, such as to
remove particle from a solid surface, to separate particle from a mixture, to
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<tr>
<td>( a )</td>
<td>radius of a rod</td>
<td>m</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>Sound speed in a fluid medium</td>
<td>m/s</td>
</tr>
<tr>
<td>( c_s )</td>
<td>Sound speed in a particle</td>
<td>m/s</td>
</tr>
<tr>
<td>( C_{tr} )</td>
<td>Trapping capability coefficient</td>
<td>( \mu \text{m}^{-1} )</td>
</tr>
<tr>
<td>( d )</td>
<td>Vibration amplitude of a transducer</td>
<td>( \mu \text{m} )</td>
</tr>
<tr>
<td>( f )</td>
<td>Operating frequency</td>
<td>kHz</td>
</tr>
<tr>
<td>( F )</td>
<td>Acoustic radiation force</td>
<td>N</td>
</tr>
<tr>
<td>( k )</td>
<td>Wave number</td>
<td>m(^{-1})</td>
</tr>
<tr>
<td>( K )</td>
<td>Kinetic energy density</td>
<td>N/m(^2)</td>
</tr>
<tr>
<td>( P )</td>
<td>Sound pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>( P_m )</td>
<td>Pressure amplitude at a sound point</td>
<td>Pa</td>
</tr>
<tr>
<td>( R )</td>
<td>radius of a sphere</td>
<td>m</td>
</tr>
<tr>
<td>( S )</td>
<td>Surface of a particle</td>
<td>m(^2)</td>
</tr>
<tr>
<td>( u )</td>
<td>Vibration velocity of a fluid medium</td>
<td>m/s</td>
</tr>
<tr>
<td>( U )</td>
<td>Potential energy density</td>
<td>N/m(^2)</td>
</tr>
<tr>
<td>( v )</td>
<td>Vibration velocity of a particle</td>
<td>m/s</td>
</tr>
<tr>
<td>( V )</td>
<td>Volume of a particle</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Attenuation coefficient</td>
<td>m(^{-1})</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>Density of a fluid medium</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>Density of a particle</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>( w )</td>
<td>Angular frequency</td>
<td>rad/s</td>
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CHAPTER 1  INTRODUCTION

1.1 Motivation

Particle trapping is the base and core of many techniques such as the separation, filtration and agglomeration of particles. The technology of trapping small particles is increasingly important in pharmaceutical industry, environmental engineering, chemical engineering, household industry, etc [1-4].

In many industries, companies are always searching for new and improved particle trapping methods for the collection, isolation, purification and concentration of many macromolecules. For example, in household industry, the suction of dust particles by the vacuum cleaner is caused by a difference in air pressure between the pressure inside the hose of machine and atmospheric pressure outside the machine. However, to reduce the pressure inside the hose from normal atmospheric pressure needs to displace the entire air inside the hose, which requires large amount of energy. Therefore an effective and economical particle trapping technique is very important in those areas.

Particle trapping by acoustic method has been developed for decades [5-22]. Coakley et al. investigated the cell manipulation in ultrasonic standing wave fields in 1989 [5]. Wu (1991) proposed acoustic tweezers using two collimated focused ultrasonic beams, in which the acoustic tweezers can trap latex particles of 270 µm diameter and clusters of frog eggs [6]. Takeuchi, Abe and Yamanouchi (1994) developed a VHF-range leaky wave transducer to trap
and transport 50 µm diameter glass spheres in water [7]. Other significant works in this area are reported in references [8-22].

For conventional trapping of particles, there are two methods: magnetic trapping and optical trapping [23]. Magnetic trapping needs the magnetization of the treated particles. For optical trapping typically controlled laser beams, its weakness is that the particles can hardly be trapped in the optical potential well if the refractive index of the particles is smaller than that of the surrounding fluid, and the reflection and absorption of the particles are not proper; also it is not suitable to trap large number of particles. In conventional acoustic methods of trapping small particles, standing ultrasonic wave or focused ultrasonic beam can be used. Standing ultrasonic wave uses ultrasonic far-field to trap particles, which may result in low utilization energy efficiency; particles can be trapped only at nodal points, so the capability of manipulating a particular particle is weak. Ultrasonic focused beams use ultrasonic far-field to trap particles as well, which may result in low utilization energy efficiency; the capability of manipulating a particular particle by focused beams is relatively strong, but it can’t trap large number of particles. Also devices in these methods are not compact.

Therefore to response to the above stated challenges, this thesis aims at a method of trapping particles which satisfies the following conditions:

(1) New operating principles use ultrasonic near-field to trap particles.

Conventionally, the boundary between acoustic near-field and far-field is defined as \( z_g = \frac{a^2}{\lambda} \), where \( a \) is the radius of source and \( \lambda \) is the wavelength [24]. In power ultrasound applications, \( z_g \) can be reduced to
around 1 mm [25]. In this thesis, ultrasonic near-field refers to the sound field within a distance of 1 mm to the acoustic radiation surface. Only the sound field near the radiation surface is used, thus the utilization energy efficiency is improved.

(2) Simple and compact structures which are portable in manipulating particles.

(3) Different shaped particles with different sizes can be trapped. In practical applications, particles have different shapes and sizes, for example dust particles which have various shapes and sizes, such as flakes, hair, etc.

1.2 Objectives

The objectives of this work are:

(1) To explore new operating principle of novel transducers.

(2) To propose the detailed structure of the transducers.

(3) To fabricate the devices and clarify the characteristics of the transducers.

(4) To develop detailed theoretical model for the proposed ultrasonic transducers and propose design guidelines to optimize the trapping capability.

1.3 Major contributions

The major contributions achieved in this study are summarized as follows:
A new operating principle to use ultrasonic field near the radiation surface to trap particles has been proposed. In conventional acoustic methods of trapping small particles, standing ultrasonic wave or focused ultrasonic beam can be used. Standing ultrasonic wave uses ultrasonic far-field to trap particles, which may result in low utilization energy efficiency; particles can be trapped only at nodal points, so the capability of manipulating a particular particle is weak. Ultrasonic focused beams use ultrasonic far-field to trap particles as well, which may result in low utilization energy efficiency; the capability of manipulating a particular particle by focused beams is relatively strong, but it can’t trap large number of particles. Also devices in these methods are not compact. To response to the above stated challenges, in the work reported by the thesis, the new operating principle has been proposed. Based on this principle, three novel ultrasonic transducers have been proposed and researched which have relatively simple and compact structure and use ultrasonic field near the radiation surface. They are W-shaped transducer, acoustic needle and vibrating rod. They can trap or suck particles onto radiation surface by using the ultrasonic field near the radiation surface. Particles with a size from micrometer to millimeter and with a density from 0.55 g/cm³ to 1.55 g/cm³ can be trapped by the above transducers in water, air and other fluid.

The structure of a W-shaped transducer has been proposed and investigated, the prototype has been fabricated, the trapping performance has been investigated and measured, a theoretical model
has been established to analyze the trapping mechanism, and guidelines for optimizing the trapping capability of the transducer have been obtained by experimental and theoretical analysis. The two sharp edges of the vibrating metal strip of the W-shaped transducer are used to trap particles onto the sharp edges. The main characteristics of this transducer include: the trapping capability increases with the increase of vibration displacement at the tip of the metal strip; the transducer can trap up to a weight of 256 milligrams per particle; particles which have the same mass and volume but different shapes and orientations have their own trapping capability. It is found that the operating frequency, the shape and dimensions of the metal strip affect the trapping capability, and the optimum design for this device can be achieved by choosing proper dimension of the length and thickness of metal strip, based on the resonance of the two metal strips.

(3) The structure of an acoustic needle has been proposed and investigated, the prototype has been fabricated, the trapping performance has been investigated and measured, a theoretical model has been established to analyze the trapping mechanism, and guidelines for optimizing the trapping capability of the transducer have been obtained by experimental and theoretical analysis. The tip of the acoustic needle which is in flexural vibration is used to trap small particles onto the tip. The main characteristics of this transducer include: small particles with a minimum radius of 5 µm can be trapped in water; the trapping capability increases with the increase of vibration at the tip of needle;
the trapped particles can be transported in water by moving the needle and the particle loss during the transportation may be prevented by using a strong enough vibration; the acoustic needle can also separate different particles in water by the difference in their densities. The optimum design can be obtained by choosing proper dimension of the length and radius of needle based on the resonance of the acoustic needle.

(4) The structure of an ultrasonically vibrating rod has been proposed and investigated, the prototype has been fabricated, the trapping performance has been investigated and measured, a theoretical model has been established to analyze the trapping mechanism, and guidelines for optimizing the trapping capability of the transducer have been obtained by experimental and theoretical analysis. The surface of the rod which is in the 0th order vibration mode (vibrating back and forth about its central axis along its length and the vibration is uniform) is used to trap small particles onto the vibrating surface along the whole length of the rod. The main characteristics of this device are: more than 50 particles such as plant seeds in water can be trapped each time by the vibrating rod onto its surface; in different fluids, the trapping capability of rod increases with the increase of fluid density; for particles with different sizes, smaller particles are easier to be trapped onto the surface of rod; the acoustic radiation force acting on a particle in water may change its direction when the sound speed in the particle is larger than some critical values; even if particles are not on the surface of the rod, it
is possible to attract them to the rod in certain distance range. The optimum design can be obtained by choosing proper dimension of the radius of the rod based on the optimization of ultrasonic field near the radiation surface and the particle.

(5) A method to calculate the acoustic radiation force and trapping capability of a particle with arbitrary shape in an arbitrary sound field has been proposed and developed in this thesis. It is based on the combination of the FEM (Finite Element Method) analysis and the energy density difference theory of acoustic radiation force. In the conventional method of calculating acoustic radiation force, there are some conditions needed to be satisfied: (i) the particle is sphere-shaped; (ii) the radius of the sphere $R$ is much smaller than the wavelength $\lambda$ of the sound wave ($kR \ll 1$); (iii) multiple scattering effect can’t be used. Multiple scattering means that after the incident wave is scattered by one object (for example: particle), the scattered wave by the object as an incident wave, is scattered again by another object (for example: radiation surface). Therefore, the conventional method is not suitable for calculating the acoustic radiation force acting on particle with other shape than sphere in complicated sound fields, such as the one near radiation surface. The calculation method developed in this thesis can be used to calculate the acoustic radiation force when the above conditions are not satisfied.
Five papers related to above work have been published in international journals; four conference papers related to this work have been published in the proceedings. Total number of published papers is nine (see author’s publications on Pages 128-129).

1.4 Organization of the thesis

This thesis includes 9 chapters.

In Chapter 1, the motivation, objectives, major contributions, and organization of the thesis are presented.

Chapter 2 presents the basic knowledge of ultrasound, acoustic radiation force, and the development of the acoustic trapping methods.

Chapter 3 presents the possibility of using ultrasonic near-field to trap particles.

In Chapter 4 and 5, the detailed operating mechanism and the performance of W-shaped transducer are clarified experimentally and theoretically, the optimum design is explored and established. And a method to calculate the acoustic radiation force acting on a particle of arbitrary shape in an arbitrary sound field is presented also.

In Chapter 6, the detailed operating mechanism and the performance of acoustic needle are clarified experimentally and theoretically, the optimum design is explored and established.
In Chapter 7, the detailed operating mechanism and the performance of vibrating rod are clarified experimentally and theoretically, the optimum design is explored and established.

Chapter 8 presents a method to measure the sound speed in solids, and the sound speed for some solid materials is listed.

Finally, the conclusions and recommendations for future work are presented in Chapter 9.
CHAPTER 2 LITERATURE REVIEW

This work is a research of novel transducers which use acoustic radiation force to trap small particles. A basic knowledge of the ultrasound, acoustic radiation force, and the development of the acoustic trapping methods are presented in this chapter.

2.1 Ultrasound

Ultrasound is the sound or wave propagation at the frequency beyond the audible range of human hearing sense (frequency > 20 kHz) [26].

Ultrasound is important for many applications because of its directivity, shorter wavelength and noiseless [27]. The directivity can be controlled by varying the frequency, where the higher the frequency, the greater the directivity. And at high frequencies, the wavelength could be shorter than the dimension of the samples to be measured. Since ultrasound is silent, it is helpful in preventing the damage of the eardrum due to operation noises. The production of ultrasound is used in many different fields, typically to penetrate a medium and measure the reflection signature or supply focused energy.

There are a vast number of applications of ultrasound, such as diagnostic imaging, detection of flaws in solids (non-destructive testing), ultrasonic cleaning, ultrasonic welding, etc [28-30].
2.2 Ultrasonic actuators

For most applications the generation of the ultrasound is achieved with a piezoelectric material, such as quartz, Rochelle salt, barium titanate (BaTiO$_3$), lead titanate (PbTiO$_3$), lead zirconate titanate (PZT), etc. The two complementary properties of piezoelectric materials are: direct piezoelectric effect (the production of an electric potential when stress is applied) and reverse piezoelectric effect (production of stress/strain when an electric field is applied) [31-32].

In engineering, ultrasonic actuators are devices which transform an input signal (mainly electrical signal) into motion. Usually they are made of piezoelectric materials. Ultrasonic actuators which use the reverse piezoelectric effect, have different types, such as ultrasonic motors, multilayer piezoelectric actuators, ultrasonic tweezers, etc.

(1) Ultrasonic motor

An ultrasonic motor is a motor which uses ultrasonic vibration – a type of elastic vibration – to obtain a driving force, which then drives the motor using friction [33]. Compared with conventional electromagnetic motor, ultrasonic motor can produce a high torque at a low speed with a high efficiency. Functionally, two types of motor are constructed: the rotary type and the linear type [34-35]. The ultrasonic motor is now used in many consumer and office electric requiring precision rotations over long periods of time, especially in the application of photographic lenses.
(2) Multilayer piezoelectric actuator

Multilayer piezoelectric actuators are small and high performance devices which produce large displacement and strong forces at low voltages [36]. It is usually co-fired multilayer ceramics typically with a height up to 2 – 3 mm and with up to 100 ceramic layers. It can be used in numerous applications, such as IT (hard disc drives), optics, telecommunication, instrumentation, automotive, nano-positioning, etc.

(3) Ultrasonic tweezer

Ultrasonic tweezer is a compact device whose vibration is excited by an ultrasonic transducer. It may have different kinds of structures, such as a twisted wire bundle, hypodermic needle, etc [37-38]. It can be used for manipulation of small particles, generation, rotation and transportation of liquid droplets, etc.

2.3 Acoustic radiation force

The acoustic radiation pressure is the time-averaged pressure acting on an obstacle in a sound field, which appears in the form of a steady unidirectional force even if there is no acoustic stream [39]. The force due to the acoustic radiation pressure is called acoustic radiation force. There are two types of radiation pressure: the Rayleigh radiation pressure and Langevin radiation pressure. The Rayleigh radiation pressure in the one-dimensional case is usually defined as the time-averaged pressure on the wall of a closed vessel
when an acoustic field is applied there. If an obstacle is placed in the path of a beam of an unconfined acoustic wave, the time-averaged force experienced by the obstacle is said to be due to the Langevin radiation pressure.

The mechanism of acoustic radiation force can be interpreted by the following two ways.

### 2.3.1 Difference in energy density

Generally, the acoustic radiation force is produced by a spatial change in the energy density of acoustic field. For example, in an acoustic sound field, on the left of the object, the energy density is $E_1$, while the energy density is $E_2$ on the right of the object. As $E_1$ is greater than $E_2$, there is a net acoustic radiation force produced due to the energy density difference.

One of the theories using the concept of energy density difference is the Hasegawa’s theory. The acoustic radiation force $\mathbf{F}$ due to the radiation pressure is given by integrating over the surface of an obstacle and averaging with respect to time [39]

$$
\mathbf{F} = \left( \iiint_{S_0} (K - U) n dS \right) - \left( \iiint_{S_0} \rho v_i v_{in} dS \right)
$$

(2.1)

where $K$ and $U$ are the kinetic energy and potential energy densities, $S_0$ is the surface of the obstacle, $v_i$ is the velocity of the obstacle where $v_{in} = v_i \cdot \mathbf{n}$, $\mathbf{n}$ is the outward-pointing unit normal vector of $dS$.

The followings are some other theories using the concept of energy density difference to calculate the acoustic radiation force.
(1) Kotani

Kotani calculated the Langevin radiation force on an immovable disk placed in the plane progressive sound field in an inviscid fluid [40].

(2) King

King presented a theoretical calculation of acoustic forces for a rigid sphere in plane standing or progressive wave field in an inviscid fluid [41]. Hasegawa verified King’s result, and he found that the radiation pressure by King’s method was a scalar [39]. Therefore King’s method is correct when both the sound field and the obstacle have a sort of symmetry [42].

(3) Westervelt

Westervelt derived a general expression for the force owing to radiation pressure acting on an object of any shape and having an arbitrary normal boundary impedance [43]. However it is very difficult to gain a correct understanding of his theory as it is [39].

(4) Yosioka and Kawasima

Yosioka and Kawasima extended King’s method to calculate the acoustic radiation force including the effects of a compressible sphere [44]. However the verification of derivation of the tensor term $\rho_0 u_1 v_1 n$ does not give explicitly.

(5) Gor’kov
Gor’kov used far-field scattering method to calculate the force acting on a sphere in an arbitrary acoustic field in ideal fluid [45]. Barmatz and Collas gave a simple expression according to Gor’kov’s method [45-46]. In this simplified method, the acoustic radiation force is calculated by a spatial change in the energy density of acoustic field. However, using this method, some conditions have to be satisfied: (1) the particle is sphere-shaped and the sphere is isotropic and compressible; (2) the radius of the sphere $R$ is much smaller than the wavelength $\lambda$ of the sound wave ($kR \ll 1$); (3) the surrounding fluid is nonviscous; (4) multiple scattering effect is not taken into account [46].

(6) **Roony and Nyborg**

Roony and Nyborg derived a simple expression of acoustic forces [47]. The force $F$ owing to the Langevin radiation pressure is measured by determining the time-averaged constraint force $Q$ which must be applied externally to maintain the obstacle at an equilibrium position. That is, they emphasize that one must take into account the external body force $Q$ to calculate the radiation force on an obstacle at an equilibrium position. Then the radiation force is defined as $F = - \langle Q \rangle$.

(7) **Chu and Apfel**

Chu and Apfel calculated the second-order force produced by sound beam directed normally at a plane target [48]. However Hasegawa considers that the paper involves not a few problems to be solved as follows [39]. According to the work of Chu and Apfel, the Rayleigh radiation pressure is not
s tensor, but a pressure. This analysis agrees with Hasegawa’s result for one-dimensional cases. For three-dimensional problems, they calculated the deformation produced by an acoustic beam impinging obliquely on an interfacial surface which separates a liquid from a gaseous medium of negligible inertia.

Other significant works of the acoustic radiation force are reported in [49-63]. It is known that most of the theories of acoustic radiation force in the above listed work are derived from the Navier-Stokes equations. Also it is the fact that there are more or less ambiguities, leaps in logic and unnecessary assumptions in many theories [39]. By a comparison, Hasegawa’s theory is relatively general which covers both the Rayleigh and Langevin radiation pressures, and is relatively convincing.

2.3.2 Phonon

In quantum mechanical representation, acoustic wave can be considered to be composed of phonons [64-65]. Like photons, phonons carry energy and momentum. The radiation pressure is interpreted as momentum transfer from phonons to the interface. Therefore, an object placed in an acoustic field experiences a force due to the momentum transfer from the wave to the object itself. When an acoustic wave strikes an object, part of its momentum is transferred to the object, giving rise to the acoustic radiation force phenomenon.
Using the phonon density \( n_p \), the energy density \( \varepsilon \) and momentum density \( \mu \) are presented as [64]

\[
\varepsilon = n_p \hbar \omega \quad (2.2)
\]

\[
\mu = n_p \hbar k \quad (2.3)
\]

where \( \omega = 2\pi f \) (\( f \) is the frequency), \( k = 2\pi / \lambda \) (\( \lambda \) is the wavelength) is the wave number, and \( h = h / 2\pi \) (\( h \) is Planck’s constant).

The change of momentum density when the acoustic wave strikes the object is

\[
\Delta \mu_{in} = (n_{in} + n_r - n_{th}) \hbar k \quad (2.4)
\]

where \( n_{in} \) is the incident phonon density, \( n_r \) is the reflected phonon density, \( n_{th} \) is the transmitted phonon density.

The acoustic radiation force \( f \) acting on the object can be written as

\[
f = \Delta \mu_{in} \frac{\Delta x}{\Delta t} = \Delta \mu_{in} c \quad (2.5)
\]

Using Eqs. (2.2) and (2.3), the momentum is represented as

\[
\mu = \varepsilon / c \quad (2.6)
\]

where \( c = \omega / k \) is the phase velocity of an acoustic wave.

The sound intensity \( I \) along the direction of wave propagation which is perpendicular to the surface of object is

\[
I = \varepsilon c = \frac{p_1^2}{2\rho c} \quad (2.7)
\]

where \( p_1 \) is the sound pressure, \( \rho \) is the density.

Therefore the acoustic radiation force is
\[ f = \frac{p_{in}^2 - p_{th}^2 + p_r^2}{2\rho c^2} \]  

(2.8)

where \( p_{in} \) is the incident wave sound pressure, \( p_r \) is the reflected wave sound pressure, and \( p_{th} \) is the transmitted wave sound pressure.

The phonon theory of the acoustic radiation force is verified by Sato [64]. For the travelling plane wave in liquid, the acoustic radiation force calculated from the phonon theory agrees with the Hasegawa’s energy density difference theory. However method using phonon theory to calculate the acoustic radiation force is not convenient in application.

### 2.4 Methods to trap small particles

#### 2.4.1 Conventional trapping methods

**(1) Magnetic trapping**

Magnetic trapping is usually achieved by a magnetic tweezer, which is a scientific instrument for exerting and measuring forces on magnetic particles using a magnetic field gradient [66]. Typical applications are sing-molecule micromanipulation, rheology of soft matter, and studies of force-regulated processes in living cells. Forces are typically on the order of pico- to nanonewtons.

The magnetic tweezers can be divided into several categories including translational or rotational, unipolar or multipolar. The simplest setup is the
unipolar translational magnetic tweezers. It consists of an electromagnet with a paramagnetic core material and a tip-shaped end. This results in a high field gradient around the tip. Any paramagnetic material within that gradient is magnetized and pulled towards the tip. The force magnitude depends on the magnitude and gradient of the magnetic field. While the magnitude can be controlled by the current that drives the electromagnet, the gradient depends on the distance to the tip of the core. Several electromagnets can be combined into a multipolar magnetic tweezers setup, allowing for three-dimensional translation, rotation and trapping of magnetic particles.

Fig. 2.1 General magnetic tweezers setup. A thin sample is observed with an inverted microscope, a CCD image is processed by a computer that drives the electromagnets to servo the bead position in real time.

Fig. 2.1 shows a general setup of magnetic tweezers [67]. The cell containing the magnetic particles in solution is held on the stage of an inverted microscope. The cell is typically a small capillary tube with a rectangular
section. Its top and bottom surfaces are of good optical quality. A system of six vertical electromagnets with their pole pieces arranged in a hexagonal pattern is placed just above the capillary tube. During the micromanipulation, a magnetic particle is located with nanometer accuracy by video analysis, and the position of particle can be determined in three spatial dimensions. The six-fold symmetry of the electromagnets allows rotation of the direction of magnetic field and hence of the magnetic particle itself. The force applied to the particle can be directly evaluated by Brownian motion analysis.

The magnetic trapping by magnetic tweezers needs the magnetization of the treated particles. And the devices in this method are not compact.

(2) Optical trapping

Optical trapping is usually achieved by an optical tweezer, which is a scientific instrument that uses a highly focused laser beam to provide an attractive or repulsive force (typically on the order of piconewtons), depending on the refractive index mismatch to physically hold and move microscopic dielectric objects [68]. Optical tweezers have been particularly successful in studying a variety of biological systems in recent years.

The basic principle behind optical tweezers is the momentum transfer associated with bending light. Light carries momentum that is proportional to its energy and in the direction of propagation. Any change in the direction of light by reflection or refraction will result in a change of the momentum of light. If an object ends the light, changing its momentum, conservation of momentum
requires that the object must undergo an equal and opposite momentum change. This gives rise to a force acting on the object.

In a typical optical tweezer setup, as shown in Fig. 2.2, the incoming light comes from a laser which has a Gaussian intensity profile. Basically, the light at the center of the beam is brighter than the light at the edges. When this light interacts with a bead, the light rays are bent according the laws of reflection and refraction. The sum of the forces from all such rays can be split into two components: the scattering force (pointing in the direction of the incident light) and the gradient force (arising from the gradient of the Gaussian intensity profile and pointing in \(x-y\) plane towards the center of the beam). The gradient force is a restoring force that pulls the bead into the center. If the contribution to the scattering force of the refracted rays is larger than that of the reflected rays then a restoring force is also created along the \(z\)-axis, and a stable trap will exist.

![Fig. 2.2 Optical tweezers setup.](image-url)
Optical tweezers have been used to trap dielectric spheres, viruses, bacteria, living cells, organelles, small metal particles and even strands of DNA. Applications include confinement and organization (e.g. for cell sorting), tracking of movement (e.g. of bacteria), application and measurement of small forces, and altering of larger structures (such as cell membranes).

The weakness of the optical trapping of micron-sized particles is that the particles can hardly be trapped in the optical potential well if the refractive index of the particles smaller than that of the surrounding fluid, and the reflection and absorption of the particles are not proper; also its is not suitable to trap large number of particles [69].

### 2.4.2 Ultrasonic trapping methods

Studying the researches about ultrasonic particle manipulation [5-22], it is seen that there are typically two methods to trap small particles: one is using focused ultrasonic beams, and the other is using standing ultrasonic wave.

**(1) Focused ultrasonic beams**

Acoustic tweezers is one device which uses the focused ultrasonic beam to trap small particles [6]. It is known that a stable force potential well at the physical focal point can be created by radiation pressure of a focused ultrasonic beam [70]. Based on this concept, acoustic tweezers can generate a force potential well by two collimated focused ultrasonic beams (3.5 MHz) propagating along opposite directions, as shown in Fig. 2.3. The trapped
particles in the potential well such as latex particles of 270 µm diameter can be moved axially or laterally by moving one of the PZT focusing transducers that generate the ultrasonic focused beams. The axial position of the trapped object can also be maneuvered by tuning the frequency of the electrical voltage applied to the transducers.

Fig. 2.3 Two focused collimated ultrasonic beams propagating along opposite directions are used to generate a force potential well to trap a spherical particle. T represents a focusing PZT transducer and W is the beam width at its focal point.

Acoustical tweezers which can trap micron-sized particles will be very useful in biomedical applications such as manipulating cells, organelles, and bacterial. Trapping of much smaller sized objects requires the acoustic beams of much higher frequency.

(2) Ultrasonic standing wave

Ultrasonic standing wave occurred near the reflecting walls will generate non-zero time averaged radiation forces. In a standing wave field with a properly designed reflector [71], as shown in Fig. 2.4, such forces can result
in migration of the particles to preferred positions separated by distances of half an acoustic wavelength. Radiation forces also give rise to either inter-particle attraction or repulsion and can exert a torque on suspended particles [72].

![Fig. 2.4 Radiation pressure in a standing wave field.](image)

This technique is able to levitate and transport objects up to 160 g (60.5 kg/m²) and transportation speed of 138 mm/s was obtained for a slider of 90 g [73]. Moreover it had experimentally proven using this technique, objects as heavy as 10 kg can be levitated [74]. However this technique is ineffective for levitating micro particles or particles with small surface area facing the radiation surface.

**(3) Effects of ultrasonic transducer on acoustic trapping**

Ultrasonic transducers used to manipulate particles come in a wide variety of materials, structures, sizes, and frequencies.
Transducer can be made of many piezoelectric materials, such as quartz, Rochelle salt, Gallium orthophosphate, Barium titanate, and etc. Important parameters that can affect the choice of material include the following: operating temperature, coupling coefficient, piezoelectric coefficient, dielectric constant, loss factor, etc. The most common material for a transducer is PZT. Acoustic tweezers is a device that uses two PZT transducers to generate a force potential well to trap latex particles [6].

Transducers with different structures have been designed to manipulate particles. A circular transducer at the end of a tube containing suspended particles (yeast or bacteria) is used to collect the sediments by applying ultrasound then removing it [75]. A standing wave sound field generated by a transducer and a properly designed reflector can levitate and transport particles up to 160 g. An h-shaped separator is composed of an inlet channel in a multi node standing wave and a flow channel divided into two outlets downstream. By balancing the outlet flows, across a flow splitter, a clear medium is withdrawn at the outlet closest to the transducer and the particle suspension is collected at the outlet furthest away from the transducer [76]. Also a capillary can be inserted in the separation chamber to act as a flow splitter to collect the separated particles [10].

The size of chamber of transducers for standing wave field has great influence on manipulating particles of different sizes. The standing wave system with an active volume of 75 mL can trap effectively and filter mammalian cells (with diameters of the order of 20 µm) in a flowing suspension. However, bacteria (with diameter of 1-2 µm) are difficult to
manipulate in such large active volumes [77], but have been harvested in smaller (2-8 mL) batch [77] or flow chamber volumes [78]. Microchip, used to move 5 µm polyamide particles in parallel bands in a 750 µm wide channel, can collect 90% of the suspended particles [79].

Wide ranges of frequencies and powers have been reported in the literature for the manipulation of particles. For manipulation in liquid media, frequencies are typically between 0.5 and 15 MHz. Lower frequencies increases the likelihood of generating acoustic cavitations (the generation of bubbles during the low pressure phase of an acoustic cycle, which may subsequently collapse and generate shock waves) at the pressure required for manipulation. Energy densities also vary, and are often not quoted as they are difficult to measure. However, to give an order of magnitude impression, a standing wave energy density of about 30 Jm\(^{-3}\) at 3 MHz would correspond to a pressure amplitude of about 0.5 MPa and generate a 100 pN radiation force on a 10 µm diameter polystyrene sphere [80].

Comparing the above conventional ultrasonic trapping methods, both ultrasonic beams and standing wave use ultrasonic far-field to trap particles, which may result in low utilization energy efficiency. For standing wave ultrasonic field, particles can be trapped only at nodal points, so the capability of manipulating a particular particle is weak. The capability of manipulating a particular particle by ultrasonic focused beams is relatively strong, but it can’t trap large number of particles. Also high frequency (megahertz) is used in these methods and devices in these methods are not compact.
CHAPTER 3 OPERATING PRINCIPLE

From Chapter 2, it is known that Hasegawa’s energy density difference theory is the most general theory to calculate the acoustic radiation force in an arbitrary sound field. The acoustic radiation force $F$ due to the radiation pressure is given by integrating over the surface of an obstacle and averaging with respect to time [39]

$$F = \left( \iint_{S_0} (K-U)ndS \right) - \left( \iint_{S_0} \rho_y v_y v_n dS \right)$$

(3.1)

where $K$ and $U$ are the kinetic energy and potential energy densities, $S_0$ is the surface of the obstacle, $v_I$ is the velocity of the obstacle where $v_{1n} = v_I \cdot n$, $n$ is the outward-pointing unit normal vector of $dS$.

For a rigid obstacle, the velocity on obstacle surface $v_I$ is 0, thus Eq. (3.1) can be written as

$$F = \left( \iint_{S_0} (K-U)ndS \right)$$

(3.2)

From Eq. (3.2), it is known that if $<K>$ is larger than $<U>$, $F$ has the same direction as $n$; if $<K>$ is less than $<U>$, $F$ has the opposite direction as $n$.

To analyze the sound field near the radiation surface, a simple model is developed using 3-dimensional FEM (COMSOL Multiphysics) analysis. A metal plate with a size of 50 mm × 10 mm ×1 mm is vibrating in air along the $y$-direction ultrasonically, as shown in Fig. 3.1. The amplitude of the vibration displacement of the metal plate is 10 µm. Boundary conditions for the sound field surrounding the metal plate are radiation boundaries.
Figure 3.2 shows FEM calculation results of the time-averaged kinetic energy and potential energy densities along the y-direction at the lower end of metal plate (\(x = 0\) and \(z = 0\)) at different operating frequencies. From Fig. 3.2, it is seen that in the ultrasonic near-field, it is possible that the time-averaged kinetic energy density is much larger than the potential energy density, for example at \(f = 25\) kHz and \(50\) kHz. At \(f = 25\) kHz and \(50\) kHz, it is seen that when \(y\) approaches 0, the time-averaged kinetic energy density is increasing and potential energy density is decreasing. Therefore the energy density difference \(<K-U>\) is increasing when \(y\) approaches 0. If a particle near the radiation surface is in such sound field (the energy density difference \(<K-U>\) on the particle surface near the radiation surface is much larger than that on the surface.
away from the radiation surface), the acoustic radiation force acting on the particle is mainly determined by the force acting on the particle surface near the radiation surface. As vector \( n \) on the surface of particle near the radiation surface is in the direction pointing to the radiation surface, and \( \langle K \rangle \gg \langle U \rangle \) in the near-field, according to Eq. (3.2), the acoustic radiation force \( F \) has the same direction as \( n \), pointing to the radiation surface. Thus particle may be pushed onto the radiation surface by the acoustic radiation force.

From the above analysis, it is known that under certain conditions, such as proper frequency and radiation surface, etc, it is possible that the acoustic radiation force can push the particles in ultrasonic near-field onto the radiation surface. To use the above mechanism to trap small particles, I have proposed novel transducers with different radiation surfaces to trap particles. The structure of the transducers and theoretical analysis are given in the following chapters.
$f = 25 \text{ kHz}$

(a)

$K_{\text{ Kinetic energy density}} (\text{N/m}^2)$

$U_{\text{ Potential energy density}} (\text{N/m}^2)$

$\Delta f = 25 \text{ kHz}$

(b)

$K_{\text{ Kinetic energy density}} (\text{N/m}^2)$

$U_{\text{ Potential energy density}} (\text{N/m}^2)$

$\Delta f = 50 \text{ kHz}$
Fig. 3.2 The $y$-directional distributions of time-averaged kinetic energy and potential energy densities. (a) At $f = 25$ kHz. (b) At $f = 50$ kHz. (c) At $f = 100$ kHz. (d) At $f = 200$ kHz.
CHAPTER 4  W-SHAPED TRANSDUCER

4.1 Introduction

In this chapter, I have proposed and investigated the structure of a W-shaped transducer, fabricated the prototype, investigated and measured the trapping performance, established a theoretical model to analyze the trapping mechanism, and obtained the guidelines for optimizing the trapping capability of the transducer by experimental and theoretical analysis.

4.2 Experiment

4.2.1 Structure and operation

The structure of the W-shaped transducer used to trap particles in our experiment is shown in Fig. 4.1. In this transducer, two identical metal strips are clamped to a Langevin transducer (FBL28452HS, Fuji Ceramics) shown in Fig. 4.1(a) and (b). The metal strips made of Aluminium have the shape and size shown in Fig. 4.1 (c) and (d). The upper part is a rectangular metal plate, and the lower part is a V-shaped metal strip. The upper part has a size of 40 mm × 45 mm × 1.5 mm with a 10 mm diameter hole at its center; the lower part has a length of 99 mm, width of 22.5 mm and thickness of 1.5 mm, tapers off from the upper end to the lower end. In this way, a triangular air gap, which has a thickness \( h_o \) of 1.5 mm at the end, is formed between the two V-shaped metal
strips, as shown in Fig. 4.1(d). Each metal plate is symmetric about its central plane, so $h_o$ is also the thickness of the metal plate of upper part. The Langevin transducer has a resonance frequency of 25.3 kHz. When an AC voltage with a frequency close to the resonance frequency of the ultrasonic transducer is applied, a flexural vibration is excited in the metal plates. Thus there is a standing wave sound field between the two V-shaped strips. Acoustic energy of the sound field leaks from the lower outlet and two sides of the gap. Near the lower end of the V-shaped metal strips, the leaky sound field has a large spatial gradient, which may generate a large enough acoustic radiation force to suck particles.
Fig. 4.1 Structure and size of the ultrasonic transducer with two V-shaped metal strips. (a) Structure of the ultrasonic transducer and metal strip. (b) Photo of the ultrasonic transducer with two metal strips. (c) Size of the metal strip. (d) Air gap formed by the two V-shaped metal strips.

4.2.2 Experimental observation
Particles used in the experiment are medicine pills. The size and shape of the particles are shown in Fig. 4.2, and the mass, volume and density of them are shown in Table 4.1.

The experimental procedure of trapping particles is shown in Fig. 4.3. The lower end of the vibrating metal strips of the transducer is inserted into the collection of particles in a container, then the transducer is lifted up. It is observed that all kinds of pills shown in Fig. 4.2 could be sucked and trapped to the lower end of the metal strips in air, as shown in Fig. 4.4.

Fig. 4.2 Photos and size of the particles.
Table 4.1 Properties of the medicine pills used in the experiment.

<table>
<thead>
<tr>
<th>Pill Type</th>
<th>Mass (g)</th>
<th>Volume (cm$^3$)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pill A</td>
<td>0.0194</td>
<td>0.0162</td>
<td>1.1975</td>
</tr>
<tr>
<td>Pill B</td>
<td>0.1200</td>
<td>0.1215</td>
<td>0.9877</td>
</tr>
<tr>
<td>Pill C</td>
<td>0.0400</td>
<td>0.0343</td>
<td>1.1662</td>
</tr>
<tr>
<td>Pill D</td>
<td>0.0784</td>
<td>0.0731</td>
<td>1.0725</td>
</tr>
<tr>
<td>Pill E</td>
<td>0.0316</td>
<td>0.0370</td>
<td>0.8541</td>
</tr>
<tr>
<td>Pill F</td>
<td>0.2560</td>
<td>0.2360</td>
<td>1.0847</td>
</tr>
</tbody>
</table>

Fig. 4.3 Experimental procedure of trapping the particles.
Fig. 4.4 Photos of trapped particles.

The optical microscope (Olympus BX51, Olympus Optical Co., Japan) is used to observe and measure the amplitude of the $y$-directional vibration displacement (0-peak) at the tip of the V-shaped metal strip. Figures 4.5(a) and (b) show the magnified images of the tip of strip when it’s stationary and vibrating, respectively. During the exposure time of optical microscope which
is around 10 milliseconds, the strip has vibrated more than 200 times. Therefore
the image of the vibrating strip becomes wider. The 0-peak vibration
displacement amplitude $d_{tip}$ is calculated as half the difference between the
image widths of the vibrating strip ($W_2$) and stationary strip ($W_1$) at the tip, as
shown in Eq. (4.1).

$$
\frac{d_{tip}}{2} = \frac{W_2 - W_1}{2}
$$

(4.1)

Fig. 4.5 Magnified images of the tip of the metal strip. (a) Stationary. (b) Vibrating.

4.3 Theoretical analyses

To analyze the trapping mechanism, leakage of the standing wave ultrasonic field in the air gap between the two vibrating V-shaped metal strips, is investigated by FEM (COMSOL Multiphysics) in two-dimension (2D). The structure in Fig. 4.1(c) and (d) is used in the FEM calculation; the calculation is conducted at 25.3 kHz, which is the resonance frequency of the transducer; the
amplitude of the $y$-directional vibration displacement of the upper part $d = 1 \, \mu m$; loss factor of the vibration in Aluminium is 0.01, which is defined as the ratio of the amount of energy dissipated as heat to that of total stored energy (COMSOL Multiphysics: Acoustic Module User’s Guide); and the radiation boundaries and Perfectly Matched Layers (PML) boundaries are used for the leaky sound field.

The calculated distribution of sound pressure near the outlet of the gap is shown in Fig. 4.6(a). Figure 4.6(b) shows the $x$-directional distributions of amplitude and phase of sound pressure in the leaky sound field on the line $y = 0$ (the initial time phase of the vibration displacement $d$ is zero). It is seen that the sound pressure attenuates with the increase of $x$, and the phase angle of sound pressure linearly changes with $x$ approximately. So the sound pressure on the line $y = 0$ in the leaky sound field is a travelling wave with attenuated amplitude. Based on the numerical results in Fig. 4.6(b), sound pressure on the line $y = 0$ in the leaky sound field is

$$
P = P_m e^{-\alpha x} \cos[kx + \varphi_0 + \omega x]
$$

(4.2)

For Fig. 4.6(b), the attenuation coefficient $\alpha$ is 234 m$^{-1}$, $P_m$ is 1524.9 Pa, $k$ is -372.7 m$^{-1}$, and $\varphi_0$ is -1.48 radian.

The relationship between the acoustic pressure and vibration velocity in the leaky sound field is [81]

$$
\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x}
$$

(4.3)

where $\rho_0$ is the density of the fluid. Therefore the vibration velocity of the fluid in the leaky sound field along the $x$-direction is
\[
\begin{align*}
\mathbf{u} &= -\frac{P_w^2 \sqrt{\alpha^2 + k^2}}{\rho_0 \omega} e^{-\alpha x} \cos(kx + \varphi_0 + \omega t + \theta)
\end{align*}
\]  
\[
(4.4)
\]

where \( \theta = \tan^{-1} \frac{\alpha}{k} \).

Gor’kov’s theory is used to calculate the acoustic radiation force acting on a sphere in this leaky sound field. It is known that the acoustic radiation force acting on a particle in the sound field is

\[
\vec{F} = -\nabla U
\]  
\[
(4.5)
\]

where \( U \) is the time-averaged force potential of the sound field.

When the wave number \( k \) and the radius of particle \( R \) satisfy \( kR \leq 1 \), according to Gor’kov’s theory, the force potential \( U \) is \([45-46]\]

\[
U = V[-D < K_E > + (1 - \gamma) < P_E >]
\]  
\[
(4.6)
\]

where \( V \) is the volume of the particle sphere, \( \langle K_E \rangle \) and \( \langle P_E \rangle \) are the time-averaged kinetic and potential energy densities of the sound field, respectively. \( D \) is a parameter determined by the densities of the particle sphere and fluid, and \( \gamma \) is the compressibility ratio between the particle sphere and fluid. \( D \) and \( \gamma \) can be calculated by

\[
D = \frac{3(\rho_s - \rho_0)}{2\rho_s + \rho_0}
\]  
\[
(4.7)
\]

\[
\gamma = \frac{\rho_0 c_0^2}{\rho_s c_s^2}
\]  
\[
(4.8)
\]

where \( \rho_s \) and \( \rho_0 \) are the densities of the particle sphere and fluid, respectively, and \( c_s \) and \( c_0 \) are the sound speed in the particle sphere and fluid, respectively.
From Eqs. (4.2), (4.4), (4.5) and (4.6), the acoustic radiation force on a sphere particle in the x-direction is

$$\vec{F} = -i V \alpha P_e^2 e^{-2\alpha x} \left( \frac{D(\alpha^2 + k^2)}{\omega^2} + \frac{1 - \gamma}{c_0^2} \right)$$  \hspace{1cm} (4.9)

In air, because $\rho_s \gg \rho_0$, $D \approx 1.5$, and $\gamma \approx 0$. Therefore acoustic radiation force in air is simplified to

$$\vec{F} = -i \frac{V \alpha P_e^2 e^{-2\alpha x}}{4 \rho_0 c_0^2}.$$  \hspace{1cm} (4.10)

Equation (4.10) shows that the force acting on a sphere particle near the lower outlet of the air gap of the transducer points to the $-x$ direction, which pushes the particles to the end of the V-shaped metal strips.

Using $\alpha = 234 \text{ m}^{-1}$ (from Fig. 4. 6(b)), $\rho_0 = 1.2 \text{ kg/m}^3$ (air), and $c_0 = 340 \text{ m/s}$ (air), the relationship between the acoustic radiation force and the vibration displacement amplitude of the upper metal plates $d$ is calculated by Eq. (4.10) for a sphere particle with radius of 1.57 mm and mass of 19.4 milligrams (Pill A). It is known that when the vibration displacement amplitude of the upper metal plates is larger than 4.9 μm, the acoustic radiation force is larger than the weight of the particle, and thus the particle can be trapped. Based on our experience, such a vibration can be excited by the Langevin transducer. Therefore the preceding analyses indicate the possibility to use the leakage of low frequency standing wave ultrasonic field to trap particles.
Fig. 4.6 Sound pressure distributions surrounding the gap. (a) Sound pressure distribution at the outlet of the air gap and leaky sound field at 25.3 kHz. (b) The $x$-directional distributions of amplitude and phase of sound pressure on the line $y = 0$ in the leaky sound field.
4.4 Results and discussions

4.4.1 Measured number of trapped pills vs. driving frequency

Figure 4.7 shows the experimental relationship between the number of trapped pills $A$ and driving frequency at different input voltages in air. It can be observed that the number of trapped pills reaches a maximum at a particular driving frequency or in a particular driving frequency range for a given voltage, which is caused by the resonance of the transducer system.

![Graph showing the experimental relationship between the number of trapped pills and driving frequency.](image)

Fig. 4.7 Experimental relationship between the number of trapped pills $A$ and the driving frequency in air under different input voltages.
4.4.2 Measured number of trapped pills vs. vibrating displacement at metal strip tip

Figure 4.8 shows the effect of the y-directional vibration displacement amplitude (0-peak) at the tip of the V-shaped metal strip $d_{tip}$ on the trapping capability for different particles. From Fig. 4.8, it is seen that the measured number of trapped particles increases as the vibration displacement amplitude at the tip increases. This is caused by the increase of sound pressure at the lower outlet of the air gap. Particle with a weight up to 256 milligrams can be sucked when the vibration displacement amplitude at the tip is larger than 34 µm. Among all the particles, it is observed that it is the easiest to trap pill C which has a cubic shape. From Fig. 4.2 and Table 4.1, it is known that pill C with cubic shape (1.1662 g/cm$^3$) has larger density than pill E with cylindrical shape (0.8541 g/cm$^3$) and similar density to pill A with spherical shape (1.1975 g/cm$^3$). So it is more difficult to suck spherical particles than cubic particles for a given vibration amplitude, and the particle shape affects the trapping capability. For the spherical particles, only the $+x$ and $-x$ direction components of the acoustic radiation force contribute to the trapping. While for the cubic particles, the acoustic radiation forces acting on the lower and bottom surfaces directly contribute to the trapping. This explains the above phenomena.
4.4.3 Effect of thickness of metal plate on the acoustic trapping capability

The effects of the upper plate thickness $h_a$ on the acoustic trapping capability are investigated theoretically and experimentally. Figure 4.9(a) shows the calculated relationship between the normalized trapping acoustic radiation force acting on pill A and the upper plate thickness $h_a$ when strip length $L$ is 89 mm and excitation frequency and vibration $d$ are 25.3 kHz and 1 µm respectively. It is seen that the trapping force can reach its peaks when the plate thicknesses are 1.50 mm and 1.75 mm. In the experiment, the number of trapped pills A is measured, when the transducers with a constant strip length of
89 mm and different plate thicknesses, are operating at an input voltage from 50 V\text{rms} to 65 V\text{rms} at resonance. The results are shown in the Fig. 4.9(b). It is seen that when the plate thicknesses are 1.50 mm and 1.75 mm, the trapping capability is relatively strong; no particles can be sucked when the plate thicknesses are 1.25 mm, 1.60 mm and 2.00 mm. These results agree well with the calculated result as shown in Fig. 4.9(a).

For a given vibration excitation at the upper part of the metal plates, the size $h_a$ affects the vibration velocity at the two tips of the metal plates. Figure 4.10 shows the calculated vibration displacement at the tip of the metal strips for different upper plate thickness $h_a$ when strip length $L$ is 89 mm under the same excitation conditions (the excitation frequency and vibration $d$ are 25.3 kHz and 1 $\mu$m respectively). From this figure, it is seen that the vibration displacement at the tip of metal strips reaches maximum when the plate thicknesses are 1.32 mm, 1.50 mm and 1.75 mm, respectively. This indicates the resonance of the metal plates. Compared Fig. 4.9 and Fig. 4.10, it is seen that the peaks of the acoustic radiation force occur when the vibration displacement at the tip of metal plate is maximum. Thus the phenomena in Fig. 4.9 are caused by the resonance of the two metal plates.
Fig. 4.9 The effect of the upper plate thickness on the trapping capability with an 89 mm long V-shaped metal strip. (a) Calculated relationship between the normalized trapping acoustic radiation force acting on pill A in the leaky sound field and the plate thickness. (b) Measured number of trapped pills A at resonance for different plate thicknesses at different input voltages.
4.4.4 Effect of length of metal strip on the acoustic trapping capability

Figure 4.11(a) shows the calculated relationship between the normalized trapping acoustic radiation force acting on pill A and the length of V-shaped metal strip \( L \) when plate thickness \( h_a \) is 1.5 mm and excitation frequency and vibration \( d \) are 25.3 kHz and 1 \( \mu \)m respectively. It is seen that the normalized trapping force reaches its peak when the strip length is 99.1 mm long, which means the acoustic trapping capability is the best in the calculated range of strip length. For a given vibration excitation at the upper part of the metal plates, it is
known that the size $L$ affects the vibration velocity at the two tips of the metal plates. Thus the resonance of the metal plates causes the peaks shown in Fig. 4.11(a).

The number of trapped pills $A$ is measured, when the transducers with a constant plate thickness of 1.5 mm and different strip lengths, are operating at an input voltage from $50 \, V_{rms}$ to $65 \, V_{rms}$ at resonance. The experimental results are shown in the Fig. 4.11(b). It is seen that the trapping capability is the best at the strip length of 99 mm; no particles can be sucked when the strip lengths are 97 mm and 100.5 mm. Comparing Fig. 4.11(a) and (b), it is known that experimental result agrees well with the calculated one.
Fig. 4.11 The effect of the length of V-shaped metal strip on the trapping capability at a constant plate thickness of 1.5 mm. (a) Calculated relationship between the normalized trapping acoustic radiation force acting on pill A in the leaky sound field and the strip length. (b) Measured number of trapped pills A at resonance for different strip lengths at different input voltages.
4.5 Conclusions

The W-shaped ultrasonic transducer can trap particles to the sharp edges of two vibrating V-shaped metal strips, which uses a method by the acoustic leakage from a low frequency standing wave ultrasonic field. The standing wave ultrasonic field is generated in a triangular air gap between two vibrating V-shaped metal strips. The acoustic radiation force acting on the particles in this method is opposite to the direction of the acoustic leakage. Particles such as medicine pills with a weight up to 256 milligrams per particle can be trapped at the lower outlet of metal strips. A physical model is developed to analyze the trapping phenomena. Theoretical analyses show that the trapping is due to the travelling wave ultrasonic field with spatially attenuated amplitude, which leaks from the standing wave ultrasonic field in the air gap. Experiments and theoretical analyses show that the trapping capability is affected by the vibration displacement amplitude at the tip of the V-shaped metal strip, the length and thickness of the metal strip, and the shape of particles.
5.1 Introduction

Studying the researches of acoustic radiation force theory and acoustic trapping method discussed in Chapter 2, it is found that all of them use the acoustic radiation force on a sphere or cylinder in plane standing and travelling wave fields. There has been no work reported on the dependence of acoustic trapping capability on the orientation and shape of objects in complicated sound fields, such as the one near radiation surface. In practical applications, objects have various shapes and can be in many orientations. From the results in Chapter 4, it is known that particle shape does affect the trapping capability for particles. In the applications, such as removing particles from a solid surface and separating particles from a mixture, it is necessary to understand the effects of particle shape and orientation on the trapping capability for it.

In this chapter, I have presented the effects of the orientation and shape of particles on the acoustic trapping capability experimentally and theoretically. In the experiment, the W-shaped transducer is used to trap particles which have the same mass and volume but different shapes. A method which combines the FEM analysis and energy density difference theory of acoustic radiation force is used to calculate the acoustic radiation force and trapping capability of a particle with arbitrary shape in an arbitrary sound field. It is found that the
acoustic trapping capability for particles depends on the orientation and shape of them. The dependence of acoustic trapping capability on the orientation and shape of some commonly seen particles such as rectangular cuboid, cylinder, cone, cube, sphere and hollow cylinder in different media is clarified.

5.2 Experiment

5.2.1 Construction and operation

Fig. 5.1 Structure and size of the ultrasonic transducer with two V-shaped Aluminium strips in air. (a) Photo of structure of the ultrasonic transducer. (b) Shape and size of the Aluminium strip. (c) Air gap formed by the two V-shaped strips.
The W-shaped transducer is used to trap particles in air in the experiment as shown in Fig. 5.1. The 2D FEM analysis by the acoustic module of COMSOL Multiphysics shows that a flexural vibration is excited in the Aluminium strips when an AC voltage with a frequency close to the resonance frequency of the ultrasonic transducer is applied. This flexural vibration will generate a sound field, and the sound field near the lower end of gap can generate an acoustic radiation force to suck the particles to the lower end of strips.

5.2.2 Experimental observation

Particles used in the experiment are made of clay. The sound speed of clay material is measured by a sound speed meter (SV-DH-7A, Guotai Electronics, China). In the method, two cylinder clay samples with different lengths ($L_1$ and $L_2$, respectively) are used, and time for sound traveling through each sample in its length direction is measured ($t_1$ and $t_2$, respectively). The sound speed in clay material is calculated by the length difference ($\Delta L = L_1 - L_2$) and time difference ($\Delta t = t_1 - t_2$), and the measured sound speed in clay material is 2716 m/s (see Chapter 9 for details). The density of clay material is calculated by using the measured mass and volume of clay particle, and the measured density is 1554 kg/m$^3$. There are six types of particles with different shapes but same mass and volume. The mass of particle is 40 milligrams each. The shapes of the six particles are sphere, cube, cylinder, cone, rectangular cuboid and hollow cylinder, respectively. The size of each particle is shown in Fig.5.2.
In the experiment, the two vibrating sharp edges of strips are moved to the top surface of particle, and then the transducer is lifted up. It is found that all of the six particles can be trapped in air by the transducer operating at a proper vibration. For the comparison of acoustic trapping capability, the acoustic trapping capability coefficient $C_{tr}$ is defined as

$$C_{tr} = \frac{1}{d_{\text{min}}} \quad \text{(5.1)}$$

where $d_{\text{min}}$ is the minimum vibration displacement at the tip of metal strip to trap one particle. The acoustic trapping capability for particle is stronger if its $C_{tr}$ is larger. Figure 5.3 shows the trapped rectangular cuboid shaped particle under the two sharp edges of the strips in air.

![Images of particles with dimensions](image)

**Sphere**
- $R = 1.79$ mm

**Cube**
- $L = 3.07$ mm

**Cylinder**
- $R = 1.64$ mm
- $H = 3.04$ mm

**Rectangular Cuboid**
- $L = 3.71$ mm
- $W = 1.53$ mm
- $H = 5.22$ mm

**Hollow Cylinder**
- $R_o = 2.81$ mm
- $R_i = 1.57$ mm
- $H = 1.74$ mm

**Cone**
- $R = 2.03$ mm
- $H = 4.57$ mm

Fig. 5.2 Photos and dimensions of six different shaped clay particles with the same mass and volume.
Fig. 5.3 A trapped rectangular cuboid shaped particle in air.

5.3 Theoretical analyses

To analyze the trapping mechanism for each particle trapped by the transducer, the following theoretical model is developed and investigated. Gor’kov’s theory is not suitable in this case, because particles have various shapes and particles are at the sound field near the radiation surface [46].

Using Hasegawa’s energy density difference theory, the acoustic radiation force $F$ on a rigid immovable object in a sound field in ideal fluid is given by the following integration over the surface of the object [39]

$$F = \left\langle \iiint_S (K - U) n dS \right\rangle$$

\hspace{1cm} \text{(5.2)}

where the notation $\langle \rangle$ denotes time average over one period, $K$ is the kinetic energy density, $U$ is the potential energy density, and $n$ is the outward normal unit vector of the surface. The kinetic and potential energy densities $K$ and $U$ can be calculated by [39]
\[ K = \frac{\rho_0 v^2}{2} \]  

\[ U = \frac{p^2}{2\rho_0 c_0^2} \]

where \( \rho_0 \) and \( c_0 \) are the density of and sound speed in the fluid, \( v \) is the velocity, and \( p \) is the sound pressure.

After the FEM analysis of the sound field surrounding a particle under the two vibrating sharp edges in air as shown in Fig. 5.1(c), it is found that when the two identical metal strips are vibrating in the same \( y \)-direction, the sound pressure in the air gap is anti-symmetric about its central plane (\( xz \)-plane), therefore the sound pressure in the \( xz \)-plane is zero. Also it is found that the vibration velocity on particle surfaces is symmetric about the \( xz \)-plane. Due to the limitation of our 32-bit computer’s addressable memory (physical RAM and disk swap space) for calculation, it is difficult to analyze the whole structure in 3-dimension. So we split the whole structure (two metal strips, a particle, and the surrounding sound field) into two parts about the \( xz \)-plane for the 3D FEM calculation. The boundary conditions for the sound field are: \( p = 0 \) is used for the \( xz \)-plane; the rest of the sound field boundaries are radiation boundary. The excitation conditions for the transducer are: the excitation frequency \( f \) is 25.3 kHz, which is the resonance frequency of the transducer; the amplitude of the \( y \)-directional vibration displacement (0-peak) of the upper part of metal plates \( d \) is 10 \( \mu \)m; the loss factor of the vibration in Aluminium is 0.01.

Figure 5.4(a) shows the mesh of the half structure (a single strip, half of a 3 mm \( \times \) 3 mm \( \times \) 3 mm cube particle and the surrounding sound field) in 3D FEM calculation, where the maximum mesh size at the particle surface...
boundaries is around 0.66% of the wavelength. Figure 5.4(b), (c) and (d) show the amplitudes of $x$-directional velocity, $y$-directional velocity and sound pressure on the top surface of the half particle. The $z$-directional velocity on top surface of the half particle is around $10^{-15}$ m/s, which is much smaller than the $x$-directional and $y$-directional velocities, thus it is not listed. It is seen that the $x$-directional velocity is very small compared with the $y$-directional velocity; the $y$-directional velocity is maximum at around $y = 0.75$ mm, which is near the sharp edge of the vibrating strip. From the 3D FEM results shown in Fig. 5.4(b), (c) and (d), it is known that $\iint_S \langle K \rangle dS$ and $\iint_S \langle U \rangle dS$ on the top surface of the cube particle are calculated to be $6.5 \times 10^{-4}$ N and $1.2 \times 10^{-5}$ N, respectively. Also it is found that $\iint_S \langle K \rangle dS$ and $\iint_S \langle U \rangle dS$ on the side and bottom surfaces of the particle are less than 1% of that on the top surface; thus they are negligible. From the calculation, it is known that on the top surface of particle, $\iint_S \langle K \rangle dS \gg \iint_S \langle U \rangle dS$, so $F$ has the same direction as $n$ [see Eq. (5.2)], and the particle may be sucked to the sharp edge of the strips. Therefore for the transducer used in our experiment, the acoustic radiation force acting on particle is determined by the force on the top surface of particle, pointing upwards; and the trapping force mainly occurs near the two vibrating sharp edges of the strips. Also, based on our FEM analysis, it is known that the negative acoustic radiation pressure is much larger in the contact area between the sharp edges of strips and top surface of particle than that at the other locations of particle. So the $x$-directional length of the contact area is defined as action line of particle.
Fig. 5.4 3D FEM analyses of the half structure (a single strip, and half of a
3mm long cube particle and surrounding sound field) in air when a vibration displacement amplitude (0-peak) of upper part of metal plate is 10 µm. (a) Mesh. (b) The x-directional velocity on top surface of the half particle. (c) The y-directional velocity on top surface of the half particle. (d) The sound pressure on top surface of the half particle.

5.4 Results and discussions

5.4.1 Effect of particle orientation on the acoustic trapping capability

The effect of orientation of rectangular cuboid particle on the acoustic trapping capability in air has been investigated both experimentally and theoretically. The rectangular cuboid particle trapped under the two sharp edges of metal strips has six possible trapping orientations, from a to f, as shown in Fig. 5.5(a).

Figure 5.5(b) shows the experimental and calculated acoustic trapping capability coefficients for the rectangular cuboid particle at the different trapping orientations. In the experiment, the particle can be trapped at five orientations (a, b, c, d & e), and experimental and calculated results agree well. Defining the contact line between the sharp edge of strip and the top surface of particle as action line, the lengths of action line of rectangular cuboid particle from orientation a to f are 5.22 mm, 5.22 mm, 3.71 mm, 3.71 mm, 1.53 mm and 1.53 mm, respectively. It is found that the acoustic trapping capability coefficient $C_{tr}$ increases mostly with the increase in the length of action line. Also, with the same length of action line, particle with larger top surface area
(shaded area in Fig. 5.5(a)) has stronger acoustic trapping capability. For example, cases \(a\) and \(b\) have the same length of action line, but case \(a\) has larger trapping capability than case \(b\). This is because the larger the top surface area, the larger the action area of upward acoustic radiation force.

Figure 5.6 shows the experimental and calculated effects of the orientation of cylinder particle on the acoustic trapping capability in air. The cylinder particle has three possible trapping orientations, as shown in Fig. 5.6(a). From Fig. 5.6(b), it is seen that cylinder can be trapped at all the orientations; for orientation \(a\) and \(b\), the experimental and calculated results agree well; for orientation \(c\), the difference between the experimental and calculated results is relatively large. It is known that the length of action line of cylinder particle for orientation \(c\) is 0 mm. The error in case \(c\) may be because when the action line of particle is very short, it needs finer mesh to reduce the calculation error and this requires a quite large memory for calculation which is not easy to be achieved by our current computer.
Fig. 5.5 The effect of orientation of rectangular cuboid clay particle on the acoustic trapping capability in air. (a) Six possible trapping orientations of the same rectangular cuboid shaped particle. (b) The experimental and calculated acoustic trapping capability coefficients for the rectangular cuboid particle at different trapping orientations.
Fig. 5.6 The effect of orientation of cylinder clay particle on the acoustic trapping capability in air. (a) Three possible trapping orientations of the same cylinder shaped particle. (b) The experimental and calculated acoustic trapping capability coefficients for the cylinder particle at different trapping orientations.
5.4.2 Effect of particle shape on the acoustic trapping capability

The acoustic trapping capability for particles in air with different shapes as shown in Fig. 5.2 has been investigated experimentally. For each particular shaped particle, there are several possible trapping orientations, and every orientation has its own trapping capability. The best experimental trapping capability for each particle is used, as shown in Fig. 5.7(a).

Figure 5.7(b) shows the experimental and calculated acoustic trapping capabilities for particles with different shapes. It is found that the experimental acoustic trapping capability for each particle becomes worse in the sequence from rectangular cuboid, cylinder, cone, cube, sphere to hollow cylinder. And the error between the experimental and calculated results is small except for sphere and hollow cylinder. The theoretical calculations are not applied to sphere and hollow cylinder for the reason given in the discussion of Fig. 5.6.
Fig. 5.7 The effect of particle shape on the acoustic trapping capability in air. (a) Six particles with different shapes but the same volume and density, at the orientation at which the experimental trapping capability is the strongest for each particle. (b) The experimental and calculated acoustic trapping capabilities for the particles at the orientations shown in (a).
5.4.3 Effect of medium on the acoustic trapping capability

The effect of orientation of the same rectangular cuboid particle on the acoustic radiation force acting on particle in water and in air has been investigated theoretically, and the results are shown in Fig. 5.8. In the calculation, the vibration excitation conditions in water are the same as that in air \((d = 10 \, \mu m \text{ and } f = 25.3 \, kHz)\). The trapping orientations of the rectangular cuboid particle as shown in Fig. 5.5(a) are used. From Fig. 5.8, it is seen that the acoustic radiation force acting on the rectangular cuboid particle decreases from orientation \(a\) to \(f\) both in air and in water; the acoustic radiation force on particle in water is larger than that in air for the same particle at same orientation, this is because of the stronger sound field in water. Considering the buoyancy force on particle in water, the trapping capability in water would be much stronger than that in air, this is because the upward trapping force is enhanced by buoyancy force. In the above analysis, the effect of acoustic stream is not taken into account. It is experimentally found that there is no acoustic stream or the effect of acoustic stream on particle trapping is negligible when the vibration velocity is less than a critical value [19, 82]. Our analysis is for the vibration range in which the effect of acoustic stream is small.

Figure 5.9 shows the calculated acoustic radiation forces acting on particle both in water and in air for the particles shown in Fig. 5.2, which have different shapes but the same volume and density. In the calculation, the vibration excitation conditions in water are the same as that in air \((d = 10 \, \mu m \text{ and } f = 25.3 \, kHz)\). The trapping orientation of each particle is as shown in Fig.
5.7(a). From Fig. 5.9, it is seen that the acoustic radiation force in water decreases from rectangular cuboid, cylinder, cone, cube, sphere, to hollow cylinder, which has the same order as that in air.

![Diagram showing the effect of orientation and particle shape on acoustic radiation force](attachment://image.png)

**Fig. 5.8** The effect of orientation of the rectangular cuboid particle on the calculated acoustic radiation force acting on particle in water and in air.

![Diagram showing the effect of particle shape on acoustic radiation force](attachment://image.png)

**Fig. 5.9** The effect of particle shape on the calculated acoustic radiation force acting on particle in water and in air.
Figure 5.10 shows the calculated acoustic radiation force on rectangular cuboid particle at the interface of water and air. In the calculation, the vibration excitation conditions are the same as that in air and in water \((d = 10 \mu m \text{ and } f = 25.3 \text{ kHz})\); the trapping orientations of the rectangular cuboid particle as shown in Fig. 5.5(a) are used, and half of the particle is in water and another half in air. From Fig. 5.10, it is seen that from orientation \(a\) to \(f\), the acoustic radiation force on particle at the interface of water and air decreases correspondingly. Compared with Fig. 5.8, it is found that the acoustic radiation force on particle at the interface of water and air is larger than that in air, and less than that in water.

![Figure 5.10](image)

Fig. 5.10 The effect of orientation of the rectangular cuboid particle on the calculated acoustic radiation force when particle is at the interface of water and air.
5.5 Conclusions

In summary, I have investigated the effects of orientation and shape of particles on the acoustic trapping capability in different media. When the density, volume and mass per particle are constant, the acoustic trapping capability for particle depends on the orientation and shape of particle. This dependence is caused by the difference in the length of action line of acoustic radiation force acting on particles. The acoustic trapping capability increases with the increase of the length of action line. For a certain particle, acoustic trapping capability is the best at the orientation where the action line is the longest. Acoustic trapping capability becomes worse in the sequence from rectangular cuboid, cylinder, cone, cube, sphere to hollow cylinder. A method which combines the FEM analysis and energy density difference theory of acoustic radiation force is employed to calculate the acoustic radiation force acting on particles with various shapes and orientations. By comparing the experimental and calculated results, it is seen that this method can explain the experimental results well. Also it is found that the acoustic radiation force on a certain particle in water is much larger than that in air; the acoustic radiation force on particle at the interface of water and air is larger than that in air, and less than that in water.
6.1 Introduction

In this chapter, I have proposed and investigated the structure of an acoustic needle, fabricated the prototype, investigated and measured the trapping performance, established a theoretical model to analyze the trapping mechanism, and obtained the guidelines for optimizing the trapping capability of the transducer by experimental and theoretical analysis.

6.2 Experiment

6.2.1 Construction and operation

Figure 6.1 shows the ultrasonic transducer used in our experiments. The acoustic needle made of stainless steel is welded onto the side of one of the two rectangular stainless steel plates in a sandwich type piezoelectric transducer. A multilayer piezoelectric vibrator consisting of four piezoelectric rings is pressed by the two plates via a bolt structure. The neighbouring piezoelectric rings are aligned with opposite poling directions. The size of each stainless plate is 20 mm × 20 mm × 2.3 mm, and outer diameter, inner diameter and thickness of each piezoelectric ring are 20 mm, 12 mm and 2.4 mm, respectively. The electromechanical quality factor $Q_m$, piezoelectric coefficient $d_{33}$ and relative dielectric constant $\varepsilon_{33}^T / \varepsilon_0$ of the piezoelectric rings are 2000, $325 \times 10^{-12}$ m/V.
and 1450, respectively. The needle has length of 34 mm from the edge of the stainless steel plate, and maximum diameter of 0.8 mm which gradually decreases down to its tip from \( z = 11 \) mm. When an ac voltage with a frequency near the resonance frequency of thickness vibration mode of the ultrasonic transducer is applied to the transducer, a thickness vibration can be excited in the transducer. The resonance frequency of the first order thickness vibration mode is about 66.75 kHz.

![Ultrasonic transducer structure](image)

**Fig. 6.1** The structure and size of ultrasonic transducer with a metal needle to trap small particles. (a) Front view. (b) Side view. (c) Photo.

### 6.2.2 Experimental observation

When the transducer operates near the first order thickness vibration mode and the driving condition is proper, small particles such as flying color
seeds, grass seeds and shrimp eggs near the tip of vibrating needle in water can be sucked to and trapped by the tip. Plastic vessels filled with water are used, and the height of the liquid in the vessels is 24 mm. The diameter of the vessels is much larger than the wavelength of the sound wave in water. The particles used in our experiment can sink to the bottom of vessel because they have absorbed sufficient water. Table 6.1 shows the radius and density of particles used in the experiment. Figure 6.2 shows the trapping of flying color seeds in water.

Fig. 6.2 Flying color seeds trapped by the tip of the vibrating needle in water.

Micron-sized particle, such as yeast cell can be sucked to the tip of the vibrating needle in water as well. Figure 6.3(a) shows the size of yeast cells in water under microscope, it is seen that the diameter of yeast cell is around 5 μm. Figures 6.3(b) and (c) show the yeast cells in water when the acoustic needle is stationary and vibrating, respectively. The needle is orientated horizontally under the microscope, and vibrates in y-direction. From Fig. 6.3(c), it is seen that the yeast cells trapped as agglomerations by the tip of the vibrating needle in water operating at a voltage of 35.8 V_\text{rms} at resonance.
Table 6.1 Properties of small particles used in the experiment.

<table>
<thead>
<tr>
<th></th>
<th>Radius (mm)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flying color seeds</td>
<td>0.500</td>
<td>0.93</td>
</tr>
<tr>
<td>Grass seeds</td>
<td>0.556</td>
<td>0.53</td>
</tr>
<tr>
<td>Shrimp eggs</td>
<td>0.117</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Fig. 6.3 Trapping of yeast cells in water by the acoustic needle under microscope. (a) The size of yeast cell. (b) When the needle is stationary. (c) When the needle is vibrating ultrasonically.
### 6.3 Theoretical analyses

To analyze the trapping mechanism of the acoustic needle, the same method as proposed in Chapter 5 is used to calculate the acoustic radiation force acting on particles in water near the tip of needle, the method which combines the FEM analysis and energy density difference theory of acoustic radiation force.

The sound field surrounding a sphere particle in water which is near the tip of the acoustic needle is analyzed by 3D FEM. The boundary condition for the sound field is radiation boundary. The excitation conditions for acoustic needle transducer are: excitation frequency is 66.75 kHz, which is the resonance frequency of the transducer; the amplitude of the $y$-directional vibration displacement (0-peak) of the upper metal plate is 1 µm; the loss factor of the vibration in stainless steel is 0.02.

A sphere particle with a radius of 0.5 mm can be trapped to the tip of the needle at five possible positions; these positions are when the center of particle is at $x = \pm 0.5$ mm, $y = \pm 0.5$ mm and $z = -0.5$ mm, respectively. The acoustic radiation forces acting on the sphere at the above five positions are calculated. Figure 6.4 shows the mesh of the structure (needle, sphere, and surrounding sound field) in 3D FEM calculation, where the maximum mesh element size of the particle surface boundaries is about 0.29% of the wavelength.
From the FEM results, it is calculated that \( \int_S \langle K \rangle dS \) and \( \int_S \langle U \rangle dS \) on the half sphere surface which is near the tip of needle are \( 6.47 \times 10^{-5} \) N and \( 3.92 \times 10^{-9} \) N. Also it is known that \( \int_S \langle K \rangle dS \) and \( \int_S \langle U \rangle dS \) on the other half surface which is away from the tip of needle are less than 0.02 % of that near the needle, thus they are negligible. According to the FEM results, the acoustic radiation forces acting on the sphere at five different positions are calculated: when the sphere center is at \( y = 0.5 \) mm and \( y = -0.5 \) mm, the \( y \)-directional acoustic radiation force on the sphere is \( -6.47 \times 10^{-5} \) N and \( 6.76 \times 10^{-5} \) N,
respectively; when the sphere center is at $x = 0.5$ mm and $x = -0.5$ mm, the $x$-directional acoustic radiation force on the sphere is $-3.82 \times 10^{-5}$ N and $3.33 \times 10^{-5}$ N, respectively; when the sphere is at $z = 0.5$ mm, the $z$-directional acoustic radiation force on the sphere is $1.27 \times 10^{-5}$ N. From the above results, it is seen that the acoustic radiation forces at the above five positions are all attractive forces, which may suck the sphere onto the tip of needle; when the particle is at $y = \pm 0.5$ mm, the acoustic radiation force is the largest, which means that the trapping force is the largest when particle is at the vibrating direction of upper metal plate of needle.

6.4 Results and discussions

6.4.1 Particle trapping in water

Figure 6.5(a) shows the relationship between the measured number of trapped flying color seeds and operating frequency at different driving voltages. For each driving voltage, the number of trapped particles reaches a maximum at a particular operating frequency or in a particular operating frequency range, which is caused by the resonance of the transducer system. Figure 6.5(b) shows the relationship between the measured number of trapped flying color seeds at resonance and the driving voltage. As the driving voltage increases, the number of trapped particles increases and tends to saturate when the voltage is too large.
Fig. 6.5 The trapping of flying color seeds in water. (a) The number of trapped flying color seeds vs. operating frequency at operating voltages of 7.5, 15 and 50 Vp-p. (b) The number of trapped flying color seeds at resonance vs. operating voltage.
6.4.2 Particle trapping in oil

Corn oil and olive oil are also used in the experiments. Figure 6.6(a) shows the dependence of the measured number of trapped flying color seeds on the operating frequency in corn oil and olive oil at an operating voltage of 10 Vp-p, and Fig. 6.6(b) shows the dependence of the measured number of trapped flying color seeds at resonance on the operating voltage. By the comparison of Fig. 6.6(b) with Fig. 6.5(b), it is seen that the trapping capability of the needle is weaker in corn oil and olive oil than that in water. The viscosity of corn oil and olive oil is 0.063 and 0.084 Pa s [83], respectively, which is much larger than that of water (0.001 Pa s). So the acoustic damping in corn oil and olive oil is much larger than that in water. This lowers the trapping capability of the needle. The viscosity of olive oil is a little bit larger than that of corn oil. This explains why the number of trapped particles in olive oil is less than that in corn oil at some frequencies, as shown in Fig. 6.6(a). The density of corn oil and olive oil is 0.87 and 0.84 g/ml, respectively, according to our measurement.
Fig. 6.6 The trapping of flying color seeds in corn oil and olive oil. (a) The number of trapped particles vs. operating frequency at an operating voltage of 10 Vp-p. (b) The number of trapped particles at resonance vs. operating voltage.
6.4.3 Particle transportation in water

The trapped particles can be transported from one location to another in water. Figure 6.7 shows the number of trapped particles at the initial position, and the number of lost particles during the transportation versus operating frequency at an average transportation speed of 0.88 cm/s, transportation distance of 17.5 cm and different voltages. In the experiment, the height of water is 2.4 cm, the distance from the tip to the bottom of vessel is 0.5 cm, the transportation direction is parallel to the vibration direction of the needle, flying color seeds are used. Figure 6.7 shows that the trapped particles can be transported in water by the needle. It also shows that there exists a frequency range near the resonance point in which there is no particle lost and this frequency range becomes wider when the operating voltage increases. These phenomena are because the sound pressure is relatively large near a resonance point and it increases when the operating voltage increases near a resonance point.
Fig. 6.7 The numbers of trapped flying color seeds and lost flying color seeds during the transportation vs. operating frequency at operating voltages of: (a) 20Vp-p. (b) 50Vp-p. The average transportation speed is 0.88 cm/s.
6.4.4 Particle separation from a mixture in water

In the experiments of investigating the possibility to separate different particles by the acoustic needle, the tip of the vibrating needle is inserted into the mixture of shrimp eggs and flying color seeds on the bottom of the vessel with water, and then lift to a 5 mm height from the bottom for observing whether both of the particles are trapped or not. Figure 6.8 shows the measured numbers of trapped shrimp eggs and flying color seeds versus operating frequency at operating voltages of 20 Vp-p. From the figure, it is seen that there exist some frequency ranges in which only shrimp eggs can be trapped and flying color seeds cannot. Due to the larger density of flying color seeds, the minimum vibration at the needle tip to trap flying color seeds must be larger than that to trap shrimp eggs. This is why a frequency range in which only shrimp eggs are trapped can be found.

Fig. 6.8 The numbers of trapped flying color seeds and shrimp eggs vs. operating frequency at operating voltages of 20Vp-p.
6.4.5 Effect of needle length on the acoustic trapping capability

The effect of needle length on the acoustic radiation force on a sphere with a radius of 0.5 mm in water is investigated experimentally and theoretically, and the results are shown in Fig. 6.9. Figure 6.9(a) shows the calculated acoustic trapping force acting on the sphere in water under the conditions that the radius of needle is 0.4 mm, the operating frequency is 66.75 kHz, the y-directional vibration displacement of upper metal plate is 1 µm, and the sphere center is at \( y = 0.5 \) mm. It is seen that in the needle length range from 24 mm to 60 mm, there are several peaks for the acoustic trapping force; at certain length, the acoustic trapping force reaches its maximum. To verify the calculated results, experiment has been conducted for one of the peaks: needle length range from 24 mm to 33 mm; and the result is shown in Fig. 6.9(b). The minimum voltage to trap one shrimp egg in water at resonance is measured for needle with length of 25.2 mm, 28.5 mm, 29.8 mm, 31.5 mm and 32.9 mm, respectively, when the radius of needle is 0.4 mm. The smaller the minimum voltage needed, the easier the particle can be trapped; thus the trapping capability is expressed as the reciprocal of the minimum voltage. From Fig. 6.9(b), it is seen that the trapping capability reaches the maximum at needle length of 28.5 mm, which agrees with the calculated results in Fig. 6.9(a).
Fig. 6.9 The effect of needle length on the acoustic trapping force on the same sphere in water when needle radius is 0.4 mm. (a) Calculated results. (b) Experimental results.

Figure 6.10 shows the calculated relationship between the acoustic trapping force in water and vibration displacement amplitude at the tip of
needle in the operating frequency range from 0 to 600 kHz. The length and radius of needle are 34 mm and 0.4 mm, the $y$-directional vibration displacement of upper metal plate is 1 µm, the radius of sphere is 0.5 mm, and the sphere center is at $y = 0.5$ mm. It is seen that the acoustic trapping force increases with the increase of the vibration displacement at the tip of needle; the acoustic radiation force reaches the maximum when the vibration displacement is the largest, which indicates the resonance of the needle. This result can explain the occurrence of peaks of the trapping force in Fig. 6.9(a).

![Graph](image)

**Fig. 6.10** The calculated relationship between the trapping acoustic force and vibration displacement amplitude at the tip of needle.

Figure 6.11 shows the calculated relationship between the acoustic trapping force and operating frequency for different needle lengths. The radius
of needle is 0.4 mm, the y-directional vibration displacement of upper metal plate is 1 μm, the radius of sphere is 0.5 mm, and the sphere center is at $y = 0.5$ mm. It is seen that in operating frequency range, the trapping force reaches the maximum at certain frequencies, these are the resonance frequencies; for different lengths of needle, the peaks of trapping force occur at different resonance frequencies, and the magnitudes of the peak are different as well. This may explain the difference in the magnitude of peaks of trapping force in Fig. 6.9(a).

![Graph showing the calculated relationship between acoustic trapping force and operating frequency for different needle lengths.](image)

Fig. 6.11 The calculated relationship between the acoustic trapping force and operating frequency for different needle lengths.
6.4.6 Effect of needle radius on the acoustic trapping capability

The effect of needle radius on the acoustic trapping force on the sphere in water is calculated, and the result is shown in Fig. 6.12. The length of needle is 34 mm, the operating frequency is 66.75 kHz, the $y$-directional vibration displacement of upper metal plate is 1 $\mu$m, the radius of sphere is 0.5 mm, and the sphere center is at $y = 0.5$ mm. From Fig. 6.12, it is seen that the acoustic trapping force reaches maximum at certain needle radii; and the magnitude of peaks increases when the needle radius increases. The peaks of the trapping force may be caused by the resonance of the needle, and when the needle radius is larger, the resonance is stronger.

![Graph showing effect of needle radius on acoustic trapping force](image)

Fig. 6.12 The effect of needle radius on the acoustic trapping force when the needle length is 34 mm.
6.5 Conclusions

The acoustic needle which is driven by a sandwich type ultrasonic transducer, can trap, transport and separate small particles in liquid by its ultrasonically vibrating tip. More particles, such as flying color seeds, shrimp eggs, can be trapped at high driving voltage; and the trapping capability in water is stronger than that in oil. The trapped particles can be transported in water by moving the needle and the particle loss during the transportation may be prevented by using a strong enough vibration. The acoustic needle can also separate different particles in water by the difference in their densities. The effects of length and radius of needle on the trapping capability are investigated theoretically and experimentally, and the strongest trapping capability is due to the resonance of the needle.
CHAPTER 7  ULTRASONICALLY VIBRATING ROD

7.1 Introduction

In this chapter, I have proposed and investigated the structure of an ultrasonically vibrating rod, fabricated the prototype, investigated and measured the trapping performance, established a theoretical model to analyze the trapping mechanism, and obtained the guidelines for optimizing the trapping capability of the transducer by experimental and theoretical analysis.

7.2 Experiment

7.2.1 Construction and operation

Figure 7.1 shows the structure and size of the actuator proposed. An Aluminium rod is welded to a rectangular metal plate base along the z-direction, and the plate base is fastened onto a Langevin transducer (FBL28452HS, Fuji Ceramics) with a resonance frequency of 25.3 kHz. The length of the rod is 120 mm and the rods with radii of 0.5 mm, 1 mm and 2 mm, respectively, are used. The metal plate has a size of 55 mm × 45 mm × 3 mm, and the length of the rod outside the metal plate is 75 mm. The transducer is 79.5 mm long, which consists of 2 piezoelectric rings 34 mm in outer diameter. When an AC voltage with a frequency close to the resonance point of the ultrasonic transducer is
applied to the piezoelectric rings, a vibration is excited on the rod and small particles nearby are trapped onto the surface of the vibrating rod.

Fig. 7.1 Structure and size of the rod actuator. (a) Schematic. (b) Photo.
7.2.2 Experimental observation and results

To verify the vibration mode of the rod, vibration displacement of the rod in the $x$-direction along its length is measured. The optical microscope (Olympus BX51, Olympus Optical Co., Japan) is used to observe and measure the vibration displacement at the measurement points shown in Fig. 7.2(a). The vibration displacements of the rods with radii of 0.5 mm, 1 mm and 2 mm are shown in Fig. 7.2(b). From Fig. 7.2(b), it is seen that the variation of vibration displacement along the rod is small. Therefore the rod is vibrating back and forth in the $x$-direction about its central axis along its length and the vibration is uniform (0th order vibration).

In this experiment, three rods with radii of 0.5 mm, 1 mm and 2 mm, are used to trap small particles, respectively. Small particles used in experiments include flying color seed, flowering cabbage and kailan seed. Their densities are 0.93 g/cm$^3$, 1.05 g/cm$^3$ and 1.09 g/cm$^3$, respectively; their radii are 0.5 mm, 0.74 mm and 1.1 mm, respectively.

To trap small particles, the whole vibrating rod orientated horizontally, is submerged into a collection of particles in water and corn oil where the particles are dense enough to cover all the surfaces of the rod, then lifts up immediately, as shown in Fig. 7.3.
Fig. 7.2 Measurement of vibration distribution along the rod. (a) Measured points. (b) Vibration distribution along the rod.
Fig. 7.3 Experimental procedure of trapping the particles.

It is observed that small particles such as flying color seeds, flowering cabbages and kailan seeds are difficult to be trapped in air, however they can be trapped onto the surface of the rod in water and coin oil. The trapped particles distributed all over the vibrating surface of the rod along all its length, and there are more particles trapped at the vibrating direction of the rod. After the first moment of trapping, the time that the vibrating rod is submerged in particles has little effect on the number of trapped particles.
Fig. 7.4 Experimental relationships between the number of trapped particles and input voltage at resonance. (a) In water. (b) In corn oil.

Figure 7.4(a) shows the measured number of trapped particles by the rods with radii of 0.5 mm, 1 mm, and 2 mm at the resonance for different
voltages. Flying color seeds are used in the experiment. It is observed that at resonance, when the input voltage increases, the number of trapped particles increases first then decreases. The former is due to the increase of the acoustic radiation force, and the latter perhaps due to the acoustic stream which can flush particles away. Also it is noticed that the 1 mm radius rod is able to trap most particles. Figure 7.4(b) shows the experimental relationship between the number of trapped particles at resonance and input voltage for 1mm radius rod in corn oil. From Fig. 7.4(b), it is seen that with the increase of input voltage, the number of trapped particles increases for flying color seed and flowering cabbage seed; while for kailan seed, the number of trapped particles starts to decrease after certain input voltage. The decrease in the number of trapped kailan seeds is due to its large surface areas, which makes the drag force resulting from acoustic stream large. Comparing Figs. 7.4(a) and 7.4(b), it is known that the trapping capability in corn oil is weaker than that in water. This is because of the high viscosity of corn oil, which causes a weak sound field.

7.3 Principle analyses and modelling

To explain the trapping mechanism of the ultrasonically vibrating rod, a physical model based on a vibrating rod is devolved to explain the acoustic radiation force generated in the sound field.

It is known that there are different methods of calculating acoustic radiation force acting on a particle in the acoustic sound field. In the acoustic trapping by a vibrating rod, the sound field near the vibrating rod is neither a standing wave nor a traveling wave. Also, the trapped particles are on the
acoustic radiation surface. According to our calculation, there has been up to several times difference in the theoretical results of acoustic radiation force obtained by different methods, and the Gor’kov’s theory is relatively suitable for our analyses. In the analyses, the following assumptions have been made: (1) the particle is sphere-shaped and the sphere is isotropic and compressible; (2) the radius of the sphere $R$ is much smaller than the wavelength $\lambda$ of the sound wave ($kR \ll 1$); (3) the surrounding fluid is nonviscous; (4) multiple scattering effect is not taken into account.

The theoretical analyses are based on the model shown in Fig. 7.5. A sound wave is generated by a rod of radius $a$ vibrating in the 0th order mode, with a velocity $v_0 e^{-ij2\pi ft}$, where $f$ is the vibrating frequency of the rod and $v_0$ is the amplitude of vibration. The 0th order vibration mode is the back and forth vibration of the rod with uniform vibration distribution along the length direction.

![Fig. 7.5 Model for the particle trapping.](image)
If the vibration plane is taken as the reference plane for $\phi$, the normal velocity of the rod surface at an angle $\phi$ has a component $v_0 \cos \phi e^{-2\pi f t}$.

Sound pressure $P$ of the radiated wave is to be [81]:

$$P = A \cos \phi H_1(kr)e^{-i\omega t}$$  \hspace{1cm} (7.1a)

$$A = \frac{2\pi^3 f^2 \rho_0 a^2 v_0}{c_0}$$  \hspace{1cm} (7.1b)

$$k = \frac{\omega}{c_0}$$  \hspace{1cm} (7.1c)

$$H_1(kr) = J_1(kr) + iN_1(kr)$$  \hspace{1cm} (7.1d)

where $\rho_0$ is the density of the fluid, $c_0$ is the sound speed in the fluid, $k$ is the wave number of the sound field, $\omega$ is angular frequency of the sound, $H_1(kr)$ is the Hankel function of the first order, $J_1(kr)$ is Bessel function of the first order and $N_1(kr)$ is Neumann function of the first order. $H_1(kr)$ can be written as

$$H_1(kr) = |H_1(kr)|e^{i\theta_1(r)}$$, where $|H_1(kr)| = \sqrt{J_1^2(kr) + N_1^2(kr)}$ and

$$\theta_1(r) = \tan^{-1} \frac{N_1(kr)}{J_1(kr)}$$. Therefore, Eq. (6.1a) is changed into:

$$P = A \cos \phi |H_1(kr)|e^{-i[\omega t - \theta_1(r)]}$$  \hspace{1cm} (7.2)

In sound field, the relationship between acoustic pressure and vibration velocity is [81]

$$\frac{\partial P}{\partial r} = -\rho_0 \frac{\partial v}{\partial t}$$  \hspace{1cm} (7.3)

So the velocity of fluid near the surface of rod is:

$$v = \frac{A \cos \phi}{\omega \rho_0} |G_1(kr)| e^{-i[\omega t - \theta_1(r) + \pi/2]}$$  \hspace{1cm} (7.4)
where \( |G_i(kr)| = \frac{k}{2} \sqrt{|J_0(kr) - J_2(kr)|^2 + |N_0(kr) - N_2(kr)|^2} \) and

\[
\theta_2(r) = \tan^{-1} \frac{N_0(kr) - N_2(kr)}{J_0(kr) - J_2(kr)}
\]

Thus the instantaneous pressure and vibration velocity can be written as:

\[
P = A \cos \phi [H_1(kr)] \cos[\omega t - \theta_2(r)] \tag{7.5}
\]

\[
v = -\frac{A \cos \phi}{\omega \rho_o} |G_i(kr)| \sin[\omega t - \theta_2(r)] \tag{7.6}
\]

The acoustic radiation force acting on a particle in the sound field is:

\[
\vec{F} = -\nabla U \tag{7.7}
\]

where \( U \) is the time-averaged force potential of the sound field. According to Gor’kov’s theory, the expression of \( U \) is:

\[
U = V[-D\langle K_E \rangle + (1 - \gamma)\langle P_E \rangle] \tag{7.8}
\]

where \( V \) is the volume of the particle sphere, \( \langle K_E \rangle \) and \( \langle P_E \rangle \) are the time-averaged kinetic energy density and potential energy density of the sound field, respectively, \( D \) is a parameter determined by the densities of the particle sphere and fluid, and \( \gamma \) is the compressibility ratio between the particle sphere and fluid. \( D \) and \( \gamma \) can be calculated by

\[
D = \frac{3(\rho_s - \rho_o)}{2\rho_s + \rho_o} \tag{7.9}
\]

\[
\gamma = \frac{\rho_o c_s^2}{\rho_s c_i^2} \tag{7.10}
\]

where \( \rho_s \) is the density of the particle sphere, and \( c_i \) is the sound speed in the particle. From Eqs. (7.5), (7.6) and (7.8), the force potential is:
\[
U = \frac{VA^2 \cos^2 \phi}{4\rho_0} \left[ -\frac{D}{\omega^2} \left| G_1(kr) \right|^2 + \frac{1-\gamma}{\epsilon_0^2} \left| H_1(kr) \right|^2 \right]
\] (7.11)

From Eqs. (7.7) and (7.11), the acoustic radiation force on a particle in the \( r \)-direction is:

\[
F_r = -\frac{VA^2 \cos^2 \phi}{4\rho_0} \left[ -\frac{D}{\omega^2} X + \frac{1-\gamma}{\epsilon_0^2} Y \right]
\] (7.12)

where \( X \) and \( Y \) can be calculated by:

\[
X = \frac{k^3}{4} \left\{ 3J_1(kr)[J_z(kr) - J_0(kr)] + J_3(kr)[J_0(kr) - J_z(kr)] \\
+ 3N_1(kr)[N_2(kr) - N_0(kr)] + N_3(kr)[N_0(kr) - N_z(kr)] \right\}
\] (7.13)

\[
Y = k \left[ J_1(kr)[J_0(kr) - J_z(kr)] + N_1(kr)[N_0(kr) - N_z(kr)] \right]
\] (7.14)

At \( \phi = 0 \) or \( \pi \), the trapping force \( (-F_r) \) is maximum. So \( \phi = 0 \) or \( \pi \) is used in the following analyses. On a small particle on the surface of the rod, there are four forces, i.e. gravity force \( (mg) \), acoustic radiation force \( (-F_r) \), buoyancy force \( (F_b) \) and frictional force \( (-\mu_F) \), as shown in Fig. 7.5.

In air, \( F_b \) is close to zero; thus it is negligible. Therefore, to trap the particle onto the rod surface, the following requirement must be satisfied:

\[-\mu_F F_r \geq mg \] (7.15)

where \( \mu_F \) is the frictional coefficient between the particle and Aluminium rod.

From Eq. (7.12), the acoustic radiation force acting on a small particle with radius \( R \) on the surface of the rod \( (r = a + R) \) is

\[-F_r = \frac{V\pi^2 f^2 \rho_0 \alpha^2 v_0^2}{c_0^2} \left[ -\frac{D}{\omega^2} X + \frac{1-\gamma}{\epsilon_0^2} Y \right] \] (7.16)
From Eqs. (7.15) and (7.16), it is known that the smallest vibration velocity \( v_{m0} \) at which the particle in air starts to be trapped is:

\[
v_{m0} = \frac{c_0}{\pi^3 f^2 a^2} \sqrt{\frac{\rho_s g}{\mu_s \rho_0 \left[ -\frac{D}{\omega^2} X + \frac{1-\gamma}{c_0^2} Y \right]}} \quad (7.17)
\]

While in water, the buoyancy force \( F_b = \rho_0 V g \) acting on particle is not negligible. So, to trap the particles onto the surface, the following relationship must be satisfied:

\[-\mu_s F_r \geq m g - F_b \quad (7.18)\]

Thus \( v_{m0} \) in water can be calculated by:

\[
v_{m0} = \frac{c_0}{\pi^3 f^2 a^2} \sqrt{\frac{(\rho_s - \rho_0) g}{\mu_s \rho_0 \left[ -\frac{D}{\omega^2} X + \frac{1-\gamma}{c_0^2} Y \right]}} \quad (7.19)
\]

Smallest vibration velocity is used as an index to express the trapping capability. The smaller the \( v_{m0} \), the stronger the trapping capability. In the above analyses, the effect of acoustic stream is not taken into account. In fact, when the vibration is strong enough, there is acoustic stream flowing from the radiation surface to far field, which may flush some of trapped particles away [19, 84]. However, it is found that there is no acoustic stream or the effect of acoustic stream on particle trapping is negligible when the vibration velocity is less than some value [19, 82]. Our analysis is for the vibration range in which the effect of acoustic stream is small.

### 7.4 Results and discussions
7.4.1 Calculation conditions

The density of the wet seeds with water absorbed is 1.05 g/cm$^3$, which is calculated by the mass and volume of a collection of the wet seeds. The measured sound speed in wet flying color seeds is around 500 m/s (see Chapter 8 for details). The friction coefficient between the seeds and the surface of Aluminium rod in water is calculated by measuring the tilt angle of the Aluminium rod when a seed on it starts to slide [85], and the friction coefficient of flying color seeds, flowering cabbage seeds and kailan seeds are 0.7439, 0.3536 and 0.4423, respectively. The density of and sound speed in different fluids used in the experiment and calculation are shown in Table 7.1 [83].

Table 7.1 Density of and sound speed in different fluids.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density (kg/m$^3$)</th>
<th>Sound speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$</td>
<td>0.089</td>
<td>1290</td>
</tr>
<tr>
<td>Water vapor</td>
<td>0.804</td>
<td>494</td>
</tr>
<tr>
<td>Air</td>
<td>1.200</td>
<td>340</td>
</tr>
<tr>
<td>O$_2$</td>
<td>1.429</td>
<td>316</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>1.977</td>
<td>259</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>Sea water</td>
<td>1025</td>
<td>1531</td>
</tr>
<tr>
<td>Corn oil</td>
<td>914</td>
<td>1463</td>
</tr>
<tr>
<td>Alcohol (ethyl)</td>
<td>785</td>
<td>1207</td>
</tr>
<tr>
<td>Alcohol (methyl)</td>
<td>786</td>
<td>1103</td>
</tr>
</tbody>
</table>
7.4.2 Force potential distributions in water

Under the conditions that the vibration velocity $v_0$ is 1 m/s, radius of the rod $a$ is 2 mm and $\cos \phi = 1$, the distribution of force potential per unit volume particle in water at different frequencies is calculated by the above method, and the result is shown in Fig. 7.6. In the figure, $r$ is the distance from the rod center to particle center. It is seen that when a particle approaches the rod surface, the force potential per unit volume reaches its minimum, which means that the particles near the rod may be pushed onto the rod surface. The stability of particle in transversal direction is achieved by the frictional force between the particle and rod [19].

Fig. 7.6 Force potential distribution at different frequencies in water.
7.4.3 Comparison of experimental and calculated results

Figure 7.7(a) shows the experimental relationship between the smallest vibration velocity to trap particle and rod radius in water for flying color seeds, flowering cabbage seeds and kailan seeds, respectively. The smallest vibration velocity is obtained from the measured ratio of vibration velocity to motional current, and the motional current at operating point, which can be calculated by the driving voltage, current and the phase difference between them. From the figure, it is seen that among 3 rods with radii of 0.5 mm, 1 mm and 2 mm, the trapping capability is the best for flying color seeds, and weakest for kailan seeds. The three particles have very close density, but different radius. Flying color seeds and kailan seeds have the smallest and largest radius, respectively. Thus small particles are easy to be trapped onto the rod surface. Figure 7.7(b) shows a comparison of the experimental and theoretical smallest vibration velocities to trap different particles when the rod radius is 2 mm. It is seen that the theoretical calculation can explain the difference in the measured smallest vibration velocities to trap the particles. However, the calculation error increases as the particles radius increases for a given rod radius. In Gor’kov’s theory of acoustic radiation force, the multiple scattering effect is not taken into account. This causes error in the calculation of acoustic radiation force when the particles are on the radiation surface [46]. As the radius ratio of the trapped particle to rod increases, the error of Gor’kov’s theory should increase.
Fig. 7.7 Smallest vibration velocity to trap different particles. (a) Experimental relationship between the smallest vibration velocity and rod radius for different particles in water. (b) Comparison of experimental and calculated smallest vibration velocity when rod radius is 2 mm.
From this conclusion, our following calculation is based on a relatively small radius ratio of particle to rod where the calculation error is small. Particle radius $R = 0.5$ mm and rod radius $a = 2$ mm are used in the calculation if no other specified.

**7.4.4 Smallest vibration velocity to trap particle**

The smallest vibration velocity for flying color seed in different fluids is calculated, the results are shown in Table 7.2. In the calculation, the rod radius $a$ is assumed to be 2 mm and the operating frequency is 150 kHz. It is seen that with the increase of fluid density, the smallest vibration velocity decreases. The trapping capability is better for fluid with higher density. Moreover, according to our calculation, this conclusion changes little with the operating frequency.

The effect of driving frequency on the trapping capability for wet flying color seeds in different gases and liquids is calculated, and the result is shown in Fig. 7.8. The particle density $\rho$, is 1.05 g/cm$^3$, the particle radius $R$ is 0.5 mm, and the rod radius $a$ is 2 mm. From Fig. 7.8(a), it is seen that the smallest vibration velocity in gas firstly increases then decreases with the increase of frequency. In liquids, as shown in Fig. 7.8(b), the smallest vibration velocity decreases with the increase of frequency. Therefore, to trap particles in gases and liquids, it’s better to choose high frequency.
Table 7.2 Smallest vibration velocity to trap particles in different gases and liquids.

<table>
<thead>
<tr>
<th>Types</th>
<th>(v_{m0}) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td></td>
</tr>
<tr>
<td>(H_2)</td>
<td>2.5448</td>
</tr>
<tr>
<td>Water vapor</td>
<td>2.0817</td>
</tr>
<tr>
<td>Air</td>
<td>0.9408</td>
</tr>
<tr>
<td>(O_2)</td>
<td>0.7744</td>
</tr>
<tr>
<td>(CO_2)</td>
<td>0.4809</td>
</tr>
<tr>
<td>Liquid</td>
<td></td>
</tr>
<tr>
<td>Alcohol (ethyl)</td>
<td>0.0396</td>
</tr>
<tr>
<td>Alcohol (methyl)</td>
<td>0.0347</td>
</tr>
<tr>
<td>Corn oil</td>
<td>0.0251</td>
</tr>
<tr>
<td>Water</td>
<td>0.0140</td>
</tr>
<tr>
<td>Sea water</td>
<td>0.0098</td>
</tr>
</tbody>
</table>
Fig. 7.8 Calculated relationships between the smallest vibration velocity to trap particles and driving frequency. (a) In gases. (b) In liquids.

7.4.5 Acoustic radiation force vs. driving frequency
The calculated relationship between acoustic radiation force in the \(-r\) direction \((-F_r)\) and driving frequency for particle with different sound speed is shown in Fig. 7.9. In the calculation, the vibration velocity \(v_0\) is 0.1 m/s, rod radius \(a\) is 2 mm, and the particle radius is 0.5 mm. From the figure, it is seen that for particles at some sound speeds, the value of acoustic radiation force in the \(-r\) direction \((-F_r)\) may become negative. When \(-F_r\) is positive, the acoustic radiation force acting on the particle is in the \(-r\) direction, which attracts the particles onto the surface of the rod; when \(-F_r\) is negative, the acoustic radiation force acting on the particle is in the \(+r\) direction, which repels the particles away from the surface of the rod. It is seen that when \(c_s\) is smaller than the sound speed of water, the acoustic radiation force is attractive force; when \(c_s\) is larger than the sound speed of water, the acoustic radiation force is repulsive force. Analyzing the terms in Eqs. (7.10) and (7.12), it is known that the above conclusion holds when the densities of the particle and fluid are close. Also, it is known that even if the densities of the particle and fluid are not close, the acoustic radiation force on a given particle changes its direction too. However, in this case, the critical sound speed of particle at which the direction of acoustic radiation force changes, is not equal to the sound speed of water.
Fig. 7.9 Calculated relationship between the acoustic radiation force in the $-r$ direction and driving frequency in water for particles with different sound speeds.

7.4.6 Effect of particle position on the trapping capability

The effects of the particle position $r$ on the acoustic radiation force acting on a particle ($-F_r$) in CO$_2$ and in water are calculated, the results are shown in Fig. 7.10. In the calculation, vibration velocity $v_0$ is 0.1 m/s and rod radius $a$ is 2 mm. The particles here are not necessarily on the surface of the rod. From the figures, it can be seen that the trapping force ($-F_r$) decreases with the increase of $r$ both in CO$_2$ and in water. From Fig. 7.10(b), it can be observed that in water within certain particle position range, the trapping force and its
spatial gradient can be quite large. So there exists a range $r < r_{cr}$, in which the trapping and sucking capability is strong. Define the solution of $r$ to equation

$$\frac{\partial}{\partial r} \left( \frac{-F_r}{V} \right) = -1 \text{ (N/m}^3\text{.mm)}$$

as the critical position $r_{cr}$, where $-F_r / V$ is the acoustic radiation force per unit volume particle. So, from the following equations,

$$\frac{\partial}{\partial r} \left( \frac{-F_r}{V} \right) = \frac{\pi^6 f^4 \rho_0 a^4 v_0^2}{c_0^2} \left[ -\frac{D}{\omega^2} \frac{\partial X}{\partial r} + \frac{1-\gamma}{c_0^2} \frac{\partial Y}{\partial r} \right] = -1 \quad (7.20)$$

$$\frac{\partial X}{\partial r} = \frac{k^4}{8} \left[ (J_0(kr) - J_2(kr))(4J_2(kr) - 3J_0(kr) - J_4(kr)) + 3J_1(kr) - J_3(kr) \right]^2 + [N_0(kr) - N_2(kr))(4N_2(kr) - 3N_0(kr) - N_4(kr)) + 3N_1(kr) - N_3(kr)]^2 \quad (7.21)$$

$$\frac{\partial Y}{\partial r} = \frac{k^2}{2} \left[ (J_0(kr) - J_2(kr))^2 + J_1(kr)[J_3(kr) - 3J_4(kr)] \right] + [N_0(kr) - N_2(kr))^2 + N_1(kr)[N_3(kr) - 3N_4(kr)] \right] \quad (7.22)$$

r_{cr} can be calculated.

Figure 7.11 shows the relationship between $r_{cr}$ and driving frequency for the trapping in water at different particle densities. In the calculation, vibration velocity $v_0$ is 0.1 m/s and rod radius $a$ is 2 mm. From Fig. 7.11, it is seen that $r_{cr}$ increases with the increase of frequency. This means that at high frequency, the range in which particles can be sucked to the rod surface may increase.
Fig. 7.10 Calculated relationships between the acoustic radiation force per unit volume particle in the $-r$ direction and particle position at different frequencies.

(a) In CO$_2$. (b) In water.
Fig. 7.11 Calculated relationship between $r_{cr}$ and driving frequency for particles with different densities in water.

7.5 Conclusions

The surface of the rod which is in the 0th order vibration mode (vibrating back and forth about its central axis along its length and the vibration is uniform) is used to trap small particles onto the vibrating surface along the whole length of the rod. The main characteristics of this device are: more than 50 particles such as plant seeds in water can be trapped each time by the vibrating rod onto its surface; in different fluids, the trapping capability of rod increases with the increase of fluid density; for particles with different sizes, smaller particles are easier to be trapped onto the surface of rod; the acoustic...
radiation force acting on a particle in water may change its direction when the
sound speed in the particle is larger than some critical value; even if particles
are not on the surface of the rod, it is possible to attract them to the rod in
certain distance range. The optimum design can be obtained by choosing proper
dimension of the radius of the rod based on the optimization of ultrasonic field
near the radiation surface and the particle.
The speed of sound is the rate of travel of a sound wave through an elastic medium. The speed of sound is variable and depends on the properties of the substance through which the wave is travelling. In solids, the speed of longitudinal waves depends on the stiffness to tensile stress, and the density of medium. In fluids, the medium’s compressibility and density are the important factors. In the calculation of acoustic radiation force acting on particles, speed of sound in particle is an important parameter. However, the sound speed of some commonly seen material has not been reported yet.

8.1 Structure of sound speed meter and measurement method

Sound speed meter (SV-DH-7A, Guotai Electronics, China) is composed by two parts: ultrasonic transmitter and ultrasonic receiver, as shown in Fig. 8.1(a). The sample material to be measured is clamped by the ultrasonic transmitter and receiver. The ultrasonic transmitter is composed of a signal generator, piezoceramics material and an ultrasonic stepped solid horn. The piezoceramics material is bonded to the stepped solid horn, and they are connected to the signal generator. The piezoceramics material will experience mechanical stress when an electric field is applied (reverse piezoelectric effect), therefore ultrasound is generated in the sample material. Because ultrasound has the properties of directivity and shorter wavelength, and the diameter of the ultrasonic transmitter terminal is much larger than the wavelength of ultrasound, the ultrasound generated by the transmitter can be considered as plane...
ultrasonic wave. After the ultrasound travels through the sample material, the ultrasonic receiver converts the received ultrasonic vibration to electric signal by using the direct piezoelectric effect. The receiver is movable, so sample material with different lengths can be measured. The sound speed meter has a resonance frequency of 35.7 kHz. Figure 8.1(b) shows a cylinder clay sample to be measured clamped by the ultrasonic transmitter and receiver.

Fig. 8.1 Sound speed meter. (a) Structure of the sound speed meter. (b) A cylinder clay sample to be measured clamped by the ultrasonic transmitter and receiver.

For sound speed measurement, two samples with different lengths \((L_1 \text{ and } L_2, \text{ respectively})\) are used, and the time for sound to travel through each sample in its length direction is measured \((t_1 \text{ and } t_2, \text{ respectively})\). The sound
speed in sample material \((c_s)\) is calculated by the length difference \((\Delta L = L_1 - L_2)\) and the time difference \((\Delta t = t_1 - t_2)\).

\[
c_s = \frac{\Delta L}{\Delta t}
\]

(8.1)

Fig. 8.2 Time for sound to travel through a cylinder clay sample with a length of 8.9 cm observed from the oscilloscope.

Figure 8.2 shows the time for sound to travel through a cylinder clay sample with a length of 8.9 cm observed from the oscilloscope. Channel 1 (yellow color curve in Fig. 8.2) shows the impulse signal generated by the signal generator; channel 2 (blue color curve in Fig. 8.2) shows the signal received by the ultrasonic receiver; the time difference between the two
channels is the time for sound to travel through this sample in its length direction. Therefore for this particular sample, \( L_1 = 8.9 \text{ cm} \), \( t_1 = 41 \mu\text{s} \).

8.2 Measured sound speed

The sound speed in different materials is measured by the above described method using the sound speed meter. Table 8.1 shows the measured sound speed for some materials which are common in our daily life but whose sound speeds are not reported before.

It is known that steel has a sound speed between 4000 m/s and 6000 m/s depending on the type of steel. From Table 8.1, the measured sound speed of magnetic material which is made of steel is 4832 m/s. Thus the result measured from the sound speed meter is reliable.

Some powder materials, such as soil and sand, are put into containers with different sizes. The containers are made of same material and have same thickness, therefore the time for sound travels in each container is constant, and the effect of the container thickness can be cancelled in the calculation. So the container has no influence on the measurement of the sound speed.

For some relatively large particles which can be clamped by the ultrasonic transmitter and receiver, such as rice, there are two ways to measure the sound speed: one way is to measure single particle only; the other way is to measure a cluster of particles in a container. By comparing sound speeds using the two methods, such as rice measured by single particle (635 m/s) and measured in container (256 m/s), it is seen that there is a difference between
them. This is because of the space among particles in the container which results in measuring a mixture of air and particles actually. For powder particles such as soil, the effect of air space among particles is negligible; however unfortunately the accuracy caused by air space between particles is unavoidable for relatively large particles.

Therefore, the sound speed meter is more accurate for long solid samples (such as cylinder clay samples with length of 8.9 cm and rectangular cuboid magnetic material with length of 6.2 cm), small sized spherical particles (such as flying color seeds with radius of 0.5 mm) and powder material (such as soil and sand); it is less accurate for relatively large and irregular shaped particles (such as rice particle).
Table 8.1 Measured sound speed in different solid materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Sound speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay (plasticine)</td>
<td>2716</td>
</tr>
<tr>
<td>Medicine pill (Panadol)</td>
<td>766</td>
</tr>
<tr>
<td>Soil (dry)</td>
<td>210</td>
</tr>
<tr>
<td>Soil (13.6 % wet)</td>
<td>233</td>
</tr>
<tr>
<td>Sand (dry)</td>
<td>352</td>
</tr>
<tr>
<td>Sand (15.8 % wet)</td>
<td>292</td>
</tr>
<tr>
<td>Flying color seed (dry)</td>
<td>384</td>
</tr>
<tr>
<td>Flying color seed (wet)</td>
<td>500</td>
</tr>
<tr>
<td>Paper 1 (printing paper)</td>
<td>217</td>
</tr>
<tr>
<td>Paper 2 (book)</td>
<td>515</td>
</tr>
<tr>
<td>Clothes (100% cotton)</td>
<td>770</td>
</tr>
<tr>
<td>Plastic foam</td>
<td>343</td>
</tr>
<tr>
<td>Plastic bag</td>
<td>643</td>
</tr>
<tr>
<td>CD</td>
<td>402</td>
</tr>
<tr>
<td>Button (polyester)</td>
<td>1379</td>
</tr>
<tr>
<td>Soya bean</td>
<td>899</td>
</tr>
<tr>
<td>Vegetable leaf (cabbage)</td>
<td>225</td>
</tr>
<tr>
<td>Vegetable stem (carrot)</td>
<td>269</td>
</tr>
<tr>
<td>Rice (single particle)</td>
<td>635</td>
</tr>
<tr>
<td>Rice (in container)</td>
<td>256</td>
</tr>
<tr>
<td>Rice (cooked)</td>
<td>250</td>
</tr>
<tr>
<td>Magnetic material (steel)</td>
<td>4832</td>
</tr>
<tr>
<td>Resin (glue stick)</td>
<td>1756</td>
</tr>
</tbody>
</table>
CHAPTER 9 CONCLUSIONS & RECOMMENDATIONS

9.1 Conclusions

A significant achievement in this thesis is that the author proposed a new operating principle of using ultrasonic field near the radiation surface to trap particles. The conventional acoustic method of trapping small particles (such as standing ultrasonic wave and focused ultrasonic beam) uses ultrasonic far-field to trap particles, which may result in low utilization energy efficiency. This can be improved by using the ultrasonic near field to trap or suck particles to the radiation surface.

Based on this principle, the author designed and fabricated three ultrasonic transducers with novel structures, developed and clarified the detailed theoretical model for each ultrasonic transducer and proposed useful design guidelines for improving the acoustic trapping capability of each transducer. The three novel ultrasonic devices are the W-shaped transducer, acoustic needle and ultrasonically vibrating rod, which have relatively simple and compact structure and use ultrasonic field near the radiation surface to trap particles in water, air and other fluid.

The W-shaped transducer has merits of trapping large and heavy particles. It uses the whole sharp edges (22.5 mm long) of the two metal strips with large vibration to trap particles. The main characteristics of this device include: the trapping capability increases with the increase of vibration displacement at the sharp edge of the metal strip; the transducer can trap up to a
weight of 256 milligrams per particle in air when the vibration of the sharp edge is larger than 34 µm; maximum number of 5 cubic particles with a length of 3.25 mm and weight of 40 milligrams each can be trapped to the sharp edges of the metals strip each time at a driving voltage of 65 V_{rms} at resonance; particles which have the same mass and volume but different shapes and orientations have their own trapping capability.

The acoustic needle with a length of 34 mm and a radius of 0.4 mm has the most compact structure among the 3 devices. It is used to trap small particles at a particular location as it uses the tip of needle to trap particles. It has advantages in transportation and separation of small particles. The main characteristics of this transducer include: the trapping capability increases with the increase of vibration at the tip of needle; small particles, such as flying color seeds (with a radius of 0.5 mm) and shrimp eggs (with a radius of 0.117 mm), can be trapped at the tip of the needle in water and oil; even micron sized particle, such as yeast powder (with a radius of 5 µm) can be trapped by the tip of vibrating needle in water operating at a voltage of 35.8 V_{rms} at resonance, where the needle is orientated horizontally under the microscope; the trapped particles can be transported in water at an average speed of 0.88 cm/s from one location to another by moving the needle and the particle loss during the transportation may be prevented by using a strong enough vibration; the acoustic needle can also separate different particles in water by the difference in their densities.

The surface of the rod which is in the 0th order vibration mode (vibrating back and forth about its central axis along its length and the vibration
is uniform) is used to trap small particles onto the vibrating surface along the whole length of the rod (the length of rod is 75 mm). It has potential capability to trap more particles. The main characteristics of this device are: small particles, such as flying color seeds, cabbage seeds (with a radius of 0.74 mm) and kailan seeds (with a radius of 1.1 mm), can be trapped to the surface along the whole length of the rod in water and corn oil; more than 50 particles such as plant seeds in water can be trapped each time by the vibrating rod onto its surface at a driving voltage of $50 \, V_{\text{rms}}$ at resonance; in different fluids, the trapping capability of rod increases with the increase of fluid density; for particles with different sizes, smaller particles are easier to be trapped onto the surface of rod; the acoustic radiation force acting on a particle in water may change its direction when the sound speed in the particle is larger than some critical value; even if particles are not on the surface of the rod, it is possible to attract them to the rod in certain distance range.

The characteristic of each transducer is summarized in the following table. From the above table, it is known that the W-shaped transducer, which uses the whole sharp edges with large vibration to trap particles, has merits of trapping large and heavy particles; acoustic needle, which has compact structure and uses the tip of needle to trap particles, has advantages in trapping small particles at a particular location, transporting the trapped particles and separating particular particles from a mixture; the ultrasonically vibrating rod has potential capability to trap more particles as the vibrating surface along the whole length of the rod is used to trap particles onto the surface.
Table 9.1 Comparison of three devices.

<table>
<thead>
<tr>
<th></th>
<th>W-shaped transducer</th>
<th>Acoustic needle</th>
<th>Vibrating rod</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency</td>
<td>22 ~ 24 kHz</td>
<td>65 ~ 68 kHz</td>
<td>22 ~ 24 kHz</td>
</tr>
<tr>
<td>Input voltage</td>
<td>50 ~ 65 $V_{\text{rms}}$</td>
<td>10 ~ 30 $V_{\text{rms}}$</td>
<td>40 ~ 60 $V_{\text{rms}}$</td>
</tr>
<tr>
<td>Size of particles trapped</td>
<td>1 ~ 10 mm</td>
<td>10 to 500 µm</td>
<td>500 ~ 1000 µm</td>
</tr>
<tr>
<td>Weight of particles trapped</td>
<td>Up to 256 mg</td>
<td>Up to 4 mg</td>
<td>Up to 30 mg</td>
</tr>
<tr>
<td>Number of particles trapped</td>
<td>Max. 5</td>
<td>Max. 8</td>
<td>Max. 55</td>
</tr>
</tbody>
</table>

Furthermore, in this thesis, author first proposed a method to calculate the acoustic radiation force on a particle with arbitrary shape in an arbitrary sound field. It is based on the combination of the FEM analysis and the energy density difference theory of acoustic radiation force. The conventional method of calculating acoustic radiation force is not suitable for calculating the acoustic radiation force acting on particle with other shape than sphere in complicated sound fields, such as the one near radiation surface. The limitations of the conventional method can be improved by the calculation method developed in this work.

In addition, an experiment has been designed to measure the sound speeds for some materials which are commonly seen in daily life but whose sound speeds are not reported yet. The sound speed in particle is very important in the calculation of the acoustic radiation force.
In summary, author first proposed a new operating principle that used acoustic radiation surface to trap small particles to improve the utilization energy efficiency. Based on this principle, three novel ultrasonic transducers were designed to trap different targeted particles, respectively. The performance of each transducer was clarified; a theoretical model was developed for each transducer; the theoretical calculations were confirmed by the experimental results. Also author first proposed a method to calculate the acoustic radiation force on an arbitrarily shaped particle in an arbitrary sound field. This research works have great potential in home appliances, such as removing dust particles, and separating a particular particle from a mixture.

9.2 Recommendations

Some recommendations for future work are proposed as followings:

(1) Metal strip array

From the W-shaped transducer, it is known that particles can be trapped to the lower end of metal strip, and maximum five particles (pill C) can be trapped one time to the lower end of strip. The possibility of trapping even more particles may be achieved by the metal strip array. Two or more metal strips can be arrayed in their width direction, as shown in Fig. 9.1. By studying the mechanism of the structure and adjusting the width of each strip and distance between two adjacent strips, the structure may be optimized and more particles may be trapped by the strip array.
(2) Repulsive force on particle

From the discussion in Chapter 7 for the ultrasonically vibrating rod, it is known that the acoustic radiation force on particle may change its direction from attractive force to repulsive force when sound speed in the particle is larger than a critical value. This conclusion may be applicable for the separation of mixture particles and cleaning purpose.
AUTHOR’S PUBLICATIONS

International journals


Conference papers

2007.


BIBLIOGRAPHY


