Acknowledgment

First and foremost, I am deeply indebted to my supervisor A/P Law Choi Look for his immense support and direction in the research undertaken. His clear explanations and great thoughts have allowed me to go through a process of deep thinking during the course of research. A/P Law also directed me on what should be the correct research direction I should be heading to. I thank him for all the education, inspirations and guidance he has shown to me. Without his invaluable help, patience, and encouragement, this project would not have reached this stage.

I am fortunate to work closely with Dr Francois Chin on my research. He encourages me to look at problems from different angles and points out areas which I should be paying attentions to during formulating of problems. Dr Francois Chin raised many constructive questions to allow me to think the problems in depth. He helps me look at practical problems and shares his ideas on where the current trend of wireless sensor networks is heading. His feedback encourages me to think and formulate problems in a very practical manner.

I would like to thank Dr Lin Zhiwei for showing me the correct attitude towards research work and his advice on how to balance between research and my academic study. He gave me guidance on how to analyze problem and build up background in many fundamental areas. Furthermore, I would like to thank all my friends for creating a conducive environment to work and making my research life an enjoyable one.

Finally, I would like to extend my thanks to my family for their love and support. It is them who are always supportive and encourage me to persevere and achieve this dream.
Table of Contents

Acknowledgment ii

Table of Contents iii

Summary viii

List of Figures x

List of Tables xiv

List of Abbreviations xiv

Notations xvi

1 Introduction 1

1.1 Motivation ........................................... 1
1.2 Objectives .......................................... 9
1.3 Contributions ...................................... 9
1.4 Organization of the thesis ......................... 11
2 Literature Review of Related Works

2.1 Ultra-WideBand Impulse Radio ........................................ 13

2.2 Different Localization Methods and Techniques .......................... 17
   2.2.1 Ranging Techniques ............................................. 17
   2.2.2 Anchor Based Localization Method ................................. 18
   2.2.3 Anchor Free Localization Method ................................. 20
   2.2.4 Scalability Issue ................................................ 25
   2.2.5 Factors Affecting Localization Accuracy .......................... 27

2.3 How to overcome time drift issue ...................................... 32
   2.3.1 Symmetric Double Sided Two-Way Ranging Method .............. 35
   2.3.2 Asymmetric Double Sided Two-Way Ranging Method .............. 38
   2.3.3 Unequal Reply Time Problem .................................... 41

2.4 Summary ................................................................. 41

3 Proposed Methods to Compensate Clock Time Drift ................. 42

3.1 Introduction ............................................................. 42

3.2 The Proposed SMWR Method ........................................... 44

3.3 SMWR Formulation ..................................................... 45
   3.3.1 Adjacent Distance Estimation ................................... 45
   3.3.2 Reply Time Estimation .......................................... 47
   3.3.3 Non Adjacent Distance Estimation ............................... 49

3.4 SMWR Performance Analysis ........................................... 50

3.5 SMWR Simulation Results ............................................. 54

3.6 The Proposed OWR method ............................................ 58
   3.6.1 Synchronized transmitter and receiver pair ...................... 58
   3.6.2 Non-Synchronized transmitter and receiver pair ................ 60
3.7 OWR Simulation Results .............................................. 62
3.8 The Proposed SDS-TWR-URT Method to Compensate Unequal Reply Time ...................................................... 66
3.8.1 Mathematics Formulation ........................................... 66
3.8.2 Performance Analysis ................................................ 69
3.9 SDS-TWR-URT Simulation Results .................................... 72
3.10 Summary ................................................................. 75

4 Localization System using Wireless Anchors .......................... 78
4.1 Introduction .................................................................. 78
4.2 TDOA Localization Scheme .............................................. 80
4.2.1 Crystal Clock Time Drift Estimation ............................... 83
4.2.2 Reader Offset Estimation ............................................. 84
4.2.3 TDOA Computation ................................................... 84
4.3 The Perturbation Error Analysis ........................................ 84
4.3.1 TOA Error Models .................................................... 85
4.3.2 Time Drift Estimation Error for Message Transmitted by ZS 86
4.3.3 Reader Offset Estimation Error ..................................... 87
4.3.4 TDOA Estimation Error .............................................. 88
4.4 Computation of Tags’ Locations Using Least Square Method ....... 91
4.5 The Cramér-Rao Lower Bound ........................................ 93
4.6 Simulation Results .......................................................... 98
4.7 Method to Reduce Blockages Effect on Ranging Accuracy ......... 101
4.7.1 Weighted TDOA Localization Scheme ......................... 102
4.7.2 The Cramér-Rao Lower Bound with weighting function .... 103
4.8 Weighted TDOA Simulation Results .................................... 108
C Derivation of TODA error variance with time drift compensation 143

References 145
Summary

Localization is becoming a major application of wireless sensor networks. Over the past few years, research in this area has seen tremendous growth and advancement, where various ranging and localization techniques are proposed. The aim of this thesis is to illustrate some of the important aspects in the design of a wireless sensor network for ranging and localization. Emphasis on the efficiency and practical aspects of different schemes are given, and novel techniques to improve on both aspects are proposed.

In a practical deployment of wireless sensor nodes, there are many factors that can affect the ranging accuracy such as noise, transmitter and receiver delays, Non Line of Sight (NLOS), synchronization among the sensor nodes, power, energy consumption, cost, crystal clock time drift, the nodes having different reply time, etc. This thesis focuses on how crystal clock time drift of the wireless sensor nodes and the sensor nodes having different reply time affect the ranging performance. Discussions are made on the different ranging techniques and why the selection of the Time of Arrival technique is chosen to determine the ranges between the individual nodes. Furthermore, the use of Time of Arrival technique to overcome wireless sensor nodes crystal clock time drift and their different reply times are analyzed.

As a typical wireless sensor network consists of a large number of sensor nodes, it poses a requirement for time and energy efficient ranging performance without sacrificing its accuracy. The Time of Arrival technique is modified to cater for large wireless sensor network and at the same time, to achieve good ranging accuracy by compensating the time drift. The base stations need not be synchronized in time with the modified ranging algorithm. This thesis further presents the error analysis on the proposed ranging techniques. Cramer-Rao Bound are derived to show the comparison of the ranging accuracy between time drift compensation and no time drift compensation. Blockages
effect on the sensor nodes is analyzed and compensated by introducing a weighting function. Furthermore, Cramer-Rao Bound performance is derived with the weighting function introduced. Robust performances of the proposed weighting function are illustrated through simulations with different network layouts and laboratory measurements.

In addition, a new time scheduling scheme for ranging packet transmission is provided to avoid packet collision when transmitted by the sensor nodes in a large network. Some general introduction on Kalman filter is given and it is used to improve the localization estimations. Finally, the main contributions of the dissertation are summarized and future research directions are provided.
List of Figures

1.1 Time Difference of Arrival Models ........................................... 4
1.2 Two Way Ranging Model .......................................................... 5

2.1 FCC UWB spectrum mask for an (a) indoor environment (b) outdoor environment ................................................................. 15
2.2 Narrowband and UWB-IR Models ............................................... 16
2.3 TDOA Localization Model .......................................................... 20
2.4 Geometric Dilution Of Precision (GDOP) Model ............................. 27
2.5 N-Way Time Transfer (NWTT) Model .......................................... 33
2.6 Symmetric Double Sided - Two Way Ranging (SDS-TWR) Model .... 37
2.7 Asymmetric Double Sided - Two Way Ranging (ADS-TWR) Model .... 39

3.1 Symmetric Multi-Way Ranging (SMWR) model ............................ 44
3.2 Comparison of Total Average Error for NWTT, SMWR Simulation and Theoretical results ............................................................... 55
3.3 SMWR Simulation and Theoretical Performance ............................ 56
3.4 SMWR Performance with unequal reply time ................................ 57
3.5 One Way Ranging transmission diagram with non-synchronized clocks . 58
3.6 Distance difference illustration between anchor and tag ................. 61
3.7 Path taken by tag ..................................................................... 63
3.8 Estimated distance between the tag and anchor with synchronized clocks 63
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Top View of Zones Layout of the Proposed Scheme</td>
<td>118</td>
</tr>
<tr>
<td>5.2</td>
<td>Zone By Zone Packet Transmission Sequence</td>
<td>119</td>
</tr>
<tr>
<td>5.3</td>
<td>Zone By Zone Timing Diagram between Zone Supervisor, readers and tags across different zones</td>
<td>120</td>
</tr>
<tr>
<td>5.4</td>
<td>Sequential Packet Transmission Sequence</td>
<td>122</td>
</tr>
<tr>
<td>5.5</td>
<td>Sequential Timing Diagram between Zone Supervisor, readers and tags across different zones</td>
<td>124</td>
</tr>
<tr>
<td>5.6</td>
<td>3-D area map using TDOA localization based method for ZBZ transmission scheme</td>
<td>125</td>
</tr>
<tr>
<td>5.7</td>
<td>Top view of the area map</td>
<td>126</td>
</tr>
<tr>
<td>5.8</td>
<td>Absolute Error Distribution</td>
<td>127</td>
</tr>
<tr>
<td>5.9</td>
<td>Block diagram of a system, a measurement model, and a discrete-time Kalman filter</td>
<td>128</td>
</tr>
<tr>
<td>5.10</td>
<td>Kalman Filter cycle</td>
<td>128</td>
</tr>
<tr>
<td>5.11</td>
<td>Localization estimation from TDOA computation compared with after Kalman filter process</td>
<td>131</td>
</tr>
<tr>
<td>5.12</td>
<td>RMSE between Kalman filter estimation and TDOA localization</td>
<td>131</td>
</tr>
</tbody>
</table>
List of Tables

4.1 Coherent Transceiver supported channel modes ..................... 112

5.1 ZS and readers coordinates ........................................... 126
List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>Analog-Digital Converter</td>
</tr>
<tr>
<td>ADS-TWR</td>
<td>Asymmetric Double Sided Two-Way Ranging</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>BPF</td>
<td>Band Pass Filter</td>
</tr>
<tr>
<td>BPPM</td>
<td>Binary Pulse Position Modulation</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>CP</td>
<td>Central Processor</td>
</tr>
<tr>
<td>CRB</td>
<td>Cramér-Rao Bound</td>
</tr>
<tr>
<td>DOA</td>
<td>Direction of Arrival</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution Of Precision</td>
</tr>
<tr>
<td>DS-UWB</td>
<td>Direct Sequence UWB</td>
</tr>
<tr>
<td>DV-distance</td>
<td>Distance vector distance</td>
</tr>
<tr>
<td>DV-hop</td>
<td>Distance vector hop</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>FIM</td>
<td>Fisher Information Matrix</td>
</tr>
<tr>
<td>GDOP</td>
<td>Geometric Dilution Of Precision</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphic User Interface</td>
</tr>
<tr>
<td>HDOP</td>
<td>Horizontal Dilution Of Precision</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Independent and Identically Distributed</td>
</tr>
<tr>
<td>Lite-MAC</td>
<td>Lite Medium Access Control</td>
</tr>
<tr>
<td>LNA</td>
<td>Low Noise Amplifier</td>
</tr>
<tr>
<td>LOS</td>
<td>Line of Sight</td>
</tr>
<tr>
<td>LPS</td>
<td>Local Positioning System</td>
</tr>
<tr>
<td>MAC</td>
<td>Medium Access Control</td>
</tr>
<tr>
<td>MDS</td>
<td>Multi Dimensional Scaling</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non Line of Sight</td>
</tr>
<tr>
<td>NWTT</td>
<td>N-Way Time Transfer</td>
</tr>
<tr>
<td>OWR</td>
<td>One Way Ranging</td>
</tr>
<tr>
<td>PDOP</td>
<td>Positional Dilution Of Precision</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>PHR</td>
<td>Physical Header</td>
</tr>
<tr>
<td>PHY</td>
<td>Physical Layer</td>
</tr>
<tr>
<td>PPM</td>
<td>Pulse Position Modulation</td>
</tr>
<tr>
<td>ppm</td>
<td>parts per million</td>
</tr>
<tr>
<td>PRF</td>
<td>Pulse Repetition Frequency</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RFID</td>
<td>Radio Frequency Identification</td>
</tr>
<tr>
<td>RSS</td>
<td>Received Signal Strength</td>
</tr>
<tr>
<td>SDS-TWR</td>
<td>Symmetric Double Sided Two-Way Ranging</td>
</tr>
<tr>
<td>SDS-TWR-URT</td>
<td>Symmetric Double Sided Two-Way Ranging with Unequal Reply Time</td>
</tr>
<tr>
<td>SFD</td>
<td>Start Frame Delimiter</td>
</tr>
<tr>
<td>SFR</td>
<td>Special Function Registers</td>
</tr>
<tr>
<td>SMWR</td>
<td>Symmetric Multi-Way Ranging</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-Noise Ratio</td>
</tr>
<tr>
<td>SOP</td>
<td>Simultaneously Operating Piconet</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time Difference of Arrival</td>
</tr>
<tr>
<td>TDOP</td>
<td>Time Dilution Of Precision</td>
</tr>
<tr>
<td>TOA</td>
<td>Time of Arrival</td>
</tr>
<tr>
<td>TWR</td>
<td>Two Way Ranging</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra-WideBand</td>
</tr>
<tr>
<td>UWB-IR</td>
<td>Ultra-WideBand Impulse Radio</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical Dilution Of Precision</td>
</tr>
<tr>
<td>VHDL</td>
<td>VHSIC (Very High Speed Integrated Circuit) Hardware Description Language</td>
</tr>
<tr>
<td>ZS</td>
<td>Zone Supervisor</td>
</tr>
</tbody>
</table>
Notations

δ_x  Crystal clock time drift of sensor x
σ   Standard deviation of Gaussian noise
σ^2  Variance of Gaussian noise
φ   Phase delay
c   Speed of light
a   Variable of type column vector
A   Variable of type matrix
d   Distance between two points
e^2  Mean square error
f_c  Center frequency
g_{xy}  Gaussian noise between sensor node x and sensor node y
n_p  Path Loss Exponent
[,]^T  Transpose operation
[,]^{-1}  Inverse operation
E{[.]}, E{.}  Statistical expectation
[\|\cdot\|]  Frobenius norm
|x|  Absolute value of x
I_{n \times m}  Identity matrix of size n \times m
11^T  Matrix of all ones
Chapter 1

Introduction

1.1 Motivation

One of the recent trends in a wireless sensor network is to allow different sensor nodes to determine the pairwise distances among themselves in an ad-hoc manner and finally based on these distances information, locate themselves. Usually, in a wireless sensor network, these sensor nodes are deployed in a large number and therefore, it is desirable to keep them at low power, low cost and simple to manage. Some of the applications that make use of wireless sensor nodes are warehouse monitoring for goods tracking, monitoring of patients in hospitals, security applications, location aware advertising, locating books inside a library, tagging of assets, search and rescue, car park maintenance and locating people. Traditionally, systems deployed in these applications require some permanent sensor nodes to be attached to the walls with wired infrastructure among them. Other sensors are placed on people or objects to be tracked which may be stationary or mobile. In general they are known as mobile stations (MSs). However, many applications need not rely on permanent nodes to be attached to the walls, but instead only need to attach ad-hoc sensors to the building infrastructures temporary or to those objects that
need to be tracked. These ad-hoc sensors communicate with one another in a wireless manner so that it is flexible to deploy and allow the localization system to be scalable in the coverage area.

Prior to locating the positions of the people or objects inside an indoor environment, distances between the sensor nodes need to be first determined and this is normally termed as ranging. There are many methods that can be used to perform ranging. Received Signal Strength (RSS), Angle of Arrival (AOA), Time Difference Of Arrival (TDOA) or Time Of Arrival (TOA) [1] are commonly used.

RSS method detects the received signals strength from a MS and converts it into distance information and uses the same algorithm as TOA to determine the location of the MSs. The major drawback for RSS is the influence of multipaths affecting the ranging accuracy [2]. An equation showing the relationship between the received signal power and distance is given by

\[ P(d) = P_0 - 10n_p \log \frac{d}{d_0}, \]  

(1.1)

where \( P(d) \) is the average received power (dBm), \( P_0 \) is the received power (dBm) with respect to a reference distance \( d_0 \), \( d \) is the distance between two devices and \( n_p \) is the path loss exponent which is normally between two to four. The biggest issue with RSS is its large fluctuations due to multipaths in indoor environment. Received signals are greatly affected by multipaths and complex receivers are required to correctly decode the wanted signals. If multipaths are constructive, they can enhance the data. On the contrary, if they are destructive, they can cancel out the data and result in detecting the noise.

AOA is used to find the direction of a device. There are two common methods to determine AOA [3]. First, AOA can be determined from the ratio of two RSS mea-
measurements from two different directional antennas. Second, AOA is estimated from the
TDOA measurements between two antenna elements for wide bandwidth signals or nar-
rowband signals time delay, τ, which relates to the phase delay given by

\[ \phi = 2\pi f_c \tau, \] (1.2)

where \( f_c \) is the center frequency. AOA is not desirable because it requires an expensive
system as the receiver needs several antennas to receive signals from different directions.
In addition, the antennas may be large resulting in large and bulky devices.

There are two modes of TDOA as shown in Figure 1.1 and four base stations (BSs)
are required to determine the position of a mobile station (MS) in 3-D (three BSs in
2-D) [4]. As shown in Figure 1.1(a), the BS sends a signal to MS and the MS calculates
its position based on the differences of the arrival of these four signals. This can be
classified as a mobile based location system [5]. This method requires a powerful MS
in order to perform computation therefore it is not suitable in a low cost and simple
wireless sensor network. In Figure 1.1(b), the MS broadcasts a signal and is received
by four BSs. The received times are timestamped at the four BSs and are sent to a
central processor (CP) to compute the position of the MS. This method is limited to a
centralized wireless network, classified as a network based location system [5], which is
not scalable. In both cases, all BSs need to be time synchronized for accurate TDOA
determination.

TOA ranging is based on measuring the time at which a signal first arrives at a re-
ceiver. The received time is the transmitted time plus the propagation time which is a
function of the distance between the transmitter and receiver. In order to have an accu-
rate measured range, the transmitter and receiver are required to be time synchronized.
Conventionally, if precise synchronization can be done, One Way Ranging (OWR) can
be performed by just taking the difference of the received and transmitted time. Normally this is difficult to achieve in a practical scenario and continuously maintaining time synchronization among the sensor nodes will cost more system resources and energy. Therefore, Two-Way Ranging (TWR) is proposed and TOA with TWR is suitable when there are a large number of sensor nodes in the network [1]. TWR has the inherent ability to eliminate the needs for clock synchronization between a pair of nodes. Figure 1.2 shows a typical TWR model. Node 1 transmits at time $T_1$ and is received by node 2 at time $R_{(2)}^1$ which is read as the received time at node 2 with respect to the signal arriving from node 1. Then node 2 waits for certain period of time denoted as $\Delta_2$ before it replies at time $T_2$. Then the signal is received by node 1 at $R^2_{(1)}$. With the timestamping of these values, the distance between two nodes can be found by using (1.3) and multiplying by the speed of light.

$$t_p = \frac{1}{2}[(R^2_{(1)} - T_1) - (T_2 - R^1_{(2)})]. \tag{1.3}$$

Eqn (1.3) shows two independent TDOA measurements: $R^2_{(1)} - T_1$ measured at node 1 and $T_2 - R^1_{(2)}$ measured at node 2. Hence there is no need for clock synchronization.
between the two nodes. However, the measurement values of $R_{(1)}^2 - T_1$ and $T_2 - R_{(2)}^1$ are taken at two different nodes which may have slightly different clock values due to the quality of the clock oscillator. As a result, there may still be slight differences from the true TDOA values depending on the clock frequency accuracy, stability and processing delay $\Delta_2$. For example, if there is a crystal clock drift of 10ppm between the two nodes, and $\Delta_2=100\text{us}$, the time difference is around 1ns. Thus for sub meter ranging accuracy, the processing delay should be much less than 100us.

After the ranging information is obtained, localization operation can be performed based on these information. Some of the common localization systems are described in the next few paragraphs. Finally, some of the practical problems faced by the ranging techniques are analyzed, which leads to the motivation of the research of this thesis.

One of the most common localization systems is the Global Positioning System
As GPS is only suitable for locating an object in an outdoor environment [1]-[17], factors such as the need for Line of Sight (LOS) communication prevent it from being used in the indoor wireless sensor networks. LOS communication is used in GPS between a pair of transmitter and receiver, and GPS devices are expensive as well as consuming high energy [1], [3], [13]. In addition, GPS requires high stability clocks to keep all the transmitters clock synchronized to make one way TOA measurements making it expensive to be used in a low cost wireless sensor network.

Another commonly used localization system is Local Positioning System (LPS) where the BSs are set up at the edges of the infrastructures to communicate with the indoor sensor nodes. These BSs maybe equipped with GPS capability to have global coordinates. Conventional GPS receivers do not work inside buildings due to the absence of line of sight to satellites and therefore have to position using the LPS system. Some differences that distinguish LPS from GPS are given as follows [17]:

- Physical devices attached to the people and assets being tracked (tags), should be as small and light as possible
- Tags should be inexpensive and simpler design as compared to GPS receivers
- Accuracy is more demanding for LPS as compared to GPS because LPS is operating in indoor environment while GPS is outdoor operation

For local positioning systems, several approaches have been developed in the past based on radio frequencies, ultrasonic waves, light, receive signal strength of communication systems such as Wireless LAN or Bluetooth and cell identification. For wireless sensor network, these solutions pose several disadvantages such as the power consumption, NLOS signals and achievable accuracy [3], [16]-[17].

UWB-IR (UWB-Impulse radio) technology for localization is becoming more pop-
ular and companies such as Multispectral Solutions-Inc (MSSI), Time Domain and Ubisense have commercialized some well known UWB localization systems. UWB is able to penetrate through walls or obstacles where these can defeat other radio localization system. This is because UWB has good multipath resolution characteristics and obstacle penetration capability compared with the existing transmission media. In addition, UWB-IR is low power and has simple hardware architecture and circuits resulting in low cost transceivers.

In many indoor localization systems, anchor nodes with known positions are attached to the infrastructure and are used to determine the locations of the tags whose locations are unknown [19], [20]. The system allows the tags and the anchor nodes to determine the distances among themselves [3], [19]. However, tags positioning accuracy depends very much on the anchor nodes positioning accuracy [21].

Relative location estimation is also one of the popular techniques for indoor localization. Some examples of relative location applications include locating the object or person and tracking in offices and warehouses [1]. In conventional ranging, distances are determined between devices with known positions and devices with unknown positions. In relative localization, distance information can be determined between devices with unknown positions [15] and it is normally deployed in an ad-hoc network [18]. All devices do not have any knowledge of their absolute coordinates, therefore they can only determine the relative pairwise distance with another device, thus obtaining the set of relative locations [20], [22], and more new devices can achieve better location estimation accuracy [23].

However, ranging accuracy is affected by many parameters such as crystal clock time drift and unequal reply time among the sensor nodes. Reference [100] provides an insight on using TWR to determine the pairwise distances among the nodes in a cooperative manner. However, this method ignores the ranging error affected by the crystal
clock time drift. Time drift is caused by the differences in the nodes oscillator frequency [24]. Other parameters that can affect time drift are the surrounding temperature and pressure [25]. One of the ways to overcome time drift is to use very stable oscillators, i.e. low parts-per-million (ppm) devices. However, this adds up to the cost of the nodes and therefore the system operating cost will increase which is not suitable for a low cost wireless sensor network. In addition, TWR is used to eliminate the need to synchronize the transmitter and receiver. References [26] and [27] use tracking information to manage clock offset, but it also increases the hardware complexity of the sensor nodes [28]. References [101] and [102] attempt to reduce the crystal clock time drift in ranging accuracy between two nodes by performing TWR twice in different directions. This is not very efficient as it requires twice the number of ranging to be performed between every pair of nodes in a large wireless sensor network. More detailed descriptions on these techniques in [101] and [102] are given in chapter 2.

Another important parameter affecting ranging accuracy beside crystal clock time drift is the nodes’ reply time. Reply time is defined as the time between a node receiving a message to the time it starts to transmit a reply message. When all the nodes have different values in their reply time, ranging accuracy will be greatly affected as discussed in [105]. One of the reasons for unequal reply time is the unpredictable response delay from the Medium Access Control (MAC) layer to the Physical (PHY) layer but this can be overcome by ensuring the nodes reply very fast [29] so that crystal clock time drift will not affect the ranging accuracy too much. However, in a practical scenario, it is difficult to control the nodes to reply instantly or keep them equal. Also, in order to keep the reply time small, it requires complex and expensive device to handle the reply time in hardware [30].
1.2 Objectives

Many error sources can affect range measurement. Examples of such error sources are noise, Non Line of Sight (NLOS) signals, transmitter and receiver delays, synchronization of the devices, crystal clock time drift, unequal reply time among the sensor nodes, etc. Since both time drift at the crystal clock and nodes’ reply time can affect the ranging accuracy significantly, it is important to deal with both of these problems when designing a practical ranging and localization algorithm for a wireless sensor network.

The objectives of this thesis focus on how crystal clock time drift and unequal reply time in the wireless sensor nodes affect the ranging performance. Methods including TWR, OWR and TDOA are proposed to compensate the ranging inaccuracy due to crystal clock time drift and to reduce the effect of unequal reply time on ranging accuracy.

Cramér-Rao Bound (CRB) is derived to show the comparison of ranging accuracy between time drift compensation and no time drift compensation in TDOA localization method. Blockage effect such as NLOS signals to the base stations are compensated using a proposed weighting function with laboratory measurements.

To extend from single cell network to large network with multiple cells, a time scheduling scheme in allocating time slots to allow different wireless sensor nodes is described. This can prevent ranging packets transmission without packet collision.

1.3 Contributions

The major contributions of the research include:

1. The effect of crystal clock time drift on ranging accuracy in OWR and TWR ranging are analyzed and methods to mitigate the effect are discussed in Chapter 3. In
addition, a method to reduce the effect of unequal reply time on ranging accuracy is also discussed. Details of this work are also published in [1] of the journal and [1]-[2] and [6] of the conference papers in the Author’s Publications.

2. When there are a large number of sensor nodes in a wireless sensor network, this poses a requirement for time and energy efficient ranging performance and with good ranging accuracy simultaneously. Therefore, the Time of Arrival technique is modified to cater for a large wireless sensor network and at the same time, it is able to achieve good ranging accuracy. This is presented in Chapter 4. The new scheme does not require the BSs to be synchronized thus eliminating the need for cable connections between the BSs. Furthermore, blockage effect is analyzed and a weighting function is proposed to reduce its effect on ranging accuracy. This chapter also discussed the error analysis on the proposed ranging techniques. CRB is derived to show the comparison of ranging accuracy between time drift compensation and no time drift compensation. In addition, CRB is also derived with the weighting function being added to reduce the effect of blockages on ranging accuracy. Finally, laboratory measurements using the proposed TDOA localization method and the weighting function are presented. Details of this work are also published in [2]-[3] of the journal and [4] of the conference papers in the Author’s Publications.

3. In order to cover a large network, multiple cells are required. One of the problems faced in multi cells network is packet collision. Therefore, a time scheduling scheme for ranging packet transmission is proposed to avoid packet collision when it is transmitted by the sensor nodes in a large network. This is given in Chapter 5. Details of this work are also published in [5] of the conference papers in the Author’s Publications.
1.4 Organization of the thesis

The remaining parts of this thesis are organized as follows:

Chapter 2 reviews the literature of related works. First, it gives an introduction on Ultra-WideBand and follow on to describe the different localization methods, their scalability and accuracy limitations and factors affecting the ranging accuracy. Next, it discusses the effect on crystal clock time drift and methods to overcome it. Finally this chapter discusses the influence of unequal reply time on the ranging accuracy.

Chapter 3 discusses the proposed method in reducing crystal clock time drift among the BSs using TWR. It presents the analysis of the theoretical performance and comparison with simulated results. In addition, it also discuss a OWR algorithm that is able to compensate crystal clock time drift and work for both synchronized and non-synchronized clocks. Subsequently, chapter 3 discuss the method in combating against unequal reply time by introducing some compensating factors. Performance analysis is derived to illustrate the impact of ranging error due to the variation of the reply time as well as crystal clock time drift.

Chapter 4 introduces another method to compensate crystal clock time drift in a scenario with a large number of users using TDOA based localization method. The objective of this proposed method is to provide localization coverage in one of the regional airports to track passengers and luggage. The system works with non-synchronized BS and MS. Analysis of the perturbation errors such as crystal clock estimation error, reader offset estimation error and TDOA estimation error are presented. Finally, derivation of the CRB is given and the results are compared with and without time drift compensation. In addition, the effect of blockages on the TOA readings and how it affects the localization performance is also discussed. A robust weighting function is proposed to reduce the blockage effect on ranging accuracy. Simulations in different network layouts are
given to show its robustness. Finally, the CRB with the weighting function is derived. Laboratory measurement which uses the proposed TDOA localization methods and the weighting function is carried out.

Chapter 5 discusses the time scheduling scheme in allocating time slots to allow different wireless sensor nodes inside a large wireless network to transmit their ranging packets without packet collision. This allows the localization system to track on a large number of people or assets at the same time either in an online or offline mode. This chapter is an extension of the single cell to multi cell operation described in chapter 4. A brief introduction on Kalman filter, it’s implementation and application to smooth the localization estimations are discussed. Simulations results show improvements after the estimated positions are passed through a Kalman filter.

Finally, conclusions and contributions of these works are summarized in Chapter 6. In addition, some possible future research directions are also presented.
Chapter 2

Literature Review of Related Works

2.1 Ultra-WideBand Impulse Radio

Applications related with UWB communications were introduced in the early 1990s but only have received wide interest after the Federal Communications Commission (FCC) allowed the use of unlicensed UWB communications. The first commercial systems developed under the IEEE 802.15.3a standard are intended for high data rate and short range personal area networks. Subsequently, another standardization group IEEE 802.15.4a are created for PHY layer for low data rate communications combined with positioning capabilities.

Ultra-WideBand (UWB) is defined as a signal with bandwidth of 500MHz or a fractional bandwidth greater than 0.2 where the fractional bandwidth is given as

\[
BW_{frac} = 2 \times (f_{HI} - f_{LO})/(f_{HI} + f_{LO}),
\]

(2.1)
where $f_{HI}$ and $f_{LO}$ are the -10dB frequencies above and below the frequency of peak emission. UWB occupies between 3.1 to 10.6 GHz frequency spectrum [31]. This implies that UWB has an equivalent time resolution of sub-nanoseconds. It utilizes short duration pulses to spread the transmitted energy over a wide bandwidth. With this precise time resolutions, UWB is very suitable for positioning in the time domain [33]. However, any clock inaccuracies in the devices such as time drift or unequal reply time (in which the next few chapters will address) can affect ranging accuracy. A very comprehensive write-up on UWB is given in [31].

Reference [32] addresses the UWB peak and average power densities for low PRF UWB system designs. It derives close form solutions for the peak and average power and plots the relationship between the permissible full bandwidth peak power against the transmission rate. It is reported that for a transmission rate of 187.5KHz at an effective bandwidth of 500MHz, the permissible full bandwidth peak power is around 20dBm. References [2] and [3] derive the Cramér-Rao Bound for the minimum accuracy in TOA ranging which depends on the signal duration, bandwidth and power. In addition, [2] stated that for a given Signal-Noise Ratio (SNR) of 0dB at an effective bandwidth of 1.5GHz, the lowest achievable ranging accuracy using TOA is 2.25cm. This works out that if the bandwidth is reduced to 500MHz at 0dB SNR, the minimum TOA accuracy will become 6.75cm. Alternatively, if the SNR is increased to 6dB for example at 500MHz bandwidth, the minimum TOA accuracy will be reduced to 3.38cm. This clearly shows that UWB-IR gives high accuracy ranging performance due to its large bandwidth and good SNR characteristic.

Figures 2.1(a) and 2.1(b) gives the FCC UWB spectrum mask for an indoor and outdoor environment respectively [34]. UWB average transmit spectrum density is limited to -41.3dBm/MHz and 0dBm/50MHz for peak transmit spectrum density under the FCC ruling [35]. By exploiting the peak power emission limit, UWB-IR individual
pulses has good SNR with very low duty cycle, thus it can reduce interferences. In addition, UWB has the potential robustness against multipaths compared to narrowband without increasing its transmit power.

Figure 2.1: FCC UWB spectrum mask for an (a) indoor environment (b) outdoor environment
A comparison between narrowband and UWB-IR signal is given in Figure 2.2. Narrowband uses modulated continuous time signal such as frequency modulation with low frequency represented by a binary '0' and high frequency represented by a binary '1'. Its frequency spectrum is centered at the carrier frequency with a high peak and narrow bandwidth. UWB-IR uses pulse based modulation scheme with different polarities to represent a binary '0' or '1'. Its frequency spectrum is spread across broad bandwidth with lower peak compared to narrowband.

The IEEE 802.15.4a standard uses a combination of burst position modulation and BPSK to modulate the symbols. Each symbol consists of an integer number of possible chip positions $N_c$ with duration $T_c=2.003\text{ns}$ which corresponds to a chipping rate of 499.2 MHz. The overall symbol duration is $N_cT_c$. A burst is formed by grouping $N_{cpb}$ consecutive chips with duration of $N_{cpb}T_c$. The total number of burst durations per symbol is $N_c/N_{cpb}$. Since the IEEE 802.15.4a standard has mandated the UWB-IR as the preferred localization technique, therefore, the UWB-IR localization techniques
starts to receive more and more attentions.

In the next few subsections, different localization methods and techniques that are often used by UWB-IR positioning system are described. It is followed by the factors that affect localization accuracy especially due to crystal clock time drift and unequal reply time because UWB has very short (sub-nanosecond) pulses duration.

### 2.2 Different Localization Methods and Techniques

After introducing the basic characteristics of the UWB-IR transmission system, this section first discusses the various typical ranging techniques. Following that, the different classifications of localization methods and how various ranging techniques can be classified under these categories are discussed. Next, the scalability issue faced in these localization methods is elaborated. Finally, the different factors affecting localization accuracy and in particularly crystal clock time drift are presented.

#### 2.2.1 Ranging Techniques

AOA determines the propagation direction from an MS using an antenna array. It requires two BSs to measure the angle between the propagation directions from the MS to the BS and triangulation method is used with respect to a reference point to locate the MS. This method requires complex set of antennas and therefore it is more expensive. Moreover, multipaths propagation greatly affects the range accuracy and it only works best in LOS environment [2].

In TOA method, three set of signals arrival time at the BSs can be used to locate the MSs in a 2-D plane. This is similar to a OWR method. It requires the MS and BS clocks to be synchronized thus resulting in a more complex synchronization system
and waste of resources to perform synchronization. Although [36] is able to use OWR without the transmitter and receiver pair to be time synchronized, crystal clock drift can still limit the ranging accuracy. Alternatively, the differences between the multiple OWR measurements can also be calculated to perform localization and this is termed as TDOA.

RSS is simple to use but it is very dependent on the environmental parameters such as objects around the environment, temperature and pressure or fading and blockage in the radio channel [8]. RSS is also dependent on the calibrations at the transmitter and receiver because different devices have different internal circuits [3] and different circuit components tolerance. A comprehensive study on the different ranging methods are discussed in [6] and [7] and how these methods can fit into the FCC requirements are discussed in [7]. Reference [5]-[12] also give a combination of TOA, TDOA, AOA and RSS localization methods.

### 2.2.2 Anchor Based Localization Method

For anchor based localization method, anchor nodes with known positions are attached to the infrastructure to localize the tags whose locations are unknown. If the anchor nodes are GPS capable, absolute coordinates of the tags can be determined, otherwise, only relative coordinates information with respect to a local coordinate system is available. The tags positioning accuracy depends very much on the anchor nodes positioning accuracy. Furthermore, if some of the anchor nodes malfunctioned or are blocked resulting in no LOS signals with the tags, the accuracy of the tags can be greatly affected. Localization accuracy of the tags can be increased by increasing the number of anchor nodes, however, it implies a higher system cost. Moreover, establishing anchor nodes is a manual deployment task and is cumbersome.
Another problem faced in anchor based localization is the scalability issue. Fixed infrastructure is not scalable to changing environment. If there are any changes in the environment layout, the anchor nodes would need to be re-arranged.

One of the most popular anchor based localization is TDOA where the MS is determined by the intersection of the three hyperbolas drawn around the BSs formed by the three difference readings of the signals arrival time at the BSs [4], [5], [33], [37]-[48]. Figure 2.3 shows a typical TDOA localization method. The three hyperbolas are the time difference of the arrival signals at the three BSs, denoted as $TDOA_{12}$, $TDOA_{13}$ and $TDOA_{23}$. The hyperbolas intersection gives the location estimate of the MS. In the absence of range measurement error, the hyperbolas will intersect at a single point. In practice, the range measurement is corrupted by noise and interference resulting in an area of intersection for the hyperbolas. The larger the area of intersection, the larger the location error. Also, TDOA requires all the BSs to be time synchronized. A good summary on TOA and TDOA is given in [44] and [45]. Reference [46] gives a very good way of solving TDOA equations in a linear fashion as compared to the traditional way of solving hyperbolic intersection to estimate the positions of the MSs which can be non-linear. Reference [49] gives an interesting concept of forming straight lines of positions intersection that is derived from the conventional circular intersection to locate the MSs. It is able to locate the MSs even when there are no circular intersection from the TOA measurements. Reference [50] gives exact solution but has limited application due to the assumption of equal number of TDOA measurements and nodes. Reference [40] presents a close form 2-D localization method that is non-iterative using two-stage weighted least squares. It is further extended in [51] for linear sensor array arrangement.
2.2.3 Anchor Free Localization Method

For anchor free localization method, the sensor nodes positions are with respect to a local coordinate system. Therefore, a relative map is formed after estimating the MSs locations. When TOA ranging method is used in anchor free localization, these sensor nodes exchange ranging packets with one another. With the timestamping of the transmitted and received packet timings, the time of flight can be calculated and eventually, localize among themselves. After that, these sensor nodes start to exchange ranging packets with the MSs in order to localize them. If some of the sensor nodes have known absolute positions (probably obtained through GPS), this relative map can be transformed from a local coordinate system to a global coordinate system. Some typical relative localization methods with their usages, advantages and disadvantages are described in the next few paragraphs.

References [53], [97] use connectivity based method to locate a MS. A fixed number
of nodes in the network serve as reference points whose positions are known. MS listens
to all the packets transmitted by these reference points and then based on a connectivity
matrix above a certain threshold, these MSs will locate themselves according to the
centroid of these reference points. This method requires a lot of reference nodes and
the computation lies with the MSs, thus increasing their complexity. Reference [54]
uses convex constraints to locate the positions of the MSs. It uses rectangular bounds
to determine the constraints and the smaller the constraints, the more accurate are the
estimated locations. This method requires the reference nodes to be placed at the edge
of the rectangular bound. In practice, it is difficult to determine the rectangular bounds
and place the reference nodes exactly at the edge.

Distance vector hop (DV-hop) localization is more scalable but it is limited only
to static small networks. It uses connectivity to infer the position of its nearest BS.
DV-distance is similar to DV-hop except that it uses the measured distance instead of
connectivity [55]. Routing link localization is good only for a small number of sensor
nodes [55]. Positioning centric routing determines a sensor node’s neighbors by assuming
that it is the next node that receives the sensor node’s packet. It can be applied to
mobile network but it requires a database to store the positions of all the neighboring
nodes. Reference [55] states one hop localization networks as those networks where
BSs can directly locate the positions of the MSs because it assumes that all MSs are
within the communication range of the BSs. In an ad-hoc network, it is normally classi-
fied as multihop network where MSs can only communicate to certain BSs. Therefore,
the estimated MSs positions are known as local coordinates as it is only known to a par-
ticular network. After the MS location is estimated, it will become a new BS and other
MSs can localize based on it and this continues to propagate to the entire network [55].
Reference [56] analyzes TOA and RSSI ranging technologies in a multihop wireless
networks and derives the variance of the range estimates for RSSI and TOA as given in
eqn (2.2). From (2.2), it can be seen that if there are more multihops between the source and destination, TOA performance is worse than RSSI.

\[
\begin{align*}
\text{Var}(\hat{d}_{\text{RSSI}}) &= \frac{(C^2 - 1)d^2}{n}, \\
\text{Var}(\hat{d}_{\text{TOA}}) &= n\sigma^2,
\end{align*}
\]

where \( C \) is a constant defined as \( e^{0.5(ln10\sigma/10p^2)} \), \( p \) is the channel path loss, \( d \) is the true distance between the source and destination, \( n \) is the number of nodes and \( \sigma^2 \) is the variance of the zero mean Gaussian distribution.

In self positioning algorithm, every node exchanges a Table containing the information of their adjacent nodes with one another. This process is propagated throughout the entire network [55]. In reference [57], it first assigns the reference nodes at the origin and the other sensor nodes lies in the same x-axis. In this way, the distance measurement with the second node eventually only needs to determine the y-coordinate. Further localization of other nodes is assumed to lie in the positive coordinate region and used a combination of distance and angle measurement. Reference [57] merges the first local map with other maps by some form of rotation and reflection and then refines all the nodes locations in other maps according to the first local map so that the entire network will only have one coordinate system.

Multi Dimensional Scaling (MDS) is another type of localization method which works well for a regular or irregular network layout [58]-[62]. MDS can be classified as connectivity or distance based. In connectivity based, proximity information between sensor nodes is used. In distance based, pairwise distances between every sensor nodes are used. MDS consists of three steps [60]-[62]:

1. Determine the pairwise distance (or connectivity information) between every sen-
2. Convert the pairwise distances (or connectivity information) into position estimation by keeping the first 2 (or 3) largest eigenvalues and eigenvectors to form a 2-D (or 3-D) relative local map.

3. If three or more reference nodes are defined and with their absolute coordinates is known, transform the local map into a global absolute map by rotation, reflection and translation.

The formulation of MDS are as follows. First, obtained a squared distances matrix, denoted by $D$, which contains the measured pairwise distances between the sensor nodes which is given as

$$
D = \begin{bmatrix}
0 & \hat{d}_{12}^2 & \hat{d}_{13}^2 & \ldots & \hat{d}_{1N}^2 \\
\hat{d}_{12}^2 & 0 & \hat{d}_{23}^2 & \ldots & \hat{d}_{2N}^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\hat{d}_{1N}^2 & \hat{d}_{2N}^2 & \hat{d}_{3N}^2 & \ldots & 0
\end{bmatrix},
$$

(2.3)

where $\hat{d}_{12}^2$ is the estimated squared distance between sensor node 1 and 2. Second, double centering is then applied to $D$ to obtain a scalar product matrix, which is denoted by $G$ given as

$$
G = -\frac{1}{2}HDH,
$$

(2.4)

where $H$ is the centering matrix, given by $H = I - \frac{1}{N+1}U$, in which $I$ is a $(N+1) \times (N+1)$ identity matrix and $U$ is a $(N+1) \times (N+1)$ matrix of ones. Now the inner
product of $G$ can be expressed as $G = XX^T$ where $X$ contains the coordinates of the anchor nodes which is the objective of the calculations.

In order to find $X$, Singular Value Decomposition (SVD) is applied to $G$ which is expressed as

$$G = V \Lambda V^T,$$  

(2.5)

where $V$ is the matrix of eigenvectors and $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{N+1})$ is the corresponding diagonal matrix of eigenvalues. Next, retained the three largest eigenvectors and eigenvalues for 3-D mapping and the final coordinates $X$ is given by

$$X = V \Lambda^{\frac{1}{2}}.$$  

(2.6)

Many variations of MDS methods are also presented in [60]. Reference [61] and [62] extends the centralized MDS in [60] into a distributed manner by performing MDS across different parts of a network to form many relative local maps, and then merge these relative local maps into a global absolute map by stitching two maps together if they have the most number of common sensor nodes. In addition, [61] and [62] proposed a refinement step to increase the location estimation accuracy by applying least squares minimization to refine the local maps before merging.

Reference [58] and [59] employ MDS to find the locations of the unknown nodes in its local area and then formed its own local map. With the help of at least two reference nodes in different local maps, it rotates and reflects the coordinates of those nodes in between the two local maps to form an absolute map. Reference [63] and [64] extends
MDS into locating a MS using TOA measurement, but [64] extends the work in [63] further in analyzing NLOS conditions. Reference [66] applied a weight factor to every distance measurement. If the measurements are believed to be more accurate, it is given a higher weight before applying MDS to localize. Then it also selects neighboring sensor nodes if they are above certain threshold distance. After the neighboring sensor nodes are selected, MDS algorithm is re-run to get the final estimated sensor nodes positions. Reference [65] derives another relative localization method called Simple Hybrid Absolute-Relative Positioning which has better performance as compared to MDS and Ad-hoc Positioning System.

Reference [67]-[69] describe localization using mobile BS instead of the MS moving around. In this case, the MS is stationary and BS moves to certain predefined locations and broadcast UWB pulses which are detected by the MS. After the MS received more than three broadcasted signals from the BSs, it will calculate the distance of itself to the BSs and then localize. After the MS had estimated its own position, it will broadcast this location information and these are received by their adjacent nodes. The new unknown MSs nearby after receiving more than three signals, will start to localize themselves. This will continue throughout the entire network. Reference [67] and [68] also describe a second stage refinement for the MS where it repeats the entire procedure again. In addition, [68] describes how a MS can choose which BS to reference to according to the lower bound on the mutual information between the transmitted and received signals.

2.2.4 Scalability Issue

Scalability is an important factor for an indoor wireless sensor network localization system. The anchor based localization system scales poorly because the infrastructure is fixed and any changes in the environment will require it to be re-arranged. Therefore,
anchor free localization system is preferred because the number of the anchor nodes can be adjusted in accordance to the network requirement. In addition, it is also scalable to localize a large number of tags and thus suitable for a large network.

Another interesting and upcoming trend in a scalable localization system is cooperative localization. As the name implies, in cooperative localization, the nodes cooperate with one another to perform localization [69] and [70]. These nodes make distance measurements with one another through broadcasted messages [3]. In a non-cooperative approach, there is no communication between MSs, only between MS and BSs. Since every MS needs to communicate with a number of BSs, this requires a high density of BSs. But in cooperative localization, MSs are able to communicate among themselves, hence, the requirement for high density BSs is no longer required [70].

Localization estimates in a single cell can normally achieve high accuracy. But in order to cover an entire area, normally a single cell is not sufficient. Therefore, people tend to use multiple cells to provide coverage to the entire area. However, using multiple cells which overlap with one another to localize the tag is not often deployed because localization operation does not need to be performed continuously. It is normally performed at a certain regular interval or as desired such as there is a necessary need to locate a specific object or person. Nevertheless, at this point, it is good to address some of the problems faced if multiple cell localization method is employed.

The basic problems with multiple cells operating at the same time are the handover and cells overlapping issues. MSs in these overlapping areas tends to give multiple position estimate solutions, but in fact there only exists one correct answer. Therefore, good algorithm in tackling these issues are very important in order to have multiple cells operation. Reference [71] states that RF receiver front-end saturation indicates that the MS is near a cell boundary. Another observation made in [71] and [72] is that as the MS moves near the cell boundary, location inaccuracy increases. Therefore, it uses the MS
previous estimated position, predicted current position and whether the MS is near a cell boundary to correctly estimate the MS position when it is within the overlapping region.

### 2.2.5 Factors Affecting Localization Accuracy

Localization estimation does not only depend on the accuracy of the ranging measurements, but also depend on the geometry of the BSs and MSs. References [52] and [73] describe how a bad layout of the BSs can result in a poor geometry in localization. This is known as Geometric Dilution Of Precision (GDOP) which is used to describe the placement of the sensors that impacts on the degree of accuracy of the positioning estimates. The smaller the GDOP value the better will be the localization performance.

![Good GDOP and Bad GDOP](image)

**Figure 2.4: Geometric Dilution Of Precision (GDOP) Model**

Figure 2.4 gives an example of a good and bad GDOP. For the good GDOP, BSs are evenly placed so that the hyperbolas intersection area is small and thus resulting a small estimated MSs location error. In the bad GDOP, the BSs are placed in a straight line that results the estimated MSs location error to be larger. Therefore, in a typical location based system, it is desired to place the BSs in an evenly manner in order to prevent bad GDOP case to happen which can give undesired location error. Reference [74]
suggested to use a combination of TOA and TDOA to achieve better ranging accuracy in a bad GDOP scenario. Although good accuracy in range and location estimation is essential, placement of BSs to give good GDOP is also important to determine the performance of the localization system [75].

There are a few terms that describe the Dilution Of Precision (DOP) such as HDOP, VDOP, PDOP and TDOP which stands for the horizontal, vertical, positional and time dilution of precision. Following next are some of the mathematical formulas that can be used to determine the different DOPs.

First, with an example of four BSs defining a matrix \( \mathbf{A} \) such that

\[
\mathbf{A} = \begin{bmatrix}
\frac{x_1-x}{R_1} & \frac{y_1-y}{R_1} & \frac{z_1-z}{R_1} & c \\
\frac{x_2-x}{R_2} & \frac{y_2-y}{R_2} & \frac{z_2-z}{R_2} & c \\
\frac{x_3-x}{R_3} & \frac{y_3-y}{R_3} & \frac{z_3-z}{R_3} & c \\
\frac{x_4-x}{R_4} & \frac{y_4-y}{R_4} & \frac{z_4-z}{R_4} & c \\
\end{bmatrix},
\]

(2.7)

where \((x, y, z)\) denote the coordinates of the MS that is to be detected and \((x_i, y_i, z_i)\) for \(i = 1, 2, 3, 4\) are the coordinates of the BSs. \(R_i\) is defined as the distance between the MS and the \(i^{th}\) BS such that

\[ R_i = \sqrt{(x_i-x)^2 + (y_i - y)^2 + (z_i - z)^2} \]

and \(c\) is the speed of light.

Next, defined the matrix \( \mathbf{Q} \) such that
\( Q = (A^T A)^{-1} \) \tag{2.8} \\
\begin{bmatrix}
  d_x^2 & d_{xy}^2 & d_{xz}^2 & d_{xt}^2 \\
  d_{xy}^2 & d_y^2 & d_{yz}^2 & d_{yt}^2 \\
  d_{xz}^2 & d_{yz}^2 & d_z^2 & d_{zt}^2 \\
  d_{xt}^2 & d_{yt}^2 & d_{zt}^2 & d_t^2
\end{bmatrix},

where \([.]^T\) and \([.]^{-1}\) denotes taking the transpose and inverse respectively.

Finally, the different DOPs can be calculated as follows

\begin{align*}
PDOP & = \sqrt{d_x^2 + d_y^2 + d_z^2} \tag{2.9} \\
TDOP & = \sqrt{d_t^2} \\
HDOP & = \sqrt{d_x^2 + d_y^2} \\
VDOP & = \sqrt{d_z^2} \\
GDOP & = \sqrt{PDOP^2 + TDOP^2}.
\end{align*}

An observation can be made from the above equation is that both the HDOP and VDOP depend on the coordinate system used. Reference [76] modified the GDOP equation such that it can evaluate faster for simple sensor computation.

Cramér-Rao Bound (CRB) can help to describe the localization error for a given network. Reference [79] and [80] give the CRB for both distance dependent and distance independent TDOA range errors as well as Non Line Of Sight (NLOS) performance. However, the BSs are assumed to be time synchronized and does not analyze the CRB with device crystal clock time drift. Reference [81] further analyzes the CRB for prior
NLOS information with position of the MS and [82] extends to multipath case. Reference [83] gives the CRB analysis for both synchronized and non-synchronized transmitter and receiver pair. In addition, [84] analyzes the CRB for TDOA localization in LOS environment. Reference [85] derives the CRB when the anchor nodes positions are known and when no information of the anchor nodes positions are given. A summary on CRB of other localization methods, such as TOA, AOA and RSS methods, can be found in [2], [3] and the references therein. The CRB derived for relative location estimation using TOA and RSS are given in [20] and [86]. Reference [87] gives a CRB analysis on the joint TOA and Direction of Arrival (DOA) ranging method which shows the impact on the network topology and the sensor selection in affecting localization accuracy.

Synchronization is one of the problems faced in localization. Synchronization requirement is desired to be in the order of tens of picoseconds [88]. In general, synchronization can be classified into three categories [89] and [90]. The first type is to determine which sensor nodes’ clocks are faster or slower with another sensor node. The second type is to maintain clock synchronization of all the sensor nodes inside a wireless network with respect to a global reference time. This is the most complex and energy consumption type of synchronization. The last type of synchronization is defined as maintaining relative clocks where every node run at their own clock and if at any instant of operation, these local clocks can be converted to a relative clocks to be used in the operation. Reference Broadcast Synchronization (RBS) [91] is one of the work that is based on the signal arrival time to calculate the relative clocks.

There are many synchronization methods available. Interested readers can refer to [89]-[90] and the references therein for more details. Reference [89] gives a survey on the different synchronization methods for a sensor and particularly both [89] and [90] describe a two way message transfer between two sensor nodes in order to achieve synchronization and estimate their clock drift. In ranging context, two way message transfer
between any two sensor nodes is not only able to synchronize these pair of sensor nodes, their recorded values of the transmit and receive times are also able to give the distance estimate between them [101]-[102] and this is termed as Two Way Ranging (TWR). TWR is a good choice if pairwise distances between any two sensor nodes are to be determined without having them to synchronize initially. Unfortunately, this is efficient if there are only a small number of sensor nodes inside a wireless network. If there is a large number of tags (e.g. MSs) operating inside a wireless network, performing TWR with every tag can result in high energy and time consumption. Alternatively, anchors (e.g. BSs) and tags can perform One Way Ranging (OWR) provided that they are time synchronized.

Another difficulty hindering accurate location estimation is blockage of signals at the sensor nodes. Blockages normally have a log normal amplitude distribution which also includes a shadowing term to account for total received multipath energy variation that results from the blockage of the line-of-sight path [77]. Blockage is common in an indoor wireless sensor network and this leads to inaccurate readings and can result in adverse localization performance. Normally to counter this, weighting functions are used. The choice of the weighting function should reflect the accuracy of the measured distances such that those measured distances that are believed to be less accurate will be weighted less and those believed to be more accurate will be weighted more. Many weighting functions are discussed in the literatures such as [78]-[92] and the references cited therein. These literature studies can range from a simple choice of unit weights which are believed to be more accurate to a zero weight given to the measured distances which are believed to be inaccurate.

If the variances of the noise measurement are known, the weight can be chosen as $1/(a\hat{d}_{ij} + b)^2$ where $a$ and $b$ are some constants and $\hat{d}_{ij}$ is the measured distance between sensor $i$ and $j$ [66]. Furthermore, if the noise measurement is Gaussian distributed and
the standard deviation varies proportionally to the true distance, then the variance can be expressed as $(ad_{ij} + b)^2$ where $d_{ij}$ is the true distance between sensor $i$ and $j$. Reference [66] discusses an attractive way of assigning the weight if a reliable noise model is unavailable. It uses a model independent weighting scheme where it assumes that the knowledge of the noise measurement is unavailable and it depends on the measured TDOA values. This works out to a weighting scheme which is similar to Welsch weight where the weighting function depends on the measured data.

In particular interest, Welsch and Huber weighting functions are considered here. Standard deviations of residuals in a sample can vary greatly from one data point to another even when the errors all have the same standard deviation. This means that the variances of the residuals at the different input variables may differ, even if the variances of the errors at these different input variables are equal. This is called Studentized residuals and Welsch weighting function is elegant in the sense that it is able to guard against Studentized residuals [93] and [94]. Welsch weighting function can be seen as a form of a decreasing exponential function. Huber weighting function is used to optimize large and small measurement errors, bad starting points, and statistical uncertainties [95] especially the variance when the error distribution is moderately heavy-tailed [93] and [96].

After discussing some of the typical factors in affecting ranging accuracy, a practical problem faced in ranging operation is discussed in the next section. In addition, some techniques used to overcome it are presented.

### 2.3 How to overcome time drift issue

Reference [100] introduces an energy efficient cooperative ranging method called N-Way Time Transfer (NWTT) method to determine the pairwise distances between any
two nodes in the network. Figure 2.5 shows a four nodes NWTT method.

\[ T_1 \] is the packet transmit time from node 1. \[ R_2^{(1)} \] is the packet received time at node 2 with respect to a packet transmitted from node 1. \[ \Delta_2 \] is the reply time at node 2. The vertical line (or y-axis) represents the time duration of packets exchange. The dotted arrows represent the broadcast message and the solid arrows represent the ranging message between two nodes. Node 1 broadcasts a packet to node 2 and all the nodes timestamp its arrival time. After a certain amount of time, node 2 starts to broadcast a packet to node 3 and all the other nodes timestamp its arrival time. This process continues until all the nodes in the network have completed transmitting.

First, the nodes will calculate their respective reply time, \[ \Delta \] and then broadcast
this information to all the other nodes at their respective transmission time. With this information, the estimated adjacent distance can then be calculated. Calculation for $\Delta$ is shown as

\[
\begin{align*}
\Delta_2 &= T_2 - R_2^{(1)} \\
\Delta_3 &= T_3 - R_3^{(2)} \\
\vdots \\
\Delta_N &= T_N - R_N^{(N-1)},
\end{align*}
\]

(2.10)

where $N$ is the number of sensor nodes. Adjacent distance is the distance between any two nodes that communicate in a sequential manner. For example in [100], estimated adjacent distance between sequential nodes is given as

\[
\begin{align*}
\frac{2\hat{d}_{12}}{c} &= \left( R_1^{(2)} - T_1 \right) - \Delta_2 \\
\frac{2\hat{d}_{23}}{c} &= \left( R_2^{(3)} - T_2 \right) - \Delta_3 \\
\vdots \\
\frac{2\hat{d}_{N-1,N}}{c} &= \left( R_{N-1}^{(N)} - T_{N-1} \right) - \Delta_N.
\end{align*}
\]

(2.11)

With the known estimated adjacent distances and reply times, the estimated non adjacent distances can be further calculated. Non adjacent distance is defined as the distance between any two nodes that do not communicate in a sequential manner. For example, the non adjacent distance between node 1 and 3 is given by
\[
\hat{d}_{13} = c(R_1^{(3)} - T_1) - \hat{d}_{12} - \hat{d}_{23} - c(\Delta_2 + \Delta_3).
\] (2.12)

Therefore, in general, the non-adjacent distances, \(\hat{d}_{Nk}\) can be calculated accordingly by using (2.13) and \(k \neq N - 1, N + 1\).

\[
\hat{d}_{Nk} = c(R_N^{(k)} - T_N) - \sum_{m=2}^{N} (\hat{d}_{m-1,m} + c\Delta_m) - \sum_{j=2}^{k} (\hat{d}_{j-1,j} + c\Delta_j).
\] (2.13)

NWTT is a modified form of TWR and by performing NWTT, it has the inherent property that allows the nodes to determine their pair-wise ranges without time synchronization. NWTT is also energy efficient because it can save communication time and power as it does not require every node to communicate with each other. The non adjacent distances can be calculated through formulae but not through communication. However, this method will fail when crystal clock time drift is present. Therefore, to eliminate crystal clock time drift, [101] performs TWR twice between any two nodes, in different direction. This method is called Symmetric Double Sided Two-Way Ranging (SDS-TWR).

### 2.3.1 Symmetric Double Sided Two-Way Ranging Method

A simple example [101] is given next to illustrate the calculation of propagation delay with effect of time drift. For example, if there is no time drift and given that \(t_p = 30\)ns and the reply time at node 2 is 1ms, using TWR between any two nodes with eqn(1.3), the round trip time at node 1 is given as
\[ 30\text{ns} = \frac{1}{2}(R_{1}^{2} - T_{1}) - 1\text{ms} \]

\[ (R_{1}^{2} - T_{1}) = 1.00006\text{ms}. \]  \hspace{1cm} (2.14)

The above answer gives the correct result. If time drift is introduced at both sensor nodes 1 and 2 such that node 1 has a time drift of +10ppm and node 2 has -10ppm, to solve for the propagation delay using (1.3),

\[ t_{p} = \frac{1}{2}[1.00006\text{ms} \times (1 + 10^{-6}) - 1\text{ms} \times (1 - 10^{-6})] \]

\[ = 40\text{ns}. \]  \hspace{1cm} (2.15)

The above answer gives an inaccurate result and there is an error of 10ns which works out to be 3 meters of ranging error. Through this simple example, it can be seen that if time drift is not properly compensated, it will greatly affect the ranging accuracy. Therefore, [101] proposed to perform TWR, twice in different direction in order to compensate the effect of crystal clock time drift.

Figure 2.6 shows a Symmetric Double Sided Two Way Ranging (SDS-TWR) model. Node A transmits first and the packet is received by node B with node B timestamps its arrival time. After certain amount of time, node B transmits to node A and node A timestamps the packet arrival time. This process is reversed with node B starts transmitting to node A and followed by node A replying to node B. \( t_{\text{roundA}} \) is the round trip time measured at node A and \( t_{\text{replyA}} \) is the difference in the received time to the time that node A starts to reply the request. Here, it assumes that the reply time is much greater than the propagation time and node A and node B reply time differs by a few microseconds.
In this case, SDS-TWR also assumes that the node’s reply time are approximated to the value of the round trip time [101]. By performing this process, the effect of time drift in the sensor node on ranging accuracy can be compensated. Reference [101] has modified TWR eqn (1.3) to

$$t_p = \frac{t_{\text{roundA}} - t_{\text{replyA}} + t_{\text{roundB}} - t_{\text{replyB}}}{4}. \tag{2.16}$$

Again, let’s repeat the previous example in determining the propagation delay between node 1 and 2. Assuming the round trip time is 1.00006ms and the reply time at node 2 is 1ms with time drift at both sensor node 1 and 2 such that node 1 has a time
drift of +10ppm and node 2 has -10ppm, the propagation delay using (2.16) is given by

\[
\begin{align*}
    t_p &= \frac{1}{4}[1.00006ms(1 + 10^{-6}) - 1ms(1 + 10^{-6}) \\
          &\quad + 1.00006ms(1 - 10^{-6}) - 1ms(1 - 10^{-6})] \\
    t_p &= 30ns.
\end{align*}
\]  

(2.17)

It can be seen that by performing SDS-TWR, the time drift effect can be compensated. However, this method involves performing ranging between every pair of nodes and is not efficient if it is applied directly to a large sensor network because the time and power consumed to perform every pairwise ranging is very demanding.

### 2.3.2 Asymmetric Double Sided Two-Way Ranging Method

Reference [102] proposes an asymmetric double-sided TWR method which is an enhancement for SDS-TWR [101]. Node B will send another additional message back to node A after performing SDS-TWR, therefore, another new parameter will be added to (2.16) as given by

\[
    t_{\text{roundC}} = 2t_p + t_{\text{replyB}}. 
\]  

(2.18)

Adding (2.18) to (2.16), it gives

\[
    t_p = \frac{t_{\text{roundA}} - t_{\text{replyA}} + t_{\text{roundB}} - t_{\text{replyB}} + t_{\text{roundC}} - t_{\text{replyB}}'}{6}. 
\]  

(2.19)
It can be seen that (2.19) tends to have greater ranging accuracy than (2.16) because there are more averaging operations. The denominator in eqn (2.19) has two more averaging operation as compared to eqn (2.16). Therefore, it can be concluded that by having one more additional ranging message exchange, two more averaging operations can be performed [102]. On the other hand, it implies that more ranging messages need to be exchanged and longer time is needed in order to extract the distance information. Therefore, there is a tradeoff in obtaining good ranging accuracy with longer waiting time.

To reduce the long waiting period, Figure 2.7 shows an Asymmetric Double Sided - Two Way Ranging (ADS-TWR) ranging model [102] where node A sends another ranging message to node B instantly after receiving the reply message. The second message from node A to B is necessary just to compensate time drift. As node A is the originator of the ranging request, it can be assumed that it is able to send the ranging message back instantly whereas node B needs more time to process the received signal at the MAC layer, therefore incurring significant processing delays.

![Figure 2.7: Asymmetric Double Sided - Two Way Ranging (ADS-TWR) Model](image-url)
The propagation delay between node A and B is given as

\[ t_p = \frac{t_{\text{roundA}} + t_{\text{roundB}} - t_{\text{replyB}}}{4}. \]  

(2.20)

Eqn (2.20) shows that ADS-TWR can achieve the same effect as SDS-TWR in compensating time drift without increasing the waiting time and a faster update on the distance information can be obtained. This method of reducing waiting time assume that one of the sensor nodes is able to reply instantly. But in actual implementation, there is bound to be some reply delays due to the internal circuit components, therefore, it is not a very good assumption for a practical deployment. Note that although \( t_p \) can be obtained from \( t_{\text{roundB}} \) directly which is the case for a normal TWR, this value of \( t_p \) is greatly affected by crystal clock time drift in node A and B. Therefore, another TWR operation in the reverse manner is needed in order to reduce the effect of time drift from affecting the ranging accuracy.

Beside the above mentioned SDS-TWR and ADS-TWR methods, [103] uses 1ppm crystal clock with calibration on the internal system delay to achieve good ranging accuracy. However, this translates to a more expensive system. Reference [104] shows an interesting method in detecting frequency, time offset and the internal delay as the signal propagates through the electronics circuits. This is the time difference between the signal arrives at or depart from the antenna and the time that the node records the time. Unfortunately, it utilizes TWR method, and therefore, it is inefficient if it is applied directly to a wireless networks with a large number of tags.
2.3.3 Unequal Reply Time Problem

The second problem is the ranging accuracy caused by unequal reply time among the sensor nodes. Reference [105] uses SDS-TWR method and shows that when the reply time increases and differs between every sensor nodes, ranging accuracy will decrease. When time drift is present, this will further increase the ranging error.

Reference [105] points out that for a crystal clock with time drift of 10ppm, the range error can be as high as 1.5 meters if the reply time is subject to 100% variation under the assumption that the reply time is 1ms. If the crystal clock time drift is increased to 40ppm, the range error can be more than 5 meters. This error of 5 meters is significant and therefore, there is a need to reduce the ranging error due to unequal reply time, and most importantly, with the presence of crystal clock time drift.

2.4 Summary

In this chapter, the UWB-IR signal and the transmission scheme are first presented. It can be seen that UWB is very suitable for ranging and localization because of the precise narrow pulses of the signal. Secondly, anchor based and anchor free localization methods are described and compared. Next, the scalability and accuracy issues faced in the different localization methods are discussed.

This chapter goes on to describe the different factors in affecting ranging accuracy. In particular interest, it addresses the crystal clock time drift problem which always exists in a practical system design. Some techniques used such as SDS-TWR and ADS-TWR to overcome crystal clock time drift in affecting ranging accuracy are described.
Chapter 3

Proposed Methods to Compensate Clock Time Drift

3.1 Introduction

In order to have efficient energy consumption, [100] proposes a method of cooperative ranging called NWTT to determine the pairwise distances. By doing so, it is energy saving because NWTT does not require transmitting between every pair of nodes which can be very demanding in terms of both transmission power and time. However, when there is crystal clock time drift, NWTT will fail. On the other hand, in order to eliminate the effect of crystal clock time drift, SDS-TWR [101] is proposed. However, SDS-TWR deals with point to point ranging and is inefficient if this technique is applied directly to perform ranging between every pair of nodes in a sensor network. Therefore, this chapter first describes a proposed Symmetric Multi-Way Ranging (SMWR) method to reduce the effect of crystal clock time drift on ranging accuracy in a two phase round robin fashion. SMWR model performs ranging in a distributed manner. SMWR is both time and energy efficient when compared to the traditional TWR which is an important
requirement in a wireless sensor network. In addition, SMWR is ad-hoc because it does not require any central system to perform calculation.

If there is no luxury of performing multiple transmissions and replying of messages and limited to having only a pair of transmitter and receiver, only OWR can be carried out. However, ranging accuracy will be greatly affected if crystal clock time drift is large. Therefore, the second method that this chapter presents is OWR with time drift compensation. This proposed OWR scheme works for both synchronized and non-synchronized transmitter and receiver pair. OWR is used mainly to reduce communication overheads in ranging. One of the applications that can make use of OWR is determining the distance travelled by vehicles inside a tunnel or people walking along the corridor.

Finally this chapter discusses the proposed Symmetric Double Sided-Two Way Ranging with Unequal Reply Time (SDS-TWR-URT) scheme which helps to reduce the ranging error due to the variation of the reply time at the sensor nodes especially when there is crystal clock time drift. This is because both SDS-TWR and SMWR have an assumption that the sensor nodes reply times are approximated to be the same. If there is large variation in the reply time, both SDS-TWR and SMWR will fail. Ranging error will further increase if crystal clock time drift becomes worse. Therefore, a form of compensation to the unequal reply time is needed which is described by the proposed SDS-TWR-URT method. Performance analysis showed that SDS-TWR-URT outperforms SDS-TWR which does not have any compensation for the unequal reply time in affecting ranging accuracy.
3.2 The Proposed SMWR Method

Figure 3.1 shows a SMWR model with four nodes as illustration. It can be extended to many nodes. SMWR is considered to be energy efficient because in a conventional point-to-point ranging (a transmitter sends a signal to the receiver and the receiver upon receiving this signal, will wait for a period of time before replying to the transmitter. This
transmitter will perform the same ranging process with all the different receivers in the network), the number of times of transmission is \( N(N - 1) \) and the number of times of reception is \( N(N - 1)^2 \) while compared with SMWR is \( 2N \) and \( 2N(N - 1) \) respectively, where \( N \) is the number of nodes in a network. For SMWR, the adjacent distances are obtained through ranging while the non-adjacent distances can be estimated through formulae by the nodes. Since the number of times to perform ranging is reduced and the nodes’ receivers only need to be turned on during receiving packets, SMWR offered an improvement in both efficiency of time and energy in performing ranging.

In Figure 3.1, \( T_1 \) is the time of transmission by node 1. \( R^{(2)}_1 \) is the reception time of the packet at node 1 from node 2. \( \Delta_2 \) is the delay at node 2 between the reception of packet and its next transmission. \( d \) is the distance between any two nodes. The values of \( T \) and \( \Delta \) will be embedded in the data transmission in the ranging process.

What this method suggests is to perform TWR in phase 1 starting between node 1 and 2 and it continues until all the nodes in the network had completed performing TWR. Next, the process is reversed during the phase 2 ranging, starting from the last node and continues until the first node had performed TWR. To avoid packets collision, SMWR is performed in a round robin fashion.

### 3.3 SMWR Formulation

#### 3.3.1 Adjacent Distance Estimation

Defining \( \delta_1, \delta_2, \ldots \) as the crystal clock drift in parts per million (ppm), \( \tilde{t}_1, \tilde{t}_2, \ldots \) as the time difference measurements and \( \tilde{\Delta}_1, \tilde{\Delta}_2, \ldots \) as the measured receive-to-transmit delays at node 1, node 2, \ldots respectively,
\[ \tilde{t}_1 = (1 + \delta_1)(R_1^{(2)} - T_1) \ldots \text{Phase 1} \]
\[ \tilde{t}_2 = (1 + \delta_2)(R_2^{(1)} - T_2) \ldots \text{Phase 2} \]
\[ \ldots \] \hspace{1cm} (3.1)
\[ \tilde{\Delta}_1 = (1 + \delta_1)\Delta_1 \ldots \text{Phase 2} \]
\[ \tilde{\Delta}_2 = (1 + \delta_2)\Delta_2 \ldots \text{Phase 1} \]
\[ \ldots \] \hspace{1cm} (3.2)

The round trip time at node 1 after two phases of ranging is

\[ R_1^{(2)} - T_1 = 2d_{12}/c + \tilde{\Delta}_2 \ldots \text{Phase 1} \] \hspace{1cm} (3.3)
\[ R_2^{(1)} - T_2 = 2d_{12}/c + \tilde{\Delta}_1 \ldots \text{Phase 2}. \] \hspace{1cm} (3.4)

Re-arranging (3.1)-(3.4), they become

\[ \frac{\tilde{t}_1}{1 + \delta_1} = 2d_{12}/c + \frac{\tilde{\Delta}_2}{1 + \delta_2} \ldots \text{Phase 1} \] \hspace{1cm} (3.5)
\[ \frac{\tilde{t}_2}{1 + \delta_2} = 2d_{12}/c + \frac{\tilde{\Delta}_1}{1 + \delta_1} \ldots \text{Phase 2}. \] \hspace{1cm} (3.6)

Combining (3.5) with (3.6) and assuming that \( \delta_1 \) and \( \delta_2 \) are much more smaller compared to unity, the adjacent distance between node 1 and 2 is given as
\[ \frac{\hat{d}_{12}}{c} = \frac{1}{4} \left\{ \frac{\tilde{t}_1 - \tilde{\Delta}_1}{1 + \delta_1} + \frac{\tilde{t}_2 - \tilde{\Delta}_2}{1 + \delta_2} \right\} \]

\[ \approx \frac{1}{4} \{ \tilde{t}_1 + \tilde{t}_2 - \tilde{\Delta}_1 - \tilde{\Delta}_2 \}. \quad (3.7) \]

### 3.3.2 Reply Time Estimation

This subsection discusses how to estimate the reply time of its adjacent node as seen by its neighboring node. For example, \( \tilde{\Delta}_2^{(1)} \) is defined as \( \Delta_2 \) seen at node 1 that runs at node 1 clock given in (3.8). Estimation of reply time is important because it allows individual node to view other nodes’ reply time with respect to its own local clock. Since the local clocks in every node are drifting and there is a need to estimate the adjacent and non-adjacent distances, it is important that all calculations are based on the individual node’s local clock.

\[
\tilde{\Delta}_2^{(1)} = (1 + \delta_1) \Delta_2
\]

\[
\tilde{\Delta}_3^{(1)} = (1 + \delta_1) \Delta_3
\]

\[ \ldots. \quad (3.8) \]

Re-arranging (3.1), (3.3) and (3.8), the estimated reply time of node 2 as seen at node 1 is given as

\[
\tilde{\Delta}_2^{(1)} = (1 + \delta_1) \Delta_2 = \tilde{t}_1 - 2(1 + \delta_1) \frac{d_{12}}{c} \approx \tilde{t}_1 - 2 \frac{\hat{d}_{12}}{c}. \quad (3.9)
\]

Hence, in general, the estimation of reply time for any adjacent pair of nodes is
\[ \tilde{\Delta}_{i+1}^{(i)} = (1 + \delta_i)\Delta_{i+1} \approx \tilde{t}_i - 2\frac{\hat{d}_{i,i+1}}{c}. \]  

(3.10)

Next is estimating the non-adjacent nodes’ reply time. During phase 1, it is pointed out that

\[ \tilde{\Delta}^{(1)}_2 = (1 + \delta_1)\Delta_2 \]
\[ \tilde{\Delta}_2 = (1 + \delta_2)\Delta_2. \]  

(3.11)

Solving these two equations, they become

\[ \tilde{\Delta}^{(1)}_2 = \frac{1 + \delta_1}{1 + \delta_2} \tilde{\Delta}_2. \]  

(3.12)

Similarly,

\[ \tilde{\Delta}^{(1)}_3 = \frac{1 + \delta_1}{1 + \delta_3} \tilde{\Delta}_3 \]  

(3.13)
\[ \tilde{\Delta}^{(2)}_3 = \frac{1 + \delta_2}{1 + \delta_3} \tilde{\Delta}_3. \]  

(3.14)

Solving (3.12)-(3.14), the non-adjacent estimated reply time is given as

\[ \tilde{\Delta}^{(1)}_3 = \frac{\tilde{\Delta}^{(1)}_2 \tilde{\Delta}^{(2)}_3}{\tilde{\Delta}_2}. \]  

(3.15)

Therefore, in general, the estimated non-adjacent reply time in phase 1 as \( \tilde{\Delta}_{(m)} \), and
phase 2 as $\bar{\Delta}_n^{(m)}$, is given as (3.16) and (3.17) respectively.

$$\bar{\Delta}_m^{(n)} = \frac{\bar{\Delta}_{n+1}^{(n+1)} \bar{\Delta}_{n+2}^{(n+2)} \cdots \bar{\Delta}_{m-1}^{(m-1)}}{\bar{\Delta}_{n+1} \cdots \bar{\Delta}_{m-1}},$$  \hspace{1cm} (3.16)$$

$$\bar{\Delta}_n^{(m)} = \frac{\bar{\Delta}_n^{(n+1)} \bar{\Delta}_{n+1}^{(n+2)} \cdots \bar{\Delta}_{m-1}^{(m)}}{\bar{\Delta}_{n+1} \cdots \bar{\Delta}_{m-1}},$$  \hspace{1cm} (3.17)$$

where $m$ and $n$ are for any non-adjacent pair of nodes in the network and such that $n+2 \leq m \leq$ total number of nodes.

### 3.3.3 Non Adjacent Distance Estimation

After calculating the adjacent distance and estimating both the adjacent and non-adjacent reply time, the final parameter that needs to be calculated is the non adjacent distance. First, the non-adjacent distance between node 1 and 3 is given as

$$t_1^{(3)} = R_1^{(3)} - T_1 = \frac{d_{12}}{c} + \Delta_2 + \frac{d_{23}}{c} + \Delta_3 + \frac{d_{13}}{c}. \hspace{1cm} (3.18)$$

Then re-arranging (3.18), it becomes

$$\frac{\bar{t}_1^{(3)}}{1 + \delta_1} = \frac{d_{12}}{c} + \frac{\bar{\Delta}_2}{1 + \delta_2} + \frac{d_{23}}{c} + \frac{\bar{\Delta}_3}{1 + \delta_3} + \frac{d_{13}}{c}. \hspace{1cm} (3.19)$$

Assuming that $\delta_1$, $\delta_2$ and $\delta_3$ are much smaller as compared to unity, the above equation can be approximated as
\[
\frac{\hat{d}_{13}}{c} = \frac{1}{1 + \delta_1} \{ \hat{r}_{1}^{(3)} - \frac{1 + \delta_1}{1 + \delta_2} \bar{\Delta}_2 - \frac{1 + \delta_1}{1 + \delta_3} \bar{\Delta}_3 \} - \frac{(d_{12} + d_{23})}{c} \\
\approx \{ \hat{r}_{1}^{(3)} - \bar{\Delta}_2^{(1)} - \bar{\Delta}_3^{(1)} \} - \frac{(\hat{d}_{12} + \hat{d}_{23})}{c}.
\] (3.20)

Therefore, in general, the estimation for the distance of any pair of non-adjacent nodes in phase 1 as \( \hat{d}_{nm} \), and phase 2 as \( \hat{d}_{mn} \) is given by

\[
\hat{d}_{nm} \approx c\{ \hat{r}_{n}^{(m)} - \bar{\Delta}_{n+1}^{(n)} - \ldots - \bar{\Delta}_{m}^{(n)} \} - (\hat{d}_{n,n+1} + \ldots + \hat{d}_{m-1,m}) ,
\] (3.21)

\[
\hat{d}_{mn} \approx c\{ \hat{r}_{m}^{(n)} - \bar{\Delta}_{n}^{(m)} - \ldots - \bar{\Delta}_{m-1}^{(m)} \} - (\hat{d}_{n,n+1} + \ldots + \hat{d}_{m-1,m}) ,
\] (3.22)

where \( n+2 \leq m \leq \) total number of nodes and assuming that the estimated adjacent distances and the measured receive-to-transmit delays are known to all the nodes after the broadcast.

### 3.4 SMWR Performance Analysis

In this section, the analysis for SMWR error variance is derived. For simplicity sake, let’s assume that all the individual node’s reply time are equal to 1ms (\( \Delta = 1\text{ms} \)). Also, there will be no time drift since SMWR can compensate the effect of crystal clock time drift on the ranging accuracy. SMWR is derived from SDS-TWR. In Chapter 2, section 2.3.1, it gives the numeric example on how SDS-TWR (which is the same idea as
SMWR) can compensate crystal clock time drift by performing a two-phase ranging.

With reference to Figure 3.1, the estimated adjacent distance is just the sum of the individual adjacent distances in the two phases and then divided by four. Let’s define the estimated distance between nodes 1 and 2 as

\[
\hat{d}_{12} = (d_{12(1)} + d_{21(1)} + g_{12(1)} + g_{21(1)} + d_{12(2)} + d_{21(2)} + g_{12(2)} + g_{21(2)})/4,
\]

where \(d_{12(1)}, d_{12(2)}\) is the true distance between node 1 and 2 and \(g_{12(1)}, g_{12(2)}\) is the equivalent Gaussian noise in phases 1 and 2 respectively.

Assuming zero mean Gaussian noise and same variance, \(\sigma^2\), for measurements from node 1 to 2 in phase 1 and from node 2 to 1 in phase 2, the error variance for the adjacent distance is given as

\[
\sigma^2_{\hat{d}_{12}} = (\sigma^2_{g_{12(1)}} + \sigma^2_{g_{21(1)}} + \sigma^2_{g_{12(2)}} + \sigma^2_{g_{21(2)}})/16 = \sigma^2/4.
\]  

Referring to Figure 3.1, it is clear that for \(N\) nodes network, there are \(L_{\text{adjacent}} = N - 1\) number of pairs of adjacent nodes. Assuming all adjacent nodes TOA measurements have the same variance, \(\sigma^2\), the normalized mean square error, \(e^2_{\text{adjacent}}\), for the adjacent pairs is given by
From eqn (2.12), the estimated non-adjacent distance between node 1 and 3 is

\[
\hat{d}_{13} = c(R_1^{(3)} - T_1) - \hat{d}_{12} - \hat{d}_{23} - c(\Delta_2 + \Delta_3)
\]

\[
= d_{12(1)} + c\Delta_2 + d_{23(1)} + c\Delta_3 + d_{13(1)} + g_{12(1)} + g_{23(1)} + g_{13(1)}
\]

\[
-\hat{d}_{12} - \hat{d}_{23} - c(\Delta_2 + \Delta_3)
\]

\[
= d_{12(1)} + d_{23(1)} + d_{13(1)} + g_{12(1)} + g_{23(1)} + g_{13(1)} - \hat{d}_{12} - \hat{d}_{23}.
\] (3.26)

Using eqn (3.23) and with changing the notation for \(\hat{d}_{23},\)

\[
\hat{d}_{13} = d_{12(1)} + d_{23(1)} + d_{13(1)} + g_{12(1)} + g_{23(1)} + g_{13(1)}
\]

\[
- \frac{1}{4}[\left(d_{12(1)} + g_{12(1)} + d_{21(1)} + g_{21(1)}
\right.
\]

\[
+ d_{12(2)} + g_{12(2)} + d_{21(2)} + g_{21(2)}
\]

\[
- (d_{23(1)} + g_{23(1)} + d_{32(1)} + g_{32(1)}
\]

\[
+ d_{23(2)} + g_{23(2)} + d_{32(2)} + g_{32(2)})\].
\] (3.27)

Again, assuming zero mean Gaussian noise, the variance \(\sigma^2\) is denoted as
\[ \sigma^2_{d_{13}} = \sigma^2_{d_{12}(1)} + \sigma^2_{d_{23}(1)} + \sigma^2_{d_{13}(1)} + \sigma^2_{d_{adj}}, \tag{3.28} \]

where \( \sigma^2_{d_{adj}} \) can be derived from the last two estimated adjacent distances in (3.20), and by using (3.24), it becomes \( \sigma^2_{d_{adj}} = \sigma^2 \).

Therefore, in general the non adjacent error variance can be written as

\[ \sigma^2_{d_{ij}} = \sum_{i=1}^{N-2} \sum_{j=i+2}^{N} \frac{1}{4}(\frac{3}{2}(j - i)\sigma^2 + \sigma^2) \]

\[ = \frac{N^3 + 2N^2 - 13N + 10}{16} \sigma^2, \tag{3.29} \]

where \( j \geq i+2 \) denoting the non adjacent nodes between \( i \) and \( j \). Derivation of (3.29) is given in appendix A.

Referring to Figure 3.1, for \( N \) nodes network, there are \( L_{non-adjacent} = \frac{1}{2}(N^2 - 3N + 2) \) pairs of non adjacent nodes. Therefore, the normalized mean square error for the non adjacent pairs is given as

\[ \sum e^2_{non-adjacent} / L_{non-adjacent} = \frac{(N^3 + 2N^2 - 13N + 10)\sigma^2/16}{L_{non-adjacent}}. \tag{3.30} \]

Lastly from Figure 3.1, the total number of pairs between all the \( N \) nodes are given by \( L_{total} = \frac{1}{2}(N^2 - N) \), thus defining the total normalized mean square error as
\[
\sum \left( \frac{e_{total}^2}{L_{total}} \right) = \frac{(\sum error_{adjacent}^2 + \sum error_{non-adjacent}^2)}{L_{total}} \\
= \frac{(4N - 4 + N^3 + 2N^2 - 13N + 10)\sigma^2}{16L_{total}} \\
= \frac{(N^3 + 2N^2 - 9N + 6)\sigma^2}{16L_{total}} \\
= \frac{(N^2 + 3N - 6)\sigma^2}{8N},
\]

where
\[
\sum error_{adjacent}^2 = (N - 1)\sigma^2 / 4 \text{ without normalization by the adjacent links.}
\]
\[
\sum error_{non-adjacent}^2 = (N^3 + 2N^2 - 13N + 10)\sigma^2 / 16 \text{ without normalization by the non-adjacent links.}
\]

### 3.5 SMWR Simulation Results

The simulation is performed in a 10m x 10m room and the locations of the 10 nodes are randomly generated. The nodes have a reply time of approximately 1ms±20us. Gaussian noise with \( \sigma = 0.95\)ns is assumed [98]-[99] and the results are averaged over 100 realizations. In this simulation, the crystal clock time drift is generated randomly between 0ppm to ±40ppm from an uniform distribution. Figure 3.2 shows the total average error of SMWR versus N-Way Time Transfer (NWTT) method [100]. In addition, theoretical analysis result for SMWR is also included for comparison. NWTT uses TWR method to calculate all the pairwise distances between the nodes, but it fails to reduce the effect of crystal clock drift on ranging. It can be seen that for the worst case, which is ±40ppm, the ranging error for NWTT increases dramatically as the crystal clock time drift is present. But for SMWR, the ranging error only increases moderately.
This shows that SMWR is able to reduce the ranging error effectively in the presence of crystal clock time drift when compared with NWTT.

Figure 3.2: Comparison of Total Average Error for NWTT, SMWR Simulation and Theoretical results

The bottom curves in Figure 3.2 consist of the theoretical analysis and simulation results for SMWR at 0ppm and ±40ppm. In addition, the error accumulation is due to the Gaussian noise added up when it performs a round robin fashion measurement while time drift has negligible effect. For completeness, Figure 3.3 shows the three SMWR results at 0ppm, ±20ppm, ±30ppm and ±40ppm which are the expanded version of the three curves at the bottom of Figure 3.2.

Figure 3.4 shows SMWR performance comparison between unequal reply time and approximate equal reply time. For the unequal reply time case, the reply time of the sensor nodes increases linearly from 1ms in node 1 to 10ms at node 10. So far, SMWR analysis is carried out with the assumption that the reply time among the sensor nodes are approximated to be equal. If there are unequal reply time among the sensor nodes,
the error increases. Therefore, this leads to the analysis in the later section on how to deal with unequal reply time by introducing a compensation factor.

![SMWR Theoretical and Simulation Performance](image)

Figure 3.3: SMWR Simulation and Theoretical Performance
Figure 3.4: SMWR Performance with unequal reply time
3.6 The Proposed OWR method

This section discusses how to perform ranging using OWR based method between a pair of transmitter and receiver in the presence of crystal clock time drift. A compensating factor is introduced to overcome the crystal clock time drift together with analysis for both synchronized and non-synchronized clocks between the transmitter and receiver. The term synchronization used in this section means that the transmitter and receiver starting time is different.

3.6.1 Synchronized transmitter and receiver pair

First, consider a simple case where the transmitter and receiver clocks are synchronized. This means that the starting times of the transmitter and receiver are the same. However, the transmitter and receiver have different crystals that produce differ-
ent timing drift between them. Without loss of generality, assume that the anchor is the
transmitter and the tag is the receiver. Figure 3.5 shows a OWR transmission diagram
between the anchor and tag. The transmitted time measured by the anchor in the first
frame is denoted by \( \tilde{T}_1 \) and the received time measured by the tag is denoted as \( \tilde{R}_1 \). The
true values of the anchor transmit time \( T_1 \) and tag received time \( R_1 \) are influenced by
their respective crystal clock time drift such that \( \tilde{T}_1 = (1 + \delta_A)T_1 \) and \( \tilde{R}_1 = (1 + \delta_T)R_1 \).
The distance between the anchor and tag is given by \( d_1 \) and \( N \) denotes the frame trans-
mition interval. \( \Delta_A \) is the anchor clock offset and \( \Delta_T \) is the tag clock offset.

For time synchronized transmitter and receiver pair, \( \Delta_A \) and \( \Delta_T = 0 \) in Figure 3.5.
The distance between the anchor and tag is formulated as

\[
\frac{\hat{d}_1}{c} = \tilde{R}_1 - \tilde{T}_1 \\
= (1 + \delta_T)R_1 - (1 + \delta_A)T_1
\]

\[
\frac{\hat{d}_2}{c} = \tilde{R}_2 - \tilde{T}_2 \\
= (1 + \delta_T)R_2 - (1 + \delta_A)T_2,
\]

where \( \delta_A \) and \( \delta_T \) are the crystal clock time drift for the anchor and tag respectively.

First, look at the effect of crystal clock time drift in OWR accuracy. For example,
assuming there is no time drift at the anchor and tag (i.e. \( \delta_A \) and \( \delta_T = 0 \)), the time of flight
between anchor and tag is 30ns (\( \frac{\hat{d}_1}{c} = 30\text{ns} \)) and the anchor transmits at \( T_1 = 1\text{ms} \). This
works out that \( R_1 = 1.00003\text{ms} \) which is correct. Now consider the case with crystal
clock time drift where \( \delta_A = -10\text{ppm} \) (parts per million), \( \delta_T = 10\text{ppm} \), \( T_1 = 1\text{ms} \) and
\( R_1 = 1.00003\text{ms} \). Substitute these information into (3.32) to find \( \frac{\hat{d}_1}{c} = 50\text{ns} \)
which is inaccurate with an error of 6 meter. Thus, it can be seen that the presence of
crystal clock time drift can adversely affect OWR accuracy.
Therefore, a compensating factor is introduced to reduce the effect of crystal clock time drift. From (3.32), it is known that \( \hat{d}_1 = \hat{d}_2 \) and by rearranging (3.32), the compensating factor can be defined as

\[
f = \frac{\tilde{R}_2 - \tilde{R}_1}{\tilde{T}_2 - \tilde{T}_1}.
\] (3.33)

The above equation can be read as the difference in the measured received time at the tag divided by the difference in the measured transmitted time at the anchor. With \( \delta_A \ll 1 \), the time of flight between the anchor and tag can be estimated as

\[
\frac{\hat{d}_1}{c} \approx \frac{\tilde{R}_1}{f} - \tilde{T}_1.
\] (3.34)

Now, let’s assume that the second frame take 5ms interval to transmit \( N = 5\text{ms} \) and with the similar assumption as before where \( \delta_A = -10\text{ppm}, \delta_T = 10\text{ppm}, T_1 = 1\text{ms} \) and \( R_1 = 1.00003\text{ms} \), where \( f = \frac{(1+\delta_T)(R_2-R_1)}{(1+\delta_A)(T_2-T_1)} = \frac{(1+\delta_T)N}{(1+\delta_A)N} \approx 1.0000200002 \). Then using eqn (3.34), \( \frac{\hat{d}_1}{c} \approx 29.9996\text{ns} \) which is near the initial assumption of the time of flight of 30ns. This shows that there is a need to introduce a compensating factor to reduce the effect of crystal clock time drift for OWR based method.

### 3.6.2 Non-Synchronized transmitter and receiver pair

For non-synchronized transmitter and receiver, it means that there are some clock offsets at both the anchor and tag (\( \Delta_A \) and \( \Delta_T \) in Figure 3.5 are not zero). In this case, only the distance difference between them with respect to the previous point can be determined.
Figure 3.6: Distance difference illustration between anchor and tag

Figure 3.6 illustrates a distance difference between the anchor and tag which are denoted by $\Delta d_1$, $\Delta d_2$ and $\Delta d_3$. It is obvious that if the initial distance $d_i$ as shown in Figure 3.6 can be evaluated through some initial calibration, the true distance between the anchor and tag can be found. However, the scope of this chapter will not touch on the initial calibration but only describes how to introduce a method to measure the distance difference with time drift compensation between the anchor and tag.

First, it can be seen that the frame transmission interval and the transmit time are related by the following equation

$$(1 + \delta_A)N = \tilde{T}_2 - \tilde{T}_1. \quad (3.35)$$

Then re-arranging (3.35), the estimated anchor crystal clock time drift is given by

$$\hat{\delta}_A = \frac{\tilde{T}_2 - \tilde{T}_1}{N} - 1. \quad (3.36)$$

Similarly, the relative tag crystal clock time drift with respect to the anchor can be estimated by
\[ \tilde{\delta}_T = \frac{\tilde{R}_2 - \tilde{R}_1}{N} - 1. \] (3.37)

Here the value of \( N \) is assumed to be transmitted from the anchor to the tag in the second frame. Then the estimated tag crystal clock time drift can be found by taking the difference of (3.37) and (3.36) as shown

\[ \hat{\delta}_T = \frac{(\tilde{R}_2 - \tilde{R}_1) - (\tilde{T}_2 - \tilde{T}_1)}{N}. \] (3.38)

And finally, the distance difference between the anchor and tag is computed as

\[ \Delta d_n \approx c \frac{(\tilde{R}_2 - \tilde{R}_1) - (1 + \hat{\delta}_T)(\tilde{T}_2 - \tilde{T}_1)}{(1 + \hat{\delta}_T)}, \] (3.39)

where \( \Delta d_n \) stands for the distance difference when the tag is at the \( n^{th} \) position such as \( \Delta d_1, \Delta d_2 \) and \( \Delta d_3 \) as shown in Figure 3.6. This is assumed that at least two transmissions from the anchor to the tag is required at every \( n^{th} \) position to compute their distance apart.

### 3.7 OWR Simulation Results

In the simulation, \( \delta_A \) and \( \delta_T \) are assumed to be uniformly distributed between -20ppm to +20ppm and the range error has a Gaussian distribution with zero mean and standard deviation of 1ns. The simulation is run over 1000 iterations and its average result is...
taken. Figure 3.7 shows the trajectory taken by the tag. The initial distance between the anchor and tag is 2m. Subsequently, the tag moves further away from the anchor to 5m, 10m and 11m. At both 10m and 11m, the tag stops for 1 frame duration. Then the tag moves towards the anchor to 8m, 2m and stops for 1 frame duration and finally goes behind the anchor at 1m.

![Diagram](image)

**Figure 3.7: Path taken by tag**

![Graph](image)

**Figure 3.8: Estimated distance between the tag and anchor with synchronized clocks**

Figure 3.8 shows the simulation result that compared the estimated distance and the actual distance between the tag and anchor when both of them are time synchronized.
The tag stops for 1 frame duration at 10m, 11m and 2m respectively. From Figure 3.8, the errors between the estimated and actual distances are small with a maximum error of 0.33m when the tag is 11m away from the anchor. This shows that the proposed method can reduce crystal clock time drift effect for OWR operation in synchronized clocks. If no compensation is done, the ranging error can be proportional to the order of the crystal clock time drift, which can be very large.

Figure 3.9 shows that the estimated and actual distance difference between the tag and anchor with respect to tags previous position with non-synchronized clocks between successive frame as the tag is moving. $\Delta_A = 5 \text{ms}$ and $\Delta_T = 2 \text{ms}$ are assumed. Similarly, the tag stops for 1 frame duration at 10m, 11m and 2m respectively which is given by $N=3$, 5 and 7. The estimated and actual distance difference are very close and with only a maximum error of 0.3m when $N=7$. This shows that the proposed method can reduce crystal clock time drift effect for OWR operation in non-synchronized clocks.

One interesting observation that can be drawn from Figure 3.9 is that if the sign of the measured distance difference is positive, it means that the tag is moving away from its initial position. On the contrary, if the sign of the measured distance difference is negative, it means that the tag is moving towards its initial position.

Figure 3.10 shows the estimated distance difference between the tag and anchor with respect to tag’s initial position on non-synchronized clocks. From Figure 3.10, both the actual distance difference and the estimated distance difference match closely. The large error from $N=7$ to 9 is due to the estimated error in the current estimated distance and the initial estimated distance which is being accumulated when taking their difference. Note that if the initial distance between the tag and anchor is known as illustrated by $d_i$ as shown in Figure 3.6, the distance between the tag and anchor can be determined.
Figure 3.9: Estimated distance difference between the tag and anchor with respect to tags previous position with non-synchronized clocks

Figure 3.10: Estimated distance difference between the tag and anchor with respect to tag’s initial position with non-synchronized clocks
3.8 The Proposed SDS-TWR-URT Method to Compensate Unequal Reply Time

Both SDS-TWR [101] and SMWR work well under the assumption that the reply time among all the sensor nodes are of fixed value or almost equal. However, they will fail if there are significant variations among the reply time [105] and as shown in Figure 3.4. Therefore, this section discusses a proposed SDS-TWR-URT method that uses a compensating factor based on the recorded timings to reduce the effect of unequal reply time on ranging accuracy especially when crystal clock time drift is present at the sensor nodes. The idea is to introduce a compensating factor to reduce the effect in the variation of the reply time from affecting the round trip time computation.

Figure 3.11 shows a Symmetric Double Sided-Two Way Ranging with Unequal Reply Time (SDS-TWR-URT) model. $t_{\text{round}1}$ is the round trip time measured at node 1, $t_p$ is the propagation time between the two nodes and $\Delta_1$ is the difference in received time and the time node 1 starts to reply to the request and it is assumed to be greater than $t_p$. $d_{12}$ is the distance between node 1 and 2.

3.8.1 Mathematics Formulation

Referring to Figure 3.11,

$$t_{\text{round}1} = 2 \frac{d_{12}}{c} + \Delta_2$$

$$t_{\text{round}2} = 2 \frac{d_{12}}{c} + \Delta_1.$$  \hspace{1cm} (3.40)

Modifying (3.1) according to (3.40),
Figure 3.11: Symmetric Double Sided-Two Way Ranging with Unequal Reply Time (SDS-TWR-URT) Model

\[ \tilde{t}_1 = (1 + \delta_1) t_{\text{round1}} \]
\[ \tilde{t}_2 = (1 + \delta_2) t_{\text{round2}}. \]  

Method 1:

First, define a compensating factor, \( f_1 \), given by

\[ f_1 = \frac{\Delta_1}{\Delta_2}, \]
where $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$ are the timestamped values of the reply time at node 1 and 2 respectively. Re-arranging (3.2) and (3.41),

\[
\frac{\tilde{t}_1}{1 + \delta_1} = 2 \frac{d_{12}}{c} + \frac{\tilde{\Delta}_2}{1 + \delta_2} \tag{3.43}
\]

\[
\frac{\tilde{t}_2}{1 + \delta_2} = 2 \frac{d_{12}}{c} + \frac{\tilde{\Delta}_1}{1 + \delta_1}. \tag{3.44}
\]

Multiplying (3.42) into (3.43) and adding to (3.44), the estimated adjacent distance is given as

\[
\frac{\hat{d}_{12}}{c} = \frac{1}{2(f_1 + 1)} \left\{ f_1 \tilde{t}_1 - \tilde{\Delta}_1 + \tilde{t}_2 - f_1 \tilde{\Delta}_2 \right\} \approx \frac{1}{2(f_1 + 1)} \left\{ f_1 \tilde{t}_1 - \tilde{\Delta}_1 + \tilde{t}_2 - f_1 \tilde{\Delta}_2 \right\}. \tag{3.45}
\]

The approximation in (3.45) is valid as $\delta_1 \rightarrow 0$, $\delta_2 \rightarrow 0$ and $(f_1 \tilde{t}_1 - \tilde{\Delta}_1)$, $(\tilde{t}_2 - f_1 \tilde{\Delta}_2)$ have the same order in magnitude as $\frac{d_{12}}{c}$.

**Method 2:**

Alternatively, a second compensating factor can also be designed as

\[
f_1 = \frac{\tilde{t}_1}{\tilde{\Delta}_1} \tag{3.46}
\]

\[
f_2 = \frac{\tilde{t}_2}{\tilde{\Delta}_2}. \tag{3.47}
\]

Multiplying $f_1$ to (3.43) and $f_2$ to (3.44) and adding them together,
\[
\hat{d}_{12} = \frac{1}{c} \frac{1}{2(f_1 + f_2 + 2)} \left\{ \frac{f_2 \tilde{t}_1 - \tilde{\Delta}_1}{1 + \delta_1} + \frac{f_1 \tilde{t}_2 - \tilde{\Delta}_2}{1 + \delta_2} \right\}
\approx \frac{1}{2(f_1 + f_2 + 2)} \left\{ f_2 \tilde{t}_1 - \tilde{\Delta}_1 + f_1 \tilde{t}_2 - \tilde{\Delta}_2 \right\}.
\] (3.48)

### 3.8.2 Performance Analysis

This subsection discusses the error analysis of SDS-TWR and the proposed SDS-TWR-URT methods. Rearranging eqn (3.40) and (3.41), the distance between node 1 and 2 performed by SDS-TWR can be written as

\[
\hat{d}_{12} = \frac{c}{4} \left[ \tilde{t}_1 - \tilde{\Delta}_1 + \tilde{t}_2 - \tilde{\Delta}_2 \right]
= \frac{c}{4} \left[ (1 + \delta_1) t_{\text{round}1} - (1 + \delta_1) \Delta_1 + (1 + \delta_2) t_{\text{round}2} - (1 + \delta_2) \Delta_2 \right],
\] (3.49)

where \(\delta_1\) and \(\delta_2\) are the crystal clock time drift at nodes 1 and 2 respectively. Then the difference between the true and estimated SDS-TWR distance can be written as

\[
\Delta d_{12} = \frac{c}{4} \left[ \delta_1 (t_{\text{round}1} - \Delta_1) + \delta_2 (t_{\text{round}2} - \Delta_2) \right].
\] (3.50)

Assuming worst case where \(\delta_2 = -\delta_1\), (3.49) can be written as

\[
\Delta d_{12} = \frac{c}{2} (\Delta_2 - \Delta_1) \delta_1.
\] (3.51)
Now assuming that $\delta_1 = 10\text{ppm}$ and $(\Delta_2 - \Delta_1) = 1\text{ms}$, $\Delta_{d_{12}}$ can be calculated to be 1.5 meters. This shows that the error in SDS-TWR is independent of distance and is affected by the difference in the reply time and crystal clock time drift. If $\delta_1$ is increased to 40ppm, $\Delta_{d_{12}}$ is calculated to be 6 meters.

The error analysis for SDS-TWR-URT is derived as follows

**Method 1:**

Expanding (3.45),

$$\frac{\hat{d}_{12}}{c} = \frac{1}{2(f_1 + 1)}[f_1(1 + \delta_1)t_1 - f_1(1 + \delta_2)\Delta_2 + (1 + \delta_2)t_2 - (1 + \delta_1)\Delta_1]$$

$$= \frac{1}{2(f_1 + 1)}[2\frac{d_{12}}{c}(f_1 + 1) + f_1\delta_1t_1 - f_1\delta_2\Delta_2 + \delta_2t_2 - \delta_1\Delta_1]. \tag{3.52}$$

Then the difference between the true and estimated SDS-TWR-URT distance can be written as

$$\Delta_{d_{12}} = \frac{c}{2(f_1 + 1)}[f_1\delta_1t_1 - f_1\delta_2\Delta_2 + \delta_2t_2 - \delta_1\Delta_1]$$

$$= \frac{c}{2[(1 + \delta_1)\Delta_1 + (1 + \delta_2)\Delta_2]}[\delta_1\Delta_1t_1 - \delta_1\Delta_1\Delta_2 + \delta_2\Delta_2t_2 - \delta_2\Delta_1\Delta_2$$

$$\delta_1^2\Delta_1t_1 + \delta_2^2\Delta_2t_2 - 2\delta_1\delta_2\Delta_1\Delta_2]. \tag{3.53}$$

Assuming the worst case where $\delta_2 = -\delta_1$, $(\Delta_2 - \Delta_1) = 1\text{ms}$, $\delta_1 = 10\text{ppm}$ and $\delta_1^2$ is negligible,
$$\Delta d_{12} = \frac{c}{2(\Delta_1 + \delta_1 \Delta_1 + \Delta_2 - \delta_1 \Delta_2)} \left[ \delta_1 \Delta_1 (t_1 - \Delta_2) - \delta_1 \Delta_2 (t_2 - \Delta_1) + \delta_1^2 (\Delta_1 t_1 + \Delta_2 t_2 + 2\Delta_1 \Delta_2) \right]$$
\[ \approx \frac{1}{2(\Delta_1 + \delta_1 \Delta_1 + \Delta_2 - \delta_1 \Delta_2)} \left[ \delta_1 \Delta_1 (2d_{12}) - \delta_1 \Delta_2 (2d_{12}) \right] \]
\[ \approx 0. \quad (3.54) \]

The order of the numerator is very small, hence the difference between the true and estimated SDS-TWR distance using method 1 can be approximated to be zero. This shows that the proposed SDS-TWR-URT method 1 is able to compensate for unequal reply time from affecting ranging accuracy.

**Method 2:**

Expanding (3.48),

$$\frac{\hat{d}_{12}}{c} = \frac{1}{2(f_1 + f_2 + 2)} \left[ f_2 t_1 + \delta_1 f_2 t_1 - \Delta_1 - \delta_1 \Delta_1 + f_1 t_2 + \delta_2 f_1 t_2 - \Delta_2 - \delta_2 \Delta_2 \right]$$
\[ = \frac{1}{2(f_1 + f_2 + 2)} \left[ f_2 t_1 - \Delta_1 + f_1 t_2 - \Delta_2 \right] + \frac{1}{2(f_1 + f_2 + 2)} \left[ \delta_1 f_2 t_1 - \delta_1 \Delta_1 + \delta_2 f_1 t_2 - \delta_2 \Delta_2 \right] \]
\[ = \frac{d_{12}}{c} + \frac{1}{2(f_1 + f_2 + 2)} \left[ \delta_1 f_2 t_1 - \delta_1 \Delta_1 + \delta_2 f_1 t_2 - \delta_2 \Delta_2 \right]. \quad (3.55) \]

Assuming the worst case where $\delta_2 = -\delta_1$, $(\Delta_2 - \Delta_1) = 1$ms, $\delta_1 = 10$ppm and $\delta_2^2$ is negligible, the difference between the true and estimated SDS-TWR-URT distance can be written as
The order of the numerator is very small, hence the difference between the true and estimated SDS-TWR distance using method 2 can be approximated to be zero. This shows that the proposed SDS-TWR-URT method 2 is able to compensate for unequal reply time from affecting ranging accuracy.

### 3.9 SDS-TWR-URT Simulation Results

The results in Figure 3.12 show a point-to-point two nodes scenario with distance of 10m to 50m. The nodes have a reply time of approximately 1ms±20us. The simulations are carried out such that the reply times are varied up to 90% difference over 1000 realizations. Gaussian noise with zero mean and standard deviation $\sigma = 0.95$ns is assumed and crystal clock drift is uniform distributed between ±10ppm and ±40ppm are used. The results are compared with those using SDS-TWR [105].

Figure 3.13 shows the range error versus nodes reply time variation for the two schemes with clock drift of ±10ppm. The results show SDS-TWR range error is 15% at 10m distance with 90% difference in reply time between the two nodes where the nominal reply time of one node is 1ms. With SDS-TWR-URT scheme, this range error is minimal.
Figure 3.12: Calculated ranges (m) for SDS-TWR and SDS-TWR-URT with variation from 1ms reply time at ±10ppm

Figure 3.13: Ranging Error (%) for SDS-TWR and SDS-TWR-URT with variation from 1ms reply time at ±10ppm

In Figures 3.14 and 3.15, the crystal clock drift is increased to ±40ppm and other parameters remain unchanged. For SDS-TWR, the calculated range and the error give an even larger deviation when time drift is present. But for SDS-TWR-URT, the calculated range and the error deviation still remain consistently small.

Figures 3.16 and 3.17 show the results of the two proposed methods of designing
Figure 3.14: Calculated ranges (m) for SDS-TWR and SDS-TWR-URT with variation from 1ms reply time ±40ppm

Figure 3.15: Ranging Error (%) for SDS-TWR and SDS-TWR-URT with variation from 1ms reply time at ±40ppm

the compensating factor to reduce the effect of unequal reply time affecting ranging accuracy. The compensating factor using method 1 is to take the ratio of the two reply times (3.42) while the compensating factor designed with method 2 is to take the ratio of the node’s round trip time with respect to its reply time (3.46) and (3.47). The simulation results give similar performance for both method 1 and 2 in reducing the effect of variations in the nodes’ reply time under the influence of crystal clock time drift.
Figure 3.16: Ranging Error (%) of SDS-TWR-URT Method 1 and Method 2 with variation from 1ms reply time at ±40ppm at 10m

Figure 3.17: Ranging Error (%) of SDS-TWR-URT Method 1 and Method 2 with variation from 1ms reply time at ±40ppm at 50m

3.10 Summary

This chapter first proposed a new cooperative ranging method called SMWR that performs TWR twice in different direction to reduce ranging error in the presence of devices’ crystal clock time drift. By transmitting in a round robin fashion, it is more efficient compared to traditional TWR and can avoid packets collision. This is because sen-
sor nodes only communicate to calculate their adjacent distances and the non-adjacent
distances can be calculated locally in the individual nodes, thus, saving in the number of
times of communication and making SMWR time efficient. To be energy efficient,
the nodes receivers are only turned on during ranging because non-adjacent distances
can be calculated without performing ranging. Finally, the system is scalable because it
does not require a centralized system to perform computation. SMWR is more efficient
than SDS-TWR [101] because SMWR perform ranging in a round robin fashion while
SDS-TWR needs to perform TWR twice between every pair of nodes in the wireless
network which is not very efficient.

The simulation results show that SMWR is able to compensate crystal clock time
drift. SMWR method outperforms NWTT because the errors in performing ranging
by SMWR are much more robust against crystal clock time drift. However, SMWR
assumes that the sensor reply time is approximated to be equal. If these reply time are
unequal, the error increases and this leads to the proposed SDS-TWR-URT that uses a
compensating factor to reduce this effect.

The second method that this chapter proposed is OWR with time drift compensa-
tion. This is used in applications if there are some limitations that require one way trans-
missions from anchors to mobile nodes or vice versa. Simulation results show that the
proposed OWR method is able to reduce the effect of crystal clock time drift from affect-
ing the range accuracy and it works well for both synchronized and non-synchronized
clocks between the transmitter and receiver.

Both SDS-TWR and SMWR have an assumption that the sensor nodes’ reply times
are approximated to be the same. If there are large variations in the sensor nodes’ reply
times as given in [105], both SDS-TWR and SMWR will fail. Therefore, this chapter
proposed a new ranging method called SDS-TWR-URT to overcome the influence of
unequal reply time when variations between reply times among the nodes are significant.
Two ways of introducing compensating factors to overcome variations in the reply time are introduced.

Performance analysis show that the ranging error in the uncompensated scheme (SDS-TWR) is due to the difference in the reply time and crystal clock time drift. Furthermore, if crystal clock time drift is increased, the ranging error will be further increased. The proposed SDS-TWR-URT method is able to overcome the effect on unequal reply time in affecting ranging error.

Recall that SWMR method is able to reduce the effect of crystal clock time drift on ranging accuracy. However, SMWR has the assumption that the sensor nodes have approximately equal reply times and will fail if there are significant variations. SDS-TWR-URT method is able to overcome the influence of significant variations among the reply times of the sensor nodes using some compensation factors, but this scheme performs point-to-point pairwise ranging which is inefficient. It is straightforward and efficient to incorporate SDS-TWR-URT method into SMWR by computing these compensation factors during SMWR operation to overcome significant variations of reply times among the sensor nodes.
Chapter 4

Localization System using Wireless Anchors

4.1 Introduction

In this chapter, a method to perform ranging and localization for a large number of sensor nodes in the network is presented. The objective is to locate passengers and luggage in the airport in a wide area. An anchor based localization system using TOA is more suited for this application. A UWB-IR tag will be carried by the passengers and luggage will be tagged. Anchors are deployed to measure TDOA for signal propagating from the anchors to the tag. Most TDOA localization systems use wired connections between anchors to solve clock synchronization issues. Here, wireless connections are used between anchors. This makes the system much easier to deploy. However, some of the challenges faced when designing this system are crystal clock time drift at the anchors and tags, collectively known as the nodes, and how to allow the system to be able to be self organized so that calibration is not required.
For TOA approaches, if two nodes are time synchronized, one way ranging (OWR) method can be applied. However, synchronization procedure requires a lot of resources and is not very efficient. If both nodes are not time synchronized, two way ranging (TWR) is proposed in [101], [102] and [106] which basically measure the round trip time between the two nodes. However, TWR requires more transmissions than OWR and this requires a considerable amount of resources. In addition, clock jitter becomes an important issue in ranging accuracy [2] and this effect becomes worse if crystal clock time drift is present. Time drift can be in the order of microseconds and it will produce significant error in the measured ranges. SMWR is able to compensate crystal clock time drift and is suitable among anchors. However, SMWR tends to have more transmissions as compared to OWR. There is a need to locate a large number of tags all at the same time in the system so SMWR is not efficient for locating the tags. Therefore, Time Difference of Arrival (TDOA) localization method based on OWR from multiple anchors to the tag is considered.

In this chapter, the application of TDOA localization method with non-synchronized anchors to simultaneously track a large number of tags, and to compensate crystal clock time drift among the anchors and tags (ie all nodes run on local low cost clocks) is discussed. This gives flexibility and simplicity in the system deployment. Furthermore, low cost devices with poor crystal clock can be used.

Analysis is made on the localization accuracy due to perturbation errors. The CRB for the TDOA localization is derived taking into account the crystal clock time drift. Simulation results demonstrated the accuracy of the proposed method, where the localization errors of the proposed TDOA method approach CRB in the presence of crystal clock time drift. Finally, blockage effects on the anchors are analyzed and simulated with different network arrangements. CRB on blockage effect together with crystal clock time drift is derived. A weighting function is proposed to reduce the localization
error and is robust to different network layouts. Laboratory measurements using the proposed TDOA localization method and the weighting function are presented at the end of this chapter.

### 4.2 TDOA Localization Scheme

This section introduces a TDOA localization scheme that is able to reduce the effect of crystal clock time drift. Figure 4.1 shows a transmission sequence diagram for the proposed scheme for a single zone operation.

A Zone Supervisor (ZS) is assigned to coordinate the transmission sequences and
network maintenances. In a network, one of the anchors is assigned as Zone Supervisor while the remaining anchors are designated as \( J \) readers \((R_1, R_2, \ldots, R_J)\). Anchors have a prior knowledge of their coordinates. There are \( K \) tags \((T_1, T_2, \ldots, T_K)\) which do not have a prior knowledge of their coordinates. The localization problem here is to find the tags locations.

Initially, the ZS broadcasts a message, which will be received by all the readers and tags. The ZS time-stamps the transmit time as \( \tilde{T}_{ZS} \) and the \( J \) readers and \( K \) tags record the message received time as \( (\tilde{R}_{R_1, ZS}, \tilde{R}_{R_2, ZS}, \ldots, \tilde{R}_{R_J, ZS}) \) and \( (\tilde{R}_{T_1, ZS}, \tilde{R}_{T_2, ZS}, \ldots, \tilde{R}_{T_K, ZS}) \) respectively. The readers are assumed to transmit at \( \gamma \) intervals as shown in Figure 4.1. For example, after waiting for a period of \( \gamma \) duration, \( R_1 \) transmits at time \( \tilde{T}_{R_1} \) and this packet is received by all the \( K \) tags and ZS where they record the message arrival time at \( \tilde{R}_{T_K, R_1} \) and \( \tilde{R}_{ZS, R_1} \) respectively. After all the \( J \) readers had transmitted, the whole procedure repeats itself at every \( \omega \) interval. The proposed localization scheme requires two frames \((m=2)\) for computation where a frame has \( \omega \) durations and it is defined as the period from the ZS to start transmission to the time for the ZS to start another transmission. A flowchart of the proposed TDOA localization method is given below.

Readers will not be able to transmit exactly at every \( \gamma \) duration because there will be some offsets when readers start to transmit due to their internal circuits delay. However, the proposed TDOA localization scheme is able to compensate for these offset values. Furthermore, based on these recorded transmitted and received timings, the reader is able to estimate the crystal clock time drift and compensate for it. The following sub sections describe how these offset and time drift values are estimated and used to correct the TDOA values. Only measurement noise and how the tags crystal clock time drift with respect to ZS affect localization accuracy are analyzed in the first part of this chapter. Blockages effect affecting ranging measurements in an indoor environment and methods
to mitigate this effect are discussed in the later part of this chapter.

Figure 4.2: Flowchart of the procedure of the proposed algorithm
4.2.1 Crystal Clock Time Drift Estimation

Readers will estimate their own time drift with respect to ZS and compensate their receive and transmit time stamps at every frame so that their crystal clock will not drift too much. It is assumed that the readers know the duration $\omega$ for every repeated frame which can be pre-programmed during initialization. After the $m^{th}$ frame, the estimated $j^{th}$ reader crystal clock time drift with respect to ZS, $\hat{\delta}_{R_j}$, where $j = 1, 2, \ldots, J$ is given by the following relationship

$$\hat{\delta}_{R_j} = \frac{\tilde{R}_{R_j,ZS}(m) - \tilde{R}_{R_j,ZS}(m-1)}{\omega} - 1,$$  \hspace{1cm} (4.1)

where $\tilde{R}_{R_j,ZS}(m)$ is the measured received time of the arrival message transmitted by ZS at the $j^{th}$ reader at the $m^{th}$ frame.

After the $m^{th}$ frame, the $K$ tags will transmit all their recorded arrival timings to ZS where it will next transfer to a central processor system to compute the positions of the tags. The idea for a central processor to compute the localization is mainly for security purpose. In the event if the tag is stolen or misplaced, intruder will not be able to retrieve the data. Alternatively, other applications can also allow the tags to compute their location such as displaying on a mobile phone. The central processor system first estimates the $k^{th}$ tag crystal clock time drift with respect to ZS from the following equation

$$\hat{\delta}_{T_k} = \frac{\tilde{R}_{T_k,ZS}(m) - \tilde{R}_{T_k,ZS}(m-1)}{\omega} - 1,$$  \hspace{1cm} (4.2)

where $\tilde{R}_{T_k,ZS}(m)$ is the measured received time of the arrival message transmitted by
Chapter 4

ZS at the \( k^{th} \) tag at the \( m^{th} \) frame.

### 4.2.2 Reader Offset Estimation

Normally the readers will not be able to transmit exactly at the time given but with certain offset values \( \Delta_j \). These offset values at the \( j^{th} \) reader at the \( m^{th} \) frame can be estimated by the central processor system as follows

\[
\hat{\Delta}_j(m) = \tilde{R}_{ZS,R_j}(m) - \tilde{T}_{ZS}(m) - j\gamma - \frac{d_{ZS,R_j}}{c},
\]  

(4.3)

where \( d_{ZS,R_j} \) is the distance between the \( j^{th} \) reader and ZS. \( \tilde{R}_{ZS,R_j} \) is the recorded message received time at the ZS transmitted by the \( j^{th} \) reader.

### 4.2.3 TDOA Computation

Finally, the central processor system estimates the TDOA for \( k^{th} \) tag at the \( j^{th} \) reader with respect to the ZS at the \( m^{th} \) frame which can be given by

\[
\tilde{t}_{R_j,ZS}(m) = \frac{\tilde{R}_{T_k,R_j}(m) - \tilde{R}_{T_k,ZS}(m)}{1 + \hat{\delta}_{T_k}} - j\gamma - \hat{\Delta}_j(m).
\]  

(4.4)

### 4.3 The Perturbation Error Analysis

In this section, the factors that influence the localization errors of the proposed method are analyzed. Some disturbances such as crystal clock time drift at the nodes and TOA measurement noise affecting the estimated TDOA values from the ideal case are men-
tioned. Without loss of generality, it only considers the estimation for a single tag, since the results would also apply to estimation of the other tags. Therefore, for notation convenience, the subscript $k$ will be dropped.

### 4.3.1 TOA Error Models

**TOA for Message Transmitted by ZS**

The received TOA message transmitted by ZS and measured at tag at the $m^{th}$ frame is given by

\[
\tilde{R}_{T,ZS}(m) = (1 + \delta_T) \left( \frac{d_{T,ZS}}{c} + n_{TZ} + (m - 1)\omega \right),
\]

(4.5)

where $d_{T,ZS}$ is the distance between ZS and tag, and $n_{TZ}$ is the measurement noise, which is zero-mean Gaussian with variance of $\sigma_{TZ}^2$. Although zero-mean Gaussian for the measurement noise is assumed, it is only for the derivation of the Cramér-Rao Lower Bound (CRLB) in the later section whereas in this perturbation analysis section, any form of distribution can be used. Without loss of generality, assumed that ZS transmits in the first frame perfectly at zero second. Even if ZS does not transmit perfectly at zero second, this will not affect the perturbation analysis as it will be cancelled out. Similarly, the received TOA message transmitted by ZS and measured at $R_j$ is given by

\[
\tilde{R}_{R_j,ZS}(m) = (1 + \delta_{R_j}) \left( \frac{d_{R_j,ZS}}{c} + n_{RZ} + (m - 1)\omega \right),
\]

(4.6)

where $d_{R_j,ZS}$ is the distance between ZS and $R_j$, and $n_{RZ}$ is the measurement noise, which is zero-mean Gaussian with variance of $\sigma_{R_jZ}^2$. 

85
TOA for Message Transmitted by $R_j$

The received TOA message transmitted by $R_j$ and measured at the tag at the $m^{th}$ frame is given by

$$\tilde{R}_{T,R_j}(m) = (1 + \delta_T) \left( \frac{d_{T,R_j}}{c} + n_{TR_j} + j\gamma + \Delta_j(m) + (m - 1)\omega \right), \quad (4.7)$$

where $d_{T,R_j}$ is the distance between tag and reader $R_j$, $n_{TR_j}$ is the measurement noise, and $\Delta_j(m)$ is the actual offset value at the $j^{th}$ reader at the $m^{th}$ frame. Similarly, the received TOA message transmitted by reader $R_j$ and measured at ZS at the $m^{th}$ frame is given by

$$\tilde{R}_{ZS,R_j}(m) = \frac{d_{ZS,R_j}}{c} + n_{ZR_j} + j\gamma + \Delta_j(m) + (m - 1)\omega, \quad (4.8)$$

where $d_{ZS,R_j}$ is the distance between zone supervisor and reader $R_j$, $n_{ZR_j}$ is the measurement noise, and $\Delta_j(m)$ is the actual offset value at the $j^{th}$ reader at the $m^{th}$ frame.

Note that the $\delta_T$ and $\delta_{R_j}$ are the relative time drifts with respect to ZS which means that the effect of ZS crystal clock time drift is already embedded inside and hence ZS is the point of reference.

### 4.3.2 Time Drift Estimation Error for Message Transmitted by ZS

Since both equation pairs, (4.1), (4.6) and (4.2), (4.5), look similar, their error estimate can be derived concurrently by substituting (4.6) into (4.1) and (4.5) into (4.2). Let’s denote the subscript $X$ as either $T$ for tag or $R_j$ for reader. Hence, the estimated drift error is given by
\[
\epsilon_{\delta X} \triangleq \hat{\delta}_X - \delta_X = \frac{(1 + \delta_X) \left( n_{XZ}(m) - n_{XZ}(m-1) \right)}{\omega},
\]  

(4.9)

where it has incorporated \((m)\) and \((m-1)\) to distinguish the noise from frame \(m\) to \(m-1\) in this equation. Derivation for the estimated drift error is given in appendix B. Finally, its error variance is given by

\[
E\left\{ \epsilon_{\delta X}^2 \right\} = \frac{(1 + \delta_X)^2}{\omega^2} \left( \sigma_{XZ}^2(m) + \sigma_{XZ}^2(m-1) \right)
\]

(4.10)

\[
= \frac{(1 + \delta_X)^2}{\omega^2} 2\sigma_{XZ}^2
\]

(4.11)

\[
\approx \frac{2\sigma_{XZ}^2}{\omega^2},
\]

(4.12)

where \(E\{\cdot\}\) denotes taking the expected values. In (4.11), the distance error from frame to frame is assumed to be independent and identically distributed (i.i.d) and \(\delta_X\) is not random, although it is unknown. The approximation in (4.12) is due to \(\delta_X \ll 1\). Furthermore, \(\omega\) is typically much larger than the order of the crystal clock time drift.

### 4.3.3 Reader Offset Estimation Error

Assuming that the first frame of ZS is transmitted at zero second, \(T_{ZS}(m) = (m-1)\omega\) in (4.3). Therefore, substituting (4.8) into (4.3), the estimation error model can be written as,

\[
E \left\{ \left( \hat{\Delta}_j(m) - \Delta_j(m) \right)^2 \right\} = \sigma_{ZR_i}^2.
\]

(4.13)
4.3.4 TDOA Estimation Error

TDOA of the Proposed Method

Here, the performance of the estimation of TDOA using the proposed method in (4.4) is analyzed. By substituting (4.5) and (4.7) into (4.4),

\[
\hat{t}_{R_j,ZS}(m) = \frac{1 + \delta_T}{1 + \hat{\delta}_T} \left( \frac{d_{T,R_j} - d_{T,ZS}}{c} + j\gamma + \Delta_j(m) + n_{TR_j} - n_{TZ} \right) - j\gamma - \hat{\Delta}_j(m).
\]  

(4.14)

Using the first order Taylor’s expansion of the above equation with respect to \( \epsilon_{\hat{\delta}_T} \) about zero gives

\[
\hat{t}_{R_j,ZS}(m) \approx \left( 1 - \frac{\epsilon_{\hat{\delta}_T}}{1 + \hat{\delta}_T} \right) \left( t_{R_j,ZS}(m) + j\gamma + \Delta_j(m) + n_{TR_j} - n_{TZ} \right) - j\gamma - \hat{\Delta}_j(m).
\]

(4.15)

The above equation does not consider higher order approximation because \( \epsilon_{\hat{\delta}_T} \) is relatively small (\( \ll 1 \)), hence the second and higher order terms vanished. Note that \( \frac{d_{T,R_j} - d_{T,ZS}}{c} = t_{R_j,ZS}(m) \) is used in the above derivation. With some simple rearrangement,

\[
\hat{t}_{R_j,ZS}(m) - t_{R_j,ZS}(m) \approx \left( 1 - \frac{\epsilon_{\hat{\delta}_T}}{1 + \hat{\delta}_T} \right) (n_{TR_j} - n_{TZ}) - \frac{\epsilon_{\hat{\delta}_T}}{1 + \hat{\delta}_T} \Omega + (\hat{\Delta}_j(m) - \Delta_j(m)).
\]  

(4.16)
where

\[ \Omega \triangleq t_{R_j,ZS}(m) + j\gamma + \Delta_j(m). \tag{4.17} \]

Finally, using (4.9) and through elaborative derivation, the TDOA’s error variance is simplified as follows

\[
E\left\{ \hat{t}_{R_j,ZS}(m) - t_{R_j,ZS}(m) \right\}^2 \approx \sigma_{ZR_j}^2 + \sigma_{TR_j}^2 + \sigma_{TZ}^2 \left( 1 + 2 \frac{\Omega}{\omega} + 2 \left( \frac{\Omega}{\omega} \right)^2 \right) + \frac{4}{\omega^2} \sigma_{TZ}^4 + \frac{2}{\omega^2} \sigma_{TZ}^2 \sigma_{TR_j}^2.
\tag{4.18}
\]

Assuming that \( \sigma_{TR_j}^2 = \sigma_{TZ}^2 = \sigma_{ZR_j}^2 = \sigma^2 \), then TDOA’s variance could be simplified further to

\[
E\left\{ \hat{t}_{R_j,ZS}(m) - t_{R_j,ZS}(m) \right\}^2 \approx \sigma^2 \left( 3 + 2 \omega \frac{\Omega}{\omega} + 2 \left( \omega \frac{\Omega}{\omega} \right)^2 \right) + \frac{6}{\omega^2} \sigma^4. \tag{4.19}
\]

The detailed derivation is given in appendix C. The variance of TDOA’s error employing the proposed method does not depend on \( \delta_T \) anymore. This can be seen next that the conventional method depends on \( \delta_T \) and can create significant error.

**TDOA without Time-Drift Compensation**

Here, the performance of the conventional TDOA estimation that does not include the time-drift compensation is analyzed. From (4.14), simply by substituting zero into \( \hat{\delta}_T \) and rearrange, it becomes
\[
\hat{t}_{R_j, ZS}(m) - t_{R_j, ZS}(m) = \delta_T(t_{R_j, ZS}(m) + \Delta_j(m) + j\gamma) \\
+ (1 + \delta_T)(n_{TR_j} - n_{TZ}) - n_{ZR_j} \\
= \delta_T\Omega + (1 + \delta_T)(n_{TR_j} - n_{TZ}) \\
- (\hat{\Delta}_j(m) - \Delta_j(m)).
\] (4.20)

The TDOA’s error variance is given by

\[
E \{\hat{t}_{R_j, ZS}(m) - t_{R_j, ZS}(m)\}^2 = (\delta_T\Omega)^2 + (1 + \delta_T)^2(\sigma_{TR_j}^2 + \sigma_{TZ}^2) + \sigma_{ZR_j}^2.
\] (4.22)

Assuming \(\sigma_{TR_j}^2 = \sigma_{TZ}^2 = \sigma_{ZR_j}^2 = \sigma^2\), the TDOA’s error variance simplifies to

\[
E \{\hat{t}_{R_j, ZS}(m) - t_{R_j, ZS}(m)\}^2 = (\delta_T\Omega)^2 + \sigma^2(3 + 4\delta_T + 2\delta_T^2).
\] (4.23)

From (4.23), it can clearly be seen that even when the noise goes to zero, the TDOA’s error variance will not be zero. The conventional TDOA estimation method generates bias due to time offset. Furthermore, the error due to minuscule time offset can be amplified significantly by \(\Omega\), particularly \(\gamma\) which is typically in the order of milliseconds. The above approximation is valid because typical time drift values for a low cost crystal clock are between \(\pm 20\) parts per million to \(\pm 40\) parts per million (ppm) (i.e. time drift of reader, \(\delta_{R_j}\) and tag, \(\delta_T\) are between \(\pm 20\)ppm to \(\pm 40\)ppm) and they are much smaller when compared to unity.
4.4 Computation of Tags’ Locations Using Least Square Method

Recall that TDOA between \( ZS \) and the \( j^{th} \) reader can be written as

\[
t_{R_j, ZS} = d_{T,R_j} - d_{T,ZS},
\]

where \( d_{T,ZS} \) and \( d_{T,R_j} \) are the distances between the tag and \( ZS \) and between the tag and the \( j^{th} \) reader respectively. Then the right hand side of (4.24) can be written as

\[
d_{T,R_j}^2 - d_{T,ZS}^2 = (x_{R_j} - x_T)^2 + (y_{R_j} - y_T)^2 + (z_{R_j} - z_T)^2 - (x_{ZS} - x_T)^2 - (y_{ZS} - y_T)^2 - (z_{ZS} - z_T)^2,
\]

where \((x_T, y_T, z_T)\), \((x_{R_j}, y_{R_j}, z_{R_j})\) and \((x_{ZS}, y_{ZS}, z_{ZS})\) denotes the tag, \( j^{th} \) reader and \( ZS \) coordinates respectively.

Further expand (4.24) and substitute (4.25) into it, this becomes

\[
(t_{R_j, ZS})^2 + 2d_{T,ZS}t_{R_j, ZS} = d_{T,R_j}^2 - d_{T,ZS}^2
\]

\[
= (x_{R_j} - x_T)^2 + (y_{R_j} - y_T)^2 + (z_{R_j} - z_T)^2 - (x_{ZS} - x_T)^2 - (y_{ZS} - y_T)^2 - (z_{ZS} - z_T)^2
\]

\[
= x_{R_j}^2 - x_{ZS}^2 - 2x_T(x_{R_j} - x_{ZS}) + y_{R_j}^2 - y_{ZS}^2 - 2y_T(y_{R_j} - y_{ZS}) + z_{R_j}^2 - z_{ZS}^2 - 2z_T(z_{R_j} - z_{ZS}).
\]

Next, group all the known terms together and defining it as
\[ f_{R_j,ZS} = 0.5(x_{R_j}^2 - x_{ZS}^2 + y_{R_j}^2 - y_{ZS}^2 + z_{R_j}^2 - z_{ZS}^2 - (\hat{t}_{R_j,ZS})^2). \] (4.27)

Then rearranged (4.26) and (4.27) to get

\[ d_{T,ZS}t_{R_j,ZS} + x_T(x_{R_j} - x_{ZS}) + y_T(y_{R_j} - y_{ZS}) + z_T(z_{R_j} - z_{ZS}) = f_{R_j,ZS}. \] (4.28)

Finally, using least squares, the estimated \( k^{th} \) tag position can be calculated as

\[ \rho = (\mathcal{A}^T \mathcal{A})^{-1} \mathcal{A}^T \mathcal{F}, \] (4.29)

where

\[
\rho = \begin{bmatrix} x_T \\ y_T \\ z_T \\ d_{T_k,ZS} \end{bmatrix}
\] (4.30)

\[
\mathcal{A} = \begin{bmatrix} x_{ZS} - x_{R_1} & y_{ZS} - y_{R_1} & z_{ZS} - z_{R_1} & \hat{t}_{R_1,ZS} \\ x_{ZS} - x_{R_2} & y_{ZS} - y_{R_2} & z_{ZS} - z_{R_2} & \hat{t}_{R_2,ZS} \\ \vdots & \vdots & \vdots & \vdots \\ x_{ZS} - x_{R_j} & y_{ZS} - y_{R_j} & z_{ZS} - z_{R_j} & \hat{t}_{R_j,ZS} \end{bmatrix}
\] (4.31)
\[ F = \begin{bmatrix} f_{R_1, ZS} \\ f_{R_2, ZS} \\ \vdots \\ f_{R_J, ZS} \end{bmatrix}. \quad (4.32) \]

### 4.5 The Cramér-Rao Lower Bound

In this section, the CRLB of this proposed TDOA localization method is derived by taking into account of crystal clock time drift, and without loss of generality, for one tag only. For multiple tags, the CRLB of each tag’s localization is the same as the CRLB of one tag. CRLB is the lowest bound for any estimators and there are many derivations by other authors, however, it depends on the distribution and hence, the model. Other authors did not consider crystal clock time drift as a nuisance parameter towards estimating tags location. However the proposed CRLB model takes into account of the tags locations as well as the crystal clock time drift.

For simplification, it is assumed the estimation uses only one frame; for \( M \) frames, the CRLB is simply \( \frac{1}{M} \) times the CRLB for one frame. Since the effect of \( \Delta_j \) is relatively small on the estimation error, it is assumed to be zero to simplify the derivation. Hence, the main factors of the CRLB to be analyzed are on location estimates and crystal clock time drift of the tag, \( \delta_T \).

Let the positions of the anchor nodes be \( L_j \) where \( j = 0, 1, 2, \ldots, J \) where \( L_0 \) represents the position of the ZS and \( L_1, L_2, \ldots, L_J \) represents the positions of the \( J \) readers. Let the position of the tag be \( p \). Next, defined the distance between the tag and the ZS or the reader as
\[ d_{T,ZS} = d_0(p) = \|L_0 - p\| \]

\[ d_{T,R_j} = d_j(p) = \|L_j - p\| \quad \text{for } j = 1, 2, \ldots, J, \quad (4.33) \]

where \(\|\|\) denotes taking the Frobenius norm. The difference in distance between the \(j^{th}\) reader with respect to the ZS for \(j = 1, 2, \ldots, J\) is \(d_j = [d_{j0} \ 0 \ldots 0]^T\) where \(d_{j0} = d_j(p) - d_0(p)\) and \([\cdot]^T\) denotes taking the transpose. In terms of time difference of arrival, \(t = [t_{R_1,ZS} \ t_{R_2,ZS} \ldots t_{R_J,ZS}]^T\) with

\[ t_{R_j,ZS} \triangleq \tau_{R_j} - \tau_{ZS} \quad (4.34) \]

\[ = (1 + \delta_T)(\frac{1}{c}d_{j0} + n_{j0}). \]

The reader dependent range difference error between the tag to \(j^{th}\) reader and ZS, \(n_{j0}\) is defined as follows

\[ n_{j0} \triangleq n_{TR_j} - n_{TZ}, \quad (4.35) \]

where \(\{n_j\}_{j=0,\ldots,J}\) are independent zero mean Gaussian with \(\sigma_j^2\) variances. Therefore, the covariance of the time difference of arrival between two readers can be computed as follows

\[ [\mathcal{C}]_{hi} \triangleq \text{cov}(t_{R_h,ZS}, t_{R_i,ZS}) = (1 + \delta_T)^2 E\{n_{h0}n_{i0}\} \quad (4.36) \]

\[ = (1 + \delta_T)^2 E\{(n_{TR_h} - n_{TZ})(n_{TR_i} - n_{TZ})\} \]

\[ = \begin{cases} 
  (1 + \delta_T)^2 (\sigma_{TR_i}^2 + \sigma_{TZ}^2) & \text{for } h = i \\
  (1 + \delta_T)^2 \sigma_{TZ}^2 & \text{for } h \neq i, 
\end{cases} \]
where $\sigma^2_\delta$ is the variance of the time drift, $\sigma^2$ is the variance of the Gaussian noise.

Rewriting (4.36) into matrix form, it becomes

$$
\mathcal{C} = (1 + \delta_T)^2 \mathcal{Q},
$$

(4.37)

where,

$$
\mathcal{Q} \triangleq \sigma^2_T \begin{bmatrix} 1 & 1 & \ldots & 1 \\
1 & 1 & \ldots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \ldots & 1 \end{bmatrix}_{J \times J} + \begin{bmatrix} \sigma^2_{TR_1} & 0 & \ldots & 0 \\
0 & \sigma^2_{TR_2} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \sigma^2_{TR_J} \end{bmatrix}
$$

(4.38)

If all the readers and ZS experience i.i.d. noises with equal variances, $\sigma^2_{TR_j} = \sigma^2_T = \sigma^2$ for all $j$, and therefore

$$
\mathcal{Q} \triangleq \sigma^2 \begin{bmatrix} 2 & 1 & \ldots & 1 \\
1 & 2 & \ldots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \ldots & 2 \end{bmatrix}_{J \times J}
$$

(4.39)

Next, recall that the definition of Fisher Information Matrix (FIM) [111] as follows

$$
\mathcal{J} \triangleq E\{(\nabla \ln f(t; \theta))(\nabla \ln f(t; \theta))^T\},
$$

(4.40)

where
\[ \theta = \begin{bmatrix} p^T, \delta_T \end{bmatrix}^T \]  
(4.41)

and

\[ f(t; \theta) = \frac{1}{(2\pi)^{\frac{d}{2}} \sqrt{\det(\mathbf{C})}} \exp \left[ -\frac{1}{2} \left( t - \frac{1 + \delta_T}{c} \right) \mathbf{C}^{-1} \left( t - \frac{1 + \delta_T}{c} \right) \right]. \]  
(4.42)

which is basically Gaussian distribution of (4.34). Hence, to get the FIM, from (4.40) it can be derived

\[
\nabla \ln f(t; \theta) = \nabla \left[ \ln \left\{ (2\pi)^{\frac{d}{2}} \sqrt{\det(\mathbf{C})} \right\} - \frac{1}{2} \left( t - \frac{1 + \delta_T}{c} \right)^T \mathbf{C}^{-1} \left( t - \frac{1 + \delta_T}{c} \right) \right]
\]

\[ = -\frac{1}{2} \nabla \left[ \left( t - \frac{1 + \delta_T}{c} \right)^T \mathbf{C}^{-1} \left( t - \frac{1 + \delta_T}{c} \right) \right] \]  
(4.43)

Defining the Jacobian as

\[ G = \frac{\partial}{\partial \theta} \left( \frac{1 + \delta_T}{c} d^T \right), \]  

it becomes

\[ \mathbf{G} = \frac{\partial}{\partial \theta} \left( \frac{1 + \delta_T}{c} d^T \right), \]
\[ G = \begin{bmatrix} \frac{\partial}{\partial \delta T} \left( \frac{1}{c} \delta \tau \right) \\ \frac{\partial}{\partial \delta p} \left( \frac{1}{c} \delta \tau \right) \end{bmatrix} \]

\[ = \begin{bmatrix} \frac{1}{c} \frac{\partial \delta \tau}{\partial \delta p} \\ \frac{\partial}{\partial \delta p} \left( \frac{1}{c} \delta \tau \right) \end{bmatrix}, \quad (4.44) \]

where

\[ \frac{\partial d_{j0}}{\partial p_i} = \frac{\partial}{\partial p_i} (d_j(p) - d_0(p)) \]

\[ = \frac{\partial}{\partial p_i} \|p - L_j\| - \frac{\partial}{\partial p_i} \|p - L_0\| \]

\[ = \frac{p_i - L_{ji}}{\|p - L_j\|} - \frac{p_i - L_{0i}}{\|p - L_0\|}, \quad (4.45) \]

where \( p_i \) is the \( i^{th} \) component of the position of the tag, and \( L_{ji} \) is the \( i^{th} \) component of \( j^{th} \) reader’s position. For three dimensional position, \( i = 1 \) denotes \( x \) component, \( i = 2 \) denotes \( y \) component and \( i = 3 \) denotes \( z \) component.

By stacking them vertically, for three dimensional localization, it forms a vector

\[ \begin{bmatrix} \frac{\partial d_{j0}}{\partial p_1} \\ \frac{\partial d_{j0}}{\partial p_2} \\ \frac{\partial d_{j0}}{\partial p_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial d_{j0}}{\partial p} \end{bmatrix} \triangleq R_{j0} \]

\[ = \frac{p - L_i}{\|p - L_i\|} - \frac{p - L_0}{\|p - L_0\|}. \quad (4.46) \]

Stacking the vectors in (4.46) horizontally gives \( \frac{\partial \delta \tau}{\partial p} \triangleq \mathcal{R} \triangleq [R_{10} R_{20} \ldots R_{J0}] \) and
Therefore, the FIM, $J$ can be re-write as

\[
J = E \left\{ G \mathbf{C}^{-1} \left( t - \frac{1 + \delta_T}{c} d \right) \left( t - \frac{1 + \delta_T}{c} d \right)^T \right\} \mathbf{C}^{-1} G^T
\]

\[
= G \mathbf{C}^{-1} E \left\{ \left( t - \frac{1 + \delta_T}{c} d \right) \left( t - \frac{1 + \delta_T}{c} d \right)^T \right\} \mathbf{C}^{-1} G^T
\]

\[
= G \mathbf{C}^{-1} G^T.
\]

Finally, CRLB is given by the diagonal elements of $J^{-1}$.

### 4.6 Simulation Results

Figure 4.3 shows a top view of a hexagon layout of the simulated area and the positions of the ZS and readers, where the layout can be changed according to user specification. In Figure 4.3, the ZS is placed in the middle of the network for maintenance of the network and coordination of the sensor nodes operating inside its network area while six readers are placed at the edges.

In order to have a clearer view of the CRLB on the localization errors, tag spaced across the diagonal of the network from coordinates (10, 0, 1.7) to (30, 34.64, 1.7) at a diagonal distance of 2m apart is analyzed. The height of the tag is maintained at 1.7m to simulate an average person’s height. Crystal clock time drift of readers and tags are randomly generated between $\pm 20$ ppm (parts per million) from an uniform distribution and a Gaussian noise with zero mean and $\sigma = 0.95$ ns is assumed. The locations of the tags are computed using eqn (4.29) in section 4.4 and the CRLB is
calculated using eqn (4.48). The TDOA values are computed using eqn (4.4) averaged over 1000 realizations. Figure 4.4 shows the localization mean square error of an area spanning across the diagonal of a network area with different crystal clock time drift values and no time drift. They approach the CRLB especially when the tags are near the center of the network. The CRLB values increases when the tag is near the edges of the network and attains the lowest value when the tag is in the middle of the network, which corresponds to the position of ZS. Difference in the simulated results and theoretical results is only $0.3 \, \text{m}^2$ in the middle of the zone area and about $1 \, \text{m}^2$ at the edge of the zone area.

Figure 4.5 shows the TDOA error performance with varying noise variance. The simulated performance matched closely with the theoretical analysis (4.19) under both time drift and no time drift compensation (4.23) conditions. The TDOA error is in the order of 1000 times lower for the proposed method as compared to if there is no time drift compensation. Although the proposed scheme is more computational efficient than
TWR and SMWR as described in Chapter 3, there are slightly more transmissions as compared to the traditional TDOA method. These additional transmission messages are needed in order to synchronize the ZS and readers. However, it has fewer transmissions as compared to the traditional TWR localization based method and SMWR. This method based on TDOA localization incurred an order of $O(J + K + 1)$ numbers of transmission which is much lesser than a traditional TWR localization based method with an order of $O(4K^2)$ and SMWR method of an order of $O(8K)$ where $J$ and $K$ are the number of readers and tags respectively. This clearly shows that this TDOA localization based method is more efficient in detecting a large number of tags at the same time.
4.7 Method to Reduce Blockages Effect on Ranging Accuracy

Blockage of signals from some anchors to the tags is common in an indoor wireless sensor network and this leads to inaccurate readings and can result in adverse localization performance. This section introduces a weighting function applied to the TDOA measurements to reduce the localization error caused by blockages. Next, the CRLB for the TDOA localization by taking into account of the weighting function is derived. Simulation results demonstrated the accuracy of the proposed method, where the localization errors with the weighting functions approach the CRLB.
4.7.1 Weighted TDOA Localization Scheme

Assuming that the weighting function is independent of noise and dependent on TDOA, it is given by

\[ w_i = e^{\exp\left(-\frac{\hat{t}_{R_j,ZS}}{h}\right)} + \frac{\hat{t}_{R_j,ZS} + 1}{\sqrt{2\hat{t}_{R_j,ZS}}}, \]  \hspace{1cm} (4.49)

where \( h = \max\{t\} \) and \( w_i \) is the assigned weight to be applied to the TDOA such that the weighted TDOA in the \( m^{th} \) frame is given as

\[ \hat{t}_{w,R_j,ZS}(m) = w_i \hat{t}_{R_j,ZS}(m). \]  \hspace{1cm} (4.50)

The superscript \( w \) denotes the weighted TDOA. The first portion of the weighting function is a Welsch weight which guards against Studentized residuals [94]. The second portion of the weighting function is a scaled version analogous to a Huber weighting function that is used to optimize in the presence of small measurement errors and help to provide a smoother, less biased estimate from estimated TDOA [95]. The weighting function described in [66] does not give enough weightage to small TDOA measurements especially if the tags are at the edges of the network. Conversely, if the TDOA measurements are large, some of the weighting functions approach zero and this might cause inaccurate weightage to the TDOA measurements. The proposed weighting function in this chapter is more robust and is able to compensate for this inaccurate weightage to the TDOA measurements. This choice of weighting function is able to reduce the error caused by blockages on the tags and to compensate for very small TDOA readings which can degrade the localization performance. The proposed weighting function has robust performance which will be shown in the simulations. The tags locations can be
computed from the equations in section 4.4 by replacing the TDOA measurements \( \hat{t}(.) \) by the weighted TDOA measurements \( \hat{t}(.)^w \).

### 4.7.2 The Cramér-Rao Lower Bound with weighting function

In this sub section, the CRLB of the proposed TDOA localization method by taking into account of the weighting function, without loss of generality, for one tag only is derived. The CRLB is derived taking into consideration the tag location, crystal clock time drift as well as the blockage effect.

Defining a vector of time difference of arrival, \( t = [t_{R1,ZS} \ t_{R2,ZS} \ldots t_{RJ,ZS}]^T \) where

\[
t_{Rj,ZS} = (1 + \delta_T)(\frac{1}{c}d_{j0} + b_{j0} + n_{j0}).
\]

The reader dependent range difference error between the tag to \( j^{th} \) reader and ZS, \( n_{j0} \), and the difference of the TOA blockage effect at the \( j^{th} \) reader and ZS, \( b_{j0} \), are defined as follows

\[
\begin{align*}
n_{j0} &\triangleq n_{TRj} - n_{TZ} \\
b_{j0} &\triangleq b_{TRj} - b_{TZ},
\end{align*}
\]

where \( \{n_j\}_{j=0,...,J} \) are independent zero mean Gaussian with \( \sigma_j^2 \) variances and \( \{b_j\}_{j=0,...,J} \) whose amplitude are log normal distributed [77] and after converting to Gaussian distributed, the variances are given as \( \sigma_{b_j}^2 \). Therefore, the covariance of the time difference of arrival between two readers can be computed as follows
The above equation can be rewrite as

\[ \mathcal{C} = (1 + \delta_T)^2 \mathcal{Q}, \]  

(4.54)

where,

\[ \mathcal{Q} \triangleq (\sigma_{TZ}^2 + \sigma_{bTR}^2) \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \ldots & 1 & 1 \end{bmatrix}_{J \times J} \]

(4.55)

\[ + \begin{bmatrix} (\sigma_{TR_2}^2 + \sigma_{bTR_2}^2) & 0 & \ldots & 0 \\ 0 & (\sigma_{TR_3}^2 + \sigma_{bTR_3}^2) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & (\sigma_{TR_J}^2 + \sigma_{bTR_J}^2) \end{bmatrix}. \]

If all the readers and ZS experience i.i.d. noises with equal variances, \( \sigma_{TR_j}^2 = \sigma_{TZ}^2 = \sigma^2 \) for all \( j \), and therefore
\[ \mathbf{Q} \triangleq \sigma_{bTZ}^2 \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \ldots & 1 \end{bmatrix}_{J \times J} + \sigma_{bTR_2}^2 \begin{bmatrix} \sigma_{bTR_2}^2 & 0 & \ldots & 0 \\ 0 & \sigma_{bTR_3}^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \sigma_{bTR_J}^2 \end{bmatrix} \]

where \( \mathbf{I} \) is an identity matrix of size \( J \times J \) and \( \mathbf{11}^T \) is a matrix of all ones. Subsequently, the FIM is defined as follows

\[ \mathbf{J} \triangleq E\{ (\nabla \ln f(t; \theta))(\nabla \ln f(t; \theta))^T \} \]

where

\[ \theta = [\mathbf{p}^T, \delta^T, \mathbf{b}^T]^T \]

and the blockage effect is given as \( \mathbf{b} = [b_{20} \ b_{30} \ \ldots \ b_{j0}]^T \) and

\[ f(t; \theta) = \frac{1}{(2\pi)^{J^2/2} \sqrt{\det(C)}} \times \exp \left[ -\frac{1}{2} (t - \frac{1 + \delta_T}{c}(\mathbf{d} + \mathbf{b}))^T \mathbf{C}^{-1} (t - \frac{1 + \delta_T}{c}(\mathbf{d} + \mathbf{b})) \right], \]

which is basically Gaussian distribution of (4.51). Hence, to get the FIM, from
(4.57) it can be derived

\[
\nabla \ln f(t; \theta) = \nabla [-\ln \{(2\pi)^{\frac{d}{2}} \sqrt{\det(C)}\}]
\]

\[
\frac{1}{2} (t - \frac{1 + \delta_T(d + b))^T C^{-1}(t - \frac{1 + \delta_T(d + b))}{c}
\]

\[
= \frac{1}{2} \nabla [(t - \frac{1 + \delta_T(d + b))^T C^{-1}(t - \frac{1 + \delta_T(d + b))}{c}]
\]

\[
= -\frac{\partial}{\partial \theta} \left[(t - \frac{1 + \delta_T(d + b))^T C^{-1}(t - \frac{1 + \delta_T(d + b))}{c}\right]
\]

\[
= \left[\frac{\partial}{\partial \theta} \left( \frac{1 + \delta_T(d + b))^T C^{-1}(t - \frac{1 + \delta_T(d + b))}{c}\right)\right].
\]

Defining the Jacobian as \( G = \frac{\partial}{\partial \theta} \left( \frac{1 + \delta_T(d + b))^T}{c}\right), \)

\[
G = \begin{bmatrix}
\frac{\partial}{\partial p} \\
\frac{\partial}{\partial \delta_T} \\
\frac{\partial}{\partial b}
\end{bmatrix} \left( \frac{1 + \delta_T(d + b))^T}{c}\right)
= \begin{bmatrix}
\frac{1 + \delta_T(d + b))^T}{c} \\
\frac{\partial}{\partial p} (d + b)^T \\
\frac{1 + \delta_T(d + b))^T}{c}
\end{bmatrix},
\]

where

\[
\frac{\partial}{\partial p_n} (d_j + b_j) = \frac{\partial}{\partial p_n} (d_j(p) - d_0(p) + b_j - b_0)
\]

\[
= \frac{\partial}{\partial p_n} \|p - L_j\| - \frac{\partial}{\partial p_n} \|p - L_0\|
\]

\[
= \frac{p_n - L_{jn}}{\|p - L_j\|} - \frac{p_n - L_{0n}}{\|p - L_0\|}.
\]
where $p_n$ is the $n^{th}$ component of the position of the tag, and $L_{jn}$ is the $n^{th}$ component of $j^{th}$ reader’s position. For three dimensional position, $n = 1$ denotes $x$ component, $n = 2$ denotes $y$ component and $n = 3$ denotes $z$ component.

By stacking them vertically, for three dimensional localization, it forms a vector

$$
\begin{bmatrix}
\frac{\partial (d_{j0} + b_{j0})}{\partial p_1} \\
\frac{\partial (d_{j0} + b_{j0})}{\partial p_2} \\
\frac{\partial (d_{j0} + b_{j0})}{\partial p_3}
\end{bmatrix} = \frac{\partial (d_{j0} + b_{j0})}{\partial p} \triangleq R_{j0} \tag{4.63}
$$

$$
= \frac{p - L_j}{\| p - L_j \|} - \frac{p - L_0}{\| p - L_0 \|}.
$$

Stacking the vectors above horizontally gives

$$
\frac{\partial (d + b)^T}{\partial p} \triangleq \mathcal{R} \triangleq [R_{20} \ R_{30} \ldots \ R_{J0}]
$$

and

$$
\mathbf{G} = \begin{bmatrix}
\frac{1+\delta_T}{c} \mathcal{R} \\
\frac{(d+b)^T}{c} \\
\frac{1+\delta_T}{c}
\end{bmatrix}. \tag{4.64}
$$

Therefore, the FIM, $\mathcal{J}$, can be rewrite as

$$
\mathcal{J} = E \left\{ \mathbf{G} \mathcal{C}^{-1} \left( t - \frac{1+\delta_T}{c} (d+b) \right) \left( t - \frac{1+\delta_T}{c} (d+b) \right)^T \mathcal{C}^{-1} \mathbf{G}^T \right\}
$$

$$
= \mathbf{G} \mathcal{C}^{-1} E \left\{ \left( t - \frac{1+\delta_T}{c} (d+b) \right) \left( t - \frac{1+\delta_T}{c} (d+b) \right)^T \right\} \mathcal{C}^{-1} \mathbf{G}^T
$$

$$
= \mathbf{G} \mathcal{C}^{-1} \mathbf{G}^T. \tag{4.65}
$$

Finally, CRLB is given by the diagonal elements of $\mathcal{J}^{-1}$. 

107
4.8 Weighted TDOA Simulation Results

Figure 4.6 shows the top view of the simulated areas in a hexagon and square layouts as well as the positions of the ZS and readers. ZS is placed in the middle of the network and the readers are placed around the boundary of the network. ZS is connected to the central processor where the crystal clock time drift of the tags and the timestamp offset of the readers are compensated. In addition, the central processor also calculates the TDOA values and estimates the positions of the tags. The solid line represents the online operation and dotted lines represent the offline operation. A Gaussian noise of zero mean and standard deviation $\sigma = 0.95$ns is assumed and the TOA blockage is log normal distributed with 3dB variance which is translated to 0.4ns of timing error [1] and [66]. Similarly, the locations of the tags are computed using eqn (4.29) in section 4.4 and the CRLB is computed using eqn (4.65). The TDOA values are computed using eqn (4.50) averaged over 1000 realizations. Crystal clock time drift at the tags, readers and ZS are uniformly distributed between $\pm 20$ppm (parts per million) and the ZS and readers are randomly blocked.

![Diagram](image)

Figure 4.6: Top view of the network layout (a) Hexagon (b) Square

Figure 4.7 shows the simulation results of the localization error compared with the
CRLB and the proposed weighting function in reference [66] for (a) hexagon layout and (b) square layout. The tag moves across the diagonal of the network from the bottom left to the top right. The height of the tag is maintained at 1.7m to simulate an average person’s height. From Figure 4.7, the localization errors with the proposed weighting function and the one mentioned in reference [66] are reduced and both approach the CRLB for both network layouts. High CRLB is expected at the boundaries of the network especially at the edges and the lowest CRLB occurs when the tag is in the middle of the network which is equidistance to all the anchors. In addition, the proposed weighting function has better performance as compared to the one mentioned in reference [66]. This shows that the proposed weighting function is robust in reducing localization errors.

4.9 Laboratory Measurements

Figure 4.8 shows the laboratory layout diagram with the placement of the readers and ZS where the ranging data are collected. ZS is placed in the middle of the laboratory with the remaining readers are placed on the benches. The tag moves around the shaded area in the middle of the laboratory surrounded by the benches.
It uses UWB-IR transmission with the specification listed in Table 4.1. A length of 31 chip base ternary code sequences is used to represent the preamble, Start Frame Delimiter (SFD), Physical Header (PHR) and data. The sequence consists of 64 symbols of preamble, 8 symbols of SFD, 5 symbols of PHY header and up to 127 bytes of data. Each symbol consists of 31 chips of ternary sequence. The PHY transmitter uses the differential clock provided by the RF module. This module uses a DCM to convert the clock from 249.6MHz to 31.2MHz and use as the system clock.

At the receiving side, coherent receivers are used. The receiver uses the clock from the ADC module on the RF board for processing. The differential clock of 61.4MHz is
used for processing. This module maintains the correct symbol boundary by counting the number of chips in each symbol and also keeps track of the number of preamble, SFD, PHY header and data. The ADC input from the RF module consists of 8 parallel stream of real (I) and imaginary (Q) signals. This parallel inputs form a byte of data used for the combining process. The Correlator Array consists of 16x16 correlator units. They are correlated with the 16 possible chip sequences. The individual correlator unit correlates one signal with a chip sequence and accumulates the value. The maximum value of the 16 maximum value is then determined and the position of the maximum value recorded. Finally, the absolute value of the I and Q correlated output from the correlator array is calculated and add up the I and Q samples.

The coherent RF board is designed by an external party, therefore only some brief explanations are given. The transceiver specifications are listed below:

- Analog-to-digital converter with 2 bit resolution, maximum sampling rate of 2 Gsps
- Reference oscillator of 40 MHz with less than 1 ppm, fractional PLL to be used to generate multiples of 41.6 MHz (41.6 MHz TCXO has to be custom-ordered)
- Pulse width control using solid-state delay line with a programmable range from 0 ns to 10 ns and a delay resolution of 11 ps
- Noise figure 5dB
- Filtered RF bandwidths: 500 MHz and 1.5 GHz
- Rx interface with baseband using 32 LVDS lines plus Clock running up to approximately 250 MHz
- TCLK running at 499.2 MHz
A summary of the supported channel modes are given in Table 4.1

Table 4.1: Coherent Transceiver supported channel modes

<table>
<thead>
<tr>
<th>Channel Number</th>
<th>3</th>
<th>4</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Freq (MHz)</td>
<td>4492.8</td>
<td>3993.6</td>
<td>7987.2</td>
<td>7987.2</td>
</tr>
<tr>
<td>Bandwidth (MHz)</td>
<td>499.2</td>
<td>1331.2</td>
<td>499.2</td>
<td>1331.2</td>
</tr>
<tr>
<td>Rx Sampling Rate (MHz)</td>
<td>499.2 or 1123.2</td>
<td>499.2*4</td>
<td>499.2 or 499.2*2</td>
<td>499.2*4</td>
</tr>
<tr>
<td>Average Tx Output (dBm)</td>
<td>-14.3</td>
<td>-9.5</td>
<td>-14.3</td>
<td>-9.5</td>
</tr>
<tr>
<td><strong>Option 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean PRF (MHz)</td>
<td>15.6</td>
<td>15.6</td>
<td>15.6</td>
<td>15.6</td>
</tr>
<tr>
<td>Average Duty Cycle</td>
<td>0.0313</td>
<td>0.0117</td>
<td>0.0313</td>
<td>0.0117</td>
</tr>
<tr>
<td>Peak Tx Output (dBm)</td>
<td>0.8</td>
<td>9.8</td>
<td>0.8</td>
<td>9.8</td>
</tr>
<tr>
<td><strong>Option 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean PRF (MHz)</td>
<td>3.9</td>
<td>3.9</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Average Duty Cycle</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Peak Tx Output (dBm)</td>
<td>6.8</td>
<td>15.8</td>
<td>6.8</td>
<td>15.8</td>
</tr>
</tbody>
</table>

Figure 4.9: Localization error for laboratory

Figure 4.9 shows the localization errors for the laboratory environment. The localization technique is the TDOA as described in eqn (4.4) averaged over 1000 realizations. The tag moves across horizontally in the middle of the walking area as shown in Figure
4.8 at a constant height of 1.17m. The x-axis indicates the number of steps the tag took to walk horizontally across the room. In addition, a weighting function as described in eqn (4.49) is used to reduce the blockages effect of the signals in affecting ranging accuracy such as NLOS signal from the readers to the tag. With the weighting function, the localization errors can be reduced by 15-23%. This shows that the proposed weighting function is robust to reduce localization error due to blockages.

### 4.10 Summary

In this chapter, both simulation and analytical results show that the proposed TDOA localization method is able to reduce the TDOA estimation errors even when it is influenced by devices’ crystal clock time drift. With reduced errors in TDOA estimation, robust location estimates in the presence of crystal clock time drifts can be obtained. This allows low cost devices to be deployed in the network with poor crystal clock. Additionally, the proposed method is able to track a large number of tags at the same time because it does not require time synchronization among the anchor nodes initially. This allows flexible and simple deployment of the localization system. This chapter also derived the Cramér-Rao Lower Bound for TDOA localization method with unknown deterministic clock time drift as nuisance parameter and showed that this proposed method performs relatively close to the CRLB. The conventional localization method applied to the practical case with crystal clock time drift is proven to be erroneous, because it ignores the need to estimate and compensate this nuisance parameter.

In addition, blockage effects on the ZS and readers are analyzed and simulated with different network arrangements. CRLB with blockage effect and crystal clock time drift is derived. A weighting function is proposed to reduce the localization error and is robust to different network layouts.
Ranging operations in laboratory are demonstrated. A tag moves inside the laboratory to capture the ranging signals. Subsequently, the proposed TDOA localization method as well as the weighting function is applied. Both results showed reasonable performance of the proposed TDOA localization method and the weighting function. In addition, it further proves the success of the coherent UWB transceiver prototype design.

In this chapter, a description on how to perform localization to detect all the tags inside a network with a single cell operation is given. If the network size is increased, single cell operation might not be able to provide localization coverage for the entire area.
Chapter 5

Scalable Localization and Tracking System

5.1 Introduction

This chapter propose a time scheduling transmission scheme for multi cells indoor localization. Multiple cells are used in order to cover the entire network which is larger than the radio range of a single cell. In order to have good ranging performance, ranging packets need to be transmitted among wireless sensors without collision. This is especially important in an indoor environment because there are a large number of sensors operating in a compact area. However, a typical wireless sensor network usually faces packet collision problem if transmission among the sensor nodes are not properly coordinated. Therefore, a good scheduling scheme is needed in order to avoid collision during packet transmission.

In this chapter, cellular transmission scheme is first described. Then a time scheduling transmission method for the anchor nodes which are also known as readers to transmit their ranging packets without collision is proposed. The scheme is scalable to track a large number of tags simultaneously. All the readers and tags in the entire network operate with the same frequency but transmit in different time slots to perform ranging.
Secondly, the TDOA localization method is modified to suit this time scheduling method so that it is capable of performing both offline and online localization for a multi-cell indoor network. TDOA method is preferred because it requires lesser number of packet transmission in order to save power consumption and processing time. If every sensor needs to perform Two Way Ranging (TWR), the number of packet transmission will be increased. Furthermore, Received Signal Strength (RSS) is not considered because RSS is greatly affected in the indoor environment by multipaths. The proposed TDOA method comes from chapter 4 where it allows the sensors to transmit periodically so that the tags can record the arrival time of these ranging packets. In this way, continuous packet transmission with minimum interferences can be achieved. When a tag crosses from one zone to another zone, it is assigned a new zone supervisor in that particular zone.

Finally, Kalman filtering is used to correct and predict the localization estimation of the tags. It discusses about the time and measurement update cycle and the related equations at each process. Next, Kalman filter estimation is applied into the TDOA localization so that it can help to smooth the trajectory taken by the tag.

5.2 Proposed Transmission Scheme

Traditional transmission method between sensors in a wireless cellular network is to use frequency reuse method [107]-[109]. A typically cellular network uses a number of hexagon cells layout to provide coverage to that area and a number of cells are combined to form a cluster. Cells within a cluster are assigned with different groups of frequency band and sensors within a cell is assigned with different frequencies so that there are no packets collision during transmission. Co-channel cells are defined as cells that can use the same set of frequency bands provided they are within certain co-channel reuse ratio. One of the drawbacks for this method is that it requires a certain number of dif-
ferent frequency bands and for them to be wide enough to accommodate a large number of sensors operating inside a cell. This method is not very scalable particularly for an indoor environment if the reuse distance is not large enough. In addition, the requirement for a frequency band large enough to cover all the sensors in a particular cell is demanding.

Reference [109] shows a variable bandwidth scheme using time-frequency slot allocation. Different frequencies is assigned to different sensors inside a group and the sensors in different groups can transmit at the same time. In addition, neighboring clusters within the reuse factor cannot have the same time-frequency slots and there is frequency guard band to separate sensors in order to minimize interferences. Reference [110] shows sensors use different codes to reduce interferences but transmit with the same frequency. However, the number of orthogonal codes are limited if there are a large number of sensors within a group.

Therefore, a good scheduling scheme is required for packet transmission especially in an indoor environment. Ranging packets between sensors need to be transmitted without collision in order to have a successful ranging operation. In addition, system cost must be kept low because of the high volumes of tags that will be operating inside a wireless sensor network.

As shown in Figure 5.1, the proposed scheme uses four hexagon zones as an illustration to cover the entire network. The number of hexagons, shapes of the zones and their layout may be varied using the same proposed transmission coordination scheme.

The area of the network is 70m x 90m x 20m. The anchor node in the middle of the zone is denoted as the Zone Supervisor (ZS) and the remaining anchors at the corners of the zone are known as readers. The ZS’s role is to maintain operational order and coordination for the readers and tags inside its zone. For reference, every ZS in a zone is denoted as $ZS_z$ where $z$ represents the zones inside the network. The readers are indexed
as $R_{z,j}$ meaning the $j^{th}$ reader inside the $z^{th}$ zone. In Figure 5.1, $z = \{1, 2, 3, 4\}$ and $j = \{1, 2, \ldots, J\}$ where $J$ is the number of readers and equals to six for every zone.

**5.2.1 Zone By Zone (ZBZ) Transmission Sequence**

Figure 5.2 shows a Zone By Zone (ZBZ) transmission sequence. The ZS of the first zone will transmit and followed by the first reader in the zone. This continues until the $j^{th}$ reader in the first zone. Subsequently, the ZS of the second zone will transmit followed by the first reader in the second zone. This continues until the $j^{th}$ reader in the second zone. The process continues till the last zone. In this way inter-cell interferences and interferences between ZS and readers are avoided. This method also allows neighboring zones to operate side by side with the same frequency.

Transmission between tags, ZS and readers are wireless and only the ZS is wired connected to the central processor where information are passed to a central processor to perform localization computation. Wired connection between ZS and central processor is preferred because this gives better security to the information transferred from ZS to the central processor. Assumed that all the ZS are time synchronized to the central pro-
cessor and readers are able to time synchronize to their respective ZS. However, detailed method on how to allow ZS to synchronize to the readers are discussed in Chapter 4. Finally, the system can use different localization method and in this case, Time Difference of Arrival (TDOA) localization based method is used. It is further assumed that a frame transmission rate is much faster (within 1ms) than the movement of the tag.

Figure 5.3 shows a timing diagram for the ZBZ transmission scheme between ZS, readers and tags across different zones. The transmission starts with the ZS in the first zone transmitting and records its transmit ranging counter value as indicated by $TRC^ZS_z$. This is interpreted as Transmit Ranging Counter value from $ZS_z$ where $z$ is the number of zones covering the network, which in this case, $z = \{1, 2, 3, 4\}$. This ranging packet will be received by the readers and tags and they will record their ranging counter values when the ranging packet arrives as $RRC^ZS_z^{R_z,j}$ and $RRC^ZS_z^{T_z,k}$ respectively. This can be interpreted as Received Ranging Counter value at $R_{z,j}$ and at $T_{z,k}$ from $ZS_z$ respectively, which in this case, $j = \{1, 2, \ldots, 6\}$ and $k = \{1, 2, \ldots, K\}$ where $K$ is the number of tags inside a particular zone. Ranging packet is described here because in practice, the recorded timings are in the form of ranging counter. However, conversion of ranging counter values to timing values can be found in eqn (5.2).
After the ZS had finished transmitting, the first reader in the first zone transmits and records its transmit ranging counter values. The Transmit Ranging Counter value from $R_z$ is $TRC^{R_z,1}$. ZS and tags will record their Received Ranging Counter values when
detecting these ranging packets. The Received Ranging Counter value at ZS and tags is $RRC_{ZS,j}^{R}$ and $RRC_{T,j,k}^{R}$ respectively. The relationship between distance and ranging counter is given as

$$RC = \frac{D \times F}{c}, \quad (5.1)$$

where $RC$ is the ranging counter values, $D$ is the distance between any two devices, $F$ is the counter frequency of the measuring device. An example for converting the Transmit Ranging Counter value at $ZS_1$ and Received Ranging Counter value at $R_{1,1}$ with respect to the ranging packet arrival from $ZS_1$ to time respectively is given as

$$\tilde{t}_{ZS_1} = \frac{TRC_{ZS_1}^{TS}}{F_{ZS_1}^{TS}},$$

$$\tilde{t}_{R_{1,1}} = \frac{RRC_{R_{1,1}}^{TS}}{F_{R_{1,1}}^{TS}}, \quad (5.2)$$

where $\tilde{t}_{ZS_1}$ and $F_{ZS_1}^{TS}$ is the time measured and the counter frequency at $ZS_1$ respectively while $\tilde{t}_{R_{1,1}}$ and $F_{R_{1,1}}^{TS}$ is the time measured of the arrival ranging packet from $ZS_1$ and the counter frequency at $R_{1,1}$ respectively.

The system can perform both offline or online localization. For offline localization, the tags store all the ranging packets arrival ranging counter values transmitted by ZS and readers. This allows the central processor to retrieve all these information and perform localization computation to trace its past trajectory. This is indicated by the first portion of Figure 5.3. An example of an offline tracking application is to track a person’s past trajectory inside a building. Initially, the person will be given a pass when he exchanges at the security counter. A tag will be incorporated inside the pass to record all the related timings of the packet arrival time from ZS and readers. When this person leaves the
building, the pass is retrieved back and the control room can monitor his past trajectory while he is inside the building.

In the second portion of Figure 5.3, the online tracking mode is shown where ZS will request the particular tag of interest to transmit its most recent received ranging counter values. After receiving the responds from the tag, ZS will forward these values, its own recorded ranging counter values of packet arrival from readers and its own transmit ranging counter values to the central processor for localization processing. An example to an online tracking application will be to monitor a person’s movement inside a building from the control room in real time.

5.2.2 Sequential Transmission Sequence

Another alternative transmission sequence can also be deployed as illustrated in Figure 5.4. In this scheme, the ZS of each zone will transmit till the last ZS and followed by the first reader in the zone. This continues until to the \( j^{th} \) reader of the \( z^{th} \) zone.

![Figure 5.4: Sequential Packet Transmission Sequence](image)

Similarly to the ZBZ transmission scheme, transmission between tags, ZS and readers are wireless and only the ZS is wired connected to the central processor where information are passed to a central processor to perform localization computation. It is also assumed that all the ZS are time synchronized to the central processor and readers are able to time synchronize to their respective ZS.

Figure 5.5 shows a timing diagram between ZS, readers and tags across different zones. The transmission starts with the ZS in the first zone transmitting and records its transmit ranging counter value as indicated by \( TRC_{ZS}^{ZS} \). This ranging packet will
be received by the readers and tags and they will record their ranging counter values when the ranging packet arrives as $RRC_{{R_{Z_j}}}^{Z_{S_{Z}}Z}$ and $RRC_{{T_{Z_k}}}^{Z_{S_{Z}}Z}$ respectively. After all the ZS had finished transmitting, the first reader in every zone will transmit. These readers record their transmit ranging counter values. The Transmit Ranging Counter value from $R_z$ is $TRC_{}^{R_{Z_j}}$. ZS and tags will record their Received Ranging Counter values when detecting these ranging packets. The Received Ranging Counter value at ZS and tags is $RRC_{{Z_{S_{Z}}}}^{R_{Z_j}}$ and $RRC_{{T_{Z_k}}}^{R_{Z_j}}$ respectively.

A minor limitation for Sequential transmission scheme is the number of time slots required to cater for all the sensors to transmit. ZS and readers are only required to transmit ranging packet which is only a few bytes of data and the duration for these ranging packets is short. However, if the number of readers or zones increases, then the duration to localize a tag will be longer than ZBZ transmission scheme. For example, if the tag is located in zone 4 as shown in Figure 5.1, then the duration to localize it is the same whether it is Sequential transmission scheme or ZBZ transmission scheme. However, if the tag is located in zone 1, then the duration to localize it will be longer for Sequential transmission scheme as compared to ZBZ transmission scheme.

Conversely, one can group these four zones together to form a cluster so that there are multiple clusters to cover a wider network. In this way, the Sequential transmission scheme can take place simultaneously in every cluster. Therefore, the duration to localize a tag still lie within the transmission timing across the four zones.
5.3 Simulation Results

Figure 5.6 shows a 3-D area map using TDOA localization based method for the ZBZ transmission scheme. There are a total of 1008 tags being placed inside the network and zero mean Gaussian noise of standard deviation $\sigma = 0.95$ns for every 10m is used. The
TDOA values are computed using eqn (4.4) and the locations of the tags are computed using eqn (4.29) averaged over 1000 realizations. Local clocks at the ZS, readers and tags are assumed with crystal clock time drift uniformly distributed between -20ppm to +20ppm. The red circle represents the actual tags positions and the blue cross represents the estimated tags positions. Green line joining the cross and circle represents the position error. The longer the green line, the larger the position error. For clarity, Figure 5.7 shows the top view of the area map of Figure 5.6. Table 5.1 shows the coordinates for the ZS and readers in the network area.

![3-D area map using TDOA localization based method for ZBZ transmission scheme](image)

Figure 5.6: 3-D area map using TDOA localization based method for ZBZ transmission scheme

Figure 5.8 shows the probability distribution of the absolute localization error ranges for the 1008 tags with a maximum error of 2.66 meters which is outside the zones covering areas. Large localization estimation errors are expected to occur for those tags near the edges of the hexagon zones or outside the zones covering areas as compared with those tags inside the zones. However, less than 1% of the nodes have an error of more than 1m and the remaining 99% have less than 1m error.
5.4 Kalman Filtering

After getting the initial estimation of the tags’ positions, sometimes, it is desired to improve the results especially when tracking a trajectory. Therefore, Kalman filtering can be used at the central processor to help to smoothen the trajectory. Kalman filter is used to estimate the instantaneous state of a linear dynamic system perturbed by white noise. The Kalman filter uses the probability distribution of its estimation errors
to assess its performance as a function of the design parameters of an estimation system, such as

- the types of sensors to be used,
- the locations and orientations of the various sensor types with respect to the system to be estimated,
- the allowable noise characteristics of the sensors,
- the prefiltering methods for smoothing sensor noise,
- the data sampling rates for the various sensor types,
- the level of model simplification to reduce implementation requirements.

The general relationship between the Kalman filter and the system is shown in Figure 5.9 [112].

From Figure 5.9, the steps for a Kalman filter operations can be listed as
1. Find the *a priori* value of the covariance matrix, $P_k^-$, of the estimation uncertainty as a function of the previous *a posteriori* value defined by $P_{k-1}$, $Φ_{k-1}$ and $Q_{k-1}$. $P_{k-1}$ is the *a posteriori* value of the covariance matrix, $Φ_{k-1}$ is the state transition and $Q_{k-1}$ is the process noise covariance respectively at time $k - 1$.

2. Find the Kalman gain matrix, $K_k$ using the computed $P_k^-$, the measurement sensitivity $H_k$ and measurement noise covariance $R_k$.

3. Find the updated error covariance $P_k$ using the computed values of $K_k$ and $P_k^-$.  

4. Finally, determine the successive state estimate update $\hat{x}_k$ recursively using the computed values of $K_k$, the initial given estimate $\hat{x}_0$ and the input data $z_k$.

---

**Figure 5.10: Kalman Filter cycle**
The Kalman filter will perform a time update where it projects the current state and error covariance estimates to obtain \textit{a priori} estimates for the next time. Secondly, the Kalman filter performs a measurement update to feedback the measurement into \textit{a priori} estimate to get an improved \textit{a posteriori} estimate. This process of Kalman filter can be shown in Figure 5.10.

Reference [112] gives a very detailed explanation on the steps and equations of using Kalman filter. Reference [113] proposes to feed the past locations estimate and sector information into Kalman filter to calculate and improve the current location estimate for a cellular network so that the requirement for a minimum number of range measurements can be relaxed.

In the simulations, zero mean Gaussian noise with a standard deviation of 1ns is assumed, the crystal clock time drift varies between -20ppm to +20ppm from an uniform distribution. The sampling interval is 1 second ($\Delta t = 1$) per update. Process error and measurement is assumed with a standard deviation of 0.0055 and 0.8 respectively. The matrix $A$ and $B$ are given as

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
The position measurement estimates $z_k$ comes from the localization system as described eqn (4.29) in chapter 4 where TDOA localization based method is used to locate the tags with the additional capability that it is able to compensate crystal clock time drift. The layout of the environment is similar to Figure 4.3.

Figure 5.11 shows the TDOA based localization estimation using eqn (4.4) in red cross compared with the Kalman filter estimation in blue solid line. Due to the symmetry characteristics, Figure 5.11 only shows half of the network area. The tag moves around the circumference of the hexagon network as shown in the Figure 4.3 starting from the left of the graph and moves along the circumference of the hexagon and finally reaches towards the center of the network. This shows that the Kalman filter is able to smooth the trajectory taken by the tag.

Figure 5.12 compared the root mean square error between the TDOA localization based method and Kalman filter estimation with the true location. The Kalman filter and TDOA localization estimation matches closely as given by the average root mean square error of 1.2601m and 1.2625m respectively. The large error occurs at between 50 to 120 steps, when the tag starts to take a turn at the edges of the hexagon network. Therefore, large error is expected when tag starts to turn at the edges of the network as compared when they are moving in a straight line. This is because when the tag is making a turn, there is a change of velocity.
Figure 5.11: Localization estimation from TDOA computation compared with after Kalman filter process

Figure 5.12: RMSE between Kalman filter estimation and TDOA localization
As expected, Figure 5.12 shows that the error reduces towards the end of the step. This is because the tag is approaching towards the center of the network which agrees to the analysis in chapter 4 where the CRLB is the lowest at the center of the network.

5.5 Summary

In this chapter, a time scheduling method is proposed for ranging packets transmission for a multi-cell indoor localization operation without collision. This scheme allows the same frequency to operate in the entire network among the tags and anchors. In addition, frequency guard band is not required, hence it is bandwidth saving. Multiple zones can operate side by side simultaneously to track a large number of tags at the same time. This gives a scalable, simple and low cost system deployment for a large indoor network area. Two transmission schemes are discussed, namely, ZBZ and Sequential. ZBZ is more attractive especially if the tag is moving because it is able to localize a tag within a shorter duration as compared to Sequential transmission scheme. For moving tag, TDOA measurements need to be taken over a short period of time because if the duration is long, the tag might have moved to another location. A TDOA localization method is modified to suit this time scheduling scheme for both offline and online localization operation in a multi-cell indoor environment. The simulation results showed that this scheme is robust and able to perform localization within sub meter error for most of the time.

After getting the initial estimates of the tags positions, it is observed that some of the estimates are poor and therefore, there is a need to smooth the tag’s trajectory. Consequently, Kalman filtering is proposed and investigated to help to smooth a tag’s trajectory. The working principle of a Kalman filter is presented. It discusses about the time and measurement update cycle and the related equations at each process. Next, Kalman filter estimation is applied into the TDOA localization so that it can help to
smooth the trajectory taken by the tag. As the tag is walking along a straight path, the error is smaller as compared to the case when the tag is making a turn at the edge of the network. In addition, the error is reduced when the tag is moving towards the center of the network. This agrees with the analysis in chapter 4 where the CRLB is the lowest at the center of the network.
Chapter 6

Conclusion and Future Work

6.1 Conclusions

The main works of this thesis looked at ranging and localization methods from a practical point of view. It described how to compensate crystal clock time drift and unequal reply time for accurate ranging among the sensor nodes. The thesis further discussed a TDOA localization based method that allowed the anchor nodes not to be time synchronized initially and able to compensate crystal clock time drift. Error analysis and CRLB are derived and showed the comparison on the ranging accuracy between time drift compensation and no time drift compensation. Blockages effect on the sensor nodes is analyzed and compensated using a weighting function. Finally, two scheduling schemes, Zone By Zone and Sequential Transmission scheme, for ranging packet transmission in order to avoid packet collision as well as Kalman filtering on how to track sensor nodes are described.

UWB and its transmission method is described and is seen to be very suitable for ranging and localization because of its fine time resolution. The different ranging and localization methods are addressed which can be classified according to anchor based and anchor free localizations method. The scalability and accuracy issues faced in the different localization methods are discussed. Some factors that affect ranging accuracy,
particularly, crystal clock time drift problem which typically exists in a practical system design as well as some techniques used such as SDS-TWR and ADS-TWR to overcome crystal clock time drift are also described.

A new cooperative ranging method SMWR that reduced the effect of crystal clock time drift is proposed. This new approach is time efficient and performed ranging in a round robin manner so that packet collision can be avoided. SMWR performed ranging twice in a two phase approach to calculate the adjacent distances while the non-adjacent distances are calculated locally in the individual node. In this way, the number of communication between the sensor nodes is reduced, therefore, SMWR is time efficient and energy saving. Comparing between SMWR and NWTT methods, simulation results clearly showed that the errors in performing ranging by using SMWR are more robust against crystal clock time drift and the algorithm is scalable because SMWR does not need a centralized system for computation.

Then an OWR crystal clock time drift compensation method between a pair of transmitter and receiver is described. This can be used in practice if there are some limitations that required one way transmission between transmitter and receiver. The proposed OWR method reduced the effect of crystal clock time drift on ranging accuracy and worked well for both synchronized and non-synchronized clocks between the transmitter and receiver.

SDS-TWR-URT ranging method is described to reduce the ranging error when there are variations in the reply time among the sensor nodes and under the influence of crystal clock time drift. Performance analysis showed the effectiveness of SDS-TWR-URT when compared with the prior SDS-TWR method. This is because both SDS-TWR and SMWR have an assumption that all the sensor nodes reply times are approximately the same and if there is large variation, then SDS-TWR and SMWR will fail. Although SDS-TWR-URT is a point to point ranging method, it is straight forward and efficient
to incorporate SDS-TWR-URT with SMWR to compensate crystal clock time drift and unequal reply time.

Subsequently, a TDOA localization method is designed such that the effect of crystal clock time drift on ranging accuracy can be reduced. This allowed low cost devices to be deployed in the network with poor crystal clock and it is able to track a large number of tags at the same time. In addition, there is no requirement for initial time synchronization among the anchor nodes. Therefore, this allowed flexible and simple deployment of the localization system. Error analysis and Cramér-Rao Lower Bound for the proposed TDOA localization method are derived and the simulation results showed that the proposed method performed relatively close to the CRLB. Furthermore, the blockage effects on the anchor nodes using different network arrangements is analyzed. CRLB with blockage effect and crystal clock time drift is derived. A weighting function is proposed to reduce the localization error and is robust to different network layouts. Finally, a ranging and localization demonstration is carried out in the laboratory where the proposed localization based method as well as the weighting function are applied. The robustness of the method is shown to reduce the localization error in the presence of blockages effect.

Moving from single cell to multiple cell operation, two time scheduling methods for ranging packets transmission for a multi-cell indoor localization operation without collision are proposed. This allowed a common operating frequency to be used in the network which is bandwidth saving as well as no frequency guard band is required. The system deployment is scalable, simple and low cost that allows multiple zones to operate side by side to track a large number of mobile nodes or tags simultaneously. TDOA localization method is modified to suit this time scheduling scheme for both offline and online localization operation in a multi-cell indoor environment. TDOA measurements are needed to be taken over a short period of time because if the duration is long, the
tag might have moved to another location. Simulation results showed that the scheme is robust and able to perform localization within an error of sub meter for most of the time.

Finally, the working principle of the Kalman filter on how it can correct and predict the location estimation is introduced. Kalman filtering is applied to the proposed TDOA localization based method that aimed to smooth the trajectory taken by the tag. Simulation results showed that as the tag walked on a straight path, the error is less compared to when the tag made a turn at the edge of the network. In addition, error is reduced when the tag moved towards the center of the network where the CRLB is lowest in the center of the network.

6.2 Future Works

6.2.1 Localization system using SMWR

At present, SMWR is only able to determine all the pairwise distances among the sensor nodes wirelessly. Therefore, one of the ongoing works is to extend SMWR with MDS to determine the relative locations of the sensor nodes in a wireless sensor network and to use MDS to transform the relative map into an absolute map. This is to analyze how anchor nodes positions inaccuracy affects localization error on tags.

6.2.2 Improved SMWR with Lesser Ranging Time and Better Accuracy

It is desirable to have a common ranging algorithm when there is a need to determine, firstly, the distances among the anchor nodes, and secondly, the distances between the anchor nodes and tags. In this way, there is no requirement to switch between ranging algorithms when the system is used in different deployments. Motivated by [114] and [115] where it discusses how to reduce the ranging time by using TWR and replying...
multiple packets to a single ranging request. However, the method is applied only to a point-to-point ranging scheme between any two nodes and is inefficient if applied directly to a large sensor network with a lot of nodes.

Therefore, current work tries to improve the proposed SMWR with SDS-TWR-URT methods to reduce ranging time and improve accuracy using a single ranging algorithm to determine the distances among the anchor nodes only, and between the anchor nodes and tags.

6.2.3 Ways to improve ranging and localization accuracy from system point of view

Motivated by the studies of GDOP, methods to improve ranging and localization accuracy from system point of view are important. Initial works carried out show that using the proposed TDOA localization based method as described in Chapter 4, the maximum height of the placement of the anchor nodes is proportional to the largest x or y coordinate of the network. The minimum height of the placement of the anchor nodes is proportional to a quarter of the largest x or y coordinates of the network. These initial findings show an interesting result and form the basis for further examination to determine this relationship and derived expressions that relate the height placement of the anchor nodes with the size of the network where the system is deployed.
Author’s Publications

Journal Papers:


Conference Papers:


Appendix A

Derivation of SMWR non-adjacent mean square error

In this appendix, it derived the SMWR non-adjacent mean square error as given by

\[ \sigma_{Dij}^2 = \sum_{i=1}^{N-2} \sum_{j=i+2}^{N} \frac{1}{4} (3(j - i)\sigma^2 + \sigma^2). \]  \hspace{1cm} (A.1)

It shall analyze the above equation in various parts. First it expands \(\sum_{i=1}^{N-2} \sum_{j=i+2}^{N} j\). Before that, there is a need to take note of these three expansions:

\[ \sum_{i=1}^{N-2} i^2 = \frac{(N-2)(N-1)(2N-3)}{6}, \quad \sum_{i=1}^{N-2} i = \frac{(N-2)(N-1)}{2} \quad \text{and} \quad \sum_{i=1}^{N} i = \frac{N(N+1)}{2}. \]

Next, using (A.2) to expand \(\sum_{i=1}^{N-2} \sum_{j=i+2}^{N} j\).

\[ \sum_{i=1}^{N-2} \sum_{j=i+2}^{N} j = \sum_{i=1}^{N-2} \sum_{j=1}^{m} j \hspace{1cm} (A.2) \]

\[ = \sum_{i=1}^{N-2} \left( \sum_{j=1}^{N} 1 - \sum_{j=1}^{m-1} j \right) \quad \text{where} \quad m = i + 2 \]

\[ = \sum_{i=1}^{N-2} \left( \frac{N(N+1)}{2} - \frac{m(m - 1)}{2} \right) \]

\[ = \sum_{i=1}^{N-2} \left( \frac{N(N+1)}{2} - \frac{i^2 + 3i + 2}{2} \right). \]

Next, using (A.2) to expand \(\sum_{i=1}^{N-2} \sum_{j=i+2}^{N} (j - i)\).
\[
\sum_{i=1}^{N-2} \sum_{j=i+2}^{N} (j - i) = \sum_{i=1}^{N-2} \left[ \frac{N(N + 1)}{2} - \frac{i^2 + 3i + 2}{2} \right] - \sum_{i=1}^{N-2} i(N - i - 1) \quad (A.3)
\]

\[
= \sum_{i=1}^{N-2} \left[ \frac{N(N + 1)}{2} - (N + \frac{1}{2})i + \frac{i^2}{2} - 1 \right]
\]

\[
= \frac{(N - 2)(N^2 + N - 2)}{2} + \frac{1}{12}(N - 2)(N - 1)(2N - 3)
\]

\[
- \frac{(N + \frac{1}{2})(N - 2)(N - 1)}{2}
\]

\[
= \frac{N - 2}{12} [6(N^2 + N - 2) + (2N^2 - 2N - 2N + 3)
\]

\[
- 6(N^2 - \frac{1}{2}N - \frac{1}{2})]
\]

\[
= \frac{N^3 - 7N + 6}{6}.
\]

Now, insert (A.3) into (A.1) and expands to get the following

\[
\sigma_{Dij}^2 = \sum_{i=1}^{N-2} \sum_{j=i+2}^{N} \frac{1}{4} (\frac{3}{2}(j - i)\sigma^2 + \sigma^2)
\]

\[
= \frac{1}{4} \left[ \frac{3}{2} \sigma^2 \frac{N^3 - 7N + 6}{6} + \frac{N^2 - 3N + 2}{2} \sigma^2 \right]
\]

\[
= \frac{N^3 + 2N^2 - 13N + 10}{16} \sigma^2.
\]
Appendix B

Derivation of estimated time drift error

\[ 1 + \tilde{\delta}_T^k = \frac{\tilde{R}_{T_k,ZS}(m) - \tilde{R}_{T_k,ZS}(m - 1)}{\omega} \]  
\[ = \frac{1}{\omega} [(d_{T_k,ZS}(m) + n_1)(1 + \delta_T^k) - (d_{T_k,ZS}(m - 1) + n_2)(1 + \delta_T^k)] \]
\[ = \frac{1}{\omega} [(d_{T_k,ZS}(m) + n_1 + d_{T_k,ZS}(m)\delta_T^k) + n_1\delta_T^k - (d_{T_k,ZS}(m - 1) + n_2 + d_{T_k,ZS}(m - 1)\delta_T^k + n_2\delta_T^k)] \]
\[ = \frac{1}{\omega} [\omega + n_1 - n_2 + \omega\delta_T^k + (n_1 - n_2)\delta_T^k] \]
\[ = (1 + \delta_T^k) + \frac{1}{\omega}(n_1 - n_2)(1 + \delta_T^k), \]

where \( n_1 \) and \( n_2 \) are the gaussian noise introduced in the measure distance in the \( m^{th} \) and \( (m - 1)^{th} \) frame respectively.

Therefore, the estimated drift error is given by

\[ \epsilon_{\delta_T^k} \triangleq \tilde{\delta}_T^k - \delta_T^k = \frac{1}{\omega} (n_1 - n_2)(1 + \delta_T^k). \]  

(B.2)
Appendix C

Derivation of TODA error variance with time drift compensation

\[ \hat{\tau}_{R_j,ZS}(m) - \tau_{R_j,ZS}(m) \approx \left(1 - \frac{\epsilon_{\delta_T}}{1 + \delta_T}\right)(n_{TR_j} - n_{TZ}) - \frac{\epsilon_{\delta_T}}{1 + \delta_T}\Omega \] (C.1)

\[(\hat{\Delta}_j(m) - \Delta_j(m)),\]

where \( \Omega \triangleq \tau_{R_j,ZS}(m) + j\gamma + \Delta_j(m) \).

\[ E\{\hat{\tau}_{R_j,ZS}(m) - \tau_{R_j,ZS}(m)\}^2 \approx E\left\{ \left(1 - \frac{\epsilon_{\delta_T}}{1 + \delta_T}\right)(n_{TR_j} - n_{TZ}) \right. \]

\[- \frac{\epsilon_{\delta_T}}{1 + \delta_T}\Omega + (\hat{\Delta}_j(m) - \Delta_j(m)) \right\}^2 \] (C.2)

\[ \approx E\{(n_{TR_j} - n_{TZ}) - \frac{\epsilon_{\delta_T}}{1 + \delta_T}(n_{TR_j} - n_{TZ}) \]

\[- \frac{\epsilon_{\delta_T}}{1 + \delta_T}\Omega + (\hat{\Delta}_j(m) - \Delta_j(m)) \right\}^2. \]

Note that \( \epsilon_{\delta_T} = \frac{1+\hat{\delta}_{Tk}}{\omega}(n_{TZ}(m) - n_{TZ}(m - 1)) \), which implies that \( \frac{\epsilon_{\delta_T}}{(1+\hat{\delta}_{Tk})} = \frac{1}{\omega}(n_{TZ}(m) - n_{TZ}(m - 1)) \). Denotes \( n_{TZ}(m - 1) \) as \( n_{TZO} \) and expands the right hand side of the approximation in (C.2), it becomes

143
\[ R H S \approx (\hat{\Delta}_j(m) - \Delta_j(m)) + n_{TR} - n_{TZ} - \frac{1}{\omega}(n_{TZ} - n_{TZO})(n_{TR} - n_{TZ}) \]  
\[ \approx \left( \hat{\Delta}_j(m) - \Delta_j(m) \right) + n_{TR} - n_{TZ}(1 + \frac{\Omega}{\omega}) - \frac{1}{\omega}n_{TZ}n_{TR} + \frac{1}{\omega}n_{TZ}^2 \]
\[ + \frac{1}{\omega}n_{TZO}n_{TR} - \frac{1}{\omega}n_{TZ}n_{TZO} + \frac{1}{\omega}n_{TZO}\Omega \]
\[ \approx \left( \hat{\Delta}_j(m) - \Delta_j(m) \right)^2 + n_{TR}^2 + n_{TZ}^2(1 + \frac{\Omega}{\omega})^2 + \frac{1}{\omega^2}n_{TZ}^2n_{TR}^2 + \frac{1}{\omega^2}n_{TZ}^4 \]
\[ + \frac{1}{\omega^2}n_{TZO}n_{TR}^2 + \frac{1}{\omega^2}n_{TZ}^2n_{TZO} + \frac{1}{\omega^2}n_{TZO}\Omega^2. \]

Taking the variance, it becomes

\[ E\{RHS\}^2 = \sigma^2_{ZR_j} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \frac{1}{\omega^2}\sigma^2_{TZ}\sigma^2_{TR} \]
\[ + \frac{1}{\omega^2}E\{n_{TZ}^4\} + \frac{1}{\omega^2}\sigma^2_{TZ}\sigma^2_{TR} + \frac{1}{\omega^2}\sigma^2_{TZ} + \frac{1}{\omega^2}\sigma^2_{TZ}\Omega^2 \]
\[ = \sigma^2_{ZR_j} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \frac{1}{\omega^2}\sigma^2_{TZ}\sigma^2_{TR} + \frac{3}{\omega^2}\sigma^2_{TZ} \]
\[ + \frac{1}{\omega^2}\sigma^2_{TZ}\sigma^2_{TR} + \frac{1}{\omega^2}\sigma^4_{TZ} + \frac{1}{\omega^2}\sigma^4_{TZ}\Omega^2 \]
\[ = \sigma^2_{ZR_j} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \frac{4}{\omega^2}\sigma^2_{TZ} + \frac{2}{\omega^2}\sigma^2_{TZ}\sigma^2_{TR} + \frac{1}{\omega^2}\sigma^2_{TZ}\Omega^2 \]
\[ = \sigma^2_{ZR_j} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \sigma^2_{\hat{\tau}_{R_j},ZS} + \frac{4}{\omega^2}\sigma^2_{TZ} + \frac{2}{\omega^2}\sigma^4_{TZ} + \frac{2}{\omega^2}\sigma^2_{TZ}\sigma^2_{TR}. \]

If assuming that \( \sigma^2_{TR} = \sigma^2_{TZ} = \sigma^2_{ZR_j} = \sigma^2 \), it becomes

\[ E\{\hat{\tau}_{R_j,ZS}(m) - \tau_{R_j,ZS}(m)\}^2 = 2\sigma^2 + \sigma^2(1 + \frac{\Omega}{\omega})^2 + \frac{4}{\omega^2}\sigma^4 + \frac{2}{\omega^2}\sigma^4 \]
\[ + \frac{1}{\omega^2}\sigma^2\Omega^2 \]
\[ = 2\sigma^2 + \sigma^2(1 + \frac{2\Omega}{\omega} + (\frac{\Omega}{\omega})^2) + \frac{6}{\omega^2}\sigma^4 + \frac{(\Omega}{\omega})^2\sigma^2 \]
\[ = \sigma^2 \left( 3 + 2\frac{\Omega}{\omega} + 2 \left(\frac{\Omega}{\omega}\right)^2 \right) + \frac{6}{\omega^2}\sigma^4. \]
References


[22] N. Correal and F. Martin, “IEEE 802.15.4a Relative Location”, Contribution 802.15-04-0228-01-004a to the IEEE 802.15.4a Ranging Subcommittee May 2004.


[26] V. Brethour, “Two way ranging using tracking information to manage crystal offsets”, contribution 802.15-05-0336-00-004a IEEE 802.15.4a Ranging Subcommittee, June 2005.


[28] V. Brethour, “Tracking systems to support two way ranging”, contribution 802.15-05-0342-00-004a IEEE 802.15.4a Ranging Subcommittee, June 2005.


[48] I. Reede, “Location for 802.15.4a UWB Phy radio systems”, Contribution to 802.15-06-0033-00-004a to the IEEE 802.15.4a Ranging Subcommittee, Jan 2006.


