INVESTIGATION INTO MECHANICALLY TUNABLE ONE-DIMENSIONAL PHOTONIC CRYSTAL

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ABSTRACT

Recent advances in photonic crystals subject focus mostly on optical properties of rigid structures of two-dimensional or three-dimensional photonic crystals (2-D or 3-D PhCs). While many interesting applications, such as waveguide bends, resonant cavity, add-drop filters, etc., are benefited from the PhCs, tuning optical properties of the PhCs is still a challenging issue. All tuning methods yield small alteration of photonic band gaps (PBGs). An exception from those is the mechanical tuning that promises the largest tuning of the PBG. However, the mechanical tunings for 2-D PhCs and 3-D PhCs have only been conceptualized. Surprisingly, mechanical tunability of one-dimensional photonic crystals (1-D PhCs) has not been investigated much although 1-D PhCs also possess interesting PBG effects. Moreover, simplicity of the 1-D PhC structure encourages easier fabrication and characterization. Therefore, an attempt to demonstrate an application of tunable 1-D PhCs is addressed in this work.

A mechanically-tunable PBG polarization splitter was proposed. The device utilized the PBG effect at inclined incidence to separate transverse-electric (TE) mode from transverse-magnetic (TM) mode. Silicon and polydimethylsiloxane (PDMS) were chosen for constructing the tunable 1-D PhC for the device. An improved plane wave expansion method and a transfer matrix method were employed to calculate PBGs of the PhC. The matrix method was further employed to design a finite tunable 1-D PhC at Brewster’s angle. Mechanical tuning of the PBG by varying the thickness of PDMS was studied using the method. Transmitted power and polarization degree were calculated. It was found that periodicity of three was appropriate for constructing the tunable 1-D PhC. The designed silicon thickness was 6 μm at operating frequency of 3.5 THz, whereas the range of PDMS thickness was 6-12 μm so that transmittance of TE mode could be varied from 0 to unity. Finite-difference time-domain simulation yielded consistent results.

To achieve the mechanical tuning, a thermal microactuator was designed and simulated. Microfabrication processes were developed for the tunable 1-D PhC device. The processes aimed at fabricating together both the 1-D PhC and the
microactuator monolithically. There were four major tasks of fabricating the whole 1-D PhC device: i) to fabricate suspended silicon structure, ii) to fabricate electric heater for device activation, iii) to fill silicon plates with PDMS in order to create the tunable PhC, and iv) to fill the silicon structure of the microactuator with SU-8. A fabricated 1-D PhC device was selected for electromechanical testing. The heater resistivity was found to be $6.0 \times 10^{-8}$ $\Omega$-m. Compression distance generated on the 1-D PhC was 0.405 $\mu$m at the maximum heater voltage of 1.989 V, which agreed with the finite-element model. At the maximum operating voltage, maximum temperature of the PDMS was 54.36 °C, and that of SU-8 was 233.75 °C. Both temperatures were less than the glass transition temperatures and the degradation temperatures of the materials.

In conclusion, the mechanically-tunable PBG polarization splitter was designed. The prototype was fabricated and tested electromechanically. To obtain the workable device, the microfabrication should be repeated and refined for the finalized device design.
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List of Abbreviations

BOE = Buffered Oxide Etch
CCD = Charge-Coupled Device
CTE = Coefficient of Thermal Expansion
CVD = Chemical Vapor Deposition
DBR = Distributed Bragg Reflector
DFB = Distributed Feedback
DRIE = Deep Reactive-Ion Etching
FDTD = Finite-Difference Time-Domain
FEA = Finite-Element Analysis
FEM = Finite-Element Method
LPCVD = Low-Pressure Chemical Vapor Deposition
MEMS = Microelectromechanical Systems
MOEMS = Micro-optoelectromechanical Systems
MUMs = Multi-User MEMS Processes
PBG = Photonic Band Gap
PDMS = Polydimethylsiloxane
PhC = Photonic Crystal
PML = Perfectly-Matched Layer
PVD = Physical Vapor Deposition
PZT = Piezoelectric lead-Zirconate Titanate
RIE = Reactive-Ion Etching
SEM = Scanning Electron Microscope
SOI = Silicon-on-Insulator
SMA = Shape Memory Alloy
SUMMiT V™ = Sandia Ultra-planar, Multi-level MEMS Technology 5
UV = Ultra-Violet
List of Symbols

Optical Quantities

\( \vec{E} = \) Electric field
\( \vec{D} = \) Displacement field
\( \vec{H} = \) Magnetic field
\( \vec{B} = \) Magnetic induction
\( \vec{k} = \) Wave vector
\( c = \) Speed of light in free space
\( \omega = \) Frequency
\( \lambda = \) Wavelength
\( \varepsilon = \) Electric permittivity
\( \mu = \) Magnetic permeability
\( n = \) Refractive index
\( \theta = \) Refracted angle
\( \vec{r}' = \) Position vector
\( t = \) Time
\( \vec{G} = \) Reciprocal lattice vector
\( \vec{g} = \) Primitive reciprocal lattice vector
\( \hat{a} = \) Fourier expansion coefficient of electromagnetic mode
\( b = \) Fourier expansion coefficient of dielectric function
\( A = \) Area
\( d = \) Thickness
\( r = \) Reflection coefficient
\( t = \) Transmission coefficient
\( R = \) Reflectance
\( T = \) Transmittance
\( I = \) Irradiance
\( P = \) Polarization degree
Microelectromachanical Quantities

$V =$ Voltage or potential difference
$U_s =$ Potential energy in spring system
$F_s =$ Spring force
$R =$ Reaction force
$M =$ Reaction moment
$U_e =$ Electrostatic potential energy
$F_e =$ Electrostatic force
$Q =$ Heat power dissipated in resistor
$R =$ Resistance
$T =$ Temperature
$\Delta T =$ Temperature difference
$\rho =$ Resistivity
$k =$ Spring constant or axial stiffness
$k^0 =$ Flexural stiffness
$E =$ Young’s modulus
$I =$ Area moment of inertia
$EI =$ Flexural rigidity
$\nu =$ Poisson ratio
$\alpha =$ Coefficient of thermal expansion
$r =$ Aspect ratio
$t =$ Thickness
$H =$ Height
$w =$ Width
$L =$ Length
$\delta x =$ X-displacement
$\delta y =$ Y-displacement
$\delta \theta =$ Angular displacement
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CHAPTER 1 INTRODUCTION

1.1 Background

Topics of micro-optics are recently investigated worldwide not only because of new potential applications for telecommunications, futuristic optical computing, biomedical sensing, etc. but also because of the pressing needs to satisfy ever-increasing demands in its respective markets. Many types of optical microelements have been developed; however, utilizations of such microelements are always in forms of static structures. Since optical properties of many optical microelements are essentially geometry-dependent, study of dimensional change of such optical microelements can in fact be an interesting problem. Flexible optical microelements are keys that lead to new functionalities of some types of optical microelements. The study could as well be classified as a part of the rapid expanding field of adaptive optics which has useful applications, for example, in astronomy. Nonetheless, the study is rather new and still needs more developments.

1.2 Flexible One-Dimensional Photonic Crystal

The optical microelement that will be discussed henceforth in this thesis is the so-called one-dimensional photonic crystal. Although one-dimensional photonic crystal (1-D PhC) has a long history dated back in 1880’s, based on the literature review reported here, it was not recognized as a photonic crystal at that time. A problem concerning 1-D PhC was merely a classical problem of dielectric stack. Treatments of the problem involve Fresnel’s equations and ray optics and mainly aimed for power transmission and phase change [3]. Half a century later the 1-D PhC was known as Bragg reflector or Bragg mirror, which is a well-known optical element among scientists and engineers. It is called such because the behavior of light reflected from such dielectric stack is analogous to reflection of X-ray from an atomic crystal that was previously discovered by William Lawrence Bragg. The
major application of the Bragg mirror is, as the name suggests, for high-reflectivity mirror. However, the reflectivity of the Bragg mirror is frequency-dependent. Therefore, it is usually designed for a specific range of frequencies of applications. Not until around 1970-1987, when quantum theory had been well-established, physics of the classical dielectric stack was explained in quantum theory language. And, the generalization of such periodic dielectric structure into a three-dimensional periodic structure was then developed in 1987. This gives a better understanding of dielectric stack from quantum theory point of view. The periodic dielectric stack is understood as a one-dimensional photonic crystal in present days.

Similar to atomic crystal, a 1-D PhC also has a lattice constant that describes periodicity of itself. This lattice constant tells the optical characteristic of 1-D PhC. When flexible dielectric materials are used to build 1-D PhC, it is possible to reversibly alter the lattice constant by mechanical means. Therefore, the optical characteristic can be changed according to change of the lattice constant. The mechanical tuning can be, for example, compressing or stretching the 1-D PhC in the direction perpendicular to the stack. Motivation of making a novel tunable optical device, such as a tunable optical filter or a polarizer, thus arises from this concept, and the main objective of this research is to develop such device.

1.3 Micro/Nanofabrication of Tunable One-Dimensional Photonic Crystal

The size of the lattice of the 1-D PhC is usually made comparable to the wavelengths of electromagnetic waves in applications. For example, applications of 1-D PhCs in terahertz regime require the lattice constant to be around 100 μm while applications in optics requires the lattice constant to be below 1 μm. To achieve such a small structure, micro/nanofabrication technique is usually needed. The micro/nanofabrication technique makes it possible to achieve miniaturized 1-D PhCs that might be useful in telecommunications, biomedical sensing devices, or integrated optical circuits.
Since the goal of this research is to develop a mechanically-tunable optical device, the mechanical tuning method for the flexible 1-D PhC thus relies on a minute device called a microactuator. The microactuator also has to be fabricated by some micro/nanofabrication techniques. A desirable fabrication approach is to develop and implement monolithic fabrication processes that incorporate both the 1-D PhC fabrication and the microactuator fabrication.

In order to make the 1-D PhC flexible, soft polymers with desired optical properties are usually the materials of choice. Polydimethylsiloxane (PDMS) is a promising material for the task because it is soft and transparent in both optical regime and terahertz regime. However, existing conventional micro/nanofabrication techniques do not offer any easy way to fabricate patterns of PDMS together with other materials. Hence, a novel micro/nanofabrication technique also has to be investigated for the purpose.

1.4 Scope and Objectives

The main objectives of this research are to design, to simulate, and to fabricate a mechanically tunable 1-D PhC-based device. The scope of the work can be identified as follows:

- To investigate the theory of 1-D PhCs and to perform a parametric study on the variation of photonic band gaps, then to propose and design a mechanically tunable PhC-based device,
- To simulate electromagnetic behaviors of the designed 1-D PhC using a finite-difference time-domain method in order to validate the theoretical design,
- To design a microactuator according to requirements from the optical design of the 1-D PhC,
- To study behaviors of the microactuator by a finite-element analysis so as to confirm theoretical expectations,
- To develop and implement a micro/nanofabrication technique in order to fabricate the device,
• To fabricate and test the device and compare its performance with the simulation results or theoretical predictions.

1.5 Organization of thesis

This thesis is comprised of 7 chapters in total. Chapter 1 gives introduction to the research as well as the objectives and scopes of works. Chapter 2 provides a comprehensive review on 1-D PhCs and MEMS technology for microactuator development. A novel tunable one-dimensional photonic crystal device is proposed based on the literature outcome. An optical design of a tunable photonic band gap polarizer is discussed in Chapter 3. After the optical design, requirements on the mechanical tuning are obtained. Then a microactuator is selected and designed in Chapter 4. A finite-element simulation is carried out in Chapter 4 to verify the theoretical design. New microfabrication techniques necessary for building each part of the device are presented in Chapter 5. A series of microfabrication processes for making the whole device and corresponding end results are reported in the same chapter. Test results obtained from the fabricated device are provided in Chapter 6. Analyses of the results and discussions are also included in the chapter. The thesis is concluded in Chapter 7.
CHAPTER 2 LITERATURE REVIEW

In this chapter a review of an optical microelement known as the one-dimensional photonic crystal is provided. The basic theory of the photonic crystal and its applications are reviewed. The chapter ends with a proposal of a novel tunable one-dimensional photonic crystal and its tuning mechanism that might be used for integrated optics applications or medical sensing.

2.1 Photonic Crystal Theory

A photonic crystal can be defined as a periodic dielectric structure which exhibits a special optical property that prevents electromagnetic waves of a specific frequency range to enter it. The forbidden range of electromagnetic frequencies is known as the photonic band gap (PBG). The name “photonic band gap” has its origin from an “electronic band gap” analog, where electron wave functions of certain energy levels cannot exist in a crystalline material [4].

2.1.1 Early Development of the Photonic Crystal Theory

Early developments of the photonic crystal theory dated back in 1917 when Lord Rayleigh reported in an article *On the Reflection of Light from a Regularly Stratified Medium* [5]. The regularly stratified dielectric medium can be in fact classified as a one-dimensional photonic crystal (1-D PhC). However, the concept of the photonic crystal was not known at that time. In his theoretical treatment, he used classical optics involving, calculation of optical path and phase differences, to derive the coefficients of reflection and transmission. This method is complicated and lengthy.

A more powerful method of characterizing electromagnetic waves within stratified media was developed by Abelès [6-7] in 1950. In his work, a formulation of a characteristic matrix for stratified media was employed. The matrix relates electric field components and magnetic field components at one interface to those at the another interface at a distance $\delta$ away. When light is incident at an angle on an
interface, the electric field can be decomposed into two vectors; one in the plane of incidence and the other in the direction perpendicular to the plane. The incident light with only the electric field vibrating in the plane of incidence is said to be in a p-polarized state (p-wave) or a transverse-magnetic (TM) state. On the other hand the incident light with only the normal electric field component is said to be in s-polarized state (s-wave) or transverse-electric (TE) state. The behaviour of reflected light and transmitted light can be analysed individually for each of the two polarization states. And, a complete description of the reflected light and the transmitted light can be obtained by combining the reflected or transmitted waves from both polarizations together [7]. Suppose that a layer of stratified media has a refractive index $n$, a permittivity of $\varepsilon$ and a magnetic permeability of $\mu$, and that an electromagnetic wave having a free-space wave vector $k_0$ is propagating inside the layer at a refracted angle $\theta$. The characteristic matrix $M$ relates the electromagnetic fields between two interfaces of the layer such that:

$$
\begin{bmatrix}
E' \\
H'
\end{bmatrix} = M \begin{bmatrix}
E \\
H
\end{bmatrix} = 
\begin{bmatrix}
\cos(k_0n\delta \cos \theta) & -i\sin(k_0n\delta \cos \theta)/p \\
-i\sin(k_0n\delta \cos \theta)p & \cos(k_0n\delta \cos \theta)
\end{bmatrix}
\begin{bmatrix}
E' \\
H'
\end{bmatrix}
$$

(2-1)

for case of TE mode, where $p = \sqrt{\varepsilon / \mu \cos \theta}$. For case of TM mode, the same equation holds true except for $p$ being replaced by $q = \sqrt{\mu / \varepsilon \cos \theta}$. The matrix technique has advantages in designing thin film devices, 1-D PhCs included, since the field expression between a complicated stack of stratified media (thin films) can be obtained via an elementary matrix manipulation, e.g.:

$$
\begin{bmatrix}
E \\
H
\end{bmatrix} = (M_1M_2M_3...)
\begin{bmatrix}
E' \\
H'
\end{bmatrix}
$$

(2-2)

Each characteristic matrix can be obtained from the optical properties and the dimensions of each corresponding stratified medium as mentioned earlier.

Another remarkable development in the matrix method was carried out by Yeh [8]. In his technique, the optical field near an interface can be represented by a superposition of an incident plane wave and a reflected plane wave. The fields of the
electromagnetic waves in each material are visualized in Fig. 2-1. The resultant electric field can be expressed as

\[
\vec{E} = \begin{cases} 
E_i e^{-\vec{k}_i \cdot \vec{r}} + E_r e^{-\vec{k}_r \cdot \vec{r}} & , x < 0 \\
E_t e^{-\vec{k}_t \cdot \vec{r}} + E_{tr} e^{-\vec{k}_{trr} \cdot \vec{r}} & , x > 0 
\end{cases}
\]

\[= \{ [E_t e^{-i(k_{tx} x + k_{tz} z)} + E_{re} e^{-i(k_{rx} x + k_{rz} z)} ] e^{i\omega t} , x < 0 \\
[ E_t e^{-i(k_{tx} x + k_{tz} z)} + E_{rr} e^{-i(k_{rrx} x + k_{rrz} z)} ] e^{i\omega t} , x > 0 \] \tag{2-3}

where the subscripts \(i, r, t, rr\) on the wave vectors refer to incident, reflected, transmitted and incident from opposite direction as shown in Fig. 2-1. In the third line and the fourth line of (2-3), each wave vector is decomposed along the \(x\)-direction and the \(z\)-direction, and the wave vector components are subscripted with \(x\) or \(z\) in order to indicate their respective directions. Note that a wave vector in an isotropic medium is related to its free-space wave vector or frequency as:

\[k = nk_0 = n\omega/c\] \tag{2-4}

Since the electric fields are parallel to the interface, the continuity of the superposition of the fields must hold. Also, the superposition of the magnetic field components parallel to the interface must be continuous. If time and \(x\)-coordinate are taken as zero, it follows that:

\[E_i + E_r = E_i + E_{rr}\] \tag{2-5}

where summation of the parallel magnetic field components in the second continuity equation have been written in terms of their corresponding electric fields according to the following Maxwell’s equation:

\[\vec{H} = \frac{i}{\omega\mu} \nabla \times \vec{E}. \] \tag{2-6}

Equation (2-5) can be written in matrix format as:
where each matrix can be written in terms of electric permittivity, magnetic permeability and ray angle, with regard to (2-5), as follows:

\[
D_{j,s} = \begin{bmatrix}
1 & 1 \\
\varepsilon_j \cos \theta_j & -\varepsilon_j \cos \theta_j \\
\mu_j & \mu_j
\end{bmatrix}
\]  (2-8)

in which the properties subscripted by \( j \) can be replaced by properties of the incident material \( i \) or those of transmitted material \( t \). The additional subscript \( s \) stands for s-wave so as to indicate all the derivations have been perform for s-polarized (or TE) light. The matrix \( D \) is called dynamical matrix according to Yeh. In case of TM mode, the matrix is given as:

\[
D_{j,\theta} = \begin{bmatrix}
\cos \theta_j & \cos \theta_j \\
\varepsilon_j \cos \theta_j & -\varepsilon_j \cos \theta_j \\
\mu_j & \mu_j
\end{bmatrix}
\]  (2-9)

When the electromagnetic wave propagates within a homogeneous layer (for example, the field travelling within material II in Fig. 2-1), the field at distance \( \delta \) away to the left and the field at a location of interest are related by the propagation matrix \( P \) so that:

\[
\begin{bmatrix}
E_i \\
E_r
\end{bmatrix} = P \begin{bmatrix}
E_i' \\
E_r'
\end{bmatrix} = \begin{bmatrix}
\exp(k_x \delta) & 0 \\
0 & \exp(-k_x \delta)
\end{bmatrix} \begin{bmatrix}
E_i' \\
E_r'
\end{bmatrix},
\]  (2-10)
where \( k_x \) is the wave vector in the medium in the positive x-direction. Using the forgoing matrix formalism, Yeh, Yariv and Hong proposed a solution for the case of an infinite stack in [9] in 1977. Owing to the similarity of a problem of electrons in crystals in quantum theory, the concept of Bloch wave, forbidden gaps, evanescent waves, and surface waves were applied. It can be said that the concept of photonic crystal was introduced at this point. However, their theory only applied for one-dimensional case. The following reviews the theory briefly.

Based on Fig. 2-1 (TE case), consider a situation where a wave in the dark gray material (I) is incident on the left of an interface and is propagating to a location one lattice period away to the right. The fields at the original location can be expressed in terms of the dynamical matrix, the propagation matrix and the field at the distance of one lattice away via:

\[
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix}_i = D_i^{-1} D_i P_x D_i P_y \begin{pmatrix}
E_x' \\
E_y'
\end{pmatrix}_i = M \begin{pmatrix}
E_x' \\
E_y'
\end{pmatrix}_i = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{pmatrix}
E_x' \\
E_y'
\end{pmatrix}_i. \tag{2-11}
\]

The expression for dynamical matrix in (2-11) can be replaced by (2-9) for TM case. If the field is to travel though \( N \) lattice periods, the fields can easily be described by

\[
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix}_i \left( D_i^{-1} D_i P_x D_i P_y \right)^N \begin{pmatrix}
E_x' \\
E_y'
\end{pmatrix}_i = M^N \begin{pmatrix}
E_x' \\
E_y'
\end{pmatrix}_i. \tag{2-12}
\]

The reverse is also true so that it can also be written:

\[
\begin{pmatrix}
E_x' \\
E_y'
\end{pmatrix}_i = M^{-N} \begin{pmatrix}
E_x \\
E_y
\end{pmatrix}_i. \tag{2-13}
\]

In the case of the infinite stack, the field distribution can be expressed as a Bloch wave according to Bloch-Floquet theorem:

\[
E_K(x, z) = E^z_K(x) e^{-ik_yz} e^{-ik_x}, \tag{2-14}
\]

where \( E^z_K(x) \) is a periodic function having the same periodicity as that of the 1-D crystal. The wave vector \( K \) is called the Bloch wave number. Since \( E^z_K(x) \) is periodic, the following is true:
\[ E_K(x+\Delta) = E_K(x), \quad (2-15) \]

where \( \Delta \) is a lattice period. Using (2-15), (2-5) and applying the column vector representation, the following is acquired according to [9]:

\[
\begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix} = e^{-iK\Delta} \begin{pmatrix}
E_i \\
E_r
\end{pmatrix}
\quad (2-16)
\]

Substituting (2-11) into the above equation yields an eigenvalue problem:

\[
M \begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix} = e^{iK\Delta} \begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix}
\]

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix} = e^{iK\Delta} \begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix}
\quad (2-17)
\]

After solving (2-17), the Bloch wave number can be found as:

\[
K(k_z, \omega) = \frac{1}{\Delta} \cos^{-1}\left(\frac{1}{2}[A+D]\right)
\quad (2-18)
\]

which yields a condition for real propagating Bloch wave number \( K \) when

\[
\frac{1}{2}(A+D) < 1.
\quad (2-19)
\]

On the other hand, the condition for evanescent wave is when

\[
\frac{1}{2}(A+D) > 1
\quad (2-20)
\]

The frequency ranges that satisfy the evanescent condition are the so-called **forbidden bands**. The frequencies which lie exactly at \( (A+D)/2 = 1 \) are called **band edges**. Note that the Bloch wave number depends on both frequency and the parallel wave vector which are conserved through every layer.

In year 1994, a matrix method developed by Lekner was proposed [10]. The matrix links electric fields and its derivatives from one location to those at another location. It has only real matrix elements and thus can be computed faster as compared to the two aforementioned methods whose matrix elements are complex. This is because
four multiplication operations and two addition operations must be performed when
two complex numbers are multiplied, i.e., \((A_1+iB_1)(A_2+iB_2)\) where \(A, B\) are some
real numbers and \(i\) is equal to \(\sqrt{-1}\). On the other hand, only one multiplication
operation is needed for multiplication of two real numbers. For an isotropic and
homogeneous layer, the matrix links electric field and its derivative at location \(A\) to
those at location \(B\) of s-wave (TE mode) as:

\[
\begin{pmatrix}
E_B \\
E'_B
\end{pmatrix}
= \begin{pmatrix}
\cos \delta & q^{-1} \sin \delta \\
-q \sin \delta & \cos \delta
\end{pmatrix}
\begin{pmatrix}
E_A \\
E'_A
\end{pmatrix}
= M \begin{pmatrix}
E_A \\
E'_A
\end{pmatrix};
\]

(2-21)

where the prime symbol indicates spatial derivative; \(q\) is the wave vector normal to
the layer, and \(\delta\) is the phase difference equal to \(qt\) with \(t\) being the thickness of the
layer. In case of p-wave (TM mode), the factor \(q, q^{-1}\) in front of the geometric
functions in (2-21) is replaced by \(Q = q/\varepsilon\) and \(Q^{-1} = q^{-1}/\varepsilon\) respectively. For an
infinitely periodic stratified structure consisting of two different isotropic and
homogeneous layers, the layer matrix becomes

\[
\begin{pmatrix}
E_B \\
E'_B
\end{pmatrix}
= \begin{pmatrix}
M_1 & M_2
\end{pmatrix}
\begin{pmatrix}
E_A \\
E'_A
\end{pmatrix}
= M_{\text{unit}} \begin{pmatrix}
E_A \\
E'_A
\end{pmatrix};
\]

(2-22)

where the subscript 1 and 2 indicates that the layer matrix given in (2-21) is written
for each of the two materials constituting the structure. The condition which dictates
the band gap of the structure is given by magnitude of trace of the effective matrix
\(M_{\text{unit}}\). If the magnitude of the trace of the matrix exceeds 2, the band gap occurs.

2.1.2 The Photonic Crystal Theory

In year 1987, Eli Yablonovitch and Sajev John independently proposed a similar
theoretical concept which later fully developed into the photonic crystal theory [11-
12]. The two theories are the generalized version of the foregoing theories of
infinitely periodic stack of dielectric. Particularly, the theories suggest that the
concept of band gap is not restricted only to one-dimensional problems of infinite
stack of dielectric layers. Two- and three-dimensional periodic structure also possess
electromagnetic band gaps. Ultimately, when a three-dimensional periodic structure
possesses a band gap, it means that electromagnetic wave is not allowed to propagate
through the structure in any direction. This is different from the one-dimensional case where the band gap usually prevents propagation only in the direction normal to the stack where periodicity prevails. Yablonovitch pointed out the existence of electromagnetic band gap in three-dimensional periodic dielectric structures that can inhibit spontaneous emission. In his paper in 1987 lies an important statement that marks the concept of photonic crystal [11]:

“We can anticipate, then, that full three dimensional spatial periodicity of $\lambda/2$ in the refractive index can result in a forbidden gap in the electromagnetic spectrum near the wavelength $\lambda$, irrespective of propagation direction, just as the electronic spectrum has a band gap in crystals. If the electromagnetic band gap overlaps the electronic band edge by at least a few kT in energy, then electron-hole radiative recombination will be severely inhibited.”

(Yablonovitch, 1987, p.2059)

Spontaneous emission is the major source of deficiency in solid-state devices, such as semiconductor lasers, heterojunction bipolar transistor, and solar cells. By creating such periodic structure surrounding those devices, tremendous improvement in performance of the devices can be expected. On the other hand, Sajev’s theory pointed out that a strong localization of photon can be found in certain disordered dielectric structures. The following subsection is dedicated to the investigation of present photonic crystal theory which is later used to design optical device in this report.

According to the guideline of photonic crystal theory in [4], the electromagnetic mode inside a periodic dielectric structure – a photonic crystal – can be represented by a plane wave modulated by a periodic function having the same periodicity as that of the structure. Such an electromagnetic mode, or the field distribution, is called a Bloch state. The modulating periodic function is comprised of an infinite set of plane waves each having its wavelength of size equal to or a fraction of the structure’s
period. A Fourier series can be used to represent such a periodic function [13]. Either electric field or magnetic field can be used to characterize the electromagnetic mode. However, working with magnetic field is technically simpler and thus it is preferred in the context of photonic crystal theory. The following shows how to derive an eigenvalue problem based on Fourier series solution according to [13].

The Maxwell’s equations in differential forms for lossless, isotropic, dielectric materials with no electrical source and free charge read:

\[ \nabla \cdot \mathbf{D} = 0 \]  
\[ \nabla \cdot \mathbf{B} = 0 \]  
\[ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \]  
\[ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0 \]

The relationship between electric field \( \mathbf{E} \), and displacement field \( \mathbf{D} \), and that between magnetic field \( \mathbf{H} \), and magnetic induction \( \mathbf{B} \), are given by:

\[ \mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_r \varepsilon_0 \mathbf{E} = (1 + \chi) \mathbf{E} \]  
\[ \mathbf{B} = \mu \mathbf{H} = \mu_r \mu_0 \mathbf{H} \]

where \( \varepsilon, \varepsilon_0, \varepsilon_r \) and \( \chi \) are permittivity of a medium, permittivity of free space, dielectric constant (or relative permittivity) and electric susceptibility, respectively. The factors, \( \mu, \mu_0 \) and \( \chi_m \) in (2-28) are permeability of a medium, permeability of free space and magnetic susceptibility, respectively. It is assumed that the dielectric materials that constitute a photonic crystal are non-dispersive, so that the permittivity is not a function of frequency. The materials are further assumed to be isotropic so that the electric susceptibility and magnetic susceptibility are independent upon direction of the fields. Therefore, the relative permittivity is simply a constant with respect to each material. Furthermore, if each material is assumed to be non-magnetic, it can be approximated that the magnetic field is simply:

\[ \mathbf{B} = \mu_0 \mathbf{H} \]
Now that the assumptions about the material characteristics of the photonic crystal are made, the following subsection will show the derivation for electromagnetic mode inside photonic crystal. The magnetic field is used to represent the electromagnetic field for simplicity reason.

**Electromagnetic Mode within a Photonic Crystal**

Substitution of (2-27) into (2-26) and then dividing both sides of (2-26) by dielectric constant function $\varepsilon_r$, (2-26) subsequently reads:

$$\frac{1}{\varepsilon_r} \nabla \times \vec{H} - \left( \frac{1}{\varepsilon_r} \right) \frac{\partial}{\partial t} (\varepsilon_r \varepsilon_0 \vec{E}) = 0$$

(2-30)

Taking the curl on both sides of (2-30) and bringing $\varepsilon_r$ out of the time derivative, (2-30) becomes:

$$\nabla \times \left( \frac{1}{\varepsilon_r} \right) \nabla \times \vec{H} - \nabla \times \frac{\partial}{\partial t} (\varepsilon_r \varepsilon_0 \vec{E}) = 0$$

(2-31)

After substituting for $\vec{B}$ from (2-28) into (2-25), the curl of electric field can be written as:

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

(2-32)

The curl operator in (2-31) can be brought into the time derivative because it does not involve the time variable. After substituting (2-32) into (2-31) to replace the curl of electric field with magnetic field, (2-29) yields:

$$\nabla \times \left( \frac{1}{\varepsilon_r} \right) \nabla \times \vec{H} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

(2-33)

Now, recall that the electromagnetic field inside a photonic crystal can be characterized by a Bloch wave. Therefore, a solution of (2-33) can be written in the following form:

$$\vec{H} = \vec{H}(\vec{r}, t) = \vec{u}(\vec{r}) \exp[i(\vec{k} \cdot \vec{r}) - \omega t]$$

(2-34)

where $\vec{u}(\vec{r})$ is the modulating periodic function of the coordinate vector $\vec{r}$, and the factor $\exp[i(\vec{k} \cdot \vec{r}) - \omega t]$ represents a plane wave. Inserting (2-34) into (2-33) and recalling that speed of light in vacuum is $c = 1/\sqrt{\varepsilon_0 \mu_0}$, (2-33) becomes:
\[ \nabla \times \left( \frac{1}{\varepsilon_r} \nabla \times (\bar{u}e^{i\vec{k} \cdot \vec{r}}) \right) + \left( \frac{1}{c^2} \right) \left( \bar{u}e^{i\vec{k} \cdot \vec{r}} \right)(-i\omega)^2 = 0 \, . \tag{2-35} \]

Both the modulating periodic function \( \bar{u}(\vec{r}) \) and the reciprocal of dielectric constant function \( (1/\varepsilon_r)(\vec{r}) \) have the same periodicity as that of the crystal and can be represented by Fourier series as follows:

\[ \bar{u}(\vec{r}) = \sum_{\vec{g}} \tilde{a}_{\vec{g}} \exp(i\vec{G} \cdot \vec{r}) , \tag{2-36} \]

\[ \frac{1}{\varepsilon_r}(\vec{r}) = \sum_{\vec{g}} \tilde{b}_{\vec{g}} \exp(i\vec{G} \cdot \vec{r}) , \tag{2-37} \]

where \( \vec{G} \) is the reciprocal lattice vector, \( \tilde{a}_{\vec{g}} \) is the unknown Fourier coefficient, and \( \tilde{b}_{\vec{g}} \) is the Fourier coefficient of dielectric constant function. The reciprocal lattice vector \( \vec{G} \) can be derived purely from the crystal geometry and is given by:

\[ \vec{G} = p\vec{g}_1 + q\vec{g}_2 + r\vec{g}_3 , \tag{2-38} \]

where \( \vec{g}_1, \vec{g}_2, \vec{g}_3 \) are the three primitive reciprocal lattice vectors, and \( p, q, r \) are integers. The three primitive reciprocal lattice vectors are derived from the three primitive lattice vectors \( \vec{d}_1, \vec{d}_2, \vec{d}_3 \) of the crystal and are given by:

\[ \vec{g}_1 = 2\pi\vec{d}_2 \times \vec{d}_3 / (\vec{d}_1 \cdot \vec{d}_2 \times \vec{d}_3) , \]

\[ \vec{g}_2 = 2\pi\vec{d}_3 \times \vec{d}_1 / (\vec{d}_1 \cdot \vec{d}_2 \times \vec{d}_3) , \tag{2-39} \]

\[ \vec{g}_3 = 2\pi\vec{d}_1 \times \vec{d}_2 / (\vec{d}_1 \cdot \vec{d}_2 \times \vec{d}_3) . \]

Note that the reciprocal lattice vectors form a reciprocal crystal in reciprocal space themselves. Substituting (2-36) and (2-37) into (2-35) and rearranging it, the following is obtained:

\[ \sum_{\vec{g}'} \sum_{\vec{g}} \nabla \times \left( b_{\vec{g}'}e^{i(\vec{G}' \cdot \vec{r})} \nabla \times (\tilde{a}_{\vec{g}}e^{i(\vec{G} \cdot \vec{r})}) \right) = \left( \frac{\omega^2}{c^2} \right) \sum_{\vec{g}} \tilde{a}_{\vec{g}} e^{i(\vec{G} + \vec{k}) \cdot \vec{r}} \tag{2-40} \]

The following vector calculus identity is now used:
\[ \nabla \times (\tilde{a}_\tilde{g} e^{i(\tilde{G} + \tilde{k}) \cdot \hat{r}}) = e^{i(\tilde{G} + \tilde{k}) \cdot \hat{r}} \left( \nabla \times \tilde{a}_\tilde{g} + \tilde{a}_\tilde{g} \times i(\tilde{G} + \tilde{k}) \right) , \]  
(2-41)

Note that \( \nabla \times \tilde{a}_\tilde{g} = 0 \) since \( \tilde{a}_\tilde{g} \) is merely a constant vector. Upon substitution of (2-41) into (2-40), (2-40) reduces to:

\[ \sum_{\tilde{g}'} \sum_{\tilde{g}} \nabla \times \left( ib_{\tilde{g}'} e^{i(\tilde{G}' + \tilde{k} + \tilde{G} \cdot \hat{r})} \tilde{a}_\tilde{g} \times (\tilde{G} + \tilde{k}) \right) = \left( \frac{\omega^2}{c^2} \right) \sum_{\tilde{g}} \tilde{a}_\tilde{g} e^{i(\tilde{G} + \tilde{k}) \cdot \hat{r}} \]  
(2-42)

Again, the same identity is applied and (2-42) further reduces to:

\[ \sum_{\tilde{g}'} \sum_{\tilde{g}} i^2 b_{\tilde{g}'} e^{i(\tilde{G}' + \tilde{k} + \tilde{G} \cdot \hat{r})} \left( \tilde{a}_\tilde{g} \times (\tilde{G} + \tilde{k}) \right) \times (\tilde{G}' + \tilde{G} + \tilde{k}) \]

\[ = \left( \frac{\omega^2}{c^2} \right) \sum_{\tilde{g}} \tilde{a}_\tilde{g} e^{i(\tilde{G} + \tilde{k}) \cdot \hat{r}} \]  
(2-43)

Since \( p, q, r \) in (2-38) run over the entire integer domain, it is possible to set:

\[ \tilde{G}' = \tilde{G} + \tilde{G}' = (p \tilde{g}_1 + q \tilde{g}_2 + r \tilde{g}_3) + (p' \tilde{g}_1 + q' \tilde{g}_2 + r' \tilde{g}_3) \]

\[ = p'' \tilde{g}_1 + q'' \tilde{g}_2 + r'' \tilde{g}_3 \]  
(2-44)

where \( p'', q'', r'' \) are some integers. Then, (2-43) can be rewritten as:

\[ \sum_{\tilde{g}''} \sum_{\tilde{g}} b_{\tilde{g}''} e^{i(\tilde{G}'' + \tilde{k}) \cdot \hat{r}} \left( \tilde{a}_\tilde{g} \times (\tilde{G} + \tilde{k}) \right) \times (\tilde{G}' + \tilde{G} + \tilde{k}) \]  
(2-45)

As a result, (2-45) becomes an eigenvalue problem, once written in a matrix format. The foregoing equation is applicable to one-, two- and three-dimensional PhCs. Once (2-45) is solved, complete solutions of all possible Bloch waves are acquired.

Since the theoretical foundation was established, a lot of attentions have been drawn on theoretical aspects, experimental confirmations and developments of PhC-based devices, not only for the case of 1-D PhCs but also for the case of 2-D PhCs and 3-D PhCs. The existence of photonic band gaps in 2-D and 3-D engineered periodic structure was confirmed experimentally in [14-15]. Nonetheless, the objective of this thesis is only to deal with 1-D PhCs. Therefore, some irrelevant topics are left out. Both matrix method by Yeh and the plane wave method for
photonic crystal theory will be used in designing optical devices as described in Chapter 3. One method may be preferred to the other depending on property of the PhC being investigated. The following section elaborates on the applications of 1-D PhCs.

2.2 Applications of One-Dimensional Photonic Crystals

A myriad of applications benefitted from 1-D PhC structures have been developed. Interesting examples are Bragg reflectors, optical filters, waveguides, harmonic generators, to mention a few. Although most of those devices were developed much earlier before the photonic crystal theory was established, they are worth mentioning and considered here in light of the photonic crystal concept.

2.2.1 Bragg Reflectors

A classical application of 1-D PhC is the Bragg reflector. Bragg reflector or Bragg mirror is a series of periodic dielectric media consisting of at least two types of materials of different refractive index. Perhaps, it is most commonly used in semiconductor lasers, such as distributed Bragg reflector (DBR) lasers \[16-17\] and distributed feedback (DFB) lasers \[18-20\]. Similar structures are also employed in fiber optics where they are called fiber Bragg gratings \[21\]. The function of Bragg reflectors is to confine light within the optical cavity of a laser system. Bragg reflectors are wavelength-sensitive; therefore, they are usually designed for a specific wavelength. In internal-cavity DBR lasers, they are designed at lasing wavelength so that optical amplification is achieved by multiple back-and-forth reflections between two Bragg reflectors placed at both end of the resonator. A common rule of designing a two-material Bragg reflector is to make the optical thickness of each layer about one-quarter of a target wavelength. Such a structure is sometimes called a quarter-wave stack \[8\]. When the optical thickness of each layer is a quarter of the wavelength, constructive interference outside the structure is attained, causing very high reflectivity. This is in other words the photonic band gap effect at normal incidence. It can be seen that utilization of 1-D PhC for this particular aspect is
restricted to PBG effect at normal incidence, and to a specific wavelength or a narrow bandwidth.

### 2.2.2 Piles of Plates as Polarizers

This is a simple device that makes use of incidence at Brewster’s angle to cumulatively polarize light. The device was first invented by D. F. J. Arago in 1812 according to [22]. At the special angle of incidence, i.e., at the Brewster’s angle, TM light is allowed to pass through the interface with no reflection at all. On the other hand, TE light can be partially reflected and transmitted. By placing one dielectric plate on top of the other, TE light undergoes successive partial reflections. On the transmitted site of the pile, no TE light can be transmitted if the number of plates is large enough, and the plates are properly design. In effect, a pile of plates acts as a linear polarizer, splitting TE and TM apart. An analysis for intensity of light going through a pile of plates was carried out by Stokes around 1860, but the PBG concept was not discussed.

### 2.2.3 Omnidirectional Reflectors

Another type of reflector exploiting the PBG effect is the omnidirectional reflector. The device was first proposed by Fink et al from MIT in 1998 [23]. It is a series of dielectric stratified media that can reflect a range of frequencies at any angle of incidence. Existence of the PBG at inclined incidence has also been verified [24]. An advantage of omnidirectional reflector is higher reflectivity, due to no absorption as compared to that of metallic mirror. A sufficient condition for the omnidirectionality to occur is that there must be no propagating mode existed in the stratified media (1-D PhC) when light is incident (from incident medium into the periodic medium) between normal incidence angle and critical angle. The mechanism responsible for nonexistence of propagating mode in the crystal is nothing but the PBG that not only presents in normal incidence but also extends to inclined incidence. With the invention of the omnidirectional reflector, light confinement in three dimensions can be achieved [25]. It can be seen that a true 3-D photonic crystal is not always necessary for the task.
2.2.4 Optical Limiters

An optical limiter is used in laser applications where highly intense light is suppressed for eye protection or optical detector protection. 1-D-PhC-based optical limiters were investigated in by Scalora [26] and Soon [27]. In Scalora’s work, intensity-dependent transmission and reflection were achieved by combining a linear dielectric and a non-linear Kerr dielectric to form a 1-D PhC. The 1-D PhC was designed for frequencies near band edge. Non-linear shift of band edge was attained such that only high-intensity modes see the PBG, and they are thus filtered out. On the other hand, Soon et al proposed a technique to filter high-intensity light by exploiting the non-linear defocusing effect of transverse modulational instability.

2.2.5 Bragg Waveguides, Bragg Fibers

A Bragg reflection waveguides was proposed in 1976 by Yeh and Yariv [28]. It was proposed that a guided mode could exist in the waveguide where a 1-D PhC is on one side of the core of the guide and a homogeneous material is on the other. Moreover, by sandwiching two 1-D PhCs around the core, existence of guided modes is tangible. Similar concept was used to construct hollow-core fibers by making use of cylindrical omnidirectional reflector. The fibers are called Bragg fibers. An advantage of this type of fibers is an ability to carry a wide bandwidth due to the non-dispersive hollow core [29]. They also better out the conventional optical fibers in terms of bend angle, because the guided modes do not depend on total internal reflection [30].

2.2.6 Tunable Photonic Crystal Devices

It can be seen from the previous subsections that extensive amount of efforts have been put into utilizing the PBG effect of 1-D PhCs. The 1-D PhC structures are integrated into devices in order to control the flow of light or to improve performance of the devices. Although many applications have been investigated, static 1-D PhC structures are mainly exploited. There are some efforts to alter PBG of 1-D, 2-D and 3-D PhCs. Those methods are, for instance, infiltrating liquid crystal into a PhC matrix and control the overall refractive index of the PhC by orienting the liquid crystal with electric field [31-33]. Although fast response can be achieved with
the technique, the change in PBG is little. Changing refractive index of PhC by thermal mean is another attempt [34].

A few methods of tuning PBG by mechanical means were explored for 2-D PhC [35-36], but only two attempts have been made for 1-D PhC by Wong et al [37] and by Kimura [38]. Wong’s work mainly involved forming a wide PBG on the transmission bandwidth of a strip waveguide and allowing a narrow bandwidth of resonant modes to exist at the center of the PBG by introducing a defect into the 1-D PhC. By doing this, only guided modes whose frequencies fall within the resonant bandwidth were allowed to resonate in the microcavity and further transmitted to the other side of the waveguide by tunneling through the 1-D PhC. The device structure can be described as follows. A short array of holes was crafted along the axis of a sub-micrometer silicon waveguide which was fabricated on top of a deformable silicon dioxide membrane. The array acted as a 1-D PhC. By removing one air hole in the middle of the array, a defect was introduced and the microcavity was formed. In this case, the 1-D PhC was used to form the high-reflectivity walls of the microcavity. Both compressive and tensile strain in the silicon dioxide membrane could be induced by four piezoelectric membranes located nearby the waveguide. A maximum shifting of cavity resonance of 1.54 nm at operating wavelength of 1.56 μm was achieved at an induced strain of 0.04%. The resonant mode was located well inside PBG. The application of this tunable 1-D-PhC-based resonance microcavity could be employed in a real-time reconfigurable device for active feedback compensation due to external disturbances or for tunable optical filtering. However, as marked by the author at the end of the paper, larger strain would be required for optical filtering application.

In Kimura’s work, polystyrene was used as a high-index material, and polyvinyl alcohol or collodion was used as a low-index material. The materials, all soft and elastic, were spin coated on a glass substrate layer by layer. The Bragg-reflection wavelength attained was around 600 nm, with layer thickness of 100 nm each. A plunger was used to compress the stack via two glass plates sandwiching it. A maximum reversible tuning range was about 45 nm, and the bandwidth was around 8 nm. Although the mechanically tunable 1-D PhC by Kimura demonstrated a large
shifting of center Bragg reflection, the tuning mechanism relied on the large plunger. Using the large plunger might not be suitable in many situations where portability and low driving energy are required. The PBG effect was also exploited only at the normal incidence because the incident light was injected in the normal direction to the plane of substrate. Moreover, the 1-D PhC needed to be fabricated on large flat substrate in order to achieve good uniformity of layer thickness. The goal of tuning was only to shift the stop band at normal incidence. However, 1-D PhC has different PBG for each of polarizations. As such, advantage of having different PBGs at different angle of incidence has never been exploited. This may be due to small field of view that makes it not very useful, unless angle of incidence is fixed. Therefore, it can be stated at this point that a miniaturized device that takes advantage of PBG at a fixed inclined incidence and that possesses tunability via geometrical change of 1-D PhC was yet to be developed. Such device will be elaborated here for the first time.

2.3 Microactuator as a Tuning Device for the One-Dimensional Photonic Crystal

In the last section only one mechanically tunable 1-D-PhC-based miniature device was found from the literature. The need to increase mechanical strain in order to achieve wider range of tuning was also addressed in the work. The mechanically tunable 1-D PhC made of polymer was developed earlier. The polymeric 1-D PhC might be tuned easier due to lower stiffness, but it was not miniaturized. It is possible that a 1-D-PhC-based miniature device made of polymer may have some potential applications that have not been realized before. In order to realize such a device, the fabrication technology that will make such a device small yet tunable can be nothing but the existing micro-electromechanical system (MEMS) technology. Specifically, the technology can enable one to fabricate the flexible 1-D PhC together with some active mechanism for tuning attached to it. In fact, such a tuning mechanism is usually called a microactuator. It is an active component that can convert one form of energy into mechanical work in micro-world. This section is dedicated to an investigation of prevailing types of microactuators. Advantages and disadvantages of
each type of microactuators are discussed, and a few suitable types of microactuators are identified at the end of this section.

2.3.1 MEMS Microactuator Overview

MEMS is a technology that was recognized around year 1959 when a famous Nobel Laureate Richard Feynman gave an idea of what could be fabricated in the micro-world by an existing laboratory equipment, such as a scanning electron microscope [39]. One year afterwards, a fabrication technique used in microelectronics at that time was applied to make the first 1/64-inch electric motor that could produce horse power of about one-millionth. In 1989 in USA, the emerging technology was coined MEMS by R. Howe and other scientists and engineers [40]. However, the name MEMS is known in Europe as Microsystems and in Japan as Micromachining. At the present, many commercial products based on MEMS technology are available, for example, accelerometers for automobile airbag systems found in middle-to-high-end automobiles [41], inkjet heads [42], Digital Micromirror Device for projectors [43], pressure sensors, etc. Depending on the fields in which MEMS involves, MEMS has other aliases, such as MOEMS for Micro-optoelectromechanical systems, when it is applied in conjunction with photonics or micro-optics, Bio-MEMS when incorporated into a biological system, RF-MEMS when integrated with radio-frequency devices, etc.

A microactuator may be considered as a heart of a MEMS device as its function is to convert energy from one form into another, making the MEMS device workable. There are a plenty of ways to categorize microactuators. For example, they can be categorized based on output motions, based on the form of input energy, or based on integration-digitalization level [44]. Categorization based on input energy is used here. Microactuators can be divided into six different major types, i.e., thermal, electrical, optical, acoustic, chemical, fluidic. Nonetheless, each microactuator has quite a unique design for a specific device and most of the time one design of a device is difficult to apply for another device. As a result, only actuation principles can be adapted while detailed designs and fabrication techniques have to be worked out. The following review focuses on the most common types of microactuators,
namely, piezoelectric microactuator, electrostatic microactuator, and thermal microactuator. These three types of micro actuators are taken into consideration, because they are the most widely applied in MEMS for good reasons. That is to say, the amount of output force generated is large enough for mechanical applications in microscale [45]. There are standard manufacturing processes, such as Multi-user MEMS Processes (MUMPs) [46], SUMMiT V™ [47], that were developed for these types of micro actuators. Decades of history of these types of micro actuators has allowed researchers to investigate them thoroughly, in terms of theoretical designs and simulations, manufacturing processes, and characterization techniques. As will be seen in the following review, some of these micro actuators were designed, fabricated and tested so thoroughly that they were employed in many commercial products nowadays.

2.3.2 Piezoelectric Microactuator

A piezoelectric material is a crystalline material that develops stress when an electric field is applied to it. Conversely, when stress is applied on it, electric polarization is generated. The piezoelectric material has an advantage in giving high stress and energy density but has a disadvantage in providing low output displacement [48]. According to [48], a 100-μm-long and 1-μm-thick lead zirconate titanate (PZT), a piezoelectric material, can produce only 0.17 μm of linear displacement upon excitation by external voltage of 10 V across its membrane thickness. Another advantage of piezoelectric is high frequency response that can be up to a few gigahertz. Piezoelectric microactuator can be categorized into two groups according to mechanical output obtained: one being a linear type and the other being a bimorph type. The linear type delivers directly the strain in the plane perpendicular to the applied electric field, whereas the piezoelectric material in the bimorph type usually induces bending on material that it is deposited on.

Many applications of piezoelectric micro actuators can be found in literatures. In hard disk drive applications, it is used in the secondary stage to position within microscale range a read/write head to a platter with high track density [49]. Two piezoelectric strips were aligned collinearly but separated by a certain distance in order to
accommodate a space for a movable load beam of the secondary stage. The load beam, which is attached to flexural pivots at one end and to the read/write head at the other end, could be rotated by the two piezoelectric strips when activated. Some types of inkjet printer heads are consisted of piezoelectric material that squeeze, bend, push, or shear the ink out during printing [50]. Micropump is another example similar to membrane-type inkjet heads in that it utilizes a membrane of piezoelectric material to squeeze liquid into microchannels [51-52]. Applications of such micropumps are found in drug delivery systems. By precisely controlling the amount of drugs released at the right timing, the efficacy of drugs can be expected. A micromirror fabricated together with a set of four piezoelectric bimorphs that rotate the micromirror for 2-D scanning were reported in [53]. The operating voltage of 7.5 V was reasonably low as compared to electrostatic microactuator which may require voltage above 100 V for the application. The frequency response up to 5,350 Hz was achieved.

Difficulties in fabricating piezoelectric films are concerned with achieving reliable and repeatable film properties and requirement of high temperature processes [54]. Fabrications of the thin–film PZT usually employ physical vapor depositions (PVDs), chemical vapor depositions (CVDs), or laser ablations, where a temperature between 500-700 °C is required. For fabrication of thick films, screen printing or jet printing is employed, and a temperature between 800-900 °C is usually needed. Although bulk piezoelectric materials can be incorporated to a micromachined structure to achieve a microactuator, epoxy bonding between the micromachined structure and the bulk piezoelectric introduces problems for mass productions.

2.3.3 Electrostatic Microactuator

Extensive designs of electrostatic microactuators can be found from literatures. In short, there are two major types of electrostatic microactuator, i.e., gap-closing microactuator, constant-gap electrostatic microactuator.

The gap-closing microactuator is nothing but a capacitor. It is constituted by a pair of parallel electrode initially separated by a certain distance (See Fig. 2-2). If one electrode is incorporated to a suspended structure and is allowed to move, the gap
between the two electrodes will tend to close, due to electrostatic force generated upon applying electric potential across the two electrodes. According to [48], the force acting on the electrode is due to gradient of potential energy stored within the gap-closing microactuator. The potential energy comes from both strain energy and the electrostatic energy. For simplicity, the strain energy is given in terms of effective spring constant of the suspended structure $k$, and position of the electrode $x$:

$$ U_s = \frac{1}{2} k (x - x_0)^2, \quad (2-46) $$

where $x_0$ is the equilibrium position of the electrode. The electrostatic energy is expressed as a function of position of the movable electrode $x$ and the applied voltage $V$, with parameters being the cross-sectional area of the electrode $A$, permittivity of free space $\varepsilon_0$ (assuming the gap between the two electrodes is air):

$$ U_e = \frac{\varepsilon_0 AV^2}{2x} \quad (2-47) $$

Fig. 2-2: Working principle of gap-closing microactuator. The microactuator is formed by two electrodes of cross-sectional area $A$. One of the two electrodes is fixed whereas the other one is attached to a suspended structure with effective stiffness $k$. When the movable electrode is at position $x$ due to applied external voltage $V$, the electrode experiences two conservative forces, one being the spring force and the other being the opposing electrostatic force.

The gradients of each of these potential fields in (2-46) & (2-47) yield two conservative forces acting on the movable electrode as:
\[ F_{\text{total}} = -\nabla U_s + \nabla U_e = k(x_0 - x) - \frac{\varepsilon_0 AV^2}{2x^2} = F_s - F_e \] (2-48)

where \( F_{\text{total}}, F_s, F_e \) are the total force acting on the electrode, the spring force and the electrostatic force, respectively. Note that the positive sign in front of the gradient of the electrostatic potential energy appears so that the direction of electrostatic force is consistent with the defined coordinate in Fig. 2-2. The two forces are always in the opposite direction to each other and thus help balance the electrode. At equilibrium the two forces are equal in magnitude so that the following is obtained:

\[ k(x_0 - x) = \frac{\varepsilon_0 AV^2}{2x^2}. \] (2-49)

It can be seen from (2-48) that the spring force is proportional to the position of the electrode, but the electrostatic force is inversely proportional to square of the position. Range of position within stability condition of the device can be analyzed by taking another derivative of the total force in (2-48) which yields the rate of change of total force acting on the movable electrode:

\[ \frac{dF_{\text{total}}}{dx} = -\frac{d^2U}{dx^2} = -k + \frac{\varepsilon_0 AV^2}{x^3} \] (2-50)

The condition for stable equilibrium position \( x_{ep} \) is attained when the rate of change of the total force is equal to or less than zero so that the change in spring force with respect to position is always larger than that of the electrostatic force:

\[ \frac{dF_{\text{total}}}{dx} = -k + \frac{\varepsilon_0 AV^2}{x^3} < 0. \] (2-51)

Substituting (2-51) into (2-49) for \( k \) yields the range of position of the electrode within stable region:

\[ x > \frac{2}{3}x_0. \] (2-52)
If the position of the electrode does not satisfy (2-52), the electrode will eventually collapse into the fixed electrode due to inability of the spring force to oppose the electrostatic force.

Earliest example of gap-closing microactuator can be found in Nathanson’s work on resonant-gate transistor in 1967 [55]. The resonant-gate transistor consists of a cantilever acting as a gate beneath which lies an input force plate and source and drain. When voltage is applied across the input force plate and the cantilever, separated from the source and drain by air, the cantilever above the input force plate is attracted towards the plate by electrostatic force, causing the current passing through source and drain to be modified. Commercial product merited from gap-closing microactuator is the Digital Mirror Device developed by Texas Instruments Inc [56]. Other recent examples of gap-closing microactuators can be found in [57-58]. The gap-closing microactuator has a major drawback in its pull-in instability when the two electrodes come closer than two-third of the original gap and thus limited travel distance. A limit stop is usually designed to prevent collapsing of the movable electrode into the fixed electrode.

![Diagram of Constant-gap microactuator](image)

Fig. 2-3: Constant-gap microactuator. Direction of motion of the constant-gap microactuator is parallel to the plane of the electrodes. Many pairs of electrodes are connected in series forming a comb-like structure. Pull-in instability is one of the drawbacks of this type of microactuator and is illustrated on the right-hand side of the figure.

The constant-gap microactuator is different from the gap-closing microactuator in that one of the two parallel electrodes is allowed to move in the direction parallel to the planes of the electrodes. Due to very small electrostatic force generated, many pairs of such electrodes are usually attached in series so that the whole structure looks like two combs whose fingers are interdigitated into each other (Fig. 2-3).
Therefore, such constant-gap microactuator is sometimes called a lateral comb-drive microactuator. By expressing the area of the electrode in (2-47) in terms of the height of the electrode $H$ (which is in the normal direction to this page) times the variable width of the electrode $y$ (See Fig. 2-3), i.e., $A=Hy$, the electrostatic potential energy between a pair of electrodes of a comb-drive microactuator becomes:

$$U_e = \epsilon_0 \frac{AV^2}{2x_0} = \frac{\epsilon_0 HV^2}{2x_0} y. \quad (2-53)$$

When the fingers of the two combs are engaged, the number of pairs of the electrodes are doubled. Taking a spatial derivative of electrostatic energy in (2-53) with respect to $y$ and multiplying it with $2N$, where $N$ is the number of fingers on each comb, gives the total electrostatic force:

$$F_e = 2N \frac{\partial}{\partial y} \left( \frac{\epsilon_0 HV^2}{2x_0} y \right) = \frac{N\epsilon_0 HV^2}{x_0} = N\epsilon_0 rV^2, \quad (2-54)$$

where $r$ is the aspect ratio of the height of the comb structure to the gap between two adjacent fingers. Advantage of the constant-gap microactuator over the gap-closing microactuator is larger travel distance. Nonetheless, the pull-in instability is still persistent in the design. Therefore, a usual design of a comb-drive microactuator incorporates a suspended structure with very high stiffness in the direction perpendicular to the plane of electrodes, to prevent the side-way pulling.

Fig. 2-4: Layout of the electrostatic comb-drive resonator. a) The layout of the electrostatic comb-drive resonator. The dashed rectangle encompasses the folded flexure which serves as both support and restraining spring for the movable combs. b) An image of the device during operation at resonant frequency. Adapted from [1-2].
The first comb-drive microactuator was proposed, fabricated and tested in 1990 by a team in University of California, Berkley [1-2]. A smart design of suspended compliant structure called folded flexure was incorporated into the comb structure as shown in Fig. 2-4a. The suspended structure allows for motion in the direction parallel to the plane of electrodes because of low stiffness of bended silicon beam. On the other hand, very high stiffness because of axial stiffness of silicon beam keeps the movable comb from smashing into the fixed comb. The microactuator structure was made of p-doped polysilicon on n-doped silicon substrate. The suspended structure was achieved using sacrificial phosphosilicate glass, which not only served as a sacrificial layer but also helped symmetrically dope the structural polysilicon. The sacrificial layer was removed by wet etching. The design was adopted widely in many MEMS devices. For example, it was used to move waveguides in an optical switch device [59-60], to drive a micro-gears and micro-linkages [61], to move the tip of micro-scanning tunneling microscopes [62].

In short, both gap-closing and constant-gap electrostatic microactuators promise easy fabrication process as compared to other type of microactuators such as piezoelectric type and shape-memory type (elaborated below). Frequency response is moderate and in the range of kilohertz. The major drawbacks of electrostatic microactuator are pull-in instability and minute output force.

### 2.3.4 Thermal Microactuator

The thermal microactuators can be divided into three groups according to [40]. They are the thermo-pneumatic type, the shape-memory type, and the bimorph-like type. However, another group of thermal microactuator should be introduced according to this present review. That is the silicon-polymer composite type. The details of this new type of microactuator are given at the end of this subsection.

The thermo-pneumatic microactuator consists of three crucial components, namely, the flexible diaphragm, the sealed cavity with confined fluid inside, and the heating element ([63]). The microactuators make use of thermal expansion of the confined fluid to push the flexible diaphragm. Force and displacement are generated at the diaphragm which in turn drives passive structure or fluid being in contact to it. Most
of these designs provide only out-of-plane actuation. Besides the aforementioned design of thermo-pneumatic microactuator, another example of thermo-pneumatic microactuator, perhaps the most prominent of all, is the bubble jet printer head developed by Shirato at Canon Inc [64] and Allen at Hewlett-Packard Inc [65]. Instead of having a diaphragm to separate the passive ink fluid from the expanding fluid, the heating element directly heats the ink until microscopic bubbles are nucleated on the heating surface within the ink nozzle. The gaseous fluid created within the nozzle then pushes the ink out. This thermo-pneumatic microactuation enables print head design to be compact, disposable, multi-nozzled, and battery-driven [65]. Without the diaphragm, the technique is nonetheless limited to fluidic applications. If the diaphragm is to be introduced so that the thermo-pneumatic actuation can be used to drive non-fluidic substance, a sealed cavity is required, causing more fabrication complication.

Shape memory effect appears in some crystalline metal such as copper. It also appears in many alloys. A shape-memory alloy (SMA) is an alloy that has two solid phases, namely, the martensite phase and austenite phase [40]. The popular alloys with shape memory effect are NiTi (called nitinol) or Ni_{x}Ti_{y}Cu_{z}. The soft martensite phase which exists at low temperature can be converted into austenite phase upon heating above its phase transition temperature. The shape of alloy changes as a result of phase transition, and thus actuation force is obtained. Upon cooling down, some austenite phase change back to its original martensite phase. The temperature difference needed to transform the martensite phase to austenite phase can be controlled by altering the composition of the alloy, and it can be as low as 10 °C.

Thin-film SMA microactuator is capable of generating large forces over long displacements by means of phase transformation [66]. A 2 mm × 1 mm, 5-μm-thick Ni-Ti film ligament can generate as much as 1.5 N at 300 MPa and 3% strain over 60 μm. By comparison, electrostatic microactuator and electromagnetic microactuator are able to produce forces in mN range which is much less than that produced by SMA microactuator. Nonetheless, its frequency response is in order of 100 Hz which is considerably low as compared to electrostatic or piezoelectric microactuators.
Another concern regarding SMA is fabrication difficulty in controlling thin-film deposition [40].

The third type of thermal microactuators is the bimorph-like type. The configuration of a typical bimorph is such that two strips of materials of different thermal expansion coefficients (CTEs) are joined side by side as shown in Fig. 2-5 [67]. When the assembly is heated up, the strip with higher CTE expands more whereas the one with lower CTE expands less. The whole assembly then bends as shown in Fig. 2-5. Joule’s heating is normally employed to generate heat on the structure. This can be done by patterning a metallic circuit on the assembly. Early bimorph microactuator designs and fabrications were reported by Benecke in 1989 [68]. A polymeric out-of-plane version of bimorph microactuator was demonstrated by Fujita in 1993 [69-70]. A series of the out-of-plane thermal microactuators were used to mimic the motion of a microorganism, cilia. The latest advance on this work is a small 2-D conveyance system made of a matrix of these microactuators [71].

Fig. 2-5: Schematic diagram of a bimorph microactuator before and after heated up. The configuration of the microactuator is such that two materials of different CTEs $\alpha_1 > \alpha_2$, are perfectly attached side by side. When temperature of the whole assembly is elevated, Material 1 which possesses higher CTE elongates more, resulting in upward bending.
Another important design of thermal microactuator of this type utilizes difference in geometrical dimensions to cause deflection instead of difference in CTE [72-73]. Such microactuator is called a heatuator. It is shown in Fig. 2-6. The structure of the microactuator can be fabricated easier than the original bimorph design because only one type of conductive material is needed. Two beams of different widths are anchored to substrate at one of their two ends, whereas the other ends are joined to each other, forming an electrical loop. A metallic film can also be deposited on top of the structure to reduce resistance of the device as required. Standard microfabrication technology such as Multi-User MEMS Processes (MUMPs, [46]) was applied in order to make the microactuator in [73]. Designs of the microactuator for both in-plane and out-of-plane actuations were achieved with the MUMPs technique. Also, optional metallization process can be introduced in the MUMPs in order to tailor resistance of the microactuator as wished. It was also demonstrated that the microactuator purely made of polysilicon was CMOS-compatible in terms of voltage (2.94 V), current (3.68 mA) and power consumption (10.8 mW). When electric potential difference is supplied across the two fixed ends, electric current flows into the structure from one anchor, through the two beams, to the other anchor. Heat generation can be expressed by the well-known Joule’s law:

\[ Q = I^2 R , \]  

(2-55)
where $I$ is an electric current, $Q$ is an amount of heat generated at a resistive element of a resistance $R$. In Fig. 2-6, the current density in *hot arm* is higher than that in *cold arm*, and it is represented by small solid red arrows in contrast to big faded arrows which represents lower current density on the cold arm. Nonetheless, the total electric current $I$ passing through the cross-sectional area of either beam remains constant. The resistance of the resistive element can be expressed in terms of material resistivity $\rho$, cross-sectional area $A$ and length $L$ as:

$$R = \frac{\rho L}{A}.$$  \hfill (2-56)

The resistance of the whole heatuator is a resultant resistance of the two arms connected in series. Since one of the two beams is narrower, more heat is generated on it than the other beam because of higher resistance (due to less cross-sectional area). Heat dissipation of the structure is a crucial factor that influences the performance of the microactuator. The generated heat is dissipated by thermal conduction to the substrate via the anchors and thin layer of air beneath the structure. Convection and electromagnetic radiation are other mechanisms that take away the heat from the structure. Increasing air gap between the microactuator structure and the substrate was employed in [74] in order to suppress total thermal conduction and thus to heighten the steady-state temperature of the device. Thermal microactuators of this type have a disadvantage in slow frequency response. Nonetheless, it was shown in [75], where the microactuator was used to move a variable grating, that frequency response up to 1 kHz without significant roll-off could be achieved and measurable response up to 10 kHz was obtained. A heatuator-based polymeric microgripper manufactured from a photo-patternable negative resist, SU-8, was demonstrated in [76]. The microgripper was capable to operate in aqueous solution. This is a big advantage of thermal microactuator over the electrostatic microactuator, which cannot operate in such environment. Another advantage of using polymer over using metal or silicon is a larger thermal expansion obtained. The SU-8 itself can be patterned easily, yet quite stable and strong, with a Young modulus of about 3.2 GPa and a tensile strength of 106 MPa ([77]).
The last group of thermal microactuator is the silicon-polymer composite type. Desirable properties of silicon for making a thermal microactuator are very high stiffness and high thermal conductivity. But silicon has a weakness of a moderate electrical resistivity and low thermal expansion coefficient. On the other hand, the SU-8 has high thermal expansion coefficient, but it has stiffness about 1/40\textsuperscript{th} of that of silicon, poor thermal conductivity, and high electrical resistivity. Except for electrical resistivity, it can be seen that the two materials have their own pros and cons that complement the other. One might anticipate that a composite that is comprised of the two materials should be ideal for making a thermal microactuator because the effective material properties of the composite are the average of the two. This is exactly what was proposed in the design of a powerful thermal microactuator in [78]. A schematic diagram of the microactuators is shown in Fig. 2-7. The
microactuator structure is consisted of a silicon skeleton, an aluminum heater on top of the skeleton, and a SU-8 block. The SU-8 is molded into the gaps between the silicon skeleton. In addition, some of the SU-8 surrounds the skeleton and help prevent heat convection during operation. The SU-8 also serves as a thermal expander that pushes the skeleton. Owing to high coefficient of thermal expansion, lesser operating temperature as compared to silicon heatuator could be expected for the same output displacement. Once the aluminum heater generates heat within the structure, silicon which is a good thermal conductor will help spread the heat to the entire structure, leading to a more uniform temperature distribution. The silicon skeleton not only enhances mechanical strength of the structure, but also delivers higher output force due to its very high stiffness. Higher frequency response as compared to a polymeric microactuator in [76] could also result, thanks to high stiffness of silicon and high thermal conductivity. High thermal conductivity is desirable as it can help rapidly dissipate away the heat from the structure during off-time. Theoretical investigations, finite-element studies and experimental characterizations of the new microactuators were reported in [79-84]. On manufacturability aspect, in constrast to the piezoelectric type and the thermal bimorph type where difficulties usually arise from combining active materials to passive structure, it is relatively easier since SU-8 can be directly patterned on silicon substrate via conventional photolithography [85-86]. At least three major variations of the microactuator have been realized so far: the normal meandering skeleton [78], the bimorph-like [85, 87], and the V-shape [88]. Schematic drawings for the designs are given in Fig. 2-7.

In summary, all the microactuators reviewed here were compared in terms of output force, output stroke, frequency response, efficiency, and manufacturability. Table 2-1 shows the advantages and disadvantages of each type of microactuators.
2.4 Summary and Outcomes of Literature Review

The literature review focuses on three topics, namely, the photonic crystal (PhC) theory for one-dimensional (1-D) problems, the applications of the 1-D PhC, and the microactuators as tuning devices for the 1-D PhC.

In the first section a brief history of development of photonic crystal theory was reported. Theoretical treatments of 1-D PhC were reported as early as in 1917 by Lord Rayleigh. Classical optics was then used to describe the PhC, but the PhC was not recognized as it is today. Later, similarity in the theory to that of X-ray crystallography pioneered by Bragg, i.e., the condition in which central band gap frequency occurs, was recognized. The 1-D PhC was then known as Bragg reflectors.
In 1987 when Yablonovitch and Sajeev independently reported on the generalized version of PhCs where they suggest the existence of photonic band gaps in 3-D dielectric structures, there began the modern study of PhCs in quantum theory language. Remarkable development in translation matrix for layered media was introduced by Abeles. The matrix method offers a systematic approach for characterizing the complex structures and enables one to design a layered media easier. Then in 1977 Yeh et al formalized the matrix method in a different way and it becomes, perhaps, the most popular method applied. Instead of writing one tedious matrix that links electric field and magnetic field between two dielectric media, they broke down the process into writing a Dynamical matrix which links the fields on both sides of an interface and Propagation matrix which links fields at different location within the same medium. The forms of the Dynamical matrix and Propagation matrix look simpler and thus easier to memorize and apply. In 1994 Lekner proposed another matrix formalism involving only real matrix elements, The technique leads to faster computation. A slight improvement in calculation speed for PBGs using plane wave method is reported at the beginning of Chapter 3.

Applications of optical devices were reviewed and divided into six groups. They were Bragg reflectors, pile-of-plates polarizers, omnidirectional reflector, optical limiter, Bragg waveguide & Bragg fibers, and tunable devices. Utilizations of inclined PBG effect were not investigated much except for the PBG waveguide by Yeh et al in 1977, and the omnidirectional reflector developed at MIT in 1998. Mechanically tunable 1-D PhC structures were discussed even lesser. There were only two works found: one by Kimura in 1979 on the tunable multilayer film distributed Bragg reflector filter, and the other by Wong in 2004 on the strain-tunable silicon photonic band gap microcavities in optical waveguides. In Kimura’s work, the Bragg filter was not miniaturized and mechanical tuning was relied on a large mechanical plunger which might result in limitation for many applications. Also, the PBG effect was utilized only at normal incidence. In Wong’s work, although the device was realized in microscale, the PBG effect was utilized only for filtering out a bandwidth of guided modes. The mechanical tuning was based on piezoelectric membranes which yielded very little mechanical strain up to 0.04%, thus leaded to
narrow maximum tuning range of about 1.5 nm. By improving the microactuator design, larger strain and thus wider tuning range could be achieved, and therefore a similar tunable optical filter device might also be designed. It was also pointed out at the end of the section that a structure that possesses a PBG to TE mode but not to TM mode might find an application such as a polarizer. Although the concept of using a pile of plates as polarizer was developed more than a century ago by Arago (as mentioned in [22]) and theoretically investigated by Stroke around 1860, the pile of plates was not known as a 1-D PhC at that time. Thus it has not yet been designed from a photonic crystal theory approach. By using one of the existing photonic crystal theories mentioned in the first section, 1-D-PhC-based polarizers might be designed. Although the end result of the design would still be a polarizer like the pile of plates, the origin of polarization profiles can be explained by PBGs which should give another perspective on the subject. Moreover, awareness of incorporating a mechanical tuning into such polarizer has not been spoken of yet, as far as this review is concerned. So the mechanical tunability of 1-D-PhC-based polarizer could also be explored in addition. Design of a tunable polarizer is reported in CHAPTER 3 in order to fill these gaps.

Having found necessities to improve mechanical tuning method for existing tunable PhCs, and to apply a new tuning mechanism in such static optical elements, the last section of this review concentrated on three major types of MEMS microactuators, namely, the piezoelectric microactuators, the electrostatic microactuators, and the thermal microactuators. The piezoelectric microactuators had advantages in wide dynamical bandwidth, high output force, but have disadvantages in minute output displacement and fabrication difficulties. Properties of thin-film piezoelectric materials were difficult to control and thus lead to manufacturing repeatability issue. Although bulk piezoelectric materials offered more reliable properties, they were difficult to be integrated to passive structure due to the presence of adhesive glue. The adhesive glue that bonds a bulk piezoelectric material to a passive structure caused reliability issues in terms of bond strength that was subject to temperature, and stress transfer. Large-scale manufacturability was another concern that had to do with using the glue to stick the material to the passive
structure one by one. Electrostatic microactuator on the other hand could be fabricated relatively easy. Because it could be made of either polysilicon or silicon substrate, both of which were also used for optical structures such as waveguides, PhCs, etc, the microactuator and the passive structures could be fabricated together with one-mask processes. Additionally, of-the-shelf silicon-on-insulator substrates could be employed to help simplify microfabrication processes. More complicated structures could be achieved by aids of standard microfabrication processs such as MUMPs and SUMMiT V™. The major drawbacks of electrostatic microactuator seemed to be low output force and pull-in instability. Thermal microactuator could be fabricated with similar processes as those for the electrostatic microactuator. The two standard microfabrication processes could also be employed. In contrast, output stroke of thermal microactuator was not limited by pull-in instability and thus could be larger as compared to gap-closing electrostatic microactuator. The weaknesses of the thermal microactuator were slow response and higher energy consumption in mW range. In conclusion, it can be seen from Table 2-1 that each type of microactuators has its own advantages and disadvantages. Nonetheless, the first deciding factor seems to be the manufacturability that will rule out some microactuator before any design process takes place. Among these common types of microactuators, the electrostatic type and the thermal type with composite structure seems to be the best candidates, provided capability of manufacturing it in our clean room laboratory. To further decide which type of microactuator to be used, it depends on what device and what application can be realized based on optical design, which is elaborated in Chapter 3. Therefore, the discussion on selection of microactuator is postponed until Chapter 4 where the microactuator design is investigated.
CHAPTER 3 TUNABLE ONE-DIMENSIONAL PHOTONIC CRYSTAL AND APPLICATIONS IN PHOTONICS

In this chapter a new application of the flexible one-dimensional photonic crystal is explored. The one-dimensional case of photonic crystal theory is first discussed. Then, an improved method for finding photonic band gap for one-dimensional photonic crystal is explained. A device, built based on flexible one-dimensional photonic crystal using alternate stacks of polymer and silicon, is investigated. The device can be used as a tunable polarizer. A detailed design for the device and a finite-different time-domain simulation are reported.

3.1 Photonic Band Structure and Band Gaps of One-Dimensional Photonic Crystal

In this section, it will be shown that there exists photonic band gaps (PBGs) for a one-dimensional photonic crystal comprised of silicon and poly(dimethylsiloxane) (PDMS). The method used for calculating the PBGs is the plane wave method described in Section 2.1.2.

A one-dimensional photonic crystal (1-D PhC) in the simplest form is a stack of alternate dielectric slabs. Consider an infinite series of dielectrics comprised of two different materials having dielectric constant $\varepsilon_{r_1}, \varepsilon_{r_2}$, respectively (Fig. 3-1). In 1-D case, (2-45) is simplified greatly. First of all, only one primitive lattice vector $\vec{a}$ and one primitive reciprocal lattice vector $\vec{g}$ are required to describe its geometry. Therefore, the reciprocal lattice vectors can be described by only one set of integer numbers and (2-38) becomes:

$$\vec{G} = p \vec{g},$$  \hspace{1cm} (3-1)
with $\vec{g} = 2\pi/|\vec{a}|$ and $p$ being a member of integer domain. Secondly, all the reciprocal lattice vectors are collinear and they are normal to the stack of dielectrics. Thirdly, the direction of the magnetic field (as well as the electric field) of electromagnetic mode is always parallel to dielectric slabs. Therefore, the cross product in (2-45) becomes:

$$
\vec{a}_G e^{i(G'' + k) \cdot x} = \frac{(\omega/c)^2}{c} \sum_G a_G e^{i(G + k) \cdot x}, \quad (3-2)
$$

where the coordinate vector $\vec{r}$ has been replaced by the coordinate $x$ which is running along the direction normal to the slabs. By matching $G$ on the right hand side with $G''$ on the left hand side of the above equation, the exponential factors on both sides can be canceled out, and the matrix form of the above equation can be written as:

$$
\begin{bmatrix}
    f_{11}a_1 & f_{12}a_2 & \cdots & \\
    f_{21}a_1 & f_{22}a_2 & \cdots & \\
    \cdots & \cdots & \cdots & \\
    \cdots & \cdots & \cdots & \\
    f_{ij}a_j & \cdots & \cdots & \\
\end{bmatrix}_k = \frac{(\omega/c)^2}{c^2} \begin{bmatrix}
    a_1 \\
    a_2 \\
    \cdots \\
    \cdots \\
    \cdots \\
    \cdots \\
\end{bmatrix}_k \quad (3-3)
$$

$$
; f_{ij} = b_{G''_{i-k}G_j} |G''_i + k||G''_j + k|, a_1 = a_{G_1} = |a_{G_1}|.
$$

It can be easily seen that the eigenvalue of (3-3) is $\omega^2/c^2$. The Fourier coefficient of the dielectric constant function $b_{G''_{i-k}G_j}$ can be found by Fourier transform, provided the dielectric constant function of the PhC is given. The subscripts $G', G''$ which were used in the foregoing equations for compactness are now replaced simply with integer numbers $i, j$. The subscript $k$ attached to the matrix and the column vector in

---

**Fig. 3-1:** A 1-D photonic crystal of a period $a$. Magnetic field of electromagnetic mode is parallel to the slabs as indicated by the vertical arrows. The wave vector $k$ of the electromagnetic mode is normal to the slabs.
(3-3) indicates that all the coefficients are derived for some wave vector $k$ only. Since the parameter $k$ arises when the form of the solution was assumed, it can be any real number for 1-D case. To completely characterize the electromagnetic modes inside a 1-D PhC, the wave vector $k$ needs to vary over its domain. Note that there are infinitely many unknowns in (3-3) which is impossible to solve. However, a finite number of plane waves with low spatial frequency are usually used to estimate the eigenvalues. A guideline for solving (3-3) is given in [89-90]. A Matlab script written for a 1-D PhC comprised of poly(dimethylsiloxane) (PDMS) layers and silicon layer is attached in Appendix A. The band structure obtained after solving (3-3) with 200 plane waves is shown in Fig. 3-2. The refractive index of silicon is taken as 3.420 and that of PDMS is 1.549.

To understand how the band structure is obtained, recall the solution of the Maxwell equations for a photonic crystal that is given by (2-34). It is nothing but a plane wave of wave vector $k$ multiplied by an envelope function which has the same periodicities as the crystal. Since the wave vector $k$ can be any number, the equation is solved repeatedly for different values of $k$. Nonetheless, according to Bloch theorem, the wave vector $k$ only needs to vary along the edge of the irreducible Brillouin zone wherein the wave vector $k$ ranges from -0.5 to 0.5 for the present case of 1-D PhC. Once all possible $k$’s are considered, the corresponding frequencies (derived from the eigenvalues) are obtained and plotted into what is called a band structure. For example, when the equation is solved with $k= -0.4$, corresponding frequencies are plotted as indicated by the star marks in Fig. 3-2. The diamond and the triangular marks indicate the corresponding frequencies obtained after solving the equation with $k= -0.3$, -0.2, respectively. A photonic band is obtained when $k$ continuously varies and forms a smooth line as shown in the same figure.

A photonic band structure holds tremendously significant information regarding the optical properties of a PhC. First of all, it reveals dispersion relation between the frequency and the wave vector. Secondly, the group velocity or energy velocity can be calculated once the dispersion relation is known [13]. Perhaps, the most interesting property of the photonic crystal is what is called photonic band gap (PBG).
When light of frequency $\omega$ enters a PhC, the behavior of the light deep inside the PhC can be understood by a summation of all possible modes encountered from drawing a horizontal line across the band structure (however, in case of 2-D or 3-D this method cannot be applied). Surprisingly, there are ranges of frequencies where there exist no corresponding wave vector $k$, e.g. at $k = -\pi/a$ (Fig. 3-2), albeit each material constituting the crystal does allow light of any frequency to enter themselves in bulk form. It means that there is no corresponding electromagnetic mode for the frequency in the crystal. When light within these frequency ranges is perpendicularly incident on a 1-D PhC, the light has nowhere to go but to bounce back. This phenomenon is analogous to what happen to electron wavefunction in some crystalline materials where atomic band structure exists. Since the ranges of energy in which electron wave function cannot exist are called atomic band gaps, similar terminology for the PhC, i.e., photonic band gaps (PBGs), is adapted for the forbidden frequency ranges of a PhC.

To conclude this section, it has been shown by calculation that there exist PBGs for a 1-D PhC consisted of silicon and PDMS dielectric slabs. The photonic band structure of the 1-D PhC is as shown in Fig. 3-2. The lowest PBG corresponds to a normalized central frequency of 0.2, which means the 1-D PhC prohibits electromagnetic wave with free-space wavelength of the size around five times of the lattice period to enter. Having explained about the band structure and the band gap of a photonic crystal, the next section elaborates on how to expedite the process of finding PBGs.
3.2 Photonic Band Gap Calculation – An Improved Plane Wave Method for One Dimensional Problem

According to the plane wave solution, a typical process of finding PBGs is to assume a value of wave vector $k$ on an edge of the irreducible Brillouin zone, then to calculate the corresponding eigenvalues for that value of $k$, and lastly to plot out all the frequencies versus that $k$. Afterwards, the next value of $k$ on the irreducible Brillouin zone’s edge nearby the first one is taken on. The same process is repeated in order to obtain the next set of eigenvalues and frequencies. Again, the frequencies versus the wave vector are plotted out. The wave vector $k$ is varied over irreducible Brillouin zone’s edges where highest frequencies and lowest frequencies are taken into account according to Bloch’s theorem. Once it is done, the band structure is obtained, and the PBGs are identified via observation of the band structure as shown.

Fig. 3-2: Photonic band structure of 1-D photonic crystal. The band structure is calculated with 200 plane waves and the thickness ratio of silicon layer to PDMS layer is 1. The dots indicate locations of minimum frequencies and maximum frequencies of each band. The star, the diamond and the triangular markers indicate corresponding frequencies obtained after solving the equation with 3 different values of $k$: -0.4, -0.3, -0.2.
in Fig. 3-2. Nonetheless, by observing the 1-D PhC band structure, it can be deduced that:

i. the PBGs always occur when the wave vector $k = \pm \pi m / a$ with $m = 1, 2, 3, \ldots$,

ii. as a result of (i), there is always a PBG between two adjacent bands,

iii. all of the PBGs are direct band gaps, i.e. the highest frequency of one band and the lowest frequency of the band just above are at the same wave vector.

As a result, the process of finding the PBGs can be expedited. That is, it is not necessary to vary the wave vector $k$ over the entire irreducible Brillouin zone’s edges. Rather, the wave vector $k$ only needs to be set to 3 values, i.e., $-\pi / a, 0, \pi / a$ (Observe the circular dot markers in Fig. 3.1). In fact, only 2 values ($0, \pi / a$) are needed because of mirror symmetry of the reciprocal lattice. The PBGs are then found by subtracting the lowest frequency of one band with the highest frequency of the band just below it. The physical interpretation can be explained by using Bragg condition where the wave vector $k$ satisfies [8]:

$$ka = m\pi ; m = \pm 1, \pm 2, \pm 3, \ldots$$  \hspace{1cm} (3-4)

The Bragg condition agrees with the observation from the band structure of 1-D PhC in Fig. 3-2. Although the foregoing condition was derived in [8] earlier, the Bragg condition only indicates the central band gap frequency. The band structure obtained here elucidates the truth that not only one frequency satisfying Bragg condition is totally reflected, but other frequencies surrounding it do so as well. Based on this observation, after solving the eigenvalue problem (3-3) for two times, one with $k = \pi / a$, the other with $k = 0$, the width of the band gaps between any two bands are acquired. The number of band gaps obtained is only one less the number of plane waves. The presented calculation was performed using 200 plane waves, as a results, 199 band gaps can be obtained after solving the equations two times. This is much faster than applying the method in [8], where one need to compute half the trace of the coefficient matrix, then test whether the value obtained is greater than, equal to, or less than unity. An optical device designed based on the technique was reported in [91].
In this section, the PBG effect at an inclined incidence is explained. At the inclined incidence any electromagnetic wave can generally be decomposed into two orthogonal polarizations, i.e., transverse-electric (TE) or transverse-magnetic (TM). The PBGs of a 1-D PhC are not necessarily the same for each polarization. Such difference in PBGs for each polarization motivates development of a photonic-crystal-based polarizer. To be more specific, the goal is to design a type of polarizers called a PBG polarization splitter. The PBG polarization splitter separates one polarization from the other by presenting PBG to one polarization but letting the other polarization passes through it. Since PBGs are dependent on dimensions of 1-D PhC, it is also possible to “switch-on” or “switch-off” the PBGs by reversibly altering the dimensions of the 1-D PhC. If it is possible to characterize how the PBGs change with respect to the dimensions of 1-D PhC, it should be possible to design a tunable polarization splitter. A miniature version of the tunable polarization splitter is preferred given that all applications in sensing, micro-photonics, telecommunication, etc., require small devices. Due to the minute size of the 1-D PhC to be made, using a microactuator to provide a mechanical work is a way to achieve robust dynamic tuning. Details on the microactuator design can be found in Chapter 4. The following subsections focus on designing the tunable polarization splitter in many aspects. The first subsections provide the working principle of the device. The second subsections

Fig. 3-3: Working principle of the PBG polarization splitter. a) The undeformed 1-D PhC possesses a PBG for TE mode at Brewster’s angle. b) During compression, the 1-D PhC is in its transition state where partial transmission and reflection of TE mode are allowed. c) The 1-D PhC is fully compressed and the PBG for TE mode disappears. The TE mode can fully propagate through the 1-D PhC.

3.3 Mechanically Tunable Photonic-Band-Gap Polarization Splitter

In this section, the PBG effect at an inclined incidence is explained. At the inclined incidence any electromagnetic wave can generally be decomposed into two orthogonal polarizations, i.e., transverse-electric (TE) or transverse-magnetic (TM). The PBGs of a 1-D PhC are not necessarily the same for each polarization. Such difference in PBGs for each polarization motivates development of a photonic-crystal-based polarizer. To be more specific, the goal is to design a type of polarizers called a PBG polarization splitter. The PBG polarization splitter separates one polarization from the other by presenting PBG to one polarization but letting the other polarization passes through it. Since PBGs are dependent on dimensions of 1-D PhC, it is also possible to “switch-on” or “switch-off” the PBGs by reversibly altering the dimensions of the 1-D PhC. If it is possible to characterize how the PBGs change with respect to the dimensions of 1-D PhC, it should be possible to design a tunable polarization splitter. A miniature version of the tunable polarization splitter is preferred given that all applications in sensing, micro-photonics, telecommunication, etc., require small devices. Due to the minute size of the 1-D PhC to be made, using a microactuator to provide a mechanical work is a way to achieve robust dynamic tuning. Details on the microactuator design can be found in Chapter 4. The following subsections focus on designing the tunable polarization splitter in many aspects. The first subsections provide the working principle of the device. The second subsections

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highlights the matrix method for calculating the PBGs at the inclined incidence. Then it provides a primary design of the device based on a PBG map obtained. The third subsection deals with studying of a dynamic tuning profile within the PBG map. The fourth subsection involves designing initial silicon thickness and PDMS thickness, selecting periodicity for the crystal based on a suitable transmittance profile obtained from a finite 1-D PhC of different periodicities. The fifth subsection elaborates on the polarization degree and total transmitted power of the output beam. The sixth subsection gives the numerical study results of the 1-D-PhC-based tunable polarizer.

3.3.1 Working Principle of Mechanically Tunable PBG Polarization Splitter

Fig. 3-3 illustrates a working principle of the tunable PBG polarization splitter. The device consists of a stack of alternate high-low dielectric layers. In this work, silicon and poly(dimethylsiloxane) (PDMS) are used to make the 1-D PhC. Mechanical aspects of the 1-D PhC are investigated in Chapter 4. Nonetheless, it can be stated here that the silicon layers can be assumed to be rigid, while PDMS layers are soft and elastic. Based on the working principle in Fig. 3-3, the 1-D PhC exhibits PBG to an incident TE light at initial state of the device. When the 1-D PhC is compressed, the PBG gradually disappears causing part of the TE light to go through. The change in PBG results from the change in the dimensions of PDMS layers only. Up to a point where the dimension of the 1-D PhC is appropriate, not only the PBG for TE light totally disappears, but the 1-D PhC also allows the TE light to pass through at unity transmittance. Tuning the PBGs of TE light generally affects the PBG of TM light as well. It is then difficult to control the degree of polarization on the transmitted and reflected sites of the 1-D PhC. However, if the angle of incidence is fixed at Brewster’s angle, unity transmittance of TM light can be achieved.

The Brewster’s angle is the angle of incidence at which TM light does not reflect off from the interface. Let \( \theta_i \) and \( \theta_t \) be the angle of incidence and the transmitted angle, respectively. The angle of incidence that satisfies the following condition,

\[
\theta_i + \theta_t = \pi / 2
\]  

(3-5)
is called the Brewster’s angle. By using Snell’s law, \( n_i \sin \theta_i = n_t \sin \theta_t \), the Brewster’s angle is given by

\[
\theta_B = \tan^{-1} \left( \frac{n_t}{n_i} \right)
\]

(3-6)

where \( n_i \) and \( n_t \) are refractive index of transmitted medium and refractive index of incident medium respectively. It can be seen that the Brewster’s angle is a function of the refractive indices. If the material surrounding the 1-D PhC is chosen to be one of the two that comprise the 1-D PhC, once the electromagnetic wave is incident at Brewster’s angle on the 1-D PhC (the leftmost interface as shown in Fig. 3-4), all TM mode will be transmitted into the first layer of the 1-D PhC. The TM mode will propagate further to the interface formed between the first layer (PDMS) and the second layer (silicon), then strikes at the transmitted angle \( \theta_t \), which now becomes the angle of incidence. Because of the Snell’s law, (3-5) is again satisfied and all power of TM mode is further transferred into the second layer which is silicon. The same phenomenon occurs until it reaches the last interface. If silicon is chosen as the surrounding medium on the right-hand side of the PhC, TM mode exits with the same energy as it went in.

Incidence at the Brewster’s angle is desirable not only because the degree of polarization on the transmitted site can be controlled, but also because the reflected light will be purely TE. This is because Brewster’s angle is a property that only depends on materials comprising the interface, not the dimensions of the materials. Therefore, the device gives out polarization-degree-controlled wave at the transmitted site of 1-D PhC, and at the same time gives purely TE wave at the reflected site, which can be used if desired.
3.3.2 Photonic Band Gap at Inclined Incidence

The first task in designing the device is to reveal PBGs of the 1-D PhC that is composed of PDMS and silicon. The 1-D PhC will be excited by a plane wave at a fixed Brewster’s angle at all time, while the thickness of PDMS layers is changed. After considering the appropriate frequency range in which refractive indices of silicon and PDMS are seemingly lossless, the operating frequency of the device is chosen to be 3.5 THz. The refractive index of silicon at this frequency is real and equal to 3.420. However, the refractive index of PDMS is complex with negligibly small loss tangent of 0.01 as reported in [92]. The refractive index is thus taken to be real and equal to 1.549. Having specified the refractive index of silicon and PDMS, (3-6) gives the Brewster’s angle of 23.367°.

It has been shown in the foregoing section that there exist PBGs for this pair of materials. The technique used for earlier PBG calculation was the plane wave method. Although the plane wave method has advantage in quickly revealing many PBGs in one calculation, it is only applicable to an ideal case of infinite stack. Moreover, it is restricted to the case of normal incidence only. Therefore, in this section the matrix method is employed. A common technique for characterizing optical behavior of dielectric layered media is to use a translation matrix. Here, the matrix formalization done by Yeh et al [9] is applied. A dielectric stack consisting of alternate layers of poly(dimethylsiloxane) (PDMS) and silicon is shown in Fig. 3-4. The surrounding medium is silicon. Subscripts 1 and 2 refer to silicon and PDMS.
respectively. The thickness of silicon layer is $a_1$ and that of PDMS layer is $a_2$. Therefore, the lattice period is equal to $a = a_1 + a_2$. Refractive index of silicon is taken to be $n_1$ and that of PDMS is $n_2$ respectively. A coordinate system is placed such that the x-axis is pointing perpendicularly to the dielectric stack, whereas, z-axis is parallel to the plane of the dielectric layers and is pointing upwards as shown in Fig. 3-4. According to (2-11), the electric fields in the surrounding medium near the first interface are related to the electric fields inside the first silicon layer as:

$$
\begin{pmatrix}
E_i \\
E_r
\end{pmatrix}
= (D_1^{-1}D_2P_2D_2^{-1}D_1P_1)
\begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix}
= M
\begin{pmatrix}
E_i' \\
E_r'
\end{pmatrix}
$$

(3-7)

where, for TE case, the coefficient $A, B, C, D$ are:

$$
\begin{align*}
A &= e^{ik_{1x}a_1}
\left[
\cos k_{2x}a_2 + \frac{i}{2}
\left(
\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}}
\right)
\sin k_{2x}a_2
\right] \\
B &= e^{-ik_{1x}a_1}
\frac{i}{2}
\left(
\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}}
\right)
\sin k_{2x}a_2 \\
C &= e^{ik_{1x}a_1}
\left[-\frac{i}{2}
\left(
\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}}
\right)
\sin k_{2x}a_2
\right] \\
D &= e^{-ik_{1x}a_1}
\left[
\cos k_{2x}a_2 - \frac{i}{2}
\left(
\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}}
\right)
\sin k_{2x}a_2
\right]
\end{align*}
$$

(3-8)

The normal wave vector components $k_{1x}, k_{2x}$ are for Si layer and PDMS layer correspondingly. The two wave vectors are related to frequency of incident light $\omega$, refractive index $n$ of medium it resides in, the ray angle in the medium $\theta$, and the speed of light in vacuum $c$, as:

$$
\begin{align*}
k_{1x} &= k_1 \cos \theta_1 = \omega n_1 \cos \theta_1 / c \\
k_{2x} &= k_2 \cos \theta_2 = \omega n_2 \cos \theta_2 / c
\end{align*}
$$

(3-9)

The dispersion relation from (2-18) can be restated again here as:

$$
K(\omega; n_1, n_2, \theta_1, \theta_2) = \frac{1}{a} \cos^{-1}\left(\frac{1}{2} \frac{A + D}{a + a_2}ight)
$$

(3-10)
where it should be noted that the Bloch wave number $K$ is now a function of frequency of the incident light, refractive indices of the two materials, and ray angle in each material. This is made possible via Snell’s law which can also be thought as a conservation law of parallel wave vector. It relates the parallel wave vector to the four new parameters:

$$k_z = k_{iz} = \frac{\omega n_1 \sin \theta_1}{c}$$

$$k_z = k_{zz} = \frac{\omega n_2 \sin \theta_2}{c}$$  \hspace{1cm} (3-11)

Since the angle of incidence is chosen to be the Brewster’s angle, the angle of incidence is not a parameter to study. If frequency is fixed to some value, the problem at hand is to study the transmittance as a function of thicknesses, $a_1$ and $a_2$. Instead of varying the two thicknesses independently, it is more systematic to put the two thicknesses in terms of lattice period and thickness ratio. The thickness ratio can be defined here as the thickness ratio of PDMS thickness to lattice period:

$$\gamma = \frac{a_2}{a_1 + a_2} = \frac{a_2}{a},$$  \hspace{1cm} (3-12)

so that the PDMS thickness and the silicon thickness become a function of the thickness ratio and the lattice period:

$$a_2 = \gamma a \quad , \quad a_1 = (1 - \gamma) a.$$  \hspace{1cm} (3-13)

The unit of length used for this study will be normalized with the free-space wavelength of the operating wave. The advantage of using this normalized unit is that the dimensions of the 1-D PhC can be scaled with the operating frequency. In other words, the results of parametric study are generalized and they can be used to design the device at any length scale. By normalizing relevant units with the fixed operating frequency, $\omega_0 = \epsilon k_0$, the relationship between the speed of the wave, frequency and wavelength can be redefined as follows:
Substitution of (3-9) and (3-14) into (3-8) yields the matrix elements in normalized units as:

\[
A = \Phi \left[ \cos(2\pi\omega'n_2 \cos \theta_2 \gamma a') + \frac{i}{2} \left( \frac{n_2 \cos \theta_2}{n_1 \sin \theta_1} - \frac{n_1 \sin \theta_1}{n_2 \cos \theta_2} \right) \sin(2\pi\omega'n_2 \cos \theta_2 \gamma a') \right]
\]

\[
B = \Phi^{-1} \left[ \frac{i}{2} \left( \frac{n_2 \cos \theta_2}{n_1 \sin \theta_1} - \frac{n_1 \sin \theta_1}{n_2 \cos \theta_2} \right) \sin(2\pi\omega'n_2 \cos \theta_2 \gamma a') \right]
\]

\[
C = -\Phi \left[ \cos(2\pi\omega'n_2 \cos \theta_2 \gamma a') - \frac{i}{2} \left( \frac{n_2 \cos \theta_2}{n_1 \sin \theta_1} + \frac{n_1 \sin \theta_1}{n_2 \cos \theta_2} \right) \sin(2\pi\omega'n_2 \cos \theta_2 \gamma a') \right]
\]

\[
D = \Phi^{-1} \left[ \cos(2\pi\omega'n_2 \cos \theta_2 \gamma a') - \frac{i}{2} \left( \frac{n_2 \cos \theta_2}{n_1 \sin \theta_1} + \frac{n_1 \sin \theta_1}{n_2 \cos \theta_2} \right) \sin(2\pi\omega'n_2 \cos \theta_2 \gamma a') \right]
\]

where \( \Phi = \exp(i2\pi n_1 \omega' \sin \theta_1 (1-\gamma)a') \). The normalized frequency \( \omega' \) is equal to 1, since the operating frequency is fixed. The ray angle \( \theta_1 \) and \( \theta_2 \) are already chosen to be Brewster’s angles. The refractive indices for PDMS and silicon are known. Therefore, the only two parameters left are the normalized lattice period \( a' \) and the thickness ratio \( \gamma \). Now that the matrix elements are formulated, the PBG can be found according to the conditions given in (2-19) & (2-20). A Matlab script used for calculating PBG map is attached in Appendix B and Fig. 3-5 shows the PBG map obtained. On the horizontal axis is thickness ratio \( \gamma \) at a fixed lattice period \( a' \). The shaded regions represent the regions where PBGs exist and thus strong reflection of TE wave is expected. It can be observed that when \( \gamma = 0 \) and when \( \gamma = 1 \), the PBGs vanish, because in both cases the structure become homogeneous consisting only of either silicon or PDMS. In other words, the structure is no longer a PhC. To calculate the lattice period of the 1-D PhC, recall that the frequency used is 3.5 THz which corresponds to the free-space wavelength of 85.65 μm. If the lowest band gap is used to design the device, the 1-D PhC would have a lattice period of about one-tenth to
one-fifth of the free-space wavelength, i.e., about 8 μm – 17 μm according to the tuning profile in Fig. 3-5.

3.3.3 Tuning the One-Dimensional Photonic Crystal

The tuning method used is preferably compression. Since the silicon is much stiffer than PDMS, it is assumed to be rigid. The lattice period is then the function of PDMS thickness only, and it can be given as:

\[ a(a_2) = a_2 + a_1. \]  \hspace{1cm} (3-16)

The thickness ratio can be expressed as:

\[ \gamma(a_2) = a_2/(a_1 + a_2). \]  \hspace{1cm} (3-17)

Substituting for \( a_2 \) from (3-16) into (3-17) leads to:
The normalized silicon thickness is now selected to be 0.07. Since the operating free-space wavelength is equal to 85.655 $\mu$m (3.5 THz), the silicon thickness is equal to 6.00 $\mu$m. Solid curve in Fig. 3-5 represents the tuning profile with normalized silicon thickness $a_1'$ of $=0.07$. The band edge condition is met when the thickness ratio is around 0.636. If the PDMS thickness is designed such that the thickness ratio (as computed from $a_1'$=0.07) is greater than 0.636, the 1-D PhC will initially prohibit TE wave to enter. As the compression progresses and the thickness ratio reduces beyond 0.636, the PBG starts to disappear causing TE wave to be able to pass through it. This silicon thickness of 6.00 $\mu$m and its corresponding profile are used as a starting point to further design PDMS thickness and periodicity of the 1-D PhC in the following subsection.

3.3.4 Electromagnetic Wave in Finite Stratified Media

Based on the obtained PBG map, a primary design of the 1-D PhC was attained. Nonetheless, the device is expected to work such that there is a gradual transition from PBG region to propagating region, so that the degree of polarization can be continuously tuned. Being able to design the 1-D PhC from the previous PBG map alone is not sufficient. For one reason, the PBG map does not provide information of the transmittance of TE mode which is needed in order to completely characterize the output wave. For another reason, the PBG map is calculated on the assumption that the 1-D PhC is infinitely long, which is impossible to fabricate. Nevertheless, the PBG map is required in order to obtain a quick view of PBG property of the periodic dielectric stack.

The matrix method is utilized again to calculate transmittance of the TE wave. If the periodicity of the crystal is equal to $N$, the electric fields at the first-encountered interface of the 1-D PhC are related to the electric fields at the last interface as:

$$\begin{bmatrix} E_i \\ E_r \end{bmatrix} = M^N \begin{bmatrix} E'_i \\ E'_r \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^N \begin{bmatrix} E'_i \\ E'_r \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} E_i \\ E_r \end{bmatrix}$$

(3-19)
The reflection and transmission coefficients of the system are defined by:

\[ r = \frac{E_r}{E_i}, \quad t = \frac{E_t'}{E_i} \]  (3-20)

The expression in (3-20) is similar to the definition of reflection and transmission coefficients obtained from Frensnel’s equations in case of single interface. For the present problem, where there is only light coming out from the right-hand side of the 1-D PhC, \( E'_t \) become 0. Therefore, the two coefficients become:

\[ r = \frac{C'}{A'}, \quad t = \frac{1}{A'} \]  (3-21)

The ratio of reflected power to the incident power (the reflectance, \( R \)) and that of the transmitted power to the incident power (the transmittance, \( T \)) are defined as:

\[ R = |r|^2, \quad T = 1 - R \]  (3-22)

with an assumption that the cross-sectional area of the transmitted beam and that of the reflected beam are approximately equal to that of the incident beam. This assumption may not be precise, because the actual cross-sectional area of the transmitted beam and reflected beam can be larger than the incident one because of multiple reflections within the dielectric stack. Still, most of the power is in fact concentrated to the first reflection and transmission wherein cross-sectional area of the reflected and that of the transmitted beam are the same as that of the incident beam. Tuning by compression is preferred to stretching, because it causes the stack to become thinner and thus suppresses beam expansion. It can be seen that transmittance of the 1-D PhC depends upon the thickness of both silicon and PDMS according to (3-15). Having designed the normalized silicon thickness to be 0.07 (6 μm), the tuning profile is determined. The only parameter left to study is the number of stack of the 1-D PhC. Upon substituting (3-15) into (3-19) and specifying an integer value for \( N \), the reflection and transmission coefficients in (3-21) are attained. By further substitution (3-21) into (3-22), transmittance is eventually found. Fig. 3-6 shows the transmittance profile of TE wave for \( N=2, 3, 4, 5 \). For each \( N \), the PDMS thickness runs from 0 μm - 25 μm.
Table 3-1: Transmittance of TE wave at different periodicities

<table>
<thead>
<tr>
<th>Periodicity N</th>
<th>Initial PDMS thickness at -3 dB (μm)</th>
<th>PDMS thickness at unity transmittance (μm)</th>
<th>Total PDMS thickness reduction (μm)</th>
<th>Total PhC thickness reduction (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>22.0</td>
<td>6.0</td>
<td>16.0</td>
<td>48</td>
</tr>
<tr>
<td>4</td>
<td>15.3</td>
<td>7.8</td>
<td>7.5</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>13.1</td>
<td>8.7</td>
<td>4.4</td>
<td>22</td>
</tr>
</tbody>
</table>

From Fig. 3-5, the band edge condition is met when the thickness ratio is 0.636. This corresponds to the normalized PDMS thickness of 0.122 or 10.5 μm. The transmittance profiles in Fig. 3-6 shows that the transmittance drops very quickly near PDMS thickness of 10.5 μm for all periodicities. As the periodicity increases, the drop becomes sharper and the first transmittance peak seems to approach the thickness value of 10.5 μm, albeit such a peak is not obvious for periodicity of 2. It can then be concluded for the present design that finite 1-D PhC exhibits strong
reflection in PBG region with periodicities as low as 3. The transmittance calculation results in Table 3-1 reveals that transmittance drops to -30dB (as compared to the first-encountered peak near the band edge) at the PDMS thickness around 22 μm for \( N=3 \) and less than or about 15 μm for \( N \) greater than 3. Since smooth transition from PBG region into propagating region is what is needed, it seems that the periodicity of 3 is the best in terms of the steepness of profile. If the 1-D PhC is to be designed such that the initial PDMS thickness is 22 μm (-30dB point) and the final thickness is 6 μm at unity transmittance, each PDMS thickness must be reduced by 16 μm. This amounts to 48 μm for three layers, which is considered a long travelling distance for a microactuator. Nevertheless, based on the capability of the designed microactuator in Chapter 4, travelling distance of 48 μm seems unachievable. The maximum achievable distance is only about 17 μm for the current microactuator design. If this microactuator is to be used, the initial PDMS thickness must be designed at about 12.3 μm. Upon observation of the profile, the TE transmittance at this thickness is 2.76 % which might be considered negligible in practice.

### 3.3.5 Polarization Degree & Total Transmitted Power versus PDMS Thickness

In accordance to general definition of polarization degree [22], the polarization degree is defined here as the ratio of irradiance of TE mode to the total irradiance of the output beam:

\[
P = \frac{I_{TE}}{I_{TE} + I_{TM}}
\]  

(3-23)

Where \( I_{TE} \) and \( I_{TM} \) are the irradiance of TE polarization and TM polarization, and the sum of the two is the total irradiance. If the assumption on equal beam size is presumed, the polarization degree can be expressed in terms of the transmittance of the two polarizations as:

\[
P = \frac{T_{TE}}{T_{TE} + T_{TM}}
\]  

(3-24)

In case where the input wave is unpolarized, the total irradiance is equally divided among the two polarizations. Since the angle of incidence is set to Brewster’s angle, the TM transmittance is always one. Therefore, the polarization degree is only a function of TE transmittance. Fig. 3-7a shows the polarization degree as a function of
PDMS thickness with $N=3$ for the case of naturally unpolarized input wave. Fig. 3-7b represents the corresponding total transmitted power. When the polarization degree is 0, it indicates zero TE irradiance in the transmitted wave. All TE power is directed to the reflected site of the 1-D PhC. When the polarization degree is 0.5, both TE and TM powers on the transmitted site are the same as those before they enter the PhC. All TM power is always transmitted through the 1-D PhC, due to incidence at Brewster’s angle, so that the transmitted never goes below 50% of the input power.

So far the 1-D PhC has been designed. The silicon thickness is designed at 6 $\mu$m as described in subsection 3.3.2 & 3.3.3. In Subsection 3.3.4, the initial PMDS thickness is chosen to be 12.3 $\mu$m, so that the TE power can be almost totally reflected from the PhC. The number of periods of the 1-D PhC is chosen to be 3 so as to obtain smooth and not too steep transmittance profile. Polarization degree and transmitted power are calculated for the case of naturally unpolarized input light. The design workflow can be summarized as shown in Fig. 3-8.
Fig. 3-8: Workflow of theoretical design accompanied by the results for each design step.
3.3.6 Numerical Results

The calculated transmittance of TE mode is verified in this subsection by the finite-difference time-domain (FDTD) method. The FDTD program used is the open-source MIT Electromagnetic Equation Propagation (MEEP) developed at Massachusetts Institute of Technology. The program is capable of performing sub-pixel smoothing for increased accuracy [93]. Fig. 3-9 shows the schematic diagram of the computational cell of the 1-D PhC in silicon substrate. The size of the computational cell is about 685 μm in height and 385 μm in width. The 1-D PhC is placed such that the line normal to the stack makes an angle of 23.367° (the Brewster’s angle) to a horizontal line. A finite vertical source plane of the height of 300 μm is to emit an approximate plane wave in the horizontal direction. The center of this emitted beam will be incident exactly at Brewster’s angle on the 1-D PhC. Due to diffraction, slight variation in the angle of incidence is expected for off-axis
portion of the beam. In order to prevent numerical reflection which naturally occurs in FDTD method, the boundaries of the computational cell are surrounded by perfectly-matched layers (PMLs) with thickness set to half of the free-space wavelength of input source. Three reflected flux planes labeled by number 1-3 and three transmitted flux planes labeled by number 4-6 are placed around the inner boundaries formed by the surrounding PMLs. The reflected flux planes are used to collect the electromagnetic power scattered back due to the presence of 1-D PhC. The transmitted flux planes are used to collect the electromagnetic power passing through the 1-D PhC. The electromagnetic power that does not go through these planes and leaks out at the locations where 1-D PhC intersects with the PMLs is defined as loss. The refractive index of the PDMS is taken to be 2.481 and that of silicon is 3.415. Both refractive indices are assumed to be real. The background material is also the silicon with the same refractive index. The computational grid is uniform and is that of rectangular shape. The grid resolution set in the program is equal to 6. A Gaussian-pulse source with central frequency of 3.5 THz is used throughout the simulation. After the source is turned off the simulation is run further to make sure that resonance phenomenon inside 1-D PhC dies off and all energy incident on the 1-D PhC is collected.

Transmittance of the system is defined as the ratio of the transmitted power in the presence of 1-D PhC to the transmitted power in the absence of the 1-D PhC. Reflectance of the system is defined as the ratio of the reflected power received by the three reflected flux planes in the presence of 1-D PhC to the transmitted power in the absence of the 1-D PhC. If there is no loss, the summation of transmittance and reflectance should be equal to one. Therefore, system loss can be calculated by taking one minus summation of the transmittance and the reflectance. To study the behavior of the 1-D PhC under compression, the PDMS thickness becomes the studied parameter and is varied from 1 μm to 26 μm at an interval of 1 μm. For each thickness of PDMS layer, two simulations are needed to be run. The first simulation is for collecting the transmitted power received by the transmitted flux 4, 5, 6 in the absence 1-D PhC. This transmitted power becomes the total power that would be incident on the 1-D PhC. The second simulation is run for collecting the transmitted
power and the reflected power in the presence of 1-D PhC. An example of the script written in Scheme language for running the simulation is attached in Appendix C. Transmittance, reflectance and loss are calculated according to the aforementioned definition and plotted in Fig. 3-10.

![Fig. 3-10](image)

**Fig. 3-10**: Transmittance, reflectance and loss profile of TE mode as functions of PDMS thickness. Transmittance value at different thickness of PDMS layer represents the amount of energy passing through the 1-D PhC as it is being tuning.

The shape of the transmittance profile in Fig. 3-10 resembles that in Fig. 3-6. In contrast to the theoretical calculation in subsection 3.3.4, the peak transmittance of TE mode of 0.955 is obtained when the PDMS thickness is equal to 6 μm. Maximum loss of about 0.026 is obtained when the PDMS thickness is 1 μm. There are two possible causes for the loss to happen. The first cause is primarily due to numerical error. The second one is due to leakage of electromagnetic power via the two narrow sides of the 1-D PhC. By increasing PML thickness, the error might be suppressed. Accuracy of the results could also be improved by increasing the resolution of the computational grid. However, either increasing PML thickness or grid resolution will lead to higher demand on computational resources and time.
<table>
<thead>
<tr>
<th>Mode</th>
<th>PDMS thickness</th>
<th>6 μm (TE-pass)</th>
<th>8 μm (Transition)</th>
<th>12 μm (PBG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE (E-field shown)</td>
<td>6 μm</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>TM (H-field shown)</td>
<td>8 μm</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Qualitative results for PDMS thickness of 6 μm, 8 μm, and 12 μm can be found in Table 3-2. At 12 μm (the initial structure) TE mode is strongly reflected due to the PBG. At 8 μm TE power is partially reflected and transmitted. Most of the TE power is transmitted at 6 μm. The results seem to be consistent with the earlier calculations. It can be seen that at every thickness, parts of TE wave is guided within the 1-D PhC. This probably contributes to the loss found in Fig. 3-10. In case of TM mode, most of the power is transmitted through the 1-D PhC as expected when angle of incidence is equal to the Brewster’s angle. Nonetheless, partial reflection of TM mode can be
observed. This could be due to variation in angle of incidence at different portions of the beam. Two possible origins of the variation are inability of the source to produce a perfect plane wave and diffraction of the beam prior to incidence on the 1-D PhC. Shifting of the transmitted beam from the axis of the incident beam can be clearly observed for TM mode and the largest shifting occurs when the PDMS thickness is 12 μm. The large shifting at large PDMS thickness may be explained by Snell’s law. When the PDMS thickness is larger, the effective refractive index of the 1-D PhC becomes lesser, leading to higher refracted angle within the 1-D PhC. The overall thickness of the 1-D PhC is also increased since it is contributed by PDMS thickness. Both high refracted angle and high thickness of the 1-D PhC result in larger shifting.

In conclusion, it has been shown in section 3.1 that a 1-D PhC made of silicon and PDMS possesses wide PBGs at normal incidence. An improved plane wave method for calculation of PBG of 1-D PhCs was introduced. The Bragg condition was applied in order to identify the location of the PBGs in the Irreducible Brillouin zone (IBZ). However, Bragg condition itself was unable to provide information on the size of the PBGs. The advantage of using the plane wave method is that it reveals as many PBGs as the number of plane waves per one eigenvalue computation at a k-point. Detailed design of the mechanically tunable photonic-band-gap polarization splitter was given in Section 3.3. The photonic band gaps at inclined incidence of a 1-D PhC made up of silicon and PDMS were obtained by employing a translation matrix method formalized by Yeh. Based on the obtained PBG map, the mechanically tunable PBG polarization splitter was designed. The same technique was used to design the device with a finite periodicity as well as to calculate the transmittance profile of TE mode. Degree of polarization and total transmitted power of the device were calculated for the case of unpolarized input wave. Finite-difference time-domain method was employed to confirm the theoretical calculations. The numerical results show slight discrepancy in the peak transmittance of TE mode which is unity in theory but only 0.955 in FDTD simulation. Numerical errors were discussed. Further FDTD investigation involving higher grid resolution and thicker PMLs layer should be carried out.
CHAPTER 4 MICROACTUATOR DESIGN

In this chapter a microactuator is designed for tuning the flexible one-dimensional photonic crystal that was designed previously in Chapter 3. The first section of this chapter explains about the microactuator selection. An adapted design of the thermal microactuator with Si-polymer composite and its working principle is elaborated in the second section. A design approach is given in the third section, and then the fourth section focuses on applying principle of superposition to analyze the microactuator. The fifth section deals with thermo-elastic deformation of the bimorph microactuator. Stiffness of the microactuator is analyzed in the sixth section. Verification of the theoretical design by finite-element simulation is provided in the seventh section and the finalized design is given at the end of the section.

4.1 Microactuator Selection

Flexible one-dimensional photonic crystal was designed in the previous chapter. Various types of microactuators were explored in the literature in Chapter 2. The type of microactuator to be chosen were considered based on five deciding factors: sufficient output force, sufficient output displacement, dynamical bandwidth, energy efficiency, and compatibility of microfabrication process. Based on those factors, many choices of microactuators in the literature were cut down to two types of microactuator: the electrostatic microactuator and the thermal microactuator with Si-polymer composite structure. The required output force which is the decisive factor between the two types of the microactuator is calculated in the following section.

4.1.1 Calculation of Stiffness of One-Dimensional Photonic Crystal & the Required Compressive Force

According to [94], stiffness of a rectangular plate under axial loading in the direction normal to the plane of the plate can be expressed as:

\[ k = \frac{E A}{d} \]  

(4-1)
where $E$ is the Young’s modulus of the plate, $A$ is the area under loading, $d$ is the thickness of the plate. If two plates of stiffness $k_1$ and $k_2$ are connected to each other in series, the effective stiffness $k_{\text{eff}}$ of the whole assembly can be found by using the following formula:

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$$

(4-2)

The 1-D PhC is nothing but a series of plates connected together, i.e. the alternate silicon and PDMS layers. When normal load is applied on the 1-D PhC of $N$ periods, the effective stiffness becomes:

$$\frac{1}{k_{\text{eff}}} = N \left( \frac{1}{k_{\text{si}}} + \frac{1}{k_{\text{pdms}}} \right)$$

(4-3)

where the subscript $\text{si}$ and $\text{pdms}$ refer to silicon layer and PDMS layer, respectively.

It can be seen from (4-1) that the stiffness is proportional to the Young’s modulus. Since the Young’s modulus of PDMS $E_{\text{pdms}}$ is 1 MP and that of silicon $E_{\text{si}}$ is about 130 GPa [84], the stiffness of silicon plates is much higher than that of PDMS and (4-3) reduces to:

$$k_{\text{eff}} = k_{\text{pdms}} / N$$

(4-4)

Substituting (4-1) into (4-4), and recall from Chapter 3 the dimensions of the 1-D PhC, the effective stiffness becomes:

$$k_{\text{eff}} = \left( E_{\text{pdms}} w_{\text{pdms}} H_{\text{pdms}} / d_{\text{pdms}} \right) / N = 2000 \text{N/m}$$

(4-5)

with $N = 3$, $d_{\text{pdms}} = 6 \mu$m, $w_{\text{pdms}} = 300 \mu$m, $H_{\text{pdms}} = 120 \mu$m. Although the initial thickness of the PDMS layer is 12 μm, the PDMS thickness used in calculation for the effective stiffness is taken from the minimum thickness during operation which is 6 μm. This is to overestimate the stiffness of the 1-D PhC so that when it is used for the microactuator design, the microactuator will have enough capability to compress the 1-D PhC. Since the maximum thickness reduction of each PDMS layer, $\Delta$, is 6 μm, the required compressive force becomes:

$$F_{\text{comp}} = k_{\text{eff}} N \Delta = 36.0 \text{ mN}$$

(4-6)
4.1.2 On the Choice of Microactuator

The tough requirement on the compressive force makes it clear that the microactuator should be that of the thermal type. Consider a design of a lateral comb-drive microactuator for the application. The maximum aspect ratio of fabricated structure, in terms of structure height to minimum feature size of etched area, can be about \((120 \, \mu m)/(2 \, \mu m) = 60\), with aids of standard photolithography process and high-aspect-ratio plasma etching. If an input voltage of about 10V is to be used to activate the comb drive, the number of fingers needed according to [1] would be:

\[
N = \frac{F}{r\varepsilon_0 V^2} = \frac{36.0 \, mN}{60(8.85 \times 10^{-12} \, C^2 / \, N \, m^2)(10V)^2} \approx \frac{677646}{.} \quad (4-7)
\]

The huge number of the interdigitated fingers poses a serious fabrication problem in terms of yield. To fabricate such a large comb with high yield, all the fingers must be perfectly made so as to not touch their counterpart comb. Even changing the target operating voltage to 50V would still require about 27000 fingers, not considering electrical breakdown of air between the fingers. It is thus considered impractical to use the comb-drive microactuator for this task. For the gap-closing electrostatic microactuator, there is a stability criterion that requires the separation between two electrodes to be greater than two-third of the initial separation. This would translate into 54 \, \mu m for initial separation, since the required compressing distance for the 1-D PhC is 18 \, \mu m. To minimize the electrode area required to produce enough output force, the opposing spring force is taken to be that of the 1-D PhC only. The equilibrium equation for force balance on the electrode in (2-48) at the minimum separation leads to:

\[
A \geq \frac{2x_{min}^2 k(x_0 - x_{min})}{\varepsilon_0 V^2} = \frac{2(36 \, \mu m)^2 (36mN)}{\varepsilon_0 (10V)^2} \approx (32.5cm)^2 \quad (4-8)
\]

The required area is too large. Thus the gap-closing microactuator is not suitable for the task. The only type of microactuator left is the thermal microactuator with Si-polymer composite. Note that since the high-stiffness bimorph-like design is adapted for effective force transfer, the designed length of the microactuator must generally be much greater than the tip displacement of the bimorph for the linear analysis to be
valid. Therefore, it could be anticipated that the length of the microactuator would roughly be several hundred microns. The intrinsic drawback of thermal microactuators is slow frequency response. Therefore, the tunable 1-D PhC device would be suitable for applications that do not require rapid tuning. The following section explains the design and the working principle of the thermal microactuator.

4.2 Working Principle of the Polymeric Thermal Microactuator

According to [81], the microactuator makes use of thermal expansion effect of confined polymer. The isotropic thermal expansion coefficients (CTE) of both silicon and polymer are isotropic when they are unconstrained. However, when a layer polymer is confined between two long silicon plates, the effective CTE of the composite structure becomes anisotropic. The CTE of polymeric layer is enhanced in the direction perpendicular to the silicon plates. In the extreme case where Poisson ratio is as much as 0.5, the enhanced CTE in the intended direction can be three times of the isotropic [79].

![Diagram of microactuator design](image)

Fig. 4-1: A new polymeric thermal microactuator design. a) The microactuator is in its inactive state. b) The microactuator in its active state. The upwards deflection of the connector is resulted from larger effective CTE of the expansion units.
A design of the thermal microactuator is adapted from [83] and is shown in Fig. 4-1. To activate the microactuator, a potential difference is maintained across 2 metal pads of each circuit, so that electrical currents are generated and flows through the circuits resulting in Joule’s heating. Since silicon is a good thermal conductor, heat will flow into the fins attached to the silicon backbone. Polymer which is filled between the fins receives the heat from silicon structure and expands. The expansion unit and the silicon backbone work like a bimorph possessing two different coefficient of thermal expansions (CTEs): one of which is purely that of silicon and the other is the effective CTE of the expansion unit. When the microactuator is heated up, bending stress is generated within the whole structure causing deflection of the connector as shown in Fig. 4-1.

To couple the microactuator to the 1-D PhC, the center of the connector is attached to the output waveguide which in turn connected to the 1-D PhC as illustrated in Fig. 4-2. The other side of the 1-D PhC is fixed to substrate. It is worth mentioning here that the output waveguide will move upwards slightly during operation of the device, due to dimensional change of the 1-D PhC. The design of the output waveguide is left to other engineers to figure out how to couple the output electromagnetic wave for their uses.

4.3 Design Approach

It is necessary to investigate the characteristics of the microactuator in Fig. 4-1 in order to properly design a robust device. The microactuator must be able to provide enough both output force and output displacement satisfying the optical requirements obtained in Chapter 3, i.e. the microactuator must be able to output 36 mN of force and travelling distance of 18 μm. Moreover, the temperature at the connection point to the output waveguide should be kept close to room temperature as much as possible. This is to avoid a change in refractive index of both polydimethylsiloxane (PDMS) and silicon, due to temperature rise.

Consider a situation where the microactuator is actuating while being connected the rigid output waveguide. In general both left support and right support of the microactuator and the support of the 1-D PhC will experience horizontal reaction
force, vertical reaction force and reaction moment. In total, there are 9 unknowns to be determined at those supports. This is a statically indeterminate problem. It is impossible to solve for the reactions simultaneously using equilibrium equations alone. Thus, the problem must be simplified first. To do this, elimination of some of the unknowns is carried out by approximating that the 1-D PhC is less stiff than the microactuator so that the vertical reaction force from the 1-D PhC can be neglected. In other words, the stiffness of the microactuator should be reasonably more than 2000 N/m which is the stiffness of the 1-D PhC calculated in subsection 4.1.1. The stiffness in x-direction and flexural stiffness of the 1-D PhC are also assumed to be practically smaller than that of the microactuator due to very soft PDMS that

Fig. 4-2: Illustration of how the microactuator is coupled to the 1-D PhC. a) The 1-D PhC is attached to the output waveguide which in turn connected to the center of the connector. b) During activation, the connector is bent upwards causing the 1-D PhC to be compressed.
comprises the 1-D PhC, so the horizontal reaction force and the reaction moment are also neglected. When the microactuator is operating, the situation is similar to heating one of the two beams (the 1-D PhC and the microactuator in this case) connected in series. If both ends of the beam assembly are fixed, and the heated beam has much higher stiffness than the other, thermal expansion from the heated beam will totally be transferred to the other beam since the heated beam feels no opposing reaction force from the other beam. With the above approximation in place, the problem is now equivalent to removing away the 1-D PhC leaving the microactuator as if coupled to nothing. Therefore, three unknowns from the reactions at the support of the 1-D PhC are approximated to be zero.

Consider again the uncoupled microactuator in Fig. 4-1. Due to mirror symmetry of the problem about vertical line at the center of the microactuator, and due to zero external forces applied, the total reaction force in vertical direction is known and is zero. Had one of the vertical reaction forces been non-zero, the other had to be of equal magnitude, but opposite sign, but this would violate the symmetry of the problem. Up to this point, there are 4 unknowns left. But, the horizontal reaction force and the reaction moment at each microactuator support must be of equal magnitude, due to the symmetry. Hence, there are now 2 unknowns to solve, i.e. the horizontal reaction force $R_x$, the reaction moment $M_0$. It can be seen that this is a statically-indeterminate problem, since the two unknowns cannot be determined from the available equilibrium equations.

Principle of superposition is then employed in order to solve for the two unknown reactions. The analysis is done only on half of the microactuator. For convenience, the expansion unit together with its supporting silicon backbones will be referred to as the bimorph. Both bimorphs, one on the left and the other on the right of the microactuator, will be referred to as the bimorph pair. After dissecting the microactuator into two identical parts, the thin silicon beam attached at the tip of the bimorph will be referred to as the cross-beam. After the unknown reactions are found, the maximum displacement at the center of the tip of cross-beam can then be determined. The maximum displacement of the microactuator can be approximately
equal to the distance of compression of the 1-D PhC, provided that the stiffness of the microactuator is much larger than that of the 1-D PhC.

Having simplified the whole problem, the major tasks of characterizing the microactuator can be summarized into three parts: determining the output displacement by applying superposition principle (section 4.4), thermo-elastic deformation of the bimorph (section 4.5), and stiffness determination (section 4.6).

Fig. 4-3: Dissection of the microactuator along the mirror symmetry plane. When heated, the cross-beam and the bimorph are deformed as shown in the second drawing from top. The apparent deformation is a result of superposition of deformations due to thermal effect, axial reaction force and reaction moment. Corresponding horizontal, vertical and angular tip displacements due to each of the effects are revealed in the third drawing to the fifth drawing after the prismatic constraint is removed.
4.4 Principle of Superposition

4.4.1 Problem Formulation

The superposition technique can be applied at the point where geometric constraints are known. At the middle of the microactuator, the slope and the x-displacement are equal to zero, due to mirror symmetry. The microactuator is then dissected along the mirror symmetry plane for analysis as shown in Fig. 4-3. The x-displacement constraint and the angular displacement constraint are placed as a result of the dissection. By using superposition principle, the apparent x-displacement, which is zero, is equal to summations of x-displacements due to thermal effect ($\delta x_T$), axial reaction force ($\delta x_R$) and reaction moment ($\delta x_M$):

$$\delta x = 0 = -\delta x_T + \delta x_R + \delta x_M$$  \hspace{1cm} (4-9)

The summation of angular displacements gives the apparent angular displacement at the tip which is zero:

$$\delta \theta = 0 = \delta \theta_T - \delta \theta_M$$  \hspace{1cm} (4-10)

Similarly, the apparent y-displacement which is the output displacement of the microactuator is equal to:

$$\delta y = 0 = \delta y_T - \delta y_M$$  \hspace{1cm} (4-11)

The problem at hand is to express the displacements in terms of the two unknown reactions and designed parameters, such as flexural rigidity, lengths and cross-sectional areas of the cross-beam and the bimorph. Due to complexity of the formulae, the displacements due to thermal effect are separately given in section 4.5. The x-displacement due to reaction force $R$ can be expressed in terms of the reaction force and the axial stiffness of the cross-beam and of the bimorph. The effective axial stiffness $k$ is derived in section 4.6.1 thus the x-displacement becomes:

$$\delta x_R = \frac{R}{k}$$  \hspace{1cm} (4-12)

The effective flexural stiffness is derived in subsection 4.6.2 and the angular displacement at tip of the cross-beam can be expressed as:
\[ \delta \theta_M = \frac{M}{k \theta} \]  

(4-13)

Since the bimorph can have much larger flexural stiffness than that of the cross-beam, the tip displacement of the bimorph is assumed to remain unchanged under the effect of the reaction moment \( M \). Thus, the profile of the cross-beam due to reaction moment \( M \) can be given by [94]:

\[ y'_{M,cr} = \frac{Mx'^2}{2E_{cr}I_{cr}}, \]  

(4-14)

where \( x' \) is the local coordinate measured from the left end of the cross-beam. Nonetheless, the maximum y-displacement (downwards deflection), \( \delta y_M \) cannot be determined since its local coordinate at the tip of the cross-beam \( x' \) is unknown. Recalling that the reaction moment causes no change in the total length of the cross-beam, the following equation is yielded:

\[ L_{cr} = \int_{0}^{X} \sqrt{1 + \left( \frac{d y'_{M,cr}}{d x'} \right)^2} \, dx' \]  

(4-15)

Differentiating (4-14) with respect to \( x' \) to obtain the slope and substituting it into the above equation, then normalizing the equation with \( L_{cr} \) yields:

\[ 1 = \int_{0}^{X'} \sqrt{1 + \left( \frac{L_{cr}M}{E_{cr}I_{cr}} \right)^2} \, d \xi \]

\[ 1 = \frac{1}{2} \left( X' \sqrt{1 + \left( X' \right)^2} + \frac{1}{X} \log(\chi X' + \sqrt{1 + \left( X X' \right)^2}) \right) ; \text{where} \quad \chi = \frac{L_{cr}M}{E_{cr}I_{cr}} \]  

(4-16)

It has been assumed in the above equation that the sign of \( \chi \) is positive which makes sense, because the magnitude of the unknown reaction moment \( M \) should be positive so that the reaction moment is opposing the thermally-induced displacement as illustrated in Fig. 4-3. It can be seen that (4-16) involves two unknowns, the reaction moment \( M \) and the normalized distance at tip of the cross-beam \( X' \). The x-displacement and the y-displacement at the tip of the cross-beam can be expressed in terms of the unknown distance \( X' \) as:
Substituting for $\delta x_R$, $\delta x_M$ and $\delta \theta_M$ from (4-17) and (4-13) into (4-9) and (4-10) leads to:

$$0 = -\delta x_T + \frac{R}{k} + (1 - X')L_{cr}$$

$$0 = \delta \theta_T - \frac{M}{k\bar{\sigma}}$$

The second line of (4-16) together with (4-18) form a system of equations engaging three unknowns: $X'$, $M$, $R$. Solving the second sub-equation of (4-18) gives the reaction moment:

$$M = k^\theta \delta \theta_T$$

The value of the reaction moment $M$ is found after substituting into (4-19) the angular displacement $\delta \theta_T$ and the flexural stiffness $m$, which are derived respectively in section 4.5 and section 4.6. Substitution of $M$ into (4-16) and numerically solving it, yield $X'$. Consequently, substituting for $X'$ into the first sub-equation (4-18), then taking the expression for $\delta x_T$ from (4-33) and that for $k$ from (4-34), yields the reaction force, $R$:

$$R = 2k\delta y'_{T,bi} \left[ \frac{1}{3L_{bi}} + \frac{L_{cr}}{L_{bi}^2} \right] + k(1 - X')L_{cr},$$

where

$$\delta y'_{T,bi} = \frac{3L_{bi}^2(1 + 1/\zeta)^2(\alpha_i - \alpha_Z)\Delta T}{(t_p + t_b)(3(1 + 1/\zeta)^2 + (1 + 1/\zeta) n(1/\zeta^2 + \zeta n))},$$

with $\zeta = \frac{t_p}{t_b}$,

and

$$k = \left( \frac{N}{k_a} + \frac{L_{cr}}{E_{st}H_{t_b}} \right)^{-1}.$$
\[ \delta y_M = \frac{mL_{cr}^2 X'^2}{E_{cr} I_{cr} L_{bi}} \delta y'_{T,bi} \]  
where \( k^0 = \left( \frac{N}{k_a^0} + \frac{L_{cr}}{E_{cr} I_{cr}} \right)^{-1} \) \( (4-21) \)

Notice that the y-displacement due to the reaction moment is proportional to temperature rise. Substituting the downwards displacement due to reaction moment and the upwards displacement due to thermal deformation of the bimorph (See (4-29) in subsection 4.5.2) into (4-11), leads to the output y-displacement of the microactuator, \( \delta y \), as:

\[ \delta y = \delta y'_{T,bi} + L_{cr} \sin \left( \frac{2\delta y'_{T,bi}}{L_{bi}} \right) - \frac{mL_{cr}^2 X'^2}{E_{cr} I_{cr} L_{bi}} \delta y'_{T,bi}. \]  
(4-22)

### 4.4.2 Physical Interpretation and Some Sample Calculations

The first term of (4-22) can be thought of as the y-displacement at tip of the free bimorph. The second term represents the amplification of the tip displacement of the bimorph due attachment of the cross-beam. The last term can be considered as the reduction in y-displacement as a result of joining the two cross-beams together. This reduction in y-displacement is caused by the reaction moment that is in turn a consequence of the thermal deflection of the bimorph. Table 4-1 provides the dimensions of a model microactuator. The parameter of interest is the length of the cross-beam. Thickness of the silicon backbone and the thickness of the cross-beam are set to the same values, which is 15 μm. Since the total thickness of the expansion unit together with the backbone is 95 μm which is more than six times of that of the cross-beam, the vertical stiffness of the microactuator subjected to vertical force at the center is estimated to be that of a clamped-clamped beam with thickness equal to that of the cross-beam and length of the whole connector (which is twice the length of the cross-beam). Corresponding estimated vertical stiffness of the microactuator for different lengths of the cross-beam is given in Table E-I of Appendix E. It can be seen that the vertical stiffness is reasonably greater than that of the 1-D PhC, for all lengths of the cross-beam. Therefore, the total output displacement in (4-22) also represents the compression distance on the 1-D PhC. With information from Table
4-1, both axial stiffness along x-direction and flexural stiffness are then calculated. Both the stiffness decreases as the beam length increases. The displacements due to thermal effect are calculated next. Since the cross-beam is only affected by the rotation due to deflection of the bimorph, increasing the length of the cross-beam does not affect the angular displacements $\delta \theta_T$. However, increment in the y-displacement $\delta y_T$ and decrement in the x-displacement $\delta x_T$ are resulted. On the effect of reaction moment, the normalized local x-distance at maximum y-deflection is less than 1 as can be expected from upward bending of the cross-beam. However, the value is almost equal to one and thus it plays a very small part on further reducing the output y-displacement in (4-22). It is noteworthy that increment of the value of $X'$ is not discernable anymore once the length of the cross-beam is more than 20 $\mu m$. This indicates that the reaction moment must decrease, since the flexural stiffness decreases while the angular displacement remains unchanged. The same cannot be said for the reaction force because the axial stiffness of the system is lower but $\delta x_T$ is higher. Nonetheless, increasing $\delta x_T$ seems to dominate lowering the axial stiffness, therefore the reaction force is larger as the length of the cross-beam is increased.

By assuming that the normalized distance $X'$ is equal to one and approximating that 

$$\sin\left(\frac{2\delta y'_{T,bi}}{L_{bi}}\right) \approx \frac{2\delta y'_{T,bi}}{L_{bi}},$$

it can be seen that the total output displacement is proportional to the output displacement of the free bimorph. In an extreme case where $L_{cr}$ approaches zero, the total output displacement reduces to that of the free bimorph alone which seems reasonable. For a fixed length of the cross-beam $L_{cr}$ and for a given deflection $\delta y'_{T,bi}$ of the free bimorph, the y-displacement may be increased by increasing the flexural stiffness of the bimorph, so that the last term in (4-22) is decreased. In the extreme case, the effective angular displacement will approach that of the cross-beam alone (refer to the second line of ), causing (4-22) to be:

$$\delta y = \delta y'_{T,bi} + L_{cr} \frac{2\delta y'_{T,bi}}{L_{bi}} - L_{cr} \frac{\delta y'_{T,bi}}{L_{bi}} = \delta y'_{T,bi} + \frac{L_{cr}}{L_{bi}} \delta y'_{T,bi}$$  \hspace{1cm} (4-23)
which implies linearization condition of the microactuator. The above equation does not provide the optimal condition for the output displacement, because as the flexural stiffness of the bimorph increases, the free tip displacement $\delta y'_{T,bi}$ will decrease. Therefore, an optimal value may be obtained somewhere in the middle of increasing process. Optimization of the output displacement should be investigated further.
Table 4-1: Fixed electromechanical parameters of the thermal microactuator

<table>
<thead>
<tr>
<th>Fixed parameters</th>
<th>$E_{cr}$ (GPa)</th>
<th>$E_p$ (GPa)</th>
<th>$E_c$ (GPa)</th>
<th>$t_b$ (μm)</th>
<th>$t_p$ (μm)</th>
<th>$w_p$ (μm)</th>
<th>$w_f$ (μm)</th>
<th>$N$</th>
<th>$L_{bi}$ (μm)</th>
<th>$H$ (μm)</th>
<th>$V$ (V)</th>
<th>$\Delta T$ (K)</th>
<th>$E_{cr}I_{cr}$ $(10^{-18} \text{ Nm}^2)$</th>
<th>$k_{bi}$ (N/m)</th>
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Table 4-2: Derived parameters and solutions

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<th>Studied parameters</th>
<th>$L_{cr}$ (μm)</th>
<th>$k^0$ (pNm/rad)</th>
<th>$k$ (N/m)</th>
<th>$\delta x_T$ (μm)</th>
<th>$\delta y_T$ (μm)</th>
<th>$\delta \theta_T$ (rad)</th>
<th>$X'^*$ (μm)</th>
<th>$\delta x_M$ (μm)</th>
<th>$\delta y_M$ (μm)</th>
<th>$R$ (μN)</th>
<th>$M$ (pNm)</th>
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4.5 Thermo-Elastic Deformation of the Bimorph

4.5.1 Y-Displacement of Free Bimorph

To obtain the output displacement of the bimorph, physical properties of the expansion unit are averaged and the expansion unit is treated as a homogeneous material. Fig. 4-4 depicts a thermally actuated bimorph and the corresponding mechanical properties of the homogenized expansion unit. The effect of confining polymeric layer between two rigid plates results in new apparent Young’s modulus $E_c$ and CTE $\alpha_c$ of the confined polymeric layer according to [85]:

$$
E_c = E_p / \left[1 + \frac{2\nu_p (\nu_p - \eta \nu_f)}{1-\nu_p + \gamma (1-\nu_p)}\right]
$$

$$
\alpha_c = \alpha_p + \frac{2\nu_p (\alpha_p - \alpha_f)}{1-\nu_p + \gamma (1-\nu_p)}
$$

(4-24)

where Young’s modulus, CTE and Poisson’s ratio of an unconfined polymeric layer are $E_p$, $\alpha_p$, $\nu_p$, respectively. The Young’s modulus and CTE of silicon fin are $E_f$, $\alpha_f$ and $\eta = E_p / E_f$, $\gamma = w_p E_p / w_f E_f$, in which $w_p$ and $w_f$ are the thickness of the polymeric layer and the silicon fin, respectively.

![Fig. 4-4: The expansion unit is treated as a homogeneous material with averaged properties.](image)

Suppose the silicon backbone has CTE $\alpha_2$, Young’s modulus $E_2$, and total thickness $t_2$, while the expansion unit has an apparent CTE $\alpha_i$, Young’s modulus $E_i$, and total thickness $t_1$. According to [85], the Young’s modulus $E_i$ and the apparent CTE $\alpha_i$ of the expansion unit are given as:
\[
E_i = E_f E_c / \left[ (1 - \phi)E_c + \phi E_f \right],
\]
\[
\alpha_i = \zeta \alpha_c + (1 - \zeta) \alpha_f
\]

where \( \phi = w_p / (w_p + w_p) \), \( \zeta = t_1/t_2 \). Then, the tip displacement \( \delta y'_{T,hi} \) of the actuated bimorph can be expressed according to [85] as:

\[
\delta y'_{T,hi} = \frac{3L_{bi}^2 (1 + 1/\zeta)^2 (\alpha_1 - \alpha_2) \Delta T}{(t_1 + t_2) \left[ 3(1 + 1/\zeta)^2 + (1 + 1/\zeta) n (1/\zeta^2 + \zeta n) \right]}
\]

where \( \Delta T \) is the average temperature rise, \( L_{bi} \) is the total length of the bimorph, \( \zeta = t_1/t_2 \), \( n = E_1/E_2 \). Note that the subscript \( hi \) designates the subscripted physical quantity as that of the bimorph. The subscript \( T \) describes the quantity as that due to thermal effect. The prime symbol indicates that the \( y \)-displacement is the local \( y \)-displacement measured with respect to the left end of the bimorph. If the temperature rise profile is known, the average temperature rise \( \Delta T \) of the whole bimorph structure is also known. The average temperature rise \( \Delta T \) is simply the difference between the average temperature and the initial temperature of the structure before actuation:

\[
\Delta T = T_{av} - T_0.
\]

**4.5.2 Displacements at the Center of the Microactuator due to Thermal Effect**

The effect of thermal expansion on the cross-beam is neglected due to very small CTE of silicon. Because the cross-beam is not under bending stress, the cross-beam is straight and makes a slope equal to that at the tip of the bimorph. The local \( y \)-displacement of tip of the dissected cross-beam is thus equal to:

\[
\delta y'_{T,xr} = L_{xr} \sin \delta \theta_T,
\]

where \( \delta \theta_T \) is the tip angle of the bimorph. The global \( y \)-displacement at the center of the microactuator is simply the summation of the two local \( y \)-displacements of both the bimorph (\( \delta y'_{T,hi} \)) and the cross-beam (\( \delta y'_{T,xr} \)). It hence becomes:

\[
\delta y_T = \delta y'_{T,hi} + \delta y'_{T,xr} = \frac{3L_{bi}^2 (1 + 1/\zeta)^2 (\alpha_1 - \alpha_2) \Delta T}{(t_1 + t_2) \left[ 3(1 + 1/\zeta)^2 + (1 + 1/\zeta) n (1/\zeta^2 + \zeta n) \right]} + L_{xr} \sin \delta \theta_T
\]
Note that the global y-displacement is linearly proportional to the length of the cross-beam and to the temperature rise. The tip angle of the bimorph can be determined as follows.

The local y-displacement of tip of the bimorph $\delta y_{T,bi}'$ is approximately equal to square of the total length of the bimorph divided by twice the average radius of curvature $\rho$:

$$\delta y_{T,bi}' = \delta y_{up} = \rho \left(1 - \cos \left(\frac{L_{bi}}{\rho}\right)\right) \approx \frac{L_{bi}^2}{2\rho}, \quad (4-30)$$

where the shape of the deformed cantilever has been assumed to be an arc of a circle of radius $\rho$ and Taylor’s expansion up to second order has been used on the cosine function. Again, the thermal expansion in the axial direction of the bimorph is ignored because the backbone of the bimorph which is purely silicon expands very little. This is why the original length of the bimorph $L_{bi}$ is used in the above equation.

The radius of curvature is thus evaluated to:

$$\rho = \frac{L_{bi}^2}{2\delta y_{T,bi}'} \quad (4-31)$$

The angle at tip $\delta \theta_T$ of the bimorph, thus of the cross-beam, is simply:

$$\delta \theta_T = \frac{L_{bi}}{\rho} = \frac{2\delta y_{T,bi}'}{L_{bi}} \quad (4-32)$$

Having found the tip angle and the radius of curvature, the global x-displacement is found as:

$$\delta x_T = \delta x_{T,bi}' + \delta x_{T,cr}'$$

$$= \left( L_{bi} - \rho \sin \delta \theta_T \right) + L_{cr} \left(1 - \cos \delta \theta_T \right)$$

$$= \left( L_{bi} - \frac{1}{2\delta y_{T,bi}'} L_{bi}^2 \sin \left(\frac{2\delta y_{T,bi}'}{L_{bi}}\right) \right) + L_{cr} \left(1 - \cos \left(\frac{2\delta y_{T,bi}'}{L_{bi}}\right)\right) \quad (4-33)$$

$$\approx \frac{2\delta y_{T,bi}'}{3L_{bi}} + \frac{2\delta y_{T,bi}'}{L_{bi}^2} L_{cr} = 2\delta y_{T,bi}' \left(\frac{1}{3L_{bi}} + \frac{L_{cr}}{L_{bi}^2}\right)$$
4.6 Determination of Stiffness of the Bimorph and the Cross-beam

4.6.1 Axial Stiffness

Subsection 4.4.1 requires determination of axial stiffness of half of the microactuator. The total axial stiffness \( k \) of the dissected microactuator can be related to that of the cross-beam, \( k_{cr} \), and that of the bimorph, \( k_{bi} \), as:

\[
\frac{1}{k} = \frac{1}{k_{bi}} + \frac{1}{k_{cr}}
\]  \hspace{1cm} (4-34)

Since the cross-beam is homogeneous and made of silicon, its corresponding parameters can be evaluated immediately from designed parameters:

\[
k_{cr} = \frac{E_{si} t_b H}{L_{cr}}
\]  \hspace{1cm} (4-35)

where \( E_{si}, H, t_b, L_{cr} \) are the Young’s modulus of silicon, the height of the structure, the thickness of the silicon backbone, and the length of the cross-beam, respectively. On the other hand, the bimorph is heterogeneous and is comprised of the silicon backbone, the silicon fins, and the confined polymeric layers. Its properties can be expressed as a resultant one that represents the whole. Fig. 4-5 illustrates how the axial stiffness of the bimorph can be calculated from the smaller parts comprising it. The stiffness of the smallest unit, i.e. \( k' \), \( k'' \) can be obtained in terms of their corresponding Young’s modulii and dimensions as follows:

\[
k' = \frac{E_c t_b H}{w_p}, \quad k'' = \frac{E_c t_p H}{w_p}
\]  \hspace{1cm} (4-36)

Recall from section 4.5 that \( E_c \) is the Young’s modulus of the confined polymeric layer. The stiffness of the active portion, \( k_a \), (see Fig. 4-5) is simply the summation of the two smallest units, i.e., \( k_a = k' + k'' \). The stiffness of the inactive portion, \( k_i \), is the stiffness of the silicon fin together with the stiffness of a portion of the backbone that attached to it. The stiffness of one expanding unit is the effective stiffness resulted from connecting the active portion and the inactive portion in series. The stiffness of the whole bimorph is the effective stiffness that comes from connecting all expanding units in series. Since each expanding unit has the same stiffness, the
stiffness of the bimorph pair is simply the stiffness of one expanding unit divided by the number of the expanding units constituting the bimorph. In short, the effective axial stiffness of the bimorph can be expressed as:

\[
k_{bi} = \left[\frac{1}{N} \left(\frac{1}{k_a} + \frac{1}{k_i} \right)^{-1} \right]^{-1}
\]  

(4-37)

where \(N\) is the number of expanding units. The stiffness of the last inactive portion has been included as the last term inside the bracket. It can be seen that the stiffness of the inactive portion is much more than that of the active portion due to very high Young’s modulus of silicon (about 40 times of the SU-8 polymer). Therefore, all the terms involving the reciprocal of the stiffness of the inactive portion can be ignored. The stiffness of the bimorph can then be simplified to:

\[
k_{bi} = \frac{k_a}{N}
\]  

(4-38)
The stiffness of the bimorph pair is therefore inversely proportional to the number of the expanding units, but proportional to the stiffness of the active portion. Upon considering the stiffness of the active portion, it can be seen that the stiffness mainly comes from the backbone portion, due to much lower Young’s modulus of the polymer as compared to that of the silicon.

### 4.6.2 Flexural Stiffness

According to [95], the flexural stiffness of a beam is given by:

$$ k^\theta = \frac{EI}{L}, $$

(4-39)

with $EI$ being the flexural rigidity and $L$ being the length of the beam. The flexural stiffness of the cross-beam can be found readily by using the above equation:

$$ k^\theta_{cr} = \frac{(EI)_{cr}}{L_{cr}} = \frac{E_{si}l_{cr}}{L_{cr}}. $$

(4-40)

However, it is more complicated for the bimorph. The following argument attempts to approximate the effective stiffness of the bimorph. Since the flexural stiffness is inversely proportional to the length of the beam, it obeys the same rule for the effective axial stiffness of springs in a series:

$$ \frac{1}{k^\theta_{\text{eff,series}}} = \sum_i \frac{1}{k^\theta_i}. $$

(4-41)

As a result the first equation of subsection 4.6.1 also applies in this case. The rule for parallel connection cannot be used for case of parallel connection here. Simply adding the stiffness is not correct because the flexural rigidity is not linearly proportional to the thickness of the beam. However, it is proved in Appendix D that by using the modified flexural rigidity, the same rule can be applied. Therefore, the effective flexural stiffness of beams in parallel connections, $k^\theta_{\text{eff,parallel}}$, becomes:
\[ k_{\text{eff,parallel}} = \sum_i k_i = \sum_i \frac{E_i I_i}{L_i} \]  

(4-42)

where the area-moment of inertia \( I_i \) is equal to

\[ I_i = I_{0,i} + y_{n,i}^2 A_i \]  

(4-43)

in which \( I_{0,i} \) is the area-moment of inertia measured from the neutral plane of beam \( I_i \), \( y_{n,i}^2 \) is the distance from the neutral plane of the beam \( i \) to that of the neutral plane of the whole assembly, and \( A_i \) is the cross-sectional area of beam \( i \). Break-down of connection of all the springs can be visualized by using the same figure in Fig. 4-5. The effective flexural stiffness shares the same formula in (4-37) and thus becomes:

\[ k_{bi}^\theta = \left[ \frac{1}{N} \left( \frac{1}{k_a} + \frac{1}{k_i} \right) \right]^{-1} + \frac{1}{k_i^\theta} \]  

(4-44)

Since the flexural stiffness of the inactive portion is much larger than that of the active portion, the effective stiffness reduces to:

\[ k_{bi}^\theta \approx \frac{k_a^\theta}{N} \]  

(4-45)

where \( k_a^\theta \) is equal to the sum of the flexural stiffness of the polymeric layer and the silicon backbone attached to it. In other words, it is equal to:

\[ k_a^\theta = (EI)_b w_p + (EI)_p w_p = \frac{E_s i (I_{0,b} + y_b^2 H_t_b)}{w_p} + \frac{E_c (I_{0,p} + y_p^2 H_t_p)}{w_p} \]  

(4-46)

By comparing \( k_a^\theta \) with \( k_{cr}^\theta \) it can be seen that \( k_a^\theta \gg k_{cr}^\theta \). As the bimorph is connected to the cross-beam in series, the effective stiffness of the whole assembly, \( k^\theta \), may be approximately equal to that of the cross-beam alone, provided that \( N \) is not large:

\[ k^\theta = \left( \frac{1}{k_{bi}^\theta} + \frac{1}{k_{cr}^\theta} \right)^{-1} \approx k_{cr}^\theta \]  

(4-47)
4.7 Verification of Design – Study of the Length of the Cross-beam

In this section, a finite-element simulation is presented. The objective of the simulation is to confirm the theoretical calculation in section 4.4 with the numerical method. The dimensions of the microactuator are taken from Table 4-1 and relevant mechanical properties are given in Table 4-3 & Table 4-4. The microactuator is consisted of silicon skeleton, SU-8 expander, thin-film Au heater, while the 1-D PhC is consisted of alternate Si-PDMS bilayers. Note that the resistivity of Au and the convection coefficient of air, which are important for equilibrium temperature of the system, are interpolated with temperature to simulate a more realistic situation. The points of interpolation are given in Table 4-4.

The studied parameters are the length of the cross-beam and the thickness of polymeric expander. The length of the cross-beam was varied from 20 μm to 100 μm at interval of 20 μm, while the thickness of the polymer was chosen at 40 μm, 60 μm, and 80 μm. In the simulation, an electric potential difference of 1.65 V was applied across both end of the Au heater causing the heat to be generated along the circuit. The boundary conditions were set on the supports of the bimorph where all kinematic degrees of freedom were set to zero. The temperature was set 27˚C on the boundaries. Natural convection coefficient of air was also set. Therefore, the equilibrium temperature should be reached when the heat generation due to the heater is balanced with heat dissipation via thermal conduction at the bimorph supports and via natural convection to surrounding air. To minimize computational resources, only half of the microactuator model was generated and the mirror symmetry boundary condition was applied on the central plane of the microactuator.

The simulation was run for each of the values of the length of the cross-beam and the polymer thickness. The results are plotted and shown in Fig. 4-6. Since the output displacement versus the length of the cross-beam for all polymer thickness are linear, it can be concluded, using (4-22) & (4-23), that the calculated flexural stiffness of the bimorph seems to be much larger than that of the cross-beam. On the other hand, the FEM results are nonlinear with the length of the cross-beam. However, as the polymer thickness increases causing increase in flexural stiffness,
Table 4-3: Thermo-mechaical properties of Si, PDMS, SU-8, and gold

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (MPa)</td>
<td>Silicon</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>PDMS</td>
</tr>
<tr>
<td>Thermal conductivity (pW/[μm][°C])</td>
<td>SU-8</td>
</tr>
<tr>
<td>CTE (°C⁻¹)</td>
<td>Gold (Au)</td>
</tr>
<tr>
<td>Electrical resistivity ([Ω][μm])</td>
<td></td>
</tr>
</tbody>
</table>

Table 4-4: Temperature-dependent resistivity of Au and convection coefficient of air

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Resistivity of Au ([Ω][μm])</th>
<th>Convection coefficient of air (pW/[μm]²[°C])</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>2.271e-14</td>
<td>197</td>
</tr>
<tr>
<td>127</td>
<td>3.107e-14</td>
<td>254</td>
</tr>
<tr>
<td>227</td>
<td>3.970e-14</td>
<td>300</td>
</tr>
<tr>
<td>337</td>
<td>4.870e-14</td>
<td>352</td>
</tr>
<tr>
<td>447</td>
<td>5.820e-14</td>
<td>393</td>
</tr>
</tbody>
</table>

-the profile becomes more linear. Again, this can be explained by (4-22) & (4-23), except that the theoretical flexural stiffness seems to grow faster so the theoretical profile becomes linear before the FEM’s does. It can be seen that all the calculated output displacements are in the same order of magnitude as those obtained from FEM. Also, the results from both methods are consistent in that they both increase with the beam length. The discrepancy which measures the ratio of the difference between the two outputs to the FEM output (see Table E-I in Appendix E), decreases when the polymer thickness is decreased. For each discrepancy profile, the value also
decreases with the length of the cross-beam. This is desirable because higher output displacement can be obtained at higher accuracy by simply increasing the length of the cross-beam. The microactuator design with $t_p = 60$ µm are used to further study the effect of the length of cross-beam at higher values, and both theoretical output and FEM output are given in Table E-I. It can be seen in the table that the discrepancy generally gets larger after the length of the cross-beam is more than 80 µm. However, the theoretical method assumes that the linear stiffness in y-direction of the microactuator must be reasonably greater than that of the 1-D PhC. The discrepancy is then not surprising since the linear stiffness in y-direction of the microactuator shown in Table E-I reduces to a value comparable to the stiffness of the 1-D PhC when $L_{cr}$ is 300 µm and goes below that of the 1-D PhC from 400 µm onwards. This

Fig. 4-6: Comparison between the theoretical results and the FEM results where the length of the cross-beam is the studied parameter. Three plots are shown for three different polymer thicknesses ($t_p$) of 40 µm (a), 60 µm (b), 80 µm (c). Discrepancy between the theory and the FEM are shown in d) for all polymer thicknesses.
hypothesis is also consistent with the sign of the value of the discrepancy, i.e., the theoretical method tends to overestimate the output displacement because the opposing vertical reaction force from the 1-D PhC is ignored.

Based on the understanding from the above study, the finalized design was obtained after adjusting the dimensions of the microactuator in a trial-and-error manner. Eventually the microactuator that can produce output displacement of 20 μm at applied voltage of 1.85 V was obtained. The number of the expanding cells is 12 in contrast to 10 cells for the model used in the foregoing study. The length of the cross-beam is 360 μm and the polymer thickness is 45 μm. Other dimensions are the same as the studied model. As shown in Fig. 4-7, the maximum temperature found on the microactuator structure is about 176˚C, or ≈ 24 °C below the glass transition temperature, and ≈ 200˚C below degradation temperature of SU -8 polymer. The problem that can be addressed here is the high temperature at the output waveguide. Design improvement for the temperature profile should be investigated further.

Having designed the microactuator, both the 1-D PhC and the microactuator can now be put together to form a device. Fig. 4-8 illustrates how the fabricated device may look like. The device shown is fabricated into a silicon substrate (gray). The 1-D PhC is located at the center of the image. The inset shows the 1-D PhC consisted of the PDMS blocks (shown in blue color) and the silicon slabs. The microactuator is located on both sides with the 1-D PhC. The microactuator is comprised of the silicon skeleton, the SU-8 expanders (shown in pink), and the Au heater (shown in yellow in the inset). The 1-D PhC is between the substrate on the left-hand side and the substrate on the right-hand side. In fact the substrates can be considered as slab waveguides. If incoming light (with both TE mode and TM mode) guided inside the substrate is incident on the 1-D PhC as shown, the polarization degree of output light can be varied by the 1-D PhC according to the dimensions of the 1-D PhC (Refer to Fig. 3-7a). The microactuator can compress the 1-D PhC to the left or let it go back to initial position. The reflected light is always purely TE since the incidence angle is at the Brewster’s angle. The reflected TE mode might be used as an optical feedback that provides information on the polarization degree of the output light.
Fig. 4-7: Nodal temperature profile of half of the microactuator. Maximum temperature of 176.216 °C can be observed at the Au heater. Most of the SU-8 polymer experienced uniform temperature of about 130 °C. The temperature of the central waveguide is around 110 °C.

Fig. 4-8: The mechanically tunable PBG polarization splitter.
CHAPTER 5 MICROFABRICATION OF 1-D PHOTONIC CRYSTAL DEVICE

This chapter deals with microfabrication processes for the mechanically tunable one-dimensional photonic crystal (1-D PhC) device. The idea of fabricating the device was to come up with a monolithic microfabrication process for a large-scale production. The microfabrication process were divided into four tasks of building four different features of the device, i.e., i) the suspended silicon structure, ii) the electric heater for the device activation, iii) the polymeric expander, and iv) the tunable 1-D PhC. Each feature is associated with the main microfabrication processes which are discussed in different sections as shown in Fig. 5-1. Both conventional and unconventional microfabrication processes were employed to make the device. The first two features were quite common in MEMS and their microfabrication processes were readily available in the literature [96]. Nonetheless, a new method of making suspended silicon structures for this particular device is reported here. A technique of embedding SU-8 resin into a silicon structure were recently developed as reported in [84]. On the other hand, a technique of patterning polydimethylsiloxane (PDMS) selectively into a silicon structures had not been explored earlier and was thus developed in this work.

As indicated in Fig. 5-1, Section 5.1 elaborates on the etching and releasing techniques for the suspended silicon structures. The structures were made into a suspended Si-PDMS hybrid structure to demonstrate the compatibility of the microfabrication processes with the polymer filling processes. Section 5.2 discusses in detail about microfabrication processes for the selective filling with PDMS. The processes were employed to fabricate a thermal microactuator made up of silicon and PDMS. The whole series of microfabrication processes for the tunable 1-D PhC device is presented in section 5.3 where the other two important microfabrication processes are presented in the section. The microfabrication results and discussions are accompanied in each section.
5.1 Microfabrication Process for Si-PDMS Hybrid Microstructures*

Perhaps the most important work in making the mechanically tunable device is to integrate the microfabrication process for suspended silicon structure to the polymer filling processes. To demonstrate how it could be done, Si-PDMS hybrid structures (illustrated in Fig. 5-2) were fabricated. The silicon structures (in gray color) were of cantilever types embedded in suspended PDMS blocks (shown in transparent aquamarine color). Two designs of the cantilevers were used for the microfabrication. The first design (Design 1) was a straight cantilever with a ring-like structure at the cantilever tip, and the second design (Design 2) was a meandering cantilever.

Generally, unless the expensive silicon-on-insulator (SOI) substrate is used, a form of deep silicon etching is involved in releasing a suspended silicon structure. The two most common processes in MEMS for deep silicon etching are the deep reactive-ion etching (DRIE) and the anisotropic wet etching using KOH base. Since the proposed 1-D PhC device structure was comprised of PDMS and SU-8, exposures of the polymers to etching agents in the etching processes had to be avoided. It was also necessary to not expose the polymers to high-temperature

* This work was presented as *Development of fabrication process for PDMS patterning for Si-PDMS hybrid microstructures* in The 4th International Conference on Technological Advances of Thin Films and Surface Coatings, 13-16 July 2008, Singapore.
processes, or to corrosive chemicals in other subsequent processes, in order to preserve the mechanical and optical properties of the polymers. The best way to overcome these problems was in fact to push the filling processes towards the end of fabrication workflow. However, performing the filling processes last means that the filling processes had to be done after the suspended silicon structure was fabricated. The problem then was about how to mold the liquid polymers into the suspended structure when there was nothing to support the polymers. It was for those reasons that the new fabrication processes were developed to surmount the obstacles. It was demonstrated that a Si-PDMS hybrid structure with a thickness more than 100 μm could be achieved via the monolithic fabrication processes.

Fig. 5-2: The Si-PDMS hybrid structures. Silicon cantilevers were fabricated with two designs: the straight cantilever design with ring-like tip (Design 1) and the meandering cantilever design (Design 2). A perspective view of Design 1 showing its cross-section reveals the difference in thickness of the suspended membrane and that of the substrate.
5.1.1 Microfabrication Processes

A series of the microfabrication processes are shown in Fig. 5-3. The first fabrication process is thermal oxidation of silicon substrate. A clean double-side-polished silicon wafer of thickness of 450 μm was first put in a furnace to grow a SiO$_2$ layer. The SiO$_2$ layer was created via a dry oxidation process in pure O$_2$ environment. After the process, a 1.5-μm-thick SiO$_2$ layer on each side of the silicon substrate was achieved (Fig. 5-3i). The back side of the substrate was then coated with AZ-9260 positive photoresist from AZ Electronic Materials. After pre-baking, the substrate was aligned with the first photolithographic mask and then exposed with an I-line UV source. After developing and obtaining the photoresist pattern, the substrate was etched in reactive CF$_4$ plasma to remove the oxide layer. After removing the photoresist, the SiO$_2$ pattern was attained as illustrated in Fig. 5-3ii. The SiO$_2$ pattern on the back side of the substrate became a hard mask against KOH base in wet anisotropic silicon etching. The back-side cavities were created as shown in Fig. 5-3iii after the etching. The wafer was sent into a furnace to grow the second SiO$_2$ layer (Fig. 5-3iv). The grown SiO$_2$ layer had a target thickness of 100 nm on the side walls and the bottom of the back-side cavities, while at other part of the substrate it was a little more than 1.5 μm because the SiO$_2$ simply grew further on top of the first
SiO₂ layer. The 100-nm-thick parts of the second SiO₂ layer overlaying the back-side cavities were important parts of the mechanical support of the suspended silicon structure. Although the second SiO₂ in fact served no purpose in strengthening the remaining thickness of the substrate, it was created for the purpose of preventing liquid PDMS mixture to fall through from the front side of the wafer. The SiO₂ layer at the front side was removed by reactive etching using CF₄ plasma (Fig. 5-3v). The back side of the substrate was later filled with the AZ-9260 photoresist and the substrate was baked until the photoresist was completely dried (Fig. 5-3vi). Then SU-8 50 negative photoresist was spin-coated on the back side such that it covered the whole surface of the back side and encapsulated the AZ-9260 photoresist. The substrate was pre-baked until the SU-8 photoresist became totally dried, then the back side of the substrate was exposed everywhere with the I-line UV to cross-link the SU-8 photoresist. So far the substrate became almost equivalent to a SOI substrate: it has a silicon device layer and a sacrificial layer made of the second SiO₂ layer and the AZ-9260 photoresist (Fig. 5-3vii). The SU-8 photoresist served as a protective layer for the AZ-9260 photoresist which could be susceptible to other chemicals in other processes. The second photolithographic mask was used to create an AZ-9260 photoresist pattern on the front side of the substrate. The patterned photoresist worked as a mask against the reactive SF₆ in the DRIE process. The DRIE process was carried out until the second SiO₂ layer was met (Fig. 5-3viii). Afterwards, a bubble-removed PDMS mixture with the ratio of PDMS pre-polymer (Sylgard 184 from Dow Corning) to curing agent = 10:1 by weight was poured on top of the front side. The wafer was left for a few minutes in order to let the PDMS mixture to flow into the etched silicon structure. The whole wafer was put inside a vacuum chamber to remove any air bubbles trapped between the silicon structures. Then a plastic blade was used to scrape off the excess PDMS mixture on the front surface of the wafer (Fig. 5-3ix). A baking oven was heated up to 60°C and the substrate with the uncured PDMS mixture was put inside it. The PDMS mixture was cured after baking in the oven for 40 mins. The SU-8 layer was broken manually to create openings to the AZ-9260 photoresist in the back-side cavities (Fig. 5-3x). Note that chemical-mechanical polishing (CMP) could have been implemented to remove
the SU-8 systematically as well. The use of CMP process to remove cross-linked SU-8 polymer was demonstrated in the laboratory in an earlier work [97]. The wafer was rinsed in acetone, isopropyl alcohol, and deionized water, respectively, to remove the AZ-9260 photoresist. The back side of the substrate was further cleaned by O₂ plasma and the structure was completely released after the second SiO₂ layer was removed by a brief etching with CF₄ plasma.

5.1.2 Microfabrication Results and Discussions

A SEM image of a cutaway structure just before the removal of the SU-8 layer (Step x) is presented in Fig. 5-4a. The image clearly reveals each layer of the materials. The cross-linked SU-8 layer is seen attached at the bottom of the substrate. The layer is covering the whole opening of the back-side cavity. The AZ-9260 photoresist was formed inside the cavity such that it appears like a pyramid in the image. On top of the AZ-9260 photoresist is a block of the cured PDMS polymer. An inset in Fig. 5-4a is showing the magnified Si-PDMS hybrid structure. The PDMS block is appearing white in the image, because electrons previously accumulated on the non-conductive PDMS were repelling other electrons incident on the object from the electron gun of the SEM. Seen embedded in the PDMS block is the silicon structure that looks like a meandering cantilever (Design 2 in Fig. 5-2). Another structure is shown in Fig. 5-4b and Fig. 5-4c. The structure was successfully released. The PDMS block in the two images so firmly adhered to the substrate and to the silicon cantilever that it did not fall down. A sagging profile on top of the PDMS block can be clearly observed in Fig. 5-4c. The profile may be caused by capillary effect of uncured liquid PDMS mixture and by shrinkage of the PDMS during the curing. A cross-sectional view in Fig. 5-4d represents a suspended structure hung on the substrate. A top view of another structure that was not cut away is shown in Fig. 5-4d. Note that the filling process would result in every trench filled with PDMS. However, in order to tune the 1-D PhC efficiently, the microactuator must be able to move freely. Therefore, a technique to selectively fill the trenches with the PDMS was later developed and it is described in the next section.
Fig. 5-4: The fabricated Si-PDMS hybrid structures. a) A cutaway view of the substrate just before removing the SU-8 layer. b) A release structure viewed from the back side. (c) The same structure looked from the front side. d) A cross-sectional view of the substrate showing a structure suspended on the substrate. e) A top view of an unbroken structure looked from the front side of the substrate.
5.2 Selective Filling with PDMS for Thermal Microactuator

Polydimethylsiloxane (PDMS) not only is the material that could comprise the tunable 1-D PhC but also possesses some desirable mechanical properties for the thermal microactuator, i.e. high Poisson ratio, high thermal resistance and high electrical resistance. Replacing the SU-8 resin originally used for thermal expanders ([79, 82-83, 85, 98]) may help simplify the overall microfabrication processes of the 1-D PhC device, because the 1-D PhC can then be fabricated simultaneously with the microactuator. Unfortunately, PDMS is difficult to pattern as it is not photosensitive; thus, the well-established UV photolithographic technique is not applicable.

In this section, microfabrication processes for selective Filling with PDMS were developed. The mold of the filling was a silicon skeleton of a thermal microactuator. Design of the microactuator was chosen to be the meandering type that was proposed in [78]. A top view, a side view and a perspective view of the design are shown in Fig. 5-5. The meandering design offered a lower skeleton stiffness as compared to other designs, such as the bimorph type. Hence, displacement of the skeleton due to thermal expansion may be observed easier. However, the meandering design may not be suitable for compressing the stiff tunable 1-D PhC. Nonetheless, the development of the microfabrication processes will enable one to build a proper Si-PDMS hybrid microstructure which may have potential applications in MOEMS.

Fig. 5-5: The thermal microactuator model used in the selective filling process using PDMS.
5.2.1 Microfabrication Processes

The microfabrication processes are illustrated step by step in Fig. 5-6. A SOI substrate was utilized in this study for simplicity of fabrication. The thickness of the device layer, the silicon dioxide (SiO₂) layer, and the handle layer were 60 μm, 1 μm, and 345 μm, respectively. In Fig. 5-6i, a photoresist pattern defined by the first photolithographic mask was created by photography. The applied photoresist was the AZ-9260 positive photoresist from AZ Electronic Materials. The photoresist was used as a mask for deep reactive-ion etching (DRIE) in the next step. The DRIE was carried out until the underlying SiO₂ layer was met (Fig. 5-6ii). The etched silicon became the skeleton of the microactuator. Afterwards, the silicon skeleton was released by submerging the whole SOI wafer in a buffered oxide etch (BOE) solution in order to etch SiO₂ beneath the skeleton (Fig. 5-6iii). Next, an electric Au heater was created by sputtering (Fig. 5-6iv). The Au layer thickness is in order of tens of nanometers. Since the microactuator structure had been released and isolated from
other part of the substrate, the thin Au layer did not cause a short circuit in the device. With the second photolithographic mask, another photoresist layer was patterned for use as a mold for PDMS. Openings were created at the gaps of the skeleton structure so that they could be filled with the PDMS (Fig. 5-6v). The PDMS used was the Sylgard 184 from Dow Corning. Before pouring the PDMS mixture onto the wafer, PDMS pre-polymer was mixed with a curing agent at the weight ratio of 10:1. After pouring the PDMS, the wafer was put into a vacuum chamber to allow bubbles in the PDMS mixture to escape. Afterwards, the excess PDMS on top of the wafer was removed by sliding a blade across the substrate (Fig. 5-6vi). The photoresist not only worked as the PDMS molds, but also helped protect the underlying structure from being scratched by the blade. Subsequently, the PDMS was cured for 72 hours at room temperature. The curing process could be expedited by raising the curing temperature, but more shrinking of PDMS might result. Finally, the photoresist was removed by rinsing with acetone. The excess PDMS on the photoresist was also lifted off, leaving the substrate with the thermal microactuators (Fig. 5-6vii).

5.2.2 Microfabrication Results and Discussions

The microfabrication of the microactuators was carried out successfully as planned in Subsection 5.2.1. Fig. 5-7a shows a released silicon skeleton after the BOE. An electrode was fabricated at each end of the meandering skeleton as shown in the figure. The width of the skeleton was roughly 30 μm. The inset shows a side view of the skeleton which is suspended above the substrate. The gap between the substrate and the skeleton can be seen by observing the shadow of the skeleton on the substrate and the “electron light” coming through the gap. Since the gap was created by etching the SiO₂ layer, the gap size was approximately equal to the thickness of the SiO₂ layer. Note that the suspended silicon skeleton was supported by the two large electrodes since the SiO₂ layer beneath the electrodes was not completely removed. A completed prototype can be viewed in Fig. 5-7b. The microactuator was comprised of the two electrodes and the suspended silicon skeleton filled with cured PDMS. The top surface of both the electrodes and the skeleton was conductive as it was sputtered by Au. This thin layer of Au worked as an electric heater that could raise the temperature of the structure and cause a thermal expansion.
Fig. 5-7: The thermal microactuator. a) The released silicon skeleton before being filled with PDMS. An air gap below the structure can be seen in the inset. b) The fabricated thermal microactuator. The microactuator was made up of the two electrodes and the PDMS-filled silicon skeleton. c) The skeleton was partially filled, because there was photoresist left at the bottom of the trenches. d) A close-up view of the fill skeleton showing the sagging profile at the top of PDMS block.
The PDMS blocks between the skeleton can be seen in Fig. 5-7c. The large gaps beneath them were intentionally created, so that the microactuator would not stick to the substrate. By not fully developing the AZ-9260 photoresist mold, the remaining photoresist prevented the PDMS mixture from contacting the substrate during the filling process. After the PDMS was cured, the photoresist was removed, leaving the gaps below the PDMS as shown. An improvement of the technique may be to control the thickness and the profile of the remaining photoresist so that the gaps could be minimized, but not vanished. Sagging profiles at the top of the PDMS blocks can be clearly observed in Fig. 5-7d. The sagging profile is undesirable because the profile could lead to performance deviation from the theoretical prediction, where the polymeric expander was usually assumed to be a perfect rectangular block [78]. A technique for eliminating the profile should be further investigated.

The microactuator was tested under a probe station (Cascade). Expansion of the PDMS was observed at a DC voltage of 20 V supplied across the two electrodes. As the voltage was increased, the greater expansion was observed. However, as the voltage was beyond about 40 V, the PDMS seemed to experience shrinkage after the voltage was switched off, and parts of the PDMS were blackened. The result indicates that the PDMS might have undergone too high a temperature rise that exceeded the degradation temperature of 200 °C [84]. The microactuator was also activated by an AC voltage. By observation under a microscope, it was found that the PDMS expanders could respond up to about 22 Hz. Nonetheless, the silicon skeleton hardly moved throughout the testing. One possible cause might be the stiffness mismatch between the PDMS expander and the silicon skeleton: the skeleton was too stiff and could not be moved by the soft PDMS expander. This problem may be eliminated by redesigning the microactuator such that the PDMS expander width is smaller, so that the stiffness of the expander can be increased. However, to achieve this, the much smaller feature size of the first layer of photoresist pattern is required. Therefore, a high-resolution Cr-coated glass mask will be needed for the fabrication of the device in order to achieve small feature size. Another possible cause of the immovable skeleton could be the stiction between the silicon skeleton and the substrate. A solution to the problem may be to release the microactuator by removing the
substrate from the back side of the wafer so as to eliminate any possible contact between the microactuator structure and the substrate.

5.2.3 Conclusions

The experiment has shown that PDMS may also be employed as an expanding polymer for actuation purpose. Nevertheless, the design of the microactuator must be revised and the technique for structure release must be improved. To fabricate the microactuator and the 1-D PhC as designed in Chapter 3 and Chapter 4, a more sophisticated microfabrication process which allows for separate patterning of SU-8 polymer and PDMS is needed. Nevertheless, the filling technique that has been developed is important for fabricating the 1-D PhC device.

5.3 Microfabrication processes for mechanically tunable 1-D photonic crystal device

5.3.1 Overview

So far, the method of building a suspended Si-PDMS hybrid structure has been discussed (Section 5.1). The microfabrication processes for selective filling with PDMS is also presented in Section 5.2. The technique in Section 5.2, can probably be used to make an actuation mechanism as well. It has been shown that a thermal microactuator made up of silicon and PDMS could be realized. However, a much more complicated fabrication process is required for the device proposed in Chapter 3 and Chapter 4. As mentioned earlier at the beginning of this chapter, the making of such device could be divided into four tasks of building i) the suspended silicon structure, ii) the electric heater, iii) the polymeric expander, and iv) the tunable 1-D PhC. In this section, all those tasks were dealt with and fabrication of prototypes of the mechanically tunable 1-D PhC device is reported. However, the designs of the prototypes were different from the finalized design proposed in the two foregoing chapters. This is because the designs and the fabrication of photolithographic masks were done much earlier before the finalized design could be finished. One of the designs is shown in Fig. 5-8. Regrettably, there was not sufficient time to redraw
and make a new set of masks for the finalized design. Nevertheless, the microfabrication method and characterization of the mechanically tunable 1-D PhC device can still be presented in this work with the earlier design.

### 5.3.2 Microfabrication processes

The complete microfabrication processes is shown in Fig. 5-9. To overcome the problem of stiction encountered in Section 5.2, the suspended silicon structure was created using a semi-SOI microfabrication process similar to what was developed in Section 5.1 ([99]). The major difference between the two microfabrication processes was that the supporting layer in Section 5.1 was the SiO₂ layer together with the AZ-9260 photoresist; however, the supporting layer in [99] was Al layer of about 3 μm in thickness. Since the microfabrication process in Section 5.1 was more complicated yet did not eliminate the problem of charging effect at the buffered SiO₂ layer (discussed in [99]), the latter was implemented instead. The electric heater was created with Au by a lift-off technique. Two types of polymers, i.e., SU-8 and PDMS, were used to make the polymeric expanders and the tunable 1-D PhC,
Fig. 5-9: Microfabrication steps for the 1-D PhC device.

<table>
<thead>
<tr>
<th>Photolithography level</th>
<th>Process name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si$_3$N$_4$ LPCVD</td>
<td>1st mask &amp; 2nd mask Si$_3$N$_4$ RIE with CF$_4$</td>
</tr>
<tr>
<td>Wet Si etching with KOH</td>
<td>3rd mask Au lift-off</td>
</tr>
<tr>
<td>Al magnetron sputtering</td>
<td>4th mask Si DRIE</td>
</tr>
<tr>
<td>Filled 1-D PhC</td>
<td>(5th mask) filling with PDMS</td>
</tr>
<tr>
<td>Filled expanding units</td>
<td>5th mask (6th mask) filling with SU-8</td>
</tr>
<tr>
<td>Wet Al etching</td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- Si
- Si$_3$N$_4$
- Al
- Au
- SU-8
- PDMS
respectively. Unfortunately, the process of filling with PDMS developed earlier could not be successfully applied in this case after a few attempts. The problem was that of creating good AZ-9260 photoresist molds for PDMS. When the AZ-9260 molds were not properly made, the subsequent PDMS lift-off would not be perfect, resulting in undesirable PDMS left on some part of the device structure. To overcome the problem, a manual filling technique was applied instead. Had the process of filling PDMS been successful, the microfabrication process would require six levels of photolithography in total. And, this microfabrication process would completely be monolithic, promising for developing into a large-scale and systematic microfabrication for industry. The steps of fabrication process can be explained in detail as follows (refer to Fig. 5-9 for an illustration of each step):

i) **Low-pressure chemical vapor deposition (LPCVD)**

A Si$_3$N$_4$ layer was deposited on a double-side-polished <100> silicon wafer of thickness of about 525±25 μm by the LPCVD process. The achieved thickness of the Si$_3$N$_4$ was 200 nm on both side of the wafer. The purpose of having the Si$_3$N$_4$ layer is twofold. The first one is to use it as a hard mask in wet anisotropic silicon etching at the back side of the wafer. And, the other is to use it as an electrical isolation between Au heater film and Si substrate to prevent a short circuit via Si substrate.

ii) **Reactive-ion etching (RIE) of Si$_3$N$_4$ with tetrafluorocarbon (CF$_4$)**

After performing photolithography on each side of the wafer using the AZ-9260 photoresist with the first and the second mask, the fabricated photoresist patterns were used as masks in the RIE process. Plasma power of 300 W was applied and gaseous CF$_4$ was supplied at a flow rate of 3 sccm. The etching time was 3 mins which was sufficient to completely etch the exposed Si$_3$N$_4$ layer. As a result, the photoresist patterns were transferred onto the both sides of the substrate. The front side pattern was for electrical isolation, and the back side pattern was for wet anisotropic silicon etching windows.

Fig. 5-10 shows the patterns of Si$_3$N$_4$ on the back side of Si substrate after the RIE with CF$_4$. The gray regions are the exposed silicon substrate above which lies the
patterned Si₃N₄ layer which appears in purple (or dark gray). The exposed silicon at the back side of the substrate was to be etched in the following wet anisotropic etching process. On the other hand, the exposed silicon at the front side of the substrate was to be etched in the DRIE process.

iii) Wet anisotropic Si etching

Since Si₃N₄ is resistant to potassium hydroxide (KOH) solution, it can be used as a hard mask against it. The KOH base of concentration of approximately 35% by weight was used to etch the back side of the wafer. The wafer was submerged into the KOH base at 80°C for 5 hrs 33 mins. The achieved etched depth was approximately 400 μm, indicating that the etching rate was about 1.2 μm/min.

iv) Al magnetron sputtering

Once the suspended Si membrane was fabricated, pure Al was sputtered onto the back side of the wafer at room temperature (°C). The achieved Al layer thickness was 2.3 μm. The purpose of having the Al layer was to use it as a mechanical support during the filling processes in steps vii) and viii).
Fig. 5-11 represents the Al-sputtered cavities created after the wet anisotropic silicon etching process followed by the Al magnetron sputtering process. Notice that the top profiles of the cavities in Fig. 5-11a are of the same shape and size as those gray regions defined by the overlaying Si$_3$N$_4$ layer in Fig. 5-10. In the etching process, as soon as the substrate was submerged into the KOH solution, the exposed silicon was etched in all directions except in the $<111>$ directions. Because the protective Si$_3$N$_4$ pattern was created such that all the edges of the gray regions (refer to Fig. 5-10) were aligned with $<110>$ directions in the plane of the substrate, the cavities were

![Fig. 5-11: The back side of the substrate after Al magnetron sputtering. a) An array of the back-side cavities are shown to be covered everywhere by Al. b) A bottom of a cavity became smaller than the etching window because of the slanted $\{111\}$ etch-stop planes.](image)
formed with the cavity side walls defined by four \{111\} planes that touched the four edges of the gray regions. Since the \{111\} planes were slanted with an angle of 54.7° with respect to the plane of the substrate, the bottom area of the cavities became smaller than the etching window. Measurements from microscope images revealed that the average bottom area was a square with one side equal to 399 ± 4 μm (Fig. 5-11b), whereas the etching window became slightly larger and equal to 949 ± 3 μm. The remaining substrate at the bottom of the cavities became the area where suspended structure was built from the other side of the wafer later. The thickness of the remaining structure could be approximated to the original thickness of the substrate minus the etch depth \((525 - 399)/\tan(54.7°) \approx 126 \, \mu\text{m}\).

v) **Au Lift-off**

The Au heater was designed to have 200 nm of Au thickness plus an adhesion layer of Ti of 20 nm between the Au layer and the silicon substrate. A negative-type photoresist (AZ nLOF 2035) for lift-off was first patterned on the front side of the wafer using the third photolithographic mask. The achieved photoresist pattern was about 2 μm in thickness. The reason for the thick pattern as compared to the thickness of the heater was to ensure that there was no interconnection between the Au patterns on the substrate and that on top of the photoresist. After the photolithography, the front side of the wafer was sputtered by Ti for 20 nm and Au for 200 nm at room temperature, respectively. Then the wafer was left in AZ 300T stripper at 80 °C for about 1 hrs 30 mins, prior to ultrasonic cleaning.

The fabricated circuit of the Au heater is shown in Fig. 5-12. An array of electric circuits of the Au heater is shown in Fig. 5-12a and that of an individual device is shown in Fig. 5-12b. In Fig. 5-12a and Fig. 5-12b, all bond pads (gold) are seen resting on top of the Si₃N₄ layer (purple) which is an electrical insulator. The gray region at the center of Fig. 5-12b is the exposed silicon. This region was used to fabricate the device structure in the following process. Fig. 5-12c and Fig. 5-12d show two U-shape heating elements of a device. Only the heating elements were directly in contact with the silicon structure so as to obtain best heat transfer from the heaters to the silicon structure.
vi) Silicon deep reactive-ion etching (Silicon DRIE)

The fourth photolithographic mask was used to create another photoresist pattern on the front side of the substrate. The photoresist used was AZ-9260 and the thickness obtained was 5 μm. The AZ-9260 photoresist was used as a mask against the reactive SF$_6$ during the DRIE process. The DRIE process applied was the Bosch process [100]. Total etching time of 47 mins was needed in order to etch the exposed substrate down to the supporting Al layer.

The etching results are provided in Fig. 5-13. Comparing Fig. 5-13a with Fig. 5-12b, it can be seen that the device structure was fabricated within the exposed silicon region. The rough white region surrounding the device is the exposed supporting Al region.
Fig. 5-13: Device structure after DRIE process. a) A top view of the device together with the bond pads for supplying potential difference. b) (Left) The 1-D PhC region was etched to form trenches and three Si plates of the 1-D PhC. (Right) Two Si plates are shown stuck together while the other one stuck to the substrate. c) (Center) The transmission beam, (Left) the Si skeleton of the upper expansion unit, and (Right) the Si skeleton of the lower expansion unit.
layer from the back side of the substrate. As shown in Fig. 5-13b, three silicon plates were fabricated in a 1-D PhC trench where PDMS was to be filled later in the following process. On the left of Fig. 5-13b, the transmission beam (coming from the right of the image) was connected to the 1-D PhC region, partially forming a side wall of the trench. Two of the three silicon plates were displaced. This was possibly caused by weak adhesion between the bottoms of silicon plates and the supporting Al layer. One of the plates stuck to the substrate, whereas the other one stuck to the central plate, probably due to electrostatic attraction generated during the DRIE process. Enlarging the dimensions of the silicon plates or reducing the thickness of the device layer could probably prevent the displacement from happening.

vii) Filling with PDMS

The filling process could be carried out by either manually filling the polymer or by the technique developed section 5.2. Unfortunately, the filling technique from section 5.2 could not be implemented successfully for this prototype due to its feature-specific nature of the method that highly depends on how the AZ-9260 would shape under capillary effect. Therefore, the manual filling technique was applied instead in this presented work. The manual filling, however, yielded non-uniform filling profiles, and it was not a monolithic process. Even so, with this manual technique, the prototype could be fabricated and tested in order to show the working principle of the device. In order to use the technique, process refinement is needed in order to achieve best selective filling for each mask pattern. Moreover, another photolithographic mask will be required for fabricating the AZ-9260 mold.

The applied PDMS liquid mass was the Sylgard-184 manufactured by Dow Corning. The PDMS liquid mass was manually mixed with its curing agent first. The ratio by weight of the PDMS liquid mass to the curing agent is 10:1. After mixing, the uncured PDMS mixture was put inside a vacuum chamber for 30 mins to remove air bubbles. Afterwards the mixture was dropped on the 1-D PhC trench (see Fig. 5-14a), that was created after DRIE, to make the tunable 1-D PhC. Later the substrate was put inside the vacuum chamber again to remove any trapped bubble in the 1-D PhC trench. The PDMS was later cured at room temperature for 85 hrs and then baked at 65 °C on a hot plate for 1 hr 50 mins to further cure the PDMS. Because PDMS is
optically transparent, it is difficult to observe the filling profile. Thus, an electron microscope was used to observe the filling profile shown in Fig. 5-14b. The manual filling technique yielded an uncontrollable spreading of the PDMS, making the technique undesirable for this particular device.

viii) Filling with SU-8

After the process of filling with PDMS, SU-8 photoresist was patterned on the substrate with the 5th (or 6th) photolithographic mask as indicated in the process flow diagram in Fig. 5-9. The SU-8 2005 from MicroChem was spin-coated with the spin speed profile shown in Fig. 5-15a. After the spin-coating, prebaking of the wafer was performed at 65°C for 5 mins and at 95 °C for 5 mins respectively. Then the substrate was brought into a hard contact with the 5th (6th) photolithographic mask and aligned to it prior to exposure to UV light from an I-line source for 55 s. Post-exposure baking was carried out at 95°C for 8 mins afterwards. The substrate was developed in the SU-8 developer for 15 mins, rinsed with isopropyl alcohol then DI water, and spin-dried. After the first photolithography was carried out, the substrate was examined under an optical microscope. It was found that the silicon structures of the expanding units were not fully filled with SU-8 polymer, so the second photolithography was performed subsequently to amend the first filling. The less viscous SU-8 2002 was poured onto the substrate and the substrate was put inside a
vacuum chamber for 3 mins to remove any possible trapped air bubbles from the structures. After removing the substrate from the vacuum chamber, SU-8 2005 was poured onto the substrate in order to increase the effective viscosity of the photoresist on the substrate before spin-coating. The photoresist was left on the substrate for 2 mins so that SU-8 2002 and SU-8 2005 could mix with each other. The second spin-coating was then carried out with the spin speed profile shown in Fig. 5-15b. After the spin-coating, the substrate was prebaked on a hot plate at 65 °C for 5 mins and at 95 °C for 5 mins respectively. Again, the substrate was exposed with the I-line source for 75 s, while in soft contact with the 5th (6th) photolithographic mask. Then, the substrate underwent another post-exposure baking at 65 °C for 1 min, then at 95 °C for 10 mins. Development was done for 10 mins afterwards.

![Fig. 5-15: Spin speed profiles of the first (a) and the second (b) spin-coating.](image)

ix) **Al etching**

The Al layer was removed by dissolving it with an aluminium etchant at room temperature. The aluminium etchant had an approximate composition ratio by weight as \( \text{H}_3\text{PO}_4 : \text{HNO}_3 : \text{CH}_3\text{COOH} : \text{H}_2\text{O} = (65-85\%) : (1-5\%) : (5-15\%) : (10-20\%) \). Time required to completely dissolve the aluminium was about 30 mins, indicating that the etching rate was about 67 nm/min. After Al etching, the wafer was rinsed by DI water and baked at 40 °C in order to dry it. Up to this point, the structure had been released and the device fabrication was completed.
5.3.3 End results and discussions

An electron microscope was employed to capture the images of a representative device as shown in Fig. 5-16. The footprint of the device excluding the bond pads was about 260 μm x 200 μm. The top view of the device is shown in Fig. 5-16a, whereas the perspective views of the device are presented in Fig. 5-16b and Fig. 5-16c. The images prove that the suspended structure was successfully created. Further examination of the same device from the back side of the substrate confirmed the success of the release. Displayed in Fig. 5-17b is a bottom view of the same device seen through the etched cavities from the back side of the substrate.

Looking from the front side of the substrate in Fig. 5-16d, the gaps by the side of the output guide 1-D PhC trench seems to be fully filled with PDMS. The bottom view of the 1-D PhC trench also reveals that the 1-D PhC trench was filled with PDMS (Fig. 5-17c), but the profile of the PDMS cannot be seen clearly. A block of SU-8 resting on top of the 1-D PhC in Fig. 5-16d is a result of the design of the 5th (6th) photolithographic mask for SU-8 patterning. As mentioned at the beginning of this section, the mask was designed and fabricated before the finalized design could be done. Using SU-8 as a material for the tunable 1-D PhC was one of the choices. The 5th (6th) mask was thus designed such that the SU-8 at the 1-D PhC trench would be exposed too. It turned out later that PDMS was a more suitable material for the 1-D PhC and the same mask was used to pattern SU-8 at the expansion units in order to save cost. Although the fabricated 1-D PhC together with the SU-8 block in Fig. 5-16 is yet applicable in optics or photonics, the device could still prove that the fabrication of the mechanically tunable 1-D PhC was at least feasible.

Fig. 5-16e represents one of the two expansion units of the device. The Au heaters were successfully fabricated on top of the expansion units. The region of patterned SU-8 encompasses the comb-like structure of the expansion unit as shown. The filling was further examined, and another SEM image was recorded and displayed in Fig. 5-16f. It can be seen that the SU-8 polymer did fill up the gaps between each pair of fingers of the comb-like structure. The reason that the SU-8 pattern at the top of the comb-like structure is hanging out from the structure was due to the imperfect first filling of SU-8 2005: the polymer probably could not completely fill the below
structure and its surrounding trench, or there were some air bubbles preventing it from going into the trench. The SU-8 2002 which was less viscous was able to flow into the comb-like structure and the surrounding trench in the second filling. However, the SU-8 2002 photoresist in liquid form had little content of SU-8 polymer; therefore, most of the SU-8 mass was dragged and concentrated into the gaps between the fingers due to capillary effect, as the photoresist was drying up. This left some space below the hanging SU-8 as shown in the figure. Nonetheless, the comb-like structure was successfully filled as expected. To confirm the quality of the filled SU-8, again images of the back side of the substrate were taken with an optical microscope and are presented in Fig. 5-17c-d. It was found that about one half of the fabricated devices possessed SU-8 with at least one hole. Seen in Fig. 5-17d is an example of the patterned SU-8 that has a hole. The occurrence of these holes may be explained by the present of air bubbles that were still trapped even after the second filling, and they became defects inside solidified SU-8 afterwards. On the other hand, an example of good expansion unit is shown in Fig. 5-17c. To ensure there is no such air bubbles during filling, more time should be given during air bubbles removal in vacuum.
Fig. 5-16: The SEM images of the fabricated tunable 1-D PhC device. a) A top view of the device. b) A perspective view of the device that shows the released transmission beam. c) Another perspective view of the device shows the released cross-beam and the expansion units. d) A close-up view at the 1-D PhC area, where a block of SU-8 is on top of the 1-D PhC trench. e) One of the expansion units. The comb-like structure is filled with cross-linked SU-8. The Au heater is seen as the white structure in u-shape. f) The expansion unit seen from a perspective view showing the SU-8 between the fingers of the silicon structure.
Fig. 5-17: Microscope images of the fabricated tunable 1-D PhC device. a) The 1-D PhC trench. b) A bottom view of the device seen from the KOH cavity. c) The bottom of the 1-D PhC trench. d) A bottom of a 1-D PhC trench of another device with PDMS seen clearer. e) An expansion unit with good SU-8 filling. f) Another expansion unit with bad SU-8 filling where a hole is present.
Measurements of dimensions of structural features were important for comparing simulation results with experimental results. Therefore, extensive measurements of dimensions of each structural feature were carried out using an optical microscope, a SEM, or a surface profiler. There are nineteen structural features in total. These structural features were parameterized, and a name was given for each feature. They are tabulated in Table 5-1. The first fourteen features are indicated in Fig. 5-18, whereas the rest cannot be shown in the figure, but self-explanatory. Dimension of each feature was sampled at least 5 times across the feature body. Then the measured dimensions were averaged and the mean value was taken as the representative value of that feature. All the measurement readings together with statistical values are shown in Appendix F. The mean values and their corresponding standard deviations of each feature are summarized in Table 5-2. Designed values are also given in the table for comparison. Discrepancy which represents the difference between the fabricated dimension and designed dimension divided by the designed dimension is also given in the table in percentage form. Negative discrepancy indicates that the
dimension of a fabricated feature is smaller than that of the designed one, while positive discrepancy indicates the opposite. The smaller the discrepancy, the closer to the designed dimension the fabricated dimension is; which is desirable.

It is noteworthy that the first fourteen features were measured from both sides of the substrate to ensure that the statistically representative dimensions were obtained. This is because the DRIE process tends to give a tapering side wall profile to an etched structure. For example, the width of transmission beam (transbmwid) at the bottom of the device structure tends to be smaller than that at the top. In contrast, the width of a fin gap (fngap) is likely to be wider than that at the top. This tapering profile is probably the major cause of the discrepancy in dimension. Among all the parameters, the width of the last fin gives the highest negative discrepancy of -59.6%, suggesting that its representative dimension is much smaller than the designed value which is 3 μm. This is in fact because the last fin was affected largely by the tapering profile created in the DRIE process: in which case the last fin vanished at the bottom of the structure (see Fig. 5-17c) as compared to the top structure of another device with the same design in Fig. 5-16e. Obviously, when the width of fins seems to be the most-affected feature, the gap is also affected in the opposite way and it gives the highest positive discrepancy as expected. The best structure fabricated in terms of discrepancy is the length of the cross-beam with the discrepancy of 1.34%. It can be noticed that larger features tend to have small absolute discrepancy, because in DRIE process whether it is the tapering or undercutting below AZ-9260 mask or side-wall scalloping, their effects are in the range of a few microns at most.
Table 5-1: Structural features and their parameter names

<table>
<thead>
<tr>
<th>No.</th>
<th>Features</th>
<th>Parameter names</th>
<th>No.</th>
<th>Features</th>
<th>Parameter names</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Width of transmission beam</td>
<td>transbmwid</td>
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<td>Width of 1-D PhC trench side gap</td>
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<td>Width of output waveguide</td>
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<td>Width of metallic heater (Au heater)</td>
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Table 5-2: Results of dimension measurements

<table>
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<th>Feature No.</th>
<th>Parameters</th>
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<th>Mean (μm)</th>
<th>Standard deviation (μm)</th>
<th>Discrepancy (%)</th>
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<tr>
<td>5</td>
<td>lastfnwid</td>
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<td>1.212</td>
<td>0.242</td>
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</tr>
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<td>11.980</td>
<td>0.400</td>
<td>-7.844</td>
</tr>
<tr>
<td>7</td>
<td>fnwid</td>
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<td>7.128</td>
<td>0.662</td>
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<td>0.568</td>
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<tr>
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<td>28.376</td>
<td>0.721</td>
<td>-5.414</td>
</tr>
<tr>
<td>10</td>
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<td>1.261</td>
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<tr>
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<td>1.204</td>
<td>30.764</td>
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<tr>
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<td>opgdwid</td>
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<td>48.461</td>
<td>0.541</td>
<td>-3.078</td>
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<tr>
<td>13</td>
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<td>51.584</td>
<td>1.140</td>
<td>-2.672</td>
</tr>
<tr>
<td>14</td>
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<td>4.465</td>
<td>0.375</td>
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<td>0.019</td>
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<td>2.654</td>
<td>2.569</td>
</tr>
<tr>
<td>19</td>
<td>bcavlen</td>
<td>N/A</td>
<td>399.025</td>
<td>3.979</td>
<td>N/A</td>
</tr>
</tbody>
</table>
5.4 Summary

Three major tasks were accomplished in this chapter.

First, the technique of microfabrication to create Si-PDMS hybrid structures was developed. The SOI-equivalent wafer for easy releasing of suspended structures was made by creating sacrificial layers from the back side of the wafer with the thermally grown SiO$_2$ layer and the AZ-9260 photoresist. The back-side cavities were first created by the wet anisotropic silicon etching in KOH base. Then the SiO$_2$ layer was grown on the substrate and the cavities were filled with the photoresist. It was further demonstrated that the silicon structures could be created from the front side of the wafer by the DRIE followed by filling them with the PDMS polymer. The final Si-PDMS hybrid structures were attained after removing the sacrificial layers.

The second development of microfabrication processes dealt with selectively filling the silicon structures with PDMS. The structures used in the study were that of the thermal microactuators with meandering silicon beams. The microactuator structures were first fabricated by the DRIE on the SOI wafer and then released by the BOE. The top surfaces of structures were made conductive by Au sputtering. Later, the AZ-9260 molds were created on the substrate by the photolithography. Afterwards, the PDMS mixture was poured onto the substrate and cured. The microactuators were tested and the thermal expansions of the PDMS were observed.

In the last task, the microfabrication processes of the 1-D PhC device were developed. The electrically insulating layer and the KOH-proof layer were first patterned on the front side and the back side of the double-sided polished Si$_3$N$_4$-grown wafer, respectively. The device layer was then defined by the wet anisotropic silicon etching process. Afterwards, Al was sputtered on the back-side cavities of the wafer so as to create the mechanical support in the subsequent filling processes. The electrical heaters were fabricated by sputtering Au and Ti on the lift-off photoresist at the front side of the wafer. After lifting off the metals, the electric heaters were achieved. Another photoresist pattern was then prepared for the DRIE. After the DRIE, the silicon structures of the devices were created. The structures were filled with PDMS polymer and SU-8 polymer, respectively. The devices were completed
after removing the supporting Al layer and rinsing of the wafer. The inspections of the fabricated devices were carried out, and the devices which were properly released and had the structure filled with the polymers were picked up for further study. The dimension measurements of each structural features of a good device were performed. The discrepancies in the dimensions of the fabricated features were also provided.
CHAPTER 6 ELECTROMECHANICAL TESTING

Microfabrication of the 1-D PhC device was discussed extensively in the last chapter. Unfortunately all of the fabricated devices could not be used for testing any optic or photonic applications due to problems occurred during deep silicon etching process and the filling processes. Nonetheless, it is still possible to perform an electromechanical testing on the devices, because the 1-D PhC trench was still filled with PDMS. Although the lack of some silicon plates inside the trench, or the presence of a SU-8 block on top of the trench, will certainly affect the mechanical tuning of the 1-D PhC, it should not prevent one from demonstrating that the microactuator was indeed capable of deforming the 1-D PhC. With this goal in mind, the fabricated devices were tested electromechanically and the results are reported in this chapter.

An analysis of the electric circuit of the test system is first discussed in Section 6.1. Measurements of relevant electrical parameters are given and the results are discussed in the same section. Then a method of electromechanical testing is described in Section 6.2. Experimental results from the testing are presented in Section 6.3. Finite-element simulation results are provided, compared and discussed with the experimental results.

6.1 Electric circuit analysis

When the microactuator is operated, the two expansion units are expected to expand identically so that the cross-beam is pushed toward the 1-D PhC without any side movement. A desirable way to activate the microactuator is to equally dissipate energy drawn from a power source to the two heating elements of the microactuator. To achieve this, the circuit of the device was designed as shown in Fig. 6-1. Since the layout of the bond pads and the heating elements was symmetric about the line joining Location A and Location D of the two bond pads, the power fractions
spent on the two heating elements would be equal when power supply was connected at Location A and Location D. This is because the power dissipated is a function of potential difference across the loads and the resistance of the loads. The symmetry of the circuit layout results in the resistance of ABCD path being equal to that of AFED path. In order to obtain the relationship between the voltage across the heating elements and the compression distance of the microactuator, the resistances of each part of the device circuit had to be measured first.

6.1.1 Measurement method for electrical resistances

Theoretical estimation of the resistance of the heating element was first performed. Using the dimensions of the heating element in Table 5-2 and the average resistivity of gold in Table 4-4, the following resistance formula yields the approximated resistance of the heating element:

\[
R = \rho \frac{I}{A} = \left(4.01 \times 10^{-8} \Omega \cdot m\right) \frac{133.1 \mu m}{(0.234 \mu m \times 4.465 \mu m)} = 5.11 \Omega
\]

(6-1)
It was found that the approximated resistance of the heating element is comparable to the resistance of the probes and the cables of the test equipments; therefore, the resistances of the probes and the cables were taken into account during device testing.

When the device was under electromechanical testing, an electric circuit diagram representing the electric circuit of the test system (see Fig. 6-5) can be illustrated as shown in Fig. 6-2. The circuit is consisted of a supplied voltage \( V_i \) connected in series with a series of resistors. The series of resistors can in fact be derived from internal resistances of the left and the right triaxial cables \( R_{Ltri}, R_{Rtri} \), resistance of the left probe \( R_{Lp} \) and resistance of the right probe \( R_{Rp} \), the left probe’s contact resistance \( R_{Lc} \) and the right probe’s contact resistance \( R_{Rc} \), and resultant resistance of the device circuit. Since the bond pads were touched by the probes at Location A and Location D indicated in Fig. 6-1, the device circuit can be divided into an upper section and a lower section defined by the horizontal line joining the two locations. The resistances of the device circuit can be further divided into three groups: i) resistances of the left bond pad \( R_{L1}, R_{L2} \), ii) resistances of the right bond pad \( R_{R1}, R_{R2} \), and iii) resistances of the heating elements \( R_{H1}, R_{H2} \). The subscripts 1 and 2 denote the resistances from the upper section or the lower section of the circuit, respectively. Because these resistances are unknown and could not be independently separated from one another for resistance measurement individually, any
measurement read from the characterization system will simply represent the effective resistance $R_{\text{eff}}$ of the system seen by the power supply:

$$R_{\text{eff}} \equiv \frac{V_i}{I_i}$$  \hspace{1cm} (6-2)

However, the important parameters are the resistances of the heating elements, $R_{H1}$ and $R_{H2}$, and the voltage fallen across the two heating elements, $V_H$, that directly cause the microactuator to move. Therefore, an experimental approach must be established in order to find out all the unknown resistances. Due to symmetry of the device circuit along the horizontal line, the resistances of the circuit are assumed as follows:

$$R_L = R_{L1} = R_{L2}, \quad R_R = R_{R1} = R_{R2}, \quad R_H = R_{H1} = R_{H2}$$  \hspace{1cm} (6-3)

where $R_L$, $R_R$, $R_H$ are the resistances of the upper or lower section of the left bond pad, the right bond pad, and the heater, respectively. The resistances of the two triaxial cables, the two probes, and the contact resistances can be combined into a single unknown resistance as:

$$R_{\text{tpc}} = R_{\text{tpcL}} + R_{\text{tpcR}} + R_{\text{tp}} + R_{\text{rp}} + R_{\text{lc}} + R_{\text{rc}}$$  \hspace{1cm} (6-4)

In total, there are four unknown resistances to be measured, i.e., $R_{\text{tpc}}$, $R_L$, $R_R$, and $R_H$. By changing the probes location, four different configurations of the electric circuit could be achieved. Thus measurements taken from the characterization system will yield four equations provided the configurations of the circuit are known. The circuit diagram of the first configuration is shown in Fig. 6-2 where the probe are at location A and D (illustrated in Fig. 6-1). This is also a configuration during operation of the device. The second circuit configuration is when one of probe tip is at location A and the other is either at location F or location B. The third circuit configuration is when one of probe tip is at location D and the other is either at location C or location E. Circuit diagrams for the second and the third configuration are shown in Fig. 6-3a and Fig. 6-3b, respectively. The fourth circuit configuration can be achieved by landing the two probe tips very near to each other ($<50 \, \mu\text{m}$) on a large area of bond pad. A circuit diagram for the fourth configuration is displayed in Fig. 6-3c. The
effective resistance as seen by the power supply can be measured by following the same procedures in Section 6.2.2, except that the locations of the probes are now different, and the motion observations can be neglected. The effective resistance obtained from the first configuration through the fourth configurations is denoted by $R_{\text{eff1}}$, $R_{\text{eff2}}$, $R_{\text{eff3}}$, and $R_{\text{eff4}}$, respectively.

### 6.1.2 Measurement results and calculations at 3.000-V supply

The resistance readings were recorded, and they are shown in Table 6-1, Table 6-2, and Table 6-3. In the tables the effective resistances seen by the power supply are $R_{\text{eff1}}$, $R_{\text{eff2}}$, $R_{\text{eff3}}$, and $R_{\text{eff4}}$, for the first, the second, the third, and the fourth configurations.
correspondingly. The effective resistances of the first, the second, the third, and the fourth circuit configurations were measured at the supplied voltage $V_i$ of 3.000V. The average of the resistance $R_{\text{eff}2}$ or $R_{\text{eff}3}$ is calculated by summing all the measurements from each probe location and dividing it by the number of all measurements performed. The average of the resistance $R_{\text{eff}4}$ is calculated in the same manner, except that all data were recorded from one location. The following calculation is given based on the measurements at the supplied voltage $V_i = 3.000 \text{ V}$. The average value is used to represent each of the resistance.

Using the four circuit diagrams and the foregoing equations (6-3) and (6-4), four algebraic equations can be written as follows:

$$R_{\text{eff}1} = R_{\text{pc}} + R_s / 2 \ ; \ R_s = R_L + R_H + R_R \quad (6-5)$$

$$\frac{1}{R_L} + \frac{1}{R_L + 2R_H + 2R_R} = \frac{1}{R_{\text{eff}2} - R_{\text{pc}}} \quad (6-6)$$

$$\frac{1}{R_R} + \frac{1}{R_R + 2R_H + 2R_L} = \frac{1}{R_{\text{eff}3} - R_{\text{pc}}} \quad (6-7)$$

$$R_{\text{pc}} = R_{\text{eff}4} \quad (6-8)$$

The resistance $R_s$ in (6-5) is defined as the resultant resistance of the whole upper (or lower) section of the device circuit. The Substituting (6-5) and (6-8) into (6-6) yields:

$$R_L^2 - 2R_s R_L + 2R_s (R_{\text{eff}2} - R_{\text{eff}4}) = 0 \quad (6-9)$$

Similarly, substituting (6-5) and (6-8) into (6-7) results in:

$$R_R^2 - 2(R_{\text{eff}1} - R_{\text{eff}4})R_R + 2(R_{\text{eff}1} - R_{\text{eff}4})(R_{\text{eff}2} - R_{\text{eff}4}) = 0 \quad (6-10)$$

Both (6-9) and (6-10) are quadratic equations involving the unknown $R_L$ and $R_R$, respectively. After solving (6-9) and (6-10), substituting $R_L$ and $R_R$ into (6-5) yields...
If the data were correctly sampled, it can be anticipated that there must exist only one set of physically correct solutions. Using the measured effective resistances from Table 6-1, Table 6-2, and Table 6-3, all possible solutions to (6-10) were obtained and shown in the first row of Table 6-4. It can be seen that Solutions 1, 3 and 4 give physically incorrect values of $R_H$, therefore they should be ignored. The only valid solution set is $R_{tpc} = 5.81 \, \Omega$, $R_L = 1.62 \, \Omega$, $R_R = 1.41 \, \Omega$, $R_H = 7.64 \, \Omega$. The measured $R_H$ is close to the approximated $R_H$ of 5.11 $\Omega$ in (6-1).

**Table 6-1:** effective resistance measurements at 3.000-V supply

<table>
<thead>
<tr>
<th>Effective resistance</th>
<th>Probe locations</th>
<th>1st measurement (Ω)</th>
<th>2nd measurement (Ω)</th>
<th>Average (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{eff2}$</td>
<td>D &amp; C</td>
<td>7.25</td>
<td>7.25</td>
<td>7.125</td>
</tr>
<tr>
<td></td>
<td>D &amp; E</td>
<td>7.05</td>
<td>6.95</td>
<td></td>
</tr>
<tr>
<td>$R_{eff3}$</td>
<td>A &amp; B</td>
<td>7.65</td>
<td>7.50</td>
<td>7.3125</td>
</tr>
<tr>
<td></td>
<td>A &amp; F</td>
<td>6.85</td>
<td>7.25</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6-2:** effective resistance measurements at 3.000-V supply

<table>
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<th>Effective resistance</th>
<th>n$^{th}$ measurement (Ω)</th>
<th>Average (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{eff4}$</td>
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<td>2</td>
</tr>
<tr>
<td></td>
<td>5.80</td>
<td>5.90</td>
</tr>
</tbody>
</table>

**Table 6-3:** effective resistance measurements

<table>
<thead>
<tr>
<th>$V_i$</th>
<th>$R_{eff}$</th>
<th>$0.5R_s = R_{eff} - R_{tpe}$</th>
<th>$R_s$</th>
<th>$V_i$</th>
<th>$R_{eff}$</th>
<th>$0.5R_s$</th>
<th>$R_s$</th>
</tr>
</thead>
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<tr>
<td>3.000</td>
<td>11.145</td>
<td>5.333</td>
<td>10.665</td>
<td>4.201</td>
<td>11.070</td>
<td>5.258</td>
<td>10.515</td>
</tr>
<tr>
<td>3.200</td>
<td>10.824</td>
<td>5.012</td>
<td>10.023</td>
<td>4.401</td>
<td>11.180</td>
<td>5.368</td>
<td>10.735</td>
</tr>
<tr>
<td>3.600</td>
<td>10.865</td>
<td>5.053</td>
<td>10.105</td>
<td>4.801</td>
<td>11.595</td>
<td>5.783</td>
<td>11.565</td>
</tr>
<tr>
<td>4.001</td>
<td>10.925</td>
<td>5.113</td>
<td>10.225</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.1.3 Measurement results and calculations at other supply voltages

Only the measurement for $R_{eff1}$ was repeated at other supply voltages, ranging from 3.000 V to 5.001 V, and the resistance readings are given in Table 6-3. It is noteworthy that the resistances $R_{eff2}$, $R_{eff3}$, and $R_{eff4}$ were approximated to be constants and the values are based on the 3.000-V supply. The resistances $R_{eff2}$ and $R_{eff3}$ are mainly contributed by the lumped equipment resistance $R_{lpc}$ (i.e., $R_{eff4}$) and by the bond pad resistances $R_L$ and $R_R$. The resistances of the bond pads, the triaxial cables and the probes were not much affected by Joule heating as compared to the resistance of the heating elements, $R_H$. The solutions for $R_L$, $R_H$, and $R_R$ are presented in Table 6-4. As expected, there is only one set of meaningful solutions for each voltage supplied. The reason for constant $R_{eff2}$, $R_{eff3}$, and $R_{eff4}$ can be supported by the following arguments:

The bond pad resistances $R_L$ and $R_R$ were simply conducting films like the heating elements. However, they were relatively much larger in area as compared to the two heating elements at the expansion units. Therefore, current density was dispersed over the large film, which means there was much smaller local Joule heating per unit area while there was equal access for convection above the film and for conduction below the film. As for $R_{lpc}$, the triaxial cable and the probe were many orders of magnitude longer than the heating elements, yet their resistances had the same order of magnitude as $R_H$, as can be seen from the solution in case of 3.000-V supply. Thus, heat generation in the same order magnitude as that occurred in the heating elements must be distributed over the length of the cables (~1 m long) and the probes, meanwhile there was enormous area (as compared to that of the heating elements) for convection to occur. It can then be anticipated that the temperature rise on the bond pads, the triaxial cables, and the probes would be insignificant as compared to that of the heating elements. The probe tip diameter of 25 μm implied a considerably large contact area as compared to the cross-sectional area of the heating elements which had the width of about 4 μm and the thickness of about 0.2 μm. Since the probes and the bond pads were not subjected to high temperature rise and large contact area allowed good thermal conduction to dissipate heat generating at the contact point, the contact resistances at the two probes were also assumed to be constant.
Table 6-4: Solutions for the quadratic equations at different $V_i$

<table>
<thead>
<tr>
<th>$V_i$ (V)</th>
<th>$R_L$</th>
<th>$R_R$</th>
<th>$R_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sol^1</td>
<td>Sol^2</td>
<td>Sol^1</td>
</tr>
<tr>
<td>4.001</td>
<td>18.820</td>
<td>1.630</td>
<td>19.040</td>
</tr>
<tr>
<td>5.001</td>
<td>23.689</td>
<td>1.601</td>
<td>23.901</td>
</tr>
</tbody>
</table>

To see why the resistances $R_{eff2}$ and $R_{eff3}$ are mainly contributed from $R_{tpc}$, $R_L$, and $R_R$, consider the parallel resistances in circuit diagrams in Fig. 6-3a and Fig. 6-3b. Using the solutions at the 3.000-V supply, a simple calculation reveals that the resistance of the upper branch is 1.624 Ω, while that of the lower branch is $1.624 + 2(1.405) + 2(7.636) = 19.706$ Ω for Fig. 6-3a. Since they are parallel, most of the current must flow into the upper branch, leaving the lower branch as if being cut open. Adding only $R_L$ and $R_{tpc}$ also gives a close result to $R_{eff2}$. Similar argument can be deduced for $R_{eff3}$. Moreover, the approximation became better at higher supply voltages, since only $R_H$ was affected by temperature rise and the temperature rise would only raise $R_H$ higher. With $R_L$, $R_R$, and $R_{tpc}$ being unaffected by Joule heating, it was then assumed that the effective resistance $R_{eff2}$, $R_{eff3}$, and $R_{eff4}$ remained unchanged with the supply voltage. Hence, the effective resistances $R_{eff2}$, $R_{eff3}$, and $R_{eff4}$ at the 3.000-V supply were using throughout solving the equations at other supply voltages.

6.1.4 Discussions on resistance measurements

Having found the resistance of each heating element, it is possible to find out the voltage across the heating elements by voltage division technique. According the circuit diagram of the test system (Fig. 6-2), the voltage across the heating element $V_H$ is given by:
\[ V_H = \frac{0.5R_{th}}{0.5(R_z + R_H + R_R) + R_{qp}}V_i. \] (6-11)

The values of \( V_H \) are tabulated in Table 6-5 at the end of this section for reference, and they are corresponding to the range of the supplied voltage \( V_i \) of 3.000 V – 5.000 V. The resistance of an individual heating element \( R_H \) is plotted against \( V_H \) as shown in Fig. 6-4. It can be seen that the resistance seems to increase non-linearly with \( V_H \).

As the displacement of the microactuator is linearly proportional to the temperature rise (as shown in (4-29)) and the displacement parabolically increases with the supply voltage (see Fig. 6-7), it could be deduced that the temperature also parabolically increases with \( V_H \). Since resistance change is linearly proportional to the temperature rise, it is convincing that the increasing of \( R_H \) is due to temperature effect. In order to check whether \( R_H \) have parabolic relationship with the \( V_H \), a regression technique was used to fit the data to a parabolic function of the form \( R_H = AV_H^2 + C \) where \( A \) and \( C \) are constant > 0. This is a parabolic function centred at \( V_H = 0 \) V. The function has a physical interpretation that the resistance is lowest when there is no heating effect, i.e., when \( V_H = 0 \) V, and that the resistance increased with the temperature rise, i.e., when \( V_H > 0 \) V. By using the Solver Tool in Microsoft Excel to fit the measured resistance to the parabolic function, the predicted trend is obtained as: \( R_H = 0.7848V_H^2 + 6.1619 \) (\( \Omega \)). The predicted trend is shown as a dotted curve in Fig. 6-4. The Solver Tool uses the Generalized Reduced Gradient nonlinear optimization code [101] to minimize the sum of the squares of residuals. The coefficient of determination (the R-squared coefficient) calculated turned out to be 0.843136 which indicates moderate accuracy of the predicted trend. It can be seen that the measured \( R_H \) and the predicted \( R_H \) are in the same order of magnitude as that of the approximated value of 5.11 \( \Omega \) from (6-1). Nonetheless, large fluctuation of the resistance within the range of 1.0 V to 1.2 V that might be caused by systemic errors seems to widen the discrepancy between the predicted value and the theoretical value by heightening the predicted trend. The fluctuation could probably be eliminated by sampling the resistance many times and then using the mean value to represent the resistance instead.
Resistivity of the conductive film of the heating element can be found by using the formula:

$$\rho = \frac{RA}{l},$$

(6-12)

where $R$, $A$, and $l$ are the resistance, the cross-sectional area, length of the film, respectively. The film was in fact comprised of an adhesion layer of titanium of 20 nm, on top of which lies an Au layer of 200 nm. Due to low resistivity of Au and titanium as compared to the silicon substrate, it can be assumed that the electric current was confined within the film only. The resistivity can be calculated by using the dimensions of the Au/Ti film from Table 5-2 and the resistance $R_H$. The cross-sectional area $A$ of the film was $\text{mthk} \times \text{mtwid} = 1.045 \, (\mu\text{m})^2$, the length of the film was approximately 133.1 $\mu\text{m}$. After calculating the resistivity for each $R_H$ value, summing them up, and dividing the sum by the number of $R_H$ values, the average resistivity over the voltage supply range thus becomes $6.045 \times 10^{-8} \, \Omega\text{-m}$. The derived
resistivity will be crucial in creating a finite-element model in the next section. This model will be used to compare the experimental results with the finite-element ones.

Power transfer from the power supply to the heating elements was calculated after all the resistances were found out. The power transfer to the device circuit is simply the ratio of the resistance of the device circuit to that of the whole circuit, since the device circuit is connected in series with the power supply:

\[
\frac{f_{\text{dev}}}{P_{\text{sys}}} = \frac{I^2 (R_v / 2)}{I^2 R_{\text{eff1}}} = \frac{(R_v / 2)}{R_{\text{eff1}}}
\]

(6-13)

where \( f_{\text{dev}} \), \( P_{\text{dev}} \), \( P_{\text{sys}} \), \( I \) are the power ratio, power transferred to the device, total power transferred to the system, total current going into the system, respectively. The power transferred to the device circuit is further divided among the bond pads and the heating elements. The power transferred to the device circuit can be divided equally among the upper section and the lower section of the device circuit due to the mirror symmetry of the circuit. For each section, the power can be divided further for the left bond pad, the right bond pad, and the heating element. Since all the loads in the upper (or lower) section share the same current, the formula in (6-13) can be applied again. In other words, the ratio of the resistance of the heating element to that of the whole upper (or lower) section, \( R_H/R_s \), is equal to the ratio of the power delivered to the heating element to that delivered to the upper (or lower) section. The ratio of power transferred to one heating element to the total power transfer to the test system, \( f \), equals:

\[
f = \frac{1}{2} \left( \frac{R_v / 2}{R_{\text{eff1}}} \right) \frac{R_H}{R_s} = \frac{1}{4} \frac{R_H}{R_{\text{eff1}}}
\]

(6-14)

The transferred-power ratio at different supply voltages is tabulated in Table 6-5. The power transferred to the system \( P_{\text{sys}} \) (or the power consumption by the system) is also presented in the table. The power consumption by one heating element \( P_H \) was in the range of 0.1-0.41 W, therefore the power consumption by the device was between 0.2-0.82 W. Much of energy was wasted mainly to the resistance of the triaxial cables, the probes, and the contact resistance. Only 30-40 % of the energy from the power supply was transferred to both heating elements and about 7-9 % of the energy
was lost in the bond pads. Wire bonding directly to the bond pads may improve the efficiency of the test system.

<table>
<thead>
<tr>
<th>$V_i$ (V)</th>
<th>$V_H$ (V)</th>
<th>$R_{eff}$ (Ω)</th>
<th>$P_{sys}$ (W)</th>
<th>$P_H$ (W)</th>
<th>$f$ (%)</th>
</tr>
</thead>
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<td>11.145</td>
<td>0.808</td>
<td>0.1383</td>
<td>17.130</td>
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<td>0.946</td>
<td>0.1525</td>
<td>16.117</td>
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<td>1.043</td>
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<td>0.1938</td>
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</tr>
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<td>1.319</td>
<td>0.2180</td>
<td>16.522</td>
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<tr>
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<td>0.2409</td>
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</tr>
<tr>
<td>4.401</td>
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<td>0.4099</td>
<td>19.890</td>
</tr>
</tbody>
</table>

6.2 Method of the electromechanical testing

6.2.1 Experimental setup for the electromechanical testing

A simple experiment setup is illustrated in Fig. 6-5. The setup was consisted of three main units: a probe station (Cascade), an I-V characterization system (Keithley System SourceMeter 2602A), and a computer (not shown in the figure). The probe station was comprised of:

- an analog-controlled vacuum chuck,
- an analog joy stick for moving the chuck,
- two electric probes for leading electric signals to a device under test,
- an optical microscope with a charge-coupled device (CCD) camera,
- a vibration isolation table.

The probes, the optical microscope, the CCD camera, the vacuum chuck were placed on the vibration isolation table in order to eliminate external mechanical disturbance that could interfere with probe contacts and microscope observations. The optical
microscope with a CCD camera was installed above the vacuum chuck for observing the device from a top view (not shown in Fig. 6-5). The CCD camera was connected to the computer for recording images and videos. The same computer was also used to control positioning of the vacuum chuck and to display real-time video signals from the CCD camera. The Keithley I-V characterization system was consisted of a power supply and I-V sensing units for monitoring electrical parameters. The characterization system had two sense/source channels. Each channel was connected
to each of the two probes via a triaxial cable to make a closed-loop circuit. Either channel could be used to source a DC current up to 1.01A at a DC voltage up to 40.4 V. It could also be used to source a DC voltage within the same working range. For this particular experiment, the internal power supply was used to source the DC voltage. While one channel was used to source the DC voltage, the other channel was used for sensing an output current. Electrical parameters such as the actual input voltage, the output current, the rated power and the electrical resistance could be read on the front panel of the instrument. After the fabricated prototype wafer was loaded on the chuck, it was secured by vacuum created between the chuck and the wafer.

Afterwards, the vacuum chuck was positioned close to the two probes which were already mounted and secured by magnets on the stationary stage of the probe station. The tips of the two probes were brought into contact with two bond pads of the device as shown in Fig. 6-6. Up to this point, the device was ready for testing and the microactuator would start working immediately when the power supply was switched on. The next subsection elaborates on the experimental procedures for testing the device.

6.2.2 Experimental procedures

- After the prototype substrate had been loaded onto the chuck and the probes had come into contact with the bond pads of the device, a microscope image of the initial structure of the tunable 1-D PhC was first taken.
- The power supply was used as a voltage source and a supply voltage of 0.200 V was set. The power supply was switched on for 30 s, and then an observation was made. While the voltage was still being supplied, another microscope image was recorded. The actual output voltage, the output current, the electrical resistance, and the rated power were recorded.
- The first step and the second step were carried out again with the applied voltage increased by 0.200 V. The same procedures were repeated up to when the voltage became 5.000 V. For every test carried out, the probes were always on the bond pads in the positions shown in Fig. 6-6. This is to make sure that the voltages
fallen across the upper and the lower expansion units were equal according to the first circuit configuration discussed in the previous section.

- The microscope images were processed by image-processing software (Adobe Photoshop CS3) in order to measure the displacement of the device.

6.3 Test results, analyses and discussions

6.3.1 Experimental results

According to the setup of the microscope and the CCD camera, the magnification of the objective lens was 100X, while that of the tube lens was about 1.333X, and that of the CCD camera was 40X. This gave a total magnification of about 4,333X. However, a difficulty arose during the experiment as the microscope system for inspecting the tested device did not have any software for measuring dimension of a sample. Therefore, all displacement observed from the images taken during the experiment had to be measured indirectly.

The microscope image was recorded in JPEG format with a size of 1200 pixels by 1600 pixels and a resolution of 96 dpi. The image size is hence (1200 pixels x 1600 pixels) / 96 dpi = 12.5 inches x 16.67 inches, which translates into an observation window of 73.275 μm x 97.700 μm. If it is assumed that the color information encoded into a digital image comes with an error of half a pixel and it is further assumed that this is the most significant source of systematic error, the accuracy of the recorded image becomes (73.275 μm/1200 pixels)/2 = 30.55 nm. Therefore any motion less than 30.55 nm could not be reliably measured from the image.

To measure the displacement, another microscope system that came with a calibrated scale inside the microscope software was used to measure in micrometers the width of a solid structure, which is the output guide width (opgdwid in Fig. 5-18) of the tested device in this case. After the dimension was taken, the width of the output guide was again measured in pixels from the image of the CCD camera. Afterwards, a conversion factor was derived by dividing the output guide width in micrometers by that in pixels. The conversion factor obtained was 0.05741μm/pixel. This conversion factor was later used to measure the displacement observed on the
recorded images. The measurements were performed based on a relative displacement of the output guide with respect to substrate. Different parts of the output guide were measured, and the mean displacement was used as a representative quantity. It was found that the standard deviation of the displacement at some data points was lesser than the accuracy of the system (30.55 nm). The error bar for those data points were thus replaced with the accuracy of the system. The detailed measurement results are attached in Appendix G.

Due to minute motion of the microactuator, the measurement could only be observed when the supply voltage $V_i \geq 4.000$ V. Nonetheless, the motion at $V_i < 4.000$ V was still observable via the CCD camera during the experiment. There are 6 measurement points spanning from 4.000 V to 5.000 V with an interval of 0.200 V. Using these supply voltages, the voltage across the heating element $V_H$ were calculated according to (6-11). The measured displacement was plotted against the voltage $V_H$ and is shown in Fig. 6-7. The standard deviation of each data point is used as the error bar in the plot. For $V_H = 1.66$ V and 1.77 V where the standard deviation was below the accuracy of the system, the accuracy of the system (30.55 nm) was used to create the

Fig. 6-7: Displacement of the microactuator versus voltage across a heating element. The measured displacements were plotted with diamond marker. Error bars for each measurement are the standard deviation of the measurements. The dashed line is curve-fitted, and its predicted trend is also shown. Displacements from FEA are provided for comparison. They are plotted with square marker and joined point to point.
error bars instead. It can be seen that the displacement non-linearly increases with $V_H$. A curve-fitted dashed line created by linear regression is added to predict the relationship between the displacement and the voltage. The predicted trend turned out to be $\delta = 0.1741V_H^2 - 0.1429V_H - 0.0057 \, (\mu m)$. The R-square value of 0.9115 suggests that the relationship is probably highly parabolic. Other R-square values obtained from exponential fitting and linear fitting are less than that of parabolic fitting. It can be noticed that when $V_H = 0 \, V$, the predicted displacement is non-zero with negative displacement of -5.7 nm which is physically invalid. This might be due to measurement errors, especially at $V_H = 1.52 \, V$, and 1.66 V, where the fitted curve barely fall inside the error bars, indicating that testing should be repeated if time allows. Since the displacement is quite small, qualitative results are difficult to record. Nevertheless, a qualitative result at $V_H = 1.77 \, V \, (V_i = 4.800 \, V)$ is provided in Fig. 6-8. The initial structure of the expansion unit together with the cross-beam and the transmission beam is shown in Fig. 6-8a. After applying the voltage, the whole structure was deformed such that the transmission beam was displaced to the

![](image1)

b)

Fig. 6-8: Microscope images of the expansion units with cross-beam before and after applying a supply voltage $V_i = 4.800 \, V \, (V_h = 1.77V)$. a) the structure before activation. b) The same structure after activation: the image has been reflected about x-axis for ease of comparison. Notice the gap between the vertical dotted line and the cross-beam on both images. The gap is slightly larger than that in a), indicating that there is a displacement in negative x-direction (towards the 1-D PhC)
negative x-direction as shown in Fig. 6-8b. Note that the image in Fig. 6-8b has been reflected about x-axis so that the displacement can be better observed by noticing the gap between the vertical dotted line and the cross-beam. The gap in Fig. 6-8a is slightly less than that of Fig. 6-8b, implying that the structure was displaced to the negative x-direction. The tunable 1-D PhC was in fact located on the left of the structure and was being compressed. The compression distance was equal to the displacement of the transmission beam because the transmission beam was much stiffer as compared to the 1-D PhC. Therefore, the displacement of the transmission beam is entirely transferred to the PhC.

6.3.2 Predictions from Finite-Element Analysis

A computer program (Ansys 11.0) was employed in a finite-element analysis (FEA) in order to predict the compression distance generated by the microactuator. The average structural parameters were taken from Table 5-2 to build the finite-element model shown in Fig. 6-9a. Note that, for features that were measured from both side of the substrate, the value of each parameter is the mean of the front-side feature dimension and the back-side features dimension. The model did not take account of the tapering thickness profile of the silicon structure caused by DRIE process. Only the suspended structure of the device was modeled so as to save computational cost while the two large bond pads on the substrate were ignored. The silicon plates that could not be fabricated properly inside the 1-D PhC trench and the SU-8 blocks sitting on top of the trench were excluded from the model. The average resistivity of the Au/Ti film (6.045x10^{-8} \, \Omega\cdot m) calculated in Subsection 6.1.4 was taken as the constant property for of the heating element in the model. Other relevant material properties were taken from Table 4-3 of section 4.7. The 3-D Coupled-Field Solid element was used in the analysis. Because of mirror symmetry along the center of the transmission beam, only half the model was actually used in the simulation. A symmetry boundary condition was applied along the central line of the beam to account for the other half of the model. The boundary conditions at the root of the expansion units and at the side walls of the PhC trench, where they connected to the substrate, were such that all the associated elements had zero displacement and were at room temperature. The voltage across the heating element $V_{HH}$ were taken
Fig. 6-9: Finite-element model of the tunable 1-D PhC device. a) The meshed model. The silicon structure, the heating elements, the SU-8 expanders, and the 1-D PhC (just PDMS block) are in orange, black, light gray, and blue, respectively. The smallest element dimension is about half of the width of the heating element. b) Temperature distribution over the model. The highest temperature at the heating elements is 201.3 °C, which is less than the degradation temperature of SU-8 polymer.
from Table 6-5, and it was applied at one of the two ends of the heating element, whereas the other end of the heating element was always grounded. After the model was created and all necessary settings were done, the simulation was run for each value of $V_H$ and the equilibrium solutions were recorded. The sought solutions were the nodal x-displacement at the end of the transmission beam $dx$, the overall maximum nodal temperature $T_{\text{max}}$, the average nodal temperature of the SU-8 expanders $T_{\text{avg}}$, and the current density at the end of the conductive film $\sigma$.

Table 6-6 summarizes all the FEA results. The nodal x-displacement at the end of the transmission beam was plotted against $V_H$ and is shown in Fig. 6-7. However, the FEA results seem to overestimate the actual the displacement (or compression distance of the 1-D PhC) of the device. Two possible causes for the overestimation can be addressed here. The first cause of the discrepancy was probably the difference in $R_H$. Since the measured resistance is higher than that of the FEA, the power delivered to each heating element in real situation would be lesser when equal $V_H$ was supplied. The lesser the power was delivered, the lesser the temperature rise and thus the displacement. The second discrepancy might be contributed by systemic errors and random errors during the displacement measurements. Despite the discrepancy, both experimental results and the FEA results display increasing trends of the displacement with the increasing of the voltage. The displacements from the FEA method are also within the same order of magnitude as that from the test. Improvements on the FEA model may thus result in less discrepancy.

On the electrical aspect, the resistance of the heating element of the FEA model was always 6.040 $\Omega$ because the resistivity set on the elements of the conductive film was taken as the average value of the property. It can be seen that the measured $R_H$ was more than the FEA $R_H$ by 1 $\Omega$ - 4 $\Omega$ approximately. Upon repeating the simulation using finer element size, the resistance difference did not alter significantly. Three possible causes to the resistance discrepancy can be thought here. One of them could be inaccuracy of the average resistivity which probably boils down to the errors in the measurement of the resistances. The second cause might be the errors of the measurements of structural dimensions. The last cause could be owing to the finite-element model that did not exactly resemble the device structure.
because of averaging of the feature dimensions and because of slightly uneven surface profile of the fabricated film.

The maximum operating voltage across the heating element $V_{th}$ should not be beyond 1.989 V as at this voltage the FEA analysis predicted that the maximum temperature of SU-8 was 233.8 °C which is almost at the glass transition temperature of fully cross-linked SU-8. The maximum temperature of the PDMS was 54.36 °C which is also less than the glass transition temperature and the degradation temperature of PDMS. The maximum nodal temperature at the maximum $V_{th}$ was 246.9 °C at the heating element. The temperature distribution at 1.771 V is shown in Fig. 6-9b, and it can be seen that the highest temperature is at the conductive film as expected. The temperature of the 1-D PhC at this voltage was only 48.70 °C, since thermal conduction was largely suppressed by the design of the transmission beam.

<table>
<thead>
<tr>
<th>$V_i$ (V)</th>
<th>$V_{th}$ (V)</th>
<th>$dx$ (μm)</th>
<th>$T_{avg}$ (°C)</th>
<th>$T_{max}$ (°C)</th>
<th>$\sigma$ (A/m²)</th>
<th>$I$ (A)</th>
<th>$R_{th}$ (Ω)</th>
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</table>
6.4 Summary

One of the fabricated devices was selected for the electromachanical characterization. The unknown resistances of the different parts of the device circuit were first identified. The effective resistance of each configuration of the test system was measured, so that four algebraic equations could be formulated. By solving these equations, the resistances became known. The resistivity of the heating elements was estimated based on the representative dimensions and the known resistance of the heating elements.

The fabricated device was put under the electromechanical test. Various supply voltages were applied to the test system at the interval of 0.200 V up to 5.000 V. However, the compression distance of the 1-D PhC device could only be observed from 3.000 V onwards. The microscope images of the deformed structure were recorded, and the compression distance of the 1-D PhC was measured using image processing software. The measured distance was plotted against each value of the voltage across the heating element.

The finite-element model was created using the structural features and the resistivity derived earlier. The compression distance obtained from the FEA was also plotted and compared with the experimental results. It was found that both the measured distances and the FEA seemed to have a parabolic relationship with the voltage across the heating element. The discrepancy of the distance between the two approaches was within the same order magnitude.
CHAPTER 7 CONCLUSIONS

All the accomplished works that were carried out so far are summarized here. Concise details of the works as well as some discussions from each chapter starting from Chapter 2 to Chapter 6 are given here in paragraphs.

7.1 Summary of Research

In Chapter 2 the theory of photonic crystal was explored from the literature. Two analytical tools were commonly used for revealing the photonic band gaps (PBGs) of dielectric periodic structures. The first one is the translation matrix which was first developed by Abeles. The matrix method was later applied in the so-called Bragg reflector – a one-dimensional photonic crystal (1-D PhC) – by Yariv and Yeh. By using the fact that a 1-D PhC possesses translational symmetry, the electromagnetic fields inside the crystal at a location one period away must be nothing but the same electromagnetic fields at the present location multiplied by some constant. Therefore, the electromagnetic fields at the location can be casted in terms of either the fields at old location multiplied by the one-period translation matrix or the old fields multiplied by a constant. Equating the two expressions for the new fields yields an eigenvalue problem. The characteristic equation obtained gives the condition for allowed eigenstates. When the condition cannot be satisfied it means no electromagnetic modes are allowed inside the crystal, leading to the condition for PBGs. Although the some concepts from quantum theory in crystallography, such as band gaps and eigenstates, were seen in the previous work of Yariv and Yeh, the theory was not generalized to other structures than layered media. In 1987, Yablonovitch and Sajeev independently proposed that there might exist a three-dimensional structure that possesses PBGs. The proposal was proved by experiments in later years. Formalism of present photonic crystal theory in quantum language can be found from Joannopoulos’ book.
According to Winn and his group in MIT it was discovered that 1-D dielectric structure can have a total omnidirectional reflectivity, i.e., PBG of 1-D PhC does exist at any inclined incidence. The discovery brought up a fact that 3-D structure is no longer a must for complete confinement of light in 3-D space. It was then proposed in this work that the PBG at some specific inclined incidence might be useful and a mechanically tunable PBG polarization splitter was proposed in order to demonstrate one of its applications. It was also found that there were not many works in the literature on mechanical tuning of photonic crystal. On fabrication aspect, to date, sub-micrometer dielectric structure fabrication is the key to realize a PhC device. Nonetheless, reliably mass-producing 3-D nanostructures is still challenging, albeit some researchers were successful in making them in laboratories. In contrast fabrication of 1-D PhC can be achieved much easier with existing planar fabrication technology borrowed from microelectronics and MEMS. Therefore, the proposal for the tunable device here fills the gap in the science and engineering community in three aspects. First, it addresses usefulness of PBG at specific inclined incidence for polarizing electromagnetic waves. Second, it widens the use of mechanical tuning for tunable PhC device. Third, it suggests possibility to miniaturize such device by the use of MEMS technology.

In search for the right tuning mechanism, major types of microactuators were reviewed. The three important types of microactuators are the thermal microactuators, the electrostatic microactuators and the piezoelectric microactuators. Each type of microactuators has its advantages and disadvantages. The piezoelectric microactuator has good dynamic response and large stress produced, but it comes at the cost of fabrication difficulty and small output displacement. The electrostatic microactuator offers ease of fabrication with moderate dynamic response and high output displacement. Standard fabrication processes for the electrostatic microactuators, such as MUMPs and SUMMiT V™, are available to help realize complicated structures. However, a very low output force is the big disadvantage of this type of microactuator. Thermal microactuator can also be fabricated with the aforementioned standard fabrication processes. In fact both thermal microactuator and electrostatic
microactuator can be fabricated with only one mask. One of advantages of using thermal microactuator is flexibility in material selection. This allows one to design a microactuator that suits the objective. High output force or high output displacement are achievable with the right choice of material. Moreover many variations of designs can be found from the literature. Nonetheless, the disadvantage of the thermal microactuator is slow response.

In Chapter 3 the soft polymer called poly(dimethylsiloxane) (PDMS) and silicon were selected for constructing the 1-D PhC. Then it was shown in Section 3.1 that there were large PBGs for this pair of materials. An improved technique of finding PBGs of 1-D PhC using plane wave expansion was proposed in section 3.2. Bragg condition was used to identify in k-space the locations of the PBGs. Then, the plane wave method was employed for calculation of the PBGs. In contrast to usual procedure of finding the PBGs, this method improves speed of calculation by directly solving the eigenvalue problem at the right location in irreducible Brillouin zone. The obtained frequencies from each allowed band are accordingly used to calculate the PBGs using the fact from Bragg condition that there are only direct band gaps for 1-D PhC.

The optical design of the mechanically tunable PBG polarization splitter was given in section 3.3. A transfer matrix method formalized by Yeh was used to reveal the PBGs of TE mode of the 1-D PhC at inclined incidence. Incidence at Brewster’s angle was used for the design. As a result, only transmittance of TE mode was tuned as a function of PDMS thickness, while all TM mode energy passed through the 1-D PhC. On the transmitted site of the PhC, transmitted power and polarization degree of previously unpolarized light was calculated as a function of PDMS thickness. A 1-D PhC made of 12-μm-thick PDMS layers and 6-μm-thick silicon layers was then proposed. The periodicity of 3 was selected so as to obtain gradual transmittance profile of TE mode. The frequency of 3.5 THz was chosen as the operating frequency of the device. Initial structure of the 1-D PhC is such that nearly all TE energy is reflected due to PBG effect and fully polarized state results. As the PDMS thickness is reduced, energy of TE mode can gradually pass through the 1-D PhC.
Therefore, mixing of TM energy and TE energy at the transmitted site is obtained. When the thickness of PDMS reaches 6 μm, TE energy is fully transferred through the 1-D PhC and the device is in its pass-all state. Finite-difference time-domain simulation was employed to confirm the theoretical design. The profile of TE transmittance agrees with that obtained from theoretical design. However, the peak transmittance at PDMS thickness of 6 μm is slightly lower by about 4.5%. Maximum loss was found to be about 2.6%. The loss indicates higher grid resolution and thicker PML layer should be used for the simulation in order to further investigate whether the cause is from numerical error or leakage from the sides of the 1-D PhC.

In Chapter 4 a thermal microactuator was selected for compressing the 1-D PhC previously designed in Chapter 3. Based on the optical requirements, the electrostatic microactuator could not cater for compressive force and thus it was not selected. Since thermal microactuator was chosen, the device should be employed in applications that do not require fast response.

The design procedure of the microactuator starts with simplifying the problem by assuming that the designed microactuator must have much higher linear stiffnesses and flexural stiffness as compared to the stiffness of the 1-D PhC. As a result, the microactuator feels no reactions from the 1-D PhC and the output displacement was assumed to be that from the uncoupled microactuator. After simplifying the problem, the structure of the microactuator was still statically indeterminate, but it was solvable by applying principle of superposition. In conjunction with applying the superposition principle, analyses for thermo-elastic deformation of the bimorph and for stiffness of the microactuator structure were carried out to help solve the problem. Once the output displacement was found analytically, a study on the length of the cross-beam was conducted so as to understand how other parameters, such as output displacement, reaction moment, etc., would be affected. Then, a finite-element simulation was carried out in order to verify the calculations. The analytical solution seemed consistent with the FEM results with acceptable discrepancy. Based on the knowledge of parametric study obtained, the design was further improved by adjusting dimensions of the
microactuator until requirements in terms of output displacement was satisfied. Temperature distribution of the microactuator was discussed. The maximum nodal temperature was found to be lower than both glass transition temperature and degradation temperature of SU-8. However, the temperature at the central output waveguide was still as high as about 90°C. This might cause a change in the refractive index both of silicon and PDMS. Improvement of the design should be investigated in order to reduce the temperature at the waveguide. One solution to the problems could be to design a thinner cross-beam in order to reduce the heat flux into the central waveguide. Moreover, an interlocking mechanism that can hold the structure in place might be designed so that the heater can be turned off when the lattice of the 1-D PhC is tuned to desired dimension.

Three major tasks were accomplished in Chapter 5.

The first one is the development of the microfabrication processes for the Si-PDMS hybrid structure. Although the processes were not implemented in the making of the prototype device, the ability to make such structures can in fact be an essential factor to the success of the fabrication.

The second task dealt with building a composite-type thermal microactuator using silicon and PDMS as the materials. The objective of the study was twofold. The main objective was to develop a process of selective filling with PDMS while the minor objective was to investigate the capability of the material for thermal actuation. The first objective was met as the liquid PDMS mixture could flow into the silicon skeletons as expected. Improvements of the technique would however be to better control the top profile of the PDMS so as to minimize the sagging profile. Although the minor objective was not totally satisfied, the fact that the PDMS material clearly thermally expanded inside the silicon skeleton did leave an implication that the material could actually drive the skeleton had the design of the microactuator was properly done.

As for the last and the most important task, the mechanically tunable 1-D PhC device was fabricated. There were however mainly three major problems encountered during the microfabrication. The first problem was the failure to
properly fabricate the silicon plates during the DRIE process: some of the standing silicon plates that build up the 1-D PhC were not attached firmly to the supporting aluminium, resulting in displacements and subsequent attachments of the plates to the side walls of the 1-D PhC trench. The second problem was with the profile of PDMS surface on top of the 1-D PhC trench. The profile was uncontrollable because of the manual filling process with PDMS. Moreover, the presence of a SU-8 block on top of the trench was undesirable as it tends to block or alter the power transmission of the input light. These problems were the major causes that prevented any optical testing possible after the fabrication. Overall, the dimensions of each successfully fabricated structural feature were quite satisfactory. An exception is probably the gaps between the comb-like structures that had a discrepancy of 67.26%. Such a large discrepancy may cause the microactuator to be unable to produce displacement as well as force exactly at expected applied voltage. Another exception might be the discrepancy of 27.57% in the width of the Au heater. The change in the width could directly result in an increase of the electrical resistance of the heater by the same proportion. Again the performance of the microactuator might not meet design expectation. Further investigations into the cause of these discrepancies are encouraged in the future work.

In Chapter 6, the electromechanical characteristic of the tunable 1-D PhC device was determined via accomplishing three tasks, i.e., the electric circuit analysis and measurements, the electromechanical experiment, and the analysis of the experimental results.

The circuit of the test system was first analyzed in detail, and four unknown resistances were identified. An experiment was designed in order to find out these unknown resistances. Four algebraic equations involving the four unknown resistances were derived by analyzing four different circuit configurations of the test system. Each circuit configuration yielded one equation with an additional parameter – the effective resistance of the circuit – that could be experimentally determined. After determining all the effective resistances, the four equations could be completely established. By solving the equations, all the unknown resistances were found. Using these resistances, the resistivity of the conductive film of the heating
element and the percentage of power transfer from the power supply to one heating element could also be deduced.

The electromechanical test was conducted at various supply voltages ranging from 0.200 V – 5.000 V and the images of the device before and after applying the voltage were recorded. The motion of the microactuator could be seen through the microscope system. However, the displacement or the compression distance of the 1-D PhC could only be measured when the supply voltage was from 4.000V to 5.000V, by analyzing the recorded images.

A finite-element model was created based on the measured dimensions of the device, the derived resistivity, and other relevant materials properties from Table 4-3. The measured displacements and the displacements predicted by the FEA were compared. Both the measured displacements and the FEA displacements were increasing with the voltage across the heating element $V_{HI}$, and they seemed to have a parabolic relationship with the voltage. The difference in the displacements from FEA and the experiment was in the same order magnitude. However, the measured displacements seemed to be lower than those from the FEA. The discrepancy could probably be contributed by the systemic errors, the random errors of the measurements of the displacements, and the averaging of the dimensions of the structural features in the FEA model.

7.2 Concluding Remarks

Based on the literature review reported in this thesis, it was found that the research in the field of mechanically tunable 1-D PhCs could be further looked into. There was a lack of the investigations from the photonic crystal theory point of view concerning what would happen when the lattice constant of a 1-D PhC was changed. Moreover, the number of researches on the mechanically tunable 1-D PhCs at the micrometer length scale or below was limited. This was probably due to many difficulties. First of all, the right materials for the right applications must be taken into account. The ability to build a useful structure from the materials at the desired length scale was the second. Third, a powerful microactuator was generally required in order to achieve the mechanical tuning of the lattice constant. Nonetheless, it was
pointed here that another application of the mechanically tunable 1-D PhC for the mechanically tunable PBG polarization splitter was possible. Thenceforth, the investigation of such device began.

Most of the objectives of this research were achieved. There were six objectives in total. First, the theory of the 1-D PhCs was investigated. A parametric study on the variations of the PBGs was carried out. It was found from the theory that the partial difference in the PBG profile of the TE mode and the PBG profile of the TM mode could be adjusted by changing the lattice constant of the 1-D PhC that was made of silicon and PDMS. The finding led to the use of the PBG effect as a polarization splitting mechanism. Second, a FDTD simulation was performed to confirm that the polarization splitting mechanism could happen. Third, a microactuator was designed in order to deliver necessary mechanical work enough for altering the lattice constant. Fourth, a FEA was employed to confirm the theoretical design. Fifth, necessary microfabrication processes were developed so as to realize both the 1-D PhC and the microactuator. Sixth, the microfabrication of a prototype and a performance testing were with done.

Some problems were encountered in the microfabrication processes: specifically, the DRIE, the Filling with PDMS process, and the SU-8 filling process. Nonetheless, the prototype was successfully built to the extent that it could be tested electromechanically. Although the design of the prototype was not the finalized design as proposed, the functioning prototype proved that the mechanically tunable 1-D PhC device could indeed be realized. By improving on the three problematic processes and by using a new set of photolithographic masks with the finalized design, it should be possible to fabricate the mechanically tunable PBG polarization splitter more perfectly, and the device should then be testable in both electromechanical aspect and optical aspect. The future works are to incorporate the device into a waveguide for an application and to set up an optical test system capable of supplying a monochromatic terahertz laser at 3.5 THz that should be available in the future soon.
List of Publications


References


43. Sampsell, J.B. *Digital micromirror device and its application to projection displays*. 1994. USA.


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APPENDIX A: Photonic Band Gap Calculation by Plane Wave Expansion Method

%%% calculation of the initial 1-D photonic band structure

\texttt{clear;}  
\texttt{hold off;}  
\texttt{newplot;}  
\texttt{epsilon\_k\_inv\_2d=[];}  
\texttt{c=299792458;}  % the speed of light in vacuum  
\texttt{N=200;}  % input an even number only  
\texttt{d=100;}  % no. of k = d+1  
\texttt{period=600*10^{-9};}  % structure period = 600 nm.  
\texttt{k=[-1/2:(1/d):1/2]*2*pi/period;}  
\texttt{G1=[-N/2:N/2]*2*pi/period;}  % there're N+1 values for G1  
\texttt{thickness\_fraction = 300/600;}  % ratio of the middle layer (silicon) to the whole cell  
\texttt{die\_const1=2.3;}  % dielectric constant of PDMS  
\texttt{die\_const2=11.68;}  % dielectric constant of bulk silicon  
\texttt{epsilon\_r=[1:N+1];}  % epsilon in physical space domain (as a func. of r)  
\texttt{epsilon\_r(:)=die\_const2;}  % this layer will be slightly thicker than the other layer  
\texttt{epsilon\_r(1:floor(N*(1-thickness\_fraction)/2))=die\_const1;}  
\texttt{epsilon\_r((N+1-floor(N*(1-thickness\_fraction)/2)+1):N+1)=die\_const1;}  
\texttt{epsilon\_r\_inv = 1./epsilon\_r;}  % epsilon in reciprocal space (as a func. of wave vector k)  
\texttt{epsilon\_k\_inv=((fft(epsilon\_r\_inv)))/(N+1);}  
\texttt{plot(1:N+1,abs(epsilon\_k\_inv),'k')}  
\texttt{disp('Please check the fourier coefficient values. Press enter to continue.'), pause;}  
\texttt{for p=1:N+1,}  
\texttt{epsilon\_k\_inv\_2d(p,:)=epsilon\_k\_inv(abs(p-q)+1);}  
\texttt{end}  
\texttt{end}  
\texttt{%% solving for omega}

\texttt{M=[]}; \texttt{OP=[];}  
\texttt{for m=1:length(k)}  
\texttt{M=epsilon\_k\_inv\_2d.*(((k(m)+G1).')*(k(m)+G1));}  
\texttt{OP(:,m)=eig(M);}  
\texttt{end}  
\texttt{disp('The calculation for the eigenvalues is finished!')}  
\texttt{%% Plotting 1-D PB structure}

\texttt{hold off;}  
\texttt{newplot;}  
\texttt{hold;}  
\texttt{OP2 = sort(sqrt((real(OP)))\*c);}
for m=1:3,
    plot(k*period/(2*pi),OP2(m,1:length(k))*period/c/2/pi)
end
set(gca,'xtick',[-0.5:0.1:0.5]);
set(gca,'ytick',[0:0.05:0.7]);
grid on;
title('1-D photonic band structure');
ylabel('Normalised frequency (\omegaa/2\pi)');
xlabel('Normalised wave vector (ka/2\pi)');
APPENDIX B: Photonic Band Gap at Inclined Incidence for TE Mode using Transfer Matrix Method

%% Normalized with designed lambda
%% The calculation will be carried out for 2 modes of EM: s-wave (TE) and p-wave (TM). The common parameters for both calculations are as follows:

%% Common parameters
%% Materials used: Si and PDMS
%% Dielectric constants

clear;

\begin{verbatim}
eps1 = 2.40;
eps2 = 3.42^2;
n1 = sqrt(eps1)
n2 = sqrt(eps2)
\end{verbatim}

%% Layer thickness
\begin{verbatim}
dL = 0.0001;
L = 0.01:dL:3;
\end{verbatim}

%% thickness ratio
\begin{verbatim}
r = 0:dr:1;
\end{verbatim}

%% Light incident from Si to PDMS:Si...
\begin{verbatim}
theta2= atan(n1/n2);  % incident at Brewster angle
theta1= asin(n2/n1*sin(theta2));
\end{verbatim}

%% Band Gap
\begin{verbatim}
PBG = []; mode = 'te';
\end{verbatim}

\begin{verbatim}
for m=1:length(r)
   p=1;
   tgl = 0;  %% PBG flag OR toggle switch
   for k=1:length(L)
      L1 = r(m)*L(k);
      L2 = L(k)-L1;
      for j=1:length(w)
         phi1=2*pi*n1*L1*w(j);
         phi2=2*pi*n2*L2*w(j);
         \endfor
         if strcmp(mode,'te')
            D1 = [1 1;n1*cos(theta1) -n1*cos(theta1)];
            D2 = [1 1;n2*cos(theta2) -n2*cos(theta2)];
            D12 = inv(D1)*D2;
            P2 = [exp(i*phi2*cos(theta2)) 0; 0 exp(-i*phi2*cos(theta2))];
            D21 = inv(D2)*D1;
            P1 = [exp(i*phi1*cos(theta1)) 0; 0 exp(-i*phi1*cos(theta1))];
            \endfor
         % TM
         else
            D1 = [cos(theta1) cos(theta1); n1 -n1];
            D2 = [cos(theta2) cos(theta2); n2 -n2];
            D12 = inv(D1)*D2;
            P2 = [exp(i*phi2*cos(theta2)) 0; 0 exp(-i*phi2*cos(theta2))];
            D21 = inv(D2)*D1;
            P1 = [exp(i*phi1*cos(theta1)) 0; 0 exp(-i*phi1*cos(theta1))];
\endverbatim

\begin{verbatim}
else
\endverbatim
\begin{verbatim}
D1 = [1 1;n1*cos(theta1) -n1*cos(theta1)];
D2 = [1 1;n2*cos(theta2) -n2*cos(theta2)];
D12 = inv(D1)*D2;
P2 = [exp(i*phi2*cos(theta2)) 0; 0 exp(-i*phi2*cos(theta2))];
D21 = inv(D2)*D1;
P1 = [exp(i*phi1*cos(theta1)) 0; 0 exp(-i*phi1*cos(theta1))];
\end{verbatim}
\begin{verbatim}
end
\end{verbatim}
\begin{verbatim}
M = (D21*P1*D12*P2);
x = abs((M(1,1)+M(2,2))/2);
if x>1
   if (tgl==0)  %% on band edge, going into PBG OR inside PBG and at the beginning of the array
      if (k==length(L))
         PBG = [PBG; r(m) w(j) L(k) p];
         tgl = 1;
      end
   end
\end{verbatim}
elseif (k==length(L))  % still inside PBG but at the end of the array
    PBG = [PBG; r(m) w(j) L(k) p];
end
elseif ((x<1) && (tgl==1))  % ~on band edge, going out of PBG
    PBG = [PBG; r(m) w(j) L(k) p];
    tgl = 0;
    p=p+1;
end
end
end
disp(r(m))
end

%% patch the 2-D band gap map (constant w)
figure
hdr = dr/2;
p=6;
C = ['r' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b' 'g' 'y' 'b'];  % color map for each band level
C2 = [[.8 .2 .2] [.2 .8 .2] [.2 .2 .8] [.8 .8 .2]];
for q=1:2:size(PBG,1)-1
    if (PBG(q,4)<=p) && (PBG(q,4)>0)
        X = [PBG(q,1)-hdr PBG(q,1)+hdr PBG(q,1)+hdr PBG(q,1)-hdr];
        Y = [PBG(q,3) PBG(q,3) PBG(q+1,3) PBG(q+1,3)];
        h=patch(X,Y,[.7 .1 .1]);%C2(PBG(q,4),:);
        set(h,'FaceAlpha',1,'EdgeAlpha',0.0,'EdgeColor',[.3 .3 .3])
    end
end

%plotting tuning profile
hold on
L0 = 0.07; %:0.1:2.1;
for j=1:length(L0)
    PDMSthk = (0:0.1:100)*L0(j);
    thkratio = PDMSthk./(PDMSthk+L0(j));
    y = PDMSthk+L0(j);
    plot(thkratio,y,'LineWidth',2.25,'Color','k')
end
set(gca,'XTick',0:0.1:1)
set(gca,'YTick',0:0.1:L(length(L)))
xlabel('Thickness ratio (\gamma)')
ylabel('Normalized lattice period (\ita\prime)')
axis([0 1 0 1])
%title('Photonic Band Gap Map for TE mode')
APPENDIX C: Script for FDTD Simulation of 1-D PhC

```scheme
(define nPDMS (sqrt 2.4))
(define nSi 3.415)
(define dSi 0.6)
(define-param dPDMS 2.6)
(define a (+ dSi dPDMS))
(define prd 3)
(define wlen 8.5655)
(define pmlthk (/ wlen 2))
(define-param phc? true)
(define-param resoln 6)
(define angl (atan (/ nPDMS nSi)))
(define ofsx 2)
(define y 60)
(define x (+ ofsx ofsx (- (* prd (/ a (cos angl))) (/ dSi (cos angl))) (* y (tan angl))))
(define cellheight (+ pmlthk y pmlthk))
(define cellwidth (+ pmlthk x pmlthk))
(define PDMSlocx (- (/ x 2) ofsx (* (/ y 2) (tan angl))) (* (/ dPDMS 2) (cos angl))))
; Flux planes
(define srcofs 1)
(define transht y)
(define transwid1 ofsx)
(define transwid2 (+ ofsx (* y (tan angl))))
(define transloc1 (/ x 2))
(define transloc2 (- (/ x 2) (/ ofsx 2)))
(define transloc3 (- (/ x 2) (/ transwid2 2)))
(define reflchgt y)
(define reflcwid1 (- x transwid1 srcofs 0.1 (- (* prd (/ a (cos angl))) (/ dSi (cos angl)))))
(define reflcwid2 (- x transwid2 srcofs 0.1 (- (* prd (/ a (cos angl))) (/ dSi (cos angli)))))
(define refllocx1 (+ (/ x -2) srcofs 0.1))
(define refllocy1 0)
(define refllocx2 (+ (/ x -2) (/ reflwid1 2) srcofs 0.1))
(define refllocy2 (- (/ x -2) (- reflcwid1 2) srcofs 0.1))
(define refllocx3 (+ (/ x -2) (/ reflwid2 2) srcofs 0.1))
(define refllocy3 (- (/ x -2) (- reflcwid2 2) srcofs 0.1))
(set! default-material (make dielectric (epsilon (* nSi nSi))))
(set! geometry-lattice (make lattice (size cellwidth cellheight no-size))))
(set! geometry
  (if phc?
    
    
```
(define f (/ 1 wlen))
(define df (* f 0.1))
(set! sources (list
(make source
  (src (make gaussian-src (frequency f) (fwidth df) ) )
  (component Ez )
  (center (+ (/ x -2) srcofs) 0)
  (size 0 (/ y 2))
))
(set! pml-layers (list (make pml (thickness pmlthk))))
(set-param! resolution resoln)
(define nfreq 11) ; make it odd so that the central freq is included.
(define trans1 ; transmitted flux region 1
(add-flux f df nfreq
  (make flux-region
    (center translocx1 translocy1) (size 0 transhgt) ))))
(define trans2 ; transmitted flux region 2
(add-flux f df nfreq
  (make flux-region
    (center translocx2 translocy2) (size transwid1 0) )))
(define trans3 ; transmitted flux region 3
(add-flux f df nfreq
  (make flux-region
    (center translocx3 translocy3) (size transwid2 0))))
(define reflc1 ; reflected flux region 1
(add-flux f df nfreq
  (make flux-region
    (center reflclocx1 reflclocy1) (size 0 reflchgt))))
(define reflc2 ; reflected flux region 2
(add-flux f df nfreq
  (make flux-region
    (center reflclocx2 reflclocy2) (size reflcwid1 0))))
(define reflc3 ; reflected flux region 3
(add-flux f df nfreq
  (make flux-region
    (center reflclocx3 reflclocy3) (size reflcwid2 0 ))))
(if (phc?) (load-minus-flux "reflc-flux" reflc1))
(if (phc?) (load-minus-flux "reflc-flux2" reflc2))
(if (phc?) (load-minus-flux "reflc-flux3" reflc3))

(if (not phc?) (save-flux "reflc-flux" reflc1))
(if (not phc?) (save-flux "reflc-flux2" reflc2))
(if (not phc?) (save-flux "reflc-flux3" reflc3))
(run-until 1500
  (at-beginning output-epsilon)
  (to-appended "ez" (at-every 50 output-efield-z)))
(display-fluxes trans1 trans2 trans3 reflc1 reflc2 reflc3)
(display resolution)
(newline)
APPENDIX D: Flexural Stiffness of Parallel Beam Assembly Subjected to Rigid Plane Boundary

As can be seen from subsection 4.6.2 of Chapter 4, the flexural stiffness of a beam is proportional to a cube of the thickness of the beam. Unlike the case of axial stiffness, the non-linearity prevents one from using the simple rule to determine the effective flexural stiffness of two beams connected in parallel. The following provide a derivation of effective flexural stiffness.

I. Axial Strain due to External moment & the Neutral Plane of Beam Assembly

The flexural stiffness of a beam can be defined by an external moment $M$ divided by the flexural displacement $\delta \theta$ found at beam tip. In most of engineering problems, the flexural displacement is small and can be approximated to the sloped at the tip $y'_{tip}$. To express the stiffness in terms of material property of the beam and beam dimensions, the externally applied moment must be related to the internal bending moment which can be written in terms of material property and dimensions of the beam. To obtain an expression of the bending moment, the strain is first determined. Then, the stress-strain relationship is used to relate the bending moment to the strain.

Consider a problem of two parallel homogeneous beams perfectly bonded together side by side. Both ends of the assembly are restrained by rigid caps. The two beams can possibly have different thicknesses of $t_1$ and $t_2$ and different Young’s modulii of $E_1$ and $E_2$, respectively. The original length and the width of one beam are taken to be the same as those of the other. The beam assembly is subjected to pure bending due to an external moment $M$. The deflected beam assembly is shown in Fig. D-I.

In order to obtain the axial strain cause by the external moment, the geometry of deformed beam elements must be deduced first. The condition of the end surfaces of the beams are such that, under the load, the end surfaces of both beams remain parallel and coincident because of the rigid plane boundary. Due to the condition of the end surfaces, the deformed beam elements on any cross-section must be subtended by the same angle, say $\phi$, as shown in a closed-up drawing of the assembly in the figure.¹

¹ Had the boundary not been a rigid plane, the elements of the two materials at the end of the beams could have bent differently as shown in another element drawing. As a result the radius of curvature of any element on the same cross-section can be discontinuous and the expression for the area moment of inertia would be more involved.
By definition, the beam elements lying in the neutral plane are not affected by the external bending moment, and thus they are not under axial strain. Let all beam elements on a cross-section have an initial length $\delta L$. When the deformation takes place, the length of those beam elements on the neutral plane remain unchanged, but form an arc of radius of curvature $\rho(y_n)$. The radius of curvature is written as a function of a $y$-distance from a reference point lying on the cross-section as shown. The subtending angle $\phi$ can be related to the initial length $\delta L$ and the radius of curvature $\rho(y_n)$ as:

$$\delta L = -\phi \rho(y_n) = \phi \rho_0$$  \hspace{1cm} (D-1)

Note that it has been assumed that there exists a single neutral plane for the beam assembly locating at some location $y_n$ inside the assembly. This location $y_n$ will be determined and the assumption will be verified afterwards. Since all the deformed elements on the same cross-section are subtended by the same angle, $\phi$, the length of other deformed element $\delta L'$ can be expressed in similar way as:

$$\delta L' = -\phi \rho(y) = \phi \rho$$  \hspace{1cm} (D-2)

, with $y$ being location of the element. The axial strain can be expressed as follows:

$$\varepsilon_B = \frac{\delta L' - \delta L}{\delta L} = \frac{\rho \phi - \rho_0 \phi}{\phi \rho_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{y - y_n}{\rho_0},$$  \hspace{1cm} (D-3)

where the difference in radius of curvature is equal to the difference of $y$-locations of the beam elements. Using stress-strain relationship, the bending stress $\sigma_B$ at a location $y$ is equal to:
With the presence of the rigid plane boundary, there must be reaction forces $R_1$ and $R_2$ acting along axes of Beam 1 and Beam 2, respectively:

\[
R_1 = \int_{\text{Beam}1} \sigma_1 dA = \int_{\text{Beam}1} \frac{E_1}{\rho_0} (y - y_n) dA = \frac{E_1}{\rho_0} \left[ \int_{\text{Beam}1} y dA - y_n A_1 \right],
\]

\[
R_2 = \int_{\text{Beam}2} \sigma_2 dA = \int_{\text{Beam}2} \frac{E_2}{\rho_0} (y - y_n) dA = \frac{E_2}{\rho_0} \left[ \int_{\text{Beam}2} y dA - y_n A_2 \right].
\]

To satisfy equilibrium condition, summation of $R_1$ and $R_2$ must be zero, leading to:

\[
R_1 + R_2 = 0 = \frac{1}{\rho_0} \left[ \int_{\text{Beam}1} E y dA - y_n E_1 A_1 \right] + \frac{1}{\rho_0} \left[ \int_{\text{Beam}2} E y dA - y_n E_2 A_2 \right],
\]

\[
y_n = \frac{y_1^* E_1 A_1 + y_2^* E_2 A_2}{E_1 A_1 + E_2 A_2} = \sum_{i=1}^{2} \frac{y_i^* E_i A_i}{E_i A_i},
\]

where $y_i^* = \int_{\text{Beam}i} E y dA / E_i A_i$; $i = 1, 2$. Therefore, the location of the neutral plane is the weighted average of $y_i^*$ with $E_i A_i$. In case of homogeneous beam, the Young’s modulus can be brought out from the integral sign and $y_i^*$ simply become the centroids of the cross-sections of each beam. It can be seen that $y_n$ is lying somewhere between $y_1^*$ and $y_2^*$. Since $y_i^*$ must be located inside its beam due to $E_i$ and $dA_i$ are associated with each point on the beam, $y_n$ must be located in either of the two beams. In case of only one homogeneous beam under external moment, $A_2$ can be set to zero, leading to $y_n$ being at the centroid of the cross-section, as is well known. It can be concluded that there must exist only one neutral plane located in either beam when the beams are connected in parallel. The proof that there is one neutral plane for the beam assembly can help one conveniently express the bending stress in terms of single spatial coordinate, the relative distance from the neutral plane, across the entire cross-section of the assembly.

**II. Stiffness of the Beam Assembly**

The total opposing internal bending moment $M_B$, caused by bending stress $\sigma_B$ on a cross-section $S$, must be equal to $M$ and can be expressed as:

\[
M = M_B = \int_S \gamma \sigma_B dA.
\]
The quantities in the bracket are the products of the Young’s modulus and the area moments of inertia of each beam measured from the neutral plane of the assembly. Using parallel axis theorem, the above area moment of inertia, $I_i$, can be related to the area moment of inertia measured with respect to original neutral plane of each beam, $I_0$, as:

$$I_i = \int_{\text{Beam } i} \gamma^2 dA = I_0 + A_i(y_{c,i} - y_n)^2 \quad ; i = 1, 2 , \quad (D-9)$$

with $y_{c,i}$ being the centroid of Beam $i$. Now, recall that the reciprocal of radius of curvature is approximately equal to second derivative of $y$-coordinate of the neutral plane with respect to $x$-coordinate. Integrating (D-8) with respect to $x$ from 0 to the whole length of the assembly, $L$, gives:

$$M = \frac{1}{\rho_0} \left[ E_1 I_1 + E_2 I_2 \right] \approx \left[ E_1 I_1 + E_2 I_2 \right] y''$$

$$\int_0^L M dx = \left[ E_1 I_1 + E_2 I_2 \right] y'' dx \quad (D-10)$$

$$M(L-0) = \left[ E_1 I_1 + E_2 I_2 \right] (y_{op}' - 0)$$

$$ML = \left[ E_1 I_1 + E_2 I_2 \right] \delta \theta$$

The effective stiffness of the assembly becomes:

$$k_{\text{eff}}^\theta = \frac{M}{\delta \theta} = \frac{E_1 I_1}{L} + \frac{E_2 I_2}{L} = k_1^\theta + k_2^\theta , \quad (D-11)$$

where the rule of calculating effective stiffness is similar to that of linear springs in parallel, except for the modification of the area moment of inertia. To confirm the correctness of the formula, the flexural stiffness of an assembly comprising of two identical beams of the Young’s modulus $E$, thickness $t$, height $H$ and length $L$ can be calculated as follows:

$$k_{\text{eff}}^\theta = \frac{EI}{L} + \frac{EI}{L} = \frac{2EI}{L} = \frac{2E}{L} \left( \frac{Ht^3}{12} + Ht(y^* - y_n)^2 \right)$$

$$= \frac{2E}{L} \left( \frac{Ht^3}{12} + Ht \left( \frac{t}{2} - 0 \right)^2 \right) = \frac{2E}{L} \frac{4Ht^3}{12} = \frac{8EI}{L} \quad , \quad (D-12)$$
in which a reference point for measuring $y^*$ and $y_n$ has been chosen at the joining surface. As in the case of one beam of thickness $2t$, the flexural stiffness increases by 8 times.
APPENDIX E: Finite-Element Simulation Results of Parametric Study of the Thermal Microactuator

Table E-I
Study of length of cross-beam & discrepancies between theory and FEM

<table>
<thead>
<tr>
<th>Studied parameters</th>
<th>Results</th>
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<tbody>
<tr>
<td></td>
<td>( t_p = 40 \mu m )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( L_{cr} ) (( \mu m ))</td>
</tr>
<tr>
<td>20</td>
<td>9.422</td>
</tr>
<tr>
<td>40</td>
<td>10.14</td>
</tr>
<tr>
<td>60</td>
<td>10.80</td>
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<tr>
<td>80</td>
<td>11.42</td>
</tr>
<tr>
<td>100</td>
<td>12.03</td>
</tr>
<tr>
<td></td>
<td>( t_p = 60 \mu m )</td>
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<tr>
<td>20</td>
<td>7.887</td>
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<tr>
<td>40</td>
<td>8.415</td>
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<td>8.917</td>
</tr>
<tr>
<td>80</td>
<td>9.410</td>
</tr>
<tr>
<td>100</td>
<td>9.899</td>
</tr>
<tr>
<td>200</td>
<td>12.32</td>
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<tr>
<td>300</td>
<td>14.73</td>
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<td>400</td>
<td>17.14</td>
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<tr>
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<td>19.55</td>
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<tr>
<td>600</td>
<td>21.96</td>
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<tr>
<td></td>
<td>( t_p = 80 \mu m )</td>
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<td>7.582</td>
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\(^2\) The estimation is based on clamped-clamped beam configuration where the length of clamped-clamped beam was taken to be the whole length of the connector (twice of the length of the cross-beam).
### Table E-II
FEM results with corresponding average temperature and maximum temperature

<table>
<thead>
<tr>
<th>Variable Parameters</th>
<th>$\delta y_{FEM}$ (μm)</th>
<th>$T_{av}$ &amp; $T_{max}$ (°C)</th>
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</thead>
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<td>$L_c$ (μm)</td>
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<td>20.0 40.0 60.0 80.0 100.0</td>
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<tr>
<td>$t_s$ (μm)</td>
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<tr>
<td>40</td>
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<td>108.6 108.6 108.5 108.5 107.3</td>
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<td>151.9 151.9 151.9 151.9 151.1</td>
</tr>
<tr>
<td>60</td>
<td>5.427 6.542 7.365 8.047 8.277</td>
<td>108.4 108.3 108.3 108.3 107.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150.6 150.5 150.5 150.5 149.9</td>
</tr>
<tr>
<td>80</td>
<td>5.105 5.811 6.347 6.802 7.203</td>
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<td></td>
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<td>149.3 149.3 149.3 149.3 149.3</td>
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APPENDIX F: Measurements of Dimensions of Structural Features of the Mechanically Tunable 1-D Photonic Crystal Device

Measurements of dimensions of the structural features were done from the front side and the back side of the device. They are averaged to give representative values of the features on each side. The average feature dimensions are given at the end of Chapter 5. Other features, such as thickness of heater, width of heater, thickness of the device, etc., are also provided. For all measurements, standard deviation and discrepancy (from designed values) are also available.

### Front-side parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Designed values</th>
<th>Nth Sample</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Discrepancy(%)</th>
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<td>4.342</td>
<td>4.083</td>
<td>4.147</td>
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**Back-side parameters**

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<th>Parameters</th>
<th>Designed values</th>
<th>Nth Sample</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Discrepancy(%)</th>
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<tr>
<td>mtwid</td>
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**Other parameters**
## APPENDIX G: Compression Distance of the Tunable 1-D Photonic Crystal

### Table G-I: Measured compression distance of the tunable 1-D PhC

<table>
<thead>
<tr>
<th>$V_i$</th>
<th>$V_H$</th>
<th>Nth Measurement</th>
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<th></th>
<th></th>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
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<td>1.4198306</td>
<td>0.11</td>
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<td>0.11</td>
<td>0.06</td>
<td>0.2</td>
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