MICROWAVE DETECTION OF EARLY TUMORS

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2010
Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

..........................................................  ..........................................................

Date                                                                                   Zhang Huiyu
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List of Abbreviations and Symbols

Abbreviations

2-D/3-D  two dimensional/three dimensional
CAD    computer aided detection
CT/CAT computer tomography/computer axial tomography
CMI    confocal microwave imaging
DAS    delay-and-sum
DBIM   distorted-Born iterative method
DCIS   ductal carcinoma in situ
EM     electromagnetic
EMI    electromagnetic interference
FDA    Food and Drug Administration (US)
FDTD   finite difference time domain
FEM    finite element method
GLRT   generalized likelihood ratio test
HFSS   High Frequency Structure Simulator
IDC    invasive ductal carcinoma
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ILC</td>
<td>invasive lobular carcinoma</td>
</tr>
<tr>
<td>LCIS</td>
<td>lobular carcinoma in situ</td>
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<td>MAMI</td>
<td>multistatic adaptive microwave imaging</td>
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<tr>
<td>MIM-SDFMM</td>
<td>multiple interaction model-steepest descent fast multipole method</td>
</tr>
<tr>
<td>MIST</td>
<td>microwave imaging via space–time</td>
</tr>
<tr>
<td>MRI</td>
<td>magnetic resonance imaging</td>
</tr>
<tr>
<td>PET</td>
<td>positron emission tomography</td>
</tr>
<tr>
<td>PPDW</td>
<td>parallel-plate dielectric waveguide probe</td>
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<tr>
<td>PPWP</td>
<td>parallel-plate waveguide probe</td>
</tr>
<tr>
<td>PVC</td>
<td>poly(vinyl chloride)</td>
</tr>
<tr>
<td>SI</td>
<td>Système Internationale</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
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<tr>
<td>SMA</td>
<td>subminiature version A</td>
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<tr>
<td>TR</td>
<td>time reversal</td>
</tr>
<tr>
<td>TSAR</td>
<td>tissue sensing adaptive radar</td>
</tr>
<tr>
<td>TEM</td>
<td>transverse electric and magnetic</td>
</tr>
<tr>
<td>TE</td>
<td>transverse electric</td>
</tr>
<tr>
<td>US</td>
<td>United States</td>
</tr>
<tr>
<td>UTD</td>
<td>uniform geometrical theory of diffraction</td>
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<td>UWB</td>
<td>ultrawideband</td>
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Symbols

\( \hat{a}_A \)  
unit vector along \( A \)

\( \alpha \)  
attenuation constant

\( \beta \)  
propagation constant

\( D_h \)  
diffraction coefficient

\( d_0 \)  
diameter of tumor

\( \delta_{mn} \)  
Kronecker delta

\( \delta \)  
Dirac delta function

\( \Delta S_{11(A,B)} \)  
difference in \( S_{11} \) measurements at positions \( A \) (\( S_{11(A)} \)) and \( B \) (\( S_{11(B)} \))

\( \varepsilon \)  
permittivity

\( \varepsilon_r \)  
relative permittivity

\( \varepsilon_r' \)  
real part of the relative complex permittivity

\( \varepsilon_r'' \)  
real part of the relative complex permittivity

\( \varepsilon_r'' \)  
imaginary part of the relative complex permittivity

\( \varepsilon_r'' \)  
imaginary part of the relative complex permittivity

\( E \)  
electric field

\( F(x) \)  
transition function

\( f \)  
frequency

\( f_r \)  
resonant frequency

\( \gamma \)  
interior wedge angle

\( \Gamma \)  
reflection coefficient
\( h \) depth of tumor embedded

\( h_n^{(1)} \) spherical Hankel function of first kind

\( H \) magnetic field

\( \tilde{H} \) Fourier transform of \( H \)

\( i \) square root of \(-1\)

\( j_n \) spherical Bessel function of first kind

\( k \) wavenumber

\( \lambda_0 \) wavelength at cut-off frequency

\( L \) distance parameter

\( n \) index of refraction

\( r \) spherical radial coordinates

\( \omega \) angular frequency \(( = 2\pi f )\)

\( P_i^n \) associated Legendre functions

\( \phi \) spherical azimuthal coordinates

\( \Phi \) diffraction angle with respect to the old face

\( \Phi' \) incident angle with respect to the old face,

\( \sigma \) conductivity

\( \tan \delta \) loss tangent

\( \tau \) transmission coefficient

\( \theta \) spherical polar coordinates
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Cancer is the leading cause of deaths worldwide. Early detection of malignant tumors reduces mortality rates because cancer can be cured if treated early. Even the widely accepted standards for tumor detection, such as breast mammography, limitations exist, however, such as having high false positive and negative rates, ionizing radiation and painful breast compression. Microwave cancer detection methods that identify the presence of a tumor by detecting the dielectric difference of normal tissues and malignant tumors is thus explored as a new modality.

Well received microwave approaches include tomographic and radar-based methods. In these computationally complex time domain techniques that depend on signal correlations between the sent and received ultrawide band pulses, accuracy depends mainly on resolving significant mutual coupling effects of the antenna array, and compensating the tissue’s dispersive effects on these pulses.

In this thesis the frequency domain approach of microwave detection of early tumors is explored. Theoretical and experimental study of early tumor detection using a novel, flanged, parallel-plate dielectric waveguide probe (PPDW) for contact S parameters measurements is presented. The detection technique is based on microwave scattering of lossy dielectric bodies. As reported in many literatures, a tumor exhibits
dielectric properties in high contrast to the healthy embedding tissue. Hence this technique identifies the presence of a tumor with significant backscattered signals ($S_{11}$) measured with the probe. The large backscattered signal from the tumor can be used to distinguish from clutter items having lower complex permittivity. Size and location of the tumor can also be estimated from the scattered signals as a function of frequency and angle; with the accuracy of localisation further enhanced with the use of both backscattered and scattered signals from the tumor ($S_{11}$ and $S_{21}$, respectively).

Firstly, interaction of electromagnetic waves radiated from a theoretical parallel-plate waveguide probe with a multilayer dielectric is studied. A detailed formulation using the Fourier transform method, for expressions of the Scattering parameters measured in the presence of a tumor, is described. To obtain a simple analytical solution, Mie scattering by a dielectric sphere is used. Using breast cancer detection as an illustration, the method of identifying, locating and characterizing a tumor is demonstrated through numerical simulations. To enhance accuracy and practicability, a novel design of a flanged PPDW that confines the fringe radiation and provides good electromagnetic isolation by the use of a specifically shaped central dielectric is presented. To the best of the author’s knowledge, this design of a flanged PPDW with a central dielectric guide to confine the radiation within the guide is original and unique, and no other publication is known. Finally, experimental verifications of the proposed technique and the PPDW are conducted on solid self-fabricated breast phantoms that simulate the real breast in terms of dielectric properties and phase (solid state). Experimental measurements of the S parameters correspond closely to the theoretical predictions, indicating the potential of the flanged PPDW to identify, localize and
characterize small spherical tumors embedded at reasonable depths of few centimeters in the breast.
Chapter 1

Introduction

1.1 Motivation

In worldwide, cancer is the leading cause of deaths. By 2030 it is predicted more than 26 million people are diagnosed with cancer [1]. World Health Organisation has also projected in the year 2030, there would be a 12 million global cancer deaths [2]. The top few killers are lung, stomach, liver, colon and breast cancer. With the increase in life expectancy, incidence of cancer and associated death rates are expected to remain high as risk increases dramatically with age. However reduction in mortality rates for cancer can be made possible. More than 30% of the cancer deaths rate could be decreased as cancers can be cured if detected and treated early. Thus early cancer screening takes on a vital role in reducing this public burden as treatment is more effective when cancer is detected earlier.

Breast cancer being one of the most frequent forms of cancer, is a leader in cancer deaths in women. On the average, patients diagnosed with Stage 1 breast cancer have a 5-year relative survival rate of close to 100% while this figure drops to less than 20%
for *Stage 4* diagnosis [3]. Hence the role of breast cancer detection becomes increasingly important, leading to a demand in effective diagnostic measures. The gold standard for breast cancer detection remains the mammography [4], or x ray imaging of a compressed breast is based on detecting the density differences between normal tissues and lesions. This method possesses high sensitivity, which is the ability to detect abnormalities; but has limited specificity, that is the ability to distinguish a benign lesion from a malignant tumor. It requires medical expertise to accurately diagnose the presence of tumor as the number of cancers found with mammography alone is very much less than that found with both mammography and physical examination [5]. Furthermore, it is difficult to identify cancers in women with dense breasts. The limitations of having high false negative and false positive rates [6, 7] lead to increased healthcare cost, unnecessary medical procedures such as biopsy, and the distress and anxiety on the part of the patient waiting for the confirmed diagnosis. Other important concerns also include the discomfort due to breast compression and ionizing radiation exposure patients undergo when taking the mammograms.

As such, other methods of detection that aim to overcome the present shortcomings of mammography have been proposed. These techniques which do not use x-rays such as ultrasound imaging, magnetic resonance imaging (MRI), and positron emission tomography (PET), are used to detect breast cancer [8-10]. Ultrasound and MRI helps to supplement mammography to detect small tumors that may not be identified with mammography [11]. However due to the high rate of false positive results, ultrasound and MRI are usually considered for screening women with high risk instead [12].
CHAPTER 1. INTRODUCTION

Moreover MRI and PET are more costly and cumbersome to use. These factors limit the use of these techniques for regular screening routine of breast cancer.

The lack of success in these methods to provide high sensitivity and specificity triggered exploration of other techniques as the new modality for tumor detection. Gaining significant attention is the technique using microwave scattering and reflection. Microwave detection method is believed to be a viable technique for accurate, non-invasive detection of malignant tumors. The ability of microwaves to penetrate deep inside dielectric materials makes this technique attractive for non-destructive detection of tumors. Based on detecting the difference in complex permittivity between a malignant tumor and the surrounding healthy tissues, microwave detection is attractive as it is non-invasive, non-ionizing, and is cheap.

The major microwave methods proposed based on this approach include ultrawideband (UWB) radar-based imaging system [13] and microwave tomography [14]. However algorithms to construct the image of the scanned breast from measurements from an array of antennas, especially using the tomographic methods, require complex, iterative and rigorous computations and processing. Hence the UWB radar-based system is more popular; although the information from this approach does not give the estimates of the values of the electrical properties of the scanned breast, it identifies the tumors from the normal surrounding tissue from the contrast in the electrical properties between these two areas of interest. The radar-based method works on the principle of sending a wide-band pulse from transmitting antenna arrays and receiving the backscattered signal. A bright spot indicating a tumor present will be produced on the processed image. However performing signal processing techniques
such as time-shifting, phase correlations in the UWB radar-based system to determine the presence and position of a tumor may not be very accurate. The major contribution of error in the method is the distorted back scattered pulse received. As the tissue medium is dispersive, the UWB pulses shape sent and received will differ significantly, and the inability to maintain the pulse shape will result in errors in the determination of the presence and position of the tumor. In this method, significant mutual coupling effects of the antennas in the array need to be addressed; to reduce this coupling effect that would affect the transmitting and receiving signal integrity, it is important to ensure the antennas are well-matched over the entire ultra-wide bandwidth. Therefore the design of the sensing elements and the reconstruction algorithms require specific attention.

In view of this, the author is motivated to develop and evaluate with experimental verifications, a technique and device that efficiently and accurately detect tumor in the region of concern, e.g. human organ, biological tissue, etc., with the following advantages:

(i) Non-invasive, without use of uncomfortable pressure on the body

(ii) Safe, involves only low microwave radiation levels

(iii) Simple, scalable design

(iv) Compact, portable and practical design.
CHAPTER 1. INTRODUCTION

1.2 Objectives

The main objective of this thesis is to design and develop a new technique for microwave detection of early tumors with a novel device with the following capabilities and advantages:

(i) *Provides clear, visible, and easy to interpret frequency responses of the tumor*

Screening techniques, such as the widely accepted method of breast cancer detection, mammography, requires medical expertise to accurately diagnose the presence of tumor as the number of malignant tumors found with mammography alone is very much less than that found with both mammography and physical examination [5]. The high positive and negative rates limit the cost-effectiveness of this method. The proposed detection technique and device overcome this by providing clear markers for any malignant tissue present in the region of concern in terms of frequency responses to reduce errors in diagnosis.

(ii) *Detect small tumors at realistic depths of embedment in tissue*

Detecting small tumors at earlier stage of cancer leads to earlier treatment and significantly higher chances of recovery.

(iii) *Simple to operate*

The algorithm for the device is simple and less complex to implement unlike some of the radar-based and tomographic methods which require complex and rigorous computation and processing to reconstruct the image of the scanned organ.
CHAPTER 1. INTRODUCTION

(iv) **Improved Accuracy**

As the tissue medium is frequently dispersive, the device sends continuous wave over the swept frequencies and frequency correlation is performed, thus providing better accuracy.

(v) **Non-invasive and non-ionizing**

The patient need not suffer from pain when undergoing the screening test performed using the device, unlike the discomfort the patient has to endure such as the case of breast cancer detection where the breast are compressed when taking mammograms. Low levels of microwave radiation are used in this technique, hence there is no risk of radio frequency heating.

1.3 **Contributions of the Thesis**

The main contributions of the thesis include the following.

(i) **Design and development of a technique for the microwave detection of tumor**

A technique for microwave detection of malignant tumor based on detecting the difference in complex permittivity between a malignant tumor and its surrounding healthy tissue with the use of flanged parallel-plate dielectric waveguide probe is developed. The flanged parallel-plate dielectric waveguide probe is designed for S parameters measurements. In the presence of an inclusion (tumor) having a
significant difference in dielectric properties from the embedding medium (surrounding healthy tissue), the S parameters alter and display unique characteristics due to the scattering of dielectric bodies. Information on the size and location of the tumor can also be derived from the S parameters measured. Additionally, heterogeneity of the tissue of concern or benign lesions can be distinguished from the malignant tumor from the S parameters.

The effectiveness of the proposed technique is illustrated with specific reference to the detection of breast cancer.

(ii) Design and development of a practical probe

The size of the sensor has to be practical in a clinical environment. The probe is designed to provide reasonable performance in terms of depth of detection, size of detection, etc., while being physically small in size. Furthermore good isolation from the external interference is achieved with this design. The simple design of the probe also enables fast and convenient sanitation of the system after one screening.

(iii) Evaluation of the performance of the probe for tumor detection through experimental studies

The probe is fabricated and tested on solid breast phantoms, simulating the dielectric properties of a breast, with conductive inclusions embedded in the phantoms representing tumors. Majority of the published experimental studies were performed on low loss nondispersive liquid phantoms, hence the dispersive effects and loss
factor that resemble an actual breast on the attenuation of accuracy of the transmitted and received signals are not investigated. However this thesis performs experimental studies on lossy solid breast phantom which better simulate an actual breast in terms of the actual complex permittivity and the phase (solid state). The results of the experimental investigations correspond to those predicted by theory and simulations, thus demonstrating the ability of the proposed technique and probe to detect and locate small tumors embedded at depths of a few centimeters.

(iv) Develop analytical solutions to the tumor signatures in terms of S parameters to be measured by the probe

This thesis approaches the microwave detection of tumor in an analytical manner which involves studying the electromagnetic interaction of the fields from the proposed probe for detection with the imaged biological tissue. Reflection and transmitted coefficients, or S parameters, at the aperture of the probe are derived to enable the identification and characterization of small tumors.

1.4 Organization of the Thesis

The thesis is organized as follows.

Chapter 2 provides a general overview of cancer imaging, focusing on breast imaging as the performance of the proposed technique and device for cancer detection is demonstrated with specific reference to breast cancer. Then the dielectric properties of
CHAPTER 1. INTRODUCTION

Biological tissues are reviewed, in particular breast tissues, to highlight the contrast in electrical properties of the malignant tumor and normal surrounding tissues, which forms the basis of microwave approaches to tumor detection. Subsequently, current techniques for microwave breast cancer detection are introduced.

Chapter 3 presents the description of the proposed device for microwave detection of tumor, a flanged parallel-plate waveguide probe. The formulation of the problem, including transmitting and receiving characteristics of the probe, is derived. Expressions of the scattered signals in the presence of a tumor in terms of S parameters, based on Mie Theory, are also provided.

The detection methodology and simulation results are described in chapter 4. The steps involved in determining the presence, size, and location of the tumor are discussed in details. Successful detection and localization of the tumor in a breast model are demonstrated through numerical simulations. The ability of the technique to discriminate a malignant tumor from the clutter items representing tissue heterogeneity is also shown.

Chapter 5 extends the concept of the flanged parallel-plate waveguide probe for detection described in chapter 3 to the novel flanged parallel-plate dielectric waveguide probe. To validate its effectiveness in clinical environment, the probe size has to be physically practical. Confinement of the fringe radiation of the flanged parallel-plate waveguide, and shaping of the radiated field for more accurate detection, are achieved with an improved design of the probe. The method and computer simulations results for the practical probe design are detailed in this chapter. Simulations are conducted with commercial software High Frequency Structure Simulator (HFSS).
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Chapter 6 gives an account of the type of material as a biological tissue mimicking phantom to study the interaction of the electromagnetic waves from the detection probe with the biological tissues. Dielectric properties of this material can be altered by varying the concentration of oil to simulate different types of tissues. The fabrication steps to undertake are also provided. A study of the dielectric properties of this phantom material over a broad microwave frequency range and the stability in terms of both mechanical and electrical properties is also presented.

Experimental works to evaluate the performance of the practical flanged parallel-plate dielectric waveguide probe are reported in chapter 7. Experimental investigations are carried out on self-fabricated solid phantom materials with conductive inclusions, simulating normal breast with tumor embedded a few centimeters beneath the skin layer.

Finally, chapter 8 summarizes the work presented in this thesis and recommends for future research.
Chapter 2

Cancer Imaging

2.1 Introduction

Effective early screening of the human organs is essential for the detection of cancer if any. Cancer, the worldwide leading cause of deaths, is projected to take away more than 12 million lives by year 2030 [1, 2]. Therefore the role in reducing this public burden is assumed with early cancer detection as cancer can be treated to increase survival, if detected early.

This chapter thus provides a brief introduction to cancer, highlighting breast cancer as it is the top killer in women. Following a general overview of existing cancer detection methods is presented, focusing on breast imaging as the effectiveness of the proposed technique and device for cancer detection in this thesis is demonstrated with specific reference to breast cancer. A review on the electrical properties of biological tissues is also given, in particular breast tissues, as a promising approach to cancer detection using microwaves that overcomes some limitations of the existing techniques.
bases on this contrast in complex permittivity of a malignant tumor and the normal
surrounding tissues. Also described in this chapter are the various proposed methods for
microwave breast cancer detection that have been explored.

### 2.2 Cancer

Cells, the building blocks of biological bodies, make up tissues and organs. Cells grow
and divide to form new cells to supplement the body’s need when a normal body
functions. When the cells grow old, they die and the new cells replace. The cycle
repeats until the systematic process is disturbed, causing redundant growth of new cells
without being utilized by the body while the old cells do not die when they are supposed
to [15, 16]. This condition whereby the cells in the body go beyond control and multiply,
resulting in the abnormal growth of cells is called *cancer* [15-17].

Cancer is a disease that can take up many forms and occur at any part of the body
due to the genetic change in the cells that permits them to spread from their usual
location into various regions of the body. As the cancer cells travel through the lymph
system into the blood stream, they start to invade organs with numerous blood vessels.
Once the cancer cells settle in theses tissues, they start to grow rampantly and form a
malignant tumor [15, 17]. However, at some cases, the extra mass of tissues form by
these genetically modified cells can be benign and surgical procedures can be performed
to remove the tumors. Generally, they do not grow back after being excised [15].
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The major differences between a benign lesion and a malignant tumor are mainly, the benign lesion is rarely life threatening as opposed to the malignant tumor; the low reoccurrence once removed for the benign lesion as compared to that for the malignant tumor; more concerning is the cells in the malignant tumor invade and attack surrounding tissues and organs and also metastasize to other body parts to form new tumors while the non-cancerous cells in the benign lesion do not.

The cancerous cell is being differentiated into two groups: carcinoma and sarcoma. A carcinoma cell consists of epithelial cells whose primary role is to line the tissues. Carcinomas can be found in organs where fluid secretion occurs such as breast (milk), lungs (mucus) and the colon (digestive juices). In general, about 80 to 90% of human cancer originated from carcinomas and they are usually slow growing [15]. Conversely, sarcomas are cancerous cells which consist of supporting connective tissues (e.g. uterus). Sarcomas are very uncommon and they only responsible for 2% of human cancer cases. However, they react and behave more aggressively than carcinomas [15].

2.2.1 Breast Anatomy

Knowledge of the anatomy of the breast provides a better understanding of how breast cancer is formed and staged. As shown in Figure 2.1, the breast is a mammary gland builds up of glandular tissues and beneath the lines of fibrous connective matter hosts around 15 to 20 lobules [18]. These lobules are cluster of sacs called acini where milk production occurs. The milk is transported and emptied via lactiferous ducts to the nipple [16, 18]. The multiple lobules empty into a single duct and is then referred to as a
lobe. Hence, the milk production system of the breast is formed and in medical perspective, the breast is also considered as a secretary gland due to the fact that it is able to secrete fluid-milk [16, 17]. The breast is also cushioned with adipose tissues (fats) to protect its important milk production system and other injuries like trauma. However, these fatty tissues only decide the size of the breast so they do not influence a women’s ability to lactate or engage in the process of milk production [16, 17].

Figure 2.1: Anatomy of a female breast [17].

Most mammary ducts end and merge directly at the nipple. Around each nipple is a region of pigmented skin called areola. The minuscule bumps found on the surface of
the areola are Montgomery glands in the skin which lubricates the nipple by oily secretions and retards bacteria growth during breast feeding.

Blood is supplied to and from the breast through arteries and veins, which nourishes the breast tissues with essentials nutrients required to produce milk [17, 18]. Conversely, a clear fluid called lymph which contains diseases fighting cells, lymphocytes, is drained through the lymph vessels located in the lymphatic system [18]. The lymphatic system and the lymph nodes play a significant role in the body’s resistance to malignant diseases such as cancer as lymph vessels are able to allow cancer cells to spread to other parts of the body, especially the auxiliary lymph nodes of the armpit as shown in Figure 2.2 where the arrows specify the major pathways of drainage of lymph from the breast to the neighboring lymph nodes.

Figure 2.2: Lymphatic drainage from the breast [18].
Nevertheless, benign conditions can also cause the lymph nodes to swell, thus such scenario of finding palpable lumps around the breast is not necessarily an indication of cancer [17, 18]. This makes distinguishing a malignant tumor from a benign challenging.

### 2.2.2 Breast Cancer

A consequence of abnormal cell growth that occurs at the ducts lining and lobules is breast cancer. The cancer cells start the metastasis process by first entering the lymph vessels and proliferate along the vessels to get to the lymph nodes. After the cells reached the lymph nodes, they begin to multiply and grow, causing the nodes to swell. The swelling of the lymph nodes can normally be found in the armpit or elsewhere. However, if the breast cancer cells proceed to infect the auxiliary lymph nodes, the possibility of the cancer cells spreading to other parts of the body will be greatly increased [19].

During a pathology report, various types of breast cancer are identified to have developed on patients. The treatment to be administered will very much depend on the type of breast cancer that the patient is suffering from. In most cases, breast cancer is regarded as relatively systematic and therefore, the whole body will be involved regardless if the tumor is localized.

The types of breast cancer are widely classified into two groups: non-invasive and invasive cancer [17]. Non-invasive cancer, or carcinoma in situ, occurs at an early stage and the cancer cells are confined within their originating location (ducts or lobules). The cancer cells have yet to infect the surrounding tissues and organs [19]. Thus, formation
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of tumor does not occur and detection by physical examination is not possible. The types of carcinoma in situ which can occur include lobular carcinoma in situ (LCIS) and ductal carcinoma in situ (DCIS) [20]. As the name suggests, LCIS starts in the lobules but the cancer cells are still trapped behind the lobule walls. DCIS originates from the ductal epithelium around the region of the terminal ductal lobular unit. DCIS is classified into three grades- low, medium and high which reflect the abnormality of the cells under microscopic analysis and determine the rate at which the cells are multiplying. Although these forms of cancer are not lethal, the cancer cells still continue to grow like other malignancies and invasive cancer might develop if left untreated [16].

It is the following type of cancer that raises great concern-invasive cancer. Invasive cancer describes the condition whereby the cancer cells penetrates the membrane and spreads beyond the ducts and lobules [19]. Being invasive in nature, the cancer cells will cause a lump to develop over a period of time [16]. Invasive breast cancers are categorized into different types and this depends on how they appear underneath the microscope and how different are they as compared to a normal cell. Then, a grading system will be drawn according to the characteristics of the cells which assist the physician in predicting the condition and outcome of the patient [19]. The two major types of invasive cancer that can occur are [16, 19-20]:

- **Invasive Lobular Carcinoma (ILC)**

  ILC accounts for typically 15% of breast cancer cases. ILC arises when the cancer cells spread from the milk producing glands to the surrounding fatty tissues around the ducts. This results in a prevalent thickening of the breast rather than developing
into a separate lump. Moreover, due to the lack of fibrous growth, the detection of ILC is much more complex and harder to see on a mammogram.

- Invasive Ductal Carcinoma (IDC)

IDC is the most common type of breast carcinoma and it relates to around 75% of all instances. IDC occurs from the milk passage and spreads to the other parts of the breast by penetrating through the walls of the ducts and into the fatty tissues. Then, the formation of fibrous, noncancerous tissues will start to encompass the cancer cells and the palpation is distinguished by a stony hardness. Alternatively, the cancer cells can also infiltrate into other parts of the body via the lymphatic channels of the breast.

Besides the two major types of invasive cancer, other infiltrating carcinomas such as medullary cancer, mucinous carcinoma, tubular carcinoma and inflammatory breast cancer have also been found [19].

### 2.2.3 Staging of Breast Cancer

Breast cancer is classified into stages, which is dependent on, whether the cancer is invasive or non-invasive, the number of lymph nodes affected, and the dimension of the tumors [19]. The staging process might include x-rays and biopsy tests to verify if the cancer cells have spread to other parts of the body. More often, during proliferation of breast cancer cells, the cells tend to target the lymph nodes found under the armpit [15]. The stage of the cancer will be determined after the tumor is surgically removed from
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the breast and lymph node with the aim of staging it to find out to what degree the cancer has progressed when it is detected. In addition, it enables the medical expertise to decide which treatment is best suited for the patients depending on the prognosis.

The system that is being used extensively to determine the stage of cancer is known as TNM staging system [19]. T refers to the size of tumor and includes categories:
(i) Tis: LCIS or DCIS
(ii) T1: tumor diameter of 2cm or smaller.
(iii) T2: tumor diameter larger than 2cm but smaller than 5cm
(iv) T3: tumor diameter larger than 5cm.
(v) T4: tumor of any size with chest wall or skin is affected.

N refers to the presence of cancer cells in lymph node identified by lymph node dissection. The categories include:
(i) N0: cancer has not spread to the lymph nodes.
(ii) N1: cancer spread to the lymph nodes under the arms; but the affected lymph nodes are not interconnected or attached to surrounding tissues.
(iii) N2: cancer spread to the lymph nodes under the arms; the affected lymph nodes are interconnected and attached to surrounding tissues.
(iv) N3: cancer spread to the lymph nodes found under the collarbone; the lymph nodes underneath the arms may or may not be affected.

M representing metastasis measures if distance metastasis to other body parts has occurred, and is categorized as either M0 (negative) or M1 (positive).
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For easier understanding of the staging system, the four stages of breast cancer (Figure 2.3) whereby each stage is associated to specific TNM groupings are listed in the following [15-17, 19].

- Stage 0- refers to carcinoma in situ, that is LCIS or DCIS. The breast cancer is still at its premature stage; no palpable signs detected but may be detected via routine mammogram. The cancer cells are well contained in the ducts or lobules and it has not been spread to other parts of the body.

- Stage 1- relatively identical to stage 0, but early signs of invasive cancer can be detected. The tumor size is approximately 2cm in size, although the cancer cells have yet to extend beyond the breast.

- Stage 2- either the tumors 2cm or smaller in the breast start to grow and spread to the lymph nodes beneath the arm; or larger tumors 2cm to 5cm can be found in the breast but they may not be present in the lymph nodes; or the tumors grow beyond 5cm but may not be present in the lymph nodes.

- Stage 3- Stage 3A can be either the tumor is smaller than 5cm and has already extended to the underarm lymph nodes; or the tumor is larger than 5cm and has already spread to the lymph nodes under the arm. Stage 3B can be either the cancer cells have spread to the chest wall tissues or the skin of the breast with underarm lymph nodes affected; or the breast is inflamed due to the lymph vessels in the skin of the breast are obstructed by the cancer cells. In stage 3C the tumor can grow to
any size and depending on the way the cancer spreads, the lymph nodes around the collarbone or beneath the breastbone could be affected.

- Stage 4 - indicates that the tumor has already spread to other tissues and organs. It is considered as a distant metastasis cancer, regardless of the tumor size and number of lymph nodes affected. Normally, the breast cancer will affect areas like the bone, liver and lungs.

![Image of breast cancer stages](image)

*Figure 2.3: Four stages of breast cancer [17].*

Although not medically precise, it is common to observe the stages 0 onwards to some of stage 3 be categorised as early stage, and the rest as advanced stage. The proposed technique and device is thus designed to identify early stage breast cancer as
studies show patients diagnosed with *Stage 1* have a 5-year relative survival rate of close to 100% while this figure drops to less than 20% for *Stage 4* diagnosis [3].

### 2.3 Breast Cancer Imaging

Technologies for cancer imaging, including positron emission tomography (PET), computer tomography (CT), magnetic resonance imaging (MRI), allow one to see into the biological tissues, into the living cells, without having to cut anything open such as in the case of biopsy. Specifically, some of these methods have been proposed as an alternative for the imaging of breast cancer.

Breast cancer is second leading cause of death in women and it is the most commonly diagnosed cancer worldwide [1-2, 21]. For the past two decades, there has been a rapid increase in the incidence and mortality rate of breast cancer; this results in a growing health burden to the general population [21]. Thus the key solution to reduce the incidence and mortality rates is early breast cancer detection as treatment can be administered at an early stage before the condition worsen. Numerous diagnostic processes provided by hospitals or clinics to allow patients to get themselves screen for early detection of breast cancer are usually non-invasive but in some cases if the sample of a breast tissue is needed, a biopsy (invasive) might be carried out. The various types of diagnosis test are included below.

A summary of the key features and issues of these methods are tabulated in Table 2.1.
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1. *Mammography*

Mammography, the gold standard of breast cancer detection, bases on natural density differences between normal tissues (such as fats, lobes, ducts, blood vessels, etc) and lesions [4]. It is an inexpensive imaging procedure whereby ionizing x-ray is passed through the breast and negative film screen, mammogram, will be produced [4]. The breast screened is compressed during the process. Among the procedures that are available, mammography is considered the most sensitive diagnostic test for early detection of breast cancer and it is the only test that can detect microcalcifications [16].

Further improvements to enhance the resolution of the mammogram have been developed, taking the forms of digital mammography and computer aided detection (CAD) [22-26]. However these enhancements are not widely used as the improvement to cancer detection rates does not justify the high cost. The discomfort and radiation exposure issues remain unaddressed.

2. *Ultrasonography*

Ultrasonography or ultrasound technique uses high frequency sound waves instead of x-rays to examine the breast for any tumor growths [27-29]. The process for ultrasound is similar to fetal scanning whereby the breast is covered in water soluble gel and a transducer is placed on the surface to detect the reflected sound waves (echoes) from the breast tissues. Echoes captured will then be processed by a computer and an image of the breast tissues will be generated on the screen for interpretation. Normally, the normal fatty and glandular tissues, ducts, tumors and cyst can be seen from the images.
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Typically ultrasound is used for differentiating whether a lump is a fluid filled cyst or a solid mass which may require a biopsy to determine if it is malignant [8,16, 27].

3. Magnetic Resonance Imaging (MRI)

In 1991, the FDA approved MRI as a complementary tool to mammography for early breast cancer detection [30]. Being a non-invasive and non-ionizing technique, MRI employs powerful magnetic and radio waves to change the alignment of the hydrogen atoms in the body tissues [9, 30-32]. When the fields pass through the breasts, the atoms in the tissues absorb the energy and re-emit them out of the body. This process is performed repetitively while the MRI scanner measures the energy level of the emitted waves. Concurrently, a sophisticated computer will be used to produce detailed images of the body through the measurements. It may take up to an hour for the scanning process to complete. Apart from the setup being massive and cumbersome to use, the patient may suffer from claustrophobia while lying within the tube-like channel during the scan [11]. Moreover to date, MRI is only recommended for patients with high risk or in situations where the breast tissues of the patients are relatively dense, or biopsy results indicating malignant conditions but no evidence of breast cancer during mammography [12, 19].

4. Positron Emission Tomography (PET)

PET is a complicated imaging technique where molecules with positron-emitting radioactive tags are injected into the body; in breast imaging, Tc-99m sestamibi is used [10, 33]. These tracers travel within the body and a large quantity of them will be
CHAPTER 2. CANCER IMAGING

absorbed by the malignant tumor than normal cells due to a difference in metabolism. A PET scanner will be used to pick up the differential gamma radiation emitted from the body to form images to determine if breast cancer has occurred [10, 33-35]. PET is rarely performed due to its availability and it is relatively expensive compared to other diagnostic techniques [32]. Radiation exposure is also a concern.

5. Computer Axial Tomography (CT or CAT)

CT scans uses radiation to construct detailed three dimensional views of the body tissues from slices of two dimensional x-ray pictures [16]. CT scan is seldom use at the early stages of breast cancer unless the cancer reaches an advanced stage [19]. In general, CT scan is expensive, its radiation level is higher than mammography, and intravenous dye have to be injected into the body prior to the scan [11, 16]. Nevertheless, CT scans are capable in determining the density of tissues in the body and are widely used for other bioimaging.
## Table 2.1: Summary of existing breast imaging methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Specific properties</th>
<th>Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Mammography</td>
<td>• Image breast densities&lt;br&gt;• Detect microcalcifications&lt;br&gt;• Inexpensive (approx. US$150)</td>
<td>• Ionizing x-rays&lt;br&gt;• Uncomfortable compression&lt;br&gt;• Low sensitivity (ability to detect tumor) due to dense breast tissues and implants&lt;br&gt;• Low specificity (ability to differentiate benign from malignant)</td>
</tr>
<tr>
<td>2 Ultrasound</td>
<td>• Non-ionizing sound waves&lt;br&gt;• Suitable for patients with dense breast tissues and implants&lt;br&gt;• Differentiate cysts from solid lesions&lt;br&gt;• Inexpensive (approx. US$70)</td>
<td>• Incapable to detect microcalcifications&lt;br&gt;• Limited depth of penetration&lt;br&gt;• Low sensitivity</td>
</tr>
<tr>
<td>3 MRI</td>
<td>• Non-ionizing radio and magnetic waves&lt;br&gt;• Detailed cross sectional images of the tissues from various angles&lt;br&gt;• Recommended for women at high risk for breast cancer&lt;br&gt;• Detects any remaining cancer after lumpectomy</td>
<td>• Expensive (approx. US$1000)&lt;br&gt;• Long process of extensive scanning and imaging&lt;br&gt;• Injection of contrast agent is required which can cause discomfort&lt;br&gt;• High false positives&lt;br&gt;• Microcalcifications may not be seen&lt;br&gt;• May cause claustrophobia</td>
</tr>
</tbody>
</table>
CHAPTER 2. CANCER IMAGING

<table>
<thead>
<tr>
<th>Method</th>
<th>Specific properties</th>
<th>Issues</th>
</tr>
</thead>
</table>
| 4 PET  | • Distinguish benign and malignant tumors thus reduces the need for biopsy  
|        | • Metabolic functions can be studied  
|        | • Staging of cancer | • Ionizing positron emitting radionuclide  
|        |                      | • Expensive (approx. US$1400)  
|        |                      | • Injection of radioactive tracer required  
|        |                      | • Not easily available  
|        |                      | • Complicated technique |
| 5 CT   | • Image tissue densities  
|        | • Reconstruct 3-D views from multiple 2-D x-ray images from various angles | • Increased ionizing x-rays  
|        |                      | • Expensive (approx. US$500)  
|        |                      | • Injection of intravenous dye required, can cause allergic reaction |

The costs are converted from Singapore Dollar to US Dollar by dividing a factor of 1.5 based on the pricing provided in [36].

The above mentioned techniques are commercially available and in use. Other alternatives undergoing further investigations includes FDA approved thermography, whereby the heat changes in a breast are detected and evaluated based on different metabolism rate between cancerous and normal cells [37]; optical mammography based on differential transmission and absorption of near-infrared light of the tissues [38-41]; impedance tomography that creates a profile of the electrical properties of the imaged organ in the kilohertz range to identify the tumor which has higher conductivity and permittivity than normal tissue [42-44]; and invasive blood test with biomarkers [19, 45-46]. Recently, techniques using microwaves for imaging have also been very much researched.
2.4 Electrical Properties of Biological Tissues

In the recent years, a significant amount of evidences are found that electrical properties of human tissues can be applied into medical uses. This includes cancer diagnostics using microwave scattering and reflection that has been put into focus lately, based on the contrast in the complex permittivity between a healthy tissue and malignant tumor.

Biological tissues can be characterized as lossy dielectrics with complex permittivity

$$\varepsilon = \varepsilon' + i\varepsilon''$$

(2.1)

where the time harmonic variation of $e^{-i\omega t}$ is assumed. And generally, the dielectric properties will fluctuate with frequency of the electric field. As a result, dispersion occurs at various frequencies and such dispersion depends on the ionic structure that surrounds the cells or the water content of the tissues [12]. Therefore the relative complex permittivity $\varepsilon_r$ can be expressed as

$$\varepsilon_r = \varepsilon'_r(\omega) + i\frac{\sigma(\omega)}{\omega\varepsilon_0}$$

(2.2)

where $\sigma$ is the conductivity of the material with SI unit of Siemens per metre (S/m), $\varepsilon_0$ is the free space permittivity, $\omega$ is the angular frequency of the field.

Frequently, a single-pole Debye equation is employed to model the dispersive biological tissue [47]

$$\varepsilon'_r(\omega) + i\frac{\sigma(\omega)}{\omega\varepsilon_0} = \varepsilon_\infty + \frac{\varepsilon_r - \varepsilon_\infty}{1 - i\omega\tau} + i\frac{\sigma_r}{\omega\varepsilon_0}$$

(2.3)
where $\varepsilon_\infty$ is the permittivity at high frequency, well above the relaxation frequency; $\varepsilon_s$ is the static permittivity; $\sigma_s$ is the static conductivity at high frequency; and $\tau$ is the relaxation time. In addition, Cole-Cole equation is also used to improve the fit of the dielectric data [47]

$$
\varepsilon'_\infty(\omega) + i \frac{\sigma(\omega)}{\omega \varepsilon_0} = \varepsilon'_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (-i \omega \tau_\infty)^{1-\alpha}} + i \frac{\sigma_\infty}{\omega \varepsilon_0}
$$

(2.4)

the subscript $c$ in the fitting parameters simply refers to “Cole-Cole”.

Studies on the dielectric properties of biological tissues began prior in the 1940s, but systematic data were few until H. Schwan [48] whose works were directed at measurements of the dielectric properties of tissues over a much broader frequency range than previously explored, and interpretation of the data using theories. Schwan’s works provide a foundation for subsequent studies, such as a review of the dielectric spectrum of biological tissues, summarized by Gabriel et al [49] in 1996, shows the dielectric constant of a tissue may reach values up to $10^6$ or $10^7$ at frequencies $100$Hz and below; following, this value decreases at high frequencies in three main steps known as the alpha, beta and gamma dispersions. The low frequency alpha dispersion results from the ionic diffusion processes at the cellular membrane, while beta dispersion in the radio frequencies range mainly results from the polarization of cellular membranes that behaves as barriers to the ion flow between the intra- and extra-cellular media, and gamma relaxation occurring in the high gigahertz region is due to the polarization of water molecules. The frequency of interest for many bioimaging methods is determined by the skin depth of the tissue of concern. Taking into consideration sufficient penetration into the tissue but not deep enough to affect the
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bones and other internal organs, and reasonable resolution; the operating frequency range lies between the beta and gamma dispersions.

Gabriel et al also performed a very broad characterization of different biological tissues over a very wide frequency range of 10Hz to 20GHz [50]. The different types of tissues were grouped according to the water content and the studies over the entire gigahertz range show that water content is a major factor to determine $\varepsilon_r$ and $\sigma$. High water content tissues (such as muscle), have higher permittivity and conductivity than low-water-content tissues (such as fat), up to the gamma dispersions.

Specifically for the basis of breast cancer detection, several investigations on the permittivity of the entire breast organ, including normal breast tissue and malignant tumor were conducted. It is generally concluded normal breast tissues displayed properties similar to fatty tissues, while tumor properties were similar to muscle.

2.4.1 Breast tissue and malignant tumor

As the breast tissue consists majorly of fat; it is categorized as low water content tissue. Hence within the useful frequency bandwidth for breast imaging, the complex permittivity and conductivity of breast tissue is lower than others higher water content tissues. On the contrary, due to the proliferation of abnormal cells leading to an increase of blood perfusion, malignant tumor tend to have higher water content than other tissues thus have larger permittivity and conductivity than normal (breast) tissues.

For the past decades a diversity of studies focusing on this contrast in dielectric properties of normal and malignant breast tissues have been carried out. Various
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measurement techniques and methodology have been adopted to record the dielectric properties from tissues samples. The tissue samples were primary excised from surgical procedure. The findings in these reports are summarized in Table 2.2.
### Table 2.2: Summary of published dielectric properties of human breast tissues.

<table>
<thead>
<tr>
<th>Research Group / Year of publication</th>
<th>Types of breast tissues</th>
<th>No. of patients involved</th>
<th>Frequency range</th>
<th>Approach and Results</th>
</tr>
</thead>
</table>
| 1 Chaudhary et al / 1984 [51]        | Normal, Malignant            | 15                       | 3MHz−100MHz − 100MHz−3GHz | - sample sandwiched by parallel plate capacitor,  
- measured by impedance meter  
- time domain reflectometry  
Normal tissues have lower $\varepsilon'$ and $\sigma$ as compared to malignant tumors.  
- Normal tissues: $\sigma=1.5−3$ mS/cm, $\varepsilon_r'=10$  
- Malignant tissues: $\sigma=7.5−12$ mS/cm, $\varepsilon_r'=50−400$ |
| 2 Surowiec et al / 1988 [52]         | Malignant, Peripheral normal | 7                        | 20kHZ−100MHz          | - end-of-line coaxial capacitive sensor  
Malignant tumor have higher $\varepsilon_r'$ and $\sigma$.  
- Normal surrounding tissues:  
  $\sigma=0.3−0.4$ mS/cm, $\varepsilon_r'=8−800$  
- Malignant tissues: $\sigma=2−8$ mS/cm, $\varepsilon_r'=80−10000$ |
| 3 Campbell and Land / 1992 [53]      | Normal, Benign, Malignant    | 37                       | 3.2GHz               | - resonant cavity perturbation  
Results did not agree with other literatures.  
No significant differences in $\varepsilon_r'$ and $\sigma$ between normal and malignant tissues. |
<table>
<thead>
<tr>
<th>Researcher Group/ Year of publication</th>
<th>Types of breast tissues</th>
<th>No. of patients involved</th>
<th>Frequency range</th>
<th>Approach and Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal, Malignant</td>
<td>12</td>
<td>50–900MHz</td>
<td>- open-ended coaxial probe</td>
</tr>
<tr>
<td>Joines et al / 1994 [54]</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Normal breast tissues:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma = 1.1–1.8$ mS/cm, $\varepsilon' = 15–21$</td>
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<tr>
<td></td>
<td>Malignant breast tissues:</td>
<td>$\sigma = 7.7–11.6$ mS/cm, $\varepsilon' = 57.1–80$</td>
<td></td>
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<tr>
<td></td>
<td>Normal nodes:</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>$\varepsilon' = 8–15$, $\varepsilon'' = 0.1–8$</td>
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<tr>
<td></td>
<td>Malignant nodes:</td>
<td></td>
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<tr>
<td></td>
<td>$\varepsilon' = 15–44$, $\varepsilon'' = 1–33$</td>
<td></td>
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<tr>
<td></td>
<td>Malignant tumor:</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$\varepsilon' = 20–57$, $\varepsilon'' = 1–32$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Choi et al / 2004 [55]</td>
<td>Lymph nodes, Malignant</td>
<td>12</td>
<td>0.5–30GHz</td>
<td>- open-ended coaxial probe</td>
</tr>
<tr>
<td>Bindu and Matthew / 2007 [56]</td>
<td>Normal, Benign, Malignant</td>
<td>6</td>
<td>2.4–2.5GHz</td>
<td>- resonant cavity perturbation</td>
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<td></td>
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<tr>
<td></td>
<td>Normal tissues:</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$\sigma = 26.4–36.1$ mS/cm, $\varepsilon' = 17.68–26.91$</td>
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<tr>
<td>Researcher Group/Year of publication</td>
<td>Types of breast tissues</td>
<td>No. of patients involved</td>
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<td>7 Lazebnik et al / 2007 [57,58]</td>
<td>Normal, Benign, Malignant (retrieved from reduction and cancerous surgeries)</td>
<td>289</td>
<td>0.5−20GHz</td>
<td>- improvised flangeless open-ended coaxial probe</td>
</tr>
</tbody>
</table>

- By far the largest scale studies of dielectric properties of breast tissues.
- The dielectric properties of normal breast tissues were found to have a wider range of values than those previously reported.
- Dependent on the fat content, normal breast tissues are categorized into adipose, fibroconnective, and glandular.
  - Normal tissues: \( \sigma = 0.3−7.8 \text{ mS/cm}, \  \varepsilon'=3.8−48 \)
  - Benign tumor: \( \sigma = 10−230 \text{ mS/cm}, \  \varepsilon'= 23−54 \)
  - Malignant tumor: \( \sigma = 15−290 \text{ mS/cm}, \  \varepsilon'= 27−62 \)
As pointed out by Hurt et al [59], out of eighteen human tissue types, the greatest uncertainty in dielectric properties at microwave frequencies exists for normal breast tissue. Discrepancies in the measured data by different research groups may be attributed to variability in the sample preparation techniques, spectroscopy methodology, and more; combined with the inhomogeneity of the breast tissue. Further investigations pending to large number of samples are essential to provide a more accurate insight to the likely permittivity of the breast tissue.

Although not all of the published data agree, these studies and measurements have consistently suggest, in general there exist a large difference in complex permittivity between the low water content normal breast tissue and the high water content malignant breast tumor. These findings provide encouraging evidences in support for microwave techniques for breast cancer detection as they are based on detecting the contrast in complex permittivity between the malignant tumor and the surrounding normal tissue.

Detection of the breast tumor requires knowledge of the dielectric properties of the skin layer. It has been widely accepted in the modeling of the skin, the skin can be regarded as having dielectric constant \( \varepsilon_r =36 \), and conductivity \( \sigma= 4.0 \, \text{S/m} \) [60-62].
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2.5 Microwave Breast Imaging

Breast cancer imaging techniques such as x-ray mammography proves as gold standard for early breast cancer detection. However, despite being the most reliable screening test, cases of missed diagnosis were discovered by a study on the false negative rates of mammograms taken from previous screening [63]. Moreover, it was also reported that the number of breast cancers diagnosed by both mammography and physical examinations is much higher than just using mammography alone [5]. These limitations of mammography has caused unnecessary apprehension in patients and also increased burden of paying higher diagnostic bills due to repeated screenings and examinations. Concerns such as the negative biological effects of ionizing x-rays on the breast tissues and the discomfort caused by the compressions on the breast during examinations have to be taken into consideration too.

Intensive diagnostic techniques aimed to address these constraints have been researched, with microwave approaches put under focus lately. Microwave imaging is non-invasive and non-ionizing in nature, and is less costly as compared to other advanced imaging process like MRI and PET scans [32, 60]. Therefore microwave imaging appears to be an attractive technique for early breast cancer detection as it fills the imperfections of current detection methods. The significant contrast in the dielectric properties between malignant and normal breast tissues when exposed to microwave frequencies forms the basis behind this imaging technique [60-62].

Methods working on this principle can be divided into: passive, active, or hybrid. Passive approaches such as microwave radiometry [64-67], identify malignant tumors
from the increase in temperature recorded as compared to the normal breast tissues, based on cancerous cells have increased metabolism and thus radiate more heat. Hybrid techniques [68-70] combine the sending of microwaves illuminating the breast and the use of ultrasound transducers to pick up pressure waves generated from expansion of the tissues; tumor and normal tissues heat up to different degree and thus have differential expansion. The most common form of microwave imaging is the active approaches which involve the external emission of electromagnetic radiation at the breast and process the received reflected and/or transmitted signals from the breast to construct images. They include microwave tomography and ultrawideband (UWB) radar-based system.

2.5.1 Microwave Tomography

Microwave tomography can be considered as the classical method of microwave imaging [14, 71-74]. It acquires data on the tumor by illuminating the organ with an antenna while other receiving antennas are scanned across it. This is repeated with different positions of the transmitting antenna. Typically, the inverse scattering problem is solved through rigorous iterative computation and processing to construct an electrical properties profile of the organ.

Such a system includes one developed by Paulsen et al [75], although prior to it Miyakawa et al [76] had modified the principles of CT to avoid diffraction and multiple scattering by using the frequency component of the signal corresponding to the distance
between the antennas to isolate the signal travelling directly from the transmitting to the receiving antennas, and reconstruct the image with CT algorithms. In the approach by Paulsen et al, an array of monopole antenna scanning in the frequency from 300 to 900MHz was used. An initial guess of the dielectric properties was performed with the use of hybrid finite-element boundary-element method, subsequently iterative Newton-Raphson method was employed to update the dielectric profile until it converges to reconstruct the image. Chew at al [77] developed a system not for bioimaging purposes, but the time-domain approach using the finite difference time domain (FDTD) method in the distorted-Born iterative method (DBIM) to reconstruct images influenced the use of UWB signals for microwave imaging.

2.5.2 UWB Radar-based Imaging

Promising results for have been obtained with these tomographic methods but alternative UWB microwave imaging which is attractive to use in terms of the simpler reconstruction algorithms gained more significant attention. UWB microwave imaging does not seek to produce profiles of the complex permittivity of the scanned organ but identifies the presence and location of the malignant tumor based on significant differential scattering compared to other tissue inhomogeneity due to the different complex permittivities. In 1998 Hagness et al [13] proposed to use a pulsed radar system primarily used for military applications, for breast cancer detection. This is a microwave confocal imaging (CMI) technique whereby an antenna sent UWB pulses to
CHAPTER 2. CANCER IMAGING

illuminate the breast and the backscattered pulses is received by the same antenna. This process is repeated at various positions and a delay-and-sum (DAS) algorithm is applied to post process the set of backscattered pulses to give a focal point signifying the presence and location of the tumor [78-82]. To account for the effects of tissue dispersion on the UWB pulse and other previous limitation of the DAS algorithm, an alternative microwave imaging via space–time (MIST) beamforming technique was proposed [83-86]. Subsequently other theories for better resolution and robustness in the reconstruction of the imaged breast, such as generalized likelihood ratio test (GLRT) [87], time reversal (TR) [88-91], multistatic adaptive microwave imaging (MAMI) [92, 93], and tissue sensing adaptive radar (TSAR) imaging [94-97] were proposed too.

Apart from focusing on the detection, efforts to characterize the shape and size of the tumor have also progressed. Davis et al [98] devised a method for shape and size classifications of the tumor based on signal-to-noise ratios derived from UWB backscatter; while El-Shenawee [99] utilized a fast algorithm, multiple interaction model combined with steepest descent fast multipole method (MIM-SDFMM), to obtain the spectra of tumor which depends on its physical characteristics. Optimization algorithm to simultaneously estimate the shape and location of the breast tumor is also available [100-102]. To improve the performance of this UWB radar-based imaging system, the estimation of the location of breast surface was also proposed. As the UWB antennas are to be placed at some positions from/on the breast, the knowledge of the breast location, shape and size when the patient lies down during the scan can be incorporated as a priori information for better accuracy [103, 104].
Last but not least, as new techniques and concepts for microwave breast imaging are ongoing and flourishing, anatomically realistic numerical breast phantoms including more accurate Debye models are developed for computational investigations to be efficiently carried out [105, 106].

Due to complexity, most of the studies on microwave confocal imaging are mainly conducted through computer simulations such as FDTD or Finite Element Method (FEM). Experimental investigations are few and far between. In these investigations, the typical size of the tumor ranges from 3 to 20mm in diameter and embedded within 2 to 6cm. It is worth to note that majority of these experimental studies were performed on very low loss nondispersive liquid phantoms instead of solid phantoms (which is the actual state of a breast); as a result the dispersive effects and significant loss factor that exist in an actual breast, on the attenuation of the transmitted and received signals and thus accuracy of the technique, are not investigated. On the contrary, the experimental validations of the proposed technique and device for the detection of tumor in this thesis were conducted on dispersive and lossy solid phantoms, with complex permittivity resembling closely to that of an actual breast.

A summary of the prototype systems and clinical/experimental setups is provided in Table 2.3.
Table 2.3: Summary of clinical/experimental investigations on microwave breast imaging.

<table>
<thead>
<tr>
<th>Research Group/Year of publication</th>
<th>System Characteristics</th>
<th>Experimental/ Clinical Setup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clinical prototype</td>
<td>Antenna array of 16</td>
</tr>
<tr>
<td></td>
<td>Frequency: 300–1000MHz</td>
<td>transceiving monopole</td>
</tr>
<tr>
<td></td>
<td></td>
<td>antennas.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Antennas and examined</td>
</tr>
<tr>
<td></td>
<td></td>
<td>breast immersed in</td>
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<tr>
<td></td>
<td></td>
<td>coupling fluid consists of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9% saline.</td>
</tr>
<tr>
<td>2 Semenov et al/2002 [71]</td>
<td>Tomography</td>
<td>Canine thorax was imaged.</td>
</tr>
<tr>
<td></td>
<td>3-D experimental system</td>
<td>Coupling fluid of salt</td>
</tr>
<tr>
<td></td>
<td>Room temperature</td>
<td>solution ((\varepsilon_r='78, \tan\delta=0.20)).</td>
</tr>
<tr>
<td></td>
<td>Frequency: 0.9GHz</td>
<td>One pair of flanged</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rectangular waveguides loaded</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with matched dielectric</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((\varepsilon_r='90, \tan\delta=10^{-5})) as</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transmitting and receiving</td>
</tr>
<tr>
<td></td>
<td></td>
<td>antennas for data acquisition.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Depolarization effects of</td>
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<td></td>
<td></td>
<td>scattered fields not</td>
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<td></td>
<td></td>
<td>considered.</td>
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<td></td>
<td></td>
<td>Fibreglass chamber with</td>
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<td></td>
<td></td>
<td>height of 1.35m and diameter</td>
</tr>
<tr>
<td></td>
<td></td>
<td>of 1.2m.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Measures attenuation up to</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120dB with SNR 40dB.</td>
</tr>
<tr>
<td>Research Group/ Year of publication</td>
<td>System Characteristics</td>
<td>Experimental/ Clinical Setup</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>Fear et al /2003 [108]</td>
<td><strong>Confocal</strong></td>
<td>Antenna at 10cm from the phantom.</td>
</tr>
<tr>
<td></td>
<td>2-D experimental system (lengths of phantom breast and tumor much greater than length of antenna)</td>
<td>Hollow PVC ($\varepsilon$'=3.3) of length 92cm and diameter 10cm as the breast phantom.</td>
</tr>
<tr>
<td></td>
<td>Frequency: 50MHz−20GHz</td>
<td>EM absorbing material at bottom.</td>
</tr>
<tr>
<td></td>
<td>Single antenna illuminates the phantom. Monopole:50MHz−20GHz Horn: 1−18GHz</td>
<td>Copper, water and wood with diameter ranging 3–16mm, simulating the tumor. Inclusion placed around 2 to 3cm from the walls of the PVC.</td>
</tr>
<tr>
<td>Li et al /2004 [85]</td>
<td><strong>Confocal</strong></td>
<td>Antenna aperture placed 1cm from the surface of the skin phantom.</td>
</tr>
<tr>
<td></td>
<td>MIST beamforming algorithm</td>
<td>Soybean oil ($\varepsilon_r$'=2.6, $\sigma$=0.05 S/m at 6GHz) as breast phantom and coupling medium.</td>
</tr>
<tr>
<td></td>
<td>3-D experimental system</td>
<td>1.5mm thick unclad FR4 glass epoxy PC board ($\varepsilon_r$'=4.34, tan $\delta$=0.016 at 1GHz) as the skin layer.</td>
</tr>
<tr>
<td></td>
<td>Frequency: 1−11GHz</td>
<td>Plastic cylindrical container having similar dielectric property as oil; with length and diameter both 4mm, holds diacetin solution ($\varepsilon_r$'=8.7, $\sigma$=1.9S/m at 6GHz), as tumor. Suspended with 0.1mm</td>
</tr>
<tr>
<td></td>
<td>Pyramidal horn with height 1.3 cm, aperture dimensions 2.5 cm by 2.0 cm.</td>
<td>Single antenna to synthesize a 49 elements array, spanned 6cm by 6cm.</td>
</tr>
</tbody>
</table>


## CHAPTER 2. CANCER IMAGING

<table>
<thead>
<tr>
<th>Research Group/ Year of publication</th>
<th>System Characteristics</th>
<th>Experimental/ Clinical Setup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>diameter nylon thread at depth 2cm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vary dilution of diacetin to simulate different dielectric contrast.</td>
</tr>
<tr>
<td>5 Sill and Fear/ 2005 [95]</td>
<td><strong>Confocal</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tissue sensing adaptive radar (TSAR) algorithm</td>
<td>PLEXIGLAS TANK OF DIAMETER 40CM, FILLED WITH IMMERSION LIQUID CANOLA OIL ($\varepsilon_r=2.5$, $\sigma=0.04S/m$ at 4GHz ).</td>
</tr>
<tr>
<td></td>
<td>3-D experimental system</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Frequency: 1−10GHz Wu-King monopole antenna of length 10.8mm, soldered to substrate with electrical properties similar to Canola oil.</td>
<td>Top of the tank covered with ground plane with holes to insert the antenna and tumor.</td>
</tr>
<tr>
<td></td>
<td>Tumors ($\varepsilon_r=54$, $\sigma=4.5S/m$ at 4GHz ) of diameters 1cm and 2cm are fabricated using Alginate powder, water and salt and attached to metal plugs; and covered with a layer of epoxy to prevent diffusion of the tumor in the oil.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Substrate of the monopole soldered to metal plug.</td>
<td>Tumor rotates around the center of the tank to synthesize antenna scanning around it.</td>
</tr>
<tr>
<td>6 Bindu et al /2006 [109]</td>
<td><strong>Confocal</strong></td>
<td>1 sample each from 4 patients with cancerous breast tissues (~1cm) within normal (~2cm diameter) tissues removed during mastectomy.</td>
</tr>
<tr>
<td></td>
<td>DAS algorithm</td>
<td></td>
</tr>
</tbody>
</table>
## TABLE 2.2. CANCER IMAGING

<table>
<thead>
<tr>
<th>Research Group/Year of publication</th>
<th>System Characteristics</th>
<th>Experimental/ Clinical Setup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single coplanar strip-line-fed bowtie antenna as transceiving antenna, suspended container containing the breast sample and coupling medium</td>
<td>Samples supported on a 2cm diameter low loss PVC holder and antenna immersed in coupling medium of corn syrup ($\varepsilon_r' = 18.7$, $\sigma = 0.64\text{S/m}$ at 2.983GHz). Antenna rotates in a radius of 6cm around the sample.</td>
</tr>
<tr>
<td>7 Shenouda and Fear/2009 [110]</td>
<td><strong>Confocal</strong></td>
<td>Canola oil as breast phantom. Tumor is attached to a 1cm diameter Plexiglas rod to position in the canola oil. Positioned 2cm, 4cm from the antenna. 6mm diameter tumor fabricated according to [95]. Cylindrical tumor of height 8mm and diameter 18mm, fabricated with epoxy ($\varepsilon_r' = 10$, $\sigma = 0.01\text{S/m}$).</td>
</tr>
</tbody>
</table>

In these time-domain approaches for detection that depend highly on signal correlations (such as phase) between the sent and received UWB pulses, there exist a few concerns to be addressed. These include the significant mutual coupling effects of the antennas that have to be challenging resolved through good matching of the antennas over the operating ultra-wide bandwidth; the accuracy of the reconstruction
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algorithms to compensate the effects of the dispersive tissue on the pulses; moreover these algorithms are generally complex and rigorous. The accuracy of these methods hence has to be enhanced through incorporation of accurate *a priori* information of the breast.

Recently frequency domain approaches for detection are explored [111-114]. In line with the operating principle of the proposed device in this thesis, continuous wave is sent over the swept frequencies, and frequency correlation is performed instead with the effects of the dispersive tissue on the scattered signals inherently accounted for. Thus the algorithms for frequency approaches are relatively simple and do not require rigorous and complex processing of the signals received as opposed to those of the time domain approaches. When compared to other existing probes such as the coaxial probe, this proposed device has a major advantage over the capacitive-natured coaxial probe in terms of penetration depth to detect tumor embedded deep; fields emanating from the coaxial probe are restricted to only subsurface while the proposed device can detect tumors embedded a few centimeters within the breast. In comparison with another common the open-ended rectangular waveguide probe, radiation from the proposed probe into the tissue can be shaped, as will be discussed in chapter 5, to more accurately identify and locate the tumor from other clutter items (tissue heterogeneity) as the scattering from the clutter items away from the tumor can be limited and thus have minimal influence on the measurements.
2.6 Conclusion

The breast anatomy and how the breast cancer forms and is staged are introduced in this chapter. Existing methods of cancer imaging are also reviewed, with the limitations of each method discussed. One promising technique to overcome these limitations is breast imaging using microwaves. As the basis of this microwave approach is on detecting the large dielectric contrast in complex permittivity of a malignant tumor and the normal surrounding tissues, a review of the electrical properties of breast tissues is also provided. The progress of microwave techniques of cancer detection, from the classical method to hybrid method, to the popular UWB confocal imaging method, is also summarized. The next chapter introduces the proposed device for microwave detection of tumor and provides a comprehensive description of the interaction of the electromagnetic signals from the device with the screened organ.
Chapter 3

Flanged Parallel-plate Waveguide Probe for Microwave Detection of Tumor

3.1 Introduction

This chapter introduces the proposed device for microwave detection of tumor, known as flanged parallel-plate waveguide probe (PPWP). A method for identifying the presence, size and location of a tumor in normal tissue is provided by performing measurements of Scattering parameters at different microwave frequencies by placing the aperture of the PPWP onto the region of concern, e.g. human organ. Description of this method will only be covered in the later chapter while the focus here is on the design of the PPWP.

Next, analytic solutions for the transmitting and receiving characteristics of the PPWP, that is the reflection coefficient and transmission coefficient at the aperture of the PPWP, are provided. The incident and scattered fields in different regions of the tissue are solved in terms of cartesian wave functions by imposing appropriate boundary conditions, on the application of mode matching techniques with Fourier transform
With knowledge of these fields, expressions for the S parameters measured by the PPWP in the presence of a tumor, based on Mie scattering [118-120] by a dielectric sphere, can be derived as shown thereafter.

3.2 Geometry of the Flanged Parallel-plate Waveguide Probe

The flanged parallel-plate waveguide probe is essentially a transverse electric and magnetic (TEM) mode-excited, cut-off section of a transmission line terminated by ground planes. It can be visualized as two perfectly conducting 90° wedges spaced apart. The geometry of the probe is given in Figure 3.1. To simplify the analysis, we consider the probe of width 2a is infinite in extent in the y-axis, with flanges infinite in extent in both the x- and y-axes.

The width 2a of the PPWP should be designed for dominant mode wave propagation in the operating frequency in accordance to the skin depth in the surveyed region of concern; such as in the case of detection of breast tumor, the frequency range is 1 to 7GHz as the skin depth of the skin (considering the skin as only 1 layer) above the breast tissue is in the order of a sub-centimetres in this frequency range [121]. This is sufficient for the signal to penetrate the skin of a few millimetres and tissue to reach the tumor and be scattered back. The skin depth is also small enough to prevent
unwanted scattered signals from bones and other organs because such signals are strongly attenuated due to increased path lengths in terms of the skin depth.

![Geometry of the flanged parallel-plate waveguide probe.](image)

Figure 3.1: Geometry of the flanged parallel-plate waveguide probe.

3.3 Radiation from the Probe

The PPWP acts as the microwave source radiating into the tissue to be screened. The radiation fields into a multi-layered dielectric from a flanged guide have been suggested by a few literatures [115, 116]. This theory is applied for the proposed microwave detection of tumor which is based on Mie scattering from the tumor. The scattered signals measured by the PPWP depend on the probe’s receiving characteristics which will be derived and shown in Section 3.4. The time harmonic variation of $e^{-j\omega t}$ is assumed and suppressed throughout the thesis.
3.3.1 Incident, Scattered and Transmitted Fields

Consider the PPWP radiating into an infinite half-space as shown in Figure 3.1, the incident fields inside the probe can be represented respectively as

\[ H^I_y(x, z) = H^I_0 e^{ik_1 z} \]  
\[ H^I_y(x, z) = \sum_{m=0}^{\infty} c_m \cos a_m(x + a) e^{-i\xi_m z} \]  

where

\[ \xi_m = \sqrt{k_1^2 - a_m^2} \]  
\[ a_m = \frac{m\pi}{2a} \]

and \( H^I_0 \) is the amplitude of the incident magnetic field with wavenumber \( k_1 \) in the waveguide probe (Region I).

Using the breast model as an illustration, in the spectral domain \( \zeta \) the transmitted fields outside the probe, unbounded in Region III (normal breast tissue), can be expressed as

\[ H^{III}_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{H}^+_3(\zeta) e^{-i\zeta x + ik_3 z} d\zeta \]  

where

\[ k_3 = \sqrt{k_3^2 - \zeta^2} \]
Then bounded Region II represents the skin layer. In Region II, the field is denoted by

\[
H_y^{II}(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \tilde{H}_x^{+}(\zeta) e^{ik_2 z} + \tilde{H}_x^{-}(\zeta) e^{-ik_2 z} \right] e^{-i\zeta z} d\zeta
\]  

where

\[
k_{z_2} = \sqrt{k_2^2 - \zeta^2}
\]

3.3.2 Imposing Boundary Conditions

To determine the unknown expansion coefficients, the boundary conditions requiring tangential E field continuity in the x-direction and tangential H field continuity at \( z = 0 \) and \( z = l \) are applied.

Matching boundary conditions at interface \( z = l \) yield

\[
\int_{-\infty}^{\infty} k_{z_2} \tilde{H}_x^{+}(\zeta) e^{ik_2 z_2} d\zeta = \int_{-\infty}^{\infty} k_{z_2} \left[ \tilde{H}_x^{+}(\zeta) e^{ik_2 z_2} - \tilde{H}_x^{+}(\zeta) e^{-ik_2 z_2} \right] e^{-i\zeta z} d\zeta
\]  

(3.10a)
\[ \int_{-\infty}^{\infty} \tilde{H}_m^+(\zeta) e^{ik_zz-\zeta} d\zeta = \int_{-\infty}^{\infty} \left[ \tilde{H}_m^+(\zeta) e^{ik_zz} + \tilde{H}_m^-(\zeta) e^{-ik_zz} \right] e^{\zeta} d\zeta \] (3.10b)

Taking integral Fourier transform of (3.10a) and (3.10b) it can be shown respectively

\[ \frac{k_{z_2}}{\omega e_1} \tilde{H}_m^+(\zeta) e^{ik_zz} = \frac{k_{z_2}}{\omega e_2} \left[ \tilde{H}_m^+(\zeta) e^{ik_zz} - \tilde{H}_m^-(\zeta) e^{-ik_zz} \right] \] (3.11a)

\[ \tilde{H}_m^+(\zeta) e^{ik_zz} = \tilde{H}_m^+(\zeta) e^{ik_zz} + \tilde{H}_m^-(\zeta) e^{-ik_zz} \] (3.11b)

Taking the ratio of (3.11a) to (3.11b) one obtains

\[ \tilde{H}_m^-(\zeta) = e^{i2k_zz} \left( \frac{e_z k_z^2 - e_z k_z^2}{e_z k_z^2 + e_z k_z^2} \right) \tilde{H}_m^+(\zeta) \] (3.12)

As a consequence,

\[ \tilde{H}_m^+(\zeta) = e^{i2k_zz} \left( 1 + \frac{e_z k_z^2 - e_z k_z^2}{e_z k_z^2 + e_z k_z^2} \right) \tilde{H}_m^+(\zeta) \] (3.13)

Following, enforcing tangential E field continuity at the aperture \((-a < x < a, z = 0)\) result

\[ \frac{k}{\omega e_1} \hat{H}_0' - \sum_{m=0}^{\infty} \frac{c_m}{\omega e_1} \cos a_m (x + a) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{k_{z_2}}{\omega e_2} \left[ \tilde{H}_m^+(\zeta) - \tilde{H}_m^+(\zeta) \right] e^{i\zeta x} d\zeta \] (3.14a)

\[ \hat{H}_0' + \sum_{m=0}^{\infty} c_m \cos a_m (x + a) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \tilde{H}_m^+(\zeta) + \tilde{H}_m^+(\zeta) \right] e^{i\zeta x} d\zeta \] (3.14b)
Performing integral Fourier transform to (3.14a) and then substituting (3.12) into the expression, it can be found that

\[
\tilde{H}_m^f(\zeta) = \left(\frac{1}{1 - \alpha}\right) \frac{\varepsilon_2}{\varepsilon_1} \left[ H_0^f \tilde{\xi}_0 K_0(\zeta) - \sum_{m=0}^{\infty} c_m \tilde{\xi}_m K_m(\zeta) \right]
\]

(3.15)

where

\[
\alpha = e^{i2kz_f} \frac{\varepsilon_1 k_{z_2} - \varepsilon_2 k_{z_3}}{\varepsilon_1 k_{z_2} + \varepsilon_2 k_{z_3}}
\]

(3.16)

\[
K_m(\zeta) = \frac{-i\zeta^m}{k_{z_2} \left(\zeta^2 - a_m^2\right)} \left[ e^{i\zeta a} (-1)^m - e^{-i\zeta a} \right]
\]

(3.17)

while conformance to tangential H field continuity at the aperture gives

\[
\frac{1}{2\pi} \int_{-a}^{a} \frac{1 + \alpha}{1 - \alpha} \frac{\varepsilon_2}{\varepsilon_1} \left[ H_0^f \tilde{\xi}_0 K_0(\zeta) - \sum_{m=0}^{\infty} c_m \tilde{\xi}_m K_m(\zeta) \right] e^{-i\zeta x} d\zeta = H_0^f + \sum_{m=0}^{\infty} c_m \cos a_m (x + a)
\]

(3.18)

Multiplying (3.18) by \(\cos a_n (x + a)\) and integrating both sides with respect to \(x\) from \(-a\) to \(a\), one obtains

\[
\frac{\varepsilon_2}{\varepsilon_1} \left[ H_0^f \tilde{\xi}_0 J_0 - \sum_{m=0}^{\infty} \xi_m c_m J_{mn} \right] = 2\pi a (H_0^f \delta_{n0} + c_n) \psi_n
\]

(3.19)

where

\(\delta_{mn}\) represents the Kronecker delta, \(\psi_0 = 2, \ \psi_1 = \psi_2 = \cdots = 1\)
And
\[
J_{\text{mm}} = \int_{-\infty}^{\infty} \left( 1 + \alpha \right) \frac{\tilde{\zeta}^2 \left[ (-1)^m e^{j\zeta_n} - e^{-j\zeta_n} \right] \left[ (-1)^n e^{-j\zeta_n} - e^{j\zeta_n} \right]}{k_{\zeta} \left( \tilde{\zeta}^2 - a_m^2 \right) \left( \tilde{\zeta}^2 - a_n^2 \right)} \, d\zeta
\]  
(3.20)

It is possible to arrange (3.19) into a system of M+1 linear equations
\[
\begin{align*}
A_{11}c_0 + A_{12}c_1 + \cdots + A_{1n}c_n &= B_1 \\
A_{21}c_0 + A_{22}c_1 + \cdots + A_{2n}c_n &= B_2 \\
& \vdots \\
A_{n1}c_0 + A_{n2}c_1 + \cdots + A_{nn}c_n &= B_n
\end{align*}
\]  
(3.21)

where \( n = M + 1 \) is the number of modes in the PPWP to be taken into consideration. With \( D \) as the determinant of the system, i.e. \( D = \text{det} \, A \), the elements \( c_m \) can be solved by applying the Cramer’s Theorem [122]
\[
c_0 = \frac{D_1}{D}, \quad c_1 = \frac{D_2}{D}, \quad \cdots, \quad c_m = \frac{D_{M+1}}{D}
\]  
(3.22)

where \( D_k \) is the determinant obtained from \( D \) by replacing in the \( k \)th column in \( D \), with the column matrix \( B \).

3.3.3 Electric Field in Tissue

Solving for the unknown coefficients \( c_m \) and substituted into (3.13) to evaluate \( \tilde{H}_{\text{III}}(\zeta) \) yields the \( x \)-direction electric field \( E^\text{III}_x \) and \( z \)-direction electric field \( E^\text{III}_z \) in Region III respectively as
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\[
E_x^{III}(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k_{n1} H_{m1}^+(\zeta, 0) e^{-i\zeta z + i k_0 z} d\zeta
\] (3.23)

\[
E_x^{III}(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\zeta}{\omega c_0} \tilde{H}_{m1}^+(\zeta, 0) e^{-i\zeta z + i k_0 z} d\zeta
\] (3.24)

These expressions would be used to evaluate the fields incident on any scatterer embedded in Region III (host tissue), which would be explained in later in Section 3.5.

In addition, the reflection coefficient (for healthy tissues without any tumor) at the aperture of the probe \( \Gamma_0 \) is given by

\[
\Gamma_0(\omega) = \frac{E_x^I}{E_x^I} = -\frac{c_0}{H_0^I}
\] (3.25)

where \( E_x^I \) and \( E_x^I \) are the incident and reflected electric fields at the aperture respectively, and \( c_0 \) is the amplitude of the dominant mode reflected magnetic field in Region I.

3.4 Receiving Characteristic

The scattered signals received by the PPWP, in the presence of a scatterer, e.g. a tumor in the case of cancer detection, are dependent on the probe’s receiving characteristic. This section is dedicated to the derivation of this characteristic. To derive a simplified
solution for this characteristic in integral form, it is assumed the scattered field arriving at the aperture of the PPWP is locally plane.

Consider a plane wave at oblique incidence into the probe (scattered field from the tumor), the incident and reflected magnetic fields in Region III are given respectively as (see Figure 3.2)

\[
H^\prime_y(x, z) = H^\prime_{0} e^{ik_3 x - ik_3 (z - l)}
\]

\[
H'_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{H}^+_y(\xi) e^{-i\xi x + ik_3 (z - l)} d\xi
\]

(3.26)

(3.27)

while the field in the bounded Region II is given as

\[
H''_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\tilde{H}^+_y(\xi) e^{ik_2 z} + \tilde{H}^-_y(\xi) e^{-ik_2 z}] e^{-i\xi x} d\xi
\]

(3.28)

and the transmitted field inside the waveguide can be represented as

\[
H^t_y(x, z) = \sum_{m=0}^{\infty} b_m \cos a_m (x + a) e^{-i\xi_m z}
\]

(3.29)

where

\[
k_x = k_3 \sin \theta_i
\]

(3.30)

\[
k_z = k_3 \cos \theta_i
\]

(3.31)

and \(H^\prime_{0}\) is the amplitude of the incident magnetic field in Region III;

\(k_1, k_2, k_3\) are the wavenumbers in Regions I to III,

\(\xi_m, a_m, k_{z2}, k_{z3}\) are as previously defined in (3.3), (3.4), (3.6), and (3.9) respectively.
Matching the boundary continuities of tangential E field and tangential H field is required to determine the unknown coefficients $b_m$. Similarly, imposing the boundary conditions, the tangential E and H field continuities at $z = l$ yield

$$\tilde{H}_{ll}(\zeta) = \alpha \tilde{H}_{ll}(\zeta) + 2\pi \beta \delta(\zeta - k_z)$$ (3.32)

where

$$\beta = e^{ik_z l} \left( \frac{\epsilon_z (k_{z_2} + k_z)}{k_{z_2} \epsilon_3 + k_z \epsilon_z} \right)$$ (3.33)

$\delta$ denotes the Dirac delta function and $\alpha$ takes the form of (3.16).
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Following, at the aperture \((-a < x < a, z = 0)\), the tangential \(E\) and \(H\) field continuities respectively yield

\[
\tilde{H}_f^+ (\zeta) = \frac{1}{1 - \alpha} \left\{ 2 \pi H_0^{\mu} \beta \delta(\zeta - k_z) - \frac{\varepsilon_2}{\varepsilon_1} \sum_{m=0}^{\infty} b_m \xi_m K_m (\zeta) \right\}
\]

\(3.34\)

\[
\sum_{m=0}^{\infty} b_m \cos a_m (x + a) e^{-i k_m z} =
\]

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ (1 + \alpha) \tilde{H}_f^+ (\zeta) + \left[ 2 \pi H_0^{\mu} \beta \delta(\zeta - k_z) - \frac{\varepsilon_2}{\varepsilon_1} \sum_{m=0}^{\infty} b_m \xi_m K_m (\zeta) \right] e^{-i k_m z} d\zeta \]

\(3.35\)

Multiplying \(3.35\) by \(\cos a_n (x + a)\) and integrating both sides with respect to \(x\) from \(-a\) to \(a\), with the use of time-delayed Dirac delta property [123],

\[
\int_{-\infty}^{\infty} f(t) \delta(t - T) dt = f(T)
\]

\(3.36\)

one obtains

\[
\sum_{m=0}^{\infty} \frac{\varepsilon_2}{\varepsilon_1} \xi_m b_m J_{m_n} = 2\pi \left( \frac{4H_0^{\mu} \varepsilon_2 k_z}{\varepsilon_3 \sqrt{k_x^2 - k_z^2} + \varepsilon_2 k_z - e^{i 2k_x a} (\varepsilon_3 \sqrt{k_x^2 - k_z^2} + \varepsilon_2 k_z)} \right) L_n - a \alpha_n b_n
\]

\(3.37\)

where

\[
L_n = \frac{ik_z \left[ (1)^n e^{ik_x a} - e^{-ik_x a} \right]}{\alpha_n^2 - k_z^2}
\]

\(3.38\)

\[
\alpha_0 = 2, \ \alpha_1 = \alpha_2 = \cdots = 1
\]

\(3.39\)

and \(J_{mn}\) takes the form of \(3.20\).

In a method similar to that highlighted in Section 3.3, the unknown coefficients \(b_m\) are solved.

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The transmission coefficient at the aperture of the probe $\tau_0$ can then be derived as

$$
\tau_0(\omega) = \frac{E^t}{E^i} = \frac{b_0}{H_0^u}
$$

(3.40)

where $E^i$ and $E^t$ are the incident (from scatterer) and transmitted (or received) electric fields at the aperture respectively, $b_0$ is the amplitude of the dominant mode transmitted magnetic field in Region I.

### 3.5 Scattering Characteristic

The basis of the detection technique is the Mie scattering of dielectric bodies [118-120]. It is assumed the incident field onto the small spherical tumor of radius $r$, embedded in the tissue, is locally plane to simplify the analysis. The exact solution for plane wave scattering by a dielectric homogeneous sphere (Mie scattering [119, 120]) is as follows.

#### 3.5.1 Mie Scattering

For a plane wave with amplitude $E_o$ polarized in the $x'_1$-direction propagating in the negative $z'_1$-direction as defined in Figure 3.3, that is

$$
E^{inc} = E_o \hat{a}_{x'_1} e^{-ikz'_1} 
$$

(3.41)
Using far-field approximations, the scattered electric field at a point \( P (r'_1, \theta'_1, \phi'_1) \) outside the sphere, is given by

\[
E^s(P, \omega) = E_o \frac{e^{ikr'_1}}{k_3 r'_1} \left[ \cos \phi'_1 S_1(\theta'_1) \hat{a}_{\theta'_1} - \sin \phi'_1 S_2(\theta'_1) \hat{a}_{\phi'_1} \right]
\]

(3.42)

where

\[
S_1(\theta'_1) = \sum_{n=1}^{\infty} (-i)^{n+1} \left[ A_n \frac{P_n'(\cos \theta'_1)}{\sin \theta'_1} + iB_n \frac{d}{d\theta'_1} P_n'(\cos \theta'_1) \right]
\]

(3.43)

and

\[
S_2(\theta'_1) = \sum_{n=1}^{\infty} (-i)^{n+1} \left[ A_n \frac{d}{d\theta'_1} P_n'(\cos \theta'_1) + iB_n \frac{P_n'(\cos \theta'_1)}{\sin \theta'_1} \right]
\]

(3.44)

\( P_n'(\cos \theta'_1) \) is the associated Legendre function [124].

Figure 3.3: Scattering geometry for a sphere.
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Denoting the medium of the sphere with a wave number

\[ k_4 = \alpha \sqrt{\varepsilon_4 \mu_0} = \alpha \sqrt{(\varepsilon_4 + i \mu_4) \mu_0} = k_3 m_4 \]  

(3.45)

\( m_4 \) is the refractive index of the medium, and is a complex number.

The coefficients of the Mie solution for numerical summations are given by

\[ A_n = (-i)^n \frac{2n + 1}{n(n+1)} \left[ j_n(k_3 r_0)[k_4 j_n(k_4 r_0)]' - j_n(k_3 r_0)[k_4 j_n(k_4 r_0)]' \right] \]  

(3.46)

and

\[ B_n = (-i)^{n+1} \frac{2n + 1}{n(n+1)} \left[ j_n(k_3 r_0)[k_4 j_n(k_4 r_0)]' - m_4^2 j_n(k_3 r_0)[k_4 j_n(k_4 r_0)]' \right] \]  

(3.47)

The primes, \([ \ ]'\), denote differentiation with respect to the argument \( k_3 r_0 \) or \( k_4 r_0 \).

For the special case of backscattering (\( \theta'_1 = 0 \)), the result simplifies to [120]

\[ E'(r'_1, \theta'_1 = 0, \omega) = E_o e^{ik_3 r'} \frac{\omega}{k_3 r'_1} \sum_{n=1}^{\infty} (-i)^{n+1} \frac{n(n+1)}{2} [A_n + iB_n] \]  

(3.48)

3.5.2 Plane Wave Incidence on Tumor

It can be proved from uniform geometrical theory of diffraction (UTD) that a field point in Region III is illuminated by the fields diffracted from the edges of the flanged parallel-plate waveguide probe at W1 and W2 respectively (see Figure 3.4) [125-127].
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From the UTD, the radiated far-zone field from a flanged parallel-plate waveguide depicted in Figure 3.1 with incident field defined in Equation (3.1), \( |H_0| \) set to 1 and Regions I, II, III identical; can be expressed as [125]

\[
H^f(\theta) = H_1^d(\theta) + H_2^d(\theta)
\]  
(3.49)

where

\[
H_1^d(\theta) = \frac{1}{2} D_h(L', L'^o, L'^n, (\pi + \theta), 0, 1.5) e^{-ik_0\sin\theta} e^{ik_0r} \sqrt{r}
\]  
(3.50)

\[
H_2^d(\theta) = \frac{1}{2} D_h(L', L'^o, L'^n, (\pi - \theta), 0, 1.5) e^{ik_0\sin\theta} e^{ik_0r} \sqrt{r}
\]  
(3.51)

in which the two dimensional UTD edge diffraction coefficient \( D_h \) is given in Appendix A.

Although the expression is dealing with far-zone, with \( r \) substituted by \( \sqrt{x^2 + z^2} \) and \( \theta = \tan^{-1} \frac{x}{z} \), it has been found the \( x- \) and \( z- \) direction electric fields fit well with that found with Equation (3.23) and (3.24), as long as the region is away from the shadow region. Therefore it is possible to resolve the incident fields at the tumor, \( E_{x}^{III}(x_0, z_0) \) and \( E_{z}^{III}(x_0, z_0) \), into components of plane wave emanating from the two edges W1 and W2; that is two electric fields propagating in the directions of \( \hat{a}_{w1} \) and \( \hat{a}_{w2} \), polarized in \( x'_{1} \)- and \( x'_{2} \)- directions respectively. With reference to Figure 3.4, these electric fields have amplitudes

\[
E_{2inc} = \frac{E_{x}^{III} \sin \theta_{w1} + E_{z}^{III} \cos \theta_{w1}}{\sin \theta_{w1} \cos \theta_{w2} - \sin \theta_{w2} \cos \theta_{w1}}
\]  
(3.52)
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\[ E^{inc} = \frac{E^I_{III} - E^{2inc} \cos \theta_{w2}}{\cos \theta_{w1}} \]  

(3.53)

where \( \theta_{w1} \) and \( \theta_{w2} \) are angles \( \hat{a}_{w1} \) and \( \hat{a}_{w2} \) made with the z axis, given by

(3.54a)

\[ \theta_{w1} = \tan^{-1} \left( \frac{x_0 - a}{z_0} \right) \]

(3.54b)

\[ \theta_{w2} = \tan^{-1} \left( \frac{x_0 + a}{z_0} \right) \]

Figure 3.4: Resolving the electric fields \( E^I_{III} (x_0, z_0) \) and \( E^{III}_{z} (x_0, z_0) \), into components of plane wave emanating from the two edges \( W_1 \) and \( W_2 \). It is to be noted, the multilayers (Region II and III) are only locally plane at the region where the PPWP is transmitting and receiving.
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With 2 plane waves incident on the tumor results in 2 contributions of scattered field received by the PPWP. By superposition, the effective scattered signals can be found.

3.5.3 Scattering Parameters

The PPWP may be used for both $S_{11}$ and $S_{21}$ measurements for better accuracy of estimating the presence and location of the tumor. In what follows the expressions for the Scattering parameters are derived. For simplicity of analysis as highlighted in Section 3.4, the scattered electric field from the scatterer is assumed to be plane wave.

Figure 3.5: Two PPWP receiving scattered signals from a tumor in terms of $S_{11}$ and $S_{21}$.
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In the presence of a tumor, the backscattered signal in $S_{11}$ measured at the aperture of the PPWP is found to be

$$S_{11} = \frac{E_x' + \tau_0^T (E_{1s} + E_{2s})}{E_x'} = \Gamma_0 + \Delta \Gamma_{bs}$$  \hspace{1cm} (3.55)

where

$E_x'$ and $E_x'$ are the incident and reflected field at the aperture

$E_{bs}$ is the backscattered field from the tumor in the direction of $\hat{a}_{bs}$

$\tau_0^T$ is the transmitting coefficient of the probe, dependent on the angle of incidence $\theta$, made with the vertical axis of the probe, and can be found with (3.40).

In addition, $S_{21}$ can be represented as

$$S_{21} = \tau_0^R \frac{(E_{1s} + E_{2s})}{E_x'} = \Delta \Gamma_s$$  \hspace{1cm} (3.56)

where

$$\Delta \Gamma_s = \tau_0^R \frac{E_x'(r_1 = r_0, \theta', \omega) + E_x'(r_2 = r_0, \theta'_2, \omega)}{E_x'}$$

$E_x'$ and $E_x'$ are the incident and reflected field at the aperture

$E_{bs}$ is the backscattered field from the tumor in the direction of $\hat{a}_{bs}$

$\tau_0^R$ is the transmission coefficient of the receiving probe, also can be found with (3.40).
3.6 Conclusion

In this chapter, the theoretical geometry of the flanged parallel-plate waveguide probe (PPWP) is introduced. The analytic solutions for the reflection and transmission coefficients at the aperture of the probe in contact with a 2-layered dielectric that simulate the skin and the biological tissue are presented. The plane wave assumption is made in the derivations in sections 3.4 and 3.5 to obtain simplified solutions. As it has been found for a width of $2a=6\text{mm}$ of the PPWP, in sweeping from frequency 1 to 7GHz, the maximum error in amplitude and phase between the backscattered signal at the aperture from a tumor embedded at 3cm beneath the skin at $x=0$ and $x=+/-a$, is less than 2% and 2.5° respectively, this assumption is made. Expressions for the Scattering parameters that are to be measured by the PPWP in the presence of a tumor, based on Mie solution of scattering by a dielectric sphere are also shown.

In the following chapter 4 section 4.3, numerical results for the detection of tumor are simulated with these analytical formulations of the Scattering parameters presented in this chapter 3.
Chapter 4

Method for Detection of Tumor

4.1 Introduction

In the earlier chapter 3, a flanged parallel-plate waveguide probe (PPWP) that may be applied to detection of cancers where the malignant tumor differs significantly in electrical properties from its host medium is introduced. Analysis of the Scattering parameters measured by the PPWP in the presence of a scatterer is presented. The knowledge of these parameters is useful in the proposed technique for tumor detection.

This chapter will thus describe in detail the proposed technique for tumor detection. Numerical examples to illustrate this technique are simulated using the analytical expressions for the Scattering parameters previously derived in chapter 3. This technique identifies the presence, size and location of a tumor in normal tissue from the S parameters measured over a range of microwave frequencies. The operating frequency range very much depends on the region of detection, such as in the case of breast cancer detection, the operating frequency is in the range of 1 to 7GHz as the skin depth of the skin above the breast tissue is in the order of sub-centimeters [121]; this is sufficient for
the signal to penetrate the skin and tissue to reach the tumor and be scattered back but small enough to prevent unwanted scattered signals from bones and internal organs as signals are attenuated due to increased path lengths in terms of the skin depth.

The working principle of this technique is based on detecting the significant difference in complex permittivity between a malignant tumor and the surrounding healthy tissue. The PPWP functions as a microwave source radiating into the region of concern. A tumor having dielectric properties in high contrast to the healthy tissue alters the \( S_{11} \) parameters hence the presence of the tumor is identified by significant resonance in the \( S_{11} \) over the swept frequencies. The greater this difference in dielectric properties between a scatterer and the healthy tissue, the greater is the contrast in the \( S_{11} \) measured. Therefore clutter items (tissue heterogeneity) having lower dielectric contrast from the healthy tissue, compared to the tumor, can be distinguished from the latter with the resonant magnitude in \( S_{11} \).

### 4.2 Methodology

Measurements for microwave detection of tumors are to be performed at a range of microwave frequencies using the PPWP, with a vector network analyzer to measure the \( S \) parameters. Depending on the type of biological tissue to be surveyed, upon calculation of the skin depth, the probe is to be designed for only the dominant mode waves propagation for the operating frequency range. To minimize reflections which can arise due to interfaces between different mediums, the PPWP is to be immersed in a
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liquid phantom material of known properties [94, 128]; in the case of, for instance
breast cancer detection, skin-mimicking phantom is selected.

4.2.1 $S_{11}$ Measurements

The PPWP is to be scanned in contact around the region of concern, e.g., human organ
or biological tissues, etc., making $S_{11}$ measurements at different positions; as shown in
Figure 4.1 using the detection of breast cancer as a specific illustration. Making $N$
measurements at different positions of the breast, there can be $N\binom{2}{2}$ pairs of differences in
backscattered signals or $S_{11}$ measured. $\Delta\Gamma$ or $\Delta S_{11}$, the difference in $S_{11}$ contained in
each pair, is dependent on the receiving characteristic of the probe $\tau_0$ as seen in
Equation (3.49). For example in Figure 4.1, three measurements are obtained from three
positions of the breast, hence there can be three pairs of differences, $\Delta\Gamma_{12}$, $\Delta\Gamma_{23}$, and
$\Delta\Gamma_{31}$ where the subscripts indicate the pair of positions where the measurements are
made.

Figure 4.1: Flanged parallel-plate waveguide probe at different positions of the breast
tissue for $S_{11}$ measurements.
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This difference in each pair of the measurements of $S_{11}$, $\Delta S_{11}$, contains information on the backscattered signal from the scatterer, in particular the tumor (if any); and thus can be used to determine the presence and size of the tumor. In the presence of a tumor, $\Delta S_{11}$ will have resonant characteristics with respect to the frequency and amplitude. On the contrary, a frequency scan of the $\Delta S_{11}$ from healthy tissues will not contain any resonant features. Therefore no presence of any tumor at the positions where the two measurements are made is indicated by an absence of resonant characteristic in the $\Delta S_{11}$. If there is a resonating response, then one of the $S_{11}$ measured by the PPWP contains the backscattered signal from the tumor. By eliminating the positions with no tumor present, the final position of the tumor can thus be predicted. The summation of the magnitudes of these differences, $\Sigma \Delta S_{11}$ ($\Delta \Gamma_{12} + \Delta \Gamma_{23} + \Delta \Gamma_{31}$, for the case of three measurements), shows a clear resonant in the presence of a tumor and thus is used to further indicate a tumor is present.

Tissue heterogeneity represented by clutter items, having a smaller difference in complex permittivity compared to the healthy tissue than that of a malignant tumor, can be distinguished from the tumor by using the resonant amplitude of the $\Delta S_{11}$. Since the contrast in dielectric properties between the clutter and the embedding host tissue is significantly smaller than that between the tumor and the tissue, the backscattered power from the clutter is significantly smaller, thus clutter items and malignant tumor may be differentiated.

As tumors of different sizes will resonate at different frequencies, the size of the tumor can be estimated from calibrated resonant frequencies $f_r$ of the $\Delta S_{11}$ which contains the backscattered signal from the tumor, that is the difference in the pair of $S_{11}$.
measurements made at the position where there is no tumor beneath and at the position where there is a tumor beneath (such as in Figure 4.1, positions 1 and 3). The larger the scatterer, the lower is the resonant frequency(ies) in the frequency scan of $\Delta S_{11}$. Once the size of the tumor is known from the resonant frequency(ies) $f_r$, the depth at which the tumor is located can be determined from the calibrated amplitudes of this $\Delta S_{11}$ at these resonant frequencies, as the total signal path loss from the PPWP to the tumor and backscattered to the PPWP depends on the tumor’s location. The difference in the amplitude of the scattered power at resonance between a tumor of the same size, embedded at different depths $h_1$ and $h_2$ (total signal path loss), can be accounted for by considering the absorption loss in the breast tissue and the spreading losses from the probe to the tumor and backscattered to the PPWP with this total loss given as

$$\text{Total Loss} = 0.5 \times \left[20 \log_{10} \left(\frac{h_1}{h_2}\right)^{3/2} + 2(h_1 - h_2)\alpha \right]$$

(4.1)

The first term in the square bracket of Equation (4.1) accounts for the spreading losses as the radiated wave from the transmitting PPWP can be shown to have an approximately cylindrical wavefront, whereas the wavefront of the scattered signal from the small tumor is approximately spherical. The second term accounts for the additional absorption losses from depth $h_1$ to $h_2$, where the attenuation constant $\alpha$ depends on the tissue dielectric properties.
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4.2.2 S_{21} Measurements

It has been highlighted in the previous section, the location of the tumor embedded in the tissue may be estimated from calibrated amplitudes of $\Delta S_{11}$ the difference in $S_{11}$ between from a tumor and that from normal tissue, as the received signal depends on the tumor's location. However, the effectiveness of this method of utilizing calibrated amplitudes to localize the tumor may be impacted by any slight changes in the tissue properties. Attenuation due to these variations in tissue properties affects the accuracy of estimating the depth of the embedded tumor using these calibrated amplitudes of the backscattered signal. Thus, an enhanced technique using the PPWP for both $S_{11}$ and $S_{21}$ measurements is proposed to overcome this. The additional information of $S_{21}$ helps to improve the accuracy of locating the tumor as slight variation in tissue properties is insignificant with the use of triangulation technique.

After obtaining \( \binom{N}{2} \) numbers of $\Delta S_{11}$ from the $N$ measurements, the region which is believed to have a tumor embedded beneath is identified by eliminating the position that yields non-resonant $\Delta S_{11}$. To zoom into the position where the tumor is located, the PPWP is to scan about this region. With reference to Figure 4.2, the PPWP is to scan about Region 2 to obtain the maximum resonating response of $\Delta S_{11}$, the difference between $S_{11}$ measurement from Region 1 where no tumor is present, and that from Region 2. The tumor is closest to the final position that gives the maximum $\Delta S_{11}$. 
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Figure 4.2: Scanning a PPWP about the breast for maximum resonating response of $\Delta S_{11}$.

Additionally, one of the flanged parallel-plate waveguide probes is used to make measurements at different positions of the breast, receiving scattered signals in terms of $S_{21}$ while another PPWP is fixed as the transmitting probe at the position yielding maximum resonating $\Delta S_{11}$. In the presence of a scatterer, there exist positions where amplitude of $S_{21}$ is minimum as the receiving PPWP is scanned around the breast. In accordance with the Mie Theory, typically the scattered signal is weakest at 90° from the direction of propagation of the incident field.
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Figure 4.3: Tumor in a breast phantom, scattering incident electric field.

Figure 4.4: Scattering pattern for a tumor at 2GHz.
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Based on Mie scattering depicted in Figure 4.3, the scattering pattern for the tumor with a dielectric contrast 5+i0.2 [80, 92] from the breast phantom, at a frequency of 2GHz, is shown in Figure 4.4. The magnitude of the electric field at r=10cm, scattered by the tumor of diameter 5mm, is normalized to that of forward scattering. It is observed the forward scattering is the greatest (θ=180°) whereas the scattering is weakest at θ = 90°, 270°.

Thus minimum $S_{21}$ occur(s) at the position(s) approximately $\theta_1 ' = \pm 90^\circ$ from the transmitting PPWP (with reference to Figure 4.5, assuming the probes are in the same plane).

![Figure 4.5: Scanning a PPWP about the breast for minimum response of $S_{21}$.](image)

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It is therefore possible to use triangulation technique to determine the depth of the tumor embedded, \( h \), beneath the transmitting PPWP as these positions correspond to the vertices of a triangle (shaded in Figure 4.5). For instance, if it is found the minimum \( S_{21} \) occurs at positions 1 and 1’, then the depth \( h \) can be calculated with

\[
h = \frac{PP'}{\tan \phi_1 + \tan \phi_1'}
\]

(4.2)

where \( PP' \) is the shortest distance between positions 1 and 1’.

Additionally, the scattered signal \( S_{21} \) for a unit incident electric field at an equidistance of \( d=3cm \) at varying scattering angle \( \theta \) is given in Figure 4.6a. Minimum \( S_{21} \) occurring at \( \theta = \pm 90^\circ \) is observed. However this is not fully in line with the theory, as shown in Figure 4.6b, if the frequency dependence of the dielectric properties of the breast tissues is modeled using a 2-pole Debye dispersion equation

\[
\varepsilon_r(\omega) + i\frac{\sigma(\omega)}{\omega\varepsilon_0} = \varepsilon_\infty + \sum_{p=1}^{2} \frac{\varepsilon_{sp} - \varepsilon_\infty}{1 - i\omega\tau_p}
\]

(4.3)

where \( \varepsilon_0 \) is the free space permittivity, and \( \omega \) is the angular frequency, with Debye parameters to fit the data of the breast tissue for the entire operating frequency range:

- for skin layer \( \epsilon_\infty = 4.62, \epsilon_{s1} = 37.10, \epsilon_{s2} = 41.22, \tau_1 = 7.51ps, \tau_2 = 0.31ns \);
- for normal tissue \( \epsilon_\infty = 2.68, \epsilon_{s1} = 5.01, \epsilon_{s2} = 3.85, \tau_1 = 15.84ps, \tau_2 = 0.10ns \); and
- for malignant tumor \( \epsilon_\infty = 11.05, \epsilon_{s1} = 51.67, \epsilon_{s2} = 43.35, \epsilon_{s3} = 43.35, \tau_1 = 8.56ps, \tau_2 = 0.23ns \) [82].
Figure 4.6: Magnitude of $S_{21}$ at different scattering angle $\theta'_1$ over a frequency range. (a) with dielectric contrast of $5+i0.2$ between the tumor and the surrounding medium. (b) with dielectric properties defined by (4.3).
This phenomenon can be explained using Equation (3.39). The minimum of the sum $S_r(\theta')$ no longer occurs at $\pm90^\circ$ in the higher frequencies with these dielectric properties defined by the Equation (4.3). Therefore it is recommended to use operating frequencies of less than 3GHz in this method of finding minimum $S_{21}$ for triangulation to determine the location of the tumor as illustrated by Figure 4.6b.
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4.3 Numerical Results and Discussions

Using the breast cancer detection as a specific illustration, numerical simulations have been conducted to evaluate the performance of the proposed technique. Consider a spherical tumor of diameter $d_0$ embedded in the breast with a skin layer of 2mm thickness [82, 85-86] (modelled as a concentric hemisphere of radius 50mm) at a depth $h$ from the surface in Figure. 4.7, with dielectric properties as defined in Equation (4.3). The breast and the PPWP of width $2a=6mm$ are immersed in a skin-mimicking phantom material. In the simulations only the first order scattering from the scatterer is taken into account. The $S$ parameters are simulated with the analytical formulations derived in chapter 3.

![Diagram showing PPWP making $S_{11}$ measurements at different positions of a breast with tumor and clutter items present.](image)

Figure 4.7: PPWP making $S_{11}$ measurements at different positions of a breast with tumor and clutter items present.
CHAPTER 4. METHOD FOR DETECTION OF TUMOR

4.3.1 Identify Presence of Tumor

Assuming three backscattered measurements of \( S_{11} \) are made at three positions given as \( S_{11(1)} \) (\( S_{11} \) detected at position 1), \( S_{11(2)} \) (\( S_{11} \) detected at position 2), and \( S_{11(3)} \) (\( S_{11} \) detected at position 3). There are 5 clutter items surrounding the tumor. The clutter items are simulated to be having a variation of +30% of the dielectric properties of normal breast tissue [90, 91]. With reference to coordinate system at position 2 as shown in figure, clutter item 1 is at \((x=6\,\text{mm}, z=39\,\text{mm})\); clutter item 2 is at \((x=15\,\text{mm}, z=24\,\text{mm})\); clutter item 3 is at \((x=12\,\text{mm}, z=9\,\text{mm})\); clutter item 4 is at \((x=-10\,\text{mm}, z=15\,\text{mm})\); clutter item 5 is at \((x=-10\,\text{mm}, z=40\,\text{mm})\); and the tumor is located at \((x=0\,\text{mm}, z=20\,\text{mm})\). Three readings available will result in three \( 3 \choose 2 \) combinations of differences: \( \Delta S_{11(2,1)} \) (between \( S_{11(2)} \) and \( S_{11(1)} \)); \( \Delta S_{11(2,3)} \) (between \( S_{11(2)} \) and \( S_{11(3)} \)) and \( \Delta S_{11(1,3)} \) (between \( S_{11(1)} \) and \( S_{11(3)} \)). It is assumed that there is no interaction between the tumor and clutters.

The three differences \( \Delta S_{11} \) are plotted in Figure 4.8 that follows. Figure 4.8a shows the plots of the magnitude of the difference in the \( S_{11}, S_{11(2)}, \) and \( S_{11(1)}, \Delta S_{11(2,1)} \); and difference in \( S_{11(2)} \) and \( S_{11(3)}, \Delta S_{11(2,3)} \) display resonance at the frequencies 5.0GHz and 4.6GHz respectively. \( S_{11(2)} \), the scattering parameter measured at position 2, containing the strongest backscatter signal from the tumor is present in both \( \Delta S_{11} \). Additionally, it is noted that the amplitude of the difference \( \Delta S_{11(3,1)} \) (between \( S_{11(3)} \) and \( S_{11(1)} \)) display no obvious resonance and is about 5 times lower than the amplitude of \( \Delta S_{11(2,1)} \) and \( \Delta S_{11(2,3)} \).

In terms of power, the difference in the magnitude of \( \Delta S_{11} \) at resonance gives a factor of
CHAPTER 4. METHOD FOR DETECTION OF TUMOR

about 25 times. Hence this factor can be used to predict the location of the tumor; one can predict a tumor is embedded around position 2.

Also, the summation of the magnitudes of the three differences $\Sigma \Delta S_{11}$ ($\Delta S_{11(2,1)} + \Delta S_{11(2,3)} + \Delta S_{11(3,1)}$) shows a very clear resonance at 4.8GHz with resonant amplitude of $2.2 \times 10^{-2}$. In Figure 4.8b, where the tumor is replaced by a clutter item of the same size, it is noted that there is no clear resonance in both $\Delta S_{11}$ and $\Sigma \Delta S_{11}$. The amplitude of $\Sigma \Delta S_{11}$ due to all clutter items is more than 10 times lower than the resonant amplitude of $\Sigma \Delta S_{11}$ in Figure 4.8a. This is essential for differentiating between the presence of a tumor or the presence of clutter of the same size.

(a)
As tumors have much higher complex permittivity than the surrounding healthy tissue compared to that of a clutter, it will backscatter larger power. The difference in resonant amplitudes therefore can be used as a tool to differentiate between a tumor and clutter items.
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4.3.2 Estimation of Size and Location of Tumor

In general, larger scatterers (tumor or clutter) result in larger backscattered power. This is observed in Figure 4.9a where the summation of the magnitudes of the various differences in $S_{11}$ at different positions, $\Sigma \Delta S_{11}$, due to a tumor of a larger diameter of 7mm, is greater than that by a tumor of smaller diameters of 5mm or 3mm. Additionally, as tumors of different sizes resonate at different frequencies, the size of the tumor can be estimated from calibrated resonant frequencies $f_r$ of the $\Delta S_{11}$. 
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Figure 4.9: Comparison of $\Sigma \Delta S_{11}$, the summation of the magnitude of the differences in $S_{11}$ for (a) Tumor of different sizes. (b) Tumor at different depths.

Furthermore, in Figure 4.9b, a comparison of $\Sigma \Delta S_{11}$ whereby the tumor is embedded at $h=20$mm, $h=25$mm, or $h=30$mm shows that the resonant frequency is about 4.8GHz. It is not much affected by the depth of the tumor in the tissue. This further illustrates that it is possible to estimate the size of the tumor using the calibrated resonant frequencies $f_r$.

In section 4.2.1, we have discussed the depth the tumor is embedded can be estimated from calibrated amplitudes of $\Delta S_{11}$ at resonant frequencies. This is shown in
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Figure 4.9b where there is a difference of about 2.4dB in the magnitudes of $\Sigma \Delta S_{11}$ at resonant frequency of 4.8GHz for a tumor of $d_0 = 5$mm embedded at different depths of 2.0cm and 2.5cm, that is $\Sigma \Delta S_{11}(h=2.0\text{cm},4.8\text{GHz}) = -33.2$dB, and $\Sigma \Delta S_{11}(h=2.5\text{cm},4.8\text{GHz}) = -35.6$dB.

Using Equation (4.2), the attenuation constant $\alpha$ is found to be approximately 2.51dB/cm at 4.8GHz. The additional spreading loss and absorption loss as defined in Equation (4.1), due to the increased depth from 2.0cm to 2.5cm, is thus given by 1.45dB and 1.25dB respectively. Therefore the total loss in resonant $\Sigma \Delta S_{11}$ at 4.8GHz is 2.7dB, which agrees with the observations. Similarly, the total loss in $\Sigma \Delta S_{11}$ at resonant frequency when the tumor is embedded at $h=30$mm can be accounted for by the same procedure. Additional spreading loss and absorption loss due to the increased depth from 2.0cm to 3.0cm is given by 2.64dB and 2.38dB respectively; hence it can be predicted that the resonant amplitude of $\Sigma \Delta S_{11}$ for the tumor embedded at $h=30$mm is 5.0dB smaller than that for a tumor embedded at $h=20$mm. This tallies with the observations from Figure 4.9b as it is noted that $\Sigma \Delta S_{11}(h=2.0\text{cm},4.8\text{GHz}) = -33.2$dB, and $\Sigma \Delta S_{11}(h=3.0\text{cm},4.6\text{GHz}) = -38.2$dB.

4.3.3 Validation of Enhanced Technique

To minimize any inaccuracies which may arise due to attenuation of the backscattered signals resulted by slight changes in tissue properties in estimating the position of the tumor with calibrated amplitudes of $\Delta S_{11}$, the additional information of $S_{21}$ to identify
the reference vertices of a triangle such that the coordinates and distance to the tumor can be found through triangular relations is recommended.

After obtaining $\binom{N}{2}$ numbers of $\Delta S_{11}$ from the $N$ measurements, the region which is believed to have a tumor embedded beneath is identified by eliminating the position that yields non-resonant $\Delta S_{11}$. To zoom into the position where the tumor is located, the PPWP is to scan about this region. With reference to Figure 4.2, the PPWP is to scan about Region 2 to obtain the maximum resonating response of $\Delta S_{11}$, the difference between $S_{11}$ measurement from Region 1 where no tumor is present, and that from Region 2. The tumor is closest to the final position that gives the maximum $\Delta S_{11}$.

Figure 4.10: Magnitude of differences $\Delta S_{11}$ obtained at various positions of the breast with clutter assuming +30% dielectric variation of normal breast tissue.
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With the tumor embedded at \( z_1' = 0, \ x_1' = 0 \), and a clutter embedded at \( z_1' = -6\text{mm}, \ x_1' = 15\text{mm} \), it is observed as shown in Figure. 4.10, there is resonating \( \Delta S_{11}(2a,1) \) (difference between \( S_{11} \) obtained at position 2a and position 1 (normal breast tissue)); indicating a tumor is embedded near position 2a. Afterwards, the PPWP is moved about position 2a until a maximum response of \( \Delta S_{11}(2,1) \) is detected at position 2. This implies the tumor is embedded beneath position 2, the reason being within the entire breast, the backscattering is the strongest (\( \theta = 0 \)) as can be found by Equation (3.39). Also shown in Figure 4.10, as the influence of the tumor is minimal in Region 1, \( \Delta S_{11}(1a,1) \) (difference between measurements at positions 1 and 1a) does not exhibit any resonating characteristics. In addition, \( \Delta S_{11}(3,1) \) displays some resonance but the amplitude is very much weaker than that of \( \Delta S_{11} \) in Region 2, about more than 5 times smaller; this is attributed to scattering from the tumor is also picked up at position 3 but the power is lower than that at positions in Region 2. This validates it is possible to zoom into the region where the tumor is located, in this case Region 2.

However recent studies on the dielectric properties of normal, benign and malignant breast tissues samples in the microwave ultra-wideband range [55, 56], and the development of anatomically realistic numerical breast phantoms [106, 106], show that the dielectric contrast between the malignant and normal glandular/fibroconnective tissues in the breast may not be as large as expected, thus presenting a more challenging scenario in microwave detection of breast tumors. Hence the effect of the clutter assuming dielectric properties of a fibroconnective/glandular tissues report in [105], defined by single-pole Debye dispersive equation:
with parameters: $\varepsilon_\infty = 12.8485$, $\varepsilon_\Delta = 24.6430$, $\tau = 13\text{ps}$, $\sigma_s = 0.2514\text{S/m}$, is investigated.

![Graph](image.png)

Figure 4.11: Magnitude of differences $\Delta S_{11}$ obtained at various positions of the breast with clutter assuming clutter with dielectric properties defined by Equation (4.4).

In Figure 4.11, it is shown that the resonating amplitude of $\Delta S_{11(2,1)}$ is still the maximum with that of $\Delta S_{11(2a,1)}$ is larger than that of $\Delta S_{11(2b,1)}$ due to constructive interferences of both the tumor, and the clutter which has now a larger dielectric contrast (having dielectric constant ranging from 21 to 24 using Equation (4.4), instead
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of 5 to 8 using Equation (4.3) and the associated parameters) to the normal breast tissue in this scenario. The amplitude of the peak of $\Delta S_{11(2b,1)}$ is still 2dB higher than that of $\Delta S_{11(3,1)}$ as scattering from the tumor is still stronger than that from the clutter, thus Region 2 is still identified as the region where the tumor is embedded.

It has been mentioned earlier that the position at which the minimum $S_{21}$ occurs is approximately $\pm 90^\circ$ from the transmitting PPWP for frequencies less than 3GHz. In the following example, it is to be validated that using the minimum $S_{21}$ helps to enhance the accuracy of locating the tumor. Recurring the steps involved in obtaining the maximum response of $S_{11}$ described in Section 4.2.2, it is found that maximum $S_{11}$ is found at position 2 (see Figure 4.2). With a PPWP transmitting at this position at 2GHz while another PPWP is scanned around the breast, it can be observed in Figure 4.12 that in the presence of only a tumor, the minimum $S_{21}$ occurs at $\theta = \pm 90^\circ$, at positions 1 and 1’.

It should be noted that if the tumor is embedded at the centre of the hemispheric breast, the result should be symmetrical about $\theta = 0^\circ$. However, as the tumor is embedded off the centre of the breast towards the right as shown in Figure 4.5 in this example, the magnitude of $S_{21}$ is asymmetrical, with magnitude for larger positive $\theta$ is higher than that of the negative $\theta$ as the receiving PPWP is further from the tumor in the latter. With this knowledge of the position(s) where minimum $S_{21}$ occur(s), it is possible to use triangulation technique to estimate more accurately the depth of the tumor embedded.
Figure 4.12: Magnitude of $S_{21}$ obtained at different scattering angles $\theta'_1$ at 2GHz.

Investigations on how the presence of clutter items affects the $S_{21}$ are also conducted. Having a clutter C1 having the same size as the tumor, at same depth below the surface of the skin, the $S_{21}$ scanned at the same positions of the breast at 2GHz is also given in Figure 4.12. It is observed that albeit the presence of a clutter, the position where the minimum $S_{21}$ remains as before in the case where only the tumor is present. As can be seen in Figure 4.12, as the clutter item is a scatterer itself, there exist constructive and destructive interference at some positions where the receiving PPWP is placed. Nonetheless, as the clutter item is of a lower dielectric contrast (+30% dielectric variations of the normal breast tissue) to the surrounding medium as compared to the
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tumor, the effect on the position where minimum $S_{21}$ occurs is not significant. However with the clutter assuming properties defined by Equation (4.4), the position where minimum $S_{21}$ occurs, shifted slightly away from $\theta_1 = 90^\circ$.

4.4 Conclusion

A method using flanged parallel-plate waveguide probes (PPWP) has been proposed for microwave detection of tumors, on the basis of detecting the difference between the complex permittivity of normal tissue and a malignant tumor or other tissue heterogeneity (clutter items). Using breast cancer detection as an illustration, PPWP is employed to perform contact measurements of $S_{11}$ and $S_{21}$ on the breast over a frequency range. The scattered signals have resonating characteristics in the presence of a tumor. Simulation studies show a tumor can be differentiated from a clutter item, within the assumption of there exists a large dielectric contrast between a tumor and the normal breast tissue. In addition, the size and the location of the tumor can be estimated from the knowledge of the $S_{11}$. On top of this, simulations also illustrated how the depth of the tumor embedded in the breast can be more effectively estimated with the additional information of the $S_{21}$ as a triangulation technique can be employed to locate the tumor from the positions where the minimum $S_{21}$ occur.

It is also highlighted, in general, larger scatterers result in larger backscattered power; scattered power also increases as the dielectric contrast between the scatterer and
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host medium increases. As recent studies on the dielectric properties of the heterogeneous breast bring attention to a challenging scenario for microwave detection of tumors if the tumor and its host medium do not differ significantly in electrical properties, it to be noted the level of difficulty to identify the region of tumor in a breast arises if the dielectric properties of the tumor and clutter become less distinguishable and the size of the clutter is dramatically larger than a tumor.
Chapter 5

Practical Flanged Parallel-plate Dielectric Waveguide Probe

5.1 Introduction

It has been derived analytically and shown numerically in the earlier chapters 3 and 4, a flanged parallel-plate waveguide probe is effective in microwave detection of tumors on the basis the malignant tumor exhibits significant difference in complex permittivity from the embedding host tissue. However this probe may be large to be a practical sensor in a clinical environment as the probe has to be of a substantial width to ignore the fringe field effects, in order to have good isolation from the external interference in the clinical environment; for example measurements would not be influenced when the sensor probe is grabbed by hand or placed near sources of electromagnetic interference (EMI). In addition, to detect tumors accurately, the radiated field from the sensor probe has to be shaped in order to minimize scattering from clutter items in other regions of the host tissue, in order to zoom into the region where the tumor is located.
CHAPTER 5. PRACTICAL FLANGED PARALLEL-PLATE DIELECTRIC WAVEGUIDE PROBE

In view of this, a new flanged parallel-plate dielectric waveguide probe (PPDW) is proposed. The above mentioned objectives can be achieved with our proposed PPDW by confining the radiation from the waveguide probe with a dielectric slab that functions as a central guide to allow propagation of transverse electric waves, embedded between two flanged parallel plates. The following of this chapter presents the design process of the PPDW to provide reasonable performance in terms of return loss, depth of detection, and size of tumor detection, while being physically small in size.

5.2 Probe Design

Essentially the flanged parallel-plate dielectric waveguide probe (PPDW) is similar to the theoretical flanged parallel-plate waveguide probe (PPWP) except it has finite flanges and width, instead of the assumed infinite dimensions for simplicity of analysis of the PPWP. The PPDW consists of a dielectric slab embedded between two flanged parallel plates. It can be visualized as two perfectly conducting wedges having spaced apart by a strip of dielectric as shown in Figure 5.1. The angle of the wedge, \( \theta \), can be varied and the ends can be contoured, in order to shape the radiation pattern; however to simplify analysis only the 90° wedge will be discussed here.
Chapter 5. Practical Flanged Parallel-Plate Dielectric Waveguide Probe

Figure 5.1: 3D view of the flanged parallel-plate dielectric waveguide probe.

This dielectric slab functions as a central guide to allow the propagation of transverse electric waves, with fundamental TE$_{10}$ mode which supports an electric field across the parallel plates. A lossless dielectric with its dielectric constant selected to be higher than that of the side medium, that is, $\epsilon_c > \epsilon_s$ as depicted in Figure 5.2, this central guide helps to minimize the parallel-plate radiation [129]. The fields are bounded or confined in the central dielectric and thus leading to a reduction in the loss. A short circuit is placed at quarter wavelength at the middle frequency of the operation bandwidth from the excitation.
5.2.1 Theory

The wave propagation in this proposed design can be translated from that in a dielectric-clad conductor [130] with the use of image theory as shown in Figure 5.2.

Figure 5.2: (a) A planar dielectric-clad conductor. (b) Top view of PPDW.
It is assumed that Region I is an isotropic, non-magnetic lossless dielectric of $\varepsilon_1$, having thickness $2l$ and is bounded at surface by free space as Region II ($\varepsilon_s = \varepsilon_0$). Consider TE waves propagating the positive $z$ direction, as the fields are $x$-independent, with the time harmonic variation suppressed, the magnetic fields can be expressed as

In Region I,
\[ H^I_z(y, z) = (A \sin k_c y + B \cos k_c y) e^{-\beta z} \] (5.1)

In Region II,
\[ H^H_z(y, z) = D e^{-h z} \] (5.2)

The transverse electric fields can be expressed using relations

In Region I,
\[ E^I_x(y, z) = \frac{i \mu_0}{k_c^2} \frac{dH^I_z}{dy} \] (5.3)

In Region II,
\[ E^H_x(y, z) = -\frac{i \mu_0}{h^2} \frac{dH^H_z}{dy} \] (5.4)

with
\[ k_c^2 = k_1^2 - \beta^2 \quad \text{and} \quad h^2 = \beta^2 - k_0^2 \] (5.5)

where $k_1$ and $k_0$ are the wavenumbers in Regions I and II respectively, selecting both $k_c$ and $h$ to be positive real.
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The solution in Region I is selected to be \( H_z^I(y, z) = A \sin k_y y \) due to symmetry of the problem with justifications that the transverse magnetic field distribution is similar to that of a planar dielectric-clad conductor. Hence the electric fields are thus found to be

\[
E_x^I(y, z) = -\frac{i \omega \mu_0}{k_c} A \cos k_y y e^{-\beta z} \tag{5.6}
\]

\[
E_z^I(y, z) = -\frac{i \omega \mu_0}{h_2} D e^{-h_y} \tag{5.7}
\]

Enforcing boundary conditions at interface \( x=l \) yields

Tangential H field:

\[ A \sin k_y l = D e^{-h_y d} \tag{5.8} \]

Tangential E field:

\[ \frac{A}{k_c} \cos k_y l = \frac{D}{h_2} e^{-h_y d} \tag{5.9} \]

Taking ratio of (5.8) to (5.9) leads to

\[ k_y l \tan k_y l = h_2 l \tag{5.10} \]

With the use of (5.5), it is also found that

\[ (k_y l)^2 + (h_2 l)^2 = (\varepsilon_e - 1)(k_0 l)^2 \tag{5.11} \]

Together, (5.10) and (5.11) define the relationship between the width of the central guide \( 2l \), and the TE_{n0} propagation.
CHAPTER 5. PRACTICAL FLANGED PARALLEL-PLATE DIELECTRIC WAVEGUIDE PROBE

5.2.2 Dimensions of the Central Dielectric

In order to ensure the waveguide operates in the desired mode(s), the values of \(a\) and \(b\) (see Figure 5.3) have to be correctly selected. Based on literature [129], the coaxial probe excitation can efficiently excite the flanged parallel-plate dielectric waveguide probe. Due to the even symmetry of the excitation about \(z=0\) the mode TE\(_{20}\) probe will not be excited. To operate the waveguide in its dominant TE\(_{10}\) mode, care must be taken not to excite the TE\(_{30}\) and TE\(_{11}\) modes. Therefore proper design of the width \(a\) (previously defined \(2l\)) and thickness \(b\) must be implemented as the cutoff frequency of TE\(_{11}\) and TE\(_{30}\) depends on the ratio \(a/b\).

![Diagram](image)

Figure 5.3: End view of the aperture plane of the PPDW.

To minimize the reflections which can arise due to interfaces between different mediums, the dielectric constant of the central guide of the PPDW is selected to be \(\varepsilon_c = 10\) as the dielectric constant of a normal breast tissue approximates this value; the side medium is air with \(\varepsilon_s = 1\). As a consequence, for TE\(_{30}\) cutoff, with the use of (5.10)
and (5.11), it is found that $a/\lambda_0 < 0.33$. Making a compromise between the size of a practical probe and the how well the radiation can be confined within the central guide, the width $a$ is selected to be $a=16\text{mm}$, with operating frequency of the PPDW not exceeding 6GHz. For a given $a$, there is a maximum value of $b$ for TE$_{11}$ cutoffs with relationship of normalized $a$ and $b$ as shown in Figure 5.4. Given $a/\lambda_0 = 0.32$ for the width selected, $b/\lambda_0 \leq 0.195$. Hence thickness $b$ is chosen to be $b=8\text{mm}$. The value for $a$ and $b$ are selected partly for the ease of machining of the dielectric, based on the consideration that the dielectric from Emerson & Cumming, Eccostock® [131], comes in standard bars.

![Figure 5.4: $b/\lambda_0$ max for TE$_{11}$ cutoffs for the PPDW.](image-url)

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**CHAPTER 5. PRACTICAL FLANGED PARALLEL-PLATE DIELECTRIC WAVEGUIDE PROBE**

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5.2.3 Shape of the Central Dielectric

It is required that the central dielectric guide to be specifically shaped near the aperture of the probe for the optimization of the return loss. Taking cue from the optical phenomenon of waves, the central dielectric guide is tapered near the aperture of the probe, to minimize total internal reflection at the aperture that leads to poor return loss.

As the angle of wave propagation $\theta$ as defined in Figure 5.5 inside the central dielectric guide increases with frequency; the degree of taper depends on the desired operating frequency range. Angle $\theta$ can be found with

$$
\theta = \tan^{-1}\left(\frac{\beta/k_0}{\sqrt{\varepsilon_s} - (\beta/k_0)^2}\right) \tag{5.12}
$$

where $\beta/k_0$ can be found from (5.5) with the use of (5.10) and (5.11).

![Figure 5.5: Top view of the central guide near the aperture of the probe.](image-url)
CHAPTER 5. PRACTICAL FLANGED PARALLEL-PLATE DIELECTRIC WAVEGUIDE PROBE

How the wave propagation angle $\theta$ varies with frequency for $\varepsilon_r = 10$ and $\varepsilon_s = 1$ is shown in Figure 5.6. Hence it is possible to design the degree of taper according to the operating frequency.

![Graph showing wave propagation angle versus frequency.](image)

Figure 5.6: Wave propagation angle versus frequency.
5.3 Simulations

The results are simulated with software High Frequency Structure Simulator (HFSS), with the dimensions of the PPDW as defined in Figure 5.7. The PPDW is simulated to be made of copper with properties defaulted by the software (i.e. relative permittivity and permeability of ‘1’ and bulk conductivity of $5.8 \times 10^7$ Siemens/m) having thickness 3mm as this is one of the nominal thickness of copper sheets in practice. The breast phantom is assumed to be nondispersive with complex permittivity $\varepsilon_r = 10 + i0.12$ while the central dielectric is assumed to be lossless with dielectric constant $\varepsilon_r = 10$; the side medium is vacuum.

![Figure 5.7: 3D view of the PPDW simulated in HFSS.](image)
Simulations have also been performed to demonstrate the radiation loss of the probe is reduced with the use of a central dielectric guide (dimensions $a=16$mm, $b=8$mm). The electric field distribution inside the PPWP at 8mm from the aperture, at 3GHz, in the $x$-$y$ plane of the flanged parallel-plate dielectric waveguide PPDW, along $y$-direction (along the overall width $w$ of the PPDW as defined in Figure 5.3) is shown in Figure 5.8. It is modeled the central dielectric is tapered with $\theta=63.4^\circ$ (the actual taper dimension is found subsequently in Figure 5.12). It is noticed that majority of the energy is confined to the central guide with the leakage problem at the sides of the plates mitigated.

Figure 5.8: Electric field distribution near the aperture of the PPDW.
It is also investigated for a smaller $a=12\text{mm}$, still satisfying the $\text{TE}_{30}$ cutoff of $a/\lambda_0 < 0.33$, the energy is not as well confined within the dielectric guide as compared to the guide of $a=16\text{mm}$. This is as shown in Fig. 5.9. It is observed within the ranges of normalized $w=0$ to $w=0.4$ and $w=0.6$ to $w=1.0$, corresponding to the region outside the central guide, the electric field amplitude is higher for $a=12\text{mm}$ than that for $a=16\text{mm}$. This implies that the leakage problem for $a=12\text{mm}$ is not as well mitigated as compared to $a=16\text{mm}$. The design with $a=12\text{mm}$ would be subjected to more electromagnetic interferences such as grabbing the probe by hand. Hence it was selected that $a=16\text{mm}$. A larger $a$ will lead to a narrower frequency range for dominant $\text{TE}_{10}$ operation and hence is not considered in the investigation.

![Figure 5.9: Comparison of electric field distribution near the aperture of the PPDW for different widths of the central dielectric.](image-url)
CHAPTER 5. PRACTICAL FLANGED PARALLEL-PLATE DIELECTRIC WAVEGUIDE PROBE

With the same width of $a=16\text{mm}$, it is also studied the effects of varying the dimension $b$ on the performance of the PPDW. The performance of the probe, in terms of depth of detection, for central dielectric of thickness $b=8\text{mm}$ and $b=6\text{mm}$ is compared. The electric field patterns are simulated for the $x$-$y$ plane (as defined in Figure 5.3) at a distance of 5cm away from the aperture of the PPDW as depicted in Figure 5.10.

![Electric field distribution in the x-y plane.](image)

The respective radiation plots for $b=8\text{mm}$ and $b=6\text{mm}$ are shown in Figure 5.11. The electric field strength is limited to 100V/m in order to enhance the visual difference of the field distribution.
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Figure 5.11: Simulated electric field distribution at 5cm away from the aperture of the PPDW in a breast phantom (a) $b=8\text{mm}$. (b) $b=6\text{mm}$.
It can be observed that with $b=8\text{mm}$ for the central dielectric, the electric field is stronger than that for $b=6\text{mm}$, with a wider red spot size; this implies the PPDW with $b=8\text{mm}$ is able to penetrate deeper into the breast phantom. Therefore the design of the central dielectric with $b=8\text{mm}$ is preferred over $b=6\text{mm}$ for this reason; apart from being easy to machine. Although an even larger $b$ will lead to a deeper penetration into the breast phantom, however the beam size of the radiation will also increase and may have increased influence on S parameters measurement contributed by clutter items away from the tumor. Hence dimension of $b$ greater than 8mm is not considered in this study.

Figure 5.12: Three varying designs of the taper of the central dielectric.

It has been mentioned the return loss improves with a taper design of the central dielectric near the aperture of the probe. Three different designs of the central dielectric
as shown in Figure 5.12 are investigated. As shown in Figure 5.13, good return loss or $S_{11}$, on the average lower than -3dB, is observed if the central dielectric guide is tapered near the aperture of the probe, as opposed to that of a central dielectric guide with a flat end, i.e. design 1 with $\theta=0^\circ$. Additionally, it is noticed that $S_{11}$ improves in the higher frequencies when the taper of the central dielectric guide changes from $\theta=45^\circ$ to $\theta=63.4^\circ$, as predicted by the theory in section 5.2.3.

![Figure 5.13: Simulated reflection coefficient of the PPDW.](image)

Figure 5.14 shows the plot of radiated electric fields into the breast phantom at 3GHz, from a PPDW (flanged parallel-plate dielectric waveguide probe, with a central dielectric) and a PPWP (flanged parallel-plate waveguide probe, without a central dielectric). The plane of view is from the top of the probe ($y$-$z$ plane), split at the half-spacing between the two flanged plates.
Figure 5.14: Simulated radiation into a breast phantom (a) from a PPDW. (b) from a PPWP.
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The central dielectric of the PPDW simulated in this figure takes on the design 3 in Figure 5.12, that is tapered with $\theta=63.4^\circ$ at the tip. The dotted line marks the interface between the aperture and the breast phantom. To enhance visual representation, the electric field strength is limited to 500V/m in order to view the field distribution.

It can be observed that the beam from the PPDW is more focused than that from the PPWP. Therefore, as the radiation beam is shaped the PPDW would be able to give a more accurate estimation of the region where the tumor is located as the scattering from the clutter items away from the tumor can be limited and thus have minimal influence on the S parameters measurements. Furthermore, Figure 5.14 validates the energy confinement within the waveguide is achieved with the design of practical PPDW using a central dielectric guide as opposed to the theoretical PPWP.

5.4 Conclusion

For the probe to be of a suitable size to be used as a practical sensor in a clinical environment, and also have good isolation from the external interferences, an improved design, a flanged parallel-plate dielectric waveguide probe (PPDW) is proposed. The design of this PPDW mitigates the radiation leakage problem by confining the radiation from the waveguide probe with a dielectric slab embedded between two flanged parallel plates, functioning as a central guide to allow propagation of transverse electric waves. The selection process of the parameters of the PPDW such as the width and thickness, the level of taper at one end, of the central dielectric has been discussed. Simulations
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with HFSS demonstrate good return loss of the probe over a useful bandwidth is achieved with a slight taper at the tip; in addition the ability of the central dielectric to confine the radiation within the PPDW is also shown. Last but not least the radiation beam into the breast phantom from the PPDW is shaped with the use of the central dielectric. Based on the simulated parameters, prototypes were being fabricated for experimental validations on breast mimicking phantom materials. These experimental results will be presented and discussed in detail in the following chapter.
Chapter 6

Experimental Studies on Flanged Parallel-plate Dielectric Waveguide Probe

6.1 Introduction

Experimental studies targeted to verify the basic concepts of proposed detection method are the next procedure. These preliminary initial verifications of tumor detection and localization were conducted in a 2-D plane. Prototypes based on the design of the flanged parallel-plate dielectric waveguide probe (PPDW) were fabricated and tested to study the probe’s practical performance in terms of return loss, the ability to detect tumors, accuracy, repeatability, etc. The experimental studies were performed on self-fabricated solid breast phantoms, mimicking the dielectric properties of a breast, with conductive inclusions embedded in the phantoms representing tumors, to validate against the theoretical predictions discussed in chapter 4. Results are presented for phantoms containing strongly scattering metal to more challenging detection of
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dielectric bodies. These preliminary experimental results correspond to those predicted by theory and simulations, thus demonstrating the ability of the proposed technique and PPDW probe to detect and locate small tumors exhibiting significant difference in dielectric properties to the host tissue, embedded at depths of a few centimeters.

Also presented in this chapter is the description of the type of material used for the breast and tumor phantom in the experimental investigation. Results of the dielectric properties measurement over a broad microwave frequency range and the long term stability of these properties are also presented. Also the construction of the breast phantom with the tumor inclusion embedded is also described.

6.2 Phantom Material

The development of phantoms that mimic closely the physical and electrical properties of various human tissues is critical for the research and evaluation of modalities for cancer detection, such as ultrasound, MRI, microwave methods, and others. These phantoms are actively used for studying the interaction of electromagnetic waves with biological tissues, particularly at microwave frequencies, before proceeding to animal/clinical trials. Hence the need for inexpensive, easily fabricated and moulded phantom materials has greatly increased. It is desirable the material respond to electromagnetic waves in a similar fashion to the anatomical areas they represent; the dielectric properties of the phantom should closely match to the actual tissue conditions.
Another important criterion of the material is the long-term stability in terms of mechanical and electrical properties.

Oil-in-gelatine dispersion proposed by Lazebnik et al. [132], approximating the dispersive dielectric properties of human soft tissues, is selected as the material for breast phantom in the experimental investigation of the PPDW. The basis for selecting this gelatine-based material is its stability in mechanical properties and easy to fabricate. By varying the concentration of oil to the gelatine solution, thus limiting the water content in the material, a wide range of dielectric properties can be constructed; simulating breast fat (low water content), breast glandular tissues and tumors (high water content), and skin (intermediate water content). To summarize, the higher the oil concentration in the material, the lower is the dielectric constant. This material also has the advantage of having long stability; phantoms of different oil concentration can be placed side by side without any change in properties due to diffusion across the interface. Therefore it is possible to construct a heterogeneous breast phantom with the use of this material.

### 6.2.1 Fabrication Procedure

The general steps in fabricating this oil-in-gelatine based material consist of bringing an aqueous solution of gelatine (gelatine powder dissolved in deionised water) and a solution of oil (made up of 50% kerosene and 50% safflower oil) together at 50°C with the use of surfactant, stirring vigorously to produce a uniform emulsion. Afterwards formaldehyde is added to create crosslinking of the gelatine molecules that surrounds
the oil droplets. And the emulsion is poured into moulds to left to set for at least 5 days into solids. The detailed procedure for the fabrication of this gelatine-based material is as follows.

(i) Dissolve 0.2g of p-toluic acid powder into 10ml of n-propanol with heat and stirring.

(ii) Pour the solution of p-toluic acid and n-propanol into 190ml of room temperature 10MΩ cm deionised water (DI water) (‘AutoCare Carbuddy’, Nam Wah Battery, Singapore).

(iii) Add 34g of 160 gelatine powder (Tosu Supplies, Singapore) with stirring into the mixture produced in (ii) at room temperature, making sure the gelatine granules are wetted.

(iv) Cover the mixture in (iii) with a cling wrap (GLAD®, Glad Products Company, USA) held in place with a rubber band; and heat the mixture in a water bath. The beaker of mixture should be surrounded by sufficient water jacket to prevent local overheating.

(v) Bring the mixture to a temperature of about 90°C. The mixture should become transparent with no air bubbles suspended beneath the surface.

(vi) Stir the mixture for uniformity and removal of surface air bubbles.

(vii) Cool the mixture to around 50°C.

(viii) Prepare a beaker of oil (a mixture of 50% kerosene and 50% safflower oil) and heat to 50°C. This amount of oil is determined by the percentage of oil-concentration desired. If a 50% oil-concentration is required, prepare 200ml of oil (i.e 100ml of kerosene and 100ml of safflower oil).
(ix) Pour the cooled mixture from (vii) into the beaker of oil and stir vigorously to break the oil droplets to smaller than about 0.2mm in diameter.

(x) With vigorous stirring, add surfactant with a syringe into the mixture, in this case dishwashing liquid, ‘Ligent’ dishwashing detergent (Yuri Distribution Co, Singapore). The amount of surfactant is 11.2ml for every 200ml of oil.

(xi) An emulsion should formed; nearly white for large percentage of oil-concentration.

(xii) For oil-concentration more than 50% and up to 80%, add the remaining volume of oil in portions of 100ml to the emulsion in (xi) and stir to uniform. No more surfactant is required.

(xiii) Cool the emulsion to 50°C and add formaldehyde solution (37% bar, JT Baker, USA) with a needle syringe. The amount of formaldehyde is 2.16g for every 200ml of gelatine solution in (iii).

(xiv) Pour the emulsion into the mould to set for at least 5 days for formaldehyde cross-linking of gelatine.

### 6.2.2 Dielectric Measurement

Samples of different oil-concentration phantoms have been constructed. Dielectric measurements were recorded using Agilent high temperature dielectric probe 85070E. Each phantom was cut into 5 smaller samples to be tested having cubic dimensions 5cm each side conforming to the recommended sample size [133] as shown in Figure 6.1. Following, Figure 6.2 shows the average complex permittivity for these varying oil-concentrations.
Figure 6.1: Dielectric measurement of a sample of breast phantom using the Agilent high temperature dielectric probe.
6.2.3 Breast Phantom

In the experimental validation of the PPDW using breast cancer detection as an illustration, the percentage of oil-concentration in the emulsion is selected to be 80% to simulate dielectric properties of breast fat having dielectric properties similar to that defined by the Equation (4.3). With 80% oil-concentration, the emulsion will set into a white, opaque solid. The breast phantom only mimics the breast fat and does not include a skin-mimicking top layer. As the phantom emulsion was stirred by hand and during the process some air will be typically trapped within the emulsion, the homogeneity of
the phantom would be decreased. Figure 6.3 shows the dielectric properties of the 80% oil-concentration phantoms in the more common representation of complex permittivity.

![Complex permittivity of 80% oil-concentration phantom.](image)

Figure 6.3: Complex permittivity of 80% oil-concentration phantom.

6.2.3.1 Long-term Stability

The stability of the breast phantom material was also investigated. Figures 6.4 show the complex permittivity for 80% oil-concentration over 8 weeks. It is observed both the dielectric constant and loss tangent decrease over time but within 2 weeks from preparation, the phantom material can still be considered stable in its dielectric properties with median of the dielectric constant, and loss tangent varying to within 2% and 3% respectively, across the swept frequency over the 2 weeks. After which due to the breakdown in the linkage between the oil and gelatine, and thus water can escape and leads to a drop in the dielectric constant. This evaporation rate is further affected
with the samples being placed in air conditioned room. Overall the phantom material remained a solid with a slight shrink in size after 8 week due to water evaporation.

Figure 6.4: (a) Dielectric constant and (b) Loss tangent of 80% oil-concentration phantom over a span of 8 weeks.
6.2.3.2 Types of Oil

![Graph showing dielectric constant and loss tangent of 80% oil-concentration phantom made from different types of oil.](image)

Figure 6.5: (a) Dielectric constant and (b) Loss tangent of 80% oil-concentration phantom made from different types of oil.
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As one of the constituents of the phantom material is safflower oil, a type of edible oil, investigations on changing this type of oil to other easily available edible types of oil such as palm, peanut, and rice bran oil. It has been found the dielectric constant and conductivity do not vary significantly, especially for frequency lower than 6GHz; with median of dielectric constant between samples from these oils not vary more than 10% and that for loss tangent vary less than 5%, as shown in Figure 6.5.

6.2.3.3 Planar Mould

For simplicity during fabrication of the breast phantoms, the breast phantoms do not assumed the exact morphology of an actual breast; such as the 2mm thick skin layer is not present and the phantoms are not hemispherical but planar. Also in a practical system, to minimize the interface mismatch at the breast phantom, the PPDW is to be immersed in liquid phantom.

Figure 6.6: A typical fabricated breast phantom in a mould.
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However to avoid unnecessary mechanical complications that may arise in immersing the probe, the probe is setup in free space. As the size of the PPDW was not fully optimized at the stage of experimental investigations and without the use of immersion liquid, to ensure good contact of the probe with the phantom, the solid breast phantom is made planar. The dimension of the mould used is approximately 18cm by 14cm by 8cm as shown in Figure 6.6.

### 6.2.4 Tumor Phantom

![Complex permittivity of 10% oil-concentration phantom.](image)

Inclusions in the breast phantoms, simulating tumors, were created using strongly scattering metal spheres having diameters from 8 to 16 millimetres. More challengingly,
solid dielectric spheres made from 10% oil-concentration phantom, to mimic malignant
tumors having significant dielectric contrast from the embedding host tissue, were used
to validate the prototype PPDW is capable to detect dielectric bodies. As been reviewed
in chapter 2, the dielectric contrast between a malignant tumor and the normal
embedding tissue in general exist in the range of 5 to 10. With this 10% oil-
concentration in the tumor phantom, the difference in dielectric properties between the
tumor phantom and the breast phantom is on average about 8 times. Figure 6.7 shows
the dielectric properties of the 10% oil-concentration phantom in the more common
representation of complex permittivity.

6.2.4.1 Long-tem Stability

Similarly the stability of the tumor phantom material was also investigated.
Likewise with the 80% oil-concentration phantom, the dielectric constant and conductivity of the 10% oil-concentration phantom also decrease over time due to the same reason water escaped and evaporated due to a breakdown in the molecular linkage between the oil and gelatine. However for a span of 2 weeks, the phantom only varied less than 2% in the dielectric constant with insignificant change in loss tangent.

6.2.4.2 Spherical Tumor Inclusion

Each inclusion was fixed to a plastic plug at the base of a mould, with a wooden stick of diameter 2mm, with varying lengths to simulate different depths of embedments, $h$, in the normal breast tissue. The length of $h$ is calculated from the surface of the phantom to the centre of the inclusion. Then the 80% oil-concentration emulsion would be

Figure 6.8: (a) Dielectric constant and (b) Loss tangent of 10% oil-concentration phantom over a span of 8 weeks.
poured into the mould to set to produce a solid breast model with tumor embedded within.

![Image of breast phantom with tumor]

Figure 6.9: Inclusions of 8mm diameter; metal and 10% oil-concentration tumor phantom.

### 6.2.5 Permittivity of Modeled Breast

The complex permittivity of the breast tissues modeled using a 2-pole Debye dispersion equation as found in Equation (4.3) [82] is plotted as follows. In the same figures, the measured data for the breast and tumor phantoms (see Figures 6.3 and 6.7) are also included for comparison.

It is shown in Figure 6.10a the dielectric constant of the fabricated breast phantom (made from 80% oil-concentration emulsion) is slightly higher (about a change of 1 unit) than that of the normal breast tissue property modeled with the Debye equation. This trend is similarly observed in Figure 6.10b where the loss tangent of the fabricated breast phantom is larger (approximately 0.5 unit) than the value computed from the
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 numerical model of the normal breast, although both loss tangents increase with frequency. This indicates that the fabricated phantom is more lossy than the “actual” breast (based on numerical model [82]), hence the attenuation of the radiated signal from the PPDW to the tumor inclusion would be larger in the experimental investigation which will be reported subsequently as compared to in clinical trials. On this basis, it is believed the actual depth of embedment to be detected may be larger, and for the minimum size of tumor to be detected to be smaller during clinical trials; as compared to the experimental results which will shown in the following section 6.4. Correspondingly, the fabricated tumor phantom also has an overall higher dielectric constant and loss tangent than the numerical model of the malignant tumor, as shown in Figure 6.11. The dielectric contrast between the tumor and breast phantom; and between the numerical model of malignant tumor and normal breast are similar, around 8 times.
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Figure 6.10: (a) Dielectric constant and (b) Loss tangent of 80% oil-concentration phantom and numerical model of the normal breast.
6.3 Prototype

The prototypes were fabricated with copper sheets of 3mm nominal thickness. The typical geometry of the prototype is given in Figure 6.12.

The prototypes have following features:

- Very low loss dielectric ECCSTOCK® Hik500F, k=10, [131] is used as the central dielectric guide. The ‘k’ factor represents the dielectric constant that is the very low loss dielectric ECCOSTOCK with dielectric constant 10. How the dimensions and

---

Figure 6.11: (a) Dielectric constant and (b) Loss tangent of 10% oil-concentration phantom and numerical model of the malignant tumor.
dielectric constant designed for this central dielectric guide come about have been discussed in chapter 5, section 5.2.2. As a consequence of these parameters, the operating frequency of the PPDW has to be 6GHz or lower for dominant $TE_{10}$ mode operation.

- SMA connector is used for coaxial excitation. The Teflon insulator of the SMA coaxial connector is flushed to the interface between top flanged copper plate (that the flange of the connector is fixed to) and the central dielectric, exposing only the inner conductor to be in good electrical contact with the bottom copper plate, through the central dielectric.

- A copper bar is placed at one end of the central dielectric guide, functioning as a short. The distance between the interface of the copper bar and central dielectric is approximately a quarter-wavelength from the inner conductor pin of the SMA coaxial connector to the flanged plates at the mid-frequency of 3GHz. Teflon screws and spacers are used at the sides of the flanged copper plates to maintain even spacing between them and to secure the central dielectric and the copper bar used for shorting in their positions.
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Figure 6.12: Geometry of the prototype PPDW.
Figure 6.13: Actual fabricated prototype. (a) side view. (b) oblique view. (c) with plates separated.
6.4 Experimental Results and Discussions

S parameters measured by the PPDW were recorded with Agilent PNA-L Network Analyzer N5230A connected to a 50Ω coaxial cable. The measurement system was set up as shown in Figure 6.14.

Figure 6.14: Experimental setup.
Figure 6.15: Markings on the PPDW prototype to align the aperture to the inclusion during measurements.

Throughout the experimental investigations, to measure the maximum backscattered signals from the inclusions, the PPDW was positioned on the phantom such that the centre axis $z_1z_2$ of the PPDW (see Figure 6.12) aligned to the centre of the spherical inclusion; which was achieved by marking the inclusion position with two intersecting lines on the phantom, and then aligned the two lines on the prototype PPDW to them (see Figure 6.15). The PPDW was hand-held and lightly pressed on the phantom surface during measurements, taking care to maintain the same pressure.

Separate phantoms were fabricated to simulate the different scenarios of the breast containing varying tumor dimensions and depth of embedment.
6.4.1 Different Taper for Central Dielectric

Return loss of the PPDW improves with a taper design of the central dielectric near the aperture of the probe. Three designs of the central dielectric shown in Figure 5.9 were fabricated (see Figure 6.16) and the performance of the PPDW with these designs was evaluated respectively. The experimental results for the return loss of PPDW in contact with the breast phantom are plotted in Figure 6.17.

![Fabricated centre dielectrics with varying degree of taper.](image)

Figure 6.16: Fabricated centre dielectrics with varying degree of taper.
In accordance with the theory and HFSS simulations in chapter 5, the measured $S_{11}$ for design 1 with $\theta=0^\circ$ (flat end) is poor; however with a taper design it is noticed that $S_{11}$ improves. And as the taper increases, $S_{11}$ becomes better in the higher frequencies; such as in the case of $\theta=45^\circ$ (design 2) to $\theta=63.4^\circ$ (design 3). It is observed, the design with $\theta=63.4^\circ$ gives an overall good performance of the return loss, a greater bandwidth of reflection coefficient lower than -3dB over the frequency of 2 to 5GHz, this design of taper of the central dielectric guide was used in the PPDW prototype throughout the experimental study of detecting scatterers embedded in breast phantoms. In addition, the
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$S_{11}$ measurements were recorded over a frequency range of 2 to 5GHz at 121 frequency points.

A comparison between the experimental results and the HFSS simulated results are shown in the following figure. As the dielectric properties of the fabricated phantom is frequency dependent as discussed and shown in Figure 6.3, HFSS simulations were performed with the phantom modeled to be having complex permittivity similar to that of the self-fabricated breast phantom using the measured data for 1 to 6GHz shown in Figure 6.3. Once again, it is observed the design of the central dielectric with taper $\theta=63.4^\circ$ gives a better performance in $S_{11}$ compared to $\theta=45^\circ$.

![Figure 6.18: Comparison in the simulated and experimental return loss performance.](image-url)
6.4.2 Identifying Presence of Tumor

It has been proposed in chapter 4, the presence of tumor is identified with clear resonance in the $\Delta S_{11}$; and the region where the tumor is located can be zoomed into by locating the position for maximum response of $\Delta S_{11}$. As a start of validation, $S_{11}$ measurements were conducted on three different positions of a breast phantom without any inclusions (referred to as Phantom 1). Three differences $\Delta S_{11}$ (between $S_{11}$ measurements obtained respectively at positions 1a and 1b; positions 1a and 1c; positions 1b and 1c) obtained are shown in Figure 6.19.

![Figure 6.19: Magnitude of $\Delta S_{11}$ for breast phantom without inclusions.](image)
It is observed there is no clear resonance $\Delta S_{11}$ response in the absence of a scatterer. As the self-fabricated breast phantom may not be entirely homogeneous, there would be small variations in $S_{11}$ measured, accounting for the small magnitude in the $\Delta S_{11}$ recorded in this case.

Subsequently, a breast phantom with a metal sphere of diameter 8mm, embedded at $h=1.5$cm from the surface, was used for study. $S_{11}$ measurements were performed on the phantom surface, at different angle displacement in the same plane, $\theta$, about the metal inclusion. Position 2 is right at top of the inclusion, whereas positions 2c and 2b are at $15^\circ$ to the left and right of the inclusion respectively, and position 2a is at $28^\circ$ off the inclusion, next to position 2b, as illustrated in Figure 6.20.

Figure 6.20: Schematic diagram of PPDW measuring $S_{11}$ at positions in the same plane, displaced at angle $\theta$ from the centre of the inclusion.
\( \Delta S_{11} \) obtained from the differences between these \( S_{11} \) measured on Phantom 2, and the \( S_{11} \) obtained earlier (at position 1b coinciding to the centre of the mould of Phantom 1), are shown in Figure 6.21.

Figure 6.21: Magnitude of \( \Delta S_{11} \) for an 8mm diameter metal sphere, embedded at 1.5cm in the breast phantom.

It can be observed the resonance scattered amplitude from the tumor inclusion is very much larger, on the average about 25dB higher, than the \( \Delta S_{11(1a,1b)} \) superimposed from Figure 6.19 that represents scattering by tissue heterogeneity. Furthermore \( \Delta S_{11(2,1b)} \) gives the maximum response. This is in line with the proposed concept in chapter 4, that is the PPDW can scan about the region of concern for maximum resonating \( \Delta S_{11} \), in
order to zoom into the position underneath the PPDW where the tumor is located. Last but not least, theoretically the magnitudes of $\Delta S_{11(2b,1b)}$ and $\Delta S_{11(2c,1b)}$ should be identical; however it is to be noted the fabricated phantom may not be homogeneous, thus different dielectric properties lead to different signal path losses. With the size of the metal inclusion changed from 8mm to 12mm, similar observations can be concluded as shown in Figure 6.22. The lateral spacing of 4mm between each positions depicted in Figure 6.20 remains with depth $h$ changed to 2.0cm.

![Plot of $\Delta S_{11}$ for various angles and frequencies](image)

Figure 6.22: Magnitude of $\Delta S_{11}$ for a 12mm diameter metal sphere, embedded at 2.0cm in the breast phantom.
6.4.3 Estimating Size of Tumor

Additionally, Figure 6.23 shows the $\Delta S_{11}$ for an 8mm diameter metal sphere, embedded at different depths, $h$, in the breast phantom. $S_{11}$ was measured at the position where the metal inclusion was directly beneath the PPDW, with PPDW receiving the strongest backscatter. Similarly, the differences are made between the $S_{11}$ from these three individual phantoms with inclusion, and that obtained at position 1b of Phantom 1 respectively.

![Figure 6.23: Magnitude of $\Delta S_{11}$ for an 8mm diameter metal sphere, embedded at different depths in the breast phantom.](image-url)
It is seen the waveforms for these three $\Delta S_{11}$ are similar; resonating at same frequencies with only the amplitudes varying according to how deep the inclusion is embedded. The deeper the tumor is embedded, that is as $h$ increases, the smaller is the scattered signal from the inclusion. This validates tumors of the same size resonate at same frequencies, and is very much independent of the depth embedded in the healthy tissue. This phenomenon can also be seen for a 12mm diameter embedded at different depths as depicted in Figure 6.24.

![Graph showing the magnitude of $\Delta S_{11}$ for different depths](image)

**Figure 6.24:** Magnitude of $\Delta S_{11}$ for a 12mm diameter metal sphere, embedded at different depths in the breast phantom.
Hence it is possible to estimate the size of the tumor from calibrated resonant frequencies $f_r$ of the $\Delta S_{11}$. Also one can deduce from the plot that once the size of the tumor is known, the depth at which the tumor is located can be estimated from the calibrated amplitudes of this $\Delta S_{11}$ at these $f_r$ as the total signal path loss from the PPDW to the tumor and backscattered to the PPDW depends on the tumor’s location.

For depths greater than 4.0cm for an 8mm diameter metal inclusion, it can be seen from Figure 6.25 that the magnitude of the scattering from the metal sphere is not very easily differentiable from that of tissue heterogeneity represented by $\Delta S_{11(1a,1b)}$. Hence the PPDW can detect a metal inclusion of around 8mm diameter to a depth of 4.0cm.

Figure 6.25: Magnitude of $\Delta S_{11}$ for an 8mm diameter metal sphere, embedded at increased depths in the breast phantom.
Following it is to be verified further the size of the tumor can be estimated from calibrated resonant frequencies $f_r$ of the $\Delta S_{11}$. Figure 6.26 shows graph of normalized $\Delta S_{11}$ for inclusions of different diameters, $d_0=8\text{mm}$, $12\text{mm}$, and $16\text{mm}$, respectively. The inclusions were embedded at $h=2\text{cm}$. $\Delta S_{11}$ are obtained between the $S_{11}$ from these individual phantoms and that obtained at position 1b of Phantom 1. These piecewise graphs are obtained by joining the resonating points (marked by ‘•’) of the original $\Delta S_{11}$.

![Figure 6.26: Normalised magnitude of $\Delta S_{11}$ for metal spheres of different diameters, embedded at a depth of 2cm in the breast phantom.](image)

Figure 6.26: Normalised magnitude of $\Delta S_{11}$ for metal spheres of different diameters, embedded at a depth of 2cm in the breast phantom.
The theoretical values of the first resonant frequency $f_r$ of a spherical perfect electrical conductor (PEC) of the diameter $d_0$ is found with [119]

$$f_r = \frac{c}{\pi d_0 \sqrt{\varepsilon_r} \left[ 1 + \frac{1}{8} \left( \frac{\varepsilon_r''}{\varepsilon_r'} \right)^2 \right]}$$

(6.1)

where $c$ is the speed of light in vacuum; and $\varepsilon_r'$, $\varepsilon_r''$ are the real and imaginary parts of the relative complex permittivity of the nondispersive host medium of the sphere. Hence it is possible to estimate the diameter of the tumor with the following expression

$$d_0 = \frac{c}{\pi f_r^* \sqrt{\varepsilon_r} \left[ 1 + \frac{1}{8} \left( \frac{\varepsilon_r''}{\varepsilon_r'} \right)^2 \right]}$$

(6.2)

where $f_r^*$ is the observed resonant frequency. At these observed $f_r^*$ with $\varepsilon_r = 6(1 + i0.31)$ found by taking the average measured values of the breast phantom over 1 to 6GHz (see Figure 6.3), the predicted diameter $d_0$ of the inclusion is tabulated in Table 6.1. From Table 6.1, one observes that the error in the estimated and actual diameter of tumor inclusion is less than around 10%.

Table 6.1: Comparison of diameter of tumor, $d_0$ (mm).

<table>
<thead>
<tr>
<th>$f_r^*$ (GHz)</th>
<th>Calculated from Equation (6.2)</th>
<th>Actual value</th>
<th>Percentage error</th>
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<td>8</td>
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<td>10.8</td>
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<tr>
<td>2.35</td>
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</tbody>
</table>
This illustrates it is possible to estimate the size of the tumor using the calibrated resonant frequencies $f_r$, but the knowledge of the dielectric properties of the breast tissue over the operating band is critical for better accuracy of the estimation. In the event the measurements and thus $f_r$ are affected by noise, the calibrated amplitudes for different sizes and depth of embedments for the tumor may be used to eliminate the possible sizes that fits Equation (6.1) to obtain the best estimated size of tumor.

It was also studied, to what small diameters can the PPDW detect, for metal tumor inclusions embedded at a depth of $h=2.0\text{cm}$.

Figure 6.27: Magnitude of $\Delta S_{11}$ for metal spheres of varying diameter, embedded at 2.0cm in the breast phantom.
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As shown in Figure 6.27, the smaller the size of the tumor, the smaller is the backscattered magnitude. At this depth of 2.0cm, the magnitude of scattering from the tumor of diameter 4mm is still distinct (on the average 16dB higher) from the scattering by tissue heterogeneity, $\Delta S_{11(1a,1b)}$. Hence the current PPDW is capable of detecting a metal inclusion of 4mm diameter at a 2.0cm depth. However it is predicted, for large depths, for example exceeding $h=4.0$ cm, this differential backscattered magnitude between tissue heterogeneity and very small tumor becomes marginal; which requires further optimization of the PPDW to detect tumors embedded deep in tissues.

6.4.4 Localization of Tumor

Utilizing calibrated amplitudes of $\Delta S_{11}$ to determine the depth of the tumor embedded in the tissue may be impacted by variations in the tissue properties between different patients as the attenuation due to these variations affects the accuracy. Making use of $S_{21}$ measurements overcomes this limitation as slight variation in tissue properties is insignificant with the use of triangulation technique.

In what follows, the investigation on how effective this proposed technique is presented. Although it has been highlighted one of the measuring probes is to be fixed as the transmitting probe at the position while another probe is used to make measurements at different positions of the tissue, receiving scattered signals in terms of $S_{21}$, this experimental setup deviated slightly in a manner the transmitting PPDW was not at the position receiving the maximum backscattered signal in terms of $S_{11}$ (centre axis $z_1z_2$ aligned with the centre of the inclusion) but radiating the inclusion at an
oblique angle. The setup is illustrated in Figure 6.28. The spherical metal inclusion of diameter 8mm was embedded at $h=4\text{cm}$ in the breast phantom. In the same plane as the transmitting PPDW, the receiving PPDW measured $S_{21}$ at 12 different positions labeled 1 to 12 in the figure, each position is 5mm away leading to angle $\theta$ ranging from $84^\circ$ to $94^\circ$.

![Figure 6.28: Setup of two PPDW for $S_{21}$ measurements at 12 positions.](image)

This setup was designed with the objective of keeping the volume of breast phantom fabricated minimum as reducing the volume of breast phantom leads to a reduction in the amount of carcinogenic formaldehyde to be used in the fabrication. As a consequence of this size of breast phantom, and the position of the transmitting PPDW, the angle of displacement from the direction of propagation of the incident field, $\theta$, is thus limited to a range of $84^\circ$ to $94^\circ$. This is due to a separation of approximately at
least 2cm between the end of the flanges of both PPDW is kept to minimize any influences of the flanges from different PPDW in close proximity on the readings.

![Graph](attachment:image.png)

Figure 6.29: Magnitude of $S_{21}$ obtained at positions at 2GHz and 3GHz respectively.

According to the Mie Theory, typically the scattered signal is weakest at approximately $90^\circ$ from the direction of propagation of the incident field for frequencies less than or equal to 3GHz. It is shown in Figure 6.29, the minimum magnitude of $S_{21}$ occurs at position 8 and position 9 for 2GHz and 3GHz respectively; corresponding to $\theta$ with values of approximately 91.4° and 92.1°. The values not corresponding exactly to theoretical 90° may be attributed to the inhomogeneity of the breast phantom, and the associated different dispersive nature of the phantom at
different regions. Nevertheless the results suggest with the use of another PPDW for $S_{21}$ measurements, knowing the position(s) where minimum occur(s), it is possible to use triangulation technique to estimate more accurately the depth of the tumor embedded.

### 6.4.5 Dielectric Inclusions

To study a more realistic and challenging problem of detecting tumor having complex permittivity of a few times larger than the surrounding normal tissue, the PPDW was used to make measurements on breast phantoms with solid spherical dielectric inclusions that mimic malignant tumors, made from 10% oil-concentration phantom. It is shown in Figure 6.30 the $\Delta S_{11}$ for dielectric inclusion of diameter 8mm, embedded at different depths $h=1.5cm, 2.0cm$, in the breast phantom.

Although it has been mentioned tumors of the same size resonate at the same frequency(ies) which is very much independent of the depth of embedment; it is observed the waveforms of the $\Delta S_{11}$ are not exactly identical, with the resonating frequencies shifted. As these solid dielectrics spheres are of small dimensions, it is challenging to duplicate them identically in terms of diameter and surface smoothness even though caution has been employed to extract them out of the alike moulds. This may attribute to the discrepancy in the waveforms. Nevertheless the conformance to the amplitude of the backscattered signals decreases as the path increases is still observed.
Figure 6.30: Magnitude of $\Delta S_{11}$ for dielectric spheres, embedded at different depths in the breast phantom.

Over and above, it can be seen the backscattered magnitude from the dielectric inclusion embedded at a reasonable depth of 2.0cm, is still very much differentiable from $\Delta S_{11}(1a,1b)$, the scattering due to phantom inhomogeneity in the absence of a scatterer. Correspondingly, larger backscattered signal is also picked up from larger dielectric scatterer, as can be observed from the magnitude of $\Delta S_{11}$ for 12mm diameter dielectric inclusion is larger than that of the 8mm diameter, both embedded at the same depth.

A study on the influence of clutter items on the detection of tumor was also conducted. The respective setups of the tumor and the clutter items are as depicted in
Figure 6.31. Dielectric tumor of 10% oil-concentration, having 8mm diameter and spherical clutter items of the same dimension made from 70% oil-concentration phantom material (simulating a dielectric contrast of 2 times between the clutter and embedding phantom) are used; with all inclusions are embedded at a depth of $h=1.5\text{cm}$.

Figure 6.31: Top view of one type of configuration of the tumor and clutter items, embedded in the same breast phantom.

In the first setup, named as Setup 1, only the tumor and clutters ‘1’ and ‘2’ are present. In Setup 2, only the tumor and clutters ‘1’, ‘2’ and ‘3’ are present; while in Setup 3, only the tumor and clutters ‘1’ to ‘4’ are present. All the clutters and tumor as depicted in Figure 6.31 are present in Setup 4.

It is observed in Figure 6.32, for all setups, the backscattered signal from the 10% oil-concentration tumor measured at position A (see Figure 6.31) is the greatest over the
swept frequencies. This further validates the previous concept that the PPDW can scan
about the region of concern for maximum response in $\Delta S_{11}$ to locate the position
underneath the PPDW where the tumor is embedded. As compared to the $\Delta S_{11}$ for the
case where only the tumor but no clutters are present, there exist slight shift in resonant
frequencies and difference in amplitude in $\Delta S_{11}$ at position A for the setups. The shift in
resonating frequencies may be attributed to the slight difference in sizes of the 10% oil-
concentration tumor used in all the 5 moulds fabricated (Setups ‘1’ to ‘4’ and another
mould with only the tumor present) and the slight variation in dielectric properties in the
separately fabricated phantoms. These complicate the attempt to analyze the effect of
interferences contributed from the clutters with the backscattering from the tumor.

![Graph with frequency and $\Delta S_{11}$ values]

(a)
CHAPTER 6. EXPERIMENTAL STUDIES ON FLANGED PARALLEL-PLATE WAVEGUIDE PROBE

(b)

(c)
Figure 6.32: Magnitude of $\Delta S_{11}$ for an 8mm dielectric sphere of 10% oil-concentration in the presence of clutter items. (a) Setup 1. (b) Setup 2. (c) Setup 3. (d) Setup 4.

For dielectric inclusion having dielectric contrast approximately 4 times to the embedding breast phantom, as investigated with 30% oil-concentration emulsion according to dielectric measurements in Figure 6.2, it is observed in Figure 6.33 the differential margin for identifying the presence of a tumor becomes notably smaller as predicted. With a 4 times difference in dielectric constant between the tumor and breast phantom, the PPDW can still detect at a depth of $h=1.5\text{cm}$. 
Figure 6.33: Magnitude of $\Delta S_{11}$ for dielectric spheres of different dielectric contrast to the embedding breast phantom, embedded at a depth of 1.5cm.
6.5 Conclusion

The material as a tissue mimicking phantom to be used in the experimental investigation of the PPDW for microwave detection of tumor has been described with procedures for the fabrication provided. This gelatine-based material can be used to simulate different types of biological tissues, for example malignant and normal breast tissues, as the dielectric properties of this material can be altered by varying the concentration of oil. This phantom material is cost-effective as it is made from cheap and easily accessible ingredients. It also has a stable performance in terms of phase (solid state) and dielectric properties. Last but not least how the inclusions are to be embedded in the solid breast phantoms are also accounted.

In addition, it has been investigated and validated the effectiveness of the parallel-plate dielectric waveguide probe (PPDW) and the proposed methodology using S parameter measurements for tumor detection and localization. Prototypes of the PPDW were fabricated and tested on self-fabricated solid breast phantoms mimicking a real breast, with conductive inclusions representing tumors embedded. Separate phantoms were fabricated to simulate the different scenarios of the breast containing varying tumor dimensions and depth of embedment. Along with the ease of fabrication, this manner introduces variability and uncertainty in the dielectric properties of the breast phantoms which provides a realistic platform to model the inhomogeneity nature of an actual breast and the varying breast properties from patients to patients.

Experimental studies with strong scattering metal inclusions correspond closely to the theoretical predictions while the more challenging investigation on the performance of the PPDW on lossy dielectric inclusions further assert tumors of significant dielectric
contrast to the healthy host tissue can be detected. It should be noted the moulds that were used in these experiments are each approximately 2 litres or more. Consequently, the dielectric properties of the self-fabricated phantom may differ from regions to regions, as inhomogeneity could arise in preparing this large amount of emulsion. Henceforth, certain discrepancies with the theoretical predictions are observed from the experimental results.

Currently, further optimization and investigation of the hardware, such as further reducing the size of the PPDW to be used on convex surfaces, the use of immersion liquid, and overcoming the difficulties previously encountered in the fabrication of the phantoms, are still in progress to demonstrate the full capabilities of the PPDW to identify the presence of tumor, and estimate the size and location of the tumor.
Chapter 7

Conclusion and Recommendations

7.1 Conclusion

Cancer is a major public health burden in every country, with breast cancer as one of the top killers. The survival rates for cancer deaths can be improved if effective early screening tests are available as cancer can be treated to increase survival, if detected early. With the gold standards of cancer detection having shortcomings and limitations, many efforts have been dedicated to develop new techniques. The objective of this thesis is to develop and evaluate with experimental verifications, a probe and technique that can efficiently detect and localize small tumors at reasonable depth in the human body. The research activities have been focused on breast cancer detection.

It has been found, in general malignant tumors have significantly higher complex permittivity than that of healthy tissues at microwave frequencies. This dielectric contrast forms the basis for the noninvasive method of detection proposed in this thesis. A parallel-plate dielectric waveguide probe (PPDW) is proposed as the device to identify the presence, size and location of a tumor. The probe is used to illuminate the
CHAPTER 7. CONCLUSION AND RECOMMENDATIONS

tissue or organ and contact measurements of the scattering parameters (S parameters) at
different positions over a range of microwave frequencies will be recorded together with
a vector network analyzer. $^{N}C_2$ pairs of difference in S parameters, $\Delta S$, can thus be
obtained. Tumor signatures can be found in terms of this difference.

Through the study of the interaction of the electromagnetic fields emanated from the
probe and the tissue, analytical solutions for the transmitting and receiving
characteristics at the aperture of a theoretical probe, parallel-plate waveguide probe
(PPWP) has been delivered in chapter 3. The fields in the multilayer tissue are solved in
terms of cartesian wave functions by imposing appropriate boundary conditions, and the
application of mode matching techniques with Fourier transform. Following based on
Mie scattering by a dielectric sphere, expressions for the S parameters in the presence of
a tumor have been formulated with the derived fields in the tissue. To obtain closed
form integral solutions, the analysis has been carried out with the assumptions of
dominant TEM mode wave propagation in the PPWP and the parallel plates and flanges
are unbounded in the directions perpendicular to the propagation.

With specific reference to breast cancer detection, a technique to identify and
localize a tumor with the use of S parameters measured by surveying the breast has been
proposed in chapter 4. This involves obtaining $^{N}C_2$ pairs of difference in $S_{11}, \Delta S_{11}$, from
a set of $N$ measurements. As a tumor having dielectric properties in high contrast to the
healthy tissue alters the $S_{11}$, a significant resonance in the $\Delta S_{11}$ over the swept
frequencies implies that a tumor is present at one of the positions where the pair of the
$S_{11}$ measurements is made. By eliminating the regions where there is no tumor present,
the final region where the tumor is located can be zoomed into. The localization of the
CHAPTER 7. CONCLUSION AND RECOMMENDATIONS

tumor is achieved by transmitting the probe at the position yielding the maximum response in $\Delta S_{11}$, and measures $S_{21}$ around the breast. The position of the transmitting probe and that where minimum $S_{21}$ occur(s), form reference vertices of a triangle such that the tumor can be localized through triangular relations. Numerical studies have been conducted to illustrate the use of this technique. The results show that the PPWP is able to differentiate a malignant tumor having large complex permittivity value, from a clutter item (tissue inhomogeneity) that has lower dielectric contrast from the normal breast tissue, due to the differential resonant scattered amplitudes. Furthermore, studies also demonstrate the potential to estimate the size of the tumor using the calibrated resonant frequencies $f_r$; and the use of $S_{21}$ to effectively locate the tumor.

In chapter 5, a prototype design of the probe suitable for clinical environment has been presented. In this design of a parallel-plate dielectric waveguide probe (PPDW), a practical size of the probe is achieved through confining the radiation from the waveguide probe with a dielectric slab, sandwiched by the flanged parallel plates, that functions as a central guide to propagation of transverse electric waves. Good isolation from the external interference in the environment is resulted. The design process to optimize the return loss performance of the PPDW, and confine the irradiated region of the imaged breast for enhanced accuracy has also been discussed.

The type of material selected as tissue mimicking phantoms to study the interaction of electromagnetic waves from the PPDW with the biological tissues has been described in chapter 6, along with the fabrication procedures. This oil-in-gelatine dispersion emulsion can be fabricated from inexpensive and easily accessible ingredients; and by varying the volume of oil used in the emulsion different dielectric
properties of the phantoms can be constructed to simulate the different types of biological tissues. It has also been studied the mechanical (ability to remain as solid) and dielectric properties of this phantom material; the material is stable for at least 2 week from preparation.

Finally, preliminary experimental investigations to verify the concepts of proposed detection technique have been performed. The experimental system, including the fabrication of the prototype PPDW and the measurement setup, has been described. As opposed to majority of the published experimental investigations of microwave detection of breast tumor where the tumor phantom is included in very low loss liquid breast phantom; solid phantoms resembling better to a real breast in terms of permittivity dispersion and the phase (solid state), are used in these verifications. Results are presented for phantoms containing strongly scattering metal to the more challenging detection of dielectric bodies. These results correspond to those predicted by theory and simulations and thus demonstrate the potential of the proposed PPDW to provide information such as tumor existence, and size and location of the tumor.

As been reviewed, some benign lesions may also have high water content and may respond in a similar manner to microwave scattering as malignant tumors [57, 58], the focus of this thesis is on detecting malignant tumors. In the event further confirmation that dielectric properties of the normal, benign and malignant lesions may not differ significantly becomes available, the proposed microwave detection for malignant tumor can be modified and optimized to identify the marginal differences between scattering from the tumor and from its surrounding heterogeneous medium.
CHAPTER 7. CONCLUSION AND RECOMMENDATIONS

7.2 Recommendations for Future Work

Future research involves extended studies with theoretical and experimental work. A number of possible extensions are proposed.

(i) Improved recipe of the phantom has to be further developed. As compared to the reported data of the dielectric properties of normal tissues, the dissipation factor of the fabricated phantom used in the experimental studies in this thesis is larger. Signals are thus attenuated. If the phantom can be fabricated with lower dissipation factor, resembling that of the actual normal tissue, it can be studied to what smaller dimensions and deeper embedment of the tumor can be detected.

Another objective to develop an improved recipe of the phantom is to reduce or replace one carcinogenic ingredient used in the current recipe, formaldehyde. With a safer non-toxic formulation, larger samples of the phantom can be fabricated and can include clutter items to better simulate a heterogeneous biological organ and provide more realistic experimental data on distinguishing a tumor from clutter. Also the surveyed area for $S_{21}$ measurements can hence be expanded and conclude to a greater degree, the feasibility of using minimum $S_{21}$ to locate the tumor.

The phantom considered in this thesis does not include the skin layer. It is definitely worthy to research on the types of phantom that can easily simulate this skin layer of a few millimeters thickness. In what manner the inclusion of a thin conductive skin layer impacts the effectiveness of the proposed PPDW can be thus be investigated.
CHAPTER 7. CONCLUSION AND RECOMMENDATIONS

(ii) It would also be beneficial to derive a recipe for liquid phantom closely mimic the dielectric properties in terms of the relative permittivity; rate of frequency dependent dispersion; and dissipation factor, of an actual biological tissue. The advantage of a liquid formulation, on the basis the liquid phantom formulated is electrically and mechanically stable over the long period of investigations, is to facilitate the experimental setup to investigate extensive different scenarios of the tumor embedment as investigations on the different tumor shapes, sizes, dielectric contrast, and depth of embedment can be conducted with the same embedding liquid phantom simply by taking out the inclusion and replace with another.

(iii) To be better suited for clinical environment, the existing dimensions of the PPDW have to be further reduced. Currently in the experimental investigations, the width and flange size of the PPDW are still not at their optimum. It can be studied to what extent the PPDW can be further miniaturized yet maintaining similar good confinement of the radiation and isolation to external EMI sources; the further optimization of the design of the wedge angle and shape; and the central dielectric guide such as the dielectric constant and associated physical dimensions, to take on for reasonable depth of penetration, beamwidth, etc., may also be considered for this purpose.

Additionally, further development of the PPDW into an array, or with spaced central dielectric guides individually excited; to achieve beamforming for increased accuracy in detection, can also be studied.
(iv) It would be interesting to study the scattering characteristics of dielectric inclusions of irregular shapes and varying surface roughness, according to the exact morphology of an actual tumor, under the interaction of electromagnetic fields radiated from the probe; to investigate the effectiveness of the device and method is under these realistic conditions.
Author’s Publications

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Conference Papers


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[133] Agilent Technical Overview, 85070E Dielectric Probe Kit.
Appendix A

Two Dimensional

UTD Diffraction Coefficients

The diffraction coefficient used in Equations (3.50) and (3.51) is given by

\[ D_n(L', L'^o, L'^n, \Phi, \Phi', n) = D_1 + D_2 + R_n(D_3 + D_4) \]  

(A.1)

The terms are defined as below.

\( R_h \) is the hard reflection coefficient of the wedge surface at the edge;

for conducting wedge, \( R_h = 1 \).

\( L', L'^o, L'^n \) are distance parameters; \( L' = L'^o = L'^n = s \), \( s \) large for plane wave incidence.

\( s \) is the distance from the diffraction point to the field point.

\( n \) is defined by \( n = \frac{2\pi - \gamma}{\pi} \), \( \gamma \) is the interior wedge angle; if \( \gamma = \frac{\pi}{2} \), \( n = 1.5 \).

\( \Phi \) is the diffraction angle with respect to the o-face; for 90° wedge, \( \Phi = \pi + \theta \).

\( \Phi' \) is the incident angle with respect to the o-face, or 90° wedge, \( \Phi' = 0 \).
Components of the diffraction coefficient are given by

\[
D_1 = \frac{-e^{i\pi/4}}{2n\sqrt{2\pi k}} \cot \left( \frac{\pi + (\Phi - \Phi')}{2n} \right) F \left[ kL^\prime a^+ (\Phi - \Phi') \right] \tag{A.2}
\]

\[
D_2 = \frac{-e^{i\pi/4}}{2n\sqrt{2\pi k}} \cot \left( \frac{\pi - (\Phi - \Phi')}{2n} \right) F \left[ kL^\prime a^- (\Phi - \Phi') \right] \tag{A.3}
\]

\[
D_3 = \frac{-e^{i\pi/4}}{2n\sqrt{2\pi k}} \cot \left( \frac{\pi + (\Phi + \Phi')}{2n} \right) F \left[ kL^\prime a^+ (\Phi + \Phi') \right] \tag{A.4}
\]

\[
D_4 = \frac{-e^{i\pi/4}}{2n\sqrt{2\pi k}} \cot \left( \frac{\pi - (\Phi + \Phi')}{2n} \right) F \left[ kL^\prime a^- (\Phi + \Phi') \right] \tag{A.5}
\]

The transition function can be expressed as

\[
F(x) = -2i\sqrt{x}e^{-ix} \int_{\sqrt{x}}^{\infty} e^{iu^2} du \quad \text{for} \quad x > 0 \tag{A.6}
\]

The arguments within the transition function \(a^\pm\) are defined as

\[
a^\pm(\beta^\pm) = 2\cos^2 \left( \frac{2n\pi N^\pm - \beta^\pm}{2} \right) \tag{A.7}
\]

with \(\beta^\pm = \Phi \pm \Phi'\) \tag{A.8}

and integers \(N^\pm\) most nearly satisfy

\[
2n\pi N^+ - (\Phi \pm \Phi') = \pi \tag{A.9}
\]

\[
2n\pi N^- - (\Phi \pm \Phi') = -\pi \tag{A.10}
\]