High Power Multi-Output Piezoelectric Transformers

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

19, Dec, 2006
Date

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Piezoelectric transformers have many excellent advantages compared with the traditional electromagnetic transformers. However, the poor power management ability of piezoelectric transformer limited its development greatly. Furthermore, in order to miniaturize the power supply further, it is very necessary to improve the power density of the piezoelectric transformer and make one transformer can supply several different outputs for the system.

The main results of this study are summarized as follows. High power piezoelectric transformers with dual and triple outputs were developed and investigated. They operate at the thickness shear vibration mode and are able to provide maximum total output power of more than 160 W with a temperature rise less than 20 °C. It is the highest output power of PTs in the world up to now. The maximum efficiency of piezoelectric transformer with dual outputs is 98%. The voltage gains of the two outputs are 0.76 and 0.51 at the frequency of 278 kHz, respectively. The maximum efficiency of piezoelectric transformer with triple outputs is 95.7%. The voltage gains of the three outputs are 0.75, 0.84 and 0.39 at the frequency of 278 kHz, respectively. Compared with most of the other low voltage piezoelectric transformers, these transformers are competitive in practical applications because of the high power, multiple outputs, simple structure and flexibility in design.
SUMMARY

In order to explore the possibility of miniaturizing the size of piezoelectric transformer further, a high power-density dual-output piezoelectric transformer operating at the thickness shear vibration mode was proposed and experimentally investigated. This transformer consists of a lead zirconate titanate (PZT) ceramic plate with a size of $30 \times 5 \times 1 \text{ mm}^3$. Lead wires are pressed on the electrodes by proper elastic force. At a temperature rise less than 20 °C, the piezoelectric transformer has a maximum output power density of 52.7 W/cm$^3$ with an efficiency of 88%. The maximum efficiency of this piezoelectric transformer is 95%. Voltage gains larger than and smaller than one are obtained by the transformer. By using a parallel combination of the two outputs of the proposed transformer, an output power density of 57.3 W/cm$^3$ can be achieved at the temperature rise less than 20 °C. This is the highest output power density of PTs in the world up to now.

As far as the design of a piezoelectric transformer is concerned, a physical model is essential by which the characteristics of the transformer can be predicated by the material and size of the transformer. In this study, a theoretical model was developed for the dual-output piezoelectric transformer operating at the thickness shear vibration mode. The equivalent circuit parameters of the piezoelectric transformer were derived, and the impedance characteristics, equivalent inductance, capacitance ratio, voltage gain and efficiency of the piezoelectric transformer were calculated. Theoretical results were verified by experimental data. Furthermore, the effect of the transformer size on the voltage gain, efficiency, output power and power density, and the effect of the load of one output on the voltage gain of another output were analyzed. Some useful guidelines were achieved by these analyses. This study will help designers to gain better physical insight and to design the driving circuit via simulation.
SUMMARY

The vibration distribution in output section of a piezoelectric transformer operating at the thickness shear vibration mode is investigated experimentally and theoretically. It is experimentally found that the vibration of output section has a spatial gradient and attenuates along the direction of vibration propagation. A theoretical model is developed to estimate the vibration attenuation in output section, which provides the guidelines of optimizing the transformer and explains some important phenomena such as the non-uniform temperature distribution in piezoelectric transformers. The larger the load resistance, the larger the vibration gradient is. At the resonance point, the vibration gradient decreases with increasing the width of input or output section, and changes little with the length and thickness of the transformer when the load resistance matches with the piezoelectric transformer. The vibration gradient increases with increasing the length or decreasing the thickness when the load resistance is constant, in which the load does not match with the transformer. Effect of load resistance and dimensions on the vibration displacement of input section is also studied by this model. At the resonance point, the vibration displacement of input section increases with decreasing the length or the width of output section, or increasing the thickness of the transformer when the load resistance is constant.

Based on the new technique developed from the multi-output piezoelectric transformer, a novel tunable low-frequency piezoelectric phase shifter was proposed and developed. The piezoelectric phase shifter is made of a piezoelectric ceramic plate operating at the thickness shear vibration mode. The ceramic plate consists of input, output and control sides. The control side is connected to a tunable inductor or capacitor, which can tune the phase shift between the output and input voltages.
smoothly. The phase shift can be increased by decreasing the inductance or capacitance at the control part. Tuning a DC bias voltage applied to the input part may also change the phase shift. A maximum phase shift of 128 deg and minimum phase shift of -10 deg are achieved by the prototype of the phase shifter. The phase shifter has a wide phase shift range, good ability to manage high power and relatively high energy transmission efficiency.

A multi-point connection and supporting structure has been developed for the high power piezoelectric devices operating at the thickness shear mode (PTs, Phase Shifters, etc.). This structure can not only make the supporting and lead wire connection of the devices stable, but also decrease the temperature rise effectively. It is very useful to improve the power management ability of the thickness shear vibration mode devices.
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1.1 Motivation

Conventionally, stepping up or down AC voltages has been achieved by using electromagnetic transformers. In these transformers, primary and secondary coils are electromagnetically coupled through a magnetic core. In the power supply application, large electromagnetic transformers are efficient, but it is difficult to miniaturize them with high efficiency. Losses, such as conductor skin effect loss, thin wire loss, and hysteresis in the core material increase rapidly when the size of the magnetic transformer is reduced. Miniaturization of power supplies has been an important issue during the last decade. The transformers and inductors in converters are usually tall and bulky compared to transistors and ICs. An alternative to magnetic transformers for transforming input voltages to a larger or smaller output voltage was initially proposed in 1956 by Rosen et al [1]. This alternative is the piezoelectric transformer (PT), which converts electrical energy into ultrasonic vibration by converse piezoelectric effect at input, and the ultrasonic vibration into electrical energy with a different voltage by piezoelectric effect at output. Continuous efforts devoted to these subjects have been carried out by many researchers [2-8]. Compared with a conventional low-profile transformer, a piezoelectric transformer has several inherent advantages such as high power density, high efficiency, low profile, small size, light weight, no windings, electromagnetic noise-free operation and non-inflammability. So,
piezoelectric transformers are very competitive in the power supplies of high-end IT and electronic products.

In about fifty years’ development, many PTs have been proposed [9-31]. However, up to now, the PTs have been mainly used to generate high voltage for the cold cathode fluorescent lamps (CCFL) in notebook computers. Although PTs for CCFL inverters requiring a power in the range of 5-10 W have been successfully achieved, the application of PTs is still limited by their low power level. In order to explore the possibility of replacing the traditional voltage transforming methods such as electromagnetic transformer or resonance tank by a RLC circuit, it is very necessary to enhance the current power management ability of PTs. Recently new structure for high power piezoelectric transformers have been proposed and developed by Face International [31]. However, the typical maximum output power of these piezoelectric transformers is less than 100 Watts, and their multilayer structures are complicated, raising production cost and decreasing reliability. Also, there is another significant research issue. The maximum output power density of the present PTs is typically less than 30 W/cm$^3$ [19-23], which is just a little larger than that of the high frequency ferrite transformers. It can not fulfill the requirements of recent miniaturized power supply systems. Furthermore in order to miniaturize the power supplies further, the transformers with multiple outputs are needed to supply the electric power to different parts in a system. However, there has been no systematic work on the problem before our study. Therefore, it is valuable and necessary to develop large power/power density multi-output PTs.
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For the mass application of PTs, decreasing the cost and improving the reliability are critical. For decreasing the cost and increasing the reliability, it is necessary to use a simple and reliable structure in the PTs.

For the practical applications of the large power/power density multi-output PTs, a theoretical model for the design and analysis is also necessary. This is because there are different requirements on the output power, size, and matching load of the PTs in practical applications and a deeper physical insight is needed to optimize the PTs.

1.2 Objectives

The need to develop the high power/power density multi-output PTs has motivated the following studies:
1) Study the material properties and vibration modes of the piezoelectric ceramic components to achieve the high power/power density multi-output PTs. Clarify the characteristics of the PTs.
2) Develop and verify a theoretical model for the proposed PT and explore the design guidelines to improve the output power and efficiency of the PT.
3) Study the vibration distribution of the proposed PTs for optimizing the mechanical structure.
4) Explore the application of the high power multi-output PTs.

1.3 Major contributions
The major contributions achieved in this study are summarized as follows:

1. High power piezoelectric transformers with dual and triple outputs were developed and investigated. They operate at the thickness shear vibration mode and are able to provide maximum total output power of more than 160 W with a temperature rise less than 20 °C. It is the highest output power of PT in the world up to now. The maximum efficiency of piezoelectric transformer with dual outputs is 98%. The voltage gains of the two outputs are 0.76 and 0.51 at the frequency of 278 kHz, respectively. The maximum efficiency of the piezoelectric transformer with triple outputs is 95.7%. The voltage gains of the three outputs are 0.75, 0.84 and 0.39 at the frequency of 278 kHz, respectively. Compared with most of the other low voltage piezoelectric transformers, these transformers are competitive in practical applications because of the high power, multiple outputs, simple structure and flexibility in design.

2. A high power-density dual-output piezoelectric transformer operating at the thickness shear vibration mode was proposed and experimentally investigated. This transformer consists of a lead zirconate titanate (PZT) ceramic plate with a size of $30 \times 5 \times 1 \text{ mm}^3$. Lead wires are pressed on the electrodes by proper elastic force. At a temperature rise less than 20 °C, the piezoelectric transformer has a maximum output power density of 52.7 W/cm$^3$ with an efficiency of 88%. The maximum efficiency of this piezoelectric transformer is 95%. Voltage gains larger than and smaller than one are obtained in the transformer. By using a parallel combination of the two outputs of the transformer, an output power density of 57.3 W/cm$^3$ can be achieved at the temperature rise less than 20 °C. This is the highest output power density of PT in the world up to now.
3. A theoretical model was developed for the dual-output piezoelectric transformer operating at the thickness shear vibration mode. The equivalent circuit parameters of the piezoelectric transformer were derived, and the impedance characteristics, equivalent inductance, capacitance ratio, voltage gain and efficiency of the piezoelectric transformer were calculated. The theoretical results were verified by experimental data. Furthermore, the effect of the transformer size on the voltage gain, efficiency, output power and power density, and the effect of the load of one output on the voltage gain of another output were analyzed. Some useful guidelines were achieved by these analyses. This study will help designers to gain better physical insight and to develop the driving circuit via simulation.

4. The vibration distribution in output section of a piezoelectric transformer operating at the thickness shear vibration mode is investigated experimentally and theoretically. It is experimentally found that the vibration of output section has a spatial gradient and attenuates along the direction of vibration propagation. A theoretical model is developed to estimate the vibration attenuation in output section, which provides the guidelines of optimizing the transformer and explains some important phenomena such as the non-uniform temperature distribution in piezoelectric transformers. The larger the load resistance, the larger the vibration gradient is. At the resonance point, the vibration gradient decreases with increasing the width of input or output section, and changes little with the length and thickness of the transformer when the load resistance matches with the piezoelectric transformer. The vibration gradient increases with increasing the length or decreasing the thickness when the load resistance is constant. Effect of load resistance and dimensions on the vibration displacement
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of input section is also studied by this model. At the resonance point, the vibration displacement of input section increases with decreasing the length or the width of output section, or increasing the thickness of the transformer when the load resistance is constant.

5. A low-frequency piezoelectric phase shifter based on a new technique was developed and investigated. The technique is derived from the multi-output piezoelectric transformer. The piezoelectric phase shifter is made of a piezoelectric ceramic plate operating at the thickness shear vibration mode. The ceramic plate consists of input, output and control sides. The control side is connected to a tunable inductor or capacitor, which can tune the phase shift between the output and input voltages smoothly. The phase shift can be increased by decreasing the inductance or capacitance connected to the control side. Tuning a DC bias voltage applied to the input side may also change the phase shift. A maximum phase shift of 128 deg and minimum phase shift of -10 deg are achieved by the prototype of phase shifter. The phase shifter has wide phase shift range, good ability to manage high power and relatively high energy transmission efficiency.

6. A multi-point connection and supporting structure have been developed for the high power thickness shear vibration mode devices (PTs, Phase Shifters, etc.). This structure can not only make the supporting and lead wire connection of the phase shifter stable, but also decrease the temperature rise effectively. It is very useful to improve the power management capability of the thickness shear vibration mode devices.

7. Seven papers related to this project have been published in or submitted to international journals; three conference papers related to this project have been
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published in the proceedings; total number of published and submitted papers is thirteen (see author's publications at Page. 134).

1.4 Organization of the Thesis

This thesis includes seven chapters.

In chapter 1, the motivation, objectives, main contributions, and organization of the thesis are presented.

Chapter 2 presents the operating principle, recent development and applications of the piezoelectric transformers. Also the basic theory used in the modeling of the thickness shear vibration mode piezoelectric transformers is introduced in this chapter.

In Chapter 3, the high power piezoelectric transformers operating at the thickness shear vibration mode with dual and triple outputs are proposed and developed. At a temperature rise less than 20 °C, a maximum output power of 169.8 W is achieved and it is the highest output power in the world up to now. Following this, the high power density dual-output piezoelectric transformer operating at the thickness shear vibration mode is proposed and developed. It has the highest maximum power density of 52.7 W/cm³ in the world with the temperature rise less than 20 °C and very simple structure.

In Chapter 4, a theoretical model is developed for the dual-output piezoelectric transformer operating at the thickness shear vibration mode. The equivalent circuit parameters of the piezoelectric transformer are derived, and the impedance
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characteristics, equivalent inductance, capacitance ratio, voltage gain and efficiency of the piezoelectric transformer are calculated. The theoretical results are verified by experimental data. Furthermore, the effect of the transformer size on the voltage gain, efficiency, output power and power density, and the effect of the load of one output on the voltage gain of another output are analyzed. Some useful guidelines are achieved by these analyses.

In Chapter 5, the vibration distribution in output section of a piezoelectric transformer operating at the thickness shear vibration mode is investigated experimentally and theoretically. It is found that the vibration of output section has a spatial gradient and attenuates along the direction of vibration propagation. The larger the load resistance, the larger the vibration gradient is. A theoretical model is developed to estimate the vibration attenuation in output section, which provides the guidelines of optimizing the transformer and explains some important phenomena in piezoelectric transformers, such as the non-uniform temperature distribution.

In Chapter 6, a low-frequency piezoelectric phase shifter based on a new technique is developed and investigated. The technique is derived from the multioutput piezoelectric transformer. A maximum phase shift of 128 deg and minimum phase shift of -10 deg are achieved by this method.

Finally, the conclusion and recommendations for future work are presented in Chapter 7.
CHAPTER 2 BACKGROUND AND BASIC THEORY

The operating principle, recent development and applications of the piezoelectric transformers are presented in the first section. The second section gives the basic theory used in the modeling of the thickness shear vibration mode piezoelectric transformers.

2.1 Piezoelectric Transformers

One of the bulkiest components in information processing equipment (such as laptop computers) is the power supply, especially the electromagnetic transformer used in the power supply. Losses such as skin effect, thin wire loss and core loss of the electromagnetic transformer increase rapidly as the size of the transformer is reduced. Therefore, it is difficult to realize the miniature low-profile electromagnetic transformers with high efficiency. On the contrary, high efficiency, small size, and absence of electromagnetic noise are some of the attractive features of piezoelectric transformers (PTs), making them more suitable for miniaturized power inverter components such as lighting up the cold cathode fluorescent lamp (CCFL) behind a color liquid crystal displays or generating high voltage for air cleaners [32].
2.1.1 Operating Principle and Structures of PTs

A PT combines the functions of an actuator and a sensor so that energy can be transformed from electrical form to electrical form via mechanical vibration. Figure 2-1 shows a fundamental type of PT, the longitudinal Rosen-type mode PT. When a periodic electric input is applied to the electrodes of left part, a mechanical deformation occurs due to the converse piezoelectric effect. The deformation is transmitted to the right side, inducing charges on its electrodes due to the direct piezoelectric effect. Those charges result in the output voltage. As a result, the input/output voltage is changed. The input side is usually called driver section, and the output side is named generator section. The voltage ratio for open-circuited output is:

\[
\frac{V_o}{V_{in}} = k_{31}k_{33}Q_m (L/t)
\]  

(2-1)

where \(L\), \(t\) are the electrode gap distances for the output and input sides, respectively. \(k_{31}\), \(k_{33}\) are the electromechanical coupling factors and \(Q_m\) is the mechanical quality factor. Note that increasing in \(L/t\) ratio, electromechanical coupling factors and/or mechanical quality factor is the key to the increase of voltage ratio. This transformer was utilized in the color TVs on trial in 1970s [32].

Figure 2-1 Construction of the Rosen-type PT.
2.1.2 Recent Development of PTs

As shown in Figure 2-1, the initial Rosen-type PT had a reliability problem, that is, easy mechanical breakdown at the center position due to the coincidence of the residual stress concentration (through the poling process) and the vibration nodal point (highest induced stress). Improvements over the years included the use of the

Figure 2-2 New PT designs improved from Rosen-type PT: (a) multilayer type, and (b) 3\textsuperscript{rd} mode type.
mechanically tough ceramic materials, of multilayer geometry that does not generate any poling direction mismatch as shown in Figure 2-2 (a) [33]; and the redesign of the electrode configuration and exciting a 3rd longitudinal vibration mode of the rectangular plate as shown in Figure 2-2 (b) [21].

In order to improve the transformer performance, various proposals have been made: improvement in piezoceramics with higher mechanical toughness and low loss; selection of a vibration mode and an electrode configuration to separate the nodal point from the maximum residual stress point; selection of a vibration mode with higher electromechanical coupling.

Figure 2-3  History of PTs in terms of power capability.

2.1.3 Applications of PTs

Basically, piezoelectric transformers are classified into two groups: resonance and non-resonance types. The non-resonance type is for step-down transformers used for
Chapter 2 Background and Basic Theory

precise measurements of high voltage such as 30 kV or high current on electric power transmission lines [34]. The resonance type is further divided into two groups: step-up types typically used as high voltage inverters for the LCD back-light, and step-down types used as AC-DC converters [35], as shown in Figure 2-3. Figure 2-4 shows a prototype 35 W, 115 VAC to 15 VDC adapter for a laptop computer application, developed by ICAT center of Pennsylvania State University. Notice 1/4 smaller size and weight in comparison with the conventional electromagnetic type adapter. Currently, leading makers in this area are reportedly focusing on the miniaturization and higher power of the transformers. PTs for power applications require lower output impedances, high power capabilities and high efficiency under step-down conditions.

Figure 2-4  A prototype AC to DC adapter for a laptop computer application.

2.2 Thickness Shear Vibration Mode
Chapter 2 Background and Basic Theory

2.2.1 Concept of the thickness shear vibration mode

Figure 2-5 illustrates the thickness shear vibration mode used in the proposed transformers. The vibration direction of the piezoelectric transformers is in parallel with its poling direction, and the applied electric field is perpendicular to its poling direction.

Figure 2-5 The thickness shear vibration mode.

2.2.2 Electromechanical Coupling Factor of the thickness shear vibration mode

The piezoelectric effect will not be activated until the material is polarized in a specified direction or several directions. The conversion between the electrical energy and the mechanical energy is measured by electromechanical coupling coefficient which is defined as:
Chapter 2 Background and Basic Theory

\[ k^2 = \frac{\text{mechanical energy converted from input electrical energy}}{\text{input electrical energy}} \], or

\[ k^2 = \frac{\text{electrical energy converted from input mechanical energy}}{\text{input mechanical energy}} \]  

Therefore, the value of the electromechanical coupling coefficient does not indicate the efficiency of the piezoelectric ceramics. The energy, which is not converted from input energy, is simply stored in the intrinsic capacitor and in the mechanical branch of the piezoelectric ceramics. The best illustration of this constant is described in [7]. Applying the appropriate piezoelectric and elastic coefficients for the conversion of mechanical energy (stress times strain) into electrical energy (displacement times electric field) gives the relation for thickness shear vibration mode.

\[ k_{15}^2 = \frac{d_{13}^2}{s_{44} \varepsilon_r^f} \]  

(2-3)

2.2.3 Equivalent circuit of a rectangular-shaped thickness shear vibration mode piezoelectric ceramic plate

2.2.3.1 Basic equations

Figure 2-6 shows a rectangular-shaped piezoelectric ceramic plate operating at thickness shear vibration mode. Its poling direction is \( z \) and the electrodes are on the surface and bottom of the ceramic plate. Because the electric excitation, \( E \), is applied to top and bottom electrodes of the piezoelectric ceramic plate in the \( x \) direction,

\[ D_1 \neq 0, D_2 = D_3 = 0. \]
Assumed there are only the shear force $F_1$ and $F_2$ at the two surface of $x = 0$ and $x = l$, the strains are

$$S_5 \neq 0, S_1 = S_2 = S_3 = S_4 = S_6 = 0.$$ 

Then the electromechanical equations can be expressed in the following equation

$$T_5 = C_{55} S_5 - h_5^s D_1, \quad (2-4 \text{ a})$$
$$E_1 = -h_5^s S_5 + \beta_{11}^s D_1. \quad (2-4 \text{ b})$$

Figure 2-6 A rectangular-shaped thickness shear vibration mode piezoelectric ceramic plate.

2.2.3.2 Electrical equations

From (2-4 a),

$$D_1 = \frac{E_1}{\beta_{11}^s} + \frac{h_5^s}{\beta_{11}^s} S_5. \quad (2-5)$$

Because there is no free charge in the ceramic, so

$$\frac{\partial D_1}{\partial x} = 0, \quad (2-6)$$
moreover because

\[ S_s = \frac{\partial \zeta}{\partial x}, \tag{2-7} \]

therefore,

\[ \frac{\partial E_i}{\partial x} = -h_s \cdot \frac{\partial^2 \zeta}{\partial x^2}, \]

\[ E_i = -h_s \cdot \frac{\partial \zeta}{\partial x} + \alpha, \]

Where \( \alpha \) is the integral constant, from the input voltage on the ceramic \( V = \int E_i \, dx \),

the integral constant \( \alpha \) is

\[ \alpha = \frac{V}{t} \cdot \frac{h_s}{t} \cdot (\zeta_2 + \zeta_1), \tag{2-8} \]

where the \( \zeta_1 \) and \( \zeta_2 \) are the shear displacements of the surfaces \( x = 0 \) and \( x = t \),

respectively. So the electric field can be expressed in the following equation:

\[ E_i = -h_s \cdot \frac{\partial \zeta}{\partial x} + \frac{V}{t} - \frac{h_s}{t} \cdot (\zeta_2 + \zeta_1), \tag{2-9} \]

Insert the (2-9) into (2-5), the electric displacement can be expressed in the following equation

\[ D_1 = -\frac{V}{\beta_{s1} \cdot t} \cdot \frac{h_s}{\beta_{s1} \cdot t} \cdot (\zeta_2 + \zeta_1), \tag{2-10} \]

The current is

\[ I = i\omega \cdot w \cdot l \cdot D_1 = i\omega \cdot C_0 \cdot V - n \cdot (\zeta_2 + \zeta_1), \tag{2-11} \]

where

\[ C_0 = \frac{w \cdot l}{t \cdot \beta_{s1}} \quad \text{and} \quad n = \frac{w \cdot l \cdot h_s}{\beta_{s1} \cdot t}. \]
2.2.3.3 Mechanical vibration equations

The kinetic equation can be simplified as

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_s}{\partial x}.$$  \hspace{1cm} (2-12)

Insert the (2-4 a) into (2-12), the previous equation becomes

$$\rho \frac{\partial^2 \xi}{\partial t^2} = C_{55} \frac{\partial S_s}{\partial x} - h_s \frac{\partial D_s}{\partial x}.$$  \hspace{1cm} (2-13)

Because $\frac{\partial D_s}{\partial x} = 0,$

$$\frac{\partial^2 \xi}{\partial t^2} = v^2 \frac{\partial^2 \xi}{\partial x^2},$$  \hspace{1cm} (2-14)

where $v = \sqrt{\frac{C_{55}}{\rho}}$ is the transverse wave velocity.

If it is the resonance excitation, (2-14) becomes

$$\frac{\partial^2 \xi}{\partial x^2} + \left( \frac{\omega}{v} \right)^2 \xi = 0.$$  \hspace{1cm} (2-15)

If we assume the solution for the previous equation is

$$\xi = A \cdot \sin \, kx + B \cos \, kx,$$  \hspace{1cm} (2-16)

where $k = \frac{\omega}{v},$ and $A$ and $B$ is the integral constants. The value of $A$ and $B$ can be derived according to $\xi_1$ and $\xi_2$; so the displacement can be expressed in the following equation

$$\xi = \frac{\xi_1 \cdot \sin \, k(t - x) - \xi_2 \cdot \sin \, kx}{\sin \, kt}.$$  \hspace{1cm} (2-17)
According to Newton's first law, $F = T \cdot S$, where $S = w \cdot l$ is the area of the surface perpendicular to the $x$ axis, as shown in Figure 2-6. The forces can be expressed by

$$ -F_1 = c_{ss} \frac{\partial^2 z}{\partial x^2} \cdot S - h_{15} \cdot D_1 \cdot S, $$

$$ -F_2 = c_{ss} \frac{\partial^2 z}{\partial x^2} \cdot S - h_{15} \cdot D_1 \cdot S. $$

(2-18)

Where $F_1$ and $F_2$ are the forces added to the surfaces at $x = 0$ and $x = t$, respectively.

From Equation (2-10),

$$ h_{15} \cdot D_1 \cdot S = n \cdot V - \frac{n^2}{i \omega \cdot C_0} \cdot (\zeta_2 + \zeta_1). $$

(2-19)

From (2-17) and (2-18), the mechanical vibration equations can be expressed in the following form

$$ F_1 = \left( \frac{\rho \cdot v \cdot S}{i \sin kt} - \frac{n^2}{i \omega \cdot C_0} \right) (\zeta_2 + \zeta_1) + i \rho \cdot v \cdot S \cdot \tan \frac{1}{2} kt \cdot \zeta_1 + n \cdot V, $$

$$ F_2 = \left( \frac{\rho \cdot v \cdot S}{i \sin kt} - \frac{n^2}{i \omega \cdot C_0} \right) (\zeta_2 + \zeta_1) + i \rho \cdot v \cdot S \cdot \tan \frac{1}{2} kt \cdot \zeta_2 + n \cdot V. $$

(2-20)

2.2.3.4 Electromechanical equivalent circuit of the ceramic plate

From the electric equation (2-11) and mechanical vibration equation (2-20), the electromechanical equivalent circuit of the ceramic plate can be shown as in Figure 2-7.
Chapter 2 Background and Basic Theory

Figure 2-7 Electromechanical equivalent circuit of a piezoelectric ceramic plate operating at the thickness shear vibration mode.

\[ Z_1 = i\rho v S \tan \frac{kt}{2}, Z_2 = \frac{\rho v S}{\sin kt}, C_0 = \frac{wl}{i\beta_{11}'} \]
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

CHAPTER 3 HIGH POWER/POWER DENSITY MULTI-OUTPUT PIEZOELECTRIC TRANSFORMERS

The poor power management ability is a big drawback to existing piezoelectric transformers (PTs) and limits their development greatly. Also, in order to miniaturize the size of PT and widen its application, it is very necessary to improve its power density and let it have several different outputs. In our study, two kinds of multi-outputs PTs were developed and investigated. One has the highest output power and another one has the highest output power density in the world up to now [36-38].

In first section of this chapter, the reasons why we choose the PZT material, thickness shear vibration mode, the rectangular thin-plate structure and the electrodes to develop the high power/power density PT were presented. In second section, the rectangular thin plate-shaped high power piezoelectric transformers with dual or triple outputs were proposed and investigated for low voltage applications. They operate at the thickness shear vibration mode and are able to provide maximum total output power of 169.8 W with a temperature rise less than 20 °C. The maximum efficiency of piezoelectric transformer with dual outputs is 98%. The voltage gains of the two outputs are 0.76 and 0.51 at the frequency of 278 kHz, respectively. The maximum efficiency of piezoelectric transformer with triple outputs is 95.7%. The voltage gains of the three outputs are 0.75, 0.84 and 0.39 at the frequency of 278 kHz, respectively.
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Compared with most of the other low voltage piezoelectric transformers, these transformers are competitive in practical applications because of the high power, multiple outputs, simple structure and flexibility in design.

In section three, a high power-density dual-output piezoelectric transformer was proposed and experimentally investigated. This transformer consists of a lead zirconate titanate (PZT) ceramic plate with a size of $30 \times 5 \times 1 \text{ mm}^3$, operating at the thickness shear vibration mode. It has a high mechanical quality factor $Q_m$ and high electromechanical coupling coefficient $k_{eff}$. The electrodes of the input and output parts are on the top and bottom surfaces of the ceramic plate and separated by narrow gaps. Lead wires are pressed on the electrodes by proper elastic force. At a temperature rise less than 20 °C, the piezoelectric transformer has a maximum output power density of 52.7 W/cm$^3$ with an efficiency of 88%. The maximum efficiency of this piezoelectric transformer is 95%. Voltage gains larger than and smaller than one are obtained in one transformer. By using a parallel combination of the two outputs of the proposed transformer, an output power density of 57.3 W/cm$^3$ can be achieved at the temperature rise less than 20 °C.

3.1 Material, Vibration Mode and Structure of the Proposed PTs

3.1.1 Material and vibration mode

Table 3-1 shows the physical properties of important applied piezoelectric materials. It is seen that the single crystal lithium niobate (LiNbO$_3$) has the high mechanical
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Table 3-1. Important physical properties of some typical piezoelectric materials

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>BaTiO₃</th>
<th>PZT</th>
<th>LiNbO₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling factors</td>
<td>k₃₁</td>
<td>%</td>
<td>46.8</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>k₃₃</td>
<td>%</td>
<td>60.8</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>k₅₁</td>
<td>%</td>
<td>43.7</td>
<td>70</td>
</tr>
<tr>
<td>Piezoelectric charge constants</td>
<td>d₃₁</td>
<td>10⁻¹² C/N</td>
<td>-78</td>
<td>-135</td>
</tr>
<tr>
<td></td>
<td>d₃₃</td>
<td>10⁻¹² C/N</td>
<td>190</td>
<td>310</td>
</tr>
<tr>
<td></td>
<td>d₁₅</td>
<td>10⁻¹² C/N</td>
<td>587</td>
<td>510</td>
</tr>
<tr>
<td>Dissipation factor</td>
<td>tan δ</td>
<td>%</td>
<td>0.01</td>
<td>0.003</td>
</tr>
<tr>
<td>Mechanical Quality factor</td>
<td>Qₘ</td>
<td></td>
<td>200</td>
<td>2500</td>
</tr>
<tr>
<td>Density</td>
<td>ρ</td>
<td>10³ kg/m³</td>
<td>5.7</td>
<td>7.8</td>
</tr>
<tr>
<td>Relative dielectric const. at 1 kHz</td>
<td>εᵣ₁</td>
<td>1</td>
<td>4400</td>
<td>1590</td>
</tr>
<tr>
<td></td>
<td>εᵣ₃₃</td>
<td>1</td>
<td>129</td>
<td>1470</td>
</tr>
<tr>
<td>Curie temperature</td>
<td>Tₜ</td>
<td>°C</td>
<td>120</td>
<td>315</td>
</tr>
</tbody>
</table>

quality factor (Qₘ>2000) and low piezoelectric loss, but its piezoelectric constants are low. The typical perovskite barium titanate (BaTiO₃) has relative high piezoelectric constants, but its Qₘ is low. Compared with the single crystal and perovskite piezoelectric materials, the lead zirconate titanate (PZT) has the high piezoelectric
constants and high mechanical quality factor. Also it is obviously that the piezoelectric constant $d_{15}$ is much larger than $d_{33}$ and $d_{31}$. Hence more effective transforming can be expected for the thickness shear vibration mode PTs than the longitudinal or extensional PTs. Furthermore, the thickness shear mode has the largest electromechanical coupling factor of the commonly used vibration modes for most piezoelectric materials. Therefore, we chose the PZT C-213 (supplied by FUJI Ceramics Corporation) as the original material to develop the high power/power density piezoelectric transformers.

3.1.2 Structure and electrodes

As we know, the temperature rise is a big problem limiting the power management ability of piezoelectric transformers. According to our previous study [39], a proper structure design can decreases the PT's temperature rise effectively. Figure 3-1 (a) shows the dependence of the temperature rise on $D$ (the ratio of perimeter to area of cross section of the transformer) at two different heat-dissipation coefficients. It is seen that the temperature rise decreases with increasing $D$. This means that a rectangular cross section is better than a square cross section, and a square cross section is better than a circular cross section as viewed from heat dissipation. Figure 3-1 (b) shows the dependence of the temperature rise on $w$ (the width of the transformer) for different cross sections. It indicates that a thin-plate structure can effectively decrease the temperature rise. Therefore, a rectangular thin-plate structure was adopted in our study.
Figure 3-1  (a) Dependence of the temperature rise on the ratio of perimeter to area of cross section of the transformer at two different heat-dissipation coefficients; (b) dependence of the temperature rise on the width of the transformer at three different shapes of cross section.

The positions of input and output electrodes in the PT also influence the performance
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

of the transformer, especially for the multi-output PTs. The PT's performance with its
input at the center of the PT is much better than that with the input at the side of the
PT [24]. Moreover, according to my experiments, when the output has a large
dimension in the vibration propagation direction (see Figure 3-2 a), the transformer
has a very small electromechanical coupling coefficient $k_{eff}$ and very poor
performance (Table 3-2). When the output has a small dimension in the vibration
propagation direction (see Figure 3-3 a), the transformer has a high electromechanical
coupling coefficient $k_{eff}$ and very good performance (Table 3-2). This is because the
vibration attenuation much more fast when the output has a large dimension in the
vibration propagation direction than that when the output has a small dimension in the
vibration propagation direction. The detailed reasons will be presented in Chapter 5.

![Configuration of PT-1](image)

(a)

![Configuration of PT-2](image)

(b)

Figure 3-2  (a) Configuration of PT-1 its output has a large dimension in the
vibration propagation direction; (b) configuration of PT-2 its output has
a small dimension in the vibration propagation direction.
Table 3-2 Performance comparison of the two PTs

<table>
<thead>
<tr>
<th></th>
<th>k_{eff}</th>
<th>Maximum Efficiency (V_{in} = 12, V_{rms})</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-1</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>PT-2</td>
<td>0.52</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Based on the PT-2 structure (as shown in Figure 3-2 (b)) and our theoretical modeling and analysis [40], part of the relationship between the dimensions and the voltage gain, maximum output power, maximum output power density, efficiency of the piezoelectric transformer is shown in Figure 3-3, 3-4, and 3-5. (The development of the theoretical method and verification will be shown in Chapter 4.) Combining the target to achieve the high power/power density piezoelectric transformers, two types of piezoelectric transformers are developed, one for high power and another for high power density.

![Graph](attachment:graph.png)

Figure 3-3 Dependence of the maximum output power and efficiency on the thickness of the transformer when the total length and widths of the input and outputs are constant.
Figure 3-4  (a) Maximum voltage gain vs. the width of the transformer when the total length, thickness and width ratio among the input, output 1 and output 2 are constant; (b) maximum voltage gain vs. the length of the transformer when the thickness and widths of the input and outputs are constant.
Figure 3-5  (a) Maximum output power and power density vs. the width of output 2 when the length, thickness and width of the input and output 1 are constant; (b) maximum output power and power density vs. the width of input when the length, thickness and width of the outputs are constant.
3.2 High Power Multi-Output PTs Operating at the Thickness Shear Vibration Mode

3.2.1 Construction and principle

Figure 3-6 shows the construction and dimensions of the proposed PTs operating at the thickness shear vibration mode with dual outputs (Figure 3-6 a) and triple outputs (Figure 3-6 b). They are made of PZT ceramic plates with a high mechanical quality factor $Q_m (=2500)$ and dimensions of $120 \text{ mm} \times 20 \text{ mm} \times 4 \text{ mm}$. The PZT ceramic plates are poled along the width direction. The electrodes of the input and output parts are on the top and bottom surfaces of the ceramic plates and separated by the narrow insulating gaps. For the transformer with dual outputs, the electrode areas of the input, output 1 and output 2 are $240 \text{ mm}^2$, $720 \text{ mm}^2$ and $1440 \text{ mm}^2$, respectively. For the transformer with triple outputs, the electrode areas of the input, output 1, output 2 and output 3 are $240 \text{ mm}^2$, $480 \text{ mm}^2$, $240 \text{ mm}^2$ and $1440 \text{ mm}^2$, respectively.

The rectangular plate structure can effectively decrease the temperature rise of the transformers, which is beneficial to increasing the maximum power of the transformers. The reason is that the heat dissipation of a transformer can be increased by increasing the ratio of perimeter to area of the cross section of the transformer.
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Figure 3-6 Rectangular plate-shaped piezoelectric transformer operating at the thickness shear vibration mode (a) with dual outputs; (b) with triple outputs.

Due to the thickness shear vibration mode, the nodal planes of the proposed
transformers are located at the middle of the ceramic plates. So, the supporting structure shown in Figure 3-7 was proposed and used. The transformer is supported by a slice of hard metal (support 3) and clamped by two thin metal bars (support 1 and 2). By this supporting construction, stable support, less energy loss and excellent heat dissipation can be attained. In this study, the transformer is perpendicular to the base. The reason of placing the transformer vertically rather than horizontally is that a vertically placed plate has a better heat dissipation than that of a horizontally placed plate [39]. Figure 3-8 is the photograph of the piezoelectric transformer with dual outputs. The lead wires with proper elasticity are chosen to keep the connection stable. By this connection, the maximum operating temperature of the transformer can be increased. Conventionally, the connection between the lead wires and electrodes of a piezoelectric transformer is realized by soldering. When the transformer operates at a large vibration, the local temperature at the solder points of the transformer may be very high. If this temperature approaches to the melting-point of the solder, the solder points fall off the electrodes. By the connection shown in Figure 3-8, the above problem is avoided. So the maximum operating temperature of the transformer can be raised. This increases the maximum output power of the transformer.

However, this supporting structure is just for the experiments in lab. It can’t endure large mechanical shock or external vibrations. But the ability to endure the large shock or vibration can be improved greatly if we make a little modification to the supporting structure, as shown in Figure 3-9. Also according to our experiments, the supporting and lead wire connection structure can decrease the temperature rise obviously compared with the structure shown in Figure 3-7 with the same output power. It will be introduced in detail in Chapter 6.
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Figure 3-7 Supporting construction of the piezoelectric transformers.

Figure 3-8 Photograph of the piezoelectric transformer with dual outputs.
3.2.2 Experimental Setup

The characteristics of the PT were measured by the experimental setup shown in Figure 3-10. The transformer was driven by an ac voltage generated by a function generator (Tektronix AFG320) and amplified by a high speed power amplifier (NF HSA4014). In high power experiments, a high frequency electromagnetic transformer was used to amplify the voltage from the power amplifier. In Figure 3-6, $R_{L1}$, $R_{L2}$ and $R_{L3}$ are pure resistors. It is known that the efficiency of a piezoelectric transformer attains a maximum value when the load resistance is equal to $1/\omega C_d$, where $C_d$ is the
clamped capacitance of the output part. This load is called the matching load. In this study, \( R_{L1} \), \( R_{L2} \) and \( R_{L3} \) are the matching loads for the output 1, output 2 and output 3, respectively. For the transformer with two outputs as shown in Figure 3-6 (a), \( R_{L1} = 305 \Omega \), \( R_{L2} = 330 \Omega \). For the transformer with three outputs as shown in Figure 3-6 (b), \( R_{L1} = 400 \Omega \), \( R_{L2} = 1190 \Omega \), \( R_{L3} = 230 \Omega \). The voltage, current and power of the input and outputs were measured by the oscilloscopes. The highest temperature in the surface of the transformer was measured by an infrared thermometer (KEYENCE IT2-50) about one minute after applying the input voltage. It was observed that the input part of a piezoelectric transformer had a higher temperature rise than the output parts, and the temperature rise at the connection points between the lead wires and the electrodes was higher than that at other points. So the measuring point of the temperature rise was chosen at the connection point of the input part [41]. Also, the temperature rise of the transformer was kept below 20 °C in the experiments, which was controlled by properly tuning the amplitude of the input voltage.

Figure 3-10 Experimental setup for measuring the characteristics of the piezoelectric transformers.
3.2.3 Experimental results and analyses

In the following discussion, PT-A is the transformer with dual outputs as shown in Figure 3-6 (a) and PT-B is the transformer with triple outputs as shown in Figure 3-6 (b).

3.2.3.1 Impedance analyses

Table 3-3 shows the measured equivalent circuit parameters of the input and output parts of PT-A and PT-B. Each part was measured under the condition that the other parts were all short circuited. HP4194A impedance analyzer was used in the measurement. From the data of the output parts of PT-A or PT-B, it is seen that the capacitance ratio $\gamma$ and the mechanical quality factor $Q_m$ decrease with increasing the area of electrode. Figure 3-11 shows the measured impedance of the input part of the piezoelectric transformers with the matching loads. Compared with the data in Table 3-3, it is seen that the resonance and anti-resonance frequencies of PT-A and PT-B all increase when the transformers have the matching loads. The impedance responses have two minimum values because of the existence of different output parts.

3.2.3.2 Characteristics of the PTs
Figure 3-12 shows the voltage gain versus driving frequency of PT-A and PT-B with the matching loads. In this experiment, the input voltages were 128 V_{rms} for PT-A and 97 V_{rms} for PT-B, and the temperature rise of the transformers was below 5 °C. For both PT-A and PT-B, the output voltage gains are below 1.0. So they can be used in voltage step-down systems. In practical applications, in order to attain specific output voltages, one may adjust the electrode areas of the input and output parts and control the driving frequency.

Table 3-3. Parameters of the input and outputs of piezoelectric transformer with dual and triple outputs.

<table>
<thead>
<tr>
<th></th>
<th>$f_r$ (kHz)</th>
<th>$f_a$ (kHz)</th>
<th>$R$ (Ω)</th>
<th>$L$ (mH)</th>
<th>$C_a$ (pF)</th>
<th>$C_b$ (pF)</th>
<th>$Q_m$</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PT-A (Dual Outputs)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input</td>
<td>264.0</td>
<td>268.3</td>
<td>10.93</td>
<td>7.537</td>
<td>48.21</td>
<td>1797.08</td>
<td>1143.8</td>
<td>37.3</td>
</tr>
<tr>
<td>Output</td>
<td>264.0</td>
<td>280.6</td>
<td>1.12</td>
<td>0.614</td>
<td>591.62</td>
<td>4567.83</td>
<td>964.9</td>
<td>7.7</td>
</tr>
<tr>
<td>Output1</td>
<td>267.0</td>
<td>275.2</td>
<td>2.79</td>
<td>1.789</td>
<td>198.51</td>
<td>3250.49</td>
<td>1109.1</td>
<td>16.4</td>
</tr>
<tr>
<td>Output2</td>
<td>264.0</td>
<td>287.3</td>
<td>1.54</td>
<td>0.797</td>
<td>455.86</td>
<td>2495.37</td>
<td>934.7</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>PT-B (Triple Outputs)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input</td>
<td>263.5</td>
<td>267.3</td>
<td>36.57</td>
<td>10.232</td>
<td>35.65</td>
<td>1411.21</td>
<td>469.7</td>
<td>39.6</td>
</tr>
<tr>
<td>Output</td>
<td>263.5</td>
<td>280.8</td>
<td>2.24</td>
<td>0.678</td>
<td>537.93</td>
<td>4075.97</td>
<td>528.0</td>
<td>7.6</td>
</tr>
<tr>
<td>Output1</td>
<td>266.3</td>
<td>274.2</td>
<td>9.29</td>
<td>5.126</td>
<td>69.68</td>
<td>1302.54</td>
<td>950.4</td>
<td>18.7</td>
</tr>
<tr>
<td>Output2</td>
<td>266.3</td>
<td>274.0</td>
<td>37.21</td>
<td>22.381</td>
<td>15.96</td>
<td>321.47</td>
<td>1034.8</td>
<td>20.1</td>
</tr>
<tr>
<td>Output3</td>
<td>263.5</td>
<td>286.9</td>
<td>3.19</td>
<td>0.943</td>
<td>386.33</td>
<td>2132.92</td>
<td>533.0</td>
<td>5.5</td>
</tr>
</tbody>
</table>

$f_r$ and $f_a$ are the resonance and anti-resonance frequencies, respectively, $R$ the equivalent resistance, $L$ the equivalent inductance, $C_a$ the equivalent capacitance, $C_b$ the clamped capacitance, $Q_m$ the mechanical quality factor, and $γ$ the capacitance ratio.
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Figure 3-11 Measured impedance of the input part of the piezoelectric transformers with the matching loads.

(a)

(b)
Figure 3-12 Voltage gain versus driving frequency of PT-A and PT-B with the matching loads.
Figure 3-13 Frequency dependence of the total output power and efficiency of PT-A and PT-B under the input voltage of $128\, V_{\text{rms}}$ and $97\, V_{\text{rms}}$, respectively.

(a) Output power $P_{\text{out}}$ and efficiency $\eta$ for PT-A with $\Delta T < 5^\circ C$.

(b) Output power $P_{\text{out}}$ and efficiency $\eta$ for PT-B with $\Delta T < 5^\circ C$. 

Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers
Figure 3-14 Output powers of different outputs measured for different driving frequencies.
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Figure 3-13 shows the frequency dependence of the total output power and the efficiency of PT-A and PT-B under the input voltages of 128 $V_{rms}$ and 97 $V_{rms}$, respectively. In this experiment, their load resistances were all matched and the maximum temperature rise was lower than 5 °C. The efficiency is calculated by $\eta = \frac{P_{out}}{P_{in}}$, and $P_{out}$ is the real output power of the PT, $P_{out} = \frac{V_{out}^2}{R_L}$, where $V_{out}$ is the output voltage of the PT, $R_L$ is the load resistance of the PT; $P_{in}$ is the real input power of the PT, $P_{in} = V_{in}I_{in}\cos(\Delta\phi)$, where $V_{in}$ is the input voltage of the PT, $I_{in}$ is the input current of the PT and $\Delta\phi$ is the phase difference between the $V_{in}$ and $I_{in}$. For PT-A, a maximum total output power of 98.1 W and maximum efficiency of 98% were obtained under an input voltage of 128 $V_{rms}$ at the driving frequency of 278 kHz and 284 kHz, respectively. For PT-B, a maximum total output power of 63.4 W and maximum efficiency of 95.7% were obtained under an input voltage of 97 $V_{rms}$ at the driving frequency of 300 kHz and 268 kHz, respectively. It is seen that the curves of both the total output power and efficiency fluctuate greatly. The first reason of this phenomenon is that there are some spurious vibration modes in the driving frequency range (see Figure 3-10). The second reason is that there are different outputs in these transformers and they arrive at their maximum output powers and efficiencies at different driving frequencies. The distributions of the output powers in Figure 3-13 were measured, and the results are shown in Figure 3-14. For PT-A, at the maximum efficiency frequency of 284 kHz, the output powers of output 1 and output 2 are 62% and 38% of the total output power, respectively. For PT-B, at the maximum efficiency frequency of 268 kHz, the output powers of output 1, output 2 and output 3 are 34%,
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

13% and 53% of the total output power, respectively. The difference of efficiency of the outputs is due to the difference of the area of these outputs.

Figure 3-15 Relationships among the maximum total output power, temperature rise and input voltage of PT-A and PT-B with the matching loads.
Figure 3-15 shows the relationship among the maximum total output power, temperature rise and input voltage of PT-A and PT-B with the matching loads. Here, the maximum total output power means the maximum value of the total output power with respect to the driving frequency for a given input voltage. It is seen that, for PT-A, a maximum output power of 169.8 W can be obtained at an input voltage of 180.1 V\textsubscript{rms} and temperature rise lower than 20 °C. For PT-B, a maximum output power of 163.1 W can be obtained at an input voltage of 176.8 V\textsubscript{rms} and temperature rise less than 20 °C. When the input voltage is greater than 180 V\textsubscript{rms}, the temperature rise of both PT-A and PT-B increases rapidly, and the characteristics of the transformer become quite unstable. When the temperature of the piezoelectric transformers is too high, the internal loss of the PZT material becomes very large. This increases the temperature of the transformers further and makes the operation unstable.
Figure 3-16  Efficiency versus input voltage when the total output power reaches the maximum with respect to the driving frequency.
Figure 3-16 shows the efficiency versus the input voltage when the total output power reaches the maximum value. The matching loads were used. It is seen that the efficiency of the transformers decrease with increasing the input voltage. The reason of this phenomenon is that the vibration of the transformers is intensified with increasing the input voltage, and the internal loss of the transformers increases with the increase of the vibration. Figure 3-17 shows the distributions of the output power in Figure 3-16. It is seen that, for PT-A, the output power is 129.5 W for output 1 and 40.3 W for output 2 when the total output power reaches the maximum value at the input voltage of 180.1 V\(_{\text{rms}}\). For PT-B, the output power is 36.9 W for output 1, 13.0 W for output 2 and 113.2 W for output 3 when the total output power reaches the maximum with respect to the driving frequency.
maximum value at the input voltage of 176.8 V\textsubscript{rms}. The difference of the power of the outputs is due to the difference of the area of the outputs.

3.2.3.3 Multiple output properties

Figure 3-18 shows the effects of the output number on the driving frequencies of the maximum efficiency and maximum total output power. In this experiment, PT-A with the two outputs in parallel was used as the single output transformer, PT-A with two separately operating outputs as the dual output transformer, and PT-B with three separately operating outputs as the triple output transformer. The matching loads were used, and the maximum temperature rise was kept below 20 °C. It is seen that the frequency of the maximum total output power increases with the increase of the output number of the piezoelectric transformer. On the contrary, the frequency of the maximum efficiency decreases with the increase of the output number. As the increase of the output number, the area of the insulating gaps between the electrodes increases. This increases the loss and decreases the effective electromechanical coupling factors.

To investigate the effect of the load of one output on the voltage gains of the other outputs, the relationship between the voltage gain of output 1 and the operating frequency was measured for PT-A when output 2 of PT-A was short circuited, open circuited and matched, respectively, and the results are shown in Figure 3-19. It is seen that the voltage gain of output 1 is affected by the load of output 2 for most of the driving frequencies, and there is a frequency range in which the voltage gain of output 1 has little change when the load of output 2 changes. For some frequencies,
the output impedance of output 2 is quite small; so the load resistance of output 2 has little effect on the voltage gain of output 1. In many power electronics applications, we want transformers with constant voltage gains. So, this result shows the competitiveness of the multi-output piezoelectric transformers in power electronic applications.

Figure 3-18 Effects of the output number on the driving frequencies of the maximum efficiency and maximum total output power.
3.3 High Power Density Dual-output PTs Operating at the Thickness Shear Vibration Mode

3.3.1 Construction and principle

Figure 3-20 shows the construction and dimensions of the proposed piezoelectric transformer operating at the thickness shear vibration mode. This transformer consists of a lead zirconate titanate (PZT) ceramic plate with a size of $30 \times 5 \times 1 \text{ mm}^3$. It has a high mechanical quality factor $Q_m$ and high electromechanical coupling coefficient $k_{eff}$. 
The PZT ceramic plate is poled along the width direction. The electrodes of input and output parts are on the top and bottom surfaces of the ceramic plate and separated by the narrow insulating gaps. The electrode areas of the input, output 1 and 2 are 30 mm², 45 mm² and 75 mm², respectively. The relevant properties of the PZT ceramic material are shown in Table 3-1.

Figure 3-20  A schematic diagram of the piezoelectric transformer operating at the thickness shear vibration mode with dual outputs.

The thin plate-shaped structure shown in Figure 3-20 can effectively decrease the temperature rise of the transformer, which is beneficial to increasing the maximum
power density of the transformer. The reason is that the heat dissipation ability of the transformer increases with increasing the ratio of perimeter to area of the cross section of the transformer.

Figure 3-21 shows the connection of the lead wires of the fabricated piezoelectric transformer with dual outputs. The lead wires with proper elasticity were chosen to keep the connection stable. By this connection, the maximum operating temperature of the transformer can be increased. Conventionally, the connection between the lead wires and electrodes of a piezoelectric transformer is realized by soldering. It is known that the local temperature at the solder points of the transformer is very high if the transformer operates at large vibration. When this local temperature approaches to the melting-point of the solder, the solder points fall off the electrodes. By the connection shown in Figure 3-21, the above problem can be avoided. So the maximum operating temperature of the transformer can be raised. This increases the maximum output power of the transformer.

![Diagram of lead wires connection](image)

Figure 3-21 Connection of the lead wires of the fabricated piezoelectric transformer.
3.3.2 Experimental results and analyses

3.3.2.1 Impedance analyses

The parameters of the input or output parts for the proposed PT were measured when the other parts were all short circuited. Table 3-4 shows the measured equivalent circuit parameters of the input part and output parts. HP4194A impedance analyzer was used in the measurement. It is seen that, for the output parts, the capacitance ratio $\gamma$ decreases when the electrode area increases, and the mechanical quality factor $Q_m$ decreases as $\gamma$ decreases. The reason is that, with the decrease of the capacitance ratio $\gamma$, the vibration of the ceramic material increases for a given input, and therefore the internal loss increases. Theoretically, the maximum power density $(P/V)_{\text{max}}$ can be approximately expressed as [26]

$$(P/V)_{\text{max}} \propto k_{\text{eff}}^2 \cdot v_{\text{max}}^2 \cdot f,$$  \hspace{1cm} (3-1)

in which $k_{\text{eff}}$ is the electromechanical coupling coefficient, $v_{\text{max}}$ is the maximum vibration velocity which is limited by the maximum temperature, and $f$ is the driving frequency. Also, we have

$$k_{\text{eff}}^2 = \frac{1}{1 + \gamma}.$$ \hspace{1cm} (3-2)

Therefore, the proposed piezoelectric transformer is expected to have a high power density because of the high operating frequency (over 1 MHz) and the relatively low capacitance ratio $\gamma$. 

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Table 3-4 Parameters of the input and outputs of the proposed piezoelectric transformer

<table>
<thead>
<tr>
<th></th>
<th>$f_r$ (MHz)</th>
<th>$f_a$ (MHz)</th>
<th>$R$ (Ω)</th>
<th>$S$ (mm$^2$)</th>
<th>$Q_m$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1.036</td>
<td>1.070</td>
<td>15.95</td>
<td>30</td>
<td>227.8</td>
<td>15.2</td>
</tr>
<tr>
<td>*Output</td>
<td>1.036</td>
<td>1.095</td>
<td>5.71</td>
<td>120</td>
<td>183.2</td>
<td>8.8</td>
</tr>
<tr>
<td>Output1</td>
<td>1.045</td>
<td>1.090</td>
<td>14.58</td>
<td>45</td>
<td>307.9</td>
<td>11.8</td>
</tr>
<tr>
<td>Output2</td>
<td>1.036</td>
<td>1.124</td>
<td>9.23</td>
<td>75</td>
<td>180</td>
<td>5.9</td>
</tr>
</tbody>
</table>

$f_r$ and $f_a$ are the resonance and anti-resonance frequencies, respectively; $R$ the equivalent resistance; $S$ the electrode area, $Q_m$ the mechanical quality factor and $\gamma$ the capacitance ratio. * represents the parallel output connection.

Figure 3-22 Measured impedance of the input part of the piezoelectric transformer with the matching loads.

Figure 3-22 shows the measured impedance characteristic of the input part of the piezoelectric transformer with the matching loads. Compared with the data in Table 3-4, the resonance and anti-resonance frequencies of the PT all increase. The frequency
range, in which the input part has a small $|Z|$ and a phase angle near to zero, is chosen as the optimum driving frequency range of the PT. The reason is that the vibration of the PT is stronger at the frequencies with small $|Z|$ than at other frequencies, and the efficiency is relatively high at the frequencies where the phase angle near to zero.

3.3.2.2 Characteristics of the PT

Figure 3-23 shows the voltage gains versus driving frequency of output 1 and 2 at their matching loads of 350 $\Omega$ and 240 $\Omega$. It is seen that the voltage gains varies with the driving frequency. The voltage gain is larger than 1.0 for output 1 and less than 1.0 for output 2. In practical applications, in order to attain specific output voltages, one may adjust the electrode areas of the input and output parts and control the driving frequency. In this experiment, the temperature rise of the transformer was below 5 °C.

![Figure 3-23 Voltage gain versus driving frequency of the proposed piezoelectric transformer with the matching loads.](image)
Figure 3-24  Voltage gain versus load resistance at the driving frequencies of 1.075MHz, 1.081MHz and 1.085 MHz: (a) output 1; (b) output 2.
Chapter 3 High Power/Power Density Multi-Output Piezoelectric Transformers

Figure 3-24 shows the voltage gain versus load resistance of output 1 and 2 at the driving frequencies of 1.075 MHz, 1.081 MHz and 1.085 MHz. When the load resistance is small, the voltage gain increases rapidly with the load resistance. At a higher load resistance, the voltage gain becomes saturated and the saturated value depends on the driving frequency. The reason can be found from the lumped constant equivalent circuit of the piezoelectric transformer. When the load resistances \( R \) is not much larger than the matching load \( R_L = \frac{1}{\alpha C_d} \), in which \( C_d \) is the clamped capacitance of the output, the output voltage increases with the load resistance \( R \) for a constant input voltage; when \( R \) is much larger than \( R_L \), the load branches can be regarded as open and the load resistances have little effect on the voltage gain.

Figure 3-25 (a) shows the frequency dependence of the total output power and the efficiency of the PT when the input voltage is 12 V\( _{\text{rms}} \). Their load resistances were all matched. In this experiment, the maximum temperature rise was less than 10 °C. A maximum output power of 2.23 W and a maximum efficiency of 95% were obtained at a driving frequency of 1.076 MHz and 1.073 MHz, respectively. It is seen that there exist a maximum output power and maximum efficiency around the anti-resonance point of the input (see Table 3-4). This is because the input impedance of the transformer has a smaller absolute value and phase angle around the anti-resonance point. It is also seen that the curves of both the total output power and efficiency fluctuate at some driving frequencies. The first reason of this is that there are some spurious vibration modes in the driving frequency range (see Figure 3-22). The second reason is that the change of output power with respect to frequency is different at the two outputs. The distributions of the output powers in Figure 3-25 (a) were measured, and the results are shown in Figure 3-25 (b). It is seen that the ratio of
output power $P_{\text{out1}}$ and $P_{\text{out2}}$ depends on the driving frequency and it changes from 2.3 to 4.5 in the experimental frequency range. This is because the voltage gains of output 1 and 2 are the functions of the driving frequency. Also it is seen that output 1 and 2 arrive at their maximum output power at the same frequency of 1.076 MHz with 1.72 W at output 1 and 0.51 W at output 2. This is because both output 1 and output 2 arrive at their maximum output power when the PT is in resonance. From Figure 3-22 and 3-25, it is seen that the frequency of the maximum output power agrees well with the resonance frequency of the PT with the matching loads.

Figure 3-26 (a) shows the relationship among the maximum output power density, temperature rise and input voltage of the PT with the matching loads. Here, the maximum output power density is with respect to the driving frequency for a given input voltage. It is seen that a maximum power density of 52.7 W/cm$^3$ can be obtained at a temperature rise lower than 20 °C. When the input voltage is larger than 20 V$_{\text{rms}}$, the temperature rise of the transformer increases rapidly, and the characteristics of the transformer become quite unstable. When the temperature of a piezoelectric transformer is too high, the internal loss of the piezoelectric material becomes very large. This increases the temperature of the transformer further and makes the operation unstable. Figure 3-26 (b) shows the distributions of the output powers when the output power density reaches the maximum in Figure 3-26 (a). It is seen that, the ratio of $P_{\text{out1}}$ and $P_{\text{out2}}$ is approximately constant (~3.8) when the input voltage is less than 10 V$_{\text{rms}}$, and this ratio increases with the increase of the input voltage when the input voltage is larger than 10 V$_{\text{rms}}$. This is because the equivalent circuit parameters of the PT change with the vibration velocity at large vibration, and the change of the equivalent parameters results in the change of the voltage gain.
2.4 Temp. rise ≤ 10°C

Aim, output power, efficiency

<table>
<thead>
<tr>
<th>Driving frequency (MHz)</th>
<th>1.06</th>
<th>1.07</th>
<th>1.08</th>
<th>1.09</th>
<th>1.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
<td>0.85</td>
<td>0.80</td>
</tr>
<tr>
<td>Output power</td>
<td>1.10</td>
<td>1.05</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Figure 3-25 (a) Frequency dependence of the total output power and the efficiency of the PT when the input voltage is constant; (b) powers of different outputs measured with respect to the driving frequencies.
Figure 3-26  (a) Relationship among the maximum output power density, temperature rise and input voltage of the PT with the matching loads.
(b) Distributions of the output powers when the total output power reaches the maximum with respect to the input voltage.
3.3.2.3 Outputs in parallel and series properties

In order to broaden the application of this piezoelectric transformer, the operation of the transformer with the outputs in parallel and series was experimentally investigated. Figure 3-27 shows the constructions of the transformer with the two output parts in parallel and series. The matching load resistance $R_L$ is 140 $\Omega$ for the parallel connection and 440 $\Omega$ for series connection. Figure 3-28 shows the characteristics of the piezoelectric transformer with the two outputs in parallel: (a) frequency dependence of the total output power and the efficiency at an input voltage of 12 V$_{\text{rms}}$; (b) relationship among the maximum output power density, temperature rise and input voltage. It is seen that the maximum output power density of this operation (57.3 W/cm$^3$) is higher than that of the dual-output transformer (52.7 W/cm$^3$) under the matching load conditions. On the impedance curve of the input part of the transformer with dual outputs, there are two anti-resonance peaks because the two outputs of the transformer are different. This results in a lower $k_{\text{eff}}$. However, the transformer with two outputs in parallel only has one anti-resonance peak. This results in a higher $k_{\text{eff}}$. So the maximum output power density of the transformer with the two outputs in parallel is higher than that of the transformer with dual outputs.
Figure 3-27 Constructions of the transformer with the two output parts: (a) in parallel; (b) in series.

Figure 3-29 shows the characteristics of the piezoelectric transformer with the two output parts in series: (a) frequency dependence of the total output power and the efficiency at an input voltage of 12 V rms; (b) relationship among the maximum output power density, temperature rise and input voltage. Contrary to the transformer with the two outputs in parallel, the maximum output power density of the transformer with the two outputs in series remarkably decreases. This is because the partial offset of the charges on the electrode A and B shown in Figure 3-27 (b).
Figure 3-28 Characteristics of the piezoelectric transformer with two outputs in parallel: (a) frequency dependence of the total output power and the efficiency with an input voltage of 12 V_{rms}; (b) relationship among the maximum output power density, temperature rise and input voltage.
Figure 3-29  Characteristics of the piezoelectric transformer with two outputs in series: (a) frequency dependence of the total output power and the efficiency with an input voltage of 12 V₉₉, (b) relationship among the maximum output power density, temperature rise and input voltage.
CHAPTER 4 MODELING AND ANALYSIS OF DUAL-OUTPUT THICKNESS SHEAR VIBRATION MODE PIEZOELECTRIC TRANSFORMER

As far as designing a desired PT is concerned, a physical model is essential so that the parameters of the model can be determined from the properties of the material and the physical size of the PT. Moreover, a good model of the PT can help designers gain better physical insight and to develop the driving and controlling circuits of PTs via simulation [42-61].

In this work, a theoretical model was developed for the dual-output piezoelectric transformer operating at the thickness shear vibration mode. The equivalent circuit parameters of the piezoelectric transformer were derived. Based on this the impedance characteristics, equivalent inductance, capacitance ratio, voltage gain and efficiency of the piezoelectric transformer were calculated. The theoretical results were verified by experimental data. Furthermore, the effect of the transformer size on the voltage gain, efficiency, output power and power density, and the effect of the load of one output on the voltage gain of another output were analyzed. Some useful guidelines of designing the transformer were achieved by these analyses.
4.1 Modeling of Dual-Output PT Operating at the Thickness Shear Vibration Mode

4.1.1 Structure for the modeling

Figure 4-1 illustrates the basic configuration of the dual-output piezoelectric transformer for the modeling. The ceramic plate is poled along the width direction. The electrodes of input and output ports on the top and bottom surfaces of the ceramic plate are separated by the narrow insulting gaps along the length direction. According to our previous experimental results, this structure has high electromechanical-coupling factor $k_{\text{eff}}$ and very good characteristics [36-38].

![Figure 4-1 Configuration of the dual-output piezoelectric transformer operating at the thickness shear vibration mode.](image)

4.1.2 Equivalent circuit and its parameters
Chapter 4 Modeling and analysis of dual-output thickness shear vibration mode

A multiterminal piezoelectric plate can be analyzed by the tensor Green's-function operations, Fourier-eigenmode expansions, and circuit-theory manipulations. Based on those analyses, the electrical equivalent circuit of the dual-output piezoelectric transformer operating at the thickness shear vibration mode can be derived in the vicinity of the resonance point [22, 24, 40, 62]. Figure 4-2 shows the equivalent circuit of the transformer, in which $L$, $C$, and $R$ are the equivalent inductance, capacitance and resistance, respectively; $C_{d1}$, $C_{d2}$ and $C_{d3}$ are the clamped capacitance of the input, output 1 and output 2, respectively; $R_{L1}$ and $R_{L2}$ are the load resistances of output 1 and output 2, respectively. The turns ratio of the ideal transformer $n_1$ and $n_2$ can be calculated by

$$n_1 = \frac{A_0}{A_1} = \sqrt{\frac{L_1}{L}} \quad \text{and} \quad n_2 = \frac{A_0}{A_2} = \sqrt{\frac{L_2}{L}} \quad (4-1)$$

where $A_0$, $A_1$, and $A_2$ are the force factors of the input, output 1 and output 2, respectively; $L_1$ and $L_2$ are the equivalent inductance of output 1 and output 2, respectively [22, 24, 40].

![Figure 4-2](image)

Figure 4-2 Equivalent circuit of a dual-output piezoelectric transformer operating at the thickness shear vibration mode.

The shear displacement of the transformer may be assumed as

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Chapter 4 Modeling and analysis of dual-output thickness shear vibration mode PT

\[ u = A \sin \beta x + B \cos \beta x \]  
(4-2a)

where \( A \) and \( B \) are two constants, and \( \beta \) is the wave number in the \( x \) direction.

\[ \beta = \frac{\omega}{v} \]  
(4-2b)

\[ v = \frac{C_{55}^P}{\sqrt{\rho}} \]  
(4-2c)

where \( \omega \) is the angular frequency, \( v \) is the velocity of sound, \( C_{55}^P \) is the shear elastic stiffness constant and \( \rho \) is the density. When the piezoelectric plate works at its resonance frequency, the following displacement may be obtained by substituting (4-2a) into the basic electromechanical equations of the ceramic plate.

\[ u = \frac{u_1 \sin \beta (t-x) - u_2 \sin \beta x}{\sin \beta t} \]  
(4-3)

where \( u_1 \) and \( u_2 \) are the displacement at \( x=0 \) and \( x=t \), respectively. Because the displacement values at \( x=0 \) and \( x=t \) are the same, we have \( u_1 = u_2 = u_0 \). Then the displacement can be expressed as

\[ u = u_0 \left[ \sin \beta (t-x) - \sin \beta x \right] \]  
(4-4)

Assume the voltage applied to the ceramic plate is \( V \). From the basic piezoelectric equation, the electric displacement of the input is

\[ D_1 = \frac{V}{\beta_{13} s_t \beta_{13} s_t} - \frac{h_{15}}{\beta_{13} s_t \beta_{13} s_t} (2u_0) \]  
(4-5)

where \( h_{15} \) and \( \beta_{13} \) are the piezoelectric and the permittivity constants, respectively.

Thus, the current \( i \) at the input is

\[ i = -j \omega \int [D_1 dydz] = -j \omega \int \left[ \frac{V}{\beta_{13} s_t \beta_{13} s_t} - \frac{h_{15}}{\beta_{13} s_t \beta_{13} s_t} (2u_0) \right] dydz \]  
(4-6)
Chapter 4 Modeling and analysis of dual-output thickness shear vibration mode PT

where $D_i$ is the electric displacement in the $x$ direction. From (4-6), the motional current $I_m$ shown in Figure 4-2 is

$$I_m = -j\omega \int \left[ -\frac{h_{ss}}{\beta_{11}^s} t (2u_0) \right] dy dz$$

$$= 2j\omega w_m l u_0 \frac{h_{ss}}{\beta_{11}^s} t$$

(4-7)

where $l$ and $t$ are the length and thickness of the piezoelectric transformer, respectively, and $w_m$ is the width of the input port.

Table 4-1. Dimensions of the dual-output piezoelectric transformer operating at the thickness shear vibration mode.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>Length of the ceramic plate</td>
<td>30</td>
</tr>
<tr>
<td>$w_m$</td>
<td>Width of the input part</td>
<td>1</td>
</tr>
<tr>
<td>$w_1$</td>
<td>Width of output 1</td>
<td>1.5</td>
</tr>
<tr>
<td>$w_2$</td>
<td>Width of output 2</td>
<td>2.5</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness</td>
<td>1</td>
</tr>
</tbody>
</table>

The FEM simulation was carried out for the piezoelectric transformer in order to investigate its vibration. The dimensions of the transformer used in the simulation are shown in Table 4-1. Figure 4-3 shows the normalized shear displacement at $x=0$ and $y=15\text{mm}$ and the stress at $x=0.5\text{mm}$ and $y=15\text{mm}$ when the two outputs are short-circuit. It is seen that half of each output has no shear stress, and even in the half output with shear stress, the stress is very weak compared to that in the input. So the
kinetic energy in the short-circuit parts can be ignored in the calculation, and the total kinetic energy of the resonator is

\[
W = \frac{1}{2} \rho \omega^2 \iiint u'^2 dV
\]

\[
= \frac{1}{4} \rho \omega^2 \omega_m u_0^2
\]

(4-8)

Figure 4-3  Normalized shear displacement and stress distributions of the transformer when its two outputs are short-circuited. The shear displacement is at \(x=0\) and \(y=15\text{mm}\) and the stress is at \(x=0.5\text{mm}\) and \(y=15\text{mm}\).

By equating the energy \(LI_m^2/2\) stored in the equivalent inductance to the total kinetic energy, we have

\[
L = \frac{\rho l^3}{8 w_m l} \left( \frac{\beta_{11} s}{h_0} \right)^2
\]

(4-9a)

By the same method, we have
where $L_1$ and $L_2$ are the equivalent inductance of output 1 and output 2, respectively; $w_1$ and $w_2$ are the width of output 1 and output 2, respectively. At the resonance frequency, the equivalent capacitance of the input is

$$C = \frac{1}{\omega^2 L}$$

(4-10a)

Substituting (4-9a) into (4-10a),

$$C = \frac{8w_1l}{\pi^2 C_{55} \rho t \left( \frac{h_{12}}{\beta_{11}^3} \right)^2}$$

(4-10b)

By the same method, we have

$$C_1 = \frac{8w_1l}{\pi^2 C_{55} \rho t \left( \frac{h_{12}}{\beta_{11}^3} \right)^2}$$

(4-10c)

$$C_2 = \frac{8w_2l}{\pi^2 C_{55} \rho t \left( \frac{h_{12}}{\beta_{11}^3} \right)^2}$$

(4-10d)

where $C_1$ and $C_2$ are the equivalent capacitance of output 1 and output 2, respectively.

In the equivalent circuit, the equivalent resistance can be calculated by [24, 40]

$$R = \frac{\omega_0 L}{Q_m}$$

(4-11)

where $\omega_0$ is the angular resonance frequency of the transformer and $Q_m$ is the mechanical quality factor.
4.2 Verifications of the Theoretical Model

In the following calculations and discussion, unless otherwise specified, the transformer has matching loads. The maximum voltage gain means the maximum value of the output voltage gain with respect to the driving frequency. The maximum output power or power density means the maximum value of the total output power or power density with respect to the driving frequency for a given input voltage.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dielectric constant $\varepsilon_{1}/\varepsilon_{0}$</td>
<td>1590</td>
</tr>
<tr>
<td>Electromechanical coupling factor $k_{15}$</td>
<td>0.70</td>
</tr>
<tr>
<td>Piezoelectric coefficient $d_{15}$, $10^{-12}$ C/N</td>
<td>510</td>
</tr>
<tr>
<td>Density, $10^{3}$ kg/m$^3$</td>
<td>7.8</td>
</tr>
<tr>
<td>Frequency constant $N_{15}$, mm-Hz</td>
<td>960</td>
</tr>
<tr>
<td>Mechanical quality factor $Q_m$</td>
<td>2500</td>
</tr>
</tbody>
</table>

A piezoelectric transformer with the structure shown in Figure 4-1, the dimensions in Table 4-1, and the material properties in Table 4-2, is taken for calculation. Table 4-3 shows the calculated equivalent circuit parameters. In Table 4-4, we illustrate a comparison of the theoretical and experimental values of the equivalent inductances and capacitance ratio. The capacitance ratio is the ratio of the clamped capacitance to the equivalent capacitance. It is $C_{dl}/C$, $C_{d2}/C_1$ and $C_{d3}/C_2$ for the input, output 1 and output 2, respectively. It is seen that the theoretical results are in agreement with the experimental ones. The small difference between the theoretical and experimental
inductances results from the fact that we ignored the kinetic energy in the short-circuit parts in the calculation of equivalent inductance. The difference between the measured and the theoretical capacitance ratio is somewhat large. This is because in the measurement of the capacitance ratio, some spurious vibration modes near the thickness shear vibration mode were taken into account by the impedance analyzer (HP4194A). Based on the calculation of these parameters, we can analyze and optimize the dual-output piezoelectric transformer operating at the thickness shear vibration mode.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ ($\Omega$)</td>
<td>1.03</td>
<td>$C_{d1}$ (nF)</td>
<td>0.422</td>
</tr>
<tr>
<td>$L$ (mH)</td>
<td>0.39</td>
<td>$C_{d2}$ (nF)</td>
<td>0.633</td>
</tr>
<tr>
<td>$C$ (pF)</td>
<td>58.3</td>
<td>$C_{d3}$ (nF)</td>
<td>1.055</td>
</tr>
<tr>
<td>$n_1$</td>
<td>0.82</td>
<td>$R_{L1}$ ($\Omega$)</td>
<td>238</td>
</tr>
<tr>
<td>$n_2$</td>
<td>0.63</td>
<td>$R_{L2}$ ($\Omega$)</td>
<td>143</td>
</tr>
</tbody>
</table>

Figure 4-4 shows the input impedance versus driving frequency of the piezoelectric transformer. It is seen that the theoretical and experimental results are in good agreement, especially in the frequency range between the resonance frequency $f_r$ and antiresonance frequency $f_a$, in which the piezoelectric transformer has a very good performance [37]. It is more reasonable to use this impedance characteristic to analyze the operation of the transformer than to use that measured with short-circuited outputs. A comparison between the theoretical and experimental voltage gains of output 1 is shown in Figure 4-5. It is seen that the theoretical and experimental results are in good
agreement, except that the theoretical frequency corresponding to the maximum voltage gain is a little higher than the experimental one. The error of the material constants used in the theoretical calculation is the reason of this difference. The efficiency vs. driving frequency of the transformer was also calculated by the equivalent circuit. Figure 4-6 shows a comparison between the calculated and experimental results. It can be observed that although the theoretical result approximately agrees with the experimental one, the theoretical frequency range with the best efficiency is a little larger than the experimental one. One possible reason of these phenomena is the influence of some spurious vibration modes in the working frequency range. Another possible reason is that the influence of the temperature rise on the efficiency of the transformer was ignored in the calculation. Actually, the temperature rise increases when the input voltage increases, especially when the input voltage is relatively high, which would aggravate the internal loss of piezoelectric transformers [39].

Table 4-4  A comparison of some theoretical and experimental parameters of the piezoelectric transformer

<table>
<thead>
<tr>
<th></th>
<th>Calculated</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent inductance</td>
<td>$L$</td>
<td>0.39</td>
</tr>
<tr>
<td>$L_m$ (mH)</td>
<td>$L_1$</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>$L_2$</td>
<td>0.156</td>
</tr>
<tr>
<td>Capacitance ratio</td>
<td>$C_{d1}/C$</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>$C_{d2}/C_1$</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>$C_{d3}/C_2$</td>
<td>7.2</td>
</tr>
<tr>
<td>Resonance frequency (MHz)</td>
<td>1.056</td>
<td>1.045</td>
</tr>
</tbody>
</table>
Figure 4-4 A comparison of the theoretical and experimental input impedances of the transformer with matching loads.

Figure 4-5 Voltage gain vs. driving frequency of the piezoelectric transformer with matching loads.
Chapter 4 Modeling and analysis of dual-output thickness shear vibration mode PT

![Graph showing efficiency vs. driving frequency](image)

Figure 4-6 Efficiency vs. driving frequency of the piezoelectric transformer with matching loads.

4.3 Numerical Results and Analyses

4.3.1 Effects of configuration on the PT's performance

Figure 4-7 shows the dependence of the maximum output power and efficiency on the thickness of the transformer with the constant length and width. The efficiency corresponds to the maximum output power. It is seen that the maximum output power and the efficiency of the transformer decrease with the increase of the thickness at a fixed input voltage. From equations (4-9a) and (4-11), it is known that the equivalent resistance is proportional to the square of the thickness. Thus the resistance increases
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rapidly with the increase of the thickness ($\omega_0 \propto t^{-1}$). Also, the electric field strength is lowered by increasing the thickness at a fixed input voltage. Therefore, it is an effective way to improve the power density of the piezoelectric transformer by properly decreasing the thickness.

Figure 4-7 Dependence of the maximum output power and efficiency on the thickness of the transformer when the total length and widths of the input and outputs are constant.

The influence of the width and length on the maximum voltage gains of the transformer is shown in Figure 4-8. It can be observed that for both output 1 and output 2 the voltage gain decreases with the increase of the width, and increases with the increase of the length when the other dimensions and the width ratio among the input, output 1 and output 2 are constant. In the former phenomenon, the mass of the transformer increases with increasing the width, and the vibration of the transformer decreases with increasing the mass. In the latter, the equivalent stiffness of the
Chapter 4 Modeling and analysis of dual-output thickness shear vibration mode PT transformer increases with increasing the length, and the vibration increases with increasing the equivalent stiffness. In this case, the influence of the equivalent stiffness change on the vibration is larger than that of the mass change. Therefore the voltage gain decreases with the increase of the width, and increases with the increase of the length.

Figure 4-9 shows the effect of the width of output 2 on the voltage gain, maximum output power and maximum power density when the other dimensions and input voltage are constant. The maximum output power is the maximum value of the total output power of output 1 and output 2 with respect to driving frequency. It can be observed that the voltage gain of output 2 decreases with the increase of \( w_2 \). But the voltage gain of output 1 is constant with the increase of \( w_2 \). The reason is that the voltage gain of an output in the transformer is mainly decided by the width ratio between the output and input [see (4-1) and (4-9)] under the computation condition shown in the figure. It also can be observed that the maximum output power is almost constant with the increase of \( w_2 \). The reason of this is that the clamped capacitance of output 2 increases with the increase of \( w_2 \). Thus the matching load of output 2 decreases with the increase of the width. Due to this, the maximum output power has little change with the width change of output 2 [voltage at output 2 decreases as \( w_2 \) increases; see Fig.4-9(a)] However, the maximum power density of the transformer decreases when \( w_2 \) increases because of the increase of the total volume of the transformer.
Figure 4-8  (a) Maximum voltage gain vs. the width of the transformer when the total length, thickness and width ratio among the input, output 1 and output 2 are constant; (b) maximum voltage gain vs. the length of the transformer when the thickness and widths of the input and outputs are constant.
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Figure 4-9  (a) Maximum voltage gain vs. the width of output 2 when the length, thickness and width of the input and output 1 are constant; (b) maximum output power and power density vs. the width of output 2 when the length, thickness and width of the input and output 1 are constant.
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Figure 4-10 shows the effect of the width of the input on the voltage gain, maximum output power and maximum power density of the transformer when the other dimensions and input voltage are constant. From Figure 4-10 (a), it is seen that the voltage gains of the two outputs all increase with the increase of $w_{in}$. This is because of the increase of $n_1$ and $n_2$ [see (4-1) and (4-9)]. From Figure 4-10 (b), it is seen that the maximum output power increases with the increase of $w_{in}$. The reason of this is that the voltage gains of the outputs increase with the increase of $w_{in}$. From Figure 4-10 (b), it is also seen that the maximum power density increases rapidly when $w_{in}$ is not much larger than $w_1$ or $w_2$, and saturates when $w_{in}$ is large enough. The reason is that $w_{in}/(w_{in} + w_1 + w_2)$ increases rapidly with the increase of $w_{in}$ when $w_{in}$ is not much larger than $w_1$ or $w_2$, and increases very slowly with the increase of $w_{in}$ when $w_{in}$ is large enough.
Figure 4-10 (a) Maximum voltage gain vs. the width of input when the length, thickness and width of the outputs are constant; (b) maximum output power and power density vs. the width of input when the length, thickness and width of the outputs are constant.

4.3.2 Effect of the load of output 1 on the voltage gain of output 2

To investigate the effect of the load of one output on the voltage gain of another output, the dependence of the voltage gain of output 2 on the driving frequencies was calculated with matched, short-circuited and open-circuited output 1, as shown in Figure 4-11. The load resistance of output 2 is matched in the calculation. In the frequency range from 1.065 MHz to 1.08 MHz, in which the proposed transformer has the best efficiency (please refer to Figure 4-6), the difference of voltage gain between the situations with matched and short-circuited output 1 is not large.
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However, the difference of voltage gain between the situations with matched and open-circuited output 1 is relatively large in the above frequency range. Therefore, to keep the voltage of one output constant, the open-circuit of another output needs be avoided.

![Figure 4-11 Effect of the load of output 1 on the voltage gain of output 2.](image)

Figure 4-11 Effect of the load of output 1 on the voltage gain of output 2.
In this chapter, the vibration distribution in output section of the piezoelectric transformer operating at the thickness shear mode is investigated experimentally and theoretically. It is experimentally found that the vibration of output section has a spatial gradient and attenuates along the direction of vibration propagation. A theoretical model is developed to estimate the vibration attenuation in output section, which provides the guidelines of optimizing the transformer and explains some important phenomena such as the non-uniform temperature distribution in piezoelectric transformers. The larger the load resistance, the larger the vibration gradient is. At the resonance point, the vibration gradient decreases with increasing the width of input or output section, and changes little with the length and thickness of the transformer when the load resistance matches with the piezoelectric transformer. The vibration gradient increases with increasing the length or decreasing the thickness when the load resistance is constant, in which the load does not match with the transformer. Effect of load resistance and dimensions on the vibration displacement of input section is also studied by this model. At the resonance point, the vibration
displacement of input section increases with decreasing the length or the width of output section, or increasing the thickness of the transformer when the load resistance is constant.

5.1 Experimental Measurement

Figure 5-1 illustrates the basic configuration of the dual-output piezoelectric transformer for the measurement and modeling. The electrodes of input and output sections on the top and bottom surfaces of the ceramic plate are separated by the narrow insulting gaps along the length direction. According to our previous experimental results, this structure has high electromechanical-coupling factor $k_{\text{eff}}$ and very good characteristics [36-38].

![Figure 5-1](image)

Figure 5-1 The basic configuration of the dual-output piezoelectric transformer.

Table 5-1 shows the dimensions of the transformer. The vibration displacement of the transformer's output section was measured by a microscope (OLYMPUS BX51, JAPAN). The experimental set-up for the measurement is shown in Figure 5-2. The
piezoelectric transformer was clamped at three points on its nodal plane. The vibration
displacement of output 1 was measured at an input voltage of 40 volts and near
resonance point. In the measurement, a prominent signed point on the PT's surface
was chosen to be the reference point. Figure 5-3 shows the pictures taken by the
microscope at \( z = 0 \text{ mm} \) when the piezoelectric transformer is (a) before vibration, (b)
in vibration. Firstly we determine the edge of blur area induced by the marked point
by the images when the transformer is in vibration. Then measure the displacement
between this edge and the marked point when the transformer is not working. The
double of this displacement is just the experimental displacement.

There are two methods in using microscope to measure ultrasonic vibration. One is to
use a high capture rate to get many images. From them, one obtains the information of
vibration. This method needs a very high speed camera. Another method is to use an
exposure time much much larger than the time period of vibration. This is the
method used in our experiments. The exposure time of BX51 has an order of ms or
larger; while the time period of the ultrasonic vibration is in micro second order \( f_r =
264 \text{ kHz} \). So during the exposure time, there are several thousands of vibration period.
This gives us a quite clear image of the blur area induced by the vibration. In practice,
people use this method to measure ultrasonic vibration because it is cheap and quite
effective for small vibrators. It is recognized by the academic society related to
ultrasonic engineering [74]. Due to the structure of microscopes, this method can only
be used to measure small vibrators.

Figure 5-4 shows the measured vibration displacement distribution along the width
direction of output 1. It can be observed that the vibration displacement decreases
Chapter 5 Vibration Distribution in Output Section Of the Piezoelectric Transformer

along the vibration propagation direction in output section. This is because the output voltage induced by the piezoelectric effect in output section counteracts the vibration propagated from input section. According to our observation, the output voltage is approximately out of phase with the input voltage by $\pi$ near the resonance point.

Table 5-1 Dimensions of the dual-output transformer

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>Length of the ceramic plate</td>
<td>60</td>
</tr>
<tr>
<td>$w_{in}$</td>
<td>Width of the input part</td>
<td>2</td>
</tr>
<tr>
<td>$w_1$</td>
<td>Width of output 1</td>
<td>9</td>
</tr>
<tr>
<td>$w_2$</td>
<td>Width of output 2</td>
<td>9</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 5-2 Experimental set-up of vibration displacement measurement for thickness shear mode.
Figure 5-3  The pictures taken by the microscope at $z=0$ mm when the piezoelectric transformer is (a) before vibration, (b) in vibration.
Chapter 5 Vibration Distribution in Output Section Of the Piezoelectric Transformer

Figure 5-4 Vibration displacement distribution along the width direction of output 1 with the matching load resistances at two outputs.

5.2 Theoretical Modeling and Verifications

5.2.1 Theoretical modeling

Figure 5-5 shows the output 1 and the coordinate system used for modeling. The basic piezoelectric equations are as follows,

\[ T_5 = C_{55}^a S_5 - h_{15} D_1 \]  
\[ E_1 = -h_{15} S_5 + \beta_{11}^5 D_1 \]  \hspace{1cm} (5-1a)

It was experimentally found that the vibration induced by the input voltage in input section is uniform [40]. So we can assume that the electric displacement of input is

\[ D_1 = D_0 \]  \hspace{1cm} (5-2)
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Figure 5-5  Output 1 of the transformer and the coordinate system used in this model.

The thickness shear stress and strain are continuous from input section to output section due to the very narrow insulating gap between them. So the electric displacement of output 1 at \( z = 0 \) is

\[
D_1|_{z=0} = D_0
\]

According to our experimental results shown in Figure 5-4, the vibration of output section decreases rapidly along the direction of vibration propagation. So we approximately assume that

\[
D_i = D_0 e^{-\alpha z},
\]

where \( \alpha \) is an attenuation factor, which expresses the vibration distribution of output 1. The charges on the surface of output 1 is

\[
Q = \iiint D_i dydz
\]

\[
= \frac{lD_0}{\alpha} \left( 1 - e^{-\alpha \xi_2} \right).
\]

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Chapter 5 Vibration Distribution in Output Section Of the Piezoelectric Transformer

Then the current at output 1 at resonance frequency \( \omega_0 \) is

\[
l = \frac{i\omega_0 lD_0}{\alpha} (1 - e^{-a_w}). \quad (5-6)
\]

From (5-1 b), the electric displacement \( D_0 \) of input is

\[
D_0 = \frac{E_0}{\beta_{11}} + \frac{h_5}{\beta_{11}} S_5. \quad (5-7)
\]

There is no free charge in the ceramic, so

\[
\frac{\partial D_0}{\partial x} = 0. \quad (5-8)
\]

Also

\[
S_5 = \frac{\partial \xi}{\partial x}, \quad (5-9)
\]

where \( \xi \) is the thickness shear displacement. From (5-7), (5-8), and (5-9),

\[
\frac{\partial E_0}{\partial x} = -h_5 \frac{\partial^2 \xi}{\partial x^2}. \quad (5-10)
\]

Thus

\[
E_0 = -h_5 \frac{\partial \xi}{\partial x} + a, \quad (5-11)
\]

where \( a \) is an integral constant. The input voltage applied to the input port is

\[
V_m = \int E_0 dx. \quad (5-12 \text{ a})
\]

From (5-11) and (5-12 a), the integral constant \( a \) is

\[
a = \frac{V_m - h_5}{t} (\xi_2 + \xi_1), \quad (5-12 \text{ b})
\]

where \( \xi_1 \) and \( \xi_2 \) are the thickness shear displacements at the surfaces \( x = 0 \) and \( x = t \), respectively, which are equal in normal operation. From (5-11) and (5-12b)

\[
E_0 = -h_5 \frac{\partial \xi}{\partial x} + \frac{V_m}{t} - \frac{h_5}{t} (\xi_2 + \xi_1), \quad (5-13)
\]

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From (5-7) and (5-13),

\[ D_0 = \frac{V_{in}}{\beta_{11} s t} - \frac{h_{15}}{\beta_{11} s t} (\xi_2 + \xi_1). \]  

(5-14)

Denoting \( \xi_1 \) and \( \xi_2 \) by \( \xi_0 \), the electric displacement at the input is

\[ D_0 = \frac{V_{in}}{\beta_{11} s t} - \frac{h_{15}}{\beta_{11} s t} (2\xi_0) \]  

(5-15)

From (5-6) and (5-15), \( \left| \frac{V_o}{V_{in}} \right| \) for a load resistance \( R_x \) is

\[ \left| \frac{V_o}{V_{in}} \right| = \frac{\omega_0 R_x (1 - e^{-\omega_0 t})}{\alpha \beta_{11} s t} \left( 1 - 2h_{15} \frac{\xi_0}{V_{in}} \right) \]  

(5-16)

Figure 5-6 Equivalent circuit of proposed piezoelectric transformer.

Figure 5-6 shows the equivalent circuit of the dual-output piezoelectric transformer, in which \( L \), \( C \) and \( R \) are the equivalent inductance, capacitance and resistance, respectively [40], [57]; \( C_d \), \( C_1 \) and \( C_2 \) are the clamped capacitance of the input, output
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1 and output 2, respectively; \( R_1 \) and \( R_2 \) are the load resistances of output 1 and output 2, respectively.

\[
L = \frac{\rho t^3}{8w_m f} \left( \frac{\beta_{1s}}{h_{15}} \right)^2 \quad (5-17a)
\]

\[
C = \frac{8w_m l}{\pi^2 C_{ss} n t} \left( \frac{h_{5s}}{\beta_{1s}} \right)^2 \quad (5-17b)
\]

\[
R = \frac{\omega_0 L}{Q_m} \quad (5-17c)
\]

where \( \beta_{1s} \) is the permittivity constant, \( h_{15} \) the piezoelectric constant and \( \omega_0 \) the angular operating frequency. The turns ratio of the ideal transformer \( n_1 \) and \( n_2 \) can be calculated by

\[
n_1 = \frac{A_0}{A_1} = \sqrt{\frac{L_1}{L}} \quad \text{and} \quad n_2 = \frac{A_0}{A_2} = \sqrt{\frac{L_2}{L}} \quad (5-18)
\]

where \( A_0, A_1 \) and \( A_2 \) are the force factors of the input, output 1 and output 2, respectively; \( L_1 \) and \( L_2 \) are the equivalent inductance of output 1 and output 2, respectively. \( Z_1 \) and \( Z_2 \) are the impedances of the two output loops referred to the motional branch and can by expressed as

\[
Z_1 = \frac{1}{n_1^2} \left( \frac{R_1}{1 + j\omega_0 C_1 R_1} \right) \quad (5-19a)
\]

\[
Z_2 = \frac{1}{n_2^2} \left( \frac{R_2}{1 + j\omega_0 C_2 R_2} \right) \quad (5-19b)
\]

Denoting

\[
Z_0 = R + j\omega_0 L + \frac{1}{j\omega_0 C}, \quad (5-20)
\]

the motional current is
The energy stored in $L$ is equal to the kinetic energy of the transformer. Therefore

$$\frac{1}{2} m_e (\omega_0 \xi_0)^2 = \frac{1}{2} L I_m^2$$

(5-22)

where $m_e$ is the equivalent mass. Assuming a sine vibration displacement distribution in the $x$ direction, it can be proved that

$$m_e = \frac{1}{2} \rho l w_m.$$  

(5-23)

From (5-22),

$$\xi_0 = \frac{1}{\omega_0} \frac{m_e}{L}.$$  

(5-24)

From (5-16), (5-21) and (5-24) that

$$\frac{V_o}{V_m} = \frac{\alpha \beta_1 t}{\alpha_0} \left(1 - e^{-\alpha_0 t}\right) \left| \frac{2k_t}{\alpha_0 (Z_0 + Z_2) L} \right| - 1.$$  

(5-25)

It can be derived from the equivalent circuit that

$$I_{o1} = \frac{V_m}{n_i (Z_0 + Z_2)}.$$  

(5-26)

Thus,

$$V_0 = \frac{-V_m}{n_i (Z_0 + Z_2)} \cdot \frac{R_x}{1 + j \alpha_0 C_i R_x}.$$  

(5-27)

From (5-19a) and (5-27),

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Chapter 5 Vibration Distribution in Output Section Of the Piezoelectric Transformer

\[
\frac{V_o}{V_i} = n_i \frac{|Z_i|}{Z_0 + \frac{Z_i Z_2}{Z_1 + Z_2}} \quad (5-28)
\]

From equation (5-25) and (5-28),

\[
\frac{2h_{i5}}{\omega_0 (Z_0 + \frac{Z_i Z_2}{Z_1 + Z_2})} \left( \frac{m_x}{L} \right) \alpha = 1 - e^{-\omega_0 t} \quad (5-29)
\]

The thickness shear vibration distribution factor \(\alpha\) can be calculated from this nonlinear equation.

From (5-1b), (5-4) and (5-15), the displacement \(\xi_{out}\) of output 1 also can be obtained.

\[
\xi_{out} = -\frac{\left( V_i - 2h_{i5} \xi_0 \right) e^{-\omega_0 t}}{\alpha \beta_{i1} s t} \frac{\left( 1 - e^{-\alpha t} \right)}{\left( 1 - 2h_{i5} \frac{\xi_0}{V_i} \right)} \quad (5-30)
\]

### 5.2.2 Verification of theoretical model

Figure 5-4 shows the vibration displacement distribution along the width direction of output 1 with matching load resistances at two output ports \((R_x = 79\Omega; R_2 = 79\Omega)\). It is seen that the calculated vibration distribution agrees quite well with the measured one. Both theoretical and experimental results show that the vibration displacement decreases along the vibration propagation direction in output section. In measuring the
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characteristics of the transformer [36], it was observed that the input section has a higher temperature than the output section. The result from Figure 5-4 can explain this phenomenon very well. It also can be seen that the measured vibration displacement is larger than the calculated one. This may be due to the error of the equivalent circuit.

Figure 5-7 shows the dependence of the vibration displacement at $z = 4.5$ mm on the load resistance $R_x$ of output 1. Both theoretical and experimental results show that the displacement decreases rapidly with the increase of load resistance and reaches a stable value when $R_x$ is larger than a certain value. The reason of this phenomenon is that the output voltage increases with the increase of $R_x$ when $R_x$ is not large enough, but becomes saturated when $R_x$ is much larger than the matching load [36].

![Figure 5-7](image)

**Figure 5-7** Dependence of the vibration displacement at $z = 4.5$ mm on the load resistance of output 1.
Table 5-2 Relevant material property constants of the piezoelectric ceramic plate

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dielectric constant $\varepsilon_{11}/\varepsilon_0$</td>
<td>1590</td>
</tr>
<tr>
<td>Electromechanical coupling factor $k_{15}$</td>
<td>0.70</td>
</tr>
<tr>
<td>Piezoelectric charge constant $d_{15}, 10^{-12}$ C/N</td>
<td>510</td>
</tr>
<tr>
<td>Youngs modulus $Y_{11}/E, 10^{10}$ N/m²</td>
<td>8.2</td>
</tr>
<tr>
<td>Piezoelectric voltage constant $g_{15}, 10^{-3}$ m²/C</td>
<td>36.4</td>
</tr>
<tr>
<td>Density, $10^3$ kg/m³</td>
<td>7.8</td>
</tr>
<tr>
<td>Frequency constant $N_{15},$ mm·Hz</td>
<td>960</td>
</tr>
<tr>
<td>Mechanical quality factor $Q_m$</td>
<td>2500</td>
</tr>
</tbody>
</table>

5.3 Numerical Results and Analyses

The dimensions shown in Table 5-1 and material property constants shown in Table 5-2 are used in calculation. Unless otherwise specified, the transformer for the modeling operates at the resonance frequency with an input voltage of 40 V$\text{rms}$; the load resistance of output 2 matches with the transformer ($R_2 = \frac{1}{\omega_0 C_2}$).

5.3.1 Effects of load resistance and dimensions on $\alpha$

Figure 5-8 (a) shows the dependence of the attenuation factor $\alpha$ on the load resistance $R_1$ of output 1. It is seen that $\alpha$ increases with increasing the load resistance.
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at the resonance point. The reason of this is that the output voltage increases with increasing the load resistance. Figure 5-8 (b) shows the vibration displacement distribution for different $\alpha$. As expected, the less $\alpha$ value, the smaller the vibration gradient is. The base quantity of the normalized displacement is the vibration displacement $\xi_0$ in input section. As shown in Figure 5-8 (a), $\xi_0$ decreases with increasing the load resistance at the resonance point.

The increase of attenuation factor $\alpha$ will increase the non-uniformity of vibration distribution in the output section, and thus increase the temperature difference between the input and the output sections. The larger the temperature difference between the input and output sections, the poorer power management ability the transformer have [24]. Therefore, in order to improve the power density of the piezoelectric transformer, we should decrease the value of $\alpha$ in the structure design.

From Figure 5-4 and 5-8 (b), it can be seen that the vibration is relatively small near the edge of output section. So the supporting structure and lead wire connection should be arranged near the edge of output section to decrease the loss of vibration energy.

Figure 5-9 shows the dependence of the attenuation factor $\alpha$ on the length of the transformer with the other dimensions constant. It is seen that $\alpha$ increases with increasing the length when the load resistance and other dimensions are constant; while $\alpha$ has little change with increasing the length when the load resistance $R_x$ is matched ($R_x = \frac{1}{\omega_0 C_1}$) and other dimensions are constant. As we know that clamped capacitance $C_i$ ($C_i = \frac{L\varepsilon}{t}$) increases with increasing the length, and the voltage gain
increases with increasing $R_s / R_t$, where $R_s = \frac{1}{\omega_0 C_1}$ is the matching load resistance of output 1. Then the output voltage $V_o$ increases with increasing the length for a given input voltage. Therefore, $\alpha$ increases with increasing the length of the transformer when the load resistance and other dimensions are constant. However, when the load resistance is matched, the phase difference between $V_o$ and $V_{in}$ is approximately constant because $|Z_1 + Z_2| \approx R$ at the resonance point. Also according to our previous study [36], the voltage gain has little change with increasing the length when the load resistance is matched. So the phase and amplitude of output voltage has little change with the length for a given input voltage. This is why $\alpha$ has little change with increasing the length when the load resistance is matched and other dimensions are constant.

The dependence of $\alpha$ on the thickness of the transformer with the other dimensions constant is shown in Figure 5-10. Due to the similar reason as Figure 5-8, the attenuation factor $\alpha$ decreases with increasing the thickness when the load resistance ($R_s = 100\Omega$) and other dimensions are constant. While $\alpha$ has little change with increasing the thickness when the load resistance is matched ($R_s = \frac{1}{\omega_0 C_1}$) and other dimensions are constant. From Figure 5-9 and 5-10, it can be observed that the change of length and thickness of the transformer has little influence on the vibration distribution of output section when the load resistance is matched. But if the load resistance is constant, the increase of length and decrease of thickness will make the vibration distribution more non-uniform. This must be taken consideration in designing the structure of high power transformers.
Figure 5-8  (a) Dependence of the attenuation factor $\alpha$ on the load resistance of output; (b) vibration displacement distribution with different $\alpha$. 

(a) Dependence of the attenuation factor $\alpha$ on the load resistance $R_x$ with $V_{in} = 40 V$ and $R_x$ is matched.

(b) Vibration displacement distribution with different $\alpha$.
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Figure 5-9 Dependence of the attenuation factor \( \alpha \) on the length of the transformer with the other dimensions constant.

Figure 5-10 Dependence of the attenuation factor \( \alpha \) on the thickness of the transformer with the other dimensions of the transformer constant.
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Figure 5-11 shows the dependence of $\alpha$ on the width of output 1 with the other dimensions constant. It is seen that $\alpha$ increases with increasing the width $w_1$ of output 1 when the load resistance $R_x$ is constant. But $\alpha$ decreases with increasing $w_1$ when $R_x$ is matched. Therefore, it is a feasible way to improve the power density of the transformer by increasing the width of output with $R_x$ matched. Figure 5-12 shows the dependence of the attenuation factor $\alpha$ on the width of input with the other dimensions constant and load resistances matched. It is seen that $\alpha$ decreases with increasing $w_{in}$. It means the larger the width of input section, the more uniform the vibration distribution of output section is.

![Graph showing the dependence of $\alpha$ on $w_{out1}$](image)

Figure 5-11 Dependence of the attenuation factor $\alpha$ on the width of output 1 with the other dimensions of the transformer constant.
5.3.2 Effects of dimensions on $\xi_0$ at the resonance point

The influence of dimensions on the vibration displacement of input section $\xi_0$ was also investigated. Figure 5-13 (a) shows the dependence of $\xi_0$ on the length of the transformer when the other dimensions of the transformer are constant. The increase of the length has little influence on $\xi_0$ when $R_x$ is matched. But $\xi_0$ decreases with increasing the length and become saturated after the length reaches a certain value when $R_x$ is constant. From equation (5-17a), (5-23) and (5-24), it can be concluded that the equivalent mass $m_e$ increases and equivalent inductance $L$ decreases with increasing the length, and the increase of $m_e$ and decrease of $L$ induces the decrease of

Figure 5-12 Dependence of the attenuation factor $\alpha$ on the width of input with the other dimensions of the transformer constant and matching load resistances.
displacement of input section $\xi_0$ when the load resistance and other dimensions are constant. But this influence will be counteracted by the decrease of impedance $Z_I$ when the load resistance is matched. So $\xi_0$ is almost constant with increasing the thickness of transformer when the load resistance is matched and other dimensions are constant.

Figure 5-13 (b) shows the dependence of $\xi_0$ on the thickness of the transformer when the other dimensions of the transformer are constant. The increase of the thickness has little influence on $\xi_0$ when $R_x$ is matched. But $\xi_0$ increases with increasing the thickness when $R_x$ is constant. From equation (5-17a), (5-23) and (5-24), it can be concluded that displacement of input section $\xi_0$ increases with the thickness when the load resistance and other dimensions are constant. But this influence will be counteracted by the increase of impedance $Z_I$ when the load resistance is matched. So $\xi_0$ is almost constant with increasing the thickness of the transformer when the load resistance is matched and other dimensions are constant.

Figure 5-13 (c) shows the dependence of $\xi_0$ on the width of output 1 with the other dimensions of the transformer constant. The increase of the width of output has little influence on $\xi_0$ when $R_x$ is matched. But $\xi_0$ decreases with increasing the width of output and become saturated after width reaches a certain value when $R_x$ is constant. Although the influence of $w_I$ on $m_e$ was omitted in the calculation, the increase of $w_I$ also causes the increases of $m_e$. So it can be observed from equation (5-24), $\xi_0$ decreases with increasing $w_I$ when the load resistance $R_x$ is matched.
Chapter 5 Vibration Distribution in Output Section Of the Piezoelectric Transformer

(a)

(b)
Chapter 5 Vibration Distribution in Output Section of the Piezoelectric Transformer

![Graph](image)

Figure 5-13 Dependence of $\xi_0$ on (a) the length; (b) the thickness and (c) the width of output 1 when the other dimensions of the transformer are constant.
Phase shifters have many applications in sensor and actuator systems, and some other AC power systems. So far, the electronic and photonic phase shifters have been widely used to shift the phase of an AC voltage [64-68]. The poor power management ability is the main disadvantage of those phase shifters. Ferroelectric film phase shifters based on the dielectric permittivity $\varepsilon(E)$ control were also developed [69-71]. The disadvantages of this type of phase shifter are that it has a narrow phase shift range, and demands a relatively high DC voltage and complicated fabrication process.

In this study, a low-frequency piezoelectric phase shifter based on a new technique was developed and investigated. The technique is derived from the multioutput piezoelectric transformer. The phase shifter is made of a piezoelectric ceramic plate operating at the thickness shear vibration mode. The ceramic plate consists of input, output and control parts. The phase shift between the output and input voltages can be smoothly tuned by an inductor or capacitor connected to the control part. The phase shift can be increased by decreasing the inductance or capacitance at the control part. Tuning a DC bias voltage applied to the input part may also change the phase shift. A maximum phase shift of 128 deg and minimum phase shift of -10 deg were achieved by this method. Compared with the commonly used phase shifters, the piezoelectric
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phase shifter not only has a very simple structure, light weight and low profile, but has a wide phase shift range, good ability to manage high power and relatively high energy transmission efficiency.

6.1 Design and Fabrication of the Piezoelectric Phase Shifter

6.1.1 Configuration of the phase shifter

![Diagram of piezoelectric phase shifter]

Figure 6-1 Construction and dimensions of the piezoelectric phase shifter.

Table 6-1. Relevant property constants of the piezoelectric material

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dielectric constant $\varepsilon_{11}/\varepsilon_0$</td>
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</tr>
<tr>
<td>Mechanical quality factor $Q_m$</td>
<td>2500</td>
</tr>
</tbody>
</table>
Chapter 6 A Tunable Low-Frequency Piezoelectric Phase Shifter

The configuration of the tunable piezoelectric phase shifter is schematically shown in Figure 6-1. It is made of a piezoelectric ceramic plate with the dimensions of 60 mm × 20 mm × 2 mm, poled along the width direction. The electrodes on the top and bottom surfaces are separated by the narrow insulating gaps along the length direction to form the input, output and control parts. The width of input, output and control parts is 2mm, 9mm and 9mm, respectively. The electric field at the input part is perpendicular to the poling direction and the ceramic plate operates at the thickness shear vibration along the poling direction. The relevant material constants of the ceramic plate are shown in Table 6-1. In order to attain the best efficiency of the phase shifter, load resistance of \( R_L = 1/(\omega C_{dl}) \) was used to match the output part. Here \( C_{dl} \) is the clamped capacitance of the output and \( \omega \) is the angular resonance frequency of the phase shifter. Tunable inductor or capacitor is used at the control part to tune the phase difference between the output and input voltages. The inductance \( L_x \) and capacitance \( C_x \) of them are called as control inductance and control capacitance, respectively. Figure 6-2 is a photograph of the phase shifter.

### 6.1.2 A multi-point connection and supporting structure

Figure 6-3 shows a multi-point connection and supporting structure developed for the piezoelectric phase shifter. The projections are made of conductive metal and work as feeding lines. The supporters are the main component to support and fix the piezoelectric component. This structure can not only makes the supporting and lead wire connection of the phase shifter stable, but also decreases the temperature rise effectively. Temperature rise of piezoelectric devices is the main obstacle to improve the power management ability of the devices [39, 41]. The size of the supporting...
structure shown in Figure 6-3 can be reduced further by decreasing the thickness and height of the plates.

Figure 6-2 Photograph of the piezoelectric phase shifter.

Figure 6-3 A multi-point connection and supporting structure for the phase shifter.
6.1.3 Operation principle of the phase shifter

Figure 6-4 shows the equivalent circuit of the piezoelectric phase shifter, which is the same as that of a multioutput piezoelectric transformer [36-38]. In Figure 6-4, \( L, C \) and \( R \) are the equivalent inductance, capacitance and resistance, respectively; \( C_{d1}, C_{d2} \) and \( C_{d3} \) are the clamped capacitance of the input, control and output parts, respectively; \( n_1 \) and \( n_2 \) are the turns ratio of the ideal transformers for the control and output parts, respectively. Experimental values of the equivalent circuit parameters are shown in Table 6-2. Here the values of \( L, C, R, C_{d1}, C_{d2}, C_{d3}, \) resonance frequency \( f_r \) and antiresonance frequency \( f_a \) were measured by the impedance analyzer HP 4194 A, and \( n_1 \) and \( n_2 \) was calculated by

\[
n_1 = \sqrt{\frac{L_2}{L}} \quad \text{and} \quad n_2 = \sqrt{\frac{L_3}{L}},
\]

(6-1)

where \( L_2 \) and \( L_3 \) are the equivalent inductance of control and output parts, respectively, which were measured by impedance analyzer. From the circuit, the phase difference between motional current \( I_m \) and the input voltage \( V_{in} \) is

\[
\angle I_m - \angle V_{in} = \arctan \left( \frac{1}{\omega C' - \omega L'} \right),
\]

(6-2a)

\[
L' = L + \Delta L
\]

(6-2b)

\[
C' = C + \Delta C
\]

(6-2c)

\[
R' = R + \Delta R,
\]

(6-2d)

where \( \Delta L, \Delta C, \) and \( \Delta R \) are the inductance, capacitance and resistance of the control and output parts referred to the motional branch. From the circuit, it is known that the phase difference between the motional current \( I_m \) and output current \( I_o \) is zero; the
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The phase difference between the output current $I_o$ and output voltage $V_o$ is approximately a constant $\phi$ under the matching load condition. So the phase difference $\Delta \phi$ between the output voltage $V_o$ and input voltage $V_{in}$ is

$$\Delta \phi = \angle V_o - \angle V_{in} = \arctan \left( \frac{1}{\omega C' - \omega L'} \right) + \phi_o.$$  

Therefore, the phase difference $\Delta \phi$ between the output voltage $V_o$ and input voltage $V_{in}$ is mainly affected by the phase difference between the motional current $I_m$ and $V_{in}$. $\Delta L$ and $\Delta C$ can be controlled by $L_x$ and $C_x$, respectively. Hence, the phase difference between the output voltage $V_o$ and input voltage $V_{in}$ can be controlled by $L_x$ or $C_x$.

![Figure 6-4](image.png)

Figure 6-4  Equivalent circuit of the piezoelectric phase shifter.

6.1.4 Simulation results

In order to validate the technique further, some theoretical simulation based on the equivalent circuit of the phase shifter was carried out. The simulation results are as
follows: Figure 6-5 (a) shows the normalized phase shift between the output and input voltage due to the varying inductance; and Figure 6-5 (b) shows the normalized phase shift between the output and input voltage due to the varying capacitance. It can be observed that the phase shift decreases with the increase of the inductance and capacitance values. Also, the phase shift changes with the varying driving frequency according to our simulation results. Therefore, it should be an effective method to tune the phase shift between the output and input voltage by changing the inductance and capacitance in the control part.

Table 6-2 The experimental values of the equivalent circuit parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(\Omega)$</td>
<td>2.8</td>
<td>$C_{d1}(nF)$</td>
<td>0.70</td>
</tr>
<tr>
<td>$L(mH)$</td>
<td>0.64</td>
<td>$C_{d2}(nF)$</td>
<td>1.7</td>
</tr>
<tr>
<td>$C(nF)$</td>
<td>0.148</td>
<td>$C_{d3}(nF)$</td>
<td>1.2</td>
</tr>
<tr>
<td>$n_1$</td>
<td>0.60</td>
<td>$R_d(\Omega)$</td>
<td>251</td>
</tr>
<tr>
<td>$n_2$</td>
<td>0.54</td>
<td>$f_a(kHz)$</td>
<td>571</td>
</tr>
<tr>
<td>$f_c(kHz)$</td>
<td>546</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
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Figure 6-5 The normalized phase shift between the output and input voltage due to (a) the varying inductor and (b) the varying capacitor.
6.2 Characteristics of the Tunable Piezoelectric Phase Shifter at a Low Input Voltage

In the following experimental results and discussion, the phase shift means the phase angle by which the output voltage leads the input voltage. Unless otherwise specified, the characteristics of the phase shifter were measured with a matching load of 251 Ω and input voltage of 10 V_{rms}.

6.2.1 Effect of control inductance on the phase shift

Figure 6-6 shows the phase shift, voltage gain and efficiency versus the control inductance at three different operating frequencies near the antiresonance point. It is seen that the phase shift can be changed smoothly by tuning the control inductance, and the phase shift range depends on the operating frequency. At 554 kHz, the phase shift changes from 120 deg to about 45 deg as \( L_x \) increases from 0.06 mH to about 2 mH. While at 558 kHz, the phase shift changes from 60 deg to 0 deg as \( L_x \) increases from 0.06 mH to about 2 mH. From the equivalent circuit and equation (6-2b), it is known that \( \Delta L \) increases with increasing the control inductance \( L_x \), and the increase of \( \Delta L \) causes the increase of \( L' \). Also, from equation (6-3), it is known that \( \Delta \phi \) decreases as \( L' \) increases. Therefore the phase difference between \( V_o \) and \( V_{in} \) decreases with increasing \( L_x \). From Figure 6-6, it is also seen that the phase shift decreases fast with increasing \( L_x \) when \( L_x \) is less than 0.25 mH and decreases little with increasing \( L_x \) when \( L_x \) is larger than 0.25 mH. This is due to the characteristic of
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arctan(x) curve. Near the antiresonance frequency, \( \frac{1}{\omega C'} - \omega L' < 0 \). When \( L' \) is small, \( \frac{1}{\omega C'} - \omega L' \) is a negative value close to zero. Thus, the phase shift decreases fast with increasing \( L' \). When \( L' \) is large, \( \frac{1}{\omega C'} - \omega L' \) is a negative value far away from zero. Thus, the phase shift decreases little with increasing \( L' \). It is also seen that the efficiency largely depends on the operating frequency and control inductance, and the maximum efficiency may be larger than 95%. So the phase shifter may have a very good tunability and efficiency when a tunable inductor is used at the control part. At the frequency of 554 kHz, the phase shift is 128.4 deg when the control part is short-circuit and 35.9 deg when the control part is open-circuit.
Figure 6-6 Phase shift, voltage gain and efficiency versus control inductance at different operating frequencies.
Figure 6-7 shows the phase shift, voltage gain and efficiency vs. operating frequency at different inductances. It is seen that as the operating frequency increases from 554 kHz to 560 kHz, the phase shift decreases from 120 deg to 40 deg at a control inductance of $1.96 \times 10^{-5}$ H, and from 55 deg to 0 deg at a control inductance of $3.22 \times 10^{-4}$ H. The efficiencies of the phase shifter are quite high (>80%) in the frequency range of 556 kHz to 559 kHz, which is around the antiresonance frequency of the phase shifter. Although the phase shift is very large when the operating frequency is lower than 554 kHz, the operation isn't recommended because of poor efficiency. The reason that the phase shift decreases with increasing the operating frequency can be obtained from the following analyses. Denoting the resonance angular frequency of the phase shifter with $L_s$ or $C_s$ by $\omega_0$ and the difference between the operating angular frequency $\omega$ and the resonance frequency by $\Delta \omega (= \omega - \omega_0)$, equation (6-3) can be changed into

$$\angle V_o - \angle V_{in} \equiv \arctan \left( \frac{-2L' \Delta \omega}{R'} \right) + \phi_0 . \quad (6-4)$$

From equation (6-4), it is seen that the phase shift between $V_o$ and $V_{in}$ decreases with increasing $\Delta \omega$. For a given $\omega_0$, $\Delta \omega$ increases with increasing $\omega$. So the phase shift decreases with increasing the operating frequency.
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(a)  

(b)
6.2.2 Effect of DC bias voltage on the phase shift

From equation (6-4), it is also known that the phase shift may be changed by adjusting the resonance frequency for a given operating frequency. There are two feasible methods to adjust the resonance frequency of the phase shift. One is to adjust the thickness of the ceramic plate and another is to apply a DC bias voltage to the input part. The latter is more convenient in real application. Figure 6-8 (a) shows the relationship between the phase shift and the DC bias voltage $V_{dc}$ applied to the input part at an operating frequency of 556 kHz and 560 kHz. The control inductance is $1.96\times10^{-5}$ H. The positive direction of the DC bias voltage is shown in Figure 6-1. It
can be observed that the phase shift increases with increasing the positive DC bias voltage and decreases with increasing the absolute value of negative DC bias voltage. These phenomena are due to the thickness change of the ceramic plate. Due to the error of fabrication process, the poling direction of the ceramic plate is not exactly in parallel with the electrodes, as shown in Figure 6-8 (b) and (c). When the positive DC bias voltage increases, the thickness of the ceramic plate decreases due to the rotation of the domains, as shown in Figure 6-8 (b). This decreases the thickness of the ceramic plate, and thus increases the resonance frequency. Therefore, as the positive DC bias voltage increases, $\Delta \omega$ decreases and thus the phase shift increases. On the contrary, when the absolute value of negative DC bias voltage increases, the thickness of the ceramic plate increases due to the rotation of the domains, as shown in Figure 6-8 (c). This increases the thickness of the ceramic plate, and thus decreases the resonance frequency. Therefore, as the negative DC bias voltage decreases, $\Delta \omega$ increases and then the phase shift decreases. With the DC bias voltage increases from -80 V to 80 V, the phase shift increases about 34 deg and 25 deg at the operating frequency of 556 kHz and 560 kHz, respectively. From Figures 6-6 (a), 6-7 (a) and 6-8, it is known that the phase shift range may be widened if we use the control inductance and the DC bias simultaneously and tune both of them.
Chapter 6 A Tunable Low-Frequency Piezoelectric Phase Shifter

(a)

Phase shift (deg)

- 556 kHz
- 560 kHz

$L_x = 1.96 \times 10^{-5} \text{ H}$

DC bias (V)

(b)

Domain directions

Poling Direction

$V_{DC}$
6.2.3 Effect of control capacitance on the phase shift

Figure 6-9 shows the phase shift, voltage gain and efficiency versus the control capacitance at different operating frequencies. It is seen that the phase shift decreases with increasing the control capacitance $C_x$. From the equivalent circuit and equation (6-2c), it is known that $C'$ increases with increasing $C_x$. From equation (6-3), it is known that the phase shift between $V_o$ and $V_{in}$ decreases with increasing $C'$. So the phase shift decreases with increasing $C_x$. It can also be seen that the phase shift is changed little with increasing $C_x$ when $C_x$ is less than 0.5 nF and decreases fast when $C_x$ is larger than 0.5 nF. This is because of the influence of the clamped capacitance $C_{d2}$. The influence of $C_x$ to the motional branch is quite small compared to that of $C_{d2}$ when $C_x << C_{d2}$.
Chapter 6 A Tunable Low-Frequency Piezoelectric Phase Shifter

(a)

(b)

\( V_{in} = 10V_{rms} \)
Figure 6-9  Phase shift, voltage gain and efficiency versus control capacitance at different operating frequencies.

Figure 6-10 shows the phase shift, voltage gain and efficiency versus operating frequency at different capacitances. It is seen that the phase shift decreases with increasing the operating frequency. The reason is similar to that for Figure 6-7. Although the phase shift range obtained by the control inductance is smaller than that obtained by the control inductance, it has a phase shift range including the positive and negative value and better efficiency.
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(a)

(b)

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6.2.4 Effect of DC bias voltage on the phase shift

Figure 6-11 shows the relationship between the phase shift and DC bias voltage at the operating frequency of 556 kHz and 560 kHz. The control capacitance is $4.17 \times 10^{-12}$ F. It can be seen that the phase shift increases with increasing the DC bias voltage. With the DC bias voltage increases from -80 V to 80 V, the phase shift increases about 25 deg and 17 deg at the operating frequency of 556 kHz and 560 kHz, respectively.
6.3 Characteristics of the Tunable Piezoelectric Phase Shifter at a High Input Voltage

In order to clarify the properties of the phase shifter at a high input voltage, the characteristics of the phase shifter with a control inductance at an input voltage of 40 \( V_{\text{rms}} \) were measured.

Figure 6-12 shows the phase shift, voltage gain and efficiency versus control inductance at different operating frequencies. It is seen that the phase shift decreases with increasing the inductance, which is the same as that at low voltage. But \( \left| \frac{d\Delta \phi}{dL_x} \right| \) is smaller than that at low voltage (Figure 6-6 (a)). From equation (6-3),
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\[
\frac{d \Delta \phi}{d L'} = \frac{-\omega R'}{R'^2 + \left( \frac{\omega L' - \frac{1}{\omega C'}}{\omega} \right)^2}.
\]  

(6-5)

From equation (6-5), it is known that \( |d \Delta \phi / d L'| \) decreases with increasing \( L' \) near the antiresonance frequency \( f_a \). Due to the mechanical nonlinear behavior of piezoelectric ceramics, \( L' \) increases with increasing the input voltage [72]. Therefore, \( |d \Delta \phi / d L_s| \) decreases with increasing the input voltage.

![Graph showing phase shift vs. inductance for different frequencies](image)

(a)
Figure 6-12 Phase shift, voltage gain and efficiency versus control inductance at different operating frequencies at an input voltage of $40 \, V_{\text{rms}}$. 
Figure 6-13 shows the phase shift, voltage gain and efficiency versus operating frequency at different inductances with an input voltage of $40 \text{ V}_{\text{rms}}$. It is observed that the phase shift decreases with increasing the operating frequency, which is the same as the characteristics at low voltage. From Figure 6-12 (c) and Figure 6-13 (c), it is seen that the frequency with the best efficiency is shifted to a lower value and the maximum efficiency becomes lower due to the increase of the input voltage. The former phenomenon results from the increase of compliance of the ceramic plate due to the mechanical nonlinearity [73], and the later results from the increase of the loss in the motional branch due to the mechanical nonlinearity [72].

![Graph showing phase shift, voltage gain, and efficiency versus operating frequency at different inductances with $40 \text{ V}_{\text{rms}}$.](image)
Figure 6-13 Phase shift, voltage gain and efficiency versus operating frequency at different inductances at an input voltage of $40 \, V_{\text{rms}}$. 

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CHAPTER 7 CONCLUSIONS AND RECOMMENDATION

7.1 Conclusions

This thesis has aimed at: developing and characterizing the high power/power density multi-output piezoelectric transformers; developing and validating the theoretical model for optimizing the high power/power density multi-output piezoelectric transformers; analyzing vibration distribution of the piezoelectric transformer to clarify some physical phenomena; exploring the possible applications of the multi-output piezoelectric transformers.

The thesis accomplished the following: 1) proposed and developed the high power multi-output piezoelectric transformers, which have the highest output power in the world up to now; 2) proposed and developed the highest power density piezoelectric transformer in the world; 3) derived and validated the theoretical model of the dual-output piezoelectric transformer operating at the thickness shear vibration mode; 4) clarified the vibration distribution of the piezoelectric transformer operating at the thickness shear vibration mode and the factors affecting this distribution experimentally and theoretically; 5) proposed and developed a novel tunable low-frequency piezoelectric phase shifter by extending the multi-output piezoelectric
Chapter 7 Conclusions and Recommendation

transformer technique; 6) developed the novel multi-point connection and supporting structure for the piezoelectric devices operating at the thickness shear vibration mode.

7.1.1 High power multi-output PTs

The high-power multi-output PTs operating at the thickness shear vibration mode have been proposed and their characteristics have been investigated experimentally. With a novel supporting construction and lead wires connection, at a temperature rise less than 20 °C and the efficiency of 90%, the piezoelectric transformer with dual outputs (PT-A) has a maximum total output power of 169.8 W with a power of 129.5 W in output 1 and 40.3 W in another, and the transformer with triple outputs (PT-B) has a maximum total output power of 163.1 W with a power of 36.9 W in output 1, 13.0 W in output 2 and 113.2 W in output 3. The maximum efficiency of PT-A and PT-B is 98% and 95.7%, respectively. The effects of the output number on the frequencies of the maximum total output power and maximum efficiency were investigated. The frequency of the maximum total output power increases with the increase of the output number and the frequency of the maximum efficiency decreases with the increase of the output number. Also, it was found that there exists a frequency range in which the voltage gain of one output is less affected by the load of another output.

7.1.2 High power density dual-output PT
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A high power-density dual-output piezoelectric transformer has been proposed and investigated. It employs a thin high-quality piezoelectric plate vibrating at the thickness shear mode. The piezoelectric transformer has a maximum output power density of 52.7 W/cm$^3$ with an efficiency of 88% at a temperature rise less than 20 °C. The maximum efficiency of this piezoelectric transformer is 95%. By using a parallel connection of the two outputs of the proposed transformer, a higher output power density of 57.3 W/cm$^3$ can be achieved at the temperature rise less than 20 °C. The power density of the piezoelectric transformer obtained by this study is much larger than the maximum power density of the piezoelectric transformers reported.

7.1.3 Modeling and analysis

A theoretical model was developed for the dual-output piezoelectric transformer operating at the thickness shear vibration mode. The equivalent circuit parameters of the piezoelectric transformer were derived. Based on this the impedance characteristics, equivalent inductance, capacitance ratio, voltage gain and efficiency of the piezoelectric transformer were calculated. The theoretical results were verified by experimental data. Furthermore, the effect of the transformer size on the voltage gain, efficiency, output power and power density were analyzed. Some useful guidelines were achieved by these analyses. By decreasing the thickness or increasing the input width of the transformer, the power density of the transformer can be improved effectively. The efficiency of the transformer decreases with the increase of its thickness. By increasing the input width or the total length or decreasing the total width, the voltage gains of the transformer can be increased. The maximum output power of the transformer can be improved effectively by increasing the input width.
Furthermore, the width change of one output has little influence on the voltage gain of another output and the maximum total output power of the transformer under the condition of matching load. Open-circuit of an output needs be avoided to stabilize the voltage of another output.

7.1.4 Vibration distribution in output section of a PT

The vibration distribution in output section of a piezoelectric transformer operating at the thickness shear vibration mode is investigated experimentally and theoretically. It is found that the vibration of output section has a spatial gradient and attenuates along the direction of vibration propagation, and the vibration is relatively small near the edge of output section. This result indicates that the support of the transformer and the lead wire connection of output section should be disposed near the edge of output section to decrease the loss of vibration energy. Also, it is found that the larger the load resistance, the larger the vibration gradient is. A theoretical model is developed to estimate the vibration attenuation in output section, which provides the guidelines of optimizing the transformer and explains some important phenomena in the piezoelectric transformer, such as the temperature distribution. At the resonance point, the vibration gradient decreases with increasing the width of input or output section, and have little change with the length and thickness of the transformer when the load resistance matches with the transformer. The increase of the length and decrease of the thickness may result in the vibration distribution more non-uniform when the load resistance is constant. These results may be used to decrease the non-uniformity of temperature distribution and increase power density of piezoelectric transformers. Effect of load resistance and dimensions on the vibration displacement of input
Chapter 7 Conclusions and Recommendation

section is also studied by this model. At the resonance point, the vibration displacement of input section increases with decreasing the length or the width of output section, or increasing the thickness of the transformer when the load resistance is constant.

7.1.5 Tunable low-frequency piezoelectric phase shifter

A low-frequency piezoelectric phase shifter based on a new technique has been developed. The technique is derived from the multi-output piezoelectric transformer. The phase shifter is made of a piezoelectric ceramic plate operating at the thickness shear vibration mode. The phase shift between the output and input voltages can be smoothly tuned by an inductor or capacitor connected to the control part of the phase shifter. The phase shift can be increased by decreasing the inductance or capacitance at the control part. Tuning the DC bias voltage applied to the input part is also an effective method of increasing the phase shift range. A maximum phase shift of 128 deg and minimum phase shift of -10 deg were achieved by this method. The phase shifter has a wide phase shift range, good ability to manage high power and relatively high energy transmission efficiency. As further study, the structure of the phase shifter will be optimized to widen the phase shift range and decreases the sensitivity of the phase shift to the operating frequency.

7.2 Recommendations
Chapter 7 Conclusions and Recommendation

High power/power density multi-output piezoelectric transformers operating at thickness shear vibration mode are very promising components in power electronics. Especially the merits such as simple structure, light weight, high reliability and high efficiency, make them have wide potential applications. Future research topics include the followings.

- The theoretical model for the multi-output PTs operating at the thickness shear vibration mode should be optimized further. To obtain a more accurate equivalent circuit, the circuit shown in Figure 2-7 should be used. Also, to consider the internal loss, $k$ and $v$ should be treated as complex numbers. In fact, there is nonlinear impedance in the motional branch which is directly proportional to the square of motional current ($I_m^2$). Although their influence on the equivalent circuit is quite small when the vibration is small, it becomes larger and larger with the increase of vibration. Therefore, the nonlinear impedance will be taken into consideration in the future model.

- It is known that the increase of operating frequency is beneficial to improve power density of PTs. But the nonlinearity of PT increases with the increase of operating frequency [42, 47, 52, 56]. As the increase of nonlinearity, the internal loss increases, and this may limit the power density of the PTs at high frequency. Further investigation is necessary to clarify the physical reasons for the nonlinear phenomena.

- The model for the tunable piezoelectric phase shifter needs to be optimized. The model proposed in the thesis can only predict the trend in the change of phase shift. To accurately calculate the phase shift, it is necessary to improve the structure of the equivalent circuit.
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• The thickness shear vibration mode, the structure and the theoretical model used in the thesis can also be applied to the step-up voltage PTs by changing the size of input and output electrodes.
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