Advanced Nonlinear Control of Complex Power Systems

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Summary

Nowadays, with increasing interconnections of power grids and transmission loading, phenomenon related with power systems becomes quite complex. Meanwhile, new generation and transmission facilities installation is more and more difficult. In addition, the transmission open access environment requires the transmission systems work close to their thermal capacity limit. More than ever, advanced controller design is called upon to guarantee the secure and reliable operation of power systems.

The objective of this thesis is to design advanced controllers for nonlinear complex power systems. The main contributions lie in proposing the following control strategies: 1) robust coordinated transient stability and voltage regulation enhancement (CTSVRE) control of generator excitation systems, which enhances both transient stability and post-fault voltage regulation; 2) robust coordinated control of AC/DC power systems with overlapping decomposition, which coordinates both AC system control and DC system control from a systematic standpoint; 3) robust constrained control of generator excitation systems, which considers input saturation from the beginning of controller design.

In case large disturbance occurs, although the most important concern is transient stability, post-fault voltage performance should not be ignored. In this thesis, robust CTSVRE control strategy is proposed to enhance post-fault voltage regulation as well as transient stability. Via direct feedback linearization (DFL) technique and voltage feedback, the original generator model is partially linearized. By treating the variables in A-matrix and B-matrix and the interconnections among generators of the partially linearized state space model as uncertain parameters, robust control theory can be employed to design the robust CTSVRE controller.
Since power systems are large scale interconnected complex systems, the dynamics of each component will influence the operation of the entire system. Thus, coordinated controllers have to be designed. In this thesis, a robust coordinated control strategy for AC/DC power systems with overlapping decomposition is proposed from a systematic standpoint. Firstly, the DFL technique is used to decouple AC and DC power systems and also partially linearize generator models, and then the robust decentralized TSE/CTSVRE controllers are designed for AC systems and overlapping decomposition controllers are designed for HVDC link for most often used control modes. The proposed controller consists of three parts: a nonlinear DFL controller for each AC power system generator excitation system and two overlapping decomposition controllers for each HVDC link. All of the controllers are coordinated to ensure the power system stability.

Similar to other physical systems, generator excitation systems also have control input saturation. Unlike most existing excitation system controller design, this work takes into account the hard-limit type input constraints from the beginning of controller design. Low and high gain linear feedback control and nonlinear feedback control are combined to design the robust excitation system controller with input constraints. The designed robust controller has theoretical guarantees for stability and performance instead of the rule of thumb.

The designed controllers are tested by numerical simulation using PSCAD/EMTDC software. Their performances are compared with those of other controllers and evaluated using different cases. Results show that the proposed control strategies are quite effective.
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List of Symbols

SMIB power systems

\[ \delta \] power angle, in radian;
\[ H \] inertia constant, in sec.;
\[ D \] damping constant, in p.u.;
\[ \omega \] rotor speed \((\omega_0 = 2\pi f_0)\), in rad/sec;
\[ P_m \] mechanical input power, in p.u.;
\[ P_e \] active electrical power; in p.u.;
\[ Q_e \] reactive electrical power, in p.u.;
\[ E_q \] EMF in quadrature axis, in p.u.;
\[ E'_q \] transient EMF in quadrature axis, in p.u.;
\[ E_f \] equivalent EMF in the excitation coil, in p.u.;
\[ T_{d0} \] direct axis transient short circuit time constant, in sec.;
\[ u_f \] input of the SCR amplifier, in p.u.;
\[ k_e \] gain of the excitation amplifier, in p.u.;
\[ I_d \] direct axis current, in p.u.;
\[ I_q \] quadrature axis current, in p.u.;
\[ I_f \] excitation current, in p.u.;
\[ x_d \] direct axis reactance, in p.u.;
\[ x'_d \] direct axis transient reactance, in p.u.;
\[ x_{ds} \] total reactance of direct axis transient reactance, transformer and transmission line reactance of the example system shown in Figure 2.1, in p.u.;
\[ x_{md} \] mutual reactance between the excitation coil and the stator coil of the generator, in p.u.;
generator terminal voltage, in \( p.u. \);

\( \alpha_i \) generator terminal voltage phase angle, in rad;

\[ \Delta \delta = \delta - \delta_0; \Delta \omega = \omega - \omega_0; \Delta P_e = P_e - P_{\infty}. \]

**Multimachine power systems (the \( n \)th generator)**

\( \alpha_i \) generator terminal voltage phase angle, in rad;

\( \delta_i \) power angle, in radian;

\( H_i \) inertia constant, in sec.;

\( D_i \) damping constant, in \( p.u. \);

\( \omega_i \) rotor speed \( (\omega_i = 2\pi f_i) \), in rad/sec;

\( P_{mi} \) mechanical input power, in \( p.u. \);

\( P_{ei} \) active electrical power; in \( p.u. \);

\( Q_{ei} \) reactive electrical power, in \( p.u. \);

\( E_{qi} \) EMF in quadrature axis, in \( p.u. \);

\( E_{\delta i} \) transient EMF in quadrature axis, in \( p.u. \);

\( E_{\beta i} \) equivalent EMF in the excitation coil, in \( p.u. \);

\( T_{di} \) direct axis transient short circuit time constant, in sec.;

\( u_{\beta i} \) input of the SCR amplifier, in \( p.u. \);

\( k_o \) gain of the excitation amplifier, in \( p.u. \);

\( I_{di} \) direct axis current, in \( p.u. \);

\( I_{qi} \) quadrature axis current, in \( p.u. \);

\( I_{\beta i} \) excitation current, in \( p.u. \);

\( x_{di} \) direct axis reactance, in \( p.u. \);

\( x'_{di} \) direct axis transient reactance, in \( p.u. \);

\( x_{ad} \) mutual reactance between the excitation coil and the stator coil of the generator, in \( p.u. \);
$B_{ij}$ the $i$th row and $j$th column element of nodal susceptance matrix at the internal nodes after eliminating all other physical buses, in $p.u.$;

$V_i$ generator terminal voltage, in $p.u.$;

$\delta_i = \delta_i - \delta_i^i \Delta \delta_i = \delta_i - \delta_i^i \Delta \omega = \omega_i - \omega_i^i : \Delta P_i = P_i - P_i^i$.

**HVDC**

$\alpha_{r,i}$ rectifier and inverter ignition delay angle, in rad;

$\gamma_i$ inverter extinction advance angle, in rad;

$I_{dc}$ DC line current, in p.u.;

$u_{re,in}$ control input at rectifier and inverter side

$B_{re,in}$ number of bridges in series at rectifier and inverter side

$k_{r,i}$ off-nominal turns of the converter transformer at rectifier and inverter side

$V_{re,ai}$ RMS phase-to-phase rectifier and inverter AC source commutation voltage which is the bus voltage on the system side of the converter transformer (primary), in $p.u.$;

$V_{di}$ inverter terminal DC voltage available to the DC line, in $p.u.$;

$R_j$ DC transmission line resistance, in $p.u.$;

$R_{re,ci}$ effective or equivalent commutation resistance at rectifier and inverter side;

$L_{re,di}$ DC smoothing reactor inductance at rectifier and inverter side;

$L_d$ DC transmission line inductance.

**Acronyms**

DFL Direct feedback linearization

FACTS Flexible alternating current transmission system;

SMIB Single machine infinite bus;
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
<tr>
<td>TSE</td>
<td>Transient stability enhancement</td>
</tr>
<tr>
<td>CTSVRE</td>
<td>Coordinated transient stability and voltage regulation enhancement</td>
</tr>
<tr>
<td>HVDC</td>
<td>High voltage direct current</td>
</tr>
<tr>
<td>CC</td>
<td>Constant current</td>
</tr>
<tr>
<td>CEA</td>
<td>Constant extinction angle</td>
</tr>
<tr>
<td>CE</td>
<td>Current error</td>
</tr>
<tr>
<td>CIA</td>
<td>Constant ignition angle</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional-Integral</td>
</tr>
<tr>
<td>SVC</td>
<td>Static var compensator;</td>
</tr>
<tr>
<td>SCR</td>
<td>Short circuit ratio;</td>
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</table>
Chapter 1 Introduction

1.1 Motivation and literature review

With the premise of continuously increasing of transmission system loading, area system interconnection and power electronics equipments installment, power systems become more and more complex. As a consequence, advanced control technique is called upon to design controllers to enhance, coordinate and/or substitute traditional control equipments or methods.

As pointed out in [1, 2], being a typical complex systems example, the power systems have four major characteristics in the meaning of "complex":

1) Substantial nonlinearity. There are various nonlinearities in power systems, such as the famous power-angle curve in rotor angle stability study and bifurcation and chaos in voltage stability study. There are also saturation or limits problems.

2) Large-scale. Area power systems are usually interconnected through AC tie lines or high voltage direct current (HVDC) transmission lines. The power systems in some countries, such as those in Europe or United States and Canada, are even interconnected.

3) Uncertainty in the model. Due to environment or measurement factors, parameters in power system model are not always exact. Furthermore, the system model may change after the external disturbance disappears or is removed or isolated.

4) Hybrid or heterogeneous form. In power systems, there is always a mixture of control actions and control requirements, possibly in different regions of operation. For example, both transient stability and post-fault voltage
regulation enhancement are always required at the same time.

To deal with the above complex characteristics of power systems, traditional linear control technique alone is usually not effective. This is because traditional controllers are designed on an approximated linear model dependent on a specific equilibrium point and are effective only in an approximately small linear region around this equilibrium point. In addition, as the transmission system loading will continue to increase, from today’s deregulated environment considerations, the trend will be toward maximum utilization of transmission systems close to thermal capacity. This means the stability margin of power systems is reduced and power systems may operate under highly stressed conditions. Furthermore, power electronics equipments have begun to be installed in modern power systems to enhance the system as well as power transfer capacity. This controllability enhancement relies on proper control strategies. All these factors require advanced nonlinear control application in power systems.

Before delving into this topic deeply, it is necessary to get a basic understanding of the configuration of power systems and associated controls. Figure 1.1 shows a typical structure and the associated controls of modern power systems. It can be seen that power systems consist of a complex array of control devices functioning at different levels. For example, the real power control (includes system generation schedule, tie line control and load frequency control) works at higher level than the generating unit controls do. Its function is to balance the total system generation against system load and losses so that the desired frequency and power interchange with neighboring systems are maintained. In addition, there are controllers operating directly on individual system elements. Generating unit controls consist of excitation control, turbine-governor control and also power system stability control. While the turbine-governor control is the primary control of generation output control and the
excitation control is the primary control to regulate generator voltage and reactive power output, the power system stability control acts as the supplementary control to improve system stability via the adjustment of the control input of excitation system control. The transmission network consists of AC network and DC network with the rectifier and inverter stations being their interface. Flexible AC transmission system (FACTS) devices control makes the power flow more flexible than ever before and plays an active role in improving power system rotor angle and voltage stability or system damping. All the controls shown in Figure 1.1 contribute to the satisfactory operation and have a profound effect on dynamic performance of power systems. It can be seen from Figure 1.1 that power systems are of complex structure.
Chapter 1 Introduction

In the following part, four aspects that play key roles in this thesis will be reviewed:

1.1.1 Decentralized control of power systems

The ideal scenario for control of a power system would be to have the capability to use the available controls to keep the system working at the optimum operating condition: maintaining a constant frequency and desired voltage profiles and line flows while supplying the system load, respecting the physical limits of the equipment, and observing a defined set of security and economy criteria.

To realize the ideal scenario, optimum operating condition should firstly be calculated, which requires instant knowledge of power flow and system topology information. Next, robust and redundant control and communications systems with adequate bandwidth are needed. Furthermore, since power systems are large scale interconnected complex systems and the controllers are dispersed geographically throughout the system, the signals for feedback and the computed actuating signals are needed travel a long distance. This is impossible for centralized controllers in a large power system to do this job. For example, to limit the signal transmission delays to 1ms one way or 2ms round-trip, only an area with a maximum radius of approximately 150 miles can be under a central control scheme [3].

Decentralized control may be a good alternative candidate for such a case. For interconnected complex systems, such as power systems and telecommunication networks, it is usually necessary to decompose the given system into a number of interconnected lower-order subsystems. After decomposition, the complexity of control problems decreases and the computational effort reduces. The controller design and implementation for each subsystem are relatively simpler and utilize only local state information. Coordinated action of the various local controllers in the system is also needed to guarantee the overall system stability and performance to some degree.
Performance of the combined decentralized controllers should be the same or at least close to that of the centralized ones. Decentralized control methods have already been applied to power systems [4-15]. Coordinated control design for power systems can be found in [16-23]. If the bounds of the dynamic interactions of one generator with other components like generators are taken into account when choosing a robust gain for the decentralized controller, the resulted controller design is a coordinated one even if the information due to other controller actions is not known [24]. The attractiveness of the decentralized controllers is that for each subsystem, the controller is dependent only on the subsystem and need not the remote signals that are not available or difficult to measure or estimate.

The standard decentralized control is usually only suitable for special structures, such as weak couplings, or when couplings can be appropriately handled. However, many large scale systems like power systems may also consist of subsystems that are strongly connected through certain dynamics (the overlapping part) but weakly connected otherwise. For such systems, conventional disjoint decompositions may fail to produce useful results. However, the overlapping decomposition technique using the "inclusion principle" [25, 26] or the "extension principle" [27, 28] is a powerful tool for such cases. This technique has already been studied in [29, 30] for decentralized design for decentralized AGC (automatic generation control) of interconnected power systems.

1.1.2 Robust control of power systems

With time elapsing, power system components endure parametric changes due to the wear and tear of natural or human factors. Moreover, in the case where a severe fault occurs, the network topology may change, so are the parameters of the network and the operating point. Furthermore, for controllers in industry like power industry, fixed gains are preferred for the simplicity and reliability. And these fixed gain controllers
are expected to operate in an operating region as large as possible. All these factors and requirements call for robust control theory.

Recently, robust control theory applications in power systems have been reported [4, 5, 9, 11, 18, 31-36]. These applications can be classified into three categories: first, the components parameters are not measured precisely or the system parameters change after the network topology changes due to some switching operation [34]; second, the obtained mathematical model has a state space form and the variables in the corresponding matrix of parameter space can be regarded as uncertain parameters [9, 36]; finally, there are some nonlinearities exist in the mathematical model which can not be easily solved by other methods, these nonlinearities are treated as model uncertainties and taken into account at the controller design stage [4, 5, 31-33, 35, 37]. A control theorem for such treatment for a class of uncertain systems is given in [38]. These applications illustrate the power of robust control theory in power systems.

Although a few of nonlinear voltage controllers have been proposed [36, 39-41], the nonlinear robust voltage controller for multimachine power systems designed in [9] is quite attractive for its simple structure. One drawback in [9] is that the technique neglects the dynamics of other generators when designing the controller for a specific generator. This is not very effective, since the control law for each generator will also be affected from other generators in the interconnected network. In fact, this problem can be solved by robust control theory if the interconnections can be regarded as model uncertainties, which is the idea in [4, 5, 33, 37].

1.1.3 Nonlinear control of power systems

Although power systems are highly nonlinear systems, traditionally, power systems are designed and operated conservatively in a region where behavior is mainly linear. These controllers are based on approximately linearized power system models and
their performance strongly depends on the specific operating point of the system. For small disturbances, linearized models around a typical equilibrium point are adequate for stability analysis and design. For example, AVR (automatic voltage regulator) and PSS (power system stabilizer) are based on linear control methods and they can effectively damp oscillations and insure asymptotic stability of the equilibrium following a small disturbance. In case of large disturbances, e.g. a three phase permanent fault, even the system can 'survive' these disturbances, the final equilibrium point changes. The original operating region that the linear controller is designed for may not cover the new equilibrium point, so the traditional linear controller is not effective any longer. In order to guarantee power system stability over a large operation region even when power systems are subjected to large disturbances, advanced nonlinear controllers should be designed, which are independent of or less dependent on specific equilibrium points.

Recently, many results of nonlinear system control design with application to power systems have been published [4-6, 9, 33-37, 40, 42-52]. Some of the nonlinear controller design methods [6, 42-44, 46] are based on the differential geometry approach [53]. This linearization procedure involves static state-feedback and a special nonlinear coordinate transformation. The problem is that in the nonlinear coordinate transformation the physical variables are often changed to non-physical ones, which makes the succeeding closed-loop design too complex to be accepted by engineers [51]. The DFL (direct feedback linearization) technique seems to be a good alternative due to its simplicity and is accepted by many researchers [4, 5, 9, 33-37, 40, 47, 48, 50, 51, 54]. DFL uses the original physical variables of the model and transforms the nonlinear system model into linear or partially linear system model directly. By using the nonlinear feedback, the inherent system nonlinearity can be cancelled or partially cancelled and this will in turn alleviates the uncertainties caused by operating point variations and, therefore, improve the system stability and performance [37].
Chapter 1 Introduction

It has long been realized that control through the excitation systems of generators is one of the most efficient and cost-effective ways to stabilize power systems. In the real world, every physical system is subject to input constraints due to physical limit of control actuator, so is the generator excitation system. While there are many researchers working on excitation system controller design, few of them have studied this nonlinearity, that is the excitation hard-limit or saturation [8, 10, 55], or it may be called "the input constraint" from a control engineer's standpoint. For most excitation system controller design, this constraint is ignored and the validity is assumed as a matter of course. However, this is not always the case. As is pointed out in [56], even though the original (unconstrained) control yields a stable closed loop system, stability cannot be guaranteed in general. In [55], it is shown that when excitation control gains are set high and when the hard-limits are taken into account, power systems may undergo global bifurcations including period-doubling cascades which lead to sustained chaotic behavior. By recognizing this fact, there has been renewed interest in the study of systems subject to input saturation especially during the last two decades [56-69]. Among them, references [63, 64] propose a composite nonlinear feedback control where nonlinear feedback is subtly combined with low gain feedback control and there is no switching element. This control strategy seems to be a good candidate for power system saturation controller design.

1.1.4 HVDC power transmission systems

HVDC power transmission systems have drawn more and more attention and developed quite fast in the last several decades. HVDC applications include underwater cables, asynchronous link between two AC systems and especially, long distance power transfer. For long distance power transfer there are both technical and economic factors. For technical consideration, unlike the AC systems where the power transfer capability is limited by the steady state and transient stability limit and is inversely proportional to transmission distance, the power transfer capability of DC
system is unaffectedly by the distance of transmission. Moreover, DC system has fast control to limit fault currents in DC lines. For economic consideration, there is a "break even" between 500-800km for the costs of AC and DC transmission with distance [70]. When the power transfer distance is larger than this "break even", it is more economical to use HVDC transmission. One of the HVDC transmission systems for long distance power transfer is Gezhouba-Shanghai HVDC link. The ±500 kV bipole HVDC line, with 1200MW rated power, interconnects the central China and East China networks over a distance of 1,080km.

One characteristic of HVDC control is that it can rapidly change power flow in response to control signals. This characteristic has been exploited by a lot of researchers to enhance power system stability [71-78]. In some research works, when designing DC controller, only second-order generator model is used, therefore the effect of the dynamics of the generator on the HVDC system is not fully considered [71, 72]. On the other hand, although some researchers use third-order generator model and try to design DC controllers from a systematic standpoint, the controllers use many remote real-time signals [75] that makes them impractical to implement due to the large scale and geographically distribution nature of power systems. Furthermore, as power systems are large-scale highly nonlinear interconnected complex systems, controllers based on approximate linearized model [73, 74] may not guarantee the power system stability over a large operation region. In this thesis the following problem is considered: how to design coordinated controllers for HVDC transmission systems and AC systems from a systematic standpoint so that the controllers of HVDC systems and generator excitation systems using local signals can work coordinately well and the entire system stability can be guaranteed?
Chapter 1 Introduction

1.2 Objectives

The broad objective of this research is to design advanced nonlinear controllers for complex power systems to deal with various nonlinearities and achieve both transient stability and voltage regulation enhancement.

These controllers should work under a wide range of operating conditions and are insensitive to various nonlinearities and parametric uncertainties, such as the generator model nonlinearities, interconnections among generators or interconnections among generators and other components, exciter input constraints, network structure changes due to large disturbance, etc. In addition, due to the characteristic of geographically distribution of power system components, e.g. rectifier and inverter stations of HVDC transmission systems, these controllers should also be decentralized and work coordinately very well.

The following are the three primary objectives of this research work.

1.2.1 Global robust control of generator excitation systems

In this work, global control idea is applied to design the excitation controllers so that both transient stability and post-fault voltage regulation are guaranteed. As the global control is actually a coordinated strategy of the transient stability enhancement (TSE) control and the coordinated transient stability and voltage regulation enhancement (CTSVRE) control, and the TSE control was studied well before, this work focused on the CTSVRE controller design. The problems for this design include linearization of generator model; dealing with interconnections among generators and the parametric uncertainties of the obtained state space model. Among them, the most difficult one is how to analyze and treat the parametric uncertainties. These parametric uncertainties include multivariables that are highly nonlinear and vary a lot with different operating
Chapter 1 Introduction

conditions. To make the CTSVRE controller work over a large wide range of operating conditions irrespective of network parameter uncertainties, detailed analysis should be carried out.

1.2.2 Robust coordinated control of AC/DC power systems

Since power systems are large scale interconnected complex systems, the dynamics of each component will influence the operation of the entire system. Thus, coordinated controllers have to be designed from a systematic standpoint. The coordinated controllers include decentralized AC system controllers and decomposed DC system controllers. The effect of the coordinated controllers should have the same or almost the same effect as those of the ideal centralized controllers. Although it is impossible to design decomposed controllers for HVDC transmission systems under all operating conditions, especially when employing the fast HVDC control capability to help stabilize the interconnected AC systems, it has the possibility to design decomposed controllers when HVDC systems are operating under “normal conditions”. New modeling and control methodology will be proposed to achieve this. The decentralization of the AC system part in AC/DC systems will also be studied in this work.

1.2.3 Robust constrained control of generator excitation systems

For most controller design in power systems, the control signals are calculated by assuming no constraints exist, and then they are simply limited in order to cope with the reality in the numerical simulation. However, the performance may deteriorate a lot and the stability can not be guaranteed in general even though the original (unconstrained) control yields a stable system with satisfactory performance. The objective of this work is to design exciter controllers for power systems which fully take into account the input constraints in the design period. The new controller design is based on rigorous constrained control theory. Comparison will be made between the
controller design considering control input constraints and that without considering control input constraints.

1.3 Major contributions of the thesis

This thesis focuses on advanced controller design for complex power systems. The major contributions of the thesis are summarized as follows:

1.3.1 Robust CTSVRE control of SMIB/multimachine power systems

Robust CTSVRE control strategy is proposed to enhance both transient stability and post-fault voltage regulation. Using DFL and voltage feedback, the original generator model is partially linearized. By treating the variables in A-matrix and B-matrix of the obtained state space model (and the interconnections among generators for multimachine case) as uncertain parameters, robust control theory can be employed. After carefully analyzing the parametric uncertainties under various typical operating conditions, it is found the parametric uncertainties have obvious different characteristics in two ranges. This characteristic is employed to partition the operating region of the generator in terms of the value of the rotor angle for SMIB power systems case or more general, the value of active power for multimachine power systems case. The partition aims to increase the accuracy of the CTSVRE controller. To avoid introducing another disturbance, smooth switch action between the obtained robust CTSVRE control laws should be guaranteed. This is realized by choosing appropriate member functions.

1.3.2 Robust coordinated control of AC/DC power systems with overlapping decomposition

Robust coordinated control strategy for AC/DC power systems with overlapping decomposition has been proposed. The DFL technique is firstly used to decouple AC and DC power systems as well as partially linearize generator models, and then the
robust TSE/CTSVRE controllers are designed for AC systems and overlapping decomposition controllers are designed for HVDC link for most widely used control modes: rectifier station at constant current mode and inverter station at constant extinction angle mode or constant voltage mode. By retaining the main dynamic characteristics of HVDC link, appropriate HVDC dynamic model is selected. Notice the special structure of the HVDC link dynamics, overlapping decomposition technique is employed to design the controllers. The original state space model of the DC link is expanded into a higher dimensional state space, where the states are rearranged and regrouped into two subsystems that are weakly connected. Also considering the uncertainties, the original overlapping decomposition control problem is converted to a standard robust decentralized output regulation problem, which is easier to be solved. After the robust decentralized control laws are obtained, they are contracted back to the original space. For the HVDC system case, the contracted control laws are exactly the same as those before contraction.

1.3.3 Robust control of generator excitation systems with input constraints

Every physical system is subject to input constraints due to physical limit of control actuator, so is the exciter. Unlike most controller designs in power systems, where the control signals are calculated as if no constraints existed, this work tries to deal with this problem in the beginning of controller design. It has theoretical guarantees for stability and performance instead of the rule of thumb. Low and high gain linear control and nonlinear control are combined to design the exciter controller subject to input constraints. Rigorous proof and detailed analysis are given. The validity of the designed controller is evaluated by numerical simulation.

1.4 Organization of the thesis

There are nine chapters in this thesis:
Chapter 1 Introduction

Chapter 1 introduces the research background and motivation, objective and the major contributions of this thesis.

Chapter 2 presents the mathematical modeling of generators, SMIB power systems, multimachine power systems, HVDC link and SVC. These models will be utilized to design and analyze generator excitation and HVDC transmission system controllers later.

Chapter 3 introduces the control methods that will be used in the following chapters. These methods include: direct feedback linearization, standard robust decentralized control, overlapping decomposition control, composite nonlinear control and global control.

In Chapter 4, robust CTSVRE controller for SMIB is designed. The generator dynamic model is linearized firstly using DFL technique. This is followed by robust TSE controller design. Finally, based on the detailed analysis of the parametric uncertainties of the state space model obtained by voltage feedback, robust CTSVRE controller is designed. Simulation results are given.

In Chapter 5, robust decentralized CTSVRE controllers for multimachine power systems are designed. Static electrical load is introduced to the multimachine power systems in order to make the controller design more close to practice. By modifying the analysis method for the parametric uncertainties of the state space model obtained by voltage feedback, the robust CTSVRE controller design method for SMIB power systems is extended to multimachine power systems. The designed robust decentralized multimachine controllers are tested on a five-machine twelve-bus example power system without infinite bus.
In Chapter 6, robust coordinated control strategy for single machine AC/DC power systems with overlapping decomposition is proposed. First, HVDC control structure, principles and related problems are introduced. Then, robust coordinated controllers for AC/DC power systems are designed. A DFL law is found to decouple the AC/DC power systems and linearize the generator model. Robust TSE controller is designed for the generator excitation system. Robust overlapping decomposition controllers are designed for the two most widely used control modes for HVDC transmission system. Finally, numerical simulation results are given.

In Chapter 7, the control methods illustrated in the previous chapters are combined to design robust nonlinear decentralized controllers for complex power systems. After the complex system is decentralized, under some assumptions and transformations, global robust controller is designed for the generator excitation system to realize both transient stability and voltage regulation enhancement. Incorporating a two terminal HVDC link, a five-machine twelve-bus example power system without infinite bus is used to evaluate the effectiveness of the proposed control strategy.

In Chapter 8, research on constrained controller design for excitation system is carried out. By considering the system uncertainties and combining the recent results on constrained control, namely, low and high gain linear feedback control and nonlinear feedback control, composite nonlinear controllers with better performance are designed for both SMIB and multimachine power systems. Simulation results illustrate the performance improvement of the proposed controller.

Chapter 9 concludes the thesis and recommends tasks for future research.
Chapter 2 Mathematical Model for Power Systems

2.1 Introduction

As mentioned in previous chapter, the objective of this thesis is to develop advanced control schemes for complex power systems. Before the control schemes are studied, power systems must be properly modeled.

As is widely known, power systems are large scale, highly nonlinear and interconnected dynamic systems. To completely represent the dynamic behavior of an entire power system, very high order dynamic nonlinear model should be used. This is certainly prohibitive for an analysis and will not be supported by simulation. In addition, the complications of the model would cause problems within the simulations, even if a powerful computer is available [79]. With the intention to make analysis feasible and simple while at the same time preserving the primary characteristic of power system transient stability, approximated simple power system models should be used.

The main components of power systems are generators, transformers, transmission lines, distribution network, electrical loads and electronics components, like HVDC converters in this study. Modeling of generator, SMIB and multimachine system, HVDC transmission system and SVC will be introduced in this chapter. The transformer magnetizing branch is ignored and only leakage reactance is represented. The AC transmission lines are represented by an equivalent $\pi$ circuit with charging capacitance at both sides. Distribution circuits and small loads are not shown in detail but are taken into account merely as lumped loads on substation bus. To reduce transmission network, electrical loads are further represented by constant impedance,
Chapter 2 Mathematical Model for Power Systems

though multiple load types maybe used for transient stability study [80]. Three phase balanced systems are assumed and represented on a per-phase basis.

This chapter is organized as follows: how to select the proper generator model is described in Section 2.2; the mathematical model of SMIB and multimachine power systems are reviewed in Sections 2.3 and 2.4 respectively; the mathematical model of HVDC transmission system is reviewed in Section 2.5; SVC model is described in Section 2.6.

2.2 Generator mathematical model selection

Synchronous generators are the principal source in large power systems and power system stability is primarily dependent on whether or how well the interconnected synchronous generators (or other synchronous machines which are not of interest here) can be kept in synchronism. Therefore, selecting a proper generator model is of great importance to the study of power system control.

Both the mechanical and electrical equations of a synchronous generator are very well established in the early literature, see [81-83]. The accuracy of the synchronous generator model depends on how many numbers of equivalent winding used. In mathematics, it depends on the order of the model. Five different synchronous machine models range from second-order to sixth-order are often used for research. While using a higher order model describes the dynamic performance precisely, it can't be used directly for system stability studies except for the analysis of very small systems due to the complexity. Moreover, too high order models don't improve the accuracy notably. A generator model with up to seven rotor windings was studied in [84]. The results have shown that using the standard machine data the more complex models do not necessarily give more accurate results.
Chapter 2 Mathematical Model for Power Systems

As a third-order generator model retains the main characteristics of power system dynamics and greatly simplifies the analysis, it is widely accepted for designing excitation controllers [4, 9, 42]. In this study, the third-order synchronous generator model will also be used. However, higher order generator model is used when doing simulation using PSCAD/EMTDC software, see Appendix C. The mathematic descriptions for single machine and multimachine power systems model are given in Sections 2.3 and 2.4 respectively.

To simplify the study of stability analysis of power systems, the following assumptions are made to the generator [85-88]:

1) The synchronous machine can be represented by a voltage source behind a transient reactance.
2) The mechanical rotor angle of a machine coincides with the angle of the voltage behind the transient reactance.
3) The transient saliency can be neglected.
4) The mechanical power input is constant during the period of the transient.
5) The stator transformer voltage terms can be neglected, so that the stator quantities contain only fundamental frequency components.
6) Distributed windings may be represented as concentrated windings.
7) Armature winding resistances of generator can be neglected.
8) Magnetic hysteresis and saturation effects are negligible.
9) While the amortisseurs are neglected, the effects of them are accounted by the constant damping ratio \( D \).
10) The three-phase stator winding is symmetrical.

2.3 Mathematical model for SMIB power systems

To understand basic concepts and effects, it is extremely useful to consider the case of
Chapter 2 Mathematical Model for Power Systems

a single machine connected to an infinite bus. Although this configuration is far simpler than actual systems, it is acceptable for some particular problems. For example, when one or more machines electrically relatively 'close' to one another that can be regarded as a single equivalent machine are connected to a node of a much larger system. The problems for such scenarios are primarily determined by the dynamic behavior of this machine or equivalent machine.

A simplified single-machine to infinite-bus (SMIB) example power system model is shown in Figure 2.1. In this model, a single synchronous generator is connected through one step-up transformer and two parallel transmission lines to a large network approximated by a constant voltage infinite bus.

![Single-Machine to infinite-bus power system](image)

Figure 2.1 Single-Machine to infinite-bus power system

Under the assumptions in Section 2.2, the motion of a generator can be described by classical model with flux decay dynamics. Such a widely accepted third-order dynamical generator model with excitation control can be written as follows [36, 89]:

Mechanical equations

\[
\Delta \dot{\delta}(t) = \Delta \omega(t) \tag{2.1}
\]

\[
\Delta \dot{\omega}(t) = -\frac{D}{2H} \Delta \omega(t) - \frac{\omega_b}{2H} (P_e(t) - P_m) \tag{2.2}
\]

Generator electrical dynamics

\[
\dot{E}_q(t) = \frac{1}{T_{q0}} (-E_q(t) + E_f(t)) \tag{2.3}
\]

Electrical equations

\[
E_q(t) = E_q(t) + (x_d - x_q) I_q(t) \tag{2.4}
\]
Chapter 2 Mathematical Model for Power Systems

\[ E_f(t) = k_n u_f(t) \]  \hspace{1cm} (2.5)

\[ I_d(t) = \frac{E'_q(t) - V_c \cos \delta(t)}{x_{by}} \]  \hspace{1cm} (2.6)

\[ I_q(t) = \frac{V_c \sin \delta(t)}{x_{by}} \]  \hspace{1cm} (2.7)

\[ P_e(t) = E'_q(t)I_q(t) \]  \hspace{1cm} (2.8)

\[ Q_e(t) = E'_q(t)I_d(t) \]  \hspace{1cm} (2.9)

\[ E_q(t) = x_{ad} I_f(t) \]  \hspace{1cm} (2.10)

\[ V'_q(t) = \left( (E'_q(t) - x_d I_d(t))^2 + (x'_q I_q(t))^2 \right)^{1/2} \]  \hspace{1cm} (2.11)

In the above equations, \( P_e(t) \) and \( Q_e(t) \) correspond to the active and reactive power behind the direct axis transient reactance. Usually, they are not measured directly. However, they can be obtained from measured variables. In practice, the active power \( P_f(t) \) and reactive power \( Q_f(t) \) at generator terminal can be measured directly. In the absence of real power losses, the relationship between \( P_f(t) \) and \( P(t) \) is

\[ P_f(t) = P(t) \]  \hspace{1cm} (2.12)

and the relationship between \( Q_f(t) \) and \( Q(t) \) is

\[ Q_f(t) = Q(t) + x'_q I(t)^2 \]  \hspace{1cm} (2.13)

The line current \( I(t) \) relates to the generator apparent power \( S_f(t) \) through

\[ I(t) = \frac{S_f(t)}{V'_q(t)} = \frac{\sqrt{P^2_f(t) + Q_f^2(t)}}{V'_q(t)} \]  \hspace{1cm} (2.14)

Substituting \( I(t) \) in (2.13) gives

\[ Q_f(t) = Q(t) + x'_q \frac{P^2_f(t) + Q_f^2(t)}{V'_q^2(t)} \]  \hspace{1cm} (2.15)

After \( P_f(t) \) and \( Q_f(t) \) are available and note \( I_f(t) \) is readily measurable variable, \( E_f(t) \) can be obtained from (2.4) and (2.9),

\[ E'_q(t) = \frac{x_{ad} I_f(t) \pm \sqrt{\left(x_{ad} I_f(t)\right)^2 - 4(x_d - x'_d)Q_f(t)}}{2} \]  \hspace{1cm} (2.16)
Chapter 2 Mathematical Model for Power Systems

Since \( E_q(t) \) must be a positive variable, thus only the following equation makes sense

\[
E_q(t) = \frac{x_m I_q(t) + \sqrt{(x_m I_q(t))^2 - 4(x_d - x'_d) Q_q(t)}}{2} \tag{2.17}
\]

So far, \( I_q(t) \) and \( I'_q(t) \) can be calculated from (2.8) and (2.9).

This method to calculate \( E_q(t) \) will be used to implement control laws for SMIB systems in later chapters.

Another method to calculate \( E_q(t) \) is given in the following. Denoting \( \theta(t) \) as the relative phase angle between \( E_q(t) \) and \( V(t) \), then there are

\[
P_q(t) = \frac{E_q(t) V(t)}{x_d} \sin \theta(t) \tag{2.18}
\]

\[
Q_q(t) = \frac{E_q(t) V(t)}{x_d} - \frac{E_q(t) V(t)}{x_d} \cos \theta(t) \tag{2.19}
\]

Eliminating \( \theta(t) \) from (2.18) and (2.19) gives

\[
E_q^4(t) - (2x'_d Q_q(t) + V_i(t)) E_q^2(t) + x_d^2 (P_q^2(t) + Q_q^2(t)) = 0 \tag{2.20}
\]

\( E_q(t) \) can then be calculated from above equation when \( P_q(t) \) and \( Q_q(t) \) are available.

Mathematically \( E_q(t) \) have four values, but physically \( E_q(t) \) can not be negative, so there are only two possible positive values available. Because synchronous generators can not operate at the lower value (underexcitation operation) permanently, only the higher value is used in this work. The relationship of \( E_q(t) \), \( V(t) \), \( P_q(t) \) and \( Q_q(t) \) can be plotted in a similar way as in [90] with some modifications, where voltage stability is studied. This method to calculate \( E_q(t) \) will be used to estimate the boundaries of parameters or variables for SMIB systems in later chapters.

The SMIB system model presented here will be used in Chapter 4 and a modified version will be used in Chapter 6, where HVDC transmission system control will be
Chapter 2 Mathematical Model for Power Systems

studied.

Remark 2.1: Note that swing equations (2.1) and (2.2) are in incremental form. This is preferred as deviations of power angle and angle speed from their equilibriums rather than their equilibrium absolute values are monitored to test stability or instability.

Remark 2.2: The static exciter is simply modeled by a constant gain in (2.5) because the time constant of the exciter is relatively short.

Remark 2.3: If the single synchronous machine system is not a SMIB system as shown in Figure 2.1, equations (2.6)-(2.7) are not applicable any longer, but other equations in this section are still applicable.

2.4 Mathematical model for multimachine power systems

Although the controllers based on SMIB model are simple and decentralized, they do not take into account dynamic phenomena in the rest of the power system and thus not suitable for industry application. For practical power system analysis, the model complications are increased not only by the number of machines but also by the characteristics of the network and electronic devices. In such a general case, multimachine power system model should be considered.

When AC transmission network transients are neglected and electrical loads are represented by constant impedance, the transmission network can be reduced into a network retaining only generator nodes. For a general $n$ multimachine AC system model without electronics devices like FACTS devices and HVDC converters, the third-order dynamic model of the $i$th ($i=1,2,...,n$) machine with excitation control can be written as follows [37, 47, 91]:

Mechanical equations:

$$\Delta\dot{\delta}_i(t) = \Delta\omega_i(t)$$  \hspace{1cm} \text{(2.21)}

$$\Delta\dot{\omega}_i(t) = -\frac{D_i}{2H_i}\Delta\omega_i(t) - \frac{\omega_n}{2H_i}(P_e(t) - P_m)$$  \hspace{1cm} \text{(2.22)}
Generator electrical dynamics:

\[ \dot{E}_q(t) = \frac{1}{T_{dii}} (E_q(t) - E_{q0}(t)) \quad (2.23) \]

Electrical equations:

\[ E_q(t) = E_{q0}(t) + (x_{ad} - x_{id}) I_d(t) \quad (2.24) \]

\[ E_s(t) = k_{ij} I_d(t) \quad (2.25) \]

\[ I_q(t) = \sum_{j=1}^{n} E_{q0}(t) B_{ji} \sin \delta_j(t) \quad (2.26) \]

\[ I_d(t) = -\sum_{j=1}^{n} E_{q0}(t) B_{ji} \cos \delta_j(t) \quad (2.27) \]

\[ P_{q0}(t) = E_{q0}(t) I_q(t) \quad (2.28) \]

\[ Q_{q0}(t) = E_{q0}(t) I_d(t) \quad (2.29) \]

\[ E_s(t) = x_{ad} I_d(t) \quad (2.30) \]

\[ V_{q0}(t) = E_{q0}(t) - \dot{x}_{ad} I_d(t) \quad (2.31) \]

\[ V_{ad}(t) = \dot{x}_{ad} I_q(t) \quad (2.32) \]

\[ V_a(t) = \sqrt{V_{q0}^2(t) + V_{ad}^2(t)} \quad (2.33) \]

Corresponding to (2.12), in the absence of real power losses, the relationship between \( P_{q0}(t) \) and the active power \( P_n(t) \) at generator terminal is

\[ P_{q0}(t) = P_n(t) \quad (2.34) \]

Corresponding to (2.15), the relationship between \( Q_{q0}(t) \) and the active power \( Q_n(t) \) at generator terminal is

\[ Q_{q0}(t) = Q_n(t) + x_{ad} \frac{P_n^2(t) + Q_n^2(t)}{V_n^2(t)} \quad (2.35) \]

Corresponding to (2.17), \( E_{q0}(t) \) is obtained as

\[ E_{q0}(t) = \frac{x_{ad} I_d(t) + \sqrt{(x_{ad} I_d(t))^2 - 4(x_{ad} - \dot{x}_{ad}) Q_{q0}(t)}}{2} \quad (2.36) \]
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\[ I_a(t) \] and \[ I_w(t) \] can then be calculated from (2.28) and (2.29).

This method to calculate \( E_p(t) \) will be used to implement control laws for multimachine power systems in later chapters.

Corresponding to (2.20), \( E_p(t) \) can be obtained from the following equation

\[
E_p^2(t) - \left( 2x_a^2 Q_w(t) + V_p^2(t) \right) E_p^2(t) + x_a^2 \left( P_w^2(t) + Q_w^2(t) \right) = 0
\]

(2.37)

The selection of \( E_p(t) \) is similar to the selection of \( E_q(t) \) as described on page 21 in Section 2.3. This method to calculate \( E_p(t) \) will be used to estimate the boundaries of parameters or variables for multimachine power systems in later chapters.

The multimachine power system model presented here will be used in Chapter 5 and a modified version will be used in Chapter 7 where a two terminal HVDC link is integrated into an AC system. Remarks 2.1-2.2 are applicable to this case too, except that the corresponding equations are multimachine ones.

2.5 Mathematical model for HVDC link

In most transient stability programs, DC links are represented by a functional model, in which the dynamics of the DC line and the converter firing angle controls are considered to be instantaneous [92].

According to [87], a detailed HVDC model should include DC line dynamics model, AC/DC interface and converter controls represented by appropriate dynamic models. In this study, while the DC transmission line dynamics is considered, the converter dynamics is not modeled. The converter characteristics are represented by equations relating average values of DC quantities and RMS values of AC fundamental.
components. This is the so-called "quasi steady-state model" [87], which accurately represents the HVDC system performance in stability studies for analysis of balanced operation.

### 2.5.1 Converter model and AC/DC Interface

Converters assume the duty to convert AC to DC and vice versa, all AC/DC interactions are realized through converters. So modeling of converters is very important for HVDC study.

To simplify analysis, following assumptions are made:

1) The forward voltage drop in a conducting valve is neglected so that the valve may be considered as a simple switch.

2) The converter transformer leakage reactance as viewed from the secondary side is equal for all the three phases.

3) The direct current is considered smooth without ripples

4) Converter transformer resistance is neglected.

With perfect filtering, or with a combination of filters and transformer phase shift, the voltage at the AC side of the converter transformer may be assumed to be sinusoidal. Neglecting the effect of local plant components at the converter terminal, the commutation reactance is equal to the transformer secondary leakage reactance. The equivalent circuits of a rectifier and an inverter are shown in Figure 2.2 and Figure 2.3,

![Figure 2.2 Rectifier equivalent circuit](image)

![Figure 2.3 Inverter equivalent circuit](image)
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see [87].

In the two figures, \( V_{dbr} = \frac{3\sqrt{2}}{\pi} B_{re} k V_{ar} \) and \( V_{d0} = \frac{3\sqrt{2}}{\pi} B_{m} k V_{a0} \).

From Figure 2.2, the DC output voltage of the rectifier can be written as

\[
V_{dc} = V_{d0} \cos \alpha_r - \frac{3}{\pi} B_{re} X_{cr} I_{dc} = \frac{3\sqrt{2}}{\pi} B_{re} k V_{ar} \cos \alpha_r - \frac{3}{\pi} B_{re} X_{cr} I_{dc} \tag{2.38}
\]

From Figure 2.3, the DC output voltage of the rectifier can be written as

\[
V_{dc} = -V_{d0} \cos \alpha_i + \frac{3}{\pi} B_{ri} X_{ci} I_{dc} = -\frac{3\sqrt{2}}{\pi} B_{re} k V_{ar} \cos \alpha_i + \frac{3}{\pi} B_{re} X_{ci} I_{dc} \tag{2.39}
\]

Then real and reactive power can be calculated

\[
P_{dc} = V_{dc} I_{dc} \tag{2.40}
\]

\[
Q_{dc} = P_{dc} \tan \phi_r \tag{2.41}
\]

\[
P_{dc} = V_{dc} I_{dc} \tag{2.42}
\]

\[
Q_{dc} = |P_{dc}| \tan \phi_i \tag{2.43}
\]

\[
\cos \phi_r = \frac{V_{dc}}{V_{d0}} \tag{2.44}
\]

\[
\cos \phi_i = \frac{V_{dc}}{V_{d0}} \tag{2.45}
\]

The AC RMS value fundamental frequency component of the total current at the converter AC side, rectifier AC side for example, is related to the direct current by the equation [87]

\[
I_p = kk \frac{\sqrt{6}}{\pi} B_{re} I_{dc} \tag{2.46}
\]

where \( kk \) is a coefficient when commutation overlap is taken into account and \( kk = 0.995 \) is sufficiently accurate for load-flow studies in most cases [88].

Alternately, under the assumption that the converter transformers are lossless, the AC RMS value fundamental frequency component can also be obtained from the relationship that the primary real power equates to the DC power

\[
I_p = V_{dc} I_{dc} / (V_{ar} \cos \phi_r) \tag{2.47}
\]
Chapter 2 Mathematical Model for Power Systems

![AC/DC interface at rectifier side](image)

**Figure 2.4** AC/DC interface at rectifier side

The above basic AC and DC quantities relationship at rectifier side is shown in Figure 2.4. The angle reference is the AC current $I_r$. Equations (2.38)-(2.47) are the equations used to convert the quantities between AC and DC systems during steady state and transient stability calculations.

![Phase-locked oscillator control system](image)

**Figure 2.5** Basic phase-locked oscillator control system [93]

In modern HVDC systems, the scheme used in the converter pole controls is the phase-locked oscillator control system, giving nominally equidistant pulse firing. Figure 2.5 [93] shows the principles of the phase-locked oscillator control for current control loop in simplified form. Its basic components are a voltage-controlled
Chapter 2 Mathematical Model for Power Systems

oscillator, and a 12-stage ring counter (for a 12-pulse converter) which feeds the 12 pulses to the respective converter valves. The oscillator runs at 12 times supply frequency, hence valve firing pulses are normally once per cycle per valve, at an accurate 30 spacing. The oscillator input has a fixed bias $V_i$, plus an error signal equal to the reference of the measured quantity for the particular loop (DC current as shown) and an order signal. However, a different method is used in the simulation software PSCAD/EMTDC, where the controller output is proper firing angles. The firing angle is then compared with the value of the ramp to trigger the actual firing pulse. The ramp is the output of phase locked oscillator and synchronized with phase A line to ground voltage.

2.5.2 DC network model

Consider a monopolar line with ground return which is terminated by converter stations with a 12-pulse bridge in each station. When DC filters and the distributed capacitance between a DC line and the ground are neglected, a DC network model is shown in Figure 2.6. The smoothing reactors are represented by lumped parameter linear elements $L_{dc}$ at the rectifier and $L_{di}$ at the inverter side.

![Figure 2.6 Equivalent circuit of a two terminal DC system](image)

If the cosines of the firing angles are controlled for simplification, the following DC
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link linear dynamics equations with a modification of those in [94] can be used.

\[ I_{dc} = \frac{1}{T_d} (-I_{ac} + V_r \cos \alpha_r + V_{ai} \cos \alpha_i) \]  
(2.48)

\[ \frac{d(\cos \alpha_r)}{dt} = \frac{1}{T_r} (-\cos \alpha_r + u_r) \]  
(2.49)

\[ \frac{d(\cos \alpha_i)}{dt} = \frac{1}{T_i} (-\cos \alpha_i + u_i) \]  
(2.50)

where

\[ T_d = \frac{L_{dc}}{R_{dc}}, \quad V_r = \frac{3\sqrt{2}B_{ac}k_i}{\pi R_{dc}}V_{ar} \quad \text{and} \quad V_i = \frac{3\sqrt{2}B_{ac}k_i}{\pi R_{ai}}V_{ai} \]

\[ L_{dc} = L_{ac} + L_{ai} + L_d \]

\[ R_{dc} = B_{ac}R_{ac} + B_{ai}R_{ai} + R_d \]

From (2.48), it can be seen that the AC system behavior affects the DC system behavior via the variations of AC side voltage \( V_{ar} \) and \( V_{ai} \).

The relationship between \( \cos \gamma_i \) and \( \cos \alpha_i \) is

\[ \cos \gamma_i = -\cos \alpha_i + k_i I_{dc} \]  
(2.51)

where

\[ k_i = \frac{\sqrt{2\pi R_{ai}}}{3k_i V_{ai}} \]

Remark 2.4: The above DC network model ignores the DC line capacitance, thus it is suitable for overhead transmission lines but not suitable for long cables. For long cables, \( T \) or \( \Pi \) equivalent models may be used.

2.6 SVC model

The SVCs are shunt connected static reactive power generators and/or absorbers that can be thought of as adjustable shunt susceptances. By using conventional thyristors, the SVC output is varied fast to achieve specific parameters. It is pointed in [95] that SVC can be used for transient stability improvement, power oscillation damping and voltage support. In practical applications, SVC is often used to regulate the voltage at weak points in a network against load variations, or to provide voltage support for the
load when the capacity of the sending end system becomes impaired. For a radial line, the end of the line is the best location for the compensator, where the largest voltage variation is experienced. By contrast, the midpoint is the most effective location for the line interconnecting two AC system buses [96].

A single line diagram of a typical SVC is given in Figure 2.7. One Thyristor controlled reactor (TCR) and two thyristor switched capacitors (TSC) are assumed. Basic elements of SVC include: 1) Fixed capacitor (FC). Fixed capacitor banks provide a permanently connected source of reactive power. They may be designed as a harmonic filters that are capacitive at fundamental frequency; 2) TCR. Each single phase unit of TCR consists of a fixed reactor in series with a bidirectional thyristor switch; 3) TSC. Each single phase unit of TSC consists of a capacitor in series with a bidirectional thyristor switch and a small surge current limiting reactor. A controller coordinates the operation of TCR and TSC to achieve some specific control objective. The auxiliary signal is used for functions such as electromechanical oscillations damping and reactive power regulation subject to voltage constraints.

![Figure 2.7 Typical static var system](image-url)
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The effective TCR susceptance \( B_l(\alpha) \) is controllable through controlling the firing angle of the thyristor. The steady state relationship between susceptance and firing angle is given in [87] as

\[
B_l(\alpha) = \frac{1}{\omega L} \frac{2\pi + \sin 2\alpha - 2\alpha}{\pi}
\]  
(2.52)

When the firing angle increases from \( 90^\circ \) to \( 180^\circ \), the susceptance value decreases from maximum to zero, and the reactive current in the reactor also decreases from maximum to zero.

Compared with mechanical switching, TSC is faster and it is possible to have transient free operation by controlling the instant of switching. The TSC itself can only provide step-like (discrete) change in the reactive current it draws. For continuous control, it is necessary to combine it with TCR of a rating slightly higher than the rating of individual capacitor bank [70]. To reduce the rating of TCR, multiple TSCs are used in practice.

![Figure 2.8 Basic SVC model 2][97]

SVC models for power flow and dynamic performance simulation was proposed by IEEE Special Stability Controls Working Group in reference [97], where two basic functional dynamic models were recommended. The basic model 2 shown in Figure

---

[97]: ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library
2.8 represents the physical structure of most installed SVCs. As SVC used in this study is just to demonstrate the reactive power compensation effect at the HVDC inverter station and SVC itself is not the primary concern, detailed SVC model is not shown here. However, it can be easily found in some references, e.g. [98, 99].

2.7 Concluding remark

Proper modeling is of great importance for power system analysis and the subsequent controller design. In this chapter, the selection of mathematical model of synchronous generator is presented at first. SMIB power system and multimachine AC power system modeling are reviewed next. HVDC and SVC modeling are described in the end. Modeling of other essential components such as transformers, transmission lines are also discussed. Appropriate assumptions and approximations are made to simplify analysis. In the rest chapters, all these models will be used to design power system controllers and study the effectiveness of these controllers on enhancing power system transient stability and performance. When designing HVDC controllers, related equations of the SMIB system and multimachine power system model will be modified in Chapters 6 and 7 respectively.
Chapter 3 Control Methods

3.1 Introduction

Power systems are large scale nonlinear complex systems. First, the electrical loads are varying from time to time and usually can not be forecasted exactly. This makes the power system operating point vary in some regions. Second, the components parameters can not be measured precisely, not to mention the network structure changes a lot in order to remove unpredictable faults or to deal with abnormal conditions. Third, because of physical limitation on the system structure, information transfer among subsystems is infeasible especially for large systems, decentralized controllers for power systems must be used in practice. Fourth, at different time scale and operating region, the performance objectives and requirements are quite different. Thus multi-objective requirement of power systems must be satisfied. There are also other factors needed to be considered, such as the well-known power angle curve that has a nonlinear characteristic, voltage stability related chaos and bifurcation problems, etc. All these problems require the application of advanced control techniques in power systems. This chapter will introduce some useful control methods that will be used in the following chapters.

The organization of this chapter is as follows: direct feedback linearization technique is reviewed in Section 3.2. In Section 3.3, robust decentralized controller design with guaranteed performance is given. Sections 3.4 and 3.5 introduce overlapping decomposition control and composite nonlinear control strategy respectively. Global control concept is introduced in Section 3.6.
3.2 Direct feedback linearization

Classical techniques used in the design of power system controllers are based on the approximately linearized model using the Taylor expansion technique. The robustness of these controllers (with respect to parameter variation and/or uncertainties in the operating conditions) is limited and the controllers are only valid within a small approximated linear region. This is due to the fact that the model is accurate only in the neighborhood of the operating point at which the system is linearized.

In order to make the controllers effective in larger operating regions, nonlinear feedback control has been applied to power systems [4, 34, 36, 40, 42, 44, 46, 49-51, 54, 94]. By employing a nonlinear feedback compensating law, a nonlinear system can be exactly or almost exactly linearized in its global state space or in a large enough region of state space. After a nonlinear system is transformed into a linear dynamic system, linear control theories can be applied to design an effective feedback control law that is effective over a wide range.

Among these techniques, DFL has widely accepted in power system application [4, 34, 36, 40, 51, 54] due to its simplicity and easy implementation. DFL uses the original physical variables of the model and transforms the nonlinear system model into linear system model directly. Without a coordinate transformation, DFL overcomes the disadvantage brought by the geometry nonlinear feedback technique, where non-physical variables are used and the design is usually too complex to be accepted by engineers.

Fundamentals of DFL technique in nonlinear controller design are illustrated in Figure 3.1 [36], where $x$ is the state vector of the controlled system; $u$ is the original input vector and $v$ is the new input vector to the feedback linearized system; $K$ is the
controller gain of the feedback linearized system, that is \( v = Kx \).

\[
\begin{align*}
\text{Feedback linearized system} & & \dot{x} = Ax + Bu \\
\text{Nonlinear feedback compensation law} & & u = h(x, v) \\
\text{Original nonlinear plant} & & \dot{x} = f(x, u)
\end{align*}
\]

Figure 3.1 Fundamentals of DFL technique in nonlinear controller design

### 3.3 Robust decentralized control

Due to the unpredictable changes in power system network parameters and operating conditions, it is obviously a practical requirement to design robust controllers. In this section, robust decentralized control with guaranteed performance of a class of linear interconnected system will be introduced and proof will be given. It will be seen in the following chapters that the particular form of uncertainties and interconnections considered in this section will be consistent with our power system model. Therefore, the robust decentralized control can be applied.

Consider a general uncertain interconnected system composed of \( n \) interconnected linear uncertain subsystems \( \Sigma_i \), \( i = 1, 2, \ldots, n \). Each \( \Sigma_i \) is described by

\[
\dot{x}_i(t) = [A_i + \Delta A_i(t)]x_i + [B_i + \Delta B_i(t)]u_i(t) + \sum_{j \neq i} G_{ij}(t)x_j(t)
\]

\[ (3.1) \]

where \( x_i(t) \in \mathbb{R}^n \) and \( u_i(t) \in \mathbb{R}^n \) are the state and control vectors of the \( i \)th subsystem respectively. \( A_i \), \( B_i \) and \( G_{ij} \) are constant matrices with appropriate dimensions and \( G_{ij} \) are interconnection matrices between the \( i \)th subsystem and the \( j \)th subsystem; \( \Delta A_i(t) \), \( \Delta B_i(t) \) and \( \Delta G_{ij}(t) \) are unknown time-varying parameter uncertainties.
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The parametric uncertainties are assumed to be of the forms

\[
\begin{bmatrix}
\Delta A(t) \\
\Delta B_i(t)
\end{bmatrix} = D_i F_i(t) [E_u E_z], \quad \Delta G_y(t) = M_y F_y(t) N_y
\]

where \( D_i, E_u, E_z, M_y, \) and \( N_y \) are known constant real matrices of appropriate dimensions and \( F_i(t) \in R^{k_i \times k_i} \) and \( F_y(t) \in R^{k_y \times k_y} \) (for all \( i, j = 1, 2, \ldots, n \)) are unknown matrix function with Lebesgue measurable elements and satisfy \( F_i^T(t) F_i(t) \leq I_i \), \( F_y^T(t) F_y(t) \leq I_y \), where \( I \) denotes the identity matrix of appropriate dimension.

Associated with (3.1) is the cost function

\[
J = \sum_{i=1}^{n} \int_0^\infty \left( x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t) \right) dt
\]  \hspace{1cm} (3.2)

where \( Q_i \) and \( R_i \) are given positive-definite symmetric matrices which are chosen at the designer’s will.

**Definition 3.1:** A control law \( u_i(t) = K_i x_i(t) \) is said to be a quadratic guaranteed cost control for the system (3.1) with cost function (3.2) if the closed-loop system is quadratically stable and the closed-loop value of the cost function (3.2) satisfies the bound \( J \leq J^* \) for some predefined scalar \( J^* \) for all admissible uncertainties.

**Theorem 3.1:** \( u_i(t) = K_i x_i(t) \) is a quadratic guaranteed cost control for the uncertain interconnected system (3.1) if there exist some symmetric positive-definite matrices \( P_i, \ R_y \), and some positive real numbers \( \varepsilon_i \) and \( \beta_y \), \( i, j = 1, 2, \ldots, n \) such that

\[
L_i = \left[ A_i + B_i K_i \right] P_i + P_i \left[ A_i + B_i K_i \right]^T + \varepsilon_i^{-1} (E_u + E_z K_i)^T (E_u + E_z K_i) + \varepsilon_i P_i F_i^T(t) F_i P_i + \sum_{j=1, j \neq i}^n \left[ R_j + P_j G_j G_j^T P_j + \beta_j^{-1} N_j N_j + \beta_j P_j M_j M_j^T P_j \right] + K_j^T R_j K_j < 0
\]  \hspace{1cm} (3.3)

Proof:

With the state feedback control law \( u_i(t) = K_i x_i(t) \), the closed loop system is

\[
\dot{x}_i(t) = \left[ A_i + \Delta A_i(t) + (B_i + \Delta B_i(t)) K_i \right] x_i(t) + \sum_{j=1, j \neq i}^n \left[ G_j + \Delta G_j(t) \right] x_j(t)
\]  \hspace{1cm} (3.4)

Construct a candidate Lyapunov function as
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\[ V(x) = \sum_{i=1}^{n} x_i^T P_i x_i(t) \quad P_i = P_i^T > 0 \]

The time derivative of \( V(x) \) along the trajectory of the closed loop system (3.4) is

\[ V = \sum_{i=1}^{n} x_i^T \left[ A_i + \Delta A(t) + (B_i + \Delta B(t)) K_i \right]^T P_i + P_i \left[ A_i + \Delta A(t) + (B_i + \Delta B(t)) K_i \right] x_i(t) \]

\[ + \sum_{j=1}^{n} \sum_{i=1}^{n} x_j^T \left[ G_{ij} + \Delta G_i(t) \right]^T P_j x_j + \sum_{j=1}^{n} G_{ij}(t) x_j \]

The first term is

\[ \sum_{i=1}^{n} x_i^T \left[ A_i + \Delta A(t) + (B_i + \Delta B(t)) K_i \right]^T P_i + P_i \left[ A_i + \Delta A(t) + (B_i + \Delta B(t)) K_i \right] x_i(t) \]

\[ = \sum_{i=1}^{n} x_i^T \left[ A_i + B_i K_i \right]^T P_i + P_i \left[ A_i + B_i K_i \right] + \Delta A(t) + \Delta B(t) K_i ]^T P_i + P_i \left[ A_i + \Delta A(t) + \Delta B(t) K_i \right] x_i(t) \]

\[ = \sum_{i=1}^{n} x_i^T \left[ A_i + B_i K_i \right]^T P_i + P_i \left[ A_i + B_i K_i \right] + D_i F_i(t)(E_{ii} + E_{22} K_i ]^T P_i \]

\[ + P_i \left[ D_i F_i(t)(E_{ii} + E_{22} K_i) \right] x_i(t) \]

\[ \leq \sum_{i=1}^{n} x_i^T \left[ A_i + B_i K_i \right]^T P_i + P_i \left[ A_i + B_i K_i \right] + \epsilon_i \left[ E_{ii} + E_{22} K_i \right]^T \left[ E_{ii} + E_{22} K_i \right] + \epsilon_i P_i D_i F_i P_i \]

where \( \epsilon_i > 0 \).

The above inequality uses one well known fact that for any matrices \( X \) and \( Y \) with appropriate dimensions, there exists \( X^T Y + Y^T X \leq \epsilon X^T X + \epsilon^{-1} Y^T Y \) for any \( \epsilon > 0 \) [100].

In fact, this equality can be easily extended to \( X^T Y + Y^T X \leq X^T R X + Y^T R^{-1} Y \) for some symmetric positive matrix \( R \). These two facts will also be used in the following proof.

The second term is

\[ \sum_{j=1}^{n} \sum_{i=1}^{n} x_j^T \left[ G_{ij} + \Delta G_i(t) \right]^T P_j x_j(t) \]

\[ = \sum_{j=1}^{n} \sum_{i=1}^{n} \left[ x_j^T G_{ij} P_j x_j(t) + x_j^T (t) P_j G_{ij} x_j(t) \right] + \sum_{j=1}^{n} \left[ x_j^T (t) \Delta G_i(t) P_j x_j(t) + x_j^T (t) P_j \Delta G_i(t) x_j(t) \right] \]

\[ = \sum_{j=1}^{n} \sum_{i=1}^{n} \left[ x_j^T G_{ij} P_j x_j(t) + x_j^T (t) P_j G_{ij} x_j(t) \right] \]
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\[ + \sum_{j=1 \atop j \neq i}^{n} \left[ x_j^T(t) \left( M_g F_g(t) N_g \right)^T P x_j(t) + x_j^T(t) P \left( M_g F_g(t) N_g \right) x_j(t) \right] \]

\[ \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} \left[ x_i^T(t) G_i^T P x_i(t) + x_i^T(t) P G_j x_j(t) \right] \leq \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} \left[ x_i^T(t) R_i^{ij} x_i(t) + x_i^T(t) P G_j R_i G_j^T P x_i(t) \right] \]

\[ = \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} x_i^T(t) \left[ R_i^{ij} + P G_j R_i G_j^T P \right] x_i(t) \]

where \( R_i = R_i^{ii} > 0 \), \( i, j = 1, 2, \ldots, n \).

\[ \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} \left[ x_i^T(t) \left( M_g F_g(t) N_g \right)^T P x_i(t) + x_i^T(t) P \left( M_g F_g(t) N_g \right) x_i(t) \right] \]

\[ = \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} \left[ x_i^T(t) N_i^T F_i^T(t) M_i^T P x_i(t) + x_i^T(t) P M_i F_i^T(t) N_i x_i(t) \right] \]

\[ \leq \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} \left[ \beta_i^{-1} x_i^T(t) N_i^T N_i x_i(t) + \beta_i x_i^T(t) P M_i M_i^T P x_i(t) \right] \]

\[ = \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} x_i^T(t) \left[ \beta_i^{-1} N_i^T N_i + \beta_i P M_i M_i^T P \right] x_i(t) \]

where \( \beta_i > 0 \), \( i, j = 1, 2, \ldots, n \).

Therefore

\[ \dot{V} \leq \sum_{i=1}^{n} x_i^T(t) \left[ \left[ A + B K_i \right]^T P + P \left[ A + B K_i \right] + \varepsilon_i^T \left( E_n + E_n K_i \right)^T \left( E_n + E_n K_i \right) + \varepsilon_i P D_i D_i^T P \right. \]

\[ + \sum_{j=1 \atop j \neq i}^{n} \left[ R_j^{ij} + P G_j R_i G_j^T P + \beta_j^{-1} N_j^T N_j + \beta_j P M_j M_j^T P \right] \left. \right] x_i(t) \]

\[ = \sum_{i=1}^{n} x_i^T(t) \left( L_i - Q_i - K_i^T R_i K_i \right) x_i(t) < \sum_{i=1}^{n} x_i^T(t) \left( -Q_i - K_i^T R_i K_i \right) x_i(t) < 0 \quad (3.5) \]

It can be seen from (3.5) the closed-loop system (3.4) is asymptotically stable with the state feedback control law \( u_i(t) = K_i x_i(t) \).

Furthermore, by integrating the inequality (3.5) over \([0, \infty)\) and considering the fact that \( V(x(t)) \to 0 \) when \( t \to \infty \),

\[ J = \sum_{i=1}^{n} \int_0^\infty x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t) dt = \sum_{i=1}^{n} \int_0^\infty x_i^T(t) \left( -Q_i - K_i^T R_i K_i \right) x_i(t) dt < \sum_{i=1}^{n} x_i^T(0) P_i x_i(0) \quad (3.6) \]

Thus, if \( L_i < 0 \), then \( u_i(t) = K_i x_i(t) \) is a quadratic guaranteed cost controller for the uncertain interconnected system (3.1).
Next, we’ll transform the inequality (3.3) into a set of LMIs that are easy to solve. Define \( X_i = P_i^{-1} \) and \( Y = K_i X_i \), pre and post-multiplying both sides of (3.3) by \( X_i \), it gives that

\[
X_iL_iX_i = \left[ A_iX_i + B_iY \right]^T + \left[ A_iX_i + B_iY \right] + \varepsilon_i^{-1} \left( E_iX_i + E_{i2}Y \right)^T (E_iX_i + E_{i2}Y) + \varepsilon_iD_iD_i^T
\]

\[+X_i \Gamma_{i2}^{-1} X_i + G_i \Gamma_{ai}^{-1} G_i^T + X_i N_i^T Y_{i2}^T N_i X_i + M_i Y_i M_i^T + X_i Q_i X_i + Y_i^T R_i Y < 0 \]  

(3.7)

Using Schur complements, the above inequality is equivalent to (3.8).

\[
\begin{bmatrix}
\Omega & (E_iX_i + E_{i2}Y)^T & X_iN_i^T & X_i & Y_i^T \\
(E_iX_i + E_{i2}Y)^T & -\varepsilon_i I & 0 & 0 & 0 \\
X_i & 0 & -\Gamma_{i2} & 0 & 0 \\
N_i X_i & 0 & 0 & -Y_{i2} & 0 \\
X_i & 0 & 0 & 0 & -G_i^{-1} \\
Y_i^T & 0 & 0 & 0 & -R_i^{-1}
\end{bmatrix} < 0
\]  

(3.8)

where

\[
\Omega = \left[ A_iX_i + B_iY \right]^T + \left[ A_iX_i + B_iY \right] + \varepsilon_iD_iD_i^T + G_i \Gamma_{ai}^{-1} G_i^T + M_i Y_i M_i^T
\]

\[
\bar{X}_i = [X_i, \ldots, X_i, \ldots, X_i] , \text{ the vector length of } \bar{X}_i \text{ in terms of } X_i \text{ is } n-1
\]

\[
G_i = \left[ G_{i1}, \ldots, G_y, \ldots, G_m \right] , \quad M_i = \left[ M_{i1}, \ldots, M_y, \ldots, M_m \right] , \quad N_i = \left[ N_{i1}, \ldots, N_y, \ldots, N_m \right]^T
\]

\[
\Gamma_{ai} = \text{diag} \left\{ R_{i1}, \ldots, R_y, \ldots, R_m \right\} , \quad \Gamma_{i2} = \text{diag} \left\{ R_{i1}, \ldots, R_y, \ldots, R_m \right\}
\]

\[
Y_{i1} = \text{diag} \left\{ \beta_{i1} I, \ldots, \beta_y I, \ldots, \beta_m I \right\} , \quad Y_{i2} = \text{diag} \left\{ \beta_{i1} I, \ldots, \beta_y I, \ldots, \beta_m I \right\}
\]

\[
i, j = 1, 2, \ldots, n, \; j \neq i
\]

Also, from (3.6) it can be seen the cost function is bounded and depended on the initial condition \( x_i(0) \). To remove this dependence, the deterministic method in [101] is employed. Suppose the initial state \( x_i(0) \) is arbitrary but belongs to the set

\[ S_i : \{ x_i(0) \in R^n : x_i(0) = \Pi_{a0} v_i^*, v_i^T v_i \leq 1 \} \].

Then it follows from (3.6) \( J \leq \sum_i \lambda_{\text{max}}(\Pi_{a0}^T X_i^{-1} \Pi_{a0}) \)

where \( \lambda_{\text{max}}(\cdot) \) denotes the maximum eigenvalue.

For any given constant \( \lambda_i > 0 \), \( \lambda_{\text{max}}(\Pi_{a0}^T X_i^{-1} \Pi_{a0}) < \lambda_i \), if and only if \( \Pi_{a0}^T X_i^{-1} \Pi_{a0} < \lambda_i I \), which is equivalent to

\[
\begin{bmatrix}
-\lambda_i I & \Pi_{a0}^T \\
\Pi_{a0} & -X_i
\end{bmatrix} < 0
\]  

(3.9)
Chapter 3 Control Methods

Up to now, it can be seen that in order to obtain the robust guaranteed performance control law

\[ u_i(t) = K_i x_i(t) = Y_i X_i^{-1} x_i(t) \]  

(3.10)

the parametric problem of minimizing \( J \leq \sum_{i=1}^{n} \lambda_i \) under the LMI constraints (3.8) and (3.9) need to be solved. Obviously, inequality (3.8) is a linear matrix inequality in \( e_i, \beta_i, R_i, X_i \) and \( Y_i \) and inequality (3.9) is also a linear one in \( X_i \), so this problem can be solved efficiently using the LMI tool [102].

**Remark 3.1:** The above proof is also suitable for designing robust performance guaranteed controllers for SMIB systems. This is realized by simply ignoring the interconnection part.

### 3.4 Overlapping decomposition control

For large scale systems, it is usually necessary to decompose the given system into a number of interconnected subsystems. Once decomposition is available, the control law for each subsystem can be designed independently and the control laws are to be combined to obtain a solution for the original problem. The decomposition technique is usually suitable for weakly interconnected systems. However, many large scale systems may consist of strongly interconnected subsystems. Power systems, socio-economic systems and freeway traffic regulation are examples of such behavior [103]. For such cases, overlapping decompositions [25, 27-30, 103-107] may produce useful solutions. In references [29, 30], this decomposition technique is applied to automatic generation control (AGC) of an electric power system.

Overlapping decomposition approach starts with expanding certain spaces of a dynamic system with overlapping subsystems into higher-dimensional spaces in which strong connection becomes weak. Then fully decentralized control laws are designed...
in the expanded spaces and contracted back to the original spaces for implementation. Any information about the motion of the original space can be extracted from the motions of the expanded space. The mathematical framework called "inclusion principle" [25, 104] or the extension principle [27, 28] developed from "inclusion principle" guarantees the correctness of this expansion/contraction procedure.

In case only the state space is needed to be expanded and contracted back, an expanded system is obtained simply by repeating states of the given system. The expanded system can be decomposed into weak connected subsystems to represent an overlapping decomposition of the original system, and standard techniques can be applied to the expanded space in order to show the stability of the system with overlapping subsystems. The conclusion for overlapping control for such a case is given below.

Consider two systems

\[ S : \dot{x} = Ax + Bu \]

with feedback law in the form \( u = Kx \)

where \( x \in \mathbb{R}^n \) is the state, \( u \in \mathbb{R}^m \) is the control input and \( K \in \mathbb{R}^{m \times n} \) is the feedback gain matrix;

and \( \tilde{S} : \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u \) with feedback law in the form \( u = \tilde{K}\tilde{x} \),

where \( \tilde{x} \in \mathbb{R}^\tilde{n} \) is the state, \( u \in \mathbb{R}^\tilde{m} \) is the control input, \( \tilde{K} \in \mathbb{R}^{\tilde{m} \times \tilde{n}} \) is the feedback gain matrix;

if a rectangular matrix \( V \in \mathbb{R}^{\tilde{n} \times n} \) can be found such that \( VA = \tilde{A}V \), \( VB = \tilde{B} \) and \( K = \tilde{KV} \), then the closed-loop system in the original space \( \tilde{S} : \dot{x} = (A + BK)x \) is included in the closed-loop system in the expanded space \( \tilde{S} : \dot{\tilde{x}} = (\tilde{A} + \tilde{B}K)\tilde{x} \).

Here a special form in the original space is considered

\[ S : \dot{x} = Ax + Bu \]
Chapter 3 Control Methods

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix} =
\begin{bmatrix}
    A_{11} & A_{12} & 0 \\
    A_{21} & A_{22} & A_{23} \\
    0 & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix} +
\begin{bmatrix}
    B_{11} & 0 \\
    0 & 0 \\
    0 & B_{31}
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]

(3.11)

Choose \( \tilde{A} \) and \( \tilde{B} \) in the form

\[
\tilde{A} =
\begin{bmatrix}
    A_{11} & A_{12} & 0 & 0 \\
    A_{21} & A_{22} & A_{23} \\
    0 & A_{32} & A_{33}
\end{bmatrix},
\]

\[
\tilde{B} =
\begin{bmatrix}
    B_{11} & 0 \\
    0 & 0 \\
    0 & B_{31}
\end{bmatrix}
\]

Then a rectangular matrix \( V \) that satisfies the expansion and inclusive principle can be found has the following form

\[
V =
\begin{bmatrix}
    I_1 & 0 & 0 \\
    0 & I_2 & 0 \\
    0 & I_2 & 0 \\
    0 & 0 & I_3
\end{bmatrix}
\]

Pre-multiplying \( V \) at both sides of \( \dot{x} = Ax + Bu \), it can be obtained

\[
V \dot{x} = VAx + VBu = \tilde{A}Vx + \tilde{Bu}
\]

Defining \( \dot{x} = Vx \), then the above equation becomes

\[
\dot{x} = \tilde{A}x + \tilde{Bu}
\]

Thus, our choices of \( \tilde{A} \), \( \tilde{B} \) and \( \dot{x} = Vx \) construct the expanded space, that is

\[
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
    A_{11} & A_{12} & 0 & 0 \\
    A_{21} & A_{22} & A_{23} \\
    A_{31} & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix} +
\begin{bmatrix}
    B_{11} & 0 \\
    0 & 0 \\
    0 & B_{31}
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]

(3.12)

Figure 3.2 and Figure 3.3 illustrate the original and expanded system in state space form. After overlapping decomposition, \( x_1, x_2, x_3 \) and \( x_4 \) can be regarded as two local subsystems that have weak connections (dashed line in Figure 3.3). Such system structure is consistent with HVDC link model which will be used for controller design in Chapter 6.

Remark 3.2: From (3.12), it can be seen, when overlapping decomposition is applied only for state space, the following three conclusions can be drawn:

1) The expanded space is formed simply by repeating the overlapping part.

2) The control law in the expanded space \( u = \hat{K}\dot{x} = \hat{K}Vx = Kx \) is exactly the same as
that in the original space, thus after the control law is obtained in the expanded space, it is not necessary to contract the control law back into the original space.

3) The state variables $x_1$, $x_2$ and $x_3$ in the state space are not necessary scalars, each can be vectors.

![Figure 3.2 Original system with overlapping](image)

![Figure 3.3 Expanded system after overlapping decomposition](image)

### 3.5 Composite nonlinear control

Due to the simplicity of linear control analysis, design and implementation, most of the existing control theories are dealt with linear controller design, especially linear feedback controller design. While in most cases linear feedback control is able to satisfy practical requirement, there are actually some cases that the linear feedback control is not. For example, such is the case for the saturation problem which will be dealt with in Chapter 8.
One outstanding disadvantage of linear feedback control is that when the controlled error vector decreases the control vector also decreases, and thus the control capacity is not fully employed especially for relatively low errors. To eliminate this advantage, one reasonable idea is to increase the controller gain to some permitted degree as the controlled error vector decreases. For problems that one controller is difficult or even impossible to solve, another one or set controllers can be designed and combined with the original controllers so that the problems are easily solved. Such a control method is a composite control method. The extra control law can be linear or nonlinear, continuous or discontinuous. When the extra control law is nonlinear, the control method is the so-called "composite nonlinear control". As is shown in Figure 3.4, a composite nonlinear control law $u(x)$ is obtained by adding an additional nonlinear control law $g(x)$ to the original linear control law $Kx$ to solve the input saturation problem. This scheme was proposed in [63, 64], where a simple nonlinear feedback law is subtly combined with low gain feedback control law to deal with the saturation problem. Details will be introduced in Chapter 8.

\[ \dot{x} = f(x,u) \]

\[ K \]

\[ g(*) \]

Figure 3.4 Composite nonlinear control block diagram

**Remark 3.3:** The composite nonlinear control concept is used to solve a single but difficult problem where the satisfaction may not be guaranteed by using only one linear control law.
3.6 Global control

In most practical situations, complex systems operate in a range of different conditions and have different goals. Besides, the systems are subjected to various kinds of external disturbances and uncertainties. Therefore, in a wide operating range, the system model and dynamic behavior are qualitatively uncertain. To guarantee the overall stability and performance for complex systems, the concept of "global control" was proposed in [1, 52, 108, 109]. This control concept is an attempt to design controls of a truly global kind which act in a coordinated way across the whole system geographically and for all operating situations, i.e. for all states and in-the-large (in the sense of stability theory) as parameters and conditions vary [1].

Figure 3.5 illustrates the concept of global control for a linear system, of which the...
behaviors are qualitatively quite different in different operating regions. It is assumed the operating regions are partitioned into \( n \) subspaces \( S_i(x) \) \( (i = 1,2,\ldots,n) \). For each subspace \( S_i(x) \), there is a special local control law \( u_i(x) \) that is able to provide high-resolution control and be analyzed by classical methods. The switch action of local control laws between different operating regions is defined by appropriate membership functions \( \lambda_i(x) \) \( (i = 1,2,\ldots,n) \), e.g., fuzzy set membership functions. The global control law is the composition of heterogeneous weighted local control laws appropriate to different operating regions, which can be written as

\[
u(x) = \sum_{i=1}^{n} \lambda_i(x) u_i(x) \quad (3.13)\]

It can be seen such a scheme is capable of controlling the system over a wide range of operating regions.

It should be pointed out that when the system operating region is not easy to partition, or there are too many regions to be partitioned, e.g., the dynamic characteristics change greatly with different operating points, or the system dynamic characteristics do not follow any obvious rules, this method may not be suitable any longer.”

As power systems are complex nonlinear systems, the requirements for transient stability, voltage stability and other factors are quite different both in time and space meaning. To achieve the global goal of satisfying these qualitatively distinctive control requirements in different operating regions, global control has been applied to power systems in the literature [1, 52, 108, 109]. Since transient stability and post-fault voltage regulation are the main concern of this study, the global control law in (3.13) only has two items for our case. Under this control law, the controller is capable of adapting itself to different operating conditions and is able to coordinate both control requirements of transient stability and voltage regulation of different state-space regions.
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3.7 Concluding remark

This chapter introduces the concept or theory of some advanced control methods. These methods will play a key role in the controller design in the following chapters. It will be seen later these methods are a good match of the practical power system requirement that the controllers should be able to deal with complex uncertainties, nonlinearities and interconnections.
Chapter 4 Robust CTSVRE control of SMIB Power Systems

4.1 Introduction

In this chapter and next chapter, robust coordinated transient stability and voltage regulation enhancement (CTSVRE) control of SMIB and multimachine power systems will be studied respectively.

In the past several decades, in order to obtain satisfactory transient performance over a large operating region, various advanced nonlinear control schemes have been proposed [4, 7, 9, 34, 36, 37, 39, 40, 42, 47, 50, 51, 54, 110]. Most of the proposed nonlinear control methods for power systems essentially address the transient stability problem by regulating rotor angle and only a few of them consider the voltage control problem [7, 9, 36, 39, 40].

While the transient stability enhancement (TSE) control can transiently stabilize power systems to some degree by regulating the post-fault rotor angle to the pre-fault angle, it may not guarantee the performance of the post-fault voltage. For example, in the case when there is a permanent fault, the network structure changes after the fault is isolated and the system restores to a new steady state. For such a case, the TSE controller is usually not effective. This motivates to add additional voltage regulation function to the original TSE controller design. In [9, 36], CTSVRE control has been proposed. The control method there is based on a preliminary analysis of the parametric uncertainties, which is quite rough and not clear. In this work, detailed analysis of the parametric uncertainties will be carried out. Based on the analysis, the operating region will be properly partitioned and LMI method will be employed to obtain the controller parameters. The modified controller is effective in a larger
operating region.

Since the robust CTSVRE controller design is based on the results of robust TSE controller design, robust TSE controller will be designed at first, followed by robust CTSVRE controller design and numerical simulation.

### 4.2 Robust TSE control of SMIB power systems

#### 4.2.1 Feedback linearization

The DFL technique introduced in Section 3.2 will be used in this section to eliminate the generator model nonlinearities. By employing DFL, the original nonlinear system can be directly transformed into a system whose closed-loop dynamics are linear over a wide range.

From equations (2.1)-(2.10), it can be seen that the synchronous generator is nonlinear through the excitation system. Denoting $\Delta P_e(t) = P_e(t) - P_n$, differentiating equation (2.8) and using (2.4)-(2.10), it gives that

$$\dot{P}_e(t) = \dot{E}_q(t)I_q(t) + E_q(t)\dot{I}_q(t)$$

$$= \frac{1}{T_{d0}}\left[ -E_q(t) - (x_d - x_d') I_q(t) + E_q(t) \right] I_q(t) + \frac{E_q(t)V_q \cos \delta(t)}{x_{d0}} \dot{\delta}(t)$$

$$= \frac{1}{T_{d0}} \Delta P_e(t) + \frac{1}{T_{d0}} \left[ k_{u}u_f(t)I_q(t) - (x_d - x_d') I_a(t)I_q(t) - P_n \right] + \left( \frac{E^2_q(t)}{x_{d0}} - Q_e(t) \right) \Delta \omega(t)$$

$$= \frac{1}{T_{d0}} \Delta P_e(t) + \frac{1}{T_{d0}} \left[ k_{u}u_f(t)I_q(t) - (x_d - x_d') I_a(t)I_q(t) - P_n + T_{d0} \left( \frac{E^2_q(t)}{x_{d0}} - Q_e(t) \right) \Delta \omega(t) \right]$$

Let

$$v_f(t) = k_{u}u_f(t)I_q(t) - (x_d - x_d') I_a(t)I_q(t) - P_n + T_{d0} \left( \frac{E^2_q(t)}{x_{d0}} - Q_e(t) \right) \Delta \omega(t)$$

there is

$$\Delta P_e(t) = \frac{1}{T_{d0}} \left[ k_{u}u_f(t)I_q(t) - (x_d - x_d') I_a(t)I_q(t) - P_n + T_{d0} \left( \frac{E^2_q(t)}{x_{d0}} - Q_e(t) \right) \Delta \omega(t) \right]$$

(4.1)
\[ \Delta P(t) = -\frac{1}{T_{d0}} \Delta P(t) + \frac{1}{T_{do}} v_j(t) \] 
(4.2)

Now, the new input is \( v_j(t) \).

The DFL compensating control law is directly obtained as follows

\[
u_j(t) = \frac{1}{k_i I_s(t)} \left[ v_j(t) + P_e + (x_d - x_d') I_d(t) I_s(t) - T_{do} \left( \frac{E_q^s(t)}{x_d} - Q_e(t) \right) \Delta \omega(t) \right]
\]
(4.3)

It can be seen from Section 2.3, \( P_e(t), Q_e(t), I_f(t), I_d(t) \) and \( I_s(t) \) are all available variables and \( \omega(t) \) is also a measurable variable. Rotor angle \( \delta(t) \) that will appear in the feedback law \( v_j(t) \) can be estimated [40] or measured [111]. Thus, the compensating law (4.3) is practically realizable.

The mapping from \( u_j(t) \) to \( v_j(t) \) is invertible except when \( I_s(t) = 0 \), which is not in the normal operating region for a generator. This can be explained like this. In the normal working region of a generator, \( 0^\circ < \delta(t) < 180^\circ \), so \( I_s(t) > 0 \). When \( I_s(t) \) decreases to some extent, the controller will fall in saturation due to the physical limit of \( u_j(t) \). This also shows that the compensating law (4.3) is practically realizable.

So far, the model (2.1) to (2.10) has been linearized. The linearized model is

\[ \Delta \delta(t) = \Delta \omega(t) \]
\[ \Delta \omega(t) = -\frac{D}{2H} \Delta \omega(t) - \frac{\phi_b}{2H} \Delta P_e(t) \]
\[ \Delta P_e(t) = -\frac{1}{T_{d0}} \Delta P(t) + \frac{1}{T_{do}} v_j(t) \]

The above three equations can be written in state space format

\[ \dot{x}(t) = Ax(t) + Bu(t) \]
(4.4)

where

\[
x(t) = \begin{bmatrix} \Delta \delta(t) \\ \Delta \omega(t) \\ \Delta P_e(t) \end{bmatrix}, \quad u(t) = v_j(t)
\]
Chapter 4 Robust CTSVRE Control of SMIB Power Systems

\[ A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{D}{2H} & -\frac{\omega_0}{2H} & 0 \\ 0 & 0 & -\frac{1}{T_{do}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{do}} \end{bmatrix} \]

To design a nonlinear TSE control law \( u_f(t) \) to stabilize the generator model (2.1)-(2.10) is equivalent to design a robust control law \( v_f(t) \) to stabilize the uncertain DFL compensated model (4.4).

4.2.2 Robust TSE controller design

After the linearization, linear control theory can then be employed to design a linear feedback control law. Here, performance guaranteed controller will be designed using robust control theory.

Consider the various parameter uncertainties in power systems, the linearized generator system model can be rewritten in the following state space form

\[ \dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) \quad (4.5) \]

where \( x(t), \ u(t), \ A \) and \( B \) are same as in (4.4).

As an illustrated example, the parametric uncertainties in \( T_{do} \) as \( \Delta T_{do} \) is considered in this work. Then the uncertainty matrices are

\[ \Delta A(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \mu(t) \end{bmatrix}, \quad \Delta B(t) = \begin{bmatrix} 0 \\ 0 \\ -\mu(t) \end{bmatrix} \]

\[ \mu(t) = \frac{1}{T_{do}} - \frac{1}{T_{do} + \Delta T_{do}} \]

Since \( \Delta T_{do} \leq |\Delta T_{do}|_{\text{max}} \), \( \mu(t) \) is norm bounded, that is \( \mu(t) \leq |\mu(t)|_{\text{max}} \).

The norm bounded parameter uncertainties are assumed to be of the following forms

\[ [\Delta A(t) \ \Delta B(t)] = DF(t)[E_1 \ E_2] \quad (4.6) \]

where \( D, \ E_1 \) and \( E_2 \) are known constant real matrices of appropriate dimensions and \( F(t) \in R^{n \times n} \) is an unknown matrix function with Lebesgue measurable elements.
and satisfies \( F(t) \leq I \), where \( I \) denotes the identity matrix of appropriate dimension.

One possible decomposition of the uncertainties can be expressed as

\[
D = \begin{bmatrix} 0 & 0 \\ \eta(t) & \eta(t) \end{bmatrix}^T, \quad F(t) = \frac{\mu(t)}{\eta(t)}, \quad E_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad E_z = -1
\]

Associated with (4.5) is the cost function

\[
J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt
\]

where \( Q \) and \( R \) are given positive-definite symmetric matrices which are chosen at the designer's will.

In the following part, a linear time-invariant feedback guaranteed cost control law \( u(t) = Kx(t) \) will be designed.

Substituting \( u(t) = Kx(t) \) into (4.5), the resulting closed-loop system is

\[
\dot{x}(t) = \left[ A + BK + DF(t)(E_i + E_zK) \right] x(t)
\]

Define \( V = x^T(t)P x(t) \), \( P = P^T > 0 \), then the time derivative of \( V(t) \) along any trajectory of the closed-loop system (4.8) is given by

\[
\dot{V} = x^T(t) \left[ P(A + BK) + (A + BK)^T P + PDF(t)(E_i + E_zK) + (E_i + E_zK)^T F^T(t)D^T P \right] x(t)
\]

Apparently, for any \( \varepsilon > 0 \), there is

\[
PDF(E_i + E_zK) + (E_i + E_zK)^T F^T(t)D^T P \\
\leq \varepsilon PDD^T P + \frac{1}{\varepsilon} (E_i + E_zK)^T F^T(t)F(t)(E_i + E_zK)
\]

\[
\leq \varepsilon PDD^T P + \frac{1}{\varepsilon} (E_i + E_zK)^T (E_i + E_zK)
\]

Thus, \( \dot{V} \leq x^T(t) \left[ P(A + BK) + (A + BK)^T P + \varepsilon PDD^T P + \frac{1}{\varepsilon} (E_i + E_zK)^T (E_i + E_zK) + Q + K^T RK \right] x(t) \)

Set \( L = P(A + BK) + (A + BK)^T P + \varepsilon PDD^T P + \frac{1}{\varepsilon} (E_i + E_zK)^T (E_i + E_zK) + Q + K^T RK \). Obviously, if \( L < 0 \), then

\[
\dot{V} \leq x^T(t) \left[ L - Q - K^T RK \right] x(t) < x^T(t) \left[ -Q - K^T RK \right] x(t) < 0
\]

that is, the closed-loop system (4.8) is asymptotically stable. Next, by integrating the
inequality (4.7) over \([0, \infty)\) and considering that \(V(x(t)) \to 0\) when \(t \to \infty\).

\[
J = \int_0^\infty x^T(t)Qx(t) + u^T(t)Ru(t)dt = \int_0^\infty x^T(t)\left(Q + K^TRK\right)x(t)dt < V(x(0)) = x^T(0)Px(0) \quad (4.10)
\]

Furthermore, by using Schur complement, it can be seen that \(L < 0\) is equivalent to

\[
\begin{bmatrix}
\Xi & (E_1 + E_2K)^T & I & K^T \\
E_1 + E_2K & -\varepsilon I & 0 & 0 \\
I & 0 & -Q^{-1} & 0 \\
K & 0 & 0 & -R^{-1}
\end{bmatrix} \prec 0
\quad (4.11)
\]

where \(\Xi = P(A + BK)^T + \varepsilon PP^T\).

Pre-multiplying and post-multiplying (4.11) by \(\text{diag}\{P^{-1}, I, I, I\}\) and denoting \(X = P^{-1}\), \(Y = KP^{-1}\) yield the matrix inequality (4.12).

\[
\begin{bmatrix}
\Phi & (E_1X + E_2Y)^T & X & Y^T \\
E_1X + E_2Y & -\varepsilon I & 0 & 0 \\
X & 0 & -Q^{-1} & 0 \\
Y & 0 & 0 & -R^{-1}
\end{bmatrix} \prec 0
\quad (4.12)
\]

where \(\Phi = AX + BY + (AX + BY)^T + \varepsilon DD^T\).

Also, from (4.10) it can be seen the cost function is bounded and depended on the initial condition \(x(0)\). To remove this dependence, the deterministic method in [101] is employed. Suppose the initial state \(x(0)\) is arbitrary but belongs to the set \(S : \{x(0) \in \mathbb{R}^n : x(0) = \Pi_0 v, \nu^Tv \leq 1\}\). Then it follows from (4.10)

\[
J \leq \lambda_{\max}(\Pi_0^TX^{-1}\Pi_0) \quad (4.13)
\]

where \(\lambda_{\max}(\cdot)\) denotes the maximum eigenvalue.

For any given constant \(\lambda > 0\), \(\lambda_{\max}(\Pi_0^TX^{-1}\Pi_0) < \lambda\), if and only if\(\Pi_0^TX^{-1}\Pi_0 < \lambda I\)

which is equivalent to

\[
\begin{bmatrix}
-\lambda I & \Pi_0^T \\
\Pi_0 & -X
\end{bmatrix} \prec 0 \quad (4.14)
\]

As yet, it can be seen that in order to obtain the robust guaranteed performance control law

\[
u(t) = \nu_f(t) = Kx(t) = YX^{-1}x(t) \quad (4.15)
\]
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the parametric problem of minimizing $\dot{\lambda}$ under the LMI constraints (4.12) and (4.14) need to be solved. Obviously, inequality (4.12) is a linear matrix inequality in $\varepsilon$, $X$ and $Y$ and inequality (4.14) is also a linear one in $X$, so both inequalities can be solved efficiently using the LMI tool [102].

The matrix $\Pi_0$ and vector $v$ may be chosen as [110]

$$\Pi_0 = \text{diag}\left\{3|\Delta\delta|_{\text{max}}, 3|\Delta\omega|_{\text{max}}, 3|\Delta P|_{\text{max}}\right\}$$

$$v = \begin{bmatrix} \Delta\delta(0) \\ 3|\Delta\delta|_{\text{max}} \\ \Delta\omega(0) \\ 3|\Delta\omega|_{\text{max}} \\ \Delta P_r(0) \\ 3|\Delta P_r|_{\text{max}} \end{bmatrix}^T$$

4.3 Robust CTSVRE control of SMIB power systems

Although TSE controller can guarantee transient stability of power systems for admissible uncertainties and during the post-fault period there are [34]

$$\lim_{t \to \infty} |\Delta\delta(t)| = 0$$

$$\lim_{t \to \infty} |\Delta\omega(t)| = 0$$

$$\lim_{t \to \infty} |\Delta P_r(t)| = 0$$

(4.16)

However, since $V_r(t)$ is a nonlinear function of not only $\delta(t)$ and $P_r(t)$ but also network parameters, any change in system structure will cause the change of $V_r(t)$. For example, when a permanent fault occurs, the network parameters change in order to remove the fault. In such a case, if the rotor angle and active power recover to the pre-fault value, the terminal voltage will reach an equilibrium that is different from the pre-fault steady state value. This is undesirable since the voltage is paid more attention to than the rotor angle is under steady-state conditions from an electrical engineer's point. In fact, in some cases, the post-voltage differs from the pre-fault value too much to be acceptable. Therefore, a CTSVRE controller should be designed.
4.3.1 Feedback linearization

In practice, we usually require the voltage recover to the pre-fault value in the post-fault state. Thus it is necessary to introduce the voltage deviation $\Delta V_r(t)$ into the feedback law to realize voltage regulation.

Equation (2.11) can be rewritten as

$$V_r(t) = \frac{1}{x_{sh}} \sqrt{x_q^2 E_q^2 + x_d^2 V_s^2 + 2x_q x_d E_q V_s \cos\delta(t)}$$

$$= \sqrt{\frac{x_q^2 P_r^2(t)}{V_r^2 \sin^2\delta(t)} + \frac{x_d^2 V_s^2}{x_{sh}} + \frac{2x_q x_d P_r(t) \cos\delta(t)}{x_{sh} \sin\delta(t)}}$$

(4.17)

Differentiating (4.17) gives

$$\dot{V}_r(t) = f_1(t)\Delta\omega(t) + f_2(t)\dot{P}_r(t)$$

(4.18)

where

$$f_1(t) = -\frac{x_q^2 P_r^2(t) \cos\delta(t)}{V_r(t) V_r^2 \sin^3\delta(t)} - \frac{x_q x_d P_r(t)}{x_{sh} V_r(t) \sin^2\delta(t)}$$

(4.19)

$$f_2(t) = \frac{x_d^2 P_r(t)}{V_r(t) V_r^2 \sin^2\delta(t)} + \frac{x_q \cos\delta(t)}{x_{sh} V_r(t) \sin\delta(t)}$$

(4.20)

Substituting (4.2) into (4.18) gives

$$\Delta\dot{V}_r(t) = f_1(t)\Delta\omega(t) - \frac{f_2(t)}{T_{so}} \Delta P_r + \frac{f_2(t)}{T_{so}} v_r(t)$$

(4.21)

where $\Delta V_r = V_r(t) - V_{r0}$

Choose $X(t) = [\Delta V_r(t) \Delta\omega(t) \Delta P_r(t)]^T$ as state variables vector, the linearized model (2.1) to (2.11) can be written in state space form as follows

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

(4.22)

where

$$x(t) = [\Delta V_r(t) \Delta\omega(t) \Delta P_r(t)]^T,$$

$$u(t) = v_r(t)$$
The mapping from $v_j(t)$ to $u_j(t)$ is exactly the same as (4.3).

If $f_1(t)$ and $f_2(t)$ are treated as variable parametric uncertainties, (4.22) can be rewritten as follows:

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t)$$

(4.23)

where $x(t)$ and $u(t)$ are the same as (4.22),

$$A = \begin{bmatrix} 0 & f_1(t) & -\frac{f_2(t)}{T_{d0}} \\ 0 & -\frac{D}{2H} & -\frac{\omega_n}{2H} \\ 0 & 0 & -\frac{1}{T_{d0}} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{f_2(t)}{T_{d0}} \\ 0 \\ -\frac{1}{T_{d0}} \end{bmatrix}$$

$$\Delta A(t) = \begin{bmatrix} 0 & \mu_1(t) & \mu_2(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta B(t) = \begin{bmatrix} -\frac{\mu_2(t)}{T_{d0}} \\ 0 \\ 0 \end{bmatrix}$$

$$\mu_1(t) = \Delta f_1(t); \quad \mu_2(t) = -\frac{\Delta f_2(t)}{T_{d0}};$$

with ' - ' representing the averaging value and 'Δ' representing the deviation from the averaging value to the respective variable.

The norm bounded uncertainties $\Delta A(t)$ and $\Delta B(t)$ can be expressed as follows:

$$[\Delta A(t), \Delta B(t)] = DF(t)[E_1, E_2]$$

where $D, E_1$, and $E_2$ are constant matrices

$$D = \begin{bmatrix} \mu_1_{\text{max}} & \mu_2_{\text{max}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad F(t) = \begin{bmatrix} \frac{\mu_1(t)}{\mu_1_{\text{max}}} & 0 \\ 0 & \frac{\mu_2(t)}{\mu_2_{\text{max}}} \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$
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It can be seen from (4.23), this model uses average values to represent the normal state space structure and treats the deviations from average values as uncertainties. To design a nonlinear voltage regulation control law \( u_r(t) \) to stabilize the generator model (2.1)-(2.11) is equivalent to design a robust control law \( v_r(t) \) to stabilize the uncertain DFL compensated model (4.23).

4.3.2 Robust CTSVRE controller design

In this section, the bounds of \( f_1(t) \) and \( f_2(t) \) will be firstly estimated and robust control theory will then be employed to design the CTSVRE controller. The example system used is shown in Figure 2.1. The system parameters will be given in Section 4.5.

After employing DFL compensating law, it can be seen from (4.22) that the control input is now a single variable \( v_r(t) \). But matrices \( A(t) \) and \( B(t) \) include \( f_1(t) \) and \( f_2(t) \), which are complex nonlinear variables dependent on operating points, so linear control methods can't be applied directly. Fortunately, the electrical power of a generator and the triangular functions of rotor angle are bounded, so are the variables of \( f_1(t) \) and \( f_2(t) \). If \( f_1(t) \) and \( f_2(t) \) are treated as bounded variable parameters, linear robust control theory can then be used to design the CTSVRE controller.

**4.3.2.1 Estimation of bounds of \( f_1(t) \) and \( f_2(t) \)**

First, let us observe the values of \( f_1(t) \) and \( f_2(t) \) under one typical operating condition, that is, \( \omega = \omega_0 \), \( V_t = 1 \), \( x_r = x_r + x_l \), where \( x_r \) is the reactance for transformer and \( x_l \) is the reactance for each transmission line. As rotor angle is an important index to denote rotor angle stability, curves of \( f_1(t) \) and \( f_2(t) \) against \( \delta(t) \) will be plotted. Let rotor angle \( \delta(t) \) vary within \((0^\circ, 180^\circ)\), after \( P_r(t) \) is calculated from (4.17), \( f_1(t) \) and \( f_2(t) \) can then be calculated from (4.19) and (4.20). Plots of \( f_1(t) \) and \( f_2(t) \) as a function of \( \delta(t) \) are shown in Figure 4.1.
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Figure 4.1 \( f_1 - \delta, \ f_2 - \delta \) when \( V_r = 1, \ x_r = x + x_i \)

From Figure 4.1, it can be seen that slopes of \( f_1(t) \) and \( f_2(t) \) are much sharper when rotor angle \( \delta(t) \) is near 0° or 180° than those when \( \delta(t) \) is at the middle range where the slopes of \( f_1(t) \) and \( f_2(t) \) are quite smooth. Normally, a synchronous generator doesn't work at very small rotor angle near 0° and it is usually impossible to work stably at very large rotor angle above 90°. Due to this reason, when calculating bounds of \( f_1(t) \) and \( f_2(t) \), it is reasonable not to consider the rotor angle range near 0° or above 90°. Even so, we partially take into account the rotor angle range above 90° to compensate the transient process of synchronous generators. Thus, the curves of \( f_1(t) \) and \( f_2(t) \) can be divided approximately into two parts according to the range of \( \delta(t) \). The curves are then approximately linearized into two piecewise lines. After the partition, the controllers can be designed piecewise. The objective of this partition is to achieve better performance when designing controllers.

In order to make the controller effective in a wide range, variations of terminal voltage and network parameters are also considered. The variations magnitude for \( V_r(t) \) is assumed to be within ±10%, namely, \( V_r(t) \in [0.9, 1.1] \). During symmetrical 3-phase short circuit period, \( u_r(t) \) usually reaches limit, so the system structure is not
considered. After the fault is removed or isolated, the network structure may change, the bound of $x_i$ can be used to reflect this change, $x_i + x_j \leq x_i \leq x_i + 2x_j$.

By considering these factors and choosing six different typical operating conditions listed in Table 4.1 ($\omega = \omega_0$), six curves of $f_1(t)$ and $f_2(t)$ when $\delta(t)\in[10^\circ, 120^\circ]$ are plotted in Figure 4.2. This rotor angle range is believed to be large enough to include the uncertainties regarding our problem. Rotor angle ranges $\delta(t) < 40^\circ$ and $\delta(t) \geq 40^\circ$ are chosen to partition $f_1(t)$ and $f_2(t)$ into two sections. The approximated envelopes of curves of $f_1(t)$ and $f_2(t)$ are plotted as straight lines at the right side in Figure 4.2. The data of the bounds of $f_1(t)$ and $f_2(t)$ are given in Table 4.2.

<table>
<thead>
<tr>
<th>Curve No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_i$ (p.u.)</td>
<td>0.9</td>
<td>1.0</td>
<td>1.1</td>
<td>0.9</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>$x_i$</td>
<td>$x_i + x_j$</td>
<td>$x_i + x_j$</td>
<td>$x_i + x_j$</td>
<td>$x_i + 2x_j$</td>
<td>$x_i + 2x_j$</td>
<td>$x_i + 2x_j$</td>
</tr>
</tbody>
</table>

Table 4.2  Bounds of $f_1(t)$ and $f_2(t)$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$&lt; 40^\circ$</th>
<th>$\geq 40^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1_{\text{max}}$</td>
<td>-0.1837</td>
<td>-0.2296</td>
</tr>
<tr>
<td>$f_1_{\text{min}}$</td>
<td>-3.3314</td>
<td>-0.8279</td>
</tr>
<tr>
<td>$\bar{f}_1$ (average value)</td>
<td>-1.7576</td>
<td>-0.5287</td>
</tr>
<tr>
<td>$f_2_{\text{max}}$</td>
<td>2.9844</td>
<td>0.7516</td>
</tr>
<tr>
<td>$f_2_{\text{min}}$</td>
<td>0.2309</td>
<td>-0.0847</td>
</tr>
<tr>
<td>$\bar{f}_2$ (average value)</td>
<td>1.6077</td>
<td>0.3334</td>
</tr>
</tbody>
</table>
4.3.2.2 Robust Controller design

After the bounds of $f_1(t)$ and $f_2(t)$ are estimated, the design of a robust guaranteed performance CTSVRE control law $u(t) = Kx(t)$ for system (4.22) can follow the procedure of designing the TSE control law in Section 4.2.2. It involves solving two LMIs corresponding to (4.12) and (4.14).

The matrix $\Pi_0$ and vector $v$ may be chosen as

$$\Pi_0 = \text{diag} \left\{ 3|\Delta V|_{\max}, 3|\Delta \omega|_{\max}, 3|\Delta P_r|_{\max} \right\}$$

$$v = \begin{bmatrix} \Delta V_r(0) \\ 3|\Delta V_r|_{\max} \\ \Delta \omega(0) \\ 3|\Delta \omega|_{\max} \\ \Delta P_r(0) \\ 3|\Delta P_r|_{\max} \end{bmatrix}^T$$

As mentioned before, parameter bounds of $f_1(t)$ or $f_2(t)$ are approximately partitioned into two parts corresponding to rotor angle ranges $\delta(t) < 40^\circ$ and $\delta(t) \geq 40^\circ$. Correspondingly, there are two CTSVRE control laws $v_{\mu_1}(t)$ and $v_{\mu_2}(t)$ for each
generator. When the rotor angle $\delta(t)$ varies across $40^\circ$, there should have a switch action between the two control laws. The switch action between the two control laws should be smooth. Otherwise, another "disturbance" will be introduced into the system.

To make the switch action smooth, appropriate control strategy should be used to coordinate the two local CTSVRE control laws. This indeed uses the global control concept. Corresponding to the rotor angle ranges $\delta(t) < 40^\circ$ and $\delta(t) \geq 40^\circ$, the following trapezoid-shaped like member functions [52] are used

$$\lambda_{\nu_1} = \left(1 - \frac{1}{1 + \exp(-120(\delta(t) - 40^\circ))}\right) \left(\frac{1}{1 + \exp(-120(\delta(t) + 40^\circ))}\right)$$  \hspace{1cm} (4.24)

$$\lambda_{\nu_2} = 1 - \lambda_{\nu_1}$$  \hspace{1cm} (4.25)

Then the combined voltage regulation control law is:

$$v_{\nu}(t) = \lambda_{\nu_1} v_{\nu_1}(t) + \lambda_{\nu_2} v_{\nu_2}(t)$$  \hspace{1cm} (4.26)

where $v_{\nu_1}$ corresponding to the rotor angle range $\delta(t) < 40^\circ$ and $v_{\nu_2}(t)$ corresponding to $\delta(t) \geq 40^\circ$.

### 4.4 Simulation study

To evaluate the proposed CTSVRE controller, the example system shown in Figure 2.1 is employed in the numerical simulation. The generator and transformer parameters of the example system under consideration are (100MVA base):

- $\omega = 314.159 \text{ rad/sec}$; $D = 5.0 \text{ p.u.}$; $H = 6.0 \text{ sec}$; $T_{\omega} = 6.9 \text{ s}$; $x_i = 0.127 \text{ p.u.}$; $x_q = 1.863 \text{ p.u.}$;
- $x_{q} = 0.657 \text{ p.u.}$; $x_{ad} = 1.712 \text{ p.u.}$; $V_r = 1.0 \text{ p.u.}$.

Based on the approach described before, the controller parameters can be obtained by solving a series of LMIs. The obtained control laws are as follows

The TSE control law is (assuming $|\Delta T_{\omega}^r| \leq 0.1T_{\omega}^r$)

$$v_{f_0}(t) = 25.85 \Delta \delta(t) + 23.19 \Delta \omega(t) + 111.15 \Delta P_r(t)$$

The CTSVRE control law corresponding to $\delta(t) < 40^\circ$ is
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\[ v_{\delta_1}(t) = -33.47\Delta V_f(t) + 16.04\Delta \omega(t) - 13.81\Delta P_f(t) \]

The CTSVRE control law corresponding to \( \delta(t) \geq 40^\circ \) is

\[ v_{\delta_2}(t) = -12.03\Delta V_f(t) + 14.77\Delta \omega(t) - 36.48\Delta P_f(t) \]

The combined CTSVRE control law is

\[ v_f(t) = \lambda_1 v_{\delta_1}(t) + \lambda_2 v_{\delta_2}(t) \]

where \( \lambda_1 \) and \( \lambda_2 \) are defined in Section 4.3 and can be obtained by local measurements or estimators. The actual control law \( u_f(t) \) can be obtained from \( v_f(t) \) through the relationship of (4.3).

It should be pointed out that the transmission line resistance is considered in the numeric simulation although it is not considered in the controller design for simplicity. Meanwhile, in order to evaluate the performance of the proposed exciter controller more accurately, the physical limit of the excitation voltage is considered:

\[ k_v u_f(t) \in [-3, 6]p.u. \]

The fault considered is a symmetrical three-phase short circuit fault, which occurs on one of the transmission lines. Two different fault sequences are considered:

**Fault Sequence 4.1** (Temporary Fault)
Stage 1 The system is in a pre-fault steady-state;
Stage 2 A fault occurs at \( t=0.1s \);
Stage 3 The fault is removed by opening the breakers of the faulted line at \( t=0.2s \);
Stage 4 The transmission lines are restored at \( t=1.2s \);
Stage 5 The system is in a post-fault state.

**Fault Sequence 4.2** (Permanent Fault)
Stage 1 The system is in a pre-fault steady-state;
Stage 2 A fault occurs at \( t=0.1s \);
Stage 3 The fault is isolated by opening the breakers of the faulted line at \( t=0.2s \);
Stage 4 The system is in a post-fault state.

First, system responses of the proposed control will be compared with those of the robust TSE control to illustrate the effectiveness of the proposed control in case of permanent fault. Next, system responses of the proposed control will be compared with conventional control. Finally, robustness of the proposed controller is shown. Different cases are used in the numerical simulation.

### 4.4.1 Comparison with robust TSE control

In this section, responses of TSE controller and robust CTSVRE controller are compared. The reason why robust CTSVRE controller is needed is given.

Figure 4.3  Power angle, relative speed, terminal voltage and active power responses for Case 4.1 with TSE control
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The following two cases are considered to evaluate the proposed control strategy.

Case 4.1:

The operating point is \( \delta_0 = 50.10^\circ \), \( V_{in} = 1.10 \text{ p.u.} \), \( P_{in} = 1.00 \text{ p.u.} \). The fault occurs on one of the parallel AC transmission lines near bus 2. The fault sequence is Fault Sequence 4.1. Parameters for each transmission line are \( r_i = 0.03 \times 3 \text{ p.u.} \); \( x_i = 0.24265 \times 3 \text{ p.u.} \).

Figure 4.3 shows power angle, relative angular speed, generator terminal voltage and active power responses of the robust TSE controller.

Case 4.2:

In this case, the operating point, fault location and network parameters are the same as Case 4.1. The fault sequence is Fault Sequence 4.2.

Figure 4.4 shows power angle, relative angular speed, generator terminal voltage and active power responses for Case 4.2 with TSE control.

Figure 4.4  Power angle, relative speed, terminal voltage and active power responses for Case 4.2 with TSE control
active power responses of the robust TSE controller; Figure 4.5 shows the four responses of the robust CTSVRE controller.

It can be seen from the simulation results in Figure 4.3 - Figure 4.5, while the robust TSE controller can transiently stabilize the example system in both the temporary fault case (Case 4.1) and permanent fault case (Case 4.2), it can only achieve satisfactory post-fault voltage performance in case of temporary fault. In case of permanent fault, although the rotor angle recovers to the pre-fault value, the voltage deviates from the pre-fault value by almost 20%, which is not allowed in practice. On the other hand, the robust CTSVRE controller can not only transiently stabilize the example system but also achieve satisfactory post-fault voltage performance.

Figure 4.5  Power angle, relative speed, terminal voltage and active power responses for Case 4.2 with CTSVRE control
The exciter input responses of the two control strategies are also compared and shown in Figure 4.6. It can be seen from Figure 4.6 that the exciter input gets out of saturation and approaches steady state faster with CTSVRE controller than TSE controller, this also illustrates the performance of CTSVRE controller is better than TSE controller.

![Exciter input for Case 4.2 with TSE and CTSVRE control strategies](image)

Figure 4.6 Exciter input for Case 4.2 with TSE and CTSVRE control strategies

### 4.4.2 Comparison with conventional control

In this section, the effectiveness of the proposed controller will be illustrated by comparing its system responses with those under conventional AVR and PSS. The conventional exciter used here is the standard IEEE static exciter type ST1A. Parameters of the AVR and PSS are selected partly based on the method described in [87, 112-114] with Ziegler-Nichols Tuning. The PSS input signal is $\Delta \omega$. Parameters are as follows:

- $K_a = 200$, $T_c = T_b = 1s$, $V_{BMAX} = 6$, $V_{BMIN} = -3$, $K_s = 5$, $T_i = T_s = 0.15s$, $T_2 = T_4 = 0.05s$, $T_5 = 10s$, $V_{SDMAX} = 0.3$, $V_{STMIN} = -0.3$

The studied case is Case 4.2. Power angle, relative speed, terminal voltage and active power responses under proposed control and conventional control are compared.
Results are shown in Figure 4.7. It can be seen although the conventional AVR+PSS control can stabilize the system successfully, it costs longer time and the oscillation magnitude is quite large. The robust voltage regulation controller has the shorter setting time and smaller magnitude variation, and all responses are satisfactory, thus has better performance.

To further illustrate the performance of the proposed controller, an extreme condition is studied in Appendix A.1. The power angle, relative speed, terminal voltage and active power responses are given for comparison.
4.4.3 Robustness of the proposed CTSVRE controller

In this section, the robustness of the proposed robust controller will be evaluated under different operating conditions, fault locations, fault sequences and network parameters.

The following three cases are considered to test the effectiveness of the proposed control strategy.

Case 4.3:
In this case, the operating point, transmission parameters and fault sequence are the same as Case 4.1. The fault occurs at the middle point of one of the parallel transmission lines. Power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in Figure 4.8.

Figure 4.8 Power angle, relative speed, terminal voltage and active power responses for Case 4.3 with CTSVRE control
Case 4.4:
In this case, the fault location, fault sequence and transmission parameters are the same as Case 4.2. The operating point is $\delta_0 = 35.94^\circ$, $V_{r0} = 1.10 \text{ p.u.}$, $P_{r0} = 0.70 \text{ p.u.}$. Power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in Figure 4.9.

![Figure 4.9](image)

**Figure 4.9** Power angle, relative speed, terminal voltage and active power responses for Case 4.4 with CTSVRE control

Case 4.5:
In this case, the fault location and fault sequence are the same as Case 4.2. The operating point is $\delta_0 = 46.52^\circ$, $V_{r0} = 1.02 \text{ p.u.}$, $P_{r0} = 0.90 \text{ p.u.}$. Parameters for each transmission line are $r_l = 0.03 \times 2 \text{ p.u.}$; $x_l = 0.24265 \times 2 \text{ p.u.}$. Power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in
Figure 4.10.

For Cases 4.3 - 4.5, the proposed robust control strategy is tested under different operating points, fault locations, fault sequences and network parameters. The power angle, relative speed, terminal voltage and active power responses are given. Figure 4.8 - Figure 4.10 show that all the responses are satisfactory.

![Graphs of power angle, relative speed, terminal voltage and active power responses](image)

Figure 4.10  Power angle, relative speed, terminal voltage and active power responses for Case 4.5 with CTSVRE control

For Case 4.4, the rotor angle of the normal operating point is $\delta_0 = 35.94^\circ$ which is below $40^\circ$, thus before the fault occurs, the proposed controller primarily operates under the control law corresponding to $\delta(t) < 40^\circ$. For other cases, before the fault occurs, the proposed controller primarily operates under the control law corresponding
Chapter 4 Robust CTSVRE Control of SMIB Power Systems

to \( \delta(t) \geq 40^\circ \).

It can be seen from the simulation results of Case 4.1 - Case 4.5, although the operating conditions, fault locations, fault sequences and network parameters vary in these cases, the power responses are all satisfactory. Note that the parameters of the proposed control strategy do not change for all these cases, thus the control strategy is a robust one.

To further illustrate the robustness of the proposed controller, the power angle, relative speed, terminal voltage and active power responses are shown in Appendix A.2 for comparison.

### 4.5 Concluding remark

This chapter gives the preliminary results of designing CTSVRE controller for power systems. Based on detailed analysis of the parametric uncertainties of the state space model obtained by introducing voltage feedback, robust control theory is employed to design the controller. Unlike the robust TSE control, the proposed robust CTSVRE control strategy is able to achieve the desired post-fault voltage level even in case permanent fault occurs and network topology changes. The controller is insensitive to operating conditions, fault locations, fault sequences and network parameters. Simulation results illustrate the effectiveness of the proposed strategy. In next chapter, more general results will be given.
Chapter 5 Robust Decentralized CTSVRE Control of Multimachine Power Systems

5.1 Introduction

In last chapter, robust CTSVRE control for SMIB systems has been studied. Its extension to multimachine power systems will be presented in this chapter. The effectiveness of the robust decentralized CTSVRE controller design method will be evaluated on a five-machine twelve-bus example system without infinite bus shown in Figure 5.1.

![Figure 5.1 The example five-machine twelve-bus system](image)

In [9], robust CTSVRE controller design for multimachine power systems has been proposed. However, the control method is based on a preliminary analysis of the parametric uncertainties. Apart from that, it ignores the dynamics of interconnections when designing the controller and the studied system does not include any type of electrical loads. In this study, detailed analysis of the parametric uncertainties will be carried out. Based on the analysis, the whole operating region will be partitioned into two ranges in terms of the active power in order to make the designed controller work
more effectively in a large operating region. The interconnection dynamics will also be taken into account and dealt with using robust control theory. Static electrical load [80] will be considered. The studied example system is more complex.

Since the robust decentralized CTSVRE controller design is based on the results of robust decentralized TSE controller design, robust decentralized TSE controller will be designed at first, followed by robust decentralized CTSVRE controller design and numerical simulation.

5.2 Robust decentralized TSE control of multimachine power systems

5.2.1 Feedback linearization

To eliminate the nonlinearities partially, the DFL technique introduced in Section 3.2 will be used in this section. The original nonlinear system will be directly transformed into a simpler system.

From equations (2.21)-(2.33), it can be seen that the synchronous generator is nonlinear through the excitation system. Denoting $\Delta P_a(t) = P_a(t) - P_m$, differentiating the active power equation (2.28) and using (2.23)-(2.33), the following result can be obtained [4],

$$\Delta P_a(t) = E'_q(t)I_q(t) + E'_q(t)\dot{I_q}(t)$$

$$= E'_q(t)I_q(t) - [E'_q(t)I_q(t) + E'_q(t)B_q]\Delta \omega(t) + \sum_{j=1}^{n} E'_q(t)E'_q(t)B_q \sin \delta_j(t)$$

$$- \sum_{j=1}^{n} E'_q(t)E'_q(t)B_q \cos \delta_j(t)\Delta \omega(t)$$

$$= \frac{1}{T_{dii}} \left[ k_{a} \dot{u}_a(t) - E'_q(t) - (x_a - x'_a)I_q(t) \right] I_q(t) - \sum_{j=1}^{n} E'_q(t)E'_q(t)B_q \sin \delta_j(t)$$

$$+ \sum_{j=1}^{n} E'_q(t)E'_q(t)B_q \cos \delta_j(t)\Delta \omega(t)$$

$$+ \sum_{j=1}^{n} E'_q(t)E'_q(t)B_q \sin \delta_j(t) - \sum_{j=1}^{n} E'_q(t)E'_q(t)B_q \cos \delta_j(t)\Delta \omega(t)$$
Chapter 5 Robust Decentralized CTSVRE Control of Multimachine Power Systems

\[
\begin{align*}
\frac{1}{T_{d0i}} \left[ k_c u_{ci}(t) - (x_{di} - x_{di}^*) I_{ai}(t) \right] I_{qi}(t) - \left[ Q_{ai}(t) + E_{q0}(t) B_{ai} \right] \Delta \omega_i(t) &= -\frac{1}{T_{d0i}} P_{mi} - \frac{1}{T_{d0i}} \Delta P_{ai}(t) \\
&+ \sum_{j=1,j \neq i}^{n} E_{qj}(t) E_{qj}(t) B_j \sin \delta_j(t) - \sum_{j=1,j \neq i}^{n} E_{qj}(t) E_{qj}(t) B_j \cos \delta_j(t) \Delta \omega_i(t)
\end{align*}
\]

(5.1)

Let

\[
v_{\beta}(t) = k_c I_{qi}(t) u_{\beta}(t) - (x_{di} - x_{di}^*) I_{ai}(t) I_{qi}(t) - P_{mi} - T_{d0i} \left( Q_{ai}(t) + E_{q0}(t) B_{ai} \right) \Delta \omega_i(t)
\]

(5.2)

It gives that

\[
\Delta P_{ai}(t) = -\frac{1}{T_{d0i}} \Delta P_{ai}(t) + \frac{1}{T_{d0i}} v_{\beta}(t) + \sum_{j=1,j \neq i}^{n} E_{qj}(t) E_{qj}(t) B_j \sin \delta_j(t) - \sum_{j=1,j \neq i}^{n} E_{qj}(t) E_{qj}(t) B_j \cos \delta_j(t) \Delta \omega_i(t)
\]

(5.3)

From (5.2), the DFL compensating law can be obtained

\[
u_{\beta}(t) = \frac{1}{k_c I_{qi}(t)} \left[ v_{\beta}(t) + P_{ai} + (x_{di} - x_{di}^*) I_{ai}(t) I_{qi}(t) + T_{d0i} \left( Q_{ai}(t) + E_{q0}(t) B_{ai} \right) \Delta \omega_i(t) \right]
\]

(5.4)

It can be seen from Section 2.4, \(P_{ai}(t), Q_{ai}(t), I_{\beta}(t), I_{mi}(t)\) and \(I_{qi}(t)\) are all available variables and \(\omega_i(t)\) is also measurable variable. Rotor angle \(\delta(t)\) that will appear in the feedback law \(v_{\beta}(t)\) can be estimated [40] or measured [111]. From (2.24)-(2.30) and (2.36) it can be seen that \(I_{mi}(t)\) and \(I_{qi}(t)\) can be calculated using these available variables. Thus, the compensating law (5.4) is practically realizable using only local measurements.

The mapping from \(u_{\beta}(t)\) to \(v_{\beta}(t)\) is invertible except when \(I_{qi}(t) = 0\), which is not in the normal operating region for a generator. When \(I_{qi}(t)\) decreases to some extent, the controller will fall in saturation due to the physical limit of \(u_{\beta}(t)\). This also shows that the compensating law (5.4) is practically realizable.

Up to present, the model (2.21)-(2.33) has been partially linearized. The partially linearized model is

\[
\Delta \delta(t) = \Delta \omega_i(t)
\]
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\[
\Delta \dot{\omega}_i(t) = -\frac{D_i}{2H_i} \Delta \omega_i(t) - \frac{\omega_m}{2H_i} \Delta P_{\alpha i}(t)
\]

\[
\Delta \dot{P}_{\alpha i}(t) = -\frac{1}{T_{d\alpha i}} \Delta P_{\alpha i}(t) + \frac{1}{T_{d\alpha i}} v_{\beta i}(t) + \sum_{j=1, j\neq i}^{n} E_{\alpha j}(t) \dot{E}_{\alpha j}(t) B_{\beta j} \sin \delta_{\beta j}(t)
\]

\[
- \sum_{j=1, j\neq i}^{n} E_{\alpha j}(t) \dot{E}_{\alpha j}(t) B_{\beta j} \cos \delta_{\beta j}(t) \Delta \omega_j(t)
\]

The above three equations can be written in state space form

\[
\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + d_i(t)
\]

where

\[
i = 1, 2, \ldots, n
\]

\[
x_i(t) = \begin{bmatrix}
\Delta \delta_i(t) \\
\Delta \omega_i(t) \\
\Delta P_{\alpha i}(t)
\end{bmatrix}^T ; \quad u_i(t) = v_{\beta i}(t)
\]

\[
A_i(t) = \begin{bmatrix}
0 & 1 & 0 \\
0 & -\frac{D_i}{2H_i} & -\frac{\omega_m}{2H_i} \\
0 & 0 & -\frac{1}{T_{d\alpha i}}
\end{bmatrix} ; \quad B_i(t) = \begin{bmatrix}
0 \\
0 \\
\frac{1}{T_{d\alpha i}}
\end{bmatrix}
\]

\[
d_i(t) = \sum_{j=1, j\neq i}^{n} E_{\alpha j}(t) \dot{E}_{\alpha j}(t) B_{\beta j} \sin \delta_{\beta j}(t) \quad + \quad \sum_{j=1, j\neq i}^{n} E_{\alpha j}(t) \dot{E}_{\alpha j}(t) B_{\beta j} \cos \delta_{\beta j}(t) \Delta \omega_j(t)
\]

To find a nonlinear control law \( u_{\beta i}(t) \) to stabilize the generator model (2.21)-(2.33) is equivalent to find a nonlinear control law \( v_{\beta i}(t) \) to stabilize the uncertain DFL compensated system model (5.5).

5.2.2 Robust TSE controller design

After the DFL compensation, there still are some nonlinearities in (5.5), which are the interconnections. There are also various parametric uncertainties in power systems, consider these factors and with the objective to eliminate the nonlinearities and decentralize the multimachine power systems, robust control theory will be used in this section.
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Considering the parametric uncertainties, the partially linearized $i$th subsystem of the nonlinear generalized generator model can be written in the following state space form:

$$
\dot{x}_i(t) = [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]v_{\mu}(t) + \sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} p_{ij}[G_{ij} + \Delta G_{ij}(t)]g_{ij}(t)
$$

\hspace{1cm} (5.6)

where $i = 1, 2, \ldots, n$, $x_i(t)$, $u_i(t)$, $A_i$ and $B_i$ are the same as those in (5.5). $g_{ij}(t)$ are unknown nonlinear vector functions representing the interconnection between the $i$th subsystem and the $j$th subsystem. $p_{ij}$ are constants of either 1 or 0 (if it is 0, it means that $i$th subsystem has no connection with the $j$th subsystem).

As an illustrated example, the parametric uncertainties in $T_{di}$ as $\Delta T_{di}$ is considered in this work. Then the uncertainty matrices are

$$
\Delta A_i(t) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \mu_i(t)
\end{bmatrix};
\quad
\Delta B_i(t) = \begin{bmatrix}
0 \\
0 \\
-\mu_i(t)
\end{bmatrix};
\quad
\Delta G_{ij}(t) = \begin{bmatrix}
0 \\
0 \\
\gamma_{ij}(t)
\end{bmatrix}
$$

where

$$
\mu_i(t) = \frac{1}{T_{di0}} - \frac{1}{T_{di} + \Delta T_{di}}; \\
\gamma_{ij}(t) = E_{pi}(t)E_{qi}(t)B_{g} ;
$$

$$
\gamma_{2ij}(t) = E_{pi}(t)E_{qi}(t)B_{g} \cos \delta_{ij}(t) \\
g_{ij}(t) = \sin \left( \delta_i(t) - \delta_j(t) \right) ;
\quad
\gamma_{2j}(t) = \Delta \omega_j(t) ;
$$

Note that $E_{pi}(t)$, $E_{qi}(t)$, $E_{pi}(t)$, $\delta_{ij}(t)$ and $B_{g}$ will change as the network parameters and electrical loads change. These changes can be regarded as nonlinear uncertainties of interconnections among generators. To estimate the bounds of the uncertainties, notice that the electrical power $P_i(t)$ of each generator and the electrical power flow through each transmission line are bounded, and the excitation voltage $E_{pi}(t)$ may rise up to 5 times of the $E_{pi}(t)$ when there is no load in the system [4]. Thus, by considering (2.23), (2.26) and (2.28) the following inequalities can be obtained [4].
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\[ |E_g'(t)E_g'(t)B_j| \leq |P_g(t)|_{\text{max}} \ ; \quad |E_g'(t)| \leq \frac{1}{T_{dij}} \left| E_g(t) - E_g(t) \right| \leq \frac{4|E_g'(t)|_{\text{max}}}{T_{dij}} \]

It follows that

\[ \gamma_{ij}(t) \leq \frac{4|P_g(t)|_{\text{max}}}{T_{dij}} \ ; \quad \gamma_{2pi}(t) \leq |P_g(t)|_{\text{max}} \]

Since \( \Delta T_{dii} \leq |\Delta T_{dij}|_{\text{max}} \), \( \mu_i(t) \) is norm bounded, that is \( \mu_i(t) \leq |\mu_i(t)|_{\text{max}} \).

The parameter uncertainties in (5.6) can be decomposed into the following form"

\[
\begin{align*}
[\Delta A_i(t) & \quad \Delta B_i(t)] = D_i F_i(t)[E_{ii} & \quad E_{2i}] \\
\end{align*}
\]

\( \Delta G_{ij}(t) = M_{ij} F_{ij}(t)N_{ij} \)

\( \|g_{ij}(x_i, x_j)\| \leq \|W_{ij} x_i(t)\| + \|W_{ij} x_j(t)\| \)

with \( F_i(t) \in R^{n_i} \) and \( F_{ij}(t) \in R^{m_{ij}} \) (for all \( i, j \)) being unknown matrix functions with

Lebesgue measurable elements and satisfying

\[
F_i^T(t)F_i(t) \leq I \ , \quad F_{ij}^T(t)F_{ij}(t) \leq I_{ij}
\]

where \( I \) denotes the identity matrix of appropriate dimension and \( D_i, \ E_{ii}, \ E_{2i}, \ D_{ij}, \ E_{ij}, \ W_{ii}, \) and \( W_{ij}, \) are known real constant matrices with appropriate dimensions.

One form of decomposition is as follows:

\[
D_i = \begin{bmatrix} 0 & 0 \ |\mu_i(t)\|_{\text{max}} \end{bmatrix}^T ; \quad F_i(t) = \frac{\mu_i(t)}{|\mu_i(t)|_{\text{max}}} ; \quad E_{ii} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} ; \quad E_{2i} = -1
\]

\[
M_{ij} = \begin{bmatrix} 0 & 0 \ |\gamma_{ij}(t)\|_{\text{max}} \end{bmatrix}^T ; \quad F_{ij}(t) = \frac{\gamma_{ij}(t)}{|\gamma_{ij}(t)|_{\text{max}}} ; \quad N_{ij} = 1
\]

\[
W_{ij} = W_{ij} = [1 & 0 & 0] ; \quad W_{2i} = [0 & 0 & 0] ; \quad W_{2ij} = [0 & 1 & 0]
\]

By solving two LMIs given in [47], the TSE controller parameters can be obtained.

Results are given in Section 5.4, where the performance of the TSE controller and CTSVRE are compared.
Chapter 5 Robust Decentralized CTSVRE Control of Multimachine Power Systems

5.3 Robust decentralized CTSVRE control of multimachine power systems

The disadvantage of robust TSE control for multimachine power systems is the same as that for SMIB power systems discussed in Chapter 4. Therefore, to achieve the post-fault voltage regulation irrespective of network parameters changes, robust CTSVRE controllers should be designed.

5.3.1 Feedback linearization

In this section, a DFL control law is developed firstly to compensate for the nonlinearities and reduce interconnections among generators.

Differentiating the voltage \( V_q(t) \) in equation (2.33) and using (2.26)-(2.27), (2.31)-(2.32) gives

\[
\dot{V}_q(t) = \frac{d}{dt} \left[ \left( E'_q(t) + x_q z \sum_{j=1}^{n_q} \sum_{j=1}^{n_q} E'_q(t)B_q \cos \delta_q(t) \right)^2 + \left( x_q z \sum_{j=1}^{n_q} E'_q(t)B_q \sin \delta_q(t) \right)^2 \right]^{1/2} 
= \frac{V_q(t)}{V_q(t)} \dot{E}'_q(t) + \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} \left( E'_q(t)B_q \cos \delta_q(t) - E'_q(t)B_q \sin \delta_q(t) \Delta \omega_q(t) \right) 
+ \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} \left( E'_q(t)B_q \sin \delta_q(t) + E'_q(t)B_q \cos \delta_q(t) \Delta \omega_q(t) \right) 
= \frac{V_q(t)}{V_q(t)} \dot{E}'_q(t) + \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} \left( E'_q(t)B_q \sin \delta_q(t) + E'_q(t)B_q \cos \delta_q(t) \Delta \omega_q(t) \right) 
- \left[ \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} E'_q(t)B_q \sin \delta_q(t) - \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} E'_q(t)B_q \cos \delta_q(t) \right] \Delta \omega_q(t) 
+ \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} E'_q(t)B_q \left( V_q(t) \sin \delta_q(t) - V_q(t) \cos \delta_q(t) \Delta \omega_q(t) \right) 
= \frac{V_q(t)}{V_q(t)} \left[ 1 + x_q z \right] \dot{E}'_q(t) - \left[ \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} E'_q(t)B_q \sin \delta_q(t) - \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} E'_q(t)B_q \cos \delta_q(t) \right] \Delta \omega_q(t) 
+ \frac{x_q z}{V_q(t)} \sum_{j=1}^{n_q} E'_q(t)B_q \left( V_q(t) \sin \delta_q(t) - V_q(t) \cos \delta_q(t) \Delta \omega_q(t) \right) \Delta \omega_q(t) (5.11)
\]

The second item in (5.11) is
\[ \begin{align*}
&\left[-\frac{x_a V_a(t)}{V_a(t)} I_a(t) + \frac{x_a V_{ab}(t)}{V_a(t)} I_{ab}(t) + \frac{x_a V_{al}(t)}{V_a(t)} E_\varphi(t) B_u \right] \Delta \omega_1(t) \\
&= \left[ V_{ab}(t) \left( E_a(t) - \frac{x_a I_{ab}(t)}{V_a(t)} \right) + \frac{x_a V_{ab}(t)}{V_a(t)} I_{ab}(t) \right] \Delta \omega_1(t) \\
&\quad + \frac{x_a V_{al}(t)}{V_a(t)} E_{\varphi}(t) B_u \Delta \omega_1(t) \\
&= -\frac{(1 + x_a B_u) V_{ab}(t) E_{\varphi}(t)}{V_a(t)} \Delta \omega_1(t) \\
&= -\frac{(1 + x_a B_u) V_{ab}(t) P_{\alpha}(t)}{V_a(t) I_{\alpha}(t)} \Delta \omega_1(t) \\
\end{align*} \] (5.12)

From (5.1) and (5.3), the following equality is obtained,

\[ \dot{E}_\varphi(t) = \frac{1}{I_{\varphi}(t)} \left[ -\frac{1}{T_{\alpha 0}} \Delta P_{\alpha}(t) + \frac{1}{T_{\alpha 0}} V_{\varphi}(t) + \left( Q_{\alpha}(t) + E_{\varphi}^2(t) B_u \right) \Delta \omega_1(t) \right] \] (5.13)

Substituting (5.12)-(5.13) into (5.11) yields

\[ \begin{align*}
&\dot{V}_a(t) = -\frac{(1 + x_a B_u) V_{ab}(t)}{V_a(t) I_{\varphi}(t) T_{\alpha 0}} \Delta P_{\alpha}(t) + \frac{(1 + x_a B_u) V_{ab}(t)}{V_a(t) I_{\alpha}(t) T_{\alpha 0}} v_{\varphi}(t) \\
&\quad + \left[ V_{ab}(t) \left( Q_{\alpha}(t) + E_{\varphi}^2(t) B_u \right) - V_{ab}(t) \Delta P_{\alpha}(t) \right] \Delta \omega_1(t) \\
&\quad + \frac{x_a}{V_a(t) E_{\varphi}(t)} \sum_{j \neq i}^n E_{\varphi}(t) E_{\varphi}(t) B_j \left( V_{ab}(t) \cos \delta_j(t) + V_{ab}(t) \sin \delta_j(t) \right) \\
&\quad + \frac{x_a}{V_a(t) E_{\varphi}(t)} \sum_{j \neq i}^n E_{\varphi}(t) E_{\varphi}(t) B_j \left( V_{ab}(t) \sin \delta_j(t) - V_{ab}(t) \cos \delta_j(t) \right) \Delta \omega_1(t) \\
\end{align*} \] (5.14)

It can be seen from (5.14) the DFL compensating law $v_{\varphi}(t)$ for CTVRE controller is exactly the same as that for TSE controller.

To simplify analysis and use the results in Section 5.2, we introduce a new variable $\theta_j$ defined as

\[ \theta_j = \cos^{-1} \left( V_{ab}(t) / V_a(t) \right) \]

Then

\[ V_{ab}(t) = V_a(t) \cos \theta_j(t), \quad V_{ab}(t) = V_a(t) \sin \theta_j(t) \]

\[ V_{ab}(t) \cos \delta_j(t) + V_{ab}(t) \sin \delta_j(t) = V_a(t) \cos \theta_j(t) \cos \delta_j(t) + V_a(t) \sin \theta_j(t) \sin \delta_j(t) \]
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\[ V_{sp}(t) \sin \delta_y(t) - V_{sl}(t) \cos \delta_y(t) = V_n(t) \cos \theta(t) \sin \delta_y(t) - V_n(t) \sin \theta(t) \cos \delta_y(t) \]

\[ = V_n(t) \sin (\delta_y(t) - \theta(t)) \]

Let \( \Delta V_n(t) = V_n(t) - V_{\omega 0} \), (5.14) can be rewritten as

\[ \Delta \dot{V}_n(t) = \frac{(1 + x_a B_a) V_{sp}(t)}{V_n(t) I_{sp}(t) T_{d\omega i}} \Delta P_n(t) + \frac{(1 + x_a B_a) V_{sp}(t)}{V_n(t) I_{sp}(t) T_{d\omega i}} V_n(t) \]

\[ + \left[ V_{sp}(t) \left( Q_a(t) + E_{sp}(t) B_a \right) - V_{sl}(t) P_n(t) \right] \cdot \frac{(1 + x_a B_a)}{V_n(t) I_{sp}(t)} \Delta \omega(t) \]

\[ + \frac{x_a}{E_{sp}(t)} \sum_{j=1,j \neq i} E_{sp}(t) E_{sp}(t) B_{ij} \cos (\delta_y(t) - \theta(t)) \]

\[ + \frac{x_a}{E_{sp}(t)} \sum_{j=1,j \neq i} E_{sp}(t) E_{sp}(t) B_{ij} \sin (\delta_y(t) - \theta(t)) \Delta \omega(t) \]  

(5.15)

As yet, it can be seen, the original multimachine power system model (2.21)-(2.33) has been compensated into the following system:

\[ \dot{x}(t) = A_i(t) x(t) + B_i(t) u_i(t) + d_i(t) \]  

(5.16)

where

\[ i = 1, 2, \ldots, n \]

\[ x(t) = [\Delta V_n(t) \Delta \omega(t) \Delta P_n(t) ] \] is the state variables vector.

\[ u_i(t) = v_{\beta i}(t) \] is the new control input of the excitation loop of the \( i \)th generator.

\[ A_i(t) = \begin{bmatrix} 0 & f_{1i}(t) & -f_{2i}(t) \\ 0 & -D_{2i} & 0 \\ 0 & 0 & - \frac{1}{T_{d\omega i}} \end{bmatrix} ; \\
B_i(t) = \begin{bmatrix} f_{2i}(t) \\ 0 \\ 1 \end{bmatrix} \]

\[ f_{1i}(t) = \frac{\left[ 1 + x_a B_a \right] \left[ V_{sp}(t) \left( Q_a(t) + E_{sp}(t) B_a \right) - V_{sl}(t) P_n(t) \right]}{V_n(t) I_{sp}(t)} \]

\[ f_{2i}(t) = \frac{(1 + x_a B_a) V_{sp}(t)}{V_n(t) I_{sp}(t)} \]
\[
\begin{align*}
\frac{d}{dt}(t) &= \begin{bmatrix}
\sum_{j=1, j \neq i}^{n} \frac{\dot{\delta}_j}{E_{qj}} (t) \dot{E}_{qj}(t) B_{ij} \sin \theta_j(t) \sin \delta_j(t) \\
0 \\
\sum_{j=1, j \neq i}^{n} \dot{E}_{qj}(t) \dot{E}_{qj}(t) B_{ij} \sin \delta_j(t) \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
&+ \begin{bmatrix}
\sum_{j=1, j \neq i}^{n} \frac{\dot{\delta}_j}{E_{qj}} (t) \dot{E}_{qj}(t) B_{ij} \sin \left(\delta_j(t) - \theta_j(t)\right) \Delta \omega_j(t) \\
0 \\
\sum_{j=1, j \neq i}^{n} \dot{E}_{qj}(t) \dot{E}_{qj}(t) B_{ij} \cos \delta_j(t) \Delta \omega_j(t) \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
&+ \begin{bmatrix}
\sum_{j=1, j \neq i}^{n} \frac{\dot{\delta}_j}{E_{qj}} (t) \dot{E}_{qj}(t) B_{ij} \cos \theta_j(t) \cos \delta_j(t) \\
0 \\
0 \\
\end{bmatrix}
\end{align*}
\]

The mapping from \( v_i(t) \) to \( u_i(t) \) is exactly the same as (5.4).

When \( f_1(t), f_2(t) \) and interconnection variables in \( d_i(t) \) are treated as variable parametric uncertainties, the DFL compensated model (5.16) can be rewritten as

\[
\dot{x}_i(t) = \left[A_i + \Delta A_i(t)\right] x_i(t) + \left[B_i + \Delta B_i(t)\right] v_i(t) + \sum_{k=1}^{n} \sum_{j=1, j \neq i}^{n} p_{ij} \left[G_{ij} + \Delta G_{ij}(t)\right] g_{ij}(t)
\]

(5.17)

where \( v_i(t) \) is the new input; \( p_{ij} \) has the same meaning as in (5.6).

\[
i = 1, 2, \ldots, n;
\]

\[
x_i(t) = \begin{bmatrix} \Delta V_i(t) & \Delta \omega_i(t) & \Delta P_i(t) \end{bmatrix}^T;
\]

\[
g_{iij} = 1;
\]

\[
g_{iij}(t) = \omega_j(t);
\]

\[
g_{ij} = 1
\]

\[
A_i(t) = \begin{bmatrix}
0 & -\frac{f_{ij}}{T_{dii}} \\
0 & -\frac{D_i}{2H_i} - \frac{\omega_0}{2H_i} \\
0 & 0 & -\frac{1}{T_{dii}}
\end{bmatrix};
\quad
B_i(t) = \begin{bmatrix}
\frac{f_{ij}}{T_{dii}} \\
0 \\
1
\end{bmatrix};
\]

\[
G_{ij} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]
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The uncertainty matrices are

\[
\Delta A(t) = \begin{bmatrix}
0 & \mu_1(t) & \mu_2(t) \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}; \\
\Delta B(t) = \begin{bmatrix}
-\mu_2(t) \\
0 \\
0 \\
\end{bmatrix}; \\
\Delta G_{ij} = \begin{bmatrix}
\gamma_{ivj}(t) \\
0 \\
\gamma_{gvj}(t) \\
\end{bmatrix}; \\
\mu_i(t) = \Delta f_i(t); \\
\mu_2(t) = -\frac{\Delta f_2}{T_{q\phi}};
\]

\[
\gamma_{ivj}(t) = \frac{x_{g}}{x_{q}} E^\prime_{q}(t) \dot{E}_{q}(t) B_{g} \sin \theta(t) \sin \delta_q(t); \\
\gamma_{ivp}(t) = E^\prime_{q}(t) \dot{E}_{q}(t) B_{g}; \\
\gamma_{givp}(t) = E_{q}(t) E^\prime_{q}(t) B_{g} \cos \left( \delta_q(t) + \theta(t) \right); \\
\gamma_{givp}(t) = E_{q}(t) E^\prime_{q}(t) B_{g} \cos \delta_q(t); \\
\gamma_{gvp}(t) = \frac{x_{g}}{x_{q}} E_{q}(t) \dot{E}_{q}(t) B_{g} \cos \theta(t) \cos \delta_q(t); \\
\gamma_{gvp}(t) = 0
\]

with ' - ' representing the averaging value and 'Δ' representing the deviation from the averaging value to the respective variable.

5.3.2 Robust decentralized CTSVRE controller design

In this section, the bounds of interconnections, \( f_{1}(t) \) and \( f_{2}(t) \) will be firstly estimated and robust control theory will next be employed to design the CTSVRE controller. The example system used is shown in Figure 5.1. System parameters are given in Section 5.4.

After the DFL compensation, it can be seen from (5.16) that the control input is now a single variable \( v_{ph}(t) \), but matrices \( A_i(t) \) and \( B_i(t) \) still include uncertainties \( f_{1}(t) \) and \( f_{2}(t) \), which are quite complex, nonlinear and dependent on operating points. Besides, (5.16) has remote dynamics and nonlinearities represented by \( \dot{E_{q}}(t) \),
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\( \delta_j(t) \) and \( \omega_j(t) \). Therefore, linear control methods can't be applied directly. Fortunately, the electrical power of a generator and the triangular functions of rotor angle are bounded. If \( f_1(t) \) and \( f_2(t) \) are bounded variables and the interconnection variables are also bounded, linear robust control theory can then be employed to design the CTSVRE controller. The robust control theory provides a possibility to eliminate all or at least a part of the nonlinearities and interconnections and guarantee system stability irrespective of network parameters.

5.3.2.1 Estimation of bounds of interconnections

As is already shown in Section 5.2.2,

\[
\begin{align*}
|E_p(t)E_p(t)B| & \leq |P_n|_{\text{max}}, \\
|E_p(t)| & \leq \frac{4|E_p(t)|_{\text{max}}}{|P_n|_{\text{min}}}
\end{align*}
\]

It follows immediately that

\[
\begin{align*}
\gamma_{10}(t) & \leq \frac{4x_{0}|P_n(t)|_{\text{max}}}{|E_p(t)|_{\text{min}}}, \\
\gamma_{1p}(t) & \leq \frac{4|P_n(t)|_{\text{max}}}{|T_{00}|_{\text{min}}}, \\
\gamma_{20}(t) & \leq \frac{x_{0}|P_n(t)|_{\text{max}}}{|E_p(t)|_{\text{min}}}, \\
\gamma_{2p}(t) & \leq |P_n(t)|_{\text{max}}, \\
\gamma_{30}(t) & \leq \frac{4x_{0}|P_n(t)|_{\text{max}}}{|E_p(t)|_{\text{min}}} = \gamma_{10}(t)_{\text{max}}
\end{align*}
\]

5.3.2.2 Estimation of bounds of \( f_1(t) \) and \( f_2(t) \)

For SMIB power systems case, the load effect is not considered. For multimachine power systems case, this effect can not be ignored. In this work, the static load model is used as a preliminary study. When the load effect is considered, the active power and reactive power can be largely influenced by the electrical load. The rotor angle is no longer suitable for the estimation of the bounds of \( f_1(t) \) and \( f_2(t) \) as the SMIB case, where the active power is assumed to be a single value. More general index should be used.

It is well known that synchronous generators are rated in terms of the maximum MVA output at a specified voltage and power factor (usually 0.85 or 0.9 lagging) which they can carry continuously without overheating. The normal voltage is around 1 p.u. While
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the active power is limited by the prime mover capability, the reactive power output capability is limited by three considerations: armature current limit, field current limit, and end region heating limit [87]. A different pair of active power $P$ and reactive power $Q$ defines a different operating point. An ideal controller should be effective under all possible $P$ and $Q$ conditions. In fact, it is not necessary to satisfy all the operating conditions. If a controller can satisfy most of the operating conditions and has good performance at the most often operated conditions, it is believed to be effective.

Let us observe the values of $f_1(t)$ and $f_2(t)$ under one typical condition, that is $\omega = \omega_0$, $V_n = 1$. A series of different typical values of $P$ and $Q$ are chosen, from (2.34)-(2.35) and (2.37), $E_{q_0}(t)$ can be calculated. Then other values can be calculated from (2.24)-(2.33). $f_1(t)$ and $f_2(t)$ can then be calculated from (5.16).

For illustration purpose, curves of the relationship of $f_1(t)$ and $f_2(t)$ with active power and reactive power for generator #1 in the example system shown in Figure 5.1 can be plotted. They are shown in Figure 5.2 and Figure 5.4 respectively. Figure 5.3 and Figure 5.5 show the relationship of $f_1(t)$ and $f_2(t)$ with only active power when reactive power is not considered. It can be seen from Figure 5.3 and Figure 5.5 when the reactive power is not of interest, $f_1(t)$ and $f_2(t)$ can be very well approximated by a cluster of curves that are only dependent on active power values. The neglect of reactive power is acceptable, as only the bounds of $f_1(t)$ and $f_2(t)$ are needed in robust control theory. In addition, the active power is paid more attention to in power industry. Observing Figure 5.3 and Figure 5.5 carefully, it can be seen slopes of $f_1(t)$ and $f_2(t)$ are sharper when active power $P$ is low and smooth when $P$ is high. The curves of $f_1(t)$ and $f_2(t)$ can thus be divided approximately into two parts according to the range of active power. The curves are then approximately linearized into two piecewise parts. After the partition, the controllers can be designed piecewise. The
objective of this partition is to achieve better performance when designing controllers.

Figure 5.2 \( f_1(P,Q) \) when \( V_i = 1 \)

Figure 5.3 \( f_1 - P \) when \( V_i = 1 \)

Figure 5.4 \( f_2(P,Q) \) when \( V_i = 1 \)

Figure 5.5 \( f_2 - P \) when \( V_i = 1 \)

In order to make the controller effective in a wide range, variations of voltage and \( B_u \) are also considered. The variations magnitude is assumed to be within \( \pm 10\% \), namely, \( 0.9 \leq V_a \leq 1.1 \) and \( 0.9B_u \leq B_u \leq 1.1B_u \), where \( B_u \) is the normal steady state value of \( B_u \). Considering these factors and choosing different active power and reactive power values, different terminal voltage values and different network parameters, curves of \( f_1(t) \) and \( f_2(t) \) against \( P(t) \) for five machines #1 - #5 of the example system are plotted in Figure 5.6 - Figure 5.10. The approximated envelopes of curves of \( f_1(t) \) and
$f_2(t)$ are plotted as straight lines at the right side in Figure 5.6 - Figure 5.10.

Figure 5.6  $f_1 - P$  $f_2 - P$ for Gen # 1 under different $Q$, $V_i$ and $B_i$

Figure 5.7  $f_1 - P$  $f_2 - P$ for Gen # 2 under different $Q$, $V_i$ and $B_i$
Figure 5.8 $f_1 - P$, $f_2 - P$ for Gen # 3 under different $Q$, $V$, and $B_i$

Figure 5.9 $f_1 - P$, $f_2 - P$ for Gen # 4 under different $Q$, $V$, and $B_i$
Figure 5.10  $f_1 - P$  $f_2 - P$  for Gen # 5 under different $Q$, $V$, and $B_u$

Table 5.1  Bounds of $f_1(t)$ and $f_2(t)$

<table>
<thead>
<tr>
<th>$P_e$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>Gen #1</th>
<th>Gen #2</th>
<th>Gen #3</th>
<th>Gen #4</th>
<th>Gen #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.4</td>
<td>$f_{1\text{min}}$</td>
<td>-4.03</td>
<td>-4.03</td>
<td>-4.03</td>
<td>-4.03</td>
<td>-4.53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{1\text{max}}$</td>
<td>-0.69</td>
<td>-0.64</td>
<td>-0.62</td>
<td>-0.52</td>
<td>-0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{2\text{min}}$</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.58</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{2\text{max}}$</td>
<td>3.51</td>
<td>5.18</td>
<td>5.18</td>
<td>5.18</td>
<td>5.18</td>
<td></td>
</tr>
<tr>
<td>≥ 0.4</td>
<td>$f_{1\text{min}}$</td>
<td>-1.09</td>
<td>-1.09</td>
<td>-1.09</td>
<td>-1.09</td>
<td>-1.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{1\text{max}}$</td>
<td>-0.33</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{2\text{min}}$</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
<td>0.41</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{2\text{max}}$</td>
<td>1.21</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from Figure 5.6 - Figure 5.10 that both $f_1(t)$ and $f_2(t)$ can be divided into two parts according to active power ranges $P_e(t) < 0.4 \text{ p.u.}$ and $P_e(t) \geq 0.4 \text{ p.u.}$. The curves are then approximately linearized into two piecewise lines with reference to the active power values $P_e = 0.8 \text{ p.u.}$ and $P_e = 0.2 \text{ p.u.}$. Notice, the active power range between $0 \text{ p.u.}$ and $0.1 \text{ p.u.}$ is not considered and shown here, which is seldom the case in practice.
The data of the bounds of \( f_1(t) \) and \( f_2(t) \) are given in Table 5.1. Note although the parameters of generators \#1 - \#5 are not the same, the bounds and characteristics of \( f_1(t) \) and \( f_2(t) \) are quite similar. This implies that the CTSVRE control method presented in next subsection can be qualitatively applied in other conditions, where the generator parameters are not necessarily the same.

### 5.3.2.3 Robust decentralized controller design

Robust LMI approach in [47] will be employed to calculate the parameters for the nonlinear robust decentralized CTSVRE controller. The difference here is that there are three items in the interconnections in (5.17) instead of two. It is easy to extend the theory in [47] to include this item. Only results are given below.

To design the performance guaranteed controller, the following cost performance index is firstly defined

\[
J = \sum_{i=1}^{n} \int_{0}^{T} (x_i^T(t)Q_i x_i(t) + u_i^T(t)R_i u_i(t)) \, dt
\]

(5.18)

where \( u_i(t) \) is the control input, \( Q_i = Q_i^T > 0 \), \( R_i = R_i^T > 0 \).

In addition, when \( g_{ai} = 1 \), the assumption in (5.9) is no longer effective, for such a case, we can simplify define \( W_u = W_{ai} = [0 \quad 0 \quad 0] \).

**Theorem 5.1**: Under the same assumptions as in (5.7)-(5.10) (except that assumption (5.9) is modified above) suppose there exist some real positive scalars \( \gamma_{ai} \), \( \gamma_{2i} \), \( \gamma_{3i} \) and real constant matrices \( X_i = X_i^T > 0 \) and \( Y_j \), \( i,j = 1,2,\ldots,n \), such that LMI (5.19) holds, then the decentralized performance guaranteed controller given by (5.20) can asymptotically stabilize the uncertain system (5.17) and render the cost performance index \( J \) defined in (5.18) satisfying (5.21).
\[
\begin{bmatrix}
\Phi_i & X \bar{Q}_{ij}^{\frac{1}{2}} & Y^T & X^T E_{ij}^T + Y^T E_{ij}^T & G_{1i} & G_{2i} & G_{3i}
\end{bmatrix} < 0 \quad (5.19)
\]

where

\[
\bar{Q}_{ij} = \sum_{k=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} p_{ij} \left( W_{ik}^T W_{kj} + W_{ji}^T W_{ji} \right) + Q,
\]

\[
G_{1i} = \begin{bmatrix} p_{11} G_{11}, \ldots, p_{1n} G_{in} \end{bmatrix}
\]

\[
G_{2i} = \begin{bmatrix} p_{21} G_{21}, \ldots, p_{2n} G_{2n} \end{bmatrix}
\]

\[
G_{3i} = \begin{bmatrix} p_{31} G_{31}, \ldots, p_{3n} G_{3n} \end{bmatrix}
\]

\[
M_{1i} = \begin{bmatrix} p_{11} M_{11}, \ldots, p_{1n} M_{1n} \end{bmatrix}
\]

\[
M_{2i} = \begin{bmatrix} p_{21} M_{21}, \ldots, p_{2n} M_{2n} \end{bmatrix}
\]

\[
M_{3i} = \begin{bmatrix} p_{31} M_{31}, \ldots, p_{3n} M_{3n} \end{bmatrix}
\]

\[
N_{1i} = \text{diag} \left\{ p_{11} N_{11}, \ldots, p_{1n} N_{1n} \right\}
\]

\[
N_{2i} = \text{diag} \left\{ p_{21} N_{21}, \ldots, p_{2n} N_{2n} \right\}
\]

\[
N_{3i} = \text{diag} \left\{ p_{31} N_{31}, \ldots, p_{3n} N_{3n} \right\}
\]

\[
\Gamma_{1i} = \text{diag} \left\{ \gamma_{11} I_{a1}, \ldots, \gamma_{1n} I_{a1} \right\}
\]

\[
\Gamma_{2i} = \text{diag} \left\{ \gamma_{11} I_{a2}, \ldots, \gamma_{1n} I_{a2} \right\}
\]

\[
\Gamma_{3i} = \text{diag} \left\{ \gamma_{11} I_{a3}, \ldots, \gamma_{1n} I_{a3} \right\}
\]

\[
\Gamma_{1i} = \text{diag} \left\{ \gamma_{11} I_{a1}, \ldots, \gamma_{1n} I_{a1} \right\}
\]

\[
\Gamma_{2i} = \text{diag} \left\{ \gamma_{11} I_{a2}, \ldots, \gamma_{1n} I_{a2} \right\}
\]
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\[ \Gamma_{3,ji} = \text{diag}\{ \gamma_{3i}^I I_{\beta 3i}^I, \ldots, \gamma_{3j}^I I_{\beta 3j}^I, \ldots, \gamma_{3n}^I I_{\beta 3n}^I \} \]

\[ i, j = 1, 2, \ldots, n, \quad j \neq i \]

\[ v_{ji}(t) = K_j x_i(t) \]  \hspace{1cm} (5.20)

where \( K_j = Y_j X_j^{-1} \)

\[ J \leq \sum_{i=1}^{n} x_i^T(0) P_i x_i(0) \]  \hspace{1cm} (5.21)

where \( P_i = X_i^{-1} \).

Notice that the bound of the performance index \( J \) in (5.21) depends on the initial condition \( x_i(0) \). To remove this dependence, the deterministic method in [101] is employed. After some assumptions and induction, the following LMI need to be solved (see Section 3.3)

\[ \begin{bmatrix} -\lambda_i I & \Pi_{0i}^T \\ \Pi_{0i} & -X_i \end{bmatrix} < 0 \]  \hspace{1cm} (5.22)

where \( \lambda_i \) is a positive constant and the cost performance index satisfies \( J \leq \sum_{i=1}^{n} \lambda_i \).

For the \( ith \) generator, the corresponding matrices can be chosen as

\[ \Pi_{0i} = \text{diag}\{ 3|\Delta V_i|_{\text{max}}, 3|\Delta \omega|_{\text{max}}, 3|\Delta P_i|_{\text{max}} \} \]

In order to obtain the robust guaranteed performance control law (5.20), the parametric problem of minimizing \( J \leq \sum_{i=1}^{n} \lambda_i \) under the LMI constraints (5.19) and (5.22) need to be solved. Obviously, inequality (5.19) is a linear matrix inequality in \( X_i, \ Y_i, \ \omega_i, \ \gamma_{1i}, \ \gamma_{2i} \) and \( \gamma_{3i} \) and inequality (5.22) is also a linear one in \( X_i \), so this problem can be solved efficiently using the LMI tool [102].

As yet, it can be seen that the exciter control laws are (5.4) and (5.20).

Now, let's have a check whether the three assumptions in (5.7)-(5.10) can be satisfied. One possible decomposition of the uncertainties of the \( ith \) generator can be expressed...
as

\[
D = \begin{bmatrix}
\mu_1 \|_{\text{max}} & \mu_2 \|_{\text{max}} \\
0 & 0 \\
0 & 0
\end{bmatrix}; \quad F_j(t) = \begin{bmatrix}
\mu_1(t) \\
0 \\
0 & \mu_2(t) \|_{\text{max}}
\end{bmatrix};
\]

\[
E_{ti} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}; \quad E_{2t} = \begin{bmatrix}
0 \\
-1
\end{bmatrix}
\]

\[
M_{ki} = \begin{bmatrix}
\gamma_{ki} \|_{\text{max}} & 0 \\
0 & 0 \\
0 & \gamma_{ki} \|_{\text{max}}
\end{bmatrix}; \quad F_{ki}(t) = \begin{bmatrix}
\gamma_{ki}(t) \\
0 \\
0 & \gamma_{ki}(t) \|_{\text{max}}
\end{bmatrix}; \quad N_{ki} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad k = 1, 2
\]

\[
M_{kj} = \begin{bmatrix}
\gamma_{kj} \|_{\text{max}} \\
0 \\
0
\end{bmatrix}, \quad F_{kj}(t) = \begin{bmatrix}
\gamma_{kj}(t) \\
\gamma_{kj} \|_{\text{max}}
\end{bmatrix}, \quad N_{kj} = 1
\]

\[
W_{t} = W_{ij} = [0 \ 0 \ 0]; \quad W_{2i} = [0 \ 0 \ 0], \ W_{2j} = [0 \ 1 \ 0]; \quad W_{3i} = W_{3j} = [0 \ 0 \ 0]
\]

Thus the three assumptions are met and the robust LMI approach can be employed.

As mentioned before, parameter bounds of \( f_1(t) \) or \( f_2(t) \) are approximately partitioned into two parts according to active power ranges \( P_e < 0.4 \, \text{p.u.} \) and \( P_e \geq 0.4 \, \text{p.u.} \). Correspondingly, there are two CTSVRE control laws \( v_{j1} \) and \( v_{j2} \) for each generator. When active power \( P_e(t) \) varies across 0.4, there should have a switch action between the two control laws. In order not to introduce another "disturbance" into the system, the switch action should be smooth. This is realized by choosing the following trapezoid-shaped like member functions [52],

\[
\lambda_{v_{ji}} = \frac{1 - \exp(-120(P_e - 0.4))}{1 + \exp(-120(P_e - 0.4))}
\]

\[
\lambda_{v_{j2i}} = 1 - \lambda_{v_{ji}} \quad (5.23)
\]

Then the composite CTSVRE control law is:

\[
v_{j}(t) = \lambda_{v_{ji}} v_{j1}(t) + \lambda_{v_{j2i}} v_{j2}(t) \quad (5.25)
\]

where \( v_{j1} \) corresponding to the active power range \( P_e < 0.4 \, \text{p.u.} \) and \( v_{j2} \)
corresponding to \( P_a \geq 0.4 \text{ p.u.} \).

### 5.4 Simulation study

In this section, the effectiveness of the proposed controller will be evaluated on the example system shown in Figure 5.1. Bus 1 in the five-machine twelve-bus system is used as the reference bus.

Parameters for all generating units and transmission lines are given in Table 5.2 and Table 5.3. It can be seen that the inertial constant differences between the generators are not very large. Thus it can be regarded as a small area power system.

#### Table 5.2 Parameters of generators in \( \text{p.u.} \) except \( H \) in sec. (100MVA base)

<table>
<thead>
<tr>
<th></th>
<th>Gen#1</th>
<th>Gen#2</th>
<th>Gen#3</th>
<th>Gen#4</th>
<th>Gen#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_d )</td>
<td>0.254</td>
<td>0.295</td>
<td>0.29</td>
<td>0.33</td>
<td>0.262</td>
</tr>
<tr>
<td>( x_q )</td>
<td>0.05</td>
<td>0.049</td>
<td>0.057</td>
<td>0.066</td>
<td>0.044</td>
</tr>
<tr>
<td>( x_f )</td>
<td>0.0224</td>
<td>0.0332</td>
<td>0.028</td>
<td>0.0295</td>
<td>0.0304</td>
</tr>
<tr>
<td>( T_{do} )</td>
<td>7.3</td>
<td>5.66</td>
<td>6.7</td>
<td>5.4</td>
<td>5.69</td>
</tr>
<tr>
<td>( H )</td>
<td>34.8</td>
<td>26.4</td>
<td>24.3</td>
<td>26</td>
<td>28.6</td>
</tr>
<tr>
<td>( D )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

#### Table 5.3 Parameters of transmission lines in \( \text{p.u.} \) (100MVA base)

<table>
<thead>
<tr>
<th>Bus</th>
<th>R</th>
<th>X</th>
<th>( B_c )</th>
<th>Bus</th>
<th>R</th>
<th>X</th>
<th>( B_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-6</td>
<td>0.0</td>
<td>0.0143</td>
<td>0.0</td>
<td>6-12</td>
<td>0.0006</td>
<td>0.0096</td>
<td>0.1846</td>
</tr>
<tr>
<td>2-7</td>
<td>0.0</td>
<td>0.0272</td>
<td>0.0</td>
<td>7-12</td>
<td>0.0043</td>
<td>0.0474</td>
<td>0.7802</td>
</tr>
<tr>
<td>3-8</td>
<td>0.0</td>
<td>0.0232</td>
<td>0.0</td>
<td>8-11</td>
<td>0.0003</td>
<td>0.0059</td>
<td>0.0680</td>
</tr>
<tr>
<td>4-9</td>
<td>0.0</td>
<td>0.018</td>
<td>0.0</td>
<td>8-12</td>
<td>0.0010</td>
<td>0.0250</td>
<td>0.7500</td>
</tr>
<tr>
<td>5-10</td>
<td>0.0</td>
<td>0.0142</td>
<td>0.0</td>
<td>9-10</td>
<td>0.0016</td>
<td>0.0195</td>
<td>0.3040</td>
</tr>
<tr>
<td>6-9</td>
<td>0.0007</td>
<td>0.0138</td>
<td>0.262</td>
<td>10-11</td>
<td>0.0035</td>
<td>0.0411</td>
<td>0.6987</td>
</tr>
<tr>
<td>6-11</td>
<td>0.0022</td>
<td>0.0350</td>
<td>0.361</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The following two operating conditions will be considered. For simplicity, the electrical loads considered are static loads [80] and are simulated as constant impedance in the numeric simulation. More detailed load model should be used in the future study.

Operating Condition 5.1

Table 5.4 Generator operating condition 5.1 in $p.u.$ except $\alpha_i$ in $\text{rad}$ (100MVA base)

<table>
<thead>
<tr>
<th></th>
<th>Gen#1</th>
<th>Gen#2</th>
<th>Gen#3</th>
<th>Gen#4</th>
<th>Gen#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>4.664</td>
<td>4.500</td>
<td>5.000</td>
<td>3.000</td>
<td>5.000</td>
</tr>
<tr>
<td>$Q$</td>
<td>2.754</td>
<td>0.830</td>
<td>2.274</td>
<td>0.969</td>
<td>2.888</td>
</tr>
<tr>
<td>$V_i$</td>
<td>1.05</td>
<td>1.02</td>
<td>1.05</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.000</td>
<td>0.225</td>
<td>0.036</td>
<td>0.005</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Loads in $p.u.$ are as follows:

$L_1 = 11 + j5$;  $L_2 = 7 + j3$;  $L_3 = 4 + j1$

Operating Condition 5.2

Table 5.5 Generator operating condition 5.2 in $p.u.$ except $\alpha_i$ in $\text{rad}$ (100MVA base)

<table>
<thead>
<tr>
<th></th>
<th>Gen#1</th>
<th>Gen#2</th>
<th>Gen#3</th>
<th>Gen#4</th>
<th>Gen#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>5.700</td>
<td>8.500</td>
<td>6.000</td>
<td>8.000</td>
<td>4.500</td>
</tr>
<tr>
<td>$Q$</td>
<td>2.925</td>
<td>3.560</td>
<td>4.367</td>
<td>5.375</td>
<td>4.932</td>
</tr>
<tr>
<td>$V_i$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.000</td>
<td>0.627</td>
<td>-0.081</td>
<td>0.070</td>
<td>-0.022</td>
</tr>
</tbody>
</table>

Loads in $p.u.$ are as follows.

$L_1 = 6 + j2$;  $L_2 = 21 + j6$;  $L_3 = 5 + j1$

The fault considered here is a symmetrical three-phase short circuit fault. Two fault locations are tested, one occurs on the transmission line 8-11 near bus 11 (Fault Location 5.1), and the other on the transmission line 6-9 near bus 9 (Fault Location 5.2).

For illustration purpose, we consider generator #1. The DFL compensated power
system model of Gen #1 is:
\[
\dot{x}_1(t) = \left[ A_1 + \Delta A_1(t) \right] x_1(t) + \left[ B_1 + \Delta B_1(t) \right] v_{j1}(t) + \sum_{j=2}^{5} \Delta G_{1j}(t) \Delta \omega_j + \sum_{j=2}^{5} \Delta G_{21}(t)
\]

Choose \( |p_n(t)|_{\max} = 1.1 \), \( i = 1, 2, \ldots, 5 \). When \( P_e \geq 0.4 \)

\[
A_1 = \begin{bmatrix}
0 & -0.7096 & 0.0 & 0 & 0 \\
-0.431 & -45.1378 & 0 & 0 & 0 \\
-0.1370 & 0 & 0 & 0 & 0 \\
0.1112 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} ;
\Delta A_1(t) = \begin{bmatrix}
0 & \mu_1(t) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} ;
\]

\[
B_1 = \begin{bmatrix}
0.1112 \\
0 \\
0.1370 \\
\end{bmatrix} ;
\Delta B_1(t) = \begin{bmatrix}
-\mu_{12}(t) \\
0 \\
0 \\
\end{bmatrix} ;
\]

\[
|\mu_1(t)| \leq 0.3775 ;
|\mu_{12}(t)| \leq 0.0541 ;
\]

\[
|\gamma_{11j}(t)| \leq 0.3134 ;
|\gamma_{1p1j}(t)| \leq 0.8148 ;
\]

\[
|\gamma_{21j}(t)| \leq 0.4231 ;
|\gamma_{2p1j}(t)| \leq 1.1000 ;
\]

\[
|\gamma_{31j}(t)| \leq 0.3134 .
\]

Next, by choosing appropriate \( Q_i \) and \( R_i \) and solving LMIs (5.19) and (5.22), the robust CTSVRE control law for generator #1 is \( (P_i(t) \geq 0.4) \)

\[
v_{j1}(t) = -56.56 \Delta V_{c1}(t) + 24.95 \Delta \omega_1(t) - 50.63 \Delta P_{e1}(t)
\]

Similarly, all the decentralized CTSVRE control laws for generators #1 - #5 can be obtained. They are listed below

When \( P_e(t) < 0.4 \)

\[
v_{j11}(t) = -45.26 \Delta V_{c1}(t) + 9.15 \Delta \omega_1(t) - 38.23 \Delta P_{e1}(t)
\]

\[
v_{j12}(t) = -47.19 \Delta V_{c2}(t) + 9.56 \Delta \omega_2(t) - 49.51 \Delta P_{e2}(t)
\]

\[
v_{j13}(t) = -47.94 \Delta V_{c3}(t) + 9.65 \Delta \omega_3(t) - 53.47 \Delta P_{e3}(t)
\]

\[
v_{j14}(t) = -44.63 \Delta V_{c4}(t) + 9.86 \Delta \omega_4(t) - 58.19 \Delta P_{e4}(t)
\]

\[
v_{j15}(t) = -48.16 \Delta V_{c5}(t) + 9.66 \Delta \omega_5(t) - 43.79 \Delta P_{e5}(t)
\]

When \( P_e(t) \geq 0.4 \)

\[
v_{j21}(t) = -56.56 \Delta V_{c1}(t) + 24.95 \Delta \omega_1(t) - 50.63 \Delta P_{e1}(t)
\]

\[
v_{j22}(t) = -45.65 \Delta V_{c2}(t) + 12.33 \Delta \omega_2(t) - 50.33 \Delta P_{e2}(t)
\]
Chapter 5 Robust Decentralized CTSVRE Control of Multimachine Power Systems

The combined control law is (5.25) and the original excitation control law is (5.4).

By solving related LMIs, the TSE control laws for the five machines are also obtained and given below (assuming \( |T_{\alpha_{\text{ref}}}^d| \leq 0.1T_{\alpha_{\text{ref}}}^d \)):

\[
\begin{align*}
v_{\alpha_{\text{ref}}}^1(t) &= 42.93 \Delta \delta_1(t) + 34.53 \Delta \omega_1(t) - 218.13 \Delta P^e_{\alpha_{\text{ref}}}(t) \\
v_{\alpha_{\text{ref}}}^2(t) &= 30.24 \Delta \delta_1(t) + 23.98 \Delta \omega_2(t) - 170.53 \Delta P^e_{\alpha_{\text{ref}}}(t) \\
v_{\alpha_{\text{ref}}}^3(t) &= 39.42 \Delta \delta_1(t) + 30.34 \Delta \omega_3(t) - 232.67 \Delta P^e_{\alpha_{\text{ref}}}(t) \\
v_{\alpha_{\text{ref}}}^4(t) &= 29.40 \Delta \delta_1(t) + 23.82 \Delta \omega_4(t) - 159.95 \Delta P^e_{\alpha_{\text{ref}}}(t) \\
v_{\alpha_{\text{ref}}}^5(t) &= 32.08 \Delta \delta_1(t) + 25.56 \Delta \omega_5(t) - 174.20 \Delta P^e_{\alpha_{\text{ref}}}(t)
\end{align*}
\]

The original excitation control law is (5.4).

In order to evaluate the performance of the proposed controller more accurately, the following physical limit of the excitation voltage is considered:

\[ k_{\text{u}} u_{\alpha_{\text{ref}}}(t) \in [-3, 6] \text{ p.u., } \quad i = 1, 2, \ldots, 5 \]

First, the proposed control will be compared with robust TSE control. Next, the proposed control will be compared with conventional control. Finally, robustness of the proposed control is shown. Different cases are used in the numerical simulation.

### 5.4.1 Comparison with robust decentralized TSE control

In this section, responses of the robust TSE controller and robust CTSVRE regulation controller are compared. The reason why robust CTSVRE controller is needed is given.

The following two cases are considered to evaluate the proposed control strategy.

Case 5.1:
Chapter 5 Robust Decentralized CTSVRE Control of Multimachine Power Systems

Operating Condition 5.1, Fault Location 5.1 and Fault Sequence 4.1.

Figure 5.11 shows the relative power angle, relative angular speed, generator terminal voltage and active power responses of the five generators with the robust TSE controller.

Case 5.2:

Operating Condition 5.1, Fault Location 5.1 and Fault Sequence 4.2.

Figure 5.12 shows the relative power angle, relative angular speed, generator terminal voltage and active power responses of the five generators with robust TSE control; Figure 5.13 shows responses of the five generators with the proposed CTSVRE control.

Figure 5.11  Relative power angle, relative speed, terminal voltage and active power responses for Case 5.1 with TSE control
It can be seen from the simulation results shown in Figure 5.11 - Figure 5.12 while the robust decentralized TSE controllers can transiently stabilize the example system in both the temporary fault case (Case 5.1) and permanent fault case (Case 5.2), it can only achieve satisfactory post-fault voltage performance in case of temporary fault. In case of permanent fault, although the rotor angle recovers to the pre-fault value, the post-fault voltage deviates from the pre-fault value greatly. This is not allowed in practice. On the other hand, it can be seen from Figure 5.13, all the four power system responses are satisfactory. The robust decentralized CTSVRE controller can not only transiently stabilize the example system but also achieve satisfactory post-fault voltage performance.
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Figure 5.13  Relative power angle, relative speed, terminal voltage and active power responses for Case 5.2 with CTSVRE control

Figure 5.14  Exciter input for Case 5.2 with TSE and CTSVRE control strategies

The exciter input responses of the two control strategies are also compared and shown
in Figure 5.14. It can be seen from Figure 5.14 that the exciter input gets out of saturation and approaches steady state faster with CTSVRE controller than TSE controller, this also illustrates the performance of CTSVRE controller is better than TSE controller.

5.4.2 Comparison with conventional control

In this section, the effectiveness of the proposed robust decentralized CTSVRE controller will be illustrated by comparing its system responses with those under conventional AVR and PSS.

![Graphs showing system responses](image)

Figure 5.15 Relative power angle, relative speed, terminal voltage and active power responses for Case 5.2 with conventional control

The studied case is Case 5.2. Relative power angle, relative speed, terminal voltage
and active power responses under conventional control are given in Figure 5.15. By comparing these responses with the responses under the proposed control shown in Figure 5.13, it can be seen that the robust decentralized CTSVRE controllers can stabilize the system and achieve satisfactory post-fault voltage performance with shorter time. With conventional controllers the system performance is not as good as the robust decentralized CTSVRE controllers. This is due to the introduction of voltage deviation from its equilibrium to the state variables of CTSVRE control.

5.4.3 Robustness of the proposed controller

In this section, the robustness of the proposed controller will be evaluated under different operating conditions, fault locations and fault sequences.

Figure 5.16  Relative power angle, relative speed, terminal voltage and active power responses for Case 5.3 with CTSVRE control
Chapter 5 Robust Decentralized CTSVRE Control of Multimachine Power Systems

The following two cases are considered to test the effectiveness of the proposed control strategy:

Case 5.3:
Operating Condition 5.1, Fault Location 5.2 and Fault Sequence 4.1.
Relative power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in Figure 5.16.

Case 5.4:
Operating Condition 5.2, Fault Location 5.2 and Fault Sequence 4.2.
Relative power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in Figure 5.17.
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For Cases 5.3 and 5.4, different fault locations, fault sequences and operating conditions are simulated. It can be seen from Figure 5.16 - Figure 5.17, in both cases the proposed robust CTSVRE controllers can not only stabilize the system but also achieve satisfactory post-fault voltage performance. Note that the parameters of the proposed control strategy do not change for all these cases, thus the control strategy is a robust one.

5.5 Concluding remark

This chapter generalizes the robust CTSVRE control for SMIB systems to multimachine power systems. Instead of using the rotor angle to partition the parametric uncertainties when designing the robust CTSVRE controller, this chapter uses the active power to partition the parametric uncertainties. This allows considering electrical load effects. The control strategy is tested on a five-machine twelve-bus power system model. Simulation results show that the proposed control strategy can not only improve the transient stability but also achieve the desired post-fault voltage level. In addition, the controller is robust against operating conditions, fault locations, fault sequences and network parameters. Although the example system is specific, the control method is general. In next chapter, control strategy for AC/DC power systems will be studied.
6.1 Introduction

In the last several decades, due to its economic advantage in long distance power transfers and the ability to interconnect AC systems with different frequencies, HVDC power transmission has become more and more popular. One outstanding characteristic of HVDC links is its ability to rapidly change power flow in response to control signals. This characteristic is conducive and has been studied and exploited to enhance the stability of power systems. On the other hand, it also complicates the operation and control of power systems.

Early research works on AC/DC controller design are mainly focused on the individual AC or DC controller design, which do not consider the generator excitation control and HVDC control simultaneously. Recently, attempts have been made to design HVDC controllers from a systematic standpoint. However, only two-order generator model is used in some research works [71, 72]. Therefore, the effect of generator dynamics on the HVDC system is not fully considered. In other research works, although three-order generator model is used, the designed controllers use many remote real-time signals [75], which makes the designed controllers impractical for implementation due to the large scale and geographically distribution nature of power systems. This motivates decentralized control of AC/DC power systems. Furthermore, as power systems are highly nonlinear interconnected complex systems, controllers based on approximate linearized model [73, 74] may not guarantee the power system stability over a wide operating region. In [77], an interesting coordinated hierarchical control scheme of generator and HVDC system has been proposed. But the dynamic
Riccati equations need to be solved in real time and the complex expression for the generator's output power when there are many DC nodes make the control strategy too complex to be implemented.

To overcome these difficulties, theories of overlapping decomposition control [115], standard decentralized control, nonlinear direct feedback control [40, 50, 51] and robust control theory will be employed in this study to design robust coordinated controllers for AC/DC power systems with overlapping decomposition. The DFL technique is firstly used to decouple AC/DC systems and also linearize generator models, and then the robust TSE controllers are designed for AC systems and overlapping decomposition controllers are designed for HVDC link for two most widely used control modes. All of the controllers are coordinated to ensure the power system stability.

For the HVDC link decomposition, as there are a series of control modes and some physical limits are violated during some control modes, besides, centralized control has to be used if high level control is required, the overlapping decomposition control is impossible to cover all the operating conditions. In this study, the overlapping decomposition control is only studied for the normal operating condition when the rectifier station work under constant current mode and the inverter station work under constant extinction angle mode (or constant voltage mode). A simple single machine AC/DC example power system is used to evaluate the proposed control strategy. Effect of additional SVC for reactive power compensation when high level DC control is used is also studied.

The organization of this chapter is as follows: HVDC low level, high level and reactive power control are reviewed at first, followed by robust coordinated controller design of AC/DC systems and numeric simulation.
6.2 HVDC low level, high level and reactive power control

As HVDC control is quite complex and flexible, it is necessary to get a basic understanding of HVDC control structure, principles and related problems before the robust coordinated controllers for AC/DC systems are designed. In this section, HVDC low level, high level and reactive power control are introduced sequently.

6.2.1 HVDC low level control

According to control structure, HVDC control can be classified into two categories, i.e. low level control and high level control. The low level HVDC control is the rectifier or inverter firing angle control whose response time is typically 5ms-50ms, thus can be considered instantaneous. The high level HVDC control is always referring to power order control, which is relatively slow, except when it is used for helping improve power system stability. Principles of low level HVDC control will be introduced in the following context. Details on this topic can be found in many literatures [70, 87, 116, 117].

An HVDC transmission system often has for low level control loops, a constant current controller at the rectifier end and a constant extinction angle controller or sometimes a constant voltage controller at the inverter end [117]. A typical inverter control diagram is shown in Figure 6.1. There are three control modes in this scheme: constant current (CC) mode, constant extinction angle (CEA) mode and current error (CE) mode. The CC control compares the current order \( I_{dref} - I_{einv} \) with the measured current \( I_c \) and then produces the firing angle order \( \alpha_{cc} \) through DC current regulator. This firing angle order will reduce the current error. Similarly, CEA control produces the firing angle \( \alpha_{cea} \) in an attempt to reduce the error between the set-point value of extinction angle \( \gamma_{ref} \) and the measured extinction angle \( \gamma_{max} \). The output values of CC control \( \alpha_{cc} \) and CEA control \( \alpha_{cea} \) are then compared and the
minimum is selected as the actual firing angle order. The CE control is used to prevent sudden changes in the operating point at the transition between CEA and CC control mode, thus achieves the effect of mode stabilization. It may be realized by modifying the extinction angle reference value with an offset which is proportional to the DC current.

![Diagram of inverter control scheme](image)

Figure 6.1 Typical inverter control scheme

The rectifier control is similar but usually uses only current control mode. In addition, the current margin $I_{\text{margin}}$ is not subtracted from the current reference. This current margin has a typical value of 0.1-0.15pu [87]. The current margin is selected to be large enough so that the rectifier and inverter CC modes do not interact to simultaneously attempt to control current because of any current harmonics which may be superimposed on the DC current. While the rectifier current control acts under normal condition, the inverter current control acts only when system condition changes. For example, when the rectifier side AC voltage reduction results in DC current reduction, the rectifier will operate under uncontrolled characteristic, i.e. constant ignition angle (CIA) mode.
The normal control modes of CC at rectifier and CEA at inverter have some advantages. For example, the CEA control mode minimizes the inverter reactive power consumption without an excessive risk of commutation failure [116]. However, for reasons of stability and recovery from system disturbances, other control modes should also be considered. The overall static voltage-current relationship for a two terminal HVDC link is shown in Figure 6.2. These characteristics are a modified version from the one given in [87]. ABCEFG and HDKLMNP represents the rectifier and inverter V-I characteristics respectively. AB is from a voltage limit loop. BC is from a minimum alpha limit loop which is the rectifier CIA mode. HD is the inverter CEA mode. CE and KL are rectifier and inverter CC mode respectively. EF and LM are the low-voltage current limit characteristics obtained by compounding a current loop with measured DC voltage. FG and MN are the minimum current limits and NP is the inverter minimum firing angle limit. DK represents the current error control CE in Figure 6.1, which is used to prevent an abrupt transition between the inverter CEA and CC control modes.
The necessity of voltage dependent current order limit (VDCOL) CEFG and KLMN shown in Figure 6.2 can be explained as follows. Under low voltage conditions, there are risks of commutation failure and even voltage instability [87]. In the context of voltage stability following clearing of short circuits, fast voltage recovery helps reacceleration of nearby induction motors [118]. VDCOL will help voltage recovery following faults [87, 116, 118, 119]. VDCOL reduces the maximum allowable direct current in case the voltage drops below a preset level. Consequently, reactive power demand is reduced during periods of depressed voltage. This helps prevent further deterioration of AC system voltage. VDCOL may be given a time delay to assist in riding through AC system faults. Its characteristics are a function of either AC voltage or DC voltage as shown in Figure 6.3 [87]. Constraints are imposed on the current order level to keep the current within an allowed band. The design and implementation of VDCOL for a practical project can be found in [120].

The basic HVDC control principles can be summarized as follows [87]:

1) The HVDC system is basically constant current controlled to limit overcurrent and minimize damage due to faults and also to prevent the system from running down due to fluctuations of the AC voltages.

2) The rectifier is provided with a current control and firing angle limit control.

3) The inverter is provided with a constant extinction angle control (or constant voltage control) and a current control.
4) Under normal conditions, the rectifier is on current control mode and the inverter is on CEA control mode. If there is a reduction in AC voltage at the rectifier end, the rectifier firing angle decreases until it hits the \( \alpha_{\text{min}} \) limit. At this point, the rectifier switches to CIA control mode and the inverter will assume current control.

5) To ensure satisfactory operation and equipment safety, several limits are recognized in establishing the current order: maximum current limit, minimum current limit, and voltage-dependent current limit.

### 6.2.2 HVDC high level control

Being the primary objective of HVDC transmission system, power transmission is of main concern. In order to transmit the scheduled power, a slow-acting power control loop is provided to adjust the current order properly. The current order is derived from the ratio of the power order and the DC voltage. As shown in Figure 6.4, the DC current reference \( I_{\text{dc ref}} \) is produced by dividing the sum of the DC power order \( P_{\text{d} \text{o}} \) and supplementary DC power order \( \Delta P_{\text{dc}} \) by the measured DC voltage \( V_d \). The upper limit of the DC current reference is further adjusted by the DC voltage. The power order \( P_{\text{d} \text{o}} \) and corresponding rate of change order could be set by the operator or be transmitted from a dispatch center.

![Figure 6.4](image)

Apart from controlling the normal DC power transmission, the power control loop can
also be employed to enhance power system stability or to help damping the oscillations of interconnected AC systems due to the characteristic of fast response of low level firing angle control to system operating condition changes. This is achieved by modulating the transmitted active power through the change of the DC power order $P_{dc,ref}$ in accordance with various system condition changes, such as the AC system frequency variations, AC bus voltage angle changes and power or current changes in the adjacent parallel AC tie. The supplementary power order $\Delta P_d$ in Figure 6.4 corresponds to the power order change produced by these system condition changes. It should be pointed out that the ultimate realization of power control is still realized by the firing angle control of the low level HVDC control loop. That is, the high level power control changes the current order $I_{dc,ref}$, which in turn changes the converter firing angle, see Figure 6.1.

The high level active power modulation control includes: AC system damping control, AC system frequency control, step change power adjustment and subsynchronous oscillation damping, etc. The effectiveness of active power modulation which can be achieved by an HVDC link is restricted by the consequent modulation of the reactive power absorption of converters and the resulting changes of the AC voltages adjacent to DC terminals. In case AC network becomes weak, variations in reactive power will cause voltage variations, which reduces the effectiveness of power modulation or even is detrimental to overall system stability [121]. One scenario is when power modulation at the rectifier station increases DC current, the reactive power demand at the inverter will increase a lot. For weak AC systems, this leads to reduction of converter bus voltage, which in turn results in reduction of power flow below the level ordered by the modulation controller [70]. Therefore, the reactive power control problem associated with active power modulation should be studied.
6.2.3 HVDC reactive power control

The converter absorbs reactive power irrespective of whether it is operating as a rectifier or an inverter. The nominal reactive power demand of the converters at full load is about 50% of the megawatt rating [93]. This reactive power demand is normally met by mechanically switched passive AC filters and shunt capacitors and/or electronically switched static var systems. The AC filters are designed to filter AC system harmonics, e.g., 11th and 13th harmonics. At fundamental frequency, they play the same role as capacitor banks. The reactive power generated by the compensator equipments and the reactive power consumed by the conversion process must be kept in balance. Any surplus or deficit in reactive power must be accommodated by the AC system and the difference needs to be kept within an accepted range to keep the AC voltage within the desired tolerance. Sudden changes of reactive power caused by line or load switching or AC system faults will result in sudden voltage changes, no matter temporary overvoltage or undervoltage. Therefore, the control of reactive power supply to match the reactive power demand of converters is essential to minimize the voltage variations at AC terminals of converter stations.

The consumption of reactive power in a DC converter is dependent on the converter control angles (alpha or gama) and terminal reactive power constraints. Although converter transformer tap changer can be used to control the reactive power, it is relatively slow (usually operates in multiples of 5 seconds [93, 119]), thus it will not be considered here. The primary method to modulate converter station's reactive power is through varying converter firing angles or switching filter banks. Various modulation control functions such as gama modulation control and voltage stabilization modulation control can be used to limit variations of AC voltage by selecting of proper control signals. For example, proper coordination of gama modulation with shunt bank switching is able to reduce step changes of reactive power caused by switching shunt capacitors and filters [121]. Another example is that when
voltage stabilization modulation control is used as a function of voltage variation, if a substantial reduction of AC voltage occurs, the DC link is ordered to reduce active power quickly, and hence a certain amount of reactive power will be released from the converters. Therefore, AC voltage restores sufficiently and AC system collapse is prevented. References for this kind of control can be found in [76, 121-131]. Among them, coordinated modulation of active power and reactive power control through firing angle control is proposed in [126].

When the DC converter controls are not sufficient or fast switching actions are required, additional compensator such as synchronous compensators or static var system control has to be used. The compensator control can be combined with the converter firing angle control to support dynamic voltage regulation. This in turn helps in the recovery of the AC system from faults and also reduces the disturbances resulting from DC load variation or from the switching of filter banks. References for this kind of control can be found in [121, 123, 127, 128, 132, 133].

Since the contribution of this study for HVDC control lies in proposing a new theoretic overlapping decomposition strategy for the normal two control modes at low level (see Section 6.3.2) and VDCOL can be effectively employed to adjust the reactive power consumption and thus help stabilize the system voltage when the converter terminal voltage is low, detailed reactive power control is not studied in the HVDC robust overlapping decomposition controller design in Section 6.3.2. Further study for reactive power optimization can be carried out using the methods introduced in this section.

6.3 Robust coordinated control of AC/DC power systems with overlapping decomposition

As power systems are interconnected complex systems, the dynamics of each
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component will influence the operation of the entire system. It is thus required to analyze the system and design controllers from a systematic standpoint. In this section, a novel robust coordinated control strategy for AC/DC systems is proposed. DFL technique is used to decouple AC/DC systems and also linearize the generator model; after that, robust TSE controller for generator excitation system is designed; next, overlapping decomposition technique introduced in Section 3.4 is used to design controllers for HVDC link for the most widely used two control modes (low level).

6.3.1 Robust TSE control of generator excitation system

When an HVDC link is integrated into AC power systems, equations of (2.6)-(2.7) are no longer applicable. However, equations (2.1)-(2.5) and (2.8)-(2.11) are still suitable to describe the generator model. Obviously, the generator model is a nonlinear one. In the mean time, it can be seen from (2.8)-(2.9) that in a AC/DC system, the generator system is coupled with the DC system through the variables of \( I_q(t) \) and \( I_d(t) \). To decouple the AC and DC power systems and at the same time to linearize the generator model, DFL technique [40, 50, 51] can be employed effectively.

Differentiating both sides of (2.8) gives

\[
\dot{P}_e(t) = E_q(t)I_q(t) + E_d(T)I_d(t)
\]

\[
= \frac{1}{T_d0} \left( k_{u}u_f(t) - E_q(t) - (x_d - \dot{x}_d)I_d(t) \right)I_q(t) + E_d(t)I_d(t)
\]

\[
= \frac{1}{T_d0} \left( -\Delta P_e(t) + P_m + k_{u}u_f(t)I_q(t) - (x_d - \dot{x}_d)I_d(t)I_q(t) \right) + E_d(t)I_d(t)
\]

\[
= -\frac{1}{T_d0} \Delta P_e(t) + \frac{1}{T_d0} v_f(t)
\]

where

\[
\Delta P_e(t) = P_e(t) - P_m; \quad v_f(t) = k_{u}u_f(t)I_q(t) - P_m - (x_d - \dot{x}_d)I_d(t)I_q(t) + T_d0E_d(t)I_d(t)
\]

The DFL compensating control law is immediately obtained as follows

\[
u_f(t) = \frac{1}{k_{u}I_q(t)} \left[ v_f(t) + P_m + (x_d - \dot{x}_d)I_d(t)I_q(t) - T_d0E_d(t)I_d(t) \right]
\]
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The nonlinear compensator (6.2) contains the differentiation of the signal \( I_c(t) \) and thus it is a dynamic DFL compensator. This differentiation can be simply approximated by using difference of \( I_c(t) \) within an enough short period. As is pointed out in Chapter 4, \( P_c(t) \), \( Q_c(t) \) and \( I_f(t) \) are all available variables in power systems, \( I_s(t) \) and \( I_f(t) \) can be calculated from these available variables. Thus, the compensating law (6.2) is practically realizable using only local measurements. As (6.2) does not use any signals from DC system, the AC/DC system is also decoupled.

Let \( \Delta \delta(t) = \delta(t) - \delta_0 \), \( \Delta \omega(t) = \omega(t) - \omega_0 \), by combining (2.1)-(2.2) and (6.1), the compensated generator model becomes

\[
\dot{x}(t) = Ax(t) + Bu(t) \tag{6.3}
\]

where

\[
x(t) = \begin{bmatrix} \Delta \delta(t) & \Delta \omega(t) & \Delta P_c(t) \end{bmatrix}^T, \quad u(t) = v_f(t)
\]

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & D/H & \omega_0/2H \\ 0 & 0 & 1/T_{\omega0} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/T_{\omega0} \end{bmatrix}
\]

At this point, it can be seen that the dynamic DFL not only linearizes the generator model but also decouples the generator with other components. Thus the generator excitation control law is a nonlinear decentralized one.

Consider the various parameter uncertainties in power systems, the linearized generator system model can be rewritten in the following state space form

\[
\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) \tag{6.4}
\]

where \( x(t) \), \( u(t) \), \( A \) and \( B \) are the same as in (6.3).

As an illustrated example, the parametric uncertainties in \( T_{\omega0} \) as \( \Delta T_{\omega0} \) will be considered in this work. Then the system uncertainty matrices are
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\[
\Delta A(t) = \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \mu(t) \end{bmatrix}, \quad \Delta B(t) = \begin{bmatrix} 0 \\
0 \\
-\mu(t) \end{bmatrix}
\]

\[
\mu(t) = \frac{1}{T_{d_0}} - \frac{1}{T_{d_0} + \Delta T_{d_0}}
\]

Since the uncertain system model (6.4) and the system uncertainty matrices are similar to (4.5), performance guaranteed robust excitation controller can be designed following the TSE control method in Section 4.2.

6.3.2 Robust overlapping decomposition control of HVDC link

After AC/DC systems are decoupled and robust generator excitation controller is designed, HVDC robust overlapping decomposition controller will be designed in this subsection.

The designed HVDC robust overlapping decomposition controller should consider the DC line dynamics and be able to function well under normal conditions. "Normal condition" here means the converter stations functions well under the most widely used control modes. That is, the rectifier station operates under CC mode and the inverter station operates under CEA mode (or constant voltage mode). Controller design for converters working under such modes is primarily considered as follows. Obviously, the overlapping decomposition control here belongs to low level control. Other control modes and high level control will be shortly briefed in the end of this section.

When rectifier station works under CC mode and inverter station works under CEA mode, the control objective is to keep the difference between the actual DC current and reference DC current at rectifier side and the difference between the actual extinction angle and reference extinction angle at the inverter side as small as possible.

Observing carefully the HVDC dynamic equations (2.48)-(2.50) and the state space form (3.11) that is suitable for overlapping decomposition technique, it can be seen the
HVDC dynamic system can be treated as a system with overlapping subsystem. Using the overlapping decomposition technique in Section 3.4 and also considering the control objective, the original HVDC system model can be expanded into the form of (3.12), which can be regarded as the combination of the following two subsystems

\[ \dot{x}_i = A_i x_i + B_i u_i + \sum_{j=i,j\neq i} G_{ij} x_j \]

\[ y_i = C_i x_i \]

\[ e_i = y_i - r_i \]

where \( i, j = 1, 2 \)

\[ x_1 = \begin{bmatrix} I_d \\ \cos \alpha_r \end{bmatrix}, \quad A_1 = \begin{bmatrix} \frac{1}{T_d} & \frac{V_i}{T_d} \\ 0 & -\frac{1}{T_r} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad G_{12} = \begin{bmatrix} 0 & \frac{V_i}{T_d} \\ 0 & 0 \end{bmatrix}, \]

\[ C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad r_1 = I_{d0} \]

\[ x_2 = \begin{bmatrix} I_d \\ \cos \alpha_i \end{bmatrix}, \quad A_2 = \begin{bmatrix} -\frac{1}{T_d} & \frac{V_i}{T_d} \\ 0 & -\frac{1}{T_i} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad G_{21} = \begin{bmatrix} 0 & \frac{V_i}{T_d} \\ 0 & 0 \end{bmatrix}, \]

\[ C_2 = \begin{bmatrix} k_r & -1 \end{bmatrix}, \quad r_2 = \cos \gamma_{i0} \]

The coefficients in the above parameter space include \( V_r, V_i \) and \( k_r \) which are time varying but bounded variables, thus these varying variables can be treated as parameter uncertainties. In addition, time constants \( T_r, T_i \) and \( T_k \) are not exactly known, robust control technique can be employed. The problem now becomes a robust decentralized regulation problem.

To eliminate the steady-state error, integral action is usually introduced for controller design. Define

\[ q_i = \int_0^t e_i dt, \quad i = 1, 2 \]

(6.6)

Then by augmenting the system (6.5) with (6.6) and considering the uncertainties, there is
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\[
\dot{z}_i = [A_i + \Delta A_i] z_i + [B_i + \Delta B_i] u_i + \sum_{j=1, j \neq i}^n [G_{ij} + \Delta G_{ij}] z_j + H_i r_i
\]  

(6.7)

where

\[
\begin{align*}
A_i &= \begin{bmatrix}
A_i & 0 \\
C_i & 0
\end{bmatrix}, & \Delta A_i &= \begin{bmatrix}
\Delta A_i & 0 \\
\Delta C_i & 0
\end{bmatrix}, \\
B_i &= \begin{bmatrix}
B_i \\
0
\end{bmatrix}, & \Delta B_i &= \begin{bmatrix}
\Delta B_i \\
0
\end{bmatrix}, \\
G_{ij} &= \begin{bmatrix}
G_{ij} & 0 \\
0 & 0
\end{bmatrix}, & H_i &= \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix},
\end{align*}
\]

If a controller can robustly stabilize the augmented system (6.7), then it also stabilizes the original uncertain system and achieves the output regulation \( \lim_{t \to \infty} e_i = 0 \), \( i = 1, 2 \).

Since the last term in (6.7) does not influence the system stability and integral control is used here, this term can be ignored. Without considering this term, the form of (6.7) agrees with that of (3.1), which can be solved by the approach given in Section 3.3.

One possible kind of decomposition of the uncertainties is given as below

\[
D_1 = \begin{bmatrix}
\Delta \left( \frac{1}{T_d} \right)_{\max} & \Delta \left( \frac{V}{T_d} \right)_{\max} & 0 \\
0 & 0 & \Delta \left( \frac{1}{T_y} \right)_{\max} \\
0 & 0 & 0
\end{bmatrix}, \quad E_{11} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad E_{21} = \begin{bmatrix}
0 \\
-1
\end{bmatrix},
\]

\[
M_{12} = \begin{bmatrix}
\Delta \left( \frac{V}{T_d} \right)_{\max} \\
0 \\
0
\end{bmatrix}, \quad N_{12} = \begin{bmatrix}
0 & 1 & 0
\end{bmatrix},
\]

\[
F_i(t) = \text{diag} \begin{bmatrix}
-\Delta \left( \frac{1}{T_d} \right)_{\max}, & \Delta \left( \frac{V}{T_d} \right)_{\max}, & -\Delta \left( \frac{1}{T_y} \right)_{\max}
\end{bmatrix}
\]

\[
D_2 = \begin{bmatrix}
\Delta \left( \frac{1}{T_d} \right)_{\max} & \Delta \left( \frac{V}{T_d} \right)_{\max} & 0 & 0 \\
0 & 0 & \Delta \left( \frac{1}{T_y} \right)_{\max} & 0 \\
0 & 0 & 0 & |\Delta k_y|_{\max}
\end{bmatrix}, \quad E_{12} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad E_{22} = \begin{bmatrix}
0 \\
0 \\
-1
\end{bmatrix}.
\]
$M_{21} = \begin{bmatrix} \Delta \left( \frac{V}{T_y} \right)_{\text{max}} \\ 0 \\ 0 \end{bmatrix}$, 

$N_{21} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$, 

$F_2(t) = \text{diag} \left( \begin{array}{ccc} -\Delta \left( \frac{1}{T_y} \right) & \Delta \left( \frac{V}{T_y} \right) & -\Delta \left( \frac{1}{T_f} \right) \\ \Delta \left( \frac{1}{T_y} \right)_{\text{max}} & \Delta \left( \frac{V}{T_y} \right)_{\text{max}} & \Delta k_{\text{max}} \\ \Delta \left( \frac{1}{T_f} \right)_{\text{max}} & \Delta k_{\text{max}} & \Delta \left( \frac{V}{T_f} \right)_{\text{max}} \end{array} \right)$, 

$F_{21} = \Delta \left( \frac{V}{T_f} \right)$.

When rectifier station works under CC mode and inverter station works under constant voltage mode, the control objective at the inverter side now changes to regulate the DC voltage to a constant value $V_{d0}$. The DC voltage at the inverter side can be expressed as

$V_d = -V_i \cos \alpha + B_{qi} R_q I_q$  \hspace{1cm} (6.8)

The output matrix and the reference for inverter voltage are now as follows

$C_2 = \begin{bmatrix} B_{qi} & -V_i \end{bmatrix}$,  \hspace{1cm} $r_2 = V_{d0}$  \hspace{1cm} (6.9)

The controller design procedure is similar to the condition when rectifier station works under CC mode and inverter station works under CEA mode.

Other control modes, such as rectifier CIA mode and inverter CC mode, can be regarded as backup control modes because such control modes are used when the system is under large disturbance and usually some physical limits are violated. Controller design under such conditions method may follow the traditional PI controller design method, which is not of the concern of this study.

For high level control, appropriate signals should be sent to remote converter station through telecommunication link. Although schemes have been operated without telecommunications, optimal performance cannot be obtained without it [93]. Therefore, centralized control has to be used. The capacity and speed of response of the telecommunication link are of importance and dependent on the control
requirement, say, damping slow oscillations or helping achieve fast transient stability.

### 6.4 Simulation results

In this section, the effectiveness of the proposed control strategy will be evaluated on the example system shown in Figure 6.5. The studied AC/DC power system contains one generator, two parallel AC transmission lines, one DC link and one infinite bus system. For simplicity, filter banks are not modeled. They are assumed to be part of the shunt capacitors, which provide the main part of reactive power support at both the rectifier and inverter ends. The DC transmission link modeled in this study is a 500KV, 12-pulse and two-terminal one. The rated frequency of the example system is 50Hz. The conventional exciter is the standard IEEE static exciter type ST1A.

![Figure 6.5  Single machine AC/DC power system example](image)

The HVDC control modes used in the simulation include: CC and CIA control at the rectifier station and CEA, CC and CE control at the inverter station. In addition, to help stabilize the system voltage when the converter terminal voltage is too low, VDCOL is also applied in the simulation. Although reactive power optimization is not studied in this work, a simple scheme was used to demonstrate the effect of SVC for reactive power compensation when HVDC power modulation control is employed to
enhance the transient stability of the example system in Section 6.4.2.

Parameters of the example system are as follows:

Parameters of the synchronous generator (p.u. 100MVA base; Rating 1, 000MW)

\[ X_d = 0.1863, \quad X_q = 0.0657, \quad X'_d = 0.0657, \quad T_{f0} = 6.9s, \quad H = 60, \quad D = 5, \]

Parameters of the AC transmission system (p.u. 100MVA base)

\[ x_i = 0.001, \quad x_i = 0.0127, \quad z_r = 0.068 \angle 72.90^\circ, \quad r_i = 0.004, \quad x_j = 0.1 \quad \text{(each AC transmission line)}. \]

Parameters of the DC link (Rating 400MW)

\[ V_{dc} = 500kV, \quad R_v = R_{d1} = 12\Omega, \quad R_d = 20\Omega, \quad L_{dc} = 0.192H, \quad L_{dc} = L_{d0} = 0.5H \quad \text{(smoothing inductor),} \quad \alpha_{d0} = 18^\circ, \quad \alpha_{r,\min} = 5^\circ, \quad \alpha_{r,\max} = 70^\circ, \quad \gamma_{r0} = 15^\circ, \quad \gamma_{r,\max} = 40^\circ, \quad \gamma_{r,\min} = 12^\circ \]

Parameters of AVR and PSS (input signal for PSS is \( \Delta \omega \)) are the same as those given in Section 4.4.2.

For illustration purpose, we consider the AC system parametric perturbation as

\[ |\Delta T_{\omega0}|_{\text{max}} = 0.1T_{\omega0,\text{nom}} \quad \text{and choose} \quad |P_{\omega}(t)|_{\text{max}} = 1.4 \quad (i = 1, 2, \ldots, 5). \]

Choose the DC system parametric perturbation as

\[ |\Delta T_r|_{\text{max}} = 0.1T_r, \quad |\Delta T_{\max} = 0.1T_{r,\text{nom}}, \quad |\Delta T_d|_{\text{max}} = 0.1T_d, \]

\[ |\Delta V_r|_{\text{max}} = 0.2V_{r,\text{nom}}, \quad |\Delta V_{\text{nom}} = 0.2V_{r,\text{nom}}, \quad |\Delta k_r|_{\text{max}} = 0.2k_{r,\text{nom}}, \quad \text{where} \quad V_{r,\text{nom}}, \quad V_{r,\text{nom}} \quad \text{and} \quad k_{r,\text{nom}} \quad \text{are the nominal value}. \]

Choose appropriate \( Q, \quad R, \quad R_i \quad (i = 1, 2) \) and solve the corresponding LMIs, the control laws can be obtained.

For exciter controller, the control law is

\[ v_j(t) = 25.85 \Delta \delta(t) + 23.19 \Delta \omega(t) - 111.15 \Delta P_j(t) \]

For DC controllers, the control law at the rectifier side (CC mode) is

\[ u_{\alpha} = -2.35I_{\alpha} - 0.59 \cos \alpha_e - 120.68 \int_{0}^{t} (I_{\alpha,\text{nom}} - I_{\alpha,\text{nom}}) dt \]

The control law at the inverter side (CEA mode) is
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\[ u_u = -0.45I_{dc} - 0.47\cos\alpha - 38.79\int_0^t (\cos\gamma - \cos\gamma_0)dt \]

For high level HVDC power modulation control, the machine speed deviation is the natural choice as input signal for transient stability enhancement for the example system. The control law for the DC power modulation control may be chosen as

\[ \Delta P_{dc} = K\Delta \omega \]

where \( \Delta P_{dc} \) is the supplementary control input shown in Figure 6.4 and \( K \) is a positive constant.

In order to evaluate the performance of the proposed exciter controller more accurately, the following physical limit of the excitation voltage is considered:

\[ k_uu(t) \in [-3, 6]\text{p.u.} \]

The fault considered here is a symmetrical three-phase temporary short circuit fault and the fault sequence is Fault Sequence 4.1. Two situations are considered as follows: without HVDC high level control and with HVDC high level control.

6.4.1 Without high level control

Effectiveness of the proposed robust coordinated control strategy is studied with different operating conditions, fault locations and network parameters. System responses with two control strategies will be compared in one simulation case, they are: (a) proposed control for generator and DC link at CC and CEA control modes (conventional PI control is used for other control modes of DC link); (b) conventional AVR and PSS with conventional PI control for DC link.

The following three cases are considered to evaluate the proposed control strategy.

Case 6.1:
\[ \delta_0 = 56.77^\circ, \quad V_{\phi} = 1.02\text{p.u.}, \quad P^{\phi} = 9.5\text{p.u.}, \quad P_{dc0} = 200\text{MW}, \quad \alpha_{\phi0} = 18^\circ, \quad \gamma_{\phi0} = 15^\circ. \] Shunt
capacitor banks provide 70MVAr reactive power at the rectifier and 65MVAr reactive power at the inverter.

For the example system, the interaction at the rectifier side is quite strong. At the inverter side, the short circuit ratio (SCR) value is $7.35 \angle -72.90^\circ$. The fault occurs at the middle point on one of the parallel AC transmission lines. Power angle, relative speed, terminal voltage and active power responses under proposed control and conventional control strategies are compared. Results are shown in Figure 6.6. The legends "proposed control" and "AVR+PSS" in Figure 6.6 correspond respectively to control strategies (a) and (b) mentioned early in this subsection.

Figure 6.6  Power angle, relative speed, terminal voltage and active power responses for Case 6.1 with different control strategies
Figure 6.7  DC power responses for Case 6.1 with different control strategies

Figure 6.8  Relative speed, terminal voltage, active power, rectifier and inverter AC voltage responses for Case 6.2 with proposed control
Case 6.2:

\[ \delta_0 = 36.55^\circ, \quad V_{in} = 1.10 \text{ p.u.}, \quad P_{o} = 8.0 \text{ p.u.}, \quad P_{dc} = 400 MW, \quad \alpha_o = 18^\circ, \quad \gamma_o = 15^\circ. \]

Shunt capacitor banks provide 170MVAr reactive power at the rectifier and 150MVAr reactive power at the inverter.

At the inverter side, the SCR value is \(3.68 \angle -72.90^\circ\). The fault occurs on one of the AC transmission lines near the rectifier terminal AC bus. The relative speed, terminal voltage, active power and rectifier and inverter AC voltage responses are shown in Figure 6.8.

Case 6.3:

\[ \delta_0 = 53.51^\circ, \quad V_{in} = 1.10 \text{ p.u.}, \quad P_{o} = 8.0 \text{ p.u.}, \quad P_{dc} = 400 MW, \quad \alpha_o = 18^\circ, \quad \gamma_o = 15^\circ. \]

Shunt
capacitor banks provide 170MVAr reactive power at the rectifier and 150MVAr reactive power at the inverter.

Parameters of the example system for this case are the same as given in the beginning of this section except that impedances for each AC transmission line are $r_i = 0.001$, $x_i = 0.025$ and the generator quadrature axis reactance is $x_q = 1.863$. The SCR value at the inverter side is the same as that in Case 6.2. The fault occurs at the inverter terminal AC bus. Relative speed, terminal voltage, active power and rectifier and inverter AC voltage responses are shown in Figure 6.9.

DC power responses are also illustrated. For Case 6.1 DC power responses are shown in Figure 6.7. For Case 6.2 and Case 6.3, DC power response is shown in the same figure of active power response.

For Case 6.1, it can be seen from Figure 6.6 although the conventional control scheme can stabilize the system successfully, it costs longer time and the oscillation magnitude is quite large for the four responses. The proposed control strategy has shorter setting time and smaller magnitude variation. The voltage performance with the proposed control strategy is also satisfactory. From Figure 6.7 it can be seen that the DC power response of the proposed control is also better than that of the conventional control and less affected by the recloser operation. Therefore, the proposed control has better performance than the conventional control.

For Case 6.2 and Case 6.3, temporary three phase short circuit fault is simulated at the rectifier and inverter respectively. The generator output power, terminal voltage and the power transmitted through DC link are all different from Case 6.1. The SCR value at the inverter side is one half of that in Case 6.1. In addition, for Case 6.3, the generator quadrature axis reactance $x_q$ is chosen as $x_q = x_d$, and AC transmission line parameters also changes. It can be seen from Figure 6.8 - Figure 6.9 that the relative
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speed, terminal voltage, active power, DC power, rectifier and inverter AC voltage responses are all satisfactory. Note that the parameters of the proposed control strategy are constants and do not change for all the three cases, thus the control strategy is a robust one.

6.4.2 With high level control

To illustrate the effect of power modulation and SVC reactive power compensation introduced in Section 6.2.2, system responses with three high level control strategies will be compared in the numeric simulation, they are: (a) no power modulation control; (b) power modulation control only; (c) power modulation with SVC. These high level power modulation control strategies are based on the proposed robust coordinated low level control discussed in Section 6.3. The power modulation control used in this simulation is to enhance the transient stability of the DC link connected AC system. The SVC used is a 12 pulse TSC and TCR combination and is modeled and controlled identically as reported in [99].

The following case is used to compare the three control strategies.

Case 6.4:

Network parameters, operating condition and fault sequence are the same as that in Case 6.1. The only difference is the fault location now is on one of the AC transmission lines near the rectifier terminal AC bus. For control strategy (c), the capacitor banks at the inverter are substituted by SVC with 100MVar TSC and 50MVar TCR. At steady state, this combination provides the same capacity of reactive power as the fixed capacitor banks do. Power angle, relative speed, terminal voltage and active power responses with different control strategies are shown in Figure 6.10. DC power responses with different control strategies are shown in Figure 6.11.

It can be seen from Figure 6.10 that system responses with modulation are improved
Figure 6.10  Power angle, relative speed, terminal voltage and active power responses for Case 6.4 with different control strategies

Figure 6.11  DC power responses for Case 6.4 with different control strategies
significantly than those without power modulation. With power modulation, when the capacitor banks at the inverter are substituted by SVC, the system performance also increases to some extent although not so significantly. Figure 6.11 shows how the DC power varies with and without power modulation.

The advantage of the DC power modulation to power system transient stability can be explained as follows. For the example system, when a short circuit fault occurs near the rectifier terminal AC bus, the transmission capacity of the generator decreases to zero. This results in acceleration of the generator rotor speed. Upon clearing the fault, the generation and the remaining transmission experience a transient swing. If the clearance time is not too long, the generator may still keep in synchronism. In such a case, the DC link has the capability to help enhancing the AC system transient stability by increasing (decreasing) the sending end DC link power in the post-fault period in response to the increase (decrease) of generator speed. This is because the DC power modulation will remove energy from the generator, reduce its speed, and thus reduce the angular displacement between the generator and the AC receiving system. In practice, the level of the modulation applied for this purpose is designed on a case by case principle and is a function of not only AC system frequency deviation but also AC system voltage support capability. That is, besides transient stability, voltage stability should also be considered, which is dependent on reactive power control.

For Case 6.4, with power modulation, the DC power variation in response to generator speed changes (Figure 6.11) helps accelerate the system to reach steady state. Although $\Delta \omega > 0$ immediately after removal of the fault at 0.2s, high level of power modulation cannot be executed in practice. The reason is that the voltage at the rectifier terminal is low at this time, high level of power modulation demand high level of reactive power, which can not be met in practice. In fact, VDCOL takes effect and limits the power modulation level for this case.
Chapter 6 Robust Coordinated Control of AC/DC Power Systems with Overlapping Decomposition

Since SVC is used at the inverter side to provide the reactive power and regulate the inverter terminal voltage and thus, it does not contribute too much to the rectifier side AC system stability. However, it does have some contributive effect, as can be seen from Figure 6.10. In fact, SVC is more helpful to the inverter AC system especially the inverter side AC system is weak, which is always related with voltage instability, long fault recovery times and commutation failure problem. Caution should be taken when using SVC for HVDC reactive power compensation as improper control may result in voltage instability [134-137].

In this section, the adopted power modulation control law is simple and not tuned to optimal function and consisted only of static gain as the intention was only to prove the effectiveness of power modulation. Similarly, the reactive power control is not studied in detail, only VDCOL is used to prevent excessive of reactive power consumption when the voltage is low. Further studies are left for future work.

6.5 Concluding remark

In this chapter, robust coordinated control strategy for single machine AC/DC power systems with overlapping decomposition is proposed. The DFL technique is firstly used to decouple AC/DC systems and also linearize generator models, and then the robust TSE controllers are designed for AC systems and overlapping decomposition controllers are designed for HVDC link for two most widely used control modes. Both the generator excitation system and HVDC link (low level at CC and CEA control modes) controllers use only local measurements. All these controllers are coordinated to ensure the power system stability. Simulation results show that the proposed control strategy is effective in enhancing power system transient stability and provides good robustness and insensitivity to different operating conditions, fault locations and network parameters. Although HVDC high level and reactive power control are not studied in detail, related control methods are reviewed and may be combined with the
proposed control strategy for the future research. In Chapter 7 it will be shown that the proposed robust coordinated control strategy for single machine AC/DC power system can be extended to multimachine AC/DC power systems.
Chapter 7 Robust Nonlinear Decentralized Control of Complex Power Systems

7.1 Introduction

In previous chapters, a series of controller design methods have been discussed, such as robust CTSVRE control for SMIB and multimachine power systems and robust coordinated control for SMIB AC/DC power systems with overlapping decomposition. When these control methods are detailed, only one or two issues are focused. However, in practice, various network structure and devices coexist, this result in high degree of complexity of power systems and hence the complexity of power system control. Such systems are regarded as "complex power systems" in this study.

A characteristic for complex systems is that the control objective is always multiple. Up to now, there has not been any general control strategy to meet all the multiple objectives simultaneously. However, there are actually some attempts to solve such problems [1, 52, 108, 109, 138, 139]. In fact, a straightforward simple idea for this problem is like this: for independent designed controllers of which there is no control requirement relationship, they can be directly applied to the complex system; on the other hand, when there is control requirement relationship, appropriate coordinated control strategy should be used to make a tradeoff among the control requirements. Of course, before the designed controllers can be applied to complex systems they should be tested on filed.

An example in practice is to require not only the system survive the transient process but also the post-fault voltage should fall in an allowed range. Thus, controllers satisfying both requirements are needed. References [7, 39, 40] proposed such a
control strategy, however, they all have some drawbacks. A nonlinear controller proposed in [7] is model-based and is sensitive to the structure of the model and the varying parameters of the transmission network. In [39], a new modeling method is proposed to express the terminal voltage of each machine in terms of local variables. The difficulty to estimate some parameters and the complexity limit the use of this method. In [40], a preset switching time between the TSE controller and voltage regulation controller is assumed that is not possible in practice. In Chapters 4 and 5, although the designed robust CTSVRE control has the ability to enhance transient stability and post-fault voltage regulation, the controller design is of local characteristic, which may not guarantee transient stability in some operating conditions, e.g., the generator active power output is very low. Therefore, new control strategies should be studied.

To achieve both transient stability and voltage regulation enhancement in the whole operating region, global control idea [1, 52, 108, 109] will be adopted in this chapter. This control scheme is capable of control over a wide range of operating regions and therefore satisfies the overall system stability and performance requirement. For each operating region of the system, there is a classical control law that is able to provide high-resolution control. The switch action between different operating regions is defined by appropriate membership functions. The global control law is the composition of heterogeneous weighted local control laws corresponding to different operating regions [140]. The key point for global control is to partition the whole operating region to subspaces and choose appropriate member functions.

In this chapter, advanced controller design for complex multimachine AC/DC power systems will be proposed by integrating the control methods discussed in previous chapters using appropriate coordinated control strategy. The designed robust nonlinear decentralized controllers can not only operate under a wide range of operating
conditions but also enhance both transient stability and voltage regulation. The HVDC robust overlapping decomposition controller design method and factors that need special attention are exactly the same as those discussed in last chapter. As is pointed out in last chapter, high level HVDC control is not the main concern of this study, only low level control will be studied in the simulation of this chapter.

7.2 Robust nonlinear decentralized control of complex power systems

The complex power system considered here is a system consisting of \( n \) synchronous machines and \( n_p \) HVDC nodes; electrical load is assumed to be of static load [80]. As the overlapping decomposition control method for HVDC transmission system has already studied in Chapter 6, this section focuses on dealing with the problem of how to decouple the complex multimachine AC/DC system.

While the HVDC link model is the same as given in Section 2.5, the generator model should be modified due to the incorporation of HVDC link. Under the same assumptions as given in Section 2.4, a synchronous generator is modeled as a voltage source behind the direct axis transient reactance. Through network reduction, only the generator internal nodes and HVDC converter (including rectifier and inverter) nodes of the complex power system are retained and all other nodes are eliminated. The resultant network has \( n+n_p \) nodes. Note while the mechanical equations of the dynamical model of the \( ith \) (\( i=1,2,...,n \)) machine with excitation control have not changed, the quadratic and direct axis current equations (2.26)-(2.27) among the generator electrical equations have been changed. This change is to include the HVDC effect. The two equations are now of the following form

\[
I_q(t) = \sum_{j=1}^{n} E_{qj}(t)B_{ij}\sin\delta_{ij}(t) + \sum_{k=1}^{2} V_{p_k}(t)B_{ik}\sin\delta_{ik}(t)
\]  

(7.1)
Chapter 7 Robust Nonlinear Decentralized Control of Complex Power Systems

\[
I_a(t) = -\sum_{j=1}^{n} E_{\phi}^{j}(t) B_j \cos \delta_j(t) - \sum_{j=1}^{n} V_{\phi}(t) B_j \cos \delta_j(t) \tag{7.2}
\]

where \( V_{\phi}(t) \) \((i = 1, 2, \cdots, n_p)\) is the DC link terminal voltage, in \textit{p.u.}.

It can be seen from Section 2.4 and equations (7.1)-(7.2) that the generator model is highly nonlinear and interconnected with other components, such as the HVDC rectifier or inverter stations. In order to decouple the AC/DC system, DFL technique will be employed. It will be seen later that the DFL control law also plays the role to partially alleviate the nonlinearities of and interconnections among generators.

To simplify analysis, it is assumed that the DC link terminal voltage \( V_{\phi} \) is constant and neglecting the effect of the angular frequency variation \( \Delta \omega_l \) at the DC link terminal bus when considering the generator dynamics, i.e. \( V_{\phi}(t) = 0 \) and \( \Delta \omega_l(t) = 0 \).

Then, by differentiating (2.28) and (7.1), and using the equations (2.24)-(2.25), (2.28)-(2.32) and (7.2), following results can be obtained

\[
\dot{I}_q(t) = \sum_{j=1}^{n} \dot{E}_{\phi}^{j}(t) B_j \sin \delta_j(t) + \sum_{j=1}^{n} E_{\phi}^{j}(t) B_j \cos \delta_j(t) \left[ \Delta \omega_l(t) - \Delta \omega_j(t) \right] + \sum_{j=1}^{n} \dot{V}_{\phi}(t) B_j \sin \delta_j(t) \\
= \sum_{j=1}^{n} \dot{E}_{\phi}^{j}(t) B_j \sin \delta_j(t) - I_a(t) \Delta \omega_l(t) - \sum_{j=1}^{n} E_{\phi}^{j}(t) B_j \cos \delta_j(t) \Delta \omega_j(t) \\
= -\left[ I_a(t) + E_{\phi}^{i}(t) B_j \right] \Delta \omega_l(t) + \sum_{j=1}^{n} \dot{E}_{\phi}^{j}(t) B_j \sin \delta_j(t) \\
- \sum_{j=1}^{n} E_{\phi}^{j}(t) B_j \cos \delta_j(t) \Delta \omega_j(t) \tag{7.3}
\]

\[
\Delta \dot{P}_a(t) = \dot{E}_{\phi}^{i}(t) I_a(t) + E_{\phi}^{i}(t) \dot{I}_q(t) \\
= \dot{E}_{\phi}^{i}(t) I_a(t) - \left[ E_{\phi}^{i}(t) I_a(t) + E_{\phi}^{i}(t) B_j \right] \Delta \omega_l(t) + \sum_{j=1}^{n} E_{\phi}^{j}(t) \dot{E}_{\phi}^{j}(t) B_j \sin \delta_j(t) \\
- \sum_{j=1}^{n} E_{\phi}^{j}(t) \dot{E}_{\phi}^{j}(t) B_j \cos \delta_j(t) \Delta \omega_j(t) \\
= \frac{1}{T_{\phi}} \left[ \frac{k_{\phi} u_{\phi}(t) - E_{\phi}^{i}(t) - (x_{\phi} - x_{\phi}^{d}) I_a(t)}{Q_{\phi}(t) + E_{\phi}^{i}(t) B_j} \right] I_a(t) - \left[ Q_{\phi}(t) + E_{\phi}^{i}(t) B_j \right] \Delta \omega_l(t)
\]
Chapter 7 Robust Nonlinear Decentralized Control of Complex Power Systems

\[ + \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \sin \delta_{j}(t) - \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \cos \delta_{j}(t) \Delta \omega_{j}(t) \]

\[ = \frac{1}{T_{d0i}} \left[ k_{u}u_{\dot{\omega}}(t) - (x_{\dot{\delta}} - \dot{x}_{\dot{\delta}})I_{q}(t) \right] I_{q}(t) - \left[ Q_{q}(t) + E_{q}^{2}(t)B_{q} \right] \Delta \omega(t) - \frac{1}{T_{d0i}} P_{m} - \frac{1}{T_{d0i}} \Delta P_{m}(t) \]

\[ + \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \sin \delta_{j}(t) - \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \cos \delta_{j}(t) \Delta \omega_{j}(t) \]  

(7.4)

Let

\[ v_{\dot{\omega}}(t) = I_{q}(t)k_{u}u_{\dot{\omega}}(t) - (x_{\dot{\delta}} - \dot{x}_{\dot{\delta}})I_{q}(t)I_{q}(t) - P_{m} - T_{d0i} \left( Q_{q}(t) + E_{q}^{2}(t)B_{q} \right) \Delta \omega(t) \]

(7.5)

It gives that

\[ \Delta \dot{P}_{a}(t) = -\frac{1}{T_{d0i}} \Delta P_{a}(t) + \frac{1}{T_{d0i}} v_{\dot{\omega}}(t) + \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \sin \delta_{j}(t) \]

\[ - \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \cos \delta_{j}(t) \Delta \omega_{j}(t) \]  

(7.6)

From (7.5), the DFL compensating control law can be obtained

\[ u_{\dot{\omega}}(t) = \frac{1}{k_{u}I_{q}(t)} \left[ v_{\dot{\omega}}(t) + P_{m0i} + (x_{\dot{\delta}} - \dot{x}_{\dot{\delta}})I_{q}(t)I_{q}(t) + T_{d0i} \left( Q_{q}(t) + E_{q}^{2}(t)B_{q} \right) \Delta \omega(t) \right] \]

(7.7)

Note (7.7) has the same form as (5.4), thus the DFL compensating control law is practically realizable using only local measurements, see Section 5.2.1 for details.

So far, the multimachine power system generator model has been compensated into the following form

\[ \Delta \dot{\delta}(t) = \Delta \omega(t) \]

(7.8)

\[ \Delta \dot{\omega}(t) = -\frac{D}{2H_{i}} \Delta \omega(t) - \frac{\omega_{m}}{2H_{i}} \Delta P_{a}(t) \]

(7.9)

\[ \Delta \dot{P}_{a}(t) = -\frac{1}{T_{d0i}} \Delta P_{a}(t) + \frac{1}{T_{d0i}} v_{\dot{\omega}}(t) + \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \sin \delta_{j}(t) \]

\[ - \sum_{j=1,j \neq i}^{n} E_{qj}(t)E_{qj}(t)B_{j} \cos \delta_{j}(t) \Delta \omega_{j}(t) \]  

(7.10)

It can be seen from (7.8)-(7.10), the multimachine power system generator model does not include converter nodes any longer. Therefore, the DFL technique decouples the generators from HVDC links. After AC/DC power systems are decoupled, similar overlapping decomposition controllers proposed in last chapter can be designed for DC systems and decentralized controllers can be designed for AC systems that will be detailed in next section.
7.3 Global robust control of generator excitation systems

In Chapters 4 and 5, although the designed robust CTVRE control has the ability to enhance transient stability, the controller design involves estimating nonlinearity bounds within a certain operating region. The local characteristic of voltage regulation cannot guarantee transient stability in some operating conditions, e.g., the generator active power output is at very low value. On the other hand, TSE control is powerful in enhancing transient stability and it can work over almost the whole operating region under ideal conditions. To guarantee both transient stability and post-fault voltage regulation in large operating regions, it is reasonable to use the global robust control strategy to combine the advantages of both TSE control and robust CTVRE control.

As the TSE and CTVRE control designed in Chapter 5 are for pure multimachine AC system, appropriate modifications and transformations of these control strategies should be carried out for control of multimachine AC/DC systems. It can be seen from last section that the forms of (7.8)-(7.10) and the control law (7.7) are exactly the same as those in Section 5.2 for the TSE controller design. Therefore, the robust decentralized TSE controller design method can be applied here directly. For the robust decentralized CTVRE controller design, modifications and transformations have to be carried out.

By differentiating (2.33) and (7.2), and using the equations (2.24)-(2.25), (2.28)-(2.32), (7.1) and (7.3), following results can be obtained

\[
\dot{I}_i(t) = -\sum_{j=1}^{n} \dot{E}_{qj}(t) B_q \cos \delta(t) + \sum_{j=1}^{n} \dot{E}_{qj}(t) B_q \sin \delta(t) \left[ \Delta \omega_i(t) - \Delta \omega_j(t) \right] - \sum_{j=1}^{n} \dot{V}_p(t) B_q \cos \delta(t) \\
n \sum_{j=1}^{n} \dot{V}_p(t) B_q \sin \delta(t) \left[ \Delta \omega_i(t) - \Delta \omega_j(t) \right] \\
\approx \sum_{j=1}^{n} \dot{E}_{qj}(t) B_q \cos \delta_j(t) + I_{qj}(t) \Delta \omega_j(t) - \sum_{j=1}^{n} \dot{E}_{qj}(t) B_q \sin \delta_j(t) \Delta \omega_j(t) \\
= I_{qj}(t) \Delta \omega_j(t) - \dot{E}_{qj}(t) B_q \cos \delta_j(t) - \sum_{j=1}^{n} \dot{E}_{qj}(t) B_q \sin \delta_j(t) \Delta \omega_j(t) \\
(7.11)
\]
\[ V_n(t) = \frac{dx_n(t)}{dt} \left( \left( E_{\phi}(t) - x_{\psi}I_{\psi}(t) \right)^2 + \left( x_{a}I_{a}(t) \right)^2 \right) \]
\[ = \frac{V_{q}(t)}{V_n(t)} E_{\phi}(t) - \frac{V_{q}(t)}{V_n(t)} x_{\psi} I_{\psi}(t) + \frac{V_{a}(t)}{V_n(t)} x_{a} I_{a}(t) \]
\[ \approx \frac{V_{q}(t)}{V_n(t)} \left( 1 + x_{\psi} B_\psi \right) E_{\phi}(t) + \frac{V_{a}(t)}{V_n(t)} \sum_{j=1}^{n} \left( E_{\phi}(t)B_j \cos \delta_j(t) - E_{\phi}(t)B_j \sin \delta_j(t) \Delta \omega_j(t) \right) \]
\[ + \frac{V_{a}(t)}{V_n(t)} \sum_{j=1}^{n} \left( E_{\phi}(t)B_j \sin \delta_j(t) + E_{\phi}(t)B_j \cos \delta_j(t) \Delta \omega_j(t) \right) \]
\[ = \frac{V_{q}(t)}{V_n(t)} \left( 1 + x_{\psi} B_\psi \right) E_{\phi}(t) - \frac{x_{\psi}}{V_n(t)} \left[ V_{q}(t)I_{\psi}(t) + V_{a}(t)I_{a}(t) + V_{a}(t)E_{\phi}(t)B_\psi \right] \Delta \omega(t) \]
\[ + \frac{x_{\psi}}{V_n(t)} \sum_{j=1}^{n} E_{\phi}(t)B_j \left( V_{q}(t) \cos \delta_j(t) + V_{a}(t) \sin \delta_j(t) \right) \]
\[ + \frac{x_{\psi}}{V_n(t)} \sum_{j=1}^{n} E_{\phi}(t)E_{\phi}(t)B_j \left( V_{q}(t) \sin \delta_j(t) - V_{a}(t) \cos \delta_j(t) \right) \Delta \omega_j(t) \]

Notice, (7.12) has the same form as (5.14) and (7.10) has the same form as (5.3), so the techniques for the robust decentralized CTSVRE control design in Chapter 5 can also be applied here. Note there are two CTSVRE control laws corresponding to two different operating regions for each generator.

After the robust TSE control laws and robust CTSVRE control laws are obtained, the global robust control laws can then be obtained. As the TSE control is more useful during transient period and the CTSVRE control is more useful during the post-fault period, the coordinate control laws should use the TSE control during transient period and then transit to CTSVRE control during post-fault period. To make the switch action smooth, following trapezoid-shaped like member functions are used for each generator

\[ \lambda_\phi = 1 - \lambda_\delta \]

\[ \lambda_\psi = \left[ 1 - \frac{1}{1 + \exp(-120(z_i - 0.08))} \right] \left[ 1 + \exp(-120(z_i + 0.08)) \right] \]

where

\[ z_i = \sqrt{(\alpha_i \Delta \omega_i)^2 + (\alpha_i \Delta V_i)^2} \]
and \( \alpha_i \), \( \alpha_{2i} \) are positive constants providing appropriate scaling, which can be chosen according to different sensitivity requirement of power frequency and voltage.

The member functions partitioned the whole operating region into two subspaces with \( S_i \) indicating the transient period and \( S_{2i} \) indicating the post transient period

\[
\begin{align*}
S_i &= \{(\Delta \omega_i, \Delta V_i) | \lambda_i \leq \lambda_{si} \} \\
S_{2i} &= \{(\Delta \omega_i, \Delta V_i) | \lambda_i > \lambda_{si} \}
\end{align*}
\]

And the robust global exciter control law for each generator is

\[
v_i(t) = \lambda_{si} v_{f,i}(t) + \lambda_i \left( \lambda_{s1} v_{f,1}(t) + \lambda_{s2} v_{f,2}(t) \right)
\]

where

\( \lambda_{si} \) and \( \lambda_i \) are defined in last chapter. \( v_{f,i} \) and \( v_{f,si} \) are the TSE control law and CTSVRE control law respectively.

The global control has the following interpretation. Immediately after the fault is removed or isolated, the deviation of system states from the equilibrium is large. This period is the so-called transient period. During this period, the TSE control law plays the role to drive the system states to the neighborhood of equilibrium and the system goes into post-transient period. The global control law switches the control role from TSE controller to CTSVRE controller smoothly. The voltage is then regulated by the CTSVRE controller to the desired steady state value.

### 7.4 Simulation results

In this section, the system model as shown in Figure 7.1 will be used to test the effectiveness of the proposed control strategy. The example system is the same as that shown in Figure 5.10 except that the AC transmission line 8-11 is substituted by a two-terminal HVDC link here. The example system includes a five-machine twelve-bus system without infinite bus and one HVDC link. Bus 1 is used as the
reference bus. The DC transmission link model is a two-terminal one with rectifier at bus 8 and inverter at bus 11. Shunt capacitors at both the rectifier and inverter ends are used to provide the main part of reactive power support.

Figure 7.1  Five-machine twelve-bus AC/DC power system example

Parameters for the DC link are the same as those given in Section 6.4. Generating units and transmission lines parameters are the same as those given in Table 5.2 and Table 5.3 except that the transmission line 8-11 is substituted by an HVDC link. The rated frequency of the example system is 50Hz. It can be seen that the inertial constant differences between the generators are not very large. Thus it can be regarded as a small area AC/DC power system.

For illustration purpose, we assume \( |P_e(t)|_{\text{max}} = 1.4 \) and consider the AC system parametric perturbation as \( |\Delta T_{d0}|_{\text{max}} = 0.1T_{d0} \quad (i = 1, 2, \ldots, 5) \) and the DC system parametric perturbation as \( |\Delta T_i|_{\text{max}} = 0.1T_i \), \( |\Delta T_d|_{\text{max}} = 0.1T_d \), \( |\Delta V_r|_{\text{max}} = 0.2V_{r0} \), \( |\Delta V_i|_{\text{max}} = 0.2V_{i0} \), \( |\Delta k_r|_{\text{max}} = 0.2k_{r0} \), where \( V_{r0} \), \( V_{i0} \) and \( k_{r0} \) are the nominal value.

The following physical limit of the excitation voltage is considered:
The obtained HVDC overlapping decomposition control laws are the same as those in Section 6.4.1. The obtained robust decentralized TSE and CTSVRE control laws for the generator excitation systems are the same as those in Section 5.4. The global robust control laws are (7.15) and the original excitation control laws are (7.7).

The following two operating conditions will be considered:

Operating Condition 7.1

Table 7.1 Generator operating condition 7.1 in p.u. except $\alpha_i$ in rad (100MVA base)

<table>
<thead>
<tr>
<th></th>
<th>Gen#1</th>
<th>Gen#2</th>
<th>Gen#3</th>
<th>Gen#4</th>
<th>Gen#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>5.309</td>
<td>8.000</td>
<td>4.500</td>
<td>7.000</td>
<td>5.000</td>
</tr>
<tr>
<td>$Q$</td>
<td>3.921</td>
<td>2.355</td>
<td>1.853</td>
<td>2.089</td>
<td>4.779</td>
</tr>
<tr>
<td>$v_i$</td>
<td>1.05</td>
<td>1.02</td>
<td>1.05</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.000</td>
<td>0.561</td>
<td>0.025</td>
<td>0.055</td>
<td>-0.026</td>
</tr>
</tbody>
</table>

HVDC link

$P_{e0} = 500$ MW, $\alpha_o = 18^\circ$, $\gamma_o = 15^\circ$. Shunt capacitor banks provide 200MVAr reactive power at the rectifier and 200MVAr reactive power at the inverter.

Electrical loads in p.u. are as follows:

$L_1 = 6 + j2$ ; $L_2 = 18 + j3$ ; $L_3 = 5 + j1$

Operating Condition 7.2

Table 7.2 Generator operating condition 7.2 in p.u. except $\alpha_i$ in rad (100MVA base)

<table>
<thead>
<tr>
<th></th>
<th>Gen#1</th>
<th>Gen#2</th>
<th>Gen#3</th>
<th>Gen#4</th>
<th>Gen#5</th>
</tr>
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<tbody>
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<td>8.000</td>
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<tr>
<td>$Q$</td>
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<td>1.414</td>
<td>4.427</td>
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</tr>
<tr>
<td>$v_i$</td>
<td>1.00</td>
<td>1.05</td>
<td>1.10</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.000</td>
<td>0.106</td>
<td>0.004</td>
<td>0.036</td>
<td>0.160</td>
</tr>
</tbody>
</table>
HVDC link

\[ P_{dc0} = 700 \text{ MW}, \quad \alpha_0 = 18^\circ, \quad \gamma_0 = 15^\circ. \]

Shunt capacitor banks provide 300MVAr reactive power at the rectifier and 400MVAr reactive power at the inverter.

Electrical loads in \( p.u. \) are as follows.

\[ L_1 = 15 + j 5; \quad L_2 = 8 + j 3; \quad L_3 = 6 + j 1 \]

The fault considered here is a symmetric three-phase short circuit fault. Two fault locations are tested, one occurs on the transmission line 6-9 near bus 9 (Fault Location 7.1), and the other on the transmission line 9-10 near bus 10 (Fault Location 7.2).

The effectiveness of the proposed robust coordinated control strategy will be evaluated under different operating conditions, fault locations and fault sequences. System responses with two control strategies will be compared in one simulation case, they are: (a) proposed control for generator and DC link at CC and CEA control modes (conventional PI control is used for other control modes of DC link); (b) conventional AVR and PSS with conventional PI control for DC link.

The following three cases are considered to evaluate the proposed control strategy:

Case 7.1:
Operating Condition 7.1, Fault Location 7.1 and Fault Sequence 4.2. Simulation results are given in Figure 7.2 - Figure 7.3.

Case 7.2:
Operating Condition 7.1, Fault Location 7.2 and Fault Sequence 4.1. Simulation results are given in Figure 7.4.

Case 7.3:
Operating Condition 7.2, Fault Location 7.1 and Fault Sequence 4.2. Simulation results are given in Figure 7.5.

For Case 7.1, relative speed and terminal voltage responses under the two control
strategies are given in Figure 7.2 - Figure 7.3 respectively. By comparing these responses, it can be seen that the proposed control strategy (a) can stabilize the system and achieve satisfactory post-fault voltage with shorter time. The conventional control

Figure 7.2  Relative power angle, active power, relative speed and terminal voltage responses for Case 7.1 with control strategy (a)

Figure 7.3  Relative speed and terminal voltage responses for Case 7.1 with control strategy (b)
strategy (b) cost longer time to reach the post-fault steady state and the performance is not as good as the global robust controller. Figure 7.2 also shows the relative power angle and active power responses, the performance for both responses are satisfactory.

For Cases 7.2 and 7.3, the proposed control strategy (a) is tested under different operating conditions, fault locations and fault sequences. It can be seen from Figure 7.4 - Figure 7.5, in both cases the robust nonlinear decentralized control can not only...
stabilize the system but also achieve satisfactory post-fault voltage. Note that the parameters of the proposed control strategy are constants and do not change for both cases, thus the control strategy is a robust one. This illustrates the robustness of the proposed controller.

### 7.5 Concluding remark

In this chapter, robust nonlinear decentralized control strategy is proposed for complex power systems. Control techniques proposed in previous chapters are integrated into this control strategy. The control techniques used include DFL technique, overlapping decomposition technique, global control theory and robust control theory. The proposed controller consists of three parts: a global robust decentralized controller for each AC power system generator excitation system and two overlapping decomposition controllers for each HVDC link. All of the controllers are coordinated to ensure the power system stability. Numerical simulations are carried out on an example system. This example system is a five-machine twelve-bus multimachine AC/DC power system without infinite bus. Simulation results illustrate the effectiveness and robustness of the proposed control scheme.
Chapter 8 Robust Constrained Control of Power Systems

8.1 Introduction

In real world, every physical system is subject to input constraints due to physical limit of control actuator. While in some situations the control vectors are required to be designed such that the input constraints are never encountered, in other cases the control vectors are desired to be close to their maximal allowable values to achieve good performance. By recognizing this fact, there has been renewed interest in the study of systems subject to input saturation especially during the last two decades.

Optimal control method seems to be an ideal candidate to solve such problems [141], but the obtaining of control laws and subsequent implementations are not easy jobs, not to mention the necessity to store intricate switching surfaces [56, 142]. Low gain linear state feedback control is employed in [56, 57] in such a way that the control constraints are never violated as the controlled error converges towards the origin for a \textit{a priori} given bounded set of initial conditions. While the low gain linear state feedback may guarantee a large region of stability, the performance may not be satisfied. This is because of the characteristic of linear control, which is when the error decreases the control vector also decreases, and thus the control capacity is not fully employed especially for relatively low errors.

Various methods have been proposed to improve system performance under constrained control. In [56], another linear state feedback control law is added to the original low gain feedback control law. The final control law is thus a composite one, the low gain state feedback control law is used to guarantee system stability that mainly takes effect when the state is far from the equilibrium and the other linear
feedback law is used to improve system performance which mainly takes effect when the state is close to the equilibrium. Following this thread, the so-called low and high gain approach is proposed in [60, 61] to improve the performance further. Another reasonable control method is to increase the linear state feedback gain as the state approaches the equilibrium. A piecewise-linear control (PLC) law based on linear quadratic theory is proposed in [142], where the state space are partitioned into a number of nested positively invariant sets and the controller gain in a gains bank switches to a higher value one as the trajectory enter into a smaller size set. This method is a logic-based switching one and the controller gains change discontinuously. The technique of combining low gain control and gain scheduling control is proposed in [58, 62], where few switching surfaces are needed. More recently, composite nonlinear feedback control is proposed in [63, 64] where nonlinear feedback is subtly combined with low gain linear feedback control and switching element is no longer necessary.

In contrast with the development of control theory regarding input constraints for quite some time, surprisingly, few researchers incorporate the control input constraints into power system controller design [8, 10], although various advanced nonlinear control approaches have been proposed for power systems [6, 33, 34, 36, 39, 40, 50, 51]. For most controller design in power systems, the control signals are calculated as if no constraints exist, and then they are simply limited in order to cope with the reality. The validity of the saturation of the controller is assumed to be as a matter of course. However, this may not be always the case. As is pointed out in [56], even though the original (unconstrained) control yields a stable closed loop system, stability cannot be guaranteed in general.

The objective of this chapter is to design controllers for excitation systems that fully take into account the input constraints in the design period. As for power systems the
initial states are impossible to be infinitely large immediately after fault removal if the post-fault system is able to regain stability, only local stability is considered in this work. By combining the recent results on constrained control, namely, low and high gain linear feedback control and nonlinear feedback control and also considering the system uncertainties in both A-matrix and B-matrix, a theorem for robust constrained control that is based on general multimachine power systems is proposed. By using nonlinear direct feedback control \[34, 50, 51\], multimachine power system model is partially linearized and then the theorem can be employed for designing constrained controllers with satisfactory performance. Although the theorem is based on multimachine power system model, it can easily be applied to SMIB power systems, which is straightforward. This research is a preliminary one in that a special form of the uncertain input matrix deviation \( \Delta B_i(t) = k_{m_i}(t)B_i \) is assumed and the input saturation is assumed to be symmetric.

The organization of this chapter is as follows: a robust constrained control theorem is proposed and proved at first, followed by a brief introduction of its application to multimachine and SMIB power systems. Next, aspects of gain selection are discussed. Finally, numeric simulation results are given for both SMIB and multimachine power systems.

### 8.2 Robust constrained control of power systems

In this section, robust constrained control method for a class of uncertain systems with input saturation is proposed at first and the controller gain selection is discussed next, followed by a brief introduction of its application into power systems. This method tries to combine the advantages of low and high gain control, nonlinear feedback control together and also to deal with uncertainties both in A-matrix and B-matrix of the system model.
8.2.1 Robust constrained controller design

Consider a general uncertain interconnected system composed of \( n \) interconnected uncertain subsystems \( \sum_i \) \( \sum_{i=1}^n \) with saturation. Each \( \sum_i \) is described by

\[
\dot{x}_i(t) = [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]sat(u_i(t)) + \sum_{k=1}^n \sum_{j=1}^n \sum_{s_{ij}} \mu_{ij} G_{ij} + \Delta G_{ij}(t) g_{ij}(t)
\]

\[x_i(0) = x_{i0}\]

The meaning of the symbols can be found in Sections 3.3 and 5.2. In addition, only single input is considered, that is \( u_i(t) \in R \). The saturation function \( sat(u_i(t)) \) is defined as

\[sat(u_i(t)) = \text{sign}(u_i(t)) \min(u_i(t), u_{i,\text{max}})\]

where \( u_{i,\text{max}} \) is the maximum amplitude of the control input \( u_i(t) \).

Before introducing the theorem for the robust constrained control method, the following three assumptions are made first:

(a) \((A_i, B_i)\) is stabilizable.

(b) \( [\Delta A_i(t) \quad \Delta B_i(t)] = D_i F_i(t) \begin{bmatrix} E_{i1} & E_{i2} \end{bmatrix} \)

\[\Delta G_{ij}(t) = M_{ij} F_i(t) N_{ij} \]

\[
\|g_{ij}(x_i, x_j)\| \leq \|W_{i1} x_i(t)\| + \|W_{i2} x_j(t)\|
\]

\[F_i^T(t) F_i(t) \leq I, \quad F_{ij}(t) F_{ij}(t) \leq I_{ij}\]

The meaning of the symbols can be found in Sections 3.3 and 5.2.

(c) \( \Delta B_i(t) = k_{ui}(t) B_i \),

where \( k_{ui}(t) \) is a single value function satisfying \( \|k_{ui}(t)\| < 1 \).

This assumption is another constraint of the form of the input matrix uncertainties \( \Delta B_i(t) \) in addition to that in assumption (b). The reason of using both assumptions is to employ the results of previous chapters. The special requirement of \( \Delta B_i(t) \) may be relaxed in the future work.
The main results on the problem of robust constrained control are stated as follows:

**Theorem 8.1:** Under the assumptions (a) – (c), suppose there exist some real positive scalars \( \varepsilon_i, \gamma_{ij}, \gamma_{2ij} \) and real constant matrices \( X_i = X_i^T > 0 \) and \( Y_i, i, j = 1, 2, \ldots, n \), such that LMI (8.7) holds, then given an initial vector \( x_0 \) and let \( \rho_i \) be the largest positive scalar such that for all \( \{x(t) : x_i^T(t) P_i x_i(t) \leq \rho_i, P_i = X_i^{-1}\} \) the linear state feedback control law (8.8) satisfies the inequality (8.9), provided \( x_0 \in G_i \), the robust constrained control law given by (8.11) - the composite feedback law of the linear state feedback control law (8.8) and the extra feedback control law (8.10), can asymptotically stabilize the uncertain system (8.1), and the set \( \bigcup_{i=1}^n G_i \) is a positively invariant set.

\[
\begin{bmatrix}
\Phi_i & X_i Q_i^{1/2} & X_i E_i^T + Y_i^T E_{2i} & G_{li} & 0 & G_{2li} & 0 \\
Q_i^{1/2} X_i & -I & 0 & 0 & 0 & 0 & 0 \\
E_i X_i + E_2 Y_i & 0 & -\varepsilon_i I & 0 & 0 & 0 & 0 \\
G_{li} & 0 & 0 & -I & N_{li}^T & 0 & 0 \\
0 & 0 & 0 & N_{li} & -\Gamma_{li} & 0 & 0 \\
G_{2li} & 0 & 0 & 0 & 0 & -I & N_{2li}^T \\
0 & 0 & 0 & 0 & 0 & N_{2li} & -\Gamma_{2li} \\
\end{bmatrix}
< 0 \quad (8.7)
\]

where

\[
\Phi_i = A_i X_i + X_i A_i + B_i Y_i + Y_i^T B_i^T + \varepsilon_i D_i D_i^T + M_{li} \Gamma_{li} + M_{li}^T \Gamma_{li} + M_{2li} \Gamma_{2li} + M_{2li}^T \Gamma_{2li}
\]

\[
Q_i = \sum_{i=1}^n \sum_{j=1, j \neq i}^n p_{ij} \left( W_{i}^T W_i + W_{j}^T W_j \right)
\]

\( i, j = 1, 2, \ldots, n \)

\( G_{li}, M_{li}, N_{li}, \Gamma_{li} \) and \( \Gamma_{kli} \) (\( i = 1, 2, \ldots, n, k = 1, 2 \)) are matrices of the same form and meaning as those given in (5.19).

\[
u_{ui}(t) = K_i x_i(t) \quad (8.8)
\]

where \( K_i = Y_i X_i^{-1} \).

\[
|u_{ui}(t)| = |K_i x_i(t)| \leq u_{\text{max}} \quad (8.9)
\]

\[
u_{uj}(t) = k_{ji} B_j^T P_j x_j(t) \quad (8.10)
\]

where \( k_{ji} \) is a nonpositive constant scalar or single value function.
Chapter 8 Robust Constrained Control of Power Systems

\[ u_i(t) = u_{i1}(t) + u_{i2}(t) \]  \hspace{1cm} (8.11)

Proof:

The equation of each subsystem \( \sum_i \) of (8.1) can be rewritten as

\[ \dot{x}_i(t) = \left[ A_i + A_i \Delta A_i(t) \right] x_i(t) + \left[ B_i + \Delta B_i(t) \right] u_{i1}(t) + \sum_{k=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} p_{ij} \left[ G_{ij} + \Delta G_{ij}(t) \right] g_{ij}(t) + \left[ B_i + \Delta B_i(t) \right] \left[ \text{sat}(u_i(t)) - u_{i1}(t) \right] \]

Denoting two series of functions \( f_{i1}(t) \) and \( f_{i2}(t) \) \((i = 1, 2, \ldots, n)\) as follows

\[ f_{i1}(t) = \left[ A_i + A_i \Delta A_i(t) \right] x_i(t) + \left[ B_i + \Delta B_i(t) \right] u_{i1}(t) + \sum_{k=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} p_{ij} \left[ G_{ij} + \Delta G_{ij}(t) \right] g_{ij}(t) \]

\[ f_{i2}(t) = \left[ B_i + \Delta B_i(t) \right] \left[ \text{sat}(u_i(t)) - u_{i1}(t) \right] \]

Then there is

\[ \dot{x}_i(t) = f_{i1}(t) + f_{i2}(t) \]

Define a Laypunov function \( V = \sum_{i=1}^{n} x_i^T(t) P_i x_i(t) \), \( P_i = P_i > 0 \), then the time derivative of \( V(\cdot) \) along any trajectory of the closed-loop system of (8.1) can be calculated as

\[ \dot{V} = \sum_{i=1}^{n} 2x_i^T(t) P_i f_{i1}(t) + \sum_{i=1}^{n} 2x_i^T(t) P_i f_{i2}(t) \]

It can be seen the form of \( f_{i1}(t) \) is similar to the right side of (5.6) in Section 5.2.

Following the method in [47] with appropriate modifications, it is easy to prove that when LMI (8.7) holds, \( \sum_{i=1}^{n} 2x_i^T(t) P_i f_{i1}(t) < 0 \). Therefore, in the following part \( \sum_{i=1}^{n} 2x_i^T(t) P_i f_{i2}(t) \leq 0 \) will be proved.

Substituting (8.6), (8.8) and (8.11) into \( \sum_{i=1}^{n} 2x_i^T(t) P_i f_{i2}(t) \), it gives

\[ \sum_{i=1}^{n} 2x_i^T(t) P_i f_{i2}(t) = \sum_{i=1}^{n} 2x_i^T(t) P_i \left[ B_i + \Delta B_i(t) \right] \left[ \text{sat}(u_i(t)) - u_{i1}(t) \right] \]

\[ = \sum_{i=1}^{n} 2\left(1 + k_{ij}(t)\right)x_i^T(t) P_i B_i \left[ \text{sat}(K_i x_i(t) + u_{i1}(t)) - K_i x_i(t) \right] \]

To further analyze the characteristic of the above equation, the property that linear control law (8.8) satisfies the inequality (8.9) will be employed. Observing the possible situations for the control input, there are three situations totally as follows.
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(a): The control input is unsaturated, there is
\[ \text{sat}(K, x_i(t) + u_{Ei}(t)) = K, x_i(t) + u_{Ei}(t) \]
then \[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) = u_{Ei}(t) = k_{Ei} B_i^T P_i x_i, \text{ thus} \]
\[ x_i^T (t) P_i B_i \left[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) \right] = k_{Ei} x_i^T (t) P_i B_i^T P_i x_i \leq 0 \]
(8.12)

(b): The control input is exceeding the upper limit, there is
\[ \text{sat}(K, x_i(t) + u_{Ei}(t)) = u_{\text{max}} \cdot \]
Since \[ -u_{\text{max}} < K, x_i(t) \leq u_{\text{max}} \], there is \[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) = u_{\text{max}} - K, x_i(t) \geq 0 \] and
\[ u_{Ei}(t) = k_{Ei} B_i^T P_i x_i \geq 0 \], which implies \[ B_i^T P_i x_i \leq 0 \]. Thus
\[ x_i^T (t) P_i B_i \left[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) \right] = x_i^T (t) P_i B_i (u_{\text{max}} - K, x_i(t)) \leq 0 \]
(8.13)

(c): The control input is exceeding the lower limit, there is
\[ \text{sat}(K, x_i(t) + u_{Ei}(t)) = -u_{\text{max}} \]
Since \[ -u_{\text{max}} < K, x_i(t) \leq u_{\text{max}} \], there is \[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) = -u_{\text{max}} - K, x_i(t) \leq 0 \] and
\[ u_{Ei}(t) = k_{Ei} B_i^T P_i x_i \leq 0 \], which implies \[ B_i^T P_i x_i \geq 0 \]. Thus
\[ x_i^T (t) P_i B_i \left[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) \right] = x_i^T (t) P_i B_i (-u_{\text{max}} - K, x_i(t)) \leq 0 \]
(8.14)

It can be seen from the above results (8.12) - (8.14) and the fact \[ 1 + k_{Bi}(t) > 0 \] that
\[ (1 + k_{Bi}(t)) x_i^T (t) P_i B_i \left[ \text{sat}(K, x_i(t) + u_{Ei}(t)) - K, x_i(t) \right] \leq 0 \] for all three situations, therefore
\[ \sum_{i=1}^{n} 2 x_i^T (t) P_{fi_i}(t) \leq 0, \text{ then } \dot{V} < 0. \] This means with the robust constrained control law (8.11), the closed-loop system is quadratically asymptotically stable.

Since in the set \[ \bigcup_{j=1}^{n} G_j \], the Lyapunov function \[ 0 < V \leq \sum_{i=1}^{n} \rho_i \] and \[ \dot{V} < 0 \], thus the set \[ \bigcup_{j=1}^{n} G_j \] is a positively invariant set for system (8.1) with the robust constrained control law (8.11), i.e., for any initial condition \[ x_{i0} = x_i(t_0) \] such that \[ \sum_{i=1}^{n} x_i^T (t_0) P_{fi_i} x_i \leq \sum_{i=1}^{n} \rho_i \], then for all \[ t \geq t_0 \] there is \[ \sum_{i=1}^{n} x_i^T P_i X_i \leq \sum_{i=1}^{n} \rho_i \], where \[ x_i(t) \] is the solution of (8.1) with the robust constrained control law (8.11).
8.2.2 Controller gain selection

In this subsection, considerations of gain selection will be discussed. The stability region and the system performance depends on the how to select the controller gain. By observing the proof of Theorem 8.1, low gain linear feedback control, high gain linear feedback control, nonlinear feedback control and the stability region are discussed below.

8.2.2.1 Low gain linear feedback control

It can be seen from the proof of Theorem 8.1, the linear feedback control law satisfies $|K_i x_i(t)| \leq u_{\text{max}}$. Usually the gain value of such a linear controller is relatively very low, therefore this part is called 'low gain' control. The major role of this low gain control is used to achieve some satisfied stability region. Under ideal condition, for stable or quasi-stable system, the stability region can cover the whole state space $R^n$ by letting $K_i \to 0$; for unstable plants, the ellipsoids tend to a limiting ellipsoid [142]. In practice, since every physical system has certain nonlinearities, this gain may not be infinitely low because of the so-called 'slow peaking' phenomenon [143]. In addition, if only local stability is of interest, the larger this gain uses the better the performance achieves. This low gain $K_i$ can be obtained by solving the LMI (8.7) using LMI toolbox. After this $K_i$ is obtained, it should be checked if the property (8.9) is satisfied for some $x_i(t)$ in the interested state space. If the property is not satisfied, a simple method is to multiply the positive symmetric matrix $P_i$ by a positive scalar $r_i$ and adjust the value of $r_i$ until the property is satisfied. It is easy to verify from the proof of Theorem 8.1 that multiplying the positive symmetric matrix $P_i$ by a scalar $r_i$ will not influence the system stability while it influences the system performance.

8.2.2.2 Extra state feedback control

From the proof of Theorem 8.1, it can be seen while the extra feedback control does not affect the stability of the uncertain system it does affect the system performance. By observing (8.12), it can be seen that the negative value of the derivative of the
Lyapunov decreases if a higher value of $|k_{Ei}|$ is chosen when $|K_i x_i(t)| \leq u_{\text{max}}$. This means the energy related to the Lyapunov function dissipates faster with a higher value of $|k_{Ei}|$, thus appropriate $k_{Ei}$ can be selected to improve system performance.

1) High gain linear feedback control
The first simple choice of $k_{Ei}$ is a constant scalar denoted by $k_{ELi}$. This feedback control law is an extra linear feedback one, which is the so called 'high gain' feedback control in [60, 61]. While increasing the value of $|k_{ELi}|$ provides good utilization of the available actuator authority and speeds up the transient response, it can not be increased arbitrarily since both the degree of disturbance rejection and the tolerance to actuator inaccuracies are decreased as feedback gain increases [58, 65]. Appropriately chosen high value of $k_{ELi}$ will improve system performance during the period when the controlled error is still large.

2) Nonlinear feedback control
The second choice of $k_{Ei}$ is a nonlinear function denoted by $k_{ENi}$. As is well known, one of the characteristic of linear state feedback is that the control input is proportional to the controlled error. Therefore, when the error decreases to some degree the control capacity will not be well utilized. Thus the nonlinear feedback control is used which mainly takes action when the control input is far away from its saturation level.

One choice of $k_{ENi}$ is as follows [64] if the initial value $x_i(0)$ is available

$$k_{ENi} = \beta \left| e^{-\alpha_i |x_i(0)|} - e^{-\alpha_i |x_i(0)|} \right|,$$

(8.15)

where $\alpha_i$ and $\beta_i$ are constant scalars and $\alpha_i > 0$, $\beta_i < 0$. This $k_{ENi}$ starts from zero and gradually decreases to a final gain of $k_{ENi} = \beta_i \left| 1 - e^{-\alpha_i |x_i(0)|} \right|$ as $x_i(t)$ approaches to zero. $\alpha_i$ is used to determine the speed of change of $k_{ENi}$. The designer could properly select the values of $\alpha_i$ and $\beta_i$ to yield desired performance.
As yet, the composite feedback control law can be written as
\[ u_i(t) = u_{iL}(t) + u_{iE}(t) = Y_iX_i^{-1}x_i(t) + (k_{LL} + k_{LE})B_i^TP_i x_i(t) \]  
(8.16)

which consists of a linear feedback law and a nonlinear feedback law without any switching element.

**8.2.2.3 Stability region**

The stability region is the positively invariant set \( \bigcup_{i=1}^{n} G_i \), it needs to determine the largest \( \rho_i \). Since the condition \( |K_i x_i(t)| \leq u_{\text{max}} \) is to be satisfied for all \( x_i(t) \in G_i \), \( \rho_i \) should be chosen such that the hyperplanes \( K_i x_i(t) = \pm u_{\text{max}} \) are tangent to the ellipse of \( G_i \). In order to determine \( \rho_i \), solve the optimization problem maximize \( x_i^T(t)P_i x_i(t) \) subject to the restrictions \( |K_i x_i(t)| \leq u_{\text{max}} \), whose solution yields the value sought
\[ \rho_i \leq \frac{u_{\text{max}}^2}{B_i^TP_i B_i} \]  
(8.17)

As ellipsoidal invariant sets are employed here, the obtained stability region may be conservative. To make the stability region less conservative, polyhedral invariant sets [144] may be used in the future research.

**8.2.3 Applications to power systems**

For multimachine power systems, when input saturation is not considered, by employing DFL technique and considering parametric uncertainties, the \( i \)th nonlinear generalized generator model can be expressed into the following state space form:
\[ \dot{x}_i(t) = [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]v_{iT}(t) + \sum_{j=1}^{n} \sum_{j 
eq i} \{p_{ij} \Delta G_{ij}(t)\} g_{ij}(t) \]  
(8.18)

The relationship between the interim input \( v_{iT}(t) \) and the original input \( u_{i}(t) \) is as follows
\[ k_{i}I_{i}(t)u_{i}(t) = v_{i}(t) + P_{a0i} + (x_{a} - x_{a})I_{i}(t)I_{i}(t) + T_{a0i} Q_{ai}(t) + E_{a}^T(t)B_{ai} \]  
(8.19)

The meaning of symbols and details can be found in Section 5.2.

Similar to the illustrated example in Section 5.2, the parametric uncertainty in \( T_{a0i} \) as
$\Delta T_{\omega_0}$ is considered. For power systems, assumption (a) is always true and is quite easy to verify. Assumption (b) also appears in Section 5.2. For assumption (c) $\Delta B_i(t) = k_i(t)B_i$, it can be observed from Section 5.2 that such a variable $k_i(t)$ for power systems does exist and satisfies $|k_i(t)| < 1$.

It can be seen from (8.19) that the relationship between $v_p(t)$ and $u_p(t)$ is nonlinear and is affected by variables of $I_{q_p}(t)$, $I_{d_p}(t)$, $Q_{q_p}(t)$, $\Delta \omega_i(t)$ and $E'_q(t)$. Usually during dynamic process, the term $v_p(t)$ dominates the right side of (8.19) and the waveforms of $v_p(t)$ and $u_p(t)$ are similar although the magnitudes are different due to the influence of $I_{q_p}(t)$. Therefore, to simplify analysis the other items at the right side of (8.19) can be ignored during dynamic process, there is ($k_i$ is normally given value 1):

$$v_p(t) \approx I_{q_p}(t)u_p(t)$$  \hspace{1cm} (8.20)

In practice, there is a physical limit for the excitation voltage. For static exciter, this is usually in the range of $[-3, 6]$ p.u., that is $u_p(t) \in [-3, 6]$ p.u.. This does not satisfy the definition of saturation function (8.2), where symmetric maximum and minimum values of input function are assumed. While more general unsymmetric maximum and minimum values of input function can be studied in the future, for simplicity here it is assumed that the maximum and minimum limits of $u_p(t)$ are symmetric. This will influence the obtained parameters of low gain linear feedback controller to some extent but not significantly. Then the symbol of $sat(u_p(t))$ can be used. In order to employ the constrained controller design results of above sections, saturation function of $sat(v_p(t))$ instead of $sat(u_p(t))$ should be used. By observing (8.20), it can be seen that
when the value of $I_w(t)$ is very low (corresponding to the period immediately after the fault is removed), a very low value of $v_p(t)$ can make $u_p(t)$ reach its limit. When the value of $I_w(t)$ is high (corresponding to the period when the transient process almost finishes), a relatively high value of $v_p(t)$ is allowed to make $u_p(t)$ reach its limit. Therefore, the limit of $v_p(t)$ varies from very low value immediately after the fault is removed to relatively high value when the system almost approaches steady state. Correspondingly, for each $I_w(t)$ there is a group of controller gains and the values of these controller gains should be very low at first and can be larger when the system approaches steady state. This makes the problem very complex. In fact, as constant gains are desired for the low gain linear feedback control and local stability is permitted, a tradeoff for this situation is to choose only one group of appropriate limits for $v_p(t)$. It will be shown in Section 8.3, when the limits of $v_p(t)$ are chosen the same as those of $u_p(t)$, the obtained low gain linear controller parameters are able to guarantee the power systems responses to some extent. If this group of gains is not satisfactory, they can be adjusted using the method in Section 8.2.2.1.

Although the above constrained controller design method is based on multimachine power systems, it is also suitable for SMIB power systems research by simply ignoring the corresponding interaction part of the related equations.

### 8.3 Simulation results

The effectiveness of the proposed control strategy will be evaluated on both SMIB and multimachine power systems.
Chapter 8 Robust Constrained Control of Power Systems

8.3.1 Robust constrained control of SMIB power systems

The example system used is shown in Figure 2.1. The proposed control will be evaluated under different operating conditions, fault locations and network parameters. System responses with the proposed control and with nonlinear feedback linearization control without considering input constraints will be compared in one simulation case. The generator parameters of the example system are the same as those given in Section 4.4.

By assuming \( |\Delta T_{\delta 0}| \leq 0.1 T_{\delta 0}, \quad \delta_f (t) \in [-3, 6] p.u. \) and solving the corresponding LMI, the low gain linear feedback control law is obtained as

\[
u_L(t) = 1.45 \Delta \delta(t) + 1.12 \Delta \omega(t) - 4.94 \Delta P_e(t)
\]

It can be seen even for a high initial value vector \( x_0^T = [2 \pi/3 \ 6 \ 1] \), \( K_x = 4.79 < v_{f,\text{max}} \).

By choosing \( K_{EL} = -20 \), \( \alpha = 0.5 \), and \( \beta = -25 \), the extra feedback control law can be written as

\[
u_E(t) = \left[ 20 + 25 e^{-a \frac{\pi}{3}} - e^{-a \frac{2 \pi}{3}} \right] \left[ 2.67 \Delta \delta(t) + 2.05 \Delta \omega(t) - 9.13 \Delta P_e(t) \right]
\]

Then the robust constrained composite control law is

\[
u_f(t) = u_L(t) + u_E(t)
\]

The nonlinear feedback control law without considering control input constraints is the robust TSE control law discussed in Section 4.2. This control law is the same as that in Section 4.4, which is

\[
u_f(t) = 25.85 \Delta \delta(t) + 23.19 \Delta \omega(t) - 111.15 \Delta P_e(t)
\]

The actual control law \( u_f(t) \) can be obtained from the relationship between \( v_f(t) \) and \( u_f(t) \).
Chapter 8 Robust Constrained Control of Power Systems

The fault considered here is a symmetrical three-phase temporary short circuit fault and the fault sequence is Fault Sequence 4.1. The following three cases are used to evaluate the proposed control strategy:

Case 8.1:

In this case, the fault occurs at one of the parallel AC transmission lines near bus 2. The operating point is $\delta_0 = 50.10^\circ$, $V_{f0} = 1.10\, \text{p.u.}$, $P_{e0} = 1.0\, \text{p.u.}$. Parameters for each transmission line are $r_i = 0.03*3\, \text{p.u.}$; $x_i = 0.24265*3\, \text{p.u.}$. Power angle, relative speed, terminal voltage and active power responses of the proposed controller and the TSE controller are shown in Figure 8.1. Exciter input responses are shown in Figure 8.2.

![Figure 8.1](image-url)  

Figure 8.1  Power angle, relative speed, terminal voltage and active power responses for Case 8.1 with different control strategies
Figure 8.2 Exciter input for Case 8.1 with different control strategies

Figure 8.3 Power angle, relative speed, terminal voltage and active power responses for Case 8.2 with proposed control
Case 8.2:
In this case, transmission line parameters are the same as Case 8.1. The fault occurs at the middle of one of the parallel AC transmission lines. The operating point is $\delta_0 = 35.94^\circ$, $V_{in} = 1.10 \text{ p.u.}$, $P_{e0} = 0.70 \text{ p.u.}$. Power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in Figure 8.3.

Case 8.3:
In this case, the operating point is $\delta_0 = 42.00^\circ$, $V_{in} = 1.02 \text{ p.u.}$, $P_{e0} = 0.80 \text{ p.u.}$. The fault location is the same as in Case 8.1. Parameters for each transmission line are $r_l = 0.03 \times 2 \text{ p.u.}$; $x_l = 0.24265 \times 2 \text{ p.u.}$. Relative Power angle, relative speed, terminal voltage and active power responses of the proposed controller are shown in Figure 8.4.

Figure 8.4  Power angle, relative speed, terminal voltage and active power responses for Case 8.3 with proposed control
Chapter 8 Robust Constrained Control of Power Systems

To illustrate the effectiveness of the proposed control strategy, Case 8.1 compares four system responses of the proposed control with those of the TSE control. It can be seen from the four responses shown in Figure 8.1, although the performance of TSE is not poor, the responses setting time is longer and the variation magnitude is larger. The proposed control strategy has better performance.

The exciter input responses of the two control strategies are also compared and shown in Figure 8.2. By observing Figure 8.2 it can be seen that before the transmission line is restored at 1.2s, the exciter input gets out of saturation with the proposed control, while it is still in the saturation zone with the TSE control. Since the system input $u_f(t)$ is primarily dependent on $v_f(t)$ that is in turn dependent on the system state vector $x(t) = [\Delta \delta(t) \, \Delta \omega(t) \, \Delta P_f(t)]^T$, this implies that the deviation of power angle, relative speed and active power from the normal value of system decreases faster with the proposed control than with the TSE control. Immediately after the transmission line is restored at 1.2s, the system is disturbed for the second time. The exciter input with the proposed control falls into saturation again. Even so, it gets out of saturation (at about 2.1s) faster with the proposed control than with the TSE control (at about 2.2s). This again illustrates the good performance of the proposed control.

Both the relative speed responses and active power responses are shown for Case 8.2 and Case 8.3, where only the proposed control is tested. The generator active power output, terminal voltage, transmission line parameters and fault location are all different from Case 8.1. Figure 8.3 - Figure 8.4 show that all the responses are satisfactory. Note that the parameters of the proposed control strategy do not change for all these cases, thus the control strategy is a robust one.

8.3.2 Robust constrained control of multimachine power systems

The example system used is a five-machine twelve-bus example system shown in
Figure 5.1 in Section 5.1. Different operating conditions and fault locations will be studied. System responses for five generators #1- #5 with the proposed control and with the nonlinear feedback linearization control without considering input constraints will be compared in the two simulation cases. The parameters of the example system and the two operating conditions are the same as those given in Section 5.4.

By assuming \( \Delta T_{\text{d}0} \leq 0.17 T_{\text{d}0} \), \( v_i(t) \in [-6,3] \) \( (i=1,2,\ldots,5) \), and solving the LMIs (8.7), the low gain linear feedback control laws for the generators are obtained as follows:

\[
\begin{align*}
  u_{E1}(t) &= 0.80\Delta \delta(t) + 0.72\Delta \omega(t) - 4.49\Delta P_{e1}(t) \\
  u_{E2}(t) &= 0.51\Delta \delta(t) + 0.46\Delta \omega(t) - 3.38\Delta P_{e2}(t) \\
  u_{E3}(t) &= 0.45\Delta \delta(t) + 0.41\Delta \omega(t) - 3.11\Delta P_{e3}(t) \\
  u_{E4}(t) &= 0.54\Delta \delta(t) + 0.49\Delta \omega(t) - 3.60\Delta P_{e4}(t) \\
  u_{E5}(t) &= 0.49\Delta \delta(t) + 0.46\Delta \omega(t) - 3.18\Delta P_{e5}(t)
\end{align*}
\]

By choosing proper values of \( K_{E1i}, \alpha_i, \) and \( \beta_i \), the extra feedback control laws can be written as:

\[
\begin{align*}
  u_{E1}(t) &= 10 + 20[e^{-0.5|\delta|} - e^{-0.5|\delta|}] [3.26\Delta \delta(t) + 2.78\Delta \omega(t) - 1.74\Delta P_{e1}(t)] \\
  u_{E2}(t) &= 10 + 20[e^{-0.5|\delta|} - e^{-0.5|\delta|}] [2.14\Delta \delta(t) + 1.85\Delta \omega(t) - 1.35\Delta P_{e2}(t)] \\
  u_{E3}(t) &= 10 + 20[e^{-0.5|\delta|} - e^{-0.5|\delta|}] [1.85\Delta \delta(t) + 1.57\Delta \omega(t) - 1.20\Delta P_{e3}(t)] \\
  u_{E4}(t) &= 10 + 20[e^{-0.5|\delta|} - e^{-0.5|\delta|}] [2.30\Delta \delta(t) + 1.98\Delta \omega(t) - 1.46\Delta P_{e4}(t)] \\
  u_{E5}(t) &= 10 + 20[e^{-0.5|\delta|} - e^{-0.5|\delta|}] [2.06\Delta \delta(t) + 1.81\Delta \omega(t) - 1.26\Delta P_{e5}(t)]
\end{align*}
\]

Then the robust decentralized constrained composite control laws are:

\[ v_i(t) = u_{Ei}(t) + u_{E}(t) \quad i = 1,2,\ldots,5 \]

The nonlinear feedback control without considering control input constraints is the TSE control discussed in Section 5.2. The obtained robust decentralized TSE control laws for generators are the same as those given in Section 5.4.

The actual control laws \( u_i(t) \) can be obtained from \( v_i(t) \) through the relationship of

\[
\begin{align*}
  u_{E1}(t) &= 0.80\Delta \delta(t) + 0.72\Delta \omega(t) - 4.49\Delta P_{e1}(t) \\
  u_{E2}(t) &= 0.51\Delta \delta(t) + 0.46\Delta \omega(t) - 3.38\Delta P_{e2}(t) \\
  u_{E3}(t) &= 0.45\Delta \delta(t) + 0.41\Delta \omega(t) - 3.11\Delta P_{e3}(t) \\
  u_{E4}(t) &= 0.54\Delta \delta(t) + 0.49\Delta \omega(t) - 3.60\Delta P_{e4}(t) \\
  u_{E5}(t) &= 0.49\Delta \delta(t) + 0.46\Delta \omega(t) - 3.18\Delta P_{e5}(t)
\end{align*}
\]
(8.19).

The fault considered here is a symmetrical three-phase temporary short circuit fault. The fault sequence is Fault Sequence 4.1.

The following two cases are used to evaluate the proposed control strategy.

Case 8.4:

This case is exactly the same as Case 5.1.

Figure 8.5 shows relative power angle, angular speed, generator terminal voltage and active power responses of the five generators with the proposed robust decentralized constrained control. The four responses of the five generators with TSE control are shown in Figure 5.11.

Figure 8.5  Relative power angle, relative speed, terminal voltage and active power responses for Case 8.4 with proposed control
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Figure 8.6 Relative power angle, relative speed, terminal voltage and active power responses for Case 8.5 with proposed control

Case 8.5:
Operating Condition 5.2, Fault Location 5.2 and Fault Sequence 4.1.

Figure 8.6 shows the relative angular speed and generator terminal voltage responses of the five generators with proposed robust decentralized constrained control; Figure 8.7 shows the two responses of the five generators with robust decentralized TSE.

It can be seen from Figure 8.5 and Figure 8.6 that the proposed constrained control method has good responses for both cases. Since for both cases, the generator active power output, terminal voltage and fault location are all different and the parameters of the proposed control strategy are the same, the control strategy is a robust control
strategy.

By comparing the relative angular speed and generator terminal voltage responses of the five generators with proposed control and with TSE control in Figure 8.5 and Figure 5.11, Figure 8.6 and Figure 8.7, it can be seen although the performance is similar with both control strategies during the transient period, the performance is better with the proposed control strategy than with TSE control strategy during the post-transient period. This is due to the introduction of the extra nonlinear feedback control law \( k_{\text{en}}B_{i}^{T}P_{k}(t) \) into the composite control law. This extra nonlinear feedback control law mainly takes effect when the control input is far away from its saturation level. In such a case, the state is close to the equilibrium, that is,
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\[ x_i(t) = [\Delta \delta(t) \Delta \omega(t) \Delta P_m(t)]^T \] approaches zero vector.

8.4 Concluding remark

Robust constrained control method for generator excitation system is proposed that fully takes into account the input constraints in the design period. By combining low and high gain linear feedback control, nonlinear feedback control and also considering power system uncertainties, an exciter controller with good performance is designed. The control method is based on rigorous constrained control theory and has theoretical guarantees for stability and performance. Simulation results show that the proposed control strategy is effective in enhancing power system transient stability and provides good robustness to different operating conditions and network parameters. The performance is better than those controllers of which the input constraints are not considered. However, this is a preliminary result because the assumption that the uncertainty matrix \( \Delta B_i(t) \) in the B-matrix agrees with the form \( \Delta B_i(t) = k_{\iota_i}(t) B_i \) is quite special. This assumption restricts the constrained control method to be only integrated with the TSE control method in this study. Further research should be carried out to remove this requirement so that the constrained control method can also be integrated with the CTSVRE control method. In addition, robust constrained control with unsymmetric input saturation should be studied in the future. The nonlinear relationship between \( \nu_{\iota_i}(t) \) and \( u_{\iota_i}(t) \) when saturation is considered should also be studied further.
Chapter 9 Conclusions and Recommendation

9.1 Conclusions

Nowadays, the increasing interconnections of power grids make power system phenomenon very complex. In the meantime, building of new generation and transmission facilities is more and more difficult and sometimes even impossible. Furthermore, the transmission open access environment has created an economical incentive to operate power systems close to their thermal limits. More than ever, advanced controller designs for power systems are called upon to guarantee the secure and reliable operation of power systems under normal, abnormal and most importantly, large disturbance such as generator unit tripping, single or three phase fault on the transmission lines, etc.

Advanced control strategies should satisfy various requirements. Some primary requirements include: 1) the controllers should work under a wide range of operating conditions and are insensitive to various nonlinearities and parametric uncertainties, such as model uncertainties, interconnections among generators, network topology variations, exciter input saturation, etc.; 2) the controllers should be reliable and simple enough for practical implementation, for example, decentralized control is required for many cases; 3) the controller should guarantee not only the transient stability of power systems to some degree but also the post-fault voltage performance.

According to these control requirements, this thesis mainly proposed three types of control strategies: 1) robust CTSVRE control with guaranteed performance for SMIB and multimachine power systems; 2) robust coordinated control for AC/DC power systems with overlapping decomposition; 3) robust constrained control for generator
excitation systems to deal with the exciter input constraints. As all control requirements should be met for a large complex power system, the first two control strategies are separately described in detail and then they are combined to design controllers for a complex example system. Since the result of robust constrained control design is still at a preliminary stage and makes some special assumption, it is not integrated into the controller design for the final complex systems in Chapter 7 in order to maintain the systematic structure of the thesis.

A summary of some significant contributions is listed as follows:

1. Robust CTSVRE control for SMIB/multimachine power systems
Robust CTSVRE control strategy is proposed to enhance the post-fault voltage regulation as well as transient stability. Using DFL and voltage feedback, the original generator model is partially linearized. By treating the variables in A-matrix and B-matrix of the state space model (and the interconnections among generators for multimachine case) as parameters, robust control theory can be employed. After carefully analyzing the parametric uncertainties under various typical operating conditions, it is found the parametric uncertainties have obvious different characteristics in two ranges. This characteristic is employed to partition the operating region of the generator in terms of the value of the rotor angle for SMIB power systems case or more general, the value of active power for multimachine power systems case. The partition aims to increase the accuracy of the CTSVRE controller. To avoid introducing another disturbance, smooth switch action between the obtained robust CTSVRE control laws should be guaranteed. This is realized by choosing appropriate member functions.

2. Robust coordinated control for AC/DC power systems with overlapping decomposition
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From systematic standpoint, coordinated control of rectifier and inverter stations needs real time signals transmitted from remote converter stations. To eliminate or at least weaken this requirement, overlapping decomposition control strategy is proposed for the most often used control modes: constant current at rectifier station and constant extinction angle (or constant voltage) at inverter station. This decomposition is realized by employing overlapping decomposition technique. By selecting appropriate HVDC dynamic model, the HVDC link dynamic structure agrees with the form that is suitable for application of overlapping decomposition technique. By expanding the original state space model of DC link to a higher dimensional state space, the states are rearranged and regrouped into two subsystems that are weakly connected. Considering the parametric uncertainties, the original overlapping decomposition control problem is converted to a standard robust decentralized output regulation problem, which is easier to be solved. The decoupling of AC/DC power systems and linearization of generator models are realized by DFL technique. By transforming the forms of generator model and making appropriate assumptions, previous robust control methods for pure AC system generator exciters are also applicable to AC/DC system generator exciters. All of the designed controllers are coordinated to ensure the power system stability.

3. Robust constrained control of generator excitation systems with input constraints

Similar to other physical systems, excitation systems also have control input saturation, which is obviously a kind of nonlinearity. This wind-up type hard limit should not be ignored like most other researchers do. Lots of control theories have been proposed to study and deal with input constraints in the last decade. In this work, exciter controller is designed that considers the control input constraints from the beginning of controller design. Low and high gain linear feedback
control and nonlinear control theory is used to cope with the input saturation and improve the controller performance. Robust control theory is used to deal with the parametric uncertainties. Rigorous proof of stability and detailed analysis are given.

To realize the objective of enhancing both transient stability and post-fault voltage regulation of power systems, global control concept is employed in Chapter 7 to coordinate the robust TSE control and robust CTSVRE control. All of the designed controllers are tested by numerical simulation using the PSCAD/EMTDC software. The proposed control strategies are not only compared with conventional control methods to show their performances improvement, but also tested by various cases including different operating conditions, fault locations, fault sequences and component parameters to illustrate their robustness. Simulation results show that the proposed control strategies are quite effective.

The proposed control strategies may provide alternative solutions to the power system control although many practical factors should be considered for such application. For example, in today’s environment of power system deregulation, power system operators prefer using resources within their own area. As the proposed control strategies use only local measurements or fewer remote signals, this requirement is satisfied to some degree. Besides, these control strategies do not have special requirements.

9.2 Recommendations for further research

In this thesis, several advanced control strategies have been proposed to enhance power system stability and performance. Their effectivenesses have been proved by numerical simulation. However, there are still many problems needed to solve. In future research, following issues should be addressed:
Chapter 9 Conclusions and Recommendations

9.2.1  Further research for robust constrained control

In Chapter 8, constrained controller is designed for generator excitation system with input saturation. However, this is a preliminary result in that it is assumed the uncertainty matrix $\Delta B_i(t)$ in the B-matrix agrees with the form $\Delta B_i(t) = k_i(t)B_i$. This assumption is an additional assumption to the general one in the form of $\Delta B_i(t) = D_iF_i(t)E_{2i}$ . This special assumption restricts the application of robust constrained control method. In this work, the robust constrained control method is only integrated with the TSE control method. To make the robust constrained control method also be integrated with the CTSVRE control method, further research should be carried out to remove this special assumption. Fundamental results can be found in [145, 146].

In addition, symmetric input limits are assumed in Theorem 8.1 for simplicity. Further research should study the unsymmetric input limits which are more common in nature. The nonlinear relationship between $v_p(t)$ and $u_q(t)$ when saturation is considered should also be studied further.

Furthermore, since the proof is based on quadratic stability, the stability region is conservative. In order to make the stability region less conservative, polyhedral invariant sets [144, 147, 148] may be used in the future research.

9.2.2  Further research for complex power systems control

Several considerations for the future research of power system control includes but not limited to:

1. Integrated rotor angle and voltage stability control

In Chapters 5 and 7, electrical loads are considered in the example power system,
where the electrical loads considered are static load and are assumed to be constant impedance in the simulation for simplicity. However, such assumptions may not be accurate [149]. During transient processes, the voltage magnitude and the frequency will both change. But the constant impedances load representation assumes the load active power and reactive power are dependent on the voltage and independent on the frequency, this always lead to pessimistic side, though optimistic results are also possible in some situations [86]. Moreover, a higher ratio of electricity consumption goes to power motors, for which dynamic models have to be used. To make the electrical loads more accurate with physical devices, multiple load types connected to a load bus is recommended in [80]. These load models always consist of a certain nonlinearities [150]. Therefore, the simplified multimachine power system model in Chapters 5 and 7 will also include load nodes. Interaction between generators and dynamic loads, such as induction or synchronous motors, should be studied and new controllers should be designed and illustrated on this more complex dynamic example system.

In fact, electrical loads are the usual driving force for voltage stability. As more and more area power systems are interconnected, the importance of voltage stability is recognized by many researchers. Voltage stability is predominately load stability [118]. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network and the connected generation. The problem of voltage stability associated with HVDC transmission system should especially be paid attention to. Research results in this field can be found in [151-155].

The stability problem studied in this work is short term stability and belongs to the "rotor angle stability" category as classified in [156]. However, for short term stability there is also voltage stability problems. These short term voltage stability
problems involve dynamics of fast acting load components such as induction motors, electronically controlled loads, and HVDC converters. As pointed in [156], the interested period of short term voltage stability is in the order of several periods, and analysis requires solution of appropriate system differential equations; this is similar to analysis of rotor angle stability. Although there are attempts to decouple voltage stability and angle stability [157], a systematic analysis distinguishing both rotor angle and voltage stability has not been reported yet. Thus, it may be necessary to analyze the rotor angle and voltage stability together. A preliminary result is given in [158] on a three node power system.

2. HVDC control

For HVDC transmission systems, reactive power is consumed at both the rectifier and inverter stations. With normally accepted rectifier ignition delay angle and inverter advance angle of 15° to 18° and commutating reactance of 15%, a converter consumes 50 to 60% reactive power [87]. When DC systems are connected to weak AC systems, both the AC and DC voltages are very sensitive to changes in loading. For such a scenario, improper control may contribute to voltage instability, especially when power modulation is employed [134, 135, 159]. In Chapters 6 and 7, VDCOL is employed when designing overlapping decomposition controllers for HVDC link. Although this may help stabilize the system voltage, it is better to consider the coordination of active power and reactive power using the converter firing angle control or even additional equipments like SVC if necessary.

In addition, when using DFL technique to decouple multimachine AC/DC systems, the dynamic voltage variation and angular frequency variation at HVDC terminal bus are neglected. This may influence the performance of the designed controller more or less depending on the electrical distance between the generator and
Chapter 9 Conclusions and Recommendations

HVDC terminal. As such dynamics at HVDC terminal bus are highly nonlinear, how to deal with such dynamics need further study.

Furthermore, for the overlapping decomposition control of HVDC link in Chapter 6, \( \cos \alpha \) and \( \cos \gamma \) in the state space model are treated as state variables and \( \cos \beta \) is treated as an output variable. Since these triangular functions are bounded, the controller design problem obviously belongs to the category of controller design with state and output constraints. Although in Chapter 6 controllers are designed without considering these constraints, rigorous proof should be given in further research.

3. Combination with FACTS control

In the numeric simulation of Section 6.4, a SVC device at the inverter terminal bus is used to provide reactive power and achieve voltage regulation. However, the reactive power is not optimized. Appropriate coordination control of SVC and inverter firing angle to minimize the reactive power consumption and enhance voltage regulation should be considered in the future. In fact, FACTS devices have already demonstrated their effectiveness in controllability and power transfer enhancement in power systems [96, 160-163]. For example, SVC alone has the compatibility of transient stability improvement, power oscillation damping and voltage support [95]. With the fast development of high power electronics technology, more and more FACTS devices will be put into use. The issue of coordination of generator unit control, HVDC control and FACTS control is a promising topic for further study. Some results on coordination of excitation and FACTS control are given in [16, 19, 23, 164]. Furthermore, the recent nonlinear control technique provides an alternative to redesign FACTS controllers with better performance [164, 165].
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APPENDIX A: Further comparison of SMIB power system performance with CTSVRE and conventional control strategies

In Sections 4.4.2 and 4.4.3, SMIB power system performance with proposed CTSVRE controller and conventional controller is compared. To illustrate the performance of the proposed CTSVRE controller further, an extreme condition is studied and simulation results are given. Also, system responses comparison with CTSVRE controller and conventional controller for Case 4.3 - 4.5 is given. The example power system and controller parameters are detailed in Chapter 4.

![Figure A.1](image-url)  
Figure A.1 Power angle, relative speed, terminal voltage and active power responses for Case A.1 with different control schemes
Appendices

A.1 Extreme condition

The studied case is as follows:

Case A.1:

The operating point is $\delta_0 = 56.87^\circ$, $V_{m0} = 1.10$ p.u., $P_{m0} = 1.00$ p.u. The fault occurs on one of the parallel AC transmission lines near bus 1. The fault sequence is Fault Sequence 4.2. Parameters for each transmission line are $r_i = 0.03*4$ p.u.; $x_i = 0.24265*4$ p.u.

Power angle, relative speed, terminal voltage and active power responses of the proposed controller and conventional controller are shown in Figure A.1. It can be seen from this figure, at a power angle large enough, the system can finally reach stable state and

![Figure A.1](image1)

![Figure A.2](image2)

Figure A.2  Power angle, relative speed, terminal voltage and active power responses for Case 4.3 with different control schemes
achieves acceptable voltage level with proposed control strategy. But it loses stability with the conventional control strategy. This illustrates the performance of the proposed controller.

A.2 Comparison of system responses with CTSVRE and conventional control strategies for Case 4.3 – 4.5

In Section 4.4.3, to illustrate the robustness of the designed CTSVRE controller, various operating points, fault locations and system parameters are studied. It can be seen the system performance is satisfactory for these different operation conditions. In this section, system responses with CTSVRE controller and conventional controllers are compared and shown in Figure A.2 – Figure A.5. The controller parameters are the same as those given in Section 4.4 and do not change during these cases except for that in Figure A.5.

![Figure A.3](image_url)

Figure A.3 Power angle, relative speed, terminal voltage and active power responses for Case 4.4 with different control schemes
It can be seen from these figures although the conventional controllers can stabilize the system for the three cases, the performance is worse compared with proposed controller. In Figure A.3, the voltage deviates from the original value too much (from 1.1p.u. to 0.9p.u.), which is not acceptable in practice. This can be partially cured by adding a TGR (transient gain reduction, see [87, 112, 113, 114]) block behind the excitation voltage regulation block. Figure A.5 shows the responses when TGR block is added to the original conventional controller. Parameters for the TGR block are: $T_c = 1s$, $T_a = 10s$.

It can be seen from Figure A.5, the voltage performance has been increased after TGR
Appendices

block is added to the exciter control loop, although the performance is still worse than that with proposed control strategy. The key point here is that the conventional controller is designed based on some specific operating point using the small signal analysis tool. While the performance is good at one operating point, it may not be acceptable for another operating point. The operating region is usually small. That is why it is used for small disturbance study and not for large disturbance study as is in this thesis.

Figure A.5  Power angle, relative speed, terminal voltage and active power responses for Case 4.4 with different control schemes
APPENDIX B: AC/DC power flow calculation principle

In this thesis, AC power system power flow is calculated using Newton-Raphson method [87]. AC/DC power flow is calculated following the sequential method [70, 116]. In this approach, the AC and DC link equations are solved separately and thus the integration into standard load-flow program is carried out without significant modifications of the AC load flow algorithm. For the AC system iterations, each converter is simply modeled as a complex power load at the AC terminal busbar. The DC link equations are solved using the latest updated value of the AC bus bar voltage. The AC and DC equations are solved separately each iteration.

As an example, the power flow data for Case 6.1 is listed as below (the bus no. is shown in Figure B.1):

| ---Line--- | ----Power at bus & line flow--- | ---Line loss--- |
|           | from | to | MW | Mvar | MVA | MW | Mvar |
| 1         | -9.341 | 1.869 | 9.526 |
| 2         | -7.384 | 1.816 | 7.604 | 0.000 | 0.058 |
| 5         | -1.957 | 0.053 | 1.957 | 0.011 | 0.038 |
| 2         | 0.000 | 0.000 | 0.000 |
| 1         | 7.384 | -1.758 | 7.591 | 0.000 | 0.058 |
| 3         | -3.692 | 0.879 | 3.795 | 0.058 | 1.446 |
| 3         | -3.692 | 0.879 | 3.795 | 0.058 | 1.446 |
| 3         | -2.000 | -0.070 | 2.001 |
| 2         | 3.750 | 0.567 | 3.793 | 0.058 | 1.446 |
| 4         | -9.500 | -1.203 | 9.576 | 0.000 | 1.170 |
| 4         | 9.500 | 2.374 | 9.792 |
| 3         | 9.500 | 2.374 | 9.792 | 0.000 | 1.170 |

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### Appendices

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<tr>
<th>No.</th>
<th>Mag.</th>
<th>Degree</th>
<th>Load</th>
<th>Generation</th>
<th>DC link</th>
<th>Injected</th>
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<td>0.000</td>
<td>0.000</td>
<td>-9.341</td>
<td>1.869</td>
</tr>
<tr>
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<td>0.000</td>
<td>-1.968</td>
<td>0.665</td>
</tr>
</tbody>
</table>

Total loss: 0.1274, 4.158

Power Flow Solution by Newton-Raphson Method

Maximum Power Mismatch = 1.9748e-005

No. of Iterations = 4

---

**Figure B.1** Example single machine AC/DC power system for power flow calculation
APPENDIX C: Synchronous generator model used in PSCAD/EMTDC

As introduced in Section 2.2, the synchronous generator model in this thesis used is a third-order one. All the controller design is based on the third-order generator model. However, the simulation is carried on using PSCAD/EMTDC software, where sixth-order generator model is used. In the following text, this sixth-order generator model will be described.

![Concept diagram of the three-phase and dq windings](image)

Figure C.1 Concept diagram of the three-phase and dq windings [166]

where:

\[ k \]: Amortisseur windings
Appendices

\begin{itemize}
  \item \textbf{f}: Field windings
  \item \textbf{abc}: Stator windings
  \item \textbf{d}: Direct-Axis (d-axis) windings
  \item \textbf{q}: Quadrature-Axis (q-axis) windings
  \item \textbf{\(\psi\)}: flux linkage
\end{itemize}

The general equivalent circuit for the synchronous machine is as shown in Figure C. 1.

The generalized generator model transforms the stator windings into equivalent commutator windings, using the dq0 transformation as follows:

\[
\begin{bmatrix}
U_d \\
U_q \\
U_0
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & \cos(\theta - 120^\circ) & \cos(\theta - 240^\circ) \\
\sin(\theta) & \sin(\theta - 120^\circ) & \sin(\theta - 240^\circ) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]

The d-axis equivalent circuit for the generalized machine is shown in Figure C. 2.

Figure C. 3 illustrates the flux paths associated with various d-axis inductances.

![Figure C. 2](image1)[166]

![Figure C. 3](image2)[166]
Appendices

In Figure C. 2 and Figure C. 3, the d-axis voltage $U_{d2}$ and current $I_{d2}$ are the field voltage and current respectively. The damper circuit consists of parameters $L_{d}$ and $R_{d}$ with $U_{d3} = 0$. The additional inductance $L_{23d}$ accounts for the mutual flux, which links only the damper and field windings and not the stator winding. The inclusion of such flux is necessary for accurate representation of transient currents in the rotor circuits.

From Figure C. 2 and Figure C. 3, the following equations can be derived:

$$
\frac{d}{dt} \begin{bmatrix} i_{d1} \\ i_{d2} \\ i_{d3} \end{bmatrix} = L_{d}^{-1} \begin{bmatrix} -v \cdot \psi_q - R_{d} \cdot i_{d1} \\ -R_{2d} \cdot i_{d2} \\ -R_{3d} \cdot i_{d3} \end{bmatrix} + L_{d}^{-1} \begin{bmatrix} U_{d1} \\ U_{d2} \\ U_{d3} \end{bmatrix}
$$

$$
\frac{d}{dt} \begin{bmatrix} i_{q1} \\ i_{q2} \\ i_{q3} \end{bmatrix} = L_{q}^{-1} \begin{bmatrix} v \cdot \psi_d - R_{q} \cdot i_{q1} \\ -R_{2q} \cdot i_{q2} \\ -R_{3q} \cdot i_{q3} \end{bmatrix} + L_{q}^{-1} \begin{bmatrix} U_{q1} \\ U_{q2} \\ U_{q3} \end{bmatrix}
$$

where

$$L_{d} = \begin{bmatrix} L_{MD} + L_{d} & L_{MD} & L_{MD} \\ L_{MD} & L_{MD} + L_{23d} + L_{2d} & L_{MD} + L_{23d} \\ L_{MD} & L_{MD} + L_{23d} & L_{MD} + L_{23d} + L_{3d} \end{bmatrix}
$$

$$\psi_q = L_{d} \cdot i_{q1} + L_{MD} \cdot (i_{q1} + i_{q2} + i_{q3})
$$

Similar equations hold for the q-axis, and

$$\psi_d = L_{d} \cdot i_{d1} + L_{MD} \cdot (i_{d1} + i_{d2} + i_{d3})
$$

$$v = \frac{d\theta}{dt}
$$

The torque equation is given as:

$$T = \psi_q \cdot i_{q1}
$$

and the mechanical dynamic equation for motor operation is:

$$\frac{dv}{dt} = \frac{T - T_{\text{mech}} - D \cdot v}{J}
$$

where

$J$ : Generator inertia
Appendices

All above equations constitute the generator model that EMTDC uses. The designed controllers are simulated on this model. That means control laws developed from simple models are evaluated on systems with complex models, which are more close to practical system. Because EMTDC is most suitable for simulating the time domain instantaneous responses (by representing and solving differential equations) of electrical systems, the satisfactory simulation results proves the feasibility and correctness of the proposed control strateg to some degree.