Wavelength Assignment Algorithms for Optical Multicast in WDM Networks

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Summary

Multicast refers to a point-to-multipoint communication, by which a source node transfers data to multiple destinations. Many bandwidth-intensive multimedia applications, such as video-on-demand, multimedia document distribution, database replication, network storage, etc., can be efficiently and economically supported by multicast, based on the huge bandwidth provided by wavelength-division-multiplexing (WDM) technology. Performing multicast in the optical WDM network, i.e., optical multicast, opens a new research frontier, which provides a strong mandate for the future backbone network to support point-to-multipoint services. Compared with traditional multicast issues, besides routing problem, optical multicast over WDM network has an extra wavelength assignment problem. In this thesis, our focus is to derive efficient wavelength assignment algorithms for optical multicast under different wavelength conversion scenarios.

A new multicast wavelength assignment algorithm has been proposed by allowing multiple available wavelengths in a link to carry the multicast signal, so-called multi-wavelength assignment strategy. For full wavelength conversion case, such new algorithm provides a good tradeoff by making significant reduction of wavelength conversion cost while keeping the additional wavelength usage within a reasonable range. Besides full wavelength conversion case, we exploit the extra benefits of multi-wavelength assignment strategy for the
network with limited wavelength conversions. We then extend the new algorithm to cover limited wavelength conversion case. Simulation results show that much more multicast requests can be accommodated by the new algorithm. To the best of our knowledge, this is the first time that the optical multicast issue is addressed for limited wavelength conversions.

We then address the problem of multicast wavelength assignment for sparse wavelength conversion in the wavelength-routed WDM networks. We first extend the random wavelength assignment algorithm from the unicast case to the multicast case. The straightforward extension leads to inefficient use of wavelength converters. In order to minimize the use of wavelength converters for a given multicast request, a new technique has been derived by making use of a novel virtual link method to map multicast tree from sparse conversion case to full conversion case. Then, a dynamic programming algorithm has been developed for wavelength assignment aiming to minimize the number of wavelength converters required. Simulation results show that our new algorithm outperforms both random and greedy algorithms with regards to minimizing the number of wavelength converters.

Multicast wavelength assignment problem has been studied in a new multicast switch model. Traditional switch model is often based on split-convert scheme which leads to high redundant wavelength conversion cost. In order to minimize the wavelength conversion cost of multicast, we employ a new multicast switch model that supports a new split-convert-split switch scheme capable of eliminating the redundant wavelength conversion. Based on this model, we adopt the multi-wavelength assignment strategy to generalize the existing algorithm to produce a new algorithm to achieve the best performance. Compared with the existing algorithm, the new algorithm not only is a more general and flexible one, but also delivers a good performance in term of minimizing the number of wavelength conversions.
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Acronyms

ATM  asynchronous transfer mode
CBT  core based trees
DVMRP  distance vector multicast routing protocol
HDTV  high-definition TV
IP  Internet protocol
LSB  light splitter bank
MAW  multicast with any wavelength
MC-OXC  multicast-capable optical crossconnect
MC-RWA  multicast routing and wavelength assignment
MILP  mixed integer linear programming
MLW  multicast with limited wavelength
MOSAD  multicast-only splitter-and-delivery
MOSPF  multicast extensions to open shortest path first
MSDW  multicast with same destination wavelength
MSW  multicast with same wavelength
NWC  number of wavelength conversions
O-E-O  optic-electronic-optic
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OXC</td>
<td>optical crossconnects</td>
</tr>
<tr>
<td>PIM</td>
<td>protocol independent multicast</td>
</tr>
<tr>
<td>PN</td>
<td>powerful node</td>
</tr>
<tr>
<td>RWA</td>
<td>routing and wavelength assignment</td>
</tr>
<tr>
<td>SAD</td>
<td>splitter-and-delivery</td>
</tr>
<tr>
<td>TaC</td>
<td>tap-and-continue</td>
</tr>
<tr>
<td>VOD</td>
<td>video-on-demand</td>
</tr>
<tr>
<td>VoIP</td>
<td>voice-over-IP</td>
</tr>
<tr>
<td>WCB</td>
<td>wavelength converter bank</td>
</tr>
<tr>
<td>WDM</td>
<td>wavelength-division multiplexing</td>
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Chapter 1

Introduction

During the first few years of the twenty-first century, we have witnessed the tremendous progress made in the field of computer and communication networks. With the increasing richness of multimedia contents and broadband applications, both network user population and network traffic are growing at an explosive rate. Optical networks using wavelength-division multiplexing (WDM) is believed to be the technology of choice for next-generation networks meeting the huge ever-growing bandwidth demands.

Multicast is a means of point-to-multipoint communication, by which a single source transmits data to multiple destinations simultaneously. Multicast is the key technology to support one-to-many bandwidth-intensive applications, such as voice-over-IP (VoIP), video-on-demand (VOD), videoconferencing, high-definition TV (HDTV), multimedia document distribution, interactive distance learning, live auctions, distributed computing, database replication, etc.. For all these applications, multicast is bandwidth-efficient as only a single copy of data is transmitted from the source to destinations, not multiple individual copies.

Optical multicast means conducting multicast communications directly on optical net-
work domain. It is an efficient way to make good use of network resources to support point-to-multipoint bandwidth-intensive applications.

In this thesis, we study the challenges of conducting multicast communications in WDM networks. We focus our studies on the multicast routing and wavelength assignment problem in wavelength-routed WDM networks. Three suites of multicast wavelength assignment algorithms together with a new multicast switch model have been developed to tackle the problems on various scenarios, taking different resource constraints into account.

The rest of this chapter is organized as follows. Section 1.1 provides the overview of optical WDM networks and optical multicast in WDM networks. Section 1.2, through 1.5 cover the motivations, objectives, contributions and organization of the thesis.

1.1 Background

1.1.1 Optical WDM Networks

Optical communications technology, the transport of communication signals via fiber optics, supports higher data rates over greater distances with better reliability than competing copper and wireless alternatives [1]. The tremendous bandwidth (up to 50 Terahertz) that an optical fiber can provide is divided into many non-overlapping channels by using wavelength-division multiplexing (WDM) technology. Each channel is characterized by the wavelength (or the frequency) on which the transmissions can be made. Transmissions in different wavelengths on the same link do not interfere with each other. With currently available commercial technology, a few tens of wavelengths can be supported within the low-loss window at 1550 nm, and each wavelength can carry up to 10 Gigabits per second of data. In a high-density
WDM system, a single fiber can have hundreds of wavelengths and this number is expected to grow rapidly in the next few years. Therefore, optical fiber links employing WDM technology have the potential of delivering an aggregate throughput in the order of Terabits per second, enough to satisfy the ever-growing demands for more bandwidth on a sustained and long-term basis.

An optical WDM network [2–4] is a network with optical fiber transmission links and with an architecture that is designed to exploit the unique features of fibers and WDM. Currently, optical fibers are deployed in point-to-point transmission links as direct substitute for copper. Optical signals arriving at a network node are converted to electronic domain, switched using an electronic crossconnect, and then converted back to the optical domain for transmission to a downstream node. On the other hand, the architecture for next-generation backbone networks is widely expected to built on the concept of wavelength routing or wavelength switching. Switching is done in the optical crossconnects (OXC) without converting incoming signal to electronic domain. This network is called a wavelength-routed network.

A wavelength-routed network, as shown in Figure 1.1, consists of two types of nodes: OXC, which connects the fiber links in the network, and access nodes which provide the interface between non-optical end systems (such as IP\textsuperscript{1} routers or ATM\textsuperscript{2} switches) and the optical core. Access nodes provide the terminating points (sources and destinations) for the optical signal paths; the communication paths may continue outside the optical part of the network in electrical form.

Wavelength-routed networks operate based on the concept of lightpath [5]. A lightpath is

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\textsuperscript{1}IP is short for Internet Protocol, which is a data-oriented network layer protocol used for communicating data across a packet-switched internetwork.

\textsuperscript{2}ATM is short for Asynchronous Transfer Mode, which is a connection-oriented network protocol encoding data traffic into small fixed-sized (53bytes) cells.
an all-optical communication channel set up between two access nodes, which may span more than one fiber link and pass through some intermediate OXCs. An OXC takes in an optical signal at each of the wavelengths at an input port, and switches it to a particular output port, independent of other wavelengths. In Figure 1.1, the solid line between access node 1 and 6 represents a lightpath using wavelength $\lambda_1$, and the dashed line connecting node 4 and 7 is another lightpath operating on wavelength $\lambda_2$.

There are some key enabling components within the OXC of wavelength-routed networks. A *Transmitter* is used to transmit data on a wavelength channel. A transmitter is tunable if it can transmit on one of several different wavelength channels. A fixed-tuned transmitter transmits on a fixed wavelength channel. A *Receiver* is deployed to receive data on a wavelength channel. The receiver can also be either tunable or fixed-tuned. *Optical amplifiers* are used to compensate fiber losses and signal attenuations due to different elements in the
network. The semiconductor optical amplifier and Erbium-doped fiber amplifier are typical optical amplifiers used in today’s transport network. *Wavelength converter* has the capability of converting an incoming wavelength to a different outgoing wavelength.

### 1.1.2 Optical Multicast

Multicast provides a means of point-to-multipoint communication in which one source sends messages to multiple destinations. To realize one-to-many communication, the simplest way is to send individual copy of message from source to each of the destinations separately and independently, that is called multiple-unicast. Different from multiple-unicast, multicast is a more efficient way to implement one-to-many communication, making better use of network resources by simply duplicating messages at the common intermediate node. A simple example to show the efficiency of multicast is illustrated in Figure 1.2. In order to send a message from the source to three destinations, three channels are required from node A to node B using multiple-unicast (Figure 1.2(a)). However, only one channel is needed by deploying multicast (Figure 1.2(b)).

![Figure 1.2: Multiple-unicast versus multicast. (a) Multiple-unicast. (b) Multicast.](image-url)
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Multicast has been extensively studied in traditional IP-centric electronic networks [6–8], and has received much attention in optical networking community in recent years [9–12]. Multicast can be supported more efficiently in optical domain by utilizing the passive optical devices known as light splitters (or power splitters). A light splitter has the ability of splitting an incoming signal into up to \( m \) outgoing signals, \( m > 2 \); \( m \) is referred to as the fanout of the light splitter.

Multicast issues in wavelength-routed WDM network can be classified into data plane issues and control plane issues. On data plane, the fundamental problem is to study architecture of multicast-capable switch networks. This building block mainly focuses on the design of multicast-capable optical crossconnects (MC-OXC). On control plane, the main problem is multicast routing and wavelength assignment (MC-RWA), which is within the scope of interest of this thesis.

Routing and wavelength assignment (RWA) problem [13] is the fundamental control problem in wavelength-routed WDM networks. A lightpath is established by selecting a path of physical links between source and destination nodes, and reserving a particular wavelength on each of these links for the lightpath. Thus, to establish a one-to-one (or unicast) optical connection we must deal with both routing (selecting a suitable path) and wavelength assignment (allocating an available wavelength for the connection). RWA problem is significantly more difficult than the routing problem in electronic networks. The additional complexity arises from the fact that routing and wavelength assignment are subject to the following two constraints [4]:

1. Wavelength continuity constraint: In the absence of wavelength converters, a lightpath must use the same wavelength on all the links along its path from source to destination
Introduction

node.

2. *Distinct wavelength constraint*: All lightpaths using the same link (fiber) must be allocated distinct wavelengths.

By extending the concept of lightpath to that of *light-tree* [10], RWA problem in multicast (one-to-many) scenario can be formalized as multicast routing and wavelength assignment problem. Like a lightpath, light-tree is a light channel originated at source node and implemented with some wavelength on each link. But unlike a lightpath, a light-tree spans multiple destination nodes. Thus, in summary, a light-tree is a point-to-multipoint all optical channel which may span multiple fiber links.

The challenges of MC-RWA problem lie in two aspects as follows.

First, MC-RWA bears many similarities to RWA problem [14]. Specifically, the tight coupling between routing and wavelength assignment remains, and even becomes stronger: in the absence of wavelength conversion, the same wavelength must be used by the multicast connection not just along the links of a single path but along the links of the whole light-tree. Since optimal solutions for the point-to-point RWA problems are not practically obtainable [13] and MC-RWA problem can be considered as a more general case of such point-to-point counterpart, one can imagine that it must be even harder to find optimal solutions to MC-RWA.

Second, once light splitting and wavelength conversion are taken into account for the MC-RWA problem, different combinations of light splitting capability and wavelength conversion capability make the problem even more sophisticated. The splitting properties of a node can be *full light splitting* and *limited light splitting*. Full splitting means that input signal can be split and forwarded to all the outputs of the node. In limited splitting, input light signal can
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only be forwarded to a limited number of outputs. A node that has full splitting capability is referred to as a multicast-capable (MC) node. Correspondingly, a node that can only forward an input signal to one of the outputs is called a multicast-incapable (MI) node. If portions of nodes in the network are MC nodes and the rest are MI nodes, such network is called sparse splitting network. Similarly, wavelength conversion capabilities can also be of full wavelength conversion, limited wavelength conversion, and sparse wavelength conversion, respectively. Different light splitting capabilities and wavelength conversion capabilities may lead to different resource constraints and optimization objectives when solving MC-RWA problem.

1.2 Motivations

To tackle MC-RWA problem, current approaches often decouple the problem into two sub-problems: routing problem and wavelength assignment problem [13]. Given a multicast request, the multicast routing problem is to find a tree-structured route, i.e., multicast tree, rooted at source node and spanning all destinations. Various schemes can be adopted to find multicast tree. One simple and practical way is to directly map the tree route computed at the IP layer by existing IP multicast routing protocols [15] such as Distance Vector Multicast Routing Protocol (DVMRP) [16], Multicast Extensions to Open Shortest Path First (MOSPF) [17], Protocol Independent Multicast (PIM) [18, 19] and Core Based Trees (CBT) [20].

These protocols may not be efficient for the optical multicast when considering optical layer constraints, such as light splitting capability and wavelength conversion capability. Thus, alternate solutions are needed to compute multicast tree directly at the optical layer. Such routing problem has been studied in the network with sparse light splitting, i.e., only
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Partial nodes in the network can perform light splitting. This method of handling routing problem leads to the Steiner tree problem [21], which is well-known to be NP-complete [22]. In order to overcome the intractableness of such problem, efficient heuristics need to be developed to solve it.

After solving the routing problem, a multicast tree is determined. Then, the wavelength assignment begins. Given the network and multicast tree for the multicast request, the multicast wavelength assignment problem aims to allocate available wavelength(s) for each link/edge of the multicast tree. Traditionally, wavelength assignment problem for unicast case is either formulated as graph-coloring problem or solved by simple heuristics, such as Random, First-Fit, etc. [13]. All these approaches mainly attempt to overcome wavelength continuity constraint and distinct wavelength constraint as mentioned above and to lower request blocking probabilities. However, these approaches may become inefficient or even unacceptable when dealing with multicast case. Especially, when introducing wavelength conversion to the problem, for example, the cost issue becomes more important and sometimes dominating as wavelength conversion, though eliminating the wavelength continuity constraint, carries a high cost.

Unfortunately, little research has been done to address multicast wavelength assignment problem considering wavelength conversions. Many interesting topics are still untouched, such as cost tradeoff issues between wavelength cost and wavelength conversion cost, multicast for limited and sparse wavelength conversions, new multi-wavelength assignment strategy, and new multicast switch schemes and models, and so on. All of these motivate us to conduct comprehensive and systematic research to fill up the gaps on these subjects.
1.3 Objectives

Optical multicast over WDM networks benefits much from light splitting and wavelength conversion capabilities of the WDM networks. For light splitting, extensive research has been done to study the impact on optical multicast. Efficient algorithms have been proposed to solve routing and wavelength assignment problem for different splitting capabilities in terms of both cost and power issues. On the other hand, however, little research has been done to address the wavelength conversion related issues for optical multicast. Traditionally, the network is simply assumed to have no wavelength conversion or full wavelength conversion capability without any cost consideration. Consequently, careful and systematic research should be conducted to address the effects of wavelength conversion cost on optical multicast, and efficient algorithms should be developed to tackle the problems under different wavelength conversion circumstances, such as full wavelength conversion, limited wavelength conversion and sparse wavelength conversion. The main objectives of this thesis are summarized as follows:

- To conduct a comprehensive review on optical multicast issues. Both control plane and as well as data plane issues should be covered. Current progress made on various related topics need to be classified and summarized. Existing and potential problems will be located for related issues. Crucial directions for future research will be pointed out and predicted.

- To investigate multicast wavelength assignment problem for full wavelength conversion in terms of wavelength conversion costs. Efficient multicast wavelength assignment algorithm need to be proposed to minimize wavelength conversion cost. The cost tradeoff
issues between wavelength cost and wavelength conversion cost should be analyzed.

- To study the impact of limited wavelength conversion on multicast wavelength assignment problem. Efficient algorithm should be developed to support limited wavelength conversion by considering wavelength conversion cost.

- To investigate multicast wavelength assignment problem for sparse wavelength conversion. Efficient algorithms will be derived to support sparse wavelength conversion. Number of wavelength converters required for multicast request is a key metric and will be minimized when developing the algorithms.

- To study multicast switch model by considering wavelength conversion effects. New multicast switch schemes and model will be developed to make full use of wavelength converters by removing unnecessary wavelength conversions. The existing multicast wavelength assignment algorithms will be generalized to achieve the best performance.

1.4 Contributions

This thesis makes the following contributions to the body of knowledge on the optical multicast in WDM networks. Three suites of multicast wavelength assignment algorithms together with a new multicast switch model have been developed to address the problems for various switching models and schemes based on different wavelength conversion capabilities. Particularly, we propose efficient algorithms to tackle the multicast wavelength assignment for limited wavelength conversion and sparse wavelength conversion cases. Our work provides strong support to employ optical layer multicast for one-to-many bandwidth-intensive applications in future backbone networks.
In summary, our main contributions are as follows:

- A new Multi-wavelength Multicast Wavelength Assignment (MMWA) algorithm has been proposed by allowing multiple available wavelengths in a link to carry the multicast signal, i.e., multi-wavelength assignment strategy. The MMWA algorithm is implemented by employing a dynamic programming algorithm. For full wavelength conversion case, such new algorithm provides a good tradeoff by making significant reduction of wavelength conversion cost while keeping the additional wavelength usage within a reasonable range. We also show that the MMWA algorithm has acceptable time complexity for online computation demands.

- We further exploit advantages provided by the multi-wavelength assignment strategy for the network with limited wavelength conversions. The MMWA algorithm has then been extended to support limited wavelength conversion case. In comparison with traditional single-wavelength assignment strategy, MMWA algorithm can accommodate more multicast requests. The improvement can be as high as 50% to 80%, which is highly significant. To the best of our knowledge, this is the first time that optical multicast issue is addressed for limited wavelength conversion case.

- The problem of multicast wavelength assignment for sparse wavelength conversion in wavelength-routed WDM networks has been studied for the first time in literature. A new technique called MWA-SWC algorithm has been derived to solve the problem. The algorithm first maps multicast tree from sparse conversion case to full conversion case, by making use of a novel virtual link method. The method provides a forward mapping to generate an auxiliary tree as well as a reverse mapping to recover the orig-
Introduction

inal tree. Applying the auxiliary tree, we propose a dynamic programming algorithm for wavelength assignment aiming to minimize the number of wavelength converters required. The MWA-SWC algorithm can serve as a baseline for dynamic heuristic algorithms. Typically, the MWA-SWC algorithm will provide great benefit when the number of available wavelengths on links of the multicast tree is relatively large and the performance advantage is significant.

- A new multicast switch scheme has been developed, which is capable of eliminating redundant wavelength conversions. In order to implement this new switch scheme, we develop a new multicast switch model based on the concept of sharing of light splitters and wavelength converters. Furthermore, by combining the new multicast switch model with multi-wavelength assignment strategy, we generalize existing algorithms to produce a new Multicast Wavelength Assignment Algorithm (MWAA) to achieve the best performance in terms of minimizing the wavelength conversion cost. In general, the MWAA algorithm is suitable for the situation where the maximum node out-degree of the tree is relatively large.

1.5 Organization

The rest of the thesis is organized as follows:

Chapter 2 conducts a literature review of optical multicast in wavelength-routed WDM networks. Both control plane design issues and data plane design issues are covered in this chapter.

Chapter 3 presents the new multi-wavelength multicast wavelength assignment (MMWA)
algorithm. The algorithm is investigated for both full wavelength conversion and limited wavelength conversion cases.

Chapter 4 studies the multicast wavelength assignment for sparse wavelength conversion in the wavelength-routed network.

Chapter 5 investigates the multicast switch model and develops a new multicast switch scheme and a new multicast switch model. A generalized multicast wavelength assignment algorithm (MWAA) is derived to support the new model and the new multi-wavelength strategy.

Chapter 6 summarizes the main achievements and draws the conclusions of the thesis. Discussions and recommendations of future research are also presented.
Chapter 2

Literature Review

In this chapter, a comprehensive literature review of optical multicast over wavelength-routed WDM networks is provided. Optical multicast over WDM networks can be realized in two different network constructions: broadcast-and-select networks and wavelength-routed networks. A WDM local area network [23, 24] can be constructed based on the broadcast-and-select manner, which exploits a star topology with a passive star coupler at the hub and each station connecting directly to it. This topic has been extensively studied in recent years and comprehensive surveys can be found in [25–28].

Wavelength-routed WDM networks have emerged as a key technology for next-generation wide area networks. With the maturing of WDM technology, research interests have been transferred from broadcast-and-select networks to wavelength-routed networks in recent years. The issues of multicast over wavelength-routed WDM networks can be divided into two building blocks: data plane design and control plane design. A classification of major issues in these building blocks is presented in Figure 2.1. On the data plane, the key issues are power-efficient design and cost-effective design of multicast-capable OXC (MC-OXC) under
various hardware constraints. On the control plane, the major issues are the efficient multicast routing and wavelength assignment (MC-RWA). These involve various modeling and optimization processes for single multicast request as well as multiple multicast requests. Moreover, this review also covers some related issues, which represent the future research directions for this topic.

The rest of this chapter is organized as follows. Section 2.1 discusses issues of data plane design in which the architecture of MC-OXC will be introduced. Section 2.2 addresses MC-RWA problem for single multicast request. Section 2.3 handles MC-RWA problem for multiple multicast requests. For the sake of completeness, Section 2.4 briefly deals with some related issues. Finally, Section 2.5 summarizes the whole chapter.
2.1 Data Plane Design for Multicast Optical Network

The key data plane component to enable multicasting in optical layer is the light splitter. A light splitter is a passive optical device that can split an input signal without any knowledge about the optical characteristics of the input signal. A so-called $1:m$ light splitter has the capability of splitting an input signal into $m$ outputs without changing the property of the signal except signal power.

By employing light splitters in OXC, optical layer multicast can be implemented. A typical multicast-capable OXC (MC-OXC) architecture was proposed in [29], which is based on splitter-and-delivery (SAD) switches, as shown in Figure 2.2(a). In an $N \times N$ SAD switch, at each input port, light signal is first split into $N$ outputs, each of which is then either switched to a fixed output port or discarded by the optical switch. In this SAD-based MC-OXC, each input light signal can be switched to none, one, multiple, or all output ports. Thus, the SAD-based architecture is strictly nonblocking.

However, the SAD-based MC-OXC architecture may not be power-efficient or cost-effective. First, after light signal passes the $1:m$ splitter, ideally, the power of output signal at each output is only $1/m$ of input signal. In practice, splitting operation introduces additional losses such that the power of each output is lower than the ideal case. For end users to receive the signal, power of light signal should be kept above a certain level. Since light splitter is the basic device contributing power loss, a power-efficient design is needed to reduce the number of splitters in MC-OXC with minimal effects on network blocking performance. Second, when constructing nonblocking MC-OXC, use of large number of optical amplifiers and wavelength converters may lead to extremely high cost and fabrication complexity. Consequently, a cost-effective design is required to reduce both number of amplifiers and converters.
when constructing nonblocking MC-OXC. For the rest of this section, we introduce the above two aspects of data plane design, respectively.

### 2.1.1 Power-efficient Design

The overall objective of power-efficient design is to reduce the number of light splitters in the network. At the MC-OXC level, the main task is to design a new MC-OXC architecture to reduce the number of light splitters in the network. In SAD-based architecture, one light splitter is used for one wavelength from one input in a dedicated manner. The alternative is to share splitters in the design in order to reduce the number used. This is *splitter-sharing* concept, formally introduced in [30]. A multicast-only splitter-and-delivery (MOSAD) architecture was developed, as shown in Figure 2.2(b). In this architecture, unicast requests are switched separately from multicast requests, and thus do not employ any splitting. On the other hand, multicast requests will share use of light splitter in the architecture. MOSAD architecture can significantly reduce number of splitters, and thus, reducing power loss. For unicast connections, MOSAD can provide a strictly nonblocking service. However, it may
cause high blocking for multicast connections as it can only accommodate a single multicast connection at a time. Consequently, the MOSAD is only suitable in the situation where multicast requests represent a relatively small fraction of the total requests in the network.

At the network level, if only some of the OXCs are multicast-capable, it is called sparse light splitting. In this case, the objective is to minimize the number of MC-OXCs in the network with minimal effects on blocking performance for multicast requests, and to optimally place MC-OXCs in the network. Since this kind of optimal placement problem is related to routing and wavelength assignment problem for multiple requests, we leave detailed discussion to Section 2.3.

2.1.2 Cost-effective Design

There are two major challenges when constructing a cost-effective nonblocking MC-OXC. The first one is how to keep the data in the optical domain in order to eliminate the costly O-E-O conversions. To deal with this problem, we can employ an identical wavelength on which the multicast data is sent and received. However, it may be impossible to always find a common wavelength that is available in all the links along the path for any given multicast connection. Hence, it is hard to obtain good blocking performance for multicast requests in this way. Alternatively, we can employ all-optical wavelength converters to significantly improve blocking performance. However, such all-optical wavelength converters are still expensive. The other challenge lies in the use of optical amplifiers, which can compensate light splitting loss and power attenuation of light signal when traveling along the path from the source to destination. However, the complexity of fabricating large number of amplifiers into OXC may be extremely high. Consequently, we may not employ these components
arbitrarily and cost-performance tradeoff should be carefully considered when constructing the MC-OXC.

The basic idea for cost-effective design comes from multistage network design in the electronic networks [31]. It has been proven that multistage design can significantly reduce the number of network components. This conclusion can be extended to optical domain accordingly. Extensive research on this topic has been done [32–36] by the research group led by Yuanyuan Yang.

Before introducing multistage design for nonblocking MC-OXC, we first investigate different multicast models in the MC-OXC. When a light signal goes through a node, there are different multicast models to assign wavelengths to its input and output. Three models were proposed in [32], as shown in Figure 2.3. The first model is to assign the same wavelength on all output links as well as the input link. This is multicast with same wavelength (MSW), as shown in Figure 2.3(a). The second way is to assign the same wavelength to all destination nodes, but the source node may use a different wavelength. This is multicast with same destination wavelength (MSDW), as shown in Figure 2.3(b). Figure 2.3(c) illustrates the third model, multicast with any wavelength (MAW), in which the source and each of the destinations may use a different wavelength.

As pointed out in [32], cost-performance tradeoff exists between MSW and MAW models, while MSDW model is not desirable since it has the same cost as MAW but its performance is inferior to that of MAW. Therefore, based on MSW and MAW, two three-stage non-blocking networks, namely, MSW-dominant and MAW-dominant were proposed. In MSW-dominant architecture, as shown in Figure 2.4, MSW model is employed in the first two stages, so-called input stage and middle stage. In the third stage, the output stage, different
models can be used. Similarly, in MAW-dominant architecture, MAW model is used in both input stage and middle stage, and different models are used in the third stage. The MSW-dominant architecture has a lower network cost in terms of number of amplifiers and number of converters than the MAW-dominant architecture, while the latter provides a larger multicast capacity. In [33], a multicast with limited wavelength (MLW) model was proposed to extend the work of [32] to realize limited wavelength conversions in the network. Such MLW model is to be put into the third stage of the MSW-dominant architecture. Performance im-

Figure 2.3: Three multicast models in a WDM network. (a) MSW. (b) MSDW. (c) MAW.

Figure 2.4: MSW-dominant three-stage constructions for nonblocking MC-OXC.
Literature Review

Improvement obtained by limited wavelength conversion with small conversion degree is comparable to that obtained by full wavelength conversion. A four-stage Clos-type wide-sense nonblocking multicast switching network was developed in [34], which has lower hardware cost than the three-stage nonblocking multicast network. On the other hand, a two-stage design with lower delay and attenuation of signal was studied in [37]. The passive arrayed waveguide grating routers are used when constructing the two-stage architecture.

2.2 MC-RWA for Single Multicast

Given the optical network and the multicast request with source and destination nodes, the basic problem of MC-RWA for single multicast request is to find a multicast tree with minimal cost, including wavelength cost, light splitting cost and wavelength conversion cost. However, to find the minimal cost tree usually leads to complex combinatorial optimization problem. Consequently, such problem is always decomposed into different models based on the various combinations of light splitting and wavelength conversion capabilities of nodes in the network. In general, MC-RWA problem for single multicast request can be categorized into four groups, light-tree model, multi-$\lambda$-light-tree model, light-forest model and multi-drop model, as shown in Table 2.1. For the rest of this section, we introduce each of the categories accordingly.

Table 2.1: Classification of MC-RWA for Single Multicast

<table>
<thead>
<tr>
<th>No wavelength conversion</th>
<th>Full conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light-forest model</td>
<td>Multi-$\lambda$-light-tree model</td>
</tr>
<tr>
<td>Sparse splitting</td>
<td>Full splitting</td>
</tr>
<tr>
<td>Light-tree model</td>
<td>Multi-drop model</td>
</tr>
</tbody>
</table>

**Table 2.1**: Classification of MC-RWA for Single Multicast
2.2.1 Single Multicast in Light-tree Model

The simplest case for single multicast is based on the assumption that the network has full light splitting capability and no wavelength conversions are supported. In this case, multicast tree can be constructed as a single light-tree. Normally, in order to alleviate optimization complexity, a common approach is to select one or two cost metrics for optimization. Suppose we consider the case that only wavelength cost is of interest. In this case, the problem of minimizing wavelength cost can be formalized as Steiner tree problem, which is NP-complete. To deal with such NP-complete problem, some shortest path tree heuristics and minimum spanning tree heuristics have been developed [21]. Since Steiner tree problem has been well studied in graph theory, we will not introduce heuristics in detail here.

In practice, in order to guarantee that the audio or video signals can be effectively used after the transmission, delay from source to all destinations should have an upper bound. In [38], a transmission delay requirement was derived. In order to find a light-tree that has low wavelength cost and its transmission delay upper-bounded, the authors proposed a simple heuristic, which first generates a light-tree with low wavelength cost by applying a minimum spanning tree algorithm, and then modifies this tree by checking the delay requirement of each destination one at a time. As no wavelength conversion is required in this case, wavelengths on different links of a light-tree must be the same. Therefore, wavelength assignment problem in this case is to choose one common wavelength that is available on each of links in the tree. In [38], a greedy strategy is proposed to assign a currently used wavelength to the light-tree whenever it is possible.
2.2.2 Single Multicast in Multi\(\lambda\)-light-tree Model

When wavelength conversion is introduced to light-tree, different wavelengths can be assigned to different links of the tree. We name it multi\(\lambda\)-light-tree, which can be considered as an extended concept of semi-lightpath in which a transmission path is obtained by establishing and chaining several lightpaths together [39]. Wavelength conversion on a multi\(\lambda\)-light-tree is required at some intermediate nodes, but generally not at every node. In a multi\(\lambda\)-light-tree, wavelength conversion may be performed at intermediate nodes. By either means of conversion (O-E-O wavelength conversion or all-optical wavelength conversion), the extra processing delay may be introduced to the multicast session due to buffering, processing, and optic/electronic and electronic/optic conversions. Moreover, wavelength conversion may affect the quality of light signal. Therefore, MC-RWA problem in multi\(\lambda\)-light-tree model should attempt to use wavelength conversions as little as possible.

Routing problem for multi\(\lambda\)-light-tree model has been studied in [40–42]. In [40], the problem to minimize wavelength cost is formalized as wavelength cover problem in which a multi\(\lambda\)-light-tree is to be found for the given multicast request such that all links can be covered by a minimal number of wavelengths. This wavelength cover problem is proven to be NP-hard, and a two-phase heuristic was developed. First, it tries to find a minimal set of wavelengths that can cover the source and all destinations, and then, it builds a multicast tree that results in a minimal number of wavelength conversions. The routing problem to minimize both wavelength cost and wavelength conversion cost was studied in [41]. Such problem was formalized as direct Steiner tree problem. The routing problem to minimize the wavelength conversion delay was investigated in [42]. All these generalized formulations can be reduced to the constrained Steiner tree problem on some auxiliary graph, and hence can
be solved using aforementioned Steiner tree heuristics.

One the other hand, wavelength assignment problem in multi\(\lambda\)-light-tree model has been addressed in [43,44]. For a given multicast request, if the multicast tree from the source to the destinations is known beforehand, wavelength assignment problem is to assign wavelength for each link of the tree. One typical objective of this problem in multi\(\lambda\)-light-tree model is to minimize wavelength conversion cost. This problem is proven not to be NP-complete and an optimal solution was obtained in [43] by employing a two-phase process. First, wavelength conversion cost for each subtree rooted at each internal node is computed in a bottom-up order, and then based on the computation result, in a top-down order, one available wavelength on incoming link for each node will be chosen if such wavelength leads to least wavelength conversion at the subtree rooted at the node. Besides minimizing the wavelength conversion cost, a multicast-capable switch model that employed transmitters and receivers to perform O-E-O wavelength conversions was considered in [44]. Based on this model, a wavelength assignment algorithm was derived for the multi-hop networks, to minimize the maximum number of hops and number of transceivers used.

2.2.3 **Single Multicast in Light-forest Model**

Until now, all the above problems are based on the network in which all nodes have full light splitting capabilities. From the point of view of network, if parts of nodes are multicast-capable (MC) and the rest are multicast-incapable (MI), such network is called sparse splitting. Sparse splitting arises from two key reasons. First, splitting operation introduces power losses. Although amplification can partly compensate for some amount of power loss, light splitting should be used as little as possible. Second, employing optical amplifiers in OXC
will increase network cost and fabrication complexity. Due to sparse splitting, a single light-tree may not satisfy the request. In this situation, a light-forest is to be formed. A light-forest consists of multiple light-trees that have the common source node. For example, in Figure 2.5, since node TX is an MI node, in order to reach GA, a separate path from CA2 to GA has to be set up.

Given the sparse splitting network (i.e., the exact number and allocation of MC nodes in the network) and multicast request, multicast routing problem is to find a light-forest, which may consist of multiple light-trees, to span all destinations. In general, existing routing schemes for sparse splitting can be classified into: Source-based routing and Powerful Node (PN)-based routing. Source-based routing schemes are based on Steiner tree construction algorithm, with modifications for adapting to the sparse splitting constraint. In PN-based routing schemes, some nodes in the network are considered as powerful nodes. The powerful node is a general definition to the node with either(both) light splitting or (and) wavelength conversion capabilities. That is to say that, the PN nodes may simply refer to the splitting nodes in the network [49] or wavelength conversion nodes [74]; and may also refer to virtual source (VS) nodes, which possess both light splitting and wavelength conversion capabilities.
For the PN-based routing schemes, tree construction is first made around such powerful nodes, and then each of the other nodes in the multicast group separately joins the tree.

### 2.2.3.1 Source-based Routing Algorithms

In this scenario, source node is assumed to have full splitting capability and the key idea of building a light-forest is to build one light-tree using the Steiner tree heuristics at a time until all destination nodes are included. Hence, different tree construction schemes lead to different solutions in this kind of source-based routing. Four approaches were developed in [45], among which Member-First and Member-Only heuristics were most cited. In each of iterations in Member-First algorithm, one tree is constructed link by link by Dijkstra’s algorithm [46] for constructing a spanning tree. The candidate links are organized in a priority queue, where a link leading to a member has a higher priority. The tree adjustment is carried out after a link is added, in order to expand the tree to MC only or leaf-MI node. If this rule is violated, then the affected nodes and links have to be detached from the multicast tree and wait for future expansion. When all the members are included, the algorithm stops spanning the tree and starts pruning those branches that do not lead to any member. This algorithm is much similar to the Prim’s algorithm [46] for constructing minimum spanning tree. On the other hand, unlike Member-First algorithm, in Member-Only algorithm, a light-tree is constructed by including members one at a time (the closest member first), and thus eliminating the need for pruning after all the members are included. The basic idea of Member-Only is similar to that of the shortest-path heuristic [46] for constructing a near-minimum multicast tree with the main feature that as long as an MI node on a tree is a non-leaf node, other members will not join the tree.
A modification of the Member-Only heuristic has been made in [47] by resorting to Tap-and-Continue (TaC) crossconnects [48] when another separate tree is to be generated in Member-Only heuristic. This TaC architecture allows light power to be tapped while forwarding data to one output. By this means, a single transmitter and a single wavelength are used for one multicast request, and thus, the resource utilization is improved.

### 2.2.3.2 Powerful Node (PN)-based Routing Algorithms

PN-based routing algorithms first construct light-tree around some powerful nodes in the network, and then each of the remaining destination nodes joins in the tree separately. Different light-tree construction algorithms for powerful nodes are available; meanwhile, there are different strategies for non-powerful node to join in the tree. The PN nodes may simply refer to the MC nodes in the network [49] and an MC-tree is constructed first by connecting all the MC nodes from the source node, and then MI nodes with the remaining members joining the tree at the nearest on-tree MC nodes.

On the other hand, PN nodes may also refer to virtual source (VS) nodes, which possess both light splitting and wavelength conversion capabilities [50]. The whole procedure is separated into two phases, namely, network partitioning phase and tree generation phase. The network is partitioned into regions based on the vicinity of the VS nodes. Then, every node in the network needs to find the nearest VS from it and establish a connection to the VS. Next, tree generation phase begins. This phase makes use of the connectivity provided in the previous phase, so it is quicker. The source chooses a VS node, primary virtual source (PVS), to establish a connection. All other VS nodes that have destinations connected to them are called secondary virtual source (SVS). The PVS and SVSs establish connections to
the destinations. The main advantage is its short setup time. The main disadvantage is that it requires reservation for pre-established paths in the first phase.

In general, compared with source-based routing, PN-based approaches can save up more wavelength cost. However, the problem is that performances of PN-based approaches sometimes are determined by the number and location of PN nodes in the network.

2.2.3.3 Routing under Power Consideration

As mentioned previously, a light signal suffers power loss when passing through an MC node due to the light splitting. Moreover, the signal attenuation along the path from source to destination may be significant. Transmitting power and routing scheme should be carefully designed to guarantee the signal to be delivered to destinations [51]. In practice, optical amplifiers can be used to compensate such splitting loss and power attenuation, however, commercial products of such devices are still expensive and the fabrication complexity may be extremely high. Consequently, multicast routing problem in sparse splitting network under power consideration should be treated as an important research topic.

The Centralized-Splitting algorithm [52] was developed based on the consideration of suitable location to place the MC nodes in a light-tree. First, when constructing a light tree, if in a particular sub-tree that has more than one MC nodes, the power loss will have a multiplicative effect. Therefore, the suggestion is that if concatenated MC nodes in the particular sub-tree exist, it is better to be replaced by single MC node. Second, the MC node should be assigned far from the source node, if possible. Third, if a node is chosen to be a splitting node, the more number of splitting assigned at this node is preferable. That is to say that, once an MC node is chosen to perform splitting, it is better to make full use of it. Based on
these considerations, the Centralized-Splitting algorithm first makes use of the Member-Only algorithm to construct a light-forest that may consist of one or more light-trees. Then, according to each light-tree in the light-forest, the above three considerations will be applied to modify each tree.

The Balance Light-tree algorithm [53] has been developed based on the concept of split ratio of a node, which is derived to represent the residual power of the light signal received at this node after all the splits along the path. The basic idea of the algorithm is similar to the Centralized-Splitting algorithm. First, the spanning tree from source node to all destination nodes will be constructed by using existing algorithm with the consideration of wavelength cost. Then, for each node in the tree, the split ratio will be considered by checking the validation of power budgets for each light-tree constructed.

2.2.4 Single Multicast in Multi-drop Model

In light-tree model, at each branching node, light signal can be split into multiple outputs. Thus, the light-tree can be used to transmit a message from a source to an arbitrary number of destinations using a single wavelength. In practice, each time the message is split at a node, a splitting loss is incurred which reduces the power of the signal at each of the outputs. Thus, while the power budget may allow a message on a given wavelength to be “dropped off” (or “delivered to”) more than one destination, it may not be possible to drop off the message at an arbitrary number of destinations using a single lightpath or light-tree [54].

Multicast in multi-drop model permits some specified maximum number of drops, $k$, per transmission. The value of $k$ is dictated by the power budget. In multi-drop path model, a message is assumed to be transmitted on a given wavelength and can be delivered to any $k$
destinations on a path. In the multi-drop tree model, the message is assumed to be transmitted on a given wavelength and may be split at intermediate nodes and can be delivered to \( k \) destinations in a tree. Under this definition, multicast routing problem becomes a problem of finding a set of lightpaths (light-trees) such that at most \( k \) destination nodes are designated to receive data in each lightpath (light-tree) and every destination node is designated in one of lightpaths (light-trees). The objective is to minimize the total cost of lightpaths (light-trees). Therefore, in this multi-drop model, for a given multicast request, the multicast tree may consist of multiple lightpaths (light-trees) \[55, 56\]. Since this topic is of limited interest, we shall not provide further review here.

### 2.3 MC-RWA for Multiple Multicasts

Instead of dealing with a single multicast request, MC-RWA problem in multiple multicasts scenario attempts to arrange a batch of multicast requests. In the case of single multicast, the problem focuses on satisfying the request itself. However, for multiple requests, the problem should try to optimize individual request and also consider the whole set of requests as a combined optimization problem. MC-RWA problem can be cast in several forms. However, the different variants of the problem can be roughly classified into two categories: static case and dynamic case. In the static scenario, the set of requests is known in advance; in the dynamic counterpart, the sequence of requests arrives in some random manner.
2.3.1 MC-RWA for Static Requests

Static MC-RWA is appropriate for provisioning a set of semi-permanent connections. Since these connections are assumed to remain in the network for a relatively long period, computation can be performed off-line. In this scenario, the most important optimization objective is to minimize total network resource consumptions. The solution to static MC-RWA problem consists of a set of long-lived multicast trees that creates a logical (or virtual) topology, which is embedded onto the physical topology of fiber links and OXCs. Consequently, static RWA problem is always referred to as virtual topology design problem in literature [57, 58]. In this problem, routing and wavelength assignment can be formulated as mixed integer linear programming (MILP) problem [10]. Usually, these MILP formulations generate very large numbers of variables, and are intractable for large networks [59]. This fact has motivated the development of heuristic approaches for finding good solutions efficiently. In fact, static MC-RWA problem can be decomposed into three subproblems as follows:

- **Topology Subproblem**: Determine the lightpaths/light-trees in terms of their source and destination nodes.

- **Routing Subproblem**: Route the lightpaths/light-trees over the physical topology.

- **Wavelength Assignment Subproblem**: Assign a wavelength to each lightpath/light-tree in the logical topology so that wavelength restrictions are obeyed for each physical link.

This decomposition may not produce satisfactory solution as solving the subproblems separately and then combining them may not produce the optimal solution for the fully integrated problem. However, such decomposition can provide insight into the structure of MC-RWA problem and simplify the complexity of solving integrated problem.
2.3.1.1 Routing for Static MC-RWA

For the given set of multicast requests, routing problem is to build light-trees for each of the requests, such that either the number of wavelengths needed to accommodate all the requests is minimized or the number of multicast requests established is maximized for a limited number of wavelengths. In [60], the problem to maximize the number of multicast requests admitted was formulated as a nonlinear integer programming problem. Two heuristic algorithms were proposed. One makes use of the idea that it first accommodates only complete multicast groups, and then tries to serve as many users as possible by allowing partial accommodation. This approach is based on the integer linear programming formulations. The other heuristic is relatively straightforward. It tries to accommodate the group having the largest number of users that can be served using certain wavelength at each of iterations.

For a specific topology, the multiple multicasts problem in WDM ring network without considering the light splitting and wavelength conversion was studied in [61]. A heuristic based genetic algorithm was developed to overcome the NP-completeness of the problem.

2.3.1.2 Wavelength Assignment for Static MC-RWA

Given a set of light-trees, assign a wavelength to each light-tree such that no two light-trees share the same wavelength on a given fiber link. In this static case, optimization objective is always to minimize the number of wavelengths used for the given light-trees. One approach to solving this problem is to formulate it as a vertex-coloring problem [62]. In this approach, each light-tree in original graph is represented by a vertex in an auxiliary graph. There is an undirected edge between two vertices in the auxiliary graph if and only if two trees share a common link in the original graph. By coloring all the vertices in the auxiliary graph
such that no two adjacent vertices have the same color, the static wavelength assignment subproblem can be reduced to vertex-coloring problem. This problem has been shown to be NP-complete. A simple heuristic based on sequential coloring approach is used in [62]. The basic idea of this approach is that for each iteration, choose a vertex that has the least degree and find a maximal set of vertices that is not adjacent to the selected vertex and each other. By reducing the static wavelength assignment problem to vertex-coloring problem, to minimize the number of wavelengths for the given light-trees becomes a pure graph theory problem. Consequently, one can apply different heuristics of vertex-coloring problem in graph theory.

2.3.1.3 Virtual Topology Design for Static MC-RWA

Traditionally, the virtual topology embedded onto the physical network is lightpath-based, in which each link represents a lightpath between two nodes. Communication between two nodes may happen either through a single lightpath (i.e., a direct link in the logical topology) or through a number of concatenated lightpaths (i.e., so-called semi-lightpath). Previous research on logical topology design with only unicast traffic has focused on lightpath-based logical topologies. In [63–65], multicast traffic has been introduced to this lightpath-based virtual topology design problem. However, their approaches simply transform the multicast traffic matrix into a pure unicast traffic matrix by assuming that each multicast request is established by multiple lightpaths from the source to each of the individual destinations. Strictly speaking, those approaches cannot be treated as genuine optical multicast as their solutions are all based on unicast scenario. The light-tree-based virtual topology design problem for multicast traffic was addressed in [66]. By considering the transmission delay of light-trees, the problem was formulated by using MILP. Unfortunately, no heuristics were proposed to
overcome the complexity of the MILP. In summary, the light-tree-based virtual topology design problem for multicast traffic is an area deserving further research in the future.

2.3.2 MC-RWA for Dynamic Traffic

Under the dynamic traffic scenario, the multicast requests arrive one by one at random. Ideally, we only need to deal with each individual multicast request at the time when it arrives, and thus, all the solutions discussed in the previous subsection for single multicast request can be adopted in this case. However, in practice, some additional constraints should be taken into account.

- **Resource Constraint:** In the single multicast case, the network resource, e.g., number of wavelengths, is an optimization objective. However, in the dynamic case, the number of wavelengths is always a resource constraint instead. That is, some of network resources may be occupied by the existing multicast requests and have not been released for the use of new incoming request. Hence, even if one can find a multicast tree to route the new request, there may not be available wavelength(s) to be assigned to the tree. If a request cannot be accepted due to the lack of resources, it is blocked.

- **Time Constraint:** Due to the real-time nature of the dynamic problem, the MC-RWA algorithms must be very simple and time-efficient. However, as discussed previously, the MC-RWA problem for single multicast and for static multiple multicasts are both very complex problems. Using either Steiner tree approach or MILP formulations will lead to large amount of computation time. Such time may be acceptable by off-line computation for static case, but not suitable for the dynamic traffic.
Based on the above two constraints, the objective of MC-RWA for dynamic traffic usually aims to minimize the blocking probability. There are two different blocking policies introduced in literature: *full destination blocking* and *partial destination blocking*. Under full destination blocking policy, a multicast request is accepted if and only if all the destinations of the request can be reached, i.e., if one arbitrary destination fails to be reached, the whole request will be blocked. This policy is suitable for applications such as distributed computing and videoconferencing in which all the destinations must be reached for the communication activity to take place. Under this policy, the appropriate performance metric is *session blocking probability*, defined as the probability that an arriving multicast request is blocked. For some other applications which may not require all multicast members to be reached for a communication activity, such as video-on-demand. In this case, the partial destination blocking policy is more reasonable, with which if part of destinations cannot be reached, the whole request will not be blocked. The performance metric is so-called *destination (user) blocking probability*, which is defined as the probability that a destination in the group is blocked. The comparison of these two policies is illustrated by a simple example with three multicast

\[
\begin{align*}
&\text{Request (A: B, C, D, E)} \quad \text{Request (B: C, D, E)} \quad \text{Request (D: B, C, A)} \\
&\text{Session blocking probability} = 1/3 \quad \text{Destination blocking probability} = 1/10
\end{align*}
\]

Figure 2.6: Comparison of two blocking policies.
sessions as shown in Figure 2.6. For session blocking probability, as node A in session (D: B, C, A) cannot be reached, the whole session will be blocked. So, we have one session out of 3 sessions blocked, the session blocking probability should be 1/3. On the other hand, for destination blocking case, only the node A will be blocked instead of the whole session. Since there is only one blocked destination among all 10 destinations of the three sessions, the destination blocking probability is 1/10.

2.3.2.1 Routing for Dynamic MC-RWA

Three routing schemes were suggested in [67], fixed routing, alternative routing (or fixed-alternate routing) and dynamic routing (or adaptive routing). Fixed routing comes from the static approach in which a multicast tree is pre-calculated for each of possible multicast requests using a certain tree construction algorithm. When a request arrives, if such route is not available, the request will be blocked. Alternate routing [68] gives more flexibility in route selection by assigning a set of routes to each possible connection request. Here, a request is blocked only if no route in the assigned set is available at the request arrival time. In both of the schemes, the computation of candidate route can be off-line, such that the result may be more cost-efficient in terms of network cost. However, the utilization of the links may change dynamically. Thus, such routing algorithms that do not make use of the current state of the network are likely to suffer a performance penalty. A dynamic routing [39] algorithm is based on the current network state when computing the route for the arriving request. Therefore, the route chosen will reflect the current utilization of the links in the network and a connection is blocked only if there is no available route to carry it.

Among the above three schemes, dynamic routing performs the best, while fixed routing
performs the worst. The reason is as follows. The two static approaches, which are based on off-line route computation, may find the optimal solution for each individual request. However, the overall result, especially the blocking performance, may not be optimal. The dynamic routing can make the best use of the wavelengths by adaptively building multicast trees according to the current wavelength usage on the links, and therefore achieves the best performance among the three. Moreover, the alternate routing outperforms fixed routing since it provides more choices in choosing the multicast tree and hence leads to a better usage of the wavelengths. In terms of time-efficiency, the dynamic routing needs highest computation time, while fixed routing is the simplest. Consequently, choosing which routing scheme in practice becomes a tradeoff between performance and efficiency.

2.3.2.2 Wavelength Assignment for Dynamic MC-RWA

For the wavelength assignment subproblem, different from minimizing the number of wavelengths used in the static scenario, the number of wavelengths is fixed (or given). By searching the set of available wavelengths in some order, we attempt to minimize the connection blocking. One can find many searching heuristics suggested in [13], such as Random, First-Fit, Least-Used, Most-Used and so on. Although these algorithms consider the unicast case, they can be employed in the multicast case. The performance of first-fit wavelength assignment scheme combined with the fixed routing approach was evaluated by simulation in [69].

Besides minimizing the connection blocking probability, the objective of dynamic wavelength assignment problem also aims to maximize the usage of network capacity which is defined as the summation of all the path capacities that are represented by the number of available wavelengths along the paths. Once the usage of network capacity is maximized,
more potential requests can be accommodated in future. Based on this concept, the wavelength assignment to maximize the usage of network capacity problem was studied in [70] and two heuristics were proposed based on the concept of wavelength cost ratio.

### 2.3.3 Placement of PN Nodes

As mentioned in the previous section, the nodes in the network can be grouped into: nodes with no splitting and wavelength conversion capabilities, nodes with only splitting capability, i.e., splitting nodes, nodes with only wavelength conversion capability, i.e., conversion nodes, and nodes with both capabilities, i.e., virtual sources. The latter three types of nodes are all considered as powerful nodes. The problem of placement of powerful nodes in optical networks is motivated by the expected high cost of the key optical components, fabrication complexity and power considerations.

Given the number of MC nodes and a set of multicast requests, the splitting node placement problem is to maximize the number of sessions routed successfully in the network by allocating all MC nodes in the network. This kind of placement problem contains the static multicast routing problem discussed in the previous subsection. Specifically, if the given number of MC nodes equals the total number of nodes in the network, such placement problem is reduced to pure virtual topology design problem, which is known as NP-complete. Therefore, this placement is also an NP-complete problem. In order to overcome the complexity of such NP-complete problem, the solution to the splitting node placement problem can be divided into two phases. First, the static multicast routing and wavelength assignment problems are solved under full light splitting situation. Second, based on solutions to MC-RWA problem, the placement heuristic is derived. Two heuristics, Most-Saturated Node
First and Simulated Annealing, have been proposed in [71]. The Multicast-ADD algorithm has been developed in [72]. These three placement algorithms do not consider the power constraints. By considering the power loss and signal attenuation, the optimal splitting nodes placement problem was formulated as an MILP in [73].

Besides allocating the splitting nodes in the network, the wavelength converter placement problem was also studied [74–76]. However, the wavelength converter problem has been considered only for unicast traffic pattern. More work is to be done for multicast traffic. Moreover, the problem of considering jointly the splitting nodes placement and wavelength converters placement, and the problem of allocating the virtual sources in the network, remain untouched and more research efforts are expected in the near future. A recent study on the problem of considering jointly the splitting nodes placement and wavelength converters placement can be found in [77].

\section*{2.4 Related Issues}

\textbf{Multicast Performance Evaluation} – To evaluate the performance of optical multicast in the wavelength-routed network, two main metrics are involved, number of wavelengths used and blocking performance. The bounds on the minimum number of wavelengths needed for nonblocking multicasting in WDM networks were addressed in [35, 36, 78]. On the other hand, the modeling of blocking performance of multicast requests was presented in [79–82]. The modeling of blocking performance of multicast in the network can provide a test bed for the evaluation of different tree construction and wavelength assignment approaches under the dynamic traffic pattern. In addition, the modeling under different wavelength conversion capabilities can provide a useful feedback to how to select the nodes and where to employ
converters. Moreover, the performance modeling with different multicast models can provide the tradeoff information between the network resources and blocking performance.

**Multicast Session Protection** – The protection and restoration problems for the survivable point-to-point requests in the WDM network have been studied extensively in recent years. However, not much work has been done to protect the multicast sessions in the wavelength-routed mesh networks [83–86]. Compared with the unicast case, it is more important to protect the multicast sessions since a single fiber cut on the network can disrupt the transmission of information to many destinations. Therefore, the light-tree based multicast session protection problem deserves a comprehensive study.

**Multicast Traffic Grooming** – The bandwidth request of a traffic stream can be much lower than the capacity of an optical channel in the WDM networks. Hence, efficiently grooming low-speed connections onto high-capacity lightpaths will improve the network throughput and reduce the network cost. This traffic grooming problem has existed in telecommunication industry for years, however, the study on traffic grooming in WDM network has been ongoing only in recent years [87–92]. The multicast traffic grooming problem was only investigated in [93–95] and further investigations and novel algorithms are required to exploit the grooming capability and optimize the network resources efficiently.

### 2.5 Summary

In this chapter, we have provided a comprehensive review on optical multicast over wavelength-routed WDM networks. Unlike existing surveys on optical multicast, which mainly focus on broadcast-and-select single-hop networks, our review considers the issues relating to the wavelength-routed networks. This represents the current interests in optical networking tech-
nology. We divide the problem domain into two building blocks: data plane and control plane. On the data plane, the key problem is to design power-efficient and cost-effective multicast-capable OXC under hardware constraints of key components, such as light splitters, optical amplifiers and wavelength converters. On the control plane, the main thrust is to solve the multicast routing and wavelength assignment (MC-RWA) problem. We first review various routing and wavelength assignment algorithms for the single multicast request. The solution to the single multicast request is crucial because it lays the foundation to tackle more complex problems such as virtual topology design, multicast session protection and multicast traffic grooming, etc. Then we address the MC-RWA problem for multiple multicast requests under static as well as dynamic traffic conditions, respectively. With regards to methodology, the techniques of solving the MC-RWA problem can be grouped into two categories. One attempts to extend the solutions of unicast problem to multicast case, and the other aims to generalize the problem formulation analytically using mathematical theories. To the best of our knowledge, it is the first time that multicast routing and wavelength assignment problem has been reviewed extensively in such breaths and depths.
Chapter 3

Multi-wavelength Multicast Wavelength Assignment

In this chapter, we study multicast wavelength assignment problem in wavelength-routed WDM networks, considering full light splitting and full/limited wavelength conversion capabilities. The multicast wavelength assignment problem is the second phase of multicast routing and wavelength assignment (MC-RWA) problem. Normally, the MC-RWA problem is solved by decomposing the problem into multicast routing subproblem and wavelength assignment subproblem. The solution to the routing subproblem is a routing tree (multicast tree). This routing tree is the input to the wavelength assignment subproblem. Since this chapter focuses on the wavelength assignment problem, we can make the assumption that the input routing tree is given spanning from the source node to multiple destination nodes. The multicast wavelength assignment problem is how to allocate available wavelengths on links of the tree under the constraints of certain cost metrics. The cost metrics [41–43] include wavelength cost, wavelength conversion cost and light splitting cost. Among them,
wavelength cost and wavelength conversion cost dominate, as analyzed in [30, 43, 70], since light splitting cost is relatively less than wavelength cost and conversion cost since light signal can always be amplified by using optical amplifiers if necessary. Thus, light splitting cost is often omitted when considering cost issues. Between wavelength cost and wavelength conversion cost, there exists a tradeoff: minimizing wavelength cost may lead to a higher wavelength conversion cost and vice versa. Therefore, tradeoff between wavelength cost and wavelength conversion cost should be carefully considered when designing a wavelength assignment scheme. In this chapter, we take a new wavelength assignment approach by allowing multiple available wavelengths in a link to carry the multicast signal. Based on this new multi-wavelength assignment strategy, we propose a new multicast wavelength assignment algorithm to minimize the wavelength conversion cost while keeping wavelength cost within a reasonable bound. The outcome of this algorithm provides a good tradeoff between wavelength cost and wavelength conversion cost. Furthermore, we realize that multi-wavelength assignment strategy provides extra advantages for the network with limited wavelength conversions. We then extend the algorithm from full wavelength conversion case to limited wavelength conversion case.

The rest of the chapter is organized as follows. In Section 3.1, we introduce the basic idea of multi-wavelength assignment strategy. In Section 3.2, we provide a formal definition of the problem. In Section 3.3, we derive the new multicast wavelength assignment algorithm based on full wavelength conversion case. In Section 3.4, we extend the algorithm to limited wavelength conversion case. In Section 3.5, we carry out a simulation study. Finally, in Section 3.6, we summarize this chapter.
3.1 Multi-wavelength Assignment Strategy

The approach to MC-RWA problem is usually based on a multicast switch model. The multicast switch model assumes that an input signal is first split into multiple output links using a light splitter, and then, wavelength conversion is applied to light signal at each output link if it is needed. Based on this switch model, multicast wavelength assignment algorithm can be derived. Current wavelength assignment algorithm [43] is based on the single-wavelength assignment strategy which assumes that only a single available wavelength can be used on a link. This assumption is rather restrictive, that in turn limits the performance of algorithm. The alternative approach is to make use of multiple available wavelengths in a link to carry the message [44]. By using the multi-wavelength assignment strategy, we can achieve a trade-off between the number of wavelengths and number of wavelength conversions of a multicast tree. Such kind of tradeoff was first recognized in [82]. Nonetheless, to the best of our knowledge, no wavelength assignment algorithm has been developed in literature for multi-wavelength assignment strategy based on full and limited wavelength conversion capabilities, considering the cost tradeoff issues between wavelength cost and wavelength conversion cost. In general, the idea of minimizing wavelength conversion cost by the tradeoff of wavelength cost comes from the comparison between these two kinds of costs. As we know, the wavelength conversion can be realized in two ways [100], the O-E-O conversion and the all-optical conversion [101].

1. The O-E-O conversion is relatively easy to implement. However, on one hand, extra processing delay is introduced due to buffering, processing and O-E and E-O conversions. For some delay-sensitive multicast applications, such as real-time remote surgery, it is crucial to minimize such processing delay. Thus, in this situation, to min-
imize the wavelength conversion has higher priority. On the other hand, the O-E-O conversion sacrifices the transparency of the data stream. We know that it is one of key issues to keep the data transmission transparent. Thus, in summary, if the wavelength conversion is performed by O-E-O conversion, it is beneficial to minimize the wavelength conversions by the tradeoff of some wavelengths.

2. The technology of all-optical conversion is not commercially-mature yet [101]. The cost of commercial all-optical wavelength converters remains prohibitively high. An approximate estimation in [109] suggests that the typical value of the ratio of the cost of a wavelength converter over a unit-length fiber with 8 wavelengths is 25. Thus, the ratio of the cost of a wavelength conversion over a wavelength can be roughly considered as $200 (= 25 \times 8)$. This is quite high. Thus, to minimize the overall cost, it is beneficial to minimize the wavelength conversions by the tradeoff of some wavelengths.

The basic advantage of multi-wavelength assignment strategy is that the use of multiple wavelengths for multicast can lead to a reduction of wavelength conversion cost. A simple example is illustrated in Figure 3.1. Consider a simple multicast tree as shown in Figure 3.1(a). The wavelengths $\lambda_1$, $\lambda_2$, and $\lambda_3$ are available on the input link of node $v$. $\lambda_1$ and $\lambda_3$ are available on the input link of node $x_1$, and $\lambda_2$ is available on the input link of node $x_2$. By single-wavelength assignment strategy, only one of the three wavelengths is assigned to the input link of node $v$, thus, one wavelength conversion at node $v$ is required. For instance as shown in Figure 3.1(b), if $\lambda_1$ is assigned to the input link of node $v$, then wavelength conversion is required between $\lambda_1$ and $\lambda_2$ for the signal transmitting from node $v$ to node $x_2$. However, if we use both $\lambda_1$ and $\lambda_2$ to carry the signal to node $v$, there should be no wavelength conversion needed, as shown in Figure 3.1(c).
Based on this idea, we develop a new multicast wavelength assignment algorithm that makes use of multiple wavelengths to carry the same light signal on the same link. This will reduce the need for wavelength conversion. On the other hand, multi-wavelength assignment strategy tends to use more wavelengths than single-wavelength assignment strategy. Thus, tradeoff between wavelength cost and wavelength conversion cost should be addressed.

To realize the multi-wavelength assignment strategy, the light signal needs to be copied to different wavelengths on a link. This type of operation can be realized by *two-stage light splitting* as suggest in [45]. In fact, there are different switch models and architectures supporting multi-wavelength assignment strategy. But this issue actually belongs to the data plane issue which is not the focus of this thesis. We only provide some necessary introduction here. In an example architecture, as shown in Figure 3.2, the input signal experiences two-stage light splitting operations. Thus, it is possible to send multiple (up to M) “copies” to the same output port using different wavelengths by letting multiple switches connected to the same input signal. For example, the thick lines in the figure show that the input signal makes two copies, $\lambda_1$ and $\lambda_M$, to the same output port 1.
3.2 Multicast Wavelength Assignment Problem

The optical networks can be represented by a directed graph \(G(V, E)\) where \(V\) is the set of nodes (vertices) and \(E\) the set of links (edges) between any two nodes. Multicast traffic in WDM networks can be described in terms of multicast sessions where a set of nodes are engaged in some activity that requires the transfer of data. In the directed graph, a pair \((s, Y)\) represents a multicast session, where \(s\) is the source and \(Y\) the set of destinations. In each multicast session, a multicast tree \(T(V_T, E_T)\) can be formed. The tree \(T\) has \(s\) as the root node and spans all destination nodes in \(Y\).

In each multicast tree \(T\), \(N\) denotes the total number of nodes, i.e., \(N = |V_T|\). Thus, the total number of links in the tree is \(N - 1\), i.e., \(|E_T| = N - 1\). Suppose all the \(N\) nodes have a topological order \(1, 2, \ldots, N\), beginning from the root node by a breadth-first search.
Let $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_W\}$ denote the set of all $W$ distinct wavelengths supported in the tree. Each link in the tree has some available wavelength(s) that can be used to transfer data. We define the available wavelength set on the input link of non-root node $v$ as $A(v)$, and use $L$ to denote the maximum number of available wavelengths on each link of the tree.

Moreover, we use $D(v)$ to denote the out-degree of node $v$ in the tree $T$ and define $D$ to be the maximum out-degree of nodes in the tree, i.e.,

$$D = \max_{v \in V_T}\{D(v)\}$$ (3.1)

As for multi-wavelength assignment strategy, we define $K$ to be the maximum number of wavelengths permitted to carry the same signal on a single link, and $1 \leq K \leq L$. $K = 1$ is the case of single-wavelength assignment and $K \geq 2$ stands for the multi-wavelength assignment case. For example, $K = 2$ means that at most two wavelengths can be used to transmit the same signal on the same link. Actually, the single-wavelength assignment strategy is a special case of multi-wavelength assignment strategy.

Let $S$ denote the wavelength set taking the number of wavelengths not larger than $K$. Assuming we have $Q$ wavelength sets, the set of all the wavelength sets is denoted as $\Phi = \{S_1, S_2, \ldots, S_Q\}$, where the number $Q$ is given by

$$Q = \sum_{j=1}^{K} \binom{W}{j}$$ (3.2)

For a given multicast tree $T$ and the available wavelength set $A(v)$ of all links in the tree, the multicast wavelength assignment is to select a wavelength subset $F(v) \subseteq A(v)$ for each link, in order to transmit the signal from the root node to all destination nodes in the tree.
Therefore, for a given multicast tree $T$, a wavelength assignment, $\pi_T$, is defined as the set of assigned wavelength sets for each link in the tree, i.e.,

$$\pi_T = \{F(v) | F(v) \subseteq A(v), v \in V_T - \{s\}\}$$ (3.3)

For a link $e$ and an assigned wavelength $\lambda_i$ on $e$, a nonnegative weight $w(e, \lambda_i)$ denotes the cost of using $\lambda_i$ on link $e$. Thus, the total wavelength cost for the tree $T$ under the wavelength assignment $\pi_T$ is,

$$C_W(\pi_T) = \sum_{v \in V_T - \{s\}} \sum_{\lambda_i \in F(v)} w(e, \lambda_i)$$ (3.4)

For simplicity, we assume that the weight for each wavelength on each link is one, i.e., $w(e, \lambda_i) = 1$, then, the total wavelength cost equals the total number of wavelengths used in the tree $T$, i.e.,

$$C_W(\pi_T) = \sum_{v \in V_T - \{s\}} |F(v)|$$ (3.5)

Take the specific case of single-wavelength assignment strategy. In this case, only one wavelength is assigned on one link. Thus,

$$C_W(\pi_T) = \sum_{v \in V_T - \{s\}} |F(v)| = \sum_{v \in V_T - \{s\}} 1 = N - 1$$ (3.6)

Next, we define the cost of wavelength conversion from input wavelength $\lambda_p$ to output wavelength $\lambda_q$ at node $v$ in the form of $c_v(\lambda_p, \lambda_q)$. The conversion between any two different
Multi-wavelength Multicast Wavelength Assignment

wavelengths has the same cost $c_c$. Thus, we have

$$c_v(\lambda_p, \lambda_q) = \begin{cases} c_c, & \text{if } \lambda_p \text{ and } \lambda_q \text{ are both available and } p \neq q \\ 0, & \text{if } \lambda_p \text{ and } \lambda_q \text{ are both available and } p = q \\ \infty, & \text{if either } \lambda_p \text{ or } \lambda_q \text{ is not available} \end{cases} \quad (3.7)$$

For simplicity, we assume that $c_c = 1$, such that the total wavelength conversion cost for tree $T$ under wavelength assignment $\pi_T$, $C_C(\pi_T)$, equals the total number of wavelength conversions.

Based on the above definitions, we formalize the definition of the multicast wavelength assignment problem as follows:

**Definition:** Given a multicast tree $T(V_T, E_T)$ with root node $s$ and available wavelength set $A(v)$ on the input link of each non-root node $v$, and the value of $K$, i.e., the maximum number of wavelengths allowed to carry the signal on a link, the multicast wavelength assignment problem is to find a wavelength assignment $\pi_T$ by assigning the wavelength set $F(v) \subseteq A(v)$ on the input link for each non-root node of the tree. For the given value of $K$, the objective is to minimize wavelength conversion cost while keeping wavelength cost below a bound. By this means, the algorithm can minimize the usage of wavelength conversions and at the same time restrict the number of wavelengths used to a reasonable range.

The notation we use is summed up in Table 3.1 and is illustrated using the example tree as shown in Figure 3.1(a).

The multicast tree in Figure 3.1(a) has 4 nodes, i.e., $N = 4$, and $V_T = \{s, v, x_1, x_2\}$, where $s$ is the source node and $v, x_1$, and $x_2$ are destination nodes, $Y = \{v, x_1, x_2\}$. Suppose three wavelengths are supported by each link in the tree ($W = 3$), thus $\Lambda = \{\lambda_1, \lambda_2, \lambda_3\}$. 
Table 3.1: Notation Used

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Multicast tree</td>
</tr>
<tr>
<td>V_T</td>
<td>Set of all nodes in the tree</td>
</tr>
<tr>
<td>E_T</td>
<td>Set of all links in the tree</td>
</tr>
<tr>
<td>(s, Y)</td>
<td>s: Source node, Y: set of destination nodes</td>
</tr>
<tr>
<td>N</td>
<td>Number of nodes in the multicast tree</td>
</tr>
<tr>
<td>W</td>
<td>Number of distinct wavelengths supported in the tree</td>
</tr>
<tr>
<td>Λ</td>
<td>Set of all supported wavelengths in the tree</td>
</tr>
<tr>
<td>A(v)</td>
<td>Set of available wavelengths on the input link of node v</td>
</tr>
<tr>
<td>L</td>
<td>Maximum number of available wavelengths on each link in the tree</td>
</tr>
<tr>
<td>D(v)</td>
<td>Out-degree of node v</td>
</tr>
<tr>
<td>D</td>
<td>Maximum out-degree of nodes in the tree</td>
</tr>
<tr>
<td>K</td>
<td>Maximum number of wavelengths permitted to carry the same message on a single link</td>
</tr>
<tr>
<td>Q</td>
<td>Number of distinct wavelength sets taking the number of wavelengths not larger than K</td>
</tr>
<tr>
<td>S</td>
<td>Wavelength set taking the number of wavelengths not larger than K</td>
</tr>
<tr>
<td>Φ</td>
<td>Set of all the wavelength sets taking the number of wavelengths not larger than K</td>
</tr>
<tr>
<td>F(v)</td>
<td>Set of assigned wavelengths on input link of node v</td>
</tr>
<tr>
<td>B(v)</td>
<td>Set of child nodes of node v</td>
</tr>
<tr>
<td>π_T</td>
<td>Wavelength assignment to tree T</td>
</tr>
<tr>
<td>C_W(π_T)</td>
<td>Total wavelength cost for tree T under wavelength assignment π_T</td>
</tr>
<tr>
<td>C_C(π_T)</td>
<td>Total wavelength conversion cost for tree T under wavelength assignment π_T</td>
</tr>
</tbody>
</table>

The available wavelength set for each link is shown in the Figure 3.1(a), thus, \( A(v) = \{\lambda_1, \lambda_2, \lambda_3\} \), \( A(x_1) = \{\lambda_1, \lambda_3\} \) and \( A(x_2) = \{\lambda_2\} \). Assume \( K = 2 \), then, on each link, one wavelength set \( S \) can be assigned, which takes no more than 2 wavelengths. The number of wavelength sets is given by

\[
Q = \sum_{j=1}^{2} \binom{3}{j} = \binom{3}{1} + \binom{3}{2} = 6
\]

Thus, there are 6 wavelength sets supported by the link. \( \Phi \) is given by \( \Phi = \{S_1, S_2, S_3, S_4, S_5, S_6\} \), where \( S_1 = \{\lambda_1\} \), \( S_2 = \{\lambda_2\} \), \( S_3 = \{\lambda_3\} \), \( S_4 = \{\lambda_1, \lambda_2\} \), \( S_5 = \{\lambda_1, \lambda_3\} \) and \( S_6 = \{\lambda_2, \lambda_3\} \).

Suppose, after applying the wavelength assignment algorithm to the tree, \( \{\lambda_1, \lambda_2\} \) is cho-
Multi-wavelength Multicast Wavelength Assignment

For the input link of node \( v \), \( \{ \lambda_1 \} \) for node \( x_1 \), and \( \{ \lambda_2 \} \) for node \( x_2 \), respectively, then, we have \( F(v) = \{ \lambda_1, \lambda_2 \} \), \( F(x_1) = \{ \lambda_1 \} \) and \( F(x_2) = \{ \lambda_2 \} \).

### 3.3 Multi-wavelength Multicast Wavelength Assignment Algorithm

In this section, we derive our new multicast wavelength assignment algorithm based on dynamic programming. We first develop the formulae for the single-wavelength assignment strategy, and then we generalize them for the multi-wavelength assignment strategy.

For each non-root node \( v \) in the tree \( T \), we define a cost function \( c_v(\lambda) \) to be the number of wavelength conversions needed in the subtree rooted at \( v \), assuming wavelength \( \lambda \) is assigned on the input link of node \( v \). Based on this, the total number of wavelength conversions of the tree, \( c_T \), can be represented by

\[
c_T = \sum_{v \in B(s)} c_v(\lambda), \lambda \in A(v)
\] (3.8)

Here, \( B(s) \) is the set of child nodes of root node \( s \). This shows that the total number of wavelength conversions of the tree \( T \) is the summation of the number of wavelength conversions of subtrees rooted at each child node of the root node. The objective of wavelength assignment algorithm is to minimize the total number of wavelength conversions of the tree \( T \), and thus, we have

\[
\min c_T = \min \sum_{v \in B(s)} c_v(\lambda), \lambda \in A(v)
\] (3.9)

From (3.9) we know that, the problem to minimize the total number of wavelength con-
versions of the tree is a combinatorial problem. For this problem, if an optimal solution can be decomposed into optimal solutions for subproblems, the principle of optimality will hold. This is the defining characteristic of problems solvable by dynamic programming [46, 96].

In the following, we will provide **Theorem 3.1** and the proof to show that the problem can be separated into subproblems which can be solved independently.

**Theorem 3.1:** For the multicast tree $T$, and available wavelength $A(v)$ for input link of each non-root node, we have

$$\min_{\lambda \in A(v)} \sum_{v \in B(s)} c_v(\lambda) = \min_{\lambda \in A(v)} \sum_{v \in B(s)} \min_{\lambda \in A(v)} c_v(\lambda) \quad (3.10)$$

**Proof:** Without loss of generality, let the root node have three child nodes, $p$, $q$ and $r$, i.e., $B(s) = \{p, q, r\}$. Thus, (3.10) can be reduced to

$$\min_{\lambda \in A(p), \lambda' \in A(q), \lambda'' \in A(r)} \{c_p(\lambda) + c_q(\lambda') + c_r(\lambda'')\} = \min_{\lambda \in A(p)} c_p(\lambda) + \min_{\lambda' \in A(q)} c_q(\lambda') + \min_{\lambda'' \in A(r)} c_r(\lambda'') \quad (3.11)$$

In order to prove (3.11), we assume that, there exists the wavelengths $\lambda_i$, $\lambda_j$, $\lambda_k$, $\lambda_x$, $\lambda_y$, and $\lambda_z$, which are not necessarily the same or different, and

$$c_p(\lambda_i) + c_q(\lambda_j) + c_r(\lambda_k) = \min_{\lambda \in A(p), \lambda' \in A(q), \lambda'' \in A(r)} \{c_p(\lambda) + c_q(\lambda') + c_r(\lambda'')\};$$

$$c_p(\lambda_x) = \min_{\lambda \in A(p)} c_p(\lambda); c_q(\lambda_y) = \min_{\lambda \in A(q)} c_q(\lambda); c_r(\lambda_z) = \min_{\lambda \in A(r)} c_r(\lambda)$$

From the assumption, we have

$$c_p(\lambda_x) = \min_{\lambda \in A(p)} c_p(\lambda) \leq c_p(\lambda_i) \Rightarrow c_p(\lambda_x) \leq c_p(\lambda_i);$$
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\[ c_q(\lambda_y) = \min_{\lambda \in \mathbf{A}(q)} c_q(\lambda) \leq c_q(\lambda_j) \Rightarrow c_q(\lambda_y) \leq c_q(\lambda_j). \]

\[ c_r(\lambda_z) = \min_{\lambda \in \mathbf{A}(r)} c_r(\lambda) \leq c_r(\lambda_k) \Rightarrow c_r(\lambda_z) \leq c_r(\lambda_k). \]

Thus,

\[ c_p(\lambda_i) + c_q(\lambda_j) + c_r(\lambda_k) \geq c_p(\lambda_x) + c_q(\lambda_y) + c_r(\lambda_z) \quad (3.12) \]

Since,

\[ \min_{\lambda \in \mathbf{A}(p), \lambda' \in \mathbf{A}(q), \lambda'' \in \mathbf{A}(r)} \{ c_p(\lambda) + c_q(\lambda') + c_r(\lambda'') \} \leq \{ c_p(\lambda_x) + c_q(\lambda_y) + c_r(\lambda_z) \} \]

Thus, from the assumption we have

\[ c_p(\lambda_i) + c_q(\lambda_j) + c_r(\lambda_k) \leq c_p(\lambda_x) + c_q(\lambda_y) + c_r(\lambda_z) \quad (3.13) \]

Combine (3.12) and (3.13), and then we have

\[ c_p(\lambda_i) + c_q(\lambda_j) + c_r(\lambda_k) = c_p(\lambda_x) + c_q(\lambda_y) + c_r(\lambda_z) \]

i.e.,

\[ \min_{\lambda \in \mathbf{A}(p), \lambda' \in \mathbf{A}(q), \lambda'' \in \mathbf{A}(r)} \{ c_p(\lambda) + c_q(\lambda') + c_r(\lambda'') \} = \min_{\lambda \in \mathbf{A}(p)} c_p(\lambda) + \min_{\lambda' \in \mathbf{A}(q)} c_q(\lambda') + \min_{\lambda'' \in \mathbf{A}(r)} c_r(\lambda'') \]

\[ \square \]

Theorem 3.1 shows that the multicast wavelength assignment problem to minimize the total number of wavelength conversions of the tree can be divided into subproblems, which
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have the property that the optimal solution to the original problem is a composition of optimal solutions to a set of subproblems. Consequently, we can employ the dynamic programming algorithm to solve this problem.

3.3.1 Dynamic Programming Algorithm

In this subsection, we derive our new multicast wavelength assignment algorithm using a dynamic programming algorithm. We first develop the algorithm for the single-wavelength assignment strategy, and then we generalize it to produce the algorithm for the multi-wavelength assignment strategy.

For the dynamic programming algorithm, specifically, we redefine function $c_v(\lambda)$ to be the minimum number of wavelength conversions needed in the subtree rooted at $v$, assuming the signal enters $v$ on wavelength $\lambda$ and can be forwarded to all of its descendant destinations. If $\lambda$ is not available on the input link or it is not possible for $v$ to reach all of the destination nodes in its subtree when the signal enters $v$ on wavelength $\lambda$, then $c_v(\lambda) = \infty$. Based on the definition, for the leaf node of the tree, we have

$$c_v(\lambda) = \begin{cases} 
0, & \text{if } \lambda \in A(v) \\
\infty, & \text{otherwise}
\end{cases} \quad (3.14)$$

As node $v$ is a leaf node of the tree, it has no child node. Thus, the subtree rooted at $v$ is the node $v$ itself, i.e., there are no descendants. Then, any available wavelength on the input link of node $v$ requires no wavelength conversion, i.e., for any $\lambda \in A(v)$, $c_v(\lambda) = 0$.

Next, consider an internal node. For an internal node $v$, the signal arriving at $v$ on the wavelength $\lambda$ can be transmitted to its children using the available wavelength(s) on the input
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link of each child. Thus, we have

\[ c_v(\lambda) = \begin{cases} 
\sum_{x \in B(v)} \min_{\lambda' \in A(x)} (c_x(\lambda') + \Delta), & \text{if } \lambda \in A(v) \\
\infty, & \text{otherwise}
\end{cases} \]  \quad (3.15)

where

\[ \Delta = \begin{cases} 
0, & \text{if } \lambda' = \lambda \\
1, & \text{if } \lambda' \neq \lambda
\end{cases} \]  \quad (3.16)

For any child node \( x \) of node \( v \), the value of \( c_x(\lambda') \) denotes the number of wavelength conversions required at the subtree rooted at node \( x \), when the input wavelength of node \( x \) is \( \lambda' \). \( \Delta \) actually stands for the number of wavelength conversions required in node \( v \) for the signal to reach node \( x \). Therefore, \((c_x(\lambda') + \Delta)\) represents the number of wavelength conversions needed for various wavelengths reaching node \( x \). Find the minimal value of \((c_x(\lambda') + \Delta)\) for each child node \( x \). Consequently, the sum the all these minimum values of \((c_x(\lambda') + \Delta)\) should give the minimum number of wavelength conversions required for the subtree rooted at node \( v \), when the input wavelength of node \( v \) is \( \lambda \). Thus, for any \( \lambda \in A(v) \),

\[ c_v(\lambda) = \sum_{x \in B(v)} \min_{\lambda' \in A(x)} (c_x(\lambda') + \Delta) \]

Finally, consider the root node \( s \). We define \( c_T \) to be the minimum number of wavelength conversions needed for the source node \( s \) to deliver the message to all its destination nodes in the multicast tree \( T \). The root node \( s \) will have no parent. It transmits a signal to each of its child nodes by using any wavelength that is available on the input link of each child node. Therefore, there will be no wavelength conversions needed at the root node \( s \). Thus, the sum
of the minimal values of $c_x(\lambda')$ of its child nodes should give the total number of wavelength conversions required in the tree rooted at $s$. Consequently, we have,

$$c_T = \sum_{x \in B(s)} \min_{\lambda' \in A(x)} c_x(\lambda')$$  \hfill (3.17)

All the above formulae are based on the single-wavelength assignment strategy, i.e., one wavelength on one link. Next, we generalize the formulae from single-wavelength case to multi-wavelength case. Similar to the function of $c_v(\lambda)$ above, for each non-root node $v$, a cost function, $c_v(S)$ is defined to be the minimum number of wavelength conversions needed in the subtree rooted at $v$, assuming the signal enters $v$ on wavelength set $S$ and can be forwarded to all of its descendant destinations. If any wavelength in set $S$ is not available on the input link or it is not possible for $v$ to reach all of the destination nodes in its subtree when the signal enters $v$ on set $S$, then $c_v(S) = \infty$.

Then, for each leaf node $v$ in the tree $T$, we have

$$c_v(S) = \begin{cases} 0, & \text{if } S \subseteq A(v) \\ \infty, & \text{otherwise} \end{cases}$$  \hfill (3.18)

For an internal node $v$ in the tree $T$,

$$c_v(S) = \begin{cases} \sum_{x \in B(v)} \min_{S' \subseteq A(x), |S'| \leq K} (c_x(S') + \Delta), & \text{if } S \subseteq A(v) \\ \infty, & \text{otherwise} \end{cases}$$  \hfill (3.19)
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where

\[ \Delta = |S' - S| = \sum_{\lambda \in S'} \begin{cases} 0, & \text{if } \lambda \in S \\ 1, & \text{if } \lambda \notin S \end{cases} \]  

(3.20)

Here, \( \Delta \) denotes the number of wavelength conversions required for the signal transmitting from node \( v \) to node \( x \), where the wavelength set \( S \) is on the input link of \( v \) and \( S' \) on the input link of \( x \). If a wavelength existing in \( S' \) does not exist in \( S \), then a wavelength conversion is needed. As illustrated in Figure 3.3, assuming \( x \) is the only child of \( v \), suppose \( S' = \{ \lambda_1, \lambda_3 \} \) and \( S = \{ \lambda_1 \} \), then \( \lambda_3 \) is in \( S' \) but not in \( S \), thus one wavelength conversion is required. Thus, \( S' - S \) gives the wavelength set that requires wavelength conversions.

Figure 3.3: Example for computing the value of \( c_v(S) \) for node \( v \).

From (3.19), we see that \( c_v(S) \) can be computed if the value of each child node \( x \) of \( v \) has been computed already. Thus, we can use a bottom-up approach to compute each value of \( c_v(S) \).

Finally, for the root node \( s \) of the tree \( T \), we still define \( c_T \) to be the minimum number of wavelength conversions needed for the source node \( s \) to deliver the message to all its destination nodes in the multicast tree \( T \). Then,

\[ c_T = \sum_{x \in B(s)} \min_{S' \subseteq A(x), |S'| \leq K} c_x(S') \]  

(3.21)

The above formulae consider full wavelength conversion and provide the description of relationship between link state (available wavelengths on links) and wavelength assignment.
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outcome. Based on these formulae, for each node in the tree $T$, we can use a bottom-up approach to apply the dynamic programming algorithm to compute the values of $c_v(S)$ for all non-root nodes of $T$ and the value of $c_T$ for the root of the tree. During the process of computations, when $c_v(S)$ is computed for a given vertex $v$, this implies that the required values for all of its children in the subtree have already been computed.

Based on all the computed values of $c_v(S)$ for the given multicast tree, the wavelength set for each link of the tree can be decided. For a non-root node $v$, each wavelength set $S$ has a value of $c_v(S)$ respectively. We define the minimum value of all the values of $c_v(S)$ of node $v$ to be $c^m_v$, i.e.,

$$c^m_v = \min_{S \subseteq A(v), |S| \leq K} c_v(S) \tag{3.22}$$

With respect to each value of $c_v(S)$ and $c^m_v$, the wavelength set on each link can be confirmed by choosing the wavelength set that satisfies

$$c_v(S) = c^m_v$$

From this rule, when one wavelength set has been selected, it implies that the total number of wavelength conversion in the subtree rooted at $v$ should be minimal. We use a top-down approach to decide which wavelength set to be chosen for each link. Therefore, if all wavelength sets are assigned under this rule, the total number of wavelength conversion in the multicast tree $T$ should be minimal. Table 3.2 shows the related pseudo codes of the new Multi-wavelength Multicast Wavelength Assignment (MMWA) algorithm that makes use of multiple wavelengths to carry the same light signal on the same link.

We give an estimate of the upper bound of the computation time for the above algo-
Table 3.2: Algorithm MMWA

| INPUT: Multicast tree \( T \) and available wavelength set \( A(v) \) for each non-root node \( v \) |
| OUTPUT: Wavelength assignment \( \pi_T \) for \( T \) |

BEGIN

Let multicast tree \( T \) have a topological ordering \( v_1,v_2,...,v_N \) based on the breadth-first search.

**STEP 1:**

FOR ( \( j = N \) down to 2) DO

Let \( v = v_j \)

FOR (each wavelength set \( S \) is nonempty) DO

IF \( v \) is a leaf node THEN compute \( c_v(S) \) using (3.18)

ELSEIF \( j > 1 \) THEN compute \( c_v(S) \) using (3.19)

ENDIF

ENDFOR

ENDFOR

**STEP 2:**

FOR (each child node \( v \) of root node) DO

Set \( F(v) \) to be any wavelength set \( S \) satisfying \( c_v(S) = c_v^m \)

ENDFOR

FOR ( \( j = 2 \) up to \( N \)) DO

FOR (each child node \( p \) of non-leaf node \( v \)) DO

IF \( F(v) \subseteq A(p) \) and \( c_p(F(v)) = c_p^m \) THEN

Set \( F(p) = F(v) \)

ELSE

Set \( F(p) \) to be any wavelength set \( S \) satisfying \( c_p(S) = c_p^m \)

ENDIF

ENDFOR

ENDFOR

END

Among the computations performed inside the “FOR” loop, the computation in (3.19) requires the largest number of steps. For given values of \( K \) and \( W \), there are \( Q (\sim O(W^K/K!)) \) distinct wavelength sets [46]. For each wavelength set \( S \), it should take \( Q \) steps to compare \( S \) with each of \( Q \) distinct wavelength sets to obtain the value of \( \Delta \).
Moreover, the minimum value of $c_x(S')$ should be chosen among all the child nodes in $B(v)$, which is of a size at most equal to the maximum out-degree, $D$. Thus, in the worst case, for an internal node, $(DQ^2)$ steps are required to compute the value of $c_v(S')$ for each set $S$. Therefore, the computation time is upper-bounded by $(DQ^2N)$. Thus, the algorithm has $O(D(W^K/K!)^2N)$ computation time, with the constant term depending on parameters, $D$, $W$ and $K$. The complexity of MMWA algorithm varies linearly with $N$, but increases exponentially with $W$ and $K$. This implies that $K$ should be kept small if we wish to use the algorithm for online computing. In fact, good performance can be obtained even for small value of $K$. This will be illustrated with simulation results later.

### 3.3.2 Upper Bound of Wavelength Cost

In our approach, more than one wavelength may be assigned on a single link for a given multicast session. By introducing multi-wavelength assignment strategy to optical multicast, number of wavelength conversions can be minimized. However, total number of wavelengths used for a multicast tree increases correspondingly. Thus, we need to investigate whether an upper bound exists for total number of wavelengths used in the tree.

Recall that, $K$ is defined as the maximum number of wavelengths permitted to be used on a single link, and $1 \leq K \leq L$. In the worst case, each link of the tree $T$ is assigned $K$ wavelengths. Therefore, the total number of wavelengths used in the tree is $(N - 1)K$, i.e.,

$$C_W(\pi_T) \leq (N - 1)K \quad (3.23)$$

Actually, in MMWA algorithm, not all links will be assigned multiple wavelengths. Thus, the
upper bound can be reduced by excluding the nodes that are assigned only one wavelength by the algorithm.

Leaf nodes in the tree should be assigned only one wavelength by the MMWA algorithm. For a leaf node \( v \), i.e., \( D(v) = 0 \), it has no output link. It requires only one wavelength. Let us consider again the example shown in Figure 3.3. Suppose node \( x \) is a leaf node, it does not serve any purpose to assign multiple wavelengths on the input link of node \( x \).

For all nodes in \( V_T \), we define \( V_0 \) to be the set of all nodes with the out-degree equals 0, i.e.,

\[
V_0 = \{ v | v \in V_T \text{ and } D(v) = 0 \}
\]

Then, based on (3.23), we have

\[
C_W(\pi_T) \leq |V_0| + (N - 1 - |V_0|) K
\]

\[
C_W(\pi_T) \leq (N - 1) K - |V_0|(K - 1)
\]

Now, we derive the formulae of \( |V_0| \) by the following theorem.

**Theorem 3.2:** For the given nontrivial tree \( T(V_T, E_T) \) with \( N \) nodes, the number of nodes with out-degree of 0 is given as

\[
|V_0| = 1 + \left( \sum_{v \in V_T} |D(v) - 1| \right)/2
\]

**Proof:** A theorem by Leonhard Euler [97] states that the sum of the degree of the vertices of a graph is equal to twice the number of its edges. We define the degree of node \( v \) to be
\( \mathbf{D}'(v) \), i.e., \( \mathbf{D}'(v) = \mathbf{D}(v) + 1 \). Thus, as for the tree \( T(V_T, E_T) \), we have

\[
\sum_{v \in V_T} \mathbf{D}'(v) = 2|E_T| = 2(N - 1) \tag{3.26}
\]

If \( v \in V_0 \), then \( \mathbf{D}'(v) = 1 \), thus, \( |\mathbf{D}'(v) - 2| = 1 = \mathbf{D}'(v) \);

If \( v \notin V_0 \), then \( \mathbf{D}'(v) > 1 \), thus, \( |\mathbf{D}'(v) - 2| = (\mathbf{D}'(v) - 2) \).

Thus,

\[
\sum_{v \in V_T} |\mathbf{D}'(v) - 2| = \sum_{v \in V_0} |\mathbf{D}(v)' - 2| + \sum_{v \in V_T - V_0} |\mathbf{D}'(v) - 2|
\]

\[
= \sum_{v \in V_0} \mathbf{D}'(v) + \sum_{v \in V_T - V_0} (\mathbf{D}'(v) - 2)
\]

\[
= \sum_{v \in V_0} \mathbf{D}'(v) + \sum_{v \in V_T - V_0} \mathbf{D}'(v) - \sum_{v \in V_T - V_0} 2
\]

\[
= \sum_{v \in V_T} \mathbf{D}'(v) - 2(N - |V_0|)
\]

\[
= 2(N - 1) - 2(N - |V_0|)
\]

\[
= 2|V_0| - 2
\]

Therefore, we have

\[
|V_0| = 1 + \left( \sum_{v \in V_T} |\mathbf{D}'(v) - 2| \right)/2
\]

\[
= 1 + \left( \sum_{v \in V_T} |\mathbf{D}(v) - 1| \right)/2
\]

Therefore, the combination of (3.24) and (3.25) gives the upper bound of total number of wavelengths used in the tree \( T \). It is a function of \( K \), \( N \) and \( D \).
3.4 MMWA Algorithm for Limited Wavelength Conversions

The advantages of full wavelength conversions have been well studied and demonstrated [98–100]. Nevertheless, on one hand, employing transceiver-based O-E-O full wavelength conversions causes much unwanted delay. On the other hand, equipping the network with all-optical full wavelength converters is too costly currently [101]. A realistic and practical solution is to use limited wavelength converters. A limited wavelength converter can only convert an input wavelength to a limited number of output wavelengths. The limited wavelength converters are relatively easy to be implemented, compared with full wavelength converters. A typical well-demonstrated method to implement a limited wavelength converter is based on four-wave mixing in semiconductor optical amplifiers [102]. The impact of limited wavelength conversion on blocking performance of WDM network has been addressed in [103–106]. The design for non-blocking WDM network with limited wavelength conversions has been studied in [107, 108]. Unfortunately, to the best of our knowledge, there is no work done to address the limited wavelength conversion issue for optical multicast.

To analyze the limited wavelength conversion problem, we adopt a popular threshold model [102, 104–106] in this chapter. In this model, \( d \) is defined to be the conversion degree of a limited wavelength converter. A converter with conversion degree, \( d \), can convert input wavelength to \( d \) adjacent wavelengths on either side of the input wavelength in addition to the input wavelength itself. Hence, any wavelength can be converted to \( 2d \) wavelengths, plus the input wavelength. Actually, the case \( d = 0 \) is equivalent to no wavelength conversion and the case \( d = (W - 1)/2 \) means full wavelength conversion, since \( W \) is the total number of wavelengths supported in the network. For example, incoming wavelength \( \lambda_i \) can be converted to any of the outgoing wavelengths \( \lambda_{i-d}, \ldots, \lambda_i, \ldots, \lambda_{i+d} \). We also assume that the
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conversions are circularly symmetric. As shown in Figure 3.4, assume a total of \( W \) wavelengths and the conversion degree \( d = 1 \), thus, for wavelength \( \lambda_1 \), it can be converted to \( \lambda_2 \), and since the conversions are circularly symmetric, \( \lambda_1 \) can also be converted into wavelength \( \lambda_W \); similarly, the wavelength \( \lambda_2 \) can be converted to \( \lambda_1 \) and \( \lambda_3 \); the wavelength \( \lambda_W \) can be converted to \( \lambda_{W-1} \) and \( \lambda_1 \), respectively.

![Figure 3.4: Possible wavelength conversion for each wavelength for \( d = 1 \). The conversions are circularly symmetric.](image)

For the network with the above limited wavelength conversions, we can also adopt multi-wavelength assignment strategy to minimize number of wavelength conversions. Furthermore, besides reducing number of wavelength conversions, such strategy can provide additional advantages for limited conversion. We give a simple example for illustration in Figure 3.5. The wavelengths \( \lambda_1 \) and \( \lambda_3 \) are available on the input link of node \( v \). \( \lambda_1 \) is available on the input link of node \( x_1 \), and \( \lambda_4 \) is available on the input link of node \( x_2 \) (see Figure 3.5(a)).

We assume \( d = 1 \) and \( W = 8 \) in this example, thus, the wavelength \( \lambda_1 \) can only be converted into \( \lambda_2 \) and \( \lambda_8 \), and \( \lambda_3 \) can only be converted into \( \lambda_2 \) and \( \lambda_4 \). By single-wavelength assignment strategy, only one of the two wavelengths is assigned to the input link of node \( v \), hence, the request cannot be satisfied, i.e., be blocked. For example, if \( \lambda_1 \) is assigned to the input link of node \( v \), then the signal cannot go through from node \( v \) to node \( x_2 \), since the conversion
between $\lambda_1$ and $\lambda_4$ is not supported. The same result will occur, if choosing $\lambda_3$ on the input link of node $v$. However, if we use both wavelengths $\lambda_1$ and $\lambda_3$ to carry the same signal to node $v$, the multicast request can be satisfied, as shown in Figure 3.5(b). This example shows that, in order to set up a multicast request, it is necessary to assign multiple wavelengths on some links of a multicast tree in certain situations. We know that, for some multicast services such as videoconferencing and real-time remote surgery, it is crucial to guarantee that all the participants can receive the data from the source. Hence, if such a multicast request cannot be satisfied by the single-wavelength assignment strategy, it is necessary to seek alternative solution from multi-wavelength assignment strategy.

Based on this idea, we can extend our multi-wavelength multicast wavelength assignment algorithm from the full wavelength conversion case to limited wavelength conversion case. We adopt dynamic programming algorithm in the previous section. The formulae are kept almost the same as (3.18)-(3.21) except with a minor modification to (3.20) as follows:
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\[ \Delta = |S' - S| \]

\[ = \sum_{\lambda_i \in S'} \begin{cases} 
1, & \text{if } \lambda_i \notin S, \text{ and } |j - i| \leq d \text{ or } |j - i| \geq W - d, \text{ for any } \lambda_j \in S \\
0, & \text{if } \lambda_i \in S \\
\infty, & \text{otherwise} 
\end{cases} \quad (3.27) \]

This formula is supported by Theorem 3.3 as shown below.

**Theorem 3.3:** For the limited wavelength conversions with conversion degree of \( d \) and total \( W \) wavelengths, a wavelength conversion is available, if two wavelengths \( \lambda_i \) and \( \lambda_j \) satisfy,

\[ |j - i| \leq d \text{ or } |j - i| \geq W - d \]

**Proof:** Since a converter with conversion degree of \( d \) can convert the input wavelength to \( d \) adjacent wavelengths on either side of the input wavelength, and the conversions are circularly symmetric, then, for \( j > i \), the wavelength \( \lambda_j \) may have two possible relationships to \( \lambda_i \) as follows:

\[ \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{W-2}, \lambda_{W-1}, \lambda_W, \parallel \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{W-2}, \lambda_{W-1}, \lambda_W \]

Thus, we have

\[ (W - j + i) \leq d \text{ or } (j - i) \leq d \quad (3.28) \]
Similarly, for \( j < i \), we have

\[
\lambda_1, \lambda_2, \lambda_3, \dotsc, \lambda_{W-2}, \lambda_{W-1}, \lambda_W \parallel \lambda_1, \lambda_2, \lambda_3, \dotsc, \lambda_{W-2}, \lambda_{W-1}, \lambda_W
\]

\[
\lambda_j \quad \lambda_i \quad \lambda_j
\]

\[
(i - j) \quad (W - i + j)
\]

Thus, we have

\[
(i - j) \leq d \text{ or } (W - i + j) \leq d
\]  \hspace{1cm} (3.29)

Therefore, by combining (3.28) and (3.29), we have

\[
|j - i| \leq d \text{ or } |j - i| \geq W - d.
\]

The main procedure of the algorithm for limited wavelength conversions is almost the same as those codes shown in Table 3.2. The only difference is that in \textbf{STEP 1}, after the computations for each value of \( c_v(S) \) for one node, if all the wavelength sets lead \( c_v(S) \) to be infinite, the algorithm will stop, which means the request cannot be satisfied or be blocked.

\section*{3.5 Simulations and Results}

In this section, a simulation study is carried out to evaluate MMWA algorithm derived in the previous sections. We will compare the results of our algorithm with the existing wavelength assignment algorithm based on single-wavelength assignment strategy proposed in [43] which we call Chen’s algorithm. One can regard this algorithm to be a special case of
our MMWA algorithm.

### 3.5.1 A Case Study of MMWA Algorithm in NSFNET

We consider the NSFNET topology and give a simple example to illustrate the difference between single-wavelength assignment and multi-wavelength assignment. The network topology with the randomly generated information of available wavelengths on each link is shown in Figure 3.6. It shows that a total of four wavelengths are supported in all links, i.e., $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$, and thus, $W = 4$. The maximum number of available wavelengths on each link is two, i.e., $L = 2$. In the Figure, the source of the multicast session is node UT, and the destinations are CA2, NE, TX, GA, NY and NJ, respectively.

![Figure 3.6: A multicast tree in NSFNET, UT is the source and, CA2, TX, GA, NE, NY and NJ are destinations.](image)

We assume the existence of a multicast tree for the multicast session. This is shown in Figure 3.7(a) with available wavelengths on each link. The tree consists of 9 nodes, in which UT is the root node. We apply unicast algorithm, Chen’s algorithm and our MMWA algorithm to the tree, and the resulting wavelength assignments are illustrated in Figure 3.7(b), 3.7(c) and 3.7(d), respectively.
Figure 3.7: Example of wavelength assignment algorithm. (a) Multicast tree; (b) Result of assignment by unicast; (c) Result of assignment by Chen’s algorithm; (d) Result of assignment by MMWA (K = 2).

For the unicast case, the multicast request is decomposed into multiple individual unicast requests. Thus, the multicast request (UT: CA2, NE, TX, GA, NY, NJ) is decomposed into six unicast requests, i.e., (UT, CA2), (UT, NE), (UT, TX), (UT, GA), (UT, NY), and (UT, NJ). The wavelength assignment is performed to these unicast requests independently and a lightpath will be setup to each of these requests. There are only two wavelengths available on the link between node UT and CO, thus, once the wavelength \(\lambda_1\) is assigned to the lightpath for (UT, TX) and \(\lambda_3\) is assigned to the lightpath for (UT, NE), there will be no available wavelengths for the lightpath (UT, CA2) and (UT, GA), i.e., these two requests will be
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blocked. This is illustrated in Figure 3.7(b). On the other hand, the request can be satisfied by multicast-based algorithms, since the node TX can be used as intermediate node to perform light splitting. This is shown in Figure 3.7(c) and 3.7(d).

Then, let us focus on the multicast-based algorithms. The result of applying Chen’s algorithm gives the number of wavelength conversions to be 4 in the multicast tree. However, if we apply our MMWA (K = 2) algorithm, the number of wavelength conversions becomes 2. This reduction is due to the following fact. When assigning wavelengths on the input link of node CO and input link of node MI, the MMWA (K = 2) algorithm assigns two wavelengths to such two links, respectively. However, Chen’s algorithm uses only one wavelength. Thus, from node CO to node TX and node NE, no wavelength conversions is required for MMWA (K = 2) and one wavelength conversion is needed for Chen’s algorithm. Table 4.3 shows the computation results of \( c_v(S) \) for each node by MMWA algorithm. For the number of wavelengths used, Chen’s algorithm gives 8 but MMWA algorithm gives 10 – an additional 2 wavelengths are needed.

Table 3.3: Computation Results of \( c_v(S) \) for All Nodes.

<table>
<thead>
<tr>
<th>Node Name</th>
<th>UT</th>
<th>CO</th>
<th>MI</th>
<th>TX</th>
<th>NE</th>
<th>NJ</th>
<th>NY</th>
<th>CA2</th>
<th>GA</th>
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</thead>
<tbody>
<tr>
<td>Topology Order</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
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<tr>
<td>( c_v({\lambda_1}) )</td>
<td>-</td>
<td>3</td>
<td>∞</td>
<td>2</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>( c_v({\lambda_2}) )</td>
<td>-</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>0</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>( c_v({\lambda_3}) )</td>
<td>-</td>
<td>3</td>
<td>1</td>
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<td>∞</td>
<td>0</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>( c_v({\lambda_4}) )</td>
<td>-</td>
<td>∞</td>
<td>1</td>
<td>∞</td>
<td>0</td>
<td>0</td>
<td>∞</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>( c_v({\lambda_1,\lambda_2}) )</td>
<td>-</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>( c_v({\lambda_1,\lambda_3}) )</td>
<td>-</td>
<td>2</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>( c_v({\lambda_1,\lambda_4}) )</td>
<td>-</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>( c_v({\lambda_2,\lambda_3}) )</td>
<td>-</td>
<td>∞</td>
<td>∞</td>
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<td>0</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>( c_v({\lambda_2,\lambda_4}) )</td>
<td>-</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
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<td>∞</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>( c_v({\lambda_3,\lambda_4}) )</td>
<td>-</td>
<td>∞</td>
<td>0</td>
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<td>0</td>
<td>∞</td>
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<td>∞</td>
</tr>
<tr>
<td>( c^m_v )</td>
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<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
3.5.2 Evaluation of Multicast Wavelength Assignment Algorithm

In order to evaluate the performance of our proposed algorithm, in our simulation, we directly adopt the experimental method from [43,44] for fair comparison. The method employs random multicast trees. Each tree has \( N \) number of nodes with the index from 1 to \( N \), where ‘1’ denotes the root node. Each node of the tree has a random number of out-degree selected uniformly between 0 and \( D \). Each link is assumed to support \( W \) distinct wavelengths and the set of available wavelengths on each link is selected at random where the size of the set is taken from the uniform \([1, L]\) distribution, where \( L \) is the maximum number of available wavelengths on each link of the tree. For each set of parameters \((N, D, W, L)\), we generate 10,000 trees and for each tree, we apply the wavelength assignment algorithm and to compute the number of wavelengths used and the number of wavelength conversions needed.

We first evaluate the cost performance of MMWA algorithm for full wavelength conversion case. Performance metrics are used in the simulation as follows:

- **NW**: Total number of wavelengths used in the tree after the wavelength assignment.

- **NWC**: Total number of wavelength conversions needed in the tree after the wavelength assignment.

- **Wavelength Overhead (%)**: This is defined as the ratio of additional number of wavelengths needed for MMWA algorithm over Chen’s algorithm, i.e.,

\[
\text{Wavelength Overhead} \% = \frac{\text{NW by Algorithm MMWA} - \text{NW by Chen’s Algorithm}}{\text{NW by Chen’s Algorithm}}
\]

- **Conversion Improvement (%)**: This gives the performance advantage of the MMWA algorithm over Chen’s algorithm in terms of total number of wavelength conversions
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Figure 3.8: Comparison of number of wavelength conversions for \( W = 8 \).

Figure 3.9: Comparison of number of wavelength conversions for \( W = 16 \).

Conversion Improvement\(^{\%}\) = \(\frac{\text{NWC by Chen’s Algorithm} - \text{NWC by MMWA Algorithm}}{\text{NWC by Chen’s Algorithm}}\)

Figure 3.8 and Figure 3.9 give comparison of wavelength conversion requirements for Chen’s algorithm and MMWA algorithm. It shows that using at most two wavelengths to transmit the same signal on the same link (\( K = 2 \)), MMWA algorithm requires less number
of wavelength conversions than Chen’s algorithm and thus, lowering the cost of wavelength conversions. Furthermore, we observe that as the maximum number of available wavelengths, $L$, the difference between two algorithms increases. When more wavelengths are available in the links, less wavelength conversions are needed. On the other hand, the MMWA ($K = 2$) tends to use more wavelengths than Chen’s algorithm but the additional number of wavelengths used is capped at 15, as shown in Figure 3.10 and Figure 3.11.
Figure 3.12: Conversion Improvement and Wavelength Overhead ($K = 2$) for $W = 8$.

Figure 3.13: Conversion Improvement and Wavelength Overhead ($K = 2$) for $W = 16$.

Figure 3.12 and Figure 3.13 show the comparison of Conversion Improvement and Wavelength Overhead. As $L$ increases from 2 to 8, the Wavelength Overhead is kept at a relative flat level. For instance in Figure 3.12, when $L$ varies in the range between $\{3, 8\}$, the Wavelength Overhead is kept in the interval of $\{12.4\%, 16.2\\%\}$. On the other hand, Conversion
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Improvement roughly increases linearly with L from 10% to 52%. In short, we only need about 15% increase in wavelength usage to produce a reduction of the number of wavelength conversions by over 50%. The benefit is substantial.

We note that in the above comparisons, the maximum number of wavelengths permitted to use on a single link, K, is fixed at 2. A natural question arises. What will happen if we increase the value of K by allowing more wavelengths in a link to carry the message? This has been investigated and the results are shown in Figure 3.14 and Figure 3.15. As the behavior of curves for the case of W = 16 is similar to that of W = 8, we omit the figures for W = 16 and concentrate on the case of W = 8.

Figure 3.14 shows Wavelength Overhead and Conversion Improvement for different cases of K. In general, when the value of K increases, both Wavelength Overhead and Conversion Improvement increase. However, Wavelength Overheads for different K values are very close. That means allowing more wavelengths to be used on the same link will not raise Wavelength Overhead very much. Figure 3.15 shows variation of Conversion Improvement

Figure 3.14: Conversion Improvement and Wavelength Overhead (K = 2, 3, 4) for W = 8.
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Figure 3.15: Conversion Improvement versus $K$

with $K$. By and large, Conversion Improvement increases as $K$ increases. For a large value of $L$ (i.e., more wavelengths are available on links), Conversion Improvement increases rapidly as $K$ increases and eventually saturates at high $K$. In other words, if more wavelengths are available on links, we can use many wavelengths to carry the same message, thus, reducing the number of wavelength conversions. For high $K$, Conversion Improvement tends to saturate. This indicates that in order to reduce number of wavelength conversions, we do not need to increase the value of $K$ all the times. For example, if 3 wavelengths are available on links ($L = 3$), $K = 2$ is sufficient; if 7 wavelengths are available ($L = 7$), $K = 3$ is acceptable. Thus, small value of $K$ in our MMWA algorithm can still produce good cost performance.

In Figure 3.16, a 3-D figure is used to show relationship among Conversion Improvement, Wavelength Overhead and maximum number of available wavelengths, $L$, for various values of $K$. This gives the tradeoff between Conversion Improvement and Wavelength Overhead. A design decision can be made from the graph. For instance, given a value of Wavelength Overhead that is acceptable, one can select a desired value of Conversion Improvement using
Figure 3.16: 3-D version of Conversion Improvement and Wavelength Overhead (K = 2, 3, 4)

a combination of L and K. Alternatively, one can choose an acceptable value of K using a combination of Conversion Improvement and L.

In the above experiments, we consider two metrics NWC and NW separately. To further illustrate the superiority of the MMWA algorithm, we will consider jointly NWC and NW. Two additional metrics are introduced as follows:

- **Tree Cost**: Total cost of multicast tree for the given multicast request, including the wavelength cost and wavelength conversion cost. In our simulation, a simplification has been made. The wavelength cost is represented by the Number of Wavelengths (NW) used and the Number of Wavelength Conversions (NWC) needed represents wavelength conversion cost. This gives, Tree Cost = NW + α NWC. Here, α is a weight factor to denote the ratio of one unit cost of wavelength conversion over one unit cost of wavelength. Precise determination of α is difficult. However, an approximate estimation can be made using the cost ratio of a unit-length fiber with W wavelengths in
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it over the cost of wavelength converter. As suggested in [109], the typical value of the ratio of the cost of a wavelength converter over a unit-length fiber with 8 wavelengths is 25. Thus, the ratio of the cost of a wavelength conversion over a wavelength can be roughly considered as, \( \alpha = 25 \times 8 = 200 \).

- **Cost Reduction (%)**: This is defined as the ratio of the tree cost reduction for MMWA algorithm over Chen’s algorithm, i.e.,

\[
\text{Cost Reduction(\%)} = \frac{\text{Tree Cost by Chen’s Algorithm} - \text{Tree Cost by MMWA Algorithm}}{\text{Tree Cost by Chen’s Algorithm}}
\]

Figure 3.17 shows the relationship of Cost Reduction versus \( L \) for various \( \alpha \). When the value of \( \alpha \) increases, the percentage of Cost Reduction also increases. This indicates that, the more the wavelength conversion cost outweighs wavelength cost, the more cost reduction can be achieved by our MMWA algorithm. The cost reduction can be as much as 50\% for \( \alpha = 200 \) (\( L = 8 \)).

![Figure 3.17: Cost Reduction vs. \( L \) for different values of \( \alpha \) (K = 2).](image)

After evaluating cost performance of MMWA algorithm for full wavelength conversion,
we shall now consider limited wavelength conversion case. A new evaluation metric, Satisfied Requests, is needed and defined as follows:

- **Satisfied Requests (%)**: This gives the ratio of the number of satisfied requests by the wavelength assignment algorithm over the total number of input multicast requests.

A multicast request may be accepted or rejected depending on availability of resources. This is determined by wavelength assignment algorithm. A comparison of Satisfied Requests metric will indicate the strength of the algorithm. Obviously, the one with higher Satisfied Requests will perform better.

The simulation results are presented in Figure 3.18-3.20 which show Satisfied Requests versus \( L \) for various algorithms with different conversion degrees. The case with no wavelength conversion, \( d = 0 \), is shown in Figure 3.18. If only one wavelength is allowed to be assigned on one link (i.e., Chen’s algorithm), the probability of locating a common wavelength to be available on all links is very low for a large tree. Thus, no requests will be satisfied. When more wavelengths are allowed to carry data on one link, e.g., \( K \) increases

![Figure 3.18: Satisfied Requests vs. \( L \) for conversion degree \( d = 0 \).]( ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library )
from 2 to 3, Satisfied Requests rises correspondingly. In particular, when the maximum number of available wavelengths is relatively large, e.g. $L = 7$ or 8, improvement of Satisfied Requests is over 50\% to 80\%. Figure 3.19 shows the case when $d = 1$, i.e., the input wavelength can only be converted into one adjacent wavelength on each side. In this case, when $L$ is large, which means there are ample available wavelengths, Satisfied Requests is quite high.
for both MMWA and Chen’s algorithm. When \( L \) equals to 4 and 5, improvement of MMWA is about 40% to 60%, which is quite significant. A similar trend is also observed in Figure 3.20 when \( d = 2 \). However, the gap between MMWA and Chen’s algorithm is narrower. The reason is that when \( d = 2 \), \( 5 (= 2d + 1) \) wavelengths out of \( 8 (= W) \) can be used for wavelength conversions. This is close to the case of full wavelength conversions. Strictly speaking, when one considers limited wavelength conversion, it means the range of wavelengths that can be used for wavelength conversions should be less than 50% or even less, for example, 25% [102]. Therefore, the case of \( d = 1 \) (Figure 3.19) can be regarded as typical limited wavelength conversion case to illustrate performance of MMWA and Chen’s algorithm. Consequently, we can conclude that MMWA algorithm outperforms Chen’s algorithm for limited wavelength conversion situations.

Another useful result that can be obtained in Figure 3.19 and Figure 3.20 is that Satisfied Requests appear to be the same for \( K = 2 \) and \( K = 3 \). This is because, all the requests that can be satisfied by \( K = 2 \) can also be covered by \( K = 3 \). This indicates that a small value of \( K \) is adequate to produce good performance.

In conclusion, compared with single-wavelength assignment strategy, for the new multi-wavelength assignment strategy based MMWA algorithm, not only can it make significant cost reduction for full wavelength conversion case, but also accept more multicast requests for limited wavelength conversion case. For small value of \( K \), the MMWA algorithm can achieve (1) good tradeoff between wavelength cost and wavelength conversion cost for full wavelength conversions; (2) good Satisfied Requests ratios for limited wavelength conversions; (3) acceptable time complexity for online computation demands.
3.6 Summary

In this chapter, we consider multicast wavelength assignment problem in wavelength-routed WDM networks with full light splitting and various wavelength conversion capabilities. In order to minimize wavelength conversion cost of multicast, we propose a new concept that allows the use of multiple available wavelengths to transmit the multicast signal in a link. Subsequently, we develop a new Multi-wavelength Multicast Wavelength Assignment (MMWA) algorithm. The simulation results show that our new algorithm outperforms the existing algorithm in [43] by greatly reducing wavelength conversion cost while keeping the additional wavelength usage within an acceptable range. Moreover, if wavelength conversion cost is higher than wavelength cost, MMWA algorithm can reduce greatly total multicast tree cost. The higher wavelength conversion cost than wavelength cost, the more cost reduction can be achieved by MMWA algorithm. In general, MMWA algorithm provides a good tradeoff between wavelength cost and wavelength conversion cost. Besides full wavelength conversion case, we exploit the extra benefits of multi-wavelength assignment strategy for the network with limited wavelength conversions. We extend MMWA algorithm to cover limited wavelength conversion case. Simulation results show that much more multicast requests can be accommodated by our new MMWA algorithm as compared with existing algorithm. The improvement can be as high as 50% to 80%. This is highly significant. Our MMWA algorithm can be used off-line to serve as the baseline for the evaluations of online heuristics. Furthermore, for small value of $K$, i.e., using relatively small number of extra wavelengths on each link, MMWA algorithm can give good cost performance, high ratio of Satisfied Requests and a reasonable time complexity for online computing.
Chapter 4

Multicast Wavelength Assignment for Sparse Wavelength Conversion

In this chapter, multicast wavelength assignment problem is studied under sparse wavelength conversion basis. As discussed in the previous chapters, for full wavelength conversion, the input wavelength can be converted into any output wavelength. A wavelength converter can be simply used per wavelength per port in a dedicated manner. However, from the commercial point of view, this may not be cost-efficient, since the costs of commercial wavelength converters are prohibitively high. One solution is to employ limited wavelength converters which is relatively cheap and easy to implement, as discussed in Chapter 3. The other alternative is try to reduce the usage of wavelength converters either at the node or in the network. At the node level, it is possible to share the use of wavelength converters in a switch node. Figure 4.1(a) and Figure 4.1(b) show two possible architectures proposed in the classic paper by Lee and Li [110], in which the wavelength converter is shared either by each link or by the node.
On the other hand, at the network level, it is feasible to allocate wavelength converters to only a certain number of nodes in the network, i.e., some nodes possess the wavelength conversion capabilities while others do not. This refers to sparse wavelength conversion [111, 112]. Such wavelength-conversion-capable nodes, for simplicity, are called WC nodes and other nodes in the network that are wavelength-conversion-incapable are named WI nodes.

In this chapter, we study the multicast wavelength assignment problem for sparse wavelength conversion in the wavelength-routed WDM networks. To our knowledge, little re-
search work has been done in this area. In the approach, we assume a full light splitting capability for each node in the network. The network has two types of nodes: WC nodes and WI nodes. The WC nodes have full wavelength conversion capabilities, and the WI nodes have no wavelength conversion capabilities. Within each WC node, we do not distinguish whether the wavelength converters are used in dedicated or shared manner. We assume that for each multicast request, each involved WC node can provide sufficient number of wavelength converters. Our focus is to explore the minimum number of wavelength converters required for a multicast request. For the multicast wavelength assignment problem, the input is triggered by the multicast request which forms a multicast tree whereas the output provides optimal wavelength assignment on each link of the tree. The following contributions have been made in this chapter. Firstly, the random wavelength assignment algorithm for the sparse wavelength conversion network has been extended from the unicast case to the multicast case. Secondly, by recognizing the inefficiency of such straightforward extension, a novel virtual link method has been proposed to map the multicast tree from the sparse conversion case to the full conversion case. Thirdly, a new multicast wavelength assignment algorithm has been developed to minimize the number of wavelength converters required to establish a light-tree.

The rest of the chapter is organized as follows. In Section 4.1, we give a formal definition of multicast wavelength assignment problem for the sparse wavelength conversion. Then, the conventional random wavelength assignment algorithm is extended from the unicast case to the multicast case, the inefficiency of which is uncovered and analyzed. In Section 4.2, we propose a new algorithm for the multicast wavelength assignment for sparse wavelength conversion (MWA-SWC). The algorithm is capable of minimizing the number of wavelength
converters required to establish a light-tree for a given multicast request. In Section 4.3, simulation experiments are carried out and results are analyzed. Finally, in Section 4.4, we summarize the results and give a conclusion.

4.1 Wavelength Assignment for Sparse Wavelength Conversion

4.1.1 Problem Definition

A multicast request is represented by a pair \((s, Y)\) with the multicast group size \(G\), where \(s\) is the source and \(Y\) is the set of destinations, and \(G = 1 + |Y|\). For each multicast request, a multicast tree \(T(V_T, E_T)\) can be formed. The tree \(T\) has \(s\) as the root node and spans all destination nodes in \(Y\). The tree \(T\) may contain non-destination nodes, i.e., intermediate nodes. All leaf nodes in the tree are destination nodes. For a tree \(T\), let \(N\) denote the total number of nodes, i.e., \(N = |V_T|\), and \(N \geq G\). Thus, the total number of links in the tree is \(N - 1\), i.e., \(|E_T| = N - 1\). For sparse wavelength conversion network, we assume that among all \(N\) nodes, \(M\) nodes are WC nodes and the rest are WI nodes. Let \(\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_W\}\) denote the set of all \(W\) distinct wavelengths supported in the tree. Each link in the tree has some available wavelength(s) that can be used to transfer data. We define the available wavelength set on the input link of non-root node \(v\) as \(A(v)\).

For each WI node in the tree, since it cannot perform wavelength conversion, the wavelength continuity constraint must be obeyed: the input and all the output links of each WI node must use the same wavelength. On the other hand, for each WC node in the tree, although it can provide wavelength conversion capabilities, our objective is to minimize the
required number of wavelength converters. Thus, we define the required number of wavelength converters from input wavelength $\lambda_p$ to output wavelength $\lambda_q$ at node $v$ in the form of $h_v(\lambda_p, \lambda_q)$,

$$h_v(\lambda_p, \lambda_q) = \begin{cases} 
1, & \text{if } \lambda_p \neq \lambda_q \\
0, & \text{if } \lambda_p = \lambda_q 
\end{cases} \quad (4.1)$$

For the given multicast tree $T$ and the available wavelength set $A(v)$ of all links in the tree, the multicast wavelength assignment is to select a wavelength $F(v) \in A(v)$ on the input link for each non-root node $v$ of the tree, in order to transmit the signal from the root node to all destination nodes in the tree. Therefore, for a given multicast tree $T$, a wavelength assignment, $\pi_T$, is defined as the set of assigned wavelength for each link in the tree, i.e.,

$$\pi_T = \{F(v) | F(v) \in A(v), v \in V_T - \{s\}\} \quad (4.2)$$

Thus, the total number of wavelength converters required for the tree $T$ under the wavelength assignment $\pi_T$ is,

$$H_T(\pi_T) = \sum_{v \in M} \left( \sum_{x \in B(v), F(v), F(x) \in \pi_T} h_v(F(v), F(x)) \right) \quad (4.3)$$

where $B(v)$ is defined as the set of child nodes of node $v$.

Based on the above definitions, we formalize the definition of the multicast wavelength assignment problem as follows:

**Definition**: Given a multicast tree $T(V_T, E_T)$ with root node $s$ and available wavelength set $A(v)$ on the input link of each non-root node $v$. The tree $T$ has $N$ nodes in total, and
Multicast Wavelength Assignment for Sparse Wavelength Conversion

in which M WC nodes have wavelength conversion capabilities and the rest are WI nodes.
The multicast wavelength assignment problem is to find a wavelength assignment \( \pi_T \) by
assigning the wavelength \( F(v) \in A(v) \) on the input link for each non-root node of the tree.
The objective is to minimize the total number of wavelength converters required.

4.1.2 Random Wavelength Assignment Algorithm

For this multicast wavelength assignment problem, we can simply extend the random wavelength
assignment algorithm from the case of point-to-point lightpath to the multicast case.
Given a tree \( T \) to which we need to assign wavelength(s), let \( S \) be the set of idle wavelengths
available on the tree, i.e., each wavelength \( \lambda \in S \) is available on each link of the tree, and
thus, \( S \subseteq A(v) \) for any node \( v \). Then, we have,

- For the case that no wavelength converters in the network: If \( S \) is nonempty, choose a random wavelength from \( S \); otherwise, the request is blocked.

- For the sparse wavelength conversion case in the network: Try to assign a wavelength
  without using wavelength conversion as above. If not possible, i.e., \( S \) is empty, divide
  the tree \( T \) into subtrees, \( t_1, t_2, \ldots, t_k \), according to the wavelength converter avail-
  ability at intermediate nodes of the tree. Let \( S_1, S_2, \ldots, S_k \) be the set of idle wave-
  lengths available on subtrees \( t_1, t_2, \ldots, t_k \), respectively. For each \( t_i, 1 \leq i \leq k \), if \( S_i \) is
  nonempty, choose a random wavelength from set \( S_i \); otherwise, the request is blocked.

We use a simple example to illustrate the above random wavelength assignment algo-

rithm, as shown in Figure 4.2. In this case, we assume node 2 is a WC node and node 4 is
Multicast Wavelength Assignment for Sparse Wavelength Conversion

Figure 4.2: An example of random wavelength assignment algorithm. (a) Multicast tree. (b) Subtrees after decomposition.

a WI node \(^1\). For the given multicast tree (Figure 4.2(a)), it is divided into three subtrees at the intermediate WC node 2, as shown in Figure 4.2(b). In this way, \(S_1 = \{\lambda_1, \lambda_2, \lambda_3\}\) for subtree \(t_1\), \(S_2 = \{\lambda_1, \lambda_2\}\) for subtree \(t_2\), and \(S_3 = \{\lambda_2\}\) for subtree \(t_3\). Then, for each subtree, one available wavelength will be selected randomly from its set of \(S\).

From the above example, we can find that this random algorithm is quite straightforward. However, it is “naïve” in the sense that it may utilize more wavelength converters than necessary to establish a light-tree. This is because that the above algorithm does not consider the possibility that certain adjacent subtrees may have common free wavelengths and hence a wavelength converter does not need to be used in going between those subtrees. Take Figure 4.2(b) as an example. Assume that, for subtree \(t_1\), \(\lambda_3\) is chosen; for \(t_2\), \(\lambda_1\) is selected; and for \(t_3\), \(\lambda_2\) is chosen. In this way, we may need two wavelength converters at node 2 to perform wavelength conversions. On the other hand, if for subtree \(t_1\) and subtree \(t_2\), we choose \(\lambda_2\), for both subtrees and then, there will be no wavelength converters required at node 2. As we

\(^1\)Since node 1 is the root and node 3, 5 and 6 are leaf nodes, whether they are WC or WI nodes does not make any difference in our explanation. Thus, we do not distinguish them in the example.
know that, if the use of wavelength converters is on a sharing basis in a node as shown in Figure 4.1, then, for a multicast request, we should try to minimize the use of converters along the route. Using less number of converters for one request will provide more opportunities for other requests, thus, benefiting overall network performance. In summary, to reduce the use of wavelength converters, is always desirable due to the cost and resource constraints. Consequently, minimizing the number of wavelength converters required to establish a light-tree should be a key objective when designing the wavelength assignment algorithm. In the next section, we propose a new multicast wavelength assignment algorithm for sparse wavelength conversion to minimize the number of wavelength converters required to establish a light-tree for the given multicast request.

4.2 A New Multicast Wavelength Assignment Algorithm

4.2.1 Motivation and Objective

As presented in the previous Section 4.1.2, the random wavelength assignment will result in using extra wavelength converters. This problem arises from the tree decomposition at the intermediate WC nodes. When the whole tree is divided into several subtrees and the wavelength assignment is done independently for each subtree, the assigned wavelengths between adjacent subtrees may lead to unnecessary wavelength conversions. In order to overcome this problem, we introduce a new idea not to decompose the tree but to map the input multicast tree from the sparse conversion case to full conversion case. In this way, the integrity of the original input tree can be secured. The objective of the new multicast wavelength assignment algorithm should be to minimize the number of wavelength converters required for the given
multicast request.

### 4.2.2 Organization

In order not to decompose the multicast tree, the basic idea of the new algorithm is to map the original multicast tree from the sparse conversion case to full conversion case by generating an auxiliary tree. Then, based on the generated auxiliary tree, a dynamic programming algorithm for wavelength assignment can be developed to minimize the number of wavelength converters required. Finally, the outcome of wavelength assignment will be mapped back to the sparse conversion case.

The organization of the new algorithm is illustrated in Figure 4.3. The input of the algorithm is the given multicast tree $T$ and the output is the wavelength assignment $\pi_T$ for the tree $T$. The whole algorithm consists of three parts, i.e., three sub-algorithms. The first algorithm is used to generate the auxiliary tree $T_a$; based on this tree $T_a$, the wavelength assignment algorithm is performed to find the wavelength assignment result $\pi_{T_a}$ for $T_a$. Finally, the result $\pi_{T_a}$ is mapped back to $\pi_T$ for $T$.

In general, the new algorithm of multicast wavelength assignment for sparse wavelength conversion (MWA-SWC) can be summarized as shown in Table 4.1. For the rest of this section, the three sub-algorithms will be introduced in details respectively.
4.2.3 Auxiliary Tree Generation

To map the input multicast tree from the sparse wavelength conversion case to the full wavelength conversion case, we make use of a new mapping method called *virtual link* method,
Multicast Wavelength Assignment for Sparse Wavelength Conversion

to produce the auxiliary tree. For different kinds of nodes of the tree, the virtual link method will take different actions. Virtual link method is a graph theory technique [97]. The so-called virtual links do not exist in the original graph and are used to represent or replace some actual links or nodes in the original graph to generate the auxiliary graph. In [5], the concept of “lightpath” was firstly introduced and the auxiliary graph was used to find routes for lightpaths, in which some virtual links are used to represent wavelength links and wavelength conversions. In [62], the virtual links are used to construct the auxiliary graph for the wavelength assignment problem by solving the vertex-coloring problem. In [90, 94], the auxiliary graph technique is employed for the traffic grooming problem. In general, virtual link based auxiliary graph is a common technique used in the optical networking research.

For each non-leaf WI node in the tree, the basic idea of the scheme is to replace this WI node together with its incident edges by some virtual links. The common wavelength(s) available on both the input and output links of this WI node will be set as the available wavelength(s) of the virtual links. On the other hand, all the leaf nodes and WC nodes in the network will be kept unchanged. In this way, since all the WI nodes of the original multicast tree are replaced by virtual links, the original tree is successfully transformed from the sparse wavelength conversion case to full wavelength conversion case, and thus the integrity of the tree is kept. Then, based on the auxiliary tree, we can find some efficient wavelength assignment algorithm for the given optimization objectives.

We define the auxiliary tree to be $T_a(V_{T_a}, E_{T_a})$, which has in total $N'$ nodes, where $N - M \leq N' \leq N$. The construction of tree $T_a$ is as follows:

- For the source node $s \in V_T$ in the original tree $T$, there is a corresponding source node $s' \in V_{T_a}$ in the auxiliary tree $T_a$;
For each leaf node $v \in V_T$ in the original tree $T$, there is a corresponding leaf node $v' \in V_{T_a}$ in the auxiliary tree $T_a$;

For each internal node $v \in V_T$ in the original tree $T$,

1. If $v$ is a WC node, there will be a corresponding internal node $v' \in V_{T_a}$ in the auxiliary tree $T_a$;

2. If $v$ is a WI node, there will be no corresponding node in the auxiliary tree $T_a$ and the corresponding child nodes of $v$ in the tree $T_a$ will be connected directly to the corresponding parent node of $v$.

From the definition, we can find that,

- The nodes of the auxiliary tree $T_a$ is a subset of the nodes of the original tree $T$, i.e., $V_{T_a} \subseteq V_T$, since some WI nodes in $T$ are removed in the auxiliary tree $T_a$;

- The links of the auxiliary tree $T_a$ is not a subset of the links of the original tree $T$, since some virtual links in $T_a$ are used to replace the incident links of the WI nodes in $T$.

We take the multicast tree in Figure 4.4(a) as an example. There are three links incident to the WI node 4, link (2, 4), link (4, 5) and link (4, 6). Let $S$ be the set of common idle wavelengths available on these three links, and then $S = \{\lambda_2\}$. Delete node 4 and add one virtual link directly between node 2 and node 5 and another one between node 2 and node 6, respectively. Then, make $S$ be the set of available wavelengths for each of the added virtual links. The outcome is as illustrated in Figure 4.4(b).
In general, the procedure of auxiliary tree generation can be summed up and presented in a flowchart as shown in Figure 4.5. Let the original tree $T$ have a topological ordering based on breadth-first traversal. Let the auxiliary tree $T_a$ be initialized to $T$, i.e., $T_a = T$. By employing a bottom-up approach, each node of the auxiliary tree will be examined and each WI node will be replaced by the virtual links accordingly. For the original tree with $N$ nodes, the time complexity of the auxiliary tree generation algorithm is $O(N)$.

### 4.2.4 Wavelength Assignment Based on Auxiliary Tree

When the auxiliary tree has been successfully generated, the wavelength assignment process starts. Since the auxiliary tree is with full wavelength conversion, we can employ a dynamic programming algorithm to obtain the wavelength assignment, which aims to minimize the number of wavelength converters required to establish the connection for the given request.

For each non-root node $v'$ in the tree $T_a$, $H_{v'}(\lambda)$ is defined to be the minimum number of wavelength converters, which are needed in the subtree rooted at $v'$, assuming that the signal
enters $v'$ on wavelength $\lambda$ and can be forwarded to all of its descendant destinations. If $\lambda$ is not available on the input link or is not possible for $v'$ to reach all of its destination nodes in
Multicast Wavelength Assignment for Sparse Wavelength Conversion

the subtree when the message enters \( v' \) on wavelength \( \lambda \), then \( H_{v'}(\lambda) = \infty \).

For a leaf node \( v' \) in the tree \( T_a \), we have

\[
H_{v'}(\lambda) = \begin{cases} 
0, & \text{if } \lambda \in A(v') \\
\infty, & \text{otherwise}
\end{cases}
\]  

(4.4)

As node \( v' \) is a leaf node of the tree, it has no child node. Thus, the subtree rooted at \( v' \) is the node \( v' \) itself, i.e., there are no descendent branches for it. Then, any available wavelength on the input link of node \( v' \) would lead the number of wavelength converters required to be zero, i.e., for any \( \lambda \in A(v') \), \( H_{v'}(\lambda) = 0 \).

Then, we come to the internal nodes. For an internal node \( v \) in the tree \( T_a \), let \( B(v') \) be the set of child nodes of node \( v' \), and we have,

\[
H_{v'}(\lambda) = \begin{cases} 
\sum_{x \in B(v')} \min_{\lambda' \in A(x)} \left( H_x(\lambda') + \Delta \right), & \text{if } \lambda \in A(v') \\
\infty, & \text{otherwise}
\end{cases}
\]  

(4.5)

where

\[
\Delta = \begin{cases} 
0, & \text{if } \lambda' = \lambda \\
1, & \text{if } \lambda' \neq \lambda
\end{cases}
\]  

(4.6)

Since node \( v' \) is an internal node, for any child node \( x \) of node \( v' \), the value of \( H_x(\lambda') \) denotes the number of wavelength converters required at the subtree rooted at node \( x \), when the input wavelength of node \( x \) is \( \lambda' \). \( \Delta \) actually stands for the number of wavelength converters required in the node \( v' \) for the input signal passing through node \( x \). Therefore, \((H_x(\lambda') + \Delta)\) would represent the number of wavelength converters required arising from the child node \( x \). Get the minimal value of \((H_x(\lambda') + \Delta)\) for each child node \( x \). Consequently, the sum of all these minimum values of \((H_x(\lambda') + \Delta)\) should give the minimum number of wavelength
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converters required for the subtree rooted at node \( v' \), when the input wavelength of node \( v' \) is \( \lambda \).

Finally, consider the root node \( s \). We define \( H_{T_a} \) to be the minimum number of wavelength converters needed for the source node \( s \) to deliver the message to all its destination nodes in the multicast tree \( T_a \). Then, we have,

\[
H_{T_a} = \sum_{x \in B(s)} \min_{\lambda' \in A(x)} H_x(\lambda')
\]  

(4.7)

Based on the above equations, for each node in the tree \( T_a \), we can use a bottom-up approach to apply the dynamic programming algorithm to compute the values of \( H_{v'}(\lambda) \) for all non-root nodes of \( T_a \), as well as the value of \( H_{T_a} \) for the root of the tree. During the process of computations, when \( H_{v'}(\lambda) \) is computed for a given vertex \( v' \), this implies that the required values for all of its children in the subtree have already been computed. When all the values of \( H_{v'}(\lambda) \) for each node in the tree \( T_a \) have been obtained, we start to select the wavelength for each link. The values of \( H_{v'}(\lambda) \) serve as the criteria when selecting wavelengths and we define that, for each node \( v' \), the minimum value of \( H_{v'}(\lambda) \) is \( H^m_{v'} \), i.e.,

\[
H^m_{v'} = \min_{\lambda \in \Lambda} H_{v'}(\lambda)
\]

When selecting wavelength for the input link of each node \( v' \) in \( T_a \), the key selection rule is to choose the wavelength \( \lambda \) such that \( H_{v'}(\lambda) = H^m_{v'} \). If the wavelength for the input link of a node is assigned in this way, it implies that the number of wavelength converters required for the subtree rooted at such node is minimum. Therefore, if the wavelengths for all the links of the tree are assigned under this rule, the total number of wavelength converters needed for
the tree can be guaranteed to be minimum.

In general, the algorithm for wavelength assignment based on the auxiliary tree can be summed up by the pseudo code as shown in Table 4.2.

Table 4.2: Algorithm for Wavelength Assignment Based on Auxiliary Tree

<table>
<thead>
<tr>
<th>INPUT:</th>
<th>Auxiliary tree $T_a$ and available wavelength set $A(v')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTPUT:</td>
<td>Wavelength assignment $\pi_{T_a}$ for the tree $T_a$</td>
</tr>
<tr>
<td>BEGIN</td>
<td>Let $v'<em>{1}, v'</em>{2},..., v'_{N}$ be the topological ordering of the N nodes in the multicast tree</td>
</tr>
<tr>
<td>FOR ($j = N$ down to 1) DO</td>
<td>Let $v' = v'_j$</td>
</tr>
<tr>
<td>IF $v'$ is a leaf node THEN</td>
<td>FOR (each wavelength $\lambda \in \Lambda$) DO Compute $H_{v'}(\lambda)$ using (4.4)</td>
</tr>
<tr>
<td>ELSEIF $j &gt; 1$ THEN</td>
<td>FOR (each wavelength $\lambda \in \Lambda$) DO Compute $H_{v'}(\lambda)$ using (4.5)</td>
</tr>
<tr>
<td>ENDIF</td>
<td>ENDFOR</td>
</tr>
<tr>
<td>ENDFOR</td>
<td>FOR (each child node $v'$ of root node) DO</td>
</tr>
<tr>
<td>Set $F(v')$ to be any wavelength $\lambda$ satisfying $H_{v'}(\lambda) = H_{v'}^{m}$</td>
<td></td>
</tr>
<tr>
<td>ENDFOR</td>
<td>FOR ($j = 2$ up to $N$) DO Let $v' = v'_j$</td>
</tr>
<tr>
<td>FOR (each child node $x$ of non-leaf node $v'$) DO</td>
<td></td>
</tr>
<tr>
<td>IF $F(v') \in A(x)$ and $H_{x}(F(v'))=H_{x}^{m}$ THEN Set $F(x)=F(v')$</td>
<td></td>
</tr>
<tr>
<td>ELSE Set $F(x)$ to be any wavelength $\lambda$ satisfying $H_{x}(\lambda)=H_{x}^{m}$</td>
<td></td>
</tr>
<tr>
<td>ENDIF</td>
<td>ENDFOR</td>
</tr>
<tr>
<td>ENDFOR</td>
<td>END</td>
</tr>
</tbody>
</table>

In the above algorithm, the computation in (4.5) requires the largest number of steps. Thus, the upper bound of computation steps required to obtain the values of $H_{v'}(\lambda)$ in (4.5) can be derived as follows: For each wavelength $\lambda \in A(v')$, at most $W$ steps are needed to confirm whether it is in the available wavelength set of $A(x)$ for each child node $x$, i.e., $W$ steps are required to obtain the value of $(H_{x}(\lambda') + \Delta)$ for each child node. Let $D$ be the
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maximum out-degree of nodes of the tree, and then for the worst case, each internal node \( v' \) would have at most \( D \) child nodes. Therefore, at most \((DW)\) steps are needed to obtain the value of \( H_{v'}(\lambda) \) for each wavelength \( \lambda \in A(v') \) for node \( v' \). Since \( A(v') \) has at most \( W \) available wavelengths, the total steps are \((DW^2)\) at most. Thus, for total \( N \) nodes in the tree, the computation steps are upper bounded by \( O(NDW^2) \).

Table 4.3 shows the computation results of the example tree in Figure 4.4(b). The underlined values indicate that the corresponding wavelengths are selected on the input link of the nodes.

![Table 4.3: Computation Results of \( H_{v'}(\lambda) \) for All Nodes](image)

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{v'}(\lambda_1) )</td>
<td>-2</td>
<td>0</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( H_{v'}(\lambda_2) )</td>
<td>-0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( H_{v'}(\lambda_3) )</td>
<td>-3</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( H_{v'}(\lambda_4) )</td>
<td>-2</td>
<td>0</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( H_{m}^{v'} )</td>
<td>-0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

4.2.5 Result Mapping Back

After applying the above algorithms to the given multicast tree, we obtain the wavelength assignment for the auxiliary tree \( T_a \). Finally, such outcome can be mapped back to the original tree \( T \). The mapping is the reverse process of auxiliary tree generation. This poses no problem as we have the full knowledge when we generate the auxiliary tree \( T_a \) from \( T \). The pseudo codes are given in Table 4.4.

In summary, this section introduces the new algorithm of multicast wavelength assignment for sparse wavelength conversion (MWA-SWC). The basic idea of this algorithm is to
Table 4.4: Algorithm for Results Mapping Back

| INPUT: Multicast tree $T$, auxiliary tree $T_a$ and wavelength assignment for $T_a$ |
| OUTPUT: Wavelength assignment for tree $T$ |
| Initialization: Let each node $v'$ for the auxiliary tree $T_a$ has the corresponding node $v$ in tree $T$ |

BEGIN

FOR (each node $v'$ in the auxiliary tree $T_a$) DO

IF the corresponding node $v$ is a leaf node in $T$ THEN

Set $F(v) = F(v')$;

ELSE

IF the corresponding node $v$ is a WC node in tree $T$ THEN

Set $F(v) = F(v')$;

ELSEIF the corresponding node $v$ is a WI node in tree $T$ THEN

Set $F(v)$ for all the deleted nodes in $T_a$ to be $F(v')$;

ENDIF

ENDIF

ENDFOR

END

keep the integrity of the tree when doing wavelength assignment. The input multicast tree is mapped from sparse wavelength conversion case to full wavelength conversion case by replacing some WI nodes of the input tree with some virtual links in the auxiliary tree. Then, based on the full-wavelength-conversion auxiliary tree, the wavelength assignment can be conducted via dynamic programming technique. Finally, the wavelength assignment results for the auxiliary tree is translated back for the original input tree.

4.3 Simulation Results

In this section, we carry out a simulation study to evaluate the performance of our proposed algorithms. There are two important metrics to be used for the evaluation. One is the ratio for Satisfied Requests, which was suggested in [44]. This metric gives the ratio of the number
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Figure 4.6: NSFNET topology. WA, UT, CO, TX, IL, PA and NY are WC nodes.

of requests that can be satisfied (i.e., not be blocked) by the wavelength assignment over the total number of requests. The higher is the ratio, the better is the performance of the algorithm. The other metric is the number of wavelength converters required to establish a connection for the multicast request. In this section, we first employ NSFNET topology to evaluate the satisfied requests ratio for different multicast wavelength assignment algorithms. Then, we will make use of random input multicast trees to compare the number of wavelength converters required by various algorithms.

For the sparse wavelength conversion, among the total 14 nodes in the NSFNET, we choose 7 nodes to be WC nodes, and their locations are indicated in the Figure 4.6. The number and the location of the WC nodes are directly adopted from the results presented in [113]. In this way, the performance of the network will be quite close to the full wavelength conversion case. For each multicast request, given the multicast group size $G$, we choose $G$ nodes randomly from the 14 nodes. Among the $G$ nodes, one node is randomly selected as the source node and the others are destinations. For each request, we employ a simple shortest path tree heuristic algorithm [21] to construct the multicast tree $T$. The algorithm starts by selecting a shortest path from the source to an arbitrary destination node.
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It then repeatedly adds a new destination by selecting the shortest path between this destination node and the current partial tree, until all destinations are included in the tree. In our simulation, we employ the Dijkstra’s algorithm [46] to compute the shortest path between any two nodes when constructing the shortest path tree. Actually, as mentioned before, there are various algorithms [21] for the multicast tree construction. As this thesis focuses mainly on the wavelength assignment, we do not pursue further the tree construction problem. In general, the heuristic solution of the routing algorithm would have some influence on the wavelength assignment. On one hand, for the same multicast request, different routing algorithms may construct different routing trees and different tree topologies constructed would affect the performance of wavelength assignment algorithm. On the other hand, for the same multicast request under the same network state, different routing algorithms may construct different trees under different link resource states. For example, for the fixed routing algorithm, each request is assigned a single fixed route. If any link has no available wavelength, the request will be blocked, and as a result, the wavelength assignment will be useless. The fixed-alternate routing provides more candidate routes, thus, increasing the chance that a request will be accepted. The adaptive routing, on the other hand, makes use of the current network state in choosing a route. As a result, the tree chosen to establish a multicast request reflects the current utilization of the links in the network. Consequently, the adaptive routing may construct a tree whose links tend to have more wavelengths available. When more wavelengths are available on links, a more optimal wavelength assignment result can be obtained.

In our simulation, we make use of simple shortest path tree algorithm and put our focus on the performance of wavelength assignment algorithms.

Each link in the tree $T$ is assumed to support 8 distinct wavelengths and the set of avail-
able wavelengths on each link is selected at random where the size of the set is taken from
the uniform \([1, L]\) distribution, where \(L\) is the maximum number of available wavelengths
on each link of the tree. For each experiment, we generate 10,000 requests, and apply the
random wavelength assignment algorithm and the MWA-SWC algorithm to the requests, re-
spectively, and then calculate the ratio for Satisfied Requests for the two algorithms.

Figure 4.7 and Figure 4.8 show the ratio of Satisfied Requests for MWA-SWC and Ran-
dom assignment algorithms at different group sizes, for case \(W = 8\) and \(W = 16\), re-
spectively. The curves for the two algorithms overlap, showing that the random assignment
algorithm has the same performance as the MWA-SWC algorithm as expected. The reason is
that, given a multicast tree and the available wavelengths on each link, the tree can be satisfied
by the wavelength assignment \textit{if and only if there exists common idle wavelength(s) available
on the links incident to any WI node}. If such condition is satisfied, the request can be estab-
lished by the wavelength assignment algorithm, and if not, the request will be blocked. Thus,
if a request is not satisfied by the random assignment algorithm, it means that there must
exist at least one WI node in the tree whose incident links do not contain common available
wavelengths and thus in this situation, this request will certainly not satisfied by any other
wavelength assignment algorithms, including MWA-SWC approach. Therefore, the ratio of
Satisfied Requests must be the same for random as well as MWA-SWC algorithm. In both
two figures, we observe that, for the same group size, the ratio of Satisfied Requests rises as
the number of available wavelengths on each link increases. This is because the more avail-
able wavelengths on the links, the lower chance that the request will be blocked. On the other
hand, we find that, as the group size \(G\) increases, the Satisfied Requests drops. It is because
for larger group size, the multicast tree is likely to span more links. This will lead to a higher
probability for the request to be blocked.

Besides the Satisfied Requests, we evaluate the number of wavelength converters (NWC) required for different algorithms. Here, we still employ a similar method as that in [43, 44] to generate the random multicast tree. Given \( N \) (\( N = 100 \) in our simulation) number of nodes with the index from 1 to \( N \), where ‘1’ denotes the root node. Each node has a random number
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of out-degree between 0 and $D$, where $D$ is the maximum out-degree of each node in the tree and we choose $D = 4$ in the simulation. Among the $N$ nodes, we randomly choose $M$ (the default value is 50) nodes to be WC nodes. Each link is assumed to support $W$ distinct wavelengths and the set of available wavelengths on each link is selected at random where the size of the set is taken from the uniform $[1, L]$ distribution, where $L$ is the maximum number of available wavelengths on each link of the tree.

For the purpose of comparison, we also introduce a greedy algorithm. For many optimization problems, greedy algorithm is a simple and efficient approach. A greedy algorithm [46] always makes the choice in the hope that looks best at the moment. That is, it makes a local optimal choice in the hope that this choice will lead to a global optimal solution. In our multicast wavelength assignment problem, we can also employ a greedy algorithm which operates from the source node as follows. For each node, the available wavelength is selected such that it can be used to reach the largest number of its child nodes, with arbitrary tie breaking. The process of the greedy algorithm is illustrated by a flowchart in Figure 4.9. The time complexity of this greedy algorithm is $O(NW)$. This greedy algorithm determines the wavelength from local point of view each time, and the global optimality of the whole tree cannot be guaranteed.

We compare the performance of the random and greedy algorithms with our proposed MWA-SWC algorithm. For each set of parameters $(N, M, D, W, L)$, we evaluate 10,000 satisfied requests and compute the average values of number of wavelength converters required.

Figure 4.10 shows the number of wavelength converters required by various algorithms for the case $W = 8$. It shows that the greedy algorithm performs better than the random
algorithm. However, our proposed MWA-SWC performs the best - better than both greedy and random algorithms in reducing the number of wavelength converters. When the maximum number of available wavelengths on each link increases, both the greedy algorithm and MWA-SWC algorithm tend to decrease. Thus, when the available wavelengths on links are
relatively large, the greedy algorithm and MWA-SWC algorithm are able to reduce, to a great extent, the number of wavelength converters. A similar plot with $W = 16$ is given in Figure 4.11, showing a similar trend in saving the number of wavelength converters.

The MWA-SWC algorithm outperforms the random and greedy algorithms by reducing the number of wavelength converters required. To further show the performance advantage of the MWA-SWC algorithm, we evaluate the improvement percentage of reduction on number
Converter Reduction (%) = \frac{\text{NWC by Random/Greedy Algorithm} - \text{NWC by MWA-SWC Algorithm}}{\text{NWC by Random/Greedy Algorithm}}

Figure 4.12 and Figure 4.13 show the Converter Reduction of MWA-SWC over random and greedy algorithms for the case \( W = 8 \) and \( W = 16 \), respectively. In general, the Converter Reduction increases as the value of \( L \) increases. This indicates that, the more...
wavelengths available on the links of the tree, the more effective is the MWA-SWC algorithm in reducing the number of wavelength converters. For the case of $W = 8$, when the maximum number of available wavelengths on links, $L$, is 4, the Converter Reduction is over 60% for MWA-SWC algorithm over random algorithm, and about 30% for MWA-SWC over greedy algorithm; when $L = 7$, such percentages increase to about 84% and 65%, respectively. For the case of $W = 16$, the Converter Reduction increases from 73% ($L = 8$) to 90% ($L = 15$) for MWA-SWC algorithm over random algorithm, and from 44% ($L = 8$) to 77% ($L = 15$) for MWA-SWC algorithm over greedy algorithm. The reduction is rather significant.

We note that in the above comparisons, the number of WC nodes in the tree, $M$, is fixed at 50. A natural question will be that whether the MWA-SWC algorithm has a consistent performance if we vary the number of WC nodes in the tree. In Figure 4.14 and Figure 4.15, we address the question by evaluating Converter Reduction of MWA-SWC algorithm over the random algorithm for varying number of WC nodes. Here, $M = 100$ represents the full wavelength conversion case. As indicated, the Converter Reduction for different values of $M$.

![Converter Reduction vs. $L$ for various No. of WC nodes, $W = 8$.](image-url)
number of WC nodes have the same steadily increasing trend, as the value of \( L \) increases. The behavior shows the consistency of the MWA-SWC algorithm. We also find that, the less the number of WC nodes is, the more Converter Reduction can be obtained. This is because, when the number of WC nodes in the network is relatively small, the number of wavelength converters incurred by a light-tree is also relatively small. Thus, a relatively small reduction of the number of wavelength converters could lead to a relatively high Converter Reduction.

### 4.4 Summary

In this chapter, we address the problem of multicast wavelength assignment for sparse wavelength conversion in the wavelength-routed WDM networks. The wavelength assignment problem has been studied extensively for point-to-point connections (i.e. unicast) but not so for multicast. To the best of our knowledge, little work has been done on the multicast wavelength assignment problem for sparse wavelength conversion in literature. In this chapter, we first extend the random wavelength assignment algorithm from the unicast case to the multi-
cast case. The straightforward extension leads to inefficient use of wavelength converters. In order to minimize the use of wavelength converters for a given multicast request, we propose a new MWA-SWC algorithm. This algorithm first maps the multicast tree from the sparse conversion case to the full conversion case. We then propose a novel virtual link method for the tree mapping. The method generates an auxiliary tree and also handles the reverse mapping. Using the auxiliary tree, we propose a dynamic programming algorithm for the wavelength assignment aiming to minimize the number of wavelength converters required. Simulation results show that our new algorithm outperforms both random and greedy algorithms with regards to minimizing the number of wavelength converters. The performance is consistent, as demonstrated by varying the number of wavelength conversion nodes in the tree. The MWA-SWC algorithm is useful primarily for static traffic. However, it can also serve as a baseline for dynamic heuristic algorithms. Typically, the MWA-SWC algorithm will provide great benefit when the number of available wavelengths on links of the multicast tree is relatively large and the performance advantage is significant.
Chapter 5

Multicast Wavelength Assignment in A New Multicast Switch Model

This chapter addresses the multicast wavelength assignment problem in a new multicast switch model in wavelength-routed WDM networks with full light splitting and wavelength conversion capabilities. Current approaches are based on the multicast switch model that supports only split-convert (S-C) switch scheme. This scheme leads to redundant wavelength conversions for a given multicast request. In this chapter, we propose a new split-convert-split (S-C-S) switch scheme capable of eliminating the redundant wavelength conversions. In order to implement this new switch scheme, we develop a new multicast switch model based on the concept of sharing of light splitters and wavelength converters. Furthermore, based on the new S-C-S multicast switch model and the new multi-wavelength assignment strategy developed in Chapter 3, we generalize the existing algorithms to produce a new Multicast Wavelength Assignment Algorithm (MWAA) to support both the new switch model and the new wavelength assignment strategy. Compared with the existing algorithm, this new algo-
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Algorithm is a more general one which makes the multicast wavelength assignment more flexible, covering different switch schemes and different assignment strategies. In addition, it delivers good performance in term of minimizing the number of wavelength conversions.

The rest of this chapter is organized as follows. Section 5.1 introduces multicast switch model and develops a new S-C-S model. Section 5.2 deals with multicast wavelength assignment algorithm which combines new switch model and multi-wavelength assignment strategy. Section 5.3 provides simulation results. Finally, Section 5.4 summarizes this chapter.

5.1 Multicast Switch Model

The approach to optical multicast is usually based on a multicast switch model. The multicast switch model assumes that a light signal on an input link may be routed to any number of output links, which is realized by employing passive light splitters. Within the switch model, if an input light signal is required to be changed from one wavelength to another, the wavelength conversion is needed. Transmitters and receivers can be adopted to perform a transceiver-based wavelength conversion. By this means, if a light signal requires conversion, it has to be firstly captured by a receiver and switched through electronic switch, and then retransmitted using a transmitter on a different wavelength. This is the O-E-O wavelength conversion [114]. The O-E-O conversion is flexible but it introduces electronic bottleneck, which may lead to a significant processing delay to the optical signal. Therefore, for the transceiver-based O-E-O wavelength conversions, it is always a crucial objective to minimize the cost or delay of wavelength conversions for the given multicast request, or simply, the number of wavelength conversions.
Wavelength conversion can also be performed by wavelength converters instead of transceiver-based conversions. A simple two-wavelength $2 \times 2$ multicast switch model employing wavelength converters is shown in Figure 5.1, in which the input signal is first demultiplexed into individual wavelength and each wavelength is then split into multiple output parts by light splitters. At the output side, wavelength converters are employed to perform wavelength conversions before the light signals enter the multiplexers. For those switch models employing wavelength converters, it is also useful to minimize the number of wavelength conversions. When the light signal enters a wavelength converter, some optical domain (or even electronic domain) operations will be introduced to the signal. These operations will affect the signal quality. In addition, performing the wavelength conversion in the converters will also introduce unwanted processing delay. For some delay-sensitive applications, such processing delay should be eliminated or minimized at least. Furthermore, as introduced in Chapter 4, there are some possible architectures, in which wavelength converter is shared either by each link or by the node. Since the use of wavelength converters is on a sharing basis in these architectures, for a multicast request, we should try to minimize the use of converters along the route. Using less number of converters for one request will provide more opportu-
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nities for other requests, thus, benefiting overall network performance. In summary, to reduce the use of wavelength converters, i.e., the number of wavelength conversions, is always desirable due to the cost and resource constraints. Consequently, *minimizing the number of wavelength conversions* is a key objective when designing multicast switch model.

Normally, in a multicast switch model, the light signal will be first split into multiple output links and then wavelength conversion is applied to the light signal at each output link. This is so-called split-convert (S-C) switch scheme. Applying the S-C scheme to the example subtree as shown in Figure 5.2(a), the input wavelength $\lambda_1$ will be first split into four signals, and then, each copy will be converted, respectively, as shown in Figure 5.2(b). As shown, the wavelength conversion from $\lambda_1$ to $\lambda_2$ is repeated two times, so does the conversion from $\lambda_1$ to $\lambda_3$. Therefore, the number of wavelength conversion at node $v$ is four. The repeated conversion causes what we call *redundant wavelength conversion*, which is very costly. In order to remove the redundant wavelength conversions, an alternative scheme can be employed to convert the input wavelength first and then split it into individual parts. This is convert-split (C-S) scheme. The tradeoff issue between S-C and C-S schemes was first recognized in [32] and the on-line wavelength assignment to support the C-S model was studied in [115].

To generalize the S-C and C-S schemes, we propose a three-stage scheme, split-convert-split (S-C-S) scheme. This scheme first splits the signal into different wavelength groups, performs the conversion and then splits it into individual wavelength. This scheme can eliminate the redundant wavelength conversion. Take the example as shown in Figure 5.2(c). The output wavelengths can be clustered as two wavelength groups, one for $\lambda_2$ and the other for $\lambda_3$. Applying the S-C-S scheme, the signal is first split into two wavelength groups. Then,
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Figure 5.2: Different switch schemes. (a) An example subtree. (b) Split-convert (S-C) scheme. (c) Split-convert-split (S-C-S) scheme.

Each copy will be converted into the desired wavelength. After that, the converted signal will be split and sent to the individual output. In this way, the number of wavelength conversions is reduced to two.

How to implement the above three-stage S-C-S scheme in the data plane is in fact beyond the scope of this thesis. However, a concern may arise about the practicality of the proposed scheme. We shall suggest some implementation consideration here. The detail architectural design will be studied in the future.

Compared with S-C scheme, if one insists of constructing a non-blocking OXC switch with dedicated splitters and converters, then the S-C-S scheme may require extra splitters in the third stage splitting operation, thus, increasing the hardware cost and switch complexity.
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One way to overcome the hardware complexity is to provide flexible sharing of splitters and converters. In this way, one can avoid the use of extra splitters and converters in the implementation. The concept of splitter sharing was first proposed in [30] in order to reduce the number of light splitters used. Likewise, the idea of sharing wavelength converters can be found in [110], in which the wavelength converter(s) can be shared in an OXC on per link or per node basis. A simple example of splitter and converter are shared within an OXC can be found in [10], as shown in Figure 5.3. In this model, the signal on each input link is first demultiplexed into separate wavelengths. Then, the separate signals are switched by the optical switch. Signals that do not need duplication are sent directly to the output switch, while those signals requiring splitting are sent to light splitter bank (LSB) and wavelength converter bank (WCB). After that, they are sent to the output switch.

By combining these ideas and extending the above model, we propose a new three-stage multicast switch model that supports the S-C-S switch scheme, in order to eliminate the redundant wavelength conversions for the multicast requests. Figure 5.4 shows a $4 \times 4$ multicast switch model, which supports four wavelengths on each fiber link. The signal on each input
Figure 5.4: $4 \times 4$ multicast switch architecture.

link is first demultiplexed into separate wavelengths. Then, the separate signals are switched by the optical switch. Signals that do not need duplication are sent directly to the output switch, while those signals requiring splitting are sent to LSB, and then to the WCB. After that, if some signals require further duplication, they are then sent to the third stage LSB, otherwise, they are sent to the output switch directly. Consequently, in this architecture, the three-stage S-C-S switch scheme can be realized. Note that, the main difference between our proposed model and the existing model is that a third stage LSB is added in the new model. As the splitters and converters are shared in the OXC, our proposed architecture is not a non-blocking switch. This means that some of the new requests may be blocked if the splitters or converters are currently occupied by existing requests. This is a tradeoff between the hardware cost and the blocking property. It is important to note that, in this three-stage model, the light signal may not always need to be split into each of the output ports, and thus, the configurable light splitter [116] can be used in the LSB, which can adjust the splitting rate of its output ports as needed.
5.2 Multicast Wavelength Assignment Algorithm

In this section, we generalize existing multicast wavelength assignment algorithms to support the new multicast switch model proposed above.

5.2.1 Problem Definition

The optical networks can be represented by a directed graph $G(V, E)$ where $V$ is the set of nodes (vertices) and $E$ the set of links (edges) between any two nodes. A multicast session is represented by a pair $(s, Y)$, where $s$ is the source and $Y$ the set of destinations. In each multicast session, a multicast tree $T(V_T, E_T)$ can be formed, rooted at $s$ and spanning all destination nodes in $Y$. A multicast tree $T$ has $N$ total number of nodes, with a topological order 1 to $N$ by a breadth-first search. $W$ distinct wavelengths are supported in each link of the tree. Each link in the tree has some available wavelength(s) that can be used to transfer data. We define the available wavelength set on the input link of non-root node $v$ as $A(v)$.

For a given multicast tree $T$ and the available wavelength set $A(v)$ of all links in the tree, we define a wavelength assignment of tree $T$ to be a function $F$, in terms of non-root node of the tree, i.e.,

$$F : V_T \setminus \{s\} \mapsto \Lambda$$

such that, for each $v \in V_T \setminus \{s\}$, $F(v) \subseteq A(v)$. The multicast wavelength assignment algorithm is to select a wavelength set $F(v) \subseteq A(v)$ for each non-root node. We also define $K$ to be the maximum number of wavelengths permitted to carry the same signal on a single link, and $1 \leq K \leq L$, where $L$ is the maximum number of available wavelengths on each link in the tree.
In general, multicast wavelength assignment problem can be defined as follows:

**Definition:** Given a multicast tree $T(V_T, E_T)$ with root node $s$ and available wavelength set $A(v)$ on the input link of each non-root node $v$, multicast wavelength assignment problem is to assign the wavelength set $F(v) \subseteq A(v)$ on the input link for each non-root node of the tree, while the total number of wavelength conversions for the tree $T$, $c_T(F)$, is minimized.

### 5.2.2 Multicast Wavelength Assignment Algorithm

A multi-wavelength multicast wavelength assignment (MMWA) algorithm has been developed in Chapter 3 to support the multi-wavelength assignment strategy. In this subsection, we show how this algorithm can be generalized to support the new proposed multicast switch model.

For each non-root node $v$ in the tree $T$, we still define the cost function $c_v(S)$ to be the minimum number of wavelength conversions needed in the subtree rooted at $v$, assuming the signal enters $v$ on wavelength set $S$ and can be forwarded to all of its descendant destinations. If any wavelength in set $S$ is not available on the input link or it is not possible for $v$ to reach all of the destination nodes in its subtree when the signal enters $v$ on set $S$, then $c_v(S) = \infty$.

Firstly, we employ a dynamic programming algorithm to compute the values of $c_v(\lambda)$ for each link of the tree. This part is similar to that for the MMWA algorithm in Chapter 3, thus we omit the detailed mathematic definitions, theorems, and proofs here. We only list the related formulae required by this chapter, for the sake of completeness.
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Based on the definition of $c_v(S)$, for the leaf node of the tree, we have

$$c_v(S) = \begin{cases} 
0, & \text{if } S \subseteq A(v) \\
\infty, & \text{otherwise} 
\end{cases} \quad (5.1)$$

For an internal node $v$ in the tree $T$,

$$c_v(S) = \begin{cases} 
\sum_{x \in B(v)} \min_{S' \subseteq A(x), |S'| \leq K} (c_x(S') + \Delta), & \text{if } S \subseteq A(v) \\
\infty, & \text{otherwise} 
\end{cases} \quad (5.2)$$

where $B(v)$ is set of child nodes of node $v$, and

$$\Delta = |S' - S| = \sum_{\lambda \in S'} \begin{cases} 
0, & \text{if } \lambda \in S \\
1, & \text{if } \lambda \notin S 
\end{cases} \quad (5.3)$$

Based on the above formulae, for each node in the tree $T$, we can use a bottom-up approach to apply the dynamic programming algorithm to compute the values of $c_v(S)$ for all non-root nodes of $T$. During the process of computations, when $c_v(S)$ is computed for a given vertex $v$, this implies that the required values for all of its children in the subtree have already been computed. The related pseudo codes for the computation of values of $c_v(S)$ are given in Table 5.1.

As analyzed in Chapter 3, the above algorithm has $O(D(W^K/K!)^2N)$ computation time, with the constant term depending on parameters, $D$, $W$ and $K$, where $D$ is the maximum out-degree of node in the tree.

Based on all the computed values of $c_v(S)$ for the given multicast tree, the wavelength set for each link of the tree can be decided. For a non-root node $v$, each wavelength set $S$ has a
Table 5.1: Algorithm for the Computation of Values of $c_v(S)$

| INPUT: Multicast tree $T$ and available wavelength set $A(v)$ |
| OUTPUT: All values of $c_v(S)$ for tree $T$ |
| BEGIN |
| \[ \begin{array}{l}
| \text{Let } v_1, v_2, \ldots, v_N \text{ be the topological ordering of the } N \text{ nodes in the multicast tree} \\
| \text{FOR (} j = N \text{ down to } 2) \text{ DO} \\
| \quad \text{Let } v = v_j \\
| \quad \text{IF } v \text{ is a leaf node THEN} \\
| \quad \quad \text{FOR (each wavelength set } S \in \Phi) \text{ DO} \\
| \quad \quad \quad \text{Compute } c_v(S) \text{ using (5.1)} \\
| \quad \quad \text{ENDFOR} \\
| \quad \text{ELSEIF } j > 1 \text{ THEN} \\
| \quad \quad \text{FOR (each wavelength set } S \in \Phi) \text{ DO} \\
| \quad \quad \quad \text{Compute } c_v(S) \text{ using (5.2)} \\
| \quad \quad \text{ENDFOR} \\
| \quad \text{ENDIF} \\
| \text{ENDFOR} \\
| \text{END} |

value of $c_v(S)$ respectively. We define the minimum value of all the values of $c_v(S)$ of node $v$ to be $c^m_v$, i.e.,

$$c^m_v = \min_{S \subseteq A(v), |S| \leq K} c_v(S) \quad (5.4)$$

With respect to each value of $c_v(S)$ and $c^m_v$, we use a top-down approach to decide which wavelength set to be chosen for each link. The following rules are employed to choose the wavelength set for the input link of each non-root node $v$ of the tree.

**Rule 1:** For the input link of a non-root node $v$, choose the wavelength set $S$, satisfying $c_v(S) = c^m_v$.

This is a straightforward rule. When one wavelength set has been selected under this rule, it implies $c_v(S) = c^m_v$, which can guarantee that the total number of wavelength conversions
in the subtree rooted at \( v \) will be minimum. Therefore, if all wavelength sets are assigned under this rule, the total number of wavelength conversions in the multicast tree \( T \) should be minimized.

**Rule 2**: Let non-root node \( p \) be the parent of node \( v \), then, for the input link of node \( v \), choose the wavelength set \( S \), satisfying \( S \subseteq F(p) \) and \( c_v(S) = c^m_v \).

This rule is useful for the case where more than one wavelength set satisfying \( c_v(S) = c^m_v \).

Take the tree in Figure 5.5(a) as an example. Assume that, for node \( x_1 \), we have \( c_{x_1}(\{\lambda_1\}) = c_{x_1}(\{\lambda_3\}) = c_{x_1}(\{\lambda_1, \lambda_3\}) = c^m_{x_1} \). Thus, under Rule 1, \( \{\lambda_1\} \), \( \{\lambda_3\} \) and \( \{\lambda_1, \lambda_3\} \) are all qualified. Rule 2 tells us that, among all the qualified wavelength sets, we select the set whose wavelengths are in its parent’s assigned wavelength set \( F(p) \). In this example, \( v \) is the parent of node \( x_1 \), and suppose \( F(v) = \{\lambda_1\} \), then, \( S = \{\lambda_1\} \) will be selected for node \( x_1 \) under Rule 2.

Rule 2 shows that, when choosing the wavelength set, we need to consider the relationship between the child node and its parent node. Besides that, the relationship between the node and its sibling nodes (the nodes that have the same parent) should also be considered. This consideration arises from the benefit of applying the S-C-S switch model. For the example
subtree in Figure 5.5(b). Suppose that, for the node \( v \), the wavelength \( \lambda_2 \) has been assigned on its input link, i.e., \( F(v) = \{ \lambda_2 \} \). Assume that, for the child nodes \( x_1 \) and \( x_2 \), we have known that,

\[
c_{x_1}(\{\lambda_1\}) = c_{x_1}(\{\lambda_3\}) = c_{x_1}^{m};
\]
\[
c_{x_2}(\{\lambda_3\}) = c_{x_2}^{m}.
\]

In this case, based on Rule 1 and Rule 2, both \( \lambda_1 \) and \( \lambda_3 \) are qualified for node \( x_1 \). On a S-C-S multicast switch model basis, choosing \( \lambda_1 \) leads to two wavelength conversions at node \( v \) (i.e., wavelength conversion from \( \lambda_2 \) to \( \lambda_1 \) and from \( \lambda_2 \) to \( \lambda_3 \), respectively), while choosing \( \lambda_3 \) for node \( x_1 \) will result in only one wavelength conversion (from \( \lambda_2 \) to \( \lambda_3 \)). Thus, we can save one wavelength conversion by selecting \( \lambda_3 \) for node \( x_1 \). This simple example shows that, Rule 1 and Rule 2 are not adequate. Therefore, we develop another rule to explore the full advantage of S-C-S switch scheme.

Rule 3: Let non-root node \( p \) be the parent of node \( v \), then, for the input link of node \( v \in B(p) \), choose the wavelength set \( S \), such that the number of child nodes in \( B(p) \), satisfying \( c_v(S) = c_v^{m} \), is maximum.

Based on the above discussions, the algorithm for the wavelength assignment can be derived, as shown in Table 5.2. In the algorithm, for each non-root node \( v \) in the tree, it has at most \( D \) child nodes. For each child node \( x \), it is required at most \( Q \) steps to confirm whether the wavelength set \( F(v) \) is in the set of \( A(x) \) and \( c_x(F(v)) = c_x^{m} \), where \( Q \sim O(W^K/K!) \) is the number of distinct wavelength sets taking the number of wavelengths not larger than \( K \) which can be calculated using equation (3.2). Then, for the worst case, the computation steps are upper-bounded by \( O(DQN) \), i.e., \( O(D(W^K/K!)N) \).

Consequently, combining the above two algorithms, we obtain the new Multicast Wave-
Table 5.2: Algorithm for Wavelength Assignment

| INPUT: Multicast tree $T$ and all values of $c_v(S)$ for tree $T$ |
| OUTPUT: Wavelength assignment for tree $T$ |
| BEGIN |
| FOR (each child node $v$ of root) DO |
| Set $F(v)$ to be any wavelength set $S$ satisfying $c_v(S) = c_v^{mn}$ |
| ENDFOR |
| FOR ($j = 2$ up to $N$) DO |
| Let $v = v_j$ |
| IF node $v$ is not a leaf node THEN |
| FOR (each child node $x$ of node $v$) DO |
| IF $F(v) \subseteq A(x)$ and $c_x(F(v)) = c_x^{mn}$ THEN |
| Set $F(x) = F(v)$ |
| ELSEIF more than one child nodes satisfy $c_x(S) = c_x^{mn}$ |
| Set $F(x) = S$ such that maximum number of child nodes all satisfying |
| $c_x(S) = c_x^{mn}$ |
| ELSE |
| Set $F(x)$ to be any wavelength set $S$ satisfying $c_x(S) = c_x^{mn}$ |
| ENDFOR |
| ENDIF |
| ENDFOR |
| ENDFOR |
| END |

Table 5.3: Multicast Wavelength Assignment Algorithm

| INPUT: Multicast tree $T$ and available wavelength set $A(v)$ |
| OUTPUT: Wavelength assignment for tree $T$ |
| BEGIN |
| Call the algorithm for the computation of the values of $c_v(S)$ for all nodes |
| in tree $T$; |
| Call the algorithm for wavelength assignment for all the links of tree $T$; |
| END |

The computation time of the MWAA algorithm is upper-bounded by $O(D(W^K/K)!^2N +$
D(W^K/K!N), i.e., O(D(W^K/K!)^2N).

Note that, compared with Chen’s algorithm developed in [43], the MWAA algorithm is
a more general one in the following aspects: (1) MWAA supports both single-wavelength
assignment strategy and multi-wavelength assignment strategy, while Chen’s algorithm sup-
ports only the single-wavelength assignment strategy; (2) MWAA supports the newly pro-
posed three-stage S-C-S multicast switch model, as well as the traditional S-C model, while
Chen’s algorithm supports only the S-C model.

5.3 Performance Evaluation

We carry out a simulation study to evaluate the performance of our proposed algorithm
MWAA and compare the results of our algorithm with the existing wavelength assignment
algorithm (Chen’s algorithm [43]). We first give a simple example to show the efficiency of
our algorithm in subsection 5.3.1. Then, statistical results obtained from the simulation will
be presented in subsection 5.3.2.

5.3.1 Example of Wavelength Assignment Algorithms

A 9-node multicast tree with fiber optic link between nodes is taken as an example as illus-
trated in Figure 5.6(a). Each link in the tree supports at most four wavelengths, \( \lambda_1, \lambda_2, \lambda_3 \)
and \( \lambda_4 \). The available wavelengths on each link that are generated randomly are shown in the
figure. We apply Chen’s algorithm as well as MWAA algorithm using the proposed S-C-S
switch model. The resulting wavelength assignments are shown in Figure 5.6(b), (c) and (d),
respectively.
Figure 5.6: Example of wavelength assignment algorithm. (a) Multicast tree; (b) Result of assignment by Chen’s algorithm; (c) Result of assignment by MWAA (K = 1); (d) Result of assignment by MWAA (K = 2).

The result of applying Chen’s algorithm gives the number of wavelength conversions to be 4 in the multicast tree. However, if we apply the MWAA (K = 1) algorithm, the number of wavelength conversions becomes 3. This reduction is due to the following fact. When assigning wavelength on the input link of node 8, choosing λ₄, instead of λ₂ or λ₃ can save one wavelength conversion at node 4. This is benefited from the S-C-S switch model which can convert wavelength λ₁ into λ₄ first and then split it into two parts. Chen’s algorithm does not have this advantage. All λ₂, λ₃ and λ₄ are qualified in Chen’s algorithm, and a wavelength will be selected randomly, and thus, the probability that λ₄ is not selected will be 2/3, which
is quite high. However, for MWAA, the Rule 3 dictates that $\lambda_4$ must be selected. Thus, the advantage of the S-C-S switch model is fully explored.

Furthermore, if we apply MWAA ($K = 2$), the number of wavelength conversions is reduced to one. The further reduction results from the following situation. When assigning wavelengths on the input link of node 2 and node 3, the MWAA ($K = 2$) algorithm assigns two wavelengths to each of the two links. However, only one wavelength is used in the case of $K = 1$. Thus, from node 2 to node 4 and node 2 to node 5, no wavelength conversions is required for $K = 2$. However, for $K = 1$, one wavelength conversion is needed.

This example shows that, (1) Compared with Chen’s algorithm, the MWAA can take full advantage of the S-C-S switch scheme to minimize the number of wavelength conversions; (2) MWAA ($K = 2$) can save up more number of wavelength conversions than MWAA ($K = 1$). In the following subsection, we shall give statistical results obtained from the simulation.

### 5.3.2 Evaluation of Wavelength Assignment Algorithms

We employ the similar simulation procedure as in [43] for the sake of getting a fair comparison. Given four parameters, $N$, $D$, $W$, $L$, we randomly generate a multicast tree using the following algorithm: First, we fix the root node and set its index to be 1. Then we repeat the following steps until the number of nodes in the graph is equal to $N$.

1. Randomly generate its out-degree between 0 and $D$ and create the corresponding number of child nodes and links.

2. For each new link added, randomly select a number between 1 and $L$ as the number of available wavelengths on that link; then randomly select that number of wavelengths
from \( W \) wavelengths as the set of available wavelengths.

The simulations are run on the PC with 3.2GHz CPU and 1GB of RAM. For each set of parameters, we generate 10,000 instances and compute the average number of wavelength conversions (NWC).

Figure 5.7 shows a comparison of the number of wavelength conversions for MWAA algorithm and Chen’s algorithm. For MWAA, both the case \( K = 1 \) and \( K = 2 \) require less number of wavelength conversions than Chen’s algorithm. The reduction of the number of wavelength conversions between Chen’s algorithm and MWAA \( (K = 1) \) demonstrates the benefit of the proposed S-C-S switch model. Furthermore, we observe that the case of \( K = 2 \) delivers a better performance than that of \( K = 1 \). This indicates that allowing more wavelengths to be used on a link will lead to less wavelength conversions.

Thus, compared with Chen’s algorithm, the MWAA algorithm can perform better, not only can it explore fully the benefit of the S-C-S switch model but also take advantage of the multi-wavelength assignment strategy in order to reduce the number of wavelength con-
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versions needed. The performance advantage of the MWAA algorithm can be expressed in terms of Improvement Percentage of the number of wavelength conversions (NWC) between different algorithms as follows:

\[
\text{Improvement Percentage} (\%) = \frac{\text{NWC by Chen’s Algorithm} - \text{NWC by MWAA Algorithm}}{\text{NWC by Chen’s Algorithm}}
\]

We carry out further analysis based on the computed data to show how the Improvement Percentage varies with the parameters of the multicast tree, including the maximum out-degree of a node, \(D\), and number of nodes, \(N\).

Figure 5.8 shows the Improvement Percentages of MWAA (\(K = 1\) and \(K = 2\)) for various maximum out-degree values of a node, \(D\). As shown in the figure, the Improvement Percentage is sensitive to \(D\). When \(D\) increases, the Improvement Percentage of MWAA algorithm also increases. Moreover, the case of \(K = 2\) increases faster than that of \(K = 1\). The increase of \(K = 2\) is almost linear for small \(D\). For instance, when \(D = 8\), Improvement Percentage = 38% and when \(D = 16\), Improvement Percentage = 73%. This is rather significant. Consequently, we can conclude that the MWAA algorithm performs better when the maximum out-degree of a node in the multicast tree is large. Thus, this result provides useful cue that when designing the routing algorithm for the multicast tree construction, large nodal out-degree is always preferred for reducing the wavelength conversion cost.

On the other hand, the variation of the Improvement Percentage with the number of nodes, \(N\), follows a different trend. It is not sensitive to the number of nodes in the tree. This is shown in Figure 5.9.
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Figure 5.8: Improvement Percentage vs. D for N = 100, W = 8 and L = 6.

Figure 5.9: Improvement Percentage vs. N for D = 8, W = 8 and L = 6.

We note that the above comparisons are all based on the case of K = 1 and K = 2. A natural question will be that what will happen if we increase the value of K by allowing more wavelengths in a link to carry the same message. This is studied by carrying out more simulation experiments. The results are shown in Figure 5.10. It can be seen that the Improvement Percentage increases as K increases. It is sensitive to L. For L = 6, it increases from 28%
Figure 5.10: Improvement Percentage vs. K for N = 100, W = 8 and D = 6.

for K = 2 to 40% for K = 5. Whereas for L = 8, it increases from 46% for K = 2 to 55% for K = 5. We find that the performance is creditable for small value of K. In fact, the performance can be raised by raising L. For instance, for K = 2, the Improvement Percentage is 28% for L = 6, which can be raised to 46% by raising L = 8. This indicates that in order to reduce number of wavelength conversions, we do not need to increase the value of K all the times. Since the computation complexity of MWAA algorithm is about $O(D(W^K/K!)^2N)$, thus, small value of K is preferred for online computation. For example, the running time of the MWAA algorithm for a 20-node multicast tree supporting 8 wavelengths is about 0.0003 seconds when K = 1, 0.006 seconds when K = 2, and 0.04 seconds when K = 3. For a 100-node multicast tree, the computation time is around 0.0015 seconds when K = 1, and 0.03 seconds when K = 2, and 0.2 seconds when K = 3. Thus, small value of K in our MWAA algorithm can produce not only good cost performance but also good time complexity.
5.4 Summary

In this chapter, we consider multicast switch model with full light splitting and wavelength conversion capabilities. Wavelength assignment in traditional switch model often leads to high redundant wavelength conversion cost. In order to minimize the wavelength conversion cost of multicast, we employ a new multicast switch model that supports a new split-converte-split (S-C-S) switch scheme capable of eliminating the redundant wavelength conversion. Based on this model, we adopt the multi-wavelength assignment strategy that allows the use of multiple available wavelengths to transmit the multicast signal in a link. Subsequently, we generalize the existing algorithm to produce the Multicast Wavelength Assignment Algorithm (MWAA) to support both the S-C-S switch model and the multi-wavelength assignment strategy. Compared with the existing algorithm, the MWAA algorithm is more general and flexible, and delivers good performance in terms of minimizing the number of wavelength conversions. For a 100-node multicast tree with the maximum out-degree of 8, the new algorithm MWAA can reduce wavelength conversions by 38%, and this percentage increases to 73% when the maximum out-degree is 16. This is highly significant. The new algorithm is suitable for the situation where the maximum node out-degree of the tree is relatively large.
Chapter 6

Conclusion and Future Work

Optical multicast over WDM network opens an important and exciting research frontier which provides a strong mandate for the future backbone network to support point-to-multipoint service. Compared with traditional IP multicast, besides routing problem, optical multicast over WDM network has an extra wavelength assignment problem. In this thesis, our focus is to derive efficient wavelength assignment algorithms for optical multicast under different wavelength conversion scenarios. This chapter summaries our main contributions and recommends some potential directions for the future research.

6.1 Research Contributions

Chapter 3 considers multicast wavelength assignment problem in wavelength-routed WDM networks with full and limited wavelength conversion capabilities. In the network with full wavelength conversions, total multicast cost consists of wavelength cost, light splitting cost and wavelength conversion cost. Among them, wavelength conversion cost dominates. Consequently, to minimize the total multicast cost, one needs to consider wavelength conver-
Conclusion and Future Work

sion cost when designing a wavelength assignment algorithm. In order to minimize wavelength conversion cost of multicast, we propose a new concept that allows the use of multiple available wavelengths to transmit the multicast signal in a link. Subsequently, we develop a new Multi-wavelength Multicast Wavelength Assignment (MMWA) algorithm. In general, MMWA algorithm provides a good tradeoff between wavelength cost and wavelength conversion cost. By considering both wavelength cost and wavelength conversion cost together, we are able to derive a scheme that can significantly reduce wavelength conversion cost while keeping the additional wavelength usage within an acceptable range.

Besides the full wavelength conversion case, we exploit the benefits of multi-wavelength assignment algorithm for the network with limited wavelength conversions. We extend MMWA algorithm to cover limited wavelength conversion case. Simulation results show that much more multicast requests can be accommodated by our new MMWA algorithm as compared with an existing algorithm. The improvement can be as high as 50% to 80%. This is highly significant. In general, MMWA algorithm can be used off-line to serve as the baseline for the evaluations of online heuristics. Furthermore, using relatively small number of extra wavelengths on each link, MMWA algorithm can give good cost performance, high ratio of satisfied requests and a reasonable time complexity for online computing.

Chapter 4 addresses the problem of multicast wavelength assignment for sparse wavelength conversion in the wavelength-routed WDM networks. We first extend the random wavelength assignment algorithm from the unicast case to the multicast case. The straightforward extension leads to inefficient use of wavelength converters. In order to minimize the use of wavelength converters for a given multicast request, we propose a new MWA-SWC algorithm. This algorithm first maps the multicast tree from the sparse conversion case to the
Conclusion and Future Work

full conversion case. We then propose a novel virtual link method for the tree mapping. The method generates an auxiliary tree and also handles the reverse mapping. Using the auxiliary tree, we propose a dynamic programming algorithm for the wavelength assignment aiming to minimize the number of wavelength converters required. Simulation results show that our new algorithm outperforms both random and greedy algorithms with regards to minimizing the number of wavelength converters. The performance is consistent, as demonstrated by varying the number of wavelength conversion nodes in the tree. The MWA-SWC algorithm is useful primarily for off-line computing. However, it can also serve as a baseline for online heuristic algorithms. Typically, the MWA-SWC algorithm will provide great benefit when the number of available wavelengths on links of the multicast tree is relatively large and the performance advantage is significant.

Chapter 5 deals with the multicast wavelength assignment in a new multicast switch model. Traditional switch model is often based on split-convert (S-C) scheme which leads to high redundant wavelength conversion cost. In order to minimize the wavelength conversion cost of multicast, we employ a new multicast switch model that supports a new split-convert-split (S-C-S) switch scheme capable of eliminating the redundant wavelength conversion. Based on this model, we adopt the multi-wavelength assignment strategy to generalize the existing algorithm to produce the Multicast Wavelength Assignment Algorithm (MWAA) to support both the S-C-S switch model and the multi-wavelength assignment strategy. Compared with the existing algorithm, MWAA algorithm not only is a more general and flexible one, but also delivers a good performance in term of minimizing the number of wavelength conversions. For a 100-node multicast tree with the maximum out-degree of 8, the new algorithm MWAA can reduce up to 38% wavelength conversions, and such percentage increases
to 73% when the maximum out-degree becomes 16. This is highly significant. The new algorithm is suitable for the situation where the maximum node out-degree of the tree is relatively large.

6.2 Recommendations for Future Research

In general, optical multicast is a relatively new research field. This thesis is mainly on the multicast wavelength assignment problem. Looking into the future, we would like to recommend the following areas:

- **Data Plane Implementation of Multicast Switch Model** – We have different multicast switch models available, such as split-convert, convert-split, and split-convert-split. How to implement these models into MC-OXC architecture deserves systematic research. This will involve several data plane design issues, such as power issue, noise issue, cost issue, fabrication issue, and so on. For such design problem, [30, 32–34] give good references.

- **Wavelength Converter Placement and Distribution Problem** – In Chapter 4, we studied the multicast wavelength assignment problem for the sparse wavelength conversion case. However, we do not consider the placement of the wavelength converters in the network. Although placement of wavelength converters has been studied for the unicast case, not much work has been done for the multicast case. The placement strategy must have impact on the algorithm performance. Furthermore, for the case that wavelength converters are shared within a node, how to distribute the converters in the network is another interesting subject. The converters may be distributed evenly...
in the network, or some nodes may be allocated more converters than the others. Consequently, converters placement and distribution problem deserves a further study for multicast case.

- **Universal Graph Model for On-line Multicast Routing and Wavelength Assignment** – The development and upgrade of network infrastructure always follow a smooth and seamless process, which determines that the future backbone network may have various node architectures which combine different splitting and conversion capabilities. That means, some key nodes may be equipped splitting or conversion capabilities first; or for those conversion nodes, some of them may be allocated limited wavelength converters first, and then be upgraded to full wavelength converters. For the more practical and complex situation, the on-line MC-RWA algorithm would better be universal to cover all the cases for different node architectures and network structures. Consequently, a new universal graph model for the wavelength-routed WDM networks may be derived to represent the network for all kinds of node architectures and network structures.

- **Multicast Session Protection and Multicast Traffic Grooming** – Compared with the unicast case, it is more important to protect the multicast sessions since a single fiber cut on the network can disrupt the transmission of information to several destinations. Therefore, the light-tree based multicast session protection problem deserves a comprehensive study. On the other hand, efficiently grooming low-speed connections onto high-capacity lightpaths will improve the network throughput and reduce the network cost. Further investigations and novel algorithms are required to exploit the grooming capability and optimize the network resources efficiently for the multicast case.
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Bibliography


