Transmission Planning in Deregulated Power Markets

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Last, but certainly not least…all my family members, colleagues and laboratory staff are acknowledged with great honor.

Thanks to All.
Summary

Transmission systems in deregulated power systems have largely been left to centralized network operators although various forms of participation by merchant operators have been proposed. In the context of evolving decentralized power markets, transmission operations and planning issues, structural organization of the sector, design of transmission market, economics of transmission investments, and regulatory policies are still evolving. It is attempted to address the tasks of transmission network expansion planning in the new environment considering all the pertinent issues.

The ground rules of network operation incorporating welfare aspects of day-ahead power markets have been analyzed relying on network constrained dispatch policies. Measures of transmission congestion in terms of "congestion cost", "congestion revenue" and "marginal value of transmission" are defined and evaluated from dispatch procedures. These attributes are utilized in the investigation of economic network expansion in the subsequent analysis.

Long-term network planning models are formulated for both centralized and decentralized network operations. A theoretical analysis of optimal transmission expansion under various market models is conducted to clearly reveal the existence of distinct optimal expansions under different market models. It was clearly demonstrated that under-investment in transmission expansion is quite likely under monopolistic network owners. Comparative studies of various models are presented to show that the social optimum expansion can be achieved under different market models by appropriately enforcing the competition among the merchant participants. Simple examples are presented to illustrate various findings of the analysis.

A detailed framework for network planning from the perspective of a central network planner in a deregulated market has been developed. The entire problem is formulated as a combination of the long term investment problem and the short term operational sub-problem. Generalized Bender’s Decomposition technique has been utilized for the solution of the problem using the flow of suitable economic attributes as signals to link the two. The operational sub-problem is designed to incorporate the data
from the competitive power markets. Two different planning criteria namely, congestion cost and congestion revenue, have been utilized to obtain the optimal planning outcomes. A comprehensive example is presented clearly illustrating how these criteria influence the planning outcome and the investment schedule.

Realizing the potential role of FACTS devices in the power system network operation, the planning procedure is extended to include these devices along with conventional network reinforcements to investigate their possible contribution in the long term congestion alleviation in the network. Appropriate models for including Phase Shifters and Series Compensators in the operational sub problem are developed and incorporated in the overall network planning framework. A number of case studies are conducted to illustrate the effects of including these flexible options on the transmission planning and investment schedules.

The proposed planning approach is extended further to incorporate the effects of uncertainties that are inherent in various operational aspects, which include - bid prices, component outages, and the hourly load variations. Such detailed representation of operational uncertainties makes the formulation quite complex, but is expected to produce more comprehensive planning alternatives. The increased complexity of the problem formulation is handled by combining the Generalized Benders Decomposition and non-sequential Monte-Carlo technique. The combined hybrid solution method is beneficially exploited to obtain near optimal solutions in addition to the optimal network expansion schedule.

In addition to the rigorous theoretical investigation of issues related to transmission planning and transmission market models, this research work provides a fairly comprehensive framework for transmission planning in deregulated power markets considering the issues pertinent to the present day power markets.
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<tr>
<td>$t_h$</td>
<td>short-term time index (usually in hours)</td>
</tr>
<tr>
<td>$t_y$</td>
<td>long-term time index (years)</td>
</tr>
<tr>
<td>$t_0$</td>
<td>base year</td>
</tr>
<tr>
<td>$T$</td>
<td>planning horizon (years)</td>
</tr>
<tr>
<td>$[,]$</td>
<td>upper and lower bounds of variables</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dual price or shadow price ($/\text{MWh}$)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>system state</td>
</tr>
<tr>
<td>$\xi$</td>
<td>component availability</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>forced outage rate (FOR)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>discount factor</td>
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### Supplier Related

<table>
<thead>
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<tr>
<td>$i$</td>
<td>supplier or generator index (Set $I$)</td>
</tr>
<tr>
<td>$b_i$</td>
<td>intercept of price bid supplier $i$ ($/\text{MWh}$)</td>
</tr>
<tr>
<td>$m_i$</td>
<td>slope of price bid supplier $i$ ($$/\text{MW}^2\text{h}$)</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>spot (nodal) price of supplier $i$ ($$/\text{MWh}$)</td>
</tr>
<tr>
<td>$g_i$</td>
<td>supply (generation) of supplier $i$ (MW)</td>
</tr>
<tr>
<td>$C(.)$</td>
<td>cost of supply ($$/\text{h}$)</td>
</tr>
<tr>
<td>$MC$</td>
<td>marginal cost of supply ($$/\text{MWh}$)</td>
</tr>
<tr>
<td>$SS$</td>
<td>supplier’s surplus ($$/\text{h}$)</td>
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### Consumer Related

<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$j$</td>
<td>consumer (load) index (Set $J$)</td>
</tr>
<tr>
<td>$b_j$</td>
<td>intercept of price bid consumer $j$ ($$/\text{MWh}$)</td>
</tr>
</tbody>
</table>
\( m_j \) slope of price bid consumer \( j \) ($/MW^2h)

\( \rho_j \) spot (nodal) price of consumer \( j \) ($/MWh)

\( d_j \) demand (load) of consumer \( j \) (MW)

\( r_j \) demand curtailed (loss of load) for \( j \) (MW)

\( B(.) \) benefit of consumption ($/h)

\( MB \) marginal benefit ($/MWh)

\( CS \) consumer’s surplus ($/h)

\( OC(.) \) outage cost of load curtailment ($/h)

\( MOC \) marginal outage cost ($/MWh)

**Network Related**

\( m \) branch or line index (Set \( M \))

\( n \) node or bus index (Set \( N \))

\( y_n \) net power injection at node \( n \) (MW)

\( z_m \) power flow in branch \( m \) (MW)

\( st(m) \) starting node of branch \( m \)

\( en(m) \) ending node of branch \( m \)

\( r_m \) resistance of branch \( m \) (p.u.)

\( X_m \) reactance of branch \( m \) (p.u.)

\( \gamma_m \) susceptance of branch \( m \) (p.u.)

\( \psi_m \) phase shifter angle of branch \( m \) (deg.)

\( Z_m \) capacity limit of branch \( m \) (MW)

\( H \) branch-node sensitivity matrix (Power Transfer Distribution Factors)

\( \Gamma \) supply/demand – node incidence matrix

\( L(.) \) network power losses (MW)

\( B_{\text{loss}} \) power loss representation matrix

\( NR \) network revenue ($/h)

\( TC \) congestion cost ($/h)
\( TR \) congestion revenue ($/h)

\( IC_m(t_y) \) investment cost of transmission for branch \( m \) in year \( t_y \) ($)

\( \beta_m(t_y) \) marginal investment cost of transmission for branch \( m \) in year \( t_y \) ($/MW)

\( n_m(t_y) \) number of circuits added for branch \( m \) in year \( t_y \)

\( I_{z,m}(t_y) \) investment cost for a circuit in branch \( m \) in year \( t_y \) ($/circuit)

\( \varphi_m(t_y) \) phase shifter capacity installation for branch \( m \) in year \( t_y \) (deg.)

\( I_{\varphi,m}(t_y) \) phase shifter unit investment cost for branch \( m \) in year \( t_y \) ($/unit)

\( Xc_m(t_y) \) series compensation capacity installation for branch \( m \) in year \( t_y \) (\( \Omega \))

\( I_{Xc,m}(t_y) \) series compensation unit investment cost for branch \( m \) in year \( t_y \) ($/unit)
### List of Acronyms

<table>
<thead>
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<tbody>
<tr>
<td>TEP</td>
<td>Transmission Expansion Planning</td>
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<tr>
<td>RTO</td>
<td>Regional Transmission Organization</td>
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<tr>
<td>ISO</td>
<td>Independent System Operator</td>
</tr>
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<td>PX</td>
<td>Power Exchange</td>
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<tr>
<td>TransCo</td>
<td>Transmission Company</td>
</tr>
<tr>
<td>GenCo</td>
<td>Generation Company</td>
</tr>
<tr>
<td>DistCo</td>
<td>Distribution Company</td>
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<tr>
<td>LSE</td>
<td>Load Serving Entities</td>
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<td>TP</td>
<td>Transmission Provider</td>
</tr>
<tr>
<td>ITC</td>
<td>Independent Transmission Company</td>
</tr>
<tr>
<td>ED</td>
<td>Economic Dispatch</td>
</tr>
<tr>
<td>NCED</td>
<td>Network-Constrained Economic Dispatch</td>
</tr>
<tr>
<td>OPF</td>
<td>Optimal Power Flow</td>
</tr>
<tr>
<td>LMP</td>
<td>Locational Marginal Price</td>
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<td>PTDF</td>
<td>Power Transfer Distribution Factors</td>
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<td>Congestion Revenue Rights</td>
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<td>GBD</td>
<td>Generalized Benders Decomposition</td>
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<td>LDC</td>
<td>Load Duration Curve</td>
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<td>FACTS</td>
<td>Flexible AC Transmission Systems</td>
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<td>PS</td>
<td>Phase Shifter</td>
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<tr>
<td>SC</td>
<td>Series Compensator</td>
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Chapter 1

Introduction

"The work I have set before me is this ... how to get rid of the evils of competition while retaining its advantages."

--- Alfred Marshall (1842-1924) ---

British Economist

1.1 Motivation

The transmission expansion planning (TEP) problem, as a prominent research topic in power systems, extends its history way back to early seventies (Garver [1], Dusonchet and El-Abiad [2]). In practice it remained one of the major utility planning activities performed by centralized power system planners in vertically integrated electricity utilities. Major network improvements were targeted under TEP to cater to the growing demand for electricity and prospective generation expansions in future years. There were well accepted TEP objectives and norms, which were established over decades among traditional electricity supply utilities. Numerous techniques and criteria have appeared in literature to solve static and/or dynamic TEP problem formulations. The progress was mostly assisted by recent advancements in mathematical programming techniques and optimization theory. Deviating from such conventional TEP models and their rigorous optimization methods, this thesis addresses TEP related issues in deregulated electricity utilities, which is timely and of critical importance, because these issues have not yet been adequately addressed in the present day deregulated electricity markets.
The rapid transitions in the power system institutional structure, popularly known as deregulation or restructuring have raised a variety of complex issues in many aspects of power system operation and planning. The most aggravated issues in electricity market operation and planning are those which pertain to the transmission sector. These issues have intensified their importance in the recent past, since they may lead to severe consequences such as total system collapses or black outs. In the United States Federal Energy Regulatory Commission (FERC) has created a number of large Regional Transmission Organizations (RTOs) to promote interstate electricity trading, and to ensure reliable transmission service. This was to fulfill one of its set missions to modernize and expand the transmission system. FERC Order No.2000 on RTO includes "transmission planning" as one of its minimum nine functions stated as – "plan and coordinate necessary transmission additions and upgrades" (FERC [3]).

As these facts testify, transmission plays a key role in bridging physically separated electricity supply and demand sectors in wholesale competitive power markets. Its economic role extends well beyond the transportation of power from generators to consumers. An effective transmission system facilitates optimum generation dispatch, allowing most efficient generators to supply the geographically as well as demographically dispersed electricity needs. Conversely, transmission constraints or bottlenecks defeat even the extremely attractive qualities of these electricity markets. These realities indicate the need for effective transmission system planning in competitive power markets. But, the traditional TEP objectives and norms may require a complete or at least major adjustment to suit the new environment, depending upon the transmission management policy envisioned to replace the existing monopolistic
In today’s electricity markets two major transmission management practices are found,

(i) System operations and transmission functions performed by a single regulated transmission company (TransCo) e.g. NGC in England and Wales; and

(ii) Transmission services provided by merchant network investors in compliance with System Operator’s guidelines e.g. Argentina, Brazil and Australia.

Network expansion objectives and their long-term planning formulations may differ significantly under these two management options. The first may adopt social objectives with system-wide approaches and the second may prefer individual objectives confining to localized areas. Accordingly, the transmission planning and management policies and methodologies may be different and lead to distinctly different expansion planning models. On the other hand, the coordinated generation and transmission expansion planning which prevailed over the past decades might not be feasible at all after the vertical unbundling. In the new environment, generation investment planning will mainly be a decision-making issue of individual generating companies or investors. This introduces additional uncertainty and complexity to TEP, which traditionally relied on known generation expansion plans for further transmission planning purposes. Moreover, the strategic behavior of suppliers and consumers is likely to make the TEP task even more difficult. These issues have emerged as additional challenges, on top of what already existed in traditional TEP problem formulations.
Despite these complexities, economists tend to believe that perfect market liberalization including transmission sector, will resolve these challenges by the invisible hand of market forces. Yet the introduction of perfect liberalization in transmission operation and planning is hindered to a greater extent by the economy of scale, scope and the "lumpiness" inherently present in transmission investments. With the advent of recent new technologies this situation has been improving in many directions. This was emphasized by Rotger and Felder [4] as – “the capacity at which modern DC transmission projects realize economies of scale begins at roughly 200 to 300 MW, comparable to modern combined-cycle gas fired generation unit. Flexible AC Transmission System (FACTS) devices reach their economies-of-scale plateau at capacities of less than 100 MW, comparable to modern peaking generation unit” (Rotger and Felder [4]). Accordingly, economies of scale and scope barriers are becoming less stringent with the newly available technologies. These issues deserve more consideration in the new ways of transmission planning, since they provide higher operating flexibility which is useful to deal with uncertain flow patterns due to strategic behavior under market conditions, at substantially lower cost compared to transmission line reinforcements. Thus, market liberalization introduces many new facets to the transmission planning problem, which did not exist in the traditional power systems.

1.2 Objectives

TEP models found in literature are mostly confined their applications to vertically integrated systems, which deal with a known generation expansion schedule and a fixed load (or demand) forecast. The cost-of-service regulation is taken as the key regulatory policy for system planning. A more detailed review of these literatures is provided in
Chapter 2. The sluggish developments in the evolution of transmission sector market and the lack of precise policy frameworks for the same has significantly curtailed the attention to transmission planning issues in liberalized markets. However, some utility technical reports, working papers and a few research papers have attempted to lay the foundations for possible future directions.

The significance of welfare economic attributes, such as network externality, economy of scale and supply and demand uncertainty, in transmission planning was first emphasized by Baldick and Khan [5], De la Torre et al. [6] and David and Wen [7]. Impact of those attributes on existing TEP models, and how models are to be modified were qualitatively analyzed in the above articles. The extensions of network related financial tools like Transmission Congestion Contracts (TCC) (Hogan [9]) for network expansion investigations with economic insights, were found in Bushnell and Stoft [8][10]. The welfare economic aspects of network expansion were focused in the above two articles based on the concept of transmission rights. The trend of network expansion policy transition from "centralized" to "de-centralized", and different transmission management options were discussed in Rotger and Felder [4], Hirst and Kirby [11], Krapels [13], Stoft and Graves [14], Hirst and Kirby [15] and Stoft [16]. These opened up a different dimension to the TEP problem, urging completely new approaches and methodologies for network expansion planning in competitive markets. However, this area still remains mostly untouched among traditional power system researchers. The challenges from new technologies such as FACTS devices to the centralized regimes leading to natural monopoly were elaborated in Rotger and Felder [4], Hirst and Kirby [11] and Cameron [12]. New TEP models for deregulated power systems were proposed
in Cruz-Rodriguez and Latorre-Bayona [17], Saenz et al. [18], Fang and Hill [19] and Gil et al. [20]. Additionally, the need for new tools and software was stressed in Cagigas and Madrigal [21] and Clayton and Mukerji [22].

The broad objective of this research is to investigate various issues related to network expansion planning in the context of new developments in deregulated power markets, and to develop an appropriate transmission planning framework. In order to achieve this overall objective the following specific issues are identified as topics to be investigated:

- Identify the differences in the network planning objectives in line with evolving nature of power markets (*e.g.* for *regulated Transco, merchant transmission* or *transmission market*).

- Develop the necessary tools, define and obtain the necessary attributes associated with various markets.

- Analyze the various market models from a common platform and identify the similarities and differences between them, so as to facilitate comprehensive analysis of various market models.

- Develop and implement transmission planning methodologies suitable for the competitive markets.

- Investigate the role of FACTS devices in the network planning as these devices have been identified as significant contenders in the competitive markets.

- Incorporate the uncertainties in the operation of competitive power markets in the process of transmission planning.
1.3 Major Contributions

Main contributions and highlights of the thesis are summarized in the following.

1.3.1 Development of Day-Ahead Power Dispatch Model

A network-constrained dispatch algorithm has been developed as the fundamental tool to analyze day-ahead power dispatch simulations and "value-based" network expansion planning. The dispatch algorithm can accommodate supply and demand side bid details representing a double-sided auction model, which includes supply and demand price bids, supply and demand limits, network losses and network flow constraints. With the aid of this model, short-term locational marginal prices (nodal prices), congestion revenue, congestion cost and marginal values of transmission capacities (transmission shadow prices) are defined and evaluated, which provide the vital information for network expansion analysis in competitive markets. A new barrier technique has been employed for the solution of the formulated dispatch model.

1.3.2 Analysis of Network Expansion Paradigms

Long-term network expansion problems have been formulated separately for "centralized" and "decentralized" transmission managements, to achieve optimal transmission investment schedules for welfare maximization and profit maximization respectively. Short-term congestion cost is balanced against network expansion cost to reach "socially optimum" network expansion under centralized approach. Short-term congestion revenue and surpluses (supplier and consumer) are balanced against network expansion cost to reach "individual optimum" network expansion under decentralized approach.
The analysis is extended to study optimal expansion under various market models by suitably modifying the objective function. It is proven that individual optimal expansion levels differ from social optimum; and network owners’ (merchant owners) optimum always tend to fall below the social optimum leading to possible network under-investment. However, it is established through a fairly rigorous analysis that social optimum is plausible under decentralized approaches and various market conditions. For example, the socially optimal expansion is achieved when (i) perfect competition exists among merchant transmission owners, or (ii) exclusive transmission right ownership is held by network users (but not by the network owner). Additionally, it is also proven that network users equilibrium capacity expansions \( (Nash \ equilibrium) \) approaches social optimum provided no entry barriers exists to enter competitive network expansion markets. This analysis justifies the penetration of \textit{merchant transmission} under transmission rights schemes. A number of examples are presented to illustrate the analytically derived solutions.

1.3.3 Dynamic Network Expansion Planning

The centralized long-term network expansion planning methodology has been developed and implemented to obtain dynamic network investment schedule for a specified planning horizon. The dynamic formulation is resolved into (i) a series of "pseudo" dynamic problems to represent short-term dispatch optimization, and (ii) investment problem using annuatised costs and benefits figures. A \textit{Generalized Benders Decomposition} technique is utilized to obtain optimum annual investment decisions considering the annual investment scheduling and short-term social cost minimization problems. Two network expansion criteria, namely (i) congestion cost saving, and (ii)
congestion revenue containing have been implemented. Congestion cost saving approach justifies network investment cost by increased social welfare (or congestion cost saving). Congestion revenue containing approach is implemented as a regulated price-cap method, which contains excessive network congestion through necessary network expansion. A comprehensive example is presented to demonstrate the entire procedure.

1.3.4 Network Reinforcements and Flexible Transmission

Newly emerged FACTS devices have the ability to influence the network flows, which affect the network capacity requirements significantly. The dynamic network expansion planning procedure has been extended to include flexible transmission options (phase shifters and series compensation) along with network reinforcements. Individual and combined expansion options have been explored to achieve the optimal network improvement strategy for long-term congestion alleviation. It was shown that these devices may not replace network expansion needed for long-term demand growth but these devices can certainly complement the network expansion in handling the short-term network flow variations, which may arise from different reasons including strategic behavior of the market participants. A set of case studies has been presented and the planning outcomes discussed.

1.3.5 Network Expansion Planning under Uncertainties

Recognizing the uncertainties inherent in bid prices, component outages (generation and transmission) and hourly load variations, the centralized network expansion problem has been modified to accommodate the stochastic nature of these attributes. The resultant overall problem formulation becomes quite complex and a
stochastic optimization method has been developed based on Generalized Benders Decomposition and non-sequential Monte-Carlo techniques for the solution of the stochastic problem. This solution provides sufficiently optimal economic network expansion in the presence of the above mentioned uncertainties. Network expansion examples are provided to show the capabilities of the method and usefulness of the results.

1.4 Organization of the Thesis

In addition to the introductory chapter this dissertation is divided into the following chapters;

Chapter 2 reviews the literature in traditional transmission expansion planning, giving a short history of market liberalization of electricity supply industry, and transmission planning initiatives in liberalized electricity markets.

Chapter 3 presents the details of the developments of double-sided power dispatch model to be used in the subsequent network expansion studies. The necessary transmission related attributes and terms are defined.

Chapter 4 formulates centralized and decentralized long-term network expansion problems and provides a theoretical analysis of optimum network expansion. The theoretical results and findings regarding optimum expansion levels are illustrated using simple examples.

Chapter 5 presents the development of the dynamic network expansion planning procedure and its solution methodology using Generalized Benders Decomposition. The complete implementation details are provided together with a comprehensive network planning example.

Chapter 6 extends the dynamic network planning framework to incorporating flexible transmission alternatives along with network reinforcements. A set of detailed case studies is provided.
Chapter 7 incorporates market operation uncertainties in to the planning model and solves the resulting problem formulation using Generalized Benders Decomposition combined with non-sequential Monte-Carlo techniques.

Chapter 8 summarizes conclusions and findings of the thesis and hints for possible extensions of this research.
Chapter 2

Review of Literature

"If I have seen further than others, it is by standing upon the shoulders of giants."

--- Isaac Newton (1642-1727) ---

English physicist and mathematician

2.1 Traditional Transmission Expansion Planning (TEP) Problem

The prime objective of transmission system planning is to determine the timing and the type of new transmission facilities that should be added to the existing network, to ensure adequate transmission network capacity considering future generating options and load requirements. It decides the optimal capacity additions to the network over the planning horizon while minimizing the capital investment cost, system operating cost and service requirement or norm violations. This was more precisely described in Dusonchet and El-Abiad [2] – "electric power system planning is the process of determining an expansion strategy, that is WHEN WHAT facilities should be provided WHERE in order to insure "adequate" electric service so as to "maximize the welfare" of the community being serviced". The primary and secondary objectives, objective oriented outcomes (expansion schedule) become highlighted for dynamic expansion planning.

TEP task emphasizes on optimality of the expansion plan, adopting load forecast and generation expansion plan as the foundation, electricity supply quality and safety as its outcomes (Wang and McDonald [23]). Often the optimality was assessed with least-cost principles and supply quality or service quality was valued with adequate network performances in terms of reliability standards. The network performances are expected to
ensure secure and safe operating conditions under normal as well as in contingency situations. Accordingly, reliability evaluations and their worth assessments were often included in comprehensive TEP activities.

2.1.1 Conventional Transmission Planning Approach

Long-term network planning process is generally divided into two stages as scheme formation, and its evaluation. The scheme formation stage explores one or more optimal or quasi-optimal network solutions for the planning horizon. Being strict optimization models, TEP formulations mostly posses an optimal and equally worth sub-optimals giving the network planners more freedom. This part of the study mostly deals with mathematical optimization techniques combined with some technical calculations and economic evaluations. The evaluation stage makes detailed economic and technical assessments for the optimum schemes (or selected schemes). This phase includes detailed engineering and economic studies such as load flow calculations, stability analysis, reliability studies, short circuit capacity calculations and cost-benefit assessments. Hence, the first stage (scheme formation) has become an interesting research problem in academia. And, the second stage is widely practiced in transmission utilities with globally accepted as well as individually defined norms.

TEP studies are identified as either static or dynamic depending on its time frame. In static network planning, problem is formulated to obtain an optimal solution for a single time period (single horizon year). However, this does not consider when to build a new transmission facility, only the location and magnitude are decided. In dynamic (multi-stage) planning, load growths, future generation opportunities and topological changes in the network over the planning horizon are considered. This finally decides
when, where, what new transmission facilities to be built, in order to satisfy the forecasted demand economically and safely. The dynamic network planning models have not matured to the extent of other contemporary TEP models because of their inherent complexity and excessive system space requirements. When dynamic formulations are applied to real world applications, formulations become huge and requires significant amount of memory space and input data. Kaltenbatch et al. [24], Dusonchet and El-Abiad [2], Meliopoulos et al. [25], Sharifnia and Aashtiani [26], Kim et al. [27] and Youssef and Hackam [28] have been categorized as dynamic network expansion models. However, most common methods of solving dynamic formulations utilize "pseudo" dynamic subproblems, which represent a sequence of static subproblems e.g. Monticelli et al. [29], Pereira et al. 1985, Levi and Calovic [30]. These "pseudo" dynamic models relieve the excessive computational time with the risk of missing the dynamic optimum.

In the context of "least cost planning" that prevailed in vertically integrated utilities, static/dynamic network expansion planning became a popular multi-criteria optimization problem. It addressed the problem of determining the optimal number of lines that should be added to an existing network to meet the forecasted load with future generation options as economically as possible, subjected to operating constraints. The key objective was to minimize the long-range (15-20 years) capital investment costs and operating costs while maintaining adequate level of reliability and service quality. In general, costs (capital investment + operating) are minimized and everything else is treated as constraints including some reliability constraints.
2.1.2 Special Features of Network Planning Problem

The long term planning is intrinsically a dynamic process and those decisions are mostly characterized by uncertainties. Therefore, planning model to be comprehensive it should adequately represent the above two characteristics.

- **Dynamic** approach optimizes the decision criteria over the planning horizon. As a consequence, problem seeks a global optimum considering the whole time domain.

- **Uncertainties** bring more stochastic nature in to the problem, and introduce the concept of minimizing the risk associated with uncertainty. However, uncertainties are beyond the utility’s foreknowledge or control.

To void these effects due to future uncertainties one of the most common appeared methods has been to generate a set of possible solutions corresponding to a set of future scenarios. And thereafter the best solution is selected in such a way that it minimizes the maximum risk, in other words the solution with minimum maximum risk (e.g. De la Torre et al. [6], Rodriguez and Latorre-Bayona [17] and Fang and Hill [19]).

2.2 Network Optimization Techniques

TEP methods (or techniques) can be broadly categorized into two as (i) heuristic methods, and (ii) mathematical optimization methods. Based on the time horizon considered optimization models can be further classified into (i) Single-stage optimization models, and (ii) Time-phased optimization models or Multi-stage optimization models (Gonen [32]).
Heuristic methods are much more straightforward and flexible but its applicability highly depends on the experience and analytical capability of the planner. In mathematical optimization, problem is formulated as an investment planning problem with decision variables, constraints and an objective function. However, TEP formulations are generally hard, non-convex and sometimes non-linear (depending on formulation) combinatorial optimization problems with large number of possible alternatives. The number of alternatives to be analyzed increases drastically with the increase of network dimensions.

2.2.1 Heuristic Models

These are primarily interactive planning models. In contrast to mathematical models, these are considered as customer made models which propose the most direct transmission network from the generation to load without causing any circuit overloading. This is relatively close to the engineer’s perception. It may give a good design scheme based on the experience and analytical capability of the planner. Even though, these are not strict mathematical optimization methods these methods have some advantages especially in terms of speed of computation, personnel involvement in decision-making and flexibility. Generally, heuristic methods consist of overload checking, sensitivity analysis and scheme formation. Sensitivity analysis has become a popular technique for heuristic network planning, for example – Serna et al. [33], Monticelli et al. [28], Benon et al. [33], Ekwue and Cory [35] and Pereira and Pinto [36].

The emergence of meta-heuristic (based on Artificial Intelligence searching techniques) techniques for the TEP problem was quite common in the recent past. Techniques such as Simulated Annealing, SA (Romero et al. [37], Gallego et al. [38]),
Genetic Algorithm, GA (Chebbo and Irving [39], Da Silva et al. [40], Abdelaziz [41]),
Tabu Search, TS (Da Silva et al. [42], Gallego et al. [43], Wen and Chang [44]), and
Evolutionary Programming, EP (Ceciliano and Neiva [45]) have been reported in
literature.

The combinatorial nature of the TEP problem and its multi-modal objective
function has influenced enormously on the introduction of global optimization techniques
such as SA, GA and TS. Romero et al. [37] and Gallego et al. [38] emphasized on SA’s
ability to avoid local optima on the multi-modal objective function by allowing
"temporarily limited deterioration" of the solution. In spite of the TEP problem having a
large number of candidate solutions even for a moderate size network (NP hard problem),
the capability of SA techniques to handle such complications was proved in both Romero
et al. [37] and Gallego et al. [38]. However, the computationally expensive nature of SA
was acknowledged. As a remedy a "parallel" SA approach was proposed in Gallego et al.
[38], which have two distinct advantages (i) faster computing time (ii) better solution
quality.

GA being an effective global optimization tool, especially, for integer and non-
convex problems has shown capabilities in Chebbo and Irving [39], Da Silva et al. [40]
and Abdelaziz [41]. The problem formulations in all three references were somewhat
similar but each one having its own modifications to handle complexities. Especially,
binary-coded or real-coded chromosomes can handle discrete network investment
decisions and the mutation process in GA can avoid trapping into local optima in multi-
modal objective function. For example, Chebbo and Irving [39] used binary-coded
chromosome and Da Silva et al. [40], Abdelaziz [41] employed real-coded chromosome representing the number of lines to be added to each transmission corridor.

TS as another member in meta-heuristic family has been applied to TEP in Da Silva et al. [42], Gallego et al. [43] and Wen and Chang [44]. Each application handles the key features in TS, such as short-term memory, Tabu list, and aspiration criteria, differently to suit their formulations. An EP approach was suggested in Ceciliano and Neiva [45] for the TEP problem, which generates offspring through mutation of parents. Accordingly, EP solutions sometimes prefer to GA solutions when the objective function is highly multi-modal.

2.2.2 Mathematical Optimization Models

Mathematical optimization models are primarily classical optimization problems representing the static (single-stage) or dynamic (multi-stage) TEP formulation. However, dynamic TEP optimization models are not commonly found in literature due to the fact they are computationally complex and susceptible to heavy future uncertainties when handling real world problems. Therefore TEP optimization models are often handled as static problems (or single stage problems).

For over three decades various kinds of mathematical programming techniques have been applied to the TEP problem, ranging from –

- Linear Programming, LP (Garver [1], Kaltenbach et al. [24], Villasana et al. [46], Kim et al. [27])
- Dynamic Programming, DP (Dusonchet and El-Abiad [2]),
- Branch-and-Bound techniques (Lee et al. [47], Haffner et al. [48])
- Non-Linear Programming, NLP (Youssef and Hackam [28])
- Benders Decomposition, BD (Pereira et al. [36], Romero and Monticelli [49], Romero and Monticelli [50], Binato et al. [52])
- Generalized Benders Decomposition (GBD) (Siddiqi and Baughman [51])
- Branch-and-Bound and Benders Decomposition (Haffner et al. [53]).

The first attempts to computer-aided transmission network planning synthesis utilized LP as the solution technique, Garver [1], Kaltenbach et al. [24]. Linear power flow estimation was used first in Garver [1] to model the circuit flows and LP was used to minimize the overloads. Villasana et al. [46] later combined a linear (DC) power flow estimation model and a transportation model, where overloads were handled by the transportation model and existing facilities were modeled with linear flows. A more elaborated TEP model is found in Kim et al. [27], which also utilized LP to solve part of its detailed formulation.

The DP approach discussed in Dusonchet and El-Abiad [2] also opened a new way of handling the problem. The method comprised a discrete dynamic optimizing procedure with probabilistic search and a heuristic stopping criteria. The techniques used here extensively utilize the planner’s experience in reducing the vast search space. An NLP solution approach to a detailed long-term transmission planning formulation was discussed in Youssef and Hackam [28]. The fixed and variable costs of transmission investments, cost of power losses were included as minimizing objective and AC load flow operating constraints and load constraints were modeled for the dynamic or static network solution.

Benders Decomposition (BD), Generalized Benders Decomposition (GBD) and some hybrid versions of them with heuristic techniques were applied to TEP in the recent
past mostly due to the following reasons. The decomposition of separable discrete investment decisions and continuous operation planning variables facilitates the solution to a greater extent and relaxes the non-linearity problem and non-convexity problem to a manageable level. A hierarchical decomposition approach was presented in Romero and Monticelli [49] for coping with the non-convexity, where algorithm utilizes different levels of network modeling such as the transportation model, hybrid model and linearized power flow model. These levels were arranged in the order of accuracy. As a result this method confines its searching space as algorithm moves to more accurate models that avoid trapping into local optimum. This method has been further improved in Romero and Monticelli [50] using a "zero-one implicit enumeration method" to obtain better solutions for the integer programming (investment subproblem) subproblem. Another BD hybridized application was found in Haffner *et al.* [53] utilizing a branch-and-bound algorithm to solve the master (investment) subproblem. Accordingly, it is seen in common that once hierarchical BD is implemented more specialized integer programming techniques can be used for solving the relaxed investment problem. At the same time more complicated programs can be used to solve the operation planning problem associated with the overall TEP.

A new BD approach was presented in Binato *et al.* [52] for TEP problem to ensure optimal solution in large-scale problems. This method included a linear disjunctive model and combination of Benders cuts and Gomory cuts for better convergence of the decomposition approach. A GBD application in the TEP problem was given in Siddiqi and Baughman [51], where the formulation is complex and having many facets. The utility operation and investment cost plus customer outage costs, AC power
flow and linearized power flow constraints have been included in the model and the final formulation is a stochastic nonlinear mixed-integer programming problem. Even though the model does not exactly fit to the theory of GBD, the paper showed consistently positive and "good" results that deserve attention in utility planning.

2.3 Move Towards “Liberalized Electricity Markets”

2.3.1 Motivation for the Change

For many decades, vertically integrated electricity supply utilities all over the world were operated within their service territories managing the three main components of the system – generation, transmission and distribution. The cost-of-service regulation was the key regulatory principle adopted, which was supposed to hold the electricity prices affordable to end-users. However, the most common argument for deregulation was the economic inefficiency of regulated or vertically integrated monopolies and long-run cost based pricing. Many believe that the competition in "liberalized electricity markets" would bring the short-run price efficiency, than the long-run cost minimization objectives practiced by regulated utilities.

Consequently, this dramatic evolution from vertically integrated monopoly to competitive business entities has been taking place at an accelerated pace. As a result unbundling of vertically integrated utilities in to generation, transmission and distribution companies has been initiated. Some electricity utilities have already been overturned and some remain untouched. Ultimately, perfect competition is envisioned among the market participants (or players) as the main goal. Within this environment electricity would be traded as a competitive service, and the participants will act to attain their business or
customer service goals. The competition would establish the market prices for electricity without regulation. In other words, one can argue that the whole theme of deregulation of electricity is the dismantling of exclusive franchise. Besides that in developing countries the following motivations too have influenced for electricity sector restructuring (Leeprchanon et al. [54]) - dismantling key state owned assets, decreasing national control over sectors of the national economy, and preparing the ground for foreign capital penetration.

2.3.2 Electricity Sector Deregulation around the World

This ongoing deregulation wave began with natural gas, railroads, financial markets and telecommunication. The high energy prices during 1970’s oil embargoes focused the attention on the need to find more efficient methods of using electricity than ever. This urged the industry’s need to shift from selling electrons to selling services (Sheble [55]). The political winds within the United States and around the world wanted to remove monopolistic pricing and achieve better prices with competition.

The first regulatory development towards deregulation in United States was the Public Utility Regulatory Policy Act (PURPA) in 1978. This created the opportunity for small Independent Power Producers (IPPs), using renewable energy or combined heat and power, to sell power to regulating utilities serving retail customers. The access to wholesale electricity market was expanded with the Electricity Policy Act (EPAct) in 1992, allowing the entities that do not own transmission facilities with rights to use the transmission system. This was termed as "transmission open access". The subsequent regulatory decisions were issued time to time by FERC. In 1996, FERC issued the Orders 888 and 889, which required all public utilities that own, control or operate transmission
facilities to make non-discriminatory transmission open access and transmission information available to all parties on equal terms. In 1998, California became the first state in United States to adopt a competitive structure and other states started gradually moving towards. Accordingly, six major market models (or ISOs) can be found in United States, California (CAISO), the Pennsylvania-New Jersey-Maryland Interconnection (PJM), New York (NYISO), New England (ISONE), Texas (Electric Reliability Council of Texas-ERCOT), and the Midwest (MISO).

In United Kingdom, with the Electricity Act of 1983 government ended the exclusive franchise over government held enterprise, even though, the government controlled monopoly prevailed till around 1989. With the electricity act passed in 1989, the transition of electrical industry ownership from government to private investors took place allowing competition among them. To implement competition, the government controlled Central Electricity Generating Board (CEGB) was first separated into four parts – National Power Company, PowerGen Company, Nuclear Electric Company and National Grid Company (NGC). Later, National Power and PowerGen were further disintegrated to new participants increasing competition and reducing market power. NGC was formed as the natural monopoly in transmission sector, with the objective as "to develop and maintain an efficient, coordinated and economical system of electricity transmission and to facilitate competition in the generation and supply of electricity".

In Canada, preliminary changes were seen in the conventional power industry with the establishment of Electric Energy Marketing Act (EEMA) in 1982. With the legislations issued under Electric Utilities Act (EUA) in 1995 electricity industry was moved to a competitive pool market, which is a non-profit corporation (Power Pool of
Alberta) that would permit fair and open competition. NordPool (The Nordic Power Exchange) was the first international power exchange in the world, established wholesale electricity market in the Nordic region operating since 1996. Sweden became a part of the exchange in 1996, Finland joined in 1998, Western Denmark in 1999 and Eastern Denmark in 2000.

The Victorian Power Exchange (Victoria Pool) was commenced operation in July 1994, as the first Australian wholesale electricity market. Later in May 1996 another state-level wholesale electricity market began operation in New South Wales. New Zealand Electricity Market (NZEM) was established in October 1996. The Singapore Electricity Pool (SEP) was started in April 1998, later in April 2001 Energy Market Authority (EMA) was established to operate the wholesale electricity market for Singapore. The new Singapore electricity market comprises many aspects of liberalized electricity markets such as supply and demand side bidding (for certain consumer categories) for the spot market, reserve market for spinning capacity, interruptible loads and vesting contracts. Other than above mentioned prominent power markets some South American countries are successfully running power markets introduced as substitution for vertically integrated electric utilities prevailed (Source: Shahidehpour and Almoush [56], Shahidehpour and Marwali [57]).

2.4 Transmission Expansion Planning in “Liberalized Electricity Markets”

The TEP literatures related to deregulated electricity markets are still limited. The economic aspects of TEP formulations in deregulated environments and their incorporation to planning models were discussed by Baldick and Khan [5] and De la
Torre et al. [6]. Baldick and Khan explained the effects of network externalities, economy of scale, and uncertainties that can occur in both the supply and the demand modeling, on the transmission planning in competitive markets. The paper also illustrated the emerging problems and conflicts faced by regulatory policy makers when moving from centralized transmission system to a competitive transmission service. The uncertainties arising due to the lack of co-ordination between generation expansion and transmission expansion were emphasized in De la Torre et al. [6] and David and Wen [7].

And, De la Torre et al. [6] proposed a decision-analysis based method to evaluate minimum-risk transmission projects under uncertain input data. The paper provided a set of definitions, such as – attributes, future, options, plan, risk, regret and robustness, which can be useful in network planning under uncertainties. A real-world example was shown to highlight the relevance of these terms and their usage. David and Wen [6] have thoroughly reviewed the problem under the following categories:

- New challenges of TEP under competitive environment
- Management and regulation of transmission projects
- Investment and cost recovery in transmission expansion
- TEP methods or techniques under competitive environment

The paper discussed qualitatively, the uncertain factors and risks associated with generation investments and operational planning; and how it complicates the network planning process. This also introduced new planning and reliability criteria in system planning in terms of operating flexibility and system robustness. Additionally this paper identified (i) monopoly management and (ii) market driven investment managements as options to investment management and emphasized on investment regulation by an
independent regulator to drive the expansion plan towards social optimum.

The incentives for grid investments under contract network regime with *Transmission Congestion Contracts*, TCC (Hogan [9]) were analyzed in Bushnell and Stoft [8]. The new TCCs generated from grid expansions as well as the value change in the exiting TCCs due to the same were investigated in contract networks as benefits with the network investment cost. Further, the paper formalized a rule for awarding new TCC to grid investors and showed how this allocation method avoids detrimental grid investments. An extended analysis of this can be found in Bushnell and Stoft [10], which includes the effects of network externalities and embedded cost recovery into the investigation. This paper also considered TCC as the base for investment analysis, which resulted due to network expansion. This could be awarded to the investor at the time of network upgrading or can be purchased in a secondary market. However, certain regulatory intervention was envisioned in the paper to establish reliability standards and to facilitate the formation of investment coalitions even amidst competitive network investments.

The obstacles that hinders transmission sector being a self-sustained competitive area in electricity business, and the possible challenges to the natural monopoly by new technologies such as High Voltage DC (HVDC) and FACTS were presented in Rotger and Felder [4], Hirst and Kirby [11] and Cameron [12]. These articles provided better ground information and possible directions for transmission planning aspects in the new environment, although these do not specifically discuss the pure technical issues. Rotger and Felder [4] examined the benefits and drawbacks of "centralized" and "decentralized" transmission investments. The technological developments were recognized as the main
cause of the transition from "centralized" to "decentralized" investment management, which might diminish the natural monopoly situations.

The key transmission planning issues and challenges were outlined in Hirst and Kirby [11] indicating the transmission inadequacy compared to load growth in many countries including the US. The paper showed the barriers existing between transmission planners and energy markets, and lack of information exchange regarding possible transmission congestions, investment hints to generation investors and information to load-serving entities. Furthermore, the worth of congestion cost as a deciding factor for transmission and generation investments was emphasized in the article. Additionally, the information problem faced by transmission planers due to the disintegration of generation and retail service from transmission was stressed.

The organization of transmission management is crucial and often subjected to debate – one camp insists on "centralization" and another on "decentralization". An integration of different transmission entities under a single umbrella, such as RTOs in the United States, for coordinated network expansion and improvements was emphasized in Hirst and Kirby [11], Krapels [13] and Stoft and Graves [14]. In particular, Hirst and Kirby [15] discussed the transmission planning issues related to - reliability vs. commerce, congestion costs, alternatives to transmission, economies of scale, advanced technologies, planning data, and centralized vs. decentralized transmission planning and expansion, concentrating on United States electricity industry and its statistics. Three different transmission managements were given in Stoft [16] - non-profit transmission administration (TA), for-profit transmission company (TransCo), transmission market, as alternatives to vertically integrated transmission system, and their investment and
planning policies and optimal network expansion.

In addition to above analytical or qualitative articles, some technical reports and recent research papers have attempted the TEP issue in competitive electricity markets. An interactive multiyear transmission planning model named 'HIPER', which deals with generation expansion uncertainties using strategic scenario analysis, was presented in Cruz-Rodriguez and Latorre-Bayona [17]. The planning tool comprised a heuristic planning model, a pseudo-dynamic investment planning scheme, and a decision-sensitivity analysis module. A risk or regret minimization procedure was adopted in the model, which represents the difference between minimum costs of investment and operation reliability of optimum and the expansion plan being considered. A set of static optimization problems was considered using annuatised quantities to represent the planning horizon. However, the main concern of the paper is generation uncertainty and searching the most robust expansion planning schedule. The generation expansion uncertainties on network planning was emphasized in Saenz et al. [18] as well, suggesting some useful economic factors to consider in network expansion studies.

A TEP model that modifies the traditional centralized TEP formulations with some market-driven flow patterns, i.e. group flow patterns – pool dispatch and point-to-point patterns – bi-lateral trading, was found in Fang and Hill [19]. The flow patterns were considered as uncertain as it happens in competitive markets, and employed a decision analysis technique to obtain risk (or regret) minimum network configurations under uncertain futures. However, determining the future flow patterns with associated probabilities is practically hard and it might lead to a large number of probable patterns, which would blur the outcome of decision analysis. But, the model implies one important
issue that the future generation and load levels will vary in a wider range which was not experienced in vertically integrated systems. Therefore it should be adequately modeled for transmission planning studies in competitive markets.

Cost minimization driven centralized and price based decentralized formulations for TEP problem were formulated in Gil et al. [20] for competitive environments. And, a price updating technique was used to obtain the network expansion requirements. Both approaches were shown to be converging to the same expansion plan – provided perfect competition prevails among transmission providers. The optimum solution given in this paper was however, the social optimum which is the objective under centralized approaches. However, this optimum cannot be always guaranteed under decentralized approaches, particularly under strategic behaviors of transmission providers.

The disparities arising when centralized cost minimization based traditional TEP models are used to perform network planning in competitive environments were examined in Cagigas and Madrigal [21] as well. The paper showed how network expansion schedule is affected by competitive bid prices. Further, the necessity for new tools and software to cope with situation was emphasized in the paper. The outlook of network planning in the new era, system planning tools needed, and their essential capabilities were discussed in Clayton and Mukerji [22] as well paying attention to the United States electricity industry.

In summary, although there is an awareness of the changes in the context of transmission planning in the deregulated market, and a number of researchers and agencies have contributed to concepts and techniques to address various issues, a broadly accepted approach has not yet been established.
Chapter 3

Development of a Network Constrained Power Dispatch Model

"The free market is the only mechanism that has ever been discovered for achieving participatory democracy."

--- Milton Friedman ---

1976 Nobel Laureate in Economics

3.1 Introduction

3.1.1 Dispatch in Traditional Power Systems

Economic Dispatch (ED), Network Constrained Economic Dispatch (NCED), and DC or AC Optimal Power Flow (OPF) models were extensively used in vertically integrated power systems to determine the optimal schedules and various operational parameters. Depending on the formulation, such short-term scheduling programs decide not only power dispatch levels but also some operational parameters such as generator scheduled voltages, transformer tap ratios and auxiliary reactive power supports. In economic dispatch or network constrained economic dispatch, minimization of production cost of generating units is considered the objective subject to distinct operational constraints. Beside the production costs, the deviations of preferred node (or bus) voltage levels as well as active power losses are considered a part of the objective in AC-OPF. All these formulations are primarily mathematical programming problems, where the primal and dual solutions provide vital information on least-cost energy supply
and its marginal cost based pricing. Accordingly, cost minimization based ED and OPF models represent the cost-of-service regime of the electricity utilities.

3.1.2 Economic Dispatch and OPF Models in Deregulated Systems

In competitive electricity markets, where supply and demand equilibria decide the dispatch quantities and prices, the necessity for dispatch programs such as ED, NCED, DC or AC-OPF has intensified. Accordingly, competitive electricity markets around the world employ DC-OPF (NCED) or AC-OPF models to clear supply and demand as well as to determine the associated prices in day-ahead or hour-ahead power dispatch. AC-OPF models are preferred over DC-OPF models due to higher accuracies on dispatch procedures and internalization of losses and congestion management. Additionally, dynamic voltage and stability constraints can be incorporated in AC-OPF. Yet DC-OPF models are still in operation, perhaps in improved versions, due to the complexity of AC-OPF models in large scale applications with higher number of independent variables and other parameters. For example, PJM and NYPOOL can be given as DC-OPF based LMP pools.

In competitive markets social welfare objectives supersede production cost minimization that prevailed in vertically integrated systems. Customers’ willingness to buy or their benefit maximization is accommodated in the dispatch program, which differs from the pure production cost minimization practiced in traditional operation. Therefore, social welfare maximization or social cost minimization constitutes the main objective of electricity dispatch. This dispatch algorithm, also termed as "power pool" model, is generally formulated as a double-sided auction model to which each supplier (GenCo) and consumer (DistCo) bid their willingness to sell or buy in the form of price
bids. A supplier may be represented as an individual generator or collection of generators representing a generation company. A consumer may be a Load Serving Entity (LSE) serving a large group of retail consumers or substantially large industrial consumer. It should be noted that there are electricity markets, which do not allow consumer side price bidding but expects consumer load information on hourly basis.

In perfect competition, price bid functions represent true marginal cost or benefit of suppliers and consumers respectively. Under strategic bidding suppliers and consumers may not bid their true marginal cost or benefit, instead we can assume that bid functions represent apparent marginal cost or benefits. Depending on market-clearing practice (day-ahead or hour-ahead) participants will submit their bids to Independent System Operator (ISO) and/or Power Exchange (PX), who will determine the market clearing supply/demand, Locational Marginal Prices (LMPs) and associated transmission pricing details corresponding to each time period ($t_h$).

3.2 Formulation of Network-Constrained Day-Ahead Dispatch Model

For our analysis, a network constrained DC-OPF with modifications to accommodate network losses is adopted to represent the day-ahead market-clearing engine. The model takes power supply and demand details including limits and price bids as the main inputs and processes with DC network details and power loss representation to decide market cleared quantities and prices. A DC network model is specifically adopted to keep the model simple within our objective of network expansion analysis. However, AC network representation may provide detailed information that may be useful for comprehensive studies. The computational and optimization difficulties in AC-
OPF for non-convex studies such as network planning persuaded us to use the DC-OPF techniques consistently.

3.2.1 Supply Function

For the analysis, supply (GenCo) bids are assumed to be linear upward sloped (Berry et al. [58], [59], Hobbs et al. [60], Weber and Overbye [61]), representing its true marginal cost. Accordingly, for a particular supplier \( i \) the price-quantity supply bid is given by, \( p_i(t_h) = b_i + m_i g_i(t_h) \) for hour \( t_h \) and \( i \in I \), as shown in Fig. 3.1. The quantity supplied \( g_i(t_h) \) is given in MWs, parameters \( b_i \) (in $/MWh) and \( m_i \) (in $/MW^{2}h) denote the price intercept and slope respectively.

\[
SS_i(g_i) = \frac{1}{2}m_i g_i^2(t_h)
\]

Figure 3.1: Supplier Economics

The apparent variable cost of generation as per the bid curve at the operating point \( (g_i(t_h), p_i(t_h)) \) is given by, \( C_i(g_i(t_h)) = b_i g_i(t_h) + (1/2)m_i g_i^2(t_h) \). The short-term supplier’s surplus from energy sales is given by, \( SS_i(g_i(t_h)) = (1/2)m_i g_i^2(t_h) \). Further, the supplier’s
bid includes its minimum and maximum capacity offer $g_i$ and $\overline{g}_i$. The details of supplier economics are shown in Fig. 3.1.

3.2.2 Demand Function

The demand bid for a particular consumer $j$ is represented with its marginal benefit received for energy consumption. This models the elastic nature of consumption with the price bid, $p_j(t_h) = b_j - m_j d_j(t_h)$ for hour $t_h$ and $j \in J$, as shown in Fig. 3.2, where $b_j$ (in $$/MWh), $m_j$ (in $$/MW^2h)$ represent the intercept and slope of the linear model respectively.

![Figure 3.2: Consumer Economics](image)

The gross benefit received by the consumer with the consumption $d_j(t_h)$ is given by $B_j(d_j(t_h)) = b_j d_j(t_h) - (1/2)m_j d_j^2(t_h)$. The short-term consumer’s surplus from the energy consumption is given by $CS_j(d_j(t_h)) = (1/2)m_j d_j^2(t_h)$. Further, the consumer’s bid includes its minimum and maximum demand representing elastic limit, i.e. $d_j$ and $\overline{d}_j$. The details of consumer economics are shown in Fig. 3.2.
3.3 Short-term Economic Power Dispatch in an Electricity Market

3.3.1 Model Formulation

Consider a centralized market maker/dispatcher managing a day-ahead electricity market (such as the Pool in UK, Singapore Electricity Pool or ISO/PX in California) on behalf of suppliers (GenCos) and consumers (DistCos) to establish the dispatch model. Maximization of social welfare or minimization of social cost (3.1) becomes the objective of this double-sided power auction, which determines the market clearance for supply and demand, Locational Marginal Prices (LMPs) for each bidding period $t_h$ (say an hour). This may be formulated as the following auction model with supply and demand constraints and network constraints, which models total apparent supplier cost and consumer benefit as the objective. It solves for the optimum short run generation $g_i(t_h)$ and demand $d_j(t_h)$ levels.

\[
\begin{align*}
\min_{g_i(t_h), d_j(t_h)} & \sum_{i \in I} C_i(g_i(t_h)) - \sum_{j \in J} B_j(d_j(t_h)) \\
\text{subject to} & \sum_{j \in J} d_j(t_h) + L(z_m(t_h)) - \sum_{i \in I} g_i(t_h) = 0 \\
& g_i \leq g_i(t_h) \leq \bar{g}_i \quad \forall i \in I \\
& d_j \leq d_j(t_h) \leq \bar{d}_j \quad \forall j \in J \\
& z_m \leq z_m(t_h) \leq \bar{z}_m \quad \forall m \in M
\end{align*}
\] (3.1)

Subject to the following constraints:

(i) Power balance,

\[\sum_{j \in J} d_j(t_h) + L(z_m(t_h)) - \sum_{i \in I} g_i(t_h) = 0 \] (3.1.a)

(ii) Supply, demand and line flow limits,

\[g_i \leq g_i(t_h) \leq \bar{g}_i \quad \forall i \in I \] (3.1.b)

\[d_j \leq d_j(t_h) \leq \bar{d}_j \quad \forall j \in J \] (3.1.c)

\[z_m \leq z_m(t_h) \leq \bar{z}_m \quad \forall m \in M \] (3.1.d)
where, \( L(z_m(t_h)) \) denotes network power losses as a function of line flows \( z_m(t_h) \).

Supply, demand and line flow limits have their minimum and maximum limits \([\underline{z}_m, \overline{z}_m], [d_j, \overline{d}_j] \) and \([\underline{z}_m, \overline{z}_m] \) respectively. Constraints (3.1.a – 3.1.d) will yield dual prices \( \mu_v(t_h), \mu_{d,j}(t_h), \mu_{z,m}(t_h) \) and \( \mu_{z,m}(t_h) \). Short-term branch flow limits \([\underline{z}_m, \overline{z}_m] \) are expressed in terms of line capacity \( Z_m \), i.e. \( \underline{z}_m = -Z_m \) and \( \overline{z}_m = Z_m \).

### 3.3.2 Solution Method for the Dispatch Problem

The mathematical formulation (3.1-3.1.d) is expressed in the Non-Linear Programming (NLP) format in (3.2) for convenient solutions.

The independent variables \( x(t_h) \in \mathbb{R}^{I+J} \) are represented as,

\[
x(t_h)_{(I+J) \times 1} = \begin{bmatrix} g_1(t_h) & \ldots & g_I(t_h) & d_1(t_h) & \ldots & d_J(t_h) & d_J(t_h) \end{bmatrix}^T
\]

The supply/demand – node incidence matrix, \( \Gamma \in \mathbb{R}^N \times \mathbb{R}^{I+J} \) is used for the mapping of \( x(t_h) \) to nodal power injection vector, \( y(t_h) \in \mathbb{R}^N \) given by,

\[
y(t_h) = \Gamma x(t_h)
\]

where,

\[
[\Gamma_{n,i} \text{ or } j]_{N \times (I+J)} = \begin{cases} 1 & \text{if supplier } i \text{ is connected to node } n \\ -1 & \text{if consumer } j \text{ is connected to node } n \\ 0 & \text{else} \end{cases}
\]

**Representation of Network Power Losses:**

The total network power losses can be written as, \( L(z_m(t_h)) = z^T(t_h)Rz(t_h) \).

where, \( [R_{i,j}]_{M \times M} = \begin{cases} r_m & \text{if } i = j \\ 0 & \text{else} \end{cases} \quad r_m = \text{resistance of the branch } m \).
The line power flow \( z(t_h) \) can be expressed as, \( z(t_h) = H \cdot y(t_h) \) where, \( H \in \mathbb{R}^M \times \mathbb{R}^N \) stands for the branch-node sensitivity (or PTDFs) matrix. Accordingly,

\[
L(z_m(t_h)) = y(t_h)^T \cdot H^T \cdot R \cdot H \cdot y(t_h)
\]

\[
L(z_m(t_h)) = y(t_h)^T \cdot B_{loss} \cdot y(t_h), \text{ where } B_{loss} = H^T \cdot R \cdot H \in \mathbb{R}^N \times \mathbb{R}^N
\]

Using \( y(t_h) = \Gamma \cdot x(t_h) \) \( \Rightarrow \) \( L(z_m(t_h)) = (1/2) \cdot x(t_h)^T \cdot 2 \cdot \Gamma^T \cdot B_{loss} \cdot \Gamma \cdot x(t_h) \)

\[
L(z_m(t_h)) = (1/2) \cdot x(t_h)^T \cdot V \cdot x(t_h) \quad \text{where } V = 2 \cdot \Gamma^T \cdot B_{loss} \cdot \Gamma \in \mathbb{R}^{I-J} \times \mathbb{R}^{I-J}
\]

The problem formulation in (3.1) can now be arranged in the following NLP format,

\[
\min_{x(t_h)} \quad (1/2) \cdot x^T(t_h) \cdot U \cdot x(t_h) + u^T \cdot x(t_h)
\]

where,

\[
U = [\text{diag}(m_i), \text{diag}(m_j)]_{(I+J) \times (I+J)} \quad \text{and } u^T = [u_k]_{(I+J)} \quad \text{where } u_k = b_i \text{ if } k \leq I, \quad u_k = -b_j \text{ if } k > I
\]

The power balance condition (3.1.a):

\[
\sum_j d_j(t_h) + L(z_m(t_h)) - \sum_i g_i(t_h) = 0 \quad \Leftrightarrow \quad (1/2) \cdot x^T(t_h) \cdot V \cdot x(t_h) + v^T \cdot x(t_h) = 0
\]

where, \( V = 2 \cdot \Gamma^T \cdot B_{loss} \cdot \Gamma \) and \( v^T = [-1 \quad \cdots \quad -1 \quad 1 \quad \cdots \quad 1]_{(I+J)} \)

Inequality constraints (3.1.b-d):

\[
C \leq (1/2) \cdot x^T(t_h) \cdot W \cdot x(t_h) + w^T \cdot x(t_h) \leq D
\]

where, \( W = 0 \in \mathbb{R}^{I+J} \times \mathbb{R}^{I+J} \);

\[
w^T = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} \in \mathbb{R}^{I+J+M}; \quad C = \begin{bmatrix} \bar{g}^r \\ \bar{d}^r \end{bmatrix} \in \mathbb{R}^{I+J+M}; \quad D = \begin{bmatrix} \bar{g}^r \\ \bar{d}^r \end{bmatrix} \in \mathbb{R}^{I+J+M}
\]
\( I_1 \) – Unit Matrix of size \( I \times I \)  
\( I_2 \) – Unit Matrix of size \( J \times J \)

This yields the final NLP problem formulation as:

\[
\text{Min} \quad x(t_h) \quad (1/2)x^T(t_h)Ux(t_h) + u^Tx(t_h) \quad (3.2)
\]

Subject to:

\[
(1/2)x^T(t_h)Vx(t_h) + v^T x(t_h) = 0 \quad \text{and} \quad C \leq (1/2)x^T(t_h)Wx(t_h) + w^T x(t_h) \leq D
\]

The new barrier function technique (Momoh [68], [69]) is adopted for the solution of this NLP formulation. The details of the algorithm are given in Appendix A-2.

### 3.4 Important Features of the Dispatch Problem

The short-term marginal pricing details emanating from the dispatch problem correctly signal the supply and demand interaction as well as the transmission network’s implication on effective power dispatch. Accordingly, the dispatch program can be used to price supply, demand and transmission usage. These transmission pricing details can be beneficially utilized for network expansion purposes. The short-term transmission pricing related attributes derived in this chapter will be extensively used in the analysis in subsequent chapters.

Formulation of the Lagrange function will facilitate the analysis of short-term pricing issues from marginal supply and demand viewpoints. For that purpose, equations (3.1-3.1.d) in the dispatch problem may be combined to form the Lagrange function \( \Omega \{ \cdot \} \) with Lagrange multipliers (which represent the dual prices at optimality).
\[
\Omega\{g_i, d_j, \mu_d, j, \mu_{g,i}, \mu_{d,j}, \mu_{z,m}, t_h, t'_y\} = \sum_{i \in I} C(g_i(t_h)) - \sum_{j \in J} B(d_j(t_h)) + \mu_c(t_h)\left[\sum_{j \in J} d_j(t_h) + L(z_m(t_h)) - \sum_{i \in I} g_i(t_h)\right] - \sum_{j \in J} \mu_{\min,d,j}(t_h)\left[\frac{d_j(t_h) - d_j}{d_j(t_h) - d_j} + \sum_{j \in J} \mu_{\max,d,j}(t_h)\left[d_j(t_h) - \bar{d}_j\right]\right] - \sum_{i \in I} \mu_{\min,g,i}(t_h)\left[g_i(t_h) - g_i\right] + \sum_{i \in I} \mu_{\max,g,i}(t_h)\left[g_i(t_h) - \bar{g}_i\right] - \sum_{m \in M} \mu_{\min,z,m}(t_h)\left[z_m(t_h) - \bar{z}_m\right] + \sum_{m \in M} \mu_{\max,z,m}(t_h)\left[z_m(t_h) - \bar{z}_m\right]
\]

where, \(\mu_{\min,\cdot,\cdot}\) and \(\mu_{\max,\cdot,\cdot}\) represent the lower and upper bound Lagrange multipliers of \(\mu\).

### 3.4.1 Short-term Locational Marginal Prices (Nodal Prices\(^1\))

**Definition 3.1:** The spatial variation of Locational Marginal Prices, LMP (Schweppe et al. [62]) for suppliers and consumers are given by,

\[
\rho_i(t_h) = \mu_c(t_h) - \mu_c(t_h)\frac{\partial L(z_m(t_h))}{\partial g_i(t_h)} - \sum_{m} \left[-\mu_{\min,z,m}(t_h) + \mu_{\max,z,m}(t_h)\frac{\partial z_m(t_h)}{\partial g_i(t_h)}\right] \forall i \in I
\]

and

\[
\rho_j(t_h) = \mu_c(t_h) + \mu_c(t_h)\frac{\partial L(z_m(t_h))}{\partial d_j(t_h)} + \sum_{m} \left[-\mu_{\min,z,m}(t_h) + \mu_{\max,z,m}(t_h)\frac{\partial z_m(t_h)}{\partial d_j(t_h)}\right] \forall j \in J
\]

**Proof:** Let the Karush-Kuhn-Tucker (KKT) first order necessary conditions of \(\Omega\{\cdot\}\) with respect to individual supply, \(g_i(t_h)\) and demand, \(d_j(t_h)\). (i.e. \(\partial\Omega\{\cdot\}/\partial g_i(t_h) = 0\) and \(\partial\Omega\{\cdot\}/\partial d_j(t_h) = 0\)). This yields the LMPs for suppliers \(\rho_i(t_h)\) and consumers \(\rho_j(t_h)\) respectively.

---

\(^1\)“Nodal Pricing” concept was first introduced by Fred Schweppe et al. in 1980’s. See Schweppe et al. [62] for details.
\[
\frac{\partial \Omega_i}{\partial g_i(t_h)} = 0 \quad \Rightarrow \quad \rho_i(t_h) = \frac{\partial C(g_i(t_h))}{\partial g_i(t_h)} - \mu_{\min,g,i}(t_h) + \mu_{\max,g,i}(t_h)
\]

\[
= \mu_e(t_h) - \mu_e(t_h) \frac{\partial L(z_m(t_h))}{\partial g_i(t_h)} - \left[ -\sum_m \mu_{\min,z,m}(t_h) \frac{\partial z_m(t_h)}{\partial g_i(t_h)} + \sum_m \mu_{\max,z,m}(t_h) \frac{\partial z_m(t_h)}{\partial g_i(t_h)} \right]
\]

for suppliers \(i = 1,2...I\)

\[
\frac{\partial \Omega_j}{\partial d_j(t_h)} = 0 \quad \Rightarrow \quad \rho_j(t_h) = \frac{\partial B(d_j(t_h))}{\partial d_j(t_h)} + \mu_{\min,d,j}(t_h) - \mu_{\max,d,j}(t_h)
\]

\[
= \mu_e(t_h) + \mu_e(t_h) \frac{\partial L(z_m(t_h))}{\partial d_j(t_h)} + \left[ -\sum_m \mu_{\min,z,m}(t_h) \frac{\partial z_m(t_h)}{\partial d_j(t_h)} + \sum_m \mu_{\max,z,m}(t_h) \frac{\partial z_m(t_h)}{\partial d_j(t_h)} \right]
\]

for consumers \(j = 1,2...J\)

The nodal prices consist of marginal cost of power at reference node \(\mu_e(t_h)\), marginal effect on network losses evaluated at reference node marginal cost \(\mu_e(t_h) \frac{\partial L(z_m(t_h))}{\partial (\cdot)}\), and the contribution to the network congestion \(\sum_m \mu_{z,m}(t_h) \frac{\partial z_m(t_h)}{\partial (\cdot)}\). Under this model, both suppliers and consumers are charged for their contribution in network losses and network congestion.

Let \(H\) be the branch-node sensitivity matrix (or Power Transfer Distribution Factors, PTDFs) for DC power flows of transmission network. The value \(H_{m,n}\) represents the change of flow in a particular branch \(m\) for a unit increment of power injection at node \(n\) (See Appendix A-1 for the derivation of PTDFs, \(H\)). Accordingly, branch power flow can be written as,

\[
z_m = \sum_{i \in I} H_{m,(i)} g_i - \sum_{j \in J} H_{m,(j)} d_j
\]

and \(\frac{\partial z_m(t_h)}{\partial g_i(t_h)} = H_{m,(i)}, \quad \frac{\partial z_m(t_h)}{\partial d_j(t_h)} = -H_{m,(j)}\).

where \((i)\) or \((j)\) refers to the node to which \(i^{th}\) supplier or \(j^{th}\) consumer is connected. These
factors denote the change of flow in a particular branch \( m \) for a unit increment of supply and demand respectively.

**Definition 3.2:** Locational Marginal Prices (Nodal prices) of suppliers and consumers connecting to the same node are equal.

**Proof:** Let the supplier \( i \) and consumer \( j \) is connected to node \( n \).

Hence, from (3.4.a and b),

\[
\rho_i(t_h) = \mu_e(t_h) - \mu_e(t_h) \frac{\partial L(z_m(t_h))}{\partial g_i(t_h)} + \left[ \sum_m \mu_{\min,z,m}(t_h)H_{m,n} - \sum_m \mu_{\max,z,m}(t_h)H_{m,n} \right]
\]

\[
\rho_j(t_h) = \mu_e(t_h) + \mu_e(t_h) \frac{\partial L(z_m(t_h))}{\partial d_j(t_h)} + \left[ \sum_m \mu_{\min,z,m}(t_h)H_{m,n} - \sum_m \mu_{\max,z,m}(t_h)H_{m,n} \right]
\]

As the network power loss representation in § 3.3.2, \( L(z_m(t_h)) = y^T B_{loss} y \)

where, \( y_n = \sum_{i \in I_{(n)}} g_i - \sum_{j \in J_{(n)}} d_j \) and \( I_{(n)}, J_{(n)} \) represent the set of suppliers and consumers connected to node \( n \). Thus,

\[
\frac{\partial L(z_m(t_h))}{\partial y_n} = \sum_{k=1}^N B_{loss} \hat{y}_k + \sum_{k=1}^N B_{loss} \hat{y}_k \hat{y}_k \text{ and } \frac{\partial y_n}{\partial g_i} = 1 \text{ and } \frac{\partial y_n}{\partial d_j} = -1.
\]

Therefore using the chain rule, it can be proven that \( \frac{\partial L(z_m(t_h))}{\partial g_i} = -\frac{\partial L(z_m(t_h))}{\partial d_j} \).

Accordingly, from the first two equations it results in \( \rho_i(t_h) = \rho_j(t_h) \). \( \square \)

### 3.4.2 Network Congestion

The presence of binding transmission constraints (i.e. \( \mu_{z,m} > 0 \)) hinders the short-term economic power dispatch – commonly known as network congestion. The physics of power systems implies that injections and extraction at all nodes contribute to the flow in the congested branch. This causes spatial differences in LMPs not only across
congested branch but also in other branches. Accordingly, at the optimal dispatch the
dual price $\mu_{z,m}$ ($\mu_{\min,z,m}$ or $\mu_{\max,z,m}$) represents the short-term marginal value of
transmission or transmission shadow price. The difference between locational prices
represents the congestion charges plus the compensation for losses that generation at low-
priced location incur to supply power to customers at high priced locations (Joskow and
Tirole [63]).

**Definition 3.3:** Market settlement at corresponding LMPs (3.4.a) and (3.4.b) for network
users creates a merchandizing surplus also known as network revenue $NR(t_h)$, which
comprises value of losses and congestion revenue or charge.

\[
NR(t_h) = \mu_v(t_h).L(z_m(t_h)) - \sum_m \mu_{\min,z,m}(t_h).z_m(t_h) + \sum_m \mu_{\max,z,m}(t_h).z_m(t_h) \tag{3.5}
\]

**Proof:** The merchandizing surplus remains with the market maker is equivalent to,

\[
\sum_{j \in J} \rho_j(t_h)d_j(t_h) - \sum_{i \in I} \rho_i(t_h)g_i(t_h).
\]

With (3.4.a) and (3.4.b) it can be proven that $NR(t_h)$ is equivalent to,

\[
NR(t_h) = \sum_j \rho_j(t_h)d_j(t_h) - \sum_j \rho_j(t_h)g_j(t_h)
\]

\[
= \mu_v(t_h)\left[\sum_j d_j(t_h) - \sum_i g_i(t_h)\right] + \\
\mu_v(t_h)\left[\sum_i \left(\frac{\partial L(z_m(t_h))}{\partial g_i(t_h)}\right)g_i(t_h) + \sum_j \left(\frac{\partial L(z_m(t_h))}{\partial d_j(t_h)}\right)d_j(t_h)\right] + \\
\sum_m \left[-\mu_{\min,z,m}(t_h) + \mu_{\max,z,m}(t_h)\right] \left[\sum_j \left(\frac{\partial z_m(t_h)}{\partial d_j(t_h)}\right)d_j(t_h) + \sum_i \left(\frac{\partial z_m(t_h)}{\partial g_i(t_h)}\right)g_i(t_h)\right]
\]

with DC power flow approximations, $\sum_j d_j(t_h) - \sum_i g_i(t_h)$ reduces to $-L(z_m(t_h))$,
and $\sum_j \left(\frac{\partial z_m(t_h)}{\partial d_j(t_h)}\right)d_j(t_h) + \sum_i \left(\frac{\partial z_m(t_h)}{\partial g_i(t_h)}\right)g_i(t_h)$ simplifies to $z_m(t_h)$.

The symmetrical power loss representation given in § 3.3.2 reduces the term
\[ \sum_i \left( \frac{\partial L(z_m(t_h))}{\partial g_i(t_h)} \right) g_i(t_h) + \sum_j \left( \frac{\partial L(z_m(t_h))}{\partial d_j(t_h)} \right) d_j(t_h) \to 2L(z_m(t_h)). \]

This further simplifies the given expression as,

\[ NR(t_h) = \mu_c(t_h)L(z_m(t_h)) + \sum_m \left[ -\mu_{\text{min},z,m} + \mu_{\text{max},z,m} \right] z_m(t_h). \]

This \textit{merchandizing surplus} (Wu et al. [64]) or mid-man revenue is termed different names in different citations as "network revenue" (Perez-Arriaga et al. [65], Rudnick et al. [66]) or "short-term transmission rent" with the sense that it represents the transportation charge under congestion pricing.

This consists of the cost of network losses evaluated at marginal cost of power at reference node \( \mu_c(t_h)L(z_m(t_h)) \) and the \textit{congestion revenue} as, \( TR(t_h) = \sum_m \mu_{z,m} z_m(t_h) \) for the hour \( t_h \), where, \( \mu_{z,m} \) represents the branch transmission shadow price. This also appears in different terminology as "congestion price", "path cost" or "marginal value of transmission capacity".

\textit{Definition 3.4: The presence of congestion causes a social cost or loss of social welfare, defined as Congestion Cost (Barmack et al. [67]) \( TC(t_h) \).}

\textit{Proof:} The loss of social welfare due to network limitations can be estimated by repeated evaluation of the short-term dispatch algorithm with and without transmission constraints. This is equivalent to net social cost due to network constraints. Hence,

\[ TC(t_h) = \left\{ \text{Max} \left( B - C \right) \right\}_{3.1.a-3.1.c} - \left\{ \text{Max} \left( B - C \right) \right\}_{3.1.a-3.1.d} \]

\[ = \left\{ \text{Min} \left( C - B \right) \right\}_{3.1.a-3.1.d} - \left\{ \text{Min} \left( C - B \right) \right\}_{3.1.a-3.1.c} \] \hspace{1cm} (3.6)

The \textit{congestion cost}, \( TC \) in (3.6) indicates the net loss of social welfare or social cost due to network congestion. This is equivalent to profit (or surplus) loss of market participants.
(suppliers and consumers) in the context of price sensitive electricity markets. In vertically integrated systems, it can be interpreted as the out-of-merit generation (OMG) cost caused by transmission congestion. This equates to the cost of out of merit dispatch or withholding of cheap generation due to transmission limitations. The short-term marginal value of transmission capacity $\mu_{z,m}(t_k)$ will indicate how much $TC$ can be reduced (or saved) for an infinitesimal (or unit) increase in bounded transmission capacity.

3.4.3 Economic Signals for Network Expansions

Large variations of LMPs indicate the transmission system’s inadequacy to promote effective supply and demand competition. The contribution of network losses to LMP variation is comparatively small, but the presence of congested branches contributes more significantly. However, the dispersion of LMPs is not an ideal measure of identifying the system bottlenecks. Compared to LMP dispersion, transmission shadow price or marginal value of transmission capacity in the dispatch program provides better or more precise signal for network bottleneck identification. Network revenue, $NR$ and its composition show how "short-term network revenue" varies with network properties and what needs to be done to avoid higher transmission prices in LMP based transmission pricing. Congestion cost, $TC$ indicates the loss of social welfare to market participants (suppliers and consumers) in the form of loss of surplus, and how it is to be avoided with "economic transmission expansion". Therefore, the marginal value of transmission capacity or transmission shadow price is considered with utmost importance through out this thesis, and it is utilized for effective network expansion to be analyzed in the following chapters.
3.5  Illustrative Example (IEEE 24-bus RTS)

The calculation of nodal prices and transmission pricing details using the procedure developed above are tested in the modified IEEE 24-bus Reliability Test System (RTS) (RTS Task Force [70], Billinton and Li [71]). The network diagram is shown in Fig. 3.3, which has 14 generating companies (Gencos) and 17 distribution companies (Distcos).

![IEEE 24-bus Reliability Test System (RTS)](image)

**Figure 3.3: IEEE 24-bus Reliability Test System (RTS)**

Supply and demand bids adopted for the case study are shown in Table 3.1. Original branch capacities have been reduced (See Appendix A-4) in order to demonstrate the effects of network congestion.
TABLE 3.1: BIDDING DETAILS (FOR IEEE 24-BUS RTS)

<table>
<thead>
<tr>
<th>No</th>
<th>Genco Bids</th>
<th>Distco Bids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b_i$</td>
<td>$g$</td>
</tr>
<tr>
<td>1</td>
<td>71</td>
<td>0.046</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>0.043</td>
</tr>
<tr>
<td>3</td>
<td>71</td>
<td>0.031</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>0.074</td>
</tr>
<tr>
<td>5</td>
<td>34</td>
<td>0.064</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
<td>0.062</td>
</tr>
<tr>
<td>7</td>
<td>41</td>
<td>0.067</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>0.070</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>0.051</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>0.073</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>0.057</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>0.013</td>
</tr>
<tr>
<td>13</td>
<td>20</td>
<td>0.044</td>
</tr>
<tr>
<td>14</td>
<td>19</td>
<td>0.056</td>
</tr>
<tr>
<td>15</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>19</td>
<td>0.040</td>
</tr>
</tbody>
</table>

* Units: $b_i, b_j$ in $$/MWh; $m_i, m_j$ in $$/MW•h; $g, \bar{g}, d, \bar{d}$ in MW

The market clearing and LMPs (nodal prices) for the given bidding details are shown in Fig. 3.4 and 3.5 respectively. This supply and demand bid clearing results in 2,759 $/h as the merchandizing surplus or short-term network revenue $NR(t_h)$, which comprises 1,536 $/h as charges for losses $\mu_c(t_h)L(z_m(t_h))$ and 1,223 $/h as congestion revenue $TR(t_h) = \sum_m \mu_c z_m - z_m(t_h)$. The congestion cost $TC(t_h)$ for this bid clearing is 14.52 $/h. These results clearly reveal the difference between congestion revenue and cost, which will be highlighted in the coming chapters. One branch ($m = 23$) shows congestion with congestion price $\mu_{z,23} = 3.06 $/MWh. This is obtained as the Lagrange multiplier of the relevant ($m = 23$) power flow constraint (3.1.d). The value of $\mu_{z,23}$ is significant because it contributes to the total congestion revenue as
3.06 $/\text{MWh} \times 400 \text{ MW}$. From another perspective, it gives the marginal saving of congestion cost $TC(t_h)$ for an infinitesimal (or unit) increase of branch capacity, $Z_{23}$. Therefore this congestion price carries a special meaning from congestion alleviation point of view. The following Table 3.2 summarizes the network related significant quantities.

<table>
<thead>
<tr>
<th>Table 3.2: Bid Clearing Results (for IEEE 24-Bus RTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>Charges for Losses</td>
</tr>
<tr>
<td>Congestion Revenue</td>
</tr>
<tr>
<td>Network Revenue</td>
</tr>
<tr>
<td>Congestion Cost</td>
</tr>
<tr>
<td>Congestion Price†</td>
</tr>
</tbody>
</table>

† for the congested branch, $m = 23$

![Figure 3.4: Supply/Demand Market-Clearing Quantities](image)

Figure 3.4: Supply/Demand Market-Clearing Quantities

This market clearing dispatch is extensively used in the investigations of network planning studies presented in the following chapters. This tool is suitably extended to incorporate specific features of the studies.
3.6 Concluding Remarks

A network-constrained power dispatch model was developed, which can be utilized in simulating a day-ahead market operations for transmission network planning studies. The model can accommodate both supply and demand side information and network details based on DC load flow techniques. A new barrier technique based method was used for the solution. The accuracy of the developed DC-OPF model is significantly enough for long-term network expansion studies amidst the mathematical complexity of AC-OPF models in such applications.

A set of attributes pertaining to the dispatch model was defined, namely (i) short-run locational marginal prices (nodal prices) (ii) marginal value of transmission (transmission shadow price) (iii) congestion revenue and (iv) congestion cost. Accordingly, the same model can be used to obtain the transmission pricing details under short-term marginal pricing paradigm. The short-term transmission revenue calculated as the merchandizing surplus when settlements are done at locational marginal prices.
consists of cost of losses and congestion revenue. This pricing method charges suppliers and consumers for their network loss and congestion share. Additionally, marginal value of transmission and congestion cost can be obtained from the dispatch model, which represents the economic implications of network bottlenecks. Congestion cost stands for the overall welfare loss due to network bottlenecks and marginal value of transmission represents the reduction of congestion cost for infinitesimal (or unit) increase in bounded transmission facility. These attributes are utilized in network planning formulations to be presented in subsequent chapters.
Chapter 4

Network Expansion Planning under Different Competitive Market Models

"It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest."

--- Adam Smith (1723-1790) ---
Scottish philosopher and economist

4.1 Introduction

The formulations of long-term "economic" network expansion planning problem is presented in this chapter from different network management perspectives. Fairly rigorous analysis of optimal transmission expansions under different transmission management systems is presented. In vertically integrated power systems, network expansion was intended to meet present and future system reliability and service quality standards at least investment and operational costs (least-cost planning). Analogous to this, the objectives of transmission expansions in present day competitive power markets would be two fold, namely, (i) to improve the economic aspects of electricity market, and (ii) to maintain system reliability/stability standards (Gil et al. [20]). The later still needs to be carried out by incumbent transmission owners under the supervision of system operator (regulated monopoly). The first would permit introduction of economically driven investments to encourage a competitive transmission service. Introduction of Independent Transmission Companies (ITCs) (Chandley and Hogan [72]) into the market may initiate this competition. Additionally, energy traders including generation as well as
distribution companies can invest in transmission capacity for added revenue benefits.

Consequently, in today’s electricity markets two major transmission management practices can be found (i) system operations and transmission functions done by a single regulated transmission company (TransCo) e.g. NGC in England and Wales, PowerGrid in Singapore, and (ii) transmission services provided by merchant network investors in compliance with System Operator’s (SO) guide lines e.g. Argentina, Brazil and Australia (Directlink and Murraylink (Cook [73])). The invitation for merchant transmission in PJM expansion planning process PJM manual [74] indicates entering into the second category. However, the structure, functional aspects and operational procedures for transmission operation and planning remain distinctly different around the world.

For a regulated monopoly transmission operator, provisions of economic incentives and regulation of economic rents on network users are the network operator’s targets. The economy of scale and economy of scope and the "lumpiness" in transmission investments are the main reasons for the existence of a regulated monopoly. The regulation is expected to be done by an independent regulator over the regulated firm(s) handling transmission function. The "regulated revenue" is guaranteed through efficient transmission pricing procedures, which covers costs related to assets depreciation, operation and maintenance, plus a sufficient rate of return. The degree of regulation and profit oriented-ness will be the deciding factors, which will move its outlook from non-profit transmission provider (TP) to a for-profit transmission provider (or ITC). The non-profit TP focuses more on system wide considerations and adopts centralized optimizations based on societal cost minimization or social welfare maximization principles for transmission pricing and planning. The cost-of-service regulation is taken
as the key regulatory policy for system planning. However it may be noted that other principles have also been in use in different systems, such as price cap regulation in England, revenue cap regulation in Norway, etc. (Yajima [94], Rothwel and Gomez [95]).

In merchant transmission (Joskow and Tirole [75], Brunekreeft [76]), regulator intervention is minimal and it is for the establishment of necessary frameworks that defines transmission property rights (Hogan [77], [78]) to stimulate efficient investment from merchant entrants. The participation of transmission companies (merchant entrants) would be for the sole objective of profit making through transmission property rights. For-profit TP or ITC may focus more on decentralized approaches on the basis of market information available to individual participants. This is certainly closer to perfect market liberalization than the regulated monopoly.

However, both economic system operation as well as reliability requirements need to be adequately guaranteed irrespective of the transmission management system. In both systems, revenue from transmission pricing is the sole income to the TP. Therefore the transmission ratemaking and expansion of network for added benefits are interlinked and critical in this process. The distinct objectives whether welfare or profit, become crucial in formulating the network-planning problem for optimal investment decisions in either of these network management systems.
4.2 Centralized System-wide Approach

Consider a centralized electricity market, where transmission system planning and operation activities are done by a regulated firm. The regulated firm may or may not hold transmission assets. For instance, a regulated firm like National Grid Company (NGC) in England and Wales holds the sole ownership of network assets, whereas Regional Transmission Organizations (RTOs) in the US do not hold the assets. But, RTOs comprise of network owners as stakeholders. The Office of Electricity Regulation (OFFER) and FERC become the two regulators respectively.

It is assumed that pricing of the transmission services and revenue requirements of the Transmission Owners (TOs) are adequately fulfilled in the design of the market mechanism. From the long-term network expansion standpoint, the central planner would maximize the expected net social welfare arising from transmission capacity usage. This welfare maximization problem or minimization of societal cost can be formulated by linking the long-term transmission investments costs and short-term societal costs into a single objective function. It should be noted that network investments in this chapter are for the sole purpose of welfare maximization without any consideration of reliability requirements. Reliability concerns will be discussed in Chapter 7.

4.2.1 Model Formulation

For the long-term "economic" transmission investment planning, the objective is to minimize the overall cost which consists of the network expansion costs and the net expectations of suppliers’ costs and consumers’ benefits. The objective function may be written as:
This objective function (4.1) is subject to long-term as well as short-term constraints listed below.

In the long run:

\[
\text{Min } IC_m(t_y), g_i(t_h), d_j(t_h) \sum_{t_y \in T} \sum_{m \in M} \frac{IC_m(t_y)}{(1 + \tau)^{t_y - t_0}} + \\
E \left\{ \sum_{t_h \in T} \sum_{i \in I} C_i(g_i(t_h)) - \sum_{t_h \in T} \sum_{j \in J} \frac{B_j(d_j(t_h))}{(1 + \tau)^{t_y - t_0}} \right\} \tag{4.1}
\]

In the short run:

Constraints (4.1.a) will yield dual prices \( \mu_{IC,m}(t_y) \).

In the short run:

Constraints would be the same as (3.1.a) – (3.1.d), where network losses are neglected for long-term network expansion considerations. \( i.e. \)

\[
\sum_j d_j(t_h) - \sum_i g_i(t_h) = 0 \quad \forall t_h \in T \tag{4.1.b}
\]

\[
g_i \leq g_i(t_h) \leq \overline{g_i} \quad \forall i \in I, \forall t_h \in T \tag{4.1.c}
\]

\[
d_j \leq d_j(t_h) \leq \overline{d}_j \quad \forall j \in J, \forall t_h \in T \tag{4.1.d}
\]

\[
z_m \leq z_m(t_h) \leq \overline{z}_m \quad \forall m \in M, \forall t_h \in T \tag{4.1.e}
\]

The long-term discounted figures of transmission investment costs \( IC_m(t_y) \), short-term expected values of suppliers’ generation costs \( C_i(g_i(t_h)) \), and consumers’ benefits \( B_j(d_j(t_h)) \) are represented in (4.1) taking into consideration all network users for the entire planning horizon \( (T) \). Discount rate \( (\tau) \) reflects the time value of money on network investment costs, supply costs and consumers’ benefits. The trade-off between
costs and benefits over the time horizon is ensured by the formulation, giving right time
signal for investment decisions. In long-term investment planning discount rates of 10 –
15 % are typically used, which reflect the economic figures of the society such as
inflation rate. Strategic behaviour by the participants is not considered. So that the
suppliers’ and consumers’ bids are their true marginal costs, \( MC \) and marginal
benefits, \( MB \) for short-term market clearing. The suppliers’ costs and consumers’
benefits are derived from their bid values. In practice, bid values not necessarily represent
the true marginal costs and benefits. This issue (uncertainty in bid prices) is handled with
probabilistic representations of marginal costs and benefits in Chapter 7.

This formulation consists of two time domains corresponding to short run power
dispatch and pricing (in hours – \( t_h \)) and long run investment scheduling (in years – \( t_y \)).
Economic transmission investment decisions bring down the apparent production costs or
raise the consumer benefits for breakeven. In this regard, a neutral system operator would
seek cheap generation options and/or benefit valued consumers for network expansion.
The investment decisions in transmission capacity additions \( IC_m(t_y) \) will affect the
transmission network capacity limits in short-term dispatch \( i.e. z_m, \tilde{z}_m \). Accordingly, the
economic balance between short-term and long-term problems decide the optimality of
capacity investments and social cost minimization of power dispatch.

4.2.2 Optimality Analysis

The Lagrange function of the optimization problem (4.1) may be formed as:
\[ \Omega \left\{ IC_m(t_y), g_i(t_h), d_j(t_h), \mu_{IC,m} \ldots \mu_{z,m}, t_y, t_h \right\} = \sum_{t_y} \sum_{m} \frac{IC_m(t_y)}{(1 + \tau)^{y-t_0}} + \]

\[ E \left\{ \sum_{t_h} \sum_{i} \frac{C_i (g_i(t_h))}{(1 + \tau)^{y-t_0}} - \sum_{t_h} \sum_{j} \frac{B_j (d_j(t_h))}{(1 + \tau)^{y-t_0}} \right\} + \sum_{t_y, t_h} \frac{1}{(1 + \tau)^{y-t_0}} \]

\[ - \sum_{m} \mu_{min,IC,m}(t_y) \left[ IC_m(t_y) - \overline{IC}_m(t_y) \right] + \sum_{m} \mu_{max,IC,m}(t_y) \left[ IC_m(t_y) - \overline{IC}_m(t_y) \right] \]

\[ + \mu_e(t_h) \left[ \sum_{j} d_j(t_h) - \sum_{i} g_i(t_h) \right] \]

\[ - \sum_{i} \mu_{min, g,i}(t_h) \left[ g_i(t_h) - \bar{g}_i \right] + \sum_{i} \mu_{max, g,i}(t_h) \left[ g_i(t_h) - \bar{g}_i \right] \]

\[ - \sum_{j} \mu_{min, d,j}(t_h) \left[ d_j(t_h) - \bar{d}_j \right] + \sum_{j} \mu_{max, d,j}(t_h) \left[ d_j(t_h) - \bar{d}_j \right] \]

\[ - \sum_{m} \mu_{min, z,m}(t_h) \left[ z_m(t_h) - \bar{z}_m \right] + \sum_{m} \mu_{max, z,m}(t_h) \left[ z_m(t_h) - \bar{z}_m \right] \] (4.2)

Consider a transmission capacity expansion in branch \( m \) in year \( t_y \) by \( \Delta Z_m(t_y) \),

this affects the network capacity limits in the dispatch program as, \( z_m = Z_m(t_0) + \Delta Z_m(t_y) \)

and \( \bar{z}_m = -Z_m(t_0) - \Delta Z_m(t_y) \), where, \( Z_m(t_0) \) is the base year branch capacity. The following assumptions are considered valid in the following analysis.

**Assumption 1: No economies of scale exist in transmission investments.**

When no economy of scale is present in transmission investment, and the marginal \((\partial IC_m(t_y)/\partial Z_m(t_y))\) or average investment cost \((\Delta IC_m(t_y)/\Delta Z_m(t_y))\) equals to \(\beta_m(t_y)\),

then

\[ IC_m(t_y) = \beta_m(t_y) Z_m(t_y) \] (A.1)

**Assumption 2: No transmission expansion bounds exist.**

When no expansion bounds exist (upper and lower), then as far as economic expansion addressed in this analysis is concerned,
\[ \mu_{IC,m}(t_y) = 0 \quad (A.2) \]

This assumption will be removed in our further analysis in subsequent chapters. Under these assumptions, two significant results can be obtained.

**Result 1:** The discounted sum of marginal value of transmission (shadow price) over the planning horizon is equal to the discounted sum of marginal transmission investment cost under optimal dynamic network expansion.

**Proof:** Consider the first order optimality condition (KKT) of (4.2) with respect to transmission capacity upgrade \( \Delta Z_m(t_y) \) \( (i.e. \partial \Omega / \partial Z_m(t_y) = 0) \) with the assumptions A.1 and A.2. This yields:

\[
\sum_{t_y \in T} \frac{\beta_m(t_y)}{(1+\tau)^{t_y-t_0}} = \sum_{t_y \in T} \frac{\mu_{z,m}(t_h)}{(1+\tau)^{t_y-t_0}} \quad \forall m \in M \quad (4.3)
\]

where, the short-term marginal value of transmission capacity \( \mu_{z,m}(t_h) \) \( (\mu_{z,m} \text{ stands for } \mu_{min,z,m} \text{ or } \mu_{max,z,m} \text{ as applicable}) \) and marginal transmission investment cost \( \beta_m(t_y) \) are dynamically related in optimal network expansion (See also Lecinq and Ilic [79], Leotard and Ilic [80]). Note that under optimal dispatch of power, short-term marginal value of transmission capacity represents the transmission shadow price.

**Result 2:** Socially optimum network expansion objective (4.1), i.e.

\[
\text{Min} \sum_m IC_m(Z_m) + E \left\{ \sum_i C_i(Z_m) - \sum_j B_j(Z_m) \right\}, \text{ and the following objective function}
\]

\[
\text{Min} \sum_m IC_m(Z_m) + E \{ TC(Z_m) \} \text{ hold the same optimum capacity expansion levels for } Z_m.
\]

**Proof:** In the presence of network limitations congestion cost is a function of branch
capacities \( Z_m \), which may be written as:

\[
TC(Z_m) = \{\text{Min } (C - B)|_{3.1.a} - \text{Min } (C - B)|_{3.1.a - 3.1.c}\} - \{\text{Min } (C - B)|_{3.1.a - 3.1.c}\}
\]

\[
= \{\text{Min } (C - B)|_{3.1.a} - \text{Min } (C - B)|_{3.1.a - 3.1.c}\} - K
\]

where, \( K \) (a constant, \( \neq f(Z_m) \)) is the short-term social cost of power dispatch ignoring network constraints.

Therefore, \( \sum_m IC_m(Z_m) + E\{TC(Z_m)\} \) and \( \sum_m IC_m(Z_m) + E\left\{\sum_i C_i(Z_m) - \sum_j B_j(Z_m)\right\} \) are convex parallel functions having same optima (minima) for \( Z_m \). Accordingly, \( \text{Min } \sum_m IC_m(Z_m) + E\{TC(Z_m)\} \) and \( \text{Min } \sum_m IC_m(Z_m) + E\left\{\sum_i C_i(Z_m) - \sum_j B_j(Z_m)\right\} \) are equivalent functions with respect to \( Z_m \) for optimal network expansion.

### 4.3 Decentralized or Merchant Transmission Approach

Decentralization of network investments is made plausible with proper congestion pricing procedures, which can be utilized in pricing network industries from power transmission to Internet services (Nasser [81]). In the context of power transmission, congestion revenue (assuming this is to be the main source of income) stands for the transportation charge for carrying power from cheap sources to generation scarce regions.

Let the following assumption be valid for decentralized network expansion analysis.

**Assumption 3:** The branches considered for decentralized expansion remains at least partially congested even at the optimum expansion level.

In assumption \( A.3 \), partial congestion implies the existence of binding constraints at least during peak load hours of the day. Then, the overloaded transmission capacity is priced.
during the congestion for the scarcity. Fixed transmission cost recovery is assumed to be done separately, which is not a subject of this chapter.

4.3.1 Decentralized Network Owner with Congestion Revenue

Consider a merchant transmission entrant building and owning a transmission line or a set of lines \( M' \subset M \). The motive of the entrant is profit maximization (the revenue collected less investment cost), and it can be formulated as given in (4.4). Investment decisions and magnitudes are decided by the optimal solution to (4.4), which in turn will reveal the expected profit for dynamic network expansion.

\[
\begin{align*}
\max \quad & \sum_{t_y, t_h \in T \cap m'} \frac{E\{\mu_{z,m}(t_h)\} Z_m(t_y)}{(1 + \tau)^{t_y-t_0}} - \sum_{t_y \in T \cap m'} \sum_{m' \in M'} \frac{IC_m(t_y)}{(1 + \tau)^{t_y-t_0}} \\
\text{s.t.} \quad & IC_m(t_y) \leq \overline{IC}_m(t_y) \quad \forall m \in M', t_y \in T \\
& \mu_{z,m}(t_h) \in \min \left\{ \sum_i C_i(g_i(t_h)) - \sum_j B_j(d_j(t_h)) \quad \forall t_h \in T, \forall m \in M' \\
& \quad + \mu_v(t_h) \left[ \sum_j d_j(t_h) - \sum_i g_i(t_h) \right] - \sum_i \mu_{\min, g,i}(t_h) \left[ g_i(t_h) - \overline{g}_i \right] + \sum_i \mu_{\max, g,i}(t_h) \left[ g_i(t_h) - \overline{g}_i \right] \\
& \quad - \sum_j \mu_{\min, d,j}(t_h) \left[ d_j(t_h) - \overline{d}_j \right] + \sum_j \mu_{\max, d,j}(t_h) \left[ d_j(t_h) - \overline{d}_j \right] \\
& \quad - \sum_m \mu_{\min, z,m}(t_h) \left[ z_m(t_h) - \overline{z}_m \right] + \sum_m \mu_{\max, z,m}(t_h) \left[ z_m(t_h) - \overline{z}_m \right] \right\}
\end{align*}
\]

The congestion revenue \( E\{TR(t_h)\} = E\{\mu_{z,m}(t_h)\} Z_m(t_y) \) in (4.4) denotes the hourly expected network revenue \( E\{NR(t_h)\} \) when network losses are neglected. Yearly investments \( IC_m(t_y) = f(Z_m(t_y)) \) decide the capacity installations \( Z_m(t_y) \), which affect
the capacity limits in dispatch algorithm, \( i.e. \ z_m \) and \( \bar{z}_m \). In the objective function (4.4) \( \mu_{z,m} \) is not known \( a \ priori \), instead it is decided by the short-term dispatch algorithm (4.4.b). Therefore, solving the above problem involves extensive future value estimates and forecasts by the merchant entrant.

The move from a centralized to a decentralized network expansion model changes the objective from \( \text{Min } \{IC + TC\} \) (result 2) to \( \text{Min } \{IC - TR\} \). Accordingly, the congestion related details \( TC \) and \( TR \) are exploited differently in distinctly different transmission management models. At the same time, the marginal value of transmission \( \mu_{z,m} \) brings different meanings to the two models. In a centralized system it is the quantity to be minimized, whereas in a decentralized system this is the quantity to be maximized. Most importantly, as shown in (4.4.b) decentralizers will not have prior knowledge of the marginal value of transmission \( \mu_{z,m} \). It is calculated by the central pool as per the dispatch model. Therefore, in the decentralized optimization (4.4) constraint (4.4.b) is externally decided while (4.4.a) is an internal constraint.

4.3.2 Decentralized Network Owner and Transmission Right Holders

Under congestion pricing principles, network congestion and its associated congestion revenue is non-zero or finite even at the optimum (social or individual) capacity. This finite mid-man revenue is hedged in some of the electricity markets using \textit{Congestion Revenue Rights} (CRR, term used by FERC in the US), in which right holders receive a rebate equivalent to the CRR value. Therefore, holding the ownership of transmission revenue rights by network users probably encourages the best possible compromise between centrally planned network and merchant network expansion.
In this context, the network owner builds the network and transfers the transmission rights to network users. Two major proposals are found for transmission rights (i) point-to-point rights (Hogan [9]), and (ii) flow-based rights (Chao et al. [82]).

The analysis in this thesis proceeds with flow-based rights. The network owners’ revenue is generated from the sales of transmission rights to network users. The purchase levels of network users are decided by their individual profit maximization objectives. This may be formulated as shown below:

$$\text{Max} \quad \sum_{t_y \in T} \sum_{m \in M'} \frac{\text{Rev}(Z_m(t_y))}{(1 + \tau)^{t_y-t_0}} - \sum_{t_y \in T} \sum_{m \in M'} \frac{IC_m(t_y)}{(1 + \tau)^{t_y-t_0}}$$

subject to:

$$IC_m(t_y) \leq IC_m(t_y) \leq IC_m(t_y) \quad \forall m \in M', t_y \in T \quad (4.5.a)$$

$$\text{Rev}(Z_m(t_y)) = \text{Max} \left\{ \sum_i \omega_i TR + SS_i, \sum_j \omega_j TR + CS_j \right\} \quad \forall m \in M' \quad (4.5.b)$$

where, $IC_m(t_y) = f(Z_m(t_y))$ and $\omega_i, \omega_j$ are the proportions of total congestion revenue held by the users. If $\sum_i \omega_i + \sum_j \omega_j = 1$, this implies network users collectively and exhaustively own the transmission rights for the congested branches. Network users decide their total willingness to pay for network rights, which in turn becomes the revenue, $\text{Rev}(Z_m(t_y))$ for the merchant transmission entrant. i.e.

$$\text{Rev}(Z_m(t_y)) = \sum_i \int p_{m,i}dZ_m + \sum_j \int p_{m,j}dZ_m$$

where, $p_{m,i}$ and $p_{m,j}$ are the marginal willingness to pay for infra-marginal capacity increase by electricity suppliers and consumers to purchase capacity rights.
\[ p_{m,i} = \frac{\partial (\omega_i TR + SS_i)}{\partial Z_m} \quad \text{and} \quad p_{m,j} = \frac{\partial (\omega_j TR + CS_j)}{\partial Z_m}. \]

In the single stage optimal planning, it is proved in § 4.4 that under the collective and exhaustive ownership assumption, (4.5) achieves the social optimal network expansion. i.e. socially optimal expansion and merchant expansions becomes identical provided capacity rights are exclusively held by network users.

### 4.4 Single Stage Optimal Network Expansion

The detailed formulations given in § 4.2 and 4.3, is applied in this section for optimal network expansion in a single time period \((t_h = t_y = 1)\) with some assumptions. For this purpose, the following assumption is made to keep the analysis simple.

**Assumption 4:** No supply and demand bounds (limits) exist within the suppliers’ and consumers’ bid prices.

Consider a system having \(I\) number of suppliers and \(J\) number of consumers connected to an electricity network having a single congested path \((m = 1)\). Under DC power flow assumptions, branch flows can be related to the supply (generation) and demand (load) with static Power Transfer Distribution Factors (PTDFs, \(H\)) as,

\[ z_m = \sum_i H_{m,(i)} g_i - \sum_j H_{m,(j)} d_j, \]

where, \(H_{m,(i)}\) and \(H_{m,(j)}\) are the PTDFs for the network node corresponding to supplier \(i\) and consumer \(j\).

#### 4.4.1 Centralized Approach

Let the branch \(m\) be congested in the single time period under consideration. The social cost optimization (4.1), with assumptions \(A.1, A.2\) and \(A.4\), becomes:
\[
\begin{align*}
\min_{Z, g_i, d_j} \alpha &= \sum_i C_i(g_i) - \sum_j B_j(d_j) + \beta Z + \mu \left[ \sum_j d_j - \sum_i g_i \right] \\
&\quad + \mu \left[ \sum_i H_{m,(i)} g_i - \sum_j H_{m,(j)} d_j - Z \right] \quad (4.6)
\end{align*}
\]

where, \(\beta, Z, \mu\) are simplified representations of \(\beta_m, Z_m, \mu_m\) with \(m = 1\) and \(t_h\) or \(t_y = 1\).

Applying the first order optimality conditions \(\partial \alpha / \partial g_i = 0\) and \(\partial \alpha / \partial d_j = 0\) on the overall social cost objective \(\alpha\), under the assumption \(\partial \alpha / \partial t_h = 0\), it can be proven that the branch capacity \(Z\) can be expressed as,

\[Z = \Phi_1 - \Phi_2 \mu,\]

where, \(\mu\) is the short-run marginal value of transmission capacity of the congested branch. Thus \(Z\) and \(\mu\) are inversely related where \(\Phi_1, \Phi_2\) are known constants (see § 4.6 for details on \(\Phi_1\) and \(\Phi_2\)). Conversely, this result may be stated as:

**Result 3:** If a branch is congested, its marginal value of transmission (or shadow price under optimal dispatch) is a decreasing function of its own capacity. i.e.

\[\mu = (\Phi_1 / \Phi_2) - (1 / \Phi_2)Z, \quad \text{and} \quad \Phi_2 > 0\]

The remaining first order optimality condition \(\partial \alpha / \partial Z\) leads to \(\mu = \beta\), which is identified earlier as **result 1**. Combining these two conditions, the socially optimal expansion capacity can be written as,

\[Z^* = \Phi_1 - \Phi_2 \beta.\]

### 4.4.2 Decentralized Approach for Merchant Transmission

For a profit maximizing transmission company the objective is derived from (4.4),
with assumptions \(A.1, A.2\) and \(A.4\), for a single stage as:

\[
\begin{align*}
\max_Z & \quad \mu Z - \beta Z \\
\text{subject to:} & \\
\mu \in \min & \left\{ \sum_i C_i(g_i) - \sum_j B_j(d_j) + \mu \left[ \sum_j d_j - \sum_i g_i \right] \\
& + \mu \left[ \sum_i H_{m,(i)} g_i - \sum_j H_{m,(j)} d_j - Z \right] \right\}
\end{align*}
\]

(4.7)

Applying first order optimality conditions \(\partial(\cdot) / \partial g_i\) and \(\partial(\cdot) / \partial d_j\) on (4.7.a) along with the assumption \(A.3\) yields, \(\mu = (\Phi_1 / \Phi_2) - (1/\Phi_2)Z\) as the value which minimizes the argument (4.7.a). Substituting this value of \(\mu\) in the (4.7) the objective function becomes,

\[
\max \{[(\Phi_1 / \Phi_2) - (1/\Phi_2)Z]Z - \beta Z\}
\]

And the optimal expansion capacity may be obtained as,

\[
Z^*_\text{tr} = (\Phi_1 - \Phi_2\beta) / 2 = \frac{1}{2} Z^*. 
\]

The optimal expansion capacity under decentralized planning by profit maximizing merchant is half of the optimal expansion capacity under centralized planning.

Result 4: Leaving the congestion revenue to profit maximizing merchant entrant (network owner) induces under-investment in the network capacity below the social optimal.

It is worth noting that the profit function of merchant investor (4.7) is concave and becomes zero at the socially optimal point \(Z^*\).

4.4.3 Perfect Competition for Merchant Investments

As seen above, if left to the interest to monopolistic merchant transmission, there would be under-investment in network expansion. This is expected to lead to capacity
expansion to reach $Z^*_tr$ for $Max \{TR-IC\}$. But, introducing competition for network capacity expansion can avert this situation with a proper regulatory framework.

For the purpose of analyzing this phenomenon let us assume that a single (incumbent) monopolistic network owner (index-1) holds all the profit maximizing transmission capacity $Z_1$ ($= Z^*_tr(1) = \frac{1}{2} Z^* \text{ } \text{ } \text{§ 4.4.2}$). A second merchant entrant entering into the transmission market would seek to maximize its own profit as:

$$\begin{align*}
Max & \mu(Z-Z_1)-\beta(Z-Z_1) \\
& (4.8)
\end{align*}$$

with $\mu = (\Phi_1 / \Phi_2) - (1 / \Phi_2) Z$ this results in a new concave profit function having its optimum (i.e. $\partial(\mu)/\partial Z = 0$) at, $\frac{1}{2}(\Phi_1 - \Phi_2 \beta + Z_1)$ or $\frac{1}{2}(Z^* + Z_1)$.

Accordingly, merchant 2 (the new entrant) would select its optimum at,

$$Z^*_tr(2) = \frac{1}{2}(Z^* + Z^*_tr(1)) = \frac{1}{2} \left(1 - \frac{1}{2^2}\right)Z^* = \frac{3}{4} Z^*.$$ 

Provided that perfect competition exists and there is no entry barrier, the analysis can be extended to the $n^{th}$ merchant to show that the optimum capacity for the $n^{th}$ merchant will occur at,

$$Z^*_tr(n) = \frac{1}{2}(Z^* + Z^*_tr(n-1)) = \left(1 - \frac{1}{2^n}\right)Z^*$$

Therefore, when $n \rightarrow \infty$ the network capacity level reaches social optimal $Z^*$.

**Result 5**: Socially optimal network capacity expansion is plausible under merchant transmission provided perfect competition exists for new merchants to enter the network expansion market.
4.4.4 Merchant Network Owner and Transmission Right Holders

4.4.4.1 Exclusive Right Holders

In the case of transmission right holders holding exclusive rights, network owners’ profit maximization function is deduced from (4.5) for a single stage as given in (4.9). In this arrangement the total transmission rights generated from capacity installation is sold back to network users, and the users decide their willingness to pay based on their net economic gains. The income from exclusive transmission right sales for network users represents the network owner’s revenue while investment costs constitute the expenses. This formulation with assumptions $A.1, A.2$ and $A.4$ becomes:

$$\max Z \int \frac{\partial}{\partial Z} \left[ \sum_i (\omega_i TR + SS_i) \right] dZ + \int \frac{\partial}{\partial Z} \left[ \sum_j (\omega_j TR + CS_j) \right] dZ - \beta Z \quad (4.9)$$

With exclusive rights allocation, i.e. $\sum_i \omega_i + \sum_j \omega_j = 1$ the function reduces to,

$$\max Z \int \frac{\partial}{\partial Z} \left[ TR + \sum_i SS_i + \sum_j CS_j \right] dZ - \beta Z ,$$

where, $TR + \sum_i SS_i + \sum_j CS_j = \sum_j B_j(d_j) - \sum_i C_i(g_i)$ represents the total social welfare.

Hence,

$$\max Z \int \frac{\partial}{\partial Z} \left[ \sum_j B_j - \sum_i C_i \right] dZ - \beta Z = \max Z \int \mu dZ - \beta Z$$

where $\mu = (\Phi_1 / \Phi_2) - (1 / \Phi_2)Z$

This produces a concave profit function having its optimum value at,

$$Z_{cr}^* = \Phi_1 - \Phi_2 \beta = Z^* \quad (\text{which is again the social optimal}).$$
Result 6: Socially optimal network capacity expansion is plausible under merchant transmission provided transmission rights are exclusively and exhaustively held by network users (suppliers and consumers).

4.4.4.2 Single User Holding Exclusive Rights

If a single network user, supplier $i$ or consumer $j$ holds the transmission rights exclusively for the congested branch, then the profit maximizing function (4.5) with assumptions $A.1$, $A.2$ and $A.4$ reduces to (4.10.a) and (4.10. b):

For a producer holder, $i$

\[
\max \int \frac{\partial}{\partial Z} [TR(Z)] dZ + \int \frac{\partial}{\partial Z} [SS_i(Z)] dZ - \beta Z 
\]

(4.10.a)

For a consumer holder, $j$

\[
\max \int \frac{\partial}{\partial Z} [TR(Z)] dZ + \int \frac{\partial}{\partial Z} [CS_j(Z)] dZ - \beta Z
\]

(4.10.b)

With $\mu = (\Phi_1 / \Phi_2) - (1 / \Phi_2)Z$ (assumption $A.3$), these give

\[
\max (\Phi_1 / \Phi_2)Z - (1 / \Phi_2)Z^2 + SS_i(Z) - \beta Z \quad \text{and}
\]

\[
\max (\Phi_1 / \Phi_2)Z - (1 / \Phi_2)Z^2 + CS_j(Z) - \beta Z \, ,
\]

for the producer holder and the consumer holder respectively.

These can be reduced to the following differential equations, which determine the optimal capacity $Z^*_s$ and $Z^*_c$:

\[
\frac{\Phi_2}{2} \frac{\partial SS_i}{\partial Z} - Z + \frac{(\Phi_1 - \Phi_2\beta)}{2} = 0 \quad \text{for } Z^*_s
\]

\[
\frac{\Phi_2}{2} \frac{\partial CS_j}{\partial Z} - Z + \frac{(\Phi_1 - \Phi_2\beta)}{2} = 0 \quad \text{for } Z^*_c
\]
This shows that the optimal capacity level when a single user holds transmission rights is different from the monopolistic optimum level \( Z^*_e = (\Phi_1 - \Phi_2 \beta) / 2 \). The optimal capacity is dependent on the particular user’s surplus sensitivity (i.e. \( \partial SS_i / \partial Z \) or \( \partial CS_j / \partial Z \)) over capacity expansion.

### 4.4.4.3 Competitive Participation of Network Users

Competitive participation of network users in the expansion process may be represented as competitive buying of revenue rights. Alternatively it can be treated as expansion of the network by the users and keeping the generated revenue rights to themselves. This market arrangement leads to the following objective functions:

For a producer holder, \( i \)

\[
\text{Max }_{Z_i} \int \frac{\partial}{\partial Z_i} [TR(Z)]dZ_i + \int \frac{\partial}{\partial Z_i} [SS_i(Z)]dZ_i - \beta Z_i \tag{4.11.a}
\]

For a consumer holder, \( j \)

\[
\text{Max }_{Z_j} \int \frac{\partial}{\partial Z_j} [TR(Z)]dZ_j + \int \frac{\partial}{\partial Z_j} [CS_j(Z)]dZ_j - \beta Z_j \tag{4.11.b}
\]

where, \( Z_i \) and \( Z_j \) represents the individual expansion levels, and \( Z = \sum_i Z_i + \sum_j Z_j \).

With \( \mu = (\Phi_1 / \Phi_2) - (1 / \Phi_2)Z \) (with assumption A.3), it gives

\[
\text{Max }_{Z_i} SS_i(Z) + Z_i[\Phi_1/\Phi_2-1/\Phi_2 Z] - \beta Z_i \quad \text{for supplier } i
\]

\[
\text{Max }_{Z_j} CS_j(Z) + Z_j[\Phi_1/\Phi_2-1/\Phi_2 Z] - \beta Z_j \quad \text{for consumer } j.
\]

Using first order optimality conditions, these become:
If all the network users $i$, $j$ act as per (4.11), the set of first order optimality equations (4.12) forms the *Nash Equilibrium* for profit maximizing capacity expansion.

**Result 7:** The equilibrium capacity expansions under competitive participation by network users collectively form the social optimal capacity expansion.

**Proof:** Consider the summation of the set of equations (4.12.a) and (4.12.b),

$$\frac{\partial SS_i(Z)}{\partial Z_i} - \frac{Z_i}{\Phi_2} + \frac{\Phi_1}{\Phi_2} \cdot \frac{Z}{\Phi_2} - \beta = 0 \quad \forall i \in I \quad (4.12.a)$$

$$\frac{\partial CS_j(Z)}{\partial Z_j} - \frac{Z_j}{\Phi_2} + \frac{\Phi_1}{\Phi_2} \cdot \frac{Z}{\Phi_2} - \beta = 0 \quad \forall j \in J \quad (4.12.b)$$

With the assumption $A.3$, it can be expressed that the decrease in marginal value of capacity is $Z/\Phi_2$ and this value is equal to the total change in marginal surpluses due to individual expansions i.e. $\sum_i \frac{\partial SS_i}{\partial Z_i} + \sum_j \frac{\partial CS_j}{\partial Z_j} = 0$ provided $Z = \sum_i Z_i + \sum_j Z_j$.

Accordingly, this can be simplified to $Z = \Phi_1 - \Phi_2 \beta$, where $Z = \sum_i Z_i^c + \sum_j Z_j^c$, which represents the *Nash Equilibrium* of the network expansion by network users. At the same time this is equal to the social optimal $Z^*$ ($= \Phi_1 - \Phi_2 \beta$).

It is clearly seen from the above analysis that social optimal expansion capacity $Z^*$ is achievable under various market arrangements:

- Central planning considering social objectives (§ 4.4.1) leads to the socially optimum value $Z^*$. 

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· Merchant transmission with perfect competition among merchant transmission providers (§ 4.4.3) also leads to the social optimum value.

· Merchant transmission with exclusive right ownership by network users (§ 4.4.4.1 and 4.4.4.3) also leads to socially optimal transmission expansion.

Thus, while different market structures on the surface seem to lead to different optimal expansion capacities, each of these market structures in fact will lead to the socially optimal expansion provided appropriate measures could be enforced to facilitate the appropriate competition among the Transcos or merchants. Ensuring such perfect competition, however, may not be easy to achieve. Therefore more realistically in all above market conditions more practical capacity expansions are likely to lie between monopolistic situation $Z_r^*$ and social optimum $Z^*$. It is plausible to direct the expansion towards the desired end in this range by adopting suitable pragmatic investment management methods.

4.5 Loop Flows and Network Externalities

The presence of multiple congested branches complicates the theoretical analysis and often takes the problem beyond the scope of traditional economics. This introduces the effects of network externalities, which often plays a significant role in actual power networks. The effects of externalities are usually negative – although they may be negative or positive in power networks (Baldick and Khan [5]). For both positive and negative network externalities "parallel flows" or "loop flow" is the main phenomenon to contend with.

The optimality investigations in § 4.4 for a system with a single congested branch
can be extended to a system with a set \( M' \) of congested branches, and include network externalities in the analysis provided assumptions \( A.1 - A.4 \) remain valid. Accordingly, the symbols used in § 4.4 need to be changed to the corresponding vectors or matrix as given in Table 4.1 for the following analysis.

<table>
<thead>
<tr>
<th>Table 4.1: Change of Notations for Multiple Congested Branches</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single Congested Branch</strong></td>
</tr>
<tr>
<td>( Z, \mu, \beta )</td>
</tr>
<tr>
<td>( \Phi_1 )</td>
</tr>
<tr>
<td>( \Phi_2 )</td>
</tr>
<tr>
<td>( (\Phi_1/\Phi_2) )</td>
</tr>
<tr>
<td>( (1/\Phi_2) )</td>
</tr>
</tbody>
</table>

4.5.1 Centralized Approach

For the single time period social cost optimization problem (4.13) is derived from (4.1) with assumptions \( A.1, A.2 \) and \( A.4 \). This would be similar to (4.6), but consists of a set of congested branches with capacities \( Z \in \mathbb{R}^{M'} \).

\[
\text{Min} \quad \alpha = \sum_i C_i(g_i) - \sum_j B_j(d_j) + \mu_i \left[ \sum_j d_j - \sum_i g_i \right]
+ \beta^T Z + \mu^T \left[ H g - H d - Z \right]
\]  

(4.13)

Applying the first order optimality conditions \( \partial \alpha / \partial g_i \), and \( \partial \alpha / \partial d_j \) on the overall social cost objective \( \alpha \), and the assumption \( A.3 \), it can be proven that congested branch capacities related to their marginal values as \( Z = \Phi_1 - \Phi_2 \mu \), where \( \Phi_1 \) and \( \Phi_2 \) values for a general network configuration is given in § 4.6. It is notable that \( \Phi_2 \) is a diagonally dominated symmetrical matrix.

The expansion capacity of a congested branch \( m' \) can be expressed as (4.14),
\[ Z_{m'} = \Phi_1(m') - \sum_{m \in M'} \Phi_2(m', m) \mu_m \quad \forall m' \in M' \quad (4.14) \]

In \( \Phi_2 \) the self-branch coefficient \( \Phi_2(m', m') \) is dominant over the other elements (mutual elements) in a particular row. The mutual elements \( \Phi_2(m', m) \) where \( m \neq m' \) represent the inter-relationship between congested branches \( m' \) and \( m \). The remaining first order optimality conditions \( \partial \alpha / \partial Z \) leads to \( \mu = \beta \) (which is result 1).

Hence, the social optimal capacity become,

\[ Z^* = \Phi_1 - \Phi_2 . \beta , \]

which can also be written as:

\[ Z'^*_m = \Phi_1(m') - \sum_{m \in M'} \Phi_2(m', m) \beta_m \quad \forall m' \in M' \quad (4.15) \]

The socially optimal branch capacities depend on network parameters, network users details (i.e. \( \Phi_1(m') \) and \( \Phi_2(m', m') \)), self and other congested branches marginal investment costs (i.e. \( \beta_m' \) and \( \beta_m \)).

4.5.2 Merchant Transmission Approach

In order to obtain the profit maximization network capacities, \( Z = \Phi_1 - \Phi_2 . \mu \) must be solved for congestion prices \( \mu \) as functions of congested branch capacities (i.e. \( \mu = f(Z) \)). This yields \( \mu = \Lambda_1 - \Lambda_2 . Z \) under the assumption 4.3, where \( \Lambda_1 = \Phi_2^{-1} \Phi_1 \) and \( \Lambda_2 = \Phi_2^{-1} \) (symmetrical). Then, \( \text{Max } \mu . Z - \beta . Z \) will produce the profit maximizing network capacity levels for congested branches.

\[
\text{Max } Z \mu . Z - \beta . Z \Rightarrow \text{Max } Z (\Lambda_1 - \Lambda_2 . Z) . Z - \beta . Z
\]
For optimality, \( \Lambda_1 - 2\Lambda_2 Z - \beta = 0 \Rightarrow Z_{tr}^* = (1/2)\Lambda_2^{-1}(\Lambda_1 - \beta) \). Substituting the values of \( \Lambda_1 \) and \( \Lambda_2 \), this yield, \[
Z_{tr}^* = (1/2)(\Phi_1 - \Phi_2 \beta).
\]

This confirms result 4, even under the presence of network externality. That is, the optimal expansion capacity from the perspective of a merchant transmission owner is one half of the socially optimal value.

The diagonal elements of \( \Lambda_2 \) are positive and confirms result 3, i.e. marginal value of transmission capacities are decreasing function of their own capacities. Whilst, off-diagonal elements can be positive or negative depending on the nature of network externality. Positive values of \( \Lambda_2 (m', m) \) indicate negative externality, where capacity increase of branch \( m \) will decrease the marginal value of branch \( m' \). Negative values of \( \Lambda_2 (m', m) \) indicate positive externality, where capacity increase of branch \( m \) will increase the marginal value of branch \( m' \).

The rest of the analysis presented in § 4.4.4.1 - § 4.4.4.3 for a single congested branch can be extended to multi-congested branches without any change except for the change in notations as, \( Z \rightarrow Z, \mu \rightarrow \mu \) and \( \beta \rightarrow \beta \).

### 4.6 Calculation of \( \Phi_1 \) and \( \Phi_2 \)

The following examples in § 4.7 require the computation of \( \Phi_1 \) and \( \Phi_2 \) which are derived based on the assumption that power dispatches are completely based on the bid prices submitted by suppliers (\( MC_i = b_i + m_i g_i \)) and consumers (\( MB_j = b_j - m_j d_j \)), and that no limits exist in the amount of supply or demand within bid price limits (assumption
A.4).

With only supply side bidding considering the demand inelastic:

For this case, the Lagrange function of the single-sided power dispatch problem becomes,

\[ \Omega(g_i, \mu_e, \mu_m) = \sum_i C_i(g_i) + \mu_e \left[ D - \sum_i g_i \right] \]

\[ + \sum_{m \in M'} \mu_m \left[ \sum_i H_{m,(i)} g_i - \sum_j H_{m,(j)} d_j - Z_m \right] \]  \hspace{1cm} (4.16)

where, \[ C_i(g_i) = b_i g_i + (1/2) m_i g_i^2 \] and \[ D = \sum_j d_j \] is the total demand.

\[ M' \] - represents the set of congested branches (\[ M' \subset M \])

Let the assumption A.3 be valid for all branches in \[ M' \].

The first order optimal condition \[ \frac{\partial \Omega}{\partial g_i} = 0 \] of (4.16) yields,

\[ b_i + m_i g_i - \mu_e + \sum_{m \in M'} \mu_m H_{m,(i)} = 0 \]

\[ \Rightarrow g_i = \left( 1 / m_i \right) \left[ \mu_e - b_i + \sum_{m \in M'} \mu_m H_{m,(i)} \right] \]

From the total power balance, \[ D = \sum_j d_j = \sum_i g_i \], it can be obtained that,

\[ D = \left[ \mu_e \sum_i (1 / m_i) - \sum_i b_i / m_i - \sum_{m \in M'} \mu_m \sum_i H_{m,(i)} / m_i \right] \]

This can be simplified to obtain \[ \mu_e \] as follows,

\[ \mu_e = \left[ D + \sum_i b_i / m_i + \sum_{m \in M'} \mu_m \sum_i H_{m,(i)} / m_i \right] / \sum_i (1 / m_i) \]

With the assumption A.3, the power flow of a congested branch (\[ m' \]) is equal to its capacity, \( i.e., \)

\[ Z_{m'} = \sum_i H_{m',(i)} g_i - \sum_j H_{m',(j)} d_j \]

\[ Z_{m'} = \mu_e \sum_i H_{m,(i)} / m_i - \sum_i H_{m,(i)} b_i / m_i - \sum_{m \in M'} \mu_m \sum_i H_{m,(i)} H_{m,(i)} / m_i \]

\[ - \sum_j H_{m',(i)} d_j \]
\[ Z_{m'} = \left[ \frac{D + \sum_i b_i / m_i}{\sum_i 1 / m_i} \right] \sum_i H_{m',(i)} / m_i - \sum_i H_{m',(i)} b_i / m_i - \sum_j H_{m',(j)} d_j + \sum_{m \in M'} \mu_m \sum_i H_{m,(i)} H_{m',(i)} / m_i \]

Accordingly, it can be written as, \( Z = \Phi_1 - \Phi_2 \mu \)

where,
\[ \Phi_1 \in \mathbb{R}^{M'} - \text{a column vector, and } \Phi_1 \in \mathbb{R}^{M'} \times \mathbb{R}^{M'} - \text{a two dimensional matrix} \]

\[ \Phi_1(m') = \left[ \frac{D + \sum_i b_i / m_i}{\sum_i 1 / m_i} \right] \sum_i H_{m',(i)} / m_i - \sum_i H_{m',(i)} b_i / m_i - \sum_j H_{m',(j)} d_j \]

\[ \Phi_2(m', m) = \sum_i H_{m',(i)} H_{m,(i)} / m_i \left[ \sum_i H_{m',(i)} / m_i \right] \left[ \sum_i H_{m,(i)} / m_i \right] \sum_i 1 / m_i \]

\( m, m' \) stand for two branches in the set \( M' \) and \( m_i, m_j \) designate the slopes of marginal cost and benefit curves.

With supply and demand side bidding:

For this case, the Lagrange function of the double-sided power dispatch problem becomes,

\[ \Omega(g_i, d_j, \mu_e, \mu_m) = \sum_i C_i(g_i) - \sum_j B_j(d_j) + \mu_e \left[ \sum_j d_j - \sum_i g_i \right] \]

\[ + \sum_{m \in M'} \mu_m \left[ \sum_i H_{m,(i)} g_i - \sum_j H_{m,(j)} d_j - Z_m \right] \quad (4.17) \]

where, \( C_i(g_i) = b_i g_i + (1/2) m_i g_i^2 \) and \( B_j(d_j) = b_j d_j - (1/2) m_j d_j^2 \).

\( M' \) - represents the set of congested branches (\( M' \subset M \) )

Let the assumption A.3 is valid for all branches in \( M' \).

The first order optimal condition \( \partial \Omega / \partial g_i = 0 \) of (4.17) yields,
\[ b_i + m_i g_i - \mu_e + \sum_{m \in M'} \mu_m H_{m,(i)} = 0 \quad \Rightarrow \]
\[ g_i = \left( \frac{1}{m_i} \right) \left[ \mu_e - b_i - \sum_{m \in M'} \mu_m H_{m,(i)} \right] \]

The first order optimal condition \( \partial(.) / \partial d_j = 0 \) of (4.17) yields,
\[ -b_j + m_j d_j + \mu_e - \sum_{m \in M'} \mu_m H_{m,(j)} = 0 \quad \Rightarrow \]
\[ d_j = \left( \frac{1}{m_j} \right) \left[ -\mu_e + b_j + \sum_{m \in M'} \mu_m H_{m,(j)} \right] \]

From the total power balance, \( \sum_j d_j = \sum_i g_i \), it can be obtained that,
\[
\left[ \mu_e \sum_i \left( \frac{1}{m_i} \right) - \sum_i b_i / m_i - \sum_{m \in M'} \mu_m \sum_i H_{m,(i)} / m_i \right] = \\
\left[ -\mu_e \sum_j \left( \frac{1}{m_j} \right) + \sum_j b_j / m_j + \sum_{m \in M'} \mu_m \sum_j H_{m,(j)} / m_j \right]
\]

This can be simplified to obtain \( \mu_e \) as follows,
\[ \mu_e = \frac{\sum_i b_i / m_i + \sum_j b_j / m_j + \sum_{m \in M'} \mu_m \left[ \sum_i H_{m,(i)} / m_i + \sum_j H_{m,(j)} / m_j \right]}{\sum_i 1 / m_i + \sum_j 1 / m_j} \]

With the assumption \( A.3 \), the power flow of a congested branch ( \( m' \) ) is equal to its capacity, i.e.,
\[ Z_{m'} = \frac{\left[ \sum_i b_i / m_i + \sum_j b_j / m_j + \sum_{m \in M'} \mu_m \left[ \sum_i H_{m,(i)} / m_i + \sum_j H_{m,(j)} / m_j \right] \right]}{\sum_i 1 / m_i + \sum_j 1 / m_j} \]
\[ \times \left[ \sum_i H_{m',(i)} / m_i + \sum_j H_{m',(j)} / m_j \right] \]
\[ - \sum_i H_{m',(i)} b_i / m_i - \sum_j H_{m',(j)} b_j / m_j \]
\[ - \sum_{m \in M'} \mu_m \sum_i H_{m',(i)} H_{m,(i)} / m_i - \sum_{m \in M'} \mu_m \sum_j H_{m',(j)} H_{m,(j)} / m_j \]

Accordingly, it can be written as, \( Z = \Phi_1 - \Phi_2 \mu \)

where,
\( \Phi_1 \in \mathbb{R}^{M'} \) - a column vector, and \( \Phi_2 \in \mathbb{R}^{M'} \times \mathbb{R}^{M'} \) - a two dimensional matrix

\[
\Phi_1(m') = \left[ \sum_i H_{m',(i)} / m_i + \sum_j H_{m',(j)} / m_j \right] \left[ \frac{\sum_j b_j / m_j + \sum_i b_i / m_i}{\sum_j 1 / m_j + \sum_i 1 / m_i} \right]
- \sum_i H_{m',(i)} b_i / m_i - \sum_j H_{m',(j)} b_j / m_j
\]

\[
\Phi_2(m', m) = \left[ \sum_i H_{m',(i)} H_{m,(i)} / m_i + \sum_j H_{m',(j)} H_{m,(j)} / m_j \right] \left[ \frac{\sum_i H_{m,(i)} / m_i + \sum_j H_{m,(j)} / m_j}{\sum_i 1 / m_i + \sum_j 1 / m_j} \right]
\]

\( m, m' \) stand for two branches in the set \( M' \) and \( m_i, m_j \) designate the slopes of marginal cost and benefit curves.
4.7 Illustrative Examples

The findings of the analysis presented above are illustrated in this section using simple examples for a clear demonstration of various results.

4.7.1 2-Node System with Demand Elasticity

Consider a simple network having a single supplier and a single buyer connected by a constrained transmission link. The economic details are shown in Fig. 4.1.

The network expansion objectives, and their optimal values from social (central planning) and individual (network owner, supplier and consumer) perspectives are given in Table 4.2. In the case of individuals (or merchant) the network owner or supplier or consumer is assumed to hold the exclusive transmission rights (i.e. 100% of TR). If competitive network expansion is expected from supplier and consumer, the Nash Equilibrium is also shown, which collectively leads to social optimal.
TABLE 4.2: 2-NODE ELASTIC DEMAND OPTIMAL EXPANSION

<table>
<thead>
<tr>
<th>Objective</th>
<th>Optimum†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social</td>
<td>Min (TC + IC) ( Z^* = (b_2 - b_1 - \beta)/(m_1 + m_2) )</td>
</tr>
<tr>
<td>Network Owner</td>
<td>Max (TR - IC) ( Z^*_n = (1/2)(b_2 - b_1 - \beta)/(m_1 + m_2) )</td>
</tr>
<tr>
<td>Supplier</td>
<td>Max (TR + SS - IC) ( Z^*_s = (b_2 - b_1 - \beta)/(m_1 + 2m_2) )</td>
</tr>
<tr>
<td>Consumer</td>
<td>Max (TR + CS - IC) ( Z^*_c = (b_2 - b_1 - \beta)/(2m_1 + m_2) )</td>
</tr>
<tr>
<td>Nash Equilibrium</td>
<td>Supplier Max (( \omega )TR + SS - IC) ( Z^*_s = m_1(b_2 - b_1 - \beta)/(m_1 + m_2)^2 )</td>
</tr>
<tr>
<td>Consumer</td>
<td>Max (( \omega )TR + CS - IC) ( Z^*_c = m_2(b_2 - b_1 - \beta)/(m_1 + m_2)^2 )</td>
</tr>
</tbody>
</table>

† The optimum values can be obtained by following the procedure given in § 4.4 and 4.6

These optimal values are graphically illustrated in Fig.4.2, for a set of data shown in the figure.

As proven in § 4.4.3, when monopoly exists, the incumbent transmission owner will wish to maintain network capacity at \( Z_1 = Z^*_n \). The possible alleviation of network monopoly was shown in § 4.4.3 and § 4.4.4 with introduction of the new entrants from independent transmission companies or from network users. This situation is illustrated for this 2-Node example in Fig. 4.3, where the optimal capacities for successive new entrants move towards the socially optimal capacity.
4.7.2 2-Node System without Demand Elasticity

The above analysis is valid for inelastic demands as well. The system and its pertinent parameters are shown in Fig. 4.4.

\[ MC_1 = b_1 + m_1 g_1 \]
\[ MC_2 = b_2 + m_2 g_2 \]

The social and individual objectives and their optimal capacities are shown in Table 4.3. The competitive expansion levels forming the Nash Equilibrium are also shown in Table 4.3. A graphical illustration is given in Fig. 4.5 for the system data indicated in the same figure.
TABLE 4.3: 2-NODE INELASTIC DEMAND OPTIMAL EXPANSION

<table>
<thead>
<tr>
<th></th>
<th>Objective</th>
<th>Optimum†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social</td>
<td>Min {TC + IC}</td>
<td>( Z^* = (b_2 - b_1 + m_2 d - \beta)/(m_1 + m_2) )</td>
</tr>
<tr>
<td>Network Owner</td>
<td>Max {TR – IC}</td>
<td>( Z^*_w = (1/2)(b_2 - b_1 + m_2 d - \beta)/(m_1 + 2 m_2) )</td>
</tr>
<tr>
<td>Supplier-1</td>
<td>Max {TR + SS_i – IC}</td>
<td>( Z^*_i = (b_2 - b_1 + m_2 d - \beta)/(m_1 + 2 m_2) )</td>
</tr>
<tr>
<td>Supplier-2</td>
<td>Max {TR + SS_j – IC}</td>
<td>( Z^*_j = (b_2 - b_1 - \beta)/(2 m_1 + m_2) )</td>
</tr>
<tr>
<td>Nash Equilibrium Supplier-1</td>
<td>Max {\omega_1 TR + SS_1 – IC}</td>
<td>( Z^*_{i1} = m_i (b_2 - b_1 + m_2 d - \beta)/(m_1 + m_2)^2 )</td>
</tr>
<tr>
<td>Supplier-2</td>
<td>Max {\omega_2 TR + SS_2 – IC}</td>
<td>( Z^*_{i2} = m_j (b_2 - b_1 - m_2 d - \beta)/(m_1 + m_2)^2 )</td>
</tr>
<tr>
<td>Consumer</td>
<td>Max {\omega_3 TR – p_3 d – IC}</td>
<td>( Z^*_c = m_2 d / (m_1 + m_2) )</td>
</tr>
</tbody>
</table>

† The optimum values can be obtained by following the procedure given in § 4.4 and 4.6

It can be seen from Fig. 4.5 that the optimal capacity expansion while supplier-2 holds transmission rights \( Z^*_{i2} \) remain significantly less than even the monopolistic level \( Z^*_{w} \). Therefore, for its self-interest supplier-2 may either enforce such a capacity expansion or influence the regulator or network owner to do so. Therefore, necessary frameworks should be in place to avoid such a detrimental network expansion alternatives under merchant investments.

The analysis so far assumed that supplier and consumer marginal cost (\( MC \)) and benefit (\( MB \)) represent their true marginal values and are not influenced by strategic motives. The strategic behavior of supply and demand bidding certainly affects the optimal capacity values. Impact of simple strategic behavior can be analyzed for a simple system like 2-node example by placing cheap and expensive generation at both ends of Fig.4.4. The strategic effects may be represented by the manipulation of the bidding intercept \( b \). It is seen that all optimal levels will increase with increase in \( b_2 \) and decreases with increase in \( b_1 \). Hence, strategic behavior by cheap generation leads to under-incentive for transmission expansion and the strategic bids by expensive generation
lead to *over-incentive* the transmission capacity under all four options in Table 4.3.

![Graph](image)

**Figure 4.5: Social and Individual Objectives (Inelastic Demand)**

### 4.7.3 3-Node Network Example

#### 4.7.3.1 3-Node Example-1 (single congested branch)

Consider the 3-node system shown in Fig. 4.6, with equal branch reactances and a single congested branch.

![Diagram](image)

**Figure 4.6: 3-Node Example-1**
Optimal capacity expansion of the system under various market models, from the perspectives of various network planners, e.g. social, network owner, supplier-1, supplier-2 or consumer, can be expressed as:

\[ Z^*_e = \frac{k_n + (\Phi_1 / \Phi_2) - \beta}{k_d + 2 / \Phi_2} \]

where,

\[ \Phi_1 = \frac{-b_1(2m_2 + m_3) + b_2(m_3 - m_1) + b_3(m_1 + 2m_2)}{3(m_1m_2 + m_2m_m + m_3m_1)} \quad \text{and} \quad \Phi_2 = \frac{m_1 + 4m_2 + m_3}{9(m_1m_2 + m_2m_m + m_3m_1)} \]

And, \( k_n , \ k_d \) are another two characteristic quantities as defined below for different optimal values, which are independent of branch capacity or cost details.

**For social optimal (\( Z^*_s \))**

\[ k_n = 0 \quad \text{and} \quad k_d = -1 / \Phi_2 \]

**For network owner (\( Z^*_n \))**

\[ k_n = 0 \quad \text{and} \quad k_d = 0 \]

**For supplier-1 (\( Z^*_s1 \))**

\[ k_n = \frac{3m_1(-b_1 + 2b_2 - b_3)(2m_2 + m_3)}{(m_1 + 4m_2 + m_3)^2} \quad \text{and} \quad k_d = \frac{-9m_1(2m_2 + m_3)^2}{(m_1 + 4m_2 + m_3)^2} \]

**For supplier-2 (\( Z^*_s2 \))**

\[ k_n = \frac{3m_2(m_1 - m_3)(m_1(b_2 - b_1) + 2m_2(b_2 - b_1) + m_3(b_2 - b_1))}{(m_1 + 4m_2 + m_3)^2} \quad \text{and} \quad k_d = \frac{-9m_2(m_1 - m_3)^2}{(m_1 + 4m_2 + m_3)^2} \]

**For consumer (\( Z^*_c \))**

\[ k_n = \frac{3m_3(m_1 + 2m_2)(b_1 - 2b_2 + b_3)}{(m_1 + 4m_2 + m_3)^2} \quad \text{and} \quad k_d = \frac{-9m_3(m_1 + 2m_2)^2}{(m_1 + 4m_2 + m_3)^2} \]
The equilibrium capacity expansions under competitive network expansion are as follows.

**Nash Equilibrium:**

\[
Z_{s1}^e = \frac{m_1(2m_2 + m_3)(3m_3(b_2 - b_1) + 3m_2(b_3 - b_1) - \beta(2m_2 + m_3))}{9(m_1m_2 + m_2m_3 + m_3m_1)^2}
\]

\[
Z_{s2}^e = \frac{m_2(m_3 - m_1)(3m_1(b_2 - b_3) + 3m_2(b_3 - b_1) - \beta(m_3 - m_1))}{9(m_1m_2 + m_2m_3 + m_3m_1)^2}
\]

\[
Z_{c}^e = \frac{m_3(m_1 + 2m_2)(3m_1(b_3 - b_2) + 3m_2(b_3 - b_1) - \beta(m_1 + 2m_2))}{9(m_1m_2 + m_2m_3 + m_3m_1)^2}
\]

and \( Z_{s1}^e + Z_{s2}^e + Z_{s3}^e = \Phi_1 - \Phi_2 \beta = Z^* \) which is the social optimum.

Fig. 4.7 illustrates the optimum capacities for the data given in the same figure.

![Figure 4.7: Social and Individual Objectives (3-Node Example-1)](image)

### 4.7.3.2 3-Node Example-2 (two congested branches)

The effects of network externalities may be illustrated considering the following 3-node example with two congested branches as shown in Fig. 4.8.
The optimal network expansion capacities corresponding to social, network owner, supplier-1, supplier-2 or consumer planner, can be derived as shown in Table 4.4.

### TABLE 4.4: 3-NODE ELASTIC DEMAND OPTIMAL EXPANSION (EXAMPLE-2)

<table>
<thead>
<tr>
<th>Objective</th>
<th>Optimum†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social</td>
<td>Min {TC + IC} ( Z' = \Phi_1 - \Phi_2 \beta )</td>
</tr>
<tr>
<td>Network Owner</td>
<td>Max {TR - IC} ( Z'_{tr} = (1/2)(\Phi_1 - \Phi_2 \beta) )</td>
</tr>
<tr>
<td>Supplier-1</td>
<td>Max {TR + SS_1 - IC} ( Z'_{s1} = \Phi_1^{(1)} - \Phi_2^{(1)} \beta )</td>
</tr>
<tr>
<td>Supplier-2</td>
<td>Max {TR + SS_2 - IC} ( Z'_{s2} = \Phi_1^{(2)} - \Phi_2^{(2)} \beta )</td>
</tr>
<tr>
<td>Consumer</td>
<td>Max {TR + CS - IC} ( Z'_{c} = \Phi_1^{(c)} - \Phi_2^{(c)} \beta )</td>
</tr>
</tbody>
</table>

Further, it can be proven that the Nash equilibrium formed by the individual network users in competitive network expansion reaches social optimum, i.e.

\[ Z'_{s1} + Z'_{s2} + Z'_{c} = \Phi_1 - \Phi_2 \beta = Z^* . \]

For the data shown in Fig. 4.8, branch capacity – marginal value relations (i.e. \( Z = \Phi_1 - \Phi_2 \mu \) and \( \mu = \Lambda_1 - \Lambda_2 \)) for this example are,
The objective function landscapes are shown in Figs. 4.9 and 4.10 confirming the conclusions drawn so far. The socially optimal and network owners optimum regions are shown in Fig. 4.9. Suppliers and Consumers profit maximizing network expansion zones are shown in Fig. 4.10.

\[ \Phi_1 = \begin{bmatrix} 128.2 \\ 87.2 \end{bmatrix} ; \quad \Phi_2 = \begin{bmatrix} 3.08 \\ 1.03 \\ 2.56 \\ 20.0 \end{bmatrix} ; \quad \Lambda_1 = \begin{bmatrix} 34.9 \\ 20.0 \end{bmatrix} ; \quad \Lambda_2 = \begin{bmatrix} 0.3752 \\ -0.1509 \\ 0.4514 \end{bmatrix} \]

**TABLE 4.5: VALUES OF \( \Phi_1 \) AND \( \Phi_2 \)**

\[
\Phi_1 = (1/3)k \begin{bmatrix} m_1(b_1 - b_2) + 2m_2(b_1 - b_2) + m_3(b_1 - b_2) \\ 2m_1(b_1 - b_2) + m_2(b_1 - b_2) + m_3(b_1 - b_2) \end{bmatrix} \\
\Phi_2 = (1/9)k \begin{bmatrix} m_1 + 4m_2 + m_3 \\ 2m_1 + 2m_2 - m_3 \end{bmatrix} ; \quad k = 1/(m_1m_2 + m_2m_3 + m_3m_1)
\]

\[
\Phi_1^{(S_1)} = (1/6)k^{(S_1)} \begin{bmatrix} m_1(b_1 - b_2) + 4m_2(b_1 - b_2) + 2m_3(b_1 - b_2) \\ 2m_1(b_1 - b_2) + 2m_2(b_1 - b_2) + 2m_3(b_1 - b_2) \end{bmatrix} \\
\Phi_2^{(S_1)} = (1/9)k^{(S_1)} \begin{bmatrix} m_1 + 4m_2 + m_3 \\ m_1 + 2m_2 - m_3 \end{bmatrix} ; \quad k^{(S_1)} = 1/(m_1m_2 + 2m_2m_3 + m_3m_1)
\]

\[
\Phi_1^{(S_2)} = (1/6)k^{(S_2)} \begin{bmatrix} 2m_1(b_1 - b_2) + 2m_2(b_1 - b_2) + m_3(b_1 - b_2) \\ 4m_1(b_1 - b_2) + m_2(b_1 - b_2) + m_3(b_1 - b_2) \end{bmatrix} \\
\Phi_2^{(S_2)} = (1/18)k^{(S_2)} \begin{bmatrix} 2m_1 + 4m_2 + m_3 \\ 4m_1 + 2m_2 - 2m_3 \end{bmatrix} ; \quad k^{(S_2)} = 1/(m_1m_2 + m_2m_3 + 2m_3m_1)
\]

\[
\Phi_1^{(C)} = (1/6)k^{(C)} \begin{bmatrix} 2m_1(b_1 - b_2) + 4m_2(b_1 - b_2) + m_3(b_1 - b_2) \\ 4m_1(b_1 - b_2) + 2m_2(b_1 - b_2) + m_3(b_1 - b_2) \end{bmatrix} \\
\Phi_2^{(C)} = (1/18)k^{(C)} \begin{bmatrix} 2m_1 + 8m_2 + m_3 \\ 4m_1 + 4m_2 - 2m_3 \end{bmatrix} ; \quad k^{(C)} = 1/(2m_1m_2 + m_2m_3 + m_3m_1)
\]

\[
\Phi_1^{(S_1,e)} = k^{(S_1,e)} \begin{bmatrix} 3m_1(b_1 - b_2) + 3m_2(b_1 - b_2) \\ m_1 + 2m_2 \\ m_2 - m_3 \end{bmatrix} \\
\Phi_2^{(S_1,e)} = k^{(S_1,e)} \begin{bmatrix} (2m_2 + m_3)^2 \\ (2m_2 + m_3)(m_2 - m_3) \\ (m_2 - m_3)^2 \end{bmatrix} ; \quad k^{(S_1,e)} = m_1/(3(m_1m_2 + m_2m_3 + m_3m_1))^2
\]

\[
\Phi_1^{(S_2,e)} = k^{(S_2,e)} \begin{bmatrix} 3m_1(b_1 - b_2) + 3m_2(b_1 - b_2) \\ m_1 - m_3 \\ 2m_1 + m_3 \end{bmatrix} \\
\Phi_2^{(S_2,e)} = k^{(S_2,e)} \begin{bmatrix} (m_1 - m_3)^2 \\ (m_1 - m_3)(2m_1 + m_3) \\ (2m_1 + m_3)^2 \end{bmatrix} ; \quad k^{(S_2,e)} = m_2/(3(m_1m_2 + m_2m_3 + m_3m_1))^2
\]

\[
\Phi_1^{(C,e)} = k^{(C,e)} \begin{bmatrix} 3m_1(b_1 - b_2) + 3m_2(b_1 - b_2) \\ m_1 + 2m_2 \\ 2m_1 + m_3 \end{bmatrix} \\
\Phi_2^{(C,e)} = k^{(C,e)} \begin{bmatrix} (m_1 + 2m_2)^2 \\ (m_1 + 2m_2)(2m_1 + m_3) \\ (2m_1 + m_3)^2 \end{bmatrix} ; \quad k^{(C,e)} = m_3/(3(m_1m_2 + m_2m_3 + m_3m_1))^2
\]
4.8 Concluding Remarks

Detailed long-term network expansion formulations were devised in this chapter separately for "centralized" and "decentralized" transmission managements combining...
long-term transmission investment scheduling and short-term welfare maximization or profit maximization problems respectively. It should be noted that these formulations start in a similar fashion, but the objective function was suitably modified to meet the interest of the planner of the corresponding market model. The dispatch model developed in Chapter 3 was utilized here to represent the short-term congestion related features into network expansion planning. The short-term congestion cost was balanced against network expansion cost to reach "socially optimum" network expansion under centralized approach; congestion revenue and surpluses (supplier and consumer) were balanced against network expansion costs to reach "individual optimum" under decentralized markets.

Fundamental analysis of different perspectives of optimum transmission network planning was presented for a single planning stage clearly revealing the conflicting interests of different parties. It was proven analytically that the optimal network expansion levels are distinct under centralized system-wide and decentralized approaches. It was further proven that the optimal capacity expansion under a single (monopolistic) transmission merchant would only be half of what would be deemed optimal by a regulated central planner. It was shown that this situation can be improved through competition among transmission merchants, and a perfect competition would lead to the social optimum expansion.

To reconcile the "centralized" and "decentralized" extremes, the analysis has further been extended to specific market models, such as (i) network users holding congestion revenue rights, (ii) network users participating for competitive revenue rights acquisition, etc. Some interesting relationships have been revealed and important
conclusions have been drawn regarding the optimal levels of network expansion under different network market models. Most importantly, it was proven that social optimum is also plausible under exclusive transmission right ownership by network users. However, in case a single network user holds the transmission rights, the optimal will not be the social optimal but will be different from the monopolistic merchant transmission optimum. Additionally, it was established that network users equilibrium capacity expansions (Nash equilibrium) achieve social optimum. The analysis has been extended further to include the effects of network externality and generalization of the results was confirmed under the same.

Simple examples were presented to illustrate the findings of the theoretical investigations. Some assumptions were made to keep the analysis simple so that the various relationships could be clearly shown. These results, although general, would be useful in the formulation of proper frameworks for transmission expansion policies.
Chapter 5

Congestion-Driven Dynamic Network Expansion Planning

"We can't solve problems by using the same kind of thinking we used when we created them."

--- Albert Einstein (1879-1955) ---

German born American physicist

5.1 Introduction

Implementation of the long-term economic network expansion planning problem formulated in Chapter 4 is pursued in this chapter for realistic power systems. The centralized system-wide approach is solved with some relaxations in the basic formulation (4.1). Dynamic formulation is handled with a series of "pseudo" dynamic yearly models for which the input data is assumed known to the network planner with some acceptable degree of accuracy. Yearly models are optimized using the Generalized Benders Decomposition (Geoffrion [83]) technique, which generalizes the Benders Decomposition (Benders [84]) algorithm for non-linear models, for optimum network expansion considering annuatised costs and benefits. Two different investment management policies have been explored (i) congestion cost saving (ii) congestion revenue containing, to implement the economic expansion philosophy in centralized market environment.

Both continuous network expansions, the ideal case, and discrete circuit additions, the only option in practice, are considered in the analysis for better insight. An illustrative application study is presented for the IEEE 24-bus Reliability Test System (RTS) with some added data designed to bring out the highlights of the proposed method.
5.2 Long-Term Expansion Planning Model

The centralized network expansion planning formulation in (5.1) is derived from (4.1) with the assumptions that (i) capacity additions are discrete circuit additions, and (ii) the supplier cost and consumer benefit functions are known to the network planner.

\[
\begin{align*}
\text{Min} & \quad \frac{\sum_{t_y \in T} \sum_{m \in M} I_{z,m}(t_y)n_m(t_y)}{(1 + \tau)^{t_y - t_0}} \\
& + \sum_{t_h \in T} \sum_{i \in I} \frac{C_i(g_i(t_h))}{(1 + \tau)^{t_h - t_0}} - \sum_{t_h \in T} \sum_{j \in J} \frac{B_j(d_j(t_h))}{(1 + \tau)^{t_h - t_0}} \\
\end{align*}
\]

(5.1)

Subject to long-term as well as short-term constraints listed below:

In the long run:

\[
\sum_{m \in M} n_m(t_y) \leq n_{m}(t_y) \leq \bar{n}_{m}(t_y) \quad \forall m \in M, t_y \in T
\]

(5.1.a)

In the short run:

\[
\sum_{j} d_j(t_h) - \sum_{i} g_i(t_h) = 0 \quad \forall t_h \in T
\]

(5.1.b)

\[
g_j \leq g_i(t_h) \leq \bar{g}_i \quad \forall i \in I, \forall t_h \in T
\]

(5.1.c)

\[
d_j \leq d_j(t_h) \leq \bar{d}_j \quad \forall j \in J, \forall t_h \in T
\]

(5.1.d)

\[
z_m \leq z_m(t_h) \leq \bar{z}_m \quad \forall m \in M, \forall t_h \in T
\]

(5.1.e)

where, \(n_m(t_y)\) – no. of circuits added for branch \(m\) in year \(t_y\)

\(I_{z,m}(t_y)\) – investment cost ($/circuit) for branch \(m\) in year \(t_y\)

\([n_m(t_y), \bar{n}_m(t_y)]\) – expansion limits

Other notations are as defined earlier. This formulation excludes the \(E\{\cdot\}\) in short term costs and benefits. Instead, theses cost and benefit values can be derived from suppliers’ and consumers’ true marginal cost and benefit details estimated (or forecasted) for the
planning horizon by the network planner. The uncertainty in estimating these details is addressed in Chapter 7 with probabilistic techniques. Furthermore, investments are considered as circuit additions (in discrete steps), which would complicate the shape of the objective function (non-smooth), and continuous capacity increase (ideal case).

5.3 Economic Network Expansion

As per the result 2 proved in Chapter 4, (5.1) is equivalent to the following objective function for "economic" network expansion.

$$\min_{n_m(t_y)} \sum_{t_y \in T} \sum_{m \in M} \frac{I_{z,m}(t_y)n_m(t_y)}{(1 + \tau)^{t_y - t_0}} + \sum_{t_h \in T} \frac{TC(n_m(t_y))}{(1 + \tau)^{t_y - t_0}}$$

(5.2)

Therefore, if regulator intervention is envisioned it may be for the right economic transmission upgrades leading to breakeven between network expansion cost and congestion cost avoidance. This is classified in some applications as "Economic Network Expansion". Our formulation excludes stability or reliability driven transmission requirements, which perhaps does not need strict economic justification even under competitive conditions, to avoid severe consequences. Accordingly, we seek the dynamic expansion schedule for (5.2) instead of (5.1) with forecasted generation expansion, load and bid price growth details for the planning horizon. This expansion scheduling approach leads to fewer line/branch additions, since the magnitude of congestion cost is not significant in some applications, yet the congestion revenue may be considerably high.

Alternatively, if congestion revenue is adopted as revenue for the network owner, the regulator may intervene whenever the expected congestion revenue exceeds the maximum allowed transmission revenue (price cap regulation). This can be implemented
with a LMP ceiling imposed on every node of the network – this ultimately turns to a ceiling on network congestion revenue. However, this approach appears more like a regulated option, which undermines the exact economic signals for efficient pricing. Implementation of this policy can be introduced with appropriate network expansion to maintain the network congestion level below a prescribed value. This prescribed value can be the "hedgeable" limit of transmission congestion revenue through existing financial rights. Both these transmission investment management options are investigated in the following sections for network expansion.

5.4 Decomposition of Long Term Network Expansion Formulation

The generalized long-term formulation given in (5.1) will determine the time, location and magnitude of transmission investments in addition to short-term details of market clearing. Nevertheless, modeling of the problem for future planning horizon is a complex task and vulnerable to a huge amount of uncertainties. Especially, in competitive markets hourly market settlement and their pricing details are highly volatile, and accounting for these details in yearly investment models are mathematically complex. For short-term optimality the formulation seeks the best way to supply the electricity demand with least supply cost. Whenever the transmission capacities become binding their expansions will be looked into for necessary expansions, where capacity investment cost is justified by the release of expensive generation cost or added consumer benefits. Hence the formulation shows the dynamic inter-dependency of long-term investment scheduling problem and short-term dispatch algorithm. Decomposition of the original formulation, first along the yearly time axis, and then between yearly investment scheduling and short-term dispatch algorithm will facilitate in yielding a feasible solution. The entire
framework of network expansion methodology including this decomposition is shown in Fig. 5.1.

\[
\text{OVERALL PROBLEM FORMULATION}
\]

\[
\begin{align*}
\text{Min} & \quad n_m(t, y), g_j(t_y), d_j(t_h) \\
& \quad \sum_{t_y \in T} \sum_{m \in M} I_{z, m}(t, y) n_m(t, y) \\
& \quad + \sum_{t_y \in T} \sum_{j \in J} (1 + \tau)^{f_j - t_0} \\
& \quad + \sum_{t_y \in T} \sum_{j \in J} B_j(d_j(t_y)) \\
& \quad \text{subject to } \sum_{t_y \in T} \sum_{j \in J} C_i(g_j(t_y)) \leq (1 + \tau)^{f_j - t_0}
\end{align*}
\]

\[
\text{EXPANSION PROBLEM, year } t_y = 1
\]

\[
\begin{align*}
\text{Min} & \quad n_m(1) \\
& \quad \sum_{m \in M} I_{z, m}(1) n_m(1) \\
& \quad \text{subject to } n_m(1) \leq \pi_m(1) \\
& \quad \text{Benders Cuts}
\end{align*}
\]

\[
\text{EXPANSION PROBLEM, year } t_y = 2
\]

\[
\begin{align*}
\text{Min} & \quad n_m(2) \\
& \quad \sum_{m \in M} I_{z, m}(2) n_m(2) \\
& \quad \text{subject to } n_m(2) \leq \pi_m(2) \\
& \quad \text{Benders Cuts}
\end{align*}
\]

\[
\text{EXPANSION PROBLEM, year } t_y = T
\]

\[
\begin{align*}
\text{Min} & \quad n_m(T) \\
& \quad \sum_{m \in M} I_{z, m}(T) n_m(T) \\
& \quad \text{subject to } n_m(T) \leq \pi_m(T) \\
& \quad \text{Benders Cuts}
\end{align*}
\]

\[
\text{OPERATIONAL PROBLEM, year } t_y = 1
\]

\[
\begin{align*}
\text{Dispatch Algorithm} \\
& \text{representing year, } t_y = 1
\end{align*}
\]

\[
\text{OPERATIONAL PROBLEM, year } t_y = 2
\]

\[
\begin{align*}
\text{Dispatch Algorithm} \\
& \text{representing year, } t_y = 2
\end{align*}
\]

\[
\text{OPERATIONAL PROBLEM, year } t_y = T
\]

\[
\begin{align*}
\text{Dispatch Algorithm} \\
& \text{representing year, } t_y = T
\end{align*}
\]

\[
\text{INPUTS}
\]

\[
\text{BIDDING DETAILS for GENCO & DISTCO } t_y = 1
\]

\[
\text{FORECAST}
\]

\[
\text{BIDDING DETAILS for GENCO & DISTCO } t_y = 2
\]

\[
\text{BIDDING DETAILS for GENCO & DISTCO } t_y = T
\]

\[
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\]

\[
\text{Figure 5.1: Overall Network Expansion Framework}
\]

The short-term dispatch algorithm is separated from the investment scheduling problem with the fixed decisions in investment scheduling for the time interval. The effects of investment decisions on the short-term dispatch algorithm are sent back to the investment problem as a feedback (Benders’ cuts). The problem space becomes quite substantial when the optimization extends its limits towards the planning horizon. To avoid this difficulty, formulation is further decoupled to yearly models that deal with annuatised quantities of investment costs and annual congestion details emanating from the dispatch algorithm. This time domain decoupling facilitates optimization efficiency, and avoids mathematical complexity without losing much of accuracy. Accordingly, the model becomes a "pseudo" dynamic expansion model. The link between the two stages
exists only in the minimum network limit requirement \( n_m(t_y) \). The link from the dispatch algorithm to the investment scheduling problem is established through the chosen investment management policy.

5.5 Investment Management Policy

5.5.1 Economic Network Expansion Approach (Congestion Cost Saving Approach)

As shown in the economic network expansion (5.2) the decisions made in the investment scheduling problem can be evaluated for expected savings in congestion cost, and its effectiveness can be passed on to investment scheduling problem as a feedback. For this comparison the annual expected congestion costs \( E\{TC(t_y)\} \), and expected values of transmission shadow prices \( E\{\mu_{z,m}(t_y)\} \) need to be estimated corresponding to the yearly investment model considered. \( E\{\mu_{z,m}(t_y)\} \) determines the expected annual saving of congestion cost or the incremental social welfare for an infinitesimal (or unit) increase in bounded transmission capacity. This concept is implemented for economic counter-balance between investment costs and savings in congestion cost or enhancement of social welfare.

5.5.2 Price-cap Regulation Approach (Congestion Revenue Containing Approach)

Due to economy of scale and the "lumpiness" of transmission investments, transmission shadow prices and marginal investment cost formulations for network expansion planning may justify lesser investments in practice. This may perhaps be
backed by the inaccuracies as well as uncertainties in future value estimations in input details such as bid prices, load and generation details. This leads to insufficient capacity investments, or undermines the important signals for network expansions. As a result the network congestion may exceed permissible levels, and may cast adverse effects on smooth market operation. Alternatively, if congestion revenue is assumed to be the only marginal transmission revenue\(^2\) for the network owner, the regulator may intervene whenever the expected congestion revenue exceeds the maximum allowed transmission revenue (price cap regulation). Therefore, it is necessary to take into consideration the total congestion revenue and its fluctuations above "hedgeable" limits.

Congestion revenue, \(TR\) will naturally become crucial during system peak hours, therefore it suffices to analyze the most probable worst case system peak conditions for the future years and to decide the network expansion strategies to avoid fluctuations of \(TR\) above "hedgeable" limits (upper bound). The upper bound (\(\varepsilon\)) of \(TR\) can be defined as, \(\varepsilon = \sum_{m \in M^*} \pi_{z,m} Z_m\), where, \(\pi_{z,m}\) is the maximum allowed short-term congestion rent for branch \(m\) and \(M^* \subseteq \{m | \mu_{z,m} \geq \pi_{z,m}\}\) is the set of branches having short-term congestion rent above its upper limit.

5.6 Solution Technique – Generalized Benders Decomposition Based Algorithm

Decomposition algorithms such as Benders Decomposition (BD) (Benders [84]) and Generalized Benders Decomposition (GBD) (Geoffrion [83]) have shown their capabilities in handling the forms of formulations discussed above. Decomposition of

\(^2\) Congestion revenue is assumed to be the marginal transmission revenue throughout the thesis – however, an additional transmission charge may be introduced to congestion pricing for revenue reconciliation purposes, which is not addressed in this thesis. This component is intended to recover the fixed cost components occurring in transmission investments. This is occasionally termed as "access charge".
discrete investment decisions and continuous variables pertaining to system operation has successfully handled the TEP problem under vertically integrated systems using BD, for example Pereira et al. [36], Romero and Monticelli [49], Romero and Monticelli [50], and Binato et al. [52]. Siddiqi and Baughman [51] have used GBD to handle the non-linear problem formulation. Such an approach requires decomposition of the formulation into master problem and operational subproblems. We decompose network expansion scheduling problem (investment problem) as the master problem and dispatch algorithm (power pool model) representing annual behavior (congestion cost method) or peak behavior (congestion revenue method) to form the operational subproblem. Iterative solutions of these problems share the decisions and cuts to move towards an acceptable solution.

5.6.1 Expansion Scheduling (Master Problem)

This determines the expansion schedule \( n_m(t_y) \) for each year \( t_y \) based on the investment criteria. Two network expansion methodologies with different criteria (i.e. congestion cost minimization and congestion revenue minimization) were considered using GBD techniques.

In the first approach, the congestion cost \( TC \) saving justified by investment cost is implemented with feasibility cuts included in the Benders algorithm. In the second approach, network expansion to contain the congestion revenue \( TR \) ceiling is implemented with infeasibility cuts included in the Benders algorithm. The second approach perhaps appears more like a regulated option, but this avoids insufficient investment, which can result if the regulator adheres to strict economic criteria. The formulation of expansion scheduling problem based on the expansion criteria are given
by (5.3) and (5.4). These are sub problems of the overall cost minimization problem (5.1). It is optimized using Linear Programming (LP) model for each scheduling period using annuatised quantities.

5.6.1.1 Congestion Cost Saving Approach (Master Problem)

Min \( \alpha \)

Subject to:

\[
\alpha \geq \sum_{t_y \in T} \sum_{m \in M} \frac{I_{z,m}(t_y) n_m(t_y)}{(1 + \tau)^{t_y - t_0}}
\]

\[
\alpha \geq \sum_{t_y} \frac{1}{(1 + \tau)^{t_y - t_0}} \left\{ E\{TC(t_y)\} + \sum_m I_{z,m}(t_y) n_m(t_y) \right. \\
- \sum_m T_h E\{ \mu_{z,m} \} Z_m \left( n_m(t_y) - n^k_m(t_y) \right) \left\} \right.
\]

\[
n_m(t_y) \leq n_m(t_y) \leq \bar{n}_m(t_y)
\]

where, \( T_h \) is the hours per year (i.e. 8760) or total sub time periods \( t_h \) analyzed per year, and \( k \) is the Benders’ iteration index. The feasibility cut is determined by analyzing a set of scenarios representing the annual load duration curve (LDC). This provides the annual expected congestion cost \( E\{TC\} \) and the expected values of transmission shadow prices \( E\{\mu_{z,m}\} \). They are compared against annuatised investment costs.

5.6.1.2 Congestion Revenue Containing Approach (Master Problem)

Min \( \alpha \)

Subject to:

\[
\alpha \geq \sum_{t_y \in T} \sum_{m \in M} I_{z,m}(t_y) n_m(t_y) \\
(1 + \tau)^{t_y - t_0}
\]

\[
TR^k(t_y) - \sum_{\mu_{TR,m} < 0} |\mu_{TR,m}| Z_m \left( n_m(t_y) - n^k_m(t_y) \right) \leq \varepsilon
\]

\[
n_m(t_y) \leq n_m(t_y) \leq \bar{n}_m(t_y)
\]
where, \( \varepsilon \) is the maximum allowed congestion level (in terms of congestion revenue).

The *infeasibility cut* is formulated for worst-case scenarios representing higher congestion revenues (e.g., during system peak). In contrast to the congestion cost saving approach, infeasibility cuts represent higher congestion situations and all parameters that form the constraint pertaining to a particular time instant analyzed. The multiplier \( \mu_{TR,m} \) which governs the incremental congestion revenue for incremental transmission capacity increase, can be defined as,

\[
\mu_{TR,m} = \mu_{z,m} - (\partial SS / \partial Z_m) - (\partial CS / \partial Z_m) \tag{5.4.a}
\]

This is derived from the objective function of the dispatch algorithm (i.e. \( B - C = SS + CS + TR \)). The multipliers \( \mu_{z,m} \) determine how transmission capacity enhancements uplift social benefit. Even though the congestion revenue is embedded in social benefit, supplier’s surplus (SS) and consumer’s surplus (CS) become the other two components that comprise the overall social benefit. Therefore, to reduce the congestion revenue with capacity enhancements (i.e., \( \partial TR / \partial Z_m < 0 \)), SS and CS should increase at a higher rate than the social benefit does, i.e., \( (\partial SS / \partial Z_m) + (\partial CS / \partial Z_m) > \mu_{z,m} \).

In this situation increase in transmission capacity, increases supplier and consumer’s surplus and at the same time decreases the congestion revenue. However, under some operating levels capacity enhancements will improve social benefit (i.e., \( \mu_{z,m} > 0 \)), but it will increase the congestion revenue when \( \mu_{TR,m} > 0 \) for this condition \( (\partial SS / \partial Z_m) + (\partial CS / \partial Z_m) < \mu_{z,m} \). Therefore, only \( \partial TR / \partial Z_m < 0 \) conditions are considered for network expansion analysis.
5.6.2 Operational Subproblem (Dispatch Model)

This is the social cost minimization problem (dispatch algorithm) for the network specified by expansion schedule and the inputs given for generation and demand bids. The dispatch algorithm formulated in Chapter 3 with double-sided auction is solved as the operational problem in order to obtain the auxiliary information required for network expansion module. The solution yields market-cleared generation and demand quantities and LMPs. It also provides all the dual multipliers (Lagrange multipliers) useful in forming feasibility and infeasibility cuts to the master problem. For network planning studies the significant dual multipliers, congestion cost ($TC$), and congestion revenue ($TR$) would be particularly useful to form the Benders cuts to be used in the investment minimization problem. Simulation of operational problem depends on the planning criteria. In the congestion cost saving approach a sufficient number of sub periods are to be analyzed in order to estimate the annual expected values of congestion cost and transmission shadow prices. In the case studies carried out in this thesis these simulations were performed as per the load variations based on an accepted load duration curve (LDC). In the congestion revenue containing approach, single or a few worst cases representing system peak conditions are simulated.

5.6.3 Supply and Demand Bid Forecasting

For minimization of investment, it suffices to solve the investment or the expansion problem in yearly intervals. In traditional network expansion, even the operational problem needs to be solved only in yearly intervals to ensure that the network satisfies the required operating criteria under the peak load conditions. However, the proposed framework requires the calculation of the congestion details, which vary each
hour with the supply and demand conditions. Therefore, strictly speaking, the operational problem needs to be solved for each hour throughout the year in order to calculate the expected values of congestion cost and transmission shadow prices. In addition to the computational problems, this requires the representation of all supply and demand bids for each individual hour. This obviously is a huge burden. This requirement may however be softened by taking the advantage of possible hourly and seasonal patterns of the bids. For planning purposes it may be possible to use typical hours in a day (peak, off-peak), typical days (weekdays, weekend days) and a few seasons (summer, winter) to substantially reduce this requirement, yet have reasonable estimate of the required attributes over the whole year. Within our scope we have adopted a constant bidding pattern from each bidder for a particular year to represent typical supply and demand bids by linear equations. The hourly variations of demand were simulated with the load duration curve (LDC), in order to estimate the expected congestion cost $E\{TC\}$ and shadow prices $E\{\mu_{z,m}\}$. The bids for the subsequent years are forecasted simply by incremental changes in the constants of the linear bids. As the network expansion depends on this representation, the network planner (RTO or ISO) needs to foresee this bid forecast with some accuracy in order to arrive realistic network expansion schedules. More realistic statistical approach to represent bid functions is presented in Chapter 7, which would handle the errors in bid forecasting reasonably well.
5.6.4 Solution Algorithm

The proposed network expansions are carried out using GBD algorithm shown below for the two methodologies separately.

Steps

1. Set time index $t_y = 1$.
2. Set iteration index $k = 1$.
3. Solve the investment problem for the period $t_y$ and obtain the network solution $n^k_m(t_y)$.
4. Solve the operational sub problem for the period $t_y$ with the network solution $n^k_m(t_y)$. Generate the relevant Benders cut to the master problem for iteration level $k$.
5. Solve the investment problem for time period $t_y$ again with the Benders cut generated in step (4) and update $n^k_m(t_y)$.
6. Check for Convergence
   a. **Congestion Cost Approach**
      The change in objective value\(^3\) of master problem is less than the tolerance given for two consecutive iterations $k$.
      - if convergence satisfied go to step (7) else $k = k + 1$, go to step (4).
   b. **Congestion Revenue Approach**
      If no infeasibility cuts were generated or $TR^k(t_y) \leq \varepsilon$, then go to step (7) else $k = k + 1$, go to step (4).
7. If $t_y = T$ go to step (8), else $t_y = t_y + 1$, $n^k_m(t_y) = n^k_m(t_y - 1)$. go to step (2).
8. Stop.

The congestion cost saving approach carries the feasibility cuts (representing average values) and the congestion revenue approach carries the infeasibility cuts (representing peak conditions). In practice the capacity upgrades are discrete in nature.

---

\(^3\)Termination Criterion: $2(\alpha^k - \alpha^{k-1})/(\alpha^k + \alpha^{k-1}) \leq \text{tolerance (1%)}$
(by circuits), then the master problem becomes an \textit{Integer Programming} (IP) problem, which should be handled with IP techniques. For the study purposes the capacity upgrades were considered for both continuous and discrete (line additions) situations. The GBD algorithm described will solve the \textit{master problem} (LP) and \textit{operational subproblem} (QP) with the corresponding decisions and Benders’ cuts. The Benders’ cut is a \textit{feasibility} cut for congestion cost criteria and an \textit{infeasibility} cut for the congestion revenue approach depending on the choice of the two approaches.

In the congestion cost saving method investment cost is automatically justified in the formulation of the problem itself. However, in the congestion revenue hedging approach the maximum allowed congestion level ($\varepsilon$) is not strictly justified with investment cost. This limit is fixed by the transmission organization and it is the maximum hedgeable congestion level.

5.6.5 Computational and Optimization Aspects of the Algorithm

The formulation is implemented on Matlab\textsuperscript{4} operating environment, all the programs and subroutines are coded to suit the algorithm mentioned. The GBD technique was adopted for the solution because of its ability to decompose the original problem into subproblems and its guided optimization towards optimum for user defined accuracy. Branch flow modeling in the short-term dispatch model can be satisfactorily insulated from the non-linearity effects of investment decisions, by decomposition of the problem. In other words transmission line additions $n_m(t_y)$ affects the PTDF matrix $H(t_y)$. But, for a set of decisions $n_m^k(t_y)$ determined by the \textit{master problem} (in $k^{th}$ iteration), the

\textsuperscript{4} Matlab® Software by The Mathworks, Inc., was used for all technical computations reported in this thesis.
PTDF to be used in the subproblem is fixed $H^k(t_y)$ keeping the optimization simple.

Once the $n^{k+1}_m(t_y)$ is modified in the next iteration this will be reflected in the new updated $H^{k+1}(t_y)$. This procedure helps in handling discrete circuit additions and their non-linear relationship by appropriately incorporating the reactance change.

\[
p = b_1 + m_1 g_1
\]

\[
p = b_2 + m_2 g_2
\]

\[
\beta_1 = 5 \$/MWh
\]

\[
\beta_2 = 3 \$/MWh
\]

**Data**

\[
b_1 = 10 \text{ ; } m_1 = 0.05 \text{ ; } b_2 = 15 \text{ ; } m_2 = 0.075 \text{ ; } b_3 = 40 \text{ ; } m_3 = 0.1;
\]

all branches have equal reactance (1.0 p.u)

**Figure 5.2: Optimization Path Towards Social Optimum**

This optimization process is illustrated in Fig.5.2, by applying the technique to determine the optimal branch capacity additions for a simple 3-node system detailed in §4.7.3.2 of Chapter 4. The left pane shows the trajectory traced by the GBD method with continuous capacity expansion without reactance changes. The right pane shows the same trajectory when capacity additions were made in discrete 40 MW circuits.
5.7 **Illustrative Example (IEEE 24-bus RTS)**

The developed network expansion methodology was applied to determine the optimal expansion schedule for the modified IEEE 24-bus Reliability Test System (RTS) (RTS Task Force [70]). The network diagram is the same as in Fig. 3.3, which has 14 generating companies (Gencos) and 17 distribution companies (Distcos). Initial bidding details corresponding to the peak hour of the base year ($t_y = 1$) are adopted as shown in Table 3.1. However, this is assumed to represent the entire year ($t_y = 1$) for network expansion study. The same bidding details are assumed to be corresponding to system peak conditions for that particular year.

As mentioned in section 5.6.3, forecasting of bidding details for the future years becomes quite involved and extremely sensitive. This aspect is not fully addressed in this thesis. We have adopted a simple model for the future supply and demand bids in order to demonstrate the application of the proposed algorithm. The planning horizon is taken as 8 years. The growth in bids are represented by annual growth rates for intercepts ($b_i, b_j$) and the maximum and minimum demand bids, shown in Table 5.1.

<table>
<thead>
<tr>
<th>Table 5.1: Growth Rates for Bid Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years</td>
</tr>
<tr>
<td>Growth rate of price intercept ($b_i, b_j$)</td>
</tr>
<tr>
<td>Growth rates for demand bid limits ($d, d_*$)</td>
</tr>
</tbody>
</table>

Increase in the maximum generation limit ($\bar{g}$) is introduced in discrete steps. For this test application we have assumed that the maximum generation capacity bid ($\bar{g}$) increases in the 3$^{rd}$ and the 6$^{th}$ year by a factor of 1.5 and 2.5 respectively. Elaborate
models to represent the growths in demand and generation bids can be easily included without affecting the proposed methodology.

Network technical details are as given in Appendix – A.4. The line costs were estimated with the specific cost of 1,000 $ per MW-km, line length, and the original capacities\(^5\). The hourly load variations were simulated with the LDC given in Billinton and Li [71] for RTS. The economic life of transmission investments was assumed as 25 years. The discount rate (\(\tau\)) was taken as 10 % for the 8-year network planning study.

5.7.1 Network Expansion with Congestion Cost Method

The results of the planning exercise where the capacity addition cost was balanced against the congestion cost saving is shown in Fig. 5.3.

![Figure 5.3: Capacity Upgrades – Congestion Cost Method](image)

If it were possible to increase the capacity of the line in a continuous scale with the increase in the congestion cost, it is found that three lines, namely lines 23, 28 and 7

---

\(^5\) Original branch details are available in RTS Task Force [70] for IEEE 24-bus Reliability Test System (RTS).
would be upgraded over the years as indicated by the solid lines. Such capacity additions would reduce system congestion and therefore the congestion cost. It should be noted that that network still would have some congestion after these line upgrades such that the congestion costs balance the cost of these upgrades.

However line capacities can only be added in discrete steps (circuits) in practice and such gradual increase in line capacities are not possible. Under such discrete capacity addition conditions, it is found that one circuit is added in each of the lines 23 and 28 in year 4 and year 6 respectively. The congestion level is expected to be quite significant prior to the circuit additions when the capacity additions are discrete. This problem may be alleviated in the congestion revenue containing method discussed in the next section. Depending on the continuous upgrade or discrete upgrade, the master problems (5.3) and (5.4) become a linear programming problem or an integer programming problem, and they are handled with standard mathematical programming tools.

5.7.2 Network Expansion with Congestion Revenue Method

The results of network capacity expansion exercise to keep the congestion revenue within a prescribed value are shown in Fig. 5.4. If it were possible to increase the line capacities in a continuous scale, it was found that a steady increase in line capacities were necessary in lines 23, 28 and 7. The figure also shows that capacity addition requirements would be lower for higher levels of congestion revenue. Although the results of Figs 5.3 and 5.4 show similar general trends, it should be noted that congestion revenue containing method cannot automatically balance the investment against the revenue.
Figure 5.4: Capacity Requirements - Congestion Revenue method

When the capacity upgrades are made discrete circuit additions, the time ordered line additions for a congestion revenue of $1,000/h is shown in Table 5.2 as Case B1. One circuit was added in lines 23 and 28 in years 1 and 3 and one circuit each in lines 7, 19 and 30 in year 6. It is found that additional circuits were needed in lines 19 and 30 compared to the continuous upgrades. This is because of the network topology changes due to line additions causing additional congestion, which were not congested initially. It is found that additional circuits were needed in lines 19 and 30 compared to the continuous upgrades.

When the allowable congestion limit was raised to $\varepsilon = 2,000$/$h$ in Case B2, it is found that the addition of circuits in lines 28 and 07 are delayed by one year in the expansion scheduling due to the relaxation of congestion level. It should be noted that in continuous capacity upgrade study, the same set of transmission bottlenecks are identified by both approaches and each method has its own advantages. Congestion cost approach has the advantage of simultaneous economic feasibility check but had a larger
computational burden while the congestion revenue method lacks simultaneous economic feasibility check but is computationally relaxed.

**TABLE 5.2: NETWORK ADDITION SCHEDULE (DISCRETE)**
*(LINES ADDED IN THE RESPECTIVE YEAR)*

<table>
<thead>
<tr>
<th>Line</th>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>N07</td>
<td>Case B1 (ε = 1,000 $/h)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N19</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N23</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N28</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>N30</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N07</td>
<td>Case B2 (ε = 2,000 $/h)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>N19</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N23</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N28</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N30</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5.8 Concluding Remarks

The implementation of long-term network expansion planning from the perspective of central planner was presented in this chapter for dynamic transmission investment scheduling. This dynamic formulation was resolved into a series of "pseudo" dynamic problems, and annuatised quantities were used in annual network planning models to reduce the complexity of the problem. The Generalized Benders Decomposition technique was utilized to obtain optimum annual investment decisions by decomposing the investment scheduling and short-term social cost minimization as per the Benders decomposition method. The GBD technique could successfully handle the complex mixed-integer problem formulation by partitioning discrete investment decisions and continuous operational variables. The procedure can be applied to formulate expansion schemes, which are sets of time ordered investment decisions for the network
planner. Theoretical development of the procedure along with the necessary algorithms and the implementation of the procedure in a test system have also been presented.

Two network expansion criteria were explored (i) congestion cost saving, and (ii) congestion revenue containing. Both these criteria are valid and feasible for centralized electricity markets concerned with the welfare of the society being served. The network investment cost was justified by the increased social welfare (or congestion cost saving) in the congestion cost saving method. Congestion revenue containing approach was implemented as a regulated price-cap method. The transmission expansion schedules obtained with different criteria were presented and discussed. The results were consistent except for the differences created by the implementation criteria.
Chapter 6

Flexible Transmission and Network Reinforcements

"Divide each difficulty into as many parts as is feasible and necessary to resolve it."

--- Rene Descartes (1596-1650) ---

French Mathematician, philosopher and scientist

6.1 Introduction

In liberalized electricity markets generating companies are free to choose their technology and location independently. This has given rise to additional burden to network expansion process – this increased uncertainty leads to network investment risk and/or reliability risk. Additions of new line capacities may not be the best way to deal with the time dependent congestion in various lines that arise due to the lack of control over grid flows in competitive markets. One possible way of addressing this uncertainty may be the employment of flexible transmission (FACTS) devices.

These devices provide flexible line parameters, which can be adjusted quickly. These new ways of transmitting electricity using FACTS devices are seen as the most decisive challenge to the traditional grid’s status as a natural monopoly (Hirst and Kirby [11], Rotger and Felder [4]). However, the need for long-term transmission capacities cannot be resolved solely by such flexible transmission. Thus a proper mix of flexible transmission devices and network reinforcements would be required for optimal operation. Accordingly, FACTS will be utilized as a supplement to capacity expansions but not as a complete replacement. The optimum mix of the two options will have to be
determined by a proper long-term investment planning formulation based on long-term cost/benefits to the network users due to increased transmission flexibility. In practical applications, FACTS help to better utilize the network capacity in short-term provided network has some spare (or under utilized) paths available. And, network expansion provides adequate network capacity when such under utilized paths are not available. An economic comparison of the two is given in (Mutale and Strbac [85], [86]). It compared different degrees of flexibility with possible delay in network reinforcement without going into specific FACTS devices. However, effective value-based transmission expansion planning considering both line reinforcements along with FACTS devices will require the consideration of both the long-term investment scheduling problem and the short-term dispatch, simultaneously. This chapter extends a framework for the long-term network planning problem presented in Chapter 5 to incorporate both the network expansion as well as flexible options of FACTS installations.

6.2 Overall Problem Formulation

For long-term transmission investment planning considering both network reinforcements and FACTS installations, the problem formulation can be derived as follows:

$$\text{Min} \quad \sum_{t_y \in T} \sum_{m \in M} \frac{IC_m(t_y)}{(1 + \tau)^{t_y - t_0}} + \\
\sum_{t_h \in T} \sum_{i \in I} \frac{C_i(g_i(t_h))}{(1 + \tau)^{t_h - t_0}} - \sum_{t_h \in T} \sum_{j \in J} \frac{B_j(d_j(t_h))}{(1 + \tau)^{t_h - t_0}}$$

(6.1)
Subject to long-term as well as short-term constraints listed below,

In the long run:

\[
\overline{IC}_m(t_y) \leq IC_m(t_y) \leq \overline{IC}_m(t_y) \quad \forall m \in M, t_y \in T \tag{6.1.a}
\]

In the short run:

\[
\sum_j d_j(t_h) - \sum_i g_i(t_h) = 0 \quad \forall t_h \in T \tag{6.1.b}
\]

\[
g_j(t_h) \leq g_i(t_h) \leq \overline{g}_i \quad \forall i \in I, \forall t_h \in T \tag{6.1.c}
\]

\[
d_j \leq d_j(t_h) \leq \overline{d}_j \quad \forall j \in J, \forall t_h \in T \tag{6.1.d}
\]

\[
z_m \leq z_m(t_h) \leq \overline{z}_m \quad \forall m \in M, \forall t_y \in T \tag{6.1.e}
\]

where, the branch flow without a FACTS devices effect is represented as,

\[
z_m = \sum_{i \in I} H_{m,(i)} g_i - \sum_{j \in J} H_{m,(j)} d_j.
\]

The changes to the natural branch flow due to a FACTS device are modeled as described in § 6.3.

6.3 Representation of FACTS Devices in Dispatch Model

In order to investigate the merits of FACTS devices to relieve congestion, these devices need to be incorporated into the dispatch algorithm. Two kinds of devices are considered here – namely Thyristor Controlled Phase Angle Regulator or Phase Shifter (PS) and Thyristor Controlled Series Compensator or Series Compensator (SC). PS is modeled in terms of compensation injections in branch terminal nodes, and SC as variations in branch admittance or reactance (Taranto et al. [87]) which in turn will affect the PTDF matrix \( H \).
6.3.1 Representation of Phase Shifters (PS)

The base power flow in branch $m$ with admittance $\gamma_m$ is given by $z_{m,b} = \gamma_m \theta_m$, where $\theta_m$ represents the difference in voltage angles. Addition of a phase shifter to branch $m'$ ($m' \in M'$ – set of branches with PSs) with phase angle $\psi_m'$ will result in the flow to be,

$$z_{m'} = \gamma_{m'}(\theta_{m'} + \psi_{m'}) = z_{m',b} + z_{m',F}$$

where $z_{m',b} = \gamma_m \theta_{m'} = \sum_i H_{m',i}(i) g_i - \sum_j H_{m',j}(j) d_j$ and $z_{m',F} = \gamma_{m'} \psi_{m'}$

The phase shifter flow component can be represented by compensation injection (Taranto et al. [87]), as shown in Fig. 6.1.

The compensation $\gamma_{m'} \psi_{m'}$ acts as an extraction at node $st(m')$ and injection at node $en(m')$ which causes flow changes not only in branch $m'$ but also in other branches (i.e. $m \in \{M \mid m \neq m'\}$). This introduces a new independent variable $\psi_{m'}$, its operational limits and modified branch flow limits are as given below (where, $H'$ – PTDF without branch $m'$):
\[
\begin{align*}
\bar{z}_m & \leq z_{m,b} + \sum_{m' \in M'} (H'_{m,en(m')} - H'_{m,st(m')}) \gamma_m' \psi_m' \leq \bar{z}_m \quad \forall m \notin M' \quad (6.1.e.1) \\
\bar{z}_m & \leq z_{m,b} + \gamma_m \psi_m + \sum_{m' \in M'} \left( H'_{m,en(m')} - H'_{m,st(m')} \right) \gamma'_m \psi'_m \leq \bar{z}_m \\
& \quad \forall m \in M' \quad (6.1.e.2) \\
\psi_m & \leq \psi_m \leq \bar{\psi}_m \quad \forall m \in M' \quad (6.1.f)
\end{align*}
\]

6.3.2 Representation of Series Compensation (SC)

Unlike phase shifter variables susceptance variables are not directly linked to power injections. But, series compensation in branch \( m'' \ (\in M'' \ - \text{set of branches with SCs}) \) will modify the branch reactance \( X_{m''} \) and susceptance \( \gamma_{m''} \). In this formulation branch flows are related to supply and demand by the linear functions,

\[
z_m = \sum_i H_{m,i} s_i - \sum_j H_{m,j} d_j.
\]

Accordingly, series compensation affects PTDFs, \( H \) which make the flow constraints non-linear \((i.e. H = f(X_{m''}, \gamma_{m''}))\). A decomposition approach is suggested in Taranto et al. [87] to deal with this situation. Optimization of \( X_{m''} \) or \( \gamma_{m''} \) is handled in the investment master problem and its decisions (temporarily fixed for the time period) are transferred to the operational subproblem (dispatch algorithm). The dispatch algorithm evaluates the effects of series compensation on its objective and additional investment hints are passed to the master problem. This avoids the non-linearity in the dispatch algorithm through decomposition and drives the investment decisions towards optimality.
6.4 Investment Cost Formulations for Network Reinforcements and FACTS Options

6.4.1 Network Reinforcement

The transmission capacity enhancements in the form of circuit additions are modeled in the following way.

\[ IC_m(t_y) = I_{z,m}(t_y)n_m(t_y) \]  

(6.2)

and,

\[ n_m(t_y) \leq n_m(t_y) \leq \bar{n}_m(t_y) \]  \( \forall m \in M, t_y \in T \)  

(6.2.a)

where,

- \( I_{z,m}(t_y) \) – transmission line investment cost of \( m \)
- \( n_m(t_y) \) – no. of circuits added to branch \( m \) in year \( t_y \)
- \([n_m(t_y), \bar{n}_m(t_y)]\) – expansion limits

6.4.2 PS Installations

Installation of PSs at pre-identified locations can redirect branch flows through non-congested branches alleviating congestions in flow gates (congested paths). The investment costs of devices are included in the overall problem formulation and their effects are reflected in the corresponding dispatch algorithm. The investment cost of a PS is rather fixed and it only depends on the capacity of the circuit where it is installed (Oliveira et al. [88]). This presumes the maximum phase angle regulating capacity is fixed, and does not vary with the location. In this investigation it is adopted that the maximum phase angle regulating capacity is variable and its value is decided by the investment scheduling problem. This makes the investment cost of PS to be linearly
proportional to its maximum phase angle regulating capacity. The capacity additions are considered both in continuous or discrete steps.

\[ IC_m(t_y) = cS_{base}Z_m\psi_m(t_y) = I_{\psi,m}(t_y)\psi_m(t_y) \] (6.3)

and \[ 0 \leq \psi_m(t_y) \leq 10^\circ \quad \forall m \in M', t_y \in T \] (6.3.a)

where, \( c \) is the specific cost of FACTS device ($/MVA) (Oliveira et al. [88], [89]), and \( S_{base} \) is the base MVA.

### 6.4.3 SC Installations

SC can also affect the flow of the line by adjusting the admittance of lightly loaded lines. The locations where SC additions are most desirable are pre-determined as in the PS case. Such pre-identification of most probable locations will reduce the problem space. The amount of series capacitive reactance \( (X_c) \) is determined in the investment problem. The investment cost function of SC is modeled as in Oliveira et al. [88], [89].

\[ IC_m(t_y) = cX_{c,m}(t_y) \frac{Z_m^2}{S_{base}} = I_{X_{c,m}}(t_y)X_{c,m}(t_y) \] (6.4)

and \[ 0 \leq X_{c,m}(t_y) \leq 0.4X_m \quad \forall m \in M^* \] (6.4.a)

where, \( 0.4X_m \) is the upper limit of series compensation in terms of branch reactance \( (X_m) \).

This formulation helps to keep the transmission investment problem linear. For fixed investment decisions the congestion cost minimization problem is quadratic in hourly time domain. Therefore it is possible to decouple the investment problem and dispatch algorithm/congestion cost problem to facilitate its solution.
Note: The selected values for phase shifter limit (± 10°) and series compensation limit (40 %) are solely for the sake of illustration purposes. Any other limits can be adopted which are feasible in practice from the view points of available sizes and other technical considerations.

6.5 Solution Technique

6.5.1 Decoupling of the Problem

The solution technique adopted is the same as the decoupled methodology used in the network expansion study (see § 5.4 - 5.6). The long-term problem formulation (6.1) can be decoupled into the master problem (investment problem) and the operational subproblem of power dispatch. This may be solved using the Generalized Benders Decomposition technique. The partitioning procedure adopted relaxes non-linearity occurring in flow equations (i.e. effect on $H$ matrix due to capacity expansion and series compensation) and non-convex nature due to discrete investment variables, and successfully leads to convergence.

The investment problem runs in yearly time domain including FACTS options and its investment decisions are used in the operational subproblem (dispatch algorithm). The operational subproblem needs solution in hourly basis, which may be represented by load variations according to an estimated annual load duration curve ($LDC$). Its expected annual effects will be transferred to investment problem as Benders’ cuts. The congestion cost saving with a particular investment cost is adopted as the investment policy.
### 6.5.1.1 Master Problem

Min \( \alpha \)

Subject to:

\[
\alpha \geq \sum_{t_y \in T} \left( \sum_{m \in M} \frac{I_{z,m}(t_y)n_m(t_y)}{(1+\tau)^{t_y-t_0}} + \sum_{m \in M'} I_{\varphi,m}(t_y)\varphi_m(t_y) + \sum_{m \in M^*} I_{\chi_m}(t_y)X_m(t_y) \right)
\]

\[
\alpha \geq \sum_{t_y} \frac{1}{(1+\tau)^{t_y-t_0}} \left( E\{TC(t_y)\} + \sum_{m \in M} I_{z,m}(t_y)n_m(t_y) \right)
\]

\[
+ \sum_{m \in M'} I_{\varphi,m}(t_y)\varphi_m(t_y) + \sum_{m \in M^*} I_{\chi_m}(t_y)X_m(t_y)
\]

\[
- \sum_{m \in M} T_h E\{\mu_{z,m}\} Z_m \left( n_m(t_y) - n^k_m(t_y) \right)
\]

\[
- \sum_{m \in M'} T_h E\{\mu_{\varphi,m}\} \left( \varphi_m(t_y) - \varphi^k_m(t_y) \right)
\]

\[
- \sum_{m \in M^*} T_h E\{\mu_{\chi,m}\} \left( X_m(t_y) - X^k_m(t_y) \right)
\]

\[
n_m(t_y) \leq n_m(t_y) \leq \bar{n}_m(t_y) \quad \forall m \in M, t_y \in T \quad (6.5.a)
\]

\[
0 \leq \varphi_m(t_y) \leq 10^0 \quad \forall m \in M', t_y \in T \quad (6.5.b)
\]

\[
0 \leq X_m(t_y) \leq 0.4X_m \quad \forall m \in M^*, t_y \in T \quad (6.5.c)
\]

where, \( T_h \) is the hours per year (i.e. 8760) or total duration of sub time periods \((t_h)\) analyzed per year, and \( k \) Benders’ iteration index. The feasibility cut is determined by analyzing total sub periods representing the annual load duration curve (LDC). This provides the annual expected congestion cost \( E\{TC\} \) and the expected values of shadow prices \( E\{\mu_{z,m}\}, E\{\mu_{\varphi,m}\} \) and \( E\{\mu_{\chi,m}\} \) corresponding to network improvements.

### 6.5.1.2 Operational Problem

The objective function \( \sum_i C_i(g_i(t_h)) - \sum_j B_j(d_j(t_h)) \) and constraints 6.1.b – 6.1.f, is solved for each sub-period in chronological LDC as the Operational Subproblem. The
resulting values of $TC$, $\mu_{z,m}$, $\mu_{\psi,m}$ and $\mu_{xc,m}$ are aggregated with the corresponding probability of occurrence of sub-period in LDC to calculate the expected $E\{\cdot\}$ values. It provides the expected values of annual congestion cost and shadow prices corresponding to line capacity, phase shifter capacity and series compensation limits to be used in the feasibility cut (or Benders’ cut) of the master problem. This allows inclusion of time varying bidding details of supply and demand, loads into the expansion planning model.

6.5.2 Calculation of Shadow Prices

The shadow prices of transmission equipment in the dispatch algorithm play a major role in this expansion planning algorithm. They represent the amount of congestion cost relief per infinitesimal (or unit) increment of the limiting transmission facility. These transmission facility upgrades may belong to one or more of the three categories, namely transmission capacity enhancement, $PS$ installation and $SC$ installation.

Transmission capacity shadow price ($\mu_{z,m}$) is derived from constraint (6.1.e) in the dispatch algorithm. The modified dispatch algorithm including PS effects includes the PS operational constraint (6.1.f). Violation of these phase angle limits in the dispatch algorithm gives rise to PS capacity shadow price ($\mu_{\psi,m}$). The estimation of the shadow price corresponding to SC capacity is not direct as in the other two cases. Since the operational capacity of SC is not explicitly included in the dispatch algorithm. However, it can be calculated for any required branch using the relationship shown in (6.6). The representation of active power flow $z_m$ to the branch-node sensitivity matrix $H$ and the supply/demand ($g_i, d_j$) is given by the relationship, $z_m = \sum_i H_{m,i}g_i - \sum_j H_{m,j}d_j$. A unit (1%) change (decrement) of reactance in branch $m''$ will change the sensitivity
matrix $H$ by $\Delta H_{m'}$. The corresponding change in the congestion cost under the same generation and demand level will represent the shadow price of the series compensation or SC capacity. This can be calculated as

$$\mu_{Xc,m'} = \sum_{m \in M'} \mu_{z,m}(-\Delta z_m) \quad \forall m' \in M'$$

(6.6)

where $\Delta z_m = \sum_i \Delta H_{m',(i)}g_i - \sum_j \Delta H_{m',(j)}d_j$

This reflects the effect of reactance change of branch $m'$ on the constrained paths (i.e. on branches with $\mu_{z,m} > 0$). Note that when PSs are present in the network, flow equations in (6.1.e.1) and (6.1.e.2) are utilized to calculate $\Delta z_m$.

### 6.5.3 Yearly Models and Input Data

This multi-year problem formulation is broken down into yearly models as discussed in Chapter 5. The input data forecast for each planning year is also done in the same fashion as in Chapter 5.

### 6.6 Case Studies

The combined network expansion algorithm with network reinforcements and FACTS options was implemented with modified IEEE 24-bus Reliability Test System (RTS) (RTS Task Force [70]). The network diagram including the candidate FACTS options is shown in Fig. 6.2. The rest of the details are same as those used in Chapters 3 and 5, including, branch cost and technical details, supply and demand price bids and quantities, load duration curve (LDC) details.
Figure 6.2: Modified IEEE 24-bus RTS – Candidate Network

The candidate lines for PS and SC installations were selected with initial test runs. For this purpose a network-constrained dispatch clearance was performed and the lightly loaded transmission corridors were identified. They were further tested for their sensitivity for PS and SC installation. Although any number of candidate lines can be selected, their appearance in the final solution depends on their individual effectiveness. For the case studies performed, 3 PSs in lines 5, 14 and 22, and 4 SCs in lines 7, 18, 22 and 27 were selected as possible candidate locations, shown in Fig. 6.2. Some good locations for PS installation in IEEE 24-bus RTS were reported in Lima et al. [90], which was used as a starting point for the selection. The maximum phase shift angle (\(\psi_m\)) capability of the PS is taken to be 10\(^\circ\).
The planning horizon \((T)\) is taken as 8 years and discount rate \((\tau)\) is 10%. The forecasting of the bidding details was represented with simple yearly increments in the bidding attributes and the generation and load demands. The growth in bids are represented by annual growth rates for intercepts \((b_i, b_j)\) and the maximum and minimum demand bids, shown in Table. 5.1 (Chapter 5). Increase in the maximum generation limit \((\bar{g})\) is introduced in discrete steps. For this test application we have assumed that the maximum generation capacity bid \((\bar{g})\) increases in the 3\textsuperscript{rd} and the 6\textsuperscript{th} year by a factor of 1.5 and 2.5 respectively. Elaborate models to represent the growths in demand and generation bids can be easily included without affecting the proposed methodology.

The network expansions were studied under three alternate capacity expansion schemes.

(a) Installation of PS with its maximum phase angle capacity varying continuously and in discrete steps of 2.5°.

(b) Installation of SC, with the maximum line compensation rating varying continuously up to 40 % and in discrete steps of 10%.

(c) Combination of network reinforcement and FACTS devices.

6.6.1 PS Installations

Three lines were selected for PS installation namely, lines 5, 14 and 22. For the value of specific cost, \(c\) reported in Oliveira et al. [88] (100 $/kVA) only one PS in line-22 becomes justified for installation in year 8. Because of the high investment cost, it is not justifiable with congestion cost savings until year 8 and not much insight was gained from this study. Therefore the planning study was carried out with the value of specific
cost reduced to 10 $/kVA. This was done for both continuous and discrete capacity upgrades. The phase angle capacity requirements are shown in Fig. 6.3.

![Figure 6.3: Phase Shifter Capacity Upgrade Schedule](image)

It should be noted that capacity additions in steps of 2.5° for the phase shifter may not be viable in practice unlike discrete transmission line additions. It is likely that capacity upgrade of ±10° as a lump investment, and it suffices to specify the investment year and the location in expansion planning studies. The lump investment also prevents technical problems arising when cascading phase shifters. But for the interest of studying the detailed behavior, we assume that installation in fractions of the total capacity (±2.5°) is possible and the investment costs are also pro rated in this analysis. The cost – benefit analysis for the discrete capacity addition is shown in Fig. 6.4. Figure shows the total benefit of network expansion \( NPV(\Delta E|TC(t_y)) \) over the investment cost \( NPV(IC_m(t_y)) \). However, it is worth mentioning that these investment decisions are justified on annuatised quantities base and investment cost is under our reduced value
(i.e. \( c = 10 \text{ S/kVA} \)). When exact cost details are used investment decisions will appear accordingly and which will be justified on annuatised basis.

![Cost-Benefit for Discrete Phase Shifter Installations](image1)

**Figure 6.4: Cost-Benefit for Discrete Phase Shifter Installations**

### 6.6.2 SC Installations

Line 7, 21, 22 and 27 were selected as the possible locations for SC installations and maximum compensation level was taken as 40% of the respective line reactance. The solution is shown in Fig. 6.5, which is found to be the same for continuous and discrete upgrades with upgrade step of 10%.

![Series Compensation Capacity Upgrade Schedule](image2)

**Figure 6.5: Series Compensation Capacity Upgrade Schedule**
Figure 6.6: Cost-Benefit for Discrete Series Compensation Installations

Lines 7 and 22 show SC installations up to its maximum allowable limit in year 3 and 5 respectively and line 27 to 10% of the branch reactance in year 5. The cost-benefit analysis given in Fig. 6.6 guarantees the economic feasibility for net present values of investment cost and congestion cost saving.

6.6.3 Network Reinforcements with FACTS Devices

In this case line reinforcements were considered simultaneously with FACTS devices. However, with actual construction costs and specific cost for FACTS as 100 $/kVA, only SCs were scheduled for entire planning horizon and the solution obtained was the same as in § 6.6.2. Therefore, further studies were conducted with different specific costs \( c \) for PS and SC. Investments were considered only in discrete steps.

6.6.3.1 The Network expansion schedule with \( c=10 \) $/kVA for PS and \( c=100 \) $/kVA for SC

Expansion Schedule obtained and the cost-benefit analysis are shown in Fig. 6.7 and Fig. 6.8 respectively.
Figure 6.7: Network Expansion Schedule –Case: 6.6.3.1

It is seen that PS installations come to network planning schedule and delays the branch capacity upgrades to later years in the planning horizon. The second circuit addition of line-23 and 28 were delayed until year 8, and two PSs were scheduled in branch 14 and 22. The incremental capacity additions of the PS justified by the avoidance of the congestion cost are as shown in Fig.6.7.

Figure 6.8: Cost-Benefit for Expansion Schedule –Case: 6.6.3.1
It is worth noting that a significant part of benefits of line additions fall beyond the planning horizon. However, line reinforcements are justified with annuatised quantities only until the end of the planning horizon.

6.6.3.2 The Network expansion schedule with $c=10 \$/kVA for PS and $c=50 \$/kVA for SC

The network expansion schedule is shown in Fig. 6.9. With the reduction in the specific cost of SC, SC installations appear in the expansion schedule. Branch capacity additions do not appear in the entire planning horizon and installation of PS and SC alone can provide congestion relief. It should be noted that the relative costs of the capacity enhancing options will decide the extent to which each option will feature in the expansion schedule. The cost-benefit analysis is shown in Fig. 6.10.

Figure 6.9: Network Expansion Schedule – Case: 6.6.3.2
When analyzing a real system, it is important to use real costs and the practical sizes of discrete steps for the FACTS devices in order to obtain meaningful expansion schedules. But it should be noted that the proposed framework is capable of selecting the optimal combination of the options available for capacity enhancements.

6.7 Concluding Remarks

The developments in FACTS technology and realization of their costs comparable to transmission reinforcement costs have made these devices strong contenders and possible alternatives to conventional network expansion. In addition, the inherent flexibilities these devices provide suit the uncertain flow patterns expected in the market environment very well. Hence, the inclusion of FACTS options along with the conventional methods to enhance network performance alleviating possible congestion was investigated. The dynamic network expansion planning procedure in Chapter 5 was extended to include flexible transmission options (phase shifters and series
compensation) along with network reinforcements.

Individual and combined options were explored, which would provide the optimal network improvement strategy for long-term congestion alleviation. The economic network expansion was guaranteed under this scheme for the break-even of long-term transmission investment cost and congestion cost saving. The modeling of two kinds of FACTS devices, namely PS and SC, has been presented for the purpose of the network dispatch problem. Appropriate models for the investment costs of these devices have also been developed and incorporated in the overall objective function of the long-term network reinforcement model.

Case studies have been conducted in a modified IEEE 24-bus system to generate network reinforcement schemes by considering the options of the line expansion and the incorporation of PS and SC devices separately as well as different combinations of these options. These studies adopted suitable capacities for the above options and considered both discrete step as well as continuously varying capacities. It has been demonstrated that the proposed methodology produces good reinforcement schemes, which balance the investment costs against the savings from the congestion costs achieved through congestion alleviation.

It is seen that these devices may not replace network expansion needed for long-term demand growth but these devices can certainly complement the network expansion in handling the short-term network flow variations, which may arise from different reasons including strategic behavior of the market participants.
Chapter 7

Consideration of Uncertainties in Network Expansion Planning

"Everything is vague to a degree you do not realize till you have tried to make it precise."

--- Bertrand Russell (1872 - 1970) ---

*British philosopher, logician, essayist, and renowned peace advocate*

7.1 Introduction

The TEP problem is a dynamic problem extending from the base-planning year to the end of planning horizon. It involves large amount of input data, which are subject to uncertainty and vagueness (David and Wen [7], la Torre *et.al.* [6], Crousillat *et.al.* [91]). In the context of deregulated markets, competition in supply as well as demand tends to make the participants not to reveal their actual cost and benefit details. The network flows in competitive markets are expected to exhibit a wide range of variations and therefore higher degree of uncertainty. It is attempted in this chapter to incorporate such uncertainties in short-term power dispatch (dispatch model) into long-term planning.

In the following problem formulation, uncertainties in bid price details, component outages, and hourly load variations are modeled using probabilistic techniques. The resulting stochastic mixed-integer optimization problem is solved using the *Generalized Benders Decomposition* (GDB) algorithm combined with non-sequential *Monte Carlo* technique. In the context of social welfare loss due to transmission constraints, the signals given by *congestion cost* were highlighted in Chapters 5 and 6. In addition to static congestion discussed in Chapters 5 and 6, temporal congestion may
occur due to uncertainties arising from various causes such as component outages, strategic behaviors of network users etc. These incidents happen in short time durations, but the magnitude of their economic implications may be significant in the long run. This may cause very short-term welfare losses which need to be added to the "cost of congestion", if it is caused by the network limitations. In addition to static (short-term) and temporal (very short-term) welfare losses due to congestion, if network limitations cause demand curtailments below their anticipated minimum levels, such effects should also be included in the "cost of congestion" from the network expansion point of view.

For this purpose, curtailments (or loss of load) will be evaluated at individual consumers’ marginal willingness to pay to avoid load curtailments (pricing of reliability). The overall economic effects of "transmission congestion" are computed on the basis of line flows, component outages and demand curtailments represented using probabilistic models. The network planning based on such an overall "cost of congestion" can be said to consider the uncertainties in the system operation, and driven jointly by economic as well as reliability criteria.

7.2 Problem Formulation

The overall TEP problem may be formulated as (7.1) by linking the long-term transmission investment costs and expected short-term societal costs into a single objective function.

\[
\begin{align*}
\text{Min} & \quad IC_y(t_y), g_l(t_h), d_j(t_h), r_j(t_h) \\
\alpha &= \sum_{t_y \in T} \sum_{m \in M} \frac{IC_y(t_y)}{(1 + \tau)^{T_y - t_0}} \\
\frac{E}{\sum_{t_h \in T} \sum_{i \in I} \frac{C_i(g_i(t_h))}{(1 + \tau)^{T_y - t_0}} - \sum_{t_i \in T} \sum_{j \in J} \frac{B_j(d_j(t_h))}{(1 + \tau)^{T_y - t_0}} + \sum_{t_h \in T} \sum_{j \in J} \frac{OC_j(r_j(t_h))}{(1 + \tau)^{T_y - t_0}}} \quad (7.1)
\end{align*}
\]
In (7.1) the long-term discounted values of social cost components are included. The investment costs \( I_{C_m}(t_y) \) have been included as long-term costs and short-term costs such as suppliers’ costs \( C_i(g_i(t_h)) \), consumers’ benefits \( B_j(d_j(t_h)) \) and consumer outage costs \( OC_j(r_j(t_h)) \) are incorporated taking the summation over the planning horizon. The objective function (7.1) is subjected to a set of long-term as well as short-term constraints listed below.

In the long run:

\[
I_{C_m}(t_y) \leq I_{C_m}(t_y) \leq \overline{I}_m(t_y) \quad \forall m \in M, \forall t_y \in T \quad (7.1.a)
\]

In the short run:

\[
\sum_j d_j(t_h) - \sum_i g_i(t_h) = 0 \quad \forall t_h \in T \quad (7.1.b)
\]

\[
d_j(t_h) + r_j(t_h) \geq d_j(\varphi) \quad \forall j \in J, \forall t_h \in T \quad (7.1.c)
\]

\[
g_j(\xi_i(\varphi)) \leq g_i(t_h) \leq \overline{g}_i(\xi_i(\varphi)) \quad \forall i \in I, \forall t_h \in T \quad (7.1.d)
\]

\[
0 \leq d_j(t_h) \leq \overline{d}_j(\varphi) \quad \forall j \in J, \forall t_h \in T \quad (7.1.e)
\]

\[
0 \leq r_j(t_h) \leq d_j(\varphi) \quad \forall j \in J, \forall t_h \in T \quad (7.1.f)
\]

\[
z_m(\xi_m(\varphi)) \leq z_m(t_h) \leq \overline{z}_m(\xi_m(\varphi)) \quad \forall m \in M, \forall t_h \in T \quad (7.1.g)
\]

Constraints 7.1.a-g will yield the dual prices \( \mu_{I_{C,m}}(t_y) \), \( \mu_e(t_h) \), \( \mu_j(t_h) \), \( \mu_{g,i}(t_h) \), \( \mu_{d,j}(t_h) \), \( \mu_{r,j}(t_h) \), and \( \mu_{z,m}(t_h) \) respectively where applicable.

### 7.2.1 Generation and Transmission Outages

It should be noted that the effects of generation and transmission outages have been incorporated in the above formulation. The outage costs to consumers are included
in the objective function and outage rates are modeled into generation and transmission limits to adequately represent those in the constraints. \( \xi_i, \xi_m \) represent stochastic 0-1 variables for generation and transmission line availability respectively depending on the operating state \( \phi \). Generation unavailability is assumed as a non-participation in the power dispatch caused by generation outage. Transmission unavailability is considered as an outage, both with known probabilities (\textit{Forced Outage Rate}, FOR).

The generation inadequacy is generally met by the reserve capacity in the short-run. Long-term reserve requirements are not considered in this analysis. Therefore, what is captured as the congestion cost under generation outages is the congestion contribution by the transmission network during such incidents. In other words, this model assumes sufficient generation sources to be available in the system. Effectively, generation reshuffling occurs during generation outages. Accordingly, high-price supply sources will replace the outage supply unit. This might cause transmission congestion if incoming generation is in transmission constrained area, including a social welfare loss, which will surface as the congestion cost.

### 7.3 Decomposition of Formulation

The overall problem formulation is a stochastic mixed-integer programming problem. This is decomposed into long-term investment scheduling problem and short-term social cost minimization problem as in Chapters 5 and 6.

#### 7.3.1 Long-Term Transmission Investment Scheduling Problem

The dynamic transmission investment scheduling problem of (7.1) for the entire planning horizon \( t, T \) can be separated out as an \textit{Integer Programming} (IP) problem
(7.2). The number of discrete circuit additions $n_m(t_y)$ along a particular right-of-way is
considered as an independent variable having its upper and lower bounds (7.2.a) fixed by
the network planner, where the investment cost per circuit addition $I_{z,m}(t_y)$ is known a
priori.

$$
\text{Min } n_m(t_y) \quad \alpha' = \sum_{t_y} \sum_{m} \frac{I_{z,m}(t_y) n_m(t_y)}{(1+\tau)t_y - t_0} 
$$

Subject to: constraint (7.1.a) as $n_m(t_y) \leq n_m(t_y) \leq \bar{n}_m(t_y) \quad \forall m \in M, \forall t_y \in T \quad (7.2.a)$

The network investments $IC_m(t_y)$ will bring down the expected social cost $E\{C-B+OC\}$
of energy dispatch in long run giving rise to a breakeven between the two cost quantities.

7.3.2 Short-term Social Cost Minimization Problem

The long-term (i.e. $t_h \in T$) representation of the short-term dispatch algorithm
$\text{Min } \{C-B+OC\}$ constitutes the rest of the formulation (7.1).

$$
\text{Min } g_i(t_h), d_j(t_h), r_j(t_h) \quad \alpha^* = \sum_{t_h} \frac{1}{(1+\tau)t_h - t_0} \left\{ \sum_i C_i(g_i(t_h)) - \sum_j B_j(d_j(t_h)) 
+ \sum_j OC_j(r_j(t_h)) \right\} 
$$

Subject to: constraints (7.1.b – 7.1.g)

In addition to supplier’s apparent production cost $C_i(g_i(t_h))$ (derived from supply bid)
and consumer benefit $B_j(d_j(t_h))$ (derived from demand bid) such an approach requires
modeling of outage cost $OC_j(r_j(t_h))$ when load curtailment occurs, into the objective
function.
The supplier cost included in the formulation is the same as in Chapters 5 and 6. However, demand modeling follows the representation given in Fig. 7.1. When consumer’s consumption is curtailed below their minimum demand requirement (or lower bound of elastic demand) $d_j$, it incurs a heavy outage cost $OC_j(r_j(t_h))$. The demand curtailment $r_j(t_h)$ from the lower bound of the elastic demand $d_j$ is evaluated at marginal outage cost (or the value of lost load, VOLL) $MOC_j$ of the respective consumer in the process of network expansion planning (See Fig. 7.1). This allows using appropriate benefit functions with corresponding outage costs to different consumers. The consumer benefit and outage cost, using the modified demand function, is incorporated in the optimal short-term supply and demand scheduling for the optimization of (7.3).

![Figure 7.1: Reliability Differentiated Consumer Demand Function](image)

7.3.3 Value of Reliability

The "reliability differentiated" (Siddiqi and Baughman [92], Siddiqi [93]) dispatch algorithm (7.3) with consumers’ willingness to pay against the service
interruptions (at marginal outage cost) will provide efficient economic signals relating to network capacity limits during outages as well as in healthy operations. The nodal prices $\rho_i(t_h)$ and $\rho_j(t_h)$ deriving from the dispatch algorithm (7.3) captures the consumers’ willingness to pay against the service interruptions in addition to their short-term benefits and suppliers’ marginal cost in supplying power during such incidents.

**Reliability Differentiated Nodal Prices**

For supplier $i$:

$$\rho_i = (\partial C_i / \partial g_i) - \mu_{\text{min},g,i} + \mu_{\text{max},g,i}$$

$$= \mu_e + \mu_{\text{min},z,m}(\partial z_m / \partial g_i) - \mu_{\text{max},z,m}(\partial z_m / \partial g_i) \quad (7.4.a)$$

For consumer $j$:

$$\rho_j = (\partial B_j / \partial d_j) + \mu_{\text{min},d,j} - \mu_{\text{max},d,j} - (\partial OC / \partial d_j) - \mu_{\text{min},r,j} + \mu_{\text{max},r,j}$$

$$= \mu_e - \mu_{\text{min},z,m}(\partial z_m / \partial d_j) + \mu_{\text{max},z,m}(\partial z_m / \partial d_j) \quad (7.4.b)$$

During outages or service interruptions nodal prices are significantly high at the nodes where demand curtailments are foreseeable. This pricing methodology contains very short-term pricing implications as well as short-term pricing signals to be useful in long-term resource planning.

From the network point of view the main reasons behind this spatial variation of nodal prices are due to (i) network losses and (ii) network congestion. Network losses are ignored in this expansion study, therefore the variation in nodal prices is entirely due to congestion. The congestion in the network and associated social welfare loss is caused by one or more of the following reasons,
a) Network capacity limit violations under normal operating conditions (including those caused by network users’ strategic behaviors).

b) Network capacity limit violation under self branch contingency (in multi-circuit branches)

c) Network capacity limit violation under other component (generator or transmission line) contingencies or the combined effect of contingencies.

These incidents, short term (a) or very short term (b and c) cause a loss of welfare in economic terms, which can be aggregated into a single attribute "congestion cost" due to network bottlenecks, $TC(t_h)$ (7.5).

$$
TC(t_h) = \left\{ \text{Max} \ (B - C - OC) \right\}_{l.h.b - l.h.f} - \left\{ \text{Max} \ (B - C - OC) \right\}_{l.h.b - l.h.g}
$$

$$
= \left\{ \text{Min} \ (C + OC - B) \right\}_{l.h.b - l.h.f} - \left\{ \text{Min} \ (C + OC - B) \right\}_{l.h.b - l.h.g}
$$

(7.5)

7.3.4 Economic Expansion

For economic network expansion the main objective function $\text{Min} \ IC + E\{C - B + OC\}$ seeks the breakeven between positive $\Delta IC$ and negative $\Delta E\{C - B + OC\}$. With regard to optimum network expansion it can be proven that the main objective function $\text{Min} \ IC + E\{C - B + OC\}$ and the modified objective of $\text{Min} \ IC + E\{TC\}$ lead to same investment decisions (extension of result 2 in Chapter 4). In other words, $IC + E\{C - B + OC\}$ and $IC + E\{TC\}$ are similarly behaved (or parallel) functions having identical optimum capacity expansions. Therefore, annual investment scheduling problem and annual short-term social cost minimization (dispatch algorithm) problem can be decoupled for economic network expansion, having $IC$ and $TC$ as
coupling information. The decoupled problem searches for the breakeven between (i) transmission investment costs, and (ii) possible savings in congestion costs towards optimal investments decisions.

It can be deduced from (7.5) that the shadow price $\mu_{z,m}$ (or marginal value) corresponds to (7.1.g) stands for the sensitivity of $TC$ to network capacity improvements. It indicates how much $TC$ is expected to reduce for infinitesimal (or unit) increase of network capacity. Therefore, the annual expected values of $E\{TC(t_y)\}$ and $E\{\mu_{z,m}\}$ from the dispatch algorithm become a significant complementary-pair for the yearly transmission expansion planning model, which drives network expansion towards optimality. However, both these attributes $TC$ and $\mu_{z,m}$ are time ($t_y$) or state ($\varphi$) dependent, and vary with (i) bidding patterns of suppliers/consumers, (ii) hourly load variations, and (iii) outages of generation and transmission. Therefore in evaluating these yearly expected values, all these variations and fluctuations need to be considered for effective network expansion.

7.4 Solution Methodology

This decoupled problem formulation is solved using the Generalized Benders Decomposition (GBD) technique. The annual investment scheduling problem becomes the master problem and annual stochastic dispatch algorithm is solved as the operational subproblem.
7.4.1 Master Problem

\[
\begin{align*}
\text{Min} & \quad \alpha \\
\text{Subject to} & \\
\alpha & \geq \sum_{t_y} \sum_{m} \frac{I_{z,m}(t_y)n_m(t_y)}{(1+\tau)^{t_y-t_0}} \\
\alpha & \geq \sum_{t_y} \frac{1}{(1+\tau)^{t_y-t_0}} \left\{ E\{TC(t_y)\} + \sum_{m} I_{z,m}(t_y)n_m(t_y) \right. \\
& \quad \quad \left. - \sum_{m} T_h E\{\mu_{z,m}\} Z_m \left( n_m(t_y) - n^k_m(t_y) \right) \right\} \\
n_m(t_y) & \leq n_m(t_y) \leq \bar{n}_m(t_y) \quad \forall m \in M, \forall t_y \in T
\end{align*}
\]

where, \( T_h \) is the hours per year (i.e. 8760), and \( k \) is the Benders’ iteration index. The master problem remains, as an integer programming problem and the feasibility cut (Benders’ cut) is formed by \( E\{TC(t_y)\} \) and \( E\{\mu_{z,m}\} \) provided by the stochastic optimization in the operational problem.

7.4.2 Operational Subproblem

The operational subproblem is solved for the stochastic bid-based dispatch algorithm representing annual expected behavior. The uncertainty in bid prices of suppliers and consumers, component outages in generation, transmission and hourly load variations generate large number of possible system states for the dispatch algorithm. However once those attributes (load level, price bids, and component availability) are fixed for a possible system state the optimization problem \( \text{Min} \ \{C - B + OC | 7.1.b - 7.1.g\} \) reduces to a deterministic Quadratic Programming (QP) problem, which can be solved using a standard QP algorithm to evaluate \( TC \) and \( \mu_{z,m} \). A non-sequential Monte Carlo
technique is employed to simulate the system states and yearly expected values of $E\{TC(t_i)\}$ and $E\{\mu_{z,m}\}$ are estimated accordingly. These attributes are exchanged with the Master Problem to perform the Benders’ iterations towards final network solution.

The branch power flows are modeled using DC power flow techniques as,

$$z_m = \sum_i H_{m,i}^i g_i - \sum_j H_{m,j}^j d_j$$

where $H$ stands for the PTDFs matrix. $H_{m,i}$ and $H_{m,j}$ determine the supplier’s ($i$) and consumer’s ($j$) sensitivity on branch flow $z_m$. $H$ is modified following every transmission branch outage to represent the state defined by the remaining transmission network. The shadow prices ($\mu_{z,m}$) of the exiting branches can be directly obtained from the dispatch algorithm, but the shadow prices of new lines ($m'$) which are not a part of the network in this iteration (but exiting in the master problem) can be approximated by the following equation (7.7).

$$\mu_{z,m'} = \begin{cases} \sum_{m \in M} \mu_{z,m} \Delta z_m & \text{if} \quad \sum_{m \in M} \mu_{z,m} \Delta z_m < 0 \quad \forall m' \in M' \\ 0 & \text{otherwise} \end{cases} \quad (7.7)$$

where $\Delta z_m = \sum_i \Delta H_{m,i}^i g_i - \sum_j \Delta H_{m,j}^j d_j$ and $\Delta H_{m'}$ is the change of $H$ when branch $m'$ is added. $M'$ represents the set of new (candidate) branches which were not selected by the master problem in the previous iteration. In order to have a significant impact from this line addition the net value of this addition $\sum_{m \in M} \mu_{z,m} \Delta z_m$ should be negative, the positive net values need not be considered. Further, this reflects the magnitude and directional effect of new branch $m'$ on the congested branch ($i.e.$ $\mu_{z,m} > 0$) flows.
7.4.3 Stochastic Simulation – Monte Carlo Technique

The following steps are followed to calculate $E \{ TC(t_j) \}$ and $E \{ \mu_{z,m} \}$.

1. Simulate the system state $\varphi$, representing load level $\varphi_d$, generation and transmission availability $\varphi_g, \varphi_m \forall i, m$ and bid levels $\varphi_i, \varphi_j \forall i, j$.

2. Run the short-term dispatch algorithm $\text{Min} \{ C - B + OC \ | \ 7.1.b - 7.1.g \}$ for $t_h$ and obtain the values, $TC$ and $\mu_{z,m}$.

3. Update the expected values $E \{ TC(t_j) \}$ and $E \{ \mu_{z,m} \}$.

4. Check for the accuracy$^6$, if accuracy is unacceptable return to step 1 for the next system state, else Stop.

7.4.3.1 Load Level Selection

The stochastic load level is selected on the basis of the normalized chronological discrete load duration curve (LDC), which has discrete number of steps with known probability durations. A uniformly generated random number $n_{LDC} \sim U[0,1]$ is used to simulate the state ($\varphi_d$ in discrete LDC) and all the demand upper bounds $\bar{d}_j$ are multiplied with the corresponding normalized load level $LDC(\varphi_d)$.

7.4.3.2 Generation and Transmission Outage Selection

For network planning purposes only single component outages (i.e. $N-1$ contingency) are considered in either generation or transmission. It may be a generator failure or transmission line failure or healthy state based on components forced outage rates. This covers the single contingency criterion adopted by most of the utilities.

---

$^6$ Accuracy is checked with the relative uncertainty in $TC$, i.e. $\beta_{TC} = \left( \frac{\text{Var}(TC)}{N} \right)^{1/2} \frac{E(TC)}{E(TC)}$.

where, $\beta_{TC}$ – relative uncertainty of $TC$, $N$ – Sample Size, $E(TC)$ – expected value of $TC$, $\text{Var}(TC)$ – variance of $TC$.

For the examples given $\beta_{TC}$ was checked for 2% accuracy.
worldwide. The stochastic contingency selection method adopted identifies the system operating state from \( I + M + 1 \) (generation failure, \( p_i \) + transmission failure, \( p_m \) + healthy, \( p_0 \)) possible states. The probability of state occupancy, \( p \) is determined by (7.8) on the basis of outage rates.

\[
p_i = \frac{\lambda_{g_i}}{\lambda_{g_k}} \prod_{k=1}^{k \neq i} \left(1 - \frac{\lambda_{g_k}}{\lambda_{g_i}}\right) \prod_{k=1}^{k=M} \left(1 - \frac{\lambda_{z_k}}{\lambda_{g_i}}\right) \quad \forall i \in I \quad (7.8.a)
\]

\[
p_m = \frac{\lambda_{z_m}}{\lambda_{g_k}} \prod_{k=1}^{k \neq m} \left(1 - \frac{\lambda_{z_k}}{\lambda_{g_k}}\right) \prod_{k=1}^{k=M} \left(1 - \frac{\lambda_{z_k}}{\lambda_{z_m}}\right) \quad \forall m \in M \quad (7.8.b)
\]

\[
p_0 = \prod_{k=1}^{k=M} \left(1 - \frac{\lambda_{g_k}}{\lambda_{g_i}}\right) \prod_{k=1}^{k=1} \left(1 - \frac{\lambda_{z_k}}{\lambda_{z_m}}\right) \quad (7.8.c)
\]

where, \( \lambda_{g_i} \) and \( \lambda_{z_m} \) are the FORs of generator \( i \) and branch \( m \) respectively.

Obviously, \( p_0 + \sum_{i \in I} p_i + \sum_{m \in M} p_m \) will not be unity due to the omission of higher order outages. Therefore, \( p_i \) and \( p_m \) are prorated keeping \( p_0 \) unchanged in order to obtain the total probability equal to one (1) without loosing much accuracy. Thereafter, the healthy state, \( p_0 \) together with single contingency states having state probabilities \( p_i' \) and \( p_m' \) collectively form an exhaustive set with commutative probability equals to one (i.e. \( \sum p' = 1 \)). Next, a uniformly distributed random number (~\( U[0,1] \)) is used to simulate the system operating state i.e. either outage state (generator/transmission failure) or healthy state based on state probabilities \( p_0 \), \( p_i' \) and \( p_m' \).

7.4.3.3 Supplier and consumer bid variations

The bid price uncertainties are incorporated by modeling \( b_i \) and \( b_j \) of marginal
cost \((MC_i = b_i + m_i g_i(t_h))\) and marginal benefit \((MB_j = b_j - m_j d_j(t_h))\) functions of suppliers and consumers as stochastic variables. The parameters \(b_i\) and \(b_j\) are modeled as normally distributed variables with known mean and variance. Accordingly, for each simulation run \(\tilde{b_i}\), \(\tilde{b_j}\) are generated randomly using (7.9) for all \(i\) and \(j\).

\[
\tilde{b_i} \sim N(b_i, \sigma_i^2) \quad \text{and} \quad \tilde{b_j} \sim N(b_j, \sigma_j^2) \quad (7.9)
\]

More specific case studies can be performed with different statistical models, like log-normal for base load plants and normal variations for peaking plants. However, this needs more studies to determine the correct long-term implication of short-term price bids.

### 7.5 Illustrative Examples

The proposed network planning method is illustrated by implementing it in the modified Garver’s 6-bus (Garver [1]) and IEEE 24-bus (RTS Task Force [70]) test networks. Some additional data have been adopted (where not available with the original system) to bring out the important features of the application. The planning study shown is performed for a single planning year (static) for both test networks. It can be extended to a longer planning horizon following similar procedures discussed in Chapters 5 and 6.

#### 7.5.1 Garver’s 6-bus Test Network

##### 7.5.1.1 Network and Other Input Details

The network diagram is shown in Fig. 7.2, which has 11 generating units belongs to 3 Gencos and 5 distribution companies (Distcos). The mean supplier and consumer bidding details for the planning year are shown in Table 7.1. However, this is assumed to represent the entire year for this network expansion study. Deviation from this mean is
simulated with probabilistic deviation as described in 7.4.3.3. The bidding uncertainties were implemented by selecting $\sigma_i = 2 \, \text{$/MWh}$ for $\forall i$ and $\sigma_j = 1 \, \text{$/MWh}$ for $\forall j$. Branch details are shown in Table 7.2. Forced Outage Rates (FORs) adopted in this study, which were not available with original data, are also shown in the table.

![Figure 7.2: Garver’s 6-bus Test Network](image)

**TABLE 7.1: BIDDING DETAILS – GARVER’S 6-BUS SYSTEM**

<table>
<thead>
<tr>
<th>No</th>
<th>Genco Bids</th>
<th>Distco Bids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b_i$</td>
<td>$m_i$</td>
</tr>
<tr>
<td>1</td>
<td>36.0</td>
<td>0.025</td>
</tr>
<tr>
<td>2</td>
<td>36.8</td>
<td>0.025</td>
</tr>
<tr>
<td>3</td>
<td>37.5</td>
<td>0.025</td>
</tr>
<tr>
<td>4</td>
<td>38.3</td>
<td>0.025</td>
</tr>
<tr>
<td>5</td>
<td>30.0</td>
<td>0.045</td>
</tr>
<tr>
<td>6</td>
<td>32.7</td>
<td>0.045</td>
</tr>
<tr>
<td>7</td>
<td>35.4</td>
<td>0.045</td>
</tr>
<tr>
<td>8</td>
<td>40.8</td>
<td>0.045</td>
</tr>
<tr>
<td>9</td>
<td>20.0</td>
<td>0.032</td>
</tr>
<tr>
<td>10</td>
<td>23.8</td>
<td>0.032</td>
</tr>
<tr>
<td>11</td>
<td>31.5</td>
<td>0.032</td>
</tr>
</tbody>
</table>

† Units: $b_i, b_j$ in $\text{$/MWh}$; $m_i, m_j$ in $\text{$/MW^2h}$; $g, \bar{g}, d, \bar{d}$ in MW

‡ $d$ is taken as 90% of $\bar{d}$
The marginal outage cost for all consumers were taken as 1,000 $/MWh. A 10-step discrete LDC adopted to represent the annual variation of load is detailed in Appendix A-5. The annuatised branch construction costs were calculated from the specific cost of $ 1,000 per MW-mile and branch capacities, assuming 25 years of economic life and 10% discount rate.

**TABLE 7.2: BRANCH DETAILS – GARVER’S 6-BUS SYSTEM**

<table>
<thead>
<tr>
<th>m</th>
<th>st(m)</th>
<th>en(m)</th>
<th>react. (p.u.)</th>
<th>capacity (MW)</th>
<th>length (miles)</th>
<th>FOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.4000</td>
<td>100</td>
<td>40</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0.6000</td>
<td>80</td>
<td>60</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>0.2000</td>
<td>100</td>
<td>20</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>0.2000</td>
<td>100</td>
<td>20</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4</td>
<td>0.4000</td>
<td>100</td>
<td>40</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>5</td>
<td>0.2000</td>
<td>100</td>
<td>20</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>2</td>
<td>0.3000</td>
<td>100</td>
<td>30</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>4</td>
<td>0.3000</td>
<td>100</td>
<td>30</td>
<td>0.05</td>
</tr>
</tbody>
</table>

7.5.1.2 Test Results

For comparison purposes two studies were performed,

A. Without supply/demand price bid uncertainties or generation/transmission outages

B. With uncertainties as described above.

The solution trajectory along Benders’ iterations is shown in Fig. 7.3 for the two cases. It was observed that case-B takes comparatively higher number of iterations compared to case-A. This is expected because of the added non-linearity present in the problem with uncertainties. These network solutions exhibit how uncertainties influence the final solution when they are incorporated in the expansion planning. $E\{TC\}$ is notably different when uncertainties are considered, and different transmission facility upgrades are selected by the algorithm in reaching the optimal. The final network
solutions obtained are shown in Table 7.3, along with the corresponding investment cost and expected congestion cost in '000 $/year. Line 3 (L-03), which links the two generating nodes in the upper part of the system is additionally strengthened. This shows that in the presence of uncertainties the expensive generation at node 1 may be required to generate more power, therefore reinforcement in line 3 (L-03) becomes desirable under such situations.

![Figure 7.3: Convergence of Benders Iterations](image)

### Figure 7.3: Convergence of Benders Iterations

#### Table 7.3: Network Solution – Garver’s 6-bus System

<table>
<thead>
<tr>
<th></th>
<th>L-03</th>
<th>L-06</th>
<th>L-07</th>
<th>L-08</th>
<th>IC$^+$</th>
<th>E{TC}$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case -A</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1,700</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Case -B</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1,900</td>
<td>301</td>
</tr>
</tbody>
</table>

† in '000 $/year
7.5.2 IEEE 24-bus Test Network

7.5.2.1 Network and Other Input Details

The network diagram for the modified IEEE 24-bus test system is shown in Fig. 7.4, which has 14 Gencos and 17 distribution companies (Distcos). The original 24-bus system consists of 32 generators. For this study the 32 generators have been remodeled as 14 generators belonging to 14 Gencos. This modification is adopted to enhance the effects of generator failures in this study: A single generator failure does not produce a noticeable effect in the presence of a large number (32) of generators. However the proposed approach can be applied to any network system in practice.

![Diagram of IEEE 24-bus Test Network](image)

Figure 7.4: Candidate IEEE 24-bus Test Network
Other input details are as follows,

- Supplier and consumer mean bidding details are as given in Table 7.4.
- FOR of generating units and branches are taken from (RTS Task Force [70]).
- Two new branches, branches 39 and 40, are added for the purpose of this study (not reported in the original network). The branch details are shown in Appendix A-8.
- Branch capacities are assumed to be half of the original values given in (RTS Task Force [70]) as shown in Appendix A-7.
- The annuatised branch construction costs were calculated from the specific cost of $1,000 per MW-mile and branch capacities.
- The marginal outage cost for all consumers are taken as 1,000 $/MWh.
- A 20-step discrete LDC given in Appendix A-6 is adopted.
- The bidding uncertainties are implemented by selecting $\sigma_i = 2 $/MWh and $\sigma_j = 1 $/MWh.

### 7.5.2.2 Test Results

The network solutions obtained for the modified IEEE 24-bus test system are shown in Table 7.5 for the two cases (i) without uncertainty – Case A (ii) with uncertainty – Case B (Solutions 1-3). Two near optimal solutions (Solution 2, 3) are shown along with the best solution obtained (Solution 1). One desirable property of Benders Decomposition algorithms is that it shows some near-optimal solutions that may be worth considering in the decision making. These sub optimal solutions differ only slightly in terms of the overall objective $IC + E\{TC\}$ but shows very different $IC$ and $E\{TC\}$ component values.
### Table 7.4: Bidding Details – IEEE 24-bus System

<table>
<thead>
<tr>
<th>No</th>
<th>node(i)</th>
<th>Genco Bids</th>
<th>Distco Bids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(b_i)</td>
<td>(g)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>70.58</td>
<td>0.046</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>24.30</td>
<td>0.043</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>70.58</td>
<td>0.031</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>24.30</td>
<td>0.074</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>33.91</td>
<td>0.064</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>32.93</td>
<td>0.062</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>41.08</td>
<td>0.067</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>19.76</td>
<td>0.07</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>19.76</td>
<td>0.051</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
<td>10.00</td>
<td>0.073</td>
</tr>
<tr>
<td>11</td>
<td>21</td>
<td>10.00</td>
<td>0.057</td>
</tr>
<tr>
<td>12</td>
<td>22</td>
<td>23.82</td>
<td>0.013</td>
</tr>
<tr>
<td>13</td>
<td>23</td>
<td>19.76</td>
<td>0.044</td>
</tr>
<tr>
<td>14</td>
<td>23</td>
<td>19.22</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† Units: \(b_i, b_j\) in $/MWh; m_i, m_j\) in $/MW^2h; \(g, g, d, \overline{d}\) in MW

‡ \(d\) is taken as 90% of \(\overline{d}\)

### Table 7.5: Network Solution – IEEE 24-bus System

<table>
<thead>
<tr>
<th>Branch (m)</th>
<th>Case- A</th>
<th>Case- B</th>
<th>Case- B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solution-1</td>
<td>Solution-2</td>
<td>Solution-3</td>
</tr>
<tr>
<td>L-10</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>L-19</td>
<td>2</td>
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<td>1</td>
</tr>
<tr>
<td>L-23</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>L-28</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>L-30</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>L-39</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>IC\dagger</td>
<td>1,955</td>
<td>1,608</td>
<td>1,357</td>
</tr>
<tr>
<td>E{TC}\dagger</td>
<td>8</td>
<td>642</td>
<td>1,035</td>
</tr>
<tr>
<td>IC+E{TC}\dagger</td>
<td>1,964</td>
<td>2,250</td>
<td>2,393</td>
</tr>
</tbody>
</table>

† in '000 $/year
Figure 7.5: Network Solutions - IEEE 24-bus Test System

Figure 7.5 shows the composition of the network solutions in terms of the investment cost, $IC$ and expected congestion cost, $E\{TC\}$. With such additional information the network planner has higher flexibility in the final network selection process. It may be preferable to select solution 3 with higher investment costs but substantially low congestion cost if the objective is to have a robust network with little congestion. On the other hand, at times of capital shortages, it may be preferable to select solution 2 with higher congestion cost but very low investment cost. Or one may simply prefer most cost effective solution such as solution-1. Thus, this network planning procedure can provide an important feature of planning alternatives, which can be valuable in the decision making process.

7.6 Concluding Remarks

A comprehensive transmission expansion planning model was presented in this chapter, which incorporates various uncertainties involved in the dispatch model. Bid price uncertainties, random failures of generators/transmission lines, and hourly
variations in loads were incorporated in the problem formulation to reflect their influence on congestion cost and marginal value of transmission. A new way of including uncertain information to network expansion planning was proposed which should be useful in the present day deregulated systems with large amount of imprecise planning details. The $N-1$ reliability criteria were used as a standard to simulate the component outages into network planning studies.

The complex problem formulation was solved using a hybrid algorithm of Generalized Benders Decomposition and non-sequential Monte-Carlo techniques. This formulation guarantees sufficiently optimal economic network expansion along with a set of near optimal solutions under the uncertainties considered. The planning studies were conducted by implementing the developed techniques in two different systems, namely (i) Garver’s 6-bus network, and (ii) IEEE 24-bus network. Results of the study were discussed showing the effectiveness of the method and the usefulness of the results obtained. It is worth noting that the methodology discussed in this chapter involves a significant computational burden. However, with the modern day super computing technologies, which are widely used in utility planning works, this barrier can be overcome even for large practical networks.
Chapter 8

Conclusion and Recommendations

"Reasoning draws a conclusion, but does not make the conclusion certain, unless the mind discovers it by the path of experience."

--- Roger Bacon (1214-1292) ---

English Scholar, Philosopher

8.1 Conclusions

In spite of the market liberalization in electric utility industry, the transmission sector has effectively remained a centrally controlled entity, due to some well-known or well-worn reasons - such as economy of scale, economy of scope, the "lumpiness" in transmission investments and heavy external costs associated with investments. This has apparently resulted in insufficient transmission investments and capacity additions in many countries around the world. Thus the transmission sector may be the weakest link which can undermine the potential benefits of deregulated industry if it continues to remain so. The transmission sector is gaining some attention recently following the mega-blackouts in some countries. These concerns motivated this research and provided the impetus to carry out analysis starting from the fundamental concepts.

First, a network constrained power dispatch model has been developed which is a basic tool in any transmission planning exercise. A number of special features have been incorporated in this dispatch model, which are suitable for day-ahead electricity markets. Many important technical and economic attributes, such as transmission congestion,
congestion cost, congestion revenue, and marginal value of transmission were defined and evaluated as a part of the network dispatch. These attributes would describe the economics of power transmission blending the technical and market model characteristics. These details would identify network bottlenecks and evaluate the appropriate price of the constrained network resources for day-ahead electricity auctions. Further, these attributes were extended or modified to suit the needs of the centralized and decentralized long-term economic network expansion formulations.

Two basic models (i) centralized network expansion carried out by a regulated network planner to maximize the welfare of the network users, and (ii) decentralized network expansion carried out by various forms of profit maximizing merchants were identified. The theoretical investigations in Chapter 4 conducted a systematic analysis of various market models under a common platform to identify the optimal network expansion levels from the perspectives of these different network planners. It was shown that the optimal capacity expansion under a single (monopolistic) transmission merchant would only be half of what would be deemed optimal by a regulated central planner. It was further shown that this situation can be improved by introducing competition among transmission merchants, and a perfect competition would lead to the social optimum expansion. It should be noted that this analysis starts with a common objective function which is suitably modified to meet the interest of the planner of the corresponding market model. The analysis has further been extended to specific market models, such as (i) network users holding congestion revenue rights, (ii) network users participating for competitive revenue rights acquisition, etc. Some interesting relationships have been revealed and important conclusions have been drawn regarding the optimal levels of
network expansion under different network market models. Simple examples were presented to illustrate the findings of the theoretical investigations. Some assumptions were made to keep the analysis simple so that the various relationships could be clearly demonstrated.

While it was shown that under-investment in transmission expansion is quite likely under some merchant transmission investment models, it was clearly shown that this situation could be averted to achieve social optimum by ensuring perfect competition and revenue rights transfers. Though it may not be easy to guarantee perfect competition, it is shown that the optimal expansion sought by various planners under perfect competition reduces to the socially optimal value.

The formulation and implementation of long-term network expansion planning from the perspective of central planner was presented in Chapter 5. This dynamic formulation is resolved into a series of "pseudo" dynamic problems and annuatised quantities are used in annual models. Two investment management policies were introduced (i) saving of congestion cost with necessary network expansions, and (ii) containing of congestion revenue under a pre-agreed "ceiling" with necessary network expansions. The network investment cost was justified by the increased social welfare (or congestion cost saving) in the congestion cost saving method. Congestion revenue containing approach was implemented as a regulated price-cap method, which contains excessive network congestion through necessary network expansion. The Generalized Benders Decomposition based technique was used as the solution method, which could successfully handle the complex mixed-integer problem formulation. The procedure can be applied to formulate expansion schemes, which are sets of time ordered investment
decisions for the network planner. Theoretical development of the procedure along with the necessary algorithms and the implementation of the procedure in a test system have been presented.

The developments in new technologies such as FACTS and realization of their costs comparable to transmission reinforcement costs have made these devices strong contenders and possible alternatives to conventional network expansion. In addition, the inherent flexibilities these devices provide suit the uncertain flow patterns expected in the market environment very well. Hence, the inclusion of FACTS options along with the conventional methods to enhance network performance alleviating possible congestion was investigated. The complete formulation and implementation of the planning problem has been presented in Chapter 6. It was shown that these devices may not replace network expansion needed for long-term demand growth but these devices can certainly complement the network expansion in handling the short-term network flow variations, which may arise from different reasons including strategic behavior of the market participants.

Finally, the transmission expansion planning model is extended in Chapter 7 to include various uncertainties involved in the dispatch model. Three different types of uncertainties were considered (i) bid price uncertainties, (ii) random failures of generators and transmission lines, and (iii) hourly variations in load. These uncertainties were incorporated in the problem formulation in various ways to reflect their influence in the congestion cost and marginal value of transmission. However, the final problem formulation became very complex and appropriate ways were adopted to obtain a reasonably optimal solution(s). The hybrid algorithm using the Generalized Benders
Decomposition with non-sequential Monte-Carlo technique is adopted to obtain a good final solution to the user specified accuracy. Some near optimal solutions were also made available along the iterative path. It was shown how these near optimal solutions provide the planner with more flexibility in selecting the most desirable network expansion.

In summary, transmission planning in the context of deregulated power markets has been addressed in this thesis in a comprehensive way. In addition to the theoretical contributions various findings detailed in different chapters are expected to enhance and contribute to the evolving practice of transmission planning in today’s transmission market environment.

8.2 Recommendations and Future Works

This research addressed several contemporary issues in the context of evolving transmission market particularly in relation to transmission expansion planning including some detailed fundamental analysis of various market models. There is ample scope to extend this research in many directions and exploit the fundamental ideas developed. Some obvious future directions and recommendations are listed below:

- The day-head double-sided power auction formulation in Chapter 3 can be extended to a more comprehensive one by using AC-OPF techniques to fully utilize the marginal cost based pricing signals to resource planning. This can be employed to derive necessary information not only on active power constraints but also on reactive power constraints. These details would be particularly useful in planning the reactive power resources in competitive power markets.
- Analysis of various market models including the decentralized network expansion with possible transmission rights was presented in Chapter 4, which certainly not
the final words on the topic. These issues are relatively new and their proper considerations will be critical in the evolution of transmission expansion market environment. Therefore, different aspects of decentralized network expansion and issues related to transmission rights for the same will continue to remain a major research topic in future, until some consensus is reached in this respect.

- The *Monte-Carlo* technique employed in Chapter 7 can be improved for higher efficiencies using better sampling techniques. This would be especially useful when planning for large practical networks.
Author’s Publications

Portions of different sections of this research work have been reported in various publications.

Journal Papers:


Invited Panel Paper:


Conference Papers:


References


Appendixes

Appendix A-1: Power Transfer Distribution Factors (PDTFs, \( H \))

Let the nodal power injection vector, \( y \in \mathbb{R}^N \)

And, susceptance matrix, \( B \in \mathbb{R}^{N \times N} \), voltage angle vector \( \theta \in \mathbb{R}^N \).

where, \( y_n = \sum_{i \in I_n} g_i - \sum_{j \in J_n} d_j \) and \( I_n, J_n \) represent the set of suppliers and consumers connected to node \( n \).

This results: \( y = B \theta \Rightarrow \theta = X.y \)

where,

\[
X = \begin{bmatrix}
B_{N-1,N-1}^{-1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0
\end{bmatrix} - \text{node } N \text{ is considered as the reference node.}
\]

The DC power flow assumptions yield, \( z_m = (1/x_m)(\theta_{st(m)} - \theta_{en(m)}) \)

where, \( x_m \) is the branch reactance in p.u..

Accordingly,

\( z = B_{\text{line}} \cdot A.\theta \in \mathbb{R}^M \) where, \( B_{\text{line}} = \text{diag}(1/x_m) \) and \( A = \mathbb{R}^M \times \mathbb{R}^N \)

\[
[A_{m,n}]_{M \times N} = \begin{cases}
1 & \text{if branch } m \text{ starts from node } n \\
-1 & \text{if branch } m \text{ ends at node } n \\
0 & \text{else}
\end{cases}
\]

Thus,

\( z = B_{\text{line}} \cdot A.X.y = H.y \) and \( H = B_{\text{line}} \cdot A.X \).
Appendix A-2: New "Barrier Method" for NLP Solution

Let the following NLP formulation:

\[
\begin{align*}
\text{Min} & \quad f(x) = \frac{1}{2} x^T(t)U(t)x(t) + u^T . x(t) \\
\text{Subject to:} & \\
& \quad g(x) = \frac{1}{2} x^T(t) . V . x(t) + v^T . x(t) = 0 \\
& \quad C \leq h(x) = \frac{1}{2} x^T(t) . W . x(t) + w^T . x(t) \leq D
\end{align*}
\]

where, \( x \) is an \( n \)-vector and \( f(x) \) is a scalar function
\( g(x) \) and \( h(x) \) are vector functions of size \( p \) and \( m \)

Steps

1. Initial value, \( x^0 = -U^{-1}u \) \( \{ \text{for } \nabla f = 0 \} \)

\( h(x) + s = D \) and \( h(x) - r = C \), where, \( s, r - \text{non negative} \ m\)-vectors

\( s^0 = r^0 = (1/2).(D-C) \)

parameters; \( 0 < \rho < 1 \quad e_m = [1, 1, \ldots, 1]^T \quad \varepsilon_1 = 10^{-3} \quad \varepsilon_2 = 10^{-6} \)

Lagrange multipliers, \( y_{p \times 1} \) for \( g(x) \)

\( (h_u)_{m \times 1} \) for \( h(x) \) upper bound; \( (h_l)_{m \times 1} \) for \( h(x) \) lower bound

\( h_u^0 = h_l^0 = [1 + \|\nabla f\|] . e_m \quad y^0 = 0 \)

2. Penalties and Lagrange function

Let, \( \mu = \frac{1}{2m} \left( h_u^T s + h_l^T r \right) \quad \alpha = \gamma = \rho . \mu . e_m \)

\( d_1 = D - h - s \quad d_2 = C - h + r \)
Gradient of Lagrange function, \( \nabla L = \nabla f + \nabla g \cdot y + \nabla h \cdot (h_u - h_l) \)

3. Computation of \( A, b \)

\[
A = U + \nabla h (\Lambda_s^{-1} \Lambda_u + \Lambda_r^{-1} \Lambda_l) \nabla h^T
\]

\[
b = \nabla L + \nabla h (\Lambda_s^{-1} (\alpha - \Lambda_u, d_1) - \Lambda_r^{-1} (\gamma + \Lambda_l, d_2) - (h_u - h_l))
\]

where, \( \Lambda_s, \Lambda_u, \Lambda_r \) and \( \Lambda_l \) – diagonal matrices with diagonal elements as \( s, h_u, r \) and \( h_l \).

4. Calculating the increments of the control variable and Lagrange vectors.

\[
\begin{bmatrix}
\Delta x \\
\Delta y
\end{bmatrix} = -A^{-1}
\begin{bmatrix}
b \\
g
\end{bmatrix}
\]

\[
\Delta s = d_1 - \nabla h^T \Delta x
\]

\[
\Delta r = \nabla h^T \Delta x - d_2
\]

\[
\Delta h_u = \Lambda_s^{-1} (\alpha - \Lambda_u, \Delta s) - h_u
\]

\[
\Delta h_l = \Lambda_r^{-1} (\gamma + \Lambda_l, \Delta r) - h_l
\]

5. Determine the size of the increment

\[
N_1 = \text{Min} \left[ \frac{\Delta s_i}{s_i}, \frac{\Delta r_i}{r_i}; \ i = 1, 2, \ldots, m \right]
\]

\[
N_2 = \text{Min} \left[ \frac{\Delta h_{u,i}}{h_{u,i}}, \frac{\Delta h_{l,i}}{h_{l,i}}; \ i = 1, 2, \ldots, m \right]
\]

\[
\beta_1 = \begin{cases} 
1 & \text{if } N_1 \geq -1 \\
-\left(1/N_1\right) & \text{if } N_1 < -1
\end{cases}
\]

\[
\beta_2 = \begin{cases} 
1 & \text{if } N_2 \geq -1 \\
-\left(1/N_2\right) & \text{if } N_2 < -1
\end{cases}
\]

6. Update variables

\[
\Delta x = \beta_1 \Delta x \quad \Delta s = \beta_1 \Delta s \quad \Delta r = \beta_1 \Delta r
\]

\[
\Delta y = \beta_2 \Delta y \quad \Delta h_u = \beta_2 \Delta h_u \quad \Delta h_l = \beta_2 \Delta h_l
\]

7. Test for termination

Compute \( \mu \) and \( \nabla L \), if \( \mu \leq \varepsilon_1 \) and \( \|\nabla L\| \leq \varepsilon_2 \) go to step 9 else go to step 8.
8. If both $N_1 \geq -0.995$ and $N_2 \geq -0.995$ go to step 2, else go to step 2 after adjusting variables as follows,

\[
x = x - 0.005 \Delta x \\
\Delta s = s - 0.005 \Delta s \\
\Delta r = r - 0.005 \Delta r \\
y = y - 0.005 \Delta y \\
\Delta h_i = h_i - 0.005 \Delta h_i \\
\Delta h_i = h_i - 0.005 \Delta h_i
\]

9. Stop with solution, $x, y, s, r, w$ and $z$
Appendix A-3: Benders Decomposition

Let the following general mixed integer problem:

$$\min_{x,y} f_1(x) + f_2(y)$$

Subject to:

$$A_1 x \leq b_1$$
$$A_2 x + A_3 y \leq b_2$$
$$y \geq 0$$ and $x$ integer variables

Partitioned problem:

$$\min_x f_1(x) + \min_y \left\{ f_2(y) \left| A_3 y \leq b_2 - A_2 x \right. \right\}$$

Subject to:

$$A_1 x \leq b_1$$

$x$ integer variables; $x \in A$

where $A$ is the set of $x$ for which constraints $A_3 y \leq b_2 - A_2 x$ is satisfied.

For each fixed $\bar{x}$ the resulting inner minimization problem is

$$\min_y f_2(y)$$

Subject to:

$$A_3 y \leq b_2 - A_2 \bar{x}$$ having dual prices (or Lagrange Multipliers) $\lambda$

$$y \geq 0$$
The partitioned Master Problem becomes:

\[
\begin{align*}
    \text{Min } & \quad z \\
    \text{Subject to } & \quad A_1 x \leq b_1 \\
    & \quad z \geq f_1(x) + \min_{y \geq 0} \left\{ f_2(y) + \alpha \left( A_3 y - b_2 - A_2 \bar{x} \right) \right\} \quad \text{(feasibility cuts)} \\
    & \quad x \text{ integer variables} \\
    & \quad x \in A
\end{align*}
\]

In order to ensure the feasibility of the sub problem we should include the 

\textit{infeasibility cut} in the form of

\[
\begin{align*}
    \min_{y \geq 0} \left\{ \beta \left( A_3 y - (b_2 - A_2 \bar{x}) \right) \right\} \leq \varepsilon.
\end{align*}
\]
Appendix A-4:

Transmission Line Parameters (For IEEE 24-bus Test System)

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*† Capacities given are the modified capacities from (RTS Task Force [70]) for studies in Chapters 3, 5 and 6.
Appendix A-5: LDC Data – Garver’s 6-bus System

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Appendix A-6: LDC Data – IEEE 24-bus System

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Transmission Line Parameters (For IEEE 24-bus Test System)

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† Capacities given are the modified capacities from (RTS Task Force [70]) for studies in Chapter 7.
Appendix A-8:

Candidate Line Parameters (For IEEE 24-bus Test System)

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<th>(st(m))</th>
<th>(en(m))</th>
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<th>React. ((x \text{ pu}))</th>
<th>Capacity MW</th>
<th>Length (miles)</th>
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