PROPERTY MARKET BUBBLES: SOME EVIDENCE FROM SEOUL AND HONG KONG

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For 林荫修, my late teacher and mentor whose wisdom has lit up the way for me;
And for Ling Chung Tak, my husband whose emotional and financial support has instilled strength in me.
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Property Market Bubbles:

Some evidence from Seoul and Hong Kong

Xiao Qin

Abstract

Expectations are central to real-estate price formation, making speculative bubbles an inherent feature of real-estate markets. The literature has developed an array of tools on the detection of bubbles. Unfortunately, because of identification problems, none of them alone can give a definitive answer to the question, "Is a bubble in the asset price of concern?" In this thesis, the author will look at the property markets in Seoul and Hong Kong and present evidences of bubbles by examining the problem with various tools. More specifically, the Markov-switching ADF test will be used to verify the existence of a bubble; the Kalman filter to extract missing fundamentals, thus infer the magnitude of a bubble; and the power-law-log-periodicity theory to predict the future trajectory of a bubble. The last method is free of the identification problems plaguing the literature. All three approaches point to the existence of bubbles in the markets examined.

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Preface

The issue of speculative bubbles in the asset markets has occupied the minds of central bankers, academics, and practitioners for some time, but especially recently, in the midst of a heated debate about whether we are in the middle of a global real-estate bubble. After decades of academic effort, many doubts remain. Do speculative bubbles exist in asset markets at all? If they do, do they arise from investor irrationality and market inefficiency? Or can bubbles arise even in an efficient market in which the players have rational expectations? If they can, is there any clear-cut, effective measure to detect them? If left on its own, will the bubble deflate gently without causing much damage, or will it burst suddenly and violently, leaving a legacy of depressions and recessions? Should the central bank step in to intervene if it detects a bubble? What are the right ways of intervening?

Theories propose that speculative bubbles are endemic in the asset market, where expectations play a central role in determining asset prices. A speculative bubble may arise from occasional investor irrationality and market inefficiency, due to human psychology and market imperfections. It also can arise in a rational, efficient market, because of the indeterminacy of expected future events.

A wide range of methods for detecting speculative bubbles has been developed in the past two decades by academics studying financial markets. Each of these
methods relies on a set of assumptions about the fundamentals, whether in their structural model, or their distributional properties. There is, therefore, an unresolved identification problem in all cases. When the null hypothesis of “no bubble” is rejected (or accepted), it could be because of problems with the hypothesis on the fundamentals rather than the absence (or presence) of a bubble.

It looks as if there is no way of resolving this indeterminacy, as a bubble is typically inferred from fundamentals rather than observed, even though many believe that the experience of the world points to the existence of speculative bubbles.

This thesis has no intention—and is not able—to answer all the questions raised above. Its aim is merely to provide some empirical evidence of speculative bubbles in the real-estate market, using some of the most advanced techniques developed in the recent past. One of these techniques, which I will argue below, is free from the identification problems facing the empirical literature. These techniques will allow me to examine the rational speculative bubble from three different angles: to verify the existence of a bubble, to quantify the amount of the bubble, and to predict the most likely time at which the bubble ends. In order to verify the bubble, an unit root test procedure is adopted, and it is assumed that the price follows an AR(p) process. The parameters of this process changes according to a state variable that follows a first order Markov chain. This will be
referred to as “the Markov-switching ADF test procedure,” or simply “the MS ADF procedure.” It has been shown by Hall et al. (1999) that this procedure is fairly effective in detecting periodically collapsing rational speculative bubbles. To serve the second purpose of quantifying the bubble, it is necessary to assume a model which relates a property price to some other economics variables supposedly to be the economic fundamentals of the price. The present value model for asset pricing is adopted by this thesis for this purpose, as it is a well established theoretical model in the literature. To mitigate the problem of misspecifying this fundamental model, the Kalman filter technique is employed. This will be referred to as “the Kalman filter procedure.” The present-value model has been used frequently as a fundamental model by empirical researchers studying the stock and property markets. But the employment of Kalman filter or the like is not common, even though misspecification is often a criticism of the empirical literature.

Neither the MS ADF procedure nor the Kalman filter procedure will allow us to predict the burst of a bubble, however. In fact, none of the other procedures in the econometric literature would service this purpose. Therefore predicting was not in my plan until I encountered, at the later stage of composing this thesis, the power-law-log-periodicity (PLL) theory developed, primarily, by Johansen and Sornette.

Furthermore, the MS ADF and the Kalman filter procedures employed by this
thesis rely on assumptions about the behavior of the fundamentals. They are not free from the identification problem facing the literature. The PLLP theory, however, does not make any assumptions about the fundamentals—something that I argue is a salient feature of this theory. It replies on a simple idea: speculative forces tend to pull the price away from the fundamentals, which is reflected by the power-law signature; that is, the price must increase in an accelerating manner in order to compensate the ever-increasing risks involved the further away the price is from the fundamental value. At the same time, the fundamental forces struggle to exert themselves to pull the price back toward its intrinsic value. The interplay of the two forces is reflected by an accelerating log-periodic cycle around an accelerating mean of the price prior to a crash. The results from these three approaches are, nevertheless, compatible as I will show in chapter VII, “Conclusions and Discussions.”

The structure of this thesis is described below. The preface, that is the current chapter, serves the introductory purpose by stating the motivation, objective and contribution of this thesis.

The role of chapter I is to serve as an argument for why the real-estate market has been chosen for scrutiny. In the past two decades, many countries have experienced large cycles, and in particular market crashes, in the real-estate sector. Although these cycles and crashes have not been as pronounced as those witnessed by the stock market, the damage they caused was, nevertheless,
much more severe, because of the close tie between the real-estate and banking sectors. Falling real-estate prices often result in a banking crisis, or even worse, a financial crisis. Today, increased international trade and cross-country investments often turn a local crisis into a global disaster. The most glaring recent example is the Asian financial crisis.

Moreover, as real estate forms more than 50% of the wealth of the world, shrinking property values have a devastating impact on consumption and investment. This, in turn, reduces national income, which, in its own turn, depresses consumption and investment even further, triggering a vicious cycle, and in many cases precipitating a deep and chronic recession. Examples of this phenomenon are Japan in the 1990s and Hong Kong after the collapse of its property market in 1998. It is, therefore, important that policy makers understand the behavior of real-estate markets, and, more importantly, that they become skillful in taming often violent real-estate cycles.

A primary force behind real-estate cycles is a speculative bubble, especially in places where the supply of land is limited. As a bubble expands, it misdirects scarce capital away from productive investment projects to hopeless ones. When it bursts, it often results in a violent market crash, a trail of economic problems, and even a deep and prolonged recession.
In chapter II, I will introduce the literature on real-estate price bubbles. Many of the empirical studies of real-estate markets deploy tools developed for studying financial asset-price bubbles. The rationale is that both real property and stock are assets that can generate either a stream of consumption services directly or a stream of cash that can be exchanged for the stream of consumption services.

In chapter III, I will display and discuss the data sets used in my empirical studies, namely, the Hong Kong office price and rent indices, the Seoul housing price and rent indices, and the Korea general construction stock price index. I will argue, backed by facts, that the behavior of these prices warrants the investigation.

In chapters IV through VI, I will apply three methodologies to examine the problem of speculative bubbles in the real-estate markets of Hong Kong and Seoul. These methodologies will allow us to look at the problem from three angles: bubble identification, bubble quantification, and bubble prediction.

In chapter IV, I will implement one of the test procedures for detecting rational bubbles. That procedure is the unit root test, which incorporates the possibility that the bubble may swell, burst, and swell again. Following Blanchard and Watson (1982), I will call it a rational bubble. Although such rational bubbles are ruled out by Diba and Grossman (1988c) as theoretically impossible under certain assumptions, it is nevertheless empirically more appealing (Blanchard and Watson 1982). The test shows that a bubble of this type does exist in each
of price investigated. Thus, in the next chapter, I will turn to the natural question of what is the magnitude of the bubble.

In chapter V, following the practice of the empirical literature, I will try to infer the rational bubble from the residuals of a fundamental model, with the help of the Kalman filter. In particular, a present-value relationship will be used as the fundamental model, and the Kalman filter will be used to capture the misspecification of this model. The estimation shows that the inferred bubble from each price is neither constant nor explosive. Rather, it fluctuates, or it collapses periodically.

In chapter VI, I will tackle the dramatic impact of a periodically collapsing rational bubble on price behavior: a market crash. In particular, I will implement a crash-prediction procedure. Accurately predicting a market crash has important implications for policy. It has been argued that bubbles, if they exist, are driven mainly by certain forms of human behavior. Sornette and Johansen (1997) demonstrate that herd behavior among investors is reflected by a power-law and log-periodic pattern in the path of a price. They show that this pattern can then be used to predict a crash. My application of their methodology shows promising results.

Chapter VII concludes the paper and points to a few paths for future research.
Chapter I the Importance of Property Markets to the Well-being of Economies

From their beginnings in the early 1980s until today, financial deregulation and the globalization of capital markets have swept the world. The world has witnessed wild swings of stock and property prices. As is typical of business cycles, such swings have frequencies ranging from 3 to 10 years. Asset-price cycles have quite a pronounced effect on the well-being of national, and sometimes international, economies. In some cases, such as in Japan and Scandinavia during the late 1980s and early 1990s, the cycles turned out to have far-reaching, disruptive effects on domestic financial systems and played an important role in prolonging their respective recessions (IMF 2000). In other cases, such as in the United Kingdom during 1990–1992 and in Hong Kong following the 1997 Asian financial crisis, the financial system withstood the asset-price collapse well, but the ensuing recession was, nevertheless, quite severe (Gerlach and Peng 2004).

Cycles in the real-estate sector can affect economic fluctuations in a number of ways. The most notable example is the wealth effect of property-price swings on consumption and investment. During periods when real estate is booming, it has often been observed in both developed and developing countries that consumption and investment rise substantially. Conversely, there are usually severe contractions in these forms of spending during the down phase of a real-
estate cycle. These contractions in spending contribute to the decline of aggregate demand, which in turn causes a decrease in national output and employment.

Cycles in the real-estate sector exert their influence on another important part of the macro economy: the financial system. Financial intermediaries are, in general, widely exposed to the real-estate sector. It is often observed that bank credit and property prices occur in tandem. This could mean that, during a property boom, price bubbles, if they exist, will be intensified by easy credit. On the other hand, when a property-market bubble bursts, it causes nonperforming loans to accumulate. Nonperforming loans, in turn, lead to credit crunches, and credit crunches further worsen an economic contraction. What is more, the collapse of real-estate prices often results in a general financial crisis.\textsuperscript{iii} As witnessed by many Southeast Asian countries in the late 1990s, this could have a devastating impact on the real side of the economy.

Various studies show that asset prices do seem to provide useful information about the pace of future economic activity, and in particular about variations in the output gap. There is extensive empirical evidence that asset-price changes tend to lead output growth in industrial countries (Fama 1999; Choi, Hauser, and Kopecky 1999; Asprem 1989; Mauro 2000). In general, stock prices are found to have a significant predictive power on output growth in many countries. In contrast, property prices tend to be less forward-looking and more
contemporaneously correlated with output growth. While real property prices have been closely related to the business cycle in the industrialized world as a whole, in some countries the association is especially striking (IMF 2000; figs. 3.1, 3.4). Recessions in Japan and European Union countries since the early 1980s have been accompanied by falling property prices in real terms. Conversely, the strong upswings in economic activity in Australia and the smaller EU countries since the mid-1990s have been associated with robust growth in property prices (IMF 2000, 91–92). Similar patterns have been observed in Hong Kong, mainland China, and some other Southeast Asian countries in the past decade. In the case studies, more details will be given about these Asian economies and the role of their real-estate sectors.

1. The Real-estate Cycle, Banking Crises, and Financial Fragility

1.1. Property-price and Bank-credit Cycles

Cycles in credit and property prices typically occur in tandem and are often mutually reinforcing. Rising property prices can stimulate economic activity. For both firms and households, rising property prices reduce the cost of borrowing and increase the availability of financing. This is because rising property prices raise the value of collateral held by financial institutions. Credit growth and additional borrowing can then feed back into higher property prices. When property prices fall and economic conditions deteriorate, the interaction between
bank credit and the property market can be even more powerful. As the fall in property prices reduces the value of collateral, it can lead to a substantial loss by the financial institutions and ultimately to a contraction in the supply of credit.

Historically, bank lending has been closely correlated with property prices in both developed and developing economies. For example, in early 2000, Australia, Norway, Sweden, and the United Kingdom recorded large gains in residential real-estate prices. During this period, lending backed by residential mortgages also grew rapidly in these countries (BIS 2001).

The causality between property prices and bank lending can occur both ways. On the one hand, property prices may influence both demand for and supply of bank credit. Since properties are often used as collateral, changes in property prices may affect the borrowing capacity and credit demand of households and firms. Changes in property prices may also affect banks' lending capacity by affecting their capital position, both directly (through valuations of their holdings of real-estate assets), and indirectly (through changes in their non-performing loans). On the other hand, credit conditions may also affect property valuations, as increases in credit availability expand the demand for a temporarily fixed supply of properties (see Bernanke and Gertler 1989; Kiyotaki and Moore 1997; Bernanke, Gertler, and Gilchrist 1998; and Gerlach and Peng 2004).
The coincidence of cycles in bank credit and property prices has been widely documented in policy-oriented literature (cf. IMF 2000; BIS 2001). On the empirical side, Collynes and Senhadji (2001) find that credit growth has had a significant contemporaneous effect on residential-property prices in a number of Asian economies. Gerlach and Peng (2004) show that the strong correlation between property prices and bank lending in Hong Kong appears to be due to bank lending adjusting to property prices, rather than the converse. Goodhart (1995) also finds that property prices significantly affect credit growth in the United Kingdom but not in the United States.

1.2. The Property-price Cycle and Banking Crisis

International experience suggests that falling property prices have often played a central role in triggering banking crises. The collapse of the land price is clearly of central importance to the recent financial crisis in Asia, especially in Thailand and Indonesia. It is also central to the problems experienced by banks in Australia, Finland, Japan, Norway, Sweden, the United Kingdom, and the United States in the late 1980s and the early 1990s.

Since the 1980s, financial liberalization has been sweeping the world. Stiffer competition among financial intermediaries has induced banks to become increasingly engaged in non-traditional lines of business. Among these, asset trading and mortgage financing stand out. Throughout the world, banks’
exposure to the property market increased markedly during the 1980s. In some countries, this trend continued in the early 1990s (IMF 2000).

The banks in Hong Kong, for example, lend on a significant scale to the corporate sector for construction and property development. They also provide residential mortgages to households. These forms of lending accounted for an increasing share of total domestic credit between 1982 and 2002, reaching 20% and 35% respectively by the end of 2001. Moreover, there is anecdotal evidence that, in the expansion period before the onset of the Asian financial crisis, part of the other loans to the corporate and household sectors were effectively for property-related investment (Gerlach and Peng 2004).

The greater exposure of the banking sector to the property market implies that sharp swings in property prices tend to have a major impact on the balance sheets of financial institutions. There are a few channels that transmit this impact. One direct channel is through changes in property agents’ commissions, which typically depend on the value of the asset transacted. A less direct but key channel is through changes in the net worth of the household and the corporate sectors. Falling property prices tend to raise the share of nonperforming loans in the portfolios of financial institutions, thereby undermining banks’ capital position and lending capacity. This is because falling property prices affect the solvency of household and corporate borrowers. Under generalized property-price deflation, the foregoing effects are reinforced by the falling values of loan
collateral that banks can usually recover in the case of outright default. The combination of these effects can create a “credit crunch,” which worsens the contracting effects of the initial drop in property prices.

Conversely, a similar mechanism tends to magnify the impact of rising property prices during cyclical upswings. This transmission channel has been proved to be particularly strong in regions where the financial system is bank-dominated, for example in continental Europe and Japan. This transmission channel is weaker in countries (such as the United Kingdom and the United States) whose financial systems are dominated by the stock and bond markets (see Drees and Pazarbaşioğlu 1998; Bayoumi 1999).

Borio and Lowe (2000) provide some empirical evidence for the working of this channel. They show that the deviation of aggregate asset prices from their long-run trend, combined with a similarly defined credit gap, is a useful indicator of the likelihood of financial distress in industrialized countries.

1.3. Case Studies

Hong Kong
In contrast to much international experience, the banking sector of Hong Kong remained sound despite the collapse of property prices in the late 1990s. This is largely due to the stringent regulations implemented in the early 1990s.

Property prices in real terms in Hong Kong underwent extraordinary swings in the 1980s and 1990s. In size, the price swings in Hong Kong have been as dramatic as those elsewhere, but in frequency they have been more dramatic.

The growth of bank lending in real terms also fluctuated wildly, though it was less pronounced. Bank credit grew rapidly in the second half of the 1980s, largely due to strong demand for mortgage finance. As a result, the banking sector increased its exposure to the property market.

To contain the risks associated with excessive concentration of bank lending for property purchases and development, a number of prudential measures were adopted over the years. First, a maximum loan-to-value ratio of 70% was adopted by the banking industry on a voluntary basis in late 1991. This was later endorsed by the Hong Kong Monetary Authority (HKMA) and incorporated in its guideline on property lending in 1994. In 1994, the HKMA issued a guideline to advise banks to keep their ratio of property lending to loans for use in Hong Kong at about 40%. The banks also are advised not to rely unduly on the value of collateral when granting loans. They should take into account the ability of the borrower to service the debt. For residential mortgage loans, a 50% debt-service-
to-income ratio is generally adopted by banks. These measures limit the risks faced by banks from fluctuations in property prices and help ensure the stability of the banking system in times of market volatility.

Thus, despite the tumbling of the property market in the late 1990s, the financial system remained sound. Nevertheless, the economy still suffered a deep recession. The recession could have been worse, however, had it not been for the stable financial system of the territory (Gerlach and Peng 2004).

*Thailand*

During the recent Asian financial crisis, the most troubled spot in Thailand was probably its housing and real-estate sectors. The troubles in the Thai housing market are attributable partly to the growth of the economy, which took off sometime in the 1960s and 1970s and continued to grow at high rates and with few interruptions. The long period of growth created an overheated housing market (Wong 2001; Property Focus 2004).

The housing market was further fueled by the liberalization of capital inflow following the passage of the Bangkok International Banking Facilities program in 1992. This liberalization of capital inflow provided opportunities for domestic financial institutions to take out foreign loans at low rates and then lend the money to local housing developers.
The property sector began to come under pressure in 1995. The pressure came from two fronts: the overproduction of residential and office space, and the central bank’s squeeze on lending. The overproduction created excess supply, but developers were reluctant to lower prices to clear the market. The lending squeeze worsened the cash-flow problems of the developers, making it more difficult for them to get financing and survive. By 1996, the sector was showing many signs of trouble; its debt in 1996 was around 800 billion baht.

In February 1997, Somprasong Land plc defaulted on interest payments on euro-convertible debentures worth $80 million. The Somprasong default was followed by defaults by other firms. These defaults then put pressure on many banks and financial institutions, especially the small ones. In June, 16 financial companies were suspended. Two months later, another 42 followed. By the end of the year, 56 more had been closed permanently.

The growth of the Thai economy before the Asian financial crisis unintentionally enhanced the impact of the housing-market problem on the rest of the economy, as it made different sectors of the economy more interrelated and interdependent. When the housing market began to experience difficulties, the banking and the financial sectors were squeezed. This squeeze, in turn, put pressure on other parts of the economy. When the conditions of the economy became worse, investors lost confidence and started to move their capital out of the country. The
reaction snowballed, and very soon, with the joining of the currency speculators, the flight of capital became unmanageable. This bank-run type capital outflow grew so huge that eventually the central bank of Thailand no longer could defend the baht. The floating of the Thai baht on 2 July 1997 marks the start of the Asian financial crisis (Wong 2001).

2. The Wealth Effect

Land and buildings form a significant portion of national wealth. Real estate composes the great majority of tangible capital stock, and housing makes up the great majority of the stock of real estate. In an estimate by Ibbotson, Siegel, and Love, real estate forms 52% of total world wealth (Malpezzi 2000; fig. 1). In some territories and countries where land is scarce, such as Hong Kong and Singapore, real estate not only forms a significant portion of the national wealth, but it also plays an important role in the whole economy (Hail 2001). For instance, transnational property and development companies are key players in Singapore and Hong Kong. They form an important part of the local stock market and have enjoyed considerable growth while providing substantial revenue for the government and wealth for individuals.

Given the importance of real estate as national wealth, fluctuations in real-estate prices have a profound impact on aggregate spending, particularly on consumption and investment.
2.1. The Effect of Wealth on Consumption

Property prices can affect private consumption through three major channels. First, inasmuch as consumption spending is a function of households’ lifetime financial resources, and insofar as property wealth is an important part of those resources, changes in the price of real estate can be expected to influence consumption (Deaton 1992). Second, consumption in any given period will be a function of people’s expectations about their future wages. To the extent that the real-estate price affects such expectations by signaling faster or slower growth of real incomes in the future, it will influence current consumption (see Otoo 1999; Poterba and Samwick 1995). Third, information asymmetries and other imperfections in the credit market entail excess sensitivity of current consumption to disposable income and the availability of external finance. The availability and cost of external finance to a household depend on the assessment of the net worth of the household by banks and other financial intermediaries. The interest rate on a consumer loan will be a function of the market value of the assets owned by the household. To the extent that the market value of these assets affects the household’s borrowing capacity to finance its current consumption, asset-price fluctuations will have a further impact on aggregate consumption.

There is evidence that changes in the prices of real property and stock have a significant impact on private consumption in most of the industrialized world.
Estimates of the magnitude of this impact, however, vary considerably across countries. The impact of stock prices on consumption appears to be strongest in the United States, where most estimates point to an elasticity of consumption spending relative to net stock-market wealth in the range of 0.03 to 0.07. In contrast, studies of France and Italy have not found any significant effect of stock prices on private consumption. In Canada, Germany, Japan, the Netherlands, and the United Kingdom, this effect is significant but smaller than that in the United States (see Bérubé and Côté 1999; Capel and Houben, 1998; and Boone, Giorno, and Richardson 1998). These facts appear to reflect the smaller share of stock ownership relative to other financial assets in these countries. They also reflect the more concentrated distribution of stock ownership across households in continental Europe compared to the United States (IMF 2000; table 3.5).

On the other hand, the impact on consumption by real property prices appears to be much stronger in the European Union countries. In the United Kingdom, the estimated elasticity is 10% within a year, and in the Netherlands 7% over two years (De Nederlandsche Bank 1999; Boone, Giorno, and Richardson 1998). There is evidence in Australia and some European countries that changes in housing prices are also an important determinant of consumption growth, far more important than stock prices (Kent and Lowe 1998; De Bondt 1999).

Case, Quigley, and Shiller (2001) find a statistically significant and rather large effect of housing wealth on household consumption. Their study uses a sample
of 14 European and North American countries observed annually for various periods between 1974 and 2000 and a sample of U.S. states observed quarterly during the 1980s and 1990s. They concluded that, in developed countries, the housing market appears to have a greater effect on consumption than the stock market. This finding is consistent with the fact that real-estate holdings are more common than stock holdings in those countries.


Some authors argue that changes in the price of an asset can even affect the spending of a household that does not own equity and property. This is because such changes affect the confidence of a consumer or the uncertainty that the consumer perceives about his future economic circumstances. Romer (1990) develops a consumer-confidence argument in her study of how the stock market affected consumer spending between 1929 and 1932. Zandi (1999) makes a similar argument with respect to the increase in consumer spending in the 1990s. Zandi suggests that a rising stock market buoys consumer confidence, thereby raising spending even among households with little or no direct exposure to the equity market. Along this line of argument, a buoyant property market would have a similar effect on consumer confidence, hence consumption spending (Case,
Quigley, and Shiller 2001). There is, however, little empirical evidence at this stage demonstrating such an effect.

James Poterba argues that “even if the marginal propensity to consume out of wealth is smaller than the estimates in many macroeconomic models suggest, the sheer magnitude of the wealth accumulation during the last decade still translates into a substantial increase in aggregate consumer spending” (Poterba 2000, 108).

### 2.2. The Effect of Wealth on Investment

Apart from affecting consumption, property prices also have an important impact on investment. Property prices can affect investment through three channels. First, a general increase in property prices lowers the cost of new capital relative to existing capital, thereby increasing investment (Tobin’s q effect). Second, empirical studies find that private fixed investment is well-explained by the expected future output growth (Mullins and Wadhwani 1989). To the extent that changes in property prices predict future GDP growth, such changes will affect current investment. Third, a general increase in property prices improves the balance sheets of firms and banks, inducing banks to charge a lower finance premium on loans, hence lowering the cost of capital and encouraging investment.
There is evidence that property prices have a more significant effect than stock prices on investment in continental Europe and Japan. This is consistent with the fact that in those countries, the use of property as collateral against a loan is more common and bank credit plays a greater role in firms’ financing. In fact, cycles in property prices have been closely linked to cycles in credit and investment in those countries, although the direction of the causality among these three variables is hard to pin down empirically (see IMF 2000; figs. 3.7, 3.8; Kent and Lowe 1998).

Zandi (1999) raises the possibility that consumers may react more rapidly when wealth contracts than when it expands. A similar effect may also exist in investment. Therefore, falling property prices may have a more powerful influence on the real economy. Once again, empirical evidence provides little guidance on the asymmetric responses of investment to wealth shocks.

3. Property Cycles and Economic Fluctuations

3.1. A Case from Hong Kong

The real-estate sector in Hong Kong composed about 10.2% of the territory’s GDP in 1996. Income from land auctions, rate and stamp duty accounted for about 24% of government revenue in 1997–1998. More than 45% of the loans
made by all banks is directly tied to properties. Over half of these loans are mortgage loans (Chan, Lee, and Woo 2000).

In the late 1990s, Hong Kong suffered a serious recession. Ho, Wong, and Tse (2003b; henceforth, HWT) show that the plunge of the property market was to blame. On December 8, 1997, soon after the eruption of the Asian financial crisis, the Housing Authority in Hong Kong announced a housing privatization plan. The property market tumbled as a result. The situation was made worse by an increased supply of housing beginning in 2000.

Following the property-market collapse, the economy immediately plunged into a deep recession. By the first quarter of 1998, the Hong Kong economy had declined by an unprecedented 12% on a quarter-to-quarter basis and lost 80,000 jobs. In a matter of a few months, Hong Kong’s unemployment rate increased from 2.2% in the fourth quarter of 1997 to 4.3% in the second quarter of 1998. By the end of 1998, the unemployment rate had reached 6%. The economy shrank by 5% in 1998, and the rebound in 1999 was extremely weak by historical standards (HWT 2003b; tables 1 and 2).

HWT argue that housing is the most important form of assets for the community. A general decline in housing prices causes domestic consumption and investment to shrink; shrinking consumption and investment hurt employment. The decline of employment further hurts housing prices. It is a vicious circle.
There is also a second channel through which declining housing prices hurt employment. When housing prices are rising, new investors are attracted to the market, either to hedge against inflation or to speculate for capital gains. Heavy turnover in the housing market provides an impetus to the economy. The immediate beneficiary is the property brokerage sector. Renovators, movers, lawyers, bankers, and retailers also benefit from the property-market boom. In addition, rising property prices encourage new property development, which raises employment by both the construction firms and the firms that sell construction materials.

The argument of HWT is supported by their empirical study. Using Johansen's procedure, HWT (2003b) show that housing prices and employment in Hong Kong are cointegrated, and that there is a bi-directional causality between these two variables.

### 3.2. A Case from South Korea

Although falling housing prices did not cause the financial crisis experienced by South Korea in 1998, it nevertheless had a devastating impact on the unemployment rate. Thus, the collapse in housing prices played an important role in exacerbating the crisis (Kim 1999).
Real estate in South Korea, as in other countries, is an important form of assets as well as an input to production. Investment in new housing accounted for 18% to 22% of total capital formation and 6% to 8% of GDP throughout the 1990s. As of 1996, about 36% of total capital stock in South Korea was in the form of housing. The amount of nonresidential real estate is also sizable, but data are not available. Furthermore, housing finance represents a substantial share of total credit. At the end of 1998, housing loans outstanding made up 27% of all loans made by banks, and the outstanding housing loans amounted to 13% of the GDP of South Korea. The share of bank loans made against real-estate collateral fluctuated between 34% and 43% in the 1990s (Kim 1999).

The economic crisis that began in November 1997 struck the real-estate sectors hard. As the unemployment rate rose, the demand for housing fell abruptly with the reduction in households' income and increased uncertainty about job security. As a result, the average housing price dropped by 20.1% while Chonsei deposits\textsuperscript{vi} dropped by 25.9%, both in real terms. In response to a reduced demand and a credit crunch caused by the crisis, the construction of new houses decreased by 48.7% in 1998. The decline of construction work made the unemployment problem even worse (Kim, 1999).

Deteriorating conditions in the housing market battered the land market. The price index of residential plots fell by 13.8%, while that of commercial and industrial sites declined by 16.4% in 1998.
At the same time, vacancy rates in large office buildings of Seoul's major business centers rose substantially. In some business districts, the vacancy rate jumped from 15.3% to 25.7%. There was a reported 50% fall in rents on some office buildings (Kim 1999).

The supply of new mortgage loans also decreased by 25% in 1998. Mortgage companies that entered the housing finance market in 1997 stopped lending at the outbreak of the crisis. The share of housing loans that went bad doubled from 10.4% to 20.5% between 1997 and 1998. The South Korea Housing Financial Cooperative went virtually bankrupt, as many of its member home builders failed (Kim 1999).

But the fall in real-estate prices did not cause the crisis in South Korea, Kim argues. This is perhaps due to the stringent rules governing the financial sector. South Korean financial institutions had been prohibited from lending to finance real-estate purchases except for land of new housing. vii Thus, South Korean financial institutions’ exposure to the real-estate market was relatively small. Moreover, by the eve of the crisis, South Korea, unlike Hong Kong, had been experiencing deflation in real-estate prices since 1991. By the time of the crisis, housing and land prices are already very low by historical standards. viii A third explanatory factor is the relatively unimportant role of collateral, due to a cross-
guaranty mechanism among corporations belonging to the same conglomerates, or Chaebol (Kim 1999).

When the economy bounced back in 1999, the real-estate sector also recovered. By the third quarter of 1999, housing prices in major submarkets in the capital region had almost returned to the pre-crisis level.
Chapter II Literature Review

In this chapter, I will discuss the theories and empirics of asset pricing and price bubbles in real-estate markets. In section 1, I will introduce the concept of a speculative bubble. In section 2, I will discuss the characteristics of a real-estate market and a suitable analytical framework for this type of market. In section 3, I will discuss why a real-estate market may be more prone to a speculative bubble than a financial market. The remaining part of this chapter will be devoted to the experiences of the world in the past few decades, especially in Japan, South Korea, and Hong Kong. A review of these historical events will provide evidence that speculative bubbles are inherent in real-estate markets. The provision of such evidence will justify my commitment to an empirical study of speculative bubbles in this thesis.

1. What Is a Bubble?

In the language of economists, a bubble exists in a price if the price is other than what is warranted by its “fundamentals.” For instance, the fundamental price of a suit is equal to the value the suit provides to the man who wears it, which includes the value of the covering, warming, and cosmetic functions it provides to the man. If, because he expects the price of the suit to increase in the future, this man is willing to pay a price that is higher than this fundamental value, and if this expectation is based on information that will turn out to be false (or correct, but
whose implications are exaggerated by this man), then the price he pays contains a bubble component. In this sense, a negative bubble—the portion of the price below the fundamental price—can exist as well as a positive bubble, which is the portion of the price that is above its fundamental price.

The key factor that contributes to the formation of a bubble is expectations. Market participants must use expectations in making their day-to-day decisions, especially when they need to commit themselves to a large purchase, such as buying a house.

The literature is divided about whether bubbles exist at all. In the past century, the world has seen many episodes of rapid inflation in prices and their ensuing collapses, in commodity markets, stock markets, foreign-exchange markets and real-estate markets. The experience of the world suggests that speculative bubbles are important forces behind such large price swings. Yet, skeptics argue that investors are rational and markets efficient, hence speculative bubbles are, theoretically, implausible. Such skepticism is reinforced by the fact that there is no perfect way of proving the existence of a bubble empirically. This is because a bubble is typically inferred, rather than observed, with the help of a fundamental model. Such a fundamental model itself is often a subject of dispute. Sometimes, the disputes center on the specification of a model. For example, most economists agree that the present-value model is a reasonable model for asset pricing, but they disagree about which discount rate should be used in empirical
studies. Sometimes, the disputes arise through disagreement on what is the fundamental. For instance, what determines the fundamental value of a house?

Even if economists can agree that a speculative bubble exists, they are also often divided about the characteristics of the bubble. To Robert Shiller, “The idea that there has been a speculative bubble … is inherently a statement about some less-than-rational aspect of investor behavior” (2001). The literature usually calls the notion of irrationality the “greater fool” theory, implying that irrationality can exist only if the majority of participants are foolish. But Shiller argues that “the kind of less-than-perfectly-rational behavior that underlies [a speculative bubble] is not abject foolishness” (2001). He chooses the phrase “less than perfectly rational” in his Cowles Foundation discussion paper (2001), rather than the word irrational, as is the case in his book Irrational Exuberance (2000). Perhaps he does so to deliberately to avoid this confusion. He shows that less than perfectly rational behavior is due to human error in evaluating the available information, and that such an error can infect the thinking of some of the most intelligent people in our society.

The key to an irrational, speculative bubble is the use of subjective probability by market participants when no better alternative is available. This is true among professional investors as well as the general public. Even though professional investors have at their disposal formal statistical models, such models are only
as good as their specific assumptions, as argued by Shiller (2001). Thus, the assessment of market fundamentals is inherently subjective.

However, the conventional theory of a rational, speculative bubble does not distinguish subjective probability from actual probability. If a bubble arises from a market where all participants are rational—a person being rational if he makes full use of available information, asymmetric and incomplete though it may be—then the bubble is defined as a rational bubble, even if some participants miscalculate the risks involved (Brunnermeier 2001). The conventional theory also does not give a clear label to a bubble that arises through the use of a wrong economic model and a bubble that arises through the misjudgment of available information. Mathematically speaking, the bubble component in a general solution to an economic model, supposing it is the correct one, is referred to as rational, speculative bubble, and simply explained as the result of uncertainties about future events. We will follow this line of definition and call the bubble we investigate in this thesis a “rational, speculative bubble.” This is because the primary object of this thesis is to provide empirical evidence of speculative bubbles in property markets, rather than to establish a comprehensive theoretical framework encompassing all types of bubbles. The latter task will probably require the combined efforts of many distinguished economists and the support of more empirical evidence.
The remaining part of this chapter reviews the existing literature on speculative bubbles in property markets.

2. The Characteristics of and the Analytical Framework for a Real-estate Market

Real estate is one commodity with two attributes. On the one hand, it is a durable good, like a car or furniture, which delivers a stream of consumption or production services; on the other hand, it is an asset, like stocks and bonds, which stores wealth. For that reason, it is referred to as a real, as opposed to a financial, asset (Diba and Groosman 1988c). That is, it is an asset which generates consumption values directly, rather than pays dividends that must be exchanged for a “real” object to be consumable.

The consumable attribute of real estate means that the price of real estate is settled the way the price of any other consumable commodity is settled. That is, the price of real estate is subject to the forces of supply, demand, and expectations. To be more specific, other things being equal, an increase in demand, a decrease in supply, and an expected price increase would increase the current price of real estate; a decrease in demand, an increase in supply, and an expected price fall would reduce the current price of real estate.
The price of real estate often behaves in the same way as any financial asset. The value of a stock, for instance, is the present value of the stream of dividends it generates in the future. Similarly, the value of a house is the present value of the stream of rents (net of operating costs) it generates, or the rent one would have paid to dwell inside the house had one not owned it (Malpezzi and Wachter 2002). For this reason, we argue that the present-value framework is a reasonable one for analyzing the real-estate market.

In fact, because of the second interpretation—i.e., that the value of a house is the present value of the stream of rent one would have paid to enjoy the stream of consumption services provided by this house, had one not owned it—I argue that the present-value framework does indeed contain the consumable attribute of the house.

There are other reasons why a financial-asset market and a real-estate market may share the same analytical framework. First, as both real and financial assets are stores of wealth, they are substitutes. Therefore, arbitrage forces will bring speculation in one market to the other. Second, the economic fundamentals that affect the formation of expectations, such as the money supply, interest rates and the condition of an economy, are common to stock and real-estate prices. Third, it has been noticed in the empirical literature that the price of real estate is often positively correlated with the price of stock. It has also been noticed that the capitalization ratio in a real-estate market (analogous to the price-earning [PE]

For those reasons, perhaps, the analytical framework developed in a financial market for detecting a speculative bubble has been widely applied to real-estate markets. In particular, many empirical studies have used the present value model as a benchmark model for testing the existence of a bubble in a real-estate market (see, for example, Ito and Iwaisako 1995; Basile and Joyce 2001; Hendershott 2000; Chan et al. 2000; Lim 2003; and Chung and Kim 2004).

3. The Theory of a Speculative Bubble in the Price of Real Estate

Researchers in various countries point to speculation as a prime mover of cycles in a real-estate market (Malpezzi and Wachter 2002). Central to speculation is the expected excess profit. If expectations are rational and the market is efficient, the price of property will follow a random walk, making it impossible for speculators to earn excess profits. Hence, there will be no incentive for speculation in the real-estate market. If, however, expectations are formed adaptively, speculators will enter the market when the price is rising, and retreat from the market when the price is falling. As a result, speculation—hence, a bubble—can occur in this market. In fact, Malpezzi and Wachter (2002) define speculation as the formation of expectations about price through extrapolation.
However, backward-looking expectations do not necessarily lead to inefficiency in the sense that excess profits can be earned. Herring and Wachter (1999, 2002) provide some rationale about why excess volatility and a bubble, usually associated with market inefficiency, may occur in a real-estate market, yet a profit opportunity does not exist. In their model, the prevailing prices are set by “optimists” who are subject to “disaster myopia” in their pricing behavior. In an efficient asset market, such pricing behavior will be countered by informed investors who profit by selling the asset short until the price falls to its fundamental value. Investors can expect to profit from such short sales until the market returns to the efficient price. But this mechanism is not effective in limiting the positive deviations of a real-estate price from its fundamental value. This is because there are few organized markets for selling real estate short. With no short sales, optimists will strongly influence real-estate prices. Even if their optimism is not borne out by fundamentals, optimists will remain in business so long as the upward trend in a price continues. This is one of the reasons why a real-estate market is more prone to a speculative bubble.

Financial institutions’ lending processes may also contribute to the formation of a speculative bubble. First, when the price of property is rising, and when property is used as collateral, an investor is likely to be able to borrow against the property, even though he earns substandard returns on his investment
project. Second, a manager of a financial institution who lends against real-
estate collateral has an incentive to overvalue the real estate. This is
equivalent to underpricing the loans. In good states, the underpricing of loans
will increase the profits accrued to the manager. This is the agency problem
discussed in Allen (2001). Moral hazard further undermines a lender’s
incentive to price a loan efficiently. Both the agent problem and moral hazard
exacerbate the problem of excessive credit provision (Herring and Wachter
1999, 2002). Evidence shows that excess credit plays an important role in the
creation and persistence of a bubble (Ito and Iwaisako 1995).

Myopic pricing behavior is another factor contributing to the formation of
bubbles (Herring and Wachter 1999, 2002). Myopic pricing is shortsighted
pricing behavior that fails to take into account negative events likely to occur
in the future. Specialists in cognitive psychology have found that a decision
maker formulates subjective probability of an event on the basis of the ease
with which he can imagine that the event will occur (Tversky and Kahneman
1982). This tendency to underestimate shock probabilities can be
exacerbated by the threshold heuristic (Simon 1978). When subjective
probability falls below some threshold amount, it is disregarded and treated
as if it were zero. The limited information for forming a subjective probability
and the low threshold likelihood of a negative event will restrict the attention
paid to the possible negative event. This is called disaster myopia. In the case
of a real-estate investment, this tendency to underestimate the probability of a
low frequency shock promotes a speculative bubble.

Researchers also point to two other contributing factors. One of them is the
high transaction cost of real estate. Such high costs will limit the ability of a
pessimist to trade on his opinion. The other is the lengthy lag required to bring
a replacement product to the real-estate market. This lengthy lag will extend
the period in which excess returns can be earned, hence superficially
justifying an exuberant property price (Hendershott 2000).

4. The Empirical Literature on Real-estate Bubbles

The real-estate market has the longest and the most reliable history of boom and bust.
That history stretches back to the early 1800s (Carrigan 2004). Because of the great
uncertainties surrounding its fundamental value, real estate has always been a major
object of speculation. It is thus unavoidably susceptible to manias, bubbles, and panics.
Hoyt (1933) quotes from a Chicago Tribune editorial of April 1890: “In the ruin of all
collapsed booms is to be found the work of men who bought property at prices they knew
perfectly well were fictitious, but who were willing to pay such prices simply because
they knew that some still greater fool could be depended on to take the property off their
hands and leave them with a profit.”

4.1. Evidence of Investor Irrationality in a Real-estate Market
Sivitanides et al. (2001) examine the real-estate data in 14 U.S. metropolitan areas for the period 1984–2000. They find a negative relationship between the real rent and the capitalization rate (CR). This suggests that when rents are high, investors expect them to go even higher. This is the opposite of what investors should do, if their expectations are formed rationally.

Hendershott (2000) also gives an example of investor irrationality. The cyclical variations in office construction, vacancies, and values were enormous throughout the world in the late 1980s and early 1990s. Hendershott argues that such wide swings were caused by the failure of investors to understand the workings of the property market. The price in a property market has to be mean-reverting, as developers will build (not build) when the price is substantially above (below) the cost of replacing the existing property stock. If investors do not incorporate the mean reversion into their forecasts of future cash flows, they will overvalue properties when prices are high and undervalue properties when prices are low. Such behavior will exacerbate the cyclical swings in office values, office constructions, and vacancies.

Hendershott (2000) shows that the Sydney office market in the late 1980s is another example of an asset price bubble. He constructed the ratio of the actual to the equilibrium real rents and the ratio of the fundamental value to the replacement cost of office buildings in Sydney during the period 1985–2002.\textsuperscript{xi}
Both ratios were expected to be unity under the “no bubble” hypothesis. The evidence, however, rejected the null.

Hendershott’s study provides direct evidence that Sydney investors did not incorporate mean reversion into their vacancy-rate forecasts and, as a result, undervalued properties at the cyclical trough. They also provide indirect evidence that these investors failed to incorporate mean reversion into the forecast of future cash flows at the cyclical peak. This failure triggered excessive construction and vacancies. One may thus infer that investors in Sydney were irrational.

More recent evidence from the U.K. office and retail markets suggests that investors do rationally build mean reversion into their expectations (Hendershott and MacGregor 2003). Thus, at least in the United Kingdom, a rational model can largely capture the boom and bust of the late 1980s and early 1990s in that country.

4.2. Evidence of Real-estate Bubbles

Bjorklund and Soderberg (1999) examine the 1985–1994 cycle in the Swedish property market and contend that the ratio of property value to rent increased too much, indicating that a bubble may have existed.
Scott (1990) and Brooks et al. (2001) apply variance-bound tests to test the rationality of real-estate share prices. Both studies suggest the existence of an irrational property bubble. Scott analyzes price indices of 13 REITs. His sample stretches from the late 1960s and early 1970s to 1985. Brooks et al. examine the prices of U.K. property stocks. They compute two fundamental price series and compare them with the share-value series. It is shown that the variances of the actual prices are far greater than those of the fundamental price series, indicating the existence of an irrational, speculative bubble.

Abraham and Hendershott (1994) argue that the metropolitan real-estate price changes can be described by a model that contains two components: an equilibrium-price term and an error-correction term. The determinants of real appreciation in housing prices can thus be divided into two groups: one that explains changes in the equilibrium price, and another that accounts for the adjustment dynamics. Abraham and Hendershott estimated the model using data from 30 cities in the United States covering 14 years (420 observations). They found that either group of variables can explain a little over two-fifths of the variation in the real movements in the price of housing in the 30 cities over the period 1977–1992; together, they explain three-fifths. They found that, as of the end of 1992, there was a 30% “above market” premium in prices in the Northeast, a 15%–20% premium in prices on the West Coast, and probably a significantly negative premium in Texas. Their findings indicate the existence of speculative bubbles in those areas.
In their model, Abraham and Hendershott incorporate a proxy for the tendency of a bubble to burst and a proxy for the tendency of a bubble to swell. This methodology is similar to that of Case and Shiller (1990). The bubble-buster proxy in their paper is the deviation of the actual metropolitan house price level from an estimated “fundamental” price level. They found that the proxy does indeed work and is especially useful in explaining the large cyclical swings in real house prices in the West. The lagged appreciation term that represents speculative pressures in the market performs admirably in soaking up volatility, but the authors cannot explain what brings about the speculative bubbles in the first place.

4.3. Case Studies

4.3.1. The Experience of Japan

The asset prices of Japan experienced large swings in the late 1980s and early 1990s. Such swings not only affected the fortunes of many corporations and individuals, but also caused severe economic problems in the country as a whole.

Japan’s economy boomed in the late 1950s and throughout the 1960s. The Nikkei stock-market index, which started at 100 in May 1949, reached 5,000 in
the early 1970s and 10,000 in 1984. It advanced 20% to 12,000 in 1986 and then took off, reaching 39,000 by the end of 1989.

All this time, the real-estate market was not far behind. A price index for residential real estate in six large cities started at 100 in 1955 and reached 4,100 in the mid-1970s, 5,800 in about 1980, and 20,600 at its peak in 1989 (Kindleberger 2000, 113).

Banks in Japan supported speculations in the stock and real-estate markets. When the stock market in Japan crashed in January 1990, real-estate prices leveled off. Real-estate prices started going down later, but slowly, perhaps largely because of the volume of transactions drying up. The blow to the financial institutions has been heavy. The value of the bad loans of Japanese banks and financial institutions is about $550 billion, most of them loans made to the real-estate market (Kindleberger 2000, 114). The economy thenceforth plunged into a deep recession lasting more than 10 years.

This experience of Japan has intrigued many researchers. Some try to find out whether the behavior of the asset prices was caused by speculative bubbles, with bubbles typically inferred from the residuals of a fundamental model; some try to identify the role of the monetary policy, especially the interest rate, in determining the movement of the asset prices; and a few try to connect the movements of stock prices to those of land prices.
Ito and Iwaisako (1995, 4–5) argue that some fundamentals, such as the interest rate and the conditions of the economy that affect the formation of expectations, are common to stock and land prices. Using semi-annual data between 1956 and 1993, they show that a 1% increase in the stock price causes a permanent increase in the land price of 0.6%.

Ito and Iwaisako explain the co-movement between stock and land price as driven by “the same fundamentals.” But there may be other reasons that the two moved together, such as price arbitrage (Basile and Joyce 2001) and mass psychology (Kindleberger 2000).

Ito and Iwaisako consider a standard present-value model. They attempt to measure how much of the asset-price variation observed in Japan in the late 1980s and early 1990s can be attributed to the changes in “fundamentals.” Their conclusion is that “it seems impossible to offer a rational explanation of the asset price inflation in the second half of the 1980s by changes in fundamentals” (1995, 10). This point is reemphasized by the authors in a study of variance decomposition (1995, 20).

They proceed to explore the possibility of the existence of a bubble with simulation, by adding to the present-value model a periodically collapsing bubble of the form described by Blanchard and Watson (1982). They conclude that even
with a “rational stochastic bubble,” it is very hard to explain the boom in the Japanese stock market in the second half of the 1980s. The actual data can be simulated better with the irrational expectations (15).

In fact, the empirical evidence they found supports the existence of an irrational, speculative bubble. Their study shows that an unexpected increase of 1% in the price of land causes an increase of 3% in the price of the land, suggesting that expectations are not formed rationally.

The Japanese experience may be a classic example of Minsky’s model. The initial shock in 1985 and 1986 came from fundamentals, such as a productivity increase in Japanese industries, and an increased demand for commercial property in Tokyo. However, the magnitude of the boom in asset prices is largely unexplained by the fundamentals. Furthermore, from 1987 to 1989, there was no fundamental news on interest-rate and productivity growth significant enough to explain further increases in stock and land prices. Even with the incorporation of a rational bubble, the boom is hard to explain without assuming irrational expectations.

Ito and Iwaisako (1995), however, question the idea that interest rates may not be the only monetary policy variable that affects asset prices. Using VAR study, they show that the growth rate of the bank loans to the real-estate sector is a very important factor for predicting excess returns in both stock and land; that the
bank lending channel is more important than the direct effect of the interest-rate changes for the land-price changes; and that the credit channel is especially important in the bubble era. Furthermore, an initial increase in loans may have some role to play in starting the boom by increasing the demand for a fixed supply of land (p.26).

Thus, in their conclusion, Ito and Iwaisako state that although evidence in their paper is suggestive of the existence of a stochastic bubble, no clear-cut, positive evidence was discovered. I, however, would argue that the availability of credit is not a fundamental. If an investor purchases property simply because of easy credit, rather than because of a good return prospect, then he is irrational. If enough people follow suit, and the price increases as a result, then the price will have been inflated by an irrational bubble.

Basile and Joyce (2001) also investigate the relationship between stock and the land market prices, and the role of the money supply and bank lending in Japan. They measure the size of the asset-price bubble as the difference between an ex-post return of an asset and the required return. The required return has two components: a risk-free return and a risk premium. Using this method, they found that the stock-market bubble begins to grow rapidly in 1986 and declines as rapidly in 1990. The land-market bubble grows more evenly into the mid-1990s before declining.
They then specify a VAR relation in which the estimated land and stock bubbles are the endogenous variables, and the call rate, the money supply, the lending variable and the output growth are the exogenous variables. Granger causality tests based on this VAR specification are applied to the two regressands and to each regressand versus each of the regressors. To avoid the criticism of specification error, their study also uses the Choleski decomposition method to decompose the forecast error variance into theoretical contributing factors. Both tests found evidence that the stock and land markets were linked in the period 1971–1985, although this result is not robust with respect to the choice of variables and the specification of the model. There is also some evidence in the causality tests that monetary policy, as measured by the call rate, affected the land-market bubble. Both methods found that changes in the stock-price bubble led to changes in the land-market bubble in the period 1986–1991. The stock-market bubble itself, on the other hand, was not systematically affected by any of the other variables.

4.3.2. The Experience of South Korea

An analysis of data from 1974 to 1989 by Kim and Suh (1993) shows that a bubble existed in the land prices of South Korea. Park et al. (1998) suggest that the bubble in housing and land stood at 58% and 40% in 1991 at its peak, respectively, and disappeared almost completely by 1997.
Lee (1997) conducts a test for bubbles using the land price of South Korea between 1964 and 1994. On the basis of a structural model with GNP, the interest rate, and the money supply as fundamentals, he rejected the hypothesis that only market fundamentals drove land prices in South Korea. That is, the land market of South Korea had been driven in part by a speculative bubble.

On the basis of the present-value relation, Lim (2003) conducted two bubble tests on the housing prices of South Korea. One is a modified volatility test (MRS test) suggested by Mankiw et al. (1985); another is a combination of the unit-root test suggested by Diba and Grossman (1988b) and the cointegration test by Campbell and Shiller (1987). His MRS test shows that the null hypothesis of market efficiency is rejected, indicating the existence of an irrational bubble. His unit-root test and cointegration test, however, suggest no sign of any kind of bubble. Lim’s study shows that the result of a test depends heavily on the type of test employed. The author also shows that the test is sensitive to the choice of data.

Kim and Lee (2000) adapt the idea that the existence of an equilibrium relationship excludes the possibility of a price bubble. Using Johansen’s maximum-likelihood test for cointegration, they show that the nominal land price of South Korea between Q1 1974 and Q4 1999 was cointegrated with the real GDP. Since real GDP is a measure of the market fundamental in their paper, they conclude that, in the long run, the nominal price of land is determined by market fundamentals. However, in the short run, the nominal
land price can deviate from this intrinsic value. When the test is applied to a shorter sample (running between 1974 and 1989), there is no cointegration relationship between the two variables. The authors stop short of concluding the existence of bubble in the sub-sample period. Next, the authors explore the cointegration relationship between the deflated land price and the real GDP and that between the nominal land price and the nominal GDP. Again, they found cointegration relationships. Furthermore, they found that, in the full sample, the land price and the stock price are also cointegrated.

Housing prices in South Korea have experienced sustained, rapid increases since 2000. It is commonly believed that the primary driving force behind this price inflation is speculation (Chung and Kim 2004). The government has thus applied a series of antispeculative measures, such as a major increase in the capital-gains tax and property tax. These measures, however, have not produced the expected results.

Chung and Kim (2004) estimate a simple model relating housing price to income and bond yield, the two representing “normal” demand. “Speculative” demand is captured by the lagged value of the housing price in the regression equation, as in Case and Shiller (1990) and Abraham and Hendershott (1994). Their results show that “what determines housing price hike in South Korea is not ‘normal’ demand but ‘speculative’ demand.” The ratio of the speculative demand to the normal demand is 1.24 for South Korea as a whole and 2.85 for Seoul. Chung
and Kim cite low interest rates and easy credit as two of the major reasons behind the increased speculation in the housing market of South Korea.

In their study, Chung and Kim (2004) use three ways to infer a speculative bubble in the housing market of South Korea. The first is a long-run equilibrium-price approach, which is based on the idea that, in long-run equilibrium, variations in the housing price must match variations in macroeconomic variables such as GDP. A bubble is thus inferred as the difference between the two. The second is a fundamental market-value approach, in which the bubble is the difference between the actual price of a house and the present value of the future cash flows generated by the house, with the present value simply calculated as the ratio of a rent series to a bond-yield series (19–20). The third is the price-income ratio (PIR) approach, which asserts that the “normal” housing price should be the average PIR plus one standard deviation, and the magnitude of a bubble is the difference between the actual price and the “normal” price. They conclude with their estimation that a bubble exists in the housing prices of South Korea; however, the magnitude of the bubble varies widely, depending upon the method used. By and large, the long-run equilibrium-price approach produces the smallest bubble among the three.

4.3.3. The Experience of Hong Kong
Residential-property prices in Hong Kong were highly volatile throughout the 1990s. In 1991, the real average price in the overall property market rose by 40%. It fell by 16.2% in 1995, and was followed by an increase of 18.9% and 20% in the next two years and a rapid fall of 50% in 1998 (Chan et al. 2000). These dramatic changes in property prices suggest the existence of a speculative bubble.

The restriction in the land supply is often cited as one of the key reasons behind speculative bubbles in a real-estate market. The population of Hong Kong is about 7 million. There is only 50 km$^2$ of residential land in the territory, making Hong Kong one of the most densely populated cities in the world (Chan et al. 2000).

Chan et al. use a standard present-value model with a constant discount price to determine whether a bubble exists in the residential market of Hong Kong. The model assumes that property is a good investment that produces a stream of rental income over its lifetime. The current value of a piece of property is, therefore, determined by the present value of the current rental income and the expected market price in the next period. There are two solutions to the price: a fundamental solution,

$$P_t = P_t^f,$$

and a bubble solution,

$$P_t = P_t^f + B_t.$$
In addition, there could be a misspecification error, $S$, which measures the deviations of the actual price data from the bubble solution. This could arise from the measurement errors in the data or from wrong assumptions about the underlying parameters of the data-generating process. Thus the total model noise is $B + S$.

Using the signal-extraction method of Durlauf and Hall (1989a, 1989b), the authors conducted a flow-and-stock test to investigate the existence of $S$ and $B$. The data they use are monthly average rentals and quarterly average prices of the private domestic properties within the class A, which is defined as apartments with sizes less than 39.9 m$^2$. The sample period runs from the first quarter of 1985 to the third quarter of 1997. Their study shows that there existed a misspecification error in the model noise as well as a bubble. The path of the bubble shows that the bubble expanded most dramatically between 1990 and 1992 and between 1995 and 1997. Their finding is compatible with ours in chapter V.

As of today, a wide range of methods for detecting speculative bubbles has been developed and applied in the past two decades by academics studying financial markets. Each of these methods relies on a set of assumptions about the fundamentals, whether in their structural model, or their distributional properties. There is, therefore, an unresolved identification problem in all cases. When the
null hypothesis of “no bubble” is rejected (or accepted), it could be a problems with the hypothesis on the fundamentals rather than with the absence (or presence) of a bubble. It looks as if there is no way of resolving this indeterminacy, as a bubble is typically inferred from fundamentals rather than observed directly.

In this thesis, I will combine three methods to examine the issue of rational speculative bubbles: a Markov-switching ADF test to identify the existence/absence of a bubble, a Kalman-filter procedure to infer the quantity of the bubble, and the power-law-log-periodicity (PLLP) model to predict the most likely time of the ending of a bubble. Both the Markov-switching ADF and the Kalman-filter procedures rely on assumptions about the behavior of the fundamentals, they are not free from the identification problem facing the empirical literature. However, the PLLP theory does not make any assumptions about the fundamentals—something that I argue is a salient feature of this theory. It replies on a simple idea: speculative forces tend to pull the price away from the fundamentals, which is reflected by the power-law signature; that is, the price must increase in an accelerating manner in order to compensate the ever-increasing risks involved the further away the price is from the fundamental value. At the same time, the fundamental forces struggle to exert themselves to pull the price back toward its intrinsic value. The interplay of the two forces is reflected by an accelerating log-periodic cycle around an accelerating mean of the price prior
to a crash. The results from these three approaches are compatible, as I will show in chapter VII, “Conclusions and Discussions.”

While studies on speculative bubbles in financial markets are abundant, there has been relatively little effort devoted to the empirical studies of real-estate price bubbles, especially in Asia. There have been a number of studies on real-estate price bubbles in Hong Kong and Korea, but the methodologies used have been, in general, crude. The empirical studies in this thesis will attempted to apply some of the most advanced methodologies developed in the financial-asset markets to the real-estate market. The arguments for this practice are given in chapter II, section 2. In short, the fundamental reason is that both stock and real estate are assets whose value can be measured by the stream of services it can provide directly or that it can “buy.” As will be shown later, the flexible combination of tools for the identification, quantification, and prediction of bubbles yields promising results which serve as a solid piece of evidence for the existence of speculative bubbles in the real estate market.
Chapter III Data

Throughout chapters IV, V, and VI, the data will use in the empirical studies are price and rent indices for building structures, as opposed to raw land, of Hong Kong and Seoul. There is no convincing reason to believe that a bubble is more likely to exist in the price of a building than in that of a plot of land. But some sectors of the building market, such as the housing market, are more likely to involve less-informed investors. Empirical studies on speculative bubbles use data both on the price of land and on the price of buildings. In fact, following the argument of Homer Hoyt (1933), if a bubble exists in the price of land, it is likely to be transmitted to the housing price, and vice versa.

We do have a few reasons for choosing data from Hong Kong and Seoul. During the 1980s and 1990s, the two cities experienced dramatic property-price swings (figure III.1-2). These swings were suspected by practitioners and academics to be the results of speculative bubbles. Although quality data on the property sector are available, there have been relatively few research papers devoted to the study of the speculative bubble in the property market of Hong Kong (Chapter II, section 4.3.3). More studies are available on property-price bubbles in South Korea. The methodologies used in these studies are, however, crude in general (chapter II, section 4.3.2).
In Hong Kong, the data available for building structures are generally divided into four categories: domestic premises, office, retail premises, and flatted factories. Since price series among these four categories have moved closely together (appendix 1, fig. III.1), as predicted by Homer Hoyt, we select, arbitrarily, the Hong Kong office-price index and its associated rent series for our study. We will use in this study the Seoul housing-price index and its associated rent series, as it is the only data available for building structures in Seoul.

In chapter VI, I will introduce a third series: the Korea general construction stock price index (KGCSP), a stock-price index that has property as its fundamental. As the Seoul housing price decayed slowly throughout the 1990s, the power-law log-periodicity model, designed to capture the extreme behavior of the market, does not work very well with this data set. The rationale for adding the KGCSP is that, because of the strict capital constraints imposed on the property market until recently, and the tight control by the government of the supply of land in South Korea (Kim 1999, 2000), speculative forces in the property market are more likely to show up in stock written on property than in the property itself. All data series come from the CEIC database, a comprehensive source of economic statistics for Asian economiesxvii.

The series are monthly data deflated by their respective CPIxviii. Each series for Seoul has 210 observations running between 1986:1 and 2003:6. That for Hong Kong makes use of two data sets of different frequencies: the first set is a
quarterly data set running from 1984:Q1 to 2000:Q3, and the second a monthly data set stretching from 1993:1 to 2003:4. In order to combine them, the first set is converted into monthly data by means of cubic splining. Thus, the first half of each series for Hong Kong, running from January 1984 to December 1992, consists of the splined output from the first data set; the remaining half of the series is drawn from the second data set. The raw data have 232 observations for both price and rent series. The KGCSP series has 287 observations running from January 1980 to November 2003.

A plot of the data in figure III.1 shows that Hong Kong office price is highly volatile. The price index more than doubled in real value in a mere 15 months between Dec. 1987 and March 1989. Another sharp increase of the price occurred in the first half of 1994, with an average value of 6.1% per month. The price index crashed after July 1997, following the Asian financial crisis. By April 2003, it stood only at about 22% of its peak value (occurred in May 1994). The Seoul housing price is less volatile (compared to Hong Kong office price). During its most significant in-sample rally, occurring between 1987:8 and 1991:4, the index advanced only 33.5% in real value. It declined gradually throughout the remaining part of the 1990s, only starting to revive again in mid 2001. The same cannot be said about KGCSP, however. Between 1987:1 and 1989:4, it has more than quadrupled in real value, with an average of 15.7% per month increase. The index collapsed as dramatically after 1990:1. By 1992:9, it stood at only 33.2% of its peak value in 1989:4. But it shot up again thereafter. The rally continued into
1994:12, gathered a total of 123.6% gain. During the same period, Seoul housing price continued its path downwards, making the rise of KGCSP looking unsubstantiated. The index has since collapsed again. At the bottom in 1998:9, the index was only 5.3% of its value in 1989:4.

Theories suggest that, in the absence of a bubble, a real-estate price and its associated rent should move more or less hand in hand. But this seems not the case in our data (figure III.3). The price-rent ratio of Hong Kong office behaved very much like the price series. It increased continuously between 1990 and 1997, with a few temporary reversals. This ratio crashed to its historical low after the late 1997. On the other hand, the price-rent ratio of Seoul housing was on a declining trend throughout the sample period, with only a few brief episodes of reversals. As argued before, KGCSP has the real estate prices of South Korea as its fundamentals. But we see very little connections between KGCSP and Seoul housing price. Does the behavior of the two price-rent ratios and the disparity between KGCSP and Seoul housing price imply the existence of a speculative bubble in the prices of interests? This is the question the current study is interested in.
Chapter IV Markov-switching ADF Test of Bubbles

1. Introduction

Hamilton and Whiteman (1985) and Diba and Grossman (1988) suggest that stationarity tests may be used to detect an explosive rational speculative bubble. The use of such a test does not preclude the possible influences of unobservable market fundamentals. The rationale of this procedure is as following. If the first-difference of a dividend and those of unobservable fundamentals are stationary in mean, and if no rational bubble exists, then the first difference of the associated stock price must be stationary. Differencing a stock price a finite number of times would not render it stationary, however, if it contains a rational bubble. Due to the possible presence of unobserved variables, the finding that the first-difference of a stock price is not stationary does not automatically establish the existence of a rational bubble. However, the converse inference is possible. That is, evidence that the first-difference of a stock price has a stationary mean would be evidence against the existence of a rational bubble in that price.

Evans (1991) shows that stationarity tests, suggested by Diba and Grossman (1988) and Hamilton and Whiteman (1985), are incapable of detecting periodically collapsing rational bubbles. Hall et al. (1999) demonstrate that the power of these tests can be improved significantly by incorporating a state
variable that follows a Markov process. They argue that testing for a periodically-collapsing bubble essentially involves distinguishing the expanding phase from the collapsing phase of the bubble. The two phases can be modeled by a two-state Markov chain. In this case, the data generating process (DGP) would take different parameters in different states. If we model this DGP with a Markov-switching AR(p) process, then a generalized ADF unit root test would detect the bubble, if it exists, quite effectively.

In this study, I will take the Markov-switching (abbreviated as MS) ADF test approach. The Markov-switching approach is appealing ex ante. This is because that the switching points are identified endogenously within a model, rather than pre-imposed by the researchers. The model is justified by a parametric encompassing test recommended by Breunig et al. (2003). Bootstrapping, which incorporates a heteroscedastic disturbance term, is used to generate critical values for conducting the MS ADF and the parametric encompassing test.

2. Review of Unit Root Testing Literature

2.1. Diba and Grossman’s Tests

In their stationarity test, Diba and Grossman (1988b) assume that the DGP can be described by the model consisting of equations IV.1 through IV.5.

\[ P_t = (1 + r)^{-1}E_t(p_{t+1} + \alpha d_{t+1} + u_{t+1}) , \]  

\[(IV.1)\]
where \( P_t \) = the real stock price at time \( t \),

\[ r = \text{the constant real discount rate}, \]

\( E_t \) = the conditional expectations operator,

\[ \alpha = \text{a positive constant that valuates expected dividends relative to expected capital gains}, \]

\( d_{t+1} \) = the real dividends payment between time \( t \) and \( t + 1 \), and

\( u_{t+1} \) = a variable that market participants either observe or construct, but that the researcher does not observe.

The fundamental solution for IV.1 is

\[
F_t = \sum_{j=1}^{\infty} (1+r)^{-j} E_t \left[ \alpha d_{t+j} + u_{t+j} \right]. \quad \text{(IV.2)}
\]

Whereas the general solution would include a rational bubble component, \( B_t \):

\[ P_t = F_t + B_t, \quad \text{(IV.3)} \]

and \( B_t \) satisfies

\[ B_{t+1} = (1+r)B_t + z_{t+1}. \quad \text{(IV.4)} \]

The random variable \( z_{t+1} \) is an innovation comprising new information available at date \( t+1 \). This information can be intrinsically irrelevant, or it can be related to relevant variables through parameters that are not present in \( F_{t+1} \). The expected future values of \( z_{t+1} \) are always zero:

\[ E_{t-j} z_{t+1} = 0 \quad \text{for all} \quad j \geq 0. \quad \text{(IV.5)} \]
Assume that $d_i$ is nonstationary in levels, but that the first differences of $d_i$ and $u_i$ are stationary. Then $P_i$ will be nonstationary in levels but stationary in first differences when rational bubbles do not exist. However, when rational bubbles are present, differentiating $P_i$ a finite number of times would not yield a stationary process, since $B_i$ would have the generating process:

$$\begin{align*}
[1 - (1 + r)L](1 - L)B_i &= (1 - L)z_i. \\
\text{(IV.6)}
\end{align*}$$

This is neither stationary nor invertable.

By examining the sample autocorrelations, and by applying the standard ADF tests, Diba and Grossman concluded that both real stock prices and dividends are nonstationary in levels but stationary in first differences. They also conducted a cointegration test on the stock prices and dividends. Rearranging IV.2 and substituting it into IV.3 yields

$$\begin{align*}
P_i - \alpha r^{-1} d_i &= B_i + \alpha r^{-1} \left[ \sum_{j=1}^{\infty} (1 + r)^{-j} E_j \Delta d_{t+j} \right] + \sum_{j=1}^{\infty} (1 + r)^{-j} E_j u_{t+j}. \\
\text{(IV.7)}
\end{align*}$$

Hence, if $u_i$ is stationary in levels, and $d_i$ is stationary in first differences, and if $B_i$ equals zero, then $P_i$ and $d_i$ are cointegrated in order $(1,1)$, with the cointegrating vector $(1, -\alpha r^{-1})$. Their tests, however, show mixed results.
The lack of cointegration in stock prices and dividends could be because of the nonstationarity of the unobservable variable, \( u_t \). They explore this possibility by using the following equation, which is implied by IV.1:

\[
P_{t+1} + \alpha d_{t+1} - (1 + r)P_t = e_{t+1} - u_{t+1},
\]

where \( e_{t+1} \) is the expectation error. That is,

\[
e_{t+1} = P_{t+1} + \alpha d_{t+1} + u_{t+1} - E_t(P_{t+1} + \alpha d_{t+1} + u_{t+1}).
\]

The assumption of rational expectations implies that \( e_{t+1} \) is not serially correlated.

Thus, if \( P_{t+1} + \alpha d_{t+1} \) and \( P_t \) are cointegrated in the order (1,1) with the cointegrating vector \((1, -[1+r])\), then \( u_t \) is stationary in level. Their tests suggest that the null hypothesis of no cointegration can be rejected.

The conclusion that \( \Delta d_{t+1}, \Delta P_{t+1} \) and \( (P_{t+1} + \alpha d_{t+1} - (1 + r)P_t) \) are stationary would imply that \( P_t - \alpha r^{-1}d_t \) is stationary. Therefore, they said that the lack of cointegration between \( P_t \) and \( d_t \) is puzzling. Given these problems, they resorted to an alternative, the Bhargava tests (Bhargave 1986), to further investigate the stationarity properties of \( P_t - \alpha r^{-1}d_t \). Bhargava tests yield the most powerful invariant tests of the random-walk hypothesis against the one-sided stationary and explosive alternatives. The existence of explosive rational bubbles would imply that \( P_t - \alpha r^{-1}d_t \) has an explosive, rather than a unit, root. The Bhargava tests strongly suggest that stock prices and dividends are cointegrated, and, thus, are consistent with the finding that any unobservable fundamental variables and the first differences of stock prices and dividends are all stationary.
To verify that their tests would detect explosive bubbles if they were present, they applied the same tests to simulated time series. Their findings are positive. Hence they concluded that explosive rational bubbles do not exist in stock prices.

2.2. Evans’ Criticism

Evans (1991) argued that, when applied to periodically collapsing rational bubbles, the test procedures suggested by Diba and Grossman can, with high probability, incorrectly lead to the conclusion that these bubbles are not present.

Suppose that the DGP for stock prices can be adequately represented by the standard present-value model given in IV.8 through IV.15.

\[
P_t = (1 + r)^{-1} E_t (P_{t+1} + d_{t+1}), \quad 0 < (1 + r)^{-1} < 1. \quad \text{(IV.8)}
\]

The variables in the equation have the same interpretations as in IV.1. This representation ignores the possibility of unobservable fundamentals, since they are not consequential to the point to be made.

The fundamental solution to IV.8 is

\[
F_t = \sum_{j=1}^{\infty} (1 + r)^{-j} E_t \left[d_{t+j} \right], \quad \text{(IV.9)}
\]

and the general solution is

\[
P_t = F_t + B_t, \quad \text{(IV.10)}
\]
where $B_t$, the rational bubble, satisfies

$$E_t B_{t+1} = (1 + r)B_t. \tag{IV.11}$$

If the first difference of the dividend series is a stationary ARMA process, and if there are no bubbles, then it can be shown that the first difference of the price series is also a stationary ARMA process, and that $P_t$ and $d_t$ are cointegrated with the cointegrating vector $(1, -r^{-1})$. If, instead, $\Delta d_{t+1}$ is stationary but $B_t$ is not absent, then for some $C_t$,

$$E_t F_{t+j} \to C_t + \lambda j \quad \text{as } j \to \infty, \tag{IV.12}$$

where $\lambda = E(\Delta F_t)$,

but

$$E_t B_{t+j} = (1 + r)^j B_t. \tag{IV.13}$$

That is, the conditional expectations of the future fundamental values of the price grow linearly in the forecast horizon $j$, reflecting the unit root in the fundamental process, whereas the conditional expectations of the future values of the bubble contain the root $(1+r) > 1$. If $B_t$ is not zero, then as $j$ increases, the conditional expectations of $P_{t+j}$ eventually will be dominated by the explosive root $(1 + r)$, if a bubble is present. Furthermore, differencing the price will not render the series stationary, since

$$\lim_{j \to \infty} E_t \Delta F_{t+j} = E(\Delta F_t), \tag{IV.14}$$

This is a constant, but
\[ E_t \Delta B_{t+j} = r(1 + r)^{j-1} B_t , \]  

(IV.15)

which is explosive, if \( B_t \neq 0 \). Hence, the conditional expectation of \( \Delta P_{t+j} \) will be stable if the bubble is absent, but explosive otherwise.

These considerations are the motivations behind the unit root and cointegration tests by Diba and Grossman (1988b).

Evans, however, demonstrate that if the bubbles collapse periodically, the tests suggested above have very little power to detect the presence of a bubble.

Consider the class of rational bubbles that are always positive but that collapse periodically:

\[ B_{t+1} = (1 + r)B_t u_{t+1} , \quad B_t \leq \alpha ; \]  

(IV.16)

\[ B_{t+1} = [\delta + \pi^{-1}(1 + r)\theta_{t+1} (B_t - (1 + r)^{-1} \delta)]u_{t+1} , \quad \text{if } B_t > \alpha ; \]  

(IV.17)

where \( \alpha \) and \( \delta \) are positive parameters with \( 0 < \delta < (1 + r)\alpha \), and

\[ u_t = \text{an exogenous i.i.d positive random variable, with } E_t u_{t+1} = 1 ; \]

\[ \theta_{t+1} = \text{an exogenous i.i.d Bernoulli process independent of } u_t , \text{ with} \]

\[ \Pr(\theta_{t+1} = 1) = \pi , \]

\[ \Pr(\theta_{t+1} = 0) = 1 - \pi , \text{ and } 0 < \pi \leq 1. \]

Assume that

\[ u_t = \exp \left( y_t - \frac{\tau^2}{2} \right) , \text{ and } y_t \xrightarrow{d} iid, N(0, \tau^2) . \]  

(IV.18)
The frequency with which a bubble of this type erupts, the average length of time
the bubble expands, and the magnitude of the bubble are affected by the process
parameters $\alpha$, $\delta$, and $\pi$.

When the Bhargava tests are applied to the 200 simulated samples of size 100,
generated by DGPs described by equations IV.8 to IV.18, Evans found that the
results of tests depend critically on $\pi$, the probability per period that the bubble
does not collapse. When $\pi$ is close to 1, the test results are similar to those
obtained by Diba and Grossman. However, for $\pi \leq 0.95$, quite different results
are obtained. In fact, when $\pi \leq 0.75$, more than 90% of the simulations reject the
null hypothesis of a unit root process in favor of stable alternatives by both N1
and N2 statistics. These results appear to be robust to moderate changes in the
other model parameters.

Evans explains that the maintained hypothesis for the Bhargava test is a first-
order autoregressive process. When $\pi$ is close to 1, the process in IV.17
converges to that in IV.16. But when $\pi \leq 1$, the bubble process in IV.17 is a
complex, nonlinear process, which falls outside the maintained hypothesis. Thus,
unless $\pi$ is close to 1, the pattern of periodic collapse generated by IV.17 looks
more like a stable AR(1) process than an explosive one, despite the explosive
root in the conditional expectation of the bubble sequence.
Evans also applied the Dickey-Fuller unit root tests and cointegration tests to the simulated stock prices and dividends, assuming that

$$d_t = \mu + d_{t-1} + \varepsilon_t, \quad \varepsilon_t \xrightarrow{d} iid, N(0, \sigma_\varepsilon^2). \quad (IV.19)$$

The results clearly show that the DF $\phi_3$ statistic is unable to find the bubble when it is present. The cointegration tests, using the Durbin-Watson statistic and the Engle and Granger (1987) $\xi_2$ and $\xi_3$ statistics, also incorrectly indicate the absence of bubbles in the majority of simulations. In short, periodically collapsing bubbles are not detected by standard unit-root and cointegration tests.

### 2.3. Markov-switching Unit Root Test

Hall et al. (1999) argue that, when rational bubbles exist, the dynamics of asset prices are driven by the dynamics of the bubbles. If the bubbles collapse periodically, then the values taken by the parameters of the price-generating process in the bubble expansion state will differ from those in the bubble-collapsing state. That is the DGP of the price experiences structural breaks. When structural breaks exist, the standard ADF tests have little power. In such cases, allowing for the ADF regression parameters to take on different values in different states will improve the power of the tests. In particular, the authors suggested making use of the class of dynamic Markov-switching models explored in Hamilton (1989, 1990) and basing the unit-root test on the following regression model:
\[ \Delta y_t = \mu_0 (1 - s_t) + \mu_1 s_t + \left[ \phi_0 (1 - s_t) + \phi_1 s_t \right] y_{t-1} + \sum_{j=1}^{k} \left[ \psi_{0j} (1 - s_t) + \psi_{1j} s_t \right] y_{t-j} + \sigma e_t, \]

where \( e_t \overset{d}{\longrightarrow} iid, N(0,1) \) \hspace{1cm} (IV.20)

and \( s_t \) is a state variable independent of \( e_m \) for all \( t \) and \( m \), and follows the first-order Markov chain on the state space \( \{0,1\} \) with transition probabilities

\[ \Pr(S_t = 1|S_{t-1} = 1) = p, \]
\[ \Pr(S_t = 0|S_{t-1} = 1) = 1 - p, \]
\[ \Pr(S_t = 0|S_{t-1} = 0) = q, \] and
\[ \Pr(S_t = 1|S_{t-1} = 0) = 1 - q. \] \hspace{1cm} (IV.21)

The coefficient on \( y_{t-1} \) provides the basis for testing. For example, the existence of an explosive rational bubble in a price series is consistent with \( \phi_0 > 0 \) or \( \phi_1 > 0 \) for the price but not for its associated dividend series. On the other hand, when \( \phi_0 = \phi_1 = 0 \) for both the price and the dividend series, there is no rational bubble.

A test of the unit-root null hypothesis may be based on the asymptotic t-ratios associated with the ML estimates of \( \phi_0 \) and \( \phi_1 \).

The authors conducted a simulation study based on 500 independent realizations of \( \{P_t\} \) from DGPs identical to those used by Evans (1991). Two alternative assumptions about the generating mechanism of real dividends are used:

\[ d_t = \mu + d_{t-1} + e_t, \] \hspace{1cm} (IV.22)
\[ \ln d_i = \mu + \ln d_{i-1} + \varepsilon_i, \quad \text{(IV.23)} \]

where

\[ \varepsilon_i \xrightarrow{d} iid, N(0, \sigma^2) \, . \]

Their results show that, unlike the standard ADF test, the MS ADF procedure has considerable power to detect the presence of bubbles in \( \{P_i\} \). They cautioned, however, that these results do not imply that switching ADF tests would successfully detect all types of periodically collapsing bubbles. For example, if the contribution of the bubble to the volatility of the prices is not substantial, or the probability of the bubble collapse \( 1 - \pi \) is relatively large, it would be difficult for any tests to confirm the presence of the bubble.

The authors went on to apply the MS ADF testing procedure to investigate the integration properties of consumer prices in Argentina. As argued in Diba and Grossman, whether or not the nonstationarity in prices reflects a rational bubble depends on the time-series properties of the economic fundamentals driving the prices. One known and observable economics fundamental to consumer prices is the money supply. The nonstationarity in prices could also be caused by the nonstationarity of other unobserved economic fundamentals, and not a rational bubble. Hence, the authors included two other time series in their tests: the monetary base and exchange rate (in terms of the US dollar) in Argentina. Since both consumer prices and exchange rates are likely to be driven by common fundamentals, evidence of simultaneous change in these two series would suggest that the nonstationarity in prices is attributable to their market
fundamentals. On the other hand, asynchronous changes across the two series may be explained by the presence of a rational bubble. For example, if both series switch simultaneously to the explosive regime, represented by \( s_i = 1 \), while the money process remains in the non-explosive regime (\( s_i = 0 \)), one can infer that the event is being driven by some unobservable economic fundamental common to the price and exchange rate, rather than by explosive rational bubbles. Conversely, when prices switch to the explosive regime while the two other series remain in the non-explosive regime, one can conclude that there is a rational bubble in the price. Using the MS ADF test, the authors were able to identify rational bubbles presented in the consumer prices and exchange rates of Argentina.

3. Methodology

3.1. Conventional Unit-root Tests

Dickey and Fuller (1979) suggested a battery of tests based on a regression of the form \( y_t = \rho y_{t-1} + u_t \) or \( y_t = \mu + \rho y_{t-1} + u_t \) or \( y_t = \mu + \delta t + \rho y_{t-1} + u_t \), and the true process being \( y_t = y_{t-1} + u_t \) or \( y_t = \mu + y_{t-1} + u_t \). In these tests, the disturbance term is assumed to be i.i.d. and normal with zero mean and constant variance (Hamilton, 1994a, 502).
Phillips (1987) and Phillips and Perron (1988) suggest some modifications to the DF test statistics to take care of serially correlated and heteroscedastic disturbance terms. The test suggested by Phillips and Perron are referred to as the PP test.

Dickey and Fuller (1979) provide an alternative approach that controls for serial correlation by including higher-order autoregressive terms in the regression. That is the model: $$y_t = \mu + \delta t + \rho y_{t-1} + \sum_{i=1}^{p-1} \zeta_i \Delta y_{t-i} + u_t$$ is to be estimated with possibly zero coefficients on the constant and the trend terms. This modified DF test is referred to as the ADF test.

Various suggestions have been proposed regarding how to proceed when the process is deemed as $AR(p)$ with $p$ unknown but finite. Hamilton (1994a) suggests a simple approach that takes $p$ to be some pre-specified upper bound $\tilde{p}$ (We set $\tilde{p} = \sqrt{T}$ in this paper. $T$ is the sample size). The OLS t-ratio of $\zeta_{p-1}$ can then be compared with the usual critical value for a t statistic. If the null hypothesis is not rejected, then the OLS F test of the joint null hypothesis (that both $\zeta_{p-1} = 0$ and $\zeta_{p-2} = 0$) can be compared with the usual $F(2,T-K)$ distribution. The procedure continues sequentially until the joint null hypothesis (that $\zeta_{p-1} = 0$, $\zeta_{p-2} = 0$, ..., $\zeta_{p-l} = 0$) is rejected for some $l$. Greene discusses the merits and flaws of this procedure (1997, 787).

A MS model assumes that time series data may display periodic changes in their observed behavior, and it accounts for such changes through switches in states. The average duration of each state is allowed to differ. Furthermore, the statistical features and identification of the states are not imposed exogenously on the data, but determined endogenously by the estimation procedure.

Consider

\[ \Delta y_t = \mu_t + \phi_t y_{t-1} + \sum_{k=1}^{K} \psi_k \Delta y_{t-k} + \varepsilon_t, \quad \varepsilon_t \sim iid, N(0, \sigma^2), \]  

(IV.24)

where \( s_t \in \{1, 2, \ldots, N\} \), a state variable following the first order Markov chain:

\[
\Pr(s_{t+1} = j | s_t = i, s_{t-1} = i_1, \ldots, \zeta_t)
\]

\[
= \Pr(s_{t+1} = j | s_t = i)
\]

\( \equiv p_{ij} \)  

(IV.25)

where \( \zeta_t = (y_t, y_{t-1}, \ldots, y_1) \), representing the information set available at time t, and \( p_{ij} \) is the probability that state \( i \) will be followed by state \( j \) given \( s_t = i, s_{t-1} = i_1, \ldots, and \zeta_t \). Equation (IV.25) says that the probability distribution of \( s_{t+1} \) depends on past events only through the value of \( s_t \). The state variable \( s_t \) is not observable, but its probability for a given sample of size \( T, \Pr(s_t = i | \zeta_T) \), can be inferred using the discrete Kalman filter (refer to Hamilton, 1989, 1994a, 1994b for detail).
In estimating, with discrete Kalman filter, the smoothed inference of the state
variable, \( \Pr(s_i = i|\zeta_T) \), we assume that the DGP parameters, \( \beta = (\mu^u, \phi^u, \psi^u_k, p_y, \sigma)' \), are known to us. In fact, these parameters need to be estimated. We can estimate them by maximizing the log likelihood function of the observed data using the EM algorithm, since the EM algorithm is efficient, simple, and stable.\(^{xx}\) The log-likelihood function to be maximized is

\[
LL = \sum_{t=1}^{T} \log f(y_t|x_t, \zeta_{t-1}), \quad \text{with} \quad f(y_t|x_t, \zeta_{t-1}) \quad \text{the density of} \quad y_t \quad \text{conditional on} \quad x_t \quad \text{and} \quad \zeta_{t-1}.
\]

The estimation steps are given below.

1. Make an arbitrary guess about the values of \( \mu^u, \phi^u, \psi^u_k, p_y \) and \( \sigma \).

2. Calculate the smoothed probabilities of \( s_i \) using the discrete Kalman filter.

3. OLS regress \( y_t \sqrt{\Pr(s_i = i|\zeta_T)} \) on \( x_t \sqrt{\Pr(s_i = i|\zeta_T)} \), \( i = 1, 2 \), which gives the ML estimates

\[
\tilde{\mu}^u, \tilde{\phi}^u, \tilde{\psi}^u_k, \quad (k = 1, 2, \ldots K).
\]

Notice that \( y_t \equiv \Delta y_t, x_t \equiv (1, y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-k})', \quad (k = 1, 2, \ldots K) \).

4. Update \( \sigma^2 \) using the OLS residuals.

\[
\tilde{\sigma}^2_{u} = \frac{(y_t - x_t' \tilde{\beta}_u)(y_t - x_t' \tilde{\beta}_u)}{(T - J)}, \quad (IV.26)
\]

where \( J = \) the number of parameters estimated in each state.

5. Update \( p_y \).
\[ p_{ij} = \frac{\sum_{t=2}^{T} \Pr(s_i = j, s_{t-1} = i \mid \sigma_T)}{\sum_{t=2}^{T} \Pr(s_{t-1} = i \mid \sigma_T)} \]  

(IV.27)

6. Update \( \pi \).

\[ \pi_i = \Pr(s_i = i \mid \sigma_T) \]  

(IV.28)

Repeat steps 2 through 6 until the parameters and the likelihood converge.

### 3.3. Specification Tests of the Markov-switching Model

An important issue pertaining to an MS model is the number of states characterizing the data. The standard distributional theory is not applicable for evaluating the Markov-switching model against some popular alternatives, such as a linear time-series model. Hamilton (1989) shows that conventional tests of a Markov-switching model would render the Markov transition matrix unidentified and the information matrix singular, under the null hypothesis of a single state.

Several authors have proposed alternative testing procedures that attempt to overcome these problems. However, the application of these procedures can be problematic. The problems arise because that we have limited knowledge of the respective powers of these tests, and these tests are, in general, computationally demanding (Raymond and Rich 1997). For these reasons, perhaps, the previous studies seldom validate their Markov-switching specifications\(^{xxi}\).

Breunig et al. (2003) argue that a Markov-switching model should be put to specification tests, like any other model. Among the four types of tests suggested,
they highly recommend a Wald test, which they call “encompassing test”. They show that this encompassing test is the most powerful way of examining the ability of the model to match the data. Therefore, this study will employ this test to validate the MS model being estimated. The encompassing test procedure is described below.

Let \( \hat{\gamma} \) be an estimate from the data. This \( \hat{\gamma} \) can be the mean, the variance, or something else. We denote a comparable quantity implied by the MS model by \( \gamma_M(\hat{\theta}) \), where \( \hat{\theta} \) is the MLE of the parameter \( \theta \) associated with the MS model. In particular, we simulate data with the estimated MS model, and estimate \( \gamma_M(\hat{\theta}) \) from the simulated series. A scaling factor of \( 1 + \frac{T}{M} \) is applied to the variance of any test statistic to make an allowance for the effect of the simulation error upon the variance of an estimator, where \( M \) is the number of replication and \( T \) the number of observations in the sample.

Consider the statistic

\[
\hat{t} = \hat{\gamma} - \gamma_M(\hat{\theta}).
\]

Under the null

\[
\tau_0 = \gamma_0 - \gamma_M(\theta_0),
\]

where \( \theta_0 \) is the true value \( \theta \) and \( \gamma_0 \) the true value of \( \gamma \), we have

\[
T^{1/2}(\hat{t} - \tau_0) \xrightarrow{d} N(0, V_\tau).
\]
Consider the test statistic
\[ R^* = \hat{\tau} \left[ \text{Var}(\hat{\tau}) \right]^{-1} \hat{\tau}, \]
which has a \( \chi^2 \) distribution with degrees of freedom equal to the dimension of \( \tau \), with
\[ \text{Var}(\hat{\tau}) = \text{Var}(\hat{\gamma}) - \text{Var}(\gamma_M(\hat{\theta})). \]
This statistic can be replaced by
\[ R = \hat{\tau} \left[ \text{Var}(\hat{\gamma}) \right]^{-1} \hat{\tau}. \]
As \( R < R^* \), if \( R \) exceeds the critical value, we would reject the null even more strongly with \( R^* \).

Under the null, that MS model is correct and characterized by parameter \( \hat{\theta} \), we can simulate data from the model and find out \( \text{Var}(\hat{\gamma}) \) from the simulated series. Alternatively, we may use asymptotic theory and compute a robust estimator of \( \text{Var}(\hat{\gamma}) \).

In the current study \( \hat{\gamma} \) corresponds to \( \text{SSE}^{xxii} \) from the MS model. \( \gamma_M(\hat{\theta}) \) is the sample mean of SSE from the simulation with 10,000 replications, and \( \text{var}(\hat{\gamma}) \) the sample variance of SSE. Under the null \( \hat{\gamma} = \gamma_M(\hat{\theta}) \), the test statistic \( R \) has a \( \chi^2(1) \) distribution. A scaling factor \( \left( 1 + \frac{T}{M} \right) \) is applied to \( \text{Var}(\hat{\gamma}) \), as discussed before.

### 3.4. Bootstrapping
The null distributions of the statistics for the linear unit-root tests are tabulated in Hamilton (1994a). Those for the Markov-switching ADF tests are unknown but can be generated by bootstrapping.\textsuperscript{xxiii}

Bootstrapping is a method for estimating the distribution of an estimator or test statistic by resampling the data. It amounts to treating the data as if they were the population for the purpose of evaluating the distribution of interest. Under mild regularity conditions, Bootstrapping yields an approximation to the distribution of an estimator or test statistic that is at least as accurate as the approximation obtained from first-order asymptotic theory. Thus, bootstrapping provides a way to substitute computation for mathematical analysis if calculating the asymptotic distribution of an estimator or statistic is difficult (Horowitz 2001).

In fact, bootstrapping is more accurate in finite samples than first-order asymptotic approximations and does not entail the algebraic complexity of higher-order expansions. The first-order asymptotic theory often gives poor approximations to the distributions of test statistics with the sample sizes available in applications. As a result, the nominal probability that a test based on an asymptotic critical value rejects a true null hypothesis can be very different from the true rejection probability (RP). Bootstrapping often provides a tractable way to reduce or eliminate finite-sample errors in the RPs of statistical tests.
The method nevertheless has its own limitations and should not be used blindly, but it works well in general. The readers are referred to the *Handbook of Econometrics*, vol. 5, chapter 52 for details on the sampling procedure and the consistency of bootstrapping.

The steps of bootstrapping are described below.

1. Save the ML parameter estimates $\tilde{\theta}$ and residuals $\{\tilde{\varepsilon}_t\}_{t=1}^T$ from the MS model.
2. Construct the random disturbance term $e$ (to be explained later in this section).
3. Take a random draw of $e$, denote as $e_1^{(1)}$, and set

$$
\Delta y_1^{(1)} = \tilde{\mu} + \sum_{k=1}^{K} \tilde{\psi}_k \Delta y_{-k} + e_1^{(1)},
$$

$$
\Delta y_2^{(1)} = \tilde{\mu} + \tilde{\psi}_1 \Delta y_1^{(1)} + \sum_{k=2}^{K} \tilde{\psi}_k \Delta y_{-k} + e_1^{(1)},
$$

$$
\cdots
$$

$$
\Delta y_T^{(1)} = \tilde{\mu} + \sum_{k=1}^{K} \tilde{\psi}_k \Delta y_{T-k}^{(1)} + e_1^{(1)},
$$

where $\Delta y_t^{(1)} =$ simulated values of $\Delta y_t$, $\Delta y_{-k} =$ actual observed values of $\Delta y_t$, and $\tilde{\mu}, \tilde{\psi}_k =$ ML estimates.

This gives a full sample $\{y_t^{(1)}\}_{t=1}^T$, where T is the number of observations in the sample.
4. Fit the artificial sample to equation (1), producing estimates of model parameters, \( \hat{\theta}^{(i)} \), and their associated \( \tau \) and rho statistics.

5. Repeat steps 3 and 4 10,000 times, giving \( \{\hat{\theta}^{(i)}\}_{i=1}^{10000} \) and the 10,000 associated \( \tau \) and rho values. The 95% confidence interval for the ML estimates of \( \hat{\theta} \), and the \( \tau \) and rho statistics constructed under the null hypothesis include 95% of the values of \( \tilde{\theta}^{(i)} \) and the associated values of \( \tau \) and rho, respectively.

In this procedure, the random disturbance term, \( e \), is not constructed as an \( i.i.d. \) process. This is because that our MS model residuals exhibit clusters when plotted against time (figures IV.4-7). We model the MS residuals, \( e_i \), with an ARCH(q) process of the form

\[
\varepsilon_i = u_i \left[ \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{i-i}^2 \right]^{1/2},
\]

where

\[
u_i \xrightarrow{d} iid(0,1)
\]

and

\[
E[\varepsilon_i | \varepsilon_{i-i}] = 0
\]

\[
Var[\varepsilon_i | \varepsilon_{i-i}] = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{i-i}^2
\]

\[
Cov[\varepsilon_i, \varepsilon_{i-i}] = 0, \quad \text{for} \quad i \geq 1
\]

where \( q \) is selected by the usual F test. This model has the feature that disturbances are heteroscedastic but serially uncorrelated, as the covariance is
zero between $\varepsilon_i$ and $\varepsilon_{t-i}$ for all $i \geq 1$ in this model. We can estimate this ARCH(q)
model using the following procedure (Greene 1997; Bollerslev 1986)

1. Regress the squared ML residuals on their corresponding four lagged values
to give the first estimates of $\alpha_i$, denoted by $a_i$, $i = 0,1,\ldots,4$.

2. Compute the conditional variances using

$$
\hat{\sigma}_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + a_2\varepsilon_{t-2}^2 + a_3\varepsilon_{t-3}^2 + a_4\varepsilon_{t-4}^2.
$$

Run the regression \( \left( \frac{\varepsilon_t^2}{\sigma_t^2} - 1 \right) = d_0 \frac{1}{\sigma_t^2} \)

$$
+ d_1 \frac{\varepsilon_{t-1}^2}{\sigma_t^2} + d_2 \frac{\varepsilon_{t-2}^2}{\sigma_t^2} + d_3 \frac{\varepsilon_{t-3}^2}{\sigma_t^2} + d_4 \frac{\varepsilon_{t-4}^2}{\sigma_t^2}.
$$

3. The asymptotically efficient estimator of $\alpha$ is given by $\hat{\alpha} = a + d$, where

$\hat{\alpha}, a, d$ are all $5 \times 1$ vectors and the $\text{Asy.Var}(\alpha) = 2(Z'Z)$, where $Z : T \times 5$ and

$$
z_t = \begin{bmatrix} 1 & \varepsilon_{t-1}^2 & \varepsilon_{t-2}^2 & \varepsilon_{t-3}^2 & \varepsilon_{t-4}^2 \end{bmatrix}.
$$

The estimated parameters of the ARCH(q) model are then used to generate the disturbance term in the bootstrapping procedure.

4. Empirical Results

The empirical literature suggests that bubbles are short-lived phenomena, and estimations using a long sample may fail to capture a bubble if the bubble does not last long enough (Rappoport and White 1994; Kim and Lee 2000). The current study will employ a Markov-switching approach to deal with this concern.
It assumes that the data generating process is AR(p) with Markov-switching. That is there are regime changes in the data generating process. The regime in which a bubble expands differs from that in which a bubble collapses. The Markov-switching approach has the advantage that the switching of a regime is determined endogenously, rather than pre-imposed by the researchers.

The number of lags, $p$, in the model is selected using F tests (Hamilton 1994, 530). The AR(p) model estimated has a constant term but no trend, that is,

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p-1} \xi_i \Delta y_{t-i} + u_t,$$

as the samples show no signs of trending in the long run. This would be the case if the bubble, if exists, is only sustainable in the short run. The number of lags selected for each sample is listed in table IV.1.

Visual observations, preliminary linear ADF and PP tests using shorter samples, and a previous estimations of a three-state MS model suggest that a two-state Markov-switching model should be fitted to the data sets. One of the states is, perhaps, a state in which a bubble is alive, the other the bubble dormant. The data sets are scanned for a proper starting point, so that the resulting smoothed probabilities show reasonable fluctuations. The starting point finally chosen for Seoul housing was 90:7. Therefore, the selected data set (90:7–03:6) has 156 observations. The complete sample for Hong Kong office sector is adopted, for reasonable movements in smoothed probabilities are obtained with this sample\textsuperscript{xxv}. The linear ADF and PP tests do not reject the null of no bubble for these data sets (table IV. 2).
The maximum likelihood estimates of the MS model parameters are listed in tables IV.3 and IV.4, alongside their Hessian and White’s t-ratios\textsuperscript{xxvi}. Notice that the White’s t-ratios are much less significant than their Hessian counterparts. The encompassing (Wald) specification test shows that the null of two-state Markov-switching model cannot be rejected for the data sets under consideration (table IV.5). The alternative is a linear model. Table IV.6 records the state transition probabilities. These transition probabilities show that whenever the Seoul housing price reaches state 2, it will switch back to state 1 with certainty, as $p_{21} = 1$; state 1 is persistent in Seoul housing rent, because $p_{11} = 0.99$; both states are likely to appear in Hong Kong office price, given the values of the switching probabilities $p_{12}$ and $p_{21}$; the rent series of Hong Kong office is more likely to switch to and stay in state 2, as $p_{12}$ is as high as 0.826, and $p_{22} = 0.853$. These observations are more or less confirmed by the unconditional state probabilities $\pi_i, i = 1, 2$ and the second eigenvalues of the state transition matrix, $\lambda_2$. $\lambda_2$ shows that the states of the two prices have negatively serial correlation. That is state 1 is likely to be followed by state 2, and vice versa, while the states of the other two time series are positively serially correlated. With positive autocorrelation, a state is likely to persist once the data generating process enters that state.

A plot of the smoothed probabilities of the states in figure IV.8 echoes the predictions of state transition probabilities and those of the eigenvalues of the
transition matrix. There are frequent switches of states in Seoul housing price. On the other hand, state one of its corresponding rent series is highly persistent. There are also frequent switches of states in Hong Kong office price, but state two is more persistent in its associated rent series.

These observations suggest that a bubble, if exists in the price of interests, will be periodically collapsing. Furthermore, the Markov chain estimated is ergodic, because one eigenvalue of the state transition matrix is unity and the other is inside the unit circle. Hence, the long term forecast of the Markov chain is given by the unconditional state probabilities (Hamilton 1994a, 682).

The parameter estimates of the ARCH model for the MS model residuals (along with their t-ratios) are summarized in table IV.7. Not all t-ratios of the estimated parameters are significant, but the F test shows that the parameters of the selected models are jointly significant at the 5% level (table IV.8). The test statistics in table IV.9 for ARCH effect, $TR^2$, are highly significant in all cases ($R^2$ is the goodness of fit measure of the regression, T the sample size). The plots of the squared ML residuals and their corresponding predicted values demonstrate that the estimated models capture quite well the patterns of the MS residuals (figure IV.4-7).

Thus, the ARCH disturbances are incorporated into the bootstrapping procedure in generating the distribution of ADF τ and rho statistics for the Markov-switching
ADF test. For each series, 10,000 replications are used. We can then compare the $\tau$ and the rho statistics of the MS models with these simulated distributions, and test the null hypothesis of “unit root” (table IV.10).

A problem arises when one tries to draw conclusions. The two statistics, $\tau$ and rho, suggest different behavior of the series on two occasions: Seoul housing prices and rent in state two, and Hong Kong Office price in state two (table 10). Therefore, if $\tau$ is used, the conclusion would be that, for most of the period between July 1990 and the end of 1992, the null of no bubble can be rejected for Seoul housing prices. However, we cannot reach the strong conclusion that there must a bubble. This is because the explosiveness of the price could have been backed up by some other fundamentals. This is the dilemma facing the literature which we discussed in the “Preface” of this thesis. Suppose we tentatively conclude that the explosiveness of the price was indeed a result of speculative bubble. This bubble disappeared at the end of 1992 but reappeared in late 2001 (table IV.11, figure IV.8). This conclusion is consistent with the observations from the plots in figures IV.1, and with the conclusion of the various researchers (Ch.4.3.2). Following this line of thinking, the $\tau$ statistic identified four possible positive bubble episodes in Hong Kong office prices: late 1987 to late 1989, early 1994, late 1997 to late 1998, and early 2001 (table IV.11, figure 8). These are largely consistent with the facts shown in the plots of figures 1, and with the conclusions of Chan et al. (2001). Statistically speaking, however, this fact cannot be established for Hong Kong office price, though can for Seoul housing
price. The strong conclusion can only be reached if the unit root hypothesis could be rejected in favor of an explosive root for the price, and rejected in favor of a stable root for the rent. The reason is that non-rejection of a unit-root does not prove the existence of a unit root.\textsuperscript{xxvii}

If rho is used, the null hypothesis of no bubble can be rejected for Seoul housing prices throughout most part of the period between July 1990 and the end of 1992. But the fact that there was a bubble cannot be established for the two problems stated in the previous paragraph: one is the unobserved fundamentals, and the other is the power of the test. Suppose the testing results point to the existence of a positive bubble. This possible-bubble disappeared after the end of 1992, reasserted itself briefly in 1997 (for about a year), and revived again in late 2001. These accounts are mostly compatible with the conclusion of $\tau$, except for the period involving late 1997 and early 1998. But the rho side of the story about Hong Kong is less interesting. It says that the city might have experienced a positive bubble throughout the entire sample period (table IV.11).
Chapter V Extracting Missing Fundamentals with the Kalman Filter

1. Introduction

In chapter IV, I have shown with Markov-switching unit-root testing that both the Seoul housing price and the Hong Kong office price have experienced speculative bubbles in the past two decades.

In this chapter, I intend to quantify the bubble elements contained in the prices of interests, namely Seoul housing price and Hong Kong office price (the same date set as in chapter IV). Two models are estimated. One assumes that a variant of the present value model correctly specifies the fundamental process. The other assumes that this model has missed out important explanatory fundamental variables. The omitted variable is captured by a latent state variable which can be inferred using the Kalman filter. The former is referred to as the present value model, whereas the later as the state space model.

2. The Theory of Asset Pricing and the Kalman Filter

In the standard asset-pricing model, individuals estimate asset values according to their expectations of future cash flows and required rates of return (the discount rate). They buy (sell) when prices fall below (rise above) their value
estimates. Knowledgeable investors recognize that their estimates contain errors, combined with risk aversion and capital constraints, therefore unwilling to buy (sell) unless prices deviate significantly from what they perceive to be true. How much a risk-averse investor is willing to commit beyond a particular asset position depends on his wealth and ability to borrow, as well as the size of his current position. Investors' flow demand (supply) schedules will systematically create an increased demand (supply) for an asset whose price falls (rises). Consequently, in the absence of new information, prices should be fairly stable.

Investors are heterogeneous in their estimates of intrinsic asset values. When investors have different opinions, the market price will reflect their willingness and ability to trade. If most investors believe an asset is undervalued, their willingness to buy causes the price of the asset to rise. Investors who are generally correct in their estimates of asset values tend to make money and accumulate wealth, thus have more influence on price setting. This will improve asset pricing over time. Hence asset prices have the tendency to return to their fundamental true value in the long run. But disturbances, which cause information noise, can drive the asset price far above or below its true value in the short run.

2.1. The Present-value Model and Rational Speculative Bubbles
The major references for the model shown below are due to Campbell and Shiller (1988 a, b). It will be used as a benchmark model in this chapter for bubble quantification. The argument for using this model in the real-estate market can be found in chapter II, section 2.

If economic agents are risk-neutral, the price of a real estate, $P_t$, will be equal to the expected discounted present value of the rent accruing to ownership of the property during the ownership period, $D_t$, plus the price at which the property can be sold at the end of the ownership period, $P_{t+1}$. Mathematically,

$$ P_t = \frac{E_t[P_{t+1} + D_t]}{1 + R_t}, \quad (V. 1) $$

where

- $P_t$ = the real price of the property asset at time $t$;
- $D_t$ = the real total rents received during the period $t$;
- $R_t$ = the time-varying real discount rate.

Define

$$ r_t \equiv \log(1 + R_t); \quad (V. 2) $$

hence,

$$ r_t \equiv \log(E_t[P_{t+1} + D_t]) - \log(P_t). \quad (V. 3) $$

In a static world, rent grows at a constant rate, $g$, and the log of rent-to-price ratio is also a constant. That is,
\[ \log \left( \frac{D_t}{D_{t-1}} \right) = d_t - d_{t-1} = \Delta d_t = g, \]  
(V.4)

and

\[ \log \left( \frac{D_{t-1}}{P_t} \right) = d_{t-1} - p_t = \delta, \]  
(V.5)

where

\[ p = \text{the log of } P; \text{ and} \]

\[ d = \text{the log of } D. \]

Equation V.4 and V.5 imply that the asset price grows at the same rate as the rent, and the ratio of the asset price to the sum of the asset price and rent is also a constant. That is,

\[ \log \left( \frac{P_{t+1}}{P_t} \right) = \log \left( \frac{D_t/\delta}{D_{t-1}/\delta} \right) = \log \left( \frac{D_t}{D_{t-1}} \right) = g, \]  
(V.6)

and

\[ \frac{P_t}{P_t + D_{t-1}} = \frac{1}{1 + \frac{D_{t-1}}{P_t}} = \frac{1}{1 + \exp(\delta)} \equiv \rho. \]  
(V.7)

In such a world, the log of the gross discount rate, \( r_t \), would also be a constant. To see this, define

\[ \tilde{\xi}_t = \kappa + \rho p_{t+1} + (1 - \rho)d_t - p_t. \]  
(V.8)

Given the characteristics of the static world, \( \tilde{\xi}_t = \kappa + g + (1 - \rho)\delta \equiv \xi \), which is a constant. If we set

\[ \kappa = -\log(\rho) - (1 - \rho)\delta, \]  
(V.9)
then

\[
\xi = -\log(\rho) + g = \log \left( \frac{E_t[P_{t+1} + D_t]}{P} \right) = r. \tag{V. 10}
\]

Thus, in a static world,

\[
r_t = \xi_t = \kappa + \rho p_{t+1} + (1 - \rho)d_t - p_t = \xi. \tag{V. 11}
\]

In a world that is evolving over time, V.11 would hold only approximately.

Solve V.11 for \( p_t \) by forward iteration:

\[
p_t = \kappa - \xi + \rho E_t[p_{t+1}] + (1 - \rho)d_t, \]

\[
= \kappa - \xi + \rho(\kappa - \xi + \rho E_t[p_{t+2}] + (1 - \rho)E_t[d_{t+1}]) + (1 - \rho)d_t, \]

\[
= \ldots \]

\[
= \frac{\kappa - \xi}{1 - \rho} + \rho^j E_t[p_{t+j}] + (1 - \rho)\sum_{j=0}^{i-1} \rho^j E_t[d_{t+j}], \tag{V. 12}
\]

If the transversality condition, \( \lim_{i+x} \rho^j E_t[p_{t+i}] = 0 \), is satisfied, we would have the fundamental solution for the price of property:

\[
p_t = p_t^f = \frac{\kappa - \xi}{1 - \rho} + (1 - \rho)\sum_{j=0}^{\infty} \rho^j E_t[d_{t+j}]. \tag{V. 1329}
\]

However, the transversality condition may fail to hold. In such a case, we would expect the bubble solution for the asset price

\[
p_t = p_t^f + b_t, \tag{V. 14}
\]

Where the bubble component is

\[
E_t[b_{t+1}] = \frac{1}{\rho^j} b_t. \tag{V. 15}
\]

This implies that
\[ E_t[b_{t+1}] = \frac{1}{\rho} b_t. \]  
(V. 16)

If the log property prices and the log property rents are \( I(1) \) process, (that is, having one unit root), the following model may be estimated instead:

\[
\Delta p_t^f = p_t^f - p_{t-1}^f = (1 - \rho) \sum_{j=0}^{\infty} \rho^j \{ E_t[d_{t+j}] - E_{t-1}[d_{t+j-1}] \}. 
\]  
(V. 17)

Suppose the growth of the rents follows an AR(1) stochastic process.

\[
\Delta d_t = \phi \Delta d_{t-1} + \delta_t; \quad E[\delta_t] = 0, \quad \text{Var}[\delta_t] = \sigma_\delta^2. \]  
(V. 18)

Then

\[
\Delta p_t^f = \frac{1}{1 - \phi \rho} \Delta d_t - \frac{\phi \rho}{1 - \phi \rho} \Delta d_{t-1} \equiv \psi \Delta d_t + (1 - \psi) \Delta d_{t-1}. \]  
(V. 19)

*Further lagged rent will appear on the right hand side of equation V.19, if the rent process is \( AR(p) \), with \( p > 1 \).* If a bubble is present,

\[
\Delta p_t = \Delta p_t^f + \Delta b_t, \]  
(V. 20)

where

\[
E_t[\Delta b_{t+1}] = \frac{1}{\rho} \Delta b_t. \]  
(V. 21)

Suppose both price and bubble processes are stochastic. Then

\[
\Delta p_t = \Delta p_t^f + \Delta b_t + \omega_t, \quad E[\omega_t] = 0, \quad \text{Var}[\omega_t] = \sigma_\omega^2; \]  
(V. 22)

\[
\Delta b_{t+1} = \frac{1}{\rho} \Delta b_t + \zeta_t, \quad E[\zeta_t] = 0, \quad \text{Var}[\zeta_t] = \sigma_\zeta^2. \]  
(V. 23)

In arriving at equation V.19, we have made some simplified assumptions about the fundamental process, such as risk neutrality and a static world. These assumptions may have resulted in a misspecified fundamental process. The equation may also fail to account for the arrival of new information not reflected
by the current and the lagged rents. Let \( s_t \) denotes a misspecification or a measurement error, so

\[
\Delta p_t' = \Delta s_t + \psi \Delta d_t + (1 - \psi) \Delta d_{t-1}
\]  
(V. 24)

And

\[
\Delta s_{t+1} = \beta \Delta s_t
\]  
(V. 25)

This error is unobservable but can be inferred with Kalman filter. To make use of the Kalman filter, we first express the model in a time invariant state space form.

2.2. State Space Form

Let \( z_t \) be an \( nz \) – vector of state variables, \( x_t \) an \( l \) – vector of inputs, and \( y_t \) an \( N \) – vector of outputs. The state space model consists of two equations: the measurement equation,

\[
y_t = H z_t + B x_t + \varepsilon_t ; \quad E(\varepsilon_t) = 0, \quad E(\varepsilon_t, \varepsilon_t') = R,
\]  
(V. 26)

and the transition equation,

\[
z_t = F z_{t-1} + A x_t + \eta_t ; \quad E(\eta_t) = 0, \quad E(\eta_t, \eta_t') = V,
\]  
(V. 27)

where the system matrices \( H, B, F, A \), the measurement variance \( R \), and the transition variance \( V \) are all time-invariant.

2.3. Extracting the State Variable with Kalman filter
The state variable, $z_t$, is not observable but can be estimated using the Kalman filter, assuming the system parameters are known. The Kalman filter consists of a set of recursive equations. Suppose we estimate the initial value of the state variable to be $z_0$, with estimation error $P_0$. The predicted value of the state variable and the prediction error at time $t$, given the information set available at time $t-1$, $\Xi_{t-1} = \{y_1, \ldots, y_{t-1}, x_1, \ldots, x_{t-1}\}$, can be calculated using the prediction equations recursively forward:

$$z_{t|t-1} = Fz_{t-1|t-1} + Ax_t,$$
and

$$P_{t|t-1} = FP_{t-1|t-1}F^* + V.$$  

When time $t$ information becomes available, we can update our estimation of the bubbles and their estimation errors using the filtering equations recursively forward:

$$z_t = z_{t|t-1} + \kappa_t e_{t|t-1},$$
and

$$P_{t|t} = P_{t|t-1} - \kappa_t HP_{t|t-1},$$

where

$$\kappa_t = P_{t|t-1}H^*D_{t|t-1}^{-1},$$

$$D_{t|t-1} = (HP_{t|t-1} + H^* + R),$$

$$e_{t|t-1} = y_t - Hx_t - Bx_{t-1}.$$  

Once obtained the sequences $\{z_{t|t-1}\}_{t=1}^{T}$, $\{P_{t|t-1}\}_{t=1}^{T}$, $\{e_{t|t-1}\}_{t=1}^{T}$, and $\{P_{t|t}\}_{t=1}^{T}$, we can have a more efficient estimation of the state variable and its estimation errors, using the full set of information, $\Xi_T = \{y_1, \ldots, y_T, x_1, \ldots, x_T\}$, and the following smoothing equations by backward recursion:
\[ z_{t|t} = z_{t|t} + J_t(z_{t+1|t} - z_{t+1|t}) \], \quad \text{and} \quad \text{(V. 35)}

\[ P_{t|t} = P_{t|t} + J_t(P_{t+1|t} - P_{t+1|t})J_t^T, \quad \text{(V. 36)} \]

where

\[ J_t = P_t^T F_p P_{t+1|t}. \quad \text{(V. 37)} \]

The starting values for smoothing are \( z_{t|t} \) and \( P_{t|t} \) obtained from the filtering process.

### 2.4. Estimating Model Parameters

Collecting the parameters to be estimated in the vector \( \theta \). These parameters are estimated, using EM algorithm (SAS/IML version 8; Dempster et al. 1977; Dellaert 2002), by maximizing the log likelihood function of \( y_t, t = 1, 2, \ldots T \), which is

\[ \log L(y; x, \theta) = -NT \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log |D_{t|t-1}| - \frac{1}{2} \sum_{t=1}^{T} e_{t|t-1}^T D_{t|t-1}^{-1} e_{t|t-1}, \quad \text{(V. 38)} \]

where

\[ y = (y_1, y_2, \ldots, y_T)^T \]
\[ x = (x_1, x_2, \ldots, x_T)^T \]

\( N=\text{dimension of } y\text{-vector} \)
\( T=\text{sample size} \)

(Harvey 1989; Hamilton 1994). It will be explained in Section 3 that the vector parameters H and A are preset to (1, 0) and (0 0) respectively. The ML estimators of the remaining vector parameters are given by:
\[
\tilde{R} = \frac{1}{T} \sum_{t=1}^{T} \left[ H P_{t|t} H^\prime + \left( y_t - H z_{t|t} - B x_t \right) \left( y_t - H z_{t|t} - B x_t \right)^\prime \right]
\]
\[
\tilde{V} = \frac{1}{T} \left[ S_t(0) - S_t(1) (S_{t-1}(0))^{-1} S_t(1)^\prime \right]
\]
\[
\tilde{F} = S_t(1) (S_{t-1}(0))^{-1}
\]
\[
\tilde{B} = \left( \sum_{t=1}^{T} y_t x_t^\prime - H \sum_{t=1}^{T} z_{t|t} x_t \right) \left( \sum_{t=1}^{T} x_t x_t^\prime \right)^{-1}
\]

where

\[
S_t(1) = \sum_{t=1}^{T} P_{t,t-1|t|t} + z_{t|t} z_{t-1|t}^\prime
\]
\[
S_t(0) = \sum_{t=1}^{T} P_{t|t} + z_{t|t} z_{t|t}^\prime
\]

\[
P_{t,t-1|t|t} = \text{Estimated covariance between } z_t \text{ and } z_{t-1}.
\]

In order to compute V.41, we need estimated values of the state variable and their estimation errors. Thus we have to provide a guess about the starting value of the system parameters. The estimation process consists of four steps:

1. Initiate the guessed system parameter values.

2. Run through equations V.29 through V.38, to obtain the sequences \( \{ \xi_{t|t-1} \}_{t=1}^{T}, \{ P_{t|t-1} \}_{t=1}^{T}, \{ \xi_{t|t} \}_{t=1}^{T}, \{ \xi_{t|t} \}_{t=1}^{T}, \{ P_{t|t} \}_{t=1}^{T}, \{ \xi_{t|t} \}_{t=1}^{T}, \{ P_{t|t} \}_{t=1}^{T} \).

3. Compute ML estimates of system parameters using V.41.

4. Repeat steps 2 and 3 until convergence occurs.

The initial system parameter values in our experiment are based on the preliminary OLS estimate of the simple present-value model without the bubble.
component. The initial values of the state variable for the \((i+1)^{th}\) iteration is updated using estimates from the \(i^{th}\) iteration by the set of equations

\[
\begin{align*}
  z_{0}^{i+1} &= F^{i} z_{0|T}^{i} \\
  P_{0}^{i+1} &= F^{i} P_{0} F^{i\top} + V^{i}
\end{align*}
\]  (V. 42)

### 2.5. Asymptotic Properties of the ML Estimators

Suppose \(\tilde{\theta}\) is the ML estimator of \(\theta\) obtained by maximizing \(V.37\). Subject to certain regularity conditions (Caines 1988), then:

\[
\sqrt{T} \varphi_{2D,T}^{\frac{1}{2}} (\tilde{\theta} - \theta_{0}) \xrightarrow{d} N(0, I),
\]  (V. 43)

that is,

\[
\tilde{\theta} \xrightarrow{d} N(\theta_{0}, T^{-1} \varphi_{2D,T}^{-1}),
\]  (V. 44)

where \(\varphi_{2D,T}\) is the information matrix from the sample of size \(T\):

\[
\varphi_{2D,T} = \left(-\frac{1}{T} \sum_{i=1}^{T} \frac{\partial^2 \text{Log} L_{i}}{\partial \theta \partial \theta'}|_{\theta = \theta_{0}} \right),
\]  (V. 45)

Where \(\theta_{0}\) is the true parameter and

\[
\text{Log} L_{i} = -\frac{N}{2} \log 2\pi - \frac{1}{2} \log |D_{i|-1}| - \frac{1}{2} \varepsilon_{i|-1}' D_{i|-1}^{-1} \varepsilon_{i|-1}, \; i = 1, 2, \ldots, T
\]  (V. 46)

and

\[
\lim_{T \to \infty} \varphi_{2D,T} \xrightarrow{p} \Phi = \left(-\frac{1}{T} \sum_{i=1}^{T} \frac{\partial^2 \text{Log} L_{i}}{\partial \theta \partial \theta'}|_{\theta = \tilde{\theta}} \right)
\]  (V. 47)
Where $\tilde{\theta}$ is the ML estimator. The reported standard errors for $\tilde{\theta}$ are the square roots of the diagonal elements of $\left(T\tilde{\theta}\right)^{-1} = \left(\sum_{i=1}^{T} \frac{\partial^2 \log L_i}{\partial \theta \partial \theta^T} \bigg| \theta = \tilde{\theta}\right)^{-1}$.

The Hessian is calculated numerically in our paper. The method is described below. First we collect the estimated parameters in an $r \times 1$ vector $\tilde{\theta}$, run through the Kalman filter and calculate the log-likelihood of the data to give $\log L(\tilde{\theta})$. We then perturb one parameter at a time by $\Delta = +0.01$, run the Kalman filter and calculate the log-likelihood of the data again to give $\log L(\tilde{\theta} + \Delta_i)$. We perturb one parameter at a time by $\Delta = -0.01$, run the Kalman filter again and recalculate the log-likelihood of the data to give $\log L(\tilde{\theta} - \Delta_i)$. The Hessian is calculated using the formula

$$
\frac{\partial^2 \log L(\tilde{\theta})}{\partial \tilde{\theta}^2} \approx \frac{\log L(\tilde{\theta} + \Delta_i) - 2 \times \log L(\tilde{\theta}) + \log L(\tilde{\theta} - \Delta_i)}{\Delta_i^2},
$$

(V. 48)

where

$$
\Delta_i = A \ r \times 1 \ vector \ with \ all \ elements, \ except \ the \ i^{th} \ which \ is \ 0.01, \ being \ zero,
$$

and

$$
\theta_i = The \ i^{th} \ element \ of \ \tilde{\theta}.
$$

(Wheatley 2004, 267.)

The standard error of $\theta_i$ is approximated using the V.47 in the equation
\[ SE(\theta_i) = \left( - \frac{\partial^2 \log L(y; \bar{\theta})}{\partial \theta_i^2} \right)^{-1}. \]  \hspace{1cm} (V. 49)

3. Empirical Applications

The data sets used in this chapter are the same as those used in chapter IV. From chapter IV, we know that each series is an I(1) process. Instead of taking the direct difference of the level variables, we take the difference of the log transformed variables, for the sake of avoiding a negative price while allowing for a negative bubble (Wu 1997).

A LR test accepted, in Hong Kong office price, the restriction imposed by equation V.19 that the coefficients on the current and the lagged changes of log real rent should add up to one (tableV.1), but rejected it in Seoul housing price.

It is argued before that further lagged rent will appear on the right hand side of equation V.19 if the rent process is \( AR(p) \), with \( p > 1 \). However, the F test shows that, although a regression of the changes in log real price on the changes in log real rent is significant with higher lags, there is no significant increase in \( R^2 \). This observation echoes the comments made by Shiller (1990, 59) on the regression of changes in log real stock price on the changes in log real rents.
Therefore the state space model estimated in this study, assuming the state variable affects the price only, has

\[
y_t = \left( \frac{\Delta p_t}{\Delta d_t} \right), \quad z_t = \Delta s_t, \quad x_t = \left( \frac{\Delta d_t}{\Delta d_{t-1}} \right), \quad B = \begin{pmatrix} \psi & \psi^{-1} \\ 0 & \phi \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 0 \end{pmatrix},
\]

\[
H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad F = \beta, \quad \epsilon_t = \begin{pmatrix} \omega_t \\ \delta_t \end{pmatrix}, \quad \eta_t = \zeta_t,
\]

Where $\Delta p_t, \Delta d_t$ are the first differences of the log real housing price and rent indices respectively. It is assumed that $\omega_t, \delta_t$ and $\zeta_t$ are uncorrelated, hence,

\[
R = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}, \quad V = \sigma^2.
\]

A model based on equation V.19 is also estimated, and referred to as the present value model. This model assumes no specification error.

The parameter to be estimated in the state space model are collected in $\theta$, where $\theta = \{\psi, \psi^{-1}, \beta, \phi, \sigma_\omega, \sigma_\delta, \sigma_\zeta\}$. The Koenker and Bassett (1982) LM test statistics show that heteroscedasticity presents in most cases. This is confirmed by the standardized plot (defined in Harvey, 1989, 257) for model residuals show clustering in volatility (figure V.3). Therefore, a Wald statistic is calculated alongside the F statistic. The Wald test is more appropriate in the presence of heteroscedasticity (Greene 1997, 548). The estimates are jointly significant in any event (table V.2 and V.3). They also passed the post-sample predictive test (table V.4b).
In the state space model, the coefficient on the current change in log real rent is highly significant for both price series. It is larger than one for Hong Kong office price. This is consistent with the findings of Shiller (1990) and Campbell (1990). They find that investors in the stock market tend to overreact to current dividend changes. There is no such overreaction in Seoul housing market, however, as shown by the value of \( \psi \) (table V.3). According to Shiller (1990), a larger than one value might be interpreted as that the rent series itself is infected by the same speculative bubble which infests the price. The coefficient on the lagged changes in log real rent is insignificant for Seoul housing price. It is significant at 5% and negative in value for Hong Kong office price. Shiller (1990) also shows that the coefficient on the lagged log real dividends is negative. The estimated \( \beta \) and \( \phi \) are insignificant, indicating neither the rent nor the state variable is systematically affected by its past behavior.

In the present value model, the coefficient on the current change in log real rent is significant for Hong Kong office price series. This coefficient is also significant for Seoul housing price series if 1% significance level is used in the LM test for homoscedasticity. But the coefficient on the lagged change in log real rent is insignificant for both series. The parameters are, however, jointly significant (table V.2).

The R-square, AIC and BIC for both models are listed in table V.4a. However, they are not directly comparable, because when calculating these statistics for
the state space model, the residuals used are defined differently (Harvey 1989, 268-270). Therefore, the in-sample sum of squared errors is listed alongside these statistics. This variable indicates that the state space model generates much smaller in-sample errors. It also generates smaller extrapolative prediction errors (table V.4b).

The R-square measure indicates that either the state space model or the present value model can explain less than 20% of the variations in Hong Kong office price, and roughly 50% of the variations in Seoul housing price. If we interpret the model residuals as the realizations of a speculative bubble, then more than 80% of the price movement is driven by a speculative bubble in the office sector of Hong Kong real-estate market. According to this interpretation, Seoul housing price is comparatively much less prone to speculative bubble.

The residual is plotted alongside the price and its predicted value in figure V.1 and V.2. These plots show that the spectacular rise of the Hong Kong office price during the 1990s is better explained by a speculative bubble (inferred from the residual). The bubble expanded rapidly between the early 1990 and the early 1994, and between the late 1996 and the mid 1997. During these time periods, the price itself is soaring high. This is largely consistent with Chan et al’s (2000) conclusions and the findings in Chapter IV of this thesis. When the prices collapsed in the second half of 1994 and in late 1997, so did the bubble.
The plots also show that the bubble explains the rise of Seoul housing price in the early 1990s better than the fundamental model. The bubble peaked in February 1991. After that it embarked on a prolonged decline. The bubble creeps up again after December 2001.
Chapter VI When Will the Bubble Burst?

1. Introduction

In chapters IV and V of this thesis, I have demonstrated that the markets under review, the Seoul and the Hong Kong property markets, have been plagued with rational speculative bubbles, that such bubbles collapse periodically, and that negative bubbles may exist as well as positive bubbles.

It has been observed that speculative bubbles occur in all the major asset markets around the globe. A natural question to pose is this: Are there any common patterns among the rational speculative bubbles that plagued different classes of assets and assets in different geographical locations? If there are, will these patterns tell us when a bubble is going to collapse or burst?

The burst of full-blown bubbles is often cited as an important cause of large market crashes. A market crash is a significant, sudden decline in prices. The world has seen a number of market crashes in the past two decades in all major asset markets, such as stock, property, and commodity markets.

A market crash is often destructive to the economy at large. If market crashes are, indeed, caused in many cases by the bursting of speculative bubbles, then understanding the behavior of speculative bubbles and the relationship between
bubbles and market crashes will help policy makers to minimize the damage of speculative bubbles to the economy at large.

In recent years, Johansen, Sornette, and their co-authors (Johansen et al.) developed an interesting analytical framework for market crashes. Their theory has a close connection to a class of bubble postulated by Blanchard and Watson (1982) that collapses periodically. Their idea is that the probability of the bubble ending may depend on how long the bubble has lasted, or by how far the price is from market fundamentals.

The framework of Johansen et al. is deeply rooted in the recent findings of behavioral economics and in the concept of criticality developed in statistical physics. Johansen et al. propose that market crashes are large draw-downs that occur because the market has entered an unstable phase. Any small disturbance or process may have triggered the instability. Because of the instability, the collapse is total. The proximate cause of the collapse is secondary. In other words, the origin of the crash is fundamentally endogenous; exogenous shocks only serve as triggers. The origin of crashes is constructed progressively by the market as a whole, as a *self-organizing* process. In this sense, the true cause of a crash could be termed a systemic instability.

Using the model developed, Johansen et al. identified a precursory pattern, namely, the power law log periodic signature (PLLS), of asset prices before
major crashes. In their papers, they verified the presence of PLLS for different asset markets in many developed countries and some emerging economies. Johansen et al. argue that this precursory pattern originates from some very fundamental and robust properties of asset markets: the herding behavior among the traders and the self-organizing markets, which lead to accelerating speculative bubbles that often end in crashes.

In *Why Stock Markets Crash* (Sornette 2003), Robert Shiller writes, “A professor of geophysics gives a very different perspective, informed by his scientific training, on the stock market. I am sure that his view will be highly controversial, but the book is fascinating, and mind-expanding, reading.”

The rest of this chapter includes four parts. In part 2, I lay out the economic rationale behind the Johansen et al. model. Part 3 contains a summary of the model. The details of estimation and optimization strategies are described in part 4. My empirical findings, the main contributions of this chapter, are presented in part 5.

2. The Economic Rationale

2.1. Self-organization, Market Efficiency and the Speculative Bubble
Macroscopic systemic self-organization can emerge from some simple rules regarding repeated actions at the microscopic level. This idea follows Adam Smith’s notion that selfish, greedy individuals, if allowed to pursue their interests largely unchecked, would interact to produce a wealthier society as if guided by an “invisible hand.” Smith himself never worked out a proof that this invisible hand existed. The mathematical proof of the existence of the invisible hand was constructed by Arrow and Debreu (1954) under a set of very restricted assumptions.

The main tool in the analysis of Arrow and Debreu is constrained optimization. However, this is not an entirely satisfactory representation of reality, as most people are not versed in the reasoning of economic optimization.

But this does not mean that we shall fail to function effectively in social and economic exchanges in life. The remarkable insight of Adam Smith is that people have natural intuitive mechanisms enabling them to read situations and the intentions and likely reactions of others without tutored cognitive analysis.

This fact has been established by experimental economists. Their experiments show that economic agents can achieve efficient outcomes that are not part of their intentions—a key principle formulated by Adam Smith. Experiments on markets with both insiders and uninformed traders show that equilibrium prices do reveal insider information after several trials of the experiments, suggesting
that the markets disseminate information efficiently (see, for example, Rozeff and Zamank 1988).

However, these results are not always present if the following conditions are not fulfilled: identical preferences, common knowledge of the dividend structure, and complete contingent claims (i.e., the existence of a full spectrum of derivative instruments allowing one to probe the expectation of future risks). In these situations, information aggregation is a more complicated process, and market efficiency, defined as full information aggregation, depends on the complexity of the market (see, for example, Agastya and Holden 2003).

Thus the “emergence of self-organization” does not imply that the market will always be equivalent to an efficient and global optimization machine. Empirical economics shows that market forces may lead to plenty of imperfections, problems, and paradoxes. In fact, rational behavior could lead to a less-than-optimal market outcome, such as the rational speculative bubble which gives false signals about the fundamental value of an asset.

2.2. The Power Law

The power law refers to the power-law acceleration of market prices before a crash.
Consider a purely speculative asset that pays no dividends. The no-arbitrage condition plus rational expectations would imply that the price of this asset should always be zero. Any deviation of the price from zero signals the presence of a speculative bubble.

A speculative bubble emerges from “self-reinforcing imitation” among traders in a self-organizing process. However, fundamental forces would make bubbles transient phenomena. The fighting between a bubble and fundamental forces leads to repeated price fluctuations around its fundamental values.

The self-reinforcing imitation process leads to the emergence of a bubble, which often (but not necessarily) ends in a market crash.

Though not certain, a crash can be characterized by its hazard rate: the probability per unit of time that the crash will happen in the next instant (provided it has not happened yet). The crash hazard rate quantifies the probability that a large group of traders will place sell orders simultaneously and create enough imbalances in the order book of market makers to make it impossible to absorb the other side without lowering prices substantially. It is computed as the ratio of scenarios that result in a crash to all possible scenarios.

Since a crash is not a certain deterministic outcome of the bubble—there is a finite probability of a soft landing—it remains rational for the traders to stay
invested, provided they are compensated by a higher rate of growth of the bubble for taking the risk of a crash.

This means that the critical time, which is defined as the time when a bubble ends, is not the time of the crash, but the time when the crash most probably will occur.

Assume the movement of a price is driven by risks. In particular, for each period (e.g., a day), there are two components, and only two compete to determine the price increment from one day to the next: a daily market return and the possibility that a crash will occur.

With the no-arbitrage condition and rational expectations, the daily return should compensate exactly the average loss due to the possibility of a crash. It implies that the total average return at any time is exactly zero. It also implies that the market return is proportional to the crash hazard rate: the higher the risk of a crash, the higher the price return.

Most traders are organized into a network, and they influence each other locally through this network. Imitation among traders (herding behavior) creates a speculative bubble. The same imitation force also brings the bubble to an end.
On approaching the critical time (the time when the bubble ends), imitation among traders strengthens in an accelerating manner (fueled by expected large capital gains, perhaps). The market becomes more and more sensitive to news or rumors. These lead to the fueling up of the crash hazard rate, which in turn leads to the power law acceleration of price upon the approaching of the critical time.

The paragraphs above lay out the “risk-driven model,” to use the terminology of Johansen et al. We may, however, assume the price drives risks rather than the other way round. We would then have a “price-driven model” according to Johansen et al.

In the price-driven model, the price drives the crash hazard rate. The price itself is driven up by the imitation and herding behavior of the “noisy” investors. The occurrence of a crash is again characterized by its hazard rate.

Let the price variation in an elementary time period be the sum of two components: a certain instantaneous return and a random return. The first embodies the remuneration due to estimated risks as well as the effect of imitation and herding. The second embodies the noise component of the price dynamics with volatility. The volatility can also have a systematic component controlled by imitation as well as many other factors.
Noisy investors (as opposed to investors who base their investment strategy on fundamentals) look at the market price going up. They speak to each other, begin to herd, buy more and more of the stock, and push prices further up. As the price variation speeds up, the no-arbitrage condition, together with rational expectations, implies that there must be an underlying risk, not yet revealed in the price dynamics that justifies this apparent free lunch.

To capture the phenomenon of speculative bubbles, we focus on the class of models with positive feedbacks. In the present context, this means that the instantaneous return as well as the volatility becomes larger and larger when past prices and/or past returns and/or past volatilities become large. Such positive feedbacks with increasing growth rate may lead to singularities in a finite time.\textsuperscript{xxviii} Here it means that, unchecked, the price would blow up without bounds. However, two effects compete to tamper with this divergence. First, the stochastic component affecting the price variations makes the price much more erratic, and the convergence to the critical time becomes a random, uncertain event. The second effect that tampers with the possible divergence of the bubble price is the impact of the price on the crash hazard rate: as the price blows up due to imitation, herding, speculation, and randomness, the crash hazard rate increases even faster, so that a crash will occur and drive the price back closer to its fundamental value. Hence this model proposes two scenarios for the end of a bubble: either a spontaneous deflation or a crash.
The risk-driven model and the price-driven model describe a system of two populations of traders, the “rational” and the “noisy” traders. The occasional imitative and herding behavior of the noisy traders may cause global cooperation among traders, causing a crash.

In the risk-driven model, the crash hazard rate determined from herding drives the bubble price. In the price-driven model, imitation and herding induce positive feedback on the price, which itself creates an increasing risk for a looming yet unrealized crash.

Both models capture a part of reality. Studying them independently is the standard strategy of dividing to conquer the complexity of the world. Both models embody the notion that the market anticipates the crash in a subtle, self-organized, and cooperative fashion, hence releasing precursory power law “fingerprints” observable in stock-market prices.

2.3. The Log-periodicity

In the previous section, we learned that a critical point in the time domain underlies stock-market crashes. A crash is not the critical point itself, but its triggering rate is strongly influenced by its proximity to the critical point. The closer to the critical time, the more probable is the crash.
The hallmark of critical behavior in this model is the *power-law acceleration* of the price, its volatility, and the crash hazard rate, upon approaching the critical time. However, in practice, due to the presence of the noise and the irregularities of the trajectories of stock-market prices, power-law acceleration is often difficult to detect.

Luckily, a looming crash has a second fingerprint: *log periodicity*, which is more robust in the presence of noise. That is, in the presence of noise, the price is not constantly accelerating. Rather, the pattern of acceleration is punctuated by oscillations whose frequency itself accelerates on approaching the critical time. This oscillation is called “log-periodic oscillation,” as it is seen as accelerating periodic oscillation in a logarithmic representation (Sornette 2003, 179 figure 6.4).

This means that the crash hazard rate and price increase dramatically when the interaction between investors becomes strong enough, but this acceleration is interrupted by and mixed with an accelerating sequence of quiescent phases in which the risk and price decrease.

The power-law and log-periodicity signatures may be explained by the interaction between trend-following traders and fundamentalists. The trend-following traders exert a positive influence on prices and enhance the previous price trend, leading to the exponential growth of the price and resulting in a price that exhibits finite time singularity. The fundamentalists exert fundamental value restoring force,
which generates oscillations around the fundamental value that are approximately log-periodic.

3. The Model

In this section, I summarize the derivation of the model developed mainly by Johansen, Ledoit and Sornette.

3.1. The Price Dynamics

Consider a purely speculative asset that pays no dividends. Ignore the interest rate, risk aversion, information asymmetry, and market-clear condition. Assume markets are efficient in the sense that all available information is reflected in current market prices. Rational expectations then imply that the price follows a martingale process:

\[ E[p(t)] = p(t) \quad \forall t', t > t, \quad (VI. 1) \]

where \( p(t) \) is the price of the asset at time \( t \) and \( E_t[\cdot] \) is the expectation operator conditional on information revealed up to time \( t \). In a market without noise,

\[ p(t) = p(t_0) = 0 \quad \forall t, \quad (VI. 2) \]

\( t_0 \) being some initial time. A positive value of \( p(t) \) would signify a speculative bubble.\textsuperscript{xxix}

A crash is likely to occur if the price deviates too far from zero. The probability of crash is characterized by the crash hazard rate, \( h(t) \), defined as the probability
per unit of time that the crash will happen in the next instant if it has not yet
happened, and \( h(t) = q(t)/(1 - Q(t)) \), where \( Q(t) \) is the cumulative probability
density of the crash. The probability density is \( q(t) = \frac{dQ}{dt} \). The crash is an
exogenous event. Similarly, the crash hazard rate is an exogenous variable.

Define a jump process, denoted by \( j \), with \( j = 0 \) before the crash, and \( j = 1 \) after
the crash. Assume that when the crash occurs, the price \( p(t) \) drops by a fixed
fraction \( \kappa \), with \( \kappa \in (0,1) \). (A more general model would have \( \kappa \) be the mean of a
distribution of possible crash sizes.) The dynamics of the asset price before the
.crash, then are governed by

\[
dp = \mu(t)p(t)dt - \kappa p(t) dj,
\]

where \( \mu(t) \) is a time dependent drift satisfying the no-arbitrage martingale
condition

\[
E_j[dp] = \mu(t)p(t)dt - \kappa p(t)h(t)dt = 0.
\] (VI. 4)

This implies that \( \mu(t) = \kappa E_j\left[\frac{dj}{dt}\right] = \kappa h(t) \). Plugging this expression into VI.3 yields

\[
dp(t) = \kappa h(t)p(t)dt - \kappa p(t) dj
\] (VI. 5)

Since \( j = 0 \) before the crash, we have \( dp(t) = \kappa h(t)p(t)dt \), hence,

\[
\ln\left(\frac{p(t)}{p(t_0)}\right) = \kappa \int_{t_0}^{t} h(t')dt',
\] (VI. 6)

The larger the value of \( h(t) \), the higher the probability of a crash, and the faster
the price must increase—investors must be compensated by the chance of a
higher return in order to be induced to hold an asset that may crash.
3.2. The Crash

VI.6 states that the evolution of prices depends on the evolution of the crash hazard rate. The macro-level probability of a crash, in turn, results from microlevel agents’ interactions—in particular, the interplay of imitation and anti-imitations among agents.

In reality, traders are organized into a network of family, friends, colleagues, etc. They influence each other locally through this network. Consider a network of investors: each one can be named by an integer \( i = 1, 2, \ldots I \). A typical trader \( i \) has \( N(i) \) neighbors. His opinion is influenced by the opinions of these neighbors and by an idiosyncratic signal that this trader alone receives. The first force will tend to create order, the latter disorder. A crash happens when order wins. Thus, the macrolevel coordinated sells that cause a crash are a result of microlevel imitation among agents.

So what determines whether order or disorder wins?

Assume that agent \( i \) can be in only one of two possible states: \( s = +1 \) if he buys and \( s = -1 \) if he sells. Based on the information of the actions \( s_j(t - 1), j = 1, 2, \ldots \), performed at time \( t - 1 \) by his \( N(i) \) “neighbors,” agent \( i \) sets \( s_i(t - 1) \) to maximize his return.
Assume each agent can either buy or sell only one unit of the asset. The selection of one of the two states (buy or sell) is determined from small and subtle initial biases as well as from the fluctuations during the evolutionary dynamics.

The asset price variation is thus proportional to the aggregate sum \( \sum_{i=1}^{I} v_i (t-1) \) of all traders’ actions. In equilibrium, when there are as many buyers as there are sellers, the sum is zero and the price does not change. Other influences affecting the price are accounted for by adding a stochastic component.

At time \( t-1 \), when the price \( p(t-1) \) has been announced, trader \( i \) defines his strategy \( s_i (t-1) \) based on information available to maximize his expected profit,

\[
P_E = E p(t) - p(t-1)
\]

Since the price moves with the general opinion, the best strategy is to buy if \( \sum_{i=1}^{I} s_i (t-1) \) is positive and sell if it is negative. However, \( \sum_{i=1}^{I} s_i (t-1) \) is unknown to a given trader. The best the trader can do is to poll the opinions of his immediate \( N(i) \) neighbor. Suppose that the a priori probability \( P_{r+} \) and \( P_{r-} \) for each trader to buy or sell is known to all. From all this information, the trader can construct his prediction of the price drift. The best guess of trader \( i \) is that the future price change will be proportional to the sum of the actions of her neighbors whom she has been able to poll. Thus the strategy that maximizes his expected profit is
\[ s_i(t-1) = \text{sign} \left( K \sum_{j \in N(i)} s_j(t-1) + \sigma \epsilon_i + G \right), \quad (\text{VI. 7}) \]

where \( \epsilon_i \) is the noise, with \( \epsilon_i \xrightarrow{d} \text{i.i.d.} N(0,1) \) and \( N(i) \) the number of neighbors with whom trader \( i \) interacts significantly. \( K \) is a positive variable measuring the strength of imitation.\(^{xxx}\) It is inversely proportional to the “market depth”: the larger the market, the smaller the relative impact of a given imbalance between buy and sell orders, hence the smaller the price change. The tendency for idiosyncratic behavior is governed by \( \sigma \). Thus the value of \( K \) relative to \( \sigma \) determines the outcome of the battle between order and disorder, and eventually the probability of a crash. \( G \) captures the global influence, which tends to favor the state \( +1(-1) \) if \( G > 0(G < 0) \).

\(^{VI.7}\) only describes the state of an agent at a given time. In the next instant, new \( \epsilon_i \)'s are realized, new influences propagate themselves to neighbors, and agents can change their decisions. The system is thus constantly changing and reorganizing itself.

In a practical implementation of a trading strategy, it is not sufficient to know or guess the overall direction of the market. A trader may want to be slightly ahead of the herd to buy at a better price, before the price is pushed up by the bullish consensus. Symmetrically, he will want to exit the market a bit before the crowd.
The behavior of agents in a real market is neither fully imitative nor fully anti-imitative. A better representation of a real market requires a combination of the two. Indeed, one should distinguish the buy and sell actions from the holding period. In general, a typical trader would ideally like to be in the minority when entering the market, in the majority while holding his position, and again in the minority when closing his position.

Traders will try to outguess each other on when to enter the market. If all traders use the same set of decision rules, they will end up doing the same thing at the same time and cannot, therefore, be in the minority. To be in the minority implies striving to be different. By adaptation, traders will learn and be forced to differentiate their entry and exit strategies according to past successes and failures.

The interaction between the forces of imitation and the forces of anti-imitation is the key to understanding market crashes.

The chance that a large group of agents finds itself in agreement is called the susceptibility of the system. Define the average state of the system as $M = \frac{1}{T} \sum_{i=1}^{N} n_i$. In the absence of the global influence, $E[M] = 0$, agents are split evenly between the two states. In the presence of a positive (negative) global influence, agents in the positive (negative) state will outnumber the
Hence the system susceptibility is defined formally as
\[
\chi = \frac{d \langle E[M] \rangle}{dG} \bigg|_{G=0}.
\] (VI.8)

That is, the susceptibility measures the sensitivity of the average state of the system to a tiny global influence, hence the degree of coordination of the overall system. The susceptibility can also be interpreted as the variance of the average state \( M \) around its zero mean caused by the random idiosyncratic shocks followed by imitation. \textit{It is precisely the emergence of this global synchronization from local imitation that can cause a crash.}

The susceptibility depends on the structure of the network and the strength of imitation. Let \( t_c \) denote the first time the imitation strength reached its critical value. That is, for the first time, \( K = K_c \). Recall that \( t_c \) is \textit{not} the time of the crash, but the time at which the crash is most likely to occur. When \( K < K_c \), disorder reigns: the sensitivity of the system to a small global influence is small, the clusters of agents who are in agreement remain small, and imitation only propagates between close neighbors. In this case, the susceptibility \( \chi \) of the system to external news is small, as many clusters of different opinions react incoherently, thus more or less canceling out their responses.

When the imitation strength \( K \) increases and approaches \( K_c \), order starts to appear: the system becomes extremely sensitive to a small global perturbation,
agents who agree with each other form large clusters, and imitation propagates over long distances. These are the characteristics of so-called critical phenomena in the natural sciences. In this case, the susceptibility $\chi$ of the system goes to infinity.

The large susceptibility means that the system is unstable. A small external perturbation may lead to a large collective reaction of the traders who may revise their decisions dramatically, which may abruptly produce a sudden imbalance between supply and demand, thus triggering a crash or a rally.

For even stronger imitation strength $K > K_c$, the imitation is so strong that the idiosyncratic signals become negligible and the traders self-organize into strong imitative behavior.

Though the susceptibility depends on the structure of the system, notwithstanding the large variety of topological structures of social networks (e.g., horizontal or hierarchical), the qualitative conclusion of the existence of a critical transition between a mostly disordered state and an ordered one, separated by a critical point, survives by and large for most possible choices of the network of interacting investors (Sornette 2003).

A solution to [VI.8] is

$$\chi \approx A(K_c - K)^{-\gamma},$$

(VI. 9)
where $A$ is a positive constant and $\gamma > 0$ is called the critical exponent of the susceptibility, which can be a real or complex number, depending on the structure of the network. A more general version of VI.9 is

$$
\chi \approx \text{Re} \left[ A_0 (K_c - K)^{-\gamma} + A_1 (K_c - K)^{-\gamma+i\omega} + \ldots \right], \quad \text{and}
$$

$$
\approx A_0' (K_c - K)^{-\gamma} + A_1' (K_c - K)^{-\gamma} \cos \left[ \omega \log(K_c - K) + \psi \right] + \ldots,
$$

where $A_0', A_1', \omega, \psi$ are real numbers, and $\text{Re}[]$ denotes the real part of a complex number. In this expression, the power law in VI.9 is corrected by oscillations whose frequency explodes as we reach the critical time. These accelerating oscillations are “log-periodic,” which have the “log frequency” $\frac{\omega}{2\pi}$.

Assume the hazard rate of a crash behaves in the same way as the susceptibility in the neighborhood of the critical point. Thus

$$
h(t) \approx B_0 (t_c - t)^{-\alpha} + B_1 (t_c - t)^{-\alpha} \cos \left[ \omega \log(t_c - t) + \psi \right] + \ldots,
$$

where $0 < \alpha < 1$, otherwise the implied price would go to infinity when approaching the critical time. Plugging it into VI.6 gives

$$
\ln[p(t)] \approx \ln[p_c] + \frac{K}{\beta} \left[ B_0 (t_c - t)^{\beta} + B_1 (t_c - t)^{\beta} \cos \left[ \omega \log(t_c - t) + \phi \right] \right] + \ldots,
$$

where $\beta = 1 - \alpha$. The key feature is that log-periodic oscillations appear in the price of the asset just before the critical state, with angular log-frequency $\omega$ associated with the preferred scaling factor

$$
\lambda = e^{\frac{\omega}{2\pi}},
$$

(VI.13)
(Sornette 2003; Johansen et al. 2000a, 229–233).

### 3.3. Empirical Findings of Johansen et al.

In Johansen et al. (2000a), VI.12 is fitted by minimizing the mean squared errors, using a combination of taboo search and downhill simplex methods. As the function is highly nonlinear, many local minima exist, and the minimization algorithm can get trapped at any of these local minima. In their estimation, when more than one minimum is produced, they select the best one according to a set of criteria in addition to the variance (Johansen et al. 2000a, 239). Two data sets were used in this paper: the S&P 500 (July 1985 to the end of 1987 [557 trading days]), and the Hang Seng Index (approximately two and half years daily data points before the October 1997 crash). Both power law and log periodicity are present in the two samples and the in-sample performances of equation VI.12 are remarkably good.

But how long before the crash can one identify the log-periodic signature using VI.12 or its variants? To investigate this, Johansen et al. (2000a) truncated the S&P 500 to an end date approximately equal to 1985. Then approximately 0.16 years were added consecutively, and the fitting was relaunched until the full time interval was recovered. These experiments show that a year or more before the crash, the data were not sufficient to give any conclusive results at all.
Approximately a year before the crash, the fit begins to lock in the date of the crash with increasing precision. However, if one wants to actually predict the time of the crash, a major obstacle is that the fitting procedure produces several possible dates (multiple minima) for the date of the crash even for the last data set. They apply the same procedure to the Dow Jones Index before the crash of 1929. For this data set, the fit locks in on the date of the crash approximately four months before the crash.

Sornette and Johansen (1997) argued that, based on the renormalization group theory (see also Sornette 2003), validating their proposed model requires that one obtain a good fit in several data sets with approximately the same parameter values. In the past few years, Johansen et al. have produced a series of papers examining various kinds of markets. Their investigations show that, for all the bubbles in the most liquid markets, for example, the United States, Hong Kong, and the foreign exchange market, the log frequency \( \frac{\omega}{2\pi} \) has consistently been close to 1. In the framework of power laws with complex exponents or equivalently, discrete scale invariance (Sornette 2003), this corresponds to a preferred scaling ratio of \( \lambda = e^{\omega/2\pi} \approx e \approx 2.7 \). The local period of the log-periodic oscillations decreases according to a geometrical series with the ratio \( \lambda \). For the emergent market, the value of \( \lambda \) shows more fluctuations, but the statistics resulting from more than 20 bubbles were quite consistent with those of the large market (Johansen and Sornette 2000c, reproduced in table VI.1). However, the “universality” of the value of the real part of the exponent \( \beta \) quantifying the
acceleration in the price has not been established. Johansen and Sornette (2000c) explain that the technical reason for this is that the determination of $\beta$ is sensitive to finite size effects as well as to errors in the value of $t_c$, the critical point. In all cases, the model shows reasonable accuracy in identifying the crash date.

In Johansen and Sornette (2000c), the authors mentioned two false signals issued by the model. On September 17, 1997, the signal was issued to predict a crash of the stock market at the end of October 1997. It turned out that the market dropped by 7% but quickly recovered. In October 1999, the model again issued a false alarm. The authors argue that these two examples of bubbles ending more or less smoothly may be explained by the finite probability $1 - Q(t_c)$ that no crash occurs over the whole time including the critical time of the end of the bubble, even through prices show the characteristics of a looming crash. This finite probability obviously reduces the accuracy of the crash prediction.

4. Optimization and Estimation

4.1. The Optimization Problem

Consider the global optimization problem: let $f : D \rightarrow \mathbb{R}$, where $D$ is a convex set in $\mathbb{R}^n$. Find a point $x^* \in D$ such that $f(x^*) \leq f(x), \forall x \in D$. When $f$ is highly nonlinear with many local optima, finding the global optima can be very tough.
Two types of methods have been developed and implemented to solve this global optimization problem: deterministic and stochastic methods. Deterministic methods attempt to generate trajectories that eventually converge to points that satisfy the criterion of local optimality. They are beneficial only when the starting point belongs to the region of attraction of the global optimum. So any deterministic method could be attracted by the local optimum instead. Stochastic methods, on the contrary, attempt to reasonably cover the whole search space and to identify all local and global optima. In stochastic methods, points that do not strictly improve the objective function can also be created and take part in the search process. Hence, stochastic methods have a better chance of reaching the global optimum.

A number of stochastic methods have been proposed and used in different types of optimization problems. Al-Harkan and Trafalis (2002) suggest a hybrid approach, which incorporates scatter search, genetic algorithm, and tabu search, for solving the unconstrained, continuous, nonlinear global-optimization problem. They named this approach the “hybrid scatter genetic tabu” (HSGT).

The scatter search (SS) approach was introduced by Glover (1977) as a heuristic to obtain a nearly optimal solution to an integer programming problem. Recently, the SS approach has been refined and used for both discrete and continuous optimization problems (Glover 1994a, 1994b, 1995; Fleurent et al.).
approach generates sequences of coordinated initializations that are performed to ensure the exploration of the various parts of the solution space. The exploration of the solution space was based on a kindred strategy suggested in Glover (1977). Based on the kindred strategy, the SS approach directs its explorations systematically relative to a collection of points called the reference points. Hence, the SS approach begins its procedure with a set of reference points that can be obtained by applying either heuristic procedures or random methods. Then, a weighted center of gravity of the reference points is determined using a linear combination of the reference-point solutions and their weights. The linear combination allows the use of negative weights that are used to allow the weighted center of gravity to go outside the area spanned by the reference points. This process is known as the diversification process, and it allows for a variety of new solutions. Next, subsets of initial reference points and the weighted centers of gravity are used to define new sub-regions as a foundation for generating subsequent points. Then, these points are evaluated and are used as the new set of reference points. At this stage, a complete iteration of the SS approach is performed. The procedure can be repeated until some preset stopping criteria are satisfied.

The genetic algorithm (GA) approach was developed by Holland in 1975 (Holland 1992). Since GA is adaptive and flexible, it has attracted attention from researchers from different fields, among them computer science, operations research, business and social science. The theory and application of GA have
been reported by several researchers (Davis 1991; Goldberg 1989; Holland 1992; Michalewicz 1994; Srinivas and Patnaik 1994). In these reports, the GA was shown to be successfully applied to several optimization problems. The GA is a stochastic search technique whose search algorithms simulate biological evolution—the strong tends to adapt and survive, while the weak tends to die out. At the beginning, a population of binary or non-binary chromosomes is initialized randomly. Then, each chromosome is evaluated using the fitness function. A set of better chromosomes is selected to produce new chromosomes. The production process is accomplished by applying the genetic operators (crossover and mutation) on the chromosomes selected. Then, each new chromosome is evaluated. At this stage, a full iteration is performed. The procedure is repeated until some preset termination criteria are satisfied.

The tabu search (TS) approach is a heuristic for solving combinatorial optimization problems. Recently, it has been applied to solve the continuous global optimization problem (Cvijovi and Klinowski 1995; Fleurent et al. 1995; Glover 1994b). In a tabu search, restrictions (tabu) are imposed to guide the search process to investigate difficult regions. It starts with an initial solution for the problem under consideration. The solution can be constructed by using either heuristics or a random solution. Then TS constructs a neighborhood from the current solution to identify adjacent solutions, and the objective function associated with each adjacent solution is evaluated. Before determining the best move, the TS approach selects the set of admissible moves which are not tabu.
For instance, recent moves will be classified as tabu to prevent the search from going back to its previous position. A recent move will be tabu for the duration of a certain number of iterations, depending on the size of the tabu list or tabu tenure. The aspiration criterion can be activated if a move that was tabu results in a solution better than any visited solution so far. In this case, the move’s tabu status is broken, and it becomes the best move. Otherwise, the best move is selected from the set of admissible moves. By then, a complete iteration of the TS approach has been performed. The procedure is repeated until the stopping criteria are met.

Harkan and Trafalis (2002) tested their proposed approach against a simulated annealing algorithm and a modified version of a hybrid scatter genetic search approach by optimizing 21 well-known test functions. They show that the HSGT approach is quite effective in identifying the global optimum.

4.2. The Estimation Strategy

We fit \( VI.12 \) by minimizing half of the sum of squared residuals.

\[
\min_{\theta} e^2 = \frac{1}{2} \sum_{t=1}^{T} (x(t) - \hat{x}(t))^2 ,
\]

\( (VI.14) \)

where \( x(t) \) is the first difference of the log price, \( \hat{x}(t) \) given by \( VI.12 \), and \( \theta = (A, B, C, t_c, \beta, \omega, \phi) \). As argued in Greene, the values of the parameters that minimize
half of the sum of the squared residuals will be the maximum likelihood estimators, as well as the nonlinear least-squares estimators. The first-order conditions for minimization of $e^2$ will be a set of nonlinear equations that do not have an explicit solution. This will typically require an iterative procedure for a solution. In particular, the HSGT described above will be used.

To reduce the number of free parameters to be estimated, following Johansen and Sornette (2000a), the three linear parameters $A, B, C$ are enslaved as functions of the nonlinear parameters $t, \beta, \omega, \phi$. This is done by requiring the objective function to have zero derivatives with respect to $A, B, C$ at the minimum. Optimizing VI.14 with respect to $A, B, C$, we get a system of three equations:

$$
\sum_{i=1}^{T} \ln[p(t)] f(t) g(t) = \sum_{i=1}^{T} \begin{bmatrix} T & f(t) & g(t) \\ f(t) & f(t)^2 & f(t)g(t) \\ g(t) & f(t)g(t) & g(t)^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}.
$$

(VI. 15)

where $f(t) = (t - t^\phi, g(t) = (t - t^\phi) \cos[\omega \log(t) - t^\phi]$. VI.15 will be solved via LU decomposition, as it is an efficient algorithm for matrix inversion (Press et al. 1992; Lee http://prosys.korea.ac.kr/~tclee/lecture/numerical/node14.html).

The nonlinear parameters are estimated using the HSGT approach proposed by Al-Harkan and Trafalis (2002). Their seven-step method is summarized below:
1. Generate a random starting point, \( X_k = \{t, \beta, \omega, \phi\} \), from a uniform distribution between the upper and lower bounds of each variable. \(^{xxxiii}\)

2. Generate a set of \( m \) random directions from a standard normal distribution, with \( m = n \times 2^3 \), where \( n \) is the number of parameters to be estimated. In our case, \( m = 4 \times 2^3 = 32 \). \(^{xxxiv}\)

3. Recalculate the \( m \) directions so that all the \( m \) vectors of dimension \( n \times 1 \) have the unit Euclidean norm. \(^{xxxv}\)

4. Generate a set of \( m \) reference points by moving from the starting point in the \( m \) directions calculated previously.

5. Assign weights to each of the \( m \) points according to the values of the objective function at these points. The largest weight is given to the point with the smallest value.

6. Generate the center of gravity, \( X_k = \{t, \beta, \omega, \phi\} \), by taking the weighted average of the \( m \) reference points obtained in step 5. This will be the new starting point for the next round of iteration if it passes the tabu test. \(^{xxxv}\) Otherwise, retain the old value of \( X_k \).

7. Generate new search directions using a genetic operator, either the whole arithmetical crossover approach or the general mutation approach (Al-Harkan and Trafalis 2002, 9–10). Normalize the \( m \) directions the same way as in step 3; repeat steps 4 to 6 until the prespecified stopping rule is satisfied.
4.3. Properties of Nonlinear Least Square Estimators

Consider the nonlinear regression model

\[ y = h(x, \theta) + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2) \quad (VI. 16) \]

The parameters \( \theta \) can be estimated by minimizing half of the sum of the squared residuals:

\[ S(\theta) = \frac{1}{2} \sum_{i=1}^{T} [y_i - h(x_i, \theta)]^2. \quad (VI. 17) \]

The asymptotic properties of the nonlinear least squares estimator of \( \theta \) are can be found in Greene (1997) and summarized below.

If the pseudoregressors defined in VI.17 are well behaved, then

\[ \hat{\theta} \xrightarrow{a} N \left[ \theta, \frac{\sigma^2}{n} Q_0^{-1} \right], \quad (VI. 18) \]

where

\[ Q_0 = p \lim_{n \to \infty} \frac{1}{n} X_0' X_0 = p \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{\partial h(x, \theta)}{\partial \theta} \frac{\partial h(x, \theta)}{\partial \theta'} \bigg|_{\theta_0} \quad (VI. 19) \]

with \( \theta_0 \) a particular value of \( \theta \). The sample estimate of the asymptotic covariance matrix is

\[ \text{Est. Asy.Var}[\hat{\theta}] = \hat{\sigma}^2 \left( X'_0 X_0 \right)^{-1}, \quad (VI. 20) \]

where

\[ \hat{\sigma}^2 = \frac{1}{T} \sum_{i=1}^{T} [y_i - h(x_i, \theta)]^2. \quad (VI. 21) \]
Under regularity conditions, this is the ML estimator of $\sigma^2$.

5. Empirical Application

5.1. Fitting the Model

The data used for this chapter are the Hong Kong office price index, the Seoul housing price index, and the Korea general construction stock price, as discussed in Chapter III. Table VI.2 lists accounts of the ex ante eyeball-identified crashes of each series. There are three pre-identified crashes in Hong Kong office price (March 1989, May 1994 and October 1997), three in Korea general construction stock price (January 1990, December 1994 and May 1996), and one in Seoul housing price (April 1991). As a stock price, the second series is far more volatile than the two property prices. The most significant crash of it occurred in the months following December 1994, with an average decline of 7.1% per month for four consecutive months.

For each data set, I found values fitted using equation VI.12 veer too far from the actual data. To examine the problem, I took the first difference of the log price. The plots show that the differentiated sets fluctuate around a mean of 0, implying that the dominant force is a linear trend rather than power law acceleration. This is confirmed later by the estimates of $\beta$, which turns out to be close to 1.
There are less than 50, rather than 100 strong as is the case in Zhou and Sornette (2003), monthly observations on price rallies before a crash in each of our data sets. This could be one of the reasons why the power-law signature could not develop fully. Fundamentalists in the markets of our concern did not wait for too long to exert checks and balances on speculators. The relatively speedy reactions of the counter-speculative force prevented the price from run-away wildly.

Nevertheless, the pattern of log-periodicity is glaring in Hong Kong office price. The frequency of the cycle increases obviously before the 1997 crash. But the intensification of log-oscillation looks delayed to the naked eyes in Korea general construction stock price index, and is difficult to tell in Seoul housing price index due to too few observations prior to the 1991 price decline (figure VI.1).

Inspired by these observations, I fit instead the following model using the HGST algorithm described before:

$$d \ln[p(t)] \approx C(t_c - t)^\phi \cos[\omega \log(t_c - t) + \phi], \quad (VI.22)$$

where $d \ln(p(t))$ is the first difference of the log price. VI.22 is essentially the same as VI.12 except for the intercept and exponential trend terms. The standard errors of parameter estimates are given by

$$SE(\theta_i) = \sqrt{\frac{\hat{\sigma}^2}{\sum_{t=1}^{T} \frac{\partial x}{\partial \theta_i} \frac{\partial x}{\partial \theta_i}}} |\hat{\theta}|,$$

Where $\hat{x}$ is given by VI.22, $\hat{\theta}$ the ML estimates of $\theta$, and
\[
\frac{\partial \hat{x}}{\partial t_c} = \left[ (c(t_c - t)^\beta \cos(\omega \ln(t_c - t) + \phi) - (c(t_c - t)^\beta \sin(\omega \ln(t_c - t) + \phi)\omega) \right],
\]

\[
\frac{\partial \hat{x}}{\partial \beta} = \left( c(t_c - t)^\beta \ln(t_c - t) \cos(\omega \ln(t_c - t) + \phi) \right),
\]

\[
\frac{\partial \hat{x}}{\partial \omega} = \left( -c(t_c - t)^\beta \sin(\omega \ln(t_c - t) + \phi) \right) \ln(t_c - t), \text{ and}
\]

\[
\frac{\partial \hat{x}}{\partial \phi} = \left( -c(t_c - t)^\beta \sin(\omega \ln(t_c - t) + \phi) \right).
\]

### 5.2. Estimation Results

I initiated randomly the nonlinear parameters \( t_c, \beta, \omega, \phi \) on a uniform distribution, with \( \beta \in [0.3, 0.98] \), \( \omega \in [1, 10] \), and \( \phi \in [0, 20] \). These values are set with reference to the empirical results found by Johansen et al. The critical time \( t_c \) is set at \( t_c \in [t_{cl}, T] \), where \( T \) is the size of the sample, and \( t_{cl} \) a random date before a crash date identified by the naked eye.

The estimates of \( \beta, \omega, \phi \) are very consistent for all three data sets, which are \{0.98, 10, 20\}, \{0.9516, 4.5984, 5.1880\}, or \{0.9509, 4.5981, 5.1881\} (tables VI.3–VI.5). This fact is consistent with the renormalization theory, which requires consistent parameter estimates from different data sets for the validation of the suggested model.
These estimates suggest that there exist two log-periodic harmonics, $\omega$. One is 10; the other is near 10/2, suggesting the theoretical formula ideally should have two cosine terms with two harmonics. A careful examination of figure VI.1 reveals that these plots indeed resemble those of some theoretical function that has two log-periodic oscillations of different frequencies, with one superimposed on the other (VI.3). But there are cons as well as pros to fitting this second term. The obvious disadvantage is the loss of a degree of freedom.

In viewing the records of the experiments, I notice that whenever I obtain the second set of estimates, the search process is trapped at the starting point. Whenever the move becomes possible, I obtain the last set of estimates. The first set of estimates is the pre-set boundaries. Henceforth, I will refer to the set of estimates that neither hits the boundary nor is trapped at the starting values as the “preferred estimates.” There are 8, 2 and 4 preferred estimates, 9, 1 and 6 trapped estimates, and 1, 2 and 1 boundary estimates for Hong Kong office price, Korea general construction stock price and Seoul housing price respectively. The boundary estimate for $\beta$ is statistically insignificant at the conventional levels. The other estimates are significant in most cases. Aside from the boundary estimates, $\beta$ is more than twice, and $\omega$ is slightly less than their corresponding values obtained by Johansen and Sornette (refer to table VI.1). Johansen and Sornette (2000c) mentioned that the “universality” of $\beta$ quantifying the price acceleration has not been established, but their estimates of $\omega$ are quite close.
The estimates of the critical time, however, depend on the lower boundary I imposed on the starting value of $t_c$ (tables VI.3–VI.5). In the estimation process, I move the lower boundary forward by 0.083 decimal years (one monthly observation) at a time until only the last sample observation remains. As a result, I obtain 18 critical times for Hong Kong office price, 11 for Seoul housing price. However, the fit for Korea general construction stock price is remarkably good. Only five values of $t_c$ are obtained out of three actual crashes—the best of which is within one month of the actual crash.

I have tried to alter the boundaries for the nonlinear parameters. Different estimates for $\beta, \omega, \phi$ are obtained as a result. However, I always get consistent estimates for the three data sets for each pair of boundaries set for a parameter, and they are close in value to the preferred estimates. As for $t_c$, when the lower boundaries (not only for the starting value but also for the entire estimation process) are pushed backward (observations taken long before the crash are admitted), I obtain more boundary estimates for the four nonlinear parameters, suggesting that information is not clear for obtaining accurate estimates at such early dates. But moving the lower boundaries forward has no impact on the results displayed in tables VI.3–VI.5.

Table VI.6 summarizes the best fitting for the three data sets, along with their standard errors. The best estimate for Hong Kong office price is within three months of the true crash date, that for Korea general construction stock price
within a month of the true date. The estimate of $\beta$ is insignificant in the best in-sample prediction ($t_c = 1994.9997$) for KGCSP. The data show that drop of KGCSP in December 1994 was slow and underwent many reversals until May 1996, when the price fell precipitously. The model missed the true date by nearly four years in Seoul housing price, however. The reason, perhaps, is that the decay of it since its peak in 1991 proceeded very slowly.

5.3. Out-of-sample Forecast

To investigate the predictive power of the model, I have truncated the data sets so that they end at March 1991 (38 months before the crash), September 1992 (27 months before), and May 1989 (23 months before) for Hong Kong office price, Korea general construction stock price, and Seoul housing price, respectively. Table VI.7 reports the preferred estimates from these truncated data, which neither hit the boundaries nor are trapped at the starting point. The extrapolative best fits of the model are also shown in figure VI.2.

With the original truncated data set, the best forecast for Hong Kong office price is 1993.9997, which misses the true crash date (May 1994) by four months. I then move the point of truncation forward by one observation at a time. When the last observation is November 1991, I obtain one more predicted critical date, 1994.9997, which overshoots the actual date by eight months. Move the
truncation point forward further. When the last observation is January 1993 (16 months before the crash), I obtain only one preferred estimate, 1993.9997.

With the original truncated data set, Korea general construction stock price obtains three preferred estimates, the best of which is 1994.9997 (the actual crash date is the decimal year 1994.92; \( \beta \), however, is insignificant.) These results are robust with respect to changes of the \( tcl \). Move the truncation point forward by one observation at a time as before. It is peculiar that increasing the sample size in this way actually worsens the result of the forecast. For instance, when the last observation is January 1994, no preferred estimate is obtained.

Both Hong Kong office price and Korea general construction stock price miss the crashes which closely follow their predecessors, namely, the October 1997 crash of HKOP and the May 1996 crash of KSCSP.

The unique forecast obtained for Seoul housing price (1989.9997) undershoots the true date by some 15 months. Again, increasing the sample size by moving the truncation point forward actually worsens the result of the forecast.

To sum up, the parameter estimates are consistent, therefore in line with the renormalization theory. There are three sets of estimates obtained. The set hits the boundary is dismissed as the \( \beta \) in it is statistically insignificant. The other two sets of estimates are almost identical. In the in-sample fitting, the model
managed to capture all of the actual time of crash, with reasonable accuracy especially in the case of Hong Kong office price which we have shown in figure VI.1 to reveal obvious patterns of log-periodicity. Nevertheless, the model produces too much false alarms. The model also managed to capture some of the actual crash times in extrapolative forecasting. Again, there are false alarms produced. Therefore, as a model for crash prediction, its use is limited. However, it is unfair to say that the model has failed such purpose. At this moment, the issue of global optima versus local optima in the estimation process is still unresolved. We do not know whether or not the false alarm is a problem of this unresolved issue. Furthermore, the theory prompts us to think outside the conventional box of speculative bubble. From chapter V, we know that the office market of Hong Kong is highly speculative, as compared to the housing market of Seoul in the sample period. According to the PLLP theory, when a speculative bubble is gathering force it will show up in the price as power-law acceleration and/or log-periodic oscillation. This is indeed the case for Hong Kong, where the log-periodic oscillation is visible even to the naked eyes (figure VI.1).

A word of caution before the passing of the chapter: the PLLP model has been fitted to the returns (the first difference of the log price) in this chapter. Feigenbaum (2001) argue that daily returns are serially correlated. The daily returns are also noisier than the prices. These may have bearings on the calibrating result (Feigenbaum 2001; Sornette and Johansen 2001). But I suspect that the monthly returns are as noisy as the daily returns.
Chapter VII Conclusions and Discussions

I have demonstrated, in chapter I of this thesis, that large swings in real-estate prices have a powerful impact on business cycles. The impact propagates via two channels. The first is the wealth effect. Real estate forms more than 50% of the wealth of the world. Hence, falling real-estate prices have devastating consequences on consumption and investment expenditures. Falling consumption and investment expenditures in turn reduce national income, triggering a vicious cycle. The second channel has to do with the close tie between the real-estate sector and the banking sector. In most market economies, a significant portion of bank loans is made directly to the financing of real-estate projects. Furthermore, loans for other purposes often use real estate as collateral. Consequently, falling real-estate prices often trigger a banking crisis, or worse still, a financial crisis. Today, a local financial crisis travels quickly from one country to the rest of the world, thanks to the globalization of information networks and increased international trade and cross-border investment.

A speculative bubble is the major driver behind many large swings in real-estate prices. As expectations are central to real-estate price formation (in addition to the opacity of information, market incompleteness, and mass psychology), it seems impossible to eradicate speculative bubbles from real-estate markets.
We may, however, learn to tame the wild beast of speculative bubbles. The literature on asset price bubbles, especially the empirical literature, has been flourishing since the early 1980s with Flood and Garber’s (1980) “Market Fundamentals versus Price-Level Bubbles: The First Tests.” After more than two decades, academics, central bankers, and practitioners still hold polarized views. Do bubbles exist at all? If they do, what is the nature of the bubble: rational or irrational? Are they destructive or constructive? Can we effectively detect and predict them? Or better still, can we tame them?

This thesis is by no means capable of answering all these questions. It has simply attempted to give some empirical evidence of periodically collapsing rational speculative bubbles, without worrying about the harder question of irrational bubbles. Periodically collapsing rational bubbles have been ruled out by Diba and Grossman (1988c) as theoretically impossible under certain assumptions. This type of bubble is nevertheless empirically appealing according to Blanchard and Watson (1982).

In this thesis, I have applied three different methodologies developed in the literature to the Hong Kong and Seoul property markets. These two markets, especially the former, have experienced large price swings in the past two decades. The next section lays out the empirical findings of this thesis. It is followed by suggestions for future research.
1. Empirical Findings

In this thesis, I have attempted to look at rational, speculative bubbles in real-estate markets from three angles: identification, quantification, and prediction. In order to serve these three purposes, I have to select one methodology for each from the most up-to-date array of tools developed in the literature. This is because that no single methodology, as far as I am aware, is capable of serving these three masters simultaneously. In chapter IV, I have applied the unit-root test methodology, proposed by Diba and Grossman (1988b) and refined by Hall, Psaradakis, and Sola (1999), to determine whether or not the real-estate prices of Hong Kong and Seoul have been plagued by speculative bubbles in the past two decades. The sectors chosen for investigation are the Hong Kong office sector and the Seoul housing sector, for reasons explained in chapter III. The test is done by comparing the properties of a price series with those of its associated rent series. Theoretically speaking, if prices exhibit explosive behavior, whereas rent does not, then one may infer that the price contains a speculative bubble. If both price and rent exhibit explosive behavior at the same time, then the explosiveness in price is being driven by fundamentals rather than by a speculative bubble. xxxviii It has been shown that such tests are typically weak in power when there are structural breaks in the time series. Thus I have taken a Markov-switching ADF test approach. The Markov-switching process is used to describe the behavior of a random state variable that might be responsible for the structural changes.
The MS ADF procedure detected a positive bubble in Seoul housing price (SHP) and in Hong Kong office price (HKOP) (table IV.11). In particular, it has detected a positive bubble in SHP between July 1990 and the end of 1992, between late 1997 and late 1998, and since late 2001 (recall the sample period is 90:7 to 03:6) and a positive bubble in Hong Kong between late 1987 and late 1989, in early 1994, between late 1997 and late 1998, and in early 2001 (the sample period is 84:1 to 03:4). Cautions must be given however. When reaching such conclusions, I have assumed that there are no other fundamentals, other than the rent, which drive the price. This is arguable as I discussed in the “Preface” of this thesis. There are also statistical issues involved. To establish the existence of a positive bubble statistically, we require the unit root hypothesis be rejected in favor of an explosive root for the price, and rejected in favor of a stable root for the rent. The reason is that non-rejection of a unit-root does not prove the existence of a unit root. This strong requirement was only met in the case of Seoul housing price when $\tau$ was used.

Results from the Markov-switching ADF test are compatible with the findings in my other two studies (chapter V and VI). In one of them, I use the Kalman filter to infer the level of the bubble, and in the other, I use the power-law-log-periodicity theory to predict the future trajectory of the bubble.
The critical values used for MS ADF unit-root tests are generated by bootstrapping with 10,000 replications. As the MS ADF residuals exhibit heteroscedasticity, I have modeled the residuals by an ARCH process and have incorporated such ARCH disturbances into the process of bootstrapping.

To examine the significance of such results, I have conducted a specification test to examine the worth of the Markov-switching AR(p) model: a parametric encompassing test suggested by Breunig, Najarian, and Pagan (2003). The test does not reject the two-state MS ADF model for the data series under consideration.

In chapter V, I have employed the Kalman filter technique to infer an unobservable speculative bubble from an observed price, using the present-value relation as the fundamental model. The data sets are those used in chapter IV. Two models have been estimated: one assuming the fundamental model is correct (the present value model), the other assuming that the model is misspecified (the state space model). The misspecification is captured by a state variable, which is not observable by the researcher, however inferable with the Kalman filter.

The study shows neither model explains the movements of the prices very well, although the parameters of each model are jointly significant. The R-square measure indicates that either model explains less than 20% of the price
variations in Hong Kong office sector, and roughly 50% of that in Seoul housing sector. The two models are not directly comparable, however, using R-square, AIC and BIC. They are defined differently (Chapter V, p. 15). The in-sample and 12-step-ahead prediction errors, nevertheless, show that the state space model is superior.

If we interpret the residuals of the state space model as a realization of a speculative bubble, then the following statement may be made. More than 80% of the price variations in Hong Kong office sector are driven by a speculative bubble. The size of the bubble is less spectacular in Seoul housing price, but nevertheless at an impressive rate of roughly 50%. A plausible explanation for this difference is that the real-estate market of South Korea was more driven by the government, whereas that of Hong Kong more by market forces. The housing market of South Korea had been very rigid before the first few years of this century. The mortgage market was freed only thereafter (refer to chapter I, section 4.2). The government of Hong Kong does exert its influence on the property market via land supply, macroeconomic policies and bank supervision, but is far less restrictive.

The study shows that the dramatic rise of the Hong Kong office price during the 1990s is better explained by a speculative bubble. The bubble expanded rapidly between the early 1990 and the early 1994, and between the late 1996 and the mid 1997. During these time periods, the price itself is soaring high. This is
largely consistent with Chan et al.’s (2000) conclusions and the findings in Chapter IV of this thesis. When the prices collapsed in the second half of 1994 and in late 1997, so did the bubble. The bubble also explains the rise of Seoul housing price in the early 1990s better than the fundamental model. The bubble peaked in February 1991. After that it embarked on a prolonged decline. The bubble creeps up again after December 2001. Again, this is consistent with the findings of Chapter IV.

In chapter VI, I have applied the power-law-log-periodicity (PLLP) theory of Johansen and Sornette et al., and have attempted to find how accurately the model can predict the burst of a rational bubble and a pending market crash. In addition to the data sets employed in chapters IV and V, I have added a Korea general construction stock price (KGCSP) index, which has the property prices as its fundamentals. This is because the SHP index had decayed slowly throughout the 1990s, whereas the PLLP model is designed to capture some extreme behavior of prices. On the other hand, KGCSP shows dramatic price swings in the sample period.

My experiments show that the original model of Johansen and Sornette does not comport completely with the three data sets under consideration. The power-law behavior is missing from each of our series and is replaced by a linear trend. After replacing the power-law trend by the linear one, however, the model works well. The results show that the modified model is far more successful for KGCSP
than for the other two series. Too many predictions of the critical time are produced by the model for Hong Kong office price and Seoul housing price, making it difficult to tell which one is the true alarm. With Korea general construction stock price, we obtain only two preferred estimates, defined as estimates that neither hit the preset boundary nor are trapped at the starting point of the iteration. Both are within reasonable range of the crash dates (table VI.6). One possible explanation is that the price swings in KGCSP are far more spectacular than those in the two other data sets (Chapter III, p. 3). Excluding the set of estimates with insignificant $\beta$, however, we capture only one out of the three crashes identified a priori. The results for Hong Kong office price, shown in table VI.6, are compatible with those from the Markov-switching ADF test (table IV.11) in chapter IV, and Kalman filter estimates (figure V.I&II) in chapter V.

The out-of-sample forecast using Hong Kong office price gives the best prediction, capturing one of the two crashes after the data truncation point with reasonable accuracy. Forecasting using both Korea general construction price and Seoul housing price missed the crash dates completely when estimates with insignificant $\beta$ are excluded.

The estimates of the nonlinear parameters $\beta, \omega, \phi$ are nearly identical in all three samples, which is consistent with the renormalization theory.
The model I have estimated, however, fails to capture the fact that variances
tend to increase around a crash point. I have tried a few ways to remedy it, but
the improvement increases the accuracy of the predicted critical time
insignificantly. One of these remedies is to allow $\beta$ exceeding unity, which is
nevertheless ruled out theoretically at the start. Furthermore, the estimates of the
model parameters suggest a second cosine term. However, the cost of adding
additional parameters is the loss of a degree of freedom. It also goes against the
principle of parsimony.

In short, I consider the PLLP theory reasonably successful in the data sets used
in my studies. Though the prediction of the critical time is not as neat as one
would prefer, the model nevertheless provides a very useful signature of price
behavior: a power-law acceleration of price, and/or an accelerating price
oscillation, is an important indication of a speculative bubble and a looming
market crash. Furthermore, the model relates the price trajectory to the crash
hazard rate, providing another useful model for extrapolating the price variations.

Combining the results of chapters IV through VI, I conclude that the large swings
in the property prices of Hong Kong and Seoul during the last two decades have
been driven by periodically collapsing, rational, speculative bubbles.

It has been argued that none of the suggested methods in the literature for
bubble testing is reliable, due to identification problems—that is, we do not know
what the true fundamental process is like (see for example, Flood, Hodrick, and Kaplan 1986). I would like, however, to share a few observations obtained from my study. In empirical studies, one should not rely on a single method. Rather, one should combine two or more methodologies before drawing conclusions. Furthermore, when the test statistics do not give overwhelmingly convincing conclusions, one's judgment must be applied. It is an art as well as a science, to repeat the wisdom of our predecessors (Enders 1995, 119).

Taking my studies in this thesis as an example, both unit-root and Kalman-filter procedures rely on a fundamental that is subject to dispute. However, the power-law-log-periodicity theory does not make any assumptions about the properties of the fundamentals—something that I argue is a salient feature of this theory, given the identification problems facing the bubble literature. It relies on a simple idea: speculative forces pull the price away from its fundamental value, while the fundamental forces pull the price back. The interplay of the two gives an identifiable signature of an accelerating bubble and a looming crash. As all three studies point to the existence of rational, speculative bubbles in the data sets concerned, and insofar as the results from these studies are compatible, I conclude that the Hong Kong and Seoul property markets have been plagued by rational speculative bubbles that collapse periodically.

While studies on speculative bubbles in financial markets are abundant, there has been relatively little effort devoted to the empirical studies of real-estate price
bubbles, especially in Asia. There have been a number of studies on real-estate price bubbles in Hong Kong and Korea, but the methodologies used have been, in general, crude. My empirical studies in this thesis have attempted to apply some of the most advanced methodologies developed in the financial-asset markets to the real-estate market. The arguments for this practice are given in chapter II, section 2. In short, the fundamental reason is that both stock and real estate are assets whose value can be measured by the stream of services it can provide directly or that it can “buy.” The findings of my thesis show that the flexible combination of tools for the identification, quantification, and prediction of bubbles yields promising results.

As of today, the theory of speculative bubbles in real-estate markets is relatively underdeveloped. For example, while empirical studies often apply the present-value framework developed in the financial markets to the real-estate market, some argue that real estate is distinctively different from stock, and that it therefore deserves a theoretical framework of its own. The author would like to argue that developing a sound theoretical framework for the real-estate market requires more empirical evidence. This thesis serves that purpose.

2. Future Research

My study reveals that, after decades of effort by academics around the world, many issues regarding speculative bubbles are still unresolved. The literature
recognizes the importance of speculative bubbles in driving asset prices. Yet, as of today, there is no clear-cut measure for detecting speculative bubbles. This is because that most methodologies suggested are subject to the criticism that there is an identification problem. If the implications of a particular fundamental model are rejected empirically, it could be a rejection of the particular model, rather than a rejection of the null hypothesis of “no bubble.”

However, as I have shown with my empirical studies in chapters IV through VI, when combined effectively and used flexibly, the methodologies developed by the literature can give an asset price a fairly reliable diagnosis—with the help of the newly developed PLLP theory. What is lacking is, perhaps, an answer to this question: Which one of those numerous methods developed so far is preferable?

The literature on Markov-switching has made large progresses in MS model specification tests in recent years, but little has been said about the relative power of these tests. I believe that this topic deserves the attention of future researchers.

The area of the literature that is much less developed concerns the policy reactions of the government in the face of a speculative bubble. This is the case perhaps, due to the lack of consensus regarding bubbles themselves. Therefore, I suggest that future researchers devote more of their efforts in this direction.
The following areas may also be worthy of investigation. The first is the wealth effect of asset-price changes on consumption in developing countries. Empirical studies on the developed countries on this topic are numerous, but little has been done for the developing countries. The second is the confidence-boosting effect of a buoyant asset market on consumption, which has been suggested but not yet proved. Also suggested but unproved are the asymmetric responses of consumption and investment to rising and falling asset prices (Zandi 1999). If these effects indeed are present and important, then rapidly falling asset prices may have a far graver effect on output and unemployment than we so far have acknowledged.
Appendix 1: Chapter III Graphs
Data

Figure III.1 *CPI Deflated Price and Rent Indices (Hong Kong)*

*Source: CEIC database*
Data

Figure III.2 CPI Deflated Price and Rent Indices (South Korea)

Source: CEIC database
Data

Figure III.3 Price-rent Ratio

The price-rent ratio of Hong Kong office is no less volatile than the price series. It increased continuously between 1990 and 1997, with a few temporary reversals. This ratio crashed to its historical low after the late 1997. On the other hand, the price-rent ratio of Seoul housing was on a declining trend throughout the sample period, with only a few brief episodes of reversals.
Appendix 2a: Chapter IV Graphs
CPI Deflated Price and Rent Indices

Figure IV. 1 Real Estate Price and Rent Indices

The Hong Kong office price is highly volatile. The price index more than doubled in a mere 15 months between Dec. 1987 and March 1989. Another sharp increase of the price occurred in the first half of 1994, with an average value of 6.1% per month. The price crashed after July 1997, following the Asian financial crisis. By April 2003, the price index stood only at about 22% of its peak value (occurred in May 1994). The Seoul housing price is less volatile (compared with the Hong Kong office price). After rising for four years between 1988 and 1992, it declined gradually throughout the remaining part of the 1990s. The price started climbing up again in 2001.
In-Sample Predictions Using Markov-switching AR(p) Model

Figure IV. 2 Fitted Values with Markov Switching Model (Seoul Housing Price and Rent in Log Differences)

Note: Y is the actual observed value, Yhat1 the fitted value using estimates of state-one parameters, and Yhat2 the fitted value using estimates of state-two parameters of the MS model. All values are in their first log differences. These notations are applicable for figure 2 and 3.
In-Sample Predictions Using Markov-switching AR(p) Model

Figure IV. 3 Fitted Values with Markov Switching Model (Hong Kong Office Price and Rent in Log Differences)
Modeling the Residuals from the Markov-switching AR(p) Model

Figure IV. 4 ARCH Modeling of MS Residuals (Seoul Housing Price)

Note: St=j: state j (j=1,2); Et_j: square of state j (j=1,2) MS residuals; Et_j hat: fitted value of square of MS residuals using ARCH model. These notations are applicable for Figure IV.4 through IV.7. Plots in Figure IV.4 through IV.7 show that the ARCH model is a reasonable description of the actual process of the MS residuals.
Modeling the Residuals from the Markov-switching AR(p) Model

Figure IV. 5 ARCH Modeling of MS Residuals (Seoul Housing Rent)

![Graph of Seoul Housing Rent (St=1)]

![Graph of Seoul Housing Rent (St=2)]

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Modeling the Residuals from the Markov-switching AR(p) Model

Figure IV. 6 ARCH Modeling of MS Residuals (Hong Kong Office Price)
Modeling the Residuals from the Markov-switching AR(p) Model

Figure IV. 7  ARCH Modeling of MS Residuals (Hong Office Rent)
Smoothed State Probabilities

**Figure IV. 8** ADF Unit Root Tests for Rational Speculative Bubble with Markov Switching

![Graph showing Seoul Housing Price and Rent](image1)

![Graph showing Hong Kong Office Price and Rent](image2)

Appendix 2b: Chapter IV Tables
Lag Selection

Table IV.1 Number of Lags Selected in an AR(p) Model

<table>
<thead>
<tr>
<th></th>
<th>Seoul Housing Sector</th>
<th>Hong Kong Office Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample periods</td>
<td>86:1 to 03:6</td>
<td>90:7 to 03:6</td>
</tr>
<tr>
<td>Lags</td>
<td>Price</td>
<td>Rent</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

The AR(p) model we estimate for each sample has a constant term but no trend, that is,

\[ y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p-1} \zeta_i \Delta y_{t-i} + u_t, \]

as the samples show no signs of trending in the long run. This is consistent with the notion that bubbles, if exists, is not sustainable in the long run. The number of lags, \( p \), is selected using F test.
Linear Unit Root Tests

Table IV.2 Linear ADF and PP Unit Root Tests

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Rent</td>
</tr>
<tr>
<td>ADF Tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: unit root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1: not unit root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5% level)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rho</td>
<td>-0.34</td>
<td>-2.05</td>
</tr>
<tr>
<td>(Pr&lt;Rho)</td>
<td>(0.614)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>τ</td>
<td>-1.30</td>
<td>-2.35</td>
</tr>
<tr>
<td>(Pr&lt;τ )</td>
<td>(0.633)</td>
<td>(0.157)</td>
</tr>
<tr>
<td>F</td>
<td>0.86</td>
<td>2.91</td>
</tr>
<tr>
<td>(Pr&gt;F)</td>
<td>(0.851)</td>
<td>(0.330)</td>
</tr>
<tr>
<td>PP Tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: unit root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1: not unit root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5% level)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rho</td>
<td>-1.25</td>
<td>-4.82</td>
</tr>
<tr>
<td>(Pr&lt;Rho)</td>
<td>(0.861)</td>
<td>(0.450)</td>
</tr>
<tr>
<td>τ</td>
<td>-0.97</td>
<td>-1.73</td>
</tr>
<tr>
<td>(Pr&lt;τ )</td>
<td>(0.765)</td>
<td>(0.414)</td>
</tr>
<tr>
<td></td>
<td>1.29</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>(0.743)</td>
<td>(0.711)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hong Kong Office Price and Rent</th>
<th>1984:1 to 2003:4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
</tr>
<tr>
<td>ADF Tests</td>
<td></td>
</tr>
<tr>
<td>H0: unit root</td>
<td></td>
</tr>
<tr>
<td>H1: not unit root</td>
<td></td>
</tr>
<tr>
<td>(5% level)</td>
<td></td>
</tr>
<tr>
<td>Rho</td>
<td>-5.67</td>
</tr>
<tr>
<td>(Pr&lt;Rho)</td>
<td>(0.371)</td>
</tr>
<tr>
<td>τ</td>
<td>-1.60</td>
</tr>
<tr>
<td>(Pr&lt;τ)</td>
<td>(0.480)</td>
</tr>
<tr>
<td>F</td>
<td>1.29</td>
</tr>
<tr>
<td>(Pr&gt;F)</td>
<td>(0.743)</td>
</tr>
<tr>
<td>PP Tests</td>
<td></td>
</tr>
<tr>
<td>H0: unit root</td>
<td></td>
</tr>
<tr>
<td>H1: not unit root</td>
<td></td>
</tr>
<tr>
<td>(5% level)</td>
<td></td>
</tr>
<tr>
<td>Rho</td>
<td>-1.61</td>
</tr>
<tr>
<td>(Pr&lt;Rho)</td>
<td>(0.823)</td>
</tr>
<tr>
<td>τ</td>
<td>-0.87</td>
</tr>
<tr>
<td>(Pr&lt;τ)</td>
<td>(0.797)</td>
</tr>
</tbody>
</table>

Note: (i) Phillips Perron Test is adopted in drawing conclusions when conflicting results arise between PP and ADF tests and PP strongly suggest H0 or H1, as PP takes care of the Heteroscedasticity existing in our data. (ii) "**" means significant in the right tail.
### Parameter Estimates of the Markov-switching Model

Table IV.3 Markov Switching Model Parameter Estimates (Seoul Housing Price and Rent)

#### Seoul Housing Price

<table>
<thead>
<tr>
<th>State one</th>
<th>State two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>t(H)</td>
</tr>
<tr>
<td>inter 1</td>
<td>1.155</td>
</tr>
<tr>
<td>lp1</td>
<td>-0.012</td>
</tr>
<tr>
<td>ldp11</td>
<td>0.588</td>
</tr>
<tr>
<td>ldp21</td>
<td>-0.164</td>
</tr>
<tr>
<td>ldp31</td>
<td>-0.074</td>
</tr>
<tr>
<td>ldp41</td>
<td>0.072</td>
</tr>
<tr>
<td>ldp51</td>
<td>0.142</td>
</tr>
<tr>
<td>Sig1</td>
<td>0.898</td>
</tr>
</tbody>
</table>

#### Seoul Housing Rent

<table>
<thead>
<tr>
<th>State one</th>
<th>State two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>t (H)</td>
</tr>
<tr>
<td>inter 1</td>
<td>2.520</td>
</tr>
<tr>
<td>lp1</td>
<td>-0.024</td>
</tr>
<tr>
<td>ldp11</td>
<td>0.620</td>
</tr>
<tr>
<td>ldp21</td>
<td>-0.121</td>
</tr>
<tr>
<td>ldp31</td>
<td>-0.167</td>
</tr>
<tr>
<td>ldp41</td>
<td>0.009</td>
</tr>
<tr>
<td>ldp51</td>
<td>0.210</td>
</tr>
<tr>
<td>Sig1</td>
<td>1.519</td>
</tr>
</tbody>
</table>

Note: (i) inter j: ML estimate of the constant term in state j; (ii) lpj: ML estimate of coefficient on \( y_{t-1} \) for state j; (iii) ldpij: ML estimate of coefficient on \( \Delta y_{t-1} \) in state j. (iv) t (H): t ratio obtained using Hessian; (v) t (W): t ratio obtained using White covariance; (vi) sig j: variance estimate in state j. These notations are applicable throughout this chapter.
Parameter Estimates of the Markov-switching Model

Table IV.4 Markov Switching Model Parameter Estimates (Hong Kong Office Price and Rent)

### Hong Kong Office Price

<table>
<thead>
<tr>
<th>State one</th>
<th>Parameter</th>
<th>t(H)</th>
<th>t(W)</th>
<th>State two</th>
<th>Parameter</th>
<th>t(H)</th>
<th>t(W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inter 1</td>
<td>0.405</td>
<td>0.636</td>
<td>0.004</td>
<td>inter 2</td>
<td>0.612</td>
<td>1.692</td>
<td>0.002</td>
</tr>
<tr>
<td>lp1</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>lp2</td>
<td>-0.008</td>
<td>-1.525</td>
<td>-0.002</td>
</tr>
<tr>
<td>ldp11</td>
<td>0.059</td>
<td>0.245</td>
<td>0.033</td>
<td>ldp12</td>
<td>0.112</td>
<td>0.771</td>
<td>0.034</td>
</tr>
<tr>
<td>ldp21</td>
<td>-0.173</td>
<td>-0.813</td>
<td>-0.064</td>
<td>ldp22</td>
<td>0.390</td>
<td>3.769</td>
<td>0.124</td>
</tr>
<tr>
<td>ldp31</td>
<td>0.124</td>
<td>0.568</td>
<td>0.043</td>
<td>ldp32</td>
<td>-0.013</td>
<td>-0.118</td>
<td>-0.004</td>
</tr>
<tr>
<td>ldp41</td>
<td>0.410</td>
<td>1.835</td>
<td>0.169</td>
<td>ldp42</td>
<td>0.040</td>
<td>0.354</td>
<td>0.012</td>
</tr>
<tr>
<td>ldp51</td>
<td>0.187</td>
<td>1.063</td>
<td>0.059</td>
<td>ldp52</td>
<td>-0.055</td>
<td>-0.496</td>
<td>-0.016</td>
</tr>
<tr>
<td>ldp61</td>
<td>0.148</td>
<td>0.715</td>
<td>0.046</td>
<td>ldp62</td>
<td>0.015</td>
<td>0.146</td>
<td>0.004</td>
</tr>
<tr>
<td>ldp71</td>
<td>0.120</td>
<td>0.546</td>
<td>0.048</td>
<td>ldp72</td>
<td>0.072</td>
<td>0.515</td>
<td>0.020</td>
</tr>
<tr>
<td>ldp81</td>
<td>0.134</td>
<td>0.699</td>
<td>0.050</td>
<td>ldp82</td>
<td>-0.253</td>
<td>-2.273</td>
<td>-0.040</td>
</tr>
<tr>
<td>Sig1</td>
<td>25.441</td>
<td>35096.060</td>
<td></td>
<td>Sig2</td>
<td>14.263</td>
<td>39909.788</td>
<td></td>
</tr>
</tbody>
</table>

### Hong Kong Office Rent

<table>
<thead>
<tr>
<th>State one</th>
<th>Parameter</th>
<th>t (H)</th>
<th>t (W)</th>
<th>State two</th>
<th>Parameter</th>
<th>t (H)</th>
<th>t (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inter 1</td>
<td>0.85</td>
<td>3.35</td>
<td>0.01</td>
<td>inter 2</td>
<td>0.14</td>
<td>1.87</td>
<td>0.00</td>
</tr>
<tr>
<td>lp1</td>
<td>-0.01</td>
<td>-4.43</td>
<td>-0.01</td>
<td>lp2</td>
<td>0.00</td>
<td>-1.09</td>
<td>0.00</td>
</tr>
<tr>
<td>ldp11</td>
<td>-0.04</td>
<td>-0.34</td>
<td>0.00</td>
<td>ldp12</td>
<td>0.77</td>
<td>16.08</td>
<td>0.10</td>
</tr>
<tr>
<td>ldp21</td>
<td>0.55</td>
<td>4.65</td>
<td>0.03</td>
<td>ldp22</td>
<td>0.32</td>
<td>6.62</td>
<td>0.03</td>
</tr>
<tr>
<td>ldp31</td>
<td>0.07</td>
<td>0.65</td>
<td>0.00</td>
<td>ldp32</td>
<td>-0.10</td>
<td>-2.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>sig1</td>
<td>1.45</td>
<td>2.30</td>
<td></td>
<td>sig2</td>
<td>0.90</td>
<td>3.95</td>
<td></td>
</tr>
</tbody>
</table>

Note: (i). inter j: ML estimate of the constant term in state j; (ii). lpj: ML estimate of coefficient on $y_{t-1}$ for state j; (iii). ldpij: ML estimate of coefficient on $\Delta y_{t-1}$ in state j. (iv). t (H): t ratio obtained using Hessian; (v). t (W): t ratio obtained using White covariance; (vi). sig j: variance estimate in state j. These notations are applicable throughout this chapter.
Specification Test for Markov-switching AR(p) Model and State Transition Probabilities of the Markov Chain

Table IV.5 Testing for the Significance of the Markov Switching Model

<table>
<thead>
<tr>
<th>Wald Test</th>
<th>Seoul Housing Sector</th>
<th>Hong Kong Office Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wald statistic</td>
<td>Price</td>
<td>Rent</td>
</tr>
<tr>
<td>H0: MS model</td>
<td>1.098</td>
<td>0.361</td>
</tr>
<tr>
<td>(Prob (χ^2_n (1) ≤ c) )</td>
<td>(0.750)</td>
<td>(0.500)</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Do not reject H0</td>
<td>Do not reject MS model</td>
</tr>
<tr>
<td></td>
<td>MS model</td>
<td>MS model</td>
</tr>
</tbody>
</table>

Note: The Wald statistic has a χ^2 (1) distribution. The null distribution of the statistic is generated using bootstrapping with 10,000 replications.

Table IV.6. State Transition Probabilities

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Seoul Housing Sector</th>
<th>Hong Kong Office Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Rent</td>
</tr>
<tr>
<td>p_{11}</td>
<td>0.510</td>
<td>2.719</td>
</tr>
<tr>
<td>p_{12}</td>
<td>0.490</td>
<td>6.250</td>
</tr>
<tr>
<td>p_{21}</td>
<td>1.000</td>
<td>6.231</td>
</tr>
<tr>
<td>p_{22}</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>π_{1}</td>
<td>0.671</td>
<td>0.964</td>
</tr>
<tr>
<td>π_{2}</td>
<td>0.329</td>
<td>0.036</td>
</tr>
<tr>
<td>λ_{1}</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>λ_{2}</td>
<td>-0.49</td>
<td>0.723</td>
</tr>
<tr>
<td>ave. duration state 1 (months)</td>
<td>2.040</td>
<td>96.421</td>
</tr>
<tr>
<td>ave. duration state 2 (months)</td>
<td>1.000</td>
<td>3.740</td>
</tr>
</tbody>
</table>

Note: (i). p_{ij} is the probability of state j in time t+1, given the time t state is i. (ii) π_{i}, i = 1,2 is the unconditional probability of state i. (iii) λ_{i}, i = 1,2 is the eigenvalue of the transition matrix. (iv) the average duration of state i (i=1,2) is given by \((I - p_{ii})^{-1}\) (Raymond and Rich, 1997, page 202).
Parameter Estimates and Specification Tests of ARCH Model

Table IV.7 Parameter Estimates of ARCH Model for MS Residuals

<table>
<thead>
<tr>
<th>Seoul Housing Price and Rent</th>
<th>State one</th>
<th>State two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Parameter</td>
<td>Parameter</td>
</tr>
<tr>
<td>Price</td>
<td>a0</td>
<td>0.387</td>
</tr>
<tr>
<td></td>
<td>a1</td>
<td>0.198</td>
</tr>
<tr>
<td></td>
<td>a2</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>a3</td>
<td>0.216</td>
</tr>
<tr>
<td></td>
<td>a4</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td>a5</td>
<td>-0.040</td>
</tr>
<tr>
<td>Rent</td>
<td>a0</td>
<td>0.493</td>
</tr>
<tr>
<td></td>
<td>a1</td>
<td>0.420</td>
</tr>
<tr>
<td></td>
<td>a2</td>
<td>0.097</td>
</tr>
<tr>
<td></td>
<td>a3</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>a4</td>
<td>0.030</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hong Kong Office Price and Rent</th>
<th>State one</th>
<th>State two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Parameter</td>
<td>Parameter</td>
</tr>
<tr>
<td>Price</td>
<td>a0</td>
<td>6.120</td>
</tr>
<tr>
<td></td>
<td>a1</td>
<td>0.227</td>
</tr>
<tr>
<td></td>
<td>a2</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>a3</td>
<td>0.409</td>
</tr>
<tr>
<td></td>
<td>a4</td>
<td>-0.128</td>
</tr>
<tr>
<td>Rent</td>
<td>a0</td>
<td>0.337</td>
</tr>
<tr>
<td></td>
<td>a1</td>
<td>0.449</td>
</tr>
<tr>
<td></td>
<td>a2</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td>a3</td>
<td>-0.089</td>
</tr>
<tr>
<td></td>
<td>a4</td>
<td>0.119</td>
</tr>
</tbody>
</table>

Table IV.8 F Test for Joint Significance of Parameters of ARCH Model

<table>
<thead>
<tr>
<th>Seoul Housing Sector</th>
<th>Price</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>State one</td>
<td>3.264</td>
<td>7.947</td>
</tr>
<tr>
<td>df</td>
<td>(5, 150)</td>
<td>(4, 151)</td>
</tr>
<tr>
<td>State two</td>
<td>4.508</td>
<td>36.267</td>
</tr>
<tr>
<td>df</td>
<td>(5, 150)</td>
<td>(4, 151)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hong Kong Office Sector</th>
<th>Price</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>State one</td>
<td>8.793</td>
<td>40.327</td>
</tr>
<tr>
<td>df</td>
<td>(4, 218)</td>
<td>(4, 223)</td>
</tr>
<tr>
<td>State two</td>
<td>15.048</td>
<td>4.626</td>
</tr>
<tr>
<td>df</td>
<td>(4, 218)</td>
<td>(4, 223)</td>
</tr>
</tbody>
</table>

Note: All test statistics are significant at 5% level, showing the ARCH model appropriately describes the MS model residuals.
Tests for ARCH Effect and Markov-switching ADF Test for Bubble

Table IV.9 TR² Test For ARCH Effect in MS Residuals

<table>
<thead>
<tr>
<th></th>
<th>Seoul Housing Sector</th>
<th>Hong Kong Office Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price ((q = 5))</td>
<td>Rent ((q = 4))</td>
</tr>
<tr>
<td>State one</td>
<td>10.688</td>
<td>20.611</td>
</tr>
<tr>
<td>State two</td>
<td>16.085</td>
<td>67.476</td>
</tr>
</tbody>
</table>

Note: The test statistic TR² has \(\chi^2(q)\) distribution. All statistics are significant at 5% level, indicating the ARCH effect is indeed present in the MS residuals.

Table IV.10 ADF Unit Root Tests with Markov Switching

<table>
<thead>
<tr>
<th></th>
<th>Seoul Housing Sector</th>
<th>Hong Kong Office Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rho ((\Pr&lt;\rho))</td>
<td></td>
</tr>
<tr>
<td>State one</td>
<td>-4.405*</td>
<td>-8.428*</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>State two</td>
<td>-0.160</td>
<td>-7.285*</td>
</tr>
<tr>
<td></td>
<td>(0.723)</td>
<td>(0.184)</td>
</tr>
</tbody>
</table>

Note: (i) Probabilities are generated using bootstrapping with 10,000 replications. (ii) “*” means significant in the left tail; “**” means significant in the right tail. 5% levels are used for all.

Table IV.11 Markov-switching ADF Tests Identified Periods Possibly Plagued by Positive Bubbles

<table>
<thead>
<tr>
<th></th>
<th>Rho</th>
<th>(\tau)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seoul Housing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>1.</td>
<td>July 90 to end 92</td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td>July 90 to end 92</td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>since late 01</td>
</tr>
<tr>
<td>Hong Kong Office</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>Through out the sample period</td>
<td>1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.</td>
</tr>
</tbody>
</table>
Appendix 3a: Chapter V Graphs
CPI Deflated Price, Prediction of the State-space Model and Model Residuals

Figure V.1 Hong Kong Office Price

![Hong Kong Office Price (Unrestricted)](image)

Note: In both figure 1 and 2, “PHAT_KF” is the predicted price using the state space model; and “Residual” is the model residual which is interpret as the realization of a speculative bubble.

Figure V.4 Seoul Housing Price

![Seoul Housing Price (Unrestricted)](image)


Standardized Plot

Figure V. 2 Standardized Plot for The Price Residuals from the State-space Model

Note: The standardized residual from the price equation of the state-space model is distributed NID(0,1) if the model is correctly specified. The plot shows that most values are within the boundary (-1,+1). However, the variances are not homoscedastic, especially for Hong Kong office price.
Appendix 3b: Chapter V Tables
Tests for Parameter Restrictions

Table V.1 LR Test for Parameter Restrictions

<table>
<thead>
<tr>
<th>LR Statistic</th>
<th>Hong Kong Office Price</th>
<th>Seoul Housing Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0: $\psi + \psi' = 1$</td>
<td>0.9269</td>
<td>186.2433*</td>
</tr>
</tbody>
</table>

Note: the LR statistic is distributed $\chi^2(1)$ with critical values 2.71, 3.84 and 6.63 at 10%, 5% and 1% respectively.
Parameter Estimates

Table V.2 the Present Value Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hong Kong Office</th>
<th>Seoul Hosing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>rent</td>
</tr>
<tr>
<td>t</td>
<td>0.00127</td>
<td>1.1519</td>
</tr>
<tr>
<td>t(NW)</td>
<td>0.50</td>
<td>5.50*</td>
</tr>
<tr>
<td>LM (H0: Homoscedasticity)</td>
<td>0.14850</td>
<td>0.05173</td>
</tr>
<tr>
<td>F (H0: model jointly insignificant)</td>
<td>5.53</td>
<td>8.35*</td>
</tr>
<tr>
<td>Wald (H0: model jointly insignificant)</td>
<td>0.02</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Note: (i) The LM statistic for Heteroscedasticity is the Koenker & Bassett (1982) statistic, which is more powerful than the Breusch-Pagan LM test in the absence of normality; (ii) If the null of the LM test cannot be rejected, refer to t and F test, otherwise refer to t(NW) and Wald statistics for the evaluation of joint significance; (iii) t(NW) are calculated using Newey-West covariance. (iv)* indicates significance at 10%; (v) Both LM and Wald statistics are distributed $\chi^2(3)$. The corresponding critical values are 11.34, 7.82 and 6.25 at 1%, 5% and 10% levels respectively;
### Parameter Estimates

Table V.3 the State Space Model

(a) Hong Kong Office Price

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\psi$</th>
<th>$\psi_{-1}$</th>
<th>$\beta$</th>
<th>$\phi$</th>
<th>$\sigma_\omega^2$</th>
<th>$\sigma_\delta^2$</th>
<th>$\sigma_\zeta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>1.14941</td>
<td>-0.27961</td>
<td>0.00104</td>
<td>0.00000</td>
<td>0.00078</td>
<td>0.00034</td>
<td>0.00065</td>
</tr>
<tr>
<td>LM</td>
<td>8.52747*</td>
<td>-2.07439*</td>
<td>0.03636</td>
<td>0.00000</td>
<td>1.12626</td>
<td>0.80108</td>
<td>0.94521</td>
</tr>
<tr>
<td>F</td>
<td>6.89*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wald</td>
<td>343.64*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Seoul Housing Price

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\psi$</th>
<th>$\psi_{-1}$</th>
<th>$\beta$</th>
<th>$\phi$</th>
<th>$\sigma_\omega^2$</th>
<th>$\sigma_\delta^2$</th>
<th>$\sigma_\zeta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>0.43105</td>
<td>0.02673</td>
<td>0.00021</td>
<td>0.00000</td>
<td>0.00005</td>
<td>0.00042</td>
<td>0.00003</td>
</tr>
<tr>
<td>LM</td>
<td>13.82469*</td>
<td>0.85735</td>
<td>0.02043</td>
<td>0.00000</td>
<td>0.14501</td>
<td>0.89859</td>
<td>0.09468</td>
</tr>
<tr>
<td>F</td>
<td>27.80*</td>
<td></td>
<td></td>
<td></td>
<td>8.04*</td>
<td>2.19</td>
<td>0.02</td>
</tr>
<tr>
<td>Wald</td>
<td>285.15*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Note: (i) The LM statistic for Heteroscedasticity is the Koenker & Bassett (1982) statistic, which is more powerful than the Breusch-Pagan LM test in the absence of normality; (ii) If the null of the LM test cannot be rejected, refer to F test, otherwise refer to Wald statistics for the evaluation of joint significance; (iii) *indicates significance at 10%; (vi) The critical values for the various test statistics in the current and the next table are listed below,

<table>
<thead>
<tr>
<th>(Hong Kong)</th>
<th>F(7,230)</th>
<th>Wald($\chi^2(5)$)</th>
<th>LM for $\sigma_\omega^2$</th>
<th>LM for $\sigma_\delta^2$ &amp; $\sigma_\zeta^2$</th>
<th>$t$ Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2.73</td>
<td>15.09</td>
<td>11.3</td>
<td>6.63</td>
<td>2.58</td>
</tr>
<tr>
<td>5%</td>
<td>2.05</td>
<td>11.07</td>
<td>7.81</td>
<td>3.84</td>
<td>1.96</td>
</tr>
<tr>
<td>10%</td>
<td>n.a.</td>
<td>9.24</td>
<td>6.25</td>
<td>2.71</td>
<td>1.65</td>
</tr>
</tbody>
</table>
Model Comparison

Table V. 4 Comparisons of Goodness of Fit

(a) Goodness of Fit

<table>
<thead>
<tr>
<th></th>
<th>Hong Kong Office</th>
<th>Seoul Housing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present value</td>
<td>Present value</td>
</tr>
<tr>
<td>R square</td>
<td>0.1793</td>
<td>0.50992</td>
</tr>
<tr>
<td>AIC</td>
<td>0.33881</td>
<td>0.01780</td>
</tr>
<tr>
<td>BIC</td>
<td>0.35448</td>
<td>0.01863</td>
</tr>
<tr>
<td>ESS</td>
<td>0.33001</td>
<td>0.01732</td>
</tr>
</tbody>
</table>

Note: R square, AIC and BIC for state space model are based on Harvey (1989, p268-270), whereas those for the present value model are as usual. These statistics are not directly comparable with those for the present value, because different formula are used. Hence the in-sample sum of squared errors (ESS) is presented as an alternative.

(b) Post Sample Predictive Test

<table>
<thead>
<tr>
<th></th>
<th>Hong Kong Office</th>
<th>Seoul Housing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ξ((l))</td>
<td>ESS</td>
</tr>
<tr>
<td>Present value</td>
<td>n.a.</td>
<td>0.00262</td>
</tr>
<tr>
<td>State space</td>
<td>0.1193</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

Note: (i) ξ(\(l\)) is the statistic for post-sample predictive test for the state-space model. It is distributed F(12,218) for Hong Kong office price, and F(12,196) for Seoul housing price. The model accepted at 10% significance level for both series; (ii) ESS is 12-step-ahead extrapolative sum of squared errors. This variable for the state space model is calculated using equation (5.6.7) in Harvey (1989, p273).
Appendix 4a: Chapter VI Graphs
Differenced Log Series

Figure VI.1 Differenced Log Price and fitted value

Note: (i) P: the differenced log price; (ii) Phat: the in-sample fitted value.
Out-Of-Sample Forecast

Figure VI.2 Forecast

Note: (i) $P$: the differenced log price; (ii) the extrapolative forecast.
Plot of Theoretical Model with Two Cosine Terms

Figure VI. 3 A Function with Two Cosine Terms
Appendix 4b: Chapter VI Tables
Empirics from Johansen and Sornette

Table VI. 1 Empirics from Johansen and Sornette (2000c)

<table>
<thead>
<tr>
<th>Crash data (time series)</th>
<th>$t_c$</th>
<th>$t_{\text{max}}$</th>
<th>$t_{\text{min}}$</th>
<th>%drop</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1929 (DJ)</td>
<td>30.22</td>
<td>29.65</td>
<td>29.87</td>
<td>47%</td>
<td>0.45</td>
<td>VI.9</td>
<td>2.2</td>
</tr>
<tr>
<td>1985(DM)</td>
<td>85.20</td>
<td>85.15</td>
<td>85.30</td>
<td>14%</td>
<td>0.28</td>
<td>6.0</td>
<td>2.8</td>
</tr>
<tr>
<td>1985(CHF)</td>
<td>85.19</td>
<td>85.18</td>
<td>85.30</td>
<td>15%</td>
<td>0.36</td>
<td>5.2</td>
<td>3.4</td>
</tr>
<tr>
<td>1987(S&amp;P)</td>
<td>87.74</td>
<td>87.65</td>
<td>87.80</td>
<td>30%</td>
<td>0.33</td>
<td>7.4</td>
<td>2.3</td>
</tr>
<tr>
<td>1987(HK)</td>
<td>87.84</td>
<td>87.75</td>
<td>87.85</td>
<td>50%</td>
<td>0.29</td>
<td>5.6</td>
<td>3.1</td>
</tr>
<tr>
<td>1994(HK)</td>
<td>94.02</td>
<td>94.01</td>
<td>94.04</td>
<td>17%</td>
<td>0.12</td>
<td>6.3</td>
<td>2.7</td>
</tr>
<tr>
<td>1997(HK)</td>
<td>97.74</td>
<td>97.60</td>
<td>97.82</td>
<td>42%</td>
<td>0.34</td>
<td>7.5</td>
<td>2.3</td>
</tr>
<tr>
<td>1998(S&amp;P)</td>
<td>98.72</td>
<td>98.55</td>
<td>98.67</td>
<td>19.4%</td>
<td>0.60</td>
<td>6.4</td>
<td>2.7</td>
</tr>
<tr>
<td>1999(IBM)</td>
<td>99.56</td>
<td>99.53</td>
<td>99.81</td>
<td>34%</td>
<td>0.24</td>
<td>5.2</td>
<td>3.4</td>
</tr>
<tr>
<td>2000(P&amp;G)</td>
<td>00.04</td>
<td>00.04</td>
<td>00.19</td>
<td>54%</td>
<td>0.35</td>
<td>6.6</td>
<td>2.6</td>
</tr>
<tr>
<td>2000(Nasdaq)</td>
<td>00.34</td>
<td>00.22</td>
<td>00.29</td>
<td>37%</td>
<td>0.27</td>
<td>7.0</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Note: This table is reproduced from Johansen and Sornette 2000c. $t_c$ is the critical time predicted with equation VI.12. The fit is performed up to the time $t_{\text{max}}$ at which the market index achieved its highest maximum before the crash. $t_{\text{min}}$ is the time of the lowest point of the market after the maximum. The percentage loss is calculated from the total loss from $t_{\text{max}}$ to $t_{\text{min}}$. 
# Actual Crashes

Table VI. 2 Facts on Crashes

<table>
<thead>
<tr>
<th>Pre-identified crash dates</th>
<th>Percentage change and time taken</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hong Kong</strong> Mar. 89</td>
<td>↓31.697% in 23 months</td>
</tr>
<tr>
<td><strong>office price</strong> May 94</td>
<td>↓41.427% in 17 months</td>
</tr>
<tr>
<td>Oct. 97</td>
<td>↓51.867% in 11 months</td>
</tr>
<tr>
<td><strong>Seoul</strong> Apr. 91</td>
<td>↓20.075% in 15 months</td>
</tr>
<tr>
<td><strong>housing price</strong> Apr. 91</td>
<td></td>
</tr>
<tr>
<td><strong>Korean general</strong> Jan. 90</td>
<td>↓36.097% in 8 months</td>
</tr>
<tr>
<td>Dec. 94</td>
<td>↓28.308% in 4 month</td>
</tr>
<tr>
<td><strong>construction stock price</strong> May 96</td>
<td>↓89.494% in 25 months</td>
</tr>
</tbody>
</table>
Model Parameter Estimates and In-Sample Predictions of Crash

Table VI. 3 Hong Kong Office Price Parameter Estimates

<table>
<thead>
<tr>
<th>Period</th>
<th>$t_c$</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\phi$</th>
<th>$C$</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(s.e.)</td>
<td>(s.e.)</td>
<td>(s.e.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apr. 83</td>
<td>1982.92</td>
<td>0.98</td>
<td>10</td>
<td>20</td>
<td>0.0027</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.2388)</td>
<td>(4.5570)</td>
<td>(4.2163)</td>
<td>(2.0603)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>May 83-Jul. 84</td>
<td>1987</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0042</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>(0.4102)</td>
<td>(0.3080)</td>
<td>(0.4391)</td>
<td>(0.6200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aug. 84-Nov. 86</td>
<td>1988</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0082</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.1576)</td>
<td>(0.1232)</td>
<td>(0.1025)</td>
<td>(0.1742)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec. 86</td>
<td>1988.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>-0.0062</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.2090)</td>
<td>(0.1638)</td>
<td>(0.1071)</td>
<td>(0.1971)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan. 87-Mar. 88</td>
<td>1990</td>
<td>0.9516#</td>
<td>4.5984</td>
<td>5.1880</td>
<td>0.0005</td>
<td>0.0018</td>
</tr>
<tr>
<td></td>
<td>(2.5067)</td>
<td>(1.3844)</td>
<td>(1.1869)</td>
<td>(2.2544)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apr. 88-Jun. 89</td>
<td>1991</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>0.0024</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>(0.4817)</td>
<td>(0.1835)</td>
<td>(0.2272)</td>
<td>(0.4335)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jul. 89-Aug. 90</td>
<td>1992</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>0.0037</td>
<td>0.0013</td>
</tr>
<tr>
<td></td>
<td>(0.2847)</td>
<td>(0.0846)</td>
<td>(0.1269)</td>
<td>(0.2471)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. 90-Nov. 91</td>
<td>1992.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>0.0042</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td>(0.2236)</td>
<td>(0.0593)</td>
<td>(0.0813)</td>
<td>(0.1701)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 $t_c$: the lower boundary set for the initial value of $t_c$.

2 In decimal years. For non-leap year,

\[ 12 \text{ months} = 1.00 \text{ year}; \ 1 \text{ month} = 0.083 \text{ years}; \ e.g. \ December \ 1998 = 1998.92. \]

3 #: estimates insignificant at the conventional levels.

4 Rows in bold show best fits.
<table>
<thead>
<tr>
<th>Period</th>
<th>C (0.2000)</th>
<th>C (0.0200)</th>
<th>C (0.0001)</th>
<th>C (0.0000)</th>
<th>C (0.0001)</th>
<th>C (0.0000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 91-Feb. 93</td>
<td>1993.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>0.0032</td>
<td>0.0012</td>
</tr>
<tr>
<td>Mar. 93-May 94</td>
<td>1994.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>0.0014</td>
<td>0.0015</td>
</tr>
<tr>
<td>Jun. 94-Jul. 95</td>
<td>1996</td>
<td>0.9516#</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0004</td>
<td>0.0017</td>
</tr>
<tr>
<td>Aug. 95-Oct. 96</td>
<td>1997</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0017</td>
<td>0.0017</td>
</tr>
<tr>
<td>Nov. 96-Jan. 98</td>
<td>1998</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0019</td>
<td>0.0019</td>
</tr>
<tr>
<td>Feb. 98-Mar. 99</td>
<td>1999</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0021</td>
<td>0.0021</td>
</tr>
<tr>
<td>Apr. 99-Jun. 00</td>
<td>1999.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>-0.0019</td>
<td>0.0020</td>
</tr>
<tr>
<td>Jul. 00-Sep. 01</td>
<td>2000.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>-0.0015</td>
<td>0.0020</td>
</tr>
<tr>
<td>Oct. 01-Dec. 02</td>
<td>2001.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>-0.0013</td>
<td>0.0020</td>
</tr>
<tr>
<td>Jan. 03-May 03</td>
<td>2002.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>-0.0010</td>
<td>0.00187</td>
</tr>
</tbody>
</table>

Note: The linear parameter C is enslaved as function of the nonlinear parameters. Hence only the standard errors of nonlinear parameters are provided.
Model Parameter Estimates and In-Sample Predictions of Crash

Table VI.4 Korea General Construction Stock Price Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>$t_c$</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\phi$</th>
<th>$C$</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>tcl</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(s.e)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>May 88-Dec 92</strong></td>
<td>1988.33</td>
<td>0.98#</td>
<td>10</td>
<td>20</td>
<td>-0.00093</td>
<td>0.0071667</td>
</tr>
<tr>
<td></td>
<td>(1.3244)</td>
<td>(1.4441)</td>
<td>(1.6693)</td>
<td>(2.9468)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Jan. 93-Feb. 94</strong></td>
<td><strong>1994.9997</strong></td>
<td>0.9509#</td>
<td>4.5981</td>
<td>5.1881</td>
<td>0.0006</td>
<td>0.0071</td>
</tr>
<tr>
<td></td>
<td>(3.4343)</td>
<td>(1.0032)</td>
<td>(0.7099)</td>
<td>(1.7461)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mar. 94-May. 95</strong></td>
<td>1996</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0017</td>
<td>0.0069</td>
</tr>
<tr>
<td></td>
<td>(1.0980)</td>
<td>(0.3088)</td>
<td>(0.2084)</td>
<td>(0.5247)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Jun.95-Aug. 96</strong></td>
<td><strong>1996.9997</strong></td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>-0.0023</td>
<td>0.0069</td>
</tr>
<tr>
<td></td>
<td>(0.7960)</td>
<td>(0.1956)</td>
<td>(0.1440)</td>
<td>(0.3681)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sep 96-Nov. 03</strong></td>
<td>1988.33</td>
<td>0.98#</td>
<td>10</td>
<td>20</td>
<td>-0.0009</td>
<td>0.0072</td>
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<td></td>
<td>(1.3244)</td>
<td>(1.4441)</td>
<td>(1.6693)</td>
<td>(2.9468)</td>
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</table>
# Model Parameter Estimates and In-Sample Predictions of Crash

## Table VI. 5 Seoul Housing Price Parameter Estimates

<table>
<thead>
<tr>
<th>Interval</th>
<th>$t_c$</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\phi$</th>
<th>$C$</th>
<th>Variance</th>
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<tr>
<td>(s.e)</td>
<td>(s.e)</td>
<td>(s.e)</td>
<td>(s.e)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>May 89-Nov. 91</strong></td>
<td>1989.33</td>
<td>0.98#</td>
<td>10#</td>
<td>20</td>
<td>-0.0002</td>
<td>0.0002</td>
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<td></td>
<td>(1.7243)</td>
<td>(8.9809)</td>
<td>(11.3343)</td>
<td>(9.7965)</td>
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<td></td>
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<tr>
<td><strong>Dec. 91-Feb 93</strong></td>
<td>1994</td>
<td>0.9516</td>
<td>4.5984</td>
<td>5.1880</td>
<td>0.0015</td>
<td>0.0002</td>
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<td>(0.2854)</td>
<td>(0.2270)</td>
<td>(0.1479)</td>
<td>(0.2711)</td>
<td></td>
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</tr>
<tr>
<td><strong>Mar. 93-Apr. 94</strong></td>
<td>1994.9997</td>
<td><strong>0.9509</strong></td>
<td><strong>4.5981</strong></td>
<td><strong>5.1881</strong></td>
<td><strong>0.0016</strong></td>
<td><strong>0.0002</strong></td>
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<td>(0.1317)</td>
<td>(0.1093)</td>
<td>(0.2074)</td>
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<tr>
<td><strong>May 94-Jul. 95</strong></td>
<td>1995.9997</td>
<td>0.9509</td>
<td>4.5981</td>
<td>5.1881</td>
<td>0.0005</td>
<td>0.0002</td>
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<td>(0.3331)</td>
<td>(0.6354)</td>
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<tr>
<td><strong>Aug. 95- Oct. 96</strong></td>
<td>1997</td>
<td>0.9516#</td>
<td>4.5984</td>
<td>5.1880</td>
<td>-0.0002</td>
<td>0.0002</td>
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<td>(2.3786)</td>
<td>(0.7175)</td>
<td>(1.0718)</td>
<td>(2.0780)</td>
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<tr>
<td><strong>Nov. 96-Dec. 97</strong></td>
<td>1998</td>
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<td>0.0001</td>
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<td>(0.2897)</td>
<td>(0.6022)</td>
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<tr>
<td><strong>Jan. 98-Mar. 99</strong></td>
<td>1999</td>
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<td>4.5984</td>
<td>5.1880</td>
<td>-0.0005</td>
<td>0.0002</td>
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<td>(0.1887)</td>
<td>(0.2066)</td>
<td>(0.4645)</td>
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<tr>
<td><strong>Apr. 99-Jun. 00</strong></td>
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<td>5.1881</td>
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<td>0.0002</td>
</tr>
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<td>(0.8442)</td>
<td>(0.2354)</td>
<td>(0.1998)</td>
<td>(0.47429)</td>
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<tr>
<td><strong>Jul. 00-Sep. 01</strong></td>
<td>2000.9997</td>
<td>0.9509#</td>
<td>4.5981251</td>
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<td>-0.0001</td>
<td>0.0002</td>
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<td>(6.3115)</td>
<td>(1.8436)</td>
<td>(1.3046)</td>
<td>(3.2089)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Oct. 01-Nov. 02</strong></td>
<td>2002</td>
<td>0.9516#</td>
<td>4.5984</td>
<td>5.1880#</td>
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<td>0.0002</td>
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<tr>
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<td>(12.371)</td>
<td>(3.4797)</td>
<td>#</td>
<td>(5.9117)</td>
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<td></td>
<td></td>
<td></td>
<td>(2.3477)</td>
<td></td>
<td></td>
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<tr>
<td><strong>Dec. 02-Jun. 03</strong></td>
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<td>4.5984</td>
<td>5.1880</td>
<td>0.0001</td>
<td>0.0002</td>
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<td>(0.5031)</td>
<td>(0.3706)</td>
<td>(0.9471)</td>
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</table>
Model Parameter Estimates and In-Sample Predictions of Crash

Table VI. 6 The Actual and the Closest In-sample Predictions of the Most-Likely Crash Time

<table>
<thead>
<tr>
<th>Actual crash time</th>
<th>In-sample predicted most likely time of crash, $t_c$ (s.e.)</th>
<th>Prediction error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong office price</td>
<td>1989.17</td>
<td>1988.9997</td>
</tr>
<tr>
<td>1994.33</td>
<td>1993.9997</td>
<td>Early by 4 months</td>
</tr>
<tr>
<td>1997.75</td>
<td>1998.00</td>
<td>Late by 3 months</td>
</tr>
<tr>
<td>Korea general construction stock price</td>
<td>1990.00</td>
<td>1988.33</td>
</tr>
<tr>
<td>1994.92</td>
<td>1994.9997</td>
<td>Late by 1 month</td>
</tr>
<tr>
<td>1996.33</td>
<td>1996.9997</td>
<td>Late by 8 months</td>
</tr>
<tr>
<td>Seoul housing price</td>
<td>1991.25</td>
<td>1994.9997</td>
</tr>
<tr>
<td>(0.2090)</td>
<td>(0.2990)</td>
<td>(0.4944)</td>
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<tr>
<td>(3.4343)</td>
<td>(0.7960)</td>
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<tr>
<td>(0.2302)</td>
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</tr>
</tbody>
</table>
Model Parameter Estimates and Out-of-Sample Predictions of Crash

Table VI. 7 Extrapolative Forecast of the Most-likely Time of Crash

<table>
<thead>
<tr>
<th></th>
<th>Actual crash date</th>
<th>Out-of-sample predicted most likely time of crash, $t_c$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong office</td>
<td>Nov. 91-Dec. 92</td>
<td>1989.17</td>
</tr>
<tr>
<td>price</td>
<td></td>
<td>1992.9997</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2365)</td>
</tr>
<tr>
<td></td>
<td>Jan. 93 –Mar. 94</td>
<td><strong>1994.33</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>1993.9997</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.3023)</td>
</tr>
<tr>
<td>Korea general</td>
<td>Sep. 92-May 93</td>
<td>1990.00</td>
</tr>
<tr>
<td>construction stock</td>
<td></td>
<td>1992.9997</td>
</tr>
<tr>
<td>price</td>
<td></td>
<td>(0.7327)</td>
</tr>
<tr>
<td></td>
<td>Jun. 93 –Aug. 94</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1993.9997</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.8231)</td>
</tr>
<tr>
<td></td>
<td>Sep. 94 –Nov. 95</td>
<td><strong>1994.92</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>1994.9997</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.8551)</td>
</tr>
<tr>
<td>Seoul housing</td>
<td>Sep. 89 -Oct. 90</td>
<td>1991.25</td>
</tr>
<tr>
<td>price</td>
<td></td>
<td>1989.9997</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.0090)</td>
</tr>
</tbody>
</table>

5 The highlighted rows are the best fits.
Bibliography One: Books, Monographs and Journal Papers


167. Lee, T., “Numerical analysis for chemical engineers,”

http://prosys.korea.ac.kr/~tclee/lecture/numerical/node14.html,
tclee@prosys.korea.ac.kr


216. SAS/IML user’s guide, version 8.


Bibliography Two: News Articles


Real estate and property: Real estate refers (1) to lands and anything permanently affixed to the land, such as buildings, fences, and those things attached to the buildings, such as light fixtures, plumbing and heating fixtures, and other such items that would be personal property if not attached. The term is generally synonymous with real property, although in some states a fine distinction may be made. (2) Real estate may refer to rights in real property as well as to the property itself. Source of the definition: online Real Estate Dictionary, Copyright © 1975, 1979, 1984, 1986, 1991, 1998, Financial Publishing Company.

In this thesis, real estate and property will be used interchangeably to refer to definition 1.

An extensive literature review on speculative bubble is available upon request. This review is left out because it is not closely related to the theme of this thesis.

Financial crises are associated with the peaks of business, and are often the culmination of a period of expansion before a downturn. Raymond Goldsmith defines a financial crisis as “a sharp, brief, ultra-cyclical deterioration of all or most of a group of financial indicators—short-term interest rates, asset (stock, real estate, land) prices, commercial insolvencies and failures of financial institutions” (Kindleberger 2000, 3).

For more examples, please refer to BIS (2001).

This guideline was withdrawn in July 1998, as the property market cooled down and banks were much more restrained in their property lending.

Under the chonsei arrangement, the tenant who signs a rental contract deposits about half the purchase price of a house to the landlord in lieu of monthly rent, and the deposit is fully returned to the tenant at the termination of the lease. The landlord is supposed to generate an income from the deposit during the contract period equivalent to rental income. Since many houses purchased for investment purposes are financed through chonsei deposits, chonsei deposits in South Korea are equivalent to mortgage financing in other countries.

This regulation was repealed in January 1998. Before that, housing financing came mainly from the National Housing Fund (NHF), the public-sector housing-finance vehicle, and the chonsei arrangement.

Park et al. (1998) suggest that the price bubble in housing and land stood at 58% and 40% respectively in 1991 at its peak, disappearing almost completely by 1997.

An extensive review of the debate about bubbles is available from the author upon request.

We will follow the literature and use rent loosely as a synonym for net operating income.

The present value of an expected real cash flow is the fundamental value.

Capozza and Helsley (1989 and 1990) present models in which real land value is the sum of four components: the real value of agricultural land rent, the cost of developing the land for urban use, the value of “accessibility,” and the value of expected real rent increases. The conversion of a stream of rents into a value introduces the real-after-tax interest rate as a determinant of real house prices.

The proxy for a bubble to swell is the price appreciation lagged one period; that for a bubble to burst is the deviation of accrual price from the equilibrium price in the previous period (Abraham and Hendershott 1994, 2, 4).

For a description of the Minsky model, please refer to Kindleberger (2000).

Details of the paper are not available. This summary is obtained from Kim (2000).

This is in contrast to our findings in chapter IV of this thesis. But the data series we employed are not identical to those used by Lim (2003).

The CEIC Economic Databases have been established since 1992. Its core economic database is the CEIC Asia Economic Database with over 190,000 data series. Its prime sources of data include over 150 major government statistical agencies, over 80 recognized non-government issuing agencies, and over 300 reference statistical publications. Please visit http://www.ceicdata.com for more information on the profile of CEIC Data Company Ltd (“CEIC”)

The rationale for using CPI as a deflator is that buying a property is an investment decision, which very much depends on one’s consumption choices.

Refer to Hamilton (1994a) page 691 for an argument of this specification.
For an introduction to EM, refer to Frank Dellaeret (2002), which also is a good source of references.

For example, Hall et al. (1999).

SSE is the sum of the squared weighted average of residuals from both states, with weights being the smoothed probabilities associated with each state.

Cavaliere (2003) derives asymptotics for unit-root tests with Markov-switching. But we will adopt the bootstrapping approach for reasons to be explained below.

The theory does not suggest a specific number of replications. The guideline is to stop when changes in the distribution are negligible (Handbook of Econometrics, vol. 5, chapter 52). We nevertheless conducted 10,000 replications to be on the safe side.

These exercises show that the results of estimation and testing are indeed data sensitive, as noted by previous researchers (Lim 2003).

The White (1980) covariance estimator is preferred in the presence of heteroscedasticity.

This comment is from one of my external examiners.

Finite-time singularity refers to the appearance of an infinite slope or infinite value within a finite time.

More generally, the price $p(t)$ can be interpreted as the price in excess of the fundamental value of the asset.

More generally, $K$ could be heterogeneous across pairs of neighbors. Some of the $K_{ij}$’s could even be negative, as long as the average of all $K_{ij}$’s is strictly positive.

In physics, critical points are widely considered to be one of the most interesting properties of complex systems. A system becomes critical when local influences propagate over long distances and the average state of the system becomes exquisitely sensitive to a small perturbation. That is, different parts of the system become highly correlated. A critical system is self-similar across scales. Critical self-similarity is why local imitation cascades through the scales into global coordination.

Scale invariance means reproducing oneself on different time or space scales. The hallmark of it is power law. Scale invariance holds exactly at the critical point. When not exactly at the critical point, only a weaker kind of scale invariance, discrete scale invariance, holds. With discrete scale invariance, the system obeys scale invariance only for specific choices of scaling ratio $\lambda$. “The signature of discrete scale invariance is the presence of a power law with complex exponent, which manifests itself in data by log-periodic oscillations providing corrections to the simple power law scaling.” (Sornette 2003, 207).

The upper and lower bounds of parameters are set initially with reference to the empirical results in the series of papers written by Johansen and Sornette.

The Euclidean norm is the square root of the sum of the absolute squares of the vector elements.

A move is tabu if it is both located close to the previous point and the change in its objective function is very small. The tabu status of a move can be overridden if the aspiration criterion is met.

Sornette, email comments, December 6th 2004.

The points of truncation are places where a certain pattern has developed.

Ito and Iwaisako (1995) suggest that bank credit accounts for a large sum of the property-price movements. However, if investors enter a purchase agreement simply because it is easier to borrow rather than because the return is good, then the resulting price increase is driven by a bubble, not a fundamental.