EFFICIENT SIGMA-DELTA
BEAMFORMING TECHNIQUES
FOR ULTRASOUND IMAGING
APPLICATION

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<tr>
<td>(\Delta)</td>
<td>Instant increment of focal depth</td>
</tr>
<tr>
<td>(\Delta x)</td>
<td>Pitch between adjacent elements</td>
</tr>
<tr>
<td>(c)</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>(d_f)</td>
<td>Focal depth</td>
</tr>
<tr>
<td>(d_r)</td>
<td>Length of the echo path</td>
</tr>
<tr>
<td>(e(t))</td>
<td>Quantization error of sigma-delta modulators</td>
</tr>
<tr>
<td>(E(z))</td>
<td>Quantization error function of sigma-delta modulators</td>
</tr>
<tr>
<td>(f_b)</td>
<td>Signal bandwidth</td>
</tr>
<tr>
<td>(f_{BF})</td>
<td>Beamforming frequency</td>
</tr>
<tr>
<td>(f_{num})</td>
<td>(f)-number</td>
</tr>
<tr>
<td>(f_s)</td>
<td>Sampling frequency</td>
</tr>
<tr>
<td>(G_{SNR})</td>
<td>Apodization power gain</td>
</tr>
<tr>
<td>(k_N)</td>
<td>Constant (x_n \cdot \sin\theta)</td>
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</table>
$L$ Number of samples in each block selected in block-based SDBF

$M$ Twice the oversampling ratio used in block-based SDBF

$m$ Sample interval

$N_b$ Number of bits

$N_{ch}$ Number of active channels in receive beamforming

$n_e^2$ Unshaped quantization noise power

$n_q^2$ Quantization noise power after noise shaping

$N_S$ Number of samples

$p$ Full pulse-echo path

$P_y$ Output power of sigma-delta modulators

$R$ Distance from the transducer to the focal point, which is also the focal depth

$r_{cor}$ Correlation coefficient

$R_e$ Auto correlation of $e[n]$

$rf$ Ratio of sampling frequency to signal bandwidth
\( R_{n'} \)  Distance between the \( n^{th} \) element and the focal point

\( R_u \)  Auto correlation of \( u[n] \)

\( R_{ue} \)  Cross correlation of \( u[n] \) and \( e[n] \)

\( T \)  Period of signal

\( t_n' \)  Time of arrival of ultrasound echoes at the \( n^{th} \) element

\( T_S \)  Sampling period

\( u(t) \)  Input signal of sigma-delta modulators

\( U(z) \)  Input function of sigma-delta modulators

\( w_n \)  Apodization weightage at the \( n^{th} \) element

\( X(z) \)  Z-transform of signal \( x \)

\( x_n \)  Distance from the centre of the scanline to the \( n^{th} \) element

\( y(t) \)  Output signal of sigma-delta modulators

\( Y(z) \)  Output function of sigma-delta modulators

\( \theta \)  Steering angle

\( \lambda \)  Wavelength
\( \mu_c \) Mean values of cyst

\( \mu_s \) Mean values of scatterers

\( \sigma_s \) Standard deviation of scatterers

\( \Omega \) Digital frequency
# GLOSSARY

| **ADC** | Analog-to-Digital Converter |
| **A-mode** | Amplitude mode |
| **Apodization** | The amplitude windowing of electrical pulses across the aperture in order to reduce side lobe level and prevent gating lobes. |
| **B-mode** | Brightness mode |
| **CE-SDBF** | Coded Excitation Sigma-Delta BeamFormer |
| **CIC filter** | Cascaded Integrator-Comb filter |
| **CNR** | Contrast-to-Noise Ratio: The ratio of the mean value difference between scatterers and cyst to the standard deviation of the scatterers. |
| **CW** | Continuous Wave |
| **DAC** | Digital-to-Analog Converter |
| **DEM** | Dynamic Element Matching |
DFT Discrete Fourier Transform

D-mode Doppler mode

DR Dynamic Range: The ratio between the maximum and minimum signal power that a system can process.

Dynamic aperture The varying of aperture size with focal depth in order to achieve uniform point spread function.

ENBW Equivalent Noise BandWidth

ENOB Equivalent Number Of Bits

FIFO First-In-First-Out

FIR filter Finite Impulse Response filter

F-mode Color Flow mode

f-number The ratio of the focal length to the size of the aperture.

FPGA Field Programmable Gate Array

IBF Interpolation BeamFormer

IFT Inverse Fourier Transform
<table>
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<tr>
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<th>Description</th>
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<tr>
<td>LSB</td>
<td>Least Significant Bit</td>
</tr>
<tr>
<td>LUT</td>
<td>LookUp Table</td>
</tr>
<tr>
<td>MI</td>
<td>Mechanical Index: A standard measure of the acoustic output in a diagnostic ultrasound system, defined as the peak rarefractional pressure of an ultrasound longitudinal wave divided by the square root of the centre frequency.</td>
</tr>
<tr>
<td>Near field</td>
<td>A non-uniform beam intensity exists in the near field (area of non-divergence). It is also called the Fresnel zone. For a non-focused transducer, the depth of the near field (D) can be estimated using ( D = \frac{r^2}{\lambda} ), where ( r ) is the radius of the transducer and ( \lambda ) is the wavelength of the ultrasound.</td>
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<tr>
<td>OSR</td>
<td>OverSampling Ratio: The ratio of the sampling frequency to two times the signal bandwidth.</td>
</tr>
<tr>
<td>PRBF</td>
<td>Phase Rotation BeamFormer</td>
</tr>
<tr>
<td>PSNR</td>
<td>Peak Signal-to-Noise Ratio: The ratio of the peak signal power to the average noise floor level of an image.</td>
</tr>
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<td>QD</td>
<td>Quadrature Demodulation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>ROM</td>
<td>Read-Only Memory</td>
</tr>
<tr>
<td>RTL</td>
<td>Register Transfer Level</td>
</tr>
<tr>
<td>Rx</td>
<td>Receive</td>
</tr>
<tr>
<td>SDBF</td>
<td>Sigma-Delta BeamFormer</td>
</tr>
<tr>
<td>SL</td>
<td>SideLobe</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SQNR</td>
<td>Signal-to-Quantization Noise Ratio: The ratio of the signal power to the quantization noise power.</td>
</tr>
<tr>
<td>TGC</td>
<td>Time Gain Control</td>
</tr>
<tr>
<td>Tx</td>
<td>Transmit</td>
</tr>
<tr>
<td>VHDL</td>
<td>Very high speed integrated circuit Hardware Description Language</td>
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<tr>
<td>ΣΔ</td>
<td>Sigma-Delta</td>
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Pre-delay reconstruction sigma-delta beamformer (SDBF) was proposed in recent years to achieve a higher level of integration in ultrasound imaging systems. The high-order reconstruction filter used in each channel of SDBF makes the beamformer highly complex. The beamformer can be simplified by reconstructing the signal only after the delay-and-sum process. It requires only one reconstruction filter for the entire beamformer. However, this post-delay reconstruction-based beamformer degrades the image quality when dynamic focusing is performed. This thesis studied the cause of the dynamic focusing artifact problem suffered by the sigma-delta beamformer. It was found that the degree of the problem is related to the pre-delay quantization noise that is present in the signal. Hence, similar performance to the conventional pre-delay reconstruction SDBF can be achieved by simply employing a simple pre-delay filter, as long as the pre-delay filter provides the required pre-delay signal-to-quantization noise ratio (SQNR). Based on this finding, a cascaded reconstruction beamformer was developed that utilizes a boxcar filter as the pre-delay filter in each channel to achieve the required pre-delay SQNR value. Simulations using real phantom data demonstrated that the proposed cascaded reconstruction beamforming method can achieve a contrast resolution comparable to that of the pre-delay reconstruction beamforming method. In addition, the savings on hardware and power can be as much as 85% and 68% respectively, as compared to the pre-delay reconstruction SDBF.
Based on the same principle, a multi-bit SDBF was developed to further improve the hardware and power performances as compared to the cascaded reconstruction SDBF. Real point and cyst phantom images show that the multi-bit SDBF provides comparable image quality as the cascaded reconstruction SDBF without any extra artifact correction processing. It also reduces the operation frequency compared to the single-bit SDBF. Hardware synthesis result shows that the proposed multi-bit SDBF can save up to 44% and 62% on hardware and power respectively, as compared to the cascaded reconstruction SDBF.

The author believes that the development of these efficient beamformer structures will assist the advancement of ultrasound imaging systems in the two current trends, that is the low-end portable handheld systems as well as the high-end 3-D machines, since the proposed beamformer structures can be applied to both types of machine.
Chapter 1

INTRODUCTION

1.1 Background

Acoustic wave is used by animals like bats and dolphins to detect objects and measure distances. This incredible idea was first utilized by human for underwater echo-ranging application after the catastrophic Titanic disaster in 1912 [1]. Its application was then extended to SONAR (Sound Navigation and Ranging) which was an important military technology during the Second World War. This most destructive tragedy in recent human history somehow brought along a rapid development of diagnostic ultrasound instrumentation. In the late 1940s and early 1950s, ultrasound signal was found useful for the detection of different tissue structures due to its change in velocity when traveling through different mediums. This finding brought forth the amplitude mode (or A-mode) application of ultrasound signal [2] in medical application and the first brightness mode (or B-mode) ultrasound scanner was demonstrated in 1952 [3]. Since then, more medical diagnostic functions have been introduced to ultrasound scanner like Doppler and color flow imaging.

Due to the breakthrough in several technological aspects (e.g. wideband transducers, digital circuitries, etc.), ultrasound image quality has improved significantly over the past decade. In particular, the introduction of various new contrast agents (e.g. Optison,
Definity, Sonovue, etc.) has facilitated the development of 3-dimensional (3-D) ultrasound imaging systems, which is very helpful in the guidance of biopsy needle. Currently, 3-D ultrasound imaging systems have already been used for the planning of complicated surgeries.

Contrasting to the development of more sophisticated 3-D ultrasound imaging system, there is another trend to develop low-end portable ultrasound imaging systems. Together with the advancement of the communication systems, the diagnosis of ultrasound imaging no longer needs to be performed locally. Remote analysis can be carried out where physicians are stationed while ultrasound scanning can be done by patients themselves using home-based ultrasound system.

The two trends of ultrasound imaging systems (i.e., high-end 3-D ultrasound imaging system and low-end portable ultrasound imaging system) directed the research towards a common goal, which is a low cost and low power beamformer without compromising image quality. The details of the 3-D ultrasound machine, portable imaging system and ultrasound beamformer are discussed in Section 1.2 to 1.4.

1.2 3-D ultrasound imaging systems

Although the development of 3-D ultrasound imaging systems has been investigated for decades, it is receiving more attention in recent years. Three dimensional ultrasound imaging systems not only provide a more accurate way of diagnosis, but also reduce the
examination time. The 3-D technology is a process to build precise 3-D image based on 2-D information, and thus helps the physicians in their diagnosis.

Most of the 3-D ultrasound imaging systems construct 3-D images from a number of 2-D image slices. A 1-D ultrasound transducer array is moved either by free-hand or mechanically to obtain slices of 2-D image for different locations. The final 3-D images are constructed by stacking up the 2-D images. Such technique not only faces the accuracy problem in the positioning of the transducer, it is also too slow to image moving structures such as heart [4], [5].

In order to develop a more accurate real-time 3-D imaging system, large channel count 2-D transducer array needs to be utilized to capture the different slices of 2-D image simultaneously. The number of front-end processing channels needs to be increased as well. Thus, it is important to design simple, low cost, low power but high performance front-end processing circuitry for the realization of a high performance 3-D ultrasound imaging system [5]. This trend constitutes the motivation of this project.

1.3 Portable ultrasound systems

In current hospital environment, expensive, large and heavy ultrasound imaging systems are used on patients. For most cases, the medical workers need to move a bedridden patient if ultrasound scanning is needed. A portable, handheld ultrasound machine would be useful in such a case. Tom Dugan, senior vice president of global marketing and US sales for SonoSite Inc. said that "with hand-carried ultrasound, the goal is not to
replace traditional systems. It is to enable physicians to perform procedures using ultrasound more efficiently and effectively, bringing the system to the patient rather than the patient to the system." [6]

Besides advancing the ultrasound imaging systems for the high-end application, there is also a trend to minimize the ultrasound systems to be portable and battery operated while maintaining reasonable image quality. Such a system will facilitate the archiving, retrieving of images and also remote consultation, which could bring a breakthrough in the medical world. The home ultrasound system proposed by Y. M. Yoo et al. is an example of a portable and battery operated ultrasound imaging system [7]. This home ultrasound system would save the hospitalization cost for mothers with high risk pregnancy. Remote monitoring can be done with such a system at home. Several portable ultrasound machines have been commercialized like Terason t3000, Sonosite 180 Plus, iLook, and My sono. Nonetheless, these systems are restricted to the usage of clinicians and sonographers and their price is still high, greater than USD8000- not affordable for home users.

The primary consideration for portable ultrasound systems is not only small size, but also low cost and low power. As receive beamformer is the most processing intensive part of an ultrasound imaging system, it is the main challenge for size, cost, and power reduction of the whole system [8], [9]. This need also motivates the research interest in this project.
1.4 Ultrasound receive beamformer

Receive beamforming has been shifted extensively into the digital domain due to the processing flexibility and the ease of handling crosstalk, noise problem [10]. The detailed beamforming process is described in Appendix A. However, high speed multi-bit analog-to-digital converters (ADC) and front-end digital circuitries are required to achieve the necessary delay accuracy. To mitigate the high speed requirement of ADCs, interpolation beamformer (IBF) [11], [12] and quadrature demodulation (QD) based phase rotation beamformer (PRBF) [13-15] have been developed and have become common practices. Although IBF and PRBF relieve the sampling frequency of the ADC, computationally demanding finite impulse response (FIR) filter is required in each channel (as interpolation filter for IBF and demodulation filter for QD-based PRBF).

To reduce the hardware complexity of digital receive beamformer, beamforming techniques based on sigma-delta (ΣΔ) ADC were proposed [16], [17]. By directly replacing the multi-bit ADCs with sigma-delta ADCs of similar resolution (including modulators and reconstruction filters), the (pre-delay reconstruction) sigma-delta beamformer (SDBF) can provide comparable image quality to the conventional multi-bit beamformer. However, the hardware is still complex due to the use of a computationally-demanding reconstruction filter in each channel and the number of channels can be from a minimum of 32 for low-end machine up to 128 channels for high-den machine. To alleviate the hardware requirement in the pre-delay reconstruction SDBF, Noujaim et al. developed a post-delay reconstruction SDBF [16], in which one
reconstruction filter is utilized after the delay-and-sum process (instead of one filter in each channel) to recover the modulated signal. The detailed structure of the pre-delay reconstruction SDBF and post-delay reconstruction SDBF is described in Appendix B. This approach significantly reduces the hardware but it was later found to be suffered from dynamic focusing artifacts introduced when samples are repeated during delay-and-sum process. Hence, there is still a need to design a SDBF that is power and hardware efficient without compromising the image quality.

1.5 Organization of thesis

The objective of this research is formed, that is to develop a sigma-delta beamformer (SDBF) that is low cost, low power and with good image quality. The success in developing such a SDBF will provide a cost effective and power efficient front-end solution for the realization of a high end, large channel count 3-D ultrasound imaging system or a low end, low cost, low power portable system. This thesis reports the development of such a SDBF based on theoretical studies that were carried out.

The organization of the thesis is such that the architecture of an ultrasound imaging system, the theory of a sigma-delta modulator, the existing SDBF are reviewed and presented in Chapter 2. Chapter 3 presents the study on the cause of dynamic focusing artifact problem. Chapter 4 describes a proposed SDBF structure - cascaded reconstruction based SDBF, which effectively reduces the dynamic focusing artifacts. Three ways of developing the delay generator (complete lookup table (LUT),
compressed lookup table (LUT), general parameter approach) are also compared in Chapter 4. Chapter 5 describes a power efficient multi-bit SDBF, which is designed based on cascaded reconstruction SDBF but provides a better solution, in terms of both hardware complexity and power consumption, to the dynamic focusing artifact problem. Finally, Chapter 6 concludes the research works done and presents possible future works that can be followed up.

1.6 Contributions

The works of this thesis have been published or submitted in the following papers:


Conference on Electron Devices and Solid-state Circuits 2008, was awarded the
Best Student Paper Award.

  Beamformer Based on Cascaded Reconstruction for Ultrasound Imaging
  Application," accepted for publication in the 3rd International Conference on
  Bioinformatics and Biomedical Engineering 2009.

- B. Yang, J. H. Cheong, Y. Y. H. Lam, and L. S. Ng, "A Comparison of the
  Lookup Table and On-The-Fly Calculation Methods for the Beamforming Control
  Unit," in the 23rd International Technical Conference on Circuits/Systems,

- Y. Deng, J. H. Cheong, Y. Y. H. Lam, and L. S. Ng, "VHDL Implementation of
  Sigma-delta Beamformers for Ultrasound Application," in the 23rd International
  Technical Conference on Circuits/Systems, Computers and Communications,
  2008.
Chapter 2

LITERATURE REVIEW

2.1 Ultrasound system architecture

The basic structure of an ultrasound system is shown in Fig. 2.1[18]. The system consists of a transducer which converts electrical signals to ultrasound signals and vice versa. The transducer can be configured for both transmitting and receiving modes, and it can either be continuous aperture or discrete array. Discrete array transducer is preferred today because of its ability to perform focusing and beam-steering. Linear array, convex array and phase array are used by the most commonly available discrete array transducer.

The transmit path of an ultrasound system comprises a beamforming control unit, a transmit beamformer, and high voltage amplifiers. The beamforming control unit calculates the delay pattern for the beamformer to generate electrical pulses. The transducer elements are then excited by the electrical pulses to trigger ultrasound pulses that focus at different depth according to the delay profile. Theoretically, the transmit beamformer also controls the amplitude of the electrical pulses across the aperture in order to reduce the sidelobe level and to prevent grating lobes. This is known as apodization.
The aperture size of transducer can be varied with the focal depth in order to achieve uniform point spread function, and this is known as dynamic aperture. A parameter, $f$-number, which is the ratio of the focal length to the size of the aperture, was found to be 2 to provide optimal focusing performance [19]. A high voltage amplifier is needed in each channel to provide high voltage signal for transducer element excitation. At the same time, the amplifiers shape the electrical pulses to increase the transmission efficiency.

The front-end of an ultrasound receiver can consist of two different paths. One path is utilized for Doppler mode (or D-mode) scanning while the other path is for brightness
mode (or B-mode) and color flow imaging (or F-mode) scanning. In the case of D-mode scanning, an additional Continuous Wave (CW) analog beamformer is needed because the Doppler signal strength is very low. To date, there is no such a high speed ADC (eg. 150MHz) commonly available that is able to provide 14-bit resolution so that the Doppler signal can be processed digitally [20]. The calculation of the resolution can be found in Appendix C.1. Digital beamformers can be used for B-mode and F-mode scanning because of their stability, reliability and the reduction in cost of digital circuitry [21]. Time gain compensation (TGC) amplifiers are placed before the beamformer to amplify the signal with increasing gain when the firing time is longer, so that the path loss is compensated. The digital beamformer then delays the received signal from each channel according to the respective delay pattern calculated by the beamformer control unit. Similarly with transmit beamforming, apodization and dynamic aperture can be executed as well, which applies different weights to the ultrasound signal transmitted by each transducer array element. It gradually decreases the weights of the transducer elements with the distance from the transducer center. The purpose is to reduce the side lobes of the transmitted beam.

Aperture size of the transducer can be varied as well with the focal depth in order to achieve homogenous signal strength and uniform point spread function. This is known as dynamic aperture, which is commonly used in beamforming. The transducer aperture is gradually opened up from the central receive processing channels according to $f$-number, which is the ratio of the instant focal depth over the size of the aperture as defined in (2.1)
\[
f_{\text{num}} = \frac{d_f}{(N-1)\Delta x}
\]  
(2.1)

where \(d_f\) is the instant focal depth, \(N\) is the number of active channels and \(\Delta x\) is the pitch between adjacent transmitting elements on the transducer array.

It has been found that optimal focusing performance can be achieved when the \(f\)-number is greater than or equal to 2 \[19\]. In other words, when the focal depth increases, the number of active channels is gradually increases to ensure that the instant focal depth is always 2 times greater than or equal to the width of the opened aperture, until all the channels have been activated.

2.2 Acquisition modes

Since the use of ultrasound in medical diagnosis in the late 1940s, several acquisition modes have been developed for different purposes. The amplitude mode (or A-mode) scanning was first used for eye examination and for echoencephalography. The amplitude of the signal is displayed against the depth of the scanned object as spikes. Only one-dimensional information is conveyed. The information obtained by this mode is useful for tissue characterization \[19\]. The interfaces of different tissues can be produced when the ultrasound signal is being reflected by them.

The three main modes for contemporary ultrasound imaging application are B-mode, D-mode and F-mode which are shown in Fig. 2.1. Unlike A-mode scanning, B-mode scanning displays the strength of the reflected echoes as the brightness of the pixels. The
scanning beam can be steered to cover across a 2-D area. Thus, this scanning mode is usually used to obtain 2-D grey scale image.

Ultrasound can also be used for color-flow imaging (F-mode) which displays blood flow as colors overlaying on the B-mode display. On the other hand, Spectral Doppler (D-mode) indicates the blood flow velocity and frequency [18]. Over the decades, the application of ultrasound imaging system has been extended greatly from obstetric sonography to treatment in stroke sufferers, acoustic targeted drug delivery and even elastography on soft tissue to detect tumors.

2.3 Discrete array transducer

There are 5 different transducer array schemes available and each having a different elevation resolution, which is the ability to resolve adjacent structures in a plane perpendicular to the image plane [22]:

1-D: One dimensional dynamic focusing. Elevation aperture is fixed and focal length is fixed.

1.25-D: Elevation aperture is variable but focusing is static.

1.5-D: Elevation aperture, shading, focusing are dynamically variable, but symmetric about the centre line.

1.75-D: Similar to 1.5-D without the constraint of symmetry. Steering angle is limited.

2-D: Full apodization, focusing, steering in elevation direction.
Transducer can also be classified into linear, convex and phased array. Linear array has its elements arranged in a straight row, as shown in Fig. 2.2(a). A group of elements are fired at the same instance to form a scanline. It is then shifted by one element at a time across the entire transducer to scan over the area covered by the transducer. The operation of convex array is the same as linear array. The only difference is that the elements of a convex array are arranged in a curvilinear shape, as shown in Fig. 2.2(b).

Fig. 2.2(c) shows the scanning area of the phased array transducer. The element arrangement is the same as linear array but all the elements are fired according to a certain timing pattern to form a scanline at a certain steering angle. The area covered by a phased array is wider than that of a linear array with the same number of elements. However, more complex beamforming control unit is required for phase array transducers.

Fig. 2.2. (a) Linear array transducer, (b) convex array transducer and (c) phased array transducer.
2.4 Receive beamforming

2.4.1 Beamforming principle

During transmission, focusing can provide better resolution for ultrasound imaging. Ultrasound signal transmitted by different elements can be directed to arrive at a certain focal point simultaneously, so that the total signal strength at that focal point is their coherent sum. This can be achieved mechanically by arranging the transducer elements in a certain shape, or electronically by delaying the firing time of the elements with certain pattern. The delay pattern can be calculated according to the distance from the transducer to the focal point. This focusing process is known as beamforming. The delay pattern for beamforming can be calculated according to the geometric diagram shown in Fig. 2.3.

![Fig. 2.3. Geometric diagram from delay calculation.](image)

From Fig. 2.3, the distance between the \( n^{th} \) element and the focal point can be calculated as below:
\[ R_n' = \sqrt{x_n^2 + R^2 - 2x_n R \cos(90^\circ - \theta)} , \quad (2.2) \]

\[ R_n' = \sqrt{x_n^2 + R^2 - 2x_n R \sin \theta} , \quad (2.3) \]

where \( x_n \) is the distance from centre of the scanline to the \( n^{th} \) element, \( R \) is the distance from the transducer to the focal point, \( \theta \) is the steering angle.

Using Taylor series expansion, letting \( f(x_n) \) to represent the function \( R_n' \), then

\[ f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \ldots \]

\[ f'(x_n) = \frac{x_n - R \sin \theta}{\sqrt{x_n^2 + R^2 - 2x_n R \sin \theta}} \Rightarrow f'(0) = -\sin \theta \]

\[ f''(x_n) = \frac{\left( x_n - R \sin \theta \right)^2 - \left( x_n^2 + R^2 - 2x_n R \sin \theta \right)}{x_n^2 + R^2 - 2x_n R \sin \theta} \Rightarrow f''(0) = \frac{\cos^2 \theta}{R} . \quad (2.6) \]

Higher order terms can neglected if \( x_n \) is small compared to \( R \), which is true when dynamic aperture is applied. Hence,

\[ R_n' \approx R - x_n \sin \theta + \frac{x_n^2 \cos^2 \theta}{2R} . \quad (2.7) \]

The traveling time can then be calculated by dividing the distance \( R_n' \) with the speed of sound \( c \) in the medium.

\[ t_n' \approx \frac{\left( R - x_n \sin \theta + \frac{x_n^2 \cos^2 \theta}{2R} \right)}{c} . \quad (2.8) \]
Transmit focusing is achieved by providing the relative time delay according to (2.7) when firing the transducer elements. To avoid signal interference, the transducer elements can only be fired again after the previously fired ultrasound signal has dispersed. For this reason, transmit focusing is normally static focusing. Each scanline is obtained by one firing only, for one focal point, so that the frame rate will not be limited by multiple firing in the case where dynamic focusing is applied.

On the other hand, receive focusing can be performed dynamically. Focusing can be done at different depths. Theoretically, the receive beamformer inserts the necessary time delay to each channel in order to compensate the arrival time difference as calculated by (2.7). In practice, the receive beamformer realizes this time delay by selecting the received samples stored in a temporary storage according to the time delay pattern from (2.7). By doing so, the signal received from the focal point is coherently summed. Noise from locations other than the focal point will be partially or even completely cancelled off. The resultant effect is that the coherently summed signal power will dominate over the noise power.

The generation of the delay can be done by pre-calculating it and storing it in a lookup table (LUT), or on-the-fly calculation. The detailed hardware comparison between the LUT and on-the-fly calculation is presented in Section 4.4.

2.4.2 Conventional digital receive beamforming

Due to the advancement of technology, the cost of designing a digital circuitry is reducing. Hence, analog beamformer has gradually been replaced by digital beamformer
as there are a numbers of advantages that digital systems have over analog systems. Digital systems have higher precision, stability and reliability compared to analog systems [21]. Crosstalk problem can be handled more easily in digital domain without the supply voltage limitation. Digital system is also more flexible in terms of its functionality. Thus, ultrasound scanners using digital beamformer are programmable and more compact [23], [24]. Apodization can also be applied more accurately as well. Digital beamformers can even perform complex signal processing with lower power dissipation, which is of increasing importance [25].

A typical digital receive beamformer structure is shown in Fig. 2.4.

![A typical digital receive beamformer system.](image)

The received ultrasound echoes are digitized by the ADCs and then stored in a temporary storage (or first-in-first-out (FIFO) memories). The length of the temporary
storage is variable depending on the delay information used to perform dynamic focusing.

Several conventional digital beamforming methods have been developed. They can be classified into time domain and frequency domain techniques [15].

2.4.2.1 **Time domain approach**

Time domain beamforming techniques include delay-sum, partial-sum, interpolation, and shifted-side-band methods which are presented below.

2.4.2.1.1 **Delay-and-sum beamformer**

Delay-and-sum beamformer, as its name implies, performs the focusing by delaying the samples and then sum them up across the channels. The minimum delay resolution is one sampling period. Thus, high sampling rate is necessary to achieve sufficient delay resolution. Large storage and large connection bandwidth between ADCs and beamformer are needed because of the amount of data transmitted between them. The computation of apodization and summation can generally operate at Nyquist rate. It is the most direct and simple beamforming technique.

2.4.2.1.2 **Partial sum beamformer**

Partial sum beamformer predefines memories for beamformed output storage. Every beam output sample is the summation of the delayed samples across the channels. It is different from delay-and-sum in that its output is not obtained at one shot. Instead, the beam output is updated every time when the required samples from a particular focal point are received. Partial sum beamformer does not require long memory to store the
received echo samples. The received samples are added directly to the corresponding beam output memory once they are received. Hence, if the number of elements is larger than the number of beams and the beam output sampling rate is lower than the input sampling rate, the memory size required will be reduced. Although partial sum reduces input data storage, high input sampling rate (usually more than 100MHz) is still needed to provide sufficient delay resolution.

2.4.2.1.3 Interpolation beamformer

Interpolation beamformer upsamples the received echoes to obtain the necessary delay resolution. It can thus reduce both the input data storage and the input sampling rate, therefore the design of ADC circuitry and the connection bandwidth are relieved. Since beamforming and interpolation filtering are linear, the sequence of them is interchangeable. The appropriate placement of the interpolation filters depends on the beamforming scenario (if the number of beam is greater than the number of elements, pre-beamforming interpolation will be less computationally expensive) as well as the choice between partial sum and delay sum.

2.4.2.1.4 Shifted sideband beamformer

Shifted sideband beamformer translates the signal to lower frequency [26]. This results in a reduction of the required input sampling rate, the area of ADCs, data transmission bandwidth and input data storage. Nevertheless, there is grating lobe phenomenon which occurs for certain array geometries, e.g. linear array when small groups of adjacent transducer element receive the same quantized steering delay.
2.4.2.2  **Frequency domain approach**

Frequency domain beamforming techniques include Discrete Fourier Transform (DFT) and phase-rotation (phase-shift) beamforming which are presented below.

2.4.2.2.1  **Discrete Fourier Transform beamformer**

Discrete Fourier Transform (DFT) beamformer accomplishes the delay task by multiplying exponential phase offset to the frequency domain input signals. Different phase offsets are applied to different input frequencies. Generally the Inverse Fourier Transform (IFT) is not required because the frequency domain beam output representation is highly suitable for post-beamforming processing.

2.4.2.2.2  **Phase shift beamformer**

Phase shift beamformer is similar to DFT beamformer except that it assumes the received echo to have only single frequency component. Therefore, Fourier Transform is not required to find the different frequency components. Phase correction exponential term can be directly multiplied to the time domain input signal.

Both DFT and phase shift beamformer using frequency domain approach eliminate the need for high input sampling rate. Larger amount of input data storage is generally required for DFT. Although phase shift beamformer requires a relatively small amount of circuitry, good results are achieved for only narrow frequency bands.

Detailed description of all the beamforming techniques mentioned above can be found in [15]. They all need extra processing to provide sufficient delay resolution except the delay-and-sum and partial sum techniques which achieve the delay resolution by high
input sampling rate. However, there is currently no such multi-bit ADCs commercially available that can operate at high sampling rate (e.g. 175MHz as calculated in Appendix C.2) for delay-and-sum and partial sum beamforming techniques.

In addition, conventional digital beamformer needs a multi-bit ADC in every receive channel. If the number of channels is large, then the number of multi-bit ADCs required becomes impractical in terms of hardware complexity, cost and power consumption [27]. It also hinders single-chip solution for digital receive beamformers.

### 2.5 Sigma-delta receive beamforming

Sigma-delta beamformer (SDBF) was first developed and patented by General Electric Company in 1993 with the objective to simplify beamforming circuitry without sacrificing the image quality. The simplified structure of this post-delay reconstruction SDBF is shown in Fig. 2.5.

![Fig. 2.5. Post-delay reconstruction sigma-delta beamformer.](image-url)
For the SDBF system shown, the received ultrasound echoes go through the TGC before they are converted to digital data by the sigma-delta modulators. The digital outputs from the modulators are then stored in delay registers which can either be shift registers or first-in-first-out (FIFO) memories. Delay control signals, generated either by real-time calculation or LUT, select the samples from the delay registers to the adder. A reconstruction filter is then used to reconstruct the signal of the summed output. The reconstruction of post-delay reconstruction SDBF is done after the delay-and-sum process. There is only one reconstruction filter needed for the whole system. In contrast, pre-delay reconstruction beamforming needs a reconstruction filter in each beamforming channel.

Post-delay reconstruction beamformer has less hardware requirement than the pre-delay reconstruction beamformer. However, there is a problem inherently suffered by the post-delay reconstruction beamforming. When dynamic focusing is performed to obtain better image quality, delay is updated every sampling period. There are times that the same sample would be selected repeatedly when the quantized time delay remains unchanged. This sample repetition introduces unexpected artifact problem in the final image produced by post-delay reconstruction SDBF, which is not addressed by General Electric patent [16]. The white speckles at the background of Fig. 2.6(b) are the resultant artifacts.
For a single-bit sigma-delta modulator, the higher the density of +1 present in the digital output, the higher the value of its reconstructed value will be. Since the value of the reconstructed digital output does not depend only on a single sample but also on the neighbouring samples, any sample insertion to sigma-delta modulator output will cause amplitude distortion in addition to signal stretching. Hence, when dynamic focusing is performed, post-delay reconstruction SDBF will produce artifacts. Such problem does not exist in multi-bit ADC beamforming, which will be discussed in Chapter 3. Repeating a sample is feasible in such case. Several techniques have been proposed to handle the dynamic focusing artifact problem suffered by SDBF, and they are discussed in the following sections.

2.5.1 Insert zero, divide-by-two and compensated SDBF

Freeman et al. brought up this dynamic focusing issue for sigma-delta post-delay reconstruction beamforming in 1999 [17], [24], [28]. Three methods were proposed to correct the artifacts: insert-zero, divide-by-two, and compensated sigma-delta modulator. For all these 3 methods, extra logic circuitries have to be introduced to detect sample repetition so that the artifacts can be corrected.
Insert-zero method replaces the repeated samples with zeros, whenever repetition is detected. On the other hand, the sample is first divided by two before it is repeated for divide-by-two method. The principle of these two methods is to avoid inserting extra energy to the sample stream so that the average value of the sample stream will not be altered. They are illustrated in Fig. 2.7. The extra samples inserted are circled by the dotted line.

The third method, compensated sigma-delta modulator, doubles the feedback signal of the modulator whenever a sample is being repeated. Thus, the extra energy injected due to sample repetition is taken into account by the modulator.
All these 3 methods have limited performance. They cannot correct the artifact problem when the delay is updated more frequently at near field.

2.5.2 Insert +1 -1 method

Following the proposal of Freeman et al., Rigby developed a patent to further improve the insert zero method [29]. As a zero level is introduced, insert-zero method requires an extra bit during the summation of the signals from all the channels. To avoid the extra bit, Rigby inserts +1 and -1 instead of zero when a sample is to be repeated. However, the insertion of two samples at a time degrades the delay resolution to twice the sampling period. Another way suggested in the patent is to insert +1 in one channel and -1 in the complementary channel so that the sum of them produces the same result as inserting zeros. This method does not suffer from delay resolution degradation.

The same technique was published by Li et al. in the same year when the patent was filed [30]. In addition, Rigby also proposed a method that always inserts +1 when a sample is to be repeated. The number of +1 is counted and then subtracted during the beam-summation stage. Equivalent result as insert-zero method is obtained.

Just as insert-zero method, the insert +1, -1 method is not able to correct the artifact properly when the samples are repeated more frequently at near field.

2.5.3 Non-uniform sampling method

Instead of performing the sample selection after sigma-delta modulation, there is another way to delay the signal in the analog domain. Non-uniform sampling samples the analog signal of each channel according to the corresponding delay profile before sending them
to the sigma-delta modulators. As a result, the input to the modulator of each channel is actually the delayed version. However, if a sample is to be repeated, the clock frequency of the sampling circuit has to be doubled in order to obtain that extra sample. Such a high frequency is difficult to realize in practice. Karaman et al. suggested a way to avoid sample repetition while using non-uniform sampling [21], [31], [32]. Only sample dropping is needed in this case, which can easily be realized by deactivating the sampling clock when the sample is to be dropped.

Non-uniform sampling beamformer uses two different clock frequencies for sampling and delay update. As shown in (2.7), the delays can be calculated as

\[
    t_{n1} \approx \frac{R - x_n \sin \theta + \frac{x_n^2 \cos^2 \theta}{2R}}{c}
\]  

(2.9)

and

\[
    t_{n2} \approx \frac{R + \Delta R - x_n \sin \theta + \frac{x_n^2 \cos^2 \theta}{2(R + \Delta R)}}{c}
\]  

(2.10)

for two consecutive delay update instant. The difference between them is

\[
    \Delta t \approx \frac{2}{c} \left[ \Delta R - \frac{x_n^2 \cos^2 \theta}{2} \frac{\Delta R}{R(R + \Delta R)} \right]
\]  

(2.11)

The minimum of this difference occurs when

\[
    \Delta t_{\text{min}} \approx \frac{2}{c} \left[ \Delta R - \frac{x_{\text{max}}^2}{2} \frac{\Delta R}{R(R + \Delta R)} \right] = \frac{1}{f_{\text{nr}}} \left[ 1 - \frac{x_{\text{max}}^2}{2} \frac{1}{R(R + \Delta R)} \right]
\]  

(2.12)
In order to avoid sample repetition, the sampling period has to be smaller than the minimum delay difference. Thus,

$$\frac{1}{f_s} \leq \frac{1}{f_{BF}} \left[ 1 - \frac{x_{\text{max}}^2}{2} \frac{1}{R(R + \Delta R)} \right]$$  \hspace{1cm} (2.13)

$$\frac{f_s}{f_{BF}} \geq \frac{1}{\left[ 1 - \frac{x_{\text{max}}^2}{2} \frac{1}{R(R + \Delta R)} \right]}$$  \hspace{1cm} (2.14)

By assuming $R \gg \Delta R$, (2.14) can be further simplified to

$$\frac{f_s}{f_{BF}} \geq \frac{8f_{\text{num}}^2}{8f_{\text{num}}^2 - 1}$$  \hspace{1cm} (2.15)

where $f_{\text{num}}$ is the $f$-number introduced in Section 2.1. If the $f$-number is always kept to the optimal value of 2, the sampling frequency needs to be only 1.04 times of the beamforming frequency to avoid sample repetition.

However, this technique needs to have a second master clock to control the beamforming frequency. It is difficult to accurately design two clocks whose frequencies are not the multiple of each other.

2.5.4 Block-based sigma-delta beamforming

The dynamic focusing artifacts mentioned above are present only in post-delay reconstruction beamforming. A direct way to avoid it is by using pre-delay reconstruction beamforming. Intuitively, the hardware cost seems to increase a lot as a reconstruction filter is needed for each channel. However, due to the 1 bit property of
the digital signal in every channel, the reconstruction filter can be realized using only accumulator. Multipliers are not essential. Han et al. proposed a simpler hardware realization for pre-delay reconstruction type beamforming [33-35], known as block-based SDBF. A block of samples is first selected according to the delay profile and then reconstructed with filter of equal length. The filter is realized using an accumulator which adds or subtracts the filter coefficients as illustrated in Fig. 2.8.

Supposed the number of samples in each block is \( L \), the operating frequency of the accumulator is \( Mf_s \), and the output data rate of the accumulator group is decimated to \( f_s \) which has to be greater than Nyquist rate, then the number of accumulators needed for each channel, \( N \), will be \( L/M \). \( L \) is normally quite large as high performance filter is needed to get rid of the high frequency quantization noise of sigma-delta modulator output. Hence, large number of accumulators is required for each channel.

Fig. 2.8. Block-based sigma-delta beamformer.
2.6 Summary

The comparison among all these methods discussed is summarized in Table 2.1. The insert-zero, divide-by-two, compensated sigma-delta modulator and non-uniform oversampling methods need only simple hardware implementation to correct the dynamic focusing artifact problem. However, the correction performance is limited. The first 3 methods still suffer from the artifact problem when the delay update is frequent especially at the near field, whereas the noise floor of the non-uniform sampling method is higher than other methods.

The only correction technique that can provide complete dynamic focusing artifact correction is the block-based method. However, extra adders and filter coefficient memory needed are still hinders to single-chip integration.
Table 2.1. Comparison of delta-sigma dynamic focusing artifact correction schemes.

<table>
<thead>
<tr>
<th></th>
<th>Insert zero (or 1 -1)</th>
<th>Divide-by-two</th>
<th>Compensated SDM</th>
<th>Non-uniform oversampling</th>
<th>Block-based SDBF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Image Quality</strong></td>
<td>Artifacts at near field</td>
<td>Better than all except block-based SDBF</td>
<td>Artifacts at near field</td>
<td>Slightly higher noise floor</td>
<td>Complete correction</td>
</tr>
<tr>
<td><strong>Control logic</strong></td>
<td>Additional logic to determine sample repetition</td>
<td>Additional logic to determine sample repetition</td>
<td>Additional logic to determine sample repetition</td>
<td>Dynamic aperture</td>
<td>Additional logic to select accumulators and sign of filter coeff.</td>
</tr>
<tr>
<td><strong>Clock frequency</strong></td>
<td>1 master clock frequency</td>
<td>1 master clock frequency</td>
<td>1 master clock frequency</td>
<td>2 master clock frequencies</td>
<td>1 master clock frequency</td>
</tr>
<tr>
<td><strong>Hardware</strong></td>
<td># 2-bit input adder for insert zero</td>
<td># 3-bit input adder</td>
<td># Additional amplifier in modulator feedback loop # Additional multiplexer in modulator feedback loop</td>
<td># Additional hardware imposed by extra master clock # No other additional hardware except LUT for delay profile</td>
<td># Accumulators, multiplexers are used for each channel # Longer delay registers # No multipliers needed for filtering # Multi-bit adder works at lower frequency</td>
</tr>
<tr>
<td><strong>Pros</strong></td>
<td>Simple hardware</td>
<td>Simple hardware</td>
<td>Simple hardware</td>
<td>No additional processing hardware required</td>
<td>Complete artifact correction</td>
</tr>
<tr>
<td><strong>Cons</strong></td>
<td>Artifacts at near field</td>
<td>Artifacts at near field</td>
<td>Artifacts at near field</td>
<td>Slightly higher noise floor</td>
<td>More complicated hardware</td>
</tr>
</tbody>
</table>
Chapter 3

DYNAMIC FOCUSING ARTIFACTS

Due to the high sampling frequency of SDBF, sufficient delay resolution can be achieved directly with delay-and-sum approach. Delay-and-sum beamforming method selects a sample in each channel according to the quantized delay profile. A dynamic delay is realized by repeating a sample. However in SDBF, when some sigma-delta modulator output samples are repeated, signal cannot be reconstructed properly and artifacts will be introduced in the final image. The author did simulations by introducing sample repetition to show the artifacts due to dynamic focusing. This effect was shown in Fig. 2.6 where Fig. 2.6(a) and (b) show a point phantom with and without dynamic focusing artifacts, respectively.

In order to find out the cause of the dynamic focusing artifacts, the operation of the sigma-delta modulator was studied and it is presented in Section 3.1. Following that, the effect of sample repetition upon sigma-delta modulated signal is analyzed in Section 3.2. The analysis shows that when samples are repeated, some amount of the out-of-band noise present in the original signal is shifted into the signal band, which causes the artifact problem. Conventional multi-bit beamformer does not suffer from dynamic focusing artifact problem because its out-of-band quantization noise level is low
whereas sigma-delta modulated signal contains large quantization noise in high frequency band.

3.1 Sigma-delta modulator

Sigma-delta modulator is able to achieve high signal-to-quantization noise ratio (SQNR) using low bit resolution by oversampling the input analog signal and noise shaping. The short bit-length output allows simpler beamforming hardware realization and integration. The high sampling rate is able to provide high delay resolution as well as high SQNR.

Single-bit 2\textsuperscript{nd}-order sigma-delta modulator has been generally selected for ultrasound beamforming application [24], [30] because any single-bit sigma-delta modulators higher than 2\textsuperscript{nd}-order are not inherently stable.

3.1.1 Sinusoidal input

In practice, a single-bit 2\textsuperscript{nd}-order sigma-delta modulator is implemented as shown in Fig. 3.1.

![Practical single-bit 2\textsuperscript{nd}-order sigma-delta modulator.](image)
The two delay integrators in this structure can be easily realized with switched capacitor circuits [36]. The signal transfer function (STF) and the noise transfer function (NTF) of such structure is shown in (3.1).

\[ Y(z) = z^{-2}U(z) + (1 - z^{-1})^2 E(z) \]  \hspace{1cm} (3.1)

where \( Y(z) \) is the output, \( U(z) \) is the input, and \( E(z) \) is the quantization error.

Signal-to-quantization noise ratio (SQNR) of a multi-bit 2\textsuperscript{nd}-order sigma-delta modulator can be calculated using the equation:

\[ SQNR = (6.02N_b + 1.76 - 12.9 + 50\log OSR) \text{ dB} \] \hspace{1cm} (3.2)

where \( N_b \) is the number of bits, \( OSR \) is the oversampling ratio [37]. The expression is derived based on the assumptions that

(i) the probability of quantization noise is uniformly distributed from \(-\text{LSB}/2\) to \(+\text{LSB}/2\) (without overloading the quantizer),

(ii) the quantization noise is white noise,

(iii) input signal is a full scale sine wave,

(iv) oversampling ratio (OSR) \( \gg 1 \) [38-40].

On the other hand, the SQNR for single-bit 2\textsuperscript{nd}-order sigma-delta modulator is

\[ SQNR = (1.76 - 12.9 + 50\log OSR) \text{ dB} \] \hspace{1cm} (3.3)
The expression is without the $6.02N_b$ term because the noise is distributed between $-\text{LSB}$ to $+\text{LSB}$ in this case, which is different from the assumption (i). Simulation using MATLAB 7.0 has shown a compatible result with the theoretical calculation as shown in Fig. 3.2. The SQNR degradation at large input level is due to the non-linear gain of the quantizer.

![Fig. 3.2. Output SQNR vs. input level of a single-bit 2nd-order sigma-delta modulator with OSR=16.](image)

While the compatibility of the simulation result has implied that the theoretical equation is valid for sinusoidal input despite the assumption that the quantizer is not overloaded is violated [40], the validity of the equation for ultrasound input will be examined in the next section. To the author’s knowledge, verification of the theoretical calculation for wideband ultrasound signal (whose relative bandwidth, the ratio of bandwidth to 2 times center frequency, is greater than 0.01) has not been done in spite of the extensive study using single-tone input signal.
3.1.2 Ultrasound input

Signal-to-quantization noise ratio (SQNR) for wideband input signal cannot be obtained simply from the output spectrum of the modulator because the signal power is not a single frequency component. Instead, it spreads over a wide range of frequency, causing the calculation of noise power over that of the frequency range to be impossible. Hence, the signal and noise power are obtained separately.

Suppose \( y(t) = u(t) + e(t) \), where \( y(t) \) is the output, \( u(t) \) is the input signal, and \( e(t) \) is the quantization error. Power of the output

\[
P_y = \frac{1}{T} \int_{T/2}^{T} y^2(t) dt = \frac{1}{T} \int_{T/2}^{T} u^2(t) dt + 2u(t)e(t) + e^2(t) dt
\]

If the term \( \frac{1}{T} \int_{T/2}^{T} 2u(t)e(t) dt \) is equal to zero, then the output power will be equal to the summation of input signal power and the noise power. The magnitude of this correlation term can be expressed in terms of correlation coefficient.

Fig. 3.3. Ultrasound signal with centre frequency at 3.5MHz and fractional bandwidth of 0.6.
Ultrasound signal shown in Fig. 3.3 with centre frequency at 3.5MHz and fractional bandwidth of 0.6 is used as the input signal to the single bit 2\textsuperscript{nd}-order sigma-delta modulator in the simulation. With an OSR of 16, the correlation coefficient, $r_{cor}$, between the input signal and the quantization error was calculated as

$$
r_{cor} = \frac{R_{we}}{\sqrt{R_u \times R_e}} = 0.003
$$

(3.5)

where $R_{we} = \sum_{i=1}^{N_S} u[n]e[n]$, $R_u = \sum_{i=1}^{N_S} u^2[n]$, $R_e = \sum_{i=1}^{N_S} e^2[n]$, and the number of samples $N_S$ is 243 which is over one period of the signal. As the correlation coefficient is small, the output power of the sigma-delta modulator can be approximated as the summation of input signal power and quantization noise power.

3.1.3 Simulation results

Time domain quantization error was obtained by

$$
e(t) = y(t) - u(t),
$$

(3.6)

and Fig. 3.4 shows the power spectra of $e(t)$ and $u(t)$ with input peak value of -4dBFS.
Signal-to-quantization noise ratio (SQNR) with OSR of 16 and 32 are depicted in Fig. 3.5, where OSR in this case is the ratio of the sampling frequency to twice the highest frequency component of the signal.

Fig. 3.5. SQNR vs. input level of a single-bit 2nd-order sigma-delta modulator with ultrasound input signal.
The simulated SQNR versus peak signal level of the ultrasound input signal presented in Fig. 3.5 tallies with the theoretical calculation of

\[
SQNR = 1.76 - 12.9 + 50 \log OSR + 10 \log(S / 0.5),
\]

(3. 7)

where the 0.5 is the power of a full scale sinusoidal waveform, S is the ultrasound signal power. There exists similar SQNR degradation when the input peak level approaches full scale value for both OSR of 16 and 32. The required OSR for a 2\textsuperscript{nd}-order sigma-delta modulator with ultrasound input signal can be estimated using the theoretical equation as well. With an OSR of 16, a 2\textsuperscript{nd}-order sigma-delta modulator achieves an equivalent number of bits (ENOB) that is better than 7.

3.1.4 Challenges

Sigma-delta modulators are more robust against analog circuit mismatches [21] but they require oversampling to achieve sufficient signal resolution. Therefore, they are normally employed for low frequency and high resolution application. The incorporation of sigma-delta modulators in ultrasound beamformer is a high frequency, low resolution application. It violates the usual application of sigma-delta modulators in exchange for the ease of signal synchronization among the beamforming channels. Although sigma-delta modulators underperform in beamforming application, two basic aims are achieved: sufficient delay resolution and sufficient signal resolution.

Due to the high frequency of ultrasound signal for medical imaging which is centered at 3.5MHz with fractional bandwidth of 0.6, the required sampling rate of the 2\textsuperscript{nd}-order sigma-delta modulator may be as high as 160MHz for an OSR of 16. To the author’s
knowledge, the highest speed sigma-delta modulators available commercially are 
ADS1610 and AD9860/9862 with sampling rate of 60MHz and 64MHz respectively. 
Although there are only a few research papers addressing the design of CMOS sigma-
delta modulators which can operate at frequency higher than 100MHz with tested results 
[41-43], nevertheless along with the development of micro-electronic technology, the 
design of a high speed sigma-delta modulator for ultrasound imaging application can be 
envisaged.

3.2 Sample repetition

During dynamic focusing beamforming process, samples will be repeated according to 
the delay profile. When a discrete time signal goes through a sample repetition process, 
the signal is delayed and stretched at the same time. If the samples are repeated at a 
constant sample interval, \( m \), the signal can be expressed as

\[
X(z) = \sum_{n=0}^{m-1} x[n]z^{-n} + x[m-1]z^{-m} + \sum_{n=m}^{2m-1} x[n]z^{-(n+1)} + x[2m-1]z^{-(2m+1)} + \sum_{n=2m}^{3m-1} x[n]z^{-(n+2)} + x[3m-1]z^{-(3m+2)} + \ldots 
\]  
(3.8)

\[
X(z) = \sum_{k=0}^{N/m-1} \sum_{n=km}^{(k+1)m-1} \{x[n]z^{-(n+k)} + x[(k+1)m-1]z^{-(m+k)}\} 
\]  
(3.9)

\[
X(z) = \sum_{k=0}^{N/m-1} \{x[(k+1)m-1]z^{-(k+1)m+k}) + x[(k+1)m-1]z^{-(m+k)} \} 
\]  
(3.10)
\[ X(z) = \sum_{k=0}^{N/m-1} z^{-k} \{ x[(k + 1)m - 1]z^{-(k + 1)m} + \sum_{n=km}^{(k+1)m-1} x[n]z^{-n} \} \quad (3.11) \]

Assuming that \( x[k-1] \approx x[k] \approx x[k+1] \), this is true when oversampling is applied.

Then

\[ X(z) \approx \sum_{k=0}^{N/m-1} z^{-k} \{ x[(k + 1)m]z^{-(k + 1)m} + \sum_{n=km}^{(k+1)m-1} x[n]z^{-n} \} \quad (3.12) \]

\[ X(z) \approx \sum_{k=0}^{N/m-1} z^{-k} \{ \sum_{n=km}^{(k+1)m-1} x[n]z^{-n} \} \quad (3.13) \]

\[ X(z) \approx \sum_{k=0}^{N/m-1} z^{-k} \{ \sum_{p=0}^{m} x[km + p]z^{-(km+p)} \} \quad (3.14) \]

\[ X(z) \approx \sum_{k=0}^{N/m-1} \sum_{p=0}^{m} x[km + p]z^{-(km+p)} \quad (3.15) \]

Let

\[ l = k(m + 1) + p \quad (3.16) \]

Then

\[ X(z) \approx \sum_{l=0}^{N} x[l-k]z^{-l} \quad (3.17) \]

\[ X(z) \approx \sum_{l=0}^{N} x[l - \left\lfloor \frac{l}{m+1} \right\rfloor]z^{-l} \quad (3.18) \]

\[ X(z) \approx \sum_{l=0}^{N} x[(m \cdot \frac{l}{m+1}) + \text{rem}(\frac{l}{m+1})]z^{-l} \quad (3.19) \]
As the remainder of \( l \) divided by \( m+1 \) is always smaller than 1, hence

\[
X(z) \approx \sum_{l=0}^{N} x\left(\frac{m}{m+1}l\right)z^{-l} \tag{3.20}
\]

The spectrum of the repeated sample signal as described by (3.20) is like the spectrum of the original signal being compressed to a frequency band that is \( \frac{m}{m+1} \) of the original fundamental spectrum and it would be repeated every cycle of the sampling frequency \( f_S \). In other words, the noise at \( if_S \) (where \( i \) is an integer and \( 0 \leq i \leq m+1 \)) would be shifted to DC. For example, if the samples are repeated every 3 sample interval, noise at \( \frac{1}{4}f_S, \frac{2}{4}f_S, \frac{3}{4}f_S, \) and \( f_S \) would be shifted to DC. This is illustrated in Fig 3.6, where Fig 3.6(a) shows the original spectrum before repetition and Fig 3.6(b) shows the spectrum after repetition every \( m \) samples. Such a process can also be modeled as non-uniform sampling process discussed in [44], where the period of the non-uniform sampling is \( m+1 \). Spurious spectra will be created at integer multiple of \( \frac{f_s}{m+1} \).
Conventional multi-bit ADC beamformer does not suffer from dynamic focusing artifact because the quantization noise level is low throughout the whole spectrum. On the other hand, sigma-delta modulated signal contains large amount of quantization noise at high frequency band. After signal repetition, this quantization noise would be shifted into the signal band.

A set of simulation was done to demonstrate the noise shifting of sigma-delta modulated signal during sample repetition. A sinusoidal signal at 3.5MHz was digitized by a single-bit 2\textsuperscript{nd} order sigma-delta modulator at 160MHz and it is passed through a low pass filter.
with cutoff frequency at \( \frac{f_s}{m+1} \), where \( m \) is the sample repetition interval that is applied on the signal. The spectrum of the signal after going through the low pass filter is shown in the dotted lines whereas the spectrum after repeating by \( m \) sample interval is shown in the solid lines of Fig. 3.7.

As shown, when the samples are repeated at a constant rate, it is possible to band limit the signal before hand to avoid signal corruption. However, when samples are repeated dynamically, noise at different frequency band would be shifted into the signal band and corrupt the signal. Hence, by reducing the out-of-band noise of the pre-delay signal, the dynamic focusing artifacts can be reduced.
Fig. 3.7. Signal spectra before and after sample repetition when samples are repeated every (a) 2 samples, (b) 3 samples, (c) 10 samples.
3.3 Time domain analysis

Sample repetition inserts extra noise, which is not properly shaped by the noise shaping function. A set of simulations was carried out to study the effect of quantization noise on dynamic focusing artifacts, and the results are shown in Fig. 3.8. For these simulations, a radio frequency (RF) signal consisting of a 0.6 fractional bandwidth, 3.5MHz Gaussian pulse with -50dB white noise was used. The RF signal was digitized at 111MHz using a single bit 2\textsuperscript{nd}-order low pass sigma-delta modulator and reconstruction was done using a 160-tap low pass finite impulse response (FIR) filter. Higher order sigma-delta modulator provides better bit resolution but a 2\textsuperscript{nd}-order sigma-delta modulator was utilized because any single-bit sigma-delta modulators higher than 2\textsuperscript{nd} order are not inherently stable [39].

Fig. 3.8(a) shows the normalized waveform of the RF signal after it goes through sigma-delta modulation, reconstruction, single sample repetition (every 50 sample interval), and then envelope detection in sequence. On the other hand, Fig. 3.8(b) shows the normalized waveform of the RF signal after it goes through the sequence of sigma-delta modulation, single sample repetition (every 50 sample interval), reconstruction, and envelope detection.
The peak signal-to-noise ratio (PSNR) of the two cases can be obtained from the results in Fig. 3.8 as the ratio of the peak signal power to the average noise floor level [45]. The ratio is based on the assumption that the signal distortion (noise floor) is low compared to the signal power. The PSNR of the two cases are respectively 45dB and 28dB for an
average of 5 runs. It demonstrates that sample repetition before signal reconstruction increases the noise floor of the final output. In practice, since the sample repetition rate is different along the imaging depth, the increment of noise floor will be different along the imaging depth.

In order to study the effect of sample repetition at different pre-delay quantization noise level, an additional 160-tap low pass FIR filter was inserted into the simulation setup as the adjustable 1st filter shown in Fig. 3.9 to adjust the quantization noise level before sample repetition.

![Diagram](attachment:image.png)

**Fig. 3.9.** Simulation setup that investigates the effect of pre-delay SQNR on PSNR.
The signal now goes through sigma-delta modulation, adjustable 1\textsuperscript{st} filter, sample repetition, 2\textsuperscript{nd} reconstruction filter and then envelope detection. By adjusting the out-of-band attenuation of the 1\textsuperscript{st} filter, we can obtain a pre-delay signal with different amount of quantization noise. The PSNR value of the envelope-detected signal after 2\textsuperscript{nd} reconstruction filter was plotted against the pre-delay signal-to-quantization noise ratio (SQNR) of the signal after 1\textsuperscript{st} filter for different sample repetition rates, as shown in Fig. 3.10.

![Fig. 3.10. Single channel PSNR obtained against different pre-delay SQNR at different sample repetition rates.](image)

As shown in Fig. 3.10, when the sample repetition is less frequent, PSNR is higher. In addition, as the pre-delay SQNR increases, the PSNR achieved at that channel after sample repetition increases as well. It shows the pre-delay quantization noise level affects the degree of dynamic focusing artifacts. The lower the quantization noise level, the lower is the dynamic artifacts caused by sample repetition. Fig. 3.10 also shows that
when the pre-delay SQNR is high, the increment of PSNR with respect to pre-delay SQNR will becomes gradual. Further increment of the pre-delay SQNR hardly improves the PSNR. It is therefore not necessary to fully reconstruct the signal before delay-and-sum process while performing partial reconstruction is sufficient.

3.4 Frequency domain analysis

Another set of frequency domain simulation was carried out to study the effect of dynamic focusing on the spectrum of sigma-delta modulated signal. A similar simulation setup to Fig. 3.9 was utilized. However, in this simulation samples were repeated dynamically to model according to the delay profile at the outermost channel of a 64-channel beamformer.

The input signal is a pre-compressed waveform which when properly delayed according to the dynamic delay profile will be a single tone sinusoidal wave at 3.5MHz. The input signal was digitized using a 2\textsuperscript{nd} order low pass modulator at 160MHz. The frequency spectra of the sigma-delta modulated signal before and after the dynamic delay without applying any pre-delay filtering are shown in Fig. 3.11(a). When an 8-tap boxcar filter is used as the pre-delay filter, the frequency spectra of the signal before and after the dynamic delay are as shown in Fig. 3.11(b).
Fig. 3.11. Frequency spectra of sigma-delta modulated signal before and after dynamic delay when (a) no pre-delay filtering is utilized, (b) when 8-tap boxcar filter was utilized as pre-delay filter.

As shown in Fig. 3.11(a), besides the compression and shifting of the signal frequency, the noise power at low frequency increases significantly and no noise shaping function...
is observed after dynamic delay is applied. The low frequency noise causes image artifacts as it cannot be removed by the low pass reconstruction filter. On the other hand, when an 8-tap boxcar filter is utilized before delay, the noise level caused by dynamic delay is 20dB lower as compared to the previous case without any pre-delay filtering. The frequency domain analysis again justifies that reducing pre-delay quantization noise can alleviate the dynamic focusing artifacts.

Based on the findings, a SDBF based on cascaded reconstruction was proposed and developed, which will be discussed in Chapter 4.
Chapter 4

CASCADED RECONSTRUCTION BASED SIGMA-DELTA BEAMFORMING

Based on the studies of dynamic focusing artifacts in Chapter 3, a cascaded reconstruction based SDBF has been proposed and developed. The cascaded reconstruction SDBF utilizes a pre-delay filter to reduce the pre-delay quantization noise, so that the dynamic focusing artifact problem is suppressed. The selection of the pre-delay filter depends on the OSR as well as the final image SNR desired. The details are discussed in this chapter. The relevant work is also reported in [46], [47].

4.1 Beamformer structure

A cascaded reconstruction process for sigma-delta modulation is shown in Fig. 4.1. The reconstruction of the sigma-delta modulated signal is performed by two filters. The first filter in Fig. 4.1 is a computationally-inexpensive filter to partially reconstruct the signal, whereas the second filter fully reconstructs the complete multi-bit signal. Both first and second filters can be composed of a single stage filter or a set of cascaded filters, as long as the total filtering effect can successfully reconstructs the signal.
Fig. 4.1. Cascaded reconstruction of a sigma-delta modulated bit stream.

The cascaded reconstruction is employed in the developed SDBF, in which the first filter is applied before dynamic focusing whereas the second filter is applied after delay-and-sum process, as illustrated in Fig. 4.2.

Fig. 4.2. Block diagram of a SDBF based on cascaded reconstruction.

The analog signal from each receive channel is digitized with a sigma-delta modulator, and then the first filter is applied to partially reconstruct the intermediate multi-bit data from the single bit modulator output data. Unlike the sophisticated filters utilized in pre-delay reconstruction SDBF to fully reconstruct the signal, this first filter can be realized
using computationally-inexpensive filter (e.g., boxcar or cascaded integrator comb (CIC)), provided that it achieves the required pre-delay SQNR as discussed in Section 3.2. Therefore, the hardware complexity can be alleviated while achieving a significant reduction in dynamic focusing artifacts. Moreover, with the lower SQNR value required, the intermediate multi-bit data can have a lower bit depth than the fully reconstructed data (after 2\textsuperscript{nd} filter). The intermediate multi-bit data generated at each channel is then temporarily stored in a multi-bit shift register or first-in-first-out (FIFO) memory. After that, focusing delay for each beamforming output is applied to select the appropriate intermediate multi-bit sample from each channel. This focusing delay can either be pre-computed and stored in a look-up-table (LUT) or calculated in real-time. The selected intermediate multi-bit data are then summed across the channels to improve spatial and contrast resolution by coherent summation. After delay-and-sum process, the second filter is applied to fully reconstruct the complete multi-bit data for back-end processing.

To further reduce the hardware complexity in the temporary storage devices (e.g., shift register or FIFO memory), the sequence of the first filter and delay focusing can be interchanged. This is explained in Fig. 4.3 where 4-tap filtering and delay focusing stages are shown.

As shown in Fig. 4.3(a), a 4-tap filter with coefficients of $C_1 \sim C_4$ is applied on a stream of 1-bit samples, $S_1$, $S_2$, $S_3$ … $S_N$ to generate $M$-bit output samples, $S'_1$, $S'_2$, $S'_3$ … $S'_{N-3}$. Each output sample is the sum of the products of four consecutive input samples with the corresponding filter coefficients.
Fig. 4.3. Block diagram showing the difference in hardware requirement by applying delay focusing (a) after the first filter, (b) before the first filter.
During the delay focusing stage, one of the output samples is selected according to the delay information. If \( S'_1 \) is selected, it is equivalent to selecting the sum of \( S_1 \times C_4, S_2 \times C_3, S_3 \times C_2 \) and \( S_4 \times C_1 \). Therefore, the same beamforming result can be attained by selecting the four samples \( S_1, S_2, S_3, S_4 \) before multiplying with the filter coefficients and summation as shown in Fig. 4.3(b). As a result, instead of storing the \( M \)-bit data, single-bit data are stored in the temporary storage devices, awaiting to be selected during dynamic focusing, thereby optimizing the hardware.

Fig. 4.4 shows the block diagram of the cascaded reconstruction SDBF where delay focusing is performed before the first reconstruction filtering.

![Cascaded reconstruction SDBF](image)

Fig. 4.4. Cascaded reconstruction SDBF where delay focusing is performed before the first filtering.

At each instant, \( K \) consecutive single-bit input samples are selected by the delay control from the shift register (or FIFO memory), where \( K \) is equal to the length of the first filter. They are then multiplied with the filter coefficients and summed together along
the same channel to obtain the intermediate multi-bit data. To eliminate computationally expensive multipliers from the beamformer, boxcar filters can be utilized as the first filters. Thus, it further simplifies the hardware.

4.2 First filter selection

As aforementioned, boxcar filters can be utilized as the first filters to minimize the hardware complexity. It is desirable to select the boxcar filters to be as simple as possible while achieving the required image quality. Chapter 3 illustrated that different pre-delay SQNR are needed before the delay-and-sum process to achieve the required PSNR after the delay-and-sum process at different sample repetition rates. In practice, the sample repetition rate is limited as dynamic aperture is usually employed to optimize focusing at different depths. Optimal focusing occurs when f-number is equal to 2 [19]. The maximum sample repetition rate caused by dynamic focusing was found to be related to the f-number only. It can be obtained using the time difference, $\Delta t_n$, between two consecutive time of arrival, $t_{n1}$ and $t_{n2}$, assuming that the aperture size is small compared to the focal depth [24],

\[ t_{n1} \approx \frac{R}{c} + \frac{R - x_n \sin \theta + \frac{x_n^2 \cos^2 \theta}{2R}}{c} \]  
\[ t_{n2} \approx \frac{R + \Delta R}{c} + \frac{R + \Delta R - x_n \sin \theta + \frac{x_n^2 \cos^2 \theta}{2(R + \Delta R)}}{c} \]
\[ \Delta t_n \approx T \left( 1 - \frac{x_n^2 \cos^2 \theta}{4} \right) \left( \frac{1}{R(R+\Delta R)} \right) \]  

(4.3)

where \( R \) is the focal depth, \( c \) is the speed of sound, \( x_n \) is the distance between the \( n \)th element to the centre of the active transducer aperture, \( \theta \) is the steering angle, \( T \) is the sampling period which is equal to \( \frac{\Delta R}{2c} \). When \( R \) is much larger than \( \Delta R \), the minimum time difference is

\[ \Delta t_{n_{\text{min}}} \approx T \left( 1 - \frac{x_{n_{\text{max}}}^2}{4} \right) \left( \frac{1}{R} \right) = T \left( 1 - \frac{1}{16f_{\text{num}}^2} \right) \]  

(4.4)

where \( f \)-number is equal to \( \frac{R}{2x_{n_{\text{max}}}} \).

The minimum number of sample interval, \( m_{\text{min}} \), between two occurrences of sample repetition can be calculated as

\[ m_{\text{min}} = \frac{T}{T - \Delta t} = 16f_{\text{num}}^2 \]  

(4.5)

Hence, when \( f \)-number is 2, samples will be repeated at the maximum rate of every 64 samples.

The required pre-delay SQNR and achievable PSNR (per channel) when samples are repeated every 64 samples were obtained by repeating the simulations described in Section 3.2 at different oversampling frequencies (OSR, which is defined as \( \frac{f_S}{2f_B} \), where \( f_S \) is the sampling frequency and \( f_B \) is the signal bandwidth). The same RF signal
as described in Section 3.2 which consists of a 0.6 fractional bandwidth, 3.5MHz Gaussian pulse was used. White noise was inserted at a level 10dB lower than the theoretical dynamic range (DR) of a 2nd order sigma-delta modulator, which can be calculated using

$$DR = \frac{15(\text{OSR})^5}{2\pi^4}$$  \hspace{1cm} (4.6)$$

assuming the input as a sine wave with amplitude of 1 [39].

The average maximum PSNR and the -3dB corner SQNR (at which the PSNR achieved is 3dB below the maximum value) for 10 runs of the repeat-every-64-sample simulation are plotted against the OSR as in Fig. 4.5. They are denoted as the achievable PSNR and required pre-delay SQNR. As the noise floor of the normalized envelope-detected RF signal is dominated by the quantization noise of the modulator output, the PSNR will be equal to the SQNR of the modulator output, which is also the dynamic range (DR) of the modulator.

The same simulation was carried out under the same condition but with the 1st adjustable filter in Fig. 3.9 be replaced by a 4-tap, 8-tap, 16-tap and 32-tap boxcar filter respectively to find the suitable OSR range for each of them to be applied. The pre-delay SQNR achieved by different filters under different OSR are plotted in Fig. 4.5 as well.
Fig. 4.5 shows that when OSR is below 9, 4-tap boxcar filter is sufficient to provide the required pre-delay SQNR for the beamformer to achieve -3dB PSNR value. The pre-delay SQNR achieved by 32-tap boxcar filter shows two notches at OSR of 6 and 11 because the signal falls into notches in its frequency response at frequency which is the multiples of \( \frac{f_s}{\text{tapsize}} \). For example, 4-tap boxcar filter has notches at \( \frac{f_s}{4} \), \( \frac{f_s}{2} \), \( \frac{3f_s}{4} \) and \( f_s \). When the tap size increases, the number of notches in the frequency response will increase, more out-of-band quantization noise can be filtered off if the signal frequency is way below the notches. Hence, using 16-tap boxcar filter as the 1\textsuperscript{st}
filter can achieve higher PSNR than using 8-tap boxcar filter and 4-tap boxcar filter for OSR above 9 as shown in Fig. 4.5.

However, the tap size of the boxcar filter should not be larger than 2 times the value of OSR otherwise it attenuates the signal. For example when OSR is 10, the tap size should be smaller than 20. This is the reason of the lower SQNR for 16-tap boxcar filter at OSR below 8 and for 32-tap boxcar filter at OSR below 16. It should be noted that for OSR above 17, 32-tap boxcar filter does not really meet the -3dB corner SQNR requirement. Hence, to obtain a better performance, more sophisticated filter needs to be used. However, if a boxcar filter is chosen to be utilized for its hardware advantage, 32-tap size is the best candidate.

Fig. 4.5 provides the condition for the design of OSR and boxcar filter length. For example, if a 40dB DR per channel is required for a SDBF, the sigma-delta modulator in each channel needs to run at an OSR of 11 as obtained from the lower graph of Fig. 4.5. Looking up to the upper graph, the pre-delay SQNR has to be higher than 14dB at OSR of 11 in order to sufficiently suppress the dynamic focusing artifacts and an 8-tap boxcar filter is sufficient to achieve the required pre-delay SQNR.

4.3 Results and discussions

The developed cascaded reconstruction SDBF was evaluated using the data acquired by Biomedical Ultrasonics Laboratory at University of Michigan (available at http://bul.eecs.umich.edu). The data were collected using Acuson, Mountain View, CA,
Model #V328, a 128 element, 3.5MHz commercial transducer for a wire target phantom consisting of 6 wires in a water tank. The same data was used in [24] and [30].

A typical ultrasound image is displayed in 60dB dynamic range to include echoes reflected from a typical interface (e.g. liver-fat) as well as the echoes from tissue scattering [48]. Hence, the transmission and reception sidelobe level cannot be higher than -30dB each, so that the overall sidelobe level is lower than -60dB, fulfilling the dynamic range specification. To achieve a maximum average sidelobe level of -30dB for both transmission and reception (for 128 channel array with rectangular windowing), the sampling frequency required to achieve sufficient delay resolution can be calculated using the peak random quantization sidelobe (voltage) level equation in [49]

$$SL_{\text{peak}} \approx \frac{\pi}{r_f} \left( \frac{4.6 \cdot \text{ENBW}}{3N_{\text{ch}}} \right)^{\frac{1}{2}}$$

(4.7)

where ENBW is the equivalent noise bandwidth which is 1 in this case when rectangular window is used for apodization, $r_f$ is the ratio of sampling frequency to signal bandwidth (which is equal to $2 \times \text{OSR}$), and $N_{\text{ch}}$ is the number of elements, under the condition that the error components are uncorrelated from channel to channel. The condition of random noise (uncorrelated with signal, hence uncorrelated among channels) is true for sigma-delta modulation when the modulator has an active input [50], which is true for the case of ultrasound imaging.

For a system with 128 channel array with rectangular windowing, $r_f$ is found to be 11 from (4.7). Hence the sampling frequency needs to be at least 11 times higher than the
central frequency of the ultrasound signal, equivalent to an OSR of 5.5. For 64 channels, it would be 16 times higher, and for 32 channels, it would be 22 times higher.

On the other hand, the sampling frequency of sigma-delta beamformer also determines the DR that can be achieved for the final image. For a 128 channel array with rectangular windowing, the apodization power gain is

\[
G_{SNR} = \left( \sum_{n=1}^{N} w_n \right)^2 \frac{\sum_{n=1}^{N} w_n^2}{\sum_{n=1}^{N} w_n^2} = 21dB
\]  

(4.8)

In order to obtain a total DR of 60dB, the DR needed for each channel is 39dB.

For a single bit 2\textsuperscript{nd}-order sigma-delta modulator, the OSR needed can be estimated using (4.6). Alternatively, the equation can be expressed in terms of dB as

\[
DR = 1.76 - 12.9 + 50 \log_{10} OSR \text{ (dB)}
\]  

(4.9)

OSR was found to be 10.1 in order to obtain a DR of 39dB. That means the sampling frequency needs to be at least 20.2 times higher than the signal bandwidth. An OSR of 11 can be selected to fulfill the DR requirement for low resolution handheld machine. The DR achieved would be 41dB. Hence, for the following simulation, the wire phantom data were converted into 1-bit data streams using typical single bit 2\textsuperscript{nd}-order low pass sigma-delta modulators at a sampling frequency of 111 MHz (OSR of 11 for 5MHz bandwidth). Nevertheless, higher OSR can be used to ensure that the quantization noise is well below input noise level and electronic noise for higher resolution machine.
From Fig. 4.5, an 8-tap boxcar filter was chosen as the first filter to provide the required pre-delay SQNR of 39dB per channel. Three different SDBF techniques (i.e., pre-delay reconstruction, post-delay reconstruction, and insert-zero) were compared with the developed method. A 160-tap FIR filter was utilized in the pre-delay reconstruction method. Post-delay reconstruction for all techniques was performed with a 3rd-order cascaded integrator-comb (CIC) decimation filter [51], [52] followed by a 30-tap FIR filter. The 160-tap FIR filter and the CIC decimator cascaded with 30-tap FIR filter can achieve similar pass-band filter characteristic as shown in Fig. 4.6, thus provide fair comparison in terms of image quality and practical comparison in terms of hardware.

![Fig. 4.6 Frequency responses of 160 tap FIR filter and CIC decimator with 30 tap FIR filter.](image)

The various SDBF techniques were simulated using MATLAB 7.0. The floating point input data from University of Michigan were interpolated to 111MHz and converted to single-bit data using a 2nd order sigma-delta modulator described in Fig. 3.1. At each
cycle, 128 channels of the sigma-delta modulated data went through the beamforming process to form a scanline. The simulation was repeated for 192 cycles to obtain 192 scanlines. The brightness information along the scanlines was then extracted through demodulation process and was matched to the pixel of the final image through scan conversion process as described in [53]. Rectangular window was utilized for apodization whereas dynamic aperture was applied at $f$-number $\geq 2$.

The images obtained are shown in Fig. 4.7 with 60dB dynamic range. Axial projection was plotted alongside to show the different noise levels which are not visually differentiable from the images.
Fig. 4.7. Wire phantom images with axial projection obtained using (a) pre-delay reconstruction, (b) post-delay reconstruction, (c) insert zero, and (d) cascaded reconstruction SDBF.
As shown in Fig. 4.7, the post-delay reconstruction method (i.e., Fig. 4.7(b)) suffers from noisy image background due to dynamic focusing artifacts, whereas the other methods including the developed cascaded reconstruction have cleaner image background. Comparing Fig. 4.7(c) and (d), the cascaded reconstruction SDBF produces less background noise than the insert zero method, as supported by the axial projection.

Peak signal-to-noise ratio (PSNR) was calculated from the images based on the ratio of the peak signal power of the image (which is at the 3rd wire target from top in Fig. 4.8) to the average background noise within the 3 boxed areas as indicated in Fig. 4.8. The results obtained for different beamforming techniques are shown in Table 4.1.

Fig. 4.8. Wire phantom image which shows the 3 areas of noise power that are used to calculate the PSNR.
Table 4.1. PSNR values of 3 different depths for different beamforming techniques at 111MHz beamforming frequency.

<table>
<thead>
<tr>
<th></th>
<th>PSNR1(dB)</th>
<th>PSNR2(dB)</th>
<th>PSNR3(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-delay reconstruction</td>
<td>62.68</td>
<td>62.56</td>
<td>62.62</td>
</tr>
<tr>
<td>Post-delay reconstruction</td>
<td>51.74</td>
<td>54.48</td>
<td>55.68</td>
</tr>
<tr>
<td>Insert zero</td>
<td>58.79</td>
<td>58.55</td>
<td>58.76</td>
</tr>
<tr>
<td>Cascaded reconstruction using different 1st filters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-tap boxcar</td>
<td>59.09</td>
<td>59.21</td>
<td>59.50</td>
</tr>
<tr>
<td>8-tap boxcar</td>
<td>60.77</td>
<td>60.53</td>
<td>60.79</td>
</tr>
<tr>
<td>16-tap boxcar</td>
<td>62.17</td>
<td>62.11</td>
<td>62.07</td>
</tr>
</tbody>
</table>

Table 4.1 shows that at 111MHz, cascaded reconstruction SDBF is able to achieve better contrast resolution than insert zero method. As mentioned before, boxcar filters have notches in their frequency response at frequency which is the multiples of \( f_s/tapsize \).

When the tap size increases, the number of notches in the frequency response will increase, more out-of-band quantization noise can be filtered off. Hence, using 16-tap boxcar filter as the 1st filter can achieve higher PSNR than using 8-tap boxcar filter and 4-tap boxcar filter. Table 4.1 also shows that using an 8-tap boxcar filter as the 1st filter, the largest PSNR difference from pre-delay reconstruction SDBF is at position 2 and it is only 2.03dB. It is within the 3dB range as predicted from Fig. 4.5.

Another set of cyst phantom data acquired by Biomedical Ultrasonics Laboratory at University of Michigan was used to evaluate the proposed cascaded reconstruction SDBF (using 8-tap boxcar filter as the 1st filter) against the same three methods. This same set of data was also used for comparison by [24] and [30]. The image formed by the data is shown in Fig. 4.9. The contrast resolution achieved by the different methods was evaluated for comparison.
Fig. 4.9. Image of the cyst phantom data acquired by Biomedical Ultrasonics Laboratory at University of Michigan.

The contrast resolution of the images was quantified using the contrast-to-noise ratio (CNR) defined as

$$CNR = \frac{|\mu_s - \mu_c|}{\sigma_s}$$  \hspace{1cm} (4.10)

where $\mu_s$ and $\mu_c$ denotes the mean values of the scatterers and of the cyst respectively, and $\sigma_s$ is the standard deviation of the scatterers [54]. The results of the CNR for 60dB dynamic range are presented in Table 4.2.

Table 4.2. Contrast-to-noise ratio (CNR) in anechoic region of the cyst phantom image obtained by various SDBF.

<table>
<thead>
<tr>
<th>Cyst</th>
<th>Pre-delay reconstruction</th>
<th>Post-delay reconstruction</th>
<th>Insert zero</th>
<th>Cascaded reconstruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (50mm)</td>
<td>6.19</td>
<td>5.07</td>
<td>5.95</td>
<td>6.15</td>
</tr>
<tr>
<td>2 (80mm)</td>
<td>6.14</td>
<td>5.30</td>
<td>5.88</td>
<td>6.01</td>
</tr>
<tr>
<td>3 (110mm)</td>
<td>4.35</td>
<td>3.55</td>
<td>4.07</td>
<td>4.30</td>
</tr>
</tbody>
</table>
The results in Table 4.2, again, indicate that the proposed cascaded reconstruction SDBF is able to provide better contrast resolution than the insert zero method and the performance is very close to the pre-delay reconstruction SDBF.

### 4.4 Hardware estimation

#### 4.4.1 Cascaded reconstruction SDBF

The hardware of the proposed cascaded reconstruction SDBF can also be greatly simplified compared to pre-delay reconstruction method by having a much simpler filter in each channel. Table 4.3 shows the hardware comparison for a 64 channel beamformer with pre-delay reconstruction and cascaded reconstruction (using 8-tap boxcar filter as 1st filter) methods. The number of multipliers needed can be reduced from 5120 (80 per channel as the filter coefficients are symmetric) to 15 (for the 30-tap FIR filter after CIC decimator), and the adders can be reduced from 10240 (160 per channel) multi-bit adders to 64 (1 per channel) 8-input-single-bit adders and 36 multi-bit adders (for CIC decimator and 30-tap FIR filter).
Table 4.3. Number of multipliers and adders for pre-delay reconstruction SDBF and cascaded reconstruction SDBF with 8-tap boxcar pre-delay filter for 64-channel system.

<table>
<thead>
<tr>
<th>Pre-delay reconstruction</th>
<th>Cascaded reconstruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Components</td>
<td>Number</td>
</tr>
<tr>
<td><strong>Pre-delay reconstruction filter</strong></td>
<td></td>
</tr>
<tr>
<td>Multi-bit multipliers</td>
<td>80×64ch = 5120</td>
</tr>
<tr>
<td>Multi-bit adders</td>
<td>160×64ch = 10240</td>
</tr>
<tr>
<td><strong>Post-delay reconstruction filter</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Beamforming summer</strong></td>
<td>64-input 8-bit adder</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

A more thorough hardware synthesis result comparison will be discussed together with the improved SDBF in Chapter 5.

4.4.2 Delay generator

As SDBF runs at a frequency that is much higher than the conventional beamformer, it requires delay information to be updated at much higher frequency as well. Hence, the power consumption and hardware complexity of the delay generator for a SDBF based ultrasound machine is of great importance as they can affect the overall power consumption and hardware complexity of the whole machine. This section studies the three basic delay generation methods including complete lookup table, compressed lookup table with delta encoding and general parametric algorithm. They were evaluated and synthesized using Very high speed integrated circuit Hardware Description Language (VHDL) for a FPGA device Spartan 3E XC3S500E. A comparison among the
three delay generation methods on their performances in terms of both logic resource usage and power consumption is reported in [55].

4.4.2.1 Complete lookup table method

The complete lookup table method is designed to replace the extensive run-time calculation of instant delay profiles by a simple lookup operation. However, this is done at the expense of the memory usage due to the storage of the pre-generated delay patterns for each processing channel. All the delay information necessary for performing dynamic focusing was pre-calculated according to the beamforming principle presented in Section 2.4, and stored in complete form into a data structure, usually implemented by register arrays or read-only memory (ROM) modules. As shown in Fig. 4.10, during delay update operation, the pre-stored delay values stored in the ROM module are accessed with read operations according to current focal point index. The ultimate output of the beamforming control unit takes the form of memory addresses as applicable delay data such that the desired echo signal samples can be properly selected from variable length delay lines, which are implemented by using memory elements.

Fig. 4.10. Block diagram of a complete lookup table approach.
4.4.2.2  Compressed lookup table with delta encoding

The amount of necessary focusing information is large, since delay information for all receive channels at all focal depths has to be stored. Keeping it in an uncompressed form requires significant size of memory, and researchers have been working on different compression approaches for compression since the introduction of digital beamforming.

In order to avoid storing large amount of address data, the delta encoding scheme was employed to compress the conventional LUT. By applying delta encoding, only the differences between the delay (index) values instead of the absolute delays were stored in the lookup table [56]. With the instant increment of focal depth, $\Delta$, specified as half of the wavelength, $\lambda$ ($\lambda = c \cdot T_s$, where $c$ is the speed of sound and $T_s$ is the sampling period), the original $n$-bit-wide delay words can be represented by single bits, thus reducing effectively the size of the LUT. To recover the $n$-bit-wide delay addresses, accumulation operation is incorporated after the read process to add up the address difference stored, as shown in Fig. 4.11.

![Block diagram of the compressed lookup table with delta encoding approach.](image)

Fig. 4.11. Block diagram of the compressed lookup table with delta encoding approach.
The run-time calculation involved in delta encoding is minimal, as only an adder is needed to add the current relative delay read from the LUT to the previous output address stored in a register.

### 4.4.2.3 General parametric approach

In contrast to the previous approaches, rather than storing the pre-generated focusing information in a LUT, the general parametric method calculates the instant delay profile with on-the-fly calculation in the run time. The instant delays that are applied to generate a dynamic receive focus can be calculated using:

\[
p = d_f + d_r = d_f + \sqrt{x_n^2 + d_f^2 - 2x_n d_f \sin \theta}
\] (4.11)

where \(p\) is the full pulse-echo path, \(d_r\) is the length of the echo path, \(d_f\) is the instant focal depth measured from the beam origin of the transducer to the focal point, \(x_n\) is the distance from the centre of the transducer array to the \(n^{th}\) element, and \(\theta\) is the steering angle.

![Delay calculation geometry for 1D linear transducer array.](image)

Fig. 4.12. Delay calculation geometry for 1D linear transducer array.
The general parametric approach utilizes the nature of the focusing delay curves by describing the delay curve using a quadratic equation:

\[ f(p_N, d_N) = p_N^2 - 2d_N(p_N - k_N) - x_N^2 = 0 \quad (4.12) \]

which was derived from (4.11) by dividing both sides with \( \Delta^2 \), where \( \Delta \) is the instant increment of the focal depth and is defined as the distance that ultrasound signal travels over a period of the sampling frequency. The term \( k_N \), which is \( x_n \sin \theta \), is constant for a given image line inclination and transmitting element. The index \( N \) denotes time index.

Ideally, the function \( f(p_N, d_N) \) is equal to zero but due to delay quantization, it is not possible to obtain a perfect zero value for the function. In the general parametric algorithm, the function \( f(p_N, d_N) \) is kept to be as close to zero as possible by adjusting the value of \( p_N \) for each unit increment of \( d_N \). It can be seen from (4.13) that if \( p_N \) is increased by 1 for each unit increment of \( d_N \), the function is reducing. On the other hand, from (4.14), the function is increasing if \( p_N \) is increased by 2.

\[
\begin{align*}
  f(p_N + 1, d_N + 1) &= f(p_N, d_N) - 2d_N + 2k_N - 1 < f(p_N, d_N) \\
  f(p_N + 2, d_N + 1) &= f(p_N, d_N) + 2p_N - 4d_N + 2k_N \\
  &> f(p_N, d_N)
\end{align*}
\quad (4.13) \quad (4.14)
\]

Therefore, a step-by-step delay algorithm can be built around the equation by numerically solving it at each increment of a leading variable, which is the instant focal depth \( d_N \) in this case.

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1. The initial values $d_N(1) = d_{\text{start}} \frac{f_s}{c}$, $p_N(1) = p_{\text{start}} \frac{f_s}{c}$ and $k_N = x_n \sin \frac{f_s}{c}$ are applied.

2. If $f(p_N + 1, d_N + 1) > 0$, then $p_N(n + 1) = p_N(n) + 1$, else $p_N(n + 1) = p_N(n) + 2$.

3. $d_N(n+1) = d_N(n) + 1$, and if the end of the line is not reached, go back to 2.

![Fig. 4.13. Block diagram of the general parametric on-the-fly calculation approach.](image)

The general parametric algorithm generates output with a maximum error of $\pm 1$ unit, which is the distance that ultrasound waves travel in half clock period. The precision is the same as the two LUT methods presented.

A pipelined design was employed for the implementation of the described algorithm in order to increase the throughput and also to ensure that the timing constraints imposed by the real-time system are satisfied. The resulting implementation requires only a few
input parameters, but it has the disadvantage of consuming power and logic resource due to the implementation of the on-the-fly calculations.

### 4.4.2.4 Implementation results and discussions

A comparison between the three delay generation methods for the delay generator was done for a 64-channel linear array. The sampling frequency was set to be 160MHz for the purpose of consuming reasonable amount of power while achieving good focusing. The occupied logic gate count and power consumption were estimated for the VHDL implementations of the three methods in an FPGA for a target device Spartan-3E XC3S500E by Xilinx. The same level of optimization was achieved for the three designs. The synthesis results for the three delay generation approaches are shown in Table 4.4.

<table>
<thead>
<tr>
<th></th>
<th>Logic gates</th>
<th>Power @ 160MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete LUT</td>
<td>1738</td>
<td>241mW</td>
</tr>
<tr>
<td>Compressed LUT</td>
<td>1539</td>
<td>261mW</td>
</tr>
<tr>
<td>General parametric</td>
<td>3369</td>
<td>261mW</td>
</tr>
</tbody>
</table>

It can be seen that the general parametric approach takes up most logic gate count and consumes most power among the three methods, due to (i) the run-time calculations of the instant delay profile, which dissipates significant computing power and (ii) the inefficiency of implementing calculation logics compared to memory modules in FPGA, as shown in Table 4.5.
Table 4.5. Device utilization of (a) complete lookup table approach, (b) compressed lookup table with delta encoding approach and (c) general parametric approach.

<table>
<thead>
<tr>
<th></th>
<th>Slices registers</th>
<th>4-input LUTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete LUT</td>
<td>16</td>
<td>177</td>
</tr>
<tr>
<td>Compressed LUT</td>
<td>28</td>
<td>191</td>
</tr>
<tr>
<td>General parametric</td>
<td>220</td>
<td>220</td>
</tr>
</tbody>
</table>

On the other hand, the compressed LUT with delta encoding approach has the best performance in terms of overall logic resources as well as power consumption. The reduction of logic gate count by applying delta encoding in FPGA is only marginal, because the same ROM module was used to store the LUTs for the two approaches after examining their respective RTL schematics. In other words, despite the difference in word width, the size of both LUTs is the same, which is equal to the total number of focal depths along an image line. However, for more sophisticated imaging systems such as 3-D transducer with higher resolution, which require considerable memory size for their more complex focusing information, the on-the-fly calculation approach might still be advantageous.

4.5 Summary

Post-delay reconstruction SDBF suffers from artifact problem when dynamic focusing is performed. It was found in Chapter 3 that the artifact problem is related to the quantization noise present in the sigma-delta modulated signal. The relation between the image quality (in terms of PSNR) and the quantization noise before sample repetition is studied in this Chapter. It shows that dynamic focusing artifacts can be effectively
suppressed when a certain SQNR is achieved for the signal before the delay focusing. Hence, we proposed and developed a cascaded reconstruction SDBF which suppresses the dynamic focusing artifacts by reducing the quantization noise before performing delay-and-sum.

Besides, this chapter also presents the condition to select different tap size boxcar filters for different OSR when dynamic aperture is applied at f-number of 2. Simulation results using real phantom data show that the dynamic focusing artifacts can be reduced by simply using an 8-tap boxcar filter as the pre-delay filter when OSR is 11. The contrast resolution of the image produced by the proposed SDBF is comparable to that produced by the pre-delay reconstruction SDBF method, and is better than that of the insert zero method. The hardware can also be greatly simplified compared to pre-delay reconstruction method by having a much simpler filter in each channel.

Due to the high beamforming frequency required for SDBF system, the selection of delay generator is important because it contributes to the overall power consumption and hardware complexity of the entire ultrasound machine. Three delay generation methods (complete LUT, compressed LUT with delta encoding and general parametric algorithm) were evaluated in terms of their power and hardware in an FPGA for a target device Spartan-3E XC3S500E by Xilinx. The result shows that the compressed LUT method is preferred for a 64-channel linear array system with advantages in both power consumption and gate counts. However, if the channel count is large (e.g. for 3-D ultrasound machine), on-the-fly calculation method (general parametric algorithm) may
be a better solution because the amount of delay information needed to store is very large for LUT methods.
Chapter 5

MULTI-BIT SIGMA-DELTA BEAMFORMER

Since reducing the quantization noise level before the delay-and-sum process can suppress the dynamic focusing artifact, a cascaded reconstruction SDBF was developed which utilizes simple boxcar filter to reduce the pre-delay quantization noise level as described in Chapter 4. Although the method has successfully resolved the artifact problem, it can be further improved if a more effective way (less complex hardware or lower power consumption) to reduce the pre-delay quantization noise level can be found. In this chapter, a multi-bit SDBF was proposed. It discards the needs of having a boxcar filter in each channel and allows simple delay-and-sum beamforming. Section 5.1 presents the theory behind this method and also the criteria of the bit-number selection. The structure of the multi-bit SDBF is described in Section 5.2 whereas the image simulation results and hardware evaluation is illustrated in Section 5.3. The work described in this chapter is reported in [57], [58].

5.1 Theory

Sigma-delta modulators achieve high ADC resolution by oversampling and noise shaping. Increasing the order of the modulator and the sampling frequency of the sigma-delta modulator would increase the ADC resolution as the in-band quantization noise is
reduced. However, the total noise content in the whole frequency band (from DC to \( \frac{f_S}{2} \), where \( f_S \) is the sampling frequency) remains unchanged. This can be shown by the theoretical quantization noise power calculation for an \( m \)th-order sigma-delta modulator [39]

\[
n_q^2 = 2\int_0^\pi 2\sin \frac{\Omega}{2} \left( \frac{n_q^2}{f_S} \right) d\Omega
\]

(5.1)

where \( n_e^2 \) is the unshaped quantization noise power, \( n_q^2 \) is the quantization noise power after noise shaping, \( \Omega \) is the digital frequency. Second order sigma-delta modulator was selected for illustration in this paper as any higher order modulators are not inherently stable [39]. For a 2nd-order sigma-delta modulator, the quantization noise power can be simplified to

\[
n_q^2 = 6n_e^2
\]

(5.2)

The total quantization noise power from DC to \( \frac{f_S}{2} \) before reconstruction is not related to the OSR nor to the order of the modulator. It is only related to the unshaped noise power. One way to reduce the pre-delay quantization noise is to increase the bit resolution of the quantizer used in sigma-delta modulator, from single-bit to multi-bit. By having a one-bit increment, the noise level can be reduced by 6dB theoretically [39]. If the number of bits is increase such that the total quantization noise power is low enough to achieve a certain pre-delay SQNR value, the effect of dynamic focusing artifacts can be neglected. This is the reason why conventional multi-bit beamformers do
not suffer from dynamic artifact problem. The number of bits for a multi-bit SDBF for a certain PSNR per channel is determined by the pre-delay SQNR required [45].

A set of simulations was carried out to determine the bit-number required for different PSNR per channel. The simulation environment was setup according to Fig. 3.9 and described in Section 3.2. A radio frequency (RF) signal consisting of a 0.6 fractional bandwidth, 3.5MHz Gaussian pulse with -50dB white noise was used as the analog input signal. The RF signal was digitized using a single-bit 2\textsuperscript{nd}-order low-pass sigma-delta modulator at 111MHz. Both 1\textsuperscript{st} and 2\textsuperscript{nd} reconstruction filters are 160-tap low-pass FIR filters but the out-of-band attenuation of the 1\textsuperscript{st} filter is adjustable. It was also found in Chapter 4 that the maximum sample repetition rate of a dynamic focusing beamformer is only related to the $f$-number of the system and when $f$-number is 2, samples will be repeated at the maximum rate of every 64 samples.

The average maximum PSNR and the -3dB corner SQNR (at which the PSNR achieved is -3dB below the maximum value) for 10 runs of the repeat-every-64-sample simulation are plotted against the OSR as in Fig. 4.5. They are denoted as the achievable PSNR and required pre-delay SQNR.

By extracting the results from Fig. 4.5 and plotting the required average pre-delay SQNR against the corresponding achievable PSNR value (-3dB from the maximum PSNR value), Fig. 5.1 was obtained. It was found that the relationship between the achievable PSNR with the required pre-delay SQNR can be approximated to a linear relation, as illustrated in Fig. 5.1.
Based on Fig. 5.1, the number of bits required by the sigma-delta modulator will be selected according to the SQNR value that it can provide. It has to fulfill the pre-delay SQNR value required. The SQNR achieved by an $N_b$-bit 2$^{nd}$-order sigma-delta modulator can be expressed as

$$SQNR = 6.02(N_b - 1) \text{ (dB)}.$$  \hfill (5.3)

and it is comparable to the results obtained from simulation for 2- to 6-bit sigma-delta modulator as shown in Table 5.1.
Table 5.1. Theoretical and simulated pre-delay SQNR of the multi-bit 2\textsuperscript{nd} order sigma-delta modulator.

<table>
<thead>
<tr>
<th>No. of bits</th>
<th>Theoretical SQNR (dB)</th>
<th>Simulated SQNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.02</td>
<td>7.51</td>
</tr>
<tr>
<td>3</td>
<td>12.04</td>
<td>12.16</td>
</tr>
<tr>
<td>4</td>
<td>18.06</td>
<td>18.13</td>
</tr>
<tr>
<td>5</td>
<td>24.08</td>
<td>24.15</td>
</tr>
<tr>
<td>6</td>
<td>30.1</td>
<td>30.13</td>
</tr>
</tbody>
</table>

The simulated pre-delay SQNR was incorporated into Fig. 5.1 to indicate the upper limits of PSNR achievable with 2- to 6-bit sigma-delta modulators. For example, if 40dB PSNR is required for each channel, 3-bit sigma-delta modulator should be utilized to achieve the pre-delay SQNR of 10dB. After the selection of the number of bits, the OSR of the system can be calculated by

\[
\text{DR} = 6.02N_b + 1.76 - 12.9 + 50\log_{10}\text{OSR(dB)},
\]

(5.4)

where DR is the dynamic range (which is also equal to the PSNR per channel). It should be noted that the OSR obtained should also fulfill the delay resolution requirement calculated by:

\[
\text{SL}_{\text{peak}} \approx \frac{\pi}{r_f} \left(\frac{4.6\text{ENBW}}{3N_{ch}}\right)^{\frac{1}{2}},
\]

(5.5)

where \(\text{SL}_{\text{peak}}\) is the peak random quantization sidelobe (voltage) level, ENBW is the equivalent noise bandwidth which is 1 when rectangular window is used for apodization, \(r_f\) is the ratio of sampling frequency to signal frequency (which is equal to 2×OSR), and \(N_{ch}\) is the number of elements, under the condition that the error components are
uncorrelated from channel to channel [49]. For example, to obtain a sidelobe level of -30dB for a system with 128 channel array with rectangular windowing, $r_f$ needs to be at least 11 from (4.7). Hence the sampling frequency needs to be at least 11 times higher than the central frequency of the ultrasound signal, which is equivalent to an OSR of 5.5.

5.2 Beamformer structure

The structure of the multi-bit SDBF is shown in Fig. 5.2 which consists of multi-bit sigma-delta modulators and multi-bit shift registers or first-in-first-out (FIFO) memories.

Fig. 5.2. Multi-bit sigma-delta beamformer using $N$-bit modulators.
Increasing the bit resolution not only corrects the dynamic focusing artifact problem, but also reduces the beamforming frequency as long as the delay resolution is fulfilled. Although the bit length is increased, the delay length is reduced because a lower beamforming frequency is used.

For a multi-bit sigma-delta modulator, the internal DAC needs to be as linear as the overall modulator in order to provide good noise suppression [59]. Nonlinearity in the DAC will cause the increase of noise floor as well as harmonics. Thus, the resulting image will be degraded [60]. However, several techniques like dual-quantization, dynamic element matching (DEM), mismatch error shaping, digital correction can be used to correct the linearity problem effectively [39]. One of these solutions, the DEM technique, converts the internal DAC element errors to high frequency noise by randomizing the DAC element switching activity. The method is able to provide sufficient linearity for the implementation of more than 14 bits sigma-delta modulators with bandwidth beyond 1MHz. The disadvantage of DEM technique is that additional delay is introduced inside the feedback loop and causing higher operating frequency to be applied on other circuit components.

Nevertheless, for a 3-bit sigma-delta modulator, it was found that linearity correction techniques is not needed as sufficiently good element matching can be achieved by simple layout techniques. Simulations have shown that the standard deviation of a 3-bit DAC element mismatch of 0.05% is sufficient not to degrade the modulator performance and a harmonic distortion 100dB below full scale can be achieved [61].
5.3 Results

5.3.1 Image simulation results

The point phantom data acquired by Biomedical Ultrasonics Laboratory at University of Michigan (available at http://bul.eecs.umich.edu), using Acuson, Mountain View, CA, Model #V328, a 128 element, 3.5MHz commercial transducer for a wire target phantom consisting of 6 wires in a water tank, was used for comparison of the proposed multi-bit SDBF with pre-delay reconstruction, post-delay reconstruction, insert zero and cascaded reconstruction methods.

For a 128 channel, rectangular windowed array, the apodization power gain can be obtained by

\[
G_{SNR} = \left( \sum_{n=1}^{N} w_n \right)^2 / \sum_{n=1}^{N} w_n^2 = 21dB,
\]

(5.6)

where \(w_n\) is the weighting of the \(n^{th}\) channel. In order to obtain a total DR of 60dB, the DR needed for each channel is 39dB. Hence, according to Fig. 5.1, the number of bits for the sigma-delta modulator should be 3 bits. An OSR of 4.4 is required for the 3-bit system as calculated using (5.4) and OSR of 11 is required for single-bit systems as calculated by

\[
DR = 1.76 - 12.9 + 50 \log_{10} OSR \text{ (dB)}.
\]

(5.7)
On the other hand, to achieve a maximum average sidelobe level of -30dB for both transmission and reception (for 128 channel array with rectangular windowing) in order to achieve a total sidelobe level of -60dB, the sampling frequency required calculated by using (5.5) showed that the OSR needed is at least 5.5. As this value is higher than the OSR required by the ADC DR, an OSR of 5.5 was selected for the 3-bit SDBF, which provides 44dB of ADC DR per channel. An OSR of 11 was used for other single-bit SDBF architectures in the following comparison.

Similarly, the various SDBF techniques were simulated using MATLAB 7.0 as described in Section 4.3. An image formed by the point phantom data is shown in Fig. 5.3. in 60dB dynamic range with dynamic aperture applied at f-number ≥2.

![Wire phantom image](image.png)

**Fig. 5.3.** Wire phantom image which shows the 3 areas of noise power that are used to calculate the PSNR.

The contrast resolution of the point phantom images obtained using pre-delay reconstruction, post-delay reconstruction, insert zero, cascaded reconstruction (with 8-
tap boxcar filters as pre-delay filters) and 3-bit SDBF was quantified by the PSNR which is defined as the ratio of the peak signal to the average noise level in the 3 boxed areas shown in Fig. 5.3. The results are presented in Table 5.2.

### Table 5.2. PSNR values of 3 different depths for different beamforming techniques.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>111</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-delay reconstruction</td>
<td>Post-delay reconstruction</td>
</tr>
<tr>
<td>Area</td>
<td>PSNR1(dB)</td>
<td>62.68</td>
</tr>
<tr>
<td></td>
<td>PSNR2(dB)</td>
<td>62.56</td>
</tr>
<tr>
<td></td>
<td>PSNR3(dB)</td>
<td>62.62</td>
</tr>
</tbody>
</table>

Table 5.2. shows that the 3-bit SDBF achieves a PSNR of 60dB, which is similar to the cascaded reconstruction method and is better than the insert zero method, with a lower beamforming frequency at 55MHz. The PSNR of the 3-bit SDBF in this case is limited by the delay resolution rather than the ADC resolution.

In addition, another cyst phantom acquired by Biomedical Ultrasonics Laboratory at University of Michigan using the same machine was used for comparison amongst the 5 methods (proposed multi-bit SDBF, pre-delay reconstruction SDBF, post-delay reconstruction SDBF, insert zero SDBF and proposed cascaded reconstruction SDBF). The image formed by the data is shown in Fig. 5.4. with 60dB dynamic range.
Fig. 5.4. Image of the cyst phantom data acquired by Biomedical Ultrasonics Laboratory at University of Michigan.

The contrast resolution of the images can be evaluated using the contrast-to-noise ratio (CNR) metric which is defined as

\[ CNR = \frac{|\mu_s - \mu_c|}{\sigma_s}, \]  

(5.8)

where \( \mu_s \) and \( \mu_c \) denotes the mean values of the scatterers and of the cyst respectively, and \( \sigma_s \) is the standard deviation of the scatterers [54]. The results of the CNR are presented in Table 5.3.
Table 5.3. Contrast-to-noise ratio (CNR) in anechoic region of the cyst phantom image obtained by various SDBF.

<table>
<thead>
<tr>
<th>Cyst</th>
<th>Pre-delay reconstruction</th>
<th>Post-delay reconstruction</th>
<th>Insert zero</th>
<th>Cascaded reconstruction</th>
<th>3-bit SDBF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (50mm)</td>
<td>6.19</td>
<td>5.07</td>
<td>5.95</td>
<td>6.15</td>
<td>6.14</td>
</tr>
<tr>
<td>2 (80mm)</td>
<td>6.14</td>
<td>5.30</td>
<td>5.88</td>
<td>6.01</td>
<td>6.10</td>
</tr>
<tr>
<td>3 (110mm)</td>
<td>4.35</td>
<td>3.55</td>
<td>4.07</td>
<td>4.30</td>
<td>4.36</td>
</tr>
</tbody>
</table>

Again, the result in Table 5.3 shows that the 3-bit SDBF effectively suppresses the dynamic focusing artifacts. It can provide better contrast resolution, close to those produced by cascaded reconstruction and pre-delay resolution methods, and is better than the pre-delay reconstruction and the insert zero method.

5.3.2 Hardware synthesis results

The five SDBF were modeled in Very high speed integrated circuit Hardware Description Language (VHDL) for a 64-channel linear array system. Hardware synthesis of the 3-bit SDBF at 80MHz (OSR of 5.5 for 7MHz input signal bandwidth) beamforming frequency, as well as pre-delay reconstruction, post-delay reconstruction, insert zero and cascaded reconstruction (with 8-tap boxcar filters as the pre-delay filters) SDBF at 160MHz (OSR of 11 for 7MHz input signal bandwidth) beamforming frequency, was performed with Synopsys Design Vision, version X-2005.9-SP2, using commercial 0.18µm process.

The beamformer implemented includes all sub-blocks used after sigma-delta modulator until the output of the reconstructed beam-summation. The I/O of the 5 different beamformers was designed to be the same. In total, there are 6 input ports for a
beamformer. The system clock signal, the asynchronous reset signal, as well as the read and write commands are the 4 control inputs to the entire beamformer. The read and write commands were designed to provide external control ability when the beamformer is used as a sub-module in the ultrasound system. For every beamforming channel, the input data is received from the sigma-delta modulators. This input data is stored in the memory before it is processed by the beamformer. There is another 8-bit (for single-bit SDBF) or 7-bit (for 3-bit SDBF) delay profile input for each beamforming channel, which is used to select the input data stored in the memory. Hence, for a 64-channel beamformer, the processing input data is 64 bits (for single-bit SDBF) or 196 bits (for 3-bit SDBF) and the delay profile data is 512 bits (for single-bit SDBF) or 448 bits (for 3-bit SDBF) in total as shown in Fig. 5.5.

Fig. 5.5. System I/O configuration of the SDBFs (the figures in bracket are meant for 3-bit SDBF).
The synthesis results are tabulated in Table 5.4. The hardware complexity is measured in terms of gate count with simple NAND gate as unit reference. Benchmark comparison is referenced to the cascaded reconstruction method as the multi-bit SDBF was proposed to provide comparable image quality to the cascaded reconstruction SDBF, which is better than the post-delay reconstruction SDBF.

Table 5.4. Hardware synthesis results of various SDBF.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>160</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td>Pre-delay reconstruction</td>
<td>Post-delay reconstruction</td>
</tr>
<tr>
<td>Total area (gates)</td>
<td>2253298</td>
<td>148769</td>
</tr>
<tr>
<td>Normalized area</td>
<td>6.42</td>
<td>0.42</td>
</tr>
<tr>
<td>Total dynamic power (mW) @ 1.8V supply</td>
<td>1747.1</td>
<td>299.34</td>
</tr>
<tr>
<td>Normalized power</td>
<td>3.11</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Although the bit-length of the 3-bit SDBF is tripled, the delay length is reduced due to lower beamforming frequency used. Hence, it can achieve a 44% the hardware saving compared to cascaded reconstruction SDBF, whereas the cascaded reconstruction SDBF saves up 85% (which is obtained from $\left(1 - \frac{1}{6.42}\right) \times 100\%$) hardware as compared to the pre-delay reconstruction SDBF. On the other hand, the reduced beamforming frequency allows the 3-bit SDBF to have a more relaxed timing requirement and also a more power efficient structure. It saves up to 62% of the power consumed by cascaded
reconstruction SDBF, whereas the cascaded reconstruction SDBF saves up 68% (which is obtained from \(1 - \frac{1}{3.11}\times100\%\)) power compared to the pre-delay reconstruction SDBF.

The hardware comparison considers only the digital beamformer. It does not include the single-bit and 3-bit sigma-delta modulators. It has been reported by R. Gaggl et al in [61] that for a 3-bit sigma-delta modulator, good layout practice is sufficient to handle the DAC linearity problem. There is no need for any additional correction techniques. Hence the only difference between the 3-bit sigma-delta modulators and the single-bit sigma-delta modulator is the internal quantizers used. The area of the 3-bit modulator in [61] is dominated by the capacitors. The internal 3-bit quantizer, which is implemented using flash converter, occupies only 15% of the entire area in Fig. 5.6, as reported in the paper. The area of a single-bit quantizer can be estimated to be \(\frac{1}{7}\) of a 3-bit quantizer as the area is mainly dominated by the comparators it used. Hence, the difference between the area of a 3-bit modulator and single-bit modulator is about 13% of the entire area of a 3-bit sigma-delta modulator. This area increment for using a 3-bit sigma-delta modulator is not significant as compared to the saving that can be achieved in the digital circuitry, which is around 44% of that of the cascaded reconstruction SDBF.
Fig. 5.6. Layout of the test-chip of a 2\textsuperscript{nd}-order 3-bit sigma-delta modulator done by R. Gaggl \textit{et al.}

### 5.4 Summary

A power and area efficient multi-bit SDBF has been developed which reduces the pre-delay quantization noise in the modulator output signal by increasing the number of bits of the sigma-delta modulators. The number of bits is selected according to the required dynamic range for each channel. For the case in which 39dB dynamic range is required in each channel, 3-bit resolution is sufficient to suppress the artifact problem. Thus, no extra dynamic focusing artifact correction technique is needed. The bit increment also allows the usage of lower beamforming frequency.
Simulation results using real point and cyst phantom data show that the multi-bit SDBF can effectively reduce the dynamic focusing artifacts. The image quality achieved is comparable to the cascaded reconstruction SDBF.

The 3-bit SDBF developed also has 44% digital hardware saving and consumes 62% less power compared to the cascaded reconstruction SDBF as shown by the synthesis result.
Chapter 6

CONCLUSIONS AND FUTURE WORK

This thesis presented a detailed study on the cause of the artifact problem when dynamic focusing is applied to a SDBF. This study was done in order to facilitate the development of a SDBF with improved image quality. Based on the study, a cascaded reconstruction SDBF was thus developed. Further investigation led to the development of an efficient multi-bit SDBF structure, which can achieve a significant improvement in terms of both hardware and power over the cascaded reconstruction SDBF. The works in these 3 areas are summarized in Section 6.1 whereas the possible future work is presented in Section 6.2.

6.1 Conclusions

The study of dynamic focusing artifacts shows that when the sequence of a sigma-delta modulated bit-stream is disrupted during dynamic focusing, some amount of the high frequency quantization noise would be shifted to the signal baseband and causes the artifacts in the final image output. The degree of the artifact problem is found to be proportional to the amount of quantization noise present in the sigma-delta modulated bit-stream. It was concluded that a possible solution to the artifact problem is to reduce the quantization noise level before the dynamic focusing is performed.
Based on the study, a cascaded reconstruction SDBF was proposed and developed by the author. It utilizes a multiplierless box-car filter to perform the pre-delay filtering in each channel, thus, reducing the dynamic focusing artifact. The tap size of the box-car filter is selected according to the pre-delay SQNR value required for each channel to achieve a certain PSNR output. For a system with 128 channels, OSR of 11, an 8-tap box-car filter is sufficient to provide a 60dB PSNR for the final image output, which is typically used in ultrasound application. Real data simulation has shown that the proposed method can provide comparable image quality compared to the pre-delay reconstruction method while having 85% of hardware savings and 68% of power savings.

Based on the same principle, the multi-bit SDBF was proposed as a better solution to the cascaded reconstruction method. It utilizes multi-bit sigma-delta modulators to digitize the received ultrasound echoes rather than the single-bit modulators. The bit increment improves the pre-delay SQNR and hence, reduces the dynamic focusing artifacts. The bit resolution is selected according to the required pre-delay SQNR for a certain PSNR output. For a system with 128 channels, OSR of 11, 3 bit resolution is sufficient to provide a 60dB PSNR for the final image output. Real image simulation has shown that the proposed multi-bit SDBF can achieve good image quality as the cascaded reconstruction SDBF. Besides, the bit increment can also reduce the beamforming frequency of the system. As a result, multi-bit SDBF has a hardware complexity that is 44% less than the cascaded reconstruction SDBF and 62% lower in terms of power consumption.
By developing these low cost, low power SDBF structures, it allows the advancement of ultrasound imaging systems in two directions: the portable handheld machine, and the high-end, high quality machine. The simple hardware and low power structure permits the integration of the whole beamformer into a single chip. When this single chip is mounted onto the transducer side, it can eliminate the expensive cable needed to transmit multiple channels of analog signal between the transducer and the console.

On the other hand, the low cost, low power SDBF proposed is also useful for the development of the high-end large channel count system (e.g. 3D ultrasound imaging system). It can significantly reduce the cost and power consumption when the channel count is large without compromising the image quality.

Simulations have shown that the cascaded reconstruction SDBF and the multi-bit SDBF developed can provide good performance in the B-mode imaging. However, further works are still needed to verify the performance of the proposed SDBF in Doppler imaging application. It would also be useful if the proposed method can be applied in coded excitation ultrasound imaging to develop a more compact ultrasound imaging system. These proposed future works are discussed in the following sections.

The works of this thesis have been published in various papers. The publication list can be found in Section 1.6.
6.2 Future work

This section suggests the possible future work that can be carried out based on the cascaded reconstruction and multi-bit SDBF proposed.

6.2.1 Doppler mode imaging

Besides displaying B-mode images, current state-of-art ultrasound system also overlays color flow images on top of the gray scale images to indicate the moving scatterers within the tissues. The system usually employs the same beamformer for both B-mode and color flow imaging.

Color flow images are generated by repeatedly firing an ensemble of pulses over a region of interest and detecting the time difference between the consecutive reflected echoes. The acquisition of color flow data are usually achieved by autocorrelation method where the amplitude and phase information can be extracted [62].

Spectral Doppler is another mode that is commonly used to investigate blood flow in ultrasound imaging system [63]. It has a higher signal-to-noise (SNR) requirement than both B-mode and color flow imaging. The echoes reflected by blood are often lower than those reflected by tissue by 30-50dB. As a result, the Spectral Doppler would require at least 100-120dB system dynamic range to resolve the blood flow signal from the tissue signal.

Hence, further works are needed to verify the sensitivity of the phase information to the SNR level provided by the proposed SDBF. It is also important to design a SDBF system that is efficient for Spectral Doppler application.
6.2.2 Coded excitation

In order to increase the imaging depth of an ultrasound imaging system, high driving voltage is used to excite the transducer elements to produce high power ultrasound signal. Currently, the pulse amplitude generated by commercial scanners is already approaching the regulatory Mechanical Index (MI) limit for the safety of patients [64]. An alternative to increase the transmitting power of ultrasound system is by increasing the pulse duration of the ultrasound signal, but it compromises the resolution. In 1992, O’Donnell developed a coded excitation system that is able to transmit a longer encoded pulse without compromising the resolution [65].

It was also reported that it is feasible to provide similar signal-to-noise ratio (SNR) by exciting the transducer elements with reduced voltage level (e.g. 10V) using coded excitation technique compared to conventional ultrasound system with high excitation voltage (e.g. 80V) [66]. Using such a low excitation voltage will ease the integration of the transmitter with other front-end circuitries. Together with the SDBF architecture, the coded excitation system can bring a new level of breakthrough to the compactness of an ultrasound machine.

A coded excitation sigma-delta beamformer (CE-SDBF) was proposed [67], and it was found that when dynamic focusing is applied, the post compression SDBF suffers not only from the artifacts due to the disruption in sigma-delta modulated signal, but also from the artifacts caused by the disruption of the coded signal. To mitigate the artifact problem, a pre-compression structure can be utilized. However, it involves a complex decoder in each beamforming channel and resulting in high hardware complexity.
It is therefore desirable to have a more efficient CE-SDBF structure that can resolve the artifacts problem. It would be reasonable to start the investigation by applying the proposed cascaded reconstruction and multi-bit SDBF to the coded excitation system.
REFERENCES


Appendix A

BEAMFORMING

In ultrasound imaging, different transducer elements are activated to receive the reflected ultrasound echoes simultaneously. In order to extract the ultrasound echo from a certain focal point out of those from other area, beamforming is applied. The beamforming process is illustrated in Fig. A1.

Fig. A1. Beamforming process.

As shown in Fig. A1, the signal from the focal point arrives at different transducer elements at different timing. To synchronize the signal, the beamformer provides the
corresponding delay to each channel. Summation is then performed to amplify the synchronized signal. As a result, the echoes from other areas will be suppressed.
Sigma-delta beamformer (SDBF) was developed by replacing the multi-bit ADCs in conventional beamformer with sigma-delta ADCs (including sigma-delta modulators and reconstruction filters). Such a SDBF is known as pre-delay reconstruction SDBF because the signal reconstruction is done before the delay focusing process. The structure of pre-delay reconstruction SDBF is shown in Fig. B1.

Fig. B1. Pre-delay reconstruction SDBF structure.
As shown in Fig. B1, each beamforming channel consists of a highly complex pre-delay reconstruction filter to reconstruction the multi-bit signal from the sigma-delta modulated signal.

In an attempt to reduce the hardware complexity of SDBF, Noujaim et al. developed a post-delay reconstruction SDBF by moving the signal reconstruction process after delay focusing process as shown in Fig. B2.

The post-delay reconstruction SDBF needs only one reconstruction filter for the entire beamformer and the depth of the memory device in each channel is reduced to single bit. It significantly reduces the hardware complexity of the SDBF. However, post-delay reconstruction SDBF suffers from artifact problem when dynamic focusing is applied.
MULTI-BIT ADC SPECIFICATION

C.1 Doppler mode

For Doppler processing, a 100dB resolution ADCs are needed if digital processing is applied because the ultrasound echoes reflected by blood flow is very weak. For a 128 channel array with rectangular windowing, the apodization power gain is

\[ G_{SNR} = \left( \frac{\sum_{n=1}^{N} w_n}{\sum_{n=1}^{N} w_n^2} \right)^2 = 21dB \] (C1)

Hence, in order to obtain a total DR of 100dB, the DR needed for each channel is 79dB.

The DR of an N-bit ADC can be expressed in terms of dB as

\[ DR = 6.02N - 1.76 \text{ (dB)} \] (C2)

N was found to be 13.4 in order to obtain a DR of 79dB. That means the ADC needs to have at least 14 bits to fulfill the DR requirement for high resolution machine.
C.2 High quality B-mode

In order to obtain high quality ultrasound image in 80dB dynamic range, the transmission and reception sidelobe level cannot be higher than -40dB each. To achieve a maximum average sidelobe level of -40dB for both transmission and reception (for 128 channel array with rectangular windowing), the sampling frequency required to achieve sufficient delay resolution can be calculated using the peak random quantization sidelobe (voltage) level equation

\[
SL_{\text{peak}} \approx \frac{\pi}{r_f} \left( \frac{4.6 \text{ENBW}}{3 N_{\text{ch}}} \right)^{1/2}
\]

where ENBW is the equivalent noise bandwidth which is 1 in this case when rectangular window is used for apodization, \(r_f\) is the ratio of sampling frequency to signal bandwidth (which is equal to 2\times OSR), and \(N_{\text{ch}}\) is the number of elements, under the condition that the error components are uncorrelated from channel to channel.

For a system with 128 channel array with rectangular windowing, \(r_f\) is found to be 35 from (C3). Hence the sampling frequency needs to be at least 35 times higher than the central frequency of the ultrasound signal, equivalent to an OSR of 17. For a signal with 5MHz bandwidth, the sampling frequency will be 175MHz.