Numerical simulation on composite properties
(C100)

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Abstract

Composites, which consist of two or more separate materials combined in a structural unit, are made from various combinations of different materials. The composite properties are governed by the characteristics of the local materials and their micro-structures. Since composite is the most important material in structural field, there is a strong desire for new composite materials to be developed with improved properties to obtain longer span lengths and longer lifetimes.

The aim of this project is to conduct a numerical simulation on a 2-phase rectangular-bar micro composite at various volumetric percentage combinations of materials with an aid of a finite element software program, Ansys9.0. The thermal properties of the modeled rectangular bar and results of the simulations are then interpreted and discussed.

The first part of the project is to perform the thermal analysis on the same composite bar. Initial and boundary conditions are set and steady state time and thermal conductivity of the bar are calculated from the experiment. The second part of the project is to perform the thermal analysis on the air-aluminum composite bar using the same initial and boundary conditions.

The findings of the experimental data are then compared with the theoretical values using theories such as Rules of Mixtures (RoM), Hatta and Taya (H&T) Model and Hesselman and Johnson (H&J) Model.
Acknowledgement

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Chapter 1: Introduction

1.1 Background
In the present world, traditional materials such as metal, ceramics and polymers cannot fulfils the need of tremendously booming science and technology in the industry. As a result, composite materials have been created by combining two or more materials, which have different properties to fulfil the needs. These new composite materials provide some unique properties or a combination of properties, which are worked together by their parent materials.

For example, a material of light density, high stiffness, good abrasion, fatigue, impact and corrosion resistance, is required in an aerospace industry. Generally, strong materials are relatively dense while increase in the strength or stiffness results a decrease in impact resistance. Therefore, a creation of material that can provide all desired properties will be inevitably necessary in this case. So, engineers have to combine two or more materials to achieve the desired properties. However, the question arises is “how many percentage of each combined material in the composite will provide optimum requirement of properties.” Thus, geometry of inhomogeneities through the Eshelby tensor (Eshelby J. , 1957) is often utilized to evaluate average properties of composites analytically.

Hence, Powerful Finite element software such as Ansys, Abacus, etc are used to do the simulations to forecast of the result of combination of various materials. Then real experiments are conducted to verify the data obtained from Finite element software and a new composite is established.
1.2 Objective
The main purpose of this project is to model and analyze a two-phase composite bar with the help of a finite element software program, Ansys.

First, thermal analysis for the aluminium-tungsten bar will be performed. Then, thermal analysis will be performed on the aluminium-air bar. And experimental results are plotted and compared with the theoretical results.

1.3 Scope
All necessary theory and law on solid mechanics as well as heat and mass transfer are studied. In addition, understand the concept and application of finite element analysis and also the utilization of finite element software, Ansys. Based on the mechanical and thermal constrains and requirement, models are constructed and analyzed in the Ansys. After that, experimental data are plotted as graphs in the Microsoft Excel.

1.3.1 Flow of project

Chapter one: Describe the introduction of the report and explains the objective and scope of the project.

Chapter two: Cover the literature review, which comprises composite materials, mechanics of materials, heat and mass transfer and finite element method.

Chapter three: Explain the methods and procedures conducted for the experiments in this project.
Chapter four: Discuss Equivalent thermal conductivity of materials in series combination and also in parallel combination. Tabulate and plot the experimental data, and discuss on the results obtained.

Chapter five: Conclude and comment on the project.
Chapter 2: Literature Review

2.1 Composite Materials

Composite materials (or composites for short) are engineered materials made from two or more constituent materials with significantly different physical or chemical properties and which remain separate and distinct on a macroscopic level within the finished structure.

Many composites are composed of just two phases; one is termed as the matrix phase, which is continuous and surrounds the other phase called the dispersed or reinforcing phase. The dispersed phase material may be in the form of fibers, particles, or flakes.

Composite can exist naturally or artificially made. Based on the commercial and industrial purpose, composite can be categorized as shown in Fig. (2.1).

![Classification of composites](image)
2.1.1 Advantages of composite materials

It is obvious that composites can offer several other advantages over conventional metal alloys ceramics and polymeric materials. These included improved strength, stiffness, fatigue and impact resistance, thermal conductivity, corrosion resistance and so on. Using composites is more efficient and cost-effective than traditional materials in many industrial applications.

2.1.2 Disadvantages of composite materials

Compared to traditional materials, composites are more expensive due to the involvement of complex manufacturing and processing techniques. In addition, owing to the complexity in nature of combination of different material, analysis on the properties of composite is experimentally and computationally more complicated. Moreover, manufacturing flaws such as voids and cracks are not easily detected.

2.1.3 Rule of mixture

For better reinforcement of a composite, the particles (the dispersed phase) should be small and evenly distributed throughout the matrix. Two mathematical equations can be formulated for dependence of the Young’s modulus on the volume fraction of constituent phases for a two-phase element.
These equations are termed as Rules of mixtures equations, which can predict that the Young’s modules fall between an upper bound or limit,

\[ E_{\text{eff}} (u) = v_m E_m + v_p E_p \]  \hspace{1cm} (2.8)

and lower bond or limit,

\[ E_{\text{eff}} (l) = \frac{E_m E_p}{v_m E_p + v_p E_m} \]  \hspace{1cm} (2.9)
2.2 Heat and Mass Transfer

Heat transfer can be defined as thermal energy in transit due to a temperature difference in a medium. Transfer of thermal energy can occur by means of conduction, convection, radiation or any combination of these.

Conduction refers to heat transfer that will occur in stationary medium, solid or fluid, where a temperature gradient exists. Energy is transferred by electron diffusion or phonon vibration.

Convection typically occurs in moving medium or fluid. Convection heat transfer comprises two mechanisms: energy transfer due to random molecular motion (diffusion) and energy transfer due to bulk or macroscopic motion of the fluid.

Radiation is the transfer of heat by electromagnetic radiation or by photons. While the transfer of thermal energy by conduction or convection requires the presence of a medium, radiation does not and actually, it can most efficiently occur in a vacuum.
2.3 Temperature

Temperature is a function of the x-coordinate only when there is conduction in one-dimension and heat is transferred in this direction. In Figure 2.8, a rectangle bar separates the two fluids of different temperatures. Heat transfer occurs by convection from the hot fluid at $T_{s,1}$ to one surface of the wall at $T_{s,1}$ by conduction through the wall, and by convection from the other surface of the wall at $T_{s,2}$ to the cold fluid at $T_{s,2}$.

**Fig 2.3) Resistances of the composite bar**
2.3.1 Temperature Distribution

The temperature distribution in the wall can be determined by solving the heat equation with the proper boundary conditions. For steady-state conditions with no distributed source or sink of energy within the wall, the appropriate form of the heat is equation 2.12

$$\frac{d}{dx}(k\frac{dT}{dx}) = 0$$

(2.12)

Hence, from equation 2.12, it follows that, for one-dimensional, steady-state conduction in a plane wall with no heat generation, the heat flux is constant, independent of x. If the thermal conductivity of the wall material is assumed to be constant, the equation may be integrated twice to obtain the general solution.

$$T(x) = C_1x + C_2$$

To obtain the constants of integration, $C_1$ and $C_2$, boundary conditions must be introduced. Conditions of the first kind at $x = 0$ and $x = L$, in which case

$$T(0) = T_{S,1} \text{ and } T(L) = T_{S,2}$$

Applying the condition at $x = 0$ to the general solution, it follows that similarly, $T_{S,1} = C_2$

at $x = L$,

$$T_{S,2} = C_1L + C_2 = C_1L + T_{S,1}$$

In which case

$$\frac{(T_{S,2} - T_{S,1})}{L} = C_1$$

(2.13)
2.3.2 Equivalent thermal conductivity in series

A composite bar of length, L and cross-sectional area, A consists of materials, A and B having thermal conductivities of $K_A$ and $K_B$ respectively, which are connected in series as shown in below diagram. Temperatures at front, middle and back surfaces are kept constant at $T_{s,1}$, $T_0$ and $T_{s,2}$ respectively.

The equivalent thermal conductivity of the bar can be derived as follow.

$$T_A = T_{s,1} - T_0$$

$$T_B = T_0 - T_{s,2}$$

Change in temperature from origin to $L$, $T_{AB} = T_{s,1} - T_{s,2} = T_A + T_B$

$$\frac{q}{A} = K_{eq} \frac{\Delta T_{AB}}{L} = K_A \frac{\Delta T_A}{L_A} = K_B \frac{\Delta T_B}{L_B}$$

$$T_{AB} = L \frac{q}{K_{eq} A}$$

$$T_A = L_A \frac{q}{K_A}$$
\[ T_B = L_B \frac{q}{K_B} \]

\[
L \frac{q}{K_{eq} A} = L_A \frac{q}{K_A A} + L_B \frac{q}{K_B A}
\]

\[ \frac{L}{K_{eq}} = \frac{L_A}{K_A} + \frac{L_B}{K_B} \]

\[ K_{eq} = \frac{L}{\left( \frac{L_A}{K_A} + \frac{L_B}{K_B} \right)} \quad (2.14) \]

For example, 90% of material A combines with 10% of material B in series. Then the equivalent thermal conductivity will be

\[ K_{eq} = \frac{1}{\left( \frac{0.9}{K_A} + \frac{0.1}{K_B} \right)} \]

### 2.3.3 Equivalent thermal conductivity in parallel

A Composite bar shown in the diagram below consists of two materials A and B in parallel, having thermal conductivity of \( K_A \) and \( K_B \). The Bar is of length \( L \) and with a cross-sectional A. Temperature at the two surfaces is kept constant at \( T_{s,1} \) and \( T_{s,2} \).
The equivalent thermal conductivity of the bar can be derived as follow

\[ q = q_A + q_B \]

\[ \frac{q}{A} = K_{\text{eq}} \frac{T_{s,1} - T_{s,2}}{L} \]

\[ \frac{q_A}{A_1} = K_A T_{s,1} - T_{s,2} \]

\[ \frac{q_B}{A_2} = K_B T_{s,1} - T_{s,2} \]

\[ A K_{\text{eq}} \frac{T_{s,1} - T_{s,2}}{L} = A_1 K_A \frac{T_{s,1} - T_{s,2}}{L} + A_2 K_B \frac{T_{s,1} - T_{s,2}}{L} \]

\[ K_{\text{eq}} = \frac{(A_1 K_A + A_2 K_B)}{A} \quad (2.15) \]

For example, 50% of material A combines with 50% of material B in parallel. Then the equivalent thermal conductivity will be

\[ K_{\text{eq}} = 0.5 K_A + 0.5 K_B \]

### 2.4 Finite element method

The finite element method was developed to treat practical problems complex in both geometric shape and material properties. This method partitions a complicated structure or system into finite number of elements having simple geometric shape and simple material properties, whose behavior can be described by appropriate governing equations, making use of substructuring technique to consider a complex problem in parts.
2.5 Concepts of finite element method

In the finite element method, the continuum is idealized as a structure consisting of a number of individual elements connected only at nodal points, as indicated in the figure below. The finite element method is extremely powerful since it enables continua with complex geometrical properties and loading conditions to be accurately analysed.

2.5.1 Fundamental requirements of finite element method

Three basic conditions for the cause of the internal forces and deformations in a structure which must be observed are listed below:

1) The equilibrium of forces;
2) The compatibility of displacements; and
3) The laws of material behaviour.

The first condition simply requires that the internal forces balance the external applied loads. Compatibility requires that the deformed structure fits together which means that
the deformations of the members are compatible. Before this 2\textsuperscript{nd} condition can be used it is essential to know the relationship between load and deformation for each component of the structure. This relationship, which in problems of linear elasticity reduces to the use of Hooke’s Law, is the third condition.

### 2.5.2 Stiffness and flexibility methods of matrix analysis

Primarily, the matrix methods of structural analysis may be formulated in two different ways:

1) Stiffness (displacement) method
2) Flexibility (force) method

These two methods differ in the order in which their basic conditions of nodal equilibrium and compatibility are treated. In the stiffness method, the displacement compatibility conditions are fulfilled and the equations of equilibrium set up and solved to yield the unknown nodal displacements. As for the flexibility method, the conditions of nodal equilibrium are first satisfied and the equations arising from the need for compatibility of nodal displacements solved to yield the unknown forces in the members.

### 2.6 Finite element analysis

The Finite Element Analysis (FEA) is a computer simulation technique used in engineering analysis. It uses a numerical technique based on the finite element method (FEM). In general, there are three phases in any computer-aided engineering task which include Pre-processing which defines the finite element model and environmental factors to be applied to it, the Analysis solver which generates solution of finite element model, and Post-processing of results using visualization tools.
Chapter 3: Thermal analysis on the composite bar for experiment 1 and 2

3.1 Experiment 1 description
The experiment is carried out on a two-phase composite material (tungsten-aluminum composite rectangular bar) to find out the change in thermal properties when subjected to constant heat flux.

![Isometric and front view of solid element of 20 nodes](image)

3.1.1 Modeling of the composite bar
i) The procedure used in experiment 1 is carried out to model and random the elements and the nodes in the bar. 20-node element (solid90) is used for meshing. The properties of the two materials are as shown in the table below.

<table>
<thead>
<tr>
<th>No</th>
<th>Material</th>
<th>Thermal conductivity (W/m.K)</th>
<th>Specific heat (kJ/kg.K)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tungsten</td>
<td>178</td>
<td>0.138</td>
<td>19300</td>
</tr>
<tr>
<td>2</td>
<td>Aluminum</td>
<td>247</td>
<td>0.900</td>
<td>2710</td>
</tr>
</tbody>
</table>

Table 3.1) Thermals properties of Tungsten and Aluminum
ii) The dimension of the bar is modeled as 0.001m x 0.001m x 0.01m.

Fig 3.2) Meshed rectangular bar

iii) The rectangular composite bar is meshed into 10000 fine elements.

iv) The first volumetric composition consist 10 % tungsten and 90 % aluminum, with a 10 % increment of tungsten in progressive composition. (Appendix A)

v) The model is applied with initial conditions and boundary as show in the diagram below.

Fig 3.3) Schematic drawing of composite rectangular bar with boundary conditions applied
vi) Using ANSYS, the back surface in the z-direction is maintained by a constant temperature of 300k

![Fig 3.4) Applying constant temperature on the back surface in Z direction](image1)

vii) Constant heat flux of $10^5$W/m² is applied to the front surface of the bar

![Fig 3.5) Applying constant temperature on the frontal surface in Z direction](image2)
v) The reference temperature of the composite bar is kept at 300k.

![Fig 3.6) Initial temperature of the composite bar kept at 300k](image)

The time set in the program is 0.01s and the sub-step defined in the program is 50. This translates to each subset being calculated and recorded at 0.0002s interval until 0.01s is reached.

### 3.1.2 Obtaining the result

After the analysis, steady state time, $T_s$ is obtained from the result viewer. Variation in the temperature can be observed by changing the sub-steps as shown in the three figures below. Once there is no change in the temperature as shown in the figure below, the sub-step time is the steady state time.
Fig 3.7) Temperature distribution of the bar at 0.003s

Fig 3.8) Temperature distribution of the bar at 0.0052s
Fig 3.9) Temperature distribution of the bar at 0.008s

Node in the centre of the front surface is selected as shown in the figure below.

Fig 3.10) Selection of the centre node of the frontal surface
The 201 nodal temperatures from the bar is extracted from result viewer. These nodal temperatures are substituted into the heat flux equation, \( K = \frac{q''L}{\Delta T} \) and the thermal conductivity of the composite bar is obtained. The mean value of the thermal conductivity is taken. (Appendix B)

For the second method, the temperature values are extracted from all the nodes at the front plane as shown in figure below. The temperature values extracted will be used to calculate the equivalent thermal conductivity for the composites. (Appendix B)
3.2 Thermal analysis on the aluminum and air composite bar for experiment 2

3.21 Experiment 2 description
The objective of this experiment is to predict how the volumetric fraction of an insulator (air) in a metal composite affects the ability of the composite to transfer heat by conduction. The thermal conductivity of air is approximate to an insulator.

<table>
<thead>
<tr>
<th>No</th>
<th>Material</th>
<th>Thermal conductivity (W/m.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aluminum</td>
<td>247</td>
</tr>
<tr>
<td>2</td>
<td>Air</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 3.2) Thermals properties of Aluminum and Air

3.22 Method and procedure
The modeling and boundary conditions is the same as the one is experiment 1. Boundary conditions set are as shown below.

- Whole bar – Initial temperature of 300K
- Front plane – Heat flux of $1 \times 10^5$ Wm$^{-2}$
- Back plane – Constant temperature of 300K

The solution is solved through running an lgw script. The underlying principle is that the composite bar is divided into segments and the nodal temperature is taken from alternate segment where there are 8 nodes on the plane.
From this process, the final averaged 101 temperature values are calculated and taken to plot the temperature profile for each volumetric fraction.

Fig 3.13) Schematic drawing of composite rectangular bar with different segment

Segment1  Segment2  Segment3
Chapter 4: Results and Discussions for Experiment 1 and 2

4.1 Discussion on Time to Reach Steady-State

As shown above, the composite of 60% tungsten and 40% aluminum illustrate that the transient heat conduction occur when the composite bar is first subjected to a heat flux of \(10^{-5}\) Wm\(^{-2}\). From equation 2.12, a plot of temperature against displacement should give a straight line at steady state. Hence when the time approaches steady-state time 0.0044s, steady-state heat conduction is reached.
Figure 4.3 illustrate time taken to achieve steady state conduction heat transfer with different volumetric fractions of Tungsten. Although material with higher thermal conductivity supposed to have a shorter steady-state time, result of the composite with 90% aluminum and 10% tungsten do not concur with the theory. Based on the experimental value, the composition is about 40% Tungsten and 60% Aluminum took the shortest time of 0.0044s. In addition, one can observe that although the time increases accordingly with the increment of Tungsten volumetric fraction, it is not a linear trend. It should be noted that each subsets is divided into 0.002s so the deviation is about ±0.002s.
4.2 Discussion on Thermal Conductivity Calculation Method

Fig 4.3) Comparison between equivalent thermal conductivities from experiment, in series, and in parallel

As shown in the graph above, the experimental graphs are located between the series and the parallel lines. As the heat conduction is in steady state, constant temperature is maintained throughout thus the deviation in thermal conductivity values between the two methods are negligible.
4.3 Comparison of Experimental Thermal Conductivity Value with Theoretical Value

As mentioned previously at section 2.4.2 and 2.4.3, different equations for heat conduction in series and parallel are used to calculate the equivalent thermal conductivity values. The RoM (Appendix C) for thermal conductivity is identical to the equation of heat conduction in parallel. Thermal conductivity values are calculated for both series and parallel (RoM) heat conduction and compare with the experimental values. The results are then tabulated (Appendix D) and used to plot Fig. 4.3. Due to the fact that the elements in the composite bar are randomly distributed, it is rational to have the experimental curve in between the series and parallel curve.

The overlapping curves that the H&J and H&T models (Appendix C) produce indicate that the values are of slight deviation to the effective thermal conductivity. We can conclude that the experimental result of the thermal conductivity is justifiable as it is of close proximity to the two models.
4.4 Results and Discussions for Experiment 2

**Figure 4.4** Graph of log (temp) vs distance (0% to 30%)

**Figure 4.5** Graph of log (temp) vs distance (30% to 100%)
One can observe that the curves of pure air and 10% of aluminum are very similar, this is because both cases behave as excellent insulators, thus a very high temperature is required to maintain the heat flux through the bar. Also, the thermal gradient is steeper than the other curves as the thermal conductivity is very low for the 10% of aluminum and 90% of air.

As the percentage of aluminum increases for the other respective curves, the temperature required to maintain the heat relatively decreases. As seen in figure 4.11, higher temperatures can be seen very near the frontal surface. A probable reason could be due to the majority of the air elements randomly distributed at the frontal surface. The thermal gradient also become gentler as aluminum, which have much higher thermal conductivity, increases accordingly.
Chapter 5: Conclusion

The main objective of the project is to research on the thermal properties of a composite bar through the employment of finite element approach. The secondary objective is to analyze whether the presence of air elements in an aluminum bar will result in any deviation on the thermal properties.

In the first experiment, thermal analysis of a rectangular composite bar is conducted. The two materials chosen are tungsten and aluminum. From the result, we can observe that the value of the thermal conductivities of the composite bar lies between theoretical series and parallel equivalent conductivities. Since the simulation is run using composite bar with Tungsten and Aluminum elements being randomized, it is logical for the experimental curve to fall between series and parallel curve. The general trend observed is that the increment of tungsten will result in a longer time to reach steady state. Ultimately, the composite still behave as a quality conductor.

The second experiment involved the thermal analysis of the same rectangular composite bar. All the boundary conditions are kept constant except that tungsten is replaced with air. The presence of insulating element strongly influences the thermal conductivities of the composite bar. With 90% of air and 10% of aluminum, the composite almost behave like a pure insulator though a linear temperature profile can still be obtained when the volumetric composition of aluminum increase. In conclusion, with adequate trial and error, an insulator of desired qualities is probable in the near future.
References

Journals cited


Textbooks

APPENDIX

Appendix A

Note: the turquoise colour shown in the bar is the tungsten while aluminum is the purple colour.

Fig A1) Composite bar with 10% Tungsten                        Fig A2) Composite bar with 20% Tungsten

Fig A3) Composite bar with 30% Tungsten                        Fig A4) Composite bar with 40% Tungsten
Fig A5) Composite bar with 50% Tungsten

Fig A6) Composite bar with 60% Tungsten

Fig A7) Composite bar with 70% Tungsten

Fig A8) Composite bar with 80% Tungsten

Fig A9) Composite bar with 90% Tungsten
Appendix B
Thermal Conductivity Calculation Method

There are two methods to obtain the thermal conductivity value, by extracting the temperature in different ways.

**Extraction of Temperature along the Central Nodes**

**Boundary Conditions:**
\[ \chi = 0, \quad q_0 = -k \frac{\partial T}{\partial \chi} = 1 \times 10^5 \frac{W}{m^2} \]
\[ \chi = L, \quad T = T_L = 300 K \]

At steady state:
\[ \frac{\partial^2 T}{\partial \chi^2} = 0 \]

Thus,
\[ T(\chi) = C_1 \chi + C_n \]  
(3.8)

\[ \frac{\partial T}{\partial \chi} = C_1 \]
\[ q_0 = -k C_1 \]

\[ \Rightarrow C_1 = \frac{-q_0}{k} \]  
(3.9)

\[ T(L) = C_1 L + C_n = T_L \]
\[ = -\frac{q_0 L}{k} + C_n \]

\[ \Rightarrow C_n = T_L + \frac{q_0 L}{k} \]  
(3.10)
By substituting equation (3.9) and (3.10) into equation (3.8), the final equation as shown below is obtained:

\[ T(\chi) = -\frac{q_a}{k} (\chi - L) + T_L \]  

(3.11)

The steady state temperature distribution along the centerline is extracted using the results viewer, by plugging in the coordinate (x-value), the thermal conductivity of the composite material is obtained. Along the centerline, there are total 202 nodes. Thus, 202 thermal conductivity values are obtained, and the 202 thermal conductivity values are averaged.

For each composition, 30 cases are run. Thus, there are 30 averaged thermal conductivity values. The final thermal conductivity value for each composition is obtained by averaging the 30 values of thermal conductivity.

**Extraction of Temperature at Front Plane’s Nodes**

In the result viewer, the temperatures of nodes on the front plane are extracted from the experiment and these data are averaged to obtain the temperature on the front plane. This temperature, \( T_1 \) is substituted into the heat flux equation (2.15) and the thermal conductivity of the composite bar is obtained.

\[ k = \frac{q_x'' L}{\Delta T} \]  

(2.15)

\[ k = \frac{30000}{(T_1 - 100)} \]  

(3.12)
Appendix C

Methods to Predict Effective Thermal Conductivity of Composites

Rules of Mixtures

Fourier’s law stated that the heat conducted (per unit time) is as below:

$$ q = A k \left( - \frac{\partial T}{\partial x} \right) $$  \hspace{1cm} (2.7)

While, normal force is related to normal strain by Hooke’s law as follows:

$$ F = A E \varepsilon $$ \hspace{1cm} (2.11)

Both the equation shows that Fourier’s law is analogue to Hooke’s law. By using this analogy, thermal conductivities of composites can be obtained. The heat-conducted q corresponds to the force F, the thermal conductivity $k$ to the Young’s modulus, $E$ and the temperature gradient $\frac{\partial T}{\partial x}$ to the strain, $\varepsilon$.

Thus, there will also be an expression for thermal conductivity which is similar to RoM. The effective thermal conductivity can be obtained by replacing $E_1$ in RoM by $k_1$ and $E_2$ by $k_2$. Below is the resulting expression.

$$ k_{eff} = \nu_1 k_1 + \nu_2 k_2 $$ \hspace{1cm} (2.12)

Hesselman & Johnson Model

Hesselman and Johnson derived a complicated expression that accounts for the conductivity of the fiber/matrix interface region that is less than perfect due to thermal
mismatch of the fiber and the host matrix (Shabana & Noda, 2008). Below is the expression:

$$k_{eff} = \frac{k_1 \left[ \left( \frac{k_2}{k_1} - 1 \right) v_2 + \left( 1 + \frac{k_2}{k_1} \right) \right]}{\left[ \left( 1 - \frac{k_2}{k_1} \right) v_2 + \left( 1 + \frac{k_2}{k_1} \right) \right]}$$  \hspace{1cm} (2.13)

**Hatta & Taya Model**

The analysis utilizes the equivalent inclusion approach for steady state heat conduction (Hatta and Taya, 1986), through which the interaction between the various reinforcing phases at finite concentrations is approximated by the Mori-Tanaka (1973) mean field approach. Hatta and Taya model is used to investigate the effective thermal conductivity of hybrid composite materials (Shabana & Noda, 2008). Below is the expression:

$$k_{eff} = k_1 + \frac{k_1 (k_2 - k_1) v_2}{k_1 + 0.5 \left[ v_2 \left( k_2 - k_1 \right) \right]}$$
Appendix D

Table of equivalent thermal conductivity at different compositions

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<th>Tungsten (%)</th>
<th>Aluminium (%)</th>
<th>Thermal Conductivity (W/m²)</th>
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