MODELING AND ANALYSIS OF MODULATION TECHNIQUES FOR BROADBAND POWER LINE COMMUNICATIONS

MA YINGHUI

School of Electrical and Electronic Engineering

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SUMMARY

With the increasing demand for data communications and home networking, broadband power line communications (PLC) has become an attractive alternative to cable and wireless modes of communications. The advantages of PLC arise mainly out of the fact that power lines are ubiquitous. Because of this unique characteristic, PLC has significant cost advantages over other means of communication solutions. However, the use of PLC still faces several challenges. These challenges are mainly caused by the fact that PLC is based on the existing network that was not explicitly designed for communications. When a signal that carries information is transmitted into the power line, it is subject to a number of noise interferences and affected by the multipath effect of the power line channel. To overcome these challenges, robust modulation techniques are required for broadband PLC systems. Among them, modulation techniques like orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) can provide very good performance for broadband PLC systems.

So far, no analytical models addressing the modeling and performance analysis of OFDM and CDMA systems for broadband PLC under the power line multipath and impulsive noise effects are available. Therefore, this research seeks to develop analytical models to analyze the performance of the OFDM and CDMA systems when they are subject to both the multipath effect and noise interference; and to verify these analytical models using simulations through the good correspondence between the analytical and simulation results.

In the modeling and analysis of the OFDM system, the effect of guard intervals on the OFDM system is analyzed by developing analytical models and simulations both with and without guard intervals. From the analysis, it is shown that the OFDM system can mitigate the adverse effect of impulsive noise; only heavily disturbed impulsive noise can seriously interfere with the bit error rate (BER) performance of the OFDM system. It is also shown that the adverse effect of multipath is more severe than that of impulsive noise. Using a suitable length of guard interval can obviously improve the BER performance of the OFDM system. As the longer guard interval tends to be
inefficient in the use of signal transmission power and system bandwidth, the system should be configured such that it uses an optimum guard interval that would achieve the best BER performance. This optimization method to estimate the optimum guard interval can be considered when implementing the OFDM system. In the modeling and analysis of the CDMA system, the CDMA systems with and without a RAKE receiver are studied, by developing analytical models and then verifying their accuracies using simulations. The optimized parameters of the RAKE receiver CDMA system to achieve the optimum performance in a power line channel are studied. There are two frequently used spread spectrum systems, namely the direct sequence code division multiple access (DS-CDMA) system and the frequency hopping spread spectrum (FHSS) system. The DS-CDMA system is studied in this thesis and by default is named as CDMA system for simplicity.

Finally, based on the analytical models that are developed and verified by simulations, a comparison is made between the BER performance and the optimum overall data rate of the OFDM and CDMA systems, using the criteria of the same bandwidth occupation, the same transmission power for each user, the same total number of users in the system and the same power line channel. The OFDM system without multiple access scheme cannot support multiple users’ access. When the comparison of OFDM and CDMA systems is made, the multiple access scheme of frequency division multiple access (FDMA) is used in the OFDM system. Through the comparison, it is found that when the bandwidth, transmission power and channel responses are given, there exists an optimum carrier number that will achieve the best BER performance for the OFDM system. Further increasing the carrier number beyond this optimum does not improve the BER performance. We could improve the BER performance by increasing the transmission power, but need to be cautious about electromagnetic interference (EMI) problem. Unlike the OFDM system without forward error correction (FEC) in which the performance of the BER is limited by the maximum transmission power, the performance of the BER in the CDMA system can be improved by sacrificing the data rate, and subsequently increasing the processing gain without increasing the risk of a potential EMI problem. Through the comparison, it is also found that the adverse effect of impulsive noise on the BER performance of the CDMA system is worse than that on the BER performance of the OFDM system.
CHAPTER 1 INTRODUCTION

This chapter provides a brief introduction to power line communications (PLC), including its applications, advantages, standards and frequency regulations before presenting the motivations, objectives and contributions of this thesis. The organization of this thesis is also listed in this chapter.

1.1 Introduction to Power Line Communications (PLC)

Power line communications (PLC) is a technique of transmitting information using distributed power line wires or cables. Digital information is modulated by carriers in the form of an analog signal with frequencies ranging from several kHz up to 30MHz (depending on the different applications). The amplitude of the analog carrier signal is usually less than 10Vp-p. It is coupled into 50 or 60Hz AC and a high voltage (110V – 240V AC) power line network, through dedicated coupling circuits and signal amplifiers.

Depending on the data rate, PLC can be divided into two categories: narrowband PLC which has lower data rates of several kbps, and broadband PLC with data rates ranging from several hundreds kbps to 200Mbps. This 200Mbps broadband rate was recently achieved both by the world’s leading PLC chipset development company, Intellon from USA [1], and Design of Systems on Silicon (DS2) from Spain [2]. Broadband PLC is widely used for high speed applications, such as Internet access, home networking and home entertainment. Narrowband PLC is used mainly in applications which do not require very high speed data rates, such as building controls, automatic meter readings and home automations. Such applications can be procured from several narrowband PLC companies, such as Yitran Communications [3].

The advantages of PLC arise mainly out of the fact that power lines are ubiquitous. Data can be transmitted to any location over power lines, as long as they are supplied with power. Because of this unique characteristic, PLC has significant cost advantages over other means of data transmission. It does not require high installation costs and is
easily modified to suit particular needs, and users have access to multiple access points across a wide range of locations. It has significant advantages in particular, over wireless communications channels which suffer from highly attenuated signals that can be obstructed by a number of barriers within buildings and whose RF circuits costs much more than the simpler coupling circuits of the PLC.

Since the idea of PLC was first developed, several competing standards have evolved. Several have been successfully accepted by segments of the industry, including X10 which was a front runner in the early stages. Current standards include HomePlug 1.0, Home Plug AV from HomePlug Power Line Alliance [4]-[5], UPA Digital Home Specification v1.0 from Universal Power Line Association [6], the European Telecommunications Standards Institute (ETSI) standards [7] and the working standard from Power System Communications Committee (PSCC), IEEE Power Engineering Society (IEEE PES) [8].

As power lines were initially designed for the transmission of power and energy, and not specifically for high-frequency PLC transmission, they are not typically shielded to prevent interference. Functioning almost like antennas, they emit radio energy and their use for PLC causes interference to radio communications. Since the end of 1991, the European CENELEC Standard EN50065 has been in force, governing the use of the narrowband PLC frequency range of 3 to 148.5kHz [9]. There are other somewhat different regulations applicable in the United States and Japan which cover frequency ranges up to 450kHz. With frequencies of up to 30MHz, broadband PLC is at the exact same frequency of the amateur and short wave radio. The Federal Communications Commission (FCC) has adopted a memorandum opinion and order on broadband over power lines, giving the go-ahead to efforts to promote PLC-driven broadband service to all Americans [10].

1.2 Research Motivations

With the ever increasing demand for data transmission capacity and the popularity of home networking, broadband PLC has become an attractive alternative to cable and wireless modes of transmission [11]. The main advantage of PLC is that power lines are already ubiquitous in many parts of the world, and their use for communications
will involve low installation costs. However, the use of PLC still faces several challenges. These challenges stem mainly from the fact that PLC is based on the existing network that was not explicitly designed for communications.

When a signal that carries information is transmitted into the power line, it is subject to a number of sources of interference or noise. These include the power supplies of electrical equipment during the power-energy conversion or the switching on and off of electrical equipment. A number of studies have shown that the noise in the power line is not simply an additive white Gaussian noise (AWGN) [12]-[18]. In fact, the noise can be categorized into five distinct types in [13]. Of these, three usually remain stationary and can be seen as background noise. The remaining two are time-variant and can be classified as impulsive noise, with a short duration and a high power spectral density (PSD).

Because the power line network is usually designed to support multiple units within a distribution transformer, and because each unit comprises a number of different branches, signals transmitted from the transmitter will propagate through several paths before reaching the receiver. The multiple signals interfere with each other in what is defined as the multipath effect of the power line channel [19]-[21]. This effect is far more pronounced in broadband PLC than in narrowband PLC. Furthermore, during peak power consumption hours, multiple types of electrical equipment are switched on, significantly reducing the impedance of power lines (sometime to less than 1 Ω). This very low impedance brings difficulties when high current are injected into the power lines to ensure the quality of the communication. To maximize its capability, PLC requires high level of drive capability as well as a low-cost line driver to inject the signal into the power line.

These are significant challenges, and researchers have put a great deal of effort into studying PLC. A comprehensive summary of these research results about the PLC channel, communications techniques, standard and regulation issues and market perspectives/applications can be found in [22]. The representative research results have been achieved in several main research areas of PLC. These research areas include the studying of noise in power lines [23]-[27], development of test beds [28] and analyzing tools [29], modeling of the power line multiple transmission channels [19]-[21], [30]-
CHAPTER 1: INTRODUCTION

[33], their modulations [34]-[36], coding [36]-[39] and medium access control (MAC) protocol design and analysis [40]-[42]. Some of these efforts have shown very good results.

This thesis aims to extend previously mentioned research works in broadband PLC and focus on modulation techniques, orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA). OFDM and CDMA are two modulation techniques relatively robust under the multipath effect and noise interference, especially the impulsive noise interference. They have been widely studied and successfully applied in wireless and wired communications. With the development of these techniques, fruitful research works of modeling and analysis of OFDM and CDMA have been published.

The study of modeling and analysis of impulsive noise effect on the wireless OFDM communication system can be found in [43]. The effect of wireless channel on the OFDM system is studied and results can be found in [44]-[46]. Considering the impulsive noise in power line communications is different from the impulsive noise in wireless communications, the findings of [43] are not completely transferable to our area of interest. Nevertheless, the authors’ methodology to analyze the effect of impulsive noise on the OFDM system still can be applied in the studies of PLC. Similarly, as the multipath channel characteristic of wireless communications is different from that of PLC, it needs to dedicatedly study the power line multipath effect on the OFDM system. Research works in [47] and [48] are such studies working on the bit error rate (BER) performance of the OFDM system under the PLC multipath effect. But in these studies, the analysis is given in terms of simulations only and no closed-form formulas are developed. In this thesis, the analytical model to analyze the effect of power line multipath on the OFDM system is developed and verified by simulations.

The results of modeling and analysis of multipath effect on the CDMA system can be found in [49]-[53]. Among them, result in [49] is one of such representative research results and is widely referred. In this study, the multiple access interference is considered and modeled as the Gaussian distribution. Results in [50] and [51] extend such study into the Nakagami multipath fading channel. In [52] and [53], the performance of the CDMA system is modeled in Rician fading channel and measured
indoor radio propagation channels. In these studies, the assumption of the Gaussian distribution for the multiple access interference and multipath interference is made quite accurate in the BER calculation. This methodology is used in the modeling and analysis of the CDMA system in the power line multipath channel which is similar but not identical with the channel in wireless communications.

The spread spectrum technique used for the CDMA system in this thesis is the direct sequence spread spectrum. The CDMA system discussed in this thesis actually means the direct sequence code division multiple access (DS-CDMA) system. But for simplicity, the term CDMA system, instead of DS-CDMA system, is used in this thesis. As the OFDM system requires a protocol to support multiple users’ access, frequency division multiple access (FDMA) is used in the system.

Mathematical modeling of PLC communication systems is possible by results achieved from research done on channel modeling. Some research has been done on OFDM [47], [54]-[62] and CDMA [34], [63] systems for PLC. Till now, no research has been published specifically on the use of the OFDM and CDMA systems under the power line channel, specifically using analytical modeling and joint verification with simulations. Therefore, the motivations of this research are:

- To work out modulations for PLC communication systems which can achieve satisfactory performance in the presence of the power line multipath effect and noise interference.
- To analyze the performance of the OFDM and CDMA systems in the presence of the multipath effect and noise interference, so as to develop more efficient modulation schemes for PLC.
- To find out how and how much the multipath effect affects the performance of the OFDM and CDMA systems.
- To find out how and how much the impulsive noise interferes with the performance of the OFDM and CDMA systems.
- To discover the key factors that determine the performance of the OFDM and CDMA systems.
- To optimize the performance of the OFDM and CDMA systems to achieve better performance in the power line channel.
• To compare the advantages and disadvantages of PLC in the OFDM and CDMA systems, using similar comparison criteria.

1.3 Objectives of This Research Work

In line with the above motivations, the primary objective of this research is to develop analytical models to analyze the performance of the OFDM and CDMA systems when they are subject to both the multipath effect and impulsive noise interference. Once the channel impulse response between the transmitter and receiver can be measured through the quick channel training scheme before real data transmission, the algorithms (using results obtained from the analytical models) for adaptively varying the parameters of the system to achieve the optimum performance can be pre-designed into the transceivers. In this way, the factors that affect the performance of the OFDM and CDMA systems can be determined using the analytical models available. It would also be possible for the algorithms to adaptively choose suitable parameters for the systems. To achieve this, the following secondary objectives are formed:

• To develop analytical models for both OFDM and CDMA systems that can be used to analyze their bit error rate (BER) under the multipath effect and impulsive noise interference of the power line channel.

• To verify these analytical models using simulations. Good correspondence between the analytical results and simulation results will be proof of the accuracy of the models developed.

• To study the effect of guard intervals on the OFDM system by developing analytical models and simulations both with and without such guard intervals. To optimize the length of guard intervals in the OFDM system to achieve better BER performance for a power line channel.

• To study the performance difference between the CDMA systems with and without a RAKE receiver, by developing analytical models and then verifying their accuracies using simulations.

• To optimize the parameters of the RAKE receiver in the CDMA system in order to achieve the optimum performance in a power line channel.

• To compare the performance of the OFDM and CDMA systems under identical conditions.
1.4 Contributions of This Research Work

The major contributions of this research work are summarized as follows:

- Analytical models are developed to analyze the performance of the OFDM system (with and without guard interval) under the power line multipath effect and impulsive noise interference. These analytical models provide key performance estimates of the OFDM system. With the performance being estimable, an algorithm could be implemented to adaptively choose the OFDM system parameters that will achieve the optimum performance.

- Simulations of the OFDM system (with and without guard intervals) under the power line multipath effect and impulsive noise interference are developed. The simulations are used to verify the accuracy of the results of the analytical models. The correspondence between the analytical and simulation results ensures the accuracy both for the analytical and simulation models.

- A method is proposed to determine the optimum length of the guard interval that will achieve the best BER performance.

- Analytical models are developed to analyze the performance of the CDMA system (with and without RAKE receiver) under the power line multipath effect and impulsive noise interference. These analytical models allow us to conveniently estimate the performance of the CDMA system.

- The CDMA system (with and without RAKE receiver) is simulated under the multipath effect and impulsive noise interference to ensure the accuracy of both the analytical and simulation models.

- The BER performance and the optimum overall data rate of the OFDM and CDMA systems are analyzed and compared. The conditions are kept identical that they are tested using the same bandwidth occupation, the same transmission power, the same total number of users and the same power line channel. The comparison is based on the analytical models that are developed and verified by simulations. In the comparison, some OFDM and CDMA system parameters which can be adjusted to achieve better performance are studied and compared.

- The analytical models of OFDM and CDMA systems developed in this thesis clearly distinguish themselves from the research work in [34], [47], [54]-[63], where the
CHAPTER 1: INTRODUCTION

analysis approach is simulation only and no analytical models are developed or provided.

1.5 Organization of This Thesis

This thesis consists of seven chapters as follows:

Chapter 1: Introduction

This chapter provides background information on the applications, advantages, standards and regulations for broadband power line communications. It also gives the motivations, objectives and contributions of this research work, as well as the organization of this thesis.

Chapter 2: Review of Power Line Channel

In this chapter, a review of previous research on modeling power line channels is presented. The well-developed channel model makes it possible to analytically model communication systems over power line channels. It is useful to review the reasons behind the multipath effect and noise interference over power line channel before introducing the modeling of OFDM and CDMA systems. Models of the multiple paths of power line transmission and of noise interference provided in this chapter form the basis for the models of communication systems introduced in the following chapters.

Chapter 3: Review of PLC Modulations – Why OFDM and CDMA Systems Are Studied

The unfavorable multipath effect and noise interference introduced in Chapter 2 raises a number of challenges for communications over power lines. The effort to develop better modulation techniques to handle the multipath effect and noise interference to achieve better performance in power line channel, write the development history of the communications over power lines. In this chapter, a brief history of the modulation techniques used in PLC is given. From the analysis of the development of the
modulation techniques, it is found that the OFDM and CDMA systems are very suitable options for broadband communications over power lines. The characteristics, advantages and disadvantages of OFDM and CDMA techniques are examined in this chapter to provide the background to the subsequent development of the analytical models.

Chapter 4: Modeling and Performance Analysis of OFDM Systems for Broadband PLC

In this chapter, the OFDM system is studied (both with and without guard interval) and an analytical model is developed to study the BER performance of the OFDM system. As introduced in Chapter 2, the impulsive noise and multipath effects are the main reasons to cause bit errors in power line communications. Thus, the BER performance of the OFDM system is theoretically analyzed under the impulsive noise and multipath effects in terms of closed form formulas and the analytical results are verified by simulations which are introduced in detail in this chapter.

As the longer guard interval is inefficient in using the transmission power of the signal, an optimum guard interval that can achieve optimum BER performance can be estimated. The method to estimate the best guard interval to achieve optimum BER performance is introduced in this chapter.

Chapter 5: Modeling and Performance Analysis of CDMA Systems for Broadband PLC

In Chapter 2, a simple introduction to the CDMA system and its advantage under the multipath effect are described. This advantage of the CDMA system makes it a suitable modulation technique for broadband PLC. In this chapter, the analytical models to analyze the BER performance of the CDMA system for broadband PLC are developed.

The BER performance of the CDMA system (with and without the RAKE receiver) in the PLC channel is theoretically analyzed in terms of closed form formulas. The analytical results are verified by simulations, which are introduced in detail in this chapter. Through the comparison with the results obtained from the simulations, the accuracy of the analytical models is verified for the CDMA system, both with and
without the RAKE receiver. The analytical models developed in this chapter will be used to compare the OFDM and CDMA systems in Chapter 6.

Chapter 6: Comparison of OFDM and CDMA Systems for Broadband PLC

In this chapter, the OFDM and CDMA systems are compared. Their BER performance and optimum overall data rate are analyzed and compared, using a similar set of parameters: the amount of bandwidth occupied, the level of transmission power for each user, and the total number of users involved in the system. In addition, the comparison is carried out over the same power line channel. The comparison is based on the analytical models that are developed and verified by simulations in Chapters 4 and 5.

Chapter 7: Conclusions and Recommendations for Future Research

An overall assessment of the research work is given in this chapter. Areas for further research are proposed as well. These include looking into ways to refine the proposed analytical models and to design and implement the discussed communications systems in the suitable platform to further verify and/or fine tune the developed analytical models.
CHAPTER 2  REVIEW OF PLC CHANNEL MODEL

2.1 Introduction

In this chapter, a review of previous research on modeling power line channels is presented. The well-developed channel model makes it possible to analytically model communication systems over power line channels. It is useful to give a detailed introduction to the rationale and sources of the multipath effect and noise interference before introducing the modeling of OFDM and CDMA systems. Models of the multiple paths of power line transmission and of noise interference are provided in this chapter and form the basis of the models of communication systems introduced in the following chapters.

2.2 Power Line Channel Model

As power lines are not typically designed specifically for communications, a number of challenges arise when they are used for this purpose. The key issues are: a) the effect of the noise generated in the power line network and b) the distortion of the signal because of the multipath effect. This distortion is caused by frequency-dependent cable losses and multipath signal propagation in the power line network.

![Figure 2.1: A typical PLC channel model](image)

Figure 2.1 shows a typical PLC channel model. In the model, the transmitted signal $s(t)$ propagates from the transmitter to the receiver through the PLC channel. From the transmitter to the receiver, the PLC channel characteristics can be described with a time domain impulse response $h(t)$. It is noted that this impulse response may vary with...
time, as the impedance of power line may change when new appliances are connected. This characteristic of time dependent variation of power line can be found in [64]. However, compared with the high speed transmission of OFDM and CDMA systems, the power line channel can be assumed to remain unchanged during the transmission. In the frequency domain, the PLC channel can be represented as the transfer function \( H(f) \). When the signal from the transmitter reaches the receiver, it is distorted by the characteristics of the PLC channel. It is a convolution of the transmitted signal from the transmitter \( s(t) \) and the impulse response of the PLC channel \( h(t) \), such that

\[
r'(t) = s(t) * h(t) = \int_{-\infty}^{\infty} s(\tau) h(t-\tau) d\tau
\]  

(2.1)

In the frequency-domain, the received signal \( R'(f) \) is obtained by taking the Fourier transform of both sides of equation (2.1), which is

\[
R'(f) = S(f)H(f)
\]  

(2.2)

Apart from the distortion of the signal caused by the PLC channel, the signal \( r'(t) \) is also affected by the noise \( n(t) \) generated from the power line. Because of this interference, the signal that finally appears at the receiver is the sum of \( r'(t) \) and \( n(t) \), which is \( r(t) \)

\[
r(t) = r'(t) + n(t) = s(t) * h(t) + n(t)
\]  

(2.3)

The hostile power line environment brings great challenges to the design of a good communication system with highly efficient modulation and coding schemes. There are two suitable modulation techniques for broadband PLC, namely OFDM and CDMA. These systems have a number of extensions and are used in various combinations. In order to design a highly efficient modulation scheme for PLC, we need to study how OFDM and CDMA-based systems perform when subject to the multipath effect and noise interference.
2.2.1 An introduction to multipath effect

Because the distribution of power lines is complicated, signals transmitted from the transmitter to the receiver will propagate not through one path, but through multiple paths. This causes the signal to be distorted through the multipath effect.

Figure 2.2: An example of the adverse multipath effect while transmitting a sinusoid signal over power lines

Figure 2.2 provides an example of the problems created by the multipath effect. The signal transmitted from the transmitter to the receiver is assumed to be a sine signal with a frequency $f$ and amplitude $A$, and the signal can be represented as

$$s(t) = A \sin(2\pi ft) \quad (2.4)$$

The signal propagates through the power line with the speed $c$ m/s as determined by the characteristics of the power lines [19]. Suppose there are two paths for the signal to propagate from the transmitter to the receiver, the direct path and the second path. Due to the propagation delay, the signal that goes through the direct path with distance $d_1$ m can be represented as

$$s_1(t) = A' \sin[2\pi f(t - d_1 / c)] \quad (2.5)$$

The delayed signal which goes through the second path with distance $d_2$ m can be represented as
\[ s_2(t) = A'' \sin[2\pi f (t - d_2 / c)] \]  

The delayed signal lags behind the signal that took the direct path by \( d = d_2 - d_1 \) m over a time lag of \( d / c \) s. If \( d / c \) is half of a sinusoid cycle \( T = 1 / f \), the lagged signal is 180 degrees behind the direct path signal. This degree of lag brings about the cancellation of the two signals when they reach the receiver. This cancellation will greatly attenuate the signal with carrier frequency \( f \), and will bring about deterioration in the BER performance at the receiver. The cancellation of these two signals causes a notch in the transfer function of the PLC channel at the frequency \( f \) between the transmitter and the receiver.

While the section above provides a brief introduction to how the multipath signal transmission affects communications over power lines, things are far more complicated in real power line networks. There are many branches along the transmitter to the receiver and once the signal is transmitted, it will propagate according to the physical structure of the power line network and according to the characteristics of the power lines.

2.2.2 Factors that affect the PLC channel

As a signal travels from the transmitter to the receiver, it suffers from

- attenuation caused by the characteristics of the power lines used in the power line network,
- reflection at the junctions of power lines, caused by the mismatch of the impedances at the connections, and
- propagation delay between the multiple paths of the signal, which can cause the cancellation of the signal or inter symbol interference (ISI) between the symbols within the same signal.

The signal attenuation, reflection and propagation delay, plus the complicated power line network structure in the mains network make power line channels complicated and difficult to model. Despite its complexity, numerous research resources have been engaged to study and further develop the modulation and network protocol for PLC.
Several researchers have developed the multipath model for power line channels, most notably [19], [20]. The accuracy of their multipath model is objectively verified by experimentation and precise measurement. This model is widely accepted and used in the modeling and analysis of PLC by [55], [56], [65]. Because it can accurately model power line channel, it will be referred to in this paper simply as the PLC channel model for the modeling and analysis of broadband PLC. Below is a summary of this power line multipath model.

**A) Signal Attenuation**

The signal transmitted through the power line tends to attenuate over the length of a dedicated path and with known frequency. The relationship between signal attenuation, length and frequency is [20]

\[
A(f, d) = e^{-(a_0 + a_i f^k) d}
\]  

(2.7)

where \( f \) is the carrier frequency, \( d \) is the length of a path, \( a_0 \) and \( a_i \) are the attenuation parameters and \( k \) is the exponent of the attenuation factor (typical values are between 0.5 and 1). The parameters of \( a_0, a_i \) and \( k \) are related to the characteristics of the power lines, which can be derived from the measured transfer function for the network.

**B) Multipath Signal Propagation**

When a signal is transmitted through the power line from a transmitter, the signal will reflect back and forth at the branches and cable junctions because of the mismatch in impedance. The signal can be seen to be propagating through multiple paths to reach the receiver. Each path \( i \) has a weighting factor \( g_i \) among these multiple paths. \( g_i \) is the product of the reflection and transmission factors along the path. When the transmitted signal reaches a joint in the network, it faces a parallel connection of two or more cables. The impedance of these parallel cables can cause lower impedance than
that of the cable which signal coming through. In this case, the reflection and transmission factors at such joint of the power line are less than one. In case that there is no change of the impedance at both sides of the power line joint, the reflection and transmission factors are equal to 1. Thus, the weighting factor $g_i$ is less than or equal to one [20],

$$|g_i| \leq 1$$  \hspace{1cm} (2.8)

Because of the fact that transition and reflection occur along a path, there could be an infinite number of paths. However, the transitions and reflections make the attenuation for each path much larger, so signals transmitted in these highly attenuated paths contribute very much less to the overall signal at the receiving point. Therefore, it is reasonable to approximate the infinite number of paths using only $N$ dominant paths. $N$ is made as small as possible to simplify the model without sacrificing accuracy.

\section*{C) Propagation Delay}

The propagation delay $\tau_i$ for the path $i$ can be calculated from the dielectric constant $\varepsilon$ of the insulation material of the power line, the speed of light $c_0$ and the length $d_i$ of the path $i$, which can be represented as [20]

$$\tau_i = d_i \sqrt{\varepsilon} / c_0$$ \hspace{1cm} (2.9)

\subsection*{2.2.3 PLC channel transfer function and impulse response}

- Transfer function of the PLC channel (frequency domain)

Summarizing the attenuation factors, the multipath signal propagation and the propagation delay, the frequency response from the transmitter to the receiver can be expressed as [20]
CHAPTER 2: REVIEW OF PLC CHANNEL MODEL

\[ H(f) = \sum_{i=1}^{N} g_i \cdot A(f, d_i) \cdot e^{-2\pi f \tau_i} \]
\[ = \sum_{i=1}^{N} g_i \cdot e^{-\left(a_i + a_i^* \tau_i \right) d_i} \cdot e^{-2\pi f d_i/v_p} \] \hspace{1cm} (2.10)

- Impulse response of PLC channel (time domain)

In the time domain, the impulse response in the PLC channel can be described as [20]

\[ h(t) = \sum_{n=1}^{N} \beta_n \delta(t - \tau_n) \] \hspace{1cm} (2.11)

where \( \beta_n \) and \( \tau_n \) are the amplitudes and the arrival times of the multipath signal components respectively, and \( N \) is the number of the multipath components of the channel impulse response.

2.2.4 Example of the multipath model

Figure 2.3 illustrates a multipath model in [19]. The length of the power line from transmitter A to the receiver B is 200m and the characteristic impedance of this power line is about 45Ω. If transmitter A and receiver B are well matched with the characteristic impedance of the power line, there will be no signal reflection at the junctions of A and B. Due to the impedance match, the signal from transmitter A will be injected to the power line without loss. The power line of length 200m is divided into two logical parts at the branch point D. In the figure, the segment between A and D is marked as branch (1), while that between D and B is marked as branch (2). The branch at D is marked as branch (3) and is 12m long with the characteristic impedance of \( Z_{L3} = 70\Omega \). At the end of the branch (3) is an open socket outlet marked C.

![Figure 2.3: An example of the multipath PLC network](image)
When a signal is transmitted from transmitter A to the power line, the signal can pump into the power line without reflection and loss because the impedance of the transmitter is matched with the characteristic impedance of the power line. When the signal is transmitted along the power line in the direction of A→D, there is no reflection until it reaches point D. The signal that is transmitted in the direction of A→D is represented as \( s_{AD} \). Because of the mismatch in the impedance at the junction of the power line, there is a reflection of the signal at point D.

Part of the signal \( s_{AD} \) will reflect back at D in the direction of D→A according to the reflection factor \( r_{ID} \) at point D [19],

\[
r_{ID} = \frac{(Z_{l2} \parallel Z_{l3}) - Z_{l1}}{(Z_{l2} \parallel Z_{l3}) + Z_{l1}}
\]  

(2.12)

where \( Z_{l1}, Z_{l2} \) and \( Z_{l3} \) are the characteristic impedances of branches (1), (2) and (3) respectively. This signal is marked as \( s_{AD}^{D} \), which represents the signal propagated in direction of A→D→A. Another part of the signal \( s_{AD} \) will propagate in the direction of D→B and becomes the signal \( s_{ADB} \). The rest of signal \( s_{AD} \) will propagate in the direction of D→C and becomes the signal \( s_{ADC} \). These signal propagations are determined by the transmission factor \( t_{ID} \) at point D [19],

\[
t_{ID} = 1 - |r_{ID}|
\]  

(2.13)

Because of the matching impedance at point A, the signal \( s_{AD}^{D} \) in the direction of D→A will not reflect back to point D. It will not reach the receiver and can be ignored in the final analysis. The signal \( s_{ADB} \) will not be reflected back in the direction of B→D and will reach the receiver B because of the impedance match between the power line and the receiver at B. The signal \( s_{ADC} \) behaves somewhat differently and will create several problems. Because of the open socket outlet, it will reflect back on reaching C according to the reflection factor at point C [19],

\[
\]
\[ r_{SC} = \frac{Z_C - Z_{L3}}{Z_C + Z_{L3}} \]  

(2.14)

where \( Z_C \) is impedance at point C. Because of the open socket at point C, the impedance is \( Z_C = +\infty \) and \( r_{SC} = 1 \). This means that the signal at point C is reflected completely back in the direction of C→D. It can thus be marked as \( s_{ADCD} \).

Due to the mismatch in the impedance, when \( s_{ADCD} \) reaches the junction of point D, part of the signal \( s_{ADCD} \) will reflect back in the direction of D→C again and is marked as \( s_{ADCDC} \). The reflection factor \( r_{SD} \) is \[ r_{SD} = \frac{(Z_{L2} || Z_{L1}) - Z_{L3}}{(Z_{L2} || Z_{L1}) + Z_{L3}} \]  

(2.15)

Part of the signal \( s_{ADCD} \) will transmit in the direction of D→A and D→B according to the transmission factor \( t_{3D} \) at point D \[ t_{3D} = 1 - |r_{3D}| \]  

(2.16)

The signal in the direction of D→A is marked as \( s_{ADCD}A \) and the signal in the direction of D→B is marked as \( s_{ADCD}B \). Signal \( s_{ADCD}A \) will sink at point A while \( s_{ADCD}B \) will reach and sink at receiver point B without reflection. Signal \( s_{ADCDC} \) will continue to reflect at point D and point C according to the transmission and reflection factors, until the signal becomes greatly attenuated and can be ignored.

From the analysis above, it is found that there are likely to be infinite paths from the transmitter to the receiver. To simplify the model without losing accuracy, it is possible to ignore signals that are highly reflected because they are greatly attenuated. The power line channel can thus be modeled using a limited number of paths. In order to demonstrate that such a limited number will accurately model a PLC channel, Figure 2.3 illustrates a model using four paths (A→D→B, A→D→C→D→B, ...).
A→[D→C]_2→D→B, A→[D→C]_3→D→B) and six paths (A→D→B, A→D→C→D→B, A→[D→C]_2→D→B, A→[D→C]_3→D→B, A→[D→C]_4→D→B, A→[D→C]_5→D→B) respectively, where [D→C]_2 means a signal will propagate through D→C→D→C and so on. In this model, the characteristic parameters of the power line are modeled as \( k = 1, \ a_0 = 0, \ a_1 = 7.8 \times 10^{-10} \text{s/m} \) and \( v_p = 1.5 \times 10^8 \text{m/s} \) [19].

The four-path and six-path models can be derived from the reflection factors and the transmission factors of the network, which are

\[
\begin{align*}
    r_{10} &= \frac{(Z_{l2} || Z_{l3}) - Z_{l1}}{(Z_{l2} || Z_{l3}) + Z_{l1}} = -0.24 \\
    r_{3C} &= \frac{Z_C - Z_{l3}}{Z_C + Z_{l3}} = -1 \\
    r_{30} &= \frac{(Z_{l2} || Z_{l3}) - Z_{l1}}{(Z_{l2} || Z_{l3}) + Z_{l1}} = -0.514 \\
    t_{3D} &= 1 - |r_{3D}| = 0.486 \\
    t_{10} &= 1 - |r_{10}| = 0.76
\end{align*}
\]

Table 2-1: Power Line Multipath Channel Parameters

<table>
<thead>
<tr>
<th>Path Number ( i )</th>
<th>( g_i )</th>
<th>( d_i ) (m)</th>
<th>( \beta_i )</th>
<th>( \tau_i ) (( \mu \text{s} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.76</td>
<td>200</td>
<td>0.18</td>
<td>1.33</td>
</tr>
<tr>
<td>2</td>
<td>0.37</td>
<td>224</td>
<td>0.08</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>-0.1</td>
<td>248</td>
<td>-0.02</td>
<td>1.65</td>
</tr>
<tr>
<td>4</td>
<td>0.025</td>
<td>272</td>
<td>0.01</td>
<td>1.81</td>
</tr>
<tr>
<td>5</td>
<td>-0.0063</td>
<td>296</td>
<td>-0.0025</td>
<td>1.973</td>
</tr>
<tr>
<td>6</td>
<td>0.0016</td>
<td>320</td>
<td>0.000625</td>
<td>2.13</td>
</tr>
</tbody>
</table>
The parameters required to model the PLC channel with six paths are presented in Table 2-1. Only the first 4 paths need to be counted when considering the 4-path model.

Figure 2.4 provides the transfer functions and the impulse responses of the PLC network model, which is presented with 6 and 4 paths respectively. The modeling results using four and six paths were found to be almost identical. The 4 path model can almost accurately model the power line channel without a loss in accuracy. Therefore, we can safely ignore those paths where the signal is extensively attenuated, and use the minimum number of paths to model the PLC network. From the amplitude response in the figure, we found notches at several frequencies that are caused by the multipath effect of the PLC channel. These notch bands are harmful to communications because they cause signals within these frequency bands to be highly attenuated.

(a) Amplitude responses
Figure 2.4: (a) Amplitude responses, (b) phase responses, and (c) impulse responses of the sample network for four and six paths.
CHAPTER 2: REVIEW OF PLC CHANNEL MODEL

The figures above illustrate an ideal power line network which is used to verify the multipath model. In actual fact, power line networks can be far more complicated. Some factors that impact the complexity are a mismatch between the impedance of the transmitter and the receiver and the power line cable; changes in the impedance of the power line networks over time, so that it becomes difficult to match it; an increase in the number of branches in the power line network, meaning more paths for signal transmission, etc. However, no matter how complicated the power line network, the power line channel between the transmitter and the receiver still can be modeled. For more information on the modeling and measurements of complicated power line networks, please refer to [19], [20].

2.3  Power Line Noise Model

2.3.1  Introduction

As power lines are not designed specifically for broadband communications, multiple forms of electrical equipment are commonly connected to residential and commercial power lines. This electrical equipment tends to introduce various noises to the power lines. These noises are can either be in-band (within the signal frequency) or out-of-band (different with the signal frequency) noises. Once injected into the power line, they travel in the network until they become highly attenuated.

While the source of noise in broadband power line communications is widely distributed, they can in fact be divided into five general classes [13]. These are:

- Colored background noise
  This type of noise is of a relatively low power spectral density that varies with the frequency. It is caused mainly by the summation of numerous noise sources with low power.

- Narrowband noise
  Noise of this category tends to look like sinusoidal signals. It is caused mainly by the ingress of broadcast stations.
• Periodic impulsive noise asynchronous to the mains frequency
This kind of noise has a repetition rate of between 50 and 400kHz. It has a discrete line spectrum space that varies according to the impulse repetition rate. It is caused mainly by the switching modes of the power supplies used in various types of electrical equipment, because of their low cost and high power conversion efficiency.

• Periodic impulsive noise synchronous to the mains frequency
This category of noise has a repetition rate of 50 or 100Hz. It is mainly caused by linear power supplies and by the switching of rectifier diodes (which occurs synchronously with the mains cycle).

• Asynchronous impulsive noise
This final category is asynchronous with the mains frequency and is caused mainly by switching transients in the power line network. The duration of this impulsive noise ranges from several microseconds to a few milliseconds, and may also occur randomly. The power spectrum density of this type of noise can reach values of more than 50dB above background noise.

Some noise types which are registered daily in the vicinity of power lines are illustrated in Figure 2.5. It shows the noise levels in a typical office environment in Singapore. The noise is measured by using the sampling rate of 1MSPS (Million Sample Per Second). Because the length of the memory to continuously record the noise is 64k points, the noise record time is thus 64ms. From the figure, we notice that every 10ms, there is a noise valley where there is a reduction in noise. These noise valleys appear synchronously with the zero crossing of the 50Hz’s mains network (i.e., every 10ms). Further analysis reveals that the noise measured includes color background noise and the periodic impulsive noises which are asynchronous and synchronous to the mains frequency, 50Hz.
Figure 2.5: Noise measured in a typical office environment in Singapore

Figure 2.6 depicts the periodic impulsive noise measured from a linear power supply. We notice that the noise is synchronized well with the mains frequency 50Hz. The maximum values of the noise appear every 10ms at the middle of two zero crossings of the impulsive noise.

Figure 2.6: A periodic impulsive noise synchronous to the mains frequency measured from an energy meter with a linear power supply
CHAPTER 2: REVIEW OF PLC CHANNEL MODEL

Figure 2.7 illustrates asynchronous impulsive noises measured in the power line when a switching transient event occurs. The noise is measured with an Agilent Infiniium 54854B oscilloscope through a coupling circuit at the sampling rate of 25MSPS. The recording of the impulsive noise is triggered by the rising edge of the waveform.

![Figure 2.7: An asynchronous impulsive noise measured from a power line when there is a switching transient](image)

2.3.2 Noise model

The research results in [13] provide an excellent summary of the noises that interfere with power line communications. The authors developed a noise model which is widely referred to in the analysis of broadband power line communications and it suggests that the power line channel presents a Non-additive White Gaussian Noise (NAWGN) environment. The noise in the broadband PLC channel can be categorized into two main groups: background noise and impulsive noise. While background noise remains stationary, impulsive noise has a short duration and a high power spectral density. For this reason it is impulsive noise and not background noise that presents the greatest challenge for PLC.

Impulsive noise can be classified as periodic and aperiodic. Periodic impulsive noise includes interference that is synchronous or asynchronous to the mains frequency.
These periodic impulsive noises are caused mainly by the kind of power supplies in the mains networks. Aperiodic impulsive noise, which is caused by the switching transient, contains considerable energy and thus seriously affects high-speed communications over power lines. The duration time of the impulsive noise may exceed the communication symbol length.

Compared to other kinds of impulsive noises, the amplitude and the duration time of aperiodic impulsive noise are significant and can seriously affect communications over power lines. For this reason, its effects on broadband PLC systems and on OFDM and CDMA systems will be studied and modeled in this thesis. From this point, any mention of impulsive noise will refer specifically to aperiodic impulsive noise.

Aperiodic impulsive noise usually occurs in bursts, and they can affect considerable portions of useful signals. Frequency analysis from [13] concludes that this kind of noise contains a broadband portion which significantly exceeds the background noise, and a narrowband portion which appears only in a certain frequency range. The broadband portions of the frequency are caused by the sharp rising edges of the impulsive noise, whereas the narrowband portions of the frequency result from the oscillations as shown in Figure 2.7.

In the frequency domain, the impulsive noise is broadband, and it affects the whole bandwidth that is occupied by the signal. In the time domain, the impulsive noise is modeled with three variables: amplitude, impulse width or duration time and inter-arrival time (the time between two impulsive noises). A typical impulsive noise wave is shown in Figure 2.8, where $A = \max\{A^+_i, A^-_i\}$ is the impulsive noise amplitude, $t_w$ is the impulsive noise width or duration time, and $t_d$ is the impulsive noise distance. The arrival time is expressed as $t_{arr,i}$. The distance between two impulsive noises is the inter-arrival time $t_{IAT}$.

$$t_{IAT} = t_w + t_d = t_{arr,i+1} - t_{arr,i}$$ (2.22)
The definitions of two random variables, impulsive noise inter-arrival time ($IAT$) and the impulsive noise duration time, are required. Most of the measurements show that these two variables always have an exponential distribution, although their mean values depend on the location at which the measurements are made.

Figure 2.8: Time-domain impulsive noise representation and envelope with characteristic parameters [13]

Three impulsive noise scenarios representing different PLC environments are measured and defined in [13]. The first scenario is a heavily disturbed noise from an industrial area; the second is a moderately disturbed scenario from a residential area with detached and terraced houses; and the third is the lightly disturbed scenario from an apartment located in a large building in night time.

This noise model and the associated scenarios are referred to in investigations of medium access control (MAC) protocols for PLC networks by [40], [41]. The analysis in [40] assumes that the impulsive noise within one period of a data frame has enough amplitude and width to destroy the frame’s transmission. The impulsive noise generation parameters are exponentially distributed, and are given in Table 2-2. In the table, $t_{IAT}$ is the inter-arrival time of the impulsive noise, which is the reciprocal of the
arrival rate $\lambda$. $T_{\text{Noise}}$ is the average noise duration time [13]. The amplitude of the impulsive noise can be described by using a parameter impulsive noise to AWGN power ratio $\mu$, where $\mu$ is the ratio of the amplitude of the impulsive noise to the amplitude of the AWGN.

Table 2-2: Parameters of Impulsive Noise Scenarios

<table>
<thead>
<tr>
<th>Impulsive Noise Scenario</th>
<th>$t_{\text{IAT}}$</th>
<th>$T_{\text{Noise}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: Heavily disturbed</td>
<td>0.0196s</td>
<td>0.0641ms</td>
</tr>
<tr>
<td>II: Medium disturbed</td>
<td>0.9600s</td>
<td>0.0607ms</td>
</tr>
<tr>
<td>III: Weakly disturbed</td>
<td>8.1967s</td>
<td>0.1107ms</td>
</tr>
</tbody>
</table>

In simulating the effect of impulsive noise on the communication system, impulsive noise is generated using $IAT$, $T_{\text{Noise}}$, $\mu$ and $E_b/N_o$, where $N_o$ is the power spectrum density of the AWGN. Once these parameters are confirmed, impulsive noise can be generated following the Poisson process. The generation of the Poisson process using Matlab is detailed in [66].

The probability that $k$ impulsive noise appearance at time $t$ is given as

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad k = 0, 1, 2... \quad (2.23)$$

Because the generation interval exponentially distributed, the probability that no impulsive noise is generated during the period 0 to $t$ is as follows:

$$P_0(t) = e^{-\lambda t} \quad (2.24)$$

The probability that impulsive noises are generated after the time $t$ is given as

$$P(t) = 1 - e^{-\lambda t} \quad (2.25)$$
Let \( x \) replace the probability of \( P(t) \) in equation (2.25), where \( x \) is a random number that has a uniform distribution in area of \((0, 1)\). It follows that

\[
t = -\frac{1}{\lambda} \log(1 - x)
\]

(2.26)

Using the parameter \( T_{\text{Noise}} \), \( \mu \) and \( E_b / N_o \), the average duration and amplitude of the impulsive noise can be determined. Thus, both the impulsive noise and the background noise can be generated.

Figure 2.9 illustrates impulsive noises programmed in Matlab for simulations according to the scenarios listed in Table 2-2. The first scenario involves a heavily disturbed impulsive noise, and the impulsive noise to AWGN power ratio \( \mu \) is set as 10. The appearance of the impulsive noise is random because of the good simulation of the Poisson process. The amplitude and the duration time are also randomly generated according to the probability distribution. The amplitude of the impulsive noise and the Gaussian noise in the figure is the relative amplitude compared with the signal amplitude, which is determined by the signal to noise ratio \( E_b / N_o \) in the simulations.

![Impulsive noise and Gaussian noise](image)

Figure 2.9: The simulated impulsive noise and Gaussian noise which are used in the simulations
CHAPTER 3 REVIEW OF PLC MODULATIONS - WHY OFDM AND CDMA SYSTEMS

3.1 Introduction

Because the applications of broadband communications are targeted at home networking and Internet access, speed and robustness are important factors which must be considered when designing a successful communication system. Great amounts of effort and resources are needed to develop better modulation techniques to overcome the multipath effect and noise interference and to achieve better performance in communications.

Before introducing the OFDM and CDMA systems, the modulation techniques for PLC are briefly analyzed. Through this analysis, the reasons why the OFDM and CDMA systems are the two options suitable for broadband PLC are explained. During the analysis of the modulation techniques, the characteristics, advantages and disadvantages of the OFDM and CDMA techniques are also introduced. In a word, the purpose of this chapter is to provide the fundamental knowledge to the development of the analytical models for the OFDM and CDMA systems in the following chapters.

3.2 Single Carrier Systems

The early PLC modulation techniques were based on signal carrier systems, such as amplitude shift keying (ASK), phase shift keying (PSK) and quadrature amplitude modulation (QAM).

ASK is a form of modulation which represents digital data as variations in the amplitude of a carrier wave. The amplitude of the carrier signal varies in accordance with the bit, keeping frequency and phase constant. The level of amplitude can be represented using binary logic (0 and 1). ASK transmits information based on the variance of the amplitude. It is rarely used in PLC because the high level of attenuation and the noisy power line channel severely impact the quality of its performance.
CHAPTER 3: REVIEW OF PLC MODULATIONS - WHY OFDM AND CDMA SYSTEMS

Single carrier PSK is a form of modulation which represents digital data as variations in the phase of a carrier wave. It includes binary phase shift keying (BPSK), quadrature phase shift keying (QPSK) and other level of modulation. In such systems, information bits are transferred over power lines by changing the phases of the carrier. Among the variations, BPSK is the simplest single carrier modulation system. This form of modulation is illustrated in Figure 3.1.

![Figure 3.1: BPSK single carrier modulation system](image)

The illustration shows a unique carrier with frequency \( f_c = \frac{1}{T} \). When a data 1 is transmitted at the boundary of the symbols, the sine wave signal is transmitted with “0” degree phase. When a data 0 is transmitted, the transmitted sine wave is of a “180” degree phase. The transmitted signal of the BPSK system can be described by

\[
S(t) = A \sin(2\pi f_c t + d_i\pi)
\]

(3.1)

where \( A \) is the amplitude of the sinusoid signal, and \( d_i \) are the bits of information to be transmitted. Various types of PSK systems hold the same unique carrier. The only difference among systems is the type of phase variance used to transmit information.

In the case of QAM modulation, data is transmitted by changing the amplitudes and phases for two sinusoid carriers. These two sinusoids that are out of phase with each other by 90 degrees are thus called quadrature carriers. For this modulation, there is also only one frequency. The difference among the variations of QAM modulation is that they use different amplitude and phase variances to transmit data. Signals of the
single carrier system have a Sinc function spectrum with center frequency $f_c$, and bandwidth $BW = \frac{2}{T}$, as shown in Figure 3.2.

![Figure 3.2: Spectrum of the single carrier modulation system](image)

3.3 Frequency Shift Keying (FSK) and Its Variations

FSK is a method of modulation that transmits data using two different frequency sinusoid waves. It can be represented as

$$S(t) = A\sin(2\pi f_i t + \phi)$$  \hspace{1cm} (3.2)$$

where $A$ is the amplitude of the carriers, $\phi$ is the phase of the carriers usually valued either as 0 or 90 degrees, and $f_i$ ($i = 0, 1$) is the frequency of the two carriers. When a data 0 is transmitted, the sinusoid wave that is transmitted is of frequency $f_0$. On the other hand, when a data 1 is transmitted, the sinusoid wave that is transmitted is of frequency $f_1$, as shown in Figure 3.3.
The relationship between the two FSK frequencies needed to meet the orthogonal requirements can be described as

\[ f_1 - f_0 = N \frac{1}{T} \]  

(3.3)

where \( N \) is an integer larger than or equal to 1. In the frequency domain, the FSK signal spectrum is the two frequency tones of Sinc shape centered at \( f_1 \) and \( f_0 \) respectively, as shown in Figure 3.4, where the two frequency distance is \( \frac{1}{T} \). The peak of one carrier spectrum is located at the zero of the other carrier spectrum, ensuring that the two carriers do not interfere with each other.

Figure 3.4: Spectrum of FSK signal
As simple FSK do not have enough noise immunity, several good modulations have been derived that use multiple orthogonal carriers to transmit digital bits of information. Among these, the most notable is introduced in [67]. This modulation uses four orthogonal frequencies and a transmission sequence as illustrated below [67]. The four orthogonal carriers $f_1$, $f_2$, $f_3$ and $f_4$, and the transmission sequence of $f_1 \rightarrow f_2 \rightarrow f_3 \rightarrow f_4$ represents two bits 00, $f_2 \rightarrow f_3 \rightarrow f_4 \rightarrow f_1$ represents 01, $f_3 \rightarrow f_4 \rightarrow f_1 \rightarrow f_2$ represents 10, and $f_4 \rightarrow f_1 \rightarrow f_2 \rightarrow f_3$ represents 11. This increases the noise immunity of the system and achieves greater performance as indicated by experimental results. Other modulations including one called Chirp modulation have also improved on the noise immunity of the FSK and these are detailed in [68] and [69]. The signal spectrums of these modulation systems are similar to that of the FSK, except that these systems have spectrums consisting of multiple frequency tones instead of just two.

3.4 Multi-carrier System and Its Advantages in the PLC Channel

The use of multiple carriers is a growing practice in PLC. The modulations introduced above achieve noise immunity by varying the carrier frequency over time such that multiple carriers are not transmitted simultaneously.

The OFDM system is a multi-carrier system that functions differently from the systems introduced above. In the OFDM system, multiple bits of information are modulated with all the sub-carriers within a symbol period and multiple sub-carriers are simultaneously transmitted. This difference can be clearly observed from Figure 3.5 (a), (b) and (c). In the figure, (a) is the BPSK signal that is plotted in the time-frequency domain where logic “1” is represented as the sine wave at the 0 degree phase, and logic “0” is the same wave at the 180 degree phase; (b) is the FSK signal at the time-frequency domain, where logic “1” is the sine wave with frequency $3/T$, and logic “0” is represented by the sine wave with frequency $2/T$; and (c) is the OFDM signal in the time-frequency domain, with 32 carriers in the system, each of which has modulation BPSK.
CHAPTER 3: REVIEW OF PLC MODULATIONS - WHY OFDM AND CDMA SYSTEMS

(a): Single carrier BPSK signal in the time-frequency domain

(b): FSK signal in the time-frequency domain

(c): OFDM signal in the time-frequency domain

Figure 3.5: Comparison of the BPSK, FSK and OFDM system signals in the time-frequency domain
From Figure 3.5, it is obvious that all the sub-carriers of the OFDM system are transmitted simultaneously within a symbol time \( T \). However, while the OFDM system can use all the sub-carriers to transmit information simultaneously within a symbol time period, the BPSK and FSK systems do not have this capability. The simultaneous use of all the sub-carriers to transmit information is implemented by using a key block IFFT, which converts \( N \) bits of information \( d_0, \ldots, d_{N-1} \) from the frequency domain into the time domain, as shown in Figure 3.6. At the receiver, a FFT block will convert the time domain signal back into the frequency domain to achieve the corresponding result \( \hat{d}_0, \ldots, \hat{d}_{N-1} \).

![OFDM system transmitter and receiver](image)

The fact that the OFDM system transmits multiple bits of information simultaneously gives it an important advantage over single carrier PSK and FSK systems. The OFDM system allows for the transmission of more bits of information if the symbol duration time \( T \) is the same. The system signals illustrated in Figure 3.5 indicate that the OFDM system is 32 times faster than the BPSK or FSK system.

Looking at the high rate of data transfer from another point of view, it can be seen to have a longer symbol time than the single carrier PSK, despite the fact that both systems transmit data at the same data rate. A simple comparison between the single carrier BPSK system and the multi-carrier OFDM system demonstrates this advantage. Supposing that the data rates of the single carrier BPSK system and OFDM system are the same as \( R \), the symbol time for the single carrier BPSK system is

\[
T_{\text{BPSK}} = \frac{1}{R} \quad (3.4)
\]
For the OFDM system with the sub-carrier number $N$, the symbol time is

$$T_{OFDM} = \frac{N}{R}$$

(3.5)

which is $N$ times the time taken using the single carrier system. The system with the longer symbol time has higher resistance to the inter symbol interference (ISI), which is caused by the multiple paths of transmission in power lines. In addition to the above mentioned advantages, compared with the single carrier system, the OFDM system has narrow side lobes which can bring higher efficiency of frequency utilization. Another advantage of the OFDM system is that it allows some frequency bands unutilized to avoid the potential interference to the radio system working in these bands of frequency.

### 3.4.1 Advantages of the OFDM system in the power line multipath channel

As previously detailed, power lines can be modeled as multipath channels. Transmitted signals propagate through multiple paths before reaching the receiver. The maximum propagation delay $\tau_{\text{max}}$ is usually within micro seconds. Figure 3.7 illustrates the ISI problem for the BPSK system using $\tau_{\text{max}} = 5\mu s$.

![Figure 3.7: The ISI problem for a BPSK system with 1Mbps data rate, in the power line channel with the maximum propagation delay of 5$\mu$s.](image)
For a 1Mbps data rate single carrier BPSK system, the symbol time is
\[ T_{\text{BPSK}} = \frac{1}{R} = \frac{1}{1 \text{Mbps}} = 1 \mu s, \]
and the maximum delay in the signal is five symbol times. The ISI presents a serious challenge in that signals from the first and second paths interfere with each other.

Turning to the multi-carrier OFDM system with the same data rate of 1Mbps, the symbol time becomes
\[ T_{\text{OFDM}} = \frac{N}{R} = \frac{32}{1 \text{Mbps}} = 32 \mu s. \]
The maximum delay in the OFDM signal is less than 5\( \mu s \) which is within the OFDM symbol time of 32\( \mu s \). Although the delayed signal also causes ISI-related challenges, these can easily be solved by adding a guard interval to the head of each OFDM symbol. When tackling problems related to inter carrier interference (ICI), the guard interval is added to the head of an OFDM symbol from part of the tail of the OFDM signal itself. In this way, the orthogonality of the OFDM system carriers can be maintained within a symbol period.

This guard interval will be removed at the receiver, before the demodulation. As long as the length of this guard interval \( T_g \) is longer than the maximum channel delay \( \tau_{\text{max}} \), all the ISI from the reflections of previous symbols are removed and the orthogonality of OFDM sub-carriers is preserved. Details on the orthogonality of OFDM sub-carriers and why a sufficient guard interval can eliminate ISI and ICI problems can be found in Chapter 4 where the modeling and performance analysis of the OFDM system is provided.

The solution of adding a guard interval to an OFDM system to solve the ISI problem is not without cost. By placing the guard interval before the useful part of length \( T_g \), this cannot be used for transmitting information and parts of the transmitted signal power can be lost. Subsequent sections of this thesis will discuss means of optimizing the guard interval of an OFDM system.
Figure 3.8 provides an example of the OFDM signal with guard intervals inserted. The signal in the figure is the time domain representation of the OFDM signal illustrated in Figure 3.5(c). It is the product of the addition of the signal of all 32 carriers and it represents the summarized signal only in the time domain. The data to modulate each sub-carrier are randomly generated and the signal waveform is generated using the OFDM simulation model which will be introduced in Chapter 4.

3.5 Spread Spectrum System and Its Advantages in the PLC Channel

A system is defined as a spread spectrum system if it fulfills the following requirements [70]:

- The signal occupies a bandwidth that is much wider than the minimum bandwidth necessary to send the information.
- Spreading is accomplished by means of pseudo-noise, such as spreading code, which is independent of the data.
- At the receiver, despreading is accomplished by correlating the received spread signal with a synchronized replica of the spreading code which is used to spread the information.
Two frequently used spread spectrum systems are the direct sequence code division multiple access (DS-CDMA) system and the frequency hopping spread spectrum (FHSS) system. The DS-CDMA system is studied in this thesis and by default we name it as CDMA system for the reason of simplicity. A basic CDMA system includes a transmitter and a receiver, and the signals and spectrum of the system are represented as in Figure 3.9. The sample CDMA system is with data rate $1/T_b$ s, where $T_b$ is the bit duration time and the processing gain $N$ (number of bits in the spreading code) is 7. Therefore, the chip period (length of the spreading code bit duration time) is $T_c = \frac{T_b}{N} = \frac{T_b}{7}$; and the carrier frequency is $f_c = 1/T_c$ Hz.

In the CDMA system transmitter, the data to be input $b(t)$ are in the binary form as they are spread, modulated and transmitted. The data $b(t)$ is directly multiplied with the PN code $a(t)$, which is also often called the spreading code. This code is independent of $b(t)$. The spread data $s_b(t) = b(t)a(t)$ is the baseband signal. The effect of multiplying the input data with the spreading code is to spread the bandwidth of $b(t)$ from $BW_b$ to $BW_s$, which is the bandwidth of the baseband signal $s_b(t)$. The baseband signal $s_b(t)$ is modulated with the carrier frequency $f_c$ and the signal $s(t)$ is transmitted to the power line through the power line driver and the coupling circuit.
At the receiver, after filtering, sampling and frequency mixing, the received signal $r(t)$ is converted into the baseband signal $r_b(t)$. At this stage, $r_b(t)$ is still the spread signal with the bandwidth $BW'_c$. It is despread by multiplying it with the local PN code $a(t)$ of the receiver. When the local PN code and the transmitter’s PN code are the same, and they are both synchronized, the transmitted binary data can be recovered at the output stage of the receiver. The effect of multiplying the received spread spectrum
signal $r_b(t)$ with the PN code $a(t)$ is to despread the spectrum of $r_b(t)$ from bandwidth $BW_i$ to $BW_b$, which is the bandwidth of the binary data $b(t)$.

### 3.5.1 Properties of the CDMA system and the PN sequence

In a CDMA system the information data is spread by multiplying it with a PN sequence before modulation. The PN code appears to be random, but is in fact generated according to a dedicated algorithm. The most frequently used PN codes are the M-sequence, the Gold sequence, the orthogonal Gold sequence, the Barker sequence and the Hadamard-Walsh codes [66]. Different PN codes have different characteristics and usages, but they hold similar properties. In order to explain why the CDMA system is advantageous in the power line channel, a brief of the properties of the spreading code, and how it is generated can be found in [66], [70].

- **Balance property**
  In each period of sequence, the number of binary ones differs from the number of binary zeros by at most one digit. (Note, when used in the system, the binary zero needs to be converted to -1)

  For example: $PN = +1 \ 1 +1 -1 +1 -1 -1$

  \[ \sum PN = +1 \]

- **Autocorrelation**
  The autocorrelation for a $N$ bits PN sequence $a_i$ where $i = 1, \ldots, N$ is defined as

  \[ R_a(k) = \sum_{i=1}^{N} a_i \tilde{a}_{i+k} \quad \text{where} \quad i = 1, \ldots, N \]  

  \[ (3.6) \]

  In the equation, $\tilde{a}_{i+k}$ is the $k$ bits cyclic shift of the PN sequence $a_i$. The PN sequence has a very good property in that it only holds the biggest value when $i = k$. When $i \neq k$ or when the two sequences are not synchronized, it will have very small values. A well-designed PN sequence requires as small an autocorrelation value as possible when $i \neq k$. 
For example: \( PN = +1 \ +1 \ +1 \ -1 \ +1 \ -1 \ -1 \)

\[
R_u(0) = \sum_{i=1}^{N} a_i \tilde{a}_i = 7
\]

\[
R_u(0) = \sum_{i=1}^{N} a_i a_{i+1} = -1
\]

This property of the PN sequence is very useful and it is widely used in synchronizing many communication systems.

- Cross correlation

Cross correlation is used to measure the agreement between any two different PN codes. The cross correlation of any two \( N \) bits PN sequences \( a_i \) and \( a'_i \) where \( i = 1, \ldots, N \) is defined as

\[
R_c(k) = \sum_{i=1}^{N} a_i a'_{i+k} \quad \text{where} \quad i = 1, \ldots, N
\]

In the equation, \( a'_{i+k} \) is the \( k \) bits cyclic shift of the PN sequence \( a'_i \). When the cross correlation value \( R_c(k) \) is zero for all \( k \), the two PN codes are called orthogonal. This property is very important in a CDMA system with multiple users transmitting at the same time as it ensures that one user’s signals do not interfere with another’s.

### 3.5.2 Advantages of the CDMA system in a power line channel

A key advantage of the CDMA system is its ability to maintain a good level of performance in the multipath environment. This advantage arises out of its PN spreading code. An unsynchronized copy of a PN sequence will cause very little interference to the synchronized PN sequence. Suppose the two bits shown in Figure 3.10 are transmitted from the transmitter to the receiver of a CDMA frame. We assume that there are two paths in the power line channel, and the second path is delayed from the first path by \( T_c \). Although a processing gain of seven is too small to achieve good
CDMA system implementation performance, we assume it to be so for the sake of simplicity in illustration.

\[
\begin{align*}
&\sum t^e \text{ path signal} = +7 \\
&+ \sum 2^{nd} \text{ path signal} = +1 \\
&= +8 > 0, \text{thus result} = +1 \\
&\sum t^e \text{ path signal} = -7 \\
&+ \sum 2^{nd} \text{ path signal} = +1 \\
&= -6 < 0, \text{thus result} = -1
\end{align*}
\]

Figure 3.10: System with PN sequence under the PLC multipath channel

In the figure, we find that the signal from the second path, which is assumed to be delayed from the first path’s signal by a one-chip duration time of \( T_c \), interferes only slightly with the receiver’s capability to recover the bit of information which is transmitted from the transmitter. As long as the receiver is synchronized with the first path’s signal, the delayed signal will not affect the information bit decision of the receiver. The signal in the first path in a power line channel is usually the strongest of the multiple paths, as longer paths tend to suffer more from attenuation during transmission. For this reason, the signal in the first path is easier to search for and to synchronize. If the receiver is attached to a RAKE receiver, more energy can be collected from each finger of the RAKE receiver to increase decision accuracy. This effect is demonstrated in the modeling and performance analysis of the CDMA system in Chapter 5.

Another property of the CDMA system, namely that the cross correlation of the two orthogonal PN sequences is zero, allows multiple users to transmit data simultaneously through the system without interfering with each other. This is the case as long as the PN sequences employed by the users are orthogonal. This ability of the CDMA system
to support multiple users is an important advantage over the OFDM system, as is demonstrated in Chapter 6

3.6 Summary of Why OFDM and CDMA Systems Are Appropriate for Broadband PLC

Power line channels are multipath channels and simple communication systems are not able to overcome the problems posed by the multipath effect. OFDM and CDMA systems have been shown to perform well in such multipath channels, making them suitable options for effective broadband PLC.

With its high transmission speed, superior levels of bandwidth efficiency and ability to combat the multipath and impulsive noise effects, OFDM is currently widely accepted by commercial broadband PLC organizations such as Homeplug Alliance [4]. The organization responsible for Homeplug Alliance has also released the career standard Homeplug Standard 1.0 and Homeplug AV in 2001 and 2004 respectively. CDMA has similarly been successfully used in wireless communication because of its good performance in combating multiple path interference, its high security standards and anti-jamming capabilities among others. Although the CDMA system is not popularly used in broadband PLC, its intrinsic ability to support multiple users of the CDMA system make it advantageous over the OFDM system which requires complicated network access protocols to support multiple access. The anti-multipath effect that is characteristic of the CDMA technique makes it a good option for broadband PLC.

Industry progress has had a close relationship with the progress of academic research and a number of studies have focused on the design, modeling and analysis of the performance analysis of the OFDM and CDMA systems for broadband PLC. In [71], [72], the OFDM system and its performance under the PLC channel are studied by simulations. In [55], the OFDM and CDMA systems are compared for downstream power line communications by simulations. In this paper, the author provided a comparison between the OFDM and CDMA systems for broadband downstream PLC. In his analysis works, the performance of the considered systems are derived by simulations under the conditions of the multipath channel which is proposed in [19],
[20] and the additive colored Gaussian noise proposed in [73]. In this work, the same multipath model and the impulsive noise model are also utilized in the analysis.

In this thesis, the performance analysis model of the OFDM and CDMA systems under the multipath and impulsive noise effects are developed in terms of closed form formulas and verified by simulations. Based on these analysis models, comparison of the OFDM and CDMA systems about the BER performance and the optimal overall data rate of these two systems are made, using the criteria of the same bandwidth occupation, the same transmission power for each user, the same total number of users involved in the system and the same power line channel.
CHAPTER 4   MODELING AND PERFORMANCE ANALYSIS OF OFDM SYSTEMS FOR BROADBAND PLC

4.1   Introduction

In this chapter, I study the orthogonal frequency division multiplexing (OFDM) system and develop analytical models to analyze the BER performance of the OFDM system for broadband PLC. I theoretically analyze the bit error rate (BER) performance of the OFDM system (with and without guard interval) under the impulsive noise and multipath effects in terms of closed form formulas. All such analytical models are verified by simulations and detailed later in the chapter.

From the analysis, it is shown that the OFDM system can mitigate the adverse effect of impulsive noise; only heavily disturbed impulsive noise can obviously interfere with the BER performance of the OFDM system. It also becomes clear that the adverse effect of multipath is more severe than that of impulsive noise. Using a suitable length of guard interval can improve the BER performance of the OFDM system. As the longer guard interval tends to be inefficient in the use of signal power and system bandwidth, the system should be configured such that it uses an optimum guard interval that would achieve the best BER performance. This optimization method to estimate the optimum guard interval can be used when implementing the OFDM system.

4.2   OFDM System

The basic principle of an OFDM system is to split a high data rate bit stream into a number of bit streams of a lower data rate, and transmit these low data rate bit streams simultaneously over a number of sub-carriers as shown in Figure 4.1. Two symbols of the OFDM signal with 32 sub-carriers are presented in the figure in both time and frequency domains with a symbol time as $T$. The sub-carrier signal at a certain frequency appears as a sinusoid wave of various phases over time. The waveform in Figure 4.1 is a special example of the OFDM system in which each sub-carrier is modulated with BPSK. The phases for each of these sub-carriers are at either 0 or 180
degrees. From the figure, we can see that for each carrier the phase may change at the boundary of the OFDM symbol within two symbol times $T$. If we observe the signal in the time domain only by summarizing each individual signal located at all the frequencies, we would not able to clearly distinguish each sub-carriers sinusoid waveform among the sum of these individual OFDM sub-carrier signals.

![OFDM signal in the time and frequency domains](image)

**Figure 4.1: OFDM signal in the time and frequency domains**

Unlike single carrier systems, there is only one carrier for each frequency; the OFDM system transmits data by using a series of parallel carriers with different frequencies which are orthogonal to each other, as shown in Figure 4.2. The left part of the figure presents the spectrum of a single carrier system with carrier frequency $f_c$. This single frequency system can be a BPSK, QPSK or QAM modulation system among others. With symbol time $T$, the spectrum is a Sinc function, and the nearest zero frequency amplitude appears at the frequency $f_c \pm \frac{1}{T}$. The right part of Figure 4.2 gives the spectrum of the multi-carrier system like the OFDM system, uses multiple carriers to simultaneously transmit information. For this system with symbol time $T$, the spectrum shows that in order to maintain the orthogonality of multiple carriers, the frequencies of any two adjacent sub-carriers (for example $f_m$ and $f_n$ as in the figure) must meet the requirement of $f_m - f_n = \frac{1}{T}$. 

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As introduced in Chapter 3, the OFDM system has many advantages over the single carrier system. The parallel transmission of the OFDM system implies a relatively lower data rate for each sub-carrier symbol time. The longer symbol time can help the OFDM system to perform better than the single carrier system in a multipath channel. If a guard interval (GI) is introduced into the system, the inter symbol interference (ISI) and inter carrier interference (ICI) can almost be eliminated.

4.2.1 Orthogonality of sub-carryers

The orthogonality of the OFDM sub-carriers is very important to the OFDM system. Before introducing the OFDM system, it is prudent to first introduce the orthogonal properties of its sub-carriers. Since the sub-carriers of OFDM system are all sine or cosine waves, the area under one cycle period of the sine or cosine wave is zero, as shown in Figure 4.3. This area under one cycle period is the integral of the sine or cosine wave over the same period.
If we take a sine wave of frequency \( m \) and multiply it by a sine or cosine wave with frequency \( n \), where both \( m \) and \( n \) are integers, the product can be represented as follows:

\[
f(t) = \sin mt \sin nt
\]  

(4.1)

By applying the trigonometric relationship, this is equal to a sum of two sinusoids of frequencies, the equation takes on the following form:

\[
f(t) = \frac{1}{2} \cos(m - n)t - \frac{1}{2} \cos(m + n)t
\]  

(4.2)

The integral of \( f(t) \) over one period is equal to zero, therefore,

\[
\int_{0}^{2\pi} f(t)dt = \int_{0}^{2\pi} \frac{1}{2} \cos(m - n)t dt - \int_{0}^{2\pi} \frac{1}{2} \cos(m + n)t dt
\]

\[= 0
\]  

(4.3)

In general, for all integers of \( m \) and \( n \), \( \sin mt \), \( \cos mt \), \( \sin nt \) and \( \cos nt \) are orthogonal to each other. That is, if we multiply a sinusoid wave with frequency \( m \)
with another sinusoid wave of frequency \( n \), the integral of these two sinusoid products is zero.

For the OFDM signal with symbol time \( T \) and sub-carrier number \( M \), the frequency of these sub-carriers should satisfy the orthogonal condition such that

\[
f_m = f_0 + \frac{m}{T} \quad \text{where } m = 0, 1, 2, \ldots, M - 1 \quad (4.4)
\]

where \( f_0 \) is the frequency of the 0-th sub-carrier. Its value is the reciprocal of the symbol duration time \( T \) and \( f_0 = 1/T \). \( f_m \) is the frequency for the \( m \)-th sub-carrier.

For any two such sub-carriers among the \( M \) sub-carriers, (for example, the \( m \)-th and \( n \)-th sub-carriers with frequencies \( f_m \) and \( f_n \)), once they meet the requirement in (4.4), the integral of the products are

\[
\begin{align*}
\int_0^T f(t) dt &= \int_0^T \sin 2\pi f_m t \cdot \sin 2\pi f_n t dt \\
&= \int_0^T \frac{1}{2} \cos 2\pi (f_m - f_n) t dt - \int_0^T \frac{1}{2} \cos 2\pi (f_m + f_n) t dt \\
&= \int_0^T \frac{1}{2} \cos 2\pi \frac{(m-n)}{T} t dt - \int_0^T \frac{1}{2} \cos 2\pi \cdot 2f_0 t dt - \int_0^T \frac{1}{2} \cos 2\pi \frac{(m+n)}{T} t dt
\end{align*}
\]

In the integral equation above, the first term is the integral over period \( T \) of a cosine wave with frequency \( \frac{(m-n)}{T} \). No matter what the \( m \) and \( n \) is, there are \( m-n \) full cycles of cosine waves within period \( T \). The integral of these \( m-n \) full cycles of cosine waves over period \( T \) is zero and the other two terms in the equation similarly come to 0. The final result of the integration is therefore zero, and we can conclude that the two sub-carriers with frequency \( f_m \) and \( f_n \) are orthogonal with each other. If we use the two sub-carriers to transmit information, they will not interfere with each other; the information transmitted through them can be corrected and recovered.
4.2.2 OFDM system structure and model

The fundamental function of the OFDM system is to use multiple orthogonal sub-carriers to transmit information in parallel over multiple paths in order to perform well even in environments with impulsive noise. A typical OFDM system, as shown in Figure 4.4, includes a transmitter and receiver with similar structures. The transmitter and receiver handle signals and data in a reverse process.

![OFDM System Diagram](image)

Data from the upper layers of a PLC system are passed to the transmitter of the OFDM system after coding and interleaving as “Input data” to be modulated and transmitted into the power line. The “Input data” is in serial in time sequence and with data rate $R$. It is processed in the “Serial to parallel” block and converted into $M$ parallel sub-channels from the high speed data rate to a lower data rate. Then data in each $M$ parallel sub-channel is modulated by PSK-based modulations. Taking a simple BPSK modulation as an example, the data rate of each $M$ parallel sub-channel is only $R/M$ of the data rate of the input data after the serial to parallel conversion and the BPSK modulations.
modulation. These modulated data \( d_i (0 \leq i \leq M - 1) \) are then fed into an inverse fast Fourier transform (IFFT) block. After the IFFT block, the OFDM signal is processed in the “Insert guard interval” block. This is done in order to handle the problem of inter symbol interference (ISI) in the multipath system, which will be discussed in detail later in this chapter. The modulated signal of the OFDM system \( s(t) \) to be transmitted is generated after this process, and is passed to the analog front end of the power line modem system to amplify and couple into the power line.

The signal \( s(t) \) will propagate in the power line channel according to the topology, structure and cable characteristics of the power line network before it reaches the receiver. As discussed in Chapter 2, the power line channel is a multipath channel which can be described using channel impulse response. Signals that appear at the receiver are the convolution result of the signals from the transmitter, the channel impulse response and the multiple sources of noise. Impulsive noise in particular tends to interfere dramatically with the signal at the receiver.

The signal received from the power line after analog filtering and amplifying becomes the received signal \( r(t) \) to the OFDM receiver. In the OFDM receiver, \( r(t) \) is processed by the “Remove guard interval” block and the fast Fourier transform (FFT) block then becomes the parallel data \( d_i (0 \leq i \leq M - 1) \). After processing in the “Parallel to serial” block, the parallel data becomes the “Output data” which is of the same data rate \( R \) as the “Input data”. We can work out the BER of the OFDM system by comparing the “Input data” with the “Output data”.

The multi-carrier transmitted OFDM signal can be modeled as

\[
s(t) = \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} d_m(i) e^{j2\pi f_m(t-iT_s)} p(t-iT_s) \quad (4.6)
\]

where \( i \) is the sequence number of the \( i \)-th OFDM symbol, \( m \) is the sequence number of \( m \)-th sub-carrier, \( f_m \) is the frequency for the \( m \)-th carrier, \( T_s \) is the OFDM symbol time, including guard interval, and
is the symbol of the $m$-th sub-carrier at the $i$-th time interval $[iT_s,(i+1)T_s]$, for BPSK and QPSK modulation ($\pm 1$ and $\pm 1 \pm j$ respectively). $p(t)$ is the response of the transmitter filter which is a rectangular pulse with duration $T_s$ and amplitude 1.

\[
p(t) = \begin{cases} 
1 & -T_g \leq t \leq T \\
0 & \text{otherwise}
\end{cases}
\]  

(4.8)

where $T_g$ is the guard interval of the OFDM signal. The time difference between the symbol duration $T_s$ and the guard interval $T_g$ is the effective symbol duration time

\[
T = T_s - T_g
\]

(4.9)

Such an OFDM symbol with guard interval (GI) is shown in Figure 4.5

\begin{center}
\begin{tabular}{|c|c|}
\hline
GI & OFDM Symbol \\
\hline
$T_g$ & $T$ \\
\hline
\end{tabular}
\end{center}

Figure 4.5: OFDM symbol with guard interval

The frequency of the $m$-th sub-carrier of the OFDM system must meet the orthogonal requirement as

\[
f_m = f_0 + \frac{m}{T} \quad \text{where } m = 0, 1, 2, \ldots, M - 1
\]

(4.10)
4.3 Analytical Model to Analyze the BER Performance of the OFDM System Under Impulsive Noise Effect

This section will analyze the effect of impulsive noise on the OFDM system. We derive an analytical model to calculate the BER of the OFDM system under the impulsive noise effect. The BER of the single carrier BPSK system under the impulsive noise effect is also derived for the sake of comparison. Some results discussed in this section are part of the paper published in [65].

When analyzing the effect of impulsive noise on the OFDM system in power line channel, the approach in [43] to analyze the effect of impulsive noise on the OFDM system in wireless channel is referred. However, the analysis work in this thesis is different from that in [43]. Recalling the impulsive noise model described in Chapter 2 of this thesis, the impulsive noise is modeled in time domain with three variables: amplitude, duration time and inter-arrival time. The impulsive noise model in this thesis is different from that in [43]. Because of this fundamental difference, the analytical model developed in this thesis for the OFDM system in power line channel is different from that in [43], especially at the final stage of model development where BER is calculated. Therefore, the contribution of this part of the thesis can be summarized as the first analytical model is developed to analyze the effect of power line impulsive noise on the OFDM system by applying the fundamental analyzing approach in [43] and extending it to the analysis of the OFDM system in power line channel.

The effect of impulsive noise on the OFDM system is studied in the context of radio communications in [43]. Because the impulsive noises in the power line are different from the impulsive noises present in radio communications, the findings of [43] are not completely transferable to our area of interest. Still, the authors’ approach to analyzing the impact of impulsive noise can be applied to the OFDM system for PLC. As introduced in Chapter 2, according to [13], noise in the power line can be divided into two categories: background noise and impulsive noise. To analyze their effects on PLC, we assume at all times that \( t = k \). The background noise is understood to be an additive white Gaussian noise (AWGN) \( w_k \) with mean zero and variance \( \sigma_w^2 \). The impulsive noise \( i_k \) is given by
where $b_k$ is the Poisson process which indicates the arrival of the impulsive noise and $g_k$ is the white Gaussian process with mean zero and variance $\sigma_i^2$. This model can be physically thought of as each transmitted data symbol being interfered by an impulsive noise with a probability distribution $b_k$ and random amplitude $g_k$. Therefore, in the time domain, the noise is the sum of the background noise and the impulsive noise with Poisson process occurrence; and in the frequency domain, both the background noise and the impulsive noise are white and will affect the whole band of transmitted signals in the system.

Let $a_k$ and $n_k$ be the discrete time domain expression for the transmitted signal and noise respectively. The discrete points of signal $a_k$ and $n_k$ can be obtained from the sampling and quantization of the transmitted signal $a(t)$ and noise $n(t)$. The received signal $r(t)$ is the sum of the transmitted signal $a(t)$ and noise $n(t)$. Its discrete time domain expression is $r_k$,

$$r_k = a_k + n_k$$  

(4.12)

where $n_k$ is the noise given by

$$n_k = w_k + i_k = w_k + b_k g_k$$  

(4.13)

The probability density function of the noise is

$$p_{n_k}(n_{kR}, n_{kI}) = (1 - \psi)G(n_{kR}, 0, \sigma_w^2)G(n_{kI}, 0, \sigma_w^2) + \psi G(n_{kR}, 0, \sigma_w^2 + \sigma_i^2)G(n_{kI}, 0, \sigma_w^2 + \sigma_i^2)$$  

(4.14)

where $n_{kR}$ and $n_{kI}$ are the real and imaginary parts of $n_k$ respectively, $\psi$ is the probability of the occurrence of the impulsive noise, and $G(x)$ is the Gaussian density defined as follows:
The occurrence of the impulsive noise has an approximately Poisson distribution, which means that the arrival of the impulsive noise follows the Poisson process with a rate of $\lambda$ units per second, so that the event of $k$ arrivals in $t$ seconds has the probability distribution as

$$P_k(t) = \frac{e^{-\lambda t} (\lambda t)^k}{k!} \quad k = 0, 1, 2, ...$$ (4.16)

Let the average duration time of each impulsive noise be $T_{\text{noise}}$ and the duration time of the symbol be $T$, as shown in Figure 4.6. There could be more than one occurrence of impulsive noise in time $T$. $P_i$ is defined as the average duration of impulsive noise in time $T$, and $P_0$ as the average duration without impulsive noise in the time $T$ during which only AWGN is present. $P_i$ can thus be presented as

$$P_i = \frac{\sum_{k=0}^{\infty} e^{-\lambda T} \frac{(\lambda T)^k}{k!} (kT_{\text{noise}})}{T}$$

$$= \lambda T_{\text{noise}} \left[ \sum_{k=1}^{\infty} e^{-\lambda T} \frac{(\lambda T)^{k-1}}{(k-1)!} \right] = \lambda T_{\text{noise}} \left[ \sum_{k=0}^{\infty} e^{-\lambda T} \frac{(\lambda T)^{k}}{k!} \right]$$

$$= \lambda T_{\text{noise}}$$ (4.17)
4.3.1 Effect of impulsive noise on single carrier BPSK system

In the study of the performance of the OFDM system, we compare the performance of the OFDM system with that of the single carrier system under impulsive interference. The single carrier system employed here is a widely known BPSK system. The BER of a BPSK system under the AWGN is detailed in [70] such that

\[ P = Q\left( \sqrt{\frac{2E_b}{N_0}} \right) \] \hspace{1cm} (4.18)

where \( E_b \) is the signal energy per bit, and \( N_0 \) is the power spectral density of the AWGN.

Compared with the OFDM system with the same data rate, the symbol time of the single carrier BPSK system is much smaller than the symbol time of the OFDM system. Usually it is assumed that the symbol time of the single carrier BPSK system is less than the duration time of the impulsive noise. Therefore when the BPSK system is interfered by the impulsive noise and the background noise, the system BER is the average result under the impulsive noise and the AWGN as follows:

\[ P_b = P_{bi}P_{bw} + P_{bi}P_{bw} = \lambda T_{\text{noise}} P_{bi} + (1 - \lambda T_{\text{noise}}) P_{bw} \] \hspace{1cm} (4.19)

where \( P_{bi} \) and \( P_{bw} \) are the BER under the impulsive noise and the AWGN respectively.

According to the BER formula of BPSK, we have

\[ P_{bi} = Q\left( \sqrt{\frac{2E_b}{N_i + N_0}} \right) \] \hspace{1cm} (4.20)

\[ P_{bw} = Q\left( \sqrt{\frac{2E_b}{N_0}} \right) \] \hspace{1cm} (4.21)
where \( E_b \) is the signal energy per bit, \( N_i \) and \( N_0 \) are the power spectral densities of the impulsive noise and the AWGN respectively. Hence, the BER of the single carrier BPSK under the impulsive noise is

\[
P_b = \lambda T_{\text{noise}} P_m + (1 - \lambda T_{\text{noise}}) P_{\text{bw}}
\]

\[
= \lambda T_{\text{noise}} Q\left(\sqrt{\frac{2E_b}{N_i + N_0}}\right) + (1 - \lambda T_{\text{noise}}) Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\]

(4.22)

### 4.3.2 Performance of the OFDM system under impulsive noise effect

Assuming perfect synchronization, timing and the availability of an ideal channel, the transmitted OFDM signal that reaches the receiver after filtering and sampling can be expressed as

\[
r_k = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} d_m e^{-j2\pi mk} + w_k + i_k \quad k = 0, 1, 2, \ldots, M - 1
\]

(4.23)

where \( d_m = \pm 1 \) is the BPSK modulation symbol for \( m \)-th sub-carrier and \( M \) is the number of total sub-carriers of the OFDM system. \( w_k \) and \( i_k \) denote the AWGN and the impulsive noise respectively. The transmitted symbols \( \{ d_m \}_{m=0}^{M-1} \) are recovered from the received sequence \( \{ r_m \}_{m=0}^{M-1} \) by performing the \( M \) points DFT,

\[
R_m = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} r_k e^{-j2\pi mk} = a_m + W_m + I_m \quad m = 0, 1, 2, \ldots, M - 1
\]

(4.24)

where \( W_m \) is once again AWGN after DFT. \( I_m \) is given by the DFT of the impulsive noise as

\[
I_m = \frac{1}{\sqrt{M}} \sum_{k=1}^{M-1} i_k e^{-j2\pi mk} \quad m = 0, 1, 2, \ldots, M - 1
\]

(4.25)
As a result of the DFT operation, the impulsive noise is spread over $M$ data symbols. This process differs somewhat from that of a single carrier system in which the impulsive noise will affect only one symbol. Because of the effect of the noise spreading, the PSD of the overall noise $N_m$ is the sum of the background noise PSD plus the product of the probability of the impulsive noise and its PSD

$$N_m = N_0 + P_{i}N_i$$  \hfill (4.26)

Let $\mu$ be the ratio of impulsive noise PSD to AWGN PSD,

$$\mu = \frac{N_i}{N_0}$$  \hfill (4.27)

The BER of the OFDM system under the AWGN and the impulsive noise interference is

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_m}}\right) = Q\left(\sqrt{\frac{2E_b/N_0}{1 + \mu\lambda T_{\text{noise}}}}\right)$$  \hfill (4.28)

When considering the effect of impulsive noise on the BER performance of the OFDM system, the signal to noise ratio $E_b/N_0$ needs to be replaced by the overall signal to noise ratio $E_b/N_m$.

### 4.4 Analytical Model to Analyze the BER Performance of the OFDM System Under Multipath Effect

In radio communications, some research on the effect of multipath on OFDM systems are studied in [44]-[46]. Because the channel characteristic of radio communications is different from that of PLC, it needs to dedicatedly study the power line multipath effect on PLC OFDM systems. [47] and [54] are two such studies concerned with the bit error rate (BER) performances of PLC OFDM systems under the multipath effect. Only simulation results and analysis are provided and closed form formulas are not available. In this section, the multipath effect on PLC OFDM systems is theoretically analyzed.
The multipath model was developed in [19]-[20] to model the power line channel. The channel can be described in the time domain as the impulse response function as

$$h(t) = \sum_{n=1}^{N} \beta_n \delta(t - \tau_n)$$ (4.29)

where $\beta_n$ and $\tau_n$ are the amplitudes and arrival times of the multipath components respectively, and $N$ is the number of path components of the impulse response of the channel. For broadband PLC, because of the fast transmission speed compared to the variance of the channel, we can assume that the channel response remains constant throughout the transmission of data, i.e., it can be regarded as a time-invariant system. Hence, $\beta_n$ and $\tau_n$ are kept constant in the analysis.

The OFDM signal transmitted into the power line is

$$s(t) = \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} d_m(i) e^{j2\pi f_m (t - iT_s)} p(t - iT_s)$$ (4.30)

When this multi-carrier signal is transmitted through the channel with the channel impulse response of $h(t)$, the received signal $r(t)$ is the convolution result of $s(t)$ and $h(t)$. Adding noise, the received signal $r(t)$ becomes

$$r(t) = s(t) * h(t) + n(t)$$ (4.31)

The convolution result is the sum of the propagation of signal $s(t)$ through a total of $N$ paths. After attenuation and delay, it is

$$s(t) * h(t) = \sum_{n=1}^{N} \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} \beta_n d_m(i) e^{j2\pi f_m (t - iT_s - \tau_n)} p(t - iT_s - \tau_n)$$ (4.32)
\( n(t) \) is the noise received from the power line at the receiver. Because the intention here is to study the multipath effect on the OFDM system, to simplify the analysis we can assume at this stage that \( n(t) \) as AWGN.

If the guard interval is used at the receiver of the system, it will be removed from received signal \( r(t) \). In the \( i \)-th OFDM symbol interval \([iT_i, (i+1)T_i]\), the recovery of the data associated with the \( k \)-th sub-carrier with frequency \( f_k \) is performed by taking the integral of the product of the received signal and the \( k \)-th sub-carrier over one OFDM symbol period \( T \).

\[
e_k = \int_{iT_i}^{iT_{i+1}} r(t)p(t)e^{-j2\pi f_k t} dt
\]  

(4.33)

### 4.4.1 BER performance of the OFDM system without guard interval

In order to progressively develop the BER analytical models of the OFDM system, we first analyze the BER performance of the simpler OFDM system without the guard interval. When the guard interval is not used, \( T_g = 0 \), thus \( T_s = T \). In the \( i \)-th signal interval \([iT_i, (i+1)T_i]\), the input signal of the \( k \)-th decision becomes

\[
e_k = \int_{iT_i}^{(i+1)T_i} r(t)p(t)e^{-j2\pi f_k t} dt
\]  

(4.34)

Assume that the first path of the multipath received signal is matched to the receiver, i.e., \( \tau_1 = 0 \). Results in [19] show that the maximum delay \( \tau_N \) for the impulse response of the power line channel is not longer than 1\,\mu s. This is usually much less than one OFDM symbol time \( T \). Thus, it is assumed that only the symbol immediately before the current symbol can cause interference to the current symbol. Only the delayed former \((i-1)\)-th symbol will interfere with the current \( i \)-th symbol.

If we use BPSK with \( d_m(i) = \pm 1 \) to modulate each sub-carrier, then
\[ e_k = \beta_k \sum_{n=0}^{M-1} d_n(0) T \delta_{m_n} + \sum_{n=2}^{N-1} \sum_{m=0}^{M-1} \beta_n d_m(-1) e^{i \phi_{m_n}} \int_0^T e^{j 2\pi (m-k)/T} dt + \sum_{n=2}^{N-1} \sum_{m=0}^{M-1} \beta_n d_m(0) e^{j \phi_{m_n}} \int_0^T e^{j 2\pi (m-k)/T} dt + \int_0^T n(t) e^{-j 2\pi k t/T} dt \] 

(4.35)

where \( \phi_{m_n} = -2\pi m r_n / T \). \( d_n(0) \) and \( d_n(-1) \) denotes the current and previous symbols transmitted with the \( m \)-th sub-carrier respectively. \( \delta_{m_n} \) is 1 when \( m = k \), and 0 otherwise. Because BPSK is used, only the real parts of \( d_n(0) \) and \( d_n(-1) \) are used, i.e., \( a_m(0) \) and \( a_m(-1) \), both of which are equal to \( \pm 1 \).

The demodulated signal is

\[ z_k = \text{Re}\{e_k\} = \beta a_k(0) T + \sum_{n=2}^{N-1} \sum_{m=0}^{M-1} a_m(-1)[x_{mk}^{(n)}(\tau_n) + x_{mk}^{(n)}(\tau_n)] + \sum_{n=2}^{N-1} \sum_{m=0}^{M-1} a_m(0)[\tilde{x}_{mk}^{(n)}(\tau_n) + \tilde{x}_{mk}^{(n)}(\tau_n)] + n_k \]

(4.36)

where

\[
\begin{align*}
x_{mk}^{(n)}(\tau_n) &= \beta_n \cos \phi_{m,n} \frac{T}{2\pi (m-k)} \sin \left[ \frac{2\pi (m-k) \tau_n}{T} \right] \\
x_{mk}^{(n)}(\tau_n) &= \beta_n \sin \phi_{m,n} \frac{T}{2\pi (m-k)} \{1 - \cos \left[ \frac{2\pi (m-k) \tau_n}{T} \right]\} \\
\tilde{x}_{mk}^{(n)}(\tau_n) &= \beta_n \cos \phi_{m,n} \frac{-T}{2\pi (m-k)} \sin \left[ \frac{2\pi (m-k) \tau_n}{T} \right] \\
\tilde{x}_{mk}^{(n)}(\tau_n) &= \beta_n \sin \phi_{m,n} \frac{-T}{2\pi (m-k)} \{1 - \cos \left[ \frac{2\pi (m-k) \tau_n}{T} \right]\}
\end{align*}
\]

(4.37)

\( n_k \) is the real part of the AWGN as demodulated by the \( k \)-th sub-carrier. It is expressed by

\[ n_k = \text{Re}\{\int_0^T n(t) e^{j 2\pi k t/T} dt\} \]

(4.38)
When the demodulated sub-carrier equals the modulated sub-carrier, i.e., $m = k$, we have

$$
\begin{align*}
  x_{nk}^c(\tau_n) &= \beta_n r_n \cos \phi_{k,n} \\
  x_{nk}^d(\tau_n) &= 0 \\
  \tilde{x}_{nk}^c(\tau_n) &= \beta_n (T - \tau_n) \cos \phi_{k,n} \\
  \tilde{x}_{nk}^d(\tau_n) &= 0
\end{align*}
$$

(4.39)

Finally, we have

$$
z_k = \beta a_k(0) T \\
+ \sum_{n=2}^{N} [a_k(-1)x_{nk}^c(\tau_n) + a_k(0)x_{nk}^d(\tau_n)] \\
+ \sum_{n=2}^{N} \sum_{m=0 \atop m \neq k}^{M-1} \{a_m(-1)[x_{mk}^c(\tau_n) + x_{mk}^d(\tau_n)] + a_m(0)[\tilde{x}_{mk}^c(\tau_n) + \tilde{x}_{mk}^d(\tau_n)]\} \\
+ n_k
$$

(4.40)

In this formula, the first term represents the desired signal component; the second term represents the inter symbol interference (ISI) caused by the multipath effect; and the third term represents the inter carrier interference (ICI) related to the loss of orthogonality between the sub-carriers. From (4.37) to (4.40), ISI and ICI can be derived as follows:

$$
ISI = \sum_{n=2}^{N} \beta_n \cos(-2\pi k \tau_n / T)[a_k(-1)\tau_n - a_k(0)\tau_n + a_k(0)T]
$$

(4.41)

$$
ICI = \sum_{m=0 \atop m \neq k}^{M-1} \sum_{n=2}^{N} \frac{T}{2\pi(m-k)} \sin(-2\pi m \frac{\tau_n}{T}) + \sin(2\pi(2m-k) \frac{\tau_n}{T}) \{a_m(-1) - a_m(0)\}
$$

(4.42)

Error occurs when the assumed current symbol $a_k(0) = 1$ has been transmitted, but the sampled received signal $z_k$ is less than 0. Hence, the error probability of $k$-th sub-carrier is
\[ P_b^k = \text{prob}(z_k < 0 \mid a_k(0) = 1) \] (4.43)

For the ISI, the value of the previous symbol \( a_k(-1) \) is 1 or \(-1\) with equal probability of 0.5. Thus, the error probability includes two parts:

\[ P_k^b = 0.5P_{b1}^k + 0.5P_{b2}^k \]
\[ = 0.5\text{prob}\{z_k < 0 \mid a_k(0) = 1, a_k(-1) = 1\} \]
\[ + 0.5\text{prob}\{z_k < 0 \mid a_k(0) = 1, a_k(-1) = -1\} \] (4.44)

Because ICI is created by a large number of paths and sub-carriers, the interference resulting from it can be assumed to be Gaussian distributed. The condition decision variable is thus a Gaussian distribution. With this assumption, the error probability \( P_{b1}^k \) is calculated as

\[ P_{b1}^k = Q\left( \frac{E[z_k]}{\text{Var}(z_k)} \right) \] (4.45)

When \( a_k(0) = 1 \) and \( a_k(-1) = 1 \), the expectation of \( z_k \) in (4.40) is

\[ E[z_k] = \beta T + \sum_{n=2}^{N} \beta_k T \cos(2\pi k \frac{T_n}{T}) \] (4.46)

The variance of \( z_k \), which contains the variance of all interference terms, is

\[ \text{Var}(z_k) = \text{Var}(\text{ICI}) + \text{Var}(n_k) \] (4.47)

where \( \text{Var}(n_k) \) is given by

\[ \text{Var}(n_k) = E[n_k^2] - E^2[n_k] = N_0 T / 4 \] (4.48)

and \( \text{Var}(\text{ICI}) \) is given by
CHAPTER 4: MODELING AND PERFORMANCE ANALYSIS OF OFDM SYSTEMS FOR BROADBAND PLC

\[ \text{Var}(\text{ICI}) = E(\text{ICI}^2) - E^2(\text{ICI}) \]
\[ = 2 \sum_{m=0}^{M-1} \sum_{n=2}^{N} \beta_n \frac{T}{\pi(m-k)} [\cos \pi(m+k) \frac{\tau_n}{T} \sin \pi(m-k) \frac{\tau_n}{T}]^2 \quad (4.49) \]
\[ = \eta T^2 \]

Let \( \delta_n = \frac{\tau_n}{T} \) be the normalized delay. Summarizing (4.45) – (4.49), we have

\[ P_{b_1}^k = Q \left( \frac{E[z_k]}{\sqrt{\text{Var}(z_k)}} \right) = Q \left( \frac{\beta_1 + \sum_{n=2}^{N} \beta_n \cos(2\pi k \delta_n)}{\sqrt{\eta + \frac{1}{2E_b/N_o}}} \right) \quad (4.50) \]

where \( E_b/N_o \) is the signal to noise ratio.

Similarly, when \( a_k(0) = 1 \) and \( a_k(-1) = -1 \), the error probability \( P_{b_2}^k \) is

\[ P_{b_2}^k = Q \left( \frac{\beta_1 + \sum_{n=2}^{N} \beta_n (1-2\delta_n) \cos(2\pi k \delta_n)}{\sqrt{\eta + \frac{1}{2E_b/N_o}}} \right) \quad (4.51) \]

Substituting equations (4.50) and (4.51) into (4.44), we can obtain the BER formula. The BER formula derived above is only for the \( k \)-th sub-carrier. It has to be averaged over all the sub-carriers.

When the received signal is only affected by AWGN, the PSD of the noise is \( N_o/2 \). However, in the PLC channel, the impulsive noise also interferes with the received signals. The impulsive noise is spread over \( M \) data symbols due to the DFT operation. According to (4.28), the signal to noise ratio now becomes \( E_b/N_m \). When the BER of the OFDM system under the impulsive noise and multipath effects is calculated, \( E_b/N_o \) in equations (4.50) and (4.51) should be replaced by \( E_b/N_m \).
4.4.2 BER performance of the OFDM system with guard interval

The adverse effect of the delayed signals can be removed or reduced by using the guard interval. It effectively eliminates the effect of both ISI and ICI and the extension of the guard interval for a wider delay spread may further improve the transmission performance.

When the guard interval is considered, the channel impulse response can be rewritten as

\[ h(t) = \sum_{n=1}^{N_g+N_2} \beta_n \delta(t-\tau_n) \quad (4.52) \]

where \( \tau_n \) are classified by the following inequalities:

\[
0 \leq \tau_n \leq T_g, \quad \text{when} \quad n = 1, \ldots, N_1
\]

\[
T_g \leq \tau_n \leq T_s, \quad \text{when} \quad n = N_1 + 1, \ldots, N_1 + N_2
\]

When \( n \) is less than \( N_1 \), the delayed time of path \( n \) is less than \( T_g \). The delayed signal will not cause ISI or ICI interference. However, when \( n \) is larger than \( N_1 \), the delayed signal will cause ISI or ICI interference.

In the \( i \)-th signal interval \([iT_s,(i+1)T_s]\), the input signal of the \( k \)-th decision can be achieved by using (4.34). Using the similar derivations method which is previously used in the OFDM system without a guard interval, we have

\[
e_k = \beta_k d_k(0)T + \sum_{n=2}^{N_g} T \beta_n \tau_n e^{-j2\pi k \tau_n / T} a_k(0)
\]

\[
+ \sum_{n=N_1+1}^{N_g+N_2} [T - (\tau_n - T_g)] \beta_n e^{-j2\pi k \tau_n / T} d_k(0)
\]

\[
+ \sum_{n=N_1+1}^{N_g+N_2} [\tau_n - T_g] \beta_n e^{-j2\pi k (\tau_n - T_s) / T} a_k(-1)
\]

\[
+ \sum_{m=0}^{M-1} \sum_{n=N_1+1}^{N_g+N_2} \beta_n a_m(-1) e^{-j2\pi m(\tau_n - T_s) / T} \int_{\tau_n-T_s}^{T} e^{j2\pi (m-k) \nu / T} dt
\]

\[
+ \int_0^T n(t) \cdot e^{-j2\pi k \nu / T} dt
\]

\( (4.53) \)
and

\[ z_k = \text{Re}\{e_k\} = \beta_k a_k(0)T + \text{ISI} + \text{ICI} + n_k \] (4.54)

where

\[ \text{ISI} = \sum_{n=2}^{N_1} T \beta_n \cos(-2\pi k \tau_n / T) a_k(0) \]
\[ + \sum_{n=N_1+1}^{N_2} \beta_n \{[T-(\tau_n - T_g)] \cos(-2\pi k \tau_n / T) a_k(0) + (\tau_n - T_g) \cos(-2\pi k (\tau_n - T_g) / T) a_k(-1)\} \] (4.55)

and

\[ \text{ICI} = \sum_{n=N_1+1}^{N_2} \sum_{m=0}^{M-1} \{a_m(-1)[x_{mk}^{\text{rec}}(\tau_n) + x_{mk}^{\text{int}}(\tau_n)] + a_m(0)[x_{mk}^{\text{rec}}(\tau_n) + \tilde{x}_{mk}^{\text{int}}(\tau_n)]\} \] (4.56)

where

\[ x_{mk}^{\text{rec}}(\tau_n) + x_{mk}^{\text{int}}(\tau_n) = \beta_n T \frac{\pi(m-k)}{\pi(m-k)} \times \cos \frac{-\pi k (\tau_n - T_g) - \pi m (\tau_n - T)}{T} \sin \frac{\pi m (\tau_n + T) - \pi k (\tau_n - T_g)}{T} \] (4.57)

and

\[ \tilde{x}_{mk}^{\text{rec}}(\tau_n) + \tilde{x}_{mk}^{\text{int}}(\tau_n) = \beta_n T \frac{\pi(m-k)(\tau_n - T_g) - 2\pi m \tau_n}{\pi(m-k)(\tau_n - T_g)} \times \sin \frac{\pi m (\tau_n - T_g)}{T} \] (4.58)

The error probability can be obtained as
\[
P_b^k = 0.5Q \left( E[z_k | a_k(0) = 1, a_k(-1) = 1] / Var(z_k) \right) + 0.5Q \left( E[z_k | a_k(0) = 1, a_k(-1) = -1] / Var(z_k) \right)
\]

where \( E[z_k | a_k(0) = 1, a_k(-1) = 1] = \beta T + \sum_{n=2}^{N_t} T \beta_n \cos(-2\pi k \tau_n / T) \)

\[
+ \sum_{n=N_t+1}^{N_t+N_r} \beta_n \{ [T - (\tau_n - T_g)]\cos(-2\pi k \tau_n / T) + (\tau_n - T_g) \cos(-2\pi k (\tau_n - T_g) / T) \}
\]

\( E[z_k | a_k(0) = 1, a_k(-1) = -1] = \beta T + \sum_{n=2}^{N_t} T \beta_n \cos(-2\pi k \tau_n / T) \)

\[
+ \sum_{n=N_t+1}^{N_t+N_r} \beta_n \{ [T - (\tau_n - T_g)]\cos(-2\pi k \tau_n / T) - (\tau_n - T_g) \cos(-2\pi k (\tau_n - T_g) / T) \}
\]

\[
Var(z_k) = \sum_{m=0}^{M-1} \sum_{n=N_t+1}^{N_t+N_r} \left[ (x^W_{mk}(\tau_n) + x^H_{mk}(\tau_n))^2 + [\tilde{x}^W_{mk}(\tau_n) + \tilde{x}^H_{mk}(\tau_n)]^2 \right] + N_0 T / 4
\]

(4.59)

(4.59) presents the BER formula of the OFDM system with guard interval under the multipath effect. Also, the BER formula derived above is only for the \(k\)-th sub-carrier. It has to be averaged over all other sub-carriers.

### 4.5 Simulation Model of OFDM System

It is necessary to compare the BER performances of the analytical formulas with the results of simulations in order to verify the accuracy of the analytical formulas derived in the previous sections.

The simulation of the BER of the OFDM system is developed using Matlab. Simulation parameters can be set in the initial stage to control the simulation and to achieve corresponding results for different parameter settings. The simulated signal is transmitted in terms of an OFDM frame, which means several OFDM symbols are transmitted sequentially in the time domain within a frame as described in Figure 4.7.
We use a sample OFDM system to describe the simulation in detail. The system parameters are
- 64 sub-carriers
- BPSK modulation conducted for each sub-carrier
- Guard interval of 1/8 symbol length
- 5 OFDM symbols transmitted within 1 frame
- PLC impulse response channel modeled using a four-path channel
- Interference of impulsive noise

The simulation can be described using the 14 steps shown in Figure 4.8. The functions of these steps are as follows:
1. Generate random data
   In order to ensure the accuracy of the simulation results, source data needs to be randomly generated to avoid having results that are correct only for specific cases. Since the example system contains 64 sub-carriers and BPSK modulation, we need to generate 64 bits of random data for each symbol. Considering that there are a total of 5 symbols in a frame, $5 \times 64$ or 320 bits of random data have to be generated in the 1/0 form.

2. Convert serial data to parallel
   The random data generated is in serial form. According to the transmission requirements of the OFDM system, the serial data needs to be converted into a parallel form before transmission. After this step, the data spans five arrays, each containing 64 bits.

3. Modulate data by using BPSK
   The parallel data is modulated with BPSK, QPSK or QAM, depending on the chosen modulation schemes. In this work, we use BPSK modulation to modulate the data. Hence, only one bit of information is transmitted for a single carrier in the OFDM system. In the sample OFDM system shown in Figure 4.9, data after this step still spans five arrays of 64 bits, but values change from the (1/0) to (1/-1) form. For ease of illustration, the figure only shows one array of data with a total of 64 sample points.
4. Process data by using IFFT

The BPSK modulated data that is in parallel, is passed to the IFFT block where the signal is converted from the frequency domain into the time domain. The length of the signal in the time domain is the symbol length of the OFDM system. For the example system, the IFFT block needs to process 64 points of IFFT. The output of the IFFT block is 64 points of complex data, within the I-channel and Q-channel.

5. Insert guard interval

The guard interval is inserted in order to eliminate the inter symbol interference (ISI) caused by the multipath effect of the power line channel. The guard interval arises from the end part of the signal and is usually between (1/16)-th and (1/4)-th of the length of the transmitted symbol. In the system illustrated, the guard interval is (1/8)-th the length of the transmitted symbol. For each symbol, the extra sample point is $\frac{64}{8} = 8$ points. Because there are a total of five symbols in a frame, the total number of sample points within an OFDM simulation frame is calculated as $5 \times (64 + 8) = 360$ sample points, as shown in Figure 4.10.
6. Process the signal according to the PLC channel

After the guard interval is inserted, the signal is transmitted into the power line. The signal propagates from the transmitter to the receiver according to characteristics of the PLC channel. As introduced earlier, the PLC channel can be represented by the
impulse response. For the example OFDM system, signal after the multipath effect is shown in Figure 4.11. We notice that the overall signal amplitudes become smaller. This is because of the attenuation arising out of propagation through the PLC channel. Further, the signal that results is a summary of signals from multiple paths.

7. Signal power level detection

As the current study requires an examination of the variance relationship between the BER and the signal to noise ratio $E_b/N_o$, it becomes necessary to calculate the signal power level needed to prepare the Gaussian noise which is generated and added to the signal in Step 8.

8. Generate Gaussian noise according to the signal power and $E_b/N_o$ and add impulsive noise

In this step, Gaussian noise is added to the signal along with impulsive noise according to the requirements of the simulation. A number of factors are considered including the impulsive noise scenario. For the example OFDM system with $E_b/N_o = 10$dB, the noise amplitude can be calculated and added to the signal as shown in Figure 4.12. When the impulsive noise is considered, it is generated according to a number of parameters, such as inter-arrival time, duration time and amplitude. The generated impulsive noise is added to the signal which is already interfered with by the multipath effect and by Gaussian noise. The process of generating impulsive noise is described in the following section of this chapter.

9. Remove guard interval

The receiver receives the signal after it propagates through the PLC channel. We assume that the transmitter and the receiver are synchronized accurately such that we eliminate the need to consider synchronization issues in the simulation model. The guard interval of the received signal is removed in the receiver for the next stage of process.

10. Process data by using FFT

11. Demodulate data by using BPSK

12. Convert parallel data to serial

Steps 10 – 12 outline the process that occurs in the receiver of the OFDM system. These steps are the reverse of those used in the transmission process. The purpose
of these is to demodulate data from the time domain into the frequency domain, and to convert data from the parallel to serial form.

![Graphs of random noise, I-channel signal + noise, Q-channel signal + noise](image)

Figure 4.12: Noise generated according to the $E_b / N_o$ and the signal which is mixed with the generated noise

13. Compare data with transmitter

The demodulated data (now in serial) is compared with the random data generated (also in serial) at the beginning of the simulation. The number of errors and the number of transmitted data points are used to calculate our targeted simulation result: BER.

14. Calculate BER

The BER is calculated by dividing the total number of errors by the total number of data points transmitted. The errors and the data are accumulated through repeated rounds of simulations. A large number round of simulations are needed in order to ensure accuracy. Our evaluation criteria or priority in this task is to seek out a
minimum number of errors, and not simply to transmit a minimum amount of data. For this reason, the results of the simulation become quite accurate even for a very small BER. To illustrate, if the system BER is less than $10^{-6}$, and our target is set as at least 100 errors, the simulation has to be run for a very long time in order to generate at least $10^8$ of data to be transmitted, processed and compared. All simulations in this thesis are guaranteed by the fact that a minimum number of errors are met in every case.

4.6 Verification of the Analytical Models Using Simulations

We compare the results of the BER performance with the results of the simulations in order to verify the accuracy of the analytical models derived in this chapter.

4.6.1 Effect of impulsive noise

Recall the parameters of the impulsive noise listed in Table 2-2. $IAT$ is the inter-arrival time of the impulsive noise, which is the reciprocal of the arrival rate $\lambda$; $T_{\text{noise}}$ is the average noise duration time. The amplitude of the impulsive noise is described by using the “noise to AWGN power ratio $\mu$”. $\mu$ is the ratio of the amplitude of the impulsive amplitude to the amplitude of the AWGN. In the simulation of the effect of the impulsive noise on the communication system, the impulsive noise can be generated according to the $IAT$, $T_{\text{noise}}$, $\mu$ and $E_b/N_0$. Once these parameters are confirmed, the impulsive noise can be generated by simulation according to the Poisson process of the impulsive noise.

Figure 4.13 shows the BER performance comparison of the OFDM and single carrier BPSK systems under the effects of three impulsive noise scenarios. In the analysis, by fixing the impulsive noise to AWGN power ratio $\mu$ and changing $E_b/N_0$, we can obtain the BER performance curves. In the figure, there is an error floor for the BER performance of the single carrier BPSK system. Continuing to increase $E_b/N_0$ does not improve the performance effectively. The reason is that in single carrier BPSK system, the impulsive noise could destroy the whole symbols in this system, because of
the large value of $\mu$ and the longer duration time of $T_{\text{noise}}$ compared with the bit duration time.

The effect of the impulsive noise on the OFDM system is spread over $M$ data symbols because of the DFT operation as described in (4.25). The BER performance can be improved rapidly with the increase of $E_b/N_o$, which means that for OFDM system, increasing signal power can improve the BER performance effectively. From this perspective point of view, OFDM system performs better than single carrier BPSK system. It is also observed that for the medium disturbed Scenario II and weakly disturbed Scenario III, OFDM always performs better than single carrier BPSK. Only the heavily disturbed impulsive noise obviously will interfere with the BER performance of the OFDM system. For heavily disturbed Scenario I, when $E_b/N_o$ is greater than 8dB, the OFDM system performs better than the single carrier BPSK system. Hence, the OFDM system can achieve good performance under the impulsive noise effect.

![Figure 4.13: Analytical BER performances of the OFDM and single carrier BPSK systems under three impulsive noise scenarios](image)

**Figure 4.13**: Analytical BER performances of the OFDM and single carrier BPSK systems under three impulsive noise scenarios
Figure 4.14 shows the comparison of analytical and simulation results of the BER performances of single carrier BPSK and OFDM systems under the heavily disturbed Scenario I environment. The impulsive noise to AWGN power ratio $\mu$ is set as 10. In Figure 4.14, it can be seen that the analytical results correspond well with the simulation results. The error floor of the single carrier BPSK is also observed in the results. The error floor is caused by the large value of $\mu$. At lower $E_b/N_o$, the performances of both the OFDM and single carrier BPSK systems are similar because in this region the white Gaussian noise plays the main role to cause bit errors.

Figure 4.14: Comparison of analytical and simulation results for the BER performances of the OFDM and single carrier BPSK systems under the heavily disturbed Scenario I environment

### 4.6.2 Effect of multipath

In this analysis, the channel response amplitude $\beta_n$, the arrival time of multipath components $\tau_n$ and the number of path components $N$ are given in Table 4-1.

In the table, the multipath number is chosen as 4. When the delay of the first path is presented as the relative delay compared with itself as 0, the relative delay of the other paths compared with it is listed respectively in the table. The bit rate $R$ is set at
10Mbps. The symbol duration time $T$ is the result of the sub-carrier number $M$ and the bit rate $R$, i.e., $T = M / R$. Figure 4.15 shows the BER performance of the OFDM system under the effect of the multipath channel with the parameters listed in Table 4-1. In the figure, the BER performances of the OFDM system with and without guard interval (GI) are presented, by using the results from both the analytical and simulation models.

Table 4-1: Parameters of PLC Multipath Channel to OFDM Systems

<table>
<thead>
<tr>
<th>Path Number</th>
<th>$\beta_n$</th>
<th>$\tau_n$ ($\mu$s)</th>
<th>$\delta_n = \tau_n / T$</th>
<th>$M = 32$</th>
<th>$M = 64$</th>
<th>$M = 128$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.4</td>
<td>0.1250</td>
<td>0.0625</td>
<td>0.0313</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.02</td>
<td>0.6</td>
<td>0.1875</td>
<td>0.0938</td>
<td>0.0469</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.01</td>
<td>0.7</td>
<td>0.2188</td>
<td>0.1094</td>
<td>0.0547</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.15: Comparison of analytical and simulation results of the BER performance of the OFDM system under the multipath effect
For the OFDM system without the guard interval (GI), the BER performances are analyzed for three scenarios with different sub-carrier numbers $M$, namely 32, 64 and 128 respectively. Through the comparison, it is shown that there is an error floor for each curve without the guard interval. These error floors are caused by the multipath effect of the PLC channel. The BER performance cannot be improved by increasing the signal power. The reason is that there are ISI and ICI interferences in the system, and increasing the signal power will also increase the power of ISI and ICI.

From the comparison of the OFDM system with and without the guard interval, it is shown that an applicable means of improving the BER performance is to increase the sub-carrier number and to expand the symbol duration time and therefore the effect of ISI and ICI can be reduced as shown in Figure 4.15. The BER performance of the OFDM system with 128 sub-carriers outperforms the system with 64 sub-carriers, which in turn outperforms the system with 32 sub-carriers. While increasing the number of sub-carriers in order to improve the BER performance would increase hardware costs and increase requirements to maintain the synchronization between the transmitter and receiver, these concerns are beyond the scope of this research work.

For the OFDM system with guard interval, the guard interval is selected as 1/8 of the symbol duration time $T$ and the sub-carrier number $M = 32$. Both analytical and simulation results are given in the figure and the good match between the two verifies well the accuracy of the developed analytical model for the OFDM system with guard interval. Figure 4.15 also compares the BER performance of the OFDM system with and without a guard interval using the same sub-carrier number $M = 32$. It is shown that, with the same carrier number, the BER performance of the OFDM system with guard interval can be greatly improved compared with that without guard interval.

4.6.3 Effects of both impulsive noise and multipath

We analyze the BER performance of the OFDM system without guard interval under both the impulsive noise and multipath effects in order to determine which of the two causes the most severe interference. Figure 4.16 shows the BER performance of the OFDM system under both the impulsive noise and multipath effects. The parameters of the multipath channel are in accordance to those detailed in Table 4-2. Because the
heavily disturbed impulsive noise interferes severely with the OFDM system, only the heavily disturbed Scenario I in the table is used in this analysis. In order to study the effect of the impulsive noise amplitude on the BER performance of the OFDM system, for the case of the OFDM system with 128 sub-carriers, we study three scenarios with different impulsive noise to AWGN power ratios $\mu$.

Table 4-2: Simulation Parameters of the OFDM System under Impulsive Noise and Multipath Effects

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Carriers $M$</td>
<td>32</td>
</tr>
<tr>
<td>Multipath Path Number</td>
<td>$\beta_n$</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.01</td>
</tr>
<tr>
<td>Impulsive Noise to AWGN Power Ratio $\mu$</td>
<td>10</td>
</tr>
<tr>
<td>Impulsive Noise $IAT$</td>
<td>0.0196s</td>
</tr>
<tr>
<td>$T_{\text{Noise}}$</td>
<td>0.0641ms</td>
</tr>
<tr>
<td>$E_b / N_o$</td>
<td>$0 \sim 50$dB</td>
</tr>
</tbody>
</table>
Figure 4.16: BER performance of the OFDM system without guard interval under the impulsive noise and multipath effects

As can be seen in Figure 4.16, the analytical results correspond well with the simulation results with 32 and 64 sub-carrier numbers. Increasing the sub-carrier number can improve the BER performance of the OFDM system under the impulsive noise and multipath effects. However, for each of these curves with 32 and 64 sub-carrier numbers, the error floors from 40dB onwards, which are caused by the multipath effect, are still limit the BER performance of the OFDM system without guard interval.

Through the comparison of the BER performance of the OFDM system with 128 sub-carrier number under different values of impulsive noise to AWGN power ratio $\mu$, it is shown that although the impulsive noise deteriorates the BER performance, but this adverse effect is only observed at $E_b / N_o$ below 45dB. The effect of impulsive noise is subsumed by the multipath effect when $E_b / N_o$ increases from 45dB. The value of the BER error floor caused by the multipath effect will not change with the impulsive noise to AWGN power ratio. Thus, it can be concluded that the multipath effect is in fact the main obstacle to achieving good BER performance and the use of a guard interval is essential.
4.7 Optimum Guard Interval for OFDM System

As demonstrated and verified in the previous section, the use of a guard interval can effectively improve the BER performance of the OFDM system. The guard interval is copied from the tail end of the OFDM symbol and added at the beginning of the same symbol. Adding a guard interval in this way requires more signal power to be used to transmit the OFDM symbol. The guard interval will be removed from the demodulation process at the receiver, thus the effective signal to demodulate an OFDM symbol is still the part of the transmitted OFDM symbol before the guard interval is added. The signal power of the guard interval is therefore wasted – the longer the guard interval, the more power is wasted.

On the other hand, the use of the guard interval is unavoidable, and decreasing its length impairs the ability of the OFDM system to eliminate the multipath effect. The solution is to use an optimum guard interval that can minimize the power wastage while maximizing the BER performance as shown in Figure 4.17. The following parameters are used in the analysis: $E_b/N_0 = 28$dB, 30dB and 32dB, and $M = 64$. The multipath channel used is the same as that given in Table 4-1.

From the figure, it can be seen that when $E_b/N_0$ is determined, there is an optimum guard interval at which the BER can reach the minimum value. When the guard interval reaches the optimum value, the BER performance can be improved rapidly because the guard interval can eliminate the multipath effect. The signal power spent in the guard interval is worthy of such improvement. However, after the optimum value, the performance of the BER performance will deteriorate with the increase of the guard interval, because the cost paid for the guard interval in terms of wasting signal power is larger than its contribution in eliminating the multipath effect. Hence, the use of the optimum guard interval is crucial to achieve the best BER performance of the OFDM system.
For the OFDM system affected by the multipath interference, adding a suitable length of guard interval can completely eliminate the effect of multipath. The length of this guard interval can be derived from the channel impulse response. As long as the selected guard interval is longer than the maximum delay of the multipath, the OFDM system can be protected from the inter symbol interference caused by the multipath effect. Once the guard interval of the system is determined, one way to improve the effective data rate of the OFDM system is by increasing the number of subcarriers, because within a limited bandwidth, if the number of subcarriers increases, the frequency spacing between any two adjacent subcarriers will decrease. This smaller frequency space will lead to a longer symbol time for the system. For the OFDM system with the constant length of guard interval, the longer symbol time can produce a smaller ratio of $T_g / T_s$, thus increasing the effective data rate of the system.

### 4.8 Conclusions

In this chapter, the BER performance of the OFDM system for broadband PLC under the impulsive noise and multipath effects has been theoretically analyzed in terms of
closed form formulas. The validity of the derived formulas is verified by comparing the analytical BER performance results with the simulation results. From the analysis, it can be concluded that the BER performance of the OFDM system under the effect of impulsive noise depends on the inter-arrival time, the average duration time and the power spectral density of the impulsive noise. Only the heavily disturbed impulsive noise will interfere with the OFDM system. The adverse effect of multipath is more serious than the effect of impulsive noise. There is an error floor of the BER performance caused by the multipath effect. The multipath effect can be effectively overcome by the use of a larger number of sub-carriers and an optimum guard interval. The optimum guard interval that can minimize the BER of the OFDM system from a trade-off between the BER performance and the guard interval in the power line channel has been demonstrated.

The analytical model of the OFDM system developed in this chapter will be applied in Chapter 6, where a comparison will be made between the OFDM and CDMA systems.
CHAPTER 5 MODELING AND PERFORMANCE ANALYSIS OF CDMA SYSTEMS FOR BROADBAND PLC

5.1 Introduction

Chapter 3 provided a simple introduction to the CDMA system and its advantages in overcoming the multipath effect. It is these advantages of the CDMA system that render it a suitable modulation technique option for broadband PLC. In this chapter, the CDMA system is studied and analytical models are developed to analyze the BER performance of the CDMA system for broadband PLC.

The bit error rate (BER) performance of the CDMA systems with and without a RAKE receiver in the PLC channel are theoretically analyzed using closed form formulas. All analytical results are verified by simulations, which are detailed later in this chapter. The accuracy of the analytical models is verified by comparison with simulation results. The analytical model developed in this chapter will be used in Chapter 6 where the OFDM and CDMA systems will be objectively compared.

5.2 CDMA System

A basic CDMA system typically includes a transmitter and a receiver, as shown in Figure 5.1. In the transmitter, binary input data are spread by the PN code, modulated with the carrier and transmitted into the power line. After propagation through power lines, the signal is distorted by the multipath effect and noise interference of the power line channel. The signal is demodulated and despread when it reaches the receiver, after being filtered and amplified through analog and digital means of signal processing. These mechanisms are not illustrated in the figure because they are beyond the focus of this research work. The output binary data are determined in this way and they are expected to be the recovery from the transmitter. The bit error rate can be calculated by counting the differences in the output and input data, and dividing this number by the number of the input data.
There are multiple users in a power line network at any one time who may need to access the network and transmit signals to different destinations. The minimum interference between the orthogonal PN sequences of these multiple users is an important characteristic of the CDMA system, and they allow for simultaneous transmission by multiple users without a complicated protocol. This differs from the OFDM system which does not allow the transmission of data from multiple users. The CDMA system to be analyzed consists of $K$ simultaneous transmitters (users). Each user is assigned a unique spreading code sequence that modulates the phase of the carrier with the data sequence. The spreading code used in this paper is the Gold Sequence which has both good autocorrelation and inter-correlation performance [66]. The modulation scheme is BPSK.

### 5.2.1 Transmitter and transmitted signals

A total of $K$ users are assumed to exist in the model under analysis. For any single user such as the $k$-th user, as shown in Figure 5.1, the transmitted signal $S^{(k)}(t)$ is

$$S^{(k)}(t) = \sqrt{2P}a^{(k)}(t)b^{(k)}(t)\cos[\omega_ft + \phi^{(k)}]$$  \hspace{1cm} (5.1)

$a^{(k)}(t)$ is the spreading code sequence of the $k$-th user and can be expressed as

$$a^{(k)}(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} p_s(t - jT_c), \quad a_j^{(k)} \in \{-1, 1\}$$  \hspace{1cm} (5.2)
$b^{(k)}(t)$ is the data waveform, which can be expressed as

$$b^{(k)}(t) = \sum_{j=-\infty}^{\infty} b_j^{(k)} p_b(t - jT), \quad b_j^{(k)} \in \{-1, 1\}$$  \hspace{1cm} (5.3)$$

where $P$ is the average transmitted power which is common to all users; $\omega_c$ is the system carrier frequency that modulates the baseband spread signal into the passband signal which is centered at frequency $\omega_c$; $\phi^{(k)}$ is the phase angle of the $k$-th modulator, which is usually set as zero for BPSK modulation; $T$ is the data bit duration and $T_c$ is the chip duration; $p_a(t)$ and $p_b(t)$ are the rectangular pulses of unit height with chip duration $T_c$ and bit duration $T$ respectively. The processing gain of the spread spectrum system can be represented as $N = T / T_c$. A larger processing gain $N$ typically brings about better BER performance in a CDMA system, but it also brings disadvantages in the form of lower data rate within the same bandwidth, and a much more complicated structure for transmitter and receiver. Therefore, choosing a suitable processing gain is an important step in using the CDMA system.

### 5.2.2 Power line channel and received signals

The power line channel model is introduced in detail in Chapter 2, and reviewed only briefly here. Noise in power lines can be divided into two categories: background noise and impulsive noise. Background noise can be assumed as the additive white Gaussian noise (AWGN) $w_k$ with mean zero and two-sided power spectral density $N_0 / 2$. Impulsive noise $i_k$ can be described as $i_k = b_k g_k$, where $b_k$ is the Poisson process which is the arrival of the impulsive noise and $g_k$ is the white Gaussian process with mean zero and two-sided power spectral density $N_i / 2$. This model can be visualized as each transmitted symbol being independently hit by an impulsive noise with a probability distribution $b_k$ and random amplitude $g_k$.

The multipath model was developed in [17] to model the power line channel. The channel can be described in the time domain as the impulse response function, namely...
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\[ h(t) = \sum_{i=0}^{L-1} \beta_i \delta(t - \tau_i) \], where \( \delta(t) \) is a unit impulse function, \( \beta_i \) and \( \tau_i \) are the amplitude and arrival times of the multipath component \( l \) respectively. \( L \) is the number of paths in the channel’s impulse response. A signal that is transmitted through the multipath channel can be illustrated as in Figure 5.2.

\[ s(t) \]
\[ \tau_0 \]
\[ \beta_0 \]
\[ \tau_1 \]
\[ \beta_1 \]
\[ \tau_{L-1} \]
\[ \beta_{L-1} \]
\[ r(t) \]
\[ n(t) \]

\[ \sum_{i=0}^{L-1} \beta_i \delta(t - \tau_i) = - \sum_{i=0}^{L-1} \beta_i \delta(t - \tau_i), \]

where \( \beta_i \) and \( \tau_i \) are the amplitude and arrival times of the multipath component \( l \) respectively. \( L \) is the number of paths in the channel’s impulse response. A signal that is transmitted through the multipath channel can be illustrated as in Figure 5.2.

In the figure, the amplitude of the transmitted signal \( s(t) \) is attenuated in \( \beta_i \) and delayed in \( \tau_i \) during the transmission in each individual path. The attenuated and delayed signals are summarized at the receiver. The power line noise \( n(t) \) interferes with the summarized signal and appears at the receiver as signal \( r(t) \). Because of the relatively fast rate of transmission between the transmitter and receiver compared with the change of channel impulse response, we can assume that the channel response remains constant throughout the period of data transmission, and that it can be regarded as a time-invariant system.

After transmission through the multiple paths of the power line, the signals transmitted by the \( K \) users reach the receivers. One of them is the target receiver for the \( k \)-th user. This receiver has the same spreading PN code as the \( k \)-th user, and will search, trace and demodulate signals from this user. To the \( k \)-th user, signals from the other \( K - 1 \)
users become the sources of interference. The interference is small however, because there is only minimal cross-correlation between the PN codes of these \( K \) users.

For the \( k \)-th user’s transmitted signal \( s^{(k)}(t) \), the number of multipath to propagate through is \( L^{(k)} \). After propagating through the \( L^{(k)} \) multipaths, the attenuated and delayed replicas of \( s^{(k)}(t) \) reach the receiver and are summarized. This received signal for the \( k \)-th user \( r^{(k)}(t) \) is

\[
r^{(k)}(t) = \sqrt{2P} \sum_{l=0}^{L^{(k)}-1} \beta_l^{(k)} a^{(k)}(t - \tau_l^{(k)}) b^{(k)}(t - \tau_l^{(k)}) \cos(\omega_l t + \phi_l^{(k)})
\]  

(5.4)

where \( \phi_l^{(k)} = \phi_l^{(k)} - \omega_l \tau_l^{(k)} \) is the phase of the \( l \)-th path of the \( k \)-th user’s carrier. Assuming that the phase angle of the \( k \)-th modulator \( \phi_l^{(k)} \) is 0, then \( \phi_l^{(k)} = \omega_l \tau_l^{(k)} \).

The total signal at the \( k \)-th receiver is the sum of all the \( K \) users’ transmitted signals which propagate through multiple paths. Including noise from the power line, the overall signal at the \( k \)-th receiver is

\[
r(t) = \sum_{k=1}^{K} r^{(k)}(t)
\]

(5.5)

\[
= \sqrt{2P} \sum_{k=1}^{K} \sum_{l=0}^{L^{(k)}-1} \beta_l^{(k)} a^{(k)}(t - \tau_l^{(k)}) b^{(k)}(t - \tau_l^{(k)}) \cos(\omega_l t + \phi_l^{(k)}) + n(t)
\]

where \( n(t) \) denotes the power line noise added to the signal at the receiver.

5.2.3 CDMA receiver

The receiver in the CDMA system is based on a coherent receiver structure. Signals received are correlated with a local reference which is a replica of the transmitted signal. This correlation process is usually implemented with a matched filter, whose impulse response is a delayed version of the mirror image of the input signal waveform [70]. In the CDMA system, the matched filter needs to match the reference user’s spreading code and is assumed to have achieved time synchronization with the initial
path of the reference signal. The output of the matched filter is the demodulated result of the CDMA system. At the receiver, due to the multipath access of the CDMA system, there may be interference from the previous symbols which are delayed by the multipath effect and interference from other users in the system.

5.2.3.1 Self interference (SI) and multiple access interference (MAI)

There are multiple replicas of the transmitted signal in the multipath channel, which arrive at the receiver later than the signal from the first path. During the synchronization process, the receiver searches the synchronization code and synchronizes with the transmitted signal. To achieve a good signal to noise ratio, the receiver is usually synchronized and locked with the strongest path of the reference user’s signal. Further discussion of synchronization is beyond the scope of this research work, but interested readers will find detailed information on synchronization methods of the CDMA system in [70], [74].

As indicated above, the signals that appear at the receiver are complicated in that they include the signal of interest as well as multiple interferences. Before developing the analysis model, we provide a brief introduction to the various signals and interferences.

Consider the case of the first user (user 1) in a system with $K$ users. The signal transmitted from the transmitter propagates through the power line multipath channel with a multipath number of $L^{(1)}$ before it reaches the receiver. At the receiver, there are $L^{(1)}$ copies of the signal which are from the same user. If we assume that the receiver is locked with the strongest path $j$, the relationship among the synchronized $j$-th path signal of user 1, the rest of user 1’s signals from $(L^{(1)} - 1)$ paths and the multipath signals of the other $K - 1$ users can be described in Figure 5.3.
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Figure 5.3: The relationship between the synchronized $j$-th path signal of user 1 and the other signals

In the figure, $b_k^{-1}$, $b_k^0$ and $b_k^1$ are the transmitted previous bits, the current bit and the next bit of the $k$-th user (user $k$) respectively; $b_1^{-1}$, $b_1^0$ and $b_1^1$ are the bits of the 1-st user, each of length $T$. Signals from the same user (1-st user) on other paths ($l \neq j$) which arrive at the receiver earlier or later than the strongest path will interfere with the correlation and decision process. This interference from signals originating from the same user yet propagating through different paths is called self interference (SI). Interference arising from the rest users in the system which is caused by the multipath access of the CDMA system is known as multiple access interference (MAI).

In addition to the interference of SI and MAI which are caused by the multipath effect and multiple access of the CDMA system, noise from the power line also interferes with the system. This form of interference is simply referred to as noise interference (NI).
Till now, we can conclude that the interference of the CDMA system includes three types, namely SI, MAI and NI. Studying how big their interference to the system is a significant part of work to develop the BER performance analysis model for the CDMA system.

### 5.2.3.2 RAKE receiver structure

In this section, we will study and compare the CDMA system with and without a RAKE receiver. Through such a comparison, we expect to achieve a better understanding of the performance of the CDMA system in a power line channel.

The RAKE receiver was first proposed by Price and Green and patented in 1956. It was designed to counter the multipath effect by using several “sub-receivers” (each slightly delayed) in order to tune in to the individual multipath components. In this research work, we assume that the number of RAKE receiver branches is equivalent to the number of multipaths. The components are first independently and coherently decoded, and then combined at a later stage in order to make the most of the different transmission characteristics of each transmission path.

The RAKE receiver model is shown in Figure 5.4. Each finger that matches a receiver branch provides a signal component. The interference associated with this signal component originates from the correlation of the matched filter code with the codes of other users, the correlation with the rest of the $k$-th user’s signal paths and the noise in the system.
The difference between the CDMA system with and without a RAKE receiver lies in the number of fingers. In the CDMA systems without a RAKE receiver, the decision is made based on correlation values from only one branch.

### 5.3 Effect of Multipath on Non-RAKE Receiver CDMA System

Because of its simplicity, we begin our BER performance analysis of the CDMA system first by examining a system without a RAKE receiver. Assuming that acquisition has been accomplished for the user of interest $k = 1$, and assuming that the receiver is synchronized with the signal propagating through the first path $l = 0$, the local pattern to be used in the demodulation process is the $\tau_0$ delayed replica of the product of the spread code and carrier. For the sake of simplicity, parameter $\tau_0^{(1)}$ which belongs to the 1-st user is represented as $\tau_0$ such that

$$a^{(1)}(t - \tau_0) \cos(\omega_1 t - \omega_c \tau_0).$$

The output of the correlation receiver at each sampling time can be written as
\[ U = \int_{t_0}^{T+t_0} r(t)a^{(1)}(t-\tau_0)\cos(\omega_t t - \omega_0 \tau_0) dt \]

\[ = \int_{t_0}^{T+t_0} [\sqrt{2P} \sum_{k=1}^{K} \sum_{l=0}^{L+1-1} \beta_l^{(k)} a^{(k)}(t-\tau_0^k) b^{(k)}(t-\tau_0^k) \cos(\omega_t t + \varphi_l^{(k)}) + n(t)] [a^{(1)}(t-\tau_0) \cos(\omega_t t - \omega_0 \tau_0)] dt \]

\[ = S + I_{mai} + I_{si} + I_{ui} \]

(5.7)

The integration period \([\tau_0, \tau_0 + T]\) in the equation can be converted to \([0, T]\) by simply replacing it in the equation, which is

\[ U = S + I_{mai} + I_{si} + I_{ui} \]

\[ = \int_{0}^{T} \left\{ \sqrt{2P} \sum_{k=1}^{K} \sum_{l=0}^{L+1-1} \beta_l^{(k)} a^{(k)}(t-\tau_0^k) b^{(k)}(t-\tau_0^k) \cos[\omega_t (t-\tau_0^k)] + n(t) \right\} [a^{(1)}(t-\tau_0^k) \cos[\omega_t (t-\tau_0^k)]] dt \]

(5.8)

where \(\tau_0^k = \tau_0^k - \tau_0\) is the relative delay between the \(l\)-th path of the \(k\)-th user and the direct path \(l = 0\) of the user \(k = 1\).

The equation above includes four terms, each of which is detailed as follows:

- \(S\) is the correlation value from the first user \(k = 1\) that is transmitted through the first path \(l = 0\). It is the value that has to be worked out in order to recover the transmitted data.

\[ S = \int_{0}^{T} \sqrt{2P} \beta_0^{(1)} a^{(1)}(t) b_0^{(1)}(t) \cos(\omega_t t) a^{(1)}(t) \cos(\omega_t t) dt \]

\[ = \sqrt{2P} \beta_0^{(1)} \int_{0}^{T} [a^{(1)}(t)]^2 b_0^{(1)}(t) [\cos(\omega_t t)]^2 dt \]

(5.9)

where \(b_0^{(1)}(t)\) is the current bit which is demodulated for the 1-st user.

Because the value of the spread code \(a^{(1)}(t)\) is either +1 or -1, the square of it is \([a^{(1)}(t)]^2 = 1\), therefore,
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\[ S = \sqrt{2P \beta_0^{(1)}} \int_0^T b_0^{(1)}(t)[\cos(\omega_c t)]^2 dt = \sqrt{2P \beta_0^{(1)}} \int_0^T \frac{1}{2} b_0^{(1)}(t)[1 + \cos(2\omega_c t)] dt \]  

\[ = \sqrt{P/2} \beta_0^{(1)} b_0^{(1)} T \]  

\( (5.10) \)

- \( I_{\text{mai}} \) is the Multiple Access Interference (MAI) due to the cross correlation of the match filter code and the codes of the other users in the system. This interference grows with the number of users in the system.

\[ I_{\text{mai}} = \int_0^T \sqrt{2P} \sum_{k=2}^K \sum_{l=0}^{d(k)-1} \beta_l^{(1)} a^{(1)}(t) d^{(1)}(t) b^{(1)}(t) dt \]

\[ = \sqrt{P/2} \sum_{k=2}^K \sum_{l=0}^{d(k)-1} \beta_l^{(1)} \int_0^T a^{(1)}(t) d^{(1)}(t) b^{(1)}(t) dt \]

\[ = \sqrt{P/2} \sum_{k=2}^K \sum_{l=0}^{d(k)-1} \beta_l^{(1)} \int_0^T \cos(\omega_c t - \omega_c \tau_{l0}^{(k)}) \cos(\omega_c t) dt \]  

\[ (5.11) \]

For the integration of \( \int_0^T \cos(\omega_c t - \omega_c \tau_{l0}^{(k)}) \cos(\omega_c t) dt \), its value is

\[ \int_0^T \cos(\omega_c t - \omega_c \tau_{l0}^{(k)}) \cos(\omega_c t) dt 
\]

\[ = \int_0^T \frac{1}{2} [\cos(\omega_c t - \omega_c \tau_{l0}^{(k)} + \omega_c t) + \cos(\omega_c t - \omega_c \tau_{l0}^{(k)} - \omega_c t)] dt 
\]

\[ = \int_0^T \frac{1}{2} \cos(2\omega_c t - \omega_c \tau_{l0}^{(k)}) dt + \int_0^T \frac{1}{2} \cos(\omega_c \tau_{l0}^{(k)}) dt 
\]

\[ = \frac{1}{2} T \cos(\omega_c \tau_{l0}^{(k)}) \]

\[ (5.12) \]

Therefore, \( I_{\text{mai}} \) can be simplified as

\[ I_{\text{mai}} = \sqrt{2P} \sum_{k=2}^K \sum_{l=0}^{d(k)-1} \beta_l^{(1)} \int_0^T a^{(1)}(t) d^{(1)}(t) b^{(1)}(t) dt \]

\[ = \sqrt{P/2} \sum_{k=2}^K \sum_{l=0}^{d(k)-1} \beta_l^{(1)} T \cos(\omega_c \tau_{l0}^{(k)}) \int_0^T a^{(1)}(t) d^{(1)}(t) b^{(1)}(t) dt \]  

\[ (5.13) \]

The integration of \( \int_0^T a^{(1)}(t) d^{(1)}(t) b^{(1)}(t) dt \) is the interference of the MAI with the 1-st user’s signal from the path \( l = 0 \). This interference would be easier to understand if we observe its relationship with the signal of interest, which is the signal
of the 1-st user propagating through \( l = 0 \) path from sampling time \( t = 0 \) to \( t = T \), as shown in Figure 5.5.

![Diagram showing multiple access interference](image)

**Figure 5.5: Multiple access interference from signal of the \( k \)-th user to the signal of interest (the 1-st user \( l = 0 \) path)**

In the figure, we find that interference from the \( k \)-th user and \( l \)-th path is composed of two parts. The first consists of interference from the preceding bit \( b^{(k)}_{i-1} \) of the \( k \)-th user, which is delayed because of the propagation delay. In this case, the length \( T_{i0}^{(k)} \) of the delayed signal from the preceding interferes with the signal of interest, as shown in the figure. Another part of the interference comes from the current bit \( b^{(k)}_0 \) of the \( k \)-th user. Because of propagation delay, only \( T - T_{i0}^{(k)} \) length of it will interfere with the signal of interest. If we represent the interferences from the preceding and current bits as \( R_{k1}(T_{i0}^{(k)}) \) and \( \hat{R}_{k1}(T_{i0}^{(k)}) \) respectively, and let

\[
\begin{align*}
R_{k1}(T_{i0}^{(k)}) &= \int_0^{T_{i0}^{(k)}} a^{(k)}(t - T_{i0}^{(k)})a^{(i)}(t)dt \\
\hat{R}_{k1}(T_{i0}^{(k)}) &= \int_{T_{i0}^{(k)}}^T a^{(k)}(t - T_{i0}^{(k)})a^{(i)}(t)dt
\end{align*}
\]

we have

\[
\int_0^T a^{(i)}(t - T_{i0}^{(k)})a^{(k)}(t)b^{(k)}(t - T_{i0}^{(k)})dt = [b^{(k)}_1 R_{k1}(T_{i0}^{(k)}) + b^{(k)}_0 \hat{R}_{k1}(T_{i0}^{(k)})]
\]

\[\text{(5.15)}\]
Finally, MAI can be simplified as

$$I_{\text{mai}} = \sqrt{P/2} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \beta_l^{(k)} T \cos(\omega c \tau_{10}^{(k)}) \int_0^T a^{(i)}(t - \tau_{10}^{(i)}) a^{(i)}(t) b^{(i)}(t - \tau_{10}^{(i)}) dt$$

$$= \sqrt{P/2} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \beta_l^{(k)} T \cos(\omega c \tau_{10}^{(k)}) \left[ b_{l-1}^{(k)} R_{1\lambda}^{(k)}(\tau_{10}^{(k)}) + b_0^{(k)} \tilde{R}_{\lambda}^{(k)}(\tau_{10}^{(k)}) \right]$$

(5.16)

$I_s$ is the self interference (SI), which is the cross correlation with the other $L^{(i)} - 1$ signal paths ($l \neq 0$) of the first user $k = 1$.

$$I_s = \int_0^T \sqrt{2P} \sum_{l=1}^{L-1} \beta_l^{(1)} (t - \tau_{10}^{(1)}) b_t^{(1)}(t - \tau_{10}^{(1)}) \cos(\omega c t - \omega c \tau_{10}^{(1)}) \cos(\omega c t) a^{(1)}(t) dt$$

$$= \sqrt{P/2} \sum_{l=1}^{L-1} \beta_l^{(1)} \int_0^T \cos(\omega c t - \omega c \tau_{10}^{(1)}) \cos(\omega c t) dt \int_0^T a^{(1)}(t) a^{(1)}(t - \tau_{10}^{(1)}) b^{(1)}(t - \tau_{10}^{(1)}) dt$$

(5.17)

The calculation $\int_0^T \cos(\omega c t - \omega c \tau_{10}^{(1)}) \cos(\omega c t) dt$ is similar to that in the calculation of MAI,

$$\int_0^T \cos(\omega c t - \omega c \tau_{10}^{(1)}) \cos(\omega c t) dt = \frac{1}{2} T \cos(\omega c \tau_{10}^{(1)})$$

(5.18)

thus, $I_s$ can be further simplified as

$$I_s = \sqrt{P/2} \sum_{l=1}^{L-1} \beta_l^{(1)} \int_0^T \cos(\omega c t - \omega c \tau_{10}^{(1)}) \cos(\omega c t) dt \int_0^T a^{(1)}(t) a^{(1)}(t - \tau_{10}^{(1)}) b^{(1)}(t - \tau_{10}^{(1)}) dt$$

$$= \sqrt{P/2} \sum_{l=1}^{L-1} \beta_l^{(1)} T \cos(\omega c \tau_{10}^{(1)}) \int_0^T a^{(1)}(t) a^{(1)}(t - \tau_{10}^{(1)}) b^{(1)}(t - \tau_{10}^{(1)}) dt$$

(5.19)

The integration $\int_0^T a^{(1)}(t) a^{(1)}(t - \tau_{10}^{(1)}) b^{(1)}(t - \tau_{10}^{(1)}) dt$ is the self interference from the $L^{(i)} - 1$ delayed signal of the 1-st user to its signal from the path $l = 0$, as in Figure 5.6.
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Figure 5.6: Self interference from the delayed signal of the 1-st user to the signal of interest (the 1-st user $l = 0$ path)

Similarly, if we represent the interferences from the preceding bit $b_{−1}^{(i)}$ and the current bit $b_0^{(i)}$ as

\[
\begin{align*}
R_{i1}(\tau_{i0}^{(i)}) &= \int_0^{\tau_{i0}^{(i)}} a^{(i)}(t - \tau_{i0}^{(i)})a^{(i)}(t)dt \\
\hat{R}_{i1}(\tau_{i0}^{(i)}) &= \int_{\tau_{i0}^{(i)}}^{T} a^{(i)}(t - \tau_{i0}^{(i)})a^{(i)}(t)dt
\end{align*}
\]

we have

\[
\int_0^T a^{(i)}(t)a^{(i)}(t - \tau_{i0}^{(i)})b^{(i)}(t - \tau_{i0}^{(i)})dt = [b_{−1}^{(i)}R_{i1}(\tau_{i0}^{(i)}) + b_0^{(i)}\hat{R}_{i1}(\tau_{i0}^{(i)})] \tag{5.21}
\]

Finally, the self interference can be represented as

\[
I_{ni} = \sqrt{P/2} \sum_{l=1}^{\rho^{(i)-1}} \beta_l^{(i)}T \cos(\omega_l \tau_{i0}^{(i)}) \int_0^T a^{(i)}(t)a^{(i)}(t - \tau_{i0}^{(i)})b^{(i)}(t - \tau_{i0}^{(i)})dt
\]

\[
= \sqrt{P/2} \sum_{l=1}^{\rho^{(i)-1}} \beta_l^{(i)} \cos(\omega_l \tau_{i0}^{(i)})[b_{−1}^{(i)}R_{i1}(\tau_{i0}^{(i)}) + b_0^{(i)}\hat{R}_{i1}(\tau_{i0}^{(i)})]
\]

$I_{ni}$ is the noise interference (NI). It is determined by the noise interference in the power line channel.

\[
I_{ni} = \int_0^T n(t - \tau_0) \cos[\omega_n(t - \tau_0)]a^{(i)}(t - \tau_0)dt \tag{5.23}
\]
In analyzing the multipath effect on the CDMA system, we first assume that the power line noise is AWGN with two-sided power spectral density $N_0/2$. Using the Gaussian assumption for the other interference components in the BER calculations is very common, since it has been found to be quite accurate [49]-[53]. The effect of impulsive noise on the CDMA system will be discussed later in this chapter. By using the Gaussian assumption, the BER of the system can be calculated as

$$P_b = Q \left[ \frac{E[U]}{\sqrt{\text{Var}(U)}} \right] = Q \left[ \frac{\sqrt{P/2} \beta_0^{(1)} T}{\text{Var}(I_{\text{mai}}) + \text{Var}(I_s) + \text{Var}(I_n)} \right]$$ (5.24)

where $E[U]$ is the expectation of the correlation receiver output $U$, $\text{Var}[U]$ is the variance of $U$. $\text{Var}[U]$ is the sum of the variances of MAI, SI and NI.

As we know, for a series of $n$ independent variables $\xi_1, \xi_2, \ldots, \xi_n$, we have

$$E(\xi_1 \xi_2 \ldots \xi_n) = E(\xi_1)E(\xi_2)\ldots E(\xi_n)$$

$$\text{Var}(\xi_1 + \xi_2 + \ldots + \xi_n) = \text{Var}(\xi_1) + \text{Var}(\xi_2) + \ldots + \text{Var}(\xi_n)$$

Since MAI is the summation of all the multiple path interferences as in (5.16) and each of this multiple path interference is independent with each other, the variance of MAI is the summation of the variances of all the terms in MAI [49], [51]

$$\text{Var}(I_{\text{mai}}) = \text{Var}(\sqrt{P/2} \sum_{k=2}^{K} \sum_{l=0}^{K-1} \beta_l^{(k)} T \cos(\omega \tau_{10}^{(k)}) [b_{l-1}^{(k)} R_1(\tau_{10}^{(k)}) + b_0^{(k)} \hat{R}_1(\tau_{10}^{(k)})]$$

$$= \frac{P}{2} \sum_{k=2}^{K} \sum_{l=0}^{K-1} \text{Var} \left\{ [\beta_l^{(k)} T \cos(\omega \tau_{10}^{(k)})] \left[ b_{l-1}^{(k)} R_1(\tau_{10}^{(k)}) + b_0^{(k)} \hat{R}_1(\tau_{10}^{(k)}) \right] \} \right. \right. \right. \right. \right. \right. (5.25)$$

The variance of any term of MAI can be expressed as
\[ \text{Var}\{[\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})]^2 [b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2] \} \]
\[ = E\{[\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})]^2 [b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2] \} \]
\[ - E^2\{[\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})][b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})] \} \] \hspace{1cm} (5.26)

The probability of \( b_{-1}^{(k)} \) and \( b_0^{(k)} \) with values of +1 and -1 is equal to 0.5, and the expectation of any term of MAI is equal to 0, thus we have

\[ E^2\{[\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})][b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})] \} = 0 \]

Finally, from (5.25) and (5.26), we have the result of (5.27) as follows:

\[ \text{Var}(I_{\text{mai}}) = P / 2 \cdot \sum_{k=2}^{K} \sum_{l=0}^{K(l)-1} \text{Var}\{[\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})]^2 [b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2] \} \]
\[ = P / 2 \cdot \sum_{k=2}^{K} \sum_{l=0}^{K(l)-1} E\{[\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})]^2 [b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2] \} \]
\[ = P / 2 \cdot \sum_{k=2}^{K} \sum_{l=0}^{K(l)-1} [\beta_i^{(k)} T \cos(\omega_i \tau_{i0}^{(k)})]^2 E\{[b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2 \} \] \hspace{1cm} (5.27)

According to [49], the value of

\[ E\{[b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2 \} \] \hspace{1cm} (5.28)

is the average correlation parameter over all the possible \( K(K-1)/2 \) combinations of the \( k \)-th sequence and the first sequence among the set, it can be approximated as

\[ E\{[b_{-1}^{(k)} R_{k1}(\tau_{i0}^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_{i0}^{(k)})]^2 \} = \frac{2}{3N} \] \hspace{1cm} (5.29)

Therefore,
\[ Var(I_{\text{mai}}) = \sigma_{\text{mai}}^2 \]
\[ = P / 2 \cdot \sum_{k=2}^{K} \sum_{l=0}^{d^{(k)}-1} [\beta^{(k)}_l T \cos(\omega \tau^{(k)}_{i_0})]^2 E \left\{ \left[ b^{(k)}_{i_1} R_{i_1}(\tau^{(k)}_{i_0}) + b^{(k)}_0 \hat{R}_{i_1}(\tau^{(k)}_{i_0}) \right]^2 \right\} \]
\[ = \frac{PT^2}{3N} \sum_{k=2}^{K} \sum_{l=0}^{d^{(k)}-1} [\beta^{(k)}_l \cos(\omega \tau^{(k)}_{i_0})]^2 \] (5.30)

The variance of SI is
\[ Var(I_{\text{si}}) = \sigma_{\text{si}}^2 = E(I_{\text{si}}^2) - E^2(I_{\text{si}}). \]

The expectation of \( I_{\text{si}}^2 \) is
\[ E(I_{\text{si}}^2) = E\left\{ \sqrt{P / 2} \sum_{l=1}^{d^{(i)}-1} T \beta^{(i)}_l \cos(\omega \tau^{(i)}_{i_0}) [b^{(i)}_{i_1} R_{i_1}(\tau^{(i)}_{i_0}) + b^{(i)}_0 \hat{R}_{i_1}(\tau^{(i)}_{i_0})]^2 \right\} \]
\[ = E \left\{ \frac{P}{2} \cdot \sum_{l=1}^{d^{(i)}-1} [T \beta^{(i)}_l \cos(\omega \tau^{(i)}_{i_0})]^2 E \left\{ [b^{(i)}_{i_1} R_{i_1}(\tau^{(i)}_{i_0}) + b^{(i)}_0 \hat{R}_{i_1}(\tau^{(i)}_{i_0})]^2 \right\} \right\} \]
\[ = \frac{P}{2} \cdot \sum_{l=1}^{d^{(i)}-1} [T \beta^{(i)}_l \cos(\omega \tau^{(i)}_{i_0})]^2 E \left\{ [b^{(i)}_{i_1} R_{i_1}(\tau^{(i)}_{i_0}) + b^{(i)}_0 \hat{R}_{i_1}(\tau^{(i)}_{i_0})]^2 \right\} \] (5.31)

Similarly, the square of the expectation of \( I_{\text{si}} \) is 0, therefore,
\[ Var(I_{\text{si}}) = \sigma_{\text{si}}^2 \]
\[ = \frac{P}{2} \cdot \sum_{l=1}^{d^{(i)}-1} [T \beta^{(i)}_l \cos(\omega \tau^{(i)}_{i_0})]^2 E \left\{ [b^{(i)}_{i_1} R_{i_1}(\tau^{(i)}_{i_0}) + b^{(i)}_0 \hat{R}_{i_1}(\tau^{(i)}_{i_0})]^2 \right\} \] (5.32)

The expectation of \( E \left\{ b^{(i)}_{i_1} R_{i_1}(\tau^{(i)}_{i_0}) + b^{(i)}_0 \hat{R}_{i_1}(\tau^{(i)}_{i_0}) \right\}^2 \) can be approximated as
\[ E \left\{ b^{(i)}_{i_1} R_{i_1}(\tau^{(i)}_{i_0}) + b^{(i)}_0 \hat{R}_{i_1}(\tau^{(i)}_{i_0}) \right\}^2 = \frac{1}{N} \] (5.33)

Therefore, the variance of self interference is
\[ Var(I_{\text{si}}) = \sigma_{\text{si}}^2 = \frac{PT^2}{2N} \sum_{l=1}^{d^{(i)}-1} [\beta^{(i)}_l \cos(\omega \tau^{(i)}_{i_0})]^2 \] (5.34)
The variance of noise is \( \text{Var}(I_{ni}) = \sigma_{ni}^2 = \frac{N_0 T}{4} \).

Summarizing the above variances, the overall interference is the sum of
\( \text{Var}(I_{mai}) = \sigma_{mai}^2 \), \( \text{Var}(I_{si}) = \sigma_{si}^2 \) and \( \text{Var}(I_{ni}) = \sigma_{ni}^2 \). Hence by using Gaussian approximation, the BER of a non-RAKE receiver CDMA system is

\[
P_b = Q\left( \frac{\sqrt{P/2} \beta_i^{(1)}}{\sqrt{\text{Var}(I_{mai}) + \text{Var}(I_{si}) + \text{Var}(I_{ni})}} \right) = Q\left( \frac{\beta_i^{(1)}}{\sqrt{\frac{2}{3N} \sum_{k=2}^{K} \sum_{l=0}^{L-1} [\beta_i^{(k)} \cos(\omega_k \tau_i^{(k)})]^2 + \frac{1}{N} \sum_{l=1}^{L-1} [\beta_i^{(l)} \cos(\omega_l \tau_i^{(l)})]^2 + \frac{1}{2(E_s / N_0)}}} \right) \tag{5.35}
\]

where \( E_s / N_0 \) is the signal to noise ratio. The BER equation (5.35) is derived for the BER of the user of interest \( k = 1 \). The overall BER for the system can be obtained by averaging the BER results of all the \( K \) users in the system.

### 5.4 Effect of Multipath on RAKE Receiver CDMA System

In the CDMA system without a RAKE receiver, only the strongest path of the signal is correlated for decision-making purposes. In the RAKE receiver CDMA system, signals from multiple paths are collected and summarized to make the decision. In the analysis of this thesis, the number of RAKE receiver branches is assumed to be the same as the number of multipaths. The BER equation of the RAKE receiver CDMA system can be developed using similar methods to those employed in the non-RAKE receiver CDMA system. Assuming that acquisition has been accomplished for the user of interest \( k = 1 \), the matched filter is then synchronized to the paths of the desired signal of user \( k = 1 \).

The number of multipaths for user \( k = 1 \) is \( L^{(1)} \), thus there are \( L^{(1)} \) branches in the RAKE receiver. The output of the RAKE receiver is the summary of the correlation values from these \( L^{(1)} \) branches. The output decision of the receiver at each sampling time can be expressed as
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\[ U = \sum_{n=0}^{L_{+}-1} \int_{\tau_{n}^{(0)}}^{T_{+}^{(0)}} r(t) \beta_{n}^{(i)} a^{(i)}(t - \tau_{n}^{(i)}) \cos(\omega_{c} t - \omega_{c} \tau_{n}^{(i)}) dt \]

\[ = \sum_{n=0}^{L_{+}-1} \int_{\tau_{n}^{(0)}}^{T_{+}^{(0)}} \left[ \sqrt{2P} \sum_{k=1}^{K} \sum_{l=0}^{l_{+}-1} \beta_{l}^{(k)} a^{(k)}(t - \tau_{l}^{(k)}) b^{(k)}(t - \tau_{l}^{(k)}) \cos(\omega_{c} t + \phi_{l}^{(k)}) + n(t) \right] \]

\[ \left[ \beta_{n}^{(i)} a^{(i)}(t - \tau_{n}^{(i)}) \cos(\omega_{c} t - \omega_{c} \tau_{n}^{(i)}) \right] dt \]

\[ = \sum_{n=0}^{L_{+}-1} \left( S^{(n)} + I_{\text{mai}}^{(n)} + I_{\text{si}}^{(n)} + I_{\text{ni}}^{(n)} \right) \]

where \( n \) is the number of branches ranging from 0 to \( L_{+} - 1 \). In order to derive the BER analysis model for the system, we need to calculate the signal component \( S^{(n)} \), the multiple access interference \( I_{\text{mai}}^{(n)} \) from other users \( k \neq 1 \), the self interference \( I_{\text{si}}^{(n)} \) from user \( k = 1 \) and the noise interference \( I_{\text{ni}}^{(n)} \) for the \( n \)-th branch in (5.36).

\( S^{(n)} \) is the signal component of user \( k = 1 \) from the \( n \)-th branch of the RAKE receiver. It is from the equation (5.36) when \( k = 1 \) and \( l = n \) as follows:

\[ S^{(n)} = \int_{\tau_{n}^{(0)}}^{T_{+}^{(0)}} \sqrt{2P} \beta_{n}^{(i)} a^{(i)}(t - \tau_{n}^{(i)}) b_{0}^{(i)}(t - \tau_{n}^{(i)}) \cos(\omega_{c} t - \omega_{c} \tau_{n}^{(i)}) \]

\[ \times \beta_{n}^{(i)} a^{(i)}(t - \tau_{n}^{(i)}) \cos(\omega_{c} t - \omega_{c} \tau_{n}^{(i)}) dt \]

\[ = \sqrt{P / 2} b_{0}^{(i)} \left[ \beta_{n}^{(i)} T \right] \]

Multiple access interference (MAI) \( I_{\text{mai}}^{(n)} \) is the interference from the other users \( k \neq 1 \) to the first user \( k = 1 \) in the \( n \)-th branch of the RAKE receiver

\[ I_{\text{mai}}^{(n)} = \int_{\tau_{n}^{(0)}}^{T_{+}^{(0)}} \left[ \sqrt{2P} \sum_{k=2}^{K} \sum_{l=0}^{l_{+}-1} \beta_{l}^{(k)} a^{(k)}(t - \tau_{l}^{(k)}) b^{(k)}(t - \tau_{l}^{(k)}) \cos(\omega_{c} t - \omega_{c} \tau_{l}^{(k)}) \right] \]

\[ \left[ \beta_{n}^{(i)} a^{(i)}(t - \tau_{n}^{(i)}) \cos(\omega_{c} t - \omega_{c} \tau_{n}^{(i)}) \right] dt \]

Let \( \tau_{n}^{(k)} = \tau_{l}^{(k)} - \tau_{n}^{(i)} \) be the relative delay between the \( l \)-th path of the \( k \)-th user and the \( n \)-th receiver branch of the 1-st user. Therefore, \( I_{\text{mai}}^{(n)} \) can be simplified as

\[ I_{\text{mai}}^{(n)} = \sqrt{P / 2} \sum_{k=2}^{K} \sum_{l=0}^{l_{+}-1} \beta_{n}^{(i)} \beta_{l}^{(k)} T \cos(\omega_{c} \tau_{n}^{(k)}) \int_{0}^{T} a^{(k)}(t - \tau_{l}^{(k)}) b^{(k)}(t - \tau_{l}^{(k)}) dt \]
The integration of \( \int_0^T a^{(k)}(t - \tau^{(k)}_{in})a^{(i)}(t)b^{(k)}(t - \tau^{(k)}_{in})dt \) is the interference to the signal of user \( k = 1 \) in the \( n \)-th branch of the receiver. This interference is from the signal of user \( k \neq 1 \) propagating through the \( l \)-th path, which is shown in Figure 5.7.

![Signal from user \( k = 1 \), propagates through the 0-th path](image)

![Signal from user \( k = 1 \), propagates through the \( n \)-th path](image)

![Signal from the \( k \)-th user, propagates through the \( l \)-th path. Compared with the 0-th path, it is delayed by \( \tau^{(k)}_{in} \)](image)

Figure 5.7: Signals of the \( k \)-th user to the 0-th branch and the \( n \)-th branch of the RAKE receiver of the user \( k = 1 \)

In the figure, we find that the interference from the \( k \)-th user which is propagated through the \( l \)-th path is composed of two parts. The first path consists of interference from the preceding bit \( b_{l-1}^{(k)} \) of the \( k \)-th user, which is delayed by the propagation delay. In this case, the length \( \tau^{(k)}_{in} \) of this preceding bit \( b_{l-1}^{(k)} \) interferes with the signal of interest, as shown in the figure. Another part of the interference comes from the current bit \( b_0^{(k)} \) of the \( k \)-th user. Because of propagation delay, only \( T - \tau^{(k)}_{in} \) length of this current bit \( b_0^{(k)} \) interferes with the signal of interest. If we represent the interferences from the preceding and current bits as \( R_{k1}(\tau^{(k)}_{in}) \) and \( \hat{R}_{k1}(\tau^{(k)}_{in}) \) respectively, and let

\[
\begin{align*}
R_{k1}(\tau^{(k)}_{in}) &= \int_0^{\tau^{(k)}_{in}} a^{(k)}(t - \tau^{(k)}_{in})a^{(i)}(t)dt \\
\hat{R}_{k1}(\tau^{(k)}_{in}) &= \int_{\tau^{(k)}_{in}}^{T} a^{(k)}(t - \tau^{(k)}_{in})a^{(i)}(t)dt
\end{align*}
\tag{5.40}
\]

we have
\[
\int_0^T a^{(i)}(t - \tau_{in}^{(i)}) a^{(i)}(t) b^{(k)}(t - \tau_{in}^{(k)}) dt = [b^{(k)}_{-1} R_{k1}(\tau_{in}^{(k)}) + b^{(k)}_0 \hat{R}_{k1}(\tau_{in}^{(k)})]
\] (5.41)

Finally, MAI can be simplified as

\[
\begin{align*}
I_{\text{MAI}}^{(n)} &= \sqrt{P/2} \sum_{k=1}^{K} \sum_{l=0}^{\ell-1} \beta_n^{(i)} \beta_l^{(i)} T \cos(\omega_c \tau_{in}^{(i)}) \int_0^T a^{(i)}(t - \tau_{in}^{(i)}) a^{(i)}(t - \tau_{in}^{(i)}) dt \\
&= \sqrt{P/2} T \sum_{k=2}^{K} \sum_{l=0}^{\ell-1} \beta_n^{(i)} \beta_l^{(i)} \cos(\omega_c \tau_{in}^{(i)}) [b^{(k)}_{-1} R_{k1}(\tau_{al}^{(k)}) + b^{(k)}_0 \hat{R}_{k1}(\tau_{al}^{(k)})] \\
&= \sqrt{P/2} T \sum_{k=2}^{K} \sum_{l=0}^{\ell-1} \beta_n^{(i)} \beta_l^{(i)} \cos(\omega_c t - \omega_c \tau_{al}^{(i)}) dt \quad (5.42)
\end{align*}
\]

Self interference (SI) \(I_{\text{SI}}^{(n)}\) is the interference of the signal of user \(k = 1\) which is propagated through the rest paths \(l \neq n\) to the signal of user \(k = 1\) which is of interest at the \(n\)-th branch of the RAKE receiver.

\[
I_{\text{SI}}^{(n)} = \int_{t(1)}^{t(n)} \left[ \sqrt{2P} \sum_{l=0}^{\ell-1} \beta_n^{(i)} \beta_l^{(i)} (t - \tau_l^{(i)}) b^{(i)}(t - \tau_l^{(i)}) \cos(\omega_c t - \omega_c \tau_l^{(i)}) \right] dt \\
\quad \left[ \beta_n^{(i)} a^{(i)}(t - \tau_n^{(i)}) \cos(\omega_c t - \omega_c \tau_n^{(i)}) \right] dt
\] (5.43)

Through calculation, it can be simplified as

\[
I_{\text{SI}}^{(n)} = \sqrt{P/2T} \sum_{l=n+1}^{\ell-1} \beta_n^{(i)} \beta_l^{(i)} \cos(\omega_c \tau_{al}^{(i)}) [R_{11}(\tau_{al}^{(i)}) + b^{(i)}_0 \hat{R}_{11}(\tau_{al}^{(i)})] \\
\] (5.44)

where

\[
\begin{align*}
R_{11}(\tau_{in}^{(i)}) &= \int_0^{\tau_{in}^{(i)}} a^{(i)}(t - \tau_{in}^{(i)}) a^{(i)}(t) dt \\
\hat{R}_{11}(\tau_{in}^{(i)}) &= \int_{\tau_{in}^{(i)}}^T a^{(i)}(t - \tau_{in}^{(i)}) a^{(i)}(t) dt
\end{align*}
\] (5.45)

and \(\tau_{l,n}^{(i)} = \tau_l^{(i)} - \tau_n^{(i)}\) is the relative delay between the \(l\)-th path and the \(n\)-th receiver branch of user \(k = 1\).
Noise interference (NI) is $I_{ni}^{(n)}$ and can be determined from the interference in the power line channel.

$$I_{ni}^{(n)} = \int_{0}^{T} n(t - \tau_{n}) \cos[\omega_{n}(t - \tau_{n})]d^{(1)}(t - \tau_{n})dt \quad (5.46)$$

In the analysis of only the multipath effect on the CDMA system, by using Gaussian assumption, the BER of the system can be calculated as

$$P_{b} = Q\left(\frac{E[U]}{\sqrt{Var(U)}}\right) = Q\left(\frac{E[U]}{\sqrt{Var(I_{mai}) + Var(I_{si}) + Var(I_{ni})}}\right) \quad (5.47)$$

where $E[U]$ is the expectation of the correlation receiver output $U$ and $VAR[U]$ is the variance of $U$. $VAR[U]$ is the sum of the variances of MAI, SI and NI.

Variances can be calculated using a method similar to the variance derivation in the non-RAKE receiver. The results of these are listed below. Details of the derivation are omitted in the following section and only results are provided for the sake of simplicity.

The variance of MAI is

$$Var(I_{mai}) = \sigma_{mai}^{2} = \frac{PT^{2}}{3N} \sum_{n=0}^{\ell_{1}-1} \sum_{k=2}^{\ell_{2}-1} \sum_{l=0}^{\ell_{3}-1} \left[ \beta_{n}^{(1)} \beta_{l}^{(k)} \cos(\omega_{n} \tau_{ln}^{(k)}) \right]^{2} \quad (5.48)$$

The variance of SI is

$$Var(I_{si}) = \sigma_{si}^{2} = \frac{PT^{2}}{2N} \sum_{n=1}^{\ell_{1}-1} \sum_{l=n}^{\ell_{3}-1} \left[ \cos(\omega_{n} \tau_{ln}^{(1)}) \beta_{n}^{(1)} \beta_{l}^{(1)} \right]^{2} \quad (5.49)$$
The variance of NI is

\[
Var(I_n) = \sigma_n^2 = \sum_{n=0}^{L_{\text{ni}}} \left[ \beta_n^{(1)} \right]^2 \frac{N_0 T}{4}
\]  

(5.50)

The expectation of \( U \) is

\[
E[U] = \sum_{n=0}^{L_{\text{ni}}} \left[ \beta_n^{(1)} \right]^2
\]  

(5.51)

Finally, the BER equation for the RAKE receiver CDMA system is

\[
P_b = \frac{Q}{\sqrt{Var(I_{\text{san}}) + Var(I_u) + Var(I_n)}} \left( \sum_{n=0}^{L_{\text{ni}}} \left[ \beta_n^{(1)} \right]^2 \right)
\]

\[
= Q \left\{ \frac{\sum_{n=0}^{L_{\text{ni}}} \left[ \beta_n^{(1)} \right]^2}{\frac{2}{3N} \sum_{n=0}^{L_{\text{ni}}} \sum_{k=2}^{L_{\text{ni}}} \sum_{l=0}^{L_{\text{ni}}} \left[ \beta_n^{(k)} \cos(\omega_l \tau_{\text{in}}) \right]^2 + \frac{1}{N} \sum_{n=0}^{L_{\text{ni}}} \sum_{l=0}^{L_{\text{ni}}} \left[ \beta_n^{(1)} \beta_l^{(1)} \cos(\omega_l \tau_{\text{in}}) \right]^2 + \sum_{n=0}^{L_{\text{ni}}} \left[ \beta_n^{(1)} \right]^2 + \frac{2}{2(E_b / N_0)}}{2(E_b / N_0)} \right\}
\]  

(5.52)

The equation above is derived for the BER of the user of interest \( k = 1 \). The overall BER for the system can be obtained by averaging the BER results of all the \( K \) users in the system.

### 5.5 Effect of Impulsive Noise on CDMA System

Unlike the multi-carrier OFDM system, the CDMA system involves only a single carrier. The modeled impulsive noise is wideband and affects the whole bandwidth of
the CDMA system. It has an arrival rate corresponding to the Poisson process. The conclusion made in [65] after comparing the impulsive noise effect on the single carrier and OFDM systems is still applicable in comparing the CDMA and OFDM systems. If we re-write the BER equations of the CDMA systems with non-RAKE and RAKE receivers in (5.35) or (5.52), the BER performance under AWGN channel noise can be expressed as

$$p_b = \psi \left( \frac{E_b}{N_0} \right)$$  \hspace{1cm} (5.53)

then, when there is impulsive noise appearance, which is assumed to be the white Gaussian process with zero mean and two-sided power spectral density $N_i / 2$, the BER expression becomes

$$p_b' = \psi \left( \frac{E_b}{(N_i + N_0)} \right)$$  \hspace{1cm} (5.54)

Hence, the BER of the CDMA system under the effect of impulsive noise and the AWGN can be expressed as

$$P_b = \lambda T_{\text{noise}} p_b' + \left( 1 - \lambda T_{\text{noise}} \right) p_b$$  \hspace{1cm} (5.55)

where $E_b$ is the signal energy per bit, and $N_0$ is the power spectral density of the impulsive noise and AWGN, $\lambda$ is the arrival rate of the impulsive noise and $T_{\text{noise}}$ is the average duration time of each instance of impulsive noise.

### 5.6 Simulation Model of CDMA System

The BER performance analytical formulas derived in the previous section have to be compared with the results of simulations in order to verify their accuracy. The simulation used to achieve the BER of the CDMA system is developed using Matlab. The simulation parameters are set in the initial stage to control the simulation.
simulated signal is transmitted using a CDMA frame, which means that several CDMA symbols will be transmitted sequentially in the time domain.

In the CDMA system, multiple users are allowed to transmit simultaneously without interfering with each other. This is possible because the spreading codes used to spread users’ data have small cross-correlation values. Multiple users transmit their data frames simultaneously in the simulation model and the number of simultaneous users in the system is configurable before each round of simulation. The simulation is developed for baseband simulation and the spread data is not modulated by the carrier frequency. The carrier frequency value should be chosen according to the requirements of the different applications and power line channels.

We use a simple and integral CDMA system as an example to illustrate the simulation. The parameters of this system are as follows:

- Five users ($K = 5$) in the system
- Orthogonal Gold Sequence is used
- Processing gain is 63
- BPSK modulation is conducted
- 100 symbols are transmitted within each frame
- The PLC impulse response channel is modeled using four paths
- $E_b / N_0 = 10$dB
- System interfered by the noise in the power line

![CDMA system simulation steps](image-url)

Figure 5.8: CDMA system simulation steps
The simulation can be described using the 12 steps shown in Figure 5.8. The functions of these steps are as follows:

1. **Generate random data**
   
The source data needs to be randomly generated in order to verify the accuracy of the simulation result and to ensure that the simulation is correct for all cases and not just specific situations. Since the system in the example above includes five simultaneous users, we need to generate five arrays of random data for all five users. Considering that there are a total of 100 symbols in a frame and that BPSK is the method of modulation, the random data to be generated is $5 \times 100 = 500$ bits in the binary form of 1 or 0.

2. **Modulate data by using BPSK**
   
The information generated can be modulated using BPSK, QPSK or QAM methods. In this thesis, we use BPSK to modulate data within each array. In the system illustrated above, data after this step still contains five arrays of 100 bits, but the values change from a (1/0) to (1/-1) pattern, as shown in Figure 5.9. We include only one array of the data in the diagram for the purpose of description, and this includes a total of 100 sample points.

![Figure 5.9: 100 bits of randomly generated data to be transmitted](image-url)
3. Generate spreading code

The spreading code is generated and used to spread the information. Since there are five users in the sample system and the spreading codes are orthogonal Gold sequences, five arrays of orthogonal Gold sequences are generated. Because the processing gain is 63, the length of these spreading sequences is also 63. Two of the spreading sequences generated are shown in Figure 5.10. The auto-correlation and cross-correlation of these two orthogonal Gold sequences are shown in Figure 5.11

![Figure 5.10: Two orthogonal Gold sequences of 63 bits length](image)
4. Spread information data with spreading code
   The information bits are spread in this process by multiplying them by the spreading code. Each information bit is multiplied by the corresponding spreading code. In this way, one bit of information becomes 63 bits of data after spreading as shown in Figure 5.12. For the sake of clarity, the figure only shows the first four bits of information that are spread over 252 (4 × 63) sample points.

5. Process signal according to the channel impulse response
   After spreading, the information bits are transmitted into the power line. The signal will propagate from the transmitter to the receiver according to the characteristics of the PLC channel. As previously introduced, the PLC channel can be represented using impulse response. The signal of the sample CDMA system after the multipath effect is shown in Figure 5.13. The overall amplitude of the signal is smaller because of the attenuations that occur during propagation through the PLC channel. Furthermore, we notice that the signal is a summation of signals from the multiple paths.

Figure 5.11: Auto-correlation of one of the orthogonal Gold sequences (top); and cross-correlation of two of the orthogonal Gold sequences (below)
Figure 5.12: Information data and spread data after spreading

Figure 5.13: CDMA system signals affected by the multipath and noise in the power line channel
6. Detect signal power level
   In order to study the variance between the BER and the signal to noise ratio $E_b / N_o$, we need to calculate the signal power level and to prepare the Gaussian noise that will be generated and added to the signal in Step 7.

7. Generate Gaussian noise and impulsive noise
   In this step, the Gaussian noise is added to the signal. The impulsive noise is also added according to the simulation requirement, such as whether the impulsive noise should be considered, and according to what scenario.

8. Use RAKE receiver to collect energy from multiple paths
   After the signals are propagated through the PLC channel, the receiver receives and processes them. For the purposes of our analysis, we assume that the transmitter and the receiver are accurately synchronized. With accurate synchronization, the impulse response of the multipath channel between the transmitter and the receiver can be adequately estimated. When the RAKE receiver is used, signals from the multiple paths are processed according to the estimated impulse response of the multipath channel before being sent for the next step in the process.

9. Despread received data with spreading code
   This step builds on the previous step by despreading the received signal. The despreading process works at each branch of RAKE receiver and the despread data are summarized before decisions are made. Figure 5.14 illustrates the workings of this and the previous step. The figure shows the accumulated value of the four RAKE fingers. The accumulated value from four fingers is much bigger than the single value of just one finger. The despread result and transmitted data are provided in Figure 5.15 for comparison. The comparison shows that the transmitted data can be correctly recovered from the despread results. Using the receiver and the decision making process of the second point (-1 is expected to be recovered) as a means of illustration, the benefits of the RAKE receiver become clear. If the decision is made based only on the first finger, because the value of -1.1 is very close to 0, thus the probability of making a correct decision is lower. The use of four fingers provides more conclusive result of -4 which is much lesser than 0 and the probability of making a correct decision is higher.
Figure 5.14: RAKE receiver and the sum of its figures

Figure 5.15: Comparison of the output of the RAKE receiver and the transmitted data
10. Demodulate data by using BPSK.

In this step, data is demodulated and converted from the +1/-1 pattern to conform to the +1/0 pattern.

11. Compare data from source and receiver

The demodulated data (serial) is compared with the random data generated at the beginning of the simulation. The number of errors can be used together with the number of transmitted bits of data to calculate the BER.

12. Calculate BER

The BER is calculated by dividing the total number of errors by the total number of data points transmitted. The error and the data points are accumulated over repeated rounds of simulations. A large number of simulation rounds are needed to ensure a high level of accuracy. Simulation is only ceased when a minimum number of errors are identified. This approach results in a high level of accuracy, even in relation to small BER simulations.

5.7 Verification of Analytical Models

In order to verify the accuracy of the analytical models of the BER performance of the CDMA system, results once again have to be compared with the results of the simulation. We obtain the analytical and simulation results for the CDMA system both with and without RAKE receivers.

There are up to 60 users in the CDMA system to be analyzed. The spreading code is 255 bits of orthogonal Gold sequences, and the bit rate for each pair of users is 100kbps. The bit duration, therefore, is $T = 10\mu s$. Since the simulation is the baseband modulation, the delay time needs to be set as a multiple of the spreading code chip duration time $T_c$, where $T_c = T / 255 \approx 0.04\mu s$. We start the analysis by assuming that all users have the same channel impulse response, as given in Table 5-1. In the table, a 4-path multipath channel model and its parameters $\beta_l$ and $\tau_l$ are produced according to the power line channel model discussed in Section 2.2. $\beta_l$ and $\tau_l$ are the amplitude and delay of the $l$-th path component of the channel. The analysis for users with different impulse responses will be introduced later in the chapter.
Table 5-1: Parameters of the Impulse Response of PLC Multipath Channel

<table>
<thead>
<tr>
<th>Path Number</th>
<th>$\beta_l$</th>
<th>$\tau_l$ ($cT$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>17</td>
</tr>
</tbody>
</table>

5.7.1 Verification of non-RAKE receiver CDMA system

The results obtained in the effort to verify the analytical model for the non-RAKE receiver CDMA system with 40 and 60 users are given in Figure 5.16. The curves for 40 and 60 users in the figure show that the BER performance of the analytical model corresponds well with that of the simulation. The well correspondence strongly verifies the analytical model for the non-RAKE receiver CDMA system.

![BER of Non-RAKE receiver CDMA system under multipath effect](image)

Figure 5.16: BER performance of the CDMA system with non-RAKE receiver under the PLC multipath channel

Figure 5.16 also compares non-RAKE receiver CDMA systems with different user numbers. From the figure, it is shown that the systems with 10 and 20 users have much better BER performance than those with 40 or 60 users. Fewer users imply better
performance, as a smaller number of users create a smaller level of access interference from other users in the system.

5.7.2 Verification of RAKE receiver CDMA system

In order to verify the analytical model for the RAKE receiver CDMA system, the similar analysis is taken as that in the previous part. The results obtained from the analytical model and simulation for the system with 40 and 60 users are given in Figure 5.17. The curves for 40 and 60 users show that the BER results from the analytical model correspond well to those from the simulation. By comparing the results of systems with different user numbers (60, 40, 20 and 10), it becomes apparent that a lower user number can dramatically improves the BER performance.

![BER of RAKE receiver CDMA system under multipath effect](image)

Figure 5.17: BER performance of the CDMA system with RAKE receiver under the PLC multipath channel

5.7.3 Verification of RAKE receiver CDMA system under multipath and impulsive noise effects

In this section, we verify the accuracy of the analytical model of the RAKE receiver CDMA system under the effects of multiple path and impulsive noise as shown in
Figure 5.18. In the analysis, the processing gain of the system is 255, and the channel impulse response parameters are the same as those given in Table 5-2. The system is studied with 30 and 40 users and both simulation and analytical results are provided. Only analytical results are shown for the system with 10 and 20 users. The BER performance analytical model of the RAKE receiver CDMA system under the multipath and impulsive noise effects is verified by comparing results from the analytical model to those from the simulation.

Table 5-2: Simulation Parameters of the RAKE Receiver CDMA System under Multipath and Impulsive Noise Effects

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Users $N$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multipath Path Number</td>
<td>$\beta_i$</td>
<td>$\tau_i(T_i)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impulsive Noise Scenario</td>
<td>IAT</td>
<td>$T_{Noise}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.0196s</td>
<td>0.0641ms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>0.9600s</td>
<td>0.0607ms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>8.1967s</td>
<td>0.1107ms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impulsive Noise to AWGN Power Ratio $\mu$</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_b / N_o$</td>
<td></td>
<td></td>
<td>0 ~ 40dB</td>
<td></td>
</tr>
</tbody>
</table>
Comparing the BER performance of the system with 40, 30, 20 and 10 users under the same impulsive noise Scenario I, we find that as the number of users in the system decreases from 40 to 10, the BER performance can be greatly improved as the error floor increases from $10^{-3}$ to $10^{-9}$. In comparing the BER performance of the system with 10 users under different impulsive noise Scenarios I, II and III, we find that although the BER performance of the system under the weakly disturbed impulsive noise Scenario III is better than that under the heavily disturbed impulsive noise Scenario I when $E_b/N_o$ is less than 35dB, there are error floors for these three curves which are very close when $E_b/N_o$ is larger than 35dB. These error floors are caused by the MAI and SI inherent in the CDMA system. When $E_b/N_o$ is large, the effect of the impulsive noise is reduced, and the MAI and SI become the main sources of interference to the system.

5.7.4 **Comparison of non-RAKE and RAKE receiver CDMA systems**

We present the BER performance results under four different scenarios to compare the performance of the CDMA system with and without RAKE receiver. Both analytical
and simulation results are illustrated for each scenario in Figure 5.19. The scenarios are:

- Non-RAKE receiver CDMA system with 40 users;
- Non-RAKE receiver CDMA system with 60 users;
- RAKE receiver CDMA system with 40 users;
- RAKE receiver CDMA system with 60 users;

Figure 5.19: BER comparison of the CDMA system with and without RAKE receiver, for users with the same PLC channel impulse response

The curves in the figure show that the BER performance results from the analytical model and the simulation correspond well, but for the non-RAKE and RAKE receiver CDMA systems. These correspondences once again verify the accuracy of the analytical models. Looking closely at the results, the curves in the figure show that the RAKE receiver really performs better than its non-RAKE counterpart. When there are 40 users in the system at the same $E_b / N_0 = 35\text{dB}$, we found that the BER performance of the RAKE receiver is 1000 times better than that of the non-RAKE receiver.
Note that there is an error floor for each curve in the figure. Because of this, the BER performance cannot be further improved simply by increasing the signal transmission power. These error floors are caused by the MAI and SI which are inherent in the CDMA system if multiple users transmit. In order to improve the BER performance, one possible solution would be to lower the number of users in the system. This effect is demonstrated by the comparison of results derived using 40 and 60 users.

The developed PLC channel model is not only accurate when all users share the same impulse response, but is also accurate for users with different amplitudes and delays of PLC channel impulse responses. Figure 5.20 illustrates the analytical and simulation results for 40 users with different amplitudes and delays of PLC channel impulse responses. Analysis in Fig. 5.20 is developed based on the condition that, for each individual user, the channel impulse response parameters amplitude $\beta$ and delay $\tau$ are randomly chosen. As such, for any $i$-th path of the channel impulse response, the values of amplitude and delay for each individual user are randomly chosen according to the uniform distribution, with the amplitude ranging from $\beta_i/2$ to $3\beta_i/2$ and the delay ranging from $\tau_i/2$ to $3\tau_i/2$, where $\beta_i$ and $\tau_i$ are the values given in Table 5-1.

The curves in the figure show a close proximity between the analytical and simulation results. The accuracy of both the analytical and simulations models is once again supported. A comparison of the results of the RAKE and non-RAKE receivers clearly shows that the RAKE receiver is the superior choice.

Comparing Figure 5.19 and Figure 5.20, it is apparent that for the same number of users in the system, the BER of the system under different PLC impulse responses is slightly worse than that of the system under the same impulse response. This is because the variances of the MAI and SI are larger, when the PLC channel impulse response is different for each user.
5.8 Conclusions

This chapter introduced the CDMA system for broadband PLC and analyzed its BER performance. Analytical models were developed to analyze the BER performance using closed form formulas. Simulation models were also developed to simulate the performance of the CDMA system in the PLC channel. The validity of the derived formulas is verified by comparing the analytical results with the results of the simulation.

The RAKE receiver is the receiver of choice for the CDMA system because of the superior BER performance it facilitates. The difference in the performance of the RAKE and non-RAKE receiver CDMA systems is compared using both analytical and simulation models. The accuracy of the results was supported by the close correspondence between the two models, and the RAKE receiver clearly outperforms its non-RAKE counterpart.
The analytical model of the RAKE receiver CDMA system developed in this chapter will be used in the comparison of the OFDM and CDMA systems in Chapter 6.
CHAPTER 6  COMPARISON OF OFDM AND CDMA SYSTEMS FOR BROADBAND PLC

6.1  Introduction

As introduced in Chapter 3, the broadband power line channel requires a special communication system because it has multipath effect. OFDM and CDMA systems are two options for broadband power line communications. There has been some research done on the performance of the OFDM and CDMA systems in power lines. In [55], the use of OFDM and CDMA systems was compared for downstream power line communications using simulations. In [65], the performance of the OFDM system was studied under the impulsive noise and multipath effects.

In this chapter, a comparison is made between the OFDM and CDMA systems. The bit error rate (BER) performance and the optimum overall data rate of the OFDM and CDMA systems are compared, using the criteria of the same bandwidth occupation, the same transmission power for each user, the same total number of users involved in the system and the same power line channel. The comparison is made using analytical models of the OFDM system with guard interval and CDMA system with RAKE receiver. These models were developed and verified in Chapters 4 and 5.

6.2  Comparison Criteria and Objectives

The following are the conditions stipulated for an objective comparison of the performance of the OFDM and CDMA systems:

- Same bandwidth: $BW = 30\text{MHz}$
- Same total number of users in the system: Users = 10
- Same $E_b / N_o$ for each user: $E_b / N_o = 30\text{dB}$
- Same system BER: BER$<10^{-5}$
- Same power line multipath channel with the same impulse response as tabulated in Table 5-1 of Chapter 5.
• Same impulsive noise Scenario I as stipulated in Table 2-2

Based on the above listed criteria, the objectives of our comparison are:
• To determine the optimum data rate of the CDMA and OFDM systems
• To compare the BER performance of the OFDM system with variations in the sub-carrier numbers
• To compare the BER performance of the CDMA system with variations in the processing gain and number of users

The CDMA system supports the simultaneous transmissions of multiple users within the same bandwidth because of its multiple access characteristics. However, the OFDM system only allows one user to transmit within the same bandwidth, in order to avoid collision between different users in the system. Since the OFDM system cannot support multiple access users transmitting together, it needs a multiple access protocol to manage the access of users to the system. Different access protocols with different efficiencies and different traffic loads thus result in the different performances of the OFDM system.

The multiple access of OFDM system can be fulfilled with combining with existing multiple access techniques, such as frequency division multiple access (OFDM-FDMA) and time division multiple access (OFDM-TDMA). Due to the characteristics of orthogonal carriers of OFDM, a more efficient multiple access could be orthogonal frequency division multiplexing access (OFDMA), which allows multiple users transmitting simultaneously through the multiple subcarriers. In OFDMA, signal transmitted from different users are overlapped with each other in frequency domain, but occupying different subcarriers according to the arrangement of the coordinator in the system (usually the base station in wireless communications). Since the subcarriers from different users are orthogonal, the transmitting and receiving for different users will not be interfered. Due to its efficiency in using less bandwidth to fulfill the simultaneous multiple access compared with other multiple access schemes, OFDMA is widely used in wireless metropolitan area network (MAN) standard IEEE 802.16a. However, the efficiency of OFDMA comes along with the complexity of the system implementation and more stringency in the system synchronization. Carrier frequency
offset between the transmitter and the receiver causes the loss of orthogonality among the subcarriers, especially in the uplink of OFDMA with different carrier frequency offsets for different users. It is crucial to mitigate the frequency synchronization error in the OFDMA uplink receiver; otherwise the transmitted information might not be correctly retrieved.

The main objective of this thesis is modeling of modulation techniques such as OFDM and CDMA for broadband PLC. In Chapters 4 and 5 of this thesis, such objective of modeling OFDM and CDMA for broadband PLC has been achieved. Modeling of OFDMA is different from that of OFDM. Further study and modeling of OFDMA is beyond the scope of this thesis. Therefore, the multiple access scheme of FDMA instead of the relatively complicated OFDMA has been chosen for the OFDM system in the comparison with the CDMA system.

To simplify the comparison, we do not want to complicate ourselves with access protocol for the OFDM system. Instead, we divide the power line bandwidth into 10 channels. Each channel can support one OFDM user transmitting without interfering users in other channels. The 30MHz bandwidth is divided into 10 channels to support 10 users; each user has a 3MHz bandwidth. Our task is to determine the optimum sub-carrier number required to obtain an optimal data rate.

The CDMA system analyzed here is implemented with a RAKE receiver for better performance as discussed in Chapter 5. We use the entire bandwidth of 30MHz for each of the 10 users. Our task here is to determine the optimum data rate, which is related to the processing gain $N$ of the system. Increasing the processing gain $N$ lowers the data rate of the CDMA system with the fixed bandwidth. The possible length of the spreading code is $2^n - 1$ and usually ranges from 15 to 511, where $n$ is integer.
6.3 Analysis of OFDM System

In the OFDM system, the adverse outcomes of the multipath effect (due to the delayed signals) can be removed or reduced by using a guard interval. This can effectively eliminate the effects of the inter symbol interference (ISI) and the inter carrier interference (ICI). Extending the guard interval for a wider delay spread can improve the transmission performance, although a longer guard interval will require more signal transmission power and more bandwidth thus reducing the data rate.

6.3.1 The optimum number of sub-carrier to achieve the best BER performance in OFDM system

The BER performance for any one of the 10 users in the OFDM system can be obtained using equation (4.59) that was developed earlier in this thesis. Each user can be treated as a sub-system within the entire OFDM system, and the following parameters are fixed for the sake of comparison:

- Bandwidth $BW = 3\text{MHz}$
- Signal to Noise Ratio $E_b/N_0 = 30\text{dB}$

As discussed in Chapter 4 concerning the modeling of the OFDM system, in order to completely eliminate the power line multipath effect on the system, the length of guard interval needs to be longer than the maximum delay of power line channel. Usually in the initial design stage of the OFDM system, the length of guard interval is predetermined according to the channel impulse response and is kept as a fixed value. As such, in the analysis, by fixing the length of guard interval as one $1\mu s$ and one $2\mu s$ respectively, the relationship between the BER performance of the OFDM system and the sub-carrier number $M$ is studied, as shown in Figure 6.1.

From the figure, it is shown that when the fixed value of guard interval $T_g = 1\mu s$ is used, the BER performance improves with the increase of the sub-carrier number $M$, especially when $M$ is smaller than 40. The figure also shows that there is an optimum number of sub-carriers for the OFDM system. This optimum number is around 60 for guard interval $T_g = 1\mu s$. When $M$ is around this optimum number, further increase of
the sub-carrier number does not obviously improve the BER performance. There is a limit in the BER performance of the OFDM system once the bandwidth and signal to noise ratio are pre-fixed. At this sub-carrier number of 60, the comparison target of BER $< 10^{-6}$ can be met. For system with longer guard interval $2 \mu s$, there is also such an optimum sub-carrier number at around 100. Beyond this value, the BER does not improve with the increase of the sub-carrier number $M$.

![Figure 6.1: BER performance vs. sub-carrier number $M$ of the OFDM system when the bandwidth and $E_b/N_o$ are fixed as 3MHz and 30dB respectively, and the GI is set to one $1 \mu s$ and one $2 \mu s$ respectively](image)

Comparison between the two curves under different lengths of guard interval $1 \mu s$ and $2 \mu s$ shows that the BER performance of the OFDM system with a shorter length of guard interval is better than its counterpart. The reason is that once $E_b/N_o$ is fixed, using a shorter length of guard interval can increase the effective transmission power of the useful information, and thus increase the BER performance.
6.3.2 Limitations in the BER performance of OFDM system and methods of improvement

For the BER performance of the OFDM system shown in Figure 6.1, there is a limit in the BER performance once the $E_b/N_0$ is fixed. It cannot be improved further by simply increasing the sub-carrier number $M$. One way to increase the BER performance is to increase the transmission power, as shown in Figure 6.2. Here, the system uses the same sub-carrier number $M = 16$, the same guard interval $GI = 0.125T_s$, and the same bandwidth $BW = 3$MHz. Two curves are given for the BER performance of the OFDM system in the figure, one with and another without impulsive noise interference. We found that increasing the $E_b/N_0$ by 2dB, from 30dB to 32dB, can increase the BER performance of the system dramatically from $5 \times 10^{-7}$ to around $10^{-9}$, even when the system is affected by impulsive noise.

We also found that the two curves of the BER performance of the OFDM system with and without impulsive noise interference are very close. The reason why these two curves are so close is because the adverse effect of multipath is more serious than the effect of impulsive noise in the OFDM system as concluded in [65]. When the system is protected with suitable guard interval from the multipath effect, it is only affected by the minor impact of impulsive noise. Although the BER performance of the OFDM system can be effectively improved by increasing the transmission power under the impulsive noise effect, the increase of transmission power may raise the electromagnetic interference (EMI) problem in relation to the wireless radio communication within the same frequency band [75]. This is a sensitive issue for broadband power line communications.
**6.3.3 Data rate performance of OFDM system**

In terms of the data rate of the OFDM system within a bandwidth of $BW = 3$MHz and the optimum sub-carrier number $M = 16$, the optimum data rate will be $(1-GI) \times BW = 2.625$Mbps if all the sub-carriers in the system are used to transmit data. However, we cannot use all the carriers to transmit data in the system, as this may cause interference to the carriers of the adjacent band’s users or other purposes. Hence, in designing an OFDM system, some carriers are usually not used to transmit data. Suppose 4 carriers are not used, then the effective data rate is 1.96Mbps. For 10 users, the overall data rate is around 19.6Mbps. It is noteworthy that each carrier is only modulated with BPSK. If each carrier is modulated with a more complicated modulation scheme, then the OFDM system would achieve a higher data rate although the higher modulation would give the worst BER performance.
6.4 Analysis of CDMA System

In the CDMA system, the relationship of bandwidth $BW$, processing gain $N$ and data rate $R$ is $R = BW / N$. When $BW$ is fixed, increasing the processing gain $N$ leads to lower the data rate $R$. Hence, in order to achieve a higher data rate, and to save the complexity of the system, we should try to use as little $N$ as possible. However, too small $N$ will limit the advantages of the CDMA system, and further limit the BER performance. With an optimum processing gain $N$, the CDMA system will not only achieve the optimum data rate, but also meet the BER requirement.

6.4.1 Optimum processing gain to meet the BER requirement of CDMA system

To determine the optimum processing gain $N$, we need to refer to the BER performance analysis equations (5.52) and (5.55) for the RAKE CDMA system in Chapter 5. The parameters for analysis were fixed as follows according to our comparison criteria:

- Number of users $K = 10$
- Bandwidth $BW = 30$MHz
- Signal to noise ratio $E_b / N_o = 30$dB
- Under the same PLC channel

We varied Processing Gain $N$ in order to examine its relationship with the BER of the system. We found that increasing the processing gain indeed dramatically improves the system’s BER performance. For example, when the processing gain is increased from 63 to 127, the BER performance without impulsive noise improves by about 400 times as shown in Figure 6.3. In order to achieve the requirement of BER $< 10^{-5}$, a processing gain of 127 is needed, such that the BER is around $2 \times 10^{-6}$. When this processing gain is chosen, the data rate for each user is $30M/127 = 0.236$Mbps. For 10 users, the total data rate of the CDMA system is around 2.36Mbps.
In Figure 6.3, the BER performance with impulsive noise is also given. When the processing gain is 127, the BER is only around $2 \times 10^{-5}$ that cannot meet our requirement. Thus, when the CDMA system is affected by both the multipath and impulsive noise, it needs a higher processing gain to meet the BER performance, and this value could be 255, which is larger than 127. At the processing gain of 255, the data rate for each user and the overall data rate for 10 users in the system are only half of those values when the system is with the processing gain of 127.

Comparing the BER performance with and without impulsive noise of the OFDM system in Figure 6.2 with the CDMA system in Figure 6.3, we found that the adverse effect of impulsive noise on the BER performance of the OFDM system is little as the two curves are very close; however, the adverse effect of impulsive noise on the BER performance of the CDMA system is worse as there is a difference between the two curves. The difference becomes more obvious when the BER performance is to be improved by increasing the processing gain.
6.4.2 Maximum number of users allowable in CDMA system

One of the advantages of the CDMA system is its ability to allow more users to transmit simultaneously within the same bandwidth. In the CDMA system designed above, where bandwidth $BW = 30\text{MHz}$, processing gain $N = 127$ and $E_b/N_o = 30\text{dB}$, the relationship of the BER to the number of users in the system is shown in Figure 6.4. The curves are obtained by introducing the fixed parameters into the BER equations and changing the number of users $K$ in the equation. The two curves in the figure show BER performance as the number of users in the system increases and the processing gain is 127 or 255.

![Figure 6.4. BER performance vs. number of users of the CDMA system when the system bandwidth and $E_b/N_o$ are fixed as 30MHz and 30dB respectively](image)

From Figure 6.4, it is found that this CDMA system can support around 12 users simultaneously ($BER < 10^{-5}$). If the BER requirement in the system is less strict, the CDMA system could support even more users. This is one of the key advantages of the CDMA system as compared to the OFDM system. The figure shows a similar relationship for processing gain $N = 255$. Comparing the two curves in the figure, it is clear that one of the ways to support more users in the system, while meeting the BER requirement ($BER < 10^{-5}$), is to increase the processing gain.
6.5 Conclusions

In this chapter, we compared the OFDM and CDMA systems using the criteria that they have the same bandwidth 30MHz, the same number of users in each system, the same signal to noise ratio for each user, the same BER requirement and the same power line channel.

Unlike the CDMA system, the OFDM system needs a protocol to support multiple access capability. In order to compare with the CDMA system, we divided the bandwidth of the OFDM system into 10 channels and did not include a complicated medium access protocol. This comparison allowed us to reach the following conclusions:

• The OFDM system is faster than the CDMA system. The OFDM system can achieve an overall data rate of 19.6Mbps, which is faster than the 2.4Mbps achieved by the CDMA system.

• Both the OFDM and CDMA systems are suitable for broadband power line communications, since both meet the BER requirements. When the bandwidth, transmission power and channel response are given, there exists an optimum sub-carrier number that will achieve the best BER performance for the OFDM system. Further increasing the sub-carrier number beyond this optimum does not improve the BER performance. We could improve the BER performance by increasing the transmission power, but considering the EMI problem, we should be very cautious about injecting transmission power into the power lines. Unlike the OFDM system in which the performance of the BER is limited by the maximum transmission power, the performance of the BER in the CDMA system can be improved by sacrificing the data rate, and subsequently increasing the processing gain without increasing the risk of a potential EMI problem. However, since the OFDM system is much faster than the CDMA system within the same limited bandwidth, the BER performance of the OFDM system can be improved with FEC scarifying part of the data rate.

• Through the comparison, we also found that the adverse effect of impulsive noise on the BER performance of the CDMA system is worse than that on the BER performance of the OFDM system.
• Considering that the CDMA system can support more than the 10 users that it was pre-designed to support, the overall data rate of the CDMA system can be slightly improved. For the CDMA system, the maximum number of users is not necessarily limited, as it depends on the BER requirement of the system. This is one area where the CDMA system outperforms the OFDM system.
CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

7.1 Conclusions

Since PLC has come to be seen as a potentially viable means of broadband communications, it has attracted more and more interests from parties interested in application areas like home networking and network access. The attraction of the PLC lies primarily in the fact that power lines are ubiquitous worldwide and installing a PLC is a relatively low cost means of providing broadband access. However, because power line networks are not designed specifically for broadband communications, they suffer the effects of multipath and noise interference. A great deal of research has been done to overcome these challenges by means of power line channel modeling, modulation, coding, signal processing and protocol design. The work in this thesis focuses on the modulation techniques of broadband PLC.

Modeling of communication systems over power line channels is not possible without an accurate model of the PLC. Satisfactory models have been achieved due to the work of a number of key researchers. The multipath effect of the power line channel and the impulsive noise model to generate impulsive noise for analysis were explained in detail early on in this thesis.

From our investigation and comparison of modulation techniques, we concluded that OFDM and CDMA are good options because they can combat the multipath and noise effects of the power line channel. It was for these reasons that the two communication systems were chosen to be analyzed, modeled and compared in this research work.

We then developed analytical models with which to analyze the BER performance of the OFDM and CDMA systems. Given that guard intervals are an important component of an OFDM system, analytical models were developed for the OFDM systems with and without guard interval, and the results were compared. Similarly, the use of the RAKE receiver was studied in relation to the CDMA system, and models were developed and compared for systems with and without such a receiver. The accuracy of
the analytical models developed for the OFDM and CDMA systems were verified using simulations of the OFDM systems (with and without guard interval) and CDMA systems (with and without RAKE receiver). Details of the simulations and their comparison between the OFDM and CDMA systems were provided in detail and the accuracy of the developed analytical models was verified.

Some conclusions can be made from our analysis of the OFDM system. OFDM techniques can mitigate the adverse effects of impulsive noise and only very heavily disturbed impulsive noise interferes with its BER performance. The presence of multiple paths creates more serious concerns than impulsive noise in the OFDM system, but the use of a guard interval can efficiently overcome inter symbol interference and inter carrier interference and help in the achievement of superior BER performance. Long guard intervals tend to be inefficient in using signal power, and an optimum guard interval that can achieve the best BER performance can be estimated. In the analysis, the method to estimate this optimum guard interval to minimize the BER performance was presented. In our analysis of the performance of the CDMA system, the importance of the RAKE receiver structure was verified and it was found to significantly improve the BER performance.

In the final section of this thesis, a comparison was made between the OFDM and CDMA systems, using the criteria that they have the same bandwidth, same number of users in each system, same signal to noise ratio for each user, same BER requirement and the same power line channel with the same impulse response. Unlike the CDMA system, the OFDM system needs a protocol to give it multiple access capability. To facilitate a fair comparison with the CDMA system, we divided the bandwidth of the OFDM system into multiple channels and did not include a complicated medium access protocol. Such a comparison allows us to reach a number of conclusions. The OFDM system has a faster data rate than the CDMA system. Both the OFDM and CDMA systems are suitable for broadband power line communications, since they both meet the BER requirements. When the bandwidth, transmission power and channel response are preset, there exists an optimum sub-carrier number that will achieve the best BER performance for the OFDM system. Further increasing the sub-carrier number beyond this optimum value does not improve the BER performance. We could improve the BER performance by increasing the transmission power, but considering the EMI
problem of power line communications, we should be very cautious about injecting transmission power into the power lines. Unlike the OFDM system, where the performance of the BER is limited by the maximum transmission power, the performance of the BER in the CDMA system can be improved by sacrificing the data rate and increasing the processing gain without increasing the risk of a potential EMI problem. In terms of flexibility, the CDMA system can support more users than it is pre-designed to cater for.

7.2 Recommendations for Future Research

In this final section, we conclude our efforts by presenting a list of recommendations for future research works for broadband PLC:

- While the OFDM system discussed in the thesis is a robust system that can meet the challenges presented by the multipath effect, it brings several difficulties in sub-carrier synchronization sensitivity to frequency offset and nonlinear amplification. The CDMA system, on the other hand, is quite robust to frequency offset and nonlinear distortion. Combining the OFDM and CDMA techniques allows us to lower the symbol rate in each sub-carrier. Longer symbol duration makes it easier to synchronize the transmission [76]. The system that results from the combination of the OFDM and CDMA is called a multi-carrier CDMA (MC-CDMA) system [77]. The modeling of the MC-CDMA system and the analysis of its performance presents an interesting area of future research [78].

- With technological improvements, it becomes applicable to convert the model based system level design into the hardware platform for validation. One of such platforms is using the Simulink from Mathworks together with the system generator toolbox for Matlab from Xilinx [79]. The development of the model in the Simulink environment allows users to develop and verify the workings of the system. Once the design is found to be satisfactory, it can be converted into HDL language and automatically implemented on a hardware platform. The implementation could involve the use of a high performance FPGA with millions of logic gates capable of implementing the complicated logic. Thus, the use of the flow to implement and verify the analytical model is another major area of research.
that should be conducted. Such an effort, however, may involve a large number of resources as it involves not only modeling and design, but other elements such as the design of suitable PCBs, coupling circuits, analog and digital filters and signal amplifier circuits. It requires also the development of a controllable test bed for experimental validation.

- In the analysis of the noise effect on the OFDM and CDMA systems, the noise model employed in this thesis is the widely acknowledged model available in this research area. It is based on field experiments that were carried out over a long period and was developed using rigorous statistics works and thus is highly dependable. Although it cannot exactly represent the real noises in power lines both from the time and frequency domains because noises in power lines appear in numerous manners and totally different, it has accurately represented the noises in power lines and can be used to develop the analytical models and reproduce noises for simulations. According to the model, noise is the sum of two categories of noise: background noise and impulsive noise. Impulsive noise is modeled in the time domain to appear randomly. Its inter arrival time, duration time and amplitude follow a probability distribution. In the frequency domain, the impulsive noise is treated as broadband and its energy covers the whole bandwidth of the communication systems. However, the impulsive noise may not necessarily affect the entire bandwidth that is being used for data transmission. Once a new noise model which can more accurately model the noises both from the time domain and frequency domain is proposed, the accuracy of the analytical and simulation models for the noise effect on the OFDM and CDMA systems can be further improved by referring the model development methods in this research work.

- This thesis focused on the modulation techniques that can be applied to a PLC system. A good medium access control (MAC) protocol is important to support the access of the network’s multiple users. In comparing the OFDM and CDMA systems, the available PLC channel in the OFDM system is simply divided into multiple sub-channels to support the multiple accesses. A complicated MAC protocol was not used, as its investigation and design would take us beyond the scope of this thesis, but such a protocol can in fact effectively improve the output of the network, enhance network utilization and reduce the average delay that users experience. Thus, an important future effort would be to design a good MAC
protocol to support the OFDM system, as well as a combined analysis model for the OFDM system that supports the MAC protocol. Such future efforts can draw on the results obtained in this research work by using the analytical models that were developed and validated herein.
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Ma Yinghui received his B. Eng. and M. Eng. degrees in Electrical Engineering from Shanghai JiaoTong University, Shanghai, P. R. China in 1999 and 2002 respectively. Currently he is working in Avago Technologies Singapore as a Senior Communications System Engineer and is pursuing a Ph.D. degree in Nanyang Technological University, Singapore. His research interest is power line communications.

AUTHOR’S PUBLICATIONS

Journal Publications


Conference Publications


Paper Award at the IEEE International Conference on Power System Technology 2004 (PowerCon 2004), Singapore.

