PERFORMANCE-BASED BLAST RESISTANT DESIGN OF REINFORCED CONCRETE FRAME STRUCTURES UNDER DISTANT EXPLOSIONS

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To my family
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ABSTRACT

Numerical finite element analyses to evaluate the structural dynamic responses are performed on a six-storey reinforced concrete frame structure under distant explosion conditions. The finite element models involved are discussed and experimentally verified. Results show that the whole dynamic response process of the frame structure can be approximately divided into two stages. Localized responses of the blast-loaded members are critical at response stage I while the global responses of the structural system dominate at response stage II.

To keep the localized responses of the blast-loaded members under control in meeting the expected performance level, a performance-based blast resistant design approach for reinforced concrete structural members is developed with the energy spectra. Given the objective performance level defined by the combination of target displacement and target displacement ductility factor, a design procedure to uniquely determine the effective depth and longitudinal reinforcement ratio of the member is addressed and implemented into the practical design examples. The accuracy of the approach in terms of controlling the responses of maximum displacement and displacement ductility factor for the designed member under the given blast loading is numerically evaluated by comparing them with their respective design targets. Two error indices quantifying the errors between the responses of designed member and the design targets are defined and their simplified formulae are derived with extensive numerical studies and statistical analyses. The applications of the error indices formulae into the modification of the developed design approach as well as the probabilistic performance assessment of the designed member under blast loadings are exhibited.

To ensure the global responses in terms of maximum inter-storey drift ratio (MIDR) at the expected global performance level, a new design method is presented for a multi-storey reinforced concrete frame structural system based on the equivalent static force (ESF). The calculation model of the ESF that produces the same MIDR effect as that under the distant blast condition is discussed. Several design examples
are employed to demonstrate the method implementation while verifications of the MIDR responses of the designed frame structures are carried out with numerical dynamic analyses
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LIST OF SYMBOLS

- $A_s$: cross sectional area of the tension reinforcement
- $a, b$: parameters
- $a_1, a_2, a_3$: parameters
- $b_w$: width of the web of the element
- $C$: non-dimensional energy factor
- COV: coefficient of variation
- $c$: depth of the equivalent rectangular stress block
- $DIF$: dynamic increase factor
- $d$: effective depth of the element measured from the extreme compression fiber to the centroid tensile reinforcement
- $d'$: distance from the extreme compression fiber to the centroid of compression reinforcement
- $d_c$: distance between the centroid of compression and tension reinforcement
- $E_c$: Young’s modulus of elasticity for concrete
- $E_{ul}$: ultimate elastic energy
- $E_{max}$: maximum strain energy
- $E_s$: Young’s modulus of elasticity for steel
- $E_{e_1}, E_{e_2}$: mean values of $e_y$ and $e_\mu$
- $e_0, e_\mu$: nominal random variables
- $e_y, r$: $n$ percentages of non-exceedance probabilities for $e_y$
- $e_\mu, m$: $m$ percentages of non-exceedance probabilities for $e_\mu$
- $F_1$: peak value of the force
- $f(t, t_d)$: a non-dimensional time function
- $F_s, F_u$: the force produced respectively by original and additional springs
\( F_{ser} \) uniformly distributed service loads
\( f_{c}, f_{dc} \) strength of concrete in compression in static and dynamic conditions
\( f_{dv} \) dynamic stirrup yield strength
\( f_{s}, f_{ds} \) strength of flexural steel in static and dynamic conditions
\( f_{t}, f_{dt} \) tensile strength of concrete in static and dynamic conditions
\( I \) average moment of inertia of the cross section
\( I_c \) moment of inertia of the cracked concrete cross section
\( I_g \) moment of inertia of the gross concrete cross section (neglecting all reinforcing steel),
\( i \) impulse
\( K_{LE}, K_{ME} \) transformation load and mass factor
\( K_{LME} \) transformation load-mass factor
\( K_s \) initial stiffness for an elastic-perfectly-plastic SDOF system
\( K_a \) elastic stiffness for an additional spring
\( k_a \) equivalent elastic stiffness for the reinforced concrete elements
\( k_e \) initial stiffness of the equivalent elastic-perfectly-plastic SDOF system of the designed member
\( l \) length of the element
\( M_{add} \) additional mass on the beam contributed by adjacent floors
\( M_N \) moment capacity at the support of one-way elements
\( M_p \) moment capacity at the mid-span of one-way elements
\( M_u \) ultimate moment
\( m \) equivalent mass of the equivalent SDOF system
\( m_a \) mass of the reinforced concrete member
\( P \) external static force
\( P_i \) probability of performance at the \( i \)th level
\( P_r \) pressure
\( P_{st} \) equivalent static force
\( p \)  
the effective pressure stress

\( q \)  
Mises equivalent deviatoric stress

\( q_u \)  
ultimate uniformly distributed force of the reinforced concrete member

\( R \)  
standoff distance

\( RF \)  
reaction force

\( R_m \)  
ultimate strength of the equivalent SDOF system

\( SC \)  
support condition

\( T \)  
period

\( T_c \)  
total depth

\( t \)  
time

\( t_d \)  
load duration

\( V_c \)  
shear capacity of the unreinforced concrete

\( V_d \)  
ultimate support reactions

\( V_u \)  
ultimate shear force

\( W \)  
explosive weight, the constant total energy

\( W_e \)  
strain energy in the additional spring

\( W_k \)  
kinetic energy

\( W_s \)  
strain energy

\( y \)  
displacement

\( y_e \)  
elastic (recoverable) displacement

\( y_{eq}^e \)  
elastic displacement response of the equivalent SDOF system of the designed reinforced concrete member under the blast force

\( y_{eq}^{rc} \)  
elastic displacement response of designed reinforced concrete member at the significant point under the blast force

\( y_m \)  
maximum displacement response of the designed reinforced concrete member under the blast force

\( y_{m}^{rc} \)  
maximum displacement response of the designed reinforced concrete member at the significant point under the blast force

\( y_t \)  
target displacement
\( y_{te} \)  
parametric design target of displacement

\( y_{td} \)  
displacement at the end of the blast loading duration \( t_d \)

\( y_m \)  
maximum displacement

\( \dot{y} \)  
velocity

\( \dot{y}_{td} \)  
velocity at time \( t_d \)

\( \ddot{y} \)  
acceleration

\( \alpha, \beta \)  
coefficient

\( \varepsilon, \varepsilon_1, \varepsilon_2 \)  
arbitrary small value

\( \dot{\varepsilon} \)  
strain rate

\( \dot{\varepsilon}_s \)  
static strain rate

\( \varphi \)  
capacity reduction factor

\( \xi_{y}^{rc} \)  
displacement error index

\( \xi_{\mu}^{rc} \)  
displacement ductility error index

\( \gamma \)  
reduction coefficient

\( \eta \)  
non-dimensional displacement

\( \lambda \)  
equivalent static force factor

\( \lambda_{samp} \)  
sample points of \( \lambda \)

\( \lambda_{upp} \)  
upper-envelop line of \( \lambda \)

\( \lambda_{low} \)  
lower-envelop line of \( \lambda \)

\( \mu^{eq} \)  
displacement ductility response of the equivalent SDOF system of the designed reinforced concrete member under the blast force

\( \mu^{rc} \)  
displacement ductility response of the designed reinforced concrete member at the significant point under the blast force

\( \mu_t \)  
target displacement ductility ratio

\( \mu_{ge} \)  
parametric design target of displacement ductility factor

\( \rho \)  
longitudinal tension reinforcement ratio

\( \rho' \)  
longitudinal compression reinforcement ratio

\( \rho_{min} \)  
minimum longitudinal tension reinforcement
\( \rho_v \)  shear reinforcement ratio

\( \sigma_{v_y}, \sigma_{v_u} \)  standard deviations

\( \sigma_c (\epsilon_{\text{uniaxial}}^{\text{pl}}) \)  the hardening (and softening) parameter

\( \xi \)  time parameter

\( \Delta_m \)  maximum inter-storey drift ratio (MIDR)

\( \Delta_m^{P_m} \)  MIDR under the external static force \( P_m \)

\( \Delta_m^{P_{st}} \)  MIDR under the equivalent static force \( P_{st} \)

\( \Delta_{\text{sd},m} \)  maximum inter-storey drift ratio at the end of the blast force

\( \Delta_t \)  target maximum inter-storey drift ratio
1.1 Problem Statements

Blast loading is a type of extraordinary dynamic load, which is characterized by its great intensity and short duration [B1, B2, B5, B7, F1, T1, O1]. Due to the devastating effects of blast loading, the dynamic responses and potential damage mechanisms of the structure that has to withstand such a blast are one of the major concerns of current research works [B6, B7, D2, E2, G1, K6, L2, L3, L5, M16]. Many explosion tests and numerical studies have been carried out in this field to date however they are predominantly concentrated upon the structural responses under the blast loadings produced by close-in detonations. Considering that the possibility of the accidental or incidental explosions of the petroleum refineries, the chemical plants, the ammunition storage areas, or nuclear devices of substantial yields, the surrounding distant civilian structures are likely to be loaded by an intense blast wave that is produced in these bomb events and spreads from the explosion source. The structural dynamic responses and corresponding damage distributions in these distant blast incidents may be quite distinct from those in the close-in explosion environments, however little research has been done on this field. The related literatures available to date have been very limited and are mainly oriented towards a single-storey and multi-bay military structure, which is common to the ammunition manufacturing and storage facilities [B1, B4, T2, N4]. Considering that most of civilian reinforced concrete frame structures have multi-storey levels, the identification of the dynamic behaviours and damage distributions of multi-storey reinforced concrete frame structures under the distant intense surface explosion conditions becomes increasingly important.
Chapter 1. Introduction

The reinforced concrete members within the frame structures that are directly loaded by the blast loadings from distant intense surface explosions may sustain severe damages or collapse entirely, but if additional reinforcements in the appropriate positions have been used for these members, the structure might have enough strength and ductility to withstand the blast more effectively, resulting in a reduction of the damages. Thus designing and detailing the reinforced concrete members to improve their dynamic behaviours in response to the blast loadings is deemed necessary. Considering the large magnitude of the blast loadings induced by intense surface explosions and the relatively low probability of their occurrence, the requirement for structural members to remain elastic under the blast loading conditions is not economically justifiable. Thus it is deemed to be more reasonable to design the members with some levels of inelastic deformation allowed. Many performance-based blast resistant design guidelines have been presented in the ASCE Manual 42 [A2], TM5-855-1 [T1], TM5-1300 [T2], Allgood [A3], Biggs [B1], Mays and Smith [M3] with the explicit consideration of inelastic deformation. The concept of the performance basis is that the design should be able to ensure that the damage of the structure will not exceed a certain limit. The permissible level of damage is termed a performance level, which can be specified by any structural dynamic response parameter such as the displacement, the displacement ductility factor, or the drift ratio [B12, F2, G8, H7, K16, K17, P4, P5, X1]. By establishing different objective performance levels for the structural member with either the target displacements or the target displacement ductility factors in these design guidelines [A2, A3, B1, M3, T1, T2], the blast resistant design is carried out so that the predicted response of the designed member under the blast loading does not exceed its corresponding target. However an important question remains as to whether a single target of the displacement or the displacement ductility factor involved in the design could provide an adequate control of the member’s responses to satisfy the expected performance under the given blast condition. Moreover these design methods are performed basically by transforming the continuous members into equivalent single-degree-of-freedom (SDOF) systems and taking the responses of the SDOF systems as those of the designed members without considerations of their differences.
As for the structural system, the blast resistant design is predominantly focused on the prevention of the progressive collapse triggered by the failure of some critical members. Many design guidelines [A4, D3, F3, F4, G3, I1] have been issued for providing the approach of evaluating and preventing the failure of structural systems in such a way. However, under distant intense blast conditions, numerical studies indicate that the global response of the structural system in terms of side-sway become more critical [L4]. As a result, it is necessary to keep the side-sway response under control within the required performance level (damage limit) during the design of reinforced concrete frame structures against the distant blast loadings. Currently, the design manual “Blast Resistant Structures” [N4] and the ASCE special publication “Design of Blast Resistant Buildings in Petrochemical Facilities” [B4] have suggested different side-sway limits corresponding to various damage levels and a trial design method based on some approximations was recommended for simple single-storey and multi-bay military structures. However, there are still no well-developed design methods available to date against distant blast loadings for the multi-storey reinforced concrete frame structures. The need for research and development will no doubt remains.

1.2 Research Objectives and Scopes

The main objective of this thesis is to analyse the dynamic responses for multi-storey reinforced concrete frame structures under distant intense surface explosion conditions and to develop performance-based blast resistant design methods for reinforced concrete frame structural members as well as structural systems to keep their responses under control thus to meet the target performance level. The research is conducted with the following focuses:

- to trace the dynamic responses and the corresponding damage distributions for a multi-storey reinforced concrete frame structure with/without exterior cladding panels under distant explosion conditions through the numerical finite element analysis.
- to develop a performance-based blast resistant design approach for a one-
way reinforced concrete member depending on energy spectra for the purpose of the adequate control of the designed member’s responses to meet the expected performance level under the given blast conditions.

• to study the differences between the responses of the reinforced concrete member designed with the developed approach and those of the corresponding SDOF system under the given blast force and incorporate these differences into the design.

• to present a method of performance-based blast resistant design for a multi-storey reinforced concrete frame structural system in keeping its global response in terms of maximum inter-storey drift ratio (MIDR) under proper control thus to satisfy the performance requirement, by transforming the blast forces from distant explosions into the equivalent static forces (ESF).

1.3 Thesis Organization, Originality and Main Contributions

The introduction of the problems, the objectives and the scopes of this study are defined in this chapter.

Chapter 2 presents the nonlinear dynamic analysis of multi-storey reinforced concrete frame structures to evaluate their dynamic responses as well as damage distributions against the blast loadings from distant explosions. This is accomplished using the program ABAQUS. The effects of the exterior cladding panels on the structural responses are identified.

Chapter 3 is concerned with the development of a new performance-based blast resistant design approach for one-way reinforced concrete structural members by using energy spectra, which are constructed from the elastic-perfectly-plastic SDOF system. The expected performance level is defined by both the target displacement and target displacement ductility factor of the member at the significant point (e.g. the mid-span of a member with two ends constrained or the free-end of a cantilever member). By controlling the responses of maximum displacement and displacement ductility factor for the equivalent SDOF system of the designed member to exactly
reach the corresponding targets, the design process is discussed to uniquely determine the effective depth and longitudinal reinforcement ratio of the member. The effectiveness of the developed blast design approach is evaluated in Chapter 4. The actual responses of maximum displacement and displacement ductility factor for designed member at the significant point under the given blast condition are investigated with numerical finite element analysis and compared with their corresponding design targets.

The developed design approach using energy spectra is accomplished by the assumption that the maximum displacement and displacement ductility responses of the designed reinforced concrete member can be represented by the corresponding responses of its equivalent SDOF system under the blast loading. In fact, due to the complexities of the nonlinear dynamic behaviours of reinforced concrete members under blast conditions, some differences between them do appear which further induce the errors of the maximum displacement and displacement ductility responses for the designed member under the given force with the respective design targets. A study of these errors is carried out in Chapter 5 where two error indices are defined and their simplified formulae are derived through extensive numerical studies and statistical analyses.

Chapter 6 applies the formulae of the error indices into the modification of the blast resistant design approach developed in Chapter 3 for the purpose of establishing more effective control of the maximum displacement and displacement ductility responses of the designed reinforced concrete member rather than those of the corresponding equivalent SDOF system in achieving the respective design targets. Several design examples of the members are exhibited with the modified design approach followed by a comparison with those obtained from the original design process. In addition, a simpler procedure of implementing these two error indices to estimate the distribution of the maximum displacement and displacement ductility responses for designed members under blast loadings is presented and applied into the probabilistic performance assessment of the members accounting for different types of uncertainties.
Chapter 1. Introduction

A new method of blast resistant design for a multi-storey reinforced concrete frame structural system based on the transformation of the distant blast loading into the equivalent static force (ESF) is discussed in Chapters 7 and 8, where the maximum inter-storey drift ratio (MIDR) response parameter is taken as the global performance indicator. The ESF is calculated in such a way that the same MIDR effect will be produced as that under the distant blast condition. Chapter 7 focuses on the model for the calculation of ESF as well as the corresponding design method based on ESF for a SDOF system with the requirement of controlling its maximum displacement response. The extension of the calculation model for ESF and the design method with ESF for a SDOF system to a performance-based design for multi-storey reinforced concrete frame structures in controlling their MIDRs under distant blast conditions is discussed in Chapter 8. The implementation of the developed design method into practical design examples is demonstrated followed by the nonlinear dynamic evaluation of the design results.

Chapters 2 to 8 are organized in the form of technical papers. Chapter 9 concludes the work of the present study and recommend some future work.

The originality and main contribution of present work are:

- evaluating and comparing the dynamic responses and corresponding damage distributions for the multi-storey reinforced concrete frame structures with/without the exterior cladding panels under the blast loadings from distant explosions.
- developing a new approach for the blast resistant design of reinforced concrete members by using energy spectra, which could overcome the disadvantages in the existing design guidance.
- assessing the accuracy of the developed design approach in controlling the maximum displacement and displacement ductility responses for the designed member by comparing them with the corresponding design target.
- deriving the simplified formulae of the two error indices which quantify the differences of the maximum displacement and displacement ductility
responses for the designed member under the given blast force with their respective design targets.

- applying the formulae of the error indices into the modification of the developed design method so as to effectively control the actual responses of the designed members rather than those of the corresponding equivalent SDOF systems.

- implementing the error indices formulae into the probabilistic performance assessment of designed members against blast loadings without numerical finite element analysis.

- presenting a new design method for a multi-storey reinforced concrete frame structure against a blast loading from distant intense explosion in controlling its MIDR response based on the ESF, which is computed to produce the same MIDR effect as that under the blast action.

- evaluating the effectiveness of the presented ESF-based blast resistant design method in terms of controlling the MIDR response of multi-storey reinforced concrete frame structures as compared to the design target.
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

CHAPTER TWO

RESPONSES OF MULTI-STOREY REINFORCED CONCRETE FRAME STRUCTURES TO DISTANT EXPLOSIONS

Synopsis

Limited research has been carried out upon the dynamic responses for multi-storey reinforced concrete frame structures subjected to blast loadings from distant intense surface explosions. This chapter presents numerical analysis with the program ABAQUS on a six-storey reinforced concrete frame structure under two blast conditions occurring at a standoff distance of 100 m produced by the explosives respectively equivalent to a 50 ton TNT and 250 ton TNT. To incorporate the effects of exterior cladding panels, two cases are involved in the analysis for the six-storey frame structure with/without exterior cladding panels. The finite element models involved are verified by comparing the numerical results of a reinforced concrete beam and slab for blast loadings with the experimental data. The results indicate that the whole dynamic response process of the frame structure with/without exterior cladding panels can be approximately divided into two stages. The localized responses of the blast-loaded members are critical at response stage I while with the increase in time, the structural global response becomes dominant at response stage II. Flexural responses are more important in the plastic deformation of the structural members as compared to the corresponding shear responses. Due to the variation of the blast forces received, the dynamic responses are much more severe for the frame structure with exterior cladding panels than the one without exterior wall panels under the same blast condition.

Keywords: Reinforced concrete frame structure, Distant explosions, Blast loadings, Local and global responses, Damage distribution, Response stages
2.1 Introduction and Background

The behaviours of civilian reinforced concrete structures under blast conditions have become a major concern to structural engineers due to the recent surge in explosive events worldwide. After the trigger of a blast, a blast wave including a high-pressure shock front is formed and expands outward from the centre of detonation [B9, G2, H5, K7, K8, K15, N1, N2, T1]. As the blast wave strikes a building, the overpressure and drag force of the blast wave load on the structure, the interaction between which is significantly complicated [A2, B1, N1, T1]. However, considering the relative distance of the detonation centre with the target structure as well as the size of the structure itself, two classes of blast wave-structure interaction can be generally identified as shown in Figure 2.1 [S1]. The first class is the interaction of a blast wave produced by the detonation of a smaller charge loading a target structure at a short standoff distance, which is typical for most terrorist attacks such as car bombings [C1, C5, H2, L1, L3]. The second class is the interaction of a blast wave on a relatively distant structure as might be present due to an accidental severe surface explosion of petroleum refineries, chemical plants and ammunition storage areas, fuel train explosions, or nuclear devices of substantial yields, and so on [C6, G2, K1, M7].

The profiles of blast loadings on the structure are different within these two classes of blast wave-structure interaction. In the first class, the blast pressures are produced locally to individual structural members and the members are likely to be loaded sequentially [S1, L1, L2]. By comparison, in the second class of blast wave-structure interaction, the target structure is engulfed due to the diffraction of the blast wave and a normal squashing force will be applied to every exposed surface. There is also a translational drag force, which tends to move the structure bodily laterally [A2, S1, T3].
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

Figure 2.1 Two classes of blast wave-structure interaction [S1]

Many explosion tests and numerical analyses have been carried out on the behaviours of structures in the first class of blast wave-structure interaction where the blast pressures are applied locally to individual structural members resulting in the possibility of an excessive local failure of several critical structural members and further the progressive collapse present in a non-redundant structure [C1, E2, L2, L3, M6]. Therefore the study is focused dominantly on two aspects: the local responses of structural members under blast loadings and the progressive collapse of the structure under gravity loads with some members destroyed or severely damaged. The first aspect has been widely covered to date in many articles where the potential and observed failure mechanisms [C2, E3, K2, K4, M12, M15, P3, R1, S6, Z1] of structural members, the constitutive models to characterize the inelastic material behaviour [B6, B10, C7, K6, K9, M8, M9, M17], and the blast resistant
design procedures of structural members in controlling their responses [A2, A3, B1, B4, M3, T1, T2] are studied. As concerning the second aspects, General Services Administration (GSA) [G3], Interagency Security Committee (ISC) security criteria [I1], and ASCE 7-98 [A4], Federal Emergency Management Agency (FEMA-273 and FEMA-274) [F3, F4], and Department of Defence (DoD) [D3] have issued the guidelines for evaluating the progressive collapse hazard which provide general information about the approach and method of evaluating the progressive collapse potential. Moreover, various analytical methods used to evaluate the possibility of progressive collapse are developed which cover greatly from simple linear elastic static procedure [H3, M10] to complex nonlinear time history analysis [K10, K11, L2, L3, V1].

However little research has been devoted to behaviours of civilian structures and their possible failure mechanisms in the second class of blast wave-structure interaction and the available literature about it is very limited. By simplifying the members and the structural system into equivalent single-degree-of-freedom (SDOF) systems, Biggs [B1] provided a blast resistant design procedure for one-storey structures, which was demonstrated with a design example of a one-storey windowless aboveground frame structure in resisting the blast loading produced by a 0.5 MT weapon at about 1.7 km. Baker [B2] examined the responses for one and two-storey planar steel frames loaded by the blast forces through numerical simulations with the method for including plasticity in the frames by Majid [M11] and Yamada [Y1]. The method was based on the formation of plastic hinges at the ends of the members and computed the rotations of these hinges during plastic deformation.

Although the blast incidents producing the second class of blast wave-structure interaction are very rare, when they do occur the consequence can be extremely severe, particularly upon the structures not specifically designed to withstand the blast effects. There have been cases of the severe damage or even collapse of the surrounding buildings within such incidents [C9, K1]. To minimize the blast consequences to the buildings, an effective blast resistant design is needed. For this
purpose, knowledge of structural dynamic behaviours and the corresponding potential damage distributions under such blast situations is imperative, however it is limited to date in contemporary released publications. Further study of the structural behaviours in the second class of blast wave-structure interaction should be employed especially for multi-storey reinforced concrete frame structures which are mostly common to the civilian buildings, accounting for the complicated interaction between the blast wave and structure as well as the nonlinear material properties of concrete and reinforcement.

This chapter aims to study the dynamic responses and damage distributions of a multi-storey reinforced concrete frame structure loaded by blast waves from distant intense surface explosions employing nonlinear dynamic finite element analysis with the program ABAQUS [A1]. The finite element models involved are verified by comparing the numerical results of a reinforced concrete beam and slab with the corresponding experimental data by Seabold [S3] and Keennan [K3]. Since the exterior cladding panels might have noticeable effects on the structural behaviours under blast conditions, two types of the multi-storey frame structure with/without exterior cladding panels are analysed. The corresponding structural dynamic responses and damage distributions are identified and compared. The findings from this study are used to reach conclusions and recommendations in the blast resistant design concerning explosive safety for civilian reinforced concrete frame structures.

### 2.2 Blast Loadings

In this investigation, the main concern is upon the dynamic responses of frame structure when subjected to blast loadings produced by distant intense surface explosions, where compared with the size of the target, the hemispherical wave front can be reasonably idealized to be planar blast wave front and essentially parallel to the front faces of the target shown in Figure 2.1 [A2, B11, S1, T1]. As the blast wave strikes on the front face of a closed target, a reflected pressure is instantly developed. The blast wave diffracts around the target, subjecting first the sides and roof and finally the rear face to pressures equal to incident overpressure
(air-blast pressure occurring in the free-field). At the same time, these faces are as well loaded by drag pressures that are a function of a drag coefficient and the dynamic pressure associated with the air-flow behind the shock causing drag or wind type loads. This drag pressure causes a push on the front face of the target followed by a suction force on the back and sides as the blast dynamic pressure passes over and round the target [A2, B11, N3, S1].

Determination of the exact blast loadings of a distant explosion on the front, top and the sides, and the rear faces of the closed target is almost unrealistic considering the complicated process of the interaction of the blast wave with the concerned target. In order to reduce the complex problem of blast loadings to reasonable terms, a computational procedure is recommended in the US manual TM5-855-1 [T1] based on two assumptions that (a) the target is generally rectangular in shape, and (b) the object being loaded is in the region of the Mach reflection [B1, B9, G2, K12, H1, N1]. These two assumptions are rational for the derivation of the blast loadings on rectangular targets in a relatively large standoff blast environment. The simplified loading configurations on various faces are shown in Figure 2.2 whose parameter can be calculated with the equations listed in Appendix 1 for a closed rectangular aboveground target, i.e. a column, a beam, or a closed structure.

The computation of the blast loadings on a frame structure is relatively more complicated than that plotted in Figure 2.2 since the whole structure could not be arbitrarily taken as a closed rectangular target. The loading profiles applied to the frame structure are dependent on the out-of-plane strength and stiffness of exterior cladding panels due to the direct action of blast loadings on them, as well as their connection with the frame members of beams and columns. In this study, two extreme cases are considered that:

- the exterior cladding panels are constructed with the material of reinforced concrete with significant strength and stiffness.
- there is no exterior cladding panels for the frame structure (bare frame structure);
In the case of frame structure with exterior cladding panels made of reinforced concrete, the cladding panels possess the significant out-of-plane strength, stiffness and strong connection with other frame members. As the blast wave hits the outer surfaces of the structure (outer surfaces of exterior cladding panels, exterior beams and columns), the blast wave could not go into the building and the blast loadings on the cladding panels would be transferred to the primary frames in terms of reaction forces. These transferred reaction forces together with the blast loadings acting directly on the outer surfaces of the exterior beams and columns will produce dynamic responses of the structure, which is different from those of the bare frame structure. For this case, it is reasonable to treat the whole structure as a closed target, based on which the blast forces upon the front, roof and rear faces are computed according to the equations in Appendix 1.
As for the second case, the blast wave front would enter the building producing a high pressure on the rear faces of the columns. Appendix 1 could not be directly used for computing the blast loadings on a bare frame structure since the whole structure could not be taken as a closed rectangular target. However, the shock wave front in the distant surface explosion condition is essentially parallel to the front faces of blast loaded columns. By taking each column as a closed rectangular target, the blast forces on the front and rear faces of the columns could be evaluated according to Appendix 1 where the parameters ($L, W_s$ and $H_s$) are equal to those of each column. The blast loadings on both side faces of each column are identical as well as on the top and bottom surfaces of each slab leading to a zero consultant force due to the blast wave approximates to a planar wave, and therefore they are not taken into account in the following analysis.

In addition to blast overpressure effects, intense surface explosions produce ground shocks as results of the directly induced ground motion propagating through the soil or rock, which might affect structural responses. However considering the fact that the target to be studied is located at a large standoff distance relative to the source of the explosion, the magnitude of the ground shock has been greatly diminished and the arrival time of the shocks to the structures is different from that of blast wave. Therefore the direct ground shocks are ignored in this analysis.

**2.3 Design of the Target Frame Structure**

For the study of the dynamic behaviours of frame structures under the distant blast conditions, a three-dimensional six-storey reinforced concrete frame structure is designed according to BS8110 code [B3]. The loads involved in the design include the dead loads (5.0 kPa), the super imposed dead loads (1.0 kPa), the live loads (4.0 kPa), and the wind loads calculated in accordance with the BS 6399 code by taking the wind speed to be 30 m/sec. The layout of the structure and the reinforcement details of structural members are shown in Figure 2.3.
Two blast waves produced by the detonations equivalent to a 50 ton TNT (blast condition I) and 250 ton TNT (blast condition II) at the ground level with a standoff distance of 100 m are considered in the study. In these blast conditions where the standoff distance is significantly larger than the size of the target structure, the blast waves can be reasonably modelled with a planar wave as displayed in Figure 2.3. The waves engulf the whole structure producing blast pressures on exposed surfaces of the structure uniformly distributed not only along the width of the structure but also along the height. Due to the symmetry of both the configuration of the target structure and the blast pressure distributions on it, a three-dimensional sub-frame including a planar frame and half of adjacent components is modelled to simulate the whole structural dynamic responses.

In order to incorporate the effects of exterior cladding panels on the structural blast responses, two extreme cases are considered for the six-storey frame structure in this study. In the first case, the exterior cladding panels are assumed made of reinforced concrete with sufficiently out-of-plane strength and stiffness to prevent the blast waves from entering the building, therefore the modelling of the structural responses have to include the exterior cladding panels (whose reinforcement arrangements are found in Figure 2.3c), together with the floor slabs, beams and columns. In the second case, the structure is formed without exterior cladding panels. As discussed previously, the frame structure in such a situation is a bare frame against the blast loadings, accordingly only the beams, columns and floor slabs are modelled for the following numerical simulations. The appearances of these two different sub-frame structures are demonstrated in Figure 2.4.
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a). Structural layout

(b). Elevation view

(c). Reinforcement details of structural members

<table>
<thead>
<tr>
<th>Beam section (1st-6th storey)</th>
<th>Column section</th>
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<tr>
<td><strong>End</strong></td>
<td><strong>Mid-span</strong></td>
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<tr>
<td>2T25+2T20</td>
<td>2T20</td>
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<tr>
<td>300</td>
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<td>(Link R10@200)</td>
<td>(Link R10@200)</td>
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Floor slab depth = 150mm; R10@ 200mm in two directions with a concrete cover of 25 mm for top and bottom reinforcement

Cladding panel depth = 150mm; R10@ 200mm in two directions with a concrete cover of 25 mm for top and bottom reinforcement

Figure 2.3 Details of the target six-storey frame structure
2.4 Finite Element Models

The three-dimensional six-storey sub-frame structures are modelled to perform the numerical simulations of the structural dynamic responses under blast conditions I and II by utilizing the program ABAQUS [A1]. The finite element models consist of the structural modelling, material modelling of concrete and reinforcement, the analysis steps and loads, plus the integration methods.

2.4.1 Structural modelling

Two types of structural members are included in the sub-frames, which are the frame members composed of beams and columns and the planar members including exterior cladding panels and floor slabs. In order to obtain more accurate simulations of the structural responses in blast conditions, three-dimensional continuum solid elements should be employed in modelling these structural members. However it will include considerably heavy computational efforts due to the complicated nonlinear behaviours of the concrete and reinforcement as well as the extremely large size of the finite element model for a multi-storey frame structural system built with the three-dimensional solid elements. For simplicity, beam elements for modelling frame beams and columns, together with shell elements for exterior cladding panels and floor slabs are adopted. The accuracy of the numerical simulation results of the responses for different structural member modelled with the beam or shear elements will be examined with available testing data in the latter part.

Two types of beam elements: Euler-Bernoulli beam element and Timoshenko beam element are generally involved in the finite element analysis. Since the Euler-Bernoulli beam element does not allow for transverse shear deformation, it is only applicable for modelling the behaviours of beams and columns in the situation where their flexural responses dominate. However as pointed out by many researchers [C2, K2, R1], the shear responses of frame members could be important in blast conditions. Accordingly, Timoshenko beam element, which allows for the
transverse shear deformation, has been employed in the analyses. The verification of this type of beam element in simulating the responses of reinforced concrete members under severe short duration dynamic loads has been proven in a lot of research [K2, K18, K19, R1].

Shell elements are generally used to model structural members in which one dimension, the thickness, is significantly smaller than the other dimensions such as the cladding panels and floor slabs. Considering the important shear deformation of cladding panels and slabs when loaded by blast forces as well as their large inelastic deformation [C8, H4, K3, K4, K13], the shell element that allows the transverse shear deformation and accounts for finite membrane strains and arbitrarily large rotations is employed in the simulation. This shell element is quite effective in the numerically predicted responses of cladding panels or slabs with the experimental verification in the latter section of this chapter.

Reinforcement in the concrete structures is modelled by means of rebars, which are one-dimensional strain theory elements (rods) defined singly or embedded in oriented surfaces. In this analysis, each rebar is placed at its corresponding location for the beam elements while the layers of rebar are defined for the shell elements with the needed data-input of reinforcing bar’s area and location, and the space length.

The structural modelling for the six-storey frame structure with/without exterior cladding panels is shown in Figure 2.4. The bottom ends of structural columns are perfectly fixed to the solid ground and symmetry boundary conditions are applied to the edges of the slabs and cladding panels.
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

2.4.2 Material modelling

The dynamic responses of reinforced concrete structures under blast conditions are dependent on the material properties of the concrete and reinforcement. To obtain valid numerical simulations of the dynamic responses of the structures, the accurate modelling of the material properties becomes essential. In the finite element analysis, three types of material behaviours including that of the concrete, reinforcing bars and the bondages between them should be taken into account. It is presumed that the bars are perfectly bonded with concrete herein. Although it will impose some errors, it is the only practical solution for the numerical simulations of the responses of reinforced concrete structural systems, considering the unmanageable size of the computational works and the lack of reliable data on this topic in blast environments.

The smeared cracking model is utilized to represent the tensile behaviour of concrete. In this model, concrete cracking appears as the maximum principal tensile stresses reach a failure surface and will affect the stress and material stiffness associated with each material point in related elements. To simulate the softening
effect of the concrete in tension after its cracking, a bilinear tension stress-strain curve is used and shown in Figure 2.5, where $\varepsilon_u^c$ is taken as $10^{-3}$. The selection of this value is based on the assumption that the strain softening after failure reduces the stress linearly to zero at a total strain of about 10 times the strain at failure of concrete in tension, which is typically $10^{-4}$ in standard concretes [A5, H6]. The concrete tensile strength $f_t$ used in the analysis is determined from the compressive strength $f_c$ [C3]

$$f_t = 0.30 f_c^{2/3} \quad (2.1)$$

In compression, concrete is simulated with an elastic-plastic model where the elastic stress state is limited by a yield surface. Once yielding has occurred, an associated flow rule with isotropic hardening is used. This yield surface can be written in terms of the first two stress invariants

$$f = q - \sqrt{3}a_o p - \sqrt{3}\sigma_c(\varepsilon^{pl}_{\text{uniaxial}}) \quad (2.2)$$

where $p$ is the effective pressure stress, $q$ is Mises equivalent deviatoric stress, and $a_o$ is a constant, which is chosen from the ratio of the ultimate stress reached in biaxial compression to the ultimate stress reached in uniaxial compression. $\sigma_c(\varepsilon^{pl}_{\text{uniaxial}})$ is the hardening (and softening) parameter, which is defined from the uniaxial compression data of the concrete as a function of the plastic strain. Poisson’s ratio is approximate to be 0.20.
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a). Concrete in compression and tension

(b). Reinforcement

Figure 2.5 Material modelling
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

The stiffness used in the analysis for unloaded concrete in tension and compression is given in Figure 2.5. When the cracked concrete is unloaded, the secant unloading modulus is utilized as stiffness so that the strain across the crack is reduced linearly to zero as the stress approaches zero. If the load is removed at some point after inelastic straining has occurred for the concrete in compression, the unloading response is softer than the initial elastic response, but this effect is ignored in this model. Thus initial elastic stiffness is used when the concrete in compression is unloaded.

The Von-Mises yield criterion is used to describe the constitutive behaviour of the reinforcement. The stress and strain relationship of reinforcement is also shown in Figure 2.5, where the reinforcement is modelled with an elasto-plastic curve. The strain hardening of reinforcement is not considered in this analysis since it is hard to define under the blast conditions in the case of lacking enough experimental data. The ultimate strain value is often not reported in current literatures due to the difficulty of determining exactly when the peak stress occurs as well as the confusion between ultimate strain and rupture strain.

In order to consider the fact that under higher loading rates both concrete and reinforcement exhibit increased strengths [D4, D5, F5, F6, G4, G5, G6, G7, M14, R2, T4], a dynamic increase factor (DIF), the ratio of the dynamic to static strength, is employed in this analysis. The expressions by Malvar and Crawford [M1, M2] are utilized which are derived from a literature review of the extensive test data about the effects of strain rate on the strength of concrete and reinforcement. For the concrete compressive strength, DIF is given as

\[
DIF = \begin{cases} 
(\dot{\varepsilon}/\dot{\varepsilon}_s)^{0.026\alpha}, & \dot{\varepsilon} \leq 30s^{-1} \\
\gamma_s (\dot{\varepsilon}/\dot{\varepsilon}_s)^{0.3}, & \dot{\varepsilon} > 30s^{-1} \end{cases} 
\tag{2.3}
\]

where \(\dot{\varepsilon}\) is the strain rate in the range of \(30\times10^{-6} \) to \(300s^{-1}\); \(\dot{\varepsilon}_s = 30\times10^{-6} s^{-1}\) (static strain rate); \(\log \gamma_s = 6.156 \alpha_s - 2\); \(\alpha_s = 1/(5 + 9f_c/f_{co})\); \(f_{co} = 10\) MPa; \(f_c\)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

is the static compressive strength of concrete. For the concrete in tension, the formula is

$$DIF = \begin{cases} \left(\frac{\dot{e}}{\dot{e}_s}\right)^\delta & \dot{e} \leq 1.0 \text{s}^{-1} \\ \beta \left(\frac{\dot{e}}{\dot{e}_s}\right)^{1/3} & \dot{e} > 1.0 \text{s}^{-1} \end{cases} \quad (2.4)$$

where $\dot{e}$ is the strain rate in the range of $10^{-6}$ to $160 \text{ s}^{-1}$; $\dot{e}_s = 10^{-6} \text{ s}^{-1}$; $\log \beta = 6\delta - 2$; $\delta = 1/(1 + 8 f_c / f_{co})$; $f_{co} = 10 \text{ MPa}$. A plot of the proposed formulae for the DIF of concrete in tension and compression is shown in Figure 2.6, which indicates that the strength enhancement is different for tension and compression.

The DIF formula for the yield stress of reinforcement [M1, M2] is

$$DIF = \left(\frac{\dot{e}/10^{-4}}{\alpha}\right)^\alpha \quad (2.5)$$

where $\alpha = \alpha_{fs}$ and $\alpha_{fs} = 0.074 - 0.04 f_s / 414$; $f_s$ is the reinforcement static yield strength in MPa. This formula is valid for reinforcement with yield stresses between 290 and 710 MPa and for strain rates between $10^{-4}$ and $225 \text{ s}^{-1}$. The plot of DIF is also shown in Figure 2.6.
Concrete in compression

Concrete in tension

Reinforcement

Figure 2.6. DIFs for concrete and reinforcement [M1, M2]

To integrate Equations (2.3) ~ (2.5) into the analysis, the user subroutine USDFLD is used which allows the user to define the field variable at a material point as a function of any of available material point quantities. Thus by taking the strain rate as a field variable, the strain rate-dependent material properties can be introduced in the analysis since such properties can be easily defined as functions of the strain rate with Equations (2.3) ~ (2.5).
2.4.3 Analysis steps and loadings

In the simulation of the structural blast responses, two steps of analyses are carried out according to different loading stages. Before the occurrence of an explosion, the service loads have been already imposed on the structure. The intensity and distribution of the stresses and strains induced by the service loads will influence the structural behaviours under blast conditions. Thus nonlinear static analysis is performed priorly for the three-dimensional sub-frame structure under service loads. The loads involved in this step include the live loads, the dead loads and the super imposed dead loads. The second step is the dynamic response simulation of frame structures loaded by blast pressures where the nonlinear dynamic analysis is performed. The functions of blast pressures on the structures are necessary in this step, which are discussed respectively as follows for the bare frame structure and the frame with exterior cladding panels.

For the frame structure with reinforced concrete exterior cladding panels, by taking the whole structure as a closed rectangular target, the blast force functions on the front, top and rear surfaces are evaluated with Appendix 1 as demonstrated in Figure 2.7. In this case, the reflected pressure dominates in the blast loadings on the front cladding panels while the blast pressures on the top and rear surfaces are composed of the incident overpressure and dynamic pressure. As for the bare frame structure, the planar wave enters the structure loading each exposed columns, thus by taking each column as a closed rectangular target, the functions of blast forces on its front and rear faces are obtained according to Appendix 1 and the results within blast conditions I and II mentioned above are shown in Figure 2.8 where the shadow area stands for the net blast pressure on each column. It is demonstrated that due to the relatively small dimension of the column width, the reflected pressure decays rapidly and the drag force correlated with dynamic pressure plays a much important role in the net blast loadings on the columns. With the columns becoming further away from the explosive source, the peak pressure lessens while the lag time rises up. The increase of the explosive weight at the same standoff distance leads to the rise of the peak pressure while some decline in the duration.
Comparison of Figure 2.8a with Figure 2.7 indicates that there are obvious differences in the spatial distribution as well as the time history for blast loadings acting on the structures with/without exterior cladding panels.

(a). Blast wave on the frame structure with exterior cladding panels

(b). Loads on the front and rear faces

(c). Loads on the roof slabs

Figure 2.7 Blast loadings on the frame structure with exterior cladding panels under condition I
Blast wave on the bare frame structure

Figure 2.8 Blast loadings on the bare frame structure under conditions I and II

(a): Under blast condition I
2.4.4 Integration method

A general implicit dynamic integration method is employed to solve the nonlinear dynamic problems. In order to ensure the accuracy of the numerical solutions with this method, an automatic incrementation scheme is adopted to calculate the length of each time increment through a half-step residual control, where the half-step residual is the equilibrium residual error (out-of-balance forces) halfway through a time increment. The acceptable half-step residual tolerance is specified by the HAFTOL option in ABAQUS [A1], which has dimensions of force and is usually chosen by comparison with typical actual force values such as applied forces; in this investigation it is taken to be nearly approximate to the maximum blast pressure.
applied to the structure. Such an action will lead to a highly accurate solution for a problem including plasticity. This automatic incrementation scheme is especially effective for the case where a sudden blast event initiates a dynamic problem and the structural response involves large amounts of energy being dissipated by plasticity effects. In such a case small time increments will be required immediately after the sudden blast event, while at later time the response can be modelled accurately with large time increments. Moreover in order to limit the range of each time step in the numerical simulations, a minimum and a maximum allowable time increment are taken as $1 \times 10^{-7}$ s and 0.01 s, respectively.

### 2.5 Verification of Finite Element Models

The verification of the finite element models as mentioned above is carried out by implementing it into the analysis of a simply supported beam subjected to blast loadings tested by Seabold [S3] and a square slab, clamped and longitudinally restrained along all edges exposed to uniform lateral pressure tested by Keenan [K3]. The computed and experimental displacement-time histories at the mid-span for the beam and the centre of the slab are compared in Figure 2.9.

It can be observed from Figure 2.9 that for the simply supported beam the peak experimental response of 28.5 mm was recorded at 19.5 ms, which agrees well with the analytical results for which the computed peak displacement of 28.8 mm is reached at 19.2 ms. The recorded permanent deformation of this beam was 20.8 mm and it also compared well with the predicted deflection of 21.7 mm. As for the reinforced concrete slab, the maximum deflection at the centre recorded in the experiment was 12.0mm appearing at time 7.4 ms, which is fairly close to the numerically predicted one of 11.9 mm at 7.2 ms. In addition, the analytical and experimental results of top compressive strain history of concrete, strain history of main reinforcement, support reaction history, and mid-span velocity history of RC structural member are also compared in Figure 2.9. It is demonstrated that the numerical analysis reasonably agrees with the observed experimental behaviours. Therefore the numerical model has the ability to simulate the failure process of
concrete and reinforcement, which are thus applied into the nonlinear analysis of the blast-loaded six-storey frame structures in Figures 2.7 and 2.8.

Figure 2.9 Verifications of finite element models by comparison of the numerical results with the experimental ones

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2.6 Numerical Results and Discussion

With the finite element models described above, three numerical case studies are carried out to evaluate the dynamic responses and damage distributions of the designed six-storey reinforced concrete frame structure with/without exterior cladding panels under blast conditions I and II with the results respectively discussed in the following.

2.6.1 First case study: frame structure with exterior cladding panels under blast condition I

2.6.1.1 A general view of structural responses

In order to obtain a general view of the dynamic responses of the six-storey frame structure with reinforced concrete exterior cladding panels under the considered blast condition I, its energy responses including the kinetic energy and the plastic strain energy are plotted in Figures 2.10 and 2.11. In these figures, the symbols of $KE_{ws}$ and $PE_{ws}$ are respectively the kinetic energy and plastic strain energy within the whole structure; $KE_{bm}$ and $PE_{bm}$ represent these two kinds of energy within the critical blast-loaded members (including the front exterior cladding panels and columns, the top-storey beams and roof slabs) while $KE_{om}$ and $PE_{om}$ stand for the kinetic energy and plastic strain energy within other structural members.

From Figure 2.10, it can be observed that from 0 ms to 30 ms, $KE_{bm}$ composes the most significant part of $KE_{ws}$ covering averagely about 85% of it, which reflects the predominant role of the localized responses of the critical blast-loaded members in the structural responses during this time. Later, the ratio of $KE_{bm} / KE_{ws}$ decreases rapidly to a value of near 23% at about 50 ms while further it keeps almost constant. This constancy after 50 ms demonstrates that the dynamic response of the structure is in a mode where the global response of the structural system as a whole dominates. As a consequence, the whole dynamic responses of the structure can be divided into two stages at an approximate time of 40 ms as shown in Figure 2.10b.
Within response stage I, the dynamic responses are concentrated on several critical blast-loaded members while as the time goes by, more structural members are motivated and the structural global responses become much more obvious at response stage II.

The plastic strain energy response of the structure shows the same stage division as that demonstrated in Figure 2.11b where the ratio of $\frac{PE_{bm}}{PE_{ws}}$ versus time is plotted. $PE_{bm}$ accounts for almost all of $PE_{ws}$ during the time of 40 ms, however with the global response of the structure motivated later, more plastic deformation occurs on the members other than the critical blast-loaded ones and thus the ratio of $\frac{PE_{bm}}{PE_{ws}}$ declines. A more detailed description of the structure responses and its damage propagation with time will be made in the following sections according to the divided two response stages.
(a). Kinetic energy versus time for various structural members

(b). Ratio of $KE_{bm} / KE_{ws}$ versus time

Figure 2.10 Response of the kinetic energy (First case study)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a). Plastic strain energy versus time for various structural members

(b). Ratio of $PE_{bm} / PE_{ws}$ versus time

Figure 2.11 Response of the plastic strain energy (First case study)
2.6.1.2 Response stage I

Since at response stage I, both kinetic energy and plastic strain energy responses are concentrated on the critical blast-loaded members, special attention is laid on their dynamic responses and corresponding damage with the plastic hinges distributions.

Figure 2.12 Damage propagation with time (First case study)
plotted in Figure 2.12. A flexural plastic hinge is assumed to initiate when the longitudinal tensile reinforcement first yields at a point along the beam element and the hinge will continue to spread over a continuous portion of the beam. Thus the occurrence time of the hinge corresponds with the first appearance of plastic strain of reinforcement whose variation with time can be found from the numerical analysis in ABAQUS [A1].

It is observed that due to the intensive localized responses on these members induced by the blast loadings during this stage, three plastic hinges have been formed on the bottom, mid-height and top sections for the front columns at 15 ms as well as the left end, mid-span and right end sections for the top-storey beams at 40 ms; this results in a typical damage mechanism similar to that on the flexural structural members. The difference between the time of forming this damage mechanism on the front columns and top-storey beams is caused by the difference of the time when the peak blast pressures acting on them is reached. As for the other frame members (beams and columns), almost no plastic deformation is present.

The flexural deformation plays a relatively important role on the dynamic responses of structural members under this distant blast condition as indicated in Figure 2.13, where typical moment-curvature and shear force-shear strain curves for different cross sections of the first-storey, fourth-storey and top-storey columns at Axis A as well as the top-storey beam between Axis B and Axis C are plotted. It can be seen that there exist linear relationships between the shear forces and the shear strains for these cross sections while all the irrecoverable deformation is induced by their curvature responses. These phenomena are reflected on the cross sectional responses of other structural members as well.
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(a). Flexural and shear responses of the first-storey column at Axis A

Figure 2.13 Responses of some critical blast-loaded members (First case study)
(b). Flexural and shear responses of the fourth-storey column at Axis A

Figure 2.13 (Continued)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(c). Flexural and shear responses of the top-storey column at Axis A

Figure 2.13 (Continued)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(d). Flexural and shear responses of the top-storey beam between Axis B and Axis C

Figure 2.13 (Continued)
Due to the direct action of the blast loadings on them as well as their large area to receive the blast loadings during response stage I, the responses of the front exterior cladding panels are significantly severe as illustrated in Figure 2.14. The lateral deformation at the centre of the first-storey exterior cladding panel has reached 653 mm at the end of response stage I, which is approximately equivalent to a support rotation of 16.5°. According to the actual resistance deflection function presented in TM5-1300 [T2], the resistance due to the tensile membrane action produced under the continuous reinforcement conditions as well as adequate lateral constraint, increases with increasing deflection up to incipient failure at approximately 12° support rotation. Based on this, the cladding panel is almost in a failure state at the end of response stage I.

![Lateral deflection of the front cladding panels (First case study)](image)

Figure 2.14 Lateral deflection of the front cladding panels (First case study)

As for the global responses of the structural system, the storey lateral displacement (storey drift) responses are plotted in Figure 2.15a. It is indicated that all floor levels have the similar drifts (about 30 mm) at the time of 40 ms, except for the roof whose drift is somewhat smaller, therefore a concentrated maximum inter-storey drift ratio (MIDR), the ratio of maximum inter-storey drift (the drift difference between adjacent storeys) to the inter-storey height, is 0.67% occurring at the first
storey. The contour of the storey drifts for the structure at this time location can be found in Figure 2.15b.

(a). Storey drift responses

(b). Contour of the storey drifts at different time stations

Figure 2.15 Storey drifts for the structure (First case study)
2.6.1.3 Response stage II

At response stage II, the structural dynamic response is in a mode where the global response of the structural system as a whole dominates, hence the distribution of damage as well as its propagation with time is in a different manner with those at response stage I. As shown in Figure 2.12, during response stage II, plastic hinges are formed firstly on both end sections of the first-storey columns at a time of about 60 ms as the inter-storey drift within this storey increases. Later, damage spreads upward with more plastic hinges successively formed on those connected cross sections around the joints located in the first and second floor levels. Since the plastic hinges formed at this stage are caused by large global sideways responses, they appear only at the end sections of structural members.

To gain a better understanding of structural global responses, the storey drift distributions with time for this frame structure under blast condition I are demonstrated in Figures 2.15. It can be seen that during the time up to 80 ms the structural global responses are obviously concentrated within the first storey while the inter-storey drifts within other storey levels are trivial. As the time lengthens, the increase gradient for the first-storey drift drops to a relatively small value and the structural global response begins to focus on the second-storey level. The structural maximum inter-storey drifts within the two critical storey levels (the first and the second storeys) have reached 129 mm and 85 mm, which are equivalent to a MIDR of 2.88 % and 1.90 % respectively at the time of 180 ms. Referring to Table 2.1 where a global damage criteria based on the inter-storey drift ratio for a frame structure under blast conditions have been suggested in the reference [B4], the frame structure at time 180 ms is damaged to a high level.
Table 2.1 Three response levels [B4]

<table>
<thead>
<tr>
<th>Response level</th>
<th>Inter-storey drift ratio</th>
<th>Damage description</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>1/50</td>
<td>localized building/component damage</td>
</tr>
<tr>
<td>medium</td>
<td>1/35</td>
<td>widespread building/component damage</td>
</tr>
<tr>
<td>high</td>
<td>1/25</td>
<td>building/component losing structural integrity and having possibility of collapse due to environment condition</td>
</tr>
</tbody>
</table>

From the structural global responses significantly concentrated in the first and second-storey levels at the time of 180 ms, it can be deduced that plastic rotations of the joints distributed in the first and second floor levels are focused on those cross sections at the ends of the connected columns and beams as shown in Figure 2.12. Typical moment-curvature and shear force-shear strain curves for these cross sections where plastic hinges are formed at response stage II are plotted in Figure 2.16. It is clear that plastic deformation is induced by their flexural responses where irrecoverable curvatures appear at the end sections of the members.
(a). Flexural and shear responses of the first-storey column at Axis B

Figure 2.16 Responses of some structural members other than blast-loaded ones
(First case study)
(b). Flexural and shear responses of the first-storey beam between Axis A and Axis B

Figure 2.16 (Continued)
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2.6.2 Second case study: bare frame structure under blast condition I

The numerical simulation results of the energy responses for the six-storey bare frame structure under blast condition I are presented in Figure 2.17. It can be seen that due to the absence of the exterior cladding panels, this bare frame will experience much slighter dynamic responses when compared with the first case study. Therefore much less damage is induced as shown in Figure 2.17b where only a small magnitude of 1.0 kNm of plastic energy appears on the floor slabs. As for the frame members (columns and beams), there is no plastic deformation on them.

The responses of structural storey drift plotted in Figure 2.18a demonstrate that the maximum inter-storey drift, which happens to the second-storey level, is only about 9.5 mm, equivalent to a MIDR of 0.21%. The MIDRs within other storey levels are smaller than this value and thus there is no global damage to the structure under this blast condition [S4]. In addition, from the storey drift contours at different time stations shown in Figure 2.18b, it is obvious that the maximum storey drift occurs firstly at the first storey whereas with the lengthening of time, it will move upwardly to the second storey and third storey until it is present on the top storey in sequence. Thus a transverse wave is formed in the frame structure.
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(a). Kinetic energy versus time for various structural members

(b). Plastic strain energy versus time for various structural members

Figure 2.17 Responses of the kinetic energy and plastic strain energy
(Second case study)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a) Storey drift responses

(b) Contour of the storey drifts at different time stations

Figure 2.18 Storey drifts for the structure (Second case study)
2.6.3 Third case study: bare frame structure under blast condition II

2.6.3.1 A general view of structural responses

In order to further study the behaviours of the bare frame structure without exterior cladding panels under a more severe distant blast condition as well as the corresponding damage distributions, the numerical analysis of the bare six-storey frame structure under blast condition II with a 250 ton equivalent TNT at a 100 m distance is performed as well. The energy responses are plotted in Figures 2.19 and 2.20 where three sets of symbols of $K_{E_{ws}}$ and $P_{E_{ws}}$, $K_{E_{bm}}$ and $P_{E_{bm}}$, $K_{E_{om}}$ and $P_{E_{om}}$ represent the kinetic energy and plastic strain energy within the whole structure, the critical blast-loaded members including all columns, and the other structural members respectively.

From the kinetic energy responses in Figure 2.19, it is indicated that $K_{E_{bm}}$ composes the most part of $K_{E_{ws}}$ at the start of the response stage. The ratio of $K_{E_{bm}} / K_{E_{ws}}$ then decreases dramatically up to a later time of about 20ms, after which it keeps almost constant with a slight high frequency vibration. Therefore, similar to the first case study, the whole dynamic response process of the bare frame structure in this blast condition can be approximately divided into two stages at about 10 ms. At response stage I, the dynamic responses are greatly localized on the blast loaded frame columns while the global responses of the structure become dominant at response stage II. However none of the plastic strain energy is developed during response stage I, as indicated from the plastic strain energy responses within various structural members in Figure 2.20. At response stage II, the plastic strain energy occurs initially on some structural columns ($P_{E_{bm}} / P_{E_{ws}}$ nearly unit) but later they spread over other members with $P_{E_{bm}}$ occupying only about 23% of $P_{E_{ws}}$ after 600 ms, The detailed dynamic responses of the bare frame structure and damage distributions at each stage are described in the following.
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a). Kinetic energy versus time for various structural members

(b). Ratio of $KE_{bm}/KE_{ws}$ versus time

Figure 2.19 Response of the kinetic energy (Third case study)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a). Plastic strain energy versus time for various structural members

(b). Ratio of $PE_{bm} / PE_{ws}$ versus time

Figure 2.20 Response of the plastic strain energy (Third case study)
2.6.3.2 Response stage I

At response stage I, although the dynamic response is concentrated on the blast loaded structural columns, no any plastic strain energy is present in them.
Accordingly, no localized damage appears to the columns within this stage as shown in the Figure 2.21.

### 2.6.3.3 Response stage II

As the kinetic energy converts into the global responses of the bare frame structure as a whole at response stage II, plastic hinges appear firstly to the bottom sections of the first-storey columns and then as time passes, the plastic hinges tend to spread gradually upward to the cross sections of other frame members around the joints located in the upper floor levers. The process of this type of structural damage in this numerical case study is displayed in Figure 2.21.

Further studies of the dynamic responses of different cross sections are performed, where the typical shear force-shear strain and moment-curvature responses for some columns and beams are plotted in Figures 2.22. It shows the linear shear responses of these cross sections while the change of the moments with curvatures nonlinearly. Certain irrecoverable flexural responses can be observed to the end sections when unloading, which accounts for the inelastic storey drift of the whole structure.

The developments of the plastic hinges at response stage II also indicate that the global damage of the bare frame structure has a tendency to disperse from the bottom storey levels to the upper storey levels. In accordance with this, significant storey drift responses will be activated. This can be further observed in Figure 2.23 where the storey drift responses and the corresponding contours at a variety of time stations are plotted. Similar to the second case study, a transverse wave is present with the maximum storey drift travelling from the bottom to the top. The maximum inter-storey drift occurs in the second-storey that is 0.123m equivalent to 2.77% of MIDR. According to Table 2.1, the bare frame structure is damaged globally to a high level.
(a). Flexural and shear responses of the first-storey column at Axis D

Figure 2.22 Responses of some critical column and beam (Third case study)
(b). Flexural and shear responses of the first-storey beam between Axis A and Axis B

Figure 2.22 (Continued)
Chapter 2. Responses of Multi-storey Reinforced Concrete Frame Structures to Distant Explosions

(a) Storey drift responses

(b). Contour of the storey drifts at different time stations

Figure 2.23 Storey drifts for the structure (Third case study)
2.6.4 Comparisons of three case studies

Comparisons of above three numerical case studies indicate some similarities and dissimilarities, which will be discussed respectively as follows.

**Similarities** — The whole process of the structural dynamic responses can be approximately divided into two stages. Localized responses of the blast-loaded members are critical at response stage I while the global responses of the structural system dominate at response stage II. In addition, the flexural responses play a more important role in the plastic deformation of the frame beams and columns in comparison to their respective shear responses.

**Dissimilarities** — The first dissimilarity existing in the first and second case studies is that the damage is much more severe for the frame structure with reinforced concrete exterior cladding panels than those for the bare frame structure under the same blast condition. Due to the diffraction of the blast waves around the columns, reflected pressure will quickly reduce to zero for the bare frame structure and only the drag forces associated with the dynamic pressure are critical in the net blast loadings on the structure. However the exterior cladding panels cause the structure to be loaded with reflected blast pressure, overpressure, and dynamic pressure, thus the blast forces received by the frame structure with cladding panels are much greater. To dissipate the work produced by such a large blast force, some plastic deformation would appear to the structure producing damage to some extent as expected in the first case study.

Comparing the first case study with the third one indicates that within response stage I, local damage on the critical blast loaded members including the front exterior cladding panels and columns, and the top-storey beams occurs for the frame structure with exterior cladding panels under blast condition I. However no local damage is present to the blast loaded members (all columns) at this stage for the bare frame structure under blast condition II. This difference is induced by the fact that the blast loadings acting on these two kinds of frame structure differ
greatly as discussed above. Due to the significantly blast force received by the front exterior cladding panels and the roof slabs in blast condition I, a large amount of plastic deformation have been developed on them within response stage I, which further produces considerably distributed reaction forces in the opposite direction to the connected frame columns and beams. This force plus the direct action of the blast loadings creates a lot of localized damage on these columns and beams.

The third dissimilarity exists in the trend of the global responses developments at response stage II. The responses of the bare frame structure have the tendency to spread from the bottom to upper storey levels due to a transverse wave formed for the structural storey drift responses as indicated in the second and third case study. However this trend is not obvious in the first case study.

2.7 Summaries

Based on the results obtained from the numerical case studies, the following conclusions are made regarding dynamic responses and damage distributions of a six-storey reinforced concrete frame structure under distant blast conditions:

1. The whole process of the frame structural dynamic responses can be approximately divided into two stages. The flexural responses are dominant in the plastic deformation of the structural members as compared to the shear responses. In the identical blast situation, the existence of reinforced concrete exterior cladding panels produces more severe dynamic responses than those of the bare frame structure.

2. Local damage is produced to the critical blast-loaded members at response stage I as indicated from the first case study. When considering the design, to keep this local damage under control at the objective performance level, the inelastic deformation of the blast-loaded members should be strictly limited. As a result, an effective performance-based blast resistant design for the reinforced concrete structural members is urgently needed and
implemented with the explicit consideration of their inelastic deformation, which will be discussed in Chapters 3 to 6.

3. Global damage due to the significantly MIDR responses is present at response stage II from the first and third case studies. Therefore, in order to ensure that the structural global damage is at the required performance level, it is necessary to control the MIDR responses during the design of the reinforced concrete frame structure in resisting such a blast loading as produced from the distant intense surface explosion. The procedures to fulfil this purpose will be addressed in Chapters 7 and 8.
CHAPTER THREE

PERFORMANCE–BASED BLAST RESISTANT DESIGN OF REINFORCED CONCRETE MEMBERS USING ENERGY SPECTRA—PART I. METHODOLOGY

Synopsis

The displacement and displacement ductility factor correlate with an expected performance level of a structural member in a blast event, thus both of these two parameters should be utilized as performance indicators in a blast resistant design. To meet an expected performance level defined by a combination of target displacement and target displacement ductility factor, a new blast resistant design approach is developed in this chapter for one-way reinforced concrete members on the basis of energy spectra, which are constructed from an elastic-perfectly-plastic single-degree-of-freedom (SDOF) system. By controlling the responses of maximum displacement and displacement ductility factor for the equivalent SDOF system of the reinforced concrete member to exactly reach the corresponding design targets, the effective depth and longitudinal reinforcement ratio of the member can be specifically determined against the given blast loading within the design. This chapter concentrates on the methodology of the developed blast resistant design using energy spectra while the evaluation of its effectiveness with the numerical analysis follows in Chapter 4. The analytical results indicate that this new approach has several advantages over the existing blast design guidance.

Keywords: Expected performance level, Target displacement, Target displacement ductility factor, Energy spectra, Equivalent SDOF system
3.1 Overview of Existing Design Guidance

Design and construction of reinforced concrete structural members against accidental or deliberate explosions is deemed necessary with the increasing importance laid upon explosion safety. For the explosions occurring at a relatively large standoff distance, the blast loadings exerting on the structural members can be properly simplified as uniformly distributed dynamic loadings characterized by their peak values and durations. In such cases, some level of inelastic deformation of structural members is allowed within the blast resistant design when subjected to severe blast loadings to dissipate energy. Therefore much blast design guidance [A2, A3, B1, M3, T1, T2] based on performance with the explicit consideration of inelastic deformation has been proposed in recent years. The purpose of these design guidance is to ensure the performance of designed member under the given blast force meet the requirement and objective of the owners, users, and society.

In a performance-based design framework, selection of the appropriate response parameter as the performance indicator to quantify the performance level of member in blast conditions is of particular importance. According to the selected parameter, the expected performance level can be established and applied to the design process in controlling member’s responses or damages. Currently either the displacement (specified in TM5-1300 [T2] and Mays and Smith [M3]) or the displacement ductility factor (specified in TM5-855-1 [T1], Biggs [B1], and ASCE Manual 42 [A2]) has been suggested as the performance indicator. Given a target displacement \( y \), or a target displacement ductility factor \( \mu_i \) for the significant point of the member to be designed (e.g. the mid-span of a member with two ends constrained or the free-end of a cantilever member), the blast design can be carried out. In these design guidance [T1, T2, M3, B1, A2], \( y \approx l \tan \theta / 2 \) is for one-way member of reinforced concrete with two ends constrained assuming plastic hinge formation at mid-span and fixed ends and a linear deflected shape between hinges, while \( y \approx l \tan \theta_i \) is for the cantilever member (\( \theta_i \) is the expected or target support rotation of the one-way member under the blast force). The other parameter of \( \mu_i \)
can be defined as the target of $y_m/y_e$ where $y_m$ denotes the peak displacement response and $y_e$ is the elastic response (recoverable displacement) for the designed member. The existing guidance [B1, T2, M3] involving $y_e$ or $\mu_e$ alone in the blast design is reviewed as follows.

### 3.1.1 Work of Biggs

Biggs [B1] suggested the displacement ductility factor as the performance indicator. Thus after a certain level of $\mu_e$ is specified, the displacement ductility response of the member to be designed should be checked with this $\mu_e$. Since the responses of structural member under the blast conditions depends upon its mass, stiffness and strength, which are all related to the cross-sectional size after specifying a constant longitudinal reinforcement ratio $\rho$, a trial and error method should be applied to the design. The related design flow chart is shown in Figure 3.1.

### 3.1.2 The TM5-1300 method

TM5-1300 [T2] is widely used in the design of protective structures against blast loadings. The selection of displacement rather than displacement ductility factor as the performance indicator is the main difference between TM5-1300 and Biggs’s method. Therefore after a certain $y_e$ is defined, the maximum displacement response of the member is to be checked with this $y_e$ by specifying a constant $\rho$ and assuming the member’s cross-sectional size. Modifications of the cross section is performed until $y_e$ is satisfied through a trial and error method.

### 3.1.3 Work of Mays and Smith

Based on TM5-1300 [T2], Mays and Smith [M3] suggested a design procedure where displacement alone is employed as performance indicator in the blast design. But in their method, by comparing $t_d$ and $T$, the blast loading is divided into an
impulsive or a quasi-static/dynamic state based upon which the design is finished. The design flowchart is given in Figure 3.2.

![Design Flowchart](image)

* Charts plotting the distribution of the displacement ductility factor ($\mu$) against $t_d/T$ with respect to $R_m/F_1$ for an elastic-perfectly-plastic SDOF system under triangular pulse loads.

Figure 3.1 The design flowchart of Biggs’s method [B1]

Figure 3.2 The design flowchart of Mays and Smith’s method [M3]
The utilization of \( \mu \) in Biggs’s and TM5-855-1’s design methods could obviously establish an effective control of the displacement ductility response of the member in blast conditions, however it may not be able to limit its maximum displacement response explicitly. In some cases if the maximum displacement response is too large, the subsequent secondary effect will be quite significant, which may cause further damage or even the destruction of the member. Besides the large displacement response may endanger the equipments and occupants within the building. On the other hand, in TM5-1300’s as well as Mays and Smith’s design procedures, using \( y_i \) alone could only control the maximum displacement response of the member, nevertheless in such cases its displacement ductility response is not restricted, which might lead to an ineffective design. For instance, in some severe blast conditions, a relatively large cross section of the member will be designed with the specified \( \rho \) to meet such a target of \( y_i \). This design action will induce an unreasonably high displacement ductility demand. However the displacement ductility capacity of this designed member is bounded by the large cross section and thus could not guarantee satisfying such a high displacement ductility demand. Based on the above discussions, it seems that adopting either \( y_i \) or \( \mu \) alone in the blast design may not be so adequate in the control of the responses of structural members fulfilling their expected performance level. In order to carry out an effective performance-based blast design, it is better to take simultaneous consideration of \( y_i \) and \( \mu \) into the design process.

In addition, some subjectivity is involved in these existing design procedures [B1, T1, T2, M3]. Firstly \( \rho \) should be specified based on the engineering experience and judgment and keep unchanged during the design. Secondly with the Mays and Smith’s method, it is assumed that the blast loadings are located in either an impulsive or a quasi-static/dynamic state thus establishing the watershed between these two states becomes critical. Many artificial division methods have been proposed in the past [M3, A3] to deal with this problem, however these methods seem to be quite empirical.
To date the appropriate response parameters as the performance indicators have not been unified in the performance-based blast design process; either the displacement or the displacement ductility factor has been adopted. However such an action could not promise an effective performance-based blast design. Both $y_i$ and $\mu_i$ should be taken into account within the design. Therefore, in order to achieve the expected performance level defined by a combination of the dual targets $y_i$ and $\mu_i$, a new approach is developed herein based on the energy spectra, which are constructed for an elastic-perfectly-plastic SDOF system subjected to blast loadings. This design procedure could also overcome the shortcomings of the subjective specification of $\rho$ at the initial design stage and could accommodate arbitrary blast loadings. The present research attempts to make contribution to the current worldwide efforts towards the development and elaboration of a basis for a genuine performance-based blast design practice.

### 3.2 Energy Spectra

Before the detailed description of this blast resistant design approach, the energy spectra need to be specified first because of the significant energy absorption during the inelastic behaviour of the structural member under blast conditions. Also they are important tools in incorporating the dual targets of $y_i$ and $\mu_i$ into the blast resistant design thus to determine the cross-sectional size and reinforcement ratio of the member. The formation of energy spectra is discussed as follows.

#### 3.2.1 Non-dimensional energy factor

When a structural member is subjected to a blast loading, the member deflects until such time as the strain energy of the member is developed sufficiently to balance the external work applied to it due to the blast loadings. Thus a rigorous approach to obtain this strain energy exactly becomes essential. To meet such an objective, a non-dimensional energy factor $C$ is introduced, expressed as the ratio of maximum strain energy $E_{\text{max}}$ to the ultimate elastic energy $E_{\text{el}}$. Thus

\[ C = \frac{E_{\text{max}}}{E_{\text{el}}} \]  \hspace{1cm} (3.1)

In this study, the non-dimensional energy factor is computed from an elastic-perfectly-plastic SDOF system with the resistance function depicted in Figure 3.3. It is evident that the term of ultimate elastic energy \( E_{\text{el}} \) can be expressed as

\[ E_{\text{el}} = k_e \frac{y_e^2}{2} \]  \hspace{1cm} (3.2)

where \( y_e \) is the elastic (recoverable) displacement and \( k_e \) is the initial stiffness. The ultimate strength \( R_m \) for the SDOF system is

\[ R_m = k_e y_e \]  \hspace{1cm} (3.3)

Figure 3.3 Resistance function for an elastic-perfectly-plastic SDOF system

3.2.2 Associated parameters

The basic equation of motion for the elastic-perfectly-plastic SDOF system in the elastic range by ignoring the damping effect [B1, K4] can be generally described as

\[ m \ddot{y} + k_y y = F_1 f(t, t_d) \]  \hspace{1cm} (3.4)

where \( F_1 \) is the peak value of the force, \( f(t, t_d) \) is a non-dimensional time function defined by a variable \( t \) and a given loading duration \( t_d \), and \( y \) is the displacement response. It is convenient to normalize the parameters in Equation (3.4) through transforming the variables by letting \( \eta = y/y_e \) and \( \xi = t/T \), therefore the above expression in the elastic range becomes

\[ \frac{1}{4\pi^2} \ddot{\eta} + \eta = \frac{F_1}{R_m} \frac{f(\xi, \frac{t_d}{T})}{T} \]  \hspace{1cm} (3.5)

Assuming \( a = F_1/R_m \) and \( b = t_d/T \) then

\[ \frac{1}{4\pi^2} \ddot{\eta} + \eta = a f(\xi, b) \]  \hspace{1cm} (3.6)

In the plastic range, the item \( k y \) in Equation (3.4) becomes constant with a value of \( R_m \) thus Equation (3.7) may be refined from Equation (3.6) as

\[ \frac{1}{4\pi^2} \ddot{\eta} + 1 = a f(\xi, b) \]  \hspace{1cm} (3.7)

As such, the factor \( \eta(\xi) \) is only related to the parameters \( a \) and \( b \) for a given loading function from Equations (3.6) and (3.7) and by solving them, \( E_{\text{max}} \) can be obtained by

\[ E_{\text{max}} = \int_0^{t_w} F_1 f(t, t_d) dy = \int_0^{t_w/T} a R_m f(\xi, b) d\eta(\xi) y_e = a k_y y_e^2 \int_0^{t_w/T} f(\xi, b) \eta'(\xi) d\xi \]  \hspace{1cm} (3.8)
where \( t_m \) is the time for the maximum displacement of the SDOF system. The non-dimensional energy factor \( C \) may be derived from substituting Equations (3.8) and (3.2) into Equation (3.1), hence

\[
C = 2 a \int_{0}^{t_m/T} f(\xi, b) \eta'(\xi) d\xi
\]  

(3.9)

The point that the ratio of \( t_m/t_d \) is a function of the factors \( a \) and \( b \), has been proven by Biggs [B1], thus \( t_m/t_d = g(a, b) \) (where \( g(\bullet) \) is a function). Substituting this expression into \( t_m/T \) leads to

\[
\frac{t_m}{T} = \frac{t_m}{t_d} \frac{t_d}{T} = g(a, b) b
\]

(3.10)

By inserting Equation (3.10) into Equation (3.9), \( C \) is expressed as

\[
C = 2 a \int_{0}^{g(a,b)b} f(\xi, b) \eta'(\xi) d\xi
\]

(3.11)

Equation (3.11) reveals that in order to obtain a response in terms of the non-dimensional energy factor \( C \), one needs only know the values of \( F_i/R_m \) and \( t_d/T \). Thus these two parameters are necessary and sufficient for computing the factor \( C \). In this chapter, a group of curves, which represents the non-dimensional energy factor \( C \) against the ratio \( t_d/T \) with respect to various \( F_i/R_m \), are defined as energy spectra; their formation is addressed as follows.

### 3.2.3 Formation procedures

Giving the discrete values for items \( a \) and \( b \), Equations (3.6) and (3.7) are initially solved with the time history analysis for \( \eta(\xi) \) to find the value of \( t_m/T \). Further, the non-dimensional energy factor \( C \) is determined by the integration of Equation
(3.9). According to the distribution of \( C \) at various \( a \) and \( b \), the energy spectra can be finally plotted.

3.2.4 Energy spectra for various loadings types

The energy spectra for an undamped elastic-perfectly-plastic SDOF system without initial motion for various patterns of dynamic loadings are illustrated in Figures 3.4 to 3.7. For the triangular pulse with a zero rise time, \( f(\xi, b) = 1 - \frac{\xi}{b} \) up to \( \xi = b \) and then zero as seen in Figure 3.4. While in the case of a rectangular pulse as shown in Figure 3.5, \( f(\xi, b) \) is unity up to \( \xi = b \) and zero thereafter. Figure 3.6 illustrates that when the force is constant with finite rise time, \( f(\xi, b) = \frac{\xi}{b} \) up to \( \xi = b \) and unity at a later time. As far as the triangular function as shown in Figure 3.7 is concerned, \( f(\xi, b) = \frac{2\xi}{b} \) up to \( \xi = b / 2 \), followed by \( f(\xi, b) = 2 - 2\xi / b \) up to \( \xi = b \) and zero thereafter. The corresponding energy spectra are constructed by inserting these expressions for \( f(\xi, b) \) into Equations (3.6) and (3.7) and obtaining the non-dimensional energy factor \( C \) through the integration of Equation (3.9) for various items \( a \) and \( b \).

The non-dimensional energy factor \( C \) of an elastic-perfectly-plastic SDOF system when subjected to a triangular load pulse in Figure 3.4 tends to rise with the increase of \( t_d / T \) or \( F_i / R_m \). For the rectangular pulse in Figure 3.5, it is observed that \( C \) is equal to unity at \( F_i / R_m = 0.5 \) for \( t_d / T > 0.5 \) and it tends to increase with \( t_d / T \) for a definite \( F_i / R_m \). Figure 3.6 illustrates the existence of a transition point occurring at the time \( t_d / T = 1 \) for the constant force with a finite rise time and before this time, \( C \) declines gradually with \( t_d / T \) for a definite \( F_i / R_m \). It should be also noticed that in this case, the peak force \( F_i \) could not be greater than the strength \( R_m \) thus here only the ratios \( F_i / R_m \) between 0 and 0.9 are listed.
Figure 3.4 Energy spectra of an undamped elastic-perfectly-plastic SDOF system due to triangular load pulses with zero rise time

Figure 3.5 Energy spectra of an undamped elastic-perfectly-plastic SDOF system due to rectangular load pulses
Figure 3.6 Energy spectra of an undamped elastic-perfectly-plastic SDOF system due to constant forces with finite rise time

Figure 3.7 Energy spectra of an undamped elastic-perfectly-plastic SDOF system due to equilateral triangular load pulses

Generally the function of blast loading on structural member from the external explosion could be simplified into a triangular loading pulse if the reflected pressure and impulse are preserved [T1, H1]. Thus the inclusion of the energy spectra under triangular load pulses in Figure 3.4 into the design of one-way reinforced concrete members under blast loadings will be discussed in the following section. As for the design of members against other types of dynamic loadings, the respective energy spectra shown in Figures 3.5 to 3.7 could be utilized.

3.3 Design Criteria

A fundamental issue in the performance-based blast design is to determine the specific response parameters as performance indicators. The previous discussions in this chapter indicated that to carry out an effective blast design, both displacement and displacement ductility factor should be adopted as the performance indicators. To achieve the expected performance level defined by the dual targets of $y_t$ and $\mu_t$, a comparatively accurate design approach for the structural members against blast forces will be discussed based on the energy spectra.

The structural member with continuous mass and stiffness is simplified by converting it into an equivalent SDOF system involving lumped mass supported by an elastic-perfectly-plastic spring [B1]. The equivalent system is selected in such a way that the deflection response of the concentrated mass is the same as that for the significant point on the structural member. Thus the responses of equivalent system under the given blast force should also achieve the expected performance of $y_t$ and $\mu_t$. By using the energy spectra, a recurrent procedure is addressed herein to determine the initial stiffness $k_e$ and ultimate strength $R_m$ of the equivalent SDOF system for the given blast loading function and dual targets of $y_t$ and $\mu_t$. The derivation of cross-sectional size and reinforcement ratio of the member from the values of $k_e$ and $R_m$ of the equivalent system will be further identified.
3.4 Design of Equivalent Elastic-Perfectly-Plastic SDOF System Using Energy Spectra

An important point that needs to be addressed at this stage is that to achieve the performance level defined by \( y_t \) and \( \mu_t \) simultaneously, \( k_e \) and \( R_m \) of the equivalent elastic-perfectly-plastic SDOF system against the blast loading could be determined specifically, which will be proven herein.

Given \( y_t \) and \( \mu_t \), the elastic displacement \( y_e \) of equivalent SDOF system is required as

\[
y_e = \frac{y_t}{\mu_t} \quad (3.12)
\]

Assume that there are three possible designs of the equivalent systems, which have satisfied the requirement of \( y_e \) but with various initial stiffness as shown in cases (a), (b) and (c) plotted in Figure 3.8 and

\[
R_{ma} = k_{ea}y_e, \quad R_{mb} = k_{eb}y_e, \quad \text{and} \quad R_{mc} = k_{ec}y_e \quad (3.13)
\]

\( (k_{ea}, R_{ma}), (k_{eb}, R_{mb}) \) and \( (k_{ec}, R_{mc}) \) are the sets of initial stiffness and ultimate strength of the equivalent system in various cases. Under the identical blast loading, the responses of these systems are compared as follows:

Case (a): Suppose that in this case the response of maximum displacement of the equivalent system having \( k_{ea} \) under the blast loading could reach \( y_t \) exactly.

Case (b): \( k_{eb} \) in this case is smaller than \( k_{ea} \) thus \( R_{mb} < R_{ma} \) with Equation (3.13). Under the same blast loading, the maximum external work on the SDOF system in case (b) will inevitably be greater than that in case (a).
larger maximum displacement response will be produced for the equivalent system in case (b) to dissipate the enlarged external work, thus \( y_{ib} > y_i \).

Case (c): \( k_{ec} \) is larger than \( k_{ea} \) and \( R_{me} > R_{ma} \). In the same blast condition the maximum displacement response of the equivalent system in case (c) would be less than that in case (a), which is understandable since in such a case the maximum external work on the SDOF system in case (c) tends to be less than that in case (a), thus \( y_{ic} < y_i \).

![Figure 3.8 Various resistance functions](image)

The above comparative analysis of various cases indicates that in the situation of the required \( y_e \) fulfilled, if supposing that maximum displacement response of the equivalent system having \( k_{ea} \) under the given blast loading reach \( y_i \) exactly, the maximum displacements of equivalent systems with initial stiffness larger or smaller than \( k_{ea} \) are not equal to \( y_i \). Hence it can be concluded that in order to achieve the dual design targets of \( y_i \) and \( \mu_i \) under the given blast loading simultaneously, there exists only one solution of the initial stiffness for the equivalent elastic-perfectly-plastic SDOF system as assumed in case (a) (see Figure 3.8). The ultimate strength of the equivalent system can be further specifically
determined from Equation (3.13). In such a case, $E_{\text{max}}$ for the equivalent system necessary to balance the external work delivered by the blast loadings is equal to

$$E_{\text{max}} = \frac{1}{2} k_{\text{ea}} y_e^2 + k_{\text{ea}} y_e (y_t - y_e)$$  \hspace{1cm} (3.14)$$

To find this specific solution for $k_e$ and $R_m$ of the equivalent SDOF system in the blast design, the process is addressed as follows. Given the blast loading function (peak value $F_t$ and duration $t_d$), dual design targets ($y_e$ and $\mu_\tau$) and the mass ($m$) of the equivalent system, a recurrent procedure is needed to complete the design process. Firstly, $y_e$ is computed with Equation (3.12). By assuming an initial stiffness $k_e$, the corresponding $E_{\text{el}}$ and $R_m$ are then obtained with Equations (3.2) and (3.3) respectively while $T = 2\pi \sqrt{m/k_e}$. Next determine the parameters $F_t/R_m$ and $t_d/T$, with which the non-dimensional energy factor $C$ can be found from the energy spectra in Figure 3.4. $E_{\text{max}}$ is further obtained with Equation (3.1) however, determination of the quantity of $E_{\text{max}}$ will result in a new stiffness $k_{\text{el}}$. By replacing $k_{\text{ea}}$ in Equation (3.14) with $k_{\text{el}}$ and transforming Equation (3.14), $k_{\text{el}}$ is gained as

$$k_{\text{el}} = \frac{E_{\text{max}}}{y_e y_t - y_e^2 / 2}$$  \hspace{1cm} (3.15)$$

Thus make $k_e = k_{\text{el}}$ and repeat the above procedures until the convergent condition $|k_{\text{el}} - k_e| < \varepsilon$ is fulfilled after a few iterations (where $\varepsilon$ is an arbitrary small value). The required strength $R_m$ is finally obtained from the expression $R_m = k_e y_e$. 

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3.5 Design of Reinforced Concrete Member Using Energy Spectra

The structural member with continuous mass and stiffness is usually reduced to an equivalent elastic-perfectly-plastic SDOF system with the equivalent mass and stiffness [B1]. According to the above recurrent procedure, $k_e$ and $R_m$ for the equivalent system could be determined specifically with the energy spectra to achieve $y_i$ and $\mu_i$ under the given blast condition. After considering some of the particular characteristics of concrete and the embedded reinforcement, ways to compute the cross-sectional size and reinforcement ratio for the reinforced concrete member from $k_e$ and $R_m$ of the corresponding equivalent SDOF system are discussed herein.

3.5.1 Determination of cross-sectional size

After computing $k_e$ of the equivalent system with the energy spectra, the equivalent elastic stiffness $k_a$ for the reinforced concrete member could be described as [T2, M3, N4]

$$k_a = \frac{k_e}{K_{LE}}$$

(3.16)

where $K_{LE}$ is the load factor and listed in Table 3.1. For the members with various boundary conditions, $k_a$ is given by

$$k_a = \alpha \frac{E_c I}{l^3}$$

(3.17)

where $\alpha$ is the coefficient at different boundary conditions, which can be also obtained from Table 3.1, $E_c$ is the Young’s modulus of concrete, $l$ is the length, and $I$ is the cross-sectional moment of inertia of the reinforced concrete member.
### Table 3.1 Factors of one-way reinforced concrete members [B1, T2, M3, N4]

<table>
<thead>
<tr>
<th>Boundary conditions and loading diagrams</th>
<th>Mass Factor</th>
<th>Load Factor</th>
<th>Factor</th>
<th>Factor</th>
<th>Ultimate Shear force $V_u$</th>
<th>Support reactions $V_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_M$</td>
<td>$K_{ME}$</td>
<td>$K_L$</td>
<td>$K_{LE}$</td>
<td>$\alpha$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$q_u$ Elastic</td>
<td>0.50</td>
<td>0.42</td>
<td>Elastic 0.64</td>
<td>0.57</td>
<td>$q_u \left( \frac{l-d}{2} \right)$</td>
<td>$\frac{q_u}{2}$</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>Plastic 0.50</td>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_u$ Elastic plastic</td>
<td>0.45</td>
<td>0.40</td>
<td>Elastic plastic 0.64</td>
<td>0.56</td>
<td>$q_u \left( \frac{l-d}{8} \right)$</td>
<td>$\frac{5q_u}{8}$ Left $q_u \left( \frac{l-d}{8} \right)$</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>Plastic 0.50</td>
<td>0.50</td>
<td></td>
<td></td>
<td>$\frac{3q_u}{8}$ Right $q_u \left( \frac{l-d}{8} \right)$ Right $\frac{3q_u}{8}$</td>
</tr>
<tr>
<td>$q_u$ Elastic</td>
<td>0.41</td>
<td>0.39</td>
<td>Elastic plastic 0.64</td>
<td>0.54</td>
<td>$q_u \left( \frac{l-d}{2} \right)$</td>
<td>$\frac{q_u}{2}$</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>Plastic 0.50</td>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_u$ Elastic</td>
<td>0.26</td>
<td>0.30</td>
<td>Elastic 0.40</td>
<td>0.45</td>
<td>$q_u (l-d)$</td>
<td>$q_u l$</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>Plastic 0.50</td>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equivalent elastic stiffness for one-way members $k_u = \alpha \frac{EI}{l^3}$;

Ultimate moment $M_u = \beta q_u l^2$ (for $M_S = M_P$, where $M_S$, $M_P$ are moment capacity at the support and mid-span of one-way members);

$K_{ME} = \left( (K_{M\text{-elastic}} + K_{M\text{-elasto-plastic}}) \times 0.5 + K_{M\text{-plastic}} \right) / 2$ and $K_{LE} = \left( (K_{L\text{-elastic}} + K_{L\text{-elasto-plastic}}) \times 0.5 + K_{L\text{-plastic}} \right)^2$. 

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Generally, the moment of inertia of the cross section along reinforced concrete member is significantly influenced by the embedded reinforcement and the cracking propagation of the concrete. To deal with this problem, the moment of inertia could be modified by a reduction coefficient $\gamma$. For a rectangular cross section, as shown in Figure 3.9, it can be written as [T2, M3, N4]

$$I = \frac{I_g + I_c}{2} = \frac{1}{2} \left( \frac{b_w T_c^3}{12} + \frac{\gamma b_w d^3}{12} \right) \approx \frac{(\gamma + 1)b_w d^3}{24}$$  \hspace{1cm} (3.18)

Where $I$ is the average moment of inertia of reinforced concrete cross section, $I_g$ is the moment of inertia of the gross concrete cross section (neglecting all reinforcing steel), $I_c$ is the moment of inertia of the cracked concrete cross section, $T_c$, $d$ and $b_w$ are the total depth, the effective depth and the width of the cross section respectively. The parameter of $\gamma$ can be obtained from Figure 3.10 and it is observed that this coefficient varies with the reinforcement ratio $\rho$ as well as the modulus ratio ($E_s / E_c$).

![Figure 3.9 Reinforced concrete cross section with tension and compression reinforcement [T2, M3, N4]](image-url)
Figure 3.10 Reduction coefficient for moment of inertia of cracked section with different reinforcement layouts [T2, M3, N4]
Substituting Equation (3.18) into Equation (3.17) and then equating Equation (3.17) with Equation (3.16), the effective depth for the reinforced concrete member can be finally determined as

\[
d = \left( \frac{24l^3k_w}{(\gamma + 1)E_c b_w \alpha} \right)^{1/3} = \left( \frac{24l^3k_w}{(\gamma + 1)\alpha K_{LE} E_c b_w} \right)^{1/3}
\]

(3.19)

It should be pointed out that the parameter of \( b_w \) is not an independent variable within the design, which is generally taken as a unit for the design of a reinforced concrete wall and a constant ratio of \( d \) for the design of a beam [T2, M3, N4].

### 3.5.2 Determination of longitudinal reinforcement ratio

After establishing \( R_m \) of the equivalent SDOF system, the ultimate uniformly distributed force \( q_u \) applied to the reinforced concrete member is given as

\[
q_u = \frac{R_m}{K_{LE}}
\]

(3.20)

At various boundary conditions, the ultimate bending moment \( M_u \) of the reinforced concrete members when subjected to the uniformly distributed loadings can be described as

\[
M_u = \beta q_u l^2
\]

(3.21)

where \( \beta \) is the factor varying with boundary situations and listed in Table 3.1.

Determining the reinforcement ratio with the ultimate bending moment \( M_u \) in Equation (3.21) depends on the reinforcement layout in the cross section of the member. If only tension reinforcement is adopted, \( M_u \) of a rectangular cross section is given by [P1]
\[ M_u = A_s f_{ds} (d - c / 2) \]  

(3.22)

in which

\[ c = A_s f_{ds} / 0.85 b_w f_{dc} \]  

(3.23)

Where \( A_s \) is the cross-sectional area of the tensile reinforcement, \( c \) is the depth of the equivalent rectangular stress block. \( f_{ds} \) and \( f_{dc} \) are the dynamic yield strength for reinforcement and the dynamic ultimate compressive strength for concrete, written as

\[ f_{ds} = DIF \times f_s \quad \text{and} \quad f_{dc} = DIF \times f_c \]  

(3.24)

Where \( f_s \) and \( f_c \) are the static yield strength for reinforcement and the static compressive ultimate strength for concrete respectively, DIF is the dynamic increase factor, which depends upon the rate of strain of the member increasing as the strain rate increases [D4, F5, G4, G5, G6, G7, M1, M2]. For the design purpose, a constant average DIF has been established in TM5-1300 [T2], where DIF equal to 1.17 is used for reinforcement in bending and a DIF equal to 1.19 is used for concrete in compression. Since the derivation of \( A_s \) from Equation (3.24) is carried out without considering the concrete tensile effects, the concrete tensile DIF is not considered within the design although the actual bending of member due to the tensile DIF effect may be higher than the predicted value.

With Equations (3.20) ~ (3.24), the area of the tension reinforcement \( A_s \) could be determined from the ultimate strength \( R_m \) thus the tension reinforcement ratio \( \rho \) is

\[ \rho = A_s / b_w d \]  

(3.25)

Consideration is made of another major type of reinforcement layout in the cross section of member in the blast design as shown in Figure 3.9, where compression reinforcement and tension reinforcement are utilized simultaneously considering the rebound effect on the member subsequent to its maximum displacement response. If the concrete is assumed to be crushed and lose effectiveness in resisting moment, then the compression reinforcement area should be equal to the tension reinforcement area [T2]. In this case, the ultimate moment $M_u$ of a rectangular section is given by

$$M_u = A_s f_{ds} d_c = A_s f_{ds} (d - d')$$  \hspace{1cm} (3.26)

where $A_s$ is the cross sectional area of tension or compression reinforcement and $d_c$ is the distance between the centroid of the compression and the tension reinforcement, $d'$ is the distance from the extreme compression fiber to the centroid of compression reinforcement. Hence, with Equation (3.26) the tension or compression reinforcement ratio ($\rho$ or $\rho'$) of the section can be determined as

$$\rho = \rho' = \frac{M_u}{f_{ds} b_u d (d - d')}$$ \hspace{1cm} (3.27)

Substituting Equations (3.20) and (3.21) into Equation (3.27), the tension or compression reinforcement ratio utilized in the section is

$$\rho = \rho' = \frac{\beta R_m l}{K_{1E} f_{ds} b_u d (d - d')}$$ \hspace{1cm} (3.28)

To ensure proper structural behaviour under blast loadings, a minimum amount of longitudinal tension reinforcement ratio should be required given by [N4]

$$\rho_{\text{min}} = 1.38 f_s$$  \hspace{1cm} (3.29)
where \( f_y \) is the reinforcement yield strength in MPa. For the reinforcement with a yield strength of 460 MPa, the minimum reinforcement ratio is 0.30%.

### 3.5.3 Determination of shear reinforcement ratio

The brittle effect of shear failure should be strictly avoided in the blast design. Thus sufficient amounts of shear reinforcement should be provided. Generally the ultimate shear is developed when the resistance reaches the ultimate values \( q_u \) and hence the shear reinforcement is a function of the resistance of member.

There are two critical locations where shear force should be considered in the design of reinforced concrete members, as discussed in the reference [T2, M3]. It is pointed out that the ultimate shear force should be calculated at a distance \( d \) from the face of the support to check the diagonal tension stress and to provide shear reinforcement in the form of stirrups. Whenever the ultimate shear force exceeds the shear capacity of the concrete, shear reinforcement must be provided to carry the excess. The required shear reinforcement ratio is calculated from

\[
\rho_v = \frac{(V_u - V_c)}{\varphi f_{dy} b_w d}
\]  

(3.30)

Where \( V_u \) is the ultimate shear force, which can be obtained from Table 3.1, \( V_c \) is the shear capacity of the unreinforced concrete, \( \rho_v \) is the shear reinforcement ratio, \( \varphi \) is the capacity reduction factor, taken as 0.85, and \( f_{dy} \) is the dynamic yield strength of the stirrup. In order to ensure the full development of flexural reinforcement, the shear reinforcement ratio should not be less than 0.15% [N4].

On the other hand, the direct shear force or ultimate support shear force \( V_s \) should be checked at the supports, which is listed in Table 3.1. To avoid the direct shear failure at the supports, it was suggested by TM5-1300 [T2] that the magnitude of
the direct shear force $V_s$ at the support that can be resisted by the member, be limited to

$$V_d = 0.18 f_{de} b_w d$$  \hspace{1cm} (3.31)

The total support shear force $V_s$ produced by the applied loadings may not exceed $V_d$. If $V_s$ exceeds $V_d$, the cross-sectional size of the member should be increased.

### 3.6 Design Flowchart

As discussed previously, Equation (3.19) demonstrates that $d$ is a function of $\gamma$. Since $\gamma$ varies with $\rho$ as shown in Figure 3.10, $d$ should be calculated with $\rho$ known. On the other hand, Equation (3.28) indicates that $\rho$ is determined by $d$. Therefore, a recurrent computational procedure needs to be employed to determine $d$ and $\rho$ of reinforced concrete structural member against the given blast loading. Combining it with the previous recurrent procedure of determining $k_s$ and $R_m$ of the equivalent SDOF system for the given blast loading and the dual targets of $\gamma_t$ and $\mu$, the design flow chart depending on energy spectra to find $d$ and $\rho$ of the structural member is shown in Figure 3.11. In this developed design procedure, an initial data of $d_0$ and $\gamma_0$ are assumed so that the value of $E_{\text{max}}$ for the equivalent SDOF system can be obtained from energy spectra. However this $E_{\text{max}}$ will result in a new solution of $d_1$ and $\gamma_1$ and hence the process must be repeated until $d$ and $\gamma$ are consistent.

Figure 3.11 Flowchart of performance-based blast resistant design of reinforced concrete members using energy spectra (where $\epsilon_1$, $\epsilon_2$ are arbitrarily small values)

3.7 Advantages of the Design Approach Using Energy Spectra

Several advantages of the developed design approach using the energy spectra are stated as follows:

1. The values of $y_i$ and $\mu_i$, which always correlate with the expected performance levels of a reinforced concrete member in blast incidents, can be effectively incorporated into the design approach. Thus with this approach a comprehensive control of the responses of designed member under blast conditions in achieving the expected performance level can be efficiently and rationally accomplished.

2. Energy spectra are important tools in building up the recurrent procedures depending on which the blast resistant reinforced concrete member can be specifically designed in a simple way to reach the expected performance level defined by $y_i$ and $\mu_i$.

3. This design procedure could lead to the solution of $d$, $\rho$, $\rho'$ and $\rho_v$ of reinforced concrete member against blast loadings by controlling the responses of the corresponding equivalent SDOF system to meet the design targets of $y_i$ and $\mu_i$.

4. Current design recommendations are empirical and quite simplified. Although they usually help to produce safe structures, the safety margins are not well controlled. As for the developed design approach based on energy spectra, the member can be specifically designed using the recurrent procedure under the given blast loading to achieve the dual targets of $y_i$ and $\mu_i$, thus the safety margin of the member could be more effectively controlled in the design by giving conservative targets $y_i$ and $\mu_i$. Moreover since the solution of the design with this approach is unique, the difference of the responses of the designed member under the blast conditions that are
obtained from the explosive testing or numerical analysis with the corresponding design targets becomes analytical, which will be beneficial for the engineers in the reliability assessment of the design.

5. This design approach can also be utilized for the design of reinforced concrete members under other types of dynamic loadings, such as the rectangular load pulses, the constant force with finite rise time and the equilateral triangular load pulses. However their accomplishment depends on the corresponding energy spectra selected from Figures 3.5 to 3.7.

6. Application of the design approach is limited to the elastic-perfectly-plastic resistance functions of the equivalent SDOF system of RC member and simplified linear blast loading.

3.8 Summaries

A new approach for blast resistant design of reinforced concrete structural members is presented. It has been demonstrated that by controlling the responses of maximum displacement and displacement ductility factor for the equivalent SDOF system of reinforced concrete member to exactly achieve the expected performance level defined by both $y_t$ and $\mu_t$, the effective depth and longitudinal reinforcement ratio of the reinforced concrete member could be specifically determined for certain blast loading with this design approach. This solution is achieved through the recurrent computation on the basis of the energy spectra, which are constructed for an elastic-perfectly-plastic SDOF system.

The design approach developed in this chapter is more rational and straight-forward in incorporating $y_t$ and $\mu_t$ simultaneously in the design of reinforced concrete members against blast loadings, providing the equivalent SDOF system of the member can be adequately characterised by an elasto-plastic rule. It provides the design engineers an attractive alternative to genuine performance-based blast resistant design of reinforced concrete structural members when the current design
guidance is inadequate. This design procedure has been implemented in a computer program and used to investigate several practical design examples, as presented in Chapter 4.
CHAPTER FOUR

PERFORMANCE–BASED BLAST RESISTANT DESIGN OF
REINFORCED CONCRETE MEMBERS USING ENERGY
SPECTRA — PART II. EVALUATION

Synopsis

This chapter concentrates upon the evaluation of the effectiveness of the performance-based blast resistant design approach using energy spectra developed in Chapter 3. The demonstration of the approach into the practical design of a reinforced concrete member illustrates its effectiveness in implementation. By comparisons of different design examples designed with the existing guidance as well as the presented approach, the effectiveness in keeping the responses of the equivalent SDOF system of the designed member under control to achieve the expected performance level is evaluated. Due to the complicated behaviours of reinforced concrete members under blast conditions, some differences exist for the actual responses of the designed member with those of the corresponding equivalent SDOF system. Thus the effectiveness of the presented design approach in controlling the actual responses of the designed member under the given blast loading to meet the expected performance level is further evaluated by performing numerical analysis. The analytical results demonstrate the existence of some errors between the responses of the designed member and their respective design targets. The errors are produced due to some simplifications in converting the continuous reinforced concrete members into equivalent SDOF systems and computing the cross-sectional moment of inertia and ultimate strength of the members.
Keywords: Expected performance level, Target displacement, Target displacement ductility factor, Equivalent SDOF system, Energy spectra, Error, Displacement ductility responses, Maximum displacement responses

4.1 Introduction

With the disastrous effects of blast loadings on structural members, an effective control of the responses of the member to reach an expected performance level within the blast resistant design is of vital importance. For this reason, a new performance-based blast resistant design approach using energy spectra is presented in Chapter 3. This chapter will focus on the evaluations of the effectiveness of the design approach from three aspects. The implementation effectiveness is illustrated from a demonstration of this approach into the practical design of a reinforced concrete member. A more critical concern about the effectiveness of this approach lies in its fundamental notion that the expected performance of structural members under blast loadings should be achieved by incorporating the dual design targets of displacement $y_t$ and displacement ductility factor $\mu_t$ into the design and controlling the maximum displacement and displacement ductility responses of the equivalent SDOF system of the designed member to reach these dual targets exactly. This will be discussed in detail in this chapter through the comparisons of different practical design examples designed using the existing guidance and the presented approach.

The design approach using energy spectra is accomplished by converting a continuous reinforced concrete member into an equivalent SDOF system. To reach the performance level defined by the dual targets of $y_t$ and $\mu_t$ for the given blast loading, the initial stiffness ($k_e$) and ultimate strength ($R_m$) of the equivalent SDOF system are computed, based on which the effective depth ($d$) and longitudinal reinforcement ratio ($\rho$) of the reinforced concrete member are further derived. Theoretically, the maximum displacement and displacement ductility responses for the equivalent SDOF system ($y_m^\text{eq}$ and $\mu_m^\text{eq}$) can be perfectly controlled to be equal.
to the respective targets of $y_t$ and $\mu_t$ with this design approach. However due to the complexities of the behaviours of the reinforced concrete member under blast conditions, the actual dynamic responses including maximum displacement ($y_{m}^{rc}$) and displacement ductility factor ($\mu_{m}^{rc}$) at the significant point for the designed member under the given blast loading should be examined and compared with $y_t$ and $\mu_t$. Thus the latter part of this chapter will be concentrated on the evaluation of the effectiveness of the approach in controlling the actual responses of the designed member. Basically, $y_{m}^{rc}$ and $\mu_{m}^{rc}$ should be measured through the explosion tests of the designed members under blast conditions. However, due to considerable costs involved in explosion tests and the difficulties in obtaining reliable data, performing numerical studies to derive $y_{m}^{rc}$ and $\mu_{m}^{rc}$ becomes an attractive option. In this chapter, nonlinear finite element analysis is performed with the program ABAQUS [A1] on the designed reinforced concrete members under the given blast loadings. Comparisons indicate some errors of $y_{m}^{rc}$ and $\mu_{m}^{rc}$ with the respective design targets of $y_t$ and $\mu_t$. The possible reasons for producing such errors are identified.

4.2 Implementation of the Design Approach

To evaluate the effectiveness of the presented design approach in implementation, a demonstration of this approach is given by involving a practical design of a reinforced concrete wall against blast loading, which was originally presented by Mays and Smith [M3]. This reinforced concrete wall to be designed resists the blast loading perpendicular to its plane that is simplified into a triangular pulse with the peak pressure and duration shown in Figure 4.1. The design gives a target support rotation ($\theta_t$) of $4^\circ$, thus $y_t$ of the wall at the free end under the given blast condition can be obtained with an approximate expression: $y_t \approx l \tan(\theta_t)$. The compression reinforcement area is taken to be equal to the tension reinforcement area considering the rebound effect of the designed member subsequent to its maximum displacement response and the possibility of the explosion occurring in the opposite side of the wall to be designed.
Figure 4.1 Sketch of a reinforced concrete wall

To perform the design with the developed approach, a reasonable $\mu_t$ should be given. For a cantilever member, the displacement ductility factor ($\mu$) is closely related to the curvature ductility factor of the fixed-end section ($\varphi_u / \varphi_y$). Their relationship can be derived based on three assumptions [P1]. Firstly, the fixed-end sectional moment-curvature behaviour for the reinforced concrete member can be idealized to be elastic perfectly plastic. Secondly, the full length of the member has a contribution to the elastic displacement $y_e$ thus $y_e = \int_0^l \phi \, dx$. Thirdly, the plastic hinge rotation $(\varphi_u - \varphi_y) l_p$ occurring in the vicinity of the fixed-end section performs the whole contribution to the plastic displacement of the free end of the cantilever member. $\varphi_u$ and $\varphi_y$ are the ultimate and yield curvatures of the fixed-end section and $l_p$ is the equivalent plastic hinge length, which is approximately equal to $d$. Based on these above assumptions, $\mu$ for a cantilever member under the uniformly distributed force is given as [P1]

$$\mu = 1 + 4 \left( \frac{\varphi_u}{\varphi_y} - 1 \right) \left( \frac{d}{l} - \frac{d^2}{2 l^2} \right) \quad (4.1)$$
Generally, the ratio of \( d/l \) for the wall is located in the range of 1/10 to 1/20. In this design, the upper value of 1/10 for \( d/l \) is chosen for the cantilever wall considering the severe blast loadings on it. Given a target \( \varphi_u / \varphi_y \) for the fixed-end section, \( \mu_i \) can be calculated with Equation (4.1). Since the given \( \theta_i = 4^\circ \) corresponds to a state in which the member is close to collapse, a large target \( \varphi_u / \varphi_y = 20 \) is selected. Thus \( \mu_i \) in this design case is approximately taken as 9.

The design procedures at each iterative step for the given \( y_i \) and \( \mu_i \) are illustrated in Table 4.1. During the design, the value of the initial effective depth \( d_o \) is taken as 3.0 m and the initial reduction coefficient \( \gamma_0 (1.0) \) is assigned as \( \rho_0 = 3.0\% \). The convergence condition, the difference between two iterative design results less than 0.001, is employed. In addition, to check whether \( y_i \) and \( \mu_i \) is effectively achieved during the design, \( y_{meq} \) and \( \mu_{meq} \) are computed and compared with \( y_i \) and \( \mu_i \).

It can be seen from Table 4.1 that this approach is insensitive to the initial data of \( d_o \) and \( \gamma_0 \) and there is no difficulty in reaching convergence for the design. In this design case, even if an unreasonable \( d_o \) and \( \gamma_0 \) are selected, it needs only seven iterative steps to satisfy \( |(d_i - d_o)/d_o| < 0.001 \) and \( |(\gamma_1 - \gamma_0)/\gamma_0| < 0.001 \). Little time is consumed during the computation and for this design case it costs only 0.12 s using a computer program written with Matlab language [M13]. Furthermore, \( y_{meq} = y_i \) and \( \mu_{meq} = \mu_i \) can be observed, which indicates that the design could perfectly meet its dual targets of \( y_i \) and \( \mu_i \) for the equivalent SDOF system of the designed member (see Figure 4.8). Thus it can be concluded that the presented blast design approach is not only efficient in implementation, but also accurate in controlling \( y_{meq} = y_i \) and \( \mu_{meq} = \mu_i \).
Table 4.1 Iterative procedures of the design approach using energy spectra for a cantilever wall at each step

| $y_i$ | $\mu_i$ | $\gamma_0$ | $\rho_0$ | $d_0$ | $m$ | $I$ | $k_e$ | $T$ | $R_n$ | $T/t_d$ | $F_t/R_n$ | $E_d$ | $E_{max}$ | $k_{el}$ | $q_a$ | $M_n$ | $d_1$ | $\rho_1$ | $\gamma_1$ | $y_{m}^{eq}$ | $\mu^{eq}$ |
|-------|---------|-------|-------|------|-----|-----|------|------|------|---------|------------|------|--------|------|-----|-----|------|-----|-----|---------|-----------|---------|
| 1.0000 | 3.000 | 2.0000 | 3979 | 0.667 | 2.21e9 | 8.43e-3 | 5.16e7 | 0.5731 | 0.048 | 6.02e5 | 3.82e-3 | 2.30e3 | 4.97e5 | 4.30e7 | 1.936e8 | 0.1216 | 9.858 | 1.0000 |
| 1.0000 | 9.858 | 0.1216 | 242 | 1.498e4 | 4.97e5 | 1.39e-1 | 1.16e4 | 0.0348 | 215.1 | 135.38 | 2889.5 | 3.91e5 | 8.45e7 | 9.67e3 | 4.351e4 | 0.6736 | 1.025 | 0.7494 |
| 0.7494 | 1.025 | 0.6736 | 1340 | 1.909e-2 | 6.33e7 | 2.89e-2 | 1.48e6 | 0.1671 | 1.689 | 1724.4 | 0.742 | 1.28e4 | 2.76e6 | 1.23e6 | 5.542e6 | 0.2371 | 2.644 | 1.0000 |
| 0.21 | 9.0000 | 2.644 | 0.2371 | 472 | 1.111e-3 | 3.69e6 | 7.11e-2 | 8.60e4 | 0.0679 | 29.01 | 1003.7 | 38.04 | 3.82e4 | 8.25e6 | 7.17e4 | 3.226e5 | 0.3102 | 1.486 | 0.8617 | 0.21 |
| 0.8617 | 1.486 | 0.3102 | 617 | 2.142e-3 | 7.11e6 | 5.85e-2 | 1.66e5 | 0.0825 | 15.05 | 1935.3 | 14.49 | 2.90e4 | 6.27e6 | 1.38e5 | 6.221e5 | 0.2974 | 1.566 | 0.8811 |
| 0.8811 | 1.566 | 0.2974 | 591 | 1.931e-3 | 6.41e6 | 6.04e-2 | 1.49e5 | 0.0800 | 16.69 | 1744.4 | 17.36 | 3.02e4 | 6.54e6 | 1.24e5 | 5.607e5 | 0.2995 | 1.550 | 0.8772 |
| 0.8772 | 1.550 | 0.2995 | 595 | 1.963e-3 | 6.51e6 | 6.01e-2 | 1.52e5 | 0.0804 | 16.42 | 1773.4 | 16.95 | 3.01e4 | 6.50e6 | 1.26e5 | 5.700e5 | 0.2992 | 1.551 | 0.8775 |

Design results: $d = 0.2992$ m; $\rho$ or $\rho' = 1.551\%$; $\rho_s = 0.15\%$;

**Parameters:** $P_r = 2080$ kPa, $t_d = 4.83$ ms, Density of concrete $\rho_{con} = 2500$ kg/m$^3$,

$E_s = 2000$ GPa, $E_c = 280$ GPa, $f_s = 460$ MPA, $f_a = 506$ MPA, $f_s = 33$ MPA, $f_a = 40$ MPA, $f_s = 250$ MPA, $f_a = 275$ MPA

$\alpha = 8$, $\beta = 1/2$, $K_{LE} = 0.45$, $K_{ME} = 0.3$ (for elements with one end fixed and the other free in Table 3.1)

**Equations:** $y_c = y_i / \mu_i$, $m_s = \rho_{con} I b_s d_0$, $m = K_{ME} m_s$, $I = (y_i + 1)b_s d_0^3 / 24$, $k_e = k_K E_i I / l^3$, $k_e = K_{LE} k_a$, $T = 2\pi \sqrt{m I k_e}$, $F_a = P b l$, $R_n = k_a y_c$, $E_d = k_e y_c^2 / 2$, $E_{max} = C E_d$

($C$ obtained from energy spectra), $k_{el} = E_{max} / (y_i y_c - y_c^2 / 2)$, $R_{st} = k_s y_c$, $q_a = R_a / K_{LE}$, $M_n = \beta q_a F_a$ (for $\rho = \rho'$), $d_1 = \left(24 k_a E_d / (y_i + 1) \alpha K_{LE} E_d b_s \right)^{1/3}$, $\rho = \rho' = \frac{M_n}{f_a b_s^2 d (d - d')}$

**Convergence conditions:** $|d_1 - d_0| / d_0 < 0.001$ and $|\gamma_i - \gamma_0| / \gamma_i < 0.001$
4.3 Effectiveness of the Design Approach in Achieving Expected Performance Level

Whether the responses of structural member can be effectively controlled to reach an expected performance in the blast design greatly depends on the selection of the proper response parameter as the performance indicator. Currently the displacement is chosen for the blast resistant design of reinforced concrete members in TM5-1300 [T2] and Mays and Smith [M3] while the displacement ductility factor is employed in TM5-855-1 [T1], Biggs [B1], and ASCE Manual 42 [A2]. However, in Chapter 3, it is pointed out that the utilization of either \( y \) or \( \mu \) alone is not adequate in controlling the responses of the member reaching its expected performance level, and thus when carrying out an effective performance-based blast design it is better to incorporate both of them. This conclusion is mainly drawn from a brief conceptual analysis in Chapter 3; here to prove it in detail, three practical design examples with different boundary conditions documented in the current design guidance [T2, M3, B1] are critically examined and evaluated. These three design examples are further implemented with the design approach using energy spectra and thus to evaluate the effectiveness of the approach.

Design Example I (Mays and Smith)

This design example given in the book of Mays and Smith [M3], as shown in Figure 4.1, demonstrates the design of a cantilever wall loaded by a blast with their design procedure. In the design the expected performance of the designed wall under the blast condition was specified by \( y = 210 \text{ mm (}\theta = 4^\circ) \). Since the blast loading only lasted for a relatively short period of time (\( t_o = 4.83\text{ms} \)), it was assumed by them that the blast force was located in the impulsive state. Thus

\[
\frac{i^2}{2m} = R_m(y - \frac{1}{2}y_c)
\]  

(4.2)
According to Mays and Smith’s design, for a unit width of the cantilever wall and by specifying $\rho = \rho' = 0.5\%$, the value of $d$ was solved with Equation (4.2), where the parameters of $R_m$, $y_c$, and $m$ for the equivalent SDOF system could be expressed with $d$. After that, the support reaction force was computed to find $\rho_v$. Mays and Smith’s results are listed in case A1 in Table 4.2.

**Design Example II (TM5-1300)**

The analysis is further extended to the second typical design example given in TM5-1300 [T2]. This example intended to design an interior beam of a roof subjected to an overhead blast load. Structural configuration and the blast loading to be resisted are shown in Figure 4.2. The objective performance of this beam in the blast design was specified by $y_i = 53$ mm ($\theta_i = 1^\circ$). The same interior beam except for the utilization of different material properties was also introduced in the design manual 2.08 (DM-2.08) by NFEC [N4].
In TM5-1300 and DM-2.08 cases, three assumptions were made during the design process. Firstly, this particular beam was continuously connected to the adjacent floors thus the mass ($m$) necessary in computing $T$ of the equivalent SDOF system of the designed beam, included the mass of the beam itself plus 20% of the slabs perpendicular to that beam. Secondly, the force applied to 50% of the adjacent floors would transfer to this interior beam. The third assumption was that the stiffness and strength contribution from adjacent floors was neglected. After a preliminary selection of a cross-sectional dimension and $\rho$ by designer’s experience, $y_m^{eq}$ was examined with $y_f$. If it exceeded the target $y_f$, the cross section was modified and the above process was repeated. The results are listed in cases B1 and B2 in Table 4.2.
Design Example III (Biggs)

The third design example given in the book of Biggs [B1] is accomplished through limiting the displacement ductility response of the member to be less than the target. As shown in Figure 4.3, a simply supported reinforced concrete beam with a span of 4.5 m was considered which had been subjected to a uniformly distributed blast loading with the magnitude and time function shown. In addition to the weight of the beam itself, there was an attached dead weight of 1.75 kN/m. In this design the expected performance of the beam in the blast conditions was stated by $\mu_r = 3$.

Since the blast loading lasted for a very long time (150 ms) in this example, then

$$R_m = F_i \frac{1}{1 - 1/2 \mu_r}$$

(4.3)
Equation (4.3) was used to obtain the first trial value of ultimate strength \( R_m \), based on which the primary selection of the cross section was finished by fixing \( \rho \) to be 1.5%. After converting this selected member into the equivalent SDOF system, \( \mu^{eq} \) was examined with \( \mu_i \). Biggs’s design results are listed in case C1 in Table 4.2. It can be seen that \( \mu^{eq} \) was a little larger than \( \mu_i \) thus it was recommended to increase the dimension of the beam cross section by about 10%.

<table>
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<tr>
<th>Case</th>
<th>( l ) (mm)</th>
<th>( P_r ) (kPa)</th>
<th>( t_d ) (ms)</th>
<th>( y_i ) (mm)</th>
<th>( f_{dc} ) (Mpa)</th>
<th>Specified ( \rho ) (%)</th>
<th>( y_c^{eq} ) (mm)</th>
<th>( y_m^{eq} ) (mm)</th>
<th>( \mu_{cd} )</th>
<th>( \mu_{cc} )</th>
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Design example II: TM5-1300’s fixed supported beam*

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<th>( P_r ) (kPa)</th>
<th>( t_d ) (ms)</th>
<th>( y_i ) (mm)</th>
<th>( f_{dc} ) (Mpa)</th>
<th>Specified ( \rho ) (%)</th>
<th>( y_c^{eq} ) (mm)</th>
<th>( y_m^{eq} ) (mm)</th>
<th>( \mu_{cd} )</th>
<th>( \mu_{cc} )</th>
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<tr>
<td>B1</td>
<td>6000</td>
<td>49.7</td>
<td>60.7</td>
<td>53</td>
<td>532</td>
<td>32.8</td>
<td>0.44</td>
<td>712</td>
<td>0.24</td>
<td>3.64</td>
</tr>
<tr>
<td>B2</td>
<td>6000</td>
<td>49.7</td>
<td>60.7</td>
<td>53</td>
<td>455</td>
<td>34.5</td>
<td>0.44</td>
<td>712</td>
<td>0.22</td>
<td>3.14</td>
</tr>
</tbody>
</table>

Design example III: Biggs’s simply supported beam

<table>
<thead>
<tr>
<th>Case</th>
<th>( l ) (mm)</th>
<th>( P_r ) (kPa)</th>
<th>( t_d ) (ms)</th>
<th>( y_i ) (mm)</th>
<th>( f_{dc} ) (Mpa)</th>
<th>Specified ( \rho ) (%)</th>
<th>( y_c^{eq} ) (mm)</th>
<th>( y_m^{eq} ) (mm)</th>
<th>( \mu_{cd} )</th>
<th>( \mu_{cc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>4500</td>
<td>291.8</td>
<td>150</td>
<td>3</td>
<td>431</td>
<td>34.5</td>
<td>1.5</td>
<td>686</td>
<td>8.5</td>
<td>3.7</td>
</tr>
<tr>
<td>C2</td>
<td>6000</td>
<td>291.8</td>
<td>150</td>
<td>3</td>
<td>575</td>
<td>34.5</td>
<td>2.2</td>
<td>550</td>
<td>33.3</td>
<td>3</td>
</tr>
</tbody>
</table>

* B1: TM5-1300; B2: DM-2.08

In TM5-1300 [T2], DM-2.08 [N4] and Mays and Smith [M3] design procedures, the expected performance of reinforced concrete members is assumed to be fulfilled by a given \( y_i \) and with the response of \( y_m^{eq} \leq y_i \). However the control of \( \mu^{eq} \) is not executed clearly in these design procedures. This may lead to such designs tending toward the unsafe side, which is illustrated by design case A2. This case was carried out with Mays and Smith’s design procedure on the cantilever wall with the same pre-assumed \( \rho \) and design target \( \theta \), but under a more severe blast loading. A
comparison of cases A2 with A1 indicates that $d$ of the designed member has an obvious increase of 435 mm, which in turn diminishes the elastic displacement $y_{eq}^e$ of the member from 23 mm to 11.1 mm and as a consequence $\mu_{eq}$ increases from 9 to 18.9. With Equation (4.1), the curvature ductility demand ($\mu_{cd} = \phi_u / \phi_y$) of the fixed-end section in design case A2 reaches 19.7. By comparing $\mu_{cd}$ with $\mu_{cc}$, the curvature ductility capacity obtained from the section analysis, it is clear that design case A2 is not adequate in resisting the blast force since $\mu_{cc}$ for this designed member has only 15.8 to be less than $\mu_{cd}$. This phenomenon also appears in design case B2. Obviously the unsafe design in these cases demonstrates that using $y_i$ alone could not provide an effective design to fulfil the expected performance. Actually such a non-effective design of the members is due to the unreasonably pre-assumed $\rho$. As a relatively small $\rho$ is specified in the early design stage, $d$ has to be enlarged to meet $y_i$, which will cause an excessive $\mu_{eq}$. This action is apparently reflected in design case A3, where the design of Mays and Smith’s cantilever wall is repeated but with a relatively small $\rho = 0.31\%$. Certainly the increase in $\rho$ will to some extent release the non-effectiveness of the design, however the problem is that a reasonable $\rho$ is not evident at the start of the design process.

In TM5-855-1 [T1], Biggs [B1], and ASCE Manual 42 [A2] design guidance, the expected performance of reinforced concrete members in the blast design is assumed to be fulfilled by a given $\mu_i$ and with the response of $\mu_{eq} \leq \mu_i$. However in these procedures, the corresponding maximum displacement $y_{eq}^m$ is not restricted explicitly and in such situations the expected performance of designed members could not also be satisfied effectively. It can be illustrated from design case C2, where a comparatively small $\mu_i = 3$ is employed to control the member’s responses at a relatively low damage level, however in this case $y_{eq}^m$ has unexpectedly reached 100 mm (support rotation of the member about $2^\circ$). Assuming that design case C2 is performed for a column, this unexpected large $y_{eq}^m$ equivalent to a lateral drift of
1/30 at the mid-height could produce a significant secondary effect, which will cause more damage or even the collapse of the member.

The above detailed comparisons indicate that to control the member’s responses in reaching their expected performance level, it is necessary to incorporate both \( y_i \) and \( \mu_i \) into the blast design and satisfy \( y_i^{eq} \leq y_i \) and \( \mu_i^{eq} \leq \mu_i \). Although \( y_i^{eq} < y_i \) and \( \mu_i^{eq} < \mu_i \) will tend the design toward the safe side, the safety margin could still not be controlled effectively. And when considering technical efficiency and cost, they often lead to the over-design of members, which is not desirable for the clients. Thus it is better to control \( y_i^{eq} = y_i \) and \( \mu_i^{eq} = \mu_i \) with a relatively conservative \( y_i \) and \( \mu_i \) to provide the necessary safety margin. The previous demonstration of the design approach involving the practical design case in Table 4.1 indicates that \( y_i \) and \( \mu_i \) could be achieved exactly since in this case \( y_i^{eq} = y_i \) and \( \mu_i^{eq} = \mu_i \). Thus the responses of the structural members against blast forces can be effectively controlled to reach the target performance in the blast design with the approach based on energy spectra. To further explain this, the above three design examples are again executed with the developed approach as follows.

Combining the different targets of \( y_i \) and \( \mu_i \) documented in the existing blast design guidance [T1, T2, B1, B4, N4, M3], four performance levels are defined according to the structural nature, function and importance as shown in Table 4.3 [S6]. It describes the damages associated with various levels of performance and suggests the corresponding \( \mu_i \) and \( D_i \) for one-way reinforced concrete members, where \( D_i = l/y_i \), the ratio of member length to target displacement. According to Table 4.3, the previous three design examples are implemented with the presented approach and the design results are listed in Table 4.4. The results indicate that \( y_i^{eq} = y_i \) and \( \mu_i^{eq} = \mu_i \) for all design cases thus the expected performance can be effectively fulfilled. Moreover the design approach overcomes the subjectivity involved in the existing design guidance where \( \rho \) is specified in somewhat arbitrary way. The implementation of this design approach into these examples also indicates
that this approach could account for arbitrary blast conditions.

Table 4.3 Performance levels for one-way reinforced concrete member defined by the combination of target displacement and displacement ductility factor [S6]

<table>
<thead>
<tr>
<th>Performance level</th>
<th>Protection level</th>
<th>Description of damage</th>
<th>$\mu_t$</th>
<th>$D_t$,*</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>High</td>
<td>Superficial</td>
<td>1</td>
<td>240</td>
</tr>
<tr>
<td>B</td>
<td>Medium</td>
<td>Repairable</td>
<td>3</td>
<td>120</td>
</tr>
<tr>
<td>C</td>
<td>Low</td>
<td>Some elements unrepairable</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>D</td>
<td>Very low</td>
<td>Overall structure unrepairable</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

*Multiply $D_t$ by 0.5 for fixed/free (cantilever) members

Table 4.4 Design results for the members at various performance levels with the design approach using energy spectra

<table>
<thead>
<tr>
<th>Performance Level</th>
<th>$\mu_t$</th>
<th>$y_t$ (mm)</th>
<th>$d$ (mm)</th>
<th>$\rho$ or $\rho'$</th>
<th>$\rho_v$</th>
<th>$\mu_{eq}$</th>
<th>$y_m^{eq}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design example I: May and Smith’s cantilever wall ($l=3000mm$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>25</td>
<td>563</td>
<td>3.21%</td>
<td>1.28%</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>50</td>
<td>482</td>
<td>1.59%</td>
<td>0.36%</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>100</td>
<td>411</td>
<td>1.21%</td>
<td>0.15%</td>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>200</td>
<td>328</td>
<td>1.19%</td>
<td>0.15%</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>Design example II: TM5-1300’s fixed support beam ($l=6000mm$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>25</td>
<td>431</td>
<td>2.93%</td>
<td>2.26%</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>50</td>
<td>414</td>
<td>1.75%</td>
<td>1.09%</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>100</td>
<td>382</td>
<td>1.61%</td>
<td>0.86%</td>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>200</td>
<td>324</td>
<td>1.79%</td>
<td>0.75%</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>Design example III: Biggs’s simply support beam ($l=4500mm$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>18.75</td>
<td>556</td>
<td>3.23%</td>
<td>1.34%</td>
<td>1</td>
<td>18.75</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>37.5</td>
<td>544</td>
<td>1.91%</td>
<td>0.63%</td>
<td>3</td>
<td>37.5</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>75</td>
<td>515</td>
<td>1.79%</td>
<td>0.53%</td>
<td>6</td>
<td>75</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>150</td>
<td>453</td>
<td>2.03%</td>
<td>0.52%</td>
<td>10</td>
<td>150</td>
</tr>
</tbody>
</table>
Chapter 4. Performance-Based Blast Resistant Design of Members Using Energy Spectra — Part II: Evaluation

4.4 Numerical Verification of the Design Approach

Theoretically, the expected performance of reinforced concrete member in certain blast condition could be fulfilled with this approach since \( y_{eq} = y_t \) and \( \mu_{eq} = \mu_t \) are achieved simultaneously in the design. However, it should be noted that \( y_{eq} \) and \( \mu_{eq} \) are obtained from the equivalent SDOF system of designed reinforced concrete member against the blast loading. Due to the complicated behaviours of continuous reinforced concrete members, some differences between the responses of equivalent SDOF system \( (y_{eq}, \mu_{eq}) \) with those of the designed member \( (y_{eq}^{rc}, \mu_{eq}^{rc}) \) under the given blast loading are conceivable and therefore will cause differences between \( y_{eq}^{rc} \) and \( \mu_{eq}^{rc} \) and the corresponding targets of \( y_t \) and \( \mu_t \). To evaluate the effectiveness of the presented approach in controlling the responses of designed member to achieve the expected performance level, \( y_{eq}^{rc} \) and \( \mu_{eq}^{rc} \) of the member under the given blast loading are computed with the program ABAQUS [A1] and compared with their respective targets \( (y_t, \mu_t) \).

4.4.1 Finite element models

The detailed finite element models can be found in Chapter 2. The concrete model, which uses concepts of smeared cracking and isotropic compressive plasticity to represent the inelastic behaviour of concrete, is employed to represent the concrete properties. Reinforcement is simulated by the Von-Mises yield criterion with associated flow and isotropic strain hardening. Furthermore, Timoshenko beam elements are assigned to model the members while the rebar option is utilized to place each reinforcement at its exact location. To ensure the computational stability, the minimum and the maximum allowable time increment necessary in the automatic incrementation are taken as \( 1 \times 10^{-7} \) s and 0.01 s, respectively. The finite element models have been effectively verified from a simply supported beam and a square slab subjected to blast loadings tested by Sebold [S3] and Keenan [K3] where numerically predicted responses have a good match with experimental ones.
Chapter 4. Performance-Based Blast Resistant Design of Members Using Energy Spectra — Part II: Evaluation

It has been pointed out by Biggs [B1] and by Krauthammer [K4] that the damping effect could be properly neglected in the dynamic analysis of structural responses under blast conditions. In addition, the aim of dynamic analysis herein is to evaluate the effectiveness of the design approach using energy spectra. Since damping has been ignored in the design, it is more reasonable to neglect the damping effect in this dynamic analysis.

4.4.2 Results and discussion

With the finite element models above, the nonlinear dynamic analysis is performed on the designed cantilever wall shown in Table 4.1 under the given blast loading condition. Free-end responses of the wall in terms of displacement versus time are illustrated in Figure 4.4. It can be seen that \( y_m^{rc} \) is slightly less than \( y_t \) with a deviation of about 10% of \( y_t \). Note that \( y_m^{rc} \) is computed from subtracting \( y_m^{rc} \) by the permanent set \( (y_m^{rc} - y_e^{rc}) \), which is equal to the neutral position for the axis of residue vibration. \( \mu^{rc} \) for the designed walls under given blast conditions is determined by \( y_m^{rc} / y_e^{rc} \), which is 5.18 to be less than \( \mu_t = 9 \). The deviation between \( \mu^{rc} \) and \( \mu_t \) is about 42.4% of \( \mu_t \). Thus it seems that the error between \( \mu^{rc} \) and \( \mu_t \) is larger than that between \( y_m^{rc} \) and \( y_t \).

The dynamic response simulations of designed members in Table 4.4, when subjected to the respective blast loadings are also performed and the results are shown in Figures 4.5 to 4.7. It can be observed from these graphs that there are different levels of errors according to the boundary conditions. For the fixed/free members in design example I, their \( y_m^{rc} \) at the free end under the given blast loadings is better controlled since the error between \( y_m^{rc} \) and \( y_t \) is slight and unaffected by the various levels of performance. However, for the fixed/fixed members in design example II, with the increase of \( y_t \) and \( \mu_t \), it seems that the design produces more conservative error between \( y_m^{rc} \) and \( y_t \).
Figure 4.4 Deflection history of the cantilever wall designed in Table 4.1 under the given blast loading.

Values of $\mu^c$ for designed members at various levels of performance in design examples I, II and III for blast loadings are also displayed in Figures 4.5 to 4.7. A comparison of $\mu^c$ with $\mu_i$ demonstrates that $\mu^c$ is generally less than $\mu_i$ except for $\mu_i=1$. Actually under blast conditions, concrete cracking will inevitably appear which leads to a slight plastic set of the designed member even if the design requires the member in the elastic limit state ($\mu_i=1$). But such a slight non-conservative error, which does not exceed 4% of $\mu_i$ in all cases, is not significant since a very stringent responses target ($\mu_i=1$) is required in the design. With an increase of $\mu_i$, the design becomes more conservative.
Figure 4.5 Deflection history of the designed cantilever walls under the given blast loading (Example I)
Chapter 4. Performance-Based Blast Resistant Design of Members Using Energy Spectra — Part II: Evaluation

(c). At performance level C

(d). At performance level D

Figure 4.5 (Continued)
(a). At performance level A

(b). At performance level B

Figure 4.6 Deflection history of the designed fixed-supported beams under the given blast loading (Example II)
Chapter 4. Performance-Based Blast Resistant Design of Members Using Energy Spectra — Part II: Evaluation

Figure 4.6 (Continued)

(c). At performance level C

(d). At performance level D

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Figure 4.7 Deflection history of the designed simply supported beams under the given blast loading (Example III)
Chapter 4. Performance-Based Blast Resistant Design of Members Using Energy Spectra — Part II: Evaluation

(c). At performance level C

(d). At performance level D

Figure 4.7 (Continued)
From the above analysis, it can be summarized that some errors exist between the responses \((y_{rc}^e, \mu_{rc}^e)\) of the designed reinforced concrete member and their respective target values \((y_t, \mu_t)\). The reasons for producing such errors are mainly due to the simplifications adopted in the blast design. Basically, the design approach using energy spectra includes two stages. The first stage involves the computation of \(k_e\) and \(R_m\) of the equivalent SDOF system for \(y_t, \mu_t\) and the given blast condition. After that, \(d\) and \(\rho\) of reinforced concrete member are derived from \(k_e\) and \(R_m\) according to the particular characteristics of concrete and reinforcement embedded. To find at which stage the errors are produced, a comparison of the dynamic responses of the equivalent SDOF system having \(k_e\) and \(R_m\) with those of designed reinforced concrete member having \(d\) and \(\rho\) under the given blast condition is performed on the cantilever wall in Table 4.1. The results are shown in Figure 4.8. It can be observed that \(y_{m}^e\) and \(\mu_{m}^e\) of the equivalent SDOF system meet their targets exactly while for reinforced concrete member, their \(y_{m}^{rc}\) and \(\mu_{m}^{rc}\) are less than the corresponding targets. This indicates that the errors are produced in the derivation of \(d\) and \(\rho\) from \(k_e\) and \(R_m\) in the design procedure.
Several points, which are involved in the derivation of $d$ and $\rho$ from $k_c$ and $R_m$ account for the errors between the responses ($y^{rc}_c$ and $\mu^{rc}$) and their respective target values ($y$ and $\mu_t$) as follows:

1. In converting the continuous reinforced concrete member into the equivalent elastic-perfectly-plastic SDOF system, the load and mass factors ($K_{LE}$ and $K_{ME}$) are necessary, but their values are obtained in an approximate way. That is because the resistance of reinforced concrete member is a continuous process, however, in order to simplify the problem, it is represented as two or three independent linear stages. Thus the utilization of $K_{LE}$ and $K_{ME}$ in the design approach will produce some errors between the responses of the designed member and design targets.
2. The second reason for errors exists in the determination of average moment of inertia \( I \) of the cross section of reinforced concrete members. It is well known that the embedded reinforcement and the cracking propagation of the concrete have much effect on the value of \( I \). To simplify this problem in the design, \( I \) obtained from the expression of \((\gamma + 1)b_n d^3/24\) is used to calculate the member stiffness and the corresponding deformation. However the adoption of \( \gamma \) which is dependant upon the experimental data fitting will incur some errors in the design of reinforced concrete members.

3. Although \( R_m \) of the equivalent SDOF system for the reinforced concrete member is known, it is still difficult to determine \( \rho \) accurately, with which the ultimate strength of the designed member equates to the anticipated value \( (R_m) \) exactly. In these design examples, using \( \rho = \rho' = \frac{M_u}{f_u b_n d (d - d')} \) to calculate the equal tension and compression reinforcement ratio is conservative since it is assumed that concrete in the compression zone provides no contribution in the ultimate strength of the reinforced concrete member. Thus the reinforcement ratio tends to be enlarged in the design, which causes some errors in fulfilling the design targets.

4.5 Summaries

The effectiveness of the performance-based blast resistant design approach using energy spectra has been evaluated in this chapter. From the demonstration of the design approach, it is found that it is efficient and effective to apply this approach into the practical design of reinforced concrete members against blast loadings without the difficulty in convergence.

The design approach presented in Chapter 3 is effective in incorporating the performance level defined by both \( y \) and \( \mu \) into the design and controlling the responses of \( y_{eq} \) and \( \mu_{eq} \) to meet the respective target values exactly. Comparisons
of this design procedure with those existing design guidance indicate that this approach can overcome the disadvantages existing in the existing design guidance in terms of the effective control of the responses of the equivalent SDOF system of the designed member to achieve the expected performance level without any subjectivity involved.

Although the presented design approach tries to keep the responses of the equivalent SDOF system under control to reach the expected performance level with \( \mu_{eq} = \mu_t \) and \( y_{eq}^{eq} = y_t \), the numerical verification shows that some errors do exist between the responses of designed reinforced concrete member (\( y_m^{rc} \) and \( \mu^{rc} \)) under the given blast force and their respective targets (\( y_t \) and \( \mu_t \)). The errors come from the derivation of \( \delta \) and \( \rho \) from \( k_e \) and \( R_m \) due to some simplifications in converting the continuous reinforced concrete members into equivalent SDOF systems and computing the cross-sectional moment of inertia and ultimate strength of the members. However since the reinforced concrete member can be specifically designed against the given blast condition with the presented approach, the errors can be further analysed. Numerical results indicate that the errors are influenced by the boundary conditions of the member as well as the performance targets of \( y_t \) and \( \mu_t \), and the error between \( \mu^{rc} \) with \( \mu_t \) seem to be greater than that between \( y_m^{rc} \) and \( y_t \). Further research in Chapters 5 and 6 will derive the formulae to quantify the errors and involve the formulae into the modification of the design approach using energy spectra.
Synopsis

One of the main advantages of the performance-based blast resistant design of reinforced concrete structural members using energy spectra addressed in Chapters 3 and 4 is that the two indices indicating the errors between the maximum displacement and displacement ductility responses of the designed member against the given blast loading and their corresponding design targets can be explicitly analysed, since with this design procedure the structural member can be uniquely designed. The knowledge about the errors will be significantly beneficial in keeping the actual responses of designed members under control rather than those of the corresponding equivalent SDOF system within the blast design. Therefore an analysis of these two error indices is carried out in an attempt to gain their simplified explicit expressions with a variety of basic variables including the peak pressure and duration of a blast loading, the target displacement and displacement ductility factor of the design, member dimensions, support conditions (SC), the strength and Young’s modulus of concrete and reinforcement, and the longitudinal reinforcement ratio ($\rho$). It is discovered through the extensive numerical studies and statistical analyses that these two error indices could be properly formulated by two items: a nonlinear fitting function correlated with $\rho$ and $SC$, and a random variable to consider the influence of the others. The formulae of error indices provide a valuable tool of modifying the performance-based blast resistant design of the members using energy spectra and assessing the performance of the designed
members probabilistically accounting for different uncertainties. These applications will be discussed in Chapter 6.

Keywords: Expected performance level, Target displacement, Target displacement ductility factor, Energy spectra, Equivalent SDOF system, Error indices Maximum displacement responses, Displacement ductility responses

5.1 Introduction and Background

To keep the responses or damages of designed members under control to satisfy an expected performance level, much blast resistant design guidance have been issued in TM5-1300 [T2], TM5-855-1 [T1], Biggs [B1], Mays and Smith [M3], and ASCE 7-42 [A2] etc. Within these available design guidance, only a single response: the displacement response or the displacement ductility response for the members at the significant point, is restricted by specifying a longitudinal reinforcement ratio in the initial stage of the design based on the designer’s experience. To control both the displacement and displacement ductility response of a reinforced concrete member under blast loading simultaneously, a blast resistant design approach using energy spectra was developed and evaluated in Chapters 3 and 4. The effective depth ($d$) and longitudinal reinforcement ratio ($\rho$) of the reinforced concrete member can be specifically obtained in this approach by converting the continuous reinforced concrete member into an equivalent elastic-perfectly-plastic single-degree-of-freedom (SDOF) system [B1] and controlling its maximum displacement and displacement ductility responses ($y^m_{rc}$ and $\mu^m_{rc}$) to be exactly equal to the corresponding targets ($y^m_{rc}$ and $\mu^m_{rc}$).

Obviously, this design approach is accomplished based on an assumption that the responses ($y^m_{rc}$ and $\mu^m_{rc}$) of the designed reinforced concrete member at the significant point can be represented by the responses of its equivalent SDOF system ($y^m_{eq}$ and $\mu^m_{eq}$) under the given blast loading. However, a difference of $y^m_{rc}$ and $\mu^m_{rc}$ with its respective $y^m_{eq}$ and $\mu^m_{eq}$ is expectable due to the complexities of the
nonlinear dynamic behaviours of reinforced concrete members under blast conditions. This further causes the errors existing between \( y_m^{rc} \) and \( \mu^{rc} \) and the design targets of \( y_t \) and \( \mu_t \), as indicated from the design and verification process for the blast resistant design of reinforced concrete members using energy spectra in Figure 5.1.

![Figure 5.1](image_url)

**Figure 5.1** Design and verification process for performance-based blast resistant design of reinforced concrete members using energy spectra

In fact a good performance-based blast resistant design should be capable of controlling the responses of designed reinforced concrete member (\( y_m^{rc} \) and \( \mu^{rc} \)) rather than those of the corresponding equivalent SDOF system (\( y_m^{eq} \) and \( \mu^{eq} \)) with the given design targets (\( y_t \) and \( \mu_t \)). To fulfil this objective, it is worthwhile to analyse the errors between \( y_m^{rc} \) and \( \mu^{rc} \) of the member designed upon the blast resistant design approach using energy spectra and the corresponding targets of \( y_t \) and \( \mu_t \), since the knowledge of the errors can be applied in the modification of the design. Moreover, the errors also represent the differences of \( y_m^{rc} \) and \( \mu^{rc} \) with the
Based on this point, the probabilistic performance of the member at various levels can be effortlessly assessed accounting for different types of uncertainties without performing nonlinear dynamic finite element analysis, which would otherwise consume a lot of computational effort.

In this chapter, after the definition of two non-dimensional error indices, the related variables affecting them are discussed. With a larger number of numerical studies of the error indices involving different variables effects and the subsequent statistical analyses, simple formulae to estimate the error indices are recommended with the applications of these formulae to be addressed in Chapter 6.

5.2 Definition of Error Indices

To obtain a consistent measurement of the degree of the errors between \( y_m^\text{rc} \) and \( \mu^\text{rc} \) of the member designed with the developed approach and their respective \( y_t \) and \( \mu_t \), two non-dimensional error indices are defined as

\[
\begin{align*}
\varepsilon_y^\text{rc} &= \frac{y_t - y_m^\text{rc}}{y_t} \quad (5.1) \\
\varepsilon_\mu^\text{rc} &= \frac{\mu_t - \mu^\text{rc}}{\mu_t} \quad (5.2)
\end{align*}
\]

where \( \varepsilon_y^\text{rc} \) is the displacement error index standing for the error between \( y_m^\text{rc} \) and \( y_t \); and \( \varepsilon_\mu^\text{rc} \) is the displacement ductility error index representing the error between \( \mu^\text{rc} \) and \( \mu_t \).

Equations (5.1) and (5.2) describe the basic relationships between \( (y_m^\text{rc}, \mu^\text{rc}), (y_t, \mu_t) \), respectively.
5.3 Symbolical Expressions of Error Indices

One of main advantages of the blast resistant design using energy spectra is that by controlling \( y_m^{eq} = y_t \) and \( \mu^{eq} = \mu_t \), the design solution of \( d \) and \( \rho \) can be specifically gained with the given design variables including the blast loadings \( (P_r, t_d) \), design targets \( (y_t, \mu_t) \), the support condition \( (SC) \), the length \( (l) \) and the strength and Young’s modulus of elasticity of concrete and reinforcement in dynamic conditions \( (f_{dc}, E_c, f_{ds}, E_s, f_{dv}) \). Therefore \( d \) and \( \rho \) could be symbolically expressed as functions of these variables

\[
d = d(P_r, t_d, y_t, \mu_t, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}) \tag{5.3}
\]

\[
\rho = \rho(P_r, t_d, y_t, \mu_t, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}) \tag{5.4}
\]

The responses \( (y_m^{rc} \text{ and } \mu^{rc}) \) of the designed member under the given blast force are also specifically determined, which can be symbolically written as

\[
y_m^{rc} = y_m^{rc}(P_r, t_d, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}, d, \rho) \tag{5.5}
\]
\[ \mu_{rc} = \mu_{rc} \left( P_r, t_d, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}, d, \rho \right) \]  

(5.6)

Substituting Equations (5.5) and (5.6) into Equations (5.1) and (5.2) respectively produces \( \zeta_{y rc} \) and \( \zeta_{\mu rc} \) to be the functions of these variables, which is given by

\[ \zeta_{y rc} = \zeta_{y rc} \left( P_r, t_d, y_1, \mu_1, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}, d, \rho \right) \]  

(5.7)

\[ \zeta_{\mu rc} = \zeta_{\mu rc} \left( P_r, t_d, y_1, \mu_1, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}, d, \rho \right) \]  

(5.8)

5.4 Analytical Approach

To derive the explicit expressions of Equations (5.7) and (5.8) is almost impractical, since under most blast conditions the behaviours of reinforced concrete members will exhibit a significant geometric and material nonlinearity. A curve fitting technique with a large number of reliable data for \( \zeta_{y rc} \) and \( \zeta_{\mu rc} \), which are determined according to Equations (5.1) and (5.2) with \( y_{m rc} \) and \( \mu_{rc} \) obtained from nonlinear finite element analyses of the designed members, is executed in this chapter together with the statistical analyses so as to find simplified explicit expressions of Equations (5.7) and (5.8). The procedures are listed as follows:

1) Select the type of SCs of reinforced concrete members to be designed;
2) Sample the design variable vector \( \{ P_r, t_d, y_1, \mu_1, l, f_{dc}, E_c, f_{ds}, E_s, f_{dv} \} \)
   where 2000 samples are taken to ensure the accuracy of the following statistical analysis;
3) Design the reinforced concrete member using energy spectra with each sample of the design variable vector to obtain \( d, \rho \) and further \( \rho', \rho_v \) and \( b_w \) of a ratio of \( d \).
4) Repeat (3) until 2000 sampled design cases are accomplished;
5) Select 500 cases with \( \rho \) ranging from 0.31% to 2.2%;
6) Perform numerical finite element analysis on the selected 500 design cases to find $y_m^{rc}$ and $\mu^{rc}$;  

7) Compute $z_y^{rc}$ and $z_{\mu}^{rc}$ of the 500 design cases with Equations (5.1) and (5.2);  

8) Plot the distributions of $z_y^{rc}$ and $z_{\mu}^{rc}$ with the basic design variables;  

9) Carry out the curve fitting of the distributions of $z_y^{rc}$ and $z_{\mu}^{rc}$ followed by statistical analyses;  

10) Recommend the simplified formulae to estimate $z_y^{rc}$ and $z_{\mu}^{rc}$.  

11) Change the type of SC's of the members and repeat above steps;  

5.4.1 Selection of variables  

In order to perform the curve fitting of the distributions of $z_y^{rc}$ and $z_{\mu}^{rc}$ to find their simplified formulae, the design variables involved in the second step of the above procedure are randomly selected within their concerned ranges respectively which are discussed as follows.  

For external explosions, the blast loadings which can be reasonably idealized into triangular pulses are basically dependent on $P_r$ and $i$ (where $t_d = 2i/P_r$) [H1, T1 T2]. Therefore the random selection of $P_r$ and $i$ is executed in their respective ranges from 150 kPa to 59 MPa and from $5.8\times10^2$ to $3.4\times10^4$ kPams for the blast resistant design and the following error indices analysis. This provides a significantly wide range of blast incidents. For the more severe blast cases, the design of the structural system with the alternate load path in redistributing loadings seems to be more reasonable and economic where the structural members immediately opposite the blast are allowed to fail. While in the situations of the slighter blasts, the blast loadings become less critical in the structural design.  

The random selection of $y_i$ and $\mu_i$ is executed in their different ranges according to the various scopes of the blast impulses as shown in Table 5.1 since within the
blast resistant design of a particular reinforced concrete member the determination of the reasonable design targets of $y_t$ and $\mu_t$ should account for possible blast intensities. Some overlaps are allowed for the ranges of $y_t$ and $\mu_t$ considering that there is no strict watershed for them at a particular impulse within the design. The maximum allowable $y_t$ and $\mu_t$ are taken to be respectively larger than the limit of 4" for non-laced reinforced concrete members recommended in TM 5-1300 [T2] and Mays and Smith [M3] and 10.0 specified in the ASCE special publication [B4] due to the possibility of the actual responses of the designed members in blast conditions located on the conservative side by comparing them with their respective targets of $y_t$ and $\mu_t$.

Table 5.1 Various design targets

<table>
<thead>
<tr>
<th>Impulse (kPa ms)</th>
<th>$\leq 10^3$</th>
<th>$10^3 \sim 10^4$</th>
<th>$10^4 \sim 2 \times 10^4$</th>
<th>$2 \times 10^4 \sim 3 \times 10^4$</th>
<th>$\geq 3 \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_t$</td>
<td>$1^\circ \sim 1.8^\circ$</td>
<td>$1.4^\circ \sim 3.2^\circ$</td>
<td>$2.8^\circ \sim 3.6^\circ$</td>
<td>$3.2^\circ \sim 4.0^\circ$</td>
<td>$3.4^\circ \sim 4.8^\circ$</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>$3.0 \sim 8.5$</td>
<td>$4.5 \sim 10.5$</td>
<td>$5.5 \sim 12.5$</td>
<td>$6.0 \sim 14.5$</td>
<td>$7.0 \sim 16.5$</td>
</tr>
</tbody>
</table>

$y_t = l \tan(\theta_t)/2$ for the member with two ends constrained;

$y_r = l \tan(\theta_r)$ for member with one end free

The parameter of $l$ is selected in the range from 4.5 to 9.0 m for the members with two ends constrained while from 2.5 to 4.5 m for members with one end free, which cover the typical cases for most reinforced concrete structures. Due to strain rate effects, the parameters of material properties of concrete and reinforcement in dynamic conditions are implemented in this analysis whose possible ranges used in the design are listed in Table 5.2. Since the structural members exist in the context of structural systems, their real SCs are complex and generally depend on the connection with the adjacent members as well as the construction requirements. However in the blast resistant design they are generally idealized into one of the four types as listed in Table 5.3. Considering that reinforced concrete members should be designed to resist the negative deflection or rebound of the members.
subsequent to its maximum positive deflection and that the possibility of the explosion occurs in the two opposite sides of the members to be designed, the type of reinforcement layout, where the compression reinforcement area is equal to the tensile reinforcement area, is employed in this study.

Table 5.2 Various material properties

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{dc}$</td>
<td>35 ~ 45 MPa</td>
</tr>
<tr>
<td>$E_c$</td>
<td>25.5 ~ 30.5 GPa</td>
</tr>
<tr>
<td>$f_{ds}$</td>
<td>550 ~ 660 MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>190 ~ 210 GPa</td>
</tr>
<tr>
<td>$f_{dv}$</td>
<td>280 ~ 360 MPa</td>
</tr>
</tbody>
</table>

Table 5.3 Various support conditions

<table>
<thead>
<tr>
<th>Type</th>
<th>Support Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply-supported</td>
<td>$q_u$</td>
</tr>
<tr>
<td>Fixed/free</td>
<td>$q_u$</td>
</tr>
<tr>
<td>Fixed/roller-supported</td>
<td>$q_u$</td>
</tr>
<tr>
<td>Fixed/fixed</td>
<td>$q_u$</td>
</tr>
</tbody>
</table>
Based on the ranges of the variables discussed above, their random selections are executed by assuming them equally frequent or equally likely to occur. Doing so ensures the gain of a more general curve fitting of $\zeta^{rc}_y$ and $\zeta^{rc}_\mu$ which could incorporate the effects of these variables as adequately as possible.

5.4.2 Numerical analysis

The step of numerical analysis on the selected design cases to find $y^{rc}_m$ and $\mu^{rc}$ is executed with the program ABAQUS [A1]. The detailed finite element models can be found in Chapter 2. The concrete model based on smeared cracking and isotropic compressive plasticity is employed to represent the concrete properties while reinforcement is simulated by the Von-mises yield criterion. Timoshenko beam elements are employed for the modelling of the designed members. As damping is neglected within the design, its effect is still not taken into consideration in this error analysis process. This finite element models has been effectively verified from the numerical predicted responses with experimental ones [S3, K3].

5.5 Distributions of Error Indices

Based on the analytical procedure mentioned above, the distributions of $\zeta^{rc}_y$ and $\zeta^{rc}_\mu$ for designed members under various SCs versus different variables ($P_r$, $t_d$, $y_t$, $\mu_t$, $d$, $l$, $E_c$, $E_s$, $f_{sd}$ and $\rho$) are obtained as plotted in Figures 5.2 to 5.5. Several observations can be drawn as follows:

Figure 5.2 Distributions of error indices with different basic variables for simply supported members

- (g). Young’s modulus of elasticity of concrete versus error indices
- (h). Young’s modulus of elasticity of reinforcement versus error indices
- (i). Strength of reinforcement versus error indices
- (j). Longitudinal reinforcement ratio versus error indices

Figure 5.2 (Continued)
Figure 5.3 Distributions of error indices with different basic variables for fixed/free members.

(g). Young’s modulus of elasticity of concrete versus error indices

(h). Young’s modulus of elasticity of reinforcement versus error indices

(i). Strength of reinforcement versus error indices

(j). Longitudinal reinforcement ratio versus error indices

Figure 5.3 (continued)

(a). Peak pressure versus error indices

(b). Loading duration versus error indices

(c). Target displacement versus error indices

(d). Target displacement ductility factor versus error indices

(e). Effective depth versus error indices

(f). Length of member versus error indices

Figure 5.4 Distributions of error indices with different basic variables for fixed/roller-supported members

2.55x10\(^2\) 10
2.70x10\(^2\) 10
2.85x10\(^2\) 10
3.00x10\(^2\) 10

-0.2 -0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Error Indices

\[ E_c (N/m^2) \]

Displacement error
Displacement ductility error

\[ E_s (N/m^2) \]

Displacement error
Displacement ductility error

\[ f_{m} (N/m^2) \]

Displacement error
Displacement ductility error

\[ \rho \]

Displacement error
Displacement ductility error

(g). Young’s modulus of elasticity of concrete versus error indices  
(h). Young’s modulus of elasticity of reinforcement versus error indices

(i). Strength of reinforcement versus error indices  
(j). Longitudinal reinforcement ratio versus error indices

Figure 5.4 (Continued)
Figure 5.5 Distributions of error indices with different basic variables for fixed/fixed members
(g). Young’s modulus of elasticity of concrete versus error indices

(h). Young’s modulus of elasticity of reinforcement versus error indices

(i). Strength of reinforcement versus error indices

(j). Longitudinal reinforcement ratio versus error indices

Figure 5.5 (Continued)
1). The distributions of $\zeta^y_{rc}$ are generally concentrated in the strips that are much narrower and closer to the X-Axis as compared to those of $\zeta^\mu_{rc}$ especially in the case of the first three types of SCs. This indicates a more effective control of $y^m_{rc}$ of the members within the presented blast resistant design method. This result is reasonable since $\mu^rc$ is determined simultaneously by $y^m_{rc}$ and the elastic (recoverable) displacement $y^e_{rc}$ of the member at the significant point. As the continuous reinforced concrete member is converted into an equivalent SDOF system with its resistance function simplified by a bilinear curve, the elastic displacement of the designed member in blast incident is somewhat underestimated as shown in Figure 5.6. Therefore a conservative control of $\mu^rc$ is performed leading to the relatively large positive values of $\zeta^\mu_{rc}$.

![Figure 5.6 Comparison of resistance functions for the reinforced concrete member and the equivalent SDOF system](image)

2). The convergent distributions of $\zeta^y_{rc}$ and $\zeta^\mu_{rc}$ with respect to $\rho$ under any SC as shown in Figures 5.2j to 5.5j indicate that $\rho$ is an important parameter for them. It
should be pointed out that although the distributions of $\xi_y^{rc}$ versus other variables seem as concentrated as that versus $\rho$ under the first three SCs conditions, it is mainly attributed to the effective control of $y_m^{rc}$ within the presented design process. If other factors are present that influence the effective control of $y_m^{rc}$ as occurring under the fourth SC condition, the convergent distributions of $\xi_y^{rc}$ will appear only with respect to $\rho$ rather than the other variables as demonstrated in Figure 5.5.

The importance of $\rho$ to $\xi_y^{rc}$ and $\xi_\mu^{rc}$ can be conceptually accounted for from two of the three aspects which induce the errors between the responses ($y_m^{rc}$, $\mu^{rc}$) and the corresponding design targets ($y$, $\mu$) as discussed in Chapter 4. In the calculation of the equal compression and tension longitudinal reinforcement ratio ($\rho$) from the ultimate strength $R_m$, the contribution of concrete to $R_m$ is ignored within the design. For the members with a small value of $\rho$ where the concrete presents an important part in resisting external force, this ignorance will lead to a significantly conservative design and therefore a large value of the conservative error indices. With the increase of $\rho$, the contribution of concrete to $R_m$ becomes smaller and so does the conservative degree of design. It also agrees quite well with the trend of $\xi_\mu^{rc}$ with respect to $\rho$ under all SCs and $\xi_y^{rc}$ to $\rho$ under the fourth SCs. The second reason that makes $\rho$ important to error indices is related to the calculation of the stiffness of reinforced concrete members where a reduction coefficient $\gamma$ is applied to obtain the moment of inertia in the cross section along reinforced concrete members. Since $\gamma$ is determined by $\rho$ from the empirical data fitting curve, it is expected that there may be various values of $\xi_y^{rc}$ and $\xi_\mu^{rc}$ for various $\rho$.

3). The support condition (SC) of a structural member also plays an important role in producing $\xi_y^{rc}$ and $\xi_\mu^{rc}$, which is demonstrated by the trends of $\xi_y^{rc}$ and $\xi_\mu^{rc}$ with $\rho$ tending to be different under various SCs. Comparing the Figures 5.2j to 5.4j with the Figure 5.5j indicates that both $\xi_y^{rc}$ and $\xi_\mu^{rc}$ for fixed/fixed members are
generally larger than those of members under other SCs with the same $\rho$. This difference is induced by the application of unreasonable transformation factors $K_{LE}$ and $K_{ME}$ in the design of fixed/fixed members. As shown in Figure 5.7, the transformation of this type of continuous reinforced concrete member into the equivalent SDOF system can be divided into three stages with different values of transformation factors in each stage: the elastic stage with no plastic hinges, the elastic-plastic stage when two plastic hinges occurs at the two ends of the member and the purely plastic stage as a third plastic hinge appears in the mid-span. In the second stage, transformation factors are taken with the same values as those for a simply supported member. However the responses of a member with two ends pinned (plastic hinges formed at both ends) will be surely smaller than those of a simply supported member. This means the responses of fixed/fixed members are overestimated during the design thus leading to relatively larger error indices. In the cases of other SCs, such phenomenon is not present.

![Figure 5.7 Stages of fixed/fixed members in the determination of transformation factors](image-url)
The variables other than $\rho$ and SC have some minor effects, which make the values of $\xi_y^{rc}$ and $\xi_{\mu}^{rc}$ slightly scattered as $\rho$ and SC are certain. Although these effects are not significant as compared to those of $\rho$ and SC, they are also incorporated in the recommended formulae trying to consider all variable effects.

5.6 Recommended Formulae of Error Indices

Based on the distributions of $\xi_y^{rc}$ and $\xi_{\mu}^{rc}$ with respect to different variables discussed above, in order to obtain the simplified expressions of Equations (5.7) and (5.8) the involved variables are divided into two groups: the important variables of $\rho$ and SC and the secondary variables including the others such as $P_t$, $t_d$, $y_t$, $\mu_t$, and $d$ etc. To incorporate the influences of $\rho$ and SC into the expressions, nonlinear curve fittings of $\xi_y^{rc}$ and $\xi_{\mu}^{rc}$ versus $\rho$ for various SCs as plotted in Figures 5.2j to 5.5j are carried out where the functions are expressed as $f(\rho, SC)$ and $g(\rho, SC)$ respectively. The effects of the secondary variables are dealt with by introducing two nominal random variables of $e_y$ and $e_{\mu}$ that are assumed representing the deviation of $\xi_y^{rc}$ and $\xi_{\mu}^{rc}$ around the fitting curves. As a consequence, Equations (5.7) and (5.8) can be simplified into

$$\xi_y^{rc} = f(\rho, SC) + e_y \quad (5.9)$$

$$\xi_{\mu}^{rc} = g(\rho, SC) + e_{\mu} \quad (5.10)$$

A quadratic function is selected for the curve fitting given by

$$f(\rho, SC) \text{ and } g(\rho, SC) = a_1 + a_2 \rho + a_3 \rho^2 \quad (5.11)$$

With the result functions of $f(\rho, SC)$ and $g(\rho, SC)$, the nominal random variables of $e_y$ and $e_{\mu}$ can be obtained with
\[ e_y = \zeta_y^{sc} - f(\rho, SC) \]  
\[ e_\mu = \zeta_\mu^{sc} - g(\rho, SC) \]

For members with various SCs, the results of the parameters \( a_1 \), \( a_2 \) and \( a_3 \) are listed in Table 5.4 while the histograms of \( e_y \) and \( e_\mu \) are shown in Figures 5.8 to 5.11. It is indicated that the mean values of \( e_y \) and \( e_\mu \) (\( E_{e_y} \) and \( E_{e_\mu} \)) are approximate to zero and the standard deviations of \( e_y \) and \( e_\mu \) (\( \sigma_{e_y} \) and \( \sigma_{e_\mu} \)) are relatively minor indicating a slight effect of the variables other than \( \rho \) and SC. To graphically diagnose whether the data in \( e_y \) and \( e_\mu \) come from a normal distribution, the corresponding normal probability plots are also listed in these figures. If those data are normal, the plot will be linear [K5]. The results display that it is generally more accurate for \( e_y \) and \( e_\mu \) to be assumed as normal distribution although some deviation appears in some cases, i.e. when \( e_\mu \) is out of the range from –0.05 to 0.05 for the fixed/fixed members.

### Table 5.4 Results of the nonlinear curve fitting

<table>
<thead>
<tr>
<th>SC</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_y^{sc} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simply-supported</td>
<td>-0.10369</td>
<td>-2.20114</td>
<td>160.30932</td>
</tr>
<tr>
<td>Fixed/free</td>
<td>-0.01712</td>
<td>0.80877</td>
<td>66.84997</td>
</tr>
<tr>
<td>Fixed/roller-supported</td>
<td>0.01276</td>
<td>-1.67032</td>
<td>100.93644</td>
</tr>
<tr>
<td>Fixed/fixed</td>
<td>0.55459</td>
<td>-34.63653</td>
<td>735.20353</td>
</tr>
<tr>
<td>( e_\mu^{sc} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simply-supported</td>
<td>0.80257</td>
<td>-50.10919</td>
<td>675.55356</td>
</tr>
<tr>
<td>Fixed/free</td>
<td>0.83085</td>
<td>-47.05548</td>
<td>911.90338</td>
</tr>
<tr>
<td>Fixed/roller-supported</td>
<td>0.84443</td>
<td>-45.30174</td>
<td>800.57689</td>
</tr>
<tr>
<td>Fixed/fixed</td>
<td>0.99826</td>
<td>-18.99113</td>
<td>-354.75711</td>
</tr>
</tbody>
</table>

(a). Histogram of $e_y$ ($\sigma_{e_y} = 0.0172$)

(b). Normal probability plot of $e_y$

Figure 5.8 The distributions of $e_y$ and $e_\mu$ for simply supported members
(c). Histogram of $e_\mu$ ($\sigma_{e_\mu} = 0.0543$)

(d). Normal probability plot of $e_\mu$

Figure 5.8 (Continued)
Figure 5.9 The distributions of $e_y$ and $e_\mu$ for fixed/free members

(a). Histogram of $e_y$ ($\sigma_{e_y} = 0.0294$)

(b). Normal probability plot of $e_y$
Figure 5.9 (Continued)
Figure 5.10 The distributions of $e_y$ and $e_\mu$ for fixed/roller-supported members
(c). Histogram of $e_\mu$ ($\sigma_{e_\mu} = 0.05$)

(d). Normal probability plot of $e_\mu$

Figure 5.10 (Continued)
Figure 5.11 The distributions of $e_y$ and $e_\mu$ for fixed/fixed members
(c). Histogram of $e_\mu$ ($\sigma_{e_\mu} = 0.039$)

(d). Normal probability plot of $e_\mu$

Figure 5.11 (Continued)
5.7 Summaries

Simple formulae have been developed for the estimation of $\xi_y^{ce}$ and $\xi_{\mu}^{ce}$ produced within the blast resistant design of reinforced concrete members using energy spectra. The distributions of $\xi_y^{ce}$ and $\xi_{\mu}^{ce}$ with different variables that have been selected randomly from their concerned ranges within the blast resistant design, are generated through a large amount of numerical analyses. By comparing these distributions, it is found that $SC$ and $\rho$ have profound effects on $\xi_y^{ce}$ and $\xi_{\mu}^{ce}$ while the variables other than $SC$ and $\rho$ have some minor influences. As a result to incorporate all variables effects, the formulae of $\xi_y^{ce}$ and $\xi_{\mu}^{ce}$ are recommended to include two items: a nonlinear fitting function correlated with $\rho$ and $SC$ and a random variable ($e_y$ and $e_{\mu}$ respectively) to consider the influence of the others. These formulae provide a reasonable tool in modifying the performance-based blast resistant design of the members using energy spectra and assessing the performance of the designed members probabilistically accounting for different uncertainties. Their applications will be specifically identified in Chapter 6.
CHAPTER SIX

PERFORMANCE–BASED BLAST RESISTANT DESIGN OF REINFORCED CONCRETE MEMBERS USING ENERGY SPECTRA — PART IV. APPLICATIONS OF ERROR INDICES

Synopsis

The formulae of the two error indices for the performance-based blast resistant design approach of reinforced concrete members using energy spectra have been established in Chapter 5. This chapter aims to apply the two error indices into the modification of the design approach in order to keep the responses of the maximum displacement and displacement ductility factor of the designed member rather than those of the corresponding equivalent SDOF system under control. The process of the modified design approach with the error indices is discussed and demonstrated from several design examples. Nonlinear dynamic analysis of the design examples indicates that the modified design approach can more effectively control the response of the designed members than the original method. Moreover, this chapter introduces a simpler procedure of implementing the two error indices into the estimation of the distribution of the maximum displacement and displacement ductility responses for the designed members under blast loadings, where different types of uncertainties are accounted for without performing nonlinear dynamic finite element analysis. Based on this simper procedure, the probabilistic performance for the designed members under blast conditions is much more practically assessed.

Keywords: Expected performance level, Target displacement, Target displacement ductility factor, Maximum displacement responses, Displacement ductility responses, Error indices formulae
6.1 Introduction and Background

Two important topics exist concerning the blast resistance of reinforced concrete members, one of which is related to the blast resistant design approaches that are capable of controlling the responses of designed member to the expected performance level under a certain blast situation. The other is the probabilistic performance evaluation of the designed member against blast loadings accounting for different types of the uncertainties.

The blast resistant design approach using energy spectra has been developed and evaluated in Chapters 3 and 4. Both the maximum displacement and displacement ductility responses ($y_{m}^{eq}$ and $\mu_{eq}$) of the equivalent SDOF system of the designed member under the given blast force can be guarantied to reach their respective targets ($y_{t}$ and $\mu_{t}$) perfectly within the design, however it is found numerically that there still exist certain degree of errors between the responses ($y_{m}^{rc}$ and $\mu_{m}^{rc}$) of the designed member at their significant point and the corresponding targets ($y_{t}$ and $\mu_{t}$). Simple equations for the estimation of these errors represented by two error indices ($\varsigma_{y}^{rc}$ and $\varsigma_{\mu}^{rc}$) have been proposed in Chapter 5. From the distributions of $\varsigma_{y}^{rc}$ and $\varsigma_{\mu}^{rc}$ obtained from a large number of numerical analyses, two types of undesirable design cases may occur with the design approach where a significantly conservative control of $\mu_{rc}$ in the case of a small value for $\rho$ or a somewhat inadequate control of $y_{m}^{rc}$ appears. Therefore modification of the design approach in keeping $y_{m}^{rc}$ and $\mu_{rc}$ of the member under control instead of those ($y_{m}^{eq}$ and $\mu_{eq}$) of the equivalent SDOF system seems necessary through the utilization of $\varsigma_{y}^{rc}$ and $\varsigma_{\mu}^{rc}$. This particular area will be concentrated upon in the first part of this chapter where the flowchart of the modified design approach is presented depending on the error indices. Several design examples are demonstrated with the modified design approach as well as the original one. Numerical simulations of their displacement
and displacement ductility responses under the given blast force are performed while the results are compared.

In the probabilistic performance assessment of reinforced concrete flexural members under potential blast conditions, calculation of the distributions of the maximum displacement response and/or displacement ductility response provides a big practical problem for structural engineers since at present this task has to be fulfilled with nonlinear dynamic finite element analysis. The probabilistic performance assessment based on a combination of nonlinear dynamic finite element analysis and Monte-Carlo simulation [K5] is computationally considerably expensive. Hence a simpler way to find the distributions of maximum displacement and displacement ductility responses for members under blast loadings becomes much more valuable. The latter section of this chapter will introduce an efficient procedure of utilizing $\zeta_y^{rc}$ and $\zeta_\mu^{rc}$ for the estimation of them followed by the combination of this procedure with the Monte-Carlo simulation into the probabilistic performance assessment. As an illustration, the performances of the members designed with the modified approach using energy spectra are probabilistically evaluated.

6.2 Modification of the Design Approach With Error Indices

6.2.1 Design modification with error indices

For the convenience of the following discussion, the design targets are distinguished from two different viewpoints. From a physical viewpoint, design targets are the final goals for designed members to be achieved under certain blast conditions therefore they should keep unchanged within a design. They are called the physical design targets (PHY-DTs). However from another point of view, design targets are only some of the important parameters involved in the design to control the responses of the designed members to reach their PHY-DTs just as those adopted in the blast resistant design approach using energy spectra presented in Chapter 3. In this sense, they are named the parametric design targets (PAR-DTs). In the
presented blast design approach the values of PAR-DTs are simply fixed to be equal to those of PHY-DTs, however this action induces some inevitable errors of the member’s responses \( (\gamma_m^{rc} \text{ and } \mu_m^{rc}) \) to the PHY-DTs. By properly adjusting the values of PAR-DTs within the design, the elimination of such errors can be simply achieved.

Denoting PHY-DTs for maximum displacement and displacement ductility responses as \( y_t \) and \( \mu_t \) while PAR-DTs as \( y_{te} \) and \( \mu_{te} \), the two error indices introduced in Chapter 5 can be expressed through another form, written as

\[
\varepsilon_y^{rec} = \frac{y_{te} - y_m^{rc}}{y_{te}} = f(\rho, SC) + e_y 
\]

\[
\varepsilon_{\mu}^{rec} = \frac{\mu_{te} - \mu_m^{rc}}{\mu_{te}} = g(\rho, SC) + e_{\mu} 
\]

The detailed expressions of \( f(\rho, SC) \), \( g(\rho, SC) \) and the distribution properties of \( e_y \), \( e_{\mu} \) can be found in Chapter 5. Since in the design the responses of \( y_m^{rc} \text{ and } \mu_m^{rc} \) are required to be controlled in achieving the PHY-DTs of \( y_t \text{ and } \mu_t \), that is \( y_m^{rc} = y_t \) and \( \mu_m^{rc} = \mu_t \), by subjecting them into Equations (6.1) and (6.2), the PAR-DTs of \( y_{te} \) and \( \mu_{te} \) are obtained as

\[
y_{te} = \frac{y_t}{1 - f(\rho, SC) - e_y} 
\]

\[
\mu_{te} = \frac{\mu_t}{1 - g(\rho, SC) - e_{\mu}} 
\]

Equations (6.3) and (6.4) can be utilized to adjust the PAR-DTs so as to get a better control of the responses of \( y_m^{rc} \text{ and } \mu_m^{rc} \) within the design. However, two points
should be mentioned. Since $e_y$ and $e_\mu$ are random variables which indicate the uncertain influences of the design variables other than $\rho$ and $SC$ on error indices, certain quantiles [K5] of $e_{y,n}$ and $e_{\mu,m}$ corresponding to $n$ and $m$ percentages of non-exceedance probabilities for $e_y$ and $e_\mu$ have to be selected, and so Equations (6.3) and (6.4) are modified into

$$y_{we} = \frac{y_t}{1 - f(\rho, SC) - e_{y,n}}$$

(6.5)

$$\mu_{we} = \frac{\mu_t}{1 - g(\rho, SC) - e_{\mu,m}}$$

(6.6)

The physical meaning of such action can be explained as follows. Subtracting Equations (6.5) and (6.6) from Equations (6.1) and (6.2) respectively and rearranging them leads to

$$y_t - y_{we} = y_{we} (e_y - e_{y,n})$$

(6.7)

$$\mu_t - \mu_{we} = \mu_{we} (e_\mu - e_{\mu,m})$$

(6.8)

The item of $e_y - e_{y,n}$ in the bracket provides a random variable with a $n$ percentage of probability whose value is less than zero as shown in Figure 6.1. Since $y_{we}$ is always positive, it can be concluded from Equation (6.7) that the maximum displacement response $y_{we}$ of the members designed according to Equations (6.5) and (6.6) will have a $(1 - n)$ percentage of probability of non-exceeding the PHY-DT of $y_t$ (or a $n$ percentage of exceeding probability). Equation (6.8) can be explained in the same way where the displacement ductility response $\mu_{we}$ will have a $(1 - m)$ percentage of probability of non-exceeding the PHY-DT of $\mu_t$. 
Another point that should be noted is that the determination of PAR-DTs of $y_{le}$ and $\mu_{le}$ from Equations (6.5) and (6.6) is dependent on the longitudinal reinforcement ratio $\rho$. Therefore an iterative approach is needed in the design. After an assumed $d$ and $\rho$, the PAR-DTs of $y_{le}$ and $\mu_{le}$ are adjusted from the given $\alpha$ and PHY-DTs of $y_{l}$ and $\mu_{l}$. However to fulfil $y_{le}$ and $\mu_{le}$, a new $d$ and $\rho$ will be produced for the member with the design procedures using energy spectra that is developed in Chapter 3 and hence the process must be repeated until $d$ and $\rho$ are consistent. The flowchart of the modified performance-based blast resistant design using energy spectra is shown in Figure 6.2.

Figure 6.2 Flowchart of the modified performance-based blast resistant design using energy spectra with error indices ($\varepsilon_1$, $\varepsilon_2$ are arbitrarily small values)

1. Start
2. Give peak pressure $P$, duration $t$, length $l$ and all parameters related to the materials properties
3. Give PHY-DTs ($y$, $\mu$), the percentages of non-exceedance probabilities for $e_y$ and $e_{\mu}$ ($n$, $m$), and calculate $e_{y,n}$ and $e_{\mu,m}$
4. Assume an initial longitudinal reinforcement ratio $\rho_0$ and sectional effective depth $d_0$
5. Adjust the PAR-DTs ($y$, $\mu$) from Equations (6.5) and (6.6)
6. Calculate a new longitudinal reinforcement ratio $\rho_i$ and sectional effective depth $d_i$ with the design method using energy spectra presented in Chapter 3
7. Make $\rho_i = \rho_i$ and $d_i = d_i$ if
   \[
   \left| \frac{(\rho_i - \rho_0)}{\rho_0} \right| < \varepsilon_2
   \]
   \[
   \left| \frac{(d_i - d_0)}{d_0} \right| < \varepsilon_1
   \]
8. Yes: Determine the maximum shear force and the design stirrup
9. End
6.2.2 Illustrative examples

In this section, the modified blast resistant design approach by incorporating error indices is applied to three illustrative examples that enable the reader to foresee its use in engineering practices. The examples plotted in Figures 6.3 to 6.5 are carried out for a simply-supported beam, a cantilever wall and a fixed/roller-supported beam in resisting the blast loadings produced by a variety of blast loadings. The example I and III tend to control the member’s responses at a very low protection level thus a relatively higher target support rotation of \( \theta_t = 4^\circ \) \( (\gamma_t \approx l \tan(\theta_t)/2) \) and \( \mu_t = 10 \) are given, while a low protective level is required for the example II where \( \theta_t = 2^\circ \) \( (\gamma_t \approx l \tan(\theta_t)) \) and \( \mu_t = 6 \). The compression reinforcement area is taken to be equal to the tension reinforcement area considering the rebound effect of the member subsequent to its maximum displacement response. In these design examples, the values for \( n \) and \( m \) are taken to be 0.05 so as to ensure a 95 percent of probability for \( y_m^{rc} \) and \( \mu_t^{rc} \) of the designed members non-exceeding the PHY-DTs of \( y_m \) and \( \mu_t \). The convergence conditions about \( \rho \) and \( d \) are defined with \( |(\rho_i - \rho_0)/\rho_0| \leq 0.001 \) and \( |(d_i - d_0)/d_0| \leq 0.001 \).

![Figure 6.3 A simply supported beam under the blast loading (Design example I)](image-url)
Figure 6.4 A cantilever wall under the blast loading (Design example II)

Figure 6.5 A fixed/roller-supported beam under the blast loading
(Design example III)
With the initial values of $\rho_0$ and $d_0$ taken as 1.00 % and 1000 mm, the whole design processes for these three design examples with the modified approach depending on the error indices are listed in Tables 6.1 to 6.3. For the purpose of comparison, the whole design procedures of the members with the original design method are also listed in these tables. It is indicated that there is no difficulty in reaching convergence for the modified design procedures however due to the adjustment of PAR-DTs from Equations (6.3) and (6.4) within the modified design, more iterative steps and computation time are needed to reach convergence than those with the original design method. Also there is an obvious decline in $\rho$ while some increase in $d$ for the member designed with the modified approach as compared to those with the original one.
Table 6.1 Iterative design procedures of the simply supported beam (Example I)

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<tr>
<th>$y_t$ (mm)</th>
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<th>$e_{y_t,0.05}$</th>
<th>$e_{\mu_t,0.05}$</th>
<th>$d_0$ (mm)</th>
<th>$\rho_0$ (%)</th>
<th>$f(\rho_0,SC)$</th>
<th>$g(\rho_0,SC)$</th>
<th>$y_{te}$ (mm)</th>
<th>$\mu_{te}$</th>
<th>$d_1$ (mm)</th>
<th>$\rho_1$ (%)</th>
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with the modified performance-based blast resistant design approach using energy spectra

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with the original performance-based blast resistant design approach using energy spectra

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$f_{\sigma} = 506\text{MPa}$, $f_{\sigma} = 40\text{MPa}$, $E_r = 40\text{GPa}$, $E_r = 200\text{GPa}$, $f_{\sigma} = 275\text{MPa}$

Convergence conditions: $|(\rho_1 - \rho_0)/\rho_0| \leq 0.001$ and $|(d_1 - d_0)/d_1| \leq 0.001$
Table 6.2 Iterative design procedures of the cantilever wall (Example II)

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with the original performance-based blast resistant design approach using energy spectra

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$f_u = 506$ MPa, $f_y = 40$ MPa, $E_u = 40$ GPa, $E_y = 200$ GPa, $f_u = 275$ MPa

Convergence conditions: $|\rho_1 - \rho_0| \leq 0.001$ and $|d_1 - d_0|/d_0 \leq 0.001$
Table 6.3 Iterative design procedures of the fixed/roller-supported beam (Example III)

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<th>$d_0$</th>
<th>$\rho_0$</th>
<th>$g(\rho_0,SC)$</th>
<th>$y_{ce}$</th>
<th>$\mu_{ce}$</th>
<th>$d_1(m)$</th>
<th>$\rho_1$</th>
<th>CPU time (s)</th>
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<tr>
<td>with the modified performance-based blast resistant design approach using energy spectra</td>
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<tr>
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<td>0.00615</td>
<td>0.47134</td>
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<td>0.00599</td>
<td>0.56262</td>
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<td>453.16</td>
<td>0.708</td>
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<td>with the original performance-based blast resistant design approach using energy spectra</td>
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<td>-0.082</td>
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<td>16.371</td>
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<tr>
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<td>3.016</td>
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<td>10</td>
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<td>360.55</td>
<td>1.401</td>
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<td>360.55</td>
<td>1.401</td>
<td>210.00</td>
<td>10</td>
<td>367.90</td>
<td>1.330</td>
<td>360.55</td>
<td>1.401</td>
<td></td>
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<tr>
<td>367.90</td>
<td>1.330</td>
<td>360.55</td>
<td>1.401</td>
<td>360.90</td>
<td>1.402</td>
<td>360.90</td>
<td>1.402</td>
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</tr>
</tbody>
</table>

$P_\sigma = 506 \text{MPa}, \ P_\sigma = 40 \text{MPa}, \ E_c = 40 \text{GPa}, \ E_v = 200 \text{GPa}, \ f_\sigma = 275 \text{MPa}$

Convergence conditions: $| (\rho - \rho_0) / \rho_0 | \leq 0.001$ and $| (d_t - d_0) / d_0 | \leq 0.001$
6.2.3 Numerical verification

In order to check whether $y^{rc}_m$ and $\mu^{rc}$ for the designed members under the given blast loadings are controlled effectively with the modified design approach, nonlinear finite element analysis is performed on these members, in which the finite element models are taken as the same discussed in Chapter 2. The simulation results are demonstrated in Figures 6.6 to 6.8 where for comparison $y^{rc}_m$ and $\mu^{rc}$ of the members designed with the original design procedure are also given.

Results indicate that the modified design approach has overcome the disadvantages existing in the control of $y^{rc}_m$ and $\mu^{rc}$ for the simply supported beam and the cantilever wall designed with the original method, where $y^{rc}_m$ slightly exceeds the target of $y_t$ and $\mu^{rc}$ is too much conservative. Values of $y^{rc}_m$ and $\mu^{rc}$ for these two members that are designed with the modified approach are both approximate to their respective targets and restricted to be a little on the conservative sides. Besides, $y^{rc}_m$ seems to be controlled much closer to its target than $\mu^{rc}$ due to the smaller standard deviation of $e_y$. As for the fixed/roller-supported beam, due to the strict requirement of 95 percent of probability for $y^{rc}_m$ non-exceeding $y_t$ within the modified design procedures, the control of $y^{rc}_m$ seems to be slightly more conservative than that from the original design procedures. However in this case $\mu^{rc}$ is still controlled much more approximate to $\mu_t$ with the modified approach. Therefore, comparison of these simulation results displays that the modified blast resistant design depending on $\varphi_y^{rc}$ and $\boldsymbol{C}_{\mu}^{rc}$ is more effective in keeping $y^{rc}_m$ and $\mu^{rc}$ under control with respect to the design targets.
Figure 6.6 Deflection history of the designed simply supported beams under the given blast loading (Example I)

(a). Designed with the modified approach

(b). Designed with the original approach

Figure 6.7 Deflection history of the designed cantilever walls under the given blast loading (Example II)
Figure 6.8 Deflection history of the designed fixed/roller-supported beams under the given blast loading (Example III)
6.3 Estimation of Response Distributions For Designed Members With Error Indices

6.3.1 Analytical procedures

Another important application of the error indices is that they provide a simpler way for the prediction of the responses \((y_m^{rc} \text{ and } \mu^{rc})\) for the designed member at its significant point under a possible blast loading without carrying out nonlinear dynamic analysis on it. By transforming Equations (6.1) and (6.2), \(y_m^{rc}\) and \(\mu^{rc}\) are obtained as

\[
y_m^{rc} = y_m (1 - f(\rho, SC) - e_y)
\]

(6.9)

\[
\mu^{rc} = \mu (1 - g(\rho, SC) - e_u)
\]

(6.10)

Since within the original performance-based blast resistant design using energy spectra developed in Chapter 3, the members are so designed that the responses of their equivalent SDOF systems \((y_m^{eq} \text{ and } \mu^{eq})\) are controlled to be exactly equal to the PAR-DTs of \(y_m^{eq} \text{ and } \mu^{eq}\), the error indices defined in Equations (6.1) and (6.2) also reflect the relationships of \(y_m^{rc} \text{ and } \mu^{rc}\) with \(y_m^{eq} \text{ and } \mu^{eq}\). As a result, substituting \(y_m^{eq} \text{ and } \mu^{eq}\) with \(y_m^{rc} \text{ and } \mu^{rc}\) respectively into Equations (6.9) and (6.10) produces

\[
y_m^{rc} = y_m^{eq} (1 - f(\rho, SC) - e_y)
\]

(6.11)

\[
\mu^{rc} = \mu^{eq} (1 - g(\rho, SC) - e_u)
\]

(6.12)

According to Equations (6.11) and (6.12), the responses of \(y_m^{rc} \text{ and } \mu^{rc}\) of the designed member under a possible blast loading can be estimated from the corresponding \(y_m^{eq} \text{ and } \mu^{eq}\), which are computationally simple and straightforward.
By converting the member into an equivalent SDOF system \([B1]\), these parameters including equivalent mass \((m_e)\), equivalent force \((F_e)\), equivalent initial stiffness \((k_e)\), equivalent ultimate strength \((R_m)\), natural period \((T)\), elastic displacement \((y_e = R_m / k_e)\), and elastic energy \((E_{el} = k y_e^2 / 2)\) can be obtained. Further, \(y_{me}^{eq}\) and \(\mu^{eq}\) are effortlessly solved with these two expressions

\[
y_{me}^{eq} = \frac{E_{max}}{k_e y_e^{eq}} + \frac{\mu^{eq}}{2}
\]

\[
\mu^{eq} = \frac{y_{me}^{eq}}{y_e^{eq}}
\]

where \(E_{max}\) is computed with the non-dimensional energy factor \((C = E_{max} / E_{el})\) found from the energy spectra according to the ratios \(t_d / T\) and \(F_1 / R_m\) in Chapter 3.

However it should be pointed out that unlike the situation where the responses of \(y_{me}^{rc}\) and \(\mu^{rc}\) could be exactly determined through nonlinear dynamic finite element analysis, Equations (6.11) and (6.12) only provide an approximate way to predict \(y_{me}^{rc}\) and \(\mu^{rc}\) from the respective \(y_{me}^{eq}\) and \(\mu^{eq}\) since \(e_y\) and \(e_\mu\) are two uncertain nominal random variables following normal distribution as described in Chapter 5. Considering the random effects of \(e_y\) and \(e_\mu\), the coefficients of variation (COV) for \(y_{me}^{rc}\) and \(\mu^{rc}\) are obtained as

\[
COV(y_{me}^{rc}) = \frac{\sigma_{e_y}}{1 - f(\rho, SC)}
\]

\[
COV(\mu^{rc}) = \frac{\sigma_{e_\mu}}{1 - g(\rho, SC)}
\]
where $\sigma_{e_y}$ and $\sigma_{e_{\mu}}$ are standard deviations of $e_y$ and $e_{\mu}$ listed in Chapter 5. The COVs for $y_{m}^{rc}$ and $\mu^{rc}$ have no relationship with their respective $y_{m}^{eq}$ and $\mu^{eq}$ and they tends to change with $\rho$ and SC of the member. The trends are shown in Figure 6.9. It is evident that Equations (6.11) and (6.12) are more effective in the prediction of $y_{m}^{rc}$ and $\mu^{rc}$ for the simply supported, fixed/roller-supported and cantilever members under blast conditions. The maximum COVs of $y_{m}^{rc}$ for these members are generally less than 0.03, representing a significant concentration of $y_{m}^{rc}$ around their mean values. The COVs of $\mu^{rc}$ for these types of members rang from 0.05 to 0.18 and rise gradually with the decline in $\rho$. As for fixed/fixed members, the predicted $y_{m}^{rc}$ is also concentrated around its mean values with the maximum COV less than 0.08, however for $\mu^{rc}$ predicted from Equation (6.12), it has some deviation when $\rho$ is less than 1.0 %.

![Figure 6.9 COVs for $y_{m}^{rc}$ and $\mu^{rc}$ due to the random effect of $e_y$ and $e_{\mu}$](image_url)
6.3.2 Implementation into probabilistic performance assessment

The combination of Equations (6.11) and (6.12) with the probabilistic methods such as the first-order second-moment (FOSM) method, or the Monte-Carlo simulation [B13, H8, K5, M18] provides a simpler solution for the probabilistic problem of reinforced concrete flexural members in blast conditions where the performance functions are defined in terms of maximum displacement and/or displacement ductility responses together with the corresponding performance criterion values. As an illustration, examples of the members designed above with modified approach are exhibited in which the probabilities of their responses in different performance levels are required. Referring to current literatures [T1, T2, M3, B4, S6], the performance levels in these examples are defined in three ways: displacement, displacement ductility factor, or both of them. Accordingly, the probability at the $i$th performance level can be calculated with any of the following expressions

(b). COV for $\mu'^{\circ}$ predicted from Equation (6.12)

Figure 6.9 (Continued)
\[ P_i = P[(y - y_i) < 0] - P[(y - y_{i-1}) < 0] \]  
\[ P_i = P[(\mu - \mu_i) < 0] - P[(\mu - \mu_{i-1}) < 0] \]  
\[ P_i = P\{(y - y_i) < 0] \cap [(\mu - \mu_i) < 0]\} - P\{(y - y_{i-1}) < 0] \cap [(\mu - \mu_{i-1}) < 0]\} \]

where \( y \) and \( \mu \) are member's maximum displacement and displacement ductility responses accounting for different types of uncertainties, \( y_i \) and \( \mu_i \) are upper limits of the \( i \)th performance level as listed in Table 4.3. \( P_i \) is the probability that the member's responses will locate within the \( i \)th performance level. The physical meanings of (6.17c) can be explained in Figure 6.10 where the shadow area displays the field of the \( i \)th performance level.

![Figure 6.10 The field of the \( i \)th performance level](image)

The uncertainties of different variables including \( f_s, f_c, E_s, E_c, A_s, d, \) and \( l \) are considered in the probabilistic performance assessment of these three design examples. Their means values are taken as those employed in the design process while their COVs [K20, M4, M5, V1] are listed in Table 6.4. The blast loadings bring out the most uncertainty to the structural responses, however to date not
enough information is available on the random properties of blast loadings applied to a structure in its lifetime. In this example it is assumed that $P_r$ and $t_d$ follow normal distributions with the mean values equal to those used in the design and the COVs shown in Table 6.4. $e_y$ and $e_\mu$ are two additional independent random variable that link $y^{rc}_m$ and $\mu^{rc}$ with the respective $y^{eq}_m$ and $\mu^{eq}$. Their distributions can be found in Chapter 5 with the zeros of mean values and the corresponding standard deviations at various SCs.

**Table 6.4 Random variables for the designed members**

<table>
<thead>
<tr>
<th>Variables</th>
<th>$f_s$</th>
<th>$E_s$</th>
<th>$f_c$</th>
<th>$E_c$</th>
<th>$A_s$</th>
<th>$d$</th>
<th>$l$</th>
<th>$P_r$</th>
<th>$t_d$</th>
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<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
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<td>0.08</td>
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<td>0.15</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
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</table>

The probabilistic performance assessment of the designed members involving the uncertainty of the above variables is accomplished through Monte-Carlo simulation method. The choice of Monte-Carlo simulation for this case is justified because the procedure to predict $y^{rc}_m$ and $\mu^{rc}$ with Equations (6.11) and (6.12) is very simple. As a consequence, the simulation sample size (20000 in these examples) can be selected relatively large so as to achieve an accurate estimation without consuming much computational time and work. The results of $P_i$ for these designed members’ responses ($y^{rc}_m$ and $\mu^{rc}$) are computed as listed in Table 6.5. For a comparison, by replacing $y$ and $\mu$ respectively with $y^{eq}_m$ and $\mu^{eq}$ into Equation (6.15), $P_i$ for the equivalent SDOF systems of these members ($y^{eq}_m$ and $\mu^{eq}$) are also evaluated with the results shown in Table 6.5.
Table 6.5 Probabilities at various performance levels for the designed members under blast conditions

<table>
<thead>
<tr>
<th>Performance level</th>
<th>$y_{m}^{rc}$</th>
<th>$\mu^{rc}$</th>
<th>$y_{m}^{\mu^{rc}}$ &amp; $\mu^{\mu^{rc}}$</th>
<th>$y_{m}^{eq}$</th>
<th>$\mu^{eq}$</th>
<th>$y_{m}^{\mu^{eq}}$ &amp; $\mu^{\mu^{eq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.0004</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0.0020</td>
<td>0.0910</td>
<td>0.0016</td>
<td>0.0076</td>
<td>0.0004</td>
<td>0.0002</td>
</tr>
<tr>
<td>D</td>
<td>0.5346</td>
<td>0.5988</td>
<td>0.4798</td>
<td>0.7014</td>
<td>0.0662</td>
<td>0.0658</td>
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<tr>
<td>E</td>
<td>0.4634</td>
<td>0.3098</td>
<td>0.5186</td>
<td>0.2910</td>
<td>0.9334</td>
<td>0.9340</td>
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Design example II: the cantilever wall

<table>
<thead>
<tr>
<th>Performance level</th>
<th>$y_{m}^{rc}$</th>
<th>$\mu^{rc}$</th>
<th>$y_{m}^{\mu^{rc}}$ &amp; $\mu^{\mu^{rc}}$</th>
<th>$y_{m}^{eq}$</th>
<th>$\mu^{eq}$</th>
<th>$y_{m}^{\mu^{eq}}$ &amp; $\mu^{\mu^{eq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.0028</td>
<td>0.0522</td>
<td>0.0014</td>
<td>0.0032</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0.5848</td>
<td>0.7232</td>
<td>0.5284</td>
<td>0.6020</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
<tr>
<td>D</td>
<td>0.4116</td>
<td>0.2176</td>
<td>0.4626</td>
<td>0.3942</td>
<td>0.1536</td>
<td>0.1536</td>
</tr>
<tr>
<td>E</td>
<td>0.0008</td>
<td>0.0070</td>
<td>0.0076</td>
<td>0.0006</td>
<td>0.8454</td>
<td>0.8454</td>
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</table>

Design example III: the fixed/roller-supported beam

<table>
<thead>
<tr>
<th>Performance level</th>
<th>$y_{m}^{rc}$</th>
<th>$\mu^{rc}$</th>
<th>$y_{m}^{\mu^{rc}}$ &amp; $\mu^{\mu^{rc}}$</th>
<th>$y_{m}^{eq}$</th>
<th>$\mu^{eq}$</th>
<th>$y_{m}^{\mu^{eq}}$ &amp; $\mu^{\mu^{eq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.0002</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0.0022</td>
<td>0.1252</td>
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<td>0.0016</td>
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</tr>
<tr>
<td>D</td>
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<td>0.5984</td>
<td>0.4858</td>
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<td>0.0052</td>
<td>0.0052</td>
</tr>
<tr>
<td>E</td>
<td>0.4578</td>
<td>0.2762</td>
<td>0.5120</td>
<td>0.4728</td>
<td>0.9948</td>
<td>0.9948</td>
</tr>
</tbody>
</table>
It is indicated from Table 6.5 that $P_i$ assessed with $y_m^{eq}$ alone for the simply supported and fixed/roller-supported beam is mainly concentrated on Level D while those with $\mu^{eq}$ alone is focused on Level E. As for the cantilever wall, $P_i$ from $y_m^{eq}$ alone is the most in Level C as compared to that in Level E from $\mu^{eq}$ alone. Such inconsistent results are mainly caused by the fact that these members’ displacement ductility responses are significantly overestimated by $\mu^{eq}$ while the maximum displacement responses slightly underestimated by $y_m^{eq}$. Accordingly, the change trends of $P_i$ assessed on the basis of both $y_m^{eq}$ and $\mu^{eq}$ are generally estimated too highly at Level E. To overcome this, it is necessary to involve the error indices in Equations (6.11) and (6.12) to predict $y_m^{rc}$ and $\mu^{rc}$ from $y_m^{eq}$ and $\mu^{eq}$, and so apply them into the assessment. The relatively reasonable $P_i$ at different performance levels are obtained as indicated in Table 6.5 where, for instance, $P_i$ evaluated from $y_m^{rc}$, $\mu^{rc}$ or both of them are all convergent in Level C for the cantilever wall. Therefore, the distributions of $y_m^{rc}$ and $\mu^{rc}$ derived with Equations (6.11) and (6.12) depending on error indices are effective in the probabilistic performance assessment of designed members under the blast conditions accounting for different uncertainties.

### 6.4 Summaries

The applications of the formula of $\xi_y^{rc}$ and $\xi_\mu^{rc}$ proposed in Chapter 5 are discussed in these two fields: modification of the performance-based blast resistant design approach using energy spectra in controlling $y_m^{rc}$ and $\mu^{rc}$, and estimation of the distributions of $y_m^{rc}$ and $\mu^{rc}$ for the designed member under blast conditions accounting for uncertainties.

Through the adjustment of PAR-DTs with $\xi_y^{rc}$ and $\xi_\mu^{rc}$, the modification of the design approach is accomplished with iterative procedures. The implementation of the modified design approach into three design examples indicates that more iterative
steps are needed to reach convergence as compared to those of the original design procedure. However the responses of $y_m^{rc}$ and $\mu^{rc}$ for the member designed with the modified design approach from the nonlinear numerical analysis are controlled more approximately to their targets and restricted to be a little on the conservative sides.

The estimation of $y_m^{rc}$ and $\mu^{rc}$ for the designed members under possible blast forces is much simpler on the basis of $\xi_y^{rc}$ and $\xi_{\mu}^{rc}$ from the corresponding $y_m^{eq}$ and $\mu^{eq}$ that are computationally straightforward. By the combination of this procedure with Monte-Carlo simulation, the probabilistic performance of the members designed with the modified method under blast conditions can be conveniently and effectively evaluated accounting for different types of uncertainties.
CHAPTER SEVEN

PERFORMANCE-BASED BLAST RESISTANT DESIGN OF
MULTI-STOREY REINFORCED CONCRETE FRAME
STRUCTURES WITH EQUIVALENT STATIC FORCE (ESF)
— PART I: THEORETICAL BASIS

Synopsis

Control of the maximum inter-storey drift ratio (MIDR) for a multi-storey reinforced concrete frame structure becomes critical when that structure is subjected to a blast loading from a distant intense surface explosion. For this purpose, a new design method is presented based on the transformation of the lateral blast force into an equivalent static force (ESF). The ESF is calculated in such a manner that the same MIDR effect will be produced as that under the blast condition. This chapter concentrates on the calculation model of ESF for a single-degree-of-freedom (SDOF) system as well as the design method based on ESF with the requirement of controlling its maximum displacement response. Implementations of the design method into several examples are demonstrated while the verifications of the displacement responses of the designed SDOF system are performed with nonlinear dynamic analyses. The analytical results indicate that the target displacement is well met for the designed SDOF system in resisting the given blast loading. The extension of the calculation model of ESF and the corresponding design method with ESF for a SDOF system to a multi-storey reinforced concrete frame structure will be discussed in Chapter 8.

Keywords: Maximum inter-storey drift ratio, Equivalent static force
Maximum displacement response, SDOF system
7.1 Introduction and Background

As discussed in Chapter 2, under a distant intense blast condition where the blast shock wave can be simplified into a planar wave, the dynamic responses for multi-storey reinforced concrete frame structures can be approximately divided into two response stages. At response stage II, a significant global response in terms of MIDR may be provoked inducing a certain degree of global damage and therefore, in the design of multi-storey reinforced concrete frame structures against the distant blast loadings, controlling their MIDR responses becomes an important consideration. Regarding this problem, in order to ensure the global response within the expected performance level, different levels of side-sway limits are given for the design of single-storey rigid frame structures in the current design guidance ([N4, B4]) according to the operational needs of the facility and the needs for reusability. Their values in terms of inter-storey drift ratio and the description of corresponding response levels and damage degrees are listed in Table 2.1.

Blast loadings act dynamically on structures with short durations, however the dynamic loadings are extremely difficult to handle within the structural design since they could not be directly implemented into the calculation of the interior forces of the structural members. In the seismic/wind resistant design procedures, the dynamic seismic/wind actions are generally transformed into the static loadings, which are deemed to be able to produce equivalent effects on structures [C10, C11, D1, J1, K14, P2, P6, R3, Z2]. Similarly if an equivalent static force (ESF) applied to the structure with the magnitude and direction that could closely represent the effects of the distant blast loadings can be found, it will greatly facilitate the blast resistant design.

The implementation of ESF into the blast resistant design for a single-storey rigid frame subjected to a relatively low blast overpressure has been suggested in the manual by NFEC [N4]. The dynamic load factors are provided for establishing equivalent static loads for the frame mechanism. Based upon the mechanism method, as employed in static plastic design, estimations are made for the required
plastic bending capacity. However these dynamic load factors were approximate and made no distinction for different end conditions. They were only expected to result in the estimation of the required resistance for a trial design. In order to confirm that a trial design meets the recommended deformation criteria, a rigorous frame analysis has to be performed.

This chapter together with Chapter 8 aims to present a new performance-based blast resistant design method for multi-storey reinforced concrete frame structural system based on the equivalent static force (ESF) of the distant blast action. For this purpose, this chapter concentrate on the description of the calculation model of the ESF and the corresponding design method based on the ESF for a SDOF system. Three illustrative examples are employed to demonstrate the implementation of the method while verifications of the displacement responses of the designed SDOF systems are carried out through nonlinear dynamic analysis. In Chapter 8, this presented design method is extended to a performance-based design for the multi-storey reinforced concrete frame structure by reasonably selecting the MIDR response as a global performance indicator, where the frame structure could not be simply idealized into a SDOF system in the blast condition.

Basically the developed design method with ESF is for the design of a multi-storey reinforced concrete frame structural system in controlling its MIDR response, which occurs at a time apparently later than the blast loading duration \( t_d \) as indicated by numerical simulations in Chapter 2. The description of the calculation model of ESF and the design method with ESF for a SDOF system herein is only to provide a theoretical basis. Therefore the assumption that the peak response (maximum displacement for a SDOF system or MIDR response for a frame structure) takes place after the duration \( t_d \) is generally adopted in Chapters 7 and 8. Only the SDOF systems that satisfy this assumption are of concern in this chapter.
7.2 ESF for a SDOF System

7.2.1 Process for the construction of ESF

Because of the short duration of the blast loading, the vibration of a SDOF system after reaching the peak response will be limited within its elastic range inducing no further cumulative damages \[B1\]. Therefore for a well-defined SDOF system, the maximum displacement response can adequately characterize its damage status in blast events. Under such conditions, if there exists a force, which makes it possible for the SDOF system to experience exactly the same maximum displacement response when statically applied, this force is called the equivalent static force (ESF) of the blast loading. To calculate the ESF of the blast loading on a SDOF system, a model is presented herein with its process plotted in Figure 7.1.

![Figure 7.1 (a). Dynamic response process of the original SDOF system under the blast force](image-url)
Chapter 7. Performance-Based Design of Frame Structures with ESF — Part I: Theoretical Basis

Figure 7.1 Process for the construction of ESF for a SDOF system

(b). Equivalent static system

(c). Releasing the strain energy in spring $K_a$

(d). Recovering to the original SDOF system plus the action of $P$

(e). Equivalent static force

Figure 7.1 Process for the construction of ESF for a SDOF system
The blast force can be reasonably simplified into a triangular pulse only if the peak pressure and impulse are preserved [H1], hence the function of the blast force is written as

$$F(t) = \begin{cases} F_1 \left(1 - \frac{t}{t_d}\right) & t \leq t_d \\ 0 & t > t_d \end{cases}$$  \hspace{1cm} (7.1)$$

where $t_d$ is the duration and $F_1$ is the peak amplitude of the blast force. Since after $t_d$, the blast force will keep zero, no additional external energy is produced and the total energy within the SDOF system consisting of the kinetic energy and the strain energy will keep constant whose magnitude determines the maximum response by ignoring the damping effects [B1, K4]. The constant total energy can be obtained from the response state of the system at $t_d$ plotted in Figure 7.1a, written as

$$W = W_{k,td} + W_{s,td}$$  \hspace{1cm} (7.2)$$

where $W$ is the constant total energy of the SDOF system; $W_{k,td}$ and $W_{s,td}$ are the kinetic and strain energy at the time $t_d$ respectively. In order to simulate the energy components of the SDOF system equally at this time step, an equivalent static SDOF system is constructed as shown in Figure 7.1b, where an additional elastic spring is added to the original SDOF system. It is proposed that under a certain external static force $P$, the equivalent static system experience the same displacement response as $y_{td}$ (the dynamic response of the original SDOF system at $t_d$) and therefore the strain energies within the spring $K_s$ in both systems are identical. In order to model the kinetic energy, the strain energy $W_a$ within the additional spring $K_a$ should be equal to $W_{k,td}$, thus

$$M \dot{y}_{td}^2 / 2 = K_a y_{td}^2 / 2$$  \hspace{1cm} (7.3)$$
and

\[ K_a = M \dot{y}_{td}^2 / y_{td}^2 \]  

(7.4)

where \( \dot{y}_{td} \) is the velocity of the original SDOF system at the time of \( t_d \) under \( F(t) \), \( K_a \) is the elastic stiffness of the additional spring. From the equilibrium of the equivalent static system

\[ P = F_s + F_a = F_s + K_a y_{td} = F_s + M \dot{y}_{td}^2 / y_{td} \]  

(7.5)

where \( F_s \) and \( F_a \) is the force produced respectively by the original and additional spring (\( K_s \) and \( K_a \)) in the equivalent static system in Figure 7.1b.

After \( t_d \), the kinetic energy \( W_{k,td} \) will be gradually transformed into the strain energy causing further displacement for the original SDOF system until the maximum response \( y_m \) is reached as shown in Figure 7.1a. For modelling this process with the equivalent static system, the strain energy \( W_a \) in the additional spring \( K_a \) needs to be released statically in such a way that this part of energy is transferred to the original spring \( K_s \) as shown in Figure 7.1c. With the support of the additional spring statically moving toward the spring until the support reaction force \( RF \) decreases to zero, the equivalent static system is recovered to the original SDOF system with the static force \( P \) exerting on it in Figure 7.1d. However it should be noted that during this process, an amount of extra positive external energy is produced by \( P \) together with the declining \( RF \); and a relatively larger displacement response \( y_{mp} \) will be induced than \( y_m \) for the original SDOF system under the blast condition. Thus an ESF factor (\( \lambda \)) less than one is applied to \( P \) producing the ESF (\( P_{st} \)) that creates the same maximum response as \( y_m \) when statically applied to the original SDOF system as shown in Figure 7.1e

\[ P_{st} = \lambda P \]  

(7.6)
7.2.2 ESF factor

For a SDOF system with the elastic-perfectly-plastic resistance function, the close form solution of the ESF factor $\lambda$ is derived with respect to three different cases according to the response states at the time $t_d$ and $t_m$ (the time for maximum displacement response) as plotted in Figure 7.2. They are

- Case I: in the elastic state at $t = t_d$ as well as $t = t_m$;
- Case II: in the elastic state at $t = t_d$ while in the plastic state at $t = t_m$;
- Case III: in the plastic state at $t = t_d$ and $t = t_m$.

![Figure 7.2](image-url)
**Case I:** In this case, since the SDOF system is still within its elastic limit during the whole response process as shown in Figure 7.2a, the constant total energy at $t_d$ and $t_m$ can be written as

\[ W = K_s y_{id}^2 / 2 + M \dot{y}_{id}^2 / 2 \]  \hspace{1cm} (7.7)

and

\[ W = K_s y_{m}^2 / 2 \]  \hspace{1cm} (7.8)

where $K_s$ is the initial stiffness of the elastic-perfectly-plastic SDOF system. Equating Equations (7.7) with (7.8) leads to

\[ M \dot{y}_{id}^2 = K_s (y_{m}^2 - y_{id}^2) \]  \hspace{1cm} (7.9)

Subjecting Equation (7.9) into Equation (7.5) and considering that the spring $K_s$ is in its elastic range at $t_d$

\[ P = K_s y_m \frac{y_m}{y_{id}} \]  \hspace{1cm} (7.10)

To meet the requirement that the same $y_m$ appear for the SDOF system under the ESF, $P_s$ should be equal to its resistance at the displacement $y_m$, which in the elastic range is given as

\[ P_s = K_s y_m \]  \hspace{1cm} (7.11)

Substituting Equations (7.10) and (7.11) into Equation (7.6), the ESF factor $\lambda$ in this case is determined as

\[ \lambda = \frac{y_{id}}{y_m} \]  \hspace{1cm} (7.12)
Case II: For the second case, where the SDOF system has entered the plastic response stage, Equation (7.8) for the calculation of the constant total energy at \( t_m \) changes into

\[
W = \frac{1}{2} K_s y_e (2y_m - y_e) \tag{7.13}
\]

where \( y_e \) is the elastic limit displacement of the SDOF system. Equating the Equations (7.7) with (7.13) obtains

\[
M \ddot{y}_{id} = K_s (2y_e y_m - y_e^2 - y_{id}^2) \tag{7.14}
\]

By subjecting Equation (7.14) into Equation (7.5) and considering that the spring \( K_s \) is in its elastic range at \( t_d \), \( P \) is given as

\[
P = K_s y_e \frac{(2y_m - y_e)}{y_{id}} \tag{7.15}
\]

In order to statically produce the same maximum displacement as \( y_m \) in the blast condition, which is beyond the elastic limit of the SDOF system with elastic perfect plastic resistance function, the ESF \( (P_{st}) \) should be identical with the ultimate strength, thus

\[
P_{st} = K_s y_e \tag{7.16}
\]

The value of \( \lambda \) is finally obtained by subjecting Equations (7.15) and (7.16) into Equation (7.6) as

\[
\lambda = \frac{y_{id}}{2y_m - y_e} \tag{7.17}
\]
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Case III: Since the SDOF system has entered its plastic response stage before $t_d$, $W$ for the elastic-perfectly-plastic SDOF system at $t_d$ is given by

$$W = \frac{1}{2} K_s y_e (2y_{td} - y_e) + \frac{1}{2} M \dot{y}_{td}^2$$  \hspace{1cm} (7.18)

In this case, Equation (7.13) is also valid for expressing $W$ at $t_m$, therefore equating Equations (7.13) with (7.18) brings out

$$M \dot{y}_{td}^2 = 2K_s y_e (y_m - y_{td})$$  \hspace{1cm} (7.19)

By taking $F_s = K_s y_e$ and subjecting Equations (7.19) into Equations (7.5), $P$ is gained as

$$P = K_s y_e \frac{(2y_m - y_{td})}{y_{td}}$$  \hspace{1cm} (7.20)

With Equation (7.16) for the evaluation of $P_{st}$, $\lambda$ is derived from Equations (7.6) and (7.20) as

$$\lambda = \frac{y_{td}}{2y_m - y_{td}}$$  \hspace{1cm} (7.21)

7.2.3 Calculation model for ESF

By summarizing the above analyses, a model to calculate the ESF for an elastic-perfectly-plastic SDOF system is presented in Equation (7.22). An extra variable $X$ is introduced, which is for the convenience of extending this model to multi-storey reinforced concrete frame structures as discussed in Chapter 8. Obviously there is a $45^\circ$ linear relationship between $\lambda$ and $X$ as shown in Figure 7.3. The physical
meanings for the other variables have been well defined previously.

\[
\begin{align*}
\lambda &= X \\
X &= \begin{cases} 
  \frac{y_{td}}{y_m} & y_m \leq y_c \\
  \frac{y_{td}}{(2y_m - y_e)} & y_{td} \leq y_c \text{ and } y_m \geq y_c \\
  \frac{y_{td}}{(2y_m - y_{td})} & y_{td} \geq y_c
\end{cases}
\end{align*}
\] (7.22)

It should be pointed out that this model does not attempt to calculate the ESF with the purpose of assessing the maximum response of a particular SDOF system under the blast condition, but provides a powerful tool in designing the ultimate strength of the system to achieve the target of the displacement response. The design method based on this model is addressed in the following section.

Figure 7.3 Distribution of the ESF factor for the SDOF system
7.3 Design of a SDOF System with the ESF

For the elastic-perfectly-plastic SDOF systems with the stiffness $K_s$, the mass $M$ and the blast loading $F_1$ and $t_d$ acting on them, the relationship between their ultimate strengths $R_m$ and the corresponding maximum displacement responses $y_m$ can be explained as follows. Supposing that there are two such SDOF systems but with different ultimate strengths $R_{m1} < R_{m2}$, two facts are considered that:

1) during any short period of time $\Delta t$ within $t_d$, the displacement response for the first system with $R_{m2}$ will not exceed that of the second system with $R_{m1}$, accordingly the total external work by the blast loading on the first system will be equal to or less than the correspondent on the second system;

2) with the same displacement response, the energy absorbing capacity by the deformation of the first system will not be less than that of the second one.

Accordingly, it can be derived that with the increase in the ultimate strength of the system, the maximum displacement response will decrease gradually as shown in Figure 7.4 where $R_{m1} < R_{m2} < R_{m3}$ and $y_{m1} > y_{m2} > y_{m3}$. However when $R_m$ reaches the critical value of $R_{mref}$ and $y_m$ declines to $y_{mref}$, the whole response of the system is located in its elastic range. A further increase in $R_m$ will have little effect on its maximum displacement response. That means that for this SDOF system with $K_s$, $M$, $F_1$ and $t_d$, its maximum response will never be smaller than $y_{mref}$, which can be obtained from dynamic analysis by assuming the SDOF system to be fully elastic.

Design of the ultimate strength for a SDOF system satisfying the given target maximum displacement response $y_t$ is to find the value of $R_m$ corresponding to $y_m = y_t$. If $y_t \geq y_{mref}$, then the maximum response of the designed SDOF system
will locate or be beyond its elastic limit state as shown in Figure 7.4. In such a case the ESF defined above is equal to the ultimate strength, that is, \( R_m = P_{st} \). Therefore \( R_m \) and \( y_t \) should satisfy Equation (7.22) by replacing \( P_{st} \) and \( y_m \) with \( R_m \) and \( y_t \) respectively

\[
\begin{align*}
R_m &= \lambda P \\
P &= F_s + F_a \\
\lambda &= X \\
X &= \begin{cases} 
    y_{td} / y_t & y_t \leq y_e \\
    y_{td} / (2y_t - y_e) & y_{td} \leq y_e \text{ and } y_t \geq y_e \\
    y_{td} / (2y_t - y_{td}) & y_{td} \geq y_e
\end{cases}
\end{align*}
\tag{7.23}
\]

On the other hand if \( y_t < y_{mref} \), there exists no solution for \( R_m \) and in order to reach such a target \( y_t \), it needs to increase \( K_s \) or \( M \). According to the discussion, an iterative procedure is presented to gain this unique \( R_m \) based on the model of Equation (7.23). The flowchart is plotted in Figure 7.5.

![Figure 7.4 Relationship between the ultimate strength and the maximum displacement response](#)
Figure 7.5 Flowchart of blast resistant design of an elastic-perfectly-plastic SDOF system with ESF (where \( \varepsilon \) is an arbitrarily small value).

7.4 Illustrative Examples

To illustrate the design procedures with ESF, three illustrative examples are presented as shown in Table 7.1 where a variety of target displacements are required for the SDOF systems in resisting different blast forces. The iterative procedures at each step of these design examples are demonstrated in Tables 7.2 to 7.4 where the initial ultimate strength \( R_{m0} \) is taken as the multiplication of \( K_s \) with \( y_{\text{movf}} \). The convergence condition of \( |(R_{m1} - R_{m0}) / R_{m0}| \leq 0.001 \) is employed.
### Table 7.1 Parameters in three design examples

<table>
<thead>
<tr>
<th>Design Example</th>
<th>$F_1$ (N)</th>
<th>$t_d$ (s)</th>
<th>$M$ (kg)</th>
<th>$K_s$ (N/mm)</th>
<th>$y_r$ (mm)</th>
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### Table 7.2 Design procedures for example I

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<th>$y_{mref}$ (mm)</th>
<th>$R_{m0}$ (N)</th>
<th>$y_r$ (mm)</th>
<th>$y_{id}$ (mm)</th>
<th>$\dot{y}_{id}$ (mm/s)</th>
<th>$P$ (N)</th>
<th>$\lambda$ (N)</th>
<th>$R_{mI}(P_{st})$ (N)</th>
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### Table 7.3 Design procedures for example II

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<th>$R_{m0}$ (N)</th>
<th>$y_r$ (mm)</th>
<th>$y_{id}$ (mm)</th>
<th>$\dot{y}_{id}$ (mm/s)</th>
<th>$P$ (N)</th>
<th>$\lambda$ (N)</th>
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Table 7.4. Design procedures for example III

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<th>$\dot{y}_{id}$</th>
<th>$P$</th>
<th>$\lambda$</th>
<th>$R_m (P_s)$</th>
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<td>(N)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm/s)</td>
<td>(N)</td>
<td></td>
<td>(N)</td>
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<td>445.08</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>445.89</td>
<td>22.30</td>
<td>22.32</td>
<td>496.24</td>
<td>1549.4</td>
<td>0.287</td>
<td>445.08</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>445.08</td>
<td>22.25</td>
<td>22.32</td>
<td>496.24</td>
<td>1548.6</td>
<td>0.287</td>
<td>444.85</td>
<td></td>
</tr>
</tbody>
</table>

* $y_i < y_{mref}$

In design examples I and II, $y_i$ is greater than $y_{mref}$ thus $R_m$ can be iteratively determined with respective to $K_s$ listed in Table 7.1. However $y_i$ is less than $y_{mref}$ in design example III where to find the solution of $R_m$ of the SDOF system, $K_s$ given in Table 7.1 should be enlarged. By taking twice of the initial stiffness, the design is carried out and the results are shown in Table 7.4. It is indicated from the iterative procedures that there is no difficulty in reaching convergence for the design of the SDOF system with the ESF so as to find the unique $R_m$.

To evaluate the displacement responses for the SDOF systems designed based on ESF, nonlinear time history analyses are executed on them against the respective blast forces and the results are demonstrated in Figures 7.6 to 7.8. It is obvious that the maximum displacements of the designed SDOF systems exactly reach their respective objective values. Thus the design based on the ESF can effectively control the maximum displacement for the designed SDOF system in resisting the given blast force.
Chapter 7. Performance-Based Design of Frame Structures with ESF — Part I: Theoretical Basis

Figure 7.6 Displacement responses of the designed SDOF system under the given blast force (Example I)

Figure 7.7 Displacement responses of the designed SDOF system under the given blast force (Example II)
Figure 7.8 Displacement responses of the designed SDOF system under the given blast force (Example III)

### 7.5 Summaries

A calculation model for the ESF of the blast force and the design method using ESF for the elastic-perfectly-plastic SDOF systems are discussed in this chapter, which provide the theoretical basis for the performance-based blast resistant design of multi-storey reinforced concrete frame structures under distant explosion conditions in Chapter 8.

The model for the computation of ESF is derived based on the assumption that the peak response of the system is present after the blast loading duration $t_d$. An equivalent static SDOF system is derived which simulates the energy components of the original SDOF system at time $t_d$. Based on the equilibrium of equivalent static system, an external static force is computed. However this force tends to produce a larger maximum displacement demand than that of the original SDOF system under the blast force. As a result, an ESF factor is introduced to reduce the
external static force to obtain the final ESF. With respect to three different response states at time \( t_d \) and \( t_m \), a closed-form solution of the ESF factor is obtained for the elastic-perfectly-plastic SDOF system. It is found that the ESF factor follows a 45° linear relationship with an X-variable that is \( y_{td} / y_m \), \( y_{td} / (2y_m - y_d) \), or \( y_{td} / (2y_m - y_{td}) \).

By replacing the ESF with the ultimate strength as well as the maximum displacement with the required design target displacement, the computational model of the ESF is implemented for the design of an elastic-perfectly-plastic SDOF system. The ultimate strength of the SDOF system is solved iteratively during the design process so as to satisfy the specified target displacement. The presented design procedures have been illustrated via three numerical examples, which show no difficulty in convergence of the iterative procedures. The maximum displacement responses of the designed SDOF systems under the given blast conditions can be controlled to be exactly equal to their corresponding targets, as verified from nonlinear dynamic analysis.


CHAPTER EIGHT

PERFORMANCE-BASED BLAST RESISTANT DESIGN OF MULTI-STOREY REINFORCED CONCRETE FRAME STRUCTURES WITH EQUIVALENT STATIC FORCE (ESF) — PART II: IMPLEMENTATION AND EVALUATION

Synopsis

The model for the calculation of equivalent static force (ESF) and the design method with ESF for single-degree-of-freedom (SDOF) systems are extended into a performance-based design for multi-storey reinforced concrete frame structures in controlling their maximum inter-storey drift ratios (MIDR) under distant explosion conditions. An empirical formula for the ESF factor involved in the ESF model is presented based on a number of sample points of this factor, which are obtained by comparing the nonlinear dynamic responses of frame structures with corresponding nonlinear static pushover analytical results. For demonstration, two six-storey reinforced concrete frame structures are designed with the developed method against different blast intensities for various required performance levels. Numerical verification of the design method indicates the MIDR responses of these two designed structures in comparison to their respective targets are somewhat conservative. The reasons that may lead to the conservative designs are discussed.

Keywords: Maximum inter-storey drift ratio, Equivalent static force, Multi-storey reinforced concrete frame structure
8.1 Introduction

A performance-based design method by using ESF is developed for a multi-storey reinforced concrete frame structure system in this study. Four basic assumptions are made as follows:

1) The maximum inter-storey drift ratio (MIDR) can be treated as a reasonable and sufficient response parameter for the identification of the global performance level, and the critical side-sway limits in term of inter-storey drift ratio documented in the ASCE special publication [B4] as listed in Table 2.1 can properly reflect different global performance levels for a frame structure;

2) The MIDR response will occur at a time later than the blast loading duration \( t_d \), therefore the further inter-storey drift ratio response after \( t_d \) occurs as a result of the transformation of kinetic energy into strain energy of the structure. For most multi-storey frame structures, this assumption is acceptable considering the relatively short duration \( t_d \) of the blast force;

3) The nonlinear relationship of displacement and resistance of the frame structure under the action of the lateral forces can be reasonably idealized an elastic-perfectly-plastic function, consequently the calculated ESF will be the minimum required ultimate strength of the structure system based on which the reinforcements of the structure are designed;

4) The model for calculation of ESF has a similar form to that of Equation (7.22) for a multi-storey reinforced concrete frame structure.

Based on above assumptions, the process for the construction of the ESF for a SDOF system can be extended to a multi-storey reinforced concrete frame structure in term of vectors, however there is a main difference in the determination of the ESF factors through numerical studies rather than closed form solutions. A simple empirical equation for the ESF factor is derived for the convenience of the blast resistant design with ESF. For illustration and evaluation of the developed design method, two design examples are given for six-storey reinforced concrete frame
structures under different blast loadings followed by the numerical verifications.

8.2 ESF for a Multi-storey Reinforced Concrete Frame Structure

8.2.1 Process for the construction of ESF

For a frame structure with \( n \) storeys, the ESF (\( P_{st} \)) is defined as the column vector of the force applied to each floor level, which produce the same MIDR response as that under the blast action when it is statically exerted on the frame structure, thus

\[
P_{st} = \{P_{st,1}, P_{st,2}, \ldots, P_{st,n}\}^T
\]  

(8.1)

where \( P_{st,i} \) represents the component of ESF at the \( i \)th floor level. The process of the construction of \( P_{st} \) is plotted in Figure 8.1. At the end of the blast loading duration \( t_d \), the frame structure responds with a distribution of displacement and velocity as shown in Figure 8.1a, where \( y_{td} = \{y_{td,1}, y_{td,2}, \ldots, y_{td,n}\}^T \) and \( y_{td,i} \) stands for the \( i \)th floor displacement at this time station. With the increase in time, the distributed kinetic energy as a function of the velocity will be gradually transformed into further deformation reaching the MIDR response \( \Delta_m \). Therefore, if the total kinetic energy within the part of the structure located halfway above and below the \( i \)th floor level at the time \( t_d \), which is denoted by \( W_{k,i,j} \) herein, is concentrated and represented by the strain energy \( W_{a,i,j} \) of an additional spring \( K_{a,i,j} \) at the corresponding floor level with the same deformation as \( y_{td,i} \), an equivalent static system in Figure 8.1b can be established with

\[
W_{a,i,j} = \frac{1}{2} K_{a,i,j} y_{td,i}^2 = W_{k,i,j}
\]  

(8.2)

or

\[
K_{a,i,j} = \frac{2W_{k,i,j}}{y_{td,i}^2}
\]  

(8.3)
where $K_{a,i}$ is the stiffness of the additional elastic spring at the $i$th floor level. According to the equilibrium of the equivalent static system, the external static force ($P = \{P_1, P_2, \ldots, P_n\}^T$) will include two parts: the recovery force of the frame structure ($F_s = \{F_{s,1}, F_{s,2}, \ldots, F_{s,n}\}^T$) with the deformation of $y_{id}$ and the force ($F_a = \{F_{a,1}, F_{a,2}, \ldots, F_{a,n}\}^T$) by additional springs, thus

$$P = F_s + F_a$$ (8.4)

where $F_{a,i}$ for the additional elastic spring at the deformation of $y_{id,i}$ is given as $F_{a,i} = K_{a,i}y_{id,i}$. Substituting Equation (8.3) into it produces

$$F_{a,i} = \frac{2W_{k,i}}{y_{id,i}}$$ (8.5)

In order to simulate the transformation of the kinetic energy into further deformation after $t_d$, the strain energy within the springs $K_{a,1}, K_{a,2}, \ldots, K_{a,n}$ are statically released in such a manner that their supports move towards the springs until the equivalent static system is recovered to the original frame structure with $P$ acting on it. However when considering the extra positive external work produced by $P$ together with the declining reaction force of the springs at their supports during the energy transformation process, a larger MIDR will be induced in this way than that of the frame structure under the blast loading ($\Delta_m$) as shown in Figure 8.1c. Thus an ESF factor $\lambda$ is introduced to decrease $P$ so as to produce the ESF ($P_{st}$), which creates the same MIDR as $\Delta_m$ when statically applied to the original frame structure as illustrated in Figure 8.1d

$$P_{st} = \lambda P$$ (8.6)
Figure 8.1 Process for the construction of ESF for a multi-storey reinforced concrete frame structure

(a). Dynamic response under the blast force

\[ \Delta_{td,m} = \max_{i=1}^{n} \left[ \frac{(y_{td,j} - y_{td,j-1})}{h_j} \right] \]

(b). Equivalent static system

\[ \Delta_s^p = \max_{i=1}^{n} \left[ \frac{(y_i^p - y_{i-1}^p)}{h_j} \right] > \Delta_m \]

(c). Static responses under \( P \)

\[ \Delta_s^s = \max_{i=1}^{n} \left[ \frac{(y_i^{s_s} - y_{i-1}^{s_s})}{h_j} \right] = \Delta_m \]

(d). Static responses under \( P_{st} \)

\[ P_{st,p} = \lambda P_n \]

\[ P_{st,1} = \lambda P_1 \]
8.2.2 ESF factor

For a multi-storey reinforced concrete frame structure under the blast condition, it is difficult to derive a closed form expression for $\lambda$ such as that in Equations (7.22) or (7.23), an empirical formula through numerical studies seems to be a practical solution. If a number of sample points (the accurate values of $\lambda$ for different frame structures under various blast loadings) are available, the empirical formula can be found with the optimum technique, which makes the closest estimation for these points. Therefore two main works are performed in the determination of the ESF factor formula, which are to get reliable sample points of $\lambda$ and to establish such a formula.

8.2.2.1 Sample points of the ESF factor

The sample point of $\lambda$ is obtained by comparing the MIDR response of a multi-storey reinforced concrete frame structure under the blast condition with the corresponding nonlinear pushover analytical result of the structure under an external lateral force with the same distribution as $P$. The procedures are listed in the following:

1. Establish a frame structure and a lateral blast loading acting on it;
2. Evaluate $y_{ad,i}$, $W_{ki}$ and MIDR within the whole dynamic response $\Delta_m$ with numerical simulation;
3. Calculate the recovery force $F_a$ by applying $y_{ad}$ to the structure through nonlinear static analysis;
4. Compute the external static force $P$ with Equation (8.4);
5. Carry out the nonlinear pushover analysis of the frame structure under a gradually increasing load vector with the same distribution as $P$;
6. Plot the resulted curves of load versus the inter-storey responses, and find the value of $\lambda$ corresponding to $\Delta_m$ as shown in Figure 8.2.
Sixty sample points of \( \lambda \) are evaluated by using above procedures, where four different planar frame structures are subjected to a variety of lateral blast forces applied to the exterior columns and the service loads uniformly distributed along the beams \( F_{ser} \) in a vertical direction. Additional masses along with the beams \( M_{add} \) are involved that are contributed by the adjacent floors. The details of the frame structures and the analytical results for \( \lambda \) are respectively shown in Tables 8.1 and 8.2, where \( \Delta_{id,m} \) (MIDR response at the time \( t_d \)) and \( \Delta_m \) (MIDR response during the whole dynamic responses) for the frame structures are obtained through the nonlinear finite element analyses with the program ABAQUS [A1, A5]. The details about the finite element models for the dynamic analyses and the static pushover analyses of the structures are described in Chapter 2.
### Table 8.1 Different frame structures involved in the determination of sample points of $\lambda$

<table>
<thead>
<tr>
<th>Structural Type</th>
<th>Storey level</th>
<th>Height (mm)</th>
<th>Width (mm)</th>
<th>Column section (mm × mm)</th>
<th>Beam section (mm × mm)</th>
<th>Column reinforcement Ratio* (%)</th>
<th>Beam reinforcement ratio** (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>End section</td>
<td>Mid-span section</td>
</tr>
<tr>
<td>I-1</td>
<td>1-2</td>
<td>4500</td>
<td>3×6000</td>
<td>600×600</td>
<td>300×600</td>
<td>1.5</td>
<td>1.1/0.5</td>
</tr>
<tr>
<td></td>
<td>3-6</td>
<td>3300</td>
<td>3×6000</td>
<td>500×500</td>
<td>300×600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-2</td>
<td>1-2</td>
<td>4500</td>
<td>3×6000</td>
<td>700×700</td>
<td>300×700</td>
<td>1.1</td>
<td>0.9/0.4</td>
</tr>
<tr>
<td></td>
<td>3-6</td>
<td>3300</td>
<td>3×6000</td>
<td>600×600</td>
<td>300×700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>1-4</td>
<td>3600</td>
<td>3×5400</td>
<td>500×500</td>
<td>300×500</td>
<td>2.0</td>
<td>1.5/0.7</td>
</tr>
<tr>
<td>III</td>
<td>1-2</td>
<td>4500</td>
<td>3×6000</td>
<td>800×800</td>
<td>300×700</td>
<td>1.0</td>
<td>0.8/0.5</td>
</tr>
<tr>
<td></td>
<td>3-9</td>
<td>3300</td>
<td>3×6000</td>
<td>700×700</td>
<td>300×700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Material properties: Concrete dynamic compressive strength of 40 MPa;

   Longitudinal reinforcement dynamic yield strength of 520 MPa;

*: Total reinforcement ratio in the cross section of the columns where the tension and compression reinforcements are equal;

**: The upper / lower longitudinal reinforcement for the beam cross section;
Table 8.2 Sixty different cases in the determination of sample points of $\lambda$

<table>
<thead>
<tr>
<th>Case</th>
<th>Structural type</th>
<th>$P_r$ (kPa)</th>
<th>$t_d$ (ms)</th>
<th>$F_{sec}$ (kN/m)</th>
<th>$M_{add}$ (kg/m)</th>
<th>$\Delta_{ud,m}$ (%)</th>
<th>$\Delta_m$ (%)</th>
<th>$\lambda$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>I-1</td>
<td>434</td>
<td>42.8</td>
<td>60</td>
<td>No</td>
<td>0.488</td>
<td>2.301</td>
<td>0.160</td>
</tr>
<tr>
<td>2</td>
<td>I-1</td>
<td>434</td>
<td>42.8</td>
<td>No</td>
<td>No</td>
<td>0.594</td>
<td>2.811</td>
<td>0.129</td>
</tr>
<tr>
<td>3</td>
<td>I-1</td>
<td>217</td>
<td>42.8</td>
<td>60</td>
<td>No</td>
<td>0.258</td>
<td>0.878</td>
<td>0.230</td>
</tr>
<tr>
<td>4</td>
<td>I-1</td>
<td>217</td>
<td>42.8</td>
<td>No</td>
<td>No</td>
<td>0.252</td>
<td>0.964</td>
<td>0.169</td>
</tr>
<tr>
<td>5</td>
<td>I-1</td>
<td>200</td>
<td>30.0</td>
<td>60</td>
<td>No</td>
<td>0.125</td>
<td>0.517</td>
<td>0.226</td>
</tr>
<tr>
<td>6</td>
<td>I-1</td>
<td>200</td>
<td>30.0</td>
<td>No</td>
<td>No</td>
<td>0.141</td>
<td>0.564</td>
<td>0.255</td>
</tr>
<tr>
<td>7</td>
<td>I-1</td>
<td>300</td>
<td>30.0</td>
<td>60</td>
<td>No</td>
<td>0.191</td>
<td>0.844</td>
<td>0.201</td>
</tr>
<tr>
<td>8</td>
<td>I-1</td>
<td>300</td>
<td>30.0</td>
<td>No</td>
<td>No</td>
<td>0.222</td>
<td>0.978</td>
<td>0.232</td>
</tr>
<tr>
<td>9</td>
<td>I-1</td>
<td>400</td>
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<td>60</td>
<td>No</td>
<td>0.257</td>
<td>1.216</td>
<td>0.170</td>
</tr>
<tr>
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<td>I-1</td>
<td>400</td>
<td>30.0</td>
<td>No</td>
<td>No</td>
<td>0.311</td>
<td>1.516</td>
<td>0.193</td>
</tr>
<tr>
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<td>I-1</td>
<td>500</td>
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<td>No</td>
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<td>1.743</td>
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<td>12</td>
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<td>0.408</td>
<td>2.101</td>
<td>0.162</td>
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<tr>
<td>13</td>
<td>I-1</td>
<td>100</td>
<td>30.0</td>
<td>60</td>
<td>No</td>
<td>0.061</td>
<td>0.203</td>
<td>0.272</td>
</tr>
<tr>
<td>14</td>
<td>I-1</td>
<td>700</td>
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<td>60</td>
<td>No</td>
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<td>0.124</td>
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<td>100</td>
<td>35.0</td>
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<td>No</td>
<td>0.077</td>
<td>0.303</td>
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Table 8.2. (Continued)

|    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 16 | I-1 | 200 | 50.0 | 60 | No | 0.290 | 1.051 | 0.301 |
| 17 | I-2 | 200 | 50.0 | 60 | No | 0.222 | 0.661 | 0.319 |
| 18 | I-1 | 200 | 50.0 | 60 | 2700 | 0.164 | 0.517 | 0.400 |
| 19 | I-1 | 300 | 50.0 | 60 | No | 0.453 | 1.970 | 0.256 |
| 20 | I-2 | 300 | 50.0 | 60 | No | 0.338 | 1.233 | 0.260 |
| 21 | I-1 | 300 | 50.0 | 60 | 2700 | 0.260 | 0.861 | 0.345 |
| 22 | I-1 | 400 | 50.0 | 60 | No | 0.655 | 3.093 | 0.211 |
| 23 | I-2 | 400 | 50.0 | 60 | No | 0.435 | 1.953 | 0.215 |
| 24 | I-1 | 400 | 50.0 | 60 | 2700 | 0.366 | 1.240 | 0.236 |
| 25 | I-1 | 500 | 50.0 | 60 | No | 0.888 | 4.478 | 0.169 |
| 26 | I-2 | 500 | 50.0 | 60 | No | 0.576 | 2.845 | 0.168 |
| 27 | I-1 | 500 | 50.0 | 60 | 2700 | 0.474 | 1.584 | 0.271 |
| 28 | I-1 | 200 | 60.0 | 60 | No | 0.406 | 1.361 | 0.244 |
| 29 | I-1 | 200 | 60.0 | 60 | 2700 | 0.180 | 0.641 | 0.199 |
| 30 | I-1 | 300 | 60.0 | 60 | No | 0.608 | 2.532 | 0.192 |
| 31 | I-1 | 300 | 60.0 | 60 | 2700 | 0.296 | 1.171 | 0.199 |
| 32 | I-1 | 400 | 60.0 | 60 | No | 0.887 | 4.068 | 0.147 |
Table 8.2. (Continued)

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<td>33</td>
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<td>60</td>
<td>2700</td>
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<td>60</td>
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</tr>
<tr>
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<td>I-1</td>
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<td>60</td>
<td>2700</td>
<td>0.126</td>
</tr>
<tr>
<td>36</td>
<td>II</td>
<td>434</td>
<td>42.8</td>
<td>54</td>
<td>2880</td>
<td>0.377</td>
</tr>
<tr>
<td>37</td>
<td>II</td>
<td>300</td>
<td>42.8</td>
<td>54</td>
<td>2880</td>
<td>0.258</td>
</tr>
<tr>
<td>38</td>
<td>II</td>
<td>200</td>
<td>42.8</td>
<td>54</td>
<td>2880</td>
<td>0.166</td>
</tr>
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<td>39</td>
<td>II</td>
<td>800</td>
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<td>54</td>
<td>2880</td>
<td>0.719</td>
</tr>
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<td>40</td>
<td>II</td>
<td>600</td>
<td>42.8</td>
<td>54</td>
<td>2880</td>
<td>0.519</td>
</tr>
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<td>II</td>
<td>300</td>
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<td>54</td>
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<td>0.371</td>
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<td>43</td>
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<td>0.875</td>
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<td>2880</td>
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<tr>
<td>45</td>
<td>II</td>
<td>500</td>
<td>50.0</td>
<td>54</td>
<td>2070</td>
<td>0.657</td>
</tr>
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<td>600</td>
<td>50.0</td>
<td>54</td>
<td>2070</td>
<td>0.825</td>
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Table 8.2. (Continued)

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<td>53</td>
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<td>60</td>
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<tr>
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<td>60</td>
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<tr>
<td>58</td>
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<td>300</td>
<td>60.0</td>
<td>60</td>
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<td>300</td>
<td>50.0</td>
<td>60</td>
<td>3360</td>
<td>0.187</td>
</tr>
</tbody>
</table>

* $P_r$: Lateral blast peak pressure.
8.2.2.2 Formulae of the ESF factor

It is assumed that the relationship of $\lambda$ with $\Delta_{td,m}$ and $\Delta_m$ follows a similar form as that for a SDOF system in Equation (7.22), thus it can be written as

$$
\begin{align*}
\lambda &= AX + B \\
X &= \begin{cases} 
\Delta_{td,m}/\Delta_m & \Delta_m \leq a \\
\Delta_{td,m}/(2\Delta_m - a) & \Delta_{td,m} \leq a \text{ and } \Delta_m \geq a \\
\Delta_{td,m}/(2\Delta_m - \Delta_{td,m}) & \Delta_{td,m} \geq a
\end{cases}
\end{align*}
$$

(8.7)

Two facts exist however in that the value of $\lambda$ is determined by the relationship of load versus the inter-storey response of the frame structure, which is a continuously curved function, and that the process of releasing the kinetic energy into further deformation after $t_d$ for a frame structure under the blast condition is more complicated than the process of releasing the strain energy of the additional springs in the equivalent static system. As a consequence, it is more appropriate to treat $A$, $B$ and $a$ in Equation (8.7) as three parameters whose values are determined in such a manner that Equation (8.7) could make the closest predictions for the available sample points of $\lambda$. In terms of mathematics, it can be expressed with the following optimum model

$$
\begin{aligned}
\text{find:} & \quad A, B \text{ and } a \\
\text{minimize:} & \quad \Phi = \sum_{j=1}^{m} [\lambda_{samp,j} - \lambda_j]^2 \\
\text{subject to:} & \quad \lambda_j = AX_j - B
\end{aligned}
$$

(8.8)

\begin{align*}
X_j &= \begin{cases} 
\Delta_{td,m}^j/\Delta_m^j & \Delta_m^j \leq a \\
\Delta_{td,m}^j/(2\Delta_m^j - a) & \Delta_{td,m}^j \leq a \text{ and } \Delta_m^j \geq a \\
\Delta_{td,m}^j/(2\Delta_m^j - \Delta_{td,m}^j) & \Delta_{td,m}^j \geq a
\end{cases}
\end{align*}

where $\lambda_{samp,j}$, $\Delta_{td,m}^j$ and $\Delta_m^j$ indicate the $j$th sample point of the ESF factor, the
corresponding MIDR at time $t_d$ and that during the whole response process as listed in Table 8.2; $\lambda_j$ is the predicted value of the ESF factor according to Equation (8.7).

A lot of algorithms are available currently, which can be applied for solving such an optimum problem. In this study, with the help of the function ‘Fminsearch’ provided in the software MATLAB [M13], a program is written for solving this particular Equation (8.8). The results are plotted in Figure 8.3, where $A \approx 1.996$, $B \approx -0.074$ and $a \approx 0$, and the line C-D indicates the optimized $\lambda - X$ function. It can be seen in Figure 8.3 that the sample points of $\lambda_{samp}$ are scattered around this optimized function within a straight strip bounded by an upper-envelop (E-F) and a lower-envelop line (G-H). By submitting the optimum results of the parameters $A$, $B$ and $a$ into Equation (8.7), the line C-D can be written as

\[
\begin{align*}
\lambda &= 1.996 X - 0.074 \\
X &= \Delta_{td,m}/(2\Delta_m - \Delta_{td,m})
\end{align*}
\]  

(8.9)

The lines E-F and G-H are expressed respectively with

\[
\begin{align*}
\lambda_{upp} &= 1.996 X + 0.023 \\
X &= \Delta_{td,m}/(2\Delta_m - \Delta_{td,m})
\end{align*}
\]  

(8.10)

and

\[
\begin{align*}
\lambda_{low} &= 1.996 X - 0.160 \\
X &= \Delta_{td,m}/(2\Delta_m - \Delta_{td,m})
\end{align*}
\]  

(8.11)
8.3 Design of a Multi-storey Reinforced Concrete Frame Structure with the ESF

Similar to the trend of ultimate strength with the maximum displacement response for a SDOF system, if the resistance of a frame structure between its inter-storey drift ratio response and an increasing later load vector distributed as $P$ (illustrated in Figure 8.2) can be idealized into an elastic-perfectly-plastic function, the MIDR response of the structure under the blast condition is closely related to the ultimate strength $R_m$. With a predefined target MIDR $\Delta_m$, the required $R_m$ of the structure should be equal to the predicted ESF according to the previous process by taking its MIDR response $\Delta_m$ as $\Delta_m$, which can be applied to determine the reinforcements after giving the dimension of the frame members.
The required $R_m$ (ESF) is predicted from the product of $P$ with the ESF factor $\lambda$. Although Equation (8.9) can provide an optimum value for $\lambda$, it may in some cases result in an inadequate estimation of $R_m$. For the purpose of design, considering the safety of the frame structure, the conservative upper-envelop line (E-F) is suggested and therefore by combining Equations (8.4), (8.6) and (8.10), and replacing $P_{sd}$ and $\Delta_m$ with $R_m$ and $\Delta_t$ respectively, it is obtained as

$$
\begin{align*}
R_m &= \lambda_{upp} P \\
P &= F_s + F_a \\
\lambda_{upp} &= 1.996 X + 0.023 \\
X &= \Delta_{sd,m} / (2\Delta_t - \Delta_{sd,m})
\end{align*}
$$

With Equation (8.12), the performance-based blast resistant design using the ESF is iteratively accomplished as follows. According to the expected global performance level for the frame structure, a target MIDR $\Delta_t$ is defined at the start of the design. Giving the dimensions of different structural members and the initial reinforcement ratios $\rho_0$, the required $R_m$ is gained from Equation (8.12), which together with service loadings will induce new reinforcement ratios $\rho_1$. Hence make $\rho_0 = \rho_1$ and repeat the above steps until the convergence in reinforcement ratios is reached.

There exists the possibility that the cross-sectional sizes of structural members are specified to be unreasonably small during the design stage. With the above iterative design procedures the reinforcement ratios could be determined as well, however they might exceed the allowable maximum value. In such cases, modifications to the dimensions of the frame members are needed. The final design flowchart is shown in Figure 8.4.
Figure 8.4 Flowchart of blast resistant design of a multi-storey reinforced concrete frame structure with ESF ($\varepsilon_i$ is an arbitrarily small value)
8.4 Illustrative Examples

For the illustration of the developed performance-based blast resistant design procedures with the ESF, two design examples are carried out for planar six-storey reinforced concrete frame structures in resisting blast loadings from distant explosions as shown in Figure 8.5. The basic objective of the design is to ensure that the global damage of the designed structural systems be within the expected performance levels defined by the target MIDR response. Three types of external actions are considered within the design, which include the blast loadings, the dead loads (DL) and live loads (LL). The blast loadings consist of the lateral pressures on the columns and vertical roof pressures. However, as discussed from the comparison study of a planar one-storey frame structure by Baker [B2], the vertical roof pressures have a slight effect on the frame sidesway response, the limitations of which are of main concern in these design examples, therefore they are ignored in these examples. The consideration of the vertical roof loadings can be involved in the blast resistant design of the roof beams with the approach addressed in Chapters 3 to 6. The detailed loading profiles are displayed in Figure 8.5, where DL=30 kN/m, LL=24 kN/m.

In the first design example, a low protection level is required for the frame structure, accordingly a relatively large target MIDR of \( \Delta_r = 3.5\% \) is given which is slightly less than the side-sway limit corresponding to the state that the structure may lose its integrity or even collapse as listed in Table 2.1. In the second example, a higher protective level is expected for the frame structure to resist the blast loading and thus a lower target MIDR of \( \Delta_r = 1.5\% \) is defined.

At the start of design, the cross-sectional sizes for frame members are specified to be 600 mm × 600 mm for the columns in the first and second storeys, 500 mm × 500 mm for the other columns and 300 mm × 600 mm for all frame beams. Giving the initial values for reinforcement ratios \( \rho_0 \), \( R_m \) is computed at each iterative step from Equation (8.12). The determination of \( \rho_1 \) is accomplished with the integrated
building design software ETABS [E4] according to four user-specified loading combinations (1.4DL+1.6LL+1.0 $R_m$, 1.4DL+1.6LL-1.0 $R_m$, 1.4DL+1.6LL, 1.4DL).

Figure 8.5 A six-storey reinforced concrete frame structure under blast forces (Examples I and II)
The whole design procedures for these two design examples with the presented method based on ESF are listed in Tables 8.3 and 8.4 where the convergence condition is defined as \( |(\rho_1 - \rho_0)/\rho_0| \leq 0.05 \). It can be observed that there is no difficulty in reaching the convergence for the design with ESF, as only three iterative steps are needed for both design examples. Although the nonlinear dynamic analysis is needed to find \( y_{id} \) and \( W_k \) at each step, little computational time is consumed to accomplish this analysis since only slight nonlinear behaviours appear to the frame structure at the time \( t_d \) due to the short duration of blast loading. Moreover, the solved reinforcement ratios for the members are all located in their suitable ranges hence there is no need for a change to the initially specified cross-sectional sizes of the frame members.
### Table 8.3 Design procedures for example I

#### The First Iterative Design Step

<table>
<thead>
<tr>
<th>i</th>
<th>$y_{id,i}$ (mm)</th>
<th>$W_{id,i}$ (Nm)</th>
<th>$F_{a,i}$ (N)</th>
<th>$F_{s,i}$ (N)</th>
<th>$P_i$ (N)</th>
<th>$\Delta_{id,m}$ (%)</th>
<th>$\lambda$</th>
<th>$P_{st,i}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10.80</td>
<td>0.93×10⁴</td>
<td>1.72×10⁶</td>
<td>-3.91×10⁵</td>
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<td></td>
<td></td>
<td>2.09×10⁵</td>
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<tr>
<td>5</td>
<td>19.25</td>
<td>1.73×10⁴</td>
<td>1.80×10⁶</td>
<td>3.99×10⁵</td>
<td>2.20×10⁶</td>
<td></td>
<td></td>
<td>3.45×10⁵</td>
</tr>
<tr>
<td>4</td>
<td>20.40</td>
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<td>0.11×10⁵</td>
<td>2.44×10⁶</td>
<td></td>
<td></td>
<td>3.84×10⁵</td>
</tr>
<tr>
<td>3</td>
<td>20.77</td>
<td>2.80×10⁴</td>
<td>2.69×10⁶</td>
<td>-2.08×10⁵</td>
<td>2.49×10⁶</td>
<td></td>
<td></td>
<td>3.91×10⁵</td>
</tr>
<tr>
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<td>2.30×10⁶</td>
<td></td>
<td></td>
<td>3.61×10⁵</td>
</tr>
</tbody>
</table>

(a). Initial reinforcement ratios (%)

(b). New reinforcement ratios (%)

**Note:** The item inside the bracket is the total required area of shear reinforcement per unit length (mm).
Table 8.3 (Continued)

The Second Iterative Design Step

<table>
<thead>
<tr>
<th>i</th>
<th>$y_{dl,i}$ (mm)</th>
<th>$W_{ik,i}$ (Nm)</th>
<th>$F_{a,i}$ (N)</th>
<th>$F_{s,i}$ (N)</th>
<th>$P_i$ (N)</th>
<th>$\Delta_{s,i,m}$ (%)</th>
<th>$X$</th>
<th>$\lambda$</th>
<th>$P_{st,i}$ (N)</th>
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<tr>
<td>6</td>
<td>11.13</td>
<td>$0.91 \times 10^4$</td>
<td>$1.64 \times 10^6$</td>
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<td>$1.22 \times 10^6$</td>
<td>1.55\times 10^5</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>19.73</td>
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<td>$1.70 \times 10^6$</td>
<td>$3.19 \times 10^5$</td>
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</tr>
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<td>$1.50 \times 10^5$</td>
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<td>2.47\times 10^5</td>
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</table>

(a). Initial reinforcement ratios (%)  
(b). New reinforcement ratios (%)
Table 8.3 (Continued)

The Third Iterative Design Step

<table>
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<tr>
<th>i</th>
<th>$y_{d,i}$ (mm)</th>
<th>$W_{k,i}$ (Nm)</th>
<th>$F_{a,i}$ (N)</th>
<th>$F_{s,i}$ (N)</th>
<th>$P_i$ (N)</th>
<th>$\Delta_{a,m}$ (%)</th>
<th>$\lambda$</th>
<th>$P_{st,i}$ (N)</th>
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<td></td>
<td>$1.37 \times 10^5$</td>
</tr>
<tr>
<td>5</td>
<td>19.65</td>
<td>$1.84 \times 10^4$</td>
<td>$1.87 \times 10^6$</td>
<td>$3.35 \times 10^5$</td>
<td>$2.21 \times 10^6$</td>
<td></td>
<td></td>
<td>$2.92 \times 10^5$</td>
</tr>
<tr>
<td>4</td>
<td>21.77</td>
<td>$2.19 \times 10^4$</td>
<td>$2.01 \times 10^6$</td>
<td>$1.42 \times 10^5$</td>
<td>$2.15 \times 10^6$</td>
<td>$0.364$</td>
<td>$0.055$</td>
<td>$0.132$</td>
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<tr>
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<td>21.58</td>
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<td>$1.88 \times 10^6$</td>
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<td>$2.92 \times 10^5$</td>
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</table>

(a). Initial reinforcement ratios (%)  
(b). New reinforcement ratios (%) (final design)
Table 8.4 Design procedures for example II

<table>
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<th>$i$</th>
<th>$y_{id,j}$ (mm)</th>
<th>$W_{k,j}$ (Nm)</th>
<th>$F_{o,j}$ (N)</th>
<th>$F_{s,j}$ (N)</th>
<th>$P_j$ (N)</th>
<th>$\Delta_{id,m}$ (%)</th>
<th>$X$</th>
<th>$\lambda$</th>
<th>$P_{s,j}$ (N)</th>
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</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td>8.88×10⁴</td>
</tr>
<tr>
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<td>12.62</td>
<td>2.84×10³</td>
<td>4.49×10⁵</td>
<td>1.05×10⁵</td>
<td>5.55×10⁵</td>
<td></td>
<td></td>
<td></td>
<td>1.10×10⁵</td>
</tr>
<tr>
<td>4</td>
<td>14.71</td>
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<td>0.198</td>
<td>1.47×10⁵</td>
</tr>
<tr>
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<td>15.02</td>
<td>7.18×10³</td>
<td>9.56×10⁵</td>
<td>-2.64×10⁵</td>
<td>6.92×10⁵</td>
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<td></td>
<td></td>
<td>1.37×10⁵</td>
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<td>17.01</td>
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<td>6.82×10⁵</td>
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<td>1.05×10⁶</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>1.37×10⁵</td>
</tr>
</tbody>
</table>

(a). Initial reinforcement ratios (%)  
(b). New reinforcement ratios (%)

**Note:** The item inside the bracket is the total required area of shear reinforcement per unit length (mm).
Table 8.4 (Continued)

The Second Iterative Design Step

<table>
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<th>$y_{id,j}$ (mm)</th>
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<th>$F_{a,j}$ (N)</th>
<th>$F_{s,j}$ (N)</th>
<th>$P_i$ (N)</th>
<th>$\Delta_{id,m}$ (%)</th>
<th>$\lambda$</th>
<th>$P_{st,j}$ (N)</th>
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</thead>
<tbody>
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<td>$3.51 \times 10^5$</td>
<td>0.233</td>
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<td>0.191</td>
</tr>
<tr>
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<td>12.65</td>
<td>$2.37 \times 10^3$</td>
<td>$3.74 \times 10^5$</td>
<td>$1.11 \times 10^6$</td>
<td>$4.85 \times 10^5$</td>
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<td>$5.96 \times 10^3$</td>
<td>$5.96 \times 10^5$</td>
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<td>$6.74 \times 10^5$</td>
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(a). Initial reinforcement ratios (%)  
(b). New reinforcement ratios (%)
Table 8.4 (Continued)

The Third Iterative Design Step

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<tr>
<th>i</th>
<th>$y_\text{j}_{i,j}$ (mm)</th>
<th>$W_{k,j}$ (Nm)</th>
<th>$F_{s,j}$ (N)</th>
<th>$P_i$ (N)</th>
<th>$\Delta_{\text{s},m}$ (%)</th>
<th>$X$</th>
<th>$\lambda$</th>
<th>$P_{\text{sl},i}$ (N)</th>
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<tr>
<td>6</td>
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<td>2.43×10³</td>
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<td>5</td>
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<td>3.60×10⁵</td>
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<td>4.71×10³</td>
<td>6.38×10⁵</td>
<td>1.07×10⁵</td>
<td>7.45×10⁵</td>
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<td>9.70×10⁵</td>
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<td>1.38×10⁵</td>
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</tr>
<tr>
<td>2</td>
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<td>5.56×10³</td>
<td>6.50×10⁵</td>
<td>4.44×10⁵</td>
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</table>

(a). Initial reinforcement ratios (%)  
(b). New reinforcement ratios (%) (final design)
8.5 Numerical Verification of the Design Method

To check whether the MIDR responses are controlled effectively with the above design method, nonlinear finite element analyses are performed on the designed frame structures against the given blast forces. Two steps of analysis are involved in which the nonlinear static analysis under the DLs and LLs is carried out followed by the nonlinear dynamic analysis of the designed structures under the blast force. The simulation results are demonstrated in Figures 8.6 and 8.7, which show a conservative MIDR response at 1.31% and 0.72% respectively for these two frames as compared to their target values.

The reasons for the conservative design with ESF can be explained from two aspects. First, the prediction of the ESF factor $\lambda$ based on its upper-envelop line induces an overestimation of $R_m$ from Equation (8.12). The second relies on the assumption that the resistance of the designed structure be an elastic-perfectly-plastic function and therefore the calculated ESF will be equal to the required $R_m$. The strengthening of the structure after reaching this $R_m$ is not considered in the design procedure. From the viewpoint of structural safety, such a conservative design result can be acceptable.
Chapter 8. Performance-Based Design of Frame Structures with ESF — Part II: Implementation and Evaluation

Figure 8.6 Responses of the designed frame structure under the given blast force
(Example I)

Interstorey drift ratio at the critical storey (%)

Time (s)

Figure 8.7 Responses of the designed frame structure under the given blast force
(Example II)

Interstorey Drift Ratio at the critical storey (%)

Time (s)
8.6 Summaries

A new performance-based blast resistant design method has been developed in this study for a multi-storey reinforced concrete frame structure with the ESF in keeping the MIDR response under proper control to meet the expected global performance level under the distant explosion conditions.

An equivalent static system is constructed for the purpose of simulating the kinetic energy at $t_d$ by the strain energy of the additional springs. According to the equilibrium of the equivalent static system, the external static force is obtained, however this force will produce a larger MIDR demand than that of the frame structure under the blast condition and thus the ESF factor $\lambda$ is introduced to reduce the external static force to finally obtain the ESF.

By comparing the nonlinear dynamic responses of frame structures with corresponding nonlinear static pushover analytical results, a number of sample points of $\lambda$ are obtained, from which a function of $\lambda$ is determined with the optimum technique for the closest estimation for these points. It shows that the sample points of $\lambda$ are scattered around this optimized function within a straight strip that is bounded by an upper-envelop and a lower-envelop line. The upper-envelop line is utilized into the design for structural safety.

By substituting the ESF with the ultimate strength and the MIDR responses with the required design target, the calculation model of the ESF is implemented into the design of a multi-storey reinforced concrete frame structure. For given cross-sectional sizes of the frame members, the reinforcement ratios are iteratively solved. Two numerical examples show that there is no difficulty in getting computational convergence for the design method based on ESF. However since the ESF factor is based on the upper-envelop line and the strengthening of the structure after reaching the ESF, some safety margin is provided as part of the design to control the MIDR responses of the frame structure as indicated from the numerical verifications.
CHAPTER NINE

CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

The conclusions drawn as a result of the theoretical work carried out in this thesis have been generally been given at the end of each chapter. A summary of those conclusions is as follows:

1. The whole dynamic response process of frame structures with/without exterior cladding panels under distant blast conditions can be approximately divided into two stages. Localized responses of the blast-loaded members are critical at response stage I while the global responses of the structural system dominate at response stage II.

2. According to the differences of the damage distributions in the two response stages, the blast resistant design for a multi-storey reinforced concrete frame structure under distant blast conditions should include two parts: the design of critical blast-loaded members capable of controlling the degree of the local damage at response stage I and the design of frame structural systems in limiting the excessive global damage at response stage II. To meet such requirements, the performance-based blast resistant design should be performed not only for the structural members but also for the frame structural systems.

3. The developed performance-based blast resistant design approach for structural members using energy spectra is accomplished in such a manner that both the displacement and the displacement ductility response
parameters are taken as the performance indicators. For the expected performance level defined by a combination of target displacement and target displacement ductility factor, the effective depth and longitudinal reinforcement ratio of structural members could be exactly solved so that the maximum displacement and displacement ductility responses of the equivalent SDOF system of the designed member can be precisely controlled to be equal to their targets.

4. Some errors do exist between the actual maximum displacement and displacement ductility responses for the designed member under the given blast loading and the respective design targets, as indicated from numerical verification analysis of the design approach using energy spectra. This error is induced by some simplifications, which are adopted in converting the continuous reinforced concrete members into the equivalent SDOF systems and computing the cross-sectional moment of inertia and ultimate strength of the members.

5. Formulae for the two error indices, which quantify the errors between the actual responses of designed member under the given blast loading and the design targets, are derived through extensive numerical studies and statistical analyses. Two items are included in the formulae: a nonlinear fitting function that is correlated with the longitudinal reinforcement ratio and the support conditions, plus a nominal random variable to consider the uncertain influence of the other parameters.

6. Modification of the performance-based blast resistant design approach using energy spectra is accomplished depending on the formulae of the two error indices, where the actual maximum displacement and displacement ductility responses of designed member rather than those of the equivalent SDOF system are controlled. Although more iterative steps are needed to reach convergence as compared to those of the original design procedure, a better agreement is achieved between the actual responses of the member designed
with the modified approach and their targets.

7. The formulae for the two error indices provide a simple way to predict the maximum displacement and displacement ductility responses of the members from those of the corresponding equivalent SDOF systems without performing the numerical finite element analysis. Consequently, by using the Monte-Carlo simulation and introducing two independent nominal random variables in the error indices, the assessment of the distributions of these two responses under blast conditions accounting for different types of uncertainties becomes more efficient.

8. Taking the MIDR response parameter as the global performance indicator, a performance-based blast resistant design approach is presented for the multi-storey reinforced concrete frame structural system by using the equivalent static force (ESF). This approach is firstly addressed for the design of a SDOF system controlling its maximum displacement response and then extended into the design of a frame structure controlling its MIDR response. An equivalent static system is built up to simulate the energy components of the SDOF/frame structural system at the time $t_d$ (blast loading duration).

Based on the equilibrium of the equivalent static system, an external static force $P$ is computed, however this force tends to produce a larger maximum displacement/MIDR response when statically applied to the SDOF/frame structural system than that under the blast actions. Therefore, an ESF factor is introduced to reduce $P$ to finally obtain the ESF.

9. The closed form expression of the ESF factor for an elastic-perfectly-plastic SDOF system is evaluated. However for multi-storey reinforced concrete frame structures, an empirical formula of the ESF factor is numerically derived from a number of sample points of the ESF factor obtained by comparing the nonlinear dynamic responses of frame structures with corresponding nonlinear static pushover analytical results. It is shown that these sample points are scattered within a straight strip bounded by upper
and lower envelop lines. For structural safety reasons, the upper line is suggested for the calculation of the ESF and applied into the design.

10. By replacing the ESF with the ultimate strength and the maximum displacement/MIDR response with the required design target, the calculation model of the ESF is implemented into the design of an elastic-perfectly-plastic SDOF system/multi-storey reinforced concrete frame structural system. The ultimate strength for the SDOF system/the reinforcement ratios of the frame members for the given cross-sectional sizes can be iteratively solved during the design in order to satisfy the target displacement/MIDR.

11. The presented performance-based design procedures using ESF have been illustrated via several design examples for the SDOF systems and multi-storey frame structures, which display that there is no difficulty in convergence of the iterative procedures. The maximum displacement responses of the designed SDOF systems under the given blast forces are controlled to be exactly equal to the design targets. However the MIDR responses of the frame structural systems are limited on the conservative sides as compared to their targets. That is because the ESF factor is predicted from the upper-envelop line and the strengthening of the structure after reaching the ESF that is assumed to be the ultimate strength is not considered within the design.

Based on the study in the thesis, the methodology of the performance-based design for a multi-storey reinforced concrete frame structure against the distant blast loadings is summarized in Figure 9.1. After defining the expected performance level according to the requirements and objectives of the owners, users, and society for the frame structure under the given distant explosion conditions, the design is carried out for the reinforced concrete structural members using energy spectra as well as for frame structural system with ESF. The more critical design results are selected as the final design.
9.2 Recommendations for Future Works

Several issues related to this study deserve further investigation. These include:

1. The effects of the interior infill walls on the dynamic responses and damage distributions for the frame structure in resisting the blast loadings from distant intense surface explosions should be investigated.

2. The explosion tests should be conducted for a frame structure with different
types of exterior cladding panels in order to verify the numerical analytical results of the damage distributions.

3. Performance-based blast resistant design using energy spectra has been proposed for the reinforced concrete members without the effects of the initial conditions such as the axial force; future studies are necessary for the blast resistant design of the members taking into consideration of these initial conditions.

4. The performance-based design procedures using energy spectra can be also utilized for the design of reinforced concrete members under other types of dynamic loadings such as the rectangular load pulses, the constant force with finite rise time, and the equilateral triangular load pulses with zero rise time, etc. Their accomplishment depends on the corresponding energy spectra through controlling the responses of equivalent SDOF system of reinforced concrete member to reach the design targets. The accuracy of the design in controlling the responses of the designed member under such loading conditions can be also assessed through numerical analysis.

5. The experimental verifications of the blast resistant design method with the ESF for the multi-storey reinforced concrete frame structures in controlling its MIDR responses should be conducted.
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APPENDIX 1

BLAST LOADINGS ON A CLOSED TARGET

Front Load

At low pressure ranges, the variation of pressure with the front faces for a closed rectangular aboveground target is shown in Figure 2.2 whose parameters can be obtained with

- \( P_{ic} = P_{io} + C_D \ q \ (C_D = 1) \)
- \( t_c = 3S / U \)
- \( S = \) clearing distance = \( W_s / 2 \) or \( H_s \), the building half-width or height respectively, whichever is less
- \( t_{of} = 2i_s / P_{io} \)

where \( P_r \) and \( P_{io} \) is peak reflected and incident pressure of the shock wave reaching the front face; \( q \) is peak dynamic pressure, \( U \) is shock front velocity, \( i_s \) is incident blast impulse.

However, at higher pressure ranges the above equations may yield a fictitious pressure-time curve due to the extremely short pressure pulse durations involved. To overcome it, a comparison is made by constructing a second curve (dashed triangular line as indicated in Figure 2.2) where the fictitious duration \( t_r \) is

- \( t_r = 2i_r / P_r \)
The parameter $i_r$ is the reflected impulse of the blast wave. The curve, which gives the smaller value of the impulse (area under curve), is used in calculating the front face loading. The reflected pressure impulse includes the effects of both the incident and dynamic pressure.

**Top and side load**

Top and side faces will experience less blast loading than the front wall face due to little or no reflection of the blast wave, assuming a generally flat roof and a horizontally traveling blast wave. The form of this load is shown in Figure 2.2, where

\[ P_{or} = C_E P_{sb} + C_D q_{sb} \]

- $C_E = \text{overpressure averaging factor depending on } L_{wb} / L \ (<1.0)$
- $C_D = \text{drag coefficient depending on } q_{sb}$
- $t_1 = D / U_D$
- $t_2 = t_b - t_f + t_{of}$

$P_{sb}$, $q_{sb}$ and $L_{wb}$ are peak incident pressure, dynamic pressure, and blast wave length occurring at point $b$, $t_f$ and $t_b$ is the arrival time of the blast wave reaching the point $f$ and $b$, $U_D$ is the wave front velocity at point $d$ while $D$ can be calculated from $D / L$ that is a function of $L_{wb} / L$.

**Rear load**

The rear or back face of a building is loaded, similar to the side faces and roof, by the averaged attenuated overpressure pulse with peak value $P_{ob}$ and rise time as shown Figure 2.2, where
\[ P_{teb} = C_E P_{soe} + C_D q_{soe} \]
\[ t_3 = t_h - t_f \]
\[ t_4 = D'/U_{D'} \]
\[ t_5 = t_3 + t_e - t_f + t_{of} \]

\( P_{soe} \), \( q_{soe} \) and \( t_e \) are blast wave parameters at point \( e \), \( D' \) can be calculated from \( D'/S \) that is a function of \( L_{we}/L \), and \( U_{D'} \) is the wave front velocity at point \( d' \).

In the above expressions these blast wave parameters including the blast incident impulse, the reflected impulse, the incident and reflected pressure, the dynamic pressure, the arrival time, and the velocity can be determined with the conventional blast loadings program of CONWEP [H1] that is developed based on TNT-equivalence models using information provided in TM5-855-1 [T1].